Computer algebra independent integration tests

0_Independent_test_suites/Stewart_Problems

Nasser M. Abbasi

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

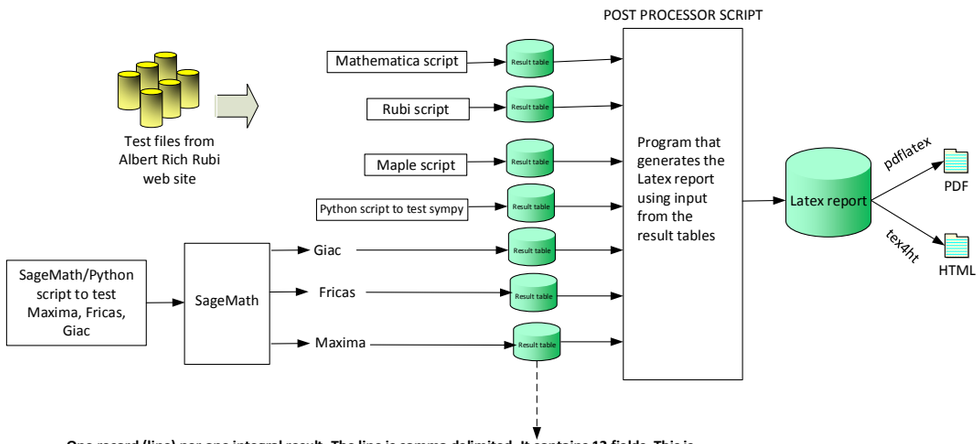
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is comma delimited. It contains 12 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expressi

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (376)	% 0. (0)
Rubi in Sympy	% 84.04 (316)	% 15.96 (60)
Mathematica	% 100. (376)	% 0. (0)
Maple	% 100. (376)	% 0. (0)
Maxima	% 98.94 (372)	% 1.06 (4)
Fricas	% 99.2 (373)	% 0.8 (3)
Sympy	% 90.96 (342)	% 9.04 (34)
Giac	% 99.47 (374)	% 0.53 (2)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

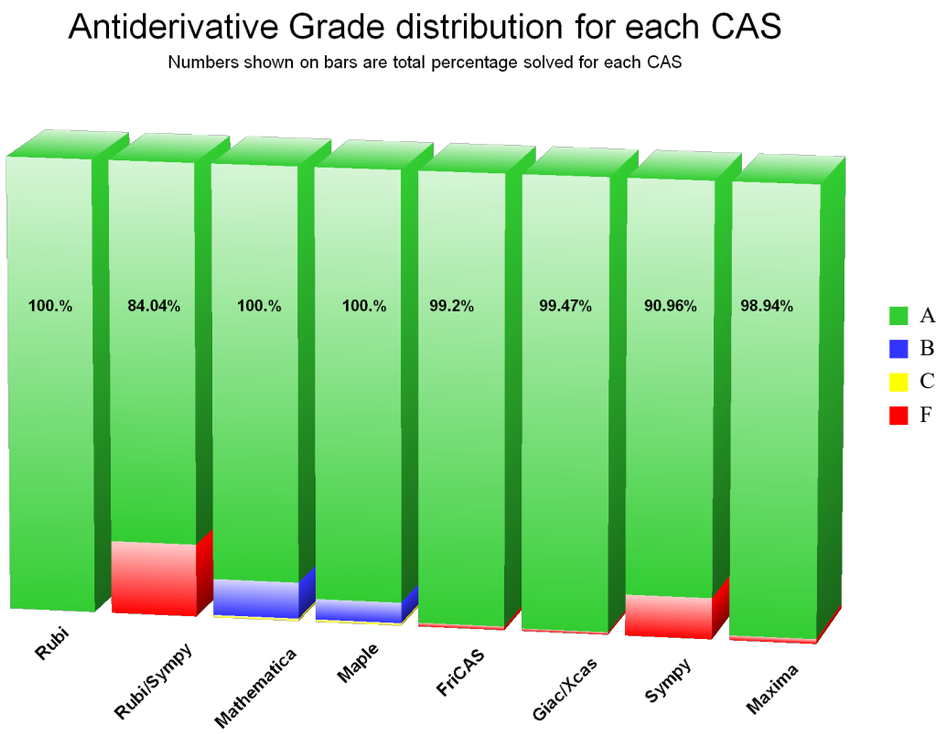
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

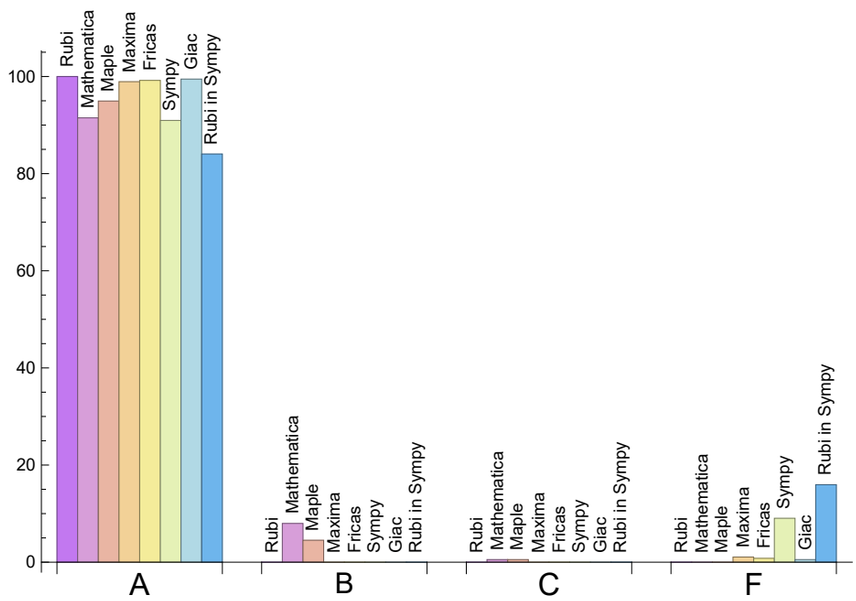
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	84.04	0.	0.	15.96
Mathematica	91.49	7.98	0.53	0.
Maple	94.95	4.52	0.53	0.
Maxima	98.94	0.	0.	1.06
Fricas	99.2	0.	0.	0.8
Sympy	90.96	0.	0.	9.04
Giac	99.47	0.	0.	0.53

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	22.97	1.	19.	1.
Rubi in Sympy	2.88	19.49	0.88	17.	0.83
Mathematica	0.02	23.94	1.15	20.	1.
Maple	0.01	21.6	1.01	18.	0.88
Maxima	1.43	29.51	1.41	22.	1.09
Fricas	0.22	34.48	1.65	24.	1.22
Sympy	0.99	24.54	1.17	19.	0.88
Giac	0.21	30.12	1.58	23.	1.1

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {10, 11, 12, 23, 29, 34, 35, 41, 67, 78, 87, 88, 89, 93, 94, 95, 96, 112, 113, 114, 115, 116, 117, 154, 156, 159, 167, 176, 183, 191, 217, 218, 220, 223, 232, 233, 237, 240, 248, 253, 256, 258, 271, 276, 285, 300, 302, 310, 314, 315, 320, 335, 337, 349, 350, 351, 354, 359, 368, 372}

Not solved by Mathematica {}

Not solved by Maple {}

Not solved by Maxima {1, 133, 250, 330}

Not solved by Fricas {220, 235, 354}

Not solved by Sympy {29, 41, 74, 123, 144, 145, 146, 147, 149, 220, 226, 227, 235, 237, 238, 247, 248, 249, 250, 251, 288, 295, 301, 308, 322, 325, 328, 337, 340, 351, 353, 359, 360, 365}

Not solved by Giac {269, 337}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	14	12	15	7
normalized size	1	1.	1.	1.09	0.	1.27	1.09	1.36	0.64
time (sec)	N/A	0.007	0.002	0.003	0.	0.213	0.031	0.206	0.613

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	3	3	2	3	2
normalized size	1	1.	1.	1.	1.	1.	0.67	1.	0.67
time (sec)	N/A	0.003	0.	0.	1.365	0.202	0.046	0.204	0.48

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	4	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	2.	1.
time (sec)	N/A	0.002	0.	0.	1.364	0.192	0.026	0.202	0.023

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	8	11	5
normalized size	1	1.	1.	1.12	1.38	1.38	1.	1.38	0.62
time (sec)	N/A	0.005	0.001	0.001	1.301	0.208	0.069	0.2	0.576

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	3	5	3
normalized size	1	1.	1.	1.25	1.25	1.25	0.75	1.25	0.75
time (sec)	N/A	0.004	0.002	0.	1.325	0.22	0.032	0.202	0.026

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	3	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.003	0.001	0.	1.368	0.247	0.03	0.207	0.023

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	9	5	3	5
normalized size	1	1.	1.	1.5	1.5	4.5	2.5	1.5	2.5
time (sec)	N/A	0.008	0.003	0.005	1.364	0.207	0.045	0.208	0.471

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	11	7	8	7
normalized size	1	1.	1.	1.25	2.	2.75	1.75	2.	1.75
time (sec)	N/A	0.007	0.003	0.006	1.421	0.206	0.044	0.21	0.474

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	5	5	3	5	3
normalized size	1	1.	1.	1.5	2.5	2.5	1.5	2.5	1.5
time (sec)	N/A	0.009	0.002	0.006	1.382	0.213	0.047	0.203	0.723

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	8	5	8	0
normalized size	1	1.	1.	1.25	2.	2.	1.25	2.	0.
time (sec)	N/A	0.011	0.002	0.009	1.375	0.208	0.049	0.203	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	15	0
normalized size	1	1.	1.	1.5	1.5	1.5	1.	7.5	0.
time (sec)	N/A	0.005	0.003	0.002	1.48	0.202	0.106	0.203	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	15	0
normalized size	1	1.	1.	1.5	1.5	1.5	1.	7.5	0.
time (sec)	N/A	0.004	0.002	0.003	1.546	0.207	0.101	0.202	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	4	15	5	8	5
normalized size	1	1.	1.	1.2	0.8	3.	1.	1.6	1.
time (sec)	N/A	0.003	0.003	0.001	1.479	0.251	0.039	0.202	0.032

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	15	3	15	3
normalized size	1	1.	1.	1.33	1.33	5.	1.	5.	1.
time (sec)	N/A	0.003	0.003	0.	1.443	0.221	0.043	0.202	0.031

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	7	11	7
normalized size	1	1.	1.	1.12	1.38	1.38	0.88	1.38	0.88
time (sec)	N/A	0.012	0.003	0.002	1.361	0.223	0.182	0.199	0.782

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	5	11	5
normalized size	1	1.	1.	1.12	1.38	1.38	0.62	1.38	0.62
time (sec)	N/A	0.003	0.001	0.	1.349	0.216	0.064	0.201	0.453

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	15	15	10	15	17
normalized size	1	1.	0.63	0.63	0.79	0.79	0.53	0.79	0.89
time (sec)	N/A	0.023	0.002	0.	1.338	0.2	0.066	0.199	1.78

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	18	15	15	15
normalized size	1	1.	0.74	0.74	0.79	0.95	0.79	0.79	0.79
time (sec)	N/A	0.013	0.016	0.	1.367	0.216	0.369	0.201	1.179

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	18	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.2	0.8
time (sec)	N/A	0.007	0.002	0.003	1.357	0.232	0.235	0.199	0.908

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	12	15	15	10	15	14
normalized size	1	1.	0.75	0.6	0.75	0.75	0.5	0.75	0.7
time (sec)	N/A	0.014	0.002	0.003	1.353	0.206	0.064	0.202	1.137

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	7	9	7
normalized size	1	1.	1.	1.14	1.29	1.29	1.	1.29	1.
time (sec)	N/A	0.014	0.004	0.	1.346	0.229	0.179	0.199	0.779

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	14	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	0.78	1.06	0.78
time (sec)	N/A	0.018	0.005	0.007	1.372	0.227	0.193	0.199	0.829

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.008	0.001	0.	1.339	0.217	0.07	0.201	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	28	27	28	27
normalized size	1	1.	0.86	0.83	0.97	0.97	0.93	0.97	0.93
time (sec)	N/A	0.039	0.028	0.008	1.373	0.228	0.398	0.201	1.538

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	28	24	28	24
normalized size	1	1.	0.86	0.83	0.97	0.97	0.83	0.97	0.83
time (sec)	N/A	0.035	0.025	0.007	1.329	0.223	0.406	0.202	1.465

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	20	15	20	15
normalized size	1	1.	1.	1.07	1.07	1.33	1.	1.33	1.
time (sec)	N/A	0.008	0.002	0.	1.46	0.217	0.082	0.2	0.562

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	19	12	19	12
normalized size	1	1.	1.	0.94	1.19	1.19	0.75	1.19	0.75
time (sec)	N/A	0.008	0.005	0.	1.502	0.232	0.132	0.202	1.062

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	19	23	24	19	19
normalized size	1	1.	0.78	0.78	0.83	1.	1.04	0.83	0.83
time (sec)	N/A	0.022	0.005	0.003	1.349	0.229	0.41	0.201	0.988

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	100	24	0	139	0
normalized size	1	1.	1.	1.12	12.5	3.	0.	17.38	0.
time (sec)	N/A	0.028	0.011	0.007	1.522	0.243	0.	0.222	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	12
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.71
time (sec)	N/A	0.013	0.002	0.003	1.416	0.22	0.082	0.201	1.129

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	17	17	22	22	15	22	26
normalized size	1	1.	0.63	0.63	0.81	0.81	0.56	0.81	0.96
time (sec)	N/A	0.047	0.003	0.004	1.41	0.205	0.07	0.198	2.646

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	26	26	26
normalized size	1	1.	0.81	0.81	0.96	1.04	0.96	0.96	0.96
time (sec)	N/A	0.021	0.029	0.009	1.403	0.218	0.373	0.201	1.447

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	23	28	20	23	20
normalized size	1	1.	0.74	0.81	0.85	1.04	0.74	0.85	0.74
time (sec)	N/A	0.022	0.019	0.01	1.353	0.223	0.794	0.2	1.449

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	12	7	23	0
normalized size	1	1.	1.	1.11	5.11	1.33	0.78	2.56	0.
time (sec)	N/A	0.019	0.005	0.003	1.356	0.211	0.178	0.198	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	77	24	20	41	0
normalized size	1	1.	1.	1.	4.05	1.26	1.05	2.16	0.
time (sec)	N/A	0.029	0.008	0.007	1.398	0.207	0.253	0.198	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	12	7	12	10
normalized size	1	1.	0.69	0.62	0.75	0.75	0.44	0.75	0.62
time (sec)	N/A	0.017	0.002	0.	1.353	0.214	0.061	0.2	1.075

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	18	19	66	18	19
normalized size	1	1.	0.71	0.67	0.86	0.9	3.14	0.86	0.9
time (sec)	N/A	0.014	0.004	0.004	1.347	0.249	3.081	0.219	1.105

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	14	19	14
normalized size	1	1.	1.	0.83	1.06	1.06	0.78	1.06	0.78
time (sec)	N/A	0.019	0.005	0.007	1.349	0.222	0.188	0.232	0.85

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	19	19	12	19	19
normalized size	1	1.	0.62	0.58	0.73	0.73	0.46	0.73	0.73
time (sec)	N/A	0.032	0.004	0.003	1.359	0.243	0.069	0.218	1.953

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	22	12	22	12
normalized size	1	1.	1.	0.94	1.22	1.22	0.67	1.22	0.67
time (sec)	N/A	0.01	0.005	0.001	1.481	0.225	0.138	0.215	1.021

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	140	27	0	70	0
normalized size	1	1.	1.	1.11	15.56	3.	0.	7.78	0.
time (sec)	N/A	0.023	0.007	0.008	1.386	0.231	0.	0.221	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	34	26	18	12
normalized size	1	1.	1.	0.82	1.06	2.	1.53	1.06	0.71
time (sec)	N/A	0.015	0.012	0.104	1.354	0.292	0.717	0.199	1.046

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	18	19	22	18	12
normalized size	1	1.	1.	0.53	1.06	1.12	1.29	1.06	0.71
time (sec)	N/A	0.015	0.01	0.014	1.331	0.237	0.777	0.201	1.015

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	10	15	10
normalized size	1	1.	1.	1.09	1.36	1.36	0.91	1.36	0.91
time (sec)	N/A	0.016	0.003	0.006	1.338	0.24	1.379	0.199	1.061

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	15	15	10	15	15
normalized size	1	1.	0.64	0.55	0.68	0.68	0.45	0.68	0.68
time (sec)	N/A	0.032	0.003	0.004	1.352	0.223	0.068	0.198	1.89

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	9	9	16	11	7	11	12
normalized size	1	1.	0.6	0.6	1.07	0.73	0.47	0.73	0.8
time (sec)	N/A	0.015	0.003	0.002	1.349	0.219	0.064	0.202	1.357

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	19	19	14	22	15
normalized size	1	1.	0.74	0.79	1.	1.	0.74	1.16	0.79
time (sec)	N/A	0.017	0.004	0.01	1.509	0.249	0.088	0.199	1.034

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	18	15	18	15
normalized size	1	1.	1.	0.82	0.82	1.06	0.88	1.06	0.88
time (sec)	N/A	0.007	0.004	0.003	1.346	0.225	0.556	0.204	0.482

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	15	20	15	20
normalized size	1	1.	0.67	0.71	0.62	0.62	0.83	0.62	0.83
time (sec)	N/A	0.015	0.004	0.001	1.35	0.211	0.219	0.201	1.084

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	12	12	8	12	8
normalized size	1	1.	0.86	0.71	0.86	0.86	0.57	0.86	0.57
time (sec)	N/A	0.005	0.001	0.001	1.363	0.213	0.068	0.199	0.559

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	18	15	18	15
normalized size	1	1.	1.	0.82	0.94	1.06	0.88	1.06	0.88
time (sec)	N/A	0.006	0.004	0.	1.404	0.22	0.573	0.2	0.497

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	20	22	46
normalized size	1	1.	1.	0.77	1.	1.	0.91	1.	2.09
time (sec)	N/A	0.018	0.009	0.	1.373	0.233	0.406	0.202	2.838

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	22	15
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.1	0.75
time (sec)	N/A	0.026	0.005	0.009	1.349	0.219	2.95	0.199	1.442

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	19	17	22	22	15	22	22
normalized size	1	1.	0.68	0.61	0.79	0.79	0.54	0.79	0.79
time (sec)	N/A	0.05	0.004	0.004	1.452	0.214	0.081	0.204	2.874

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	18	15	20	15
normalized size	1	1.	1.	0.76	0.95	0.86	0.71	0.95	0.71
time (sec)	N/A	0.018	0.003	0.	1.548	0.215	0.329	0.204	1.892

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	22	15	24	15
normalized size	1	1.	1.	0.94	1.22	1.22	0.83	1.33	0.83
time (sec)	N/A	0.023	0.01	0.009	1.424	0.222	0.207	0.2	0.874

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	18	19	66	18	19
normalized size	1	1.	0.71	0.67	0.86	0.9	3.14	0.86	0.9
time (sec)	N/A	0.011	0.004	0.001	1.548	0.212	3.091	0.204	1.072

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	14	15	14	19	14	14	14
normalized size	1	1.	0.78	0.83	0.78	1.06	0.78	0.78	0.78
time (sec)	N/A	0.012	0.004	0.008	1.415	0.233	0.041	0.2	0.508

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.011	0.003	0.	1.404	0.216	0.036	0.2	0.487

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	26	24	22	24
normalized size	1	1.	0.92	0.75	0.92	1.08	1.	0.92	1.
time (sec)	N/A	0.019	0.003	0.001	1.435	0.22	0.037	0.204	0.57

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	15	8	15	8
normalized size	1	1.	1.15	0.85	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.011	0.003	0.	1.333	0.223	0.042	0.2	0.632

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	18	18	18	12	18	12
normalized size	1	1.	1.82	1.06	1.06	1.06	0.71	1.06	0.71
time (sec)	N/A	0.036	0.016	0.01	1.325	0.221	0.053	0.202	2.33

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	30	18	30	12	18	12
normalized size	1	1.	1.82	1.76	1.06	1.76	0.71	1.06	0.71
time (sec)	N/A	0.037	0.016	0.01	1.337	0.231	0.045	0.199	2.342

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	29	24	34	31	30	32
normalized size	1	1.	0.83	0.81	0.67	0.94	0.86	0.83	0.89
time (sec)	N/A	0.07	0.013	0.01	1.35	0.259	0.041	0.205	2.659

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	26	14	14	20
normalized size	1	1.	0.58	0.79	0.58	1.08	0.58	0.58	0.83
time (sec)	N/A	0.039	0.006	0.002	1.348	0.222	0.053	0.22	1.625

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	19	24	37	19	20
normalized size	1	1.	0.82	0.86	0.86	1.09	1.68	0.86	0.91
time (sec)	N/A	0.016	0.007	0.039	1.335	0.22	0.21	0.212	0.6

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	77	37	19	0
normalized size	1	1.	1.	0.75	0.95	3.85	1.85	0.95	0.
time (sec)	N/A	0.027	0.012	0.014	1.37	0.245	0.76	0.203	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	26	26	19	26	19
normalized size	1	1.	1.	1.12	1.04	1.04	0.76	1.04	0.76
time (sec)	N/A	0.06	0.016	0.01	1.333	0.231	0.053	0.213	3.445

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	34	36	30	36
normalized size	1	1.	0.88	0.71	0.94	1.	1.06	0.88	1.06
time (sec)	N/A	0.026	0.003	0.	1.328	0.227	0.04	0.214	0.655

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	34	36	30	36
normalized size	1	1.	0.88	0.71	0.94	1.	1.06	0.88	1.06
time (sec)	N/A	0.029	0.003	0.056	1.51	0.247	0.038	0.2	0.676

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	36	24	45	41	30	41
normalized size	1	1.	0.65	0.78	0.52	0.98	0.89	0.65	0.89
time (sec)	N/A	0.058	0.021	0.019	1.332	0.235	0.046	0.2	1.898

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	23	23	17	23	17
normalized size	1	1.	1.1	0.81	1.1	1.1	0.81	1.1	0.81
time (sec)	N/A	0.013	0.003	0.001	1.326	0.223	0.047	0.205	0.724

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	22	42	31	22	46
normalized size	1	1.	0.48	0.78	0.48	0.91	0.67	0.48	1.
time (sec)	N/A	0.079	0.009	0.011	1.369	0.231	0.055	0.206	2.724

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	22	39	18	23	0	18	19
normalized size	1	1.	1.05	1.86	0.86	1.1	0.	0.86	0.9
time (sec)	N/A	0.036	0.034	0.091	1.363	0.237	0.	0.213	2.258

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	22	14	18	19	167	18	19
normalized size	1	1.	1.05	0.67	0.86	0.9	7.95	0.86	0.9
time (sec)	N/A	0.036	0.028	0.037	1.492	0.227	103.477	0.202	2.256

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	16	18	39	16	17
normalized size	1	1.	0.95	0.74	0.84	0.95	2.05	0.84	0.89
time (sec)	N/A	0.032	0.01	0.017	1.336	0.226	0.456	0.199	1.688

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	20	20	22	20	14
normalized size	1	1.	1.	0.79	1.05	1.05	1.16	1.05	0.74
time (sec)	N/A	0.022	0.005	0.007	1.36	0.228	0.823	0.201	1.355

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	19	10	19	0
normalized size	1	1.	1.	0.93	1.57	1.36	0.71	1.36	0.
time (sec)	N/A	0.022	0.007	0.017	1.484	0.235	0.078	0.203	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	29	27	50	20	49	26
normalized size	1	1.	1.	1.32	1.23	2.27	0.91	2.23	1.18
time (sec)	N/A	0.054	0.009	0.02	1.338	0.233	0.098	0.209	3.63

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	7	7	19	7	5
normalized size	1	1.	7.2	1.2	1.4	1.4	3.8	1.4	1.
time (sec)	N/A	0.022	0.011	0.036	1.351	0.224	0.481	0.214	1.555

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	20	23	8	14	7
normalized size	1	1.	2.27	1.	1.82	2.09	0.73	1.27	0.64
time (sec)	N/A	0.016	0.01	0.003	1.358	0.211	0.714	0.216	0.516

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	7	8	3
normalized size	1	1.	1.	1.17	1.33	1.33	1.17	1.33	0.5
time (sec)	N/A	0.009	0.004	0.	1.6	0.218	0.044	0.205	0.047

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	16	19	16	10
normalized size	1	1.	1.29	0.93	1.14	1.14	1.36	1.14	0.71
time (sec)	N/A	0.015	0.005	0.003	1.526	0.224	0.052	0.2	0.49

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	13	12	22	19	12	19
normalized size	1	1.	1.55	1.18	1.09	2.	1.73	1.09	1.73
time (sec)	N/A	0.012	0.004	0.043	1.342	0.206	0.045	0.209	0.524

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	19	20	30	31	20	31
normalized size	1	1.	1.42	1.	1.05	1.58	1.63	1.05	1.63
time (sec)	N/A	0.016	0.005	0.043	1.352	0.208	0.048	0.202	0.607

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	8	27	29	8	5
normalized size	1	1.	1.	1.38	1.	3.38	3.62	1.	0.62
time (sec)	N/A	0.032	0.003	0.021	1.343	0.222	0.052	0.199	1.783

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	18	27	29	18	0
normalized size	1	1.	1.59	1.29	1.06	1.59	1.71	1.06	0.
time (sec)	N/A	0.036	0.017	0.022	1.343	0.212	0.053	0.199	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.019	0.003	0.01	1.398	0.214	0.054	0.202	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	19	15	19	0
normalized size	1	1.	1.	2.47	1.12	1.12	0.88	1.12	0.
time (sec)	N/A	0.036	0.017	0.02	1.349	0.214	0.107	0.202	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	23	46	32	20	30	17
normalized size	1	1.	0.91	1.05	2.09	1.45	0.91	1.36	0.77
time (sec)	N/A	0.018	0.005	0.004	1.346	0.224	0.121	0.205	0.517

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	19	24	24	31	24	17
normalized size	1	1.	1.36	0.86	1.09	1.09	1.41	1.09	0.77
time (sec)	N/A	0.02	0.006	0.005	1.497	0.218	0.063	0.205	0.526

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	48	27	27	20	27	20
normalized size	1	1.	1.	2.53	1.42	1.42	1.05	1.42	1.05
time (sec)	N/A	0.025	0.006	0.016	1.355	0.23	0.127	0.201	1.825

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	27	27	20	27	0
normalized size	1	1.	1.	2.32	1.08	1.08	0.8	1.08	0.
time (sec)	N/A	0.047	0.008	0.019	1.36	0.23	0.152	0.205	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	14	8	7	8	0
normalized size	1	1.	1.	0.88	1.75	1.	0.88	1.	0.
time (sec)	N/A	0.019	0.003	0.01	1.363	0.216	0.051	0.199	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	32	49	19	15	19	0
normalized size	1	1.	1.	1.88	2.88	1.12	0.88	1.12	0.
time (sec)	N/A	0.037	0.01	0.02	1.374	0.253	0.132	0.204	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	14	8	7	8	0
normalized size	1	1.	1.	0.88	1.75	1.	0.88	1.	0.
time (sec)	N/A	0.02	0.003	0.018	1.372	0.212	0.057	0.2	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	42	24	36	46	27	39	14
normalized size	1	1.	2.62	1.5	2.25	2.88	1.69	2.44	0.88
time (sec)	N/A	0.022	0.073	0.017	1.373	0.263	0.126	0.206	1.655

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	14	27	8	24	7
normalized size	1	1.	1.	1.5	1.75	3.38	1.	3.	0.88
time (sec)	N/A	0.01	0.004	0.003	1.503	0.224	0.047	0.206	0.475

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	19	38	14	30	14
normalized size	1	1.	1.	1.21	1.36	2.71	1.	2.14	1.
time (sec)	N/A	0.014	0.005	0.005	1.347	0.244	0.092	0.206	0.497

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	22	19	53	41	19	44
normalized size	1	1.	2.18	1.29	1.12	3.12	2.41	1.12	2.59
time (sec)	N/A	0.04	0.02	0.019	1.458	0.235	0.058	0.213	2.84

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	19	41	14	24	22
normalized size	1	1.	1.	1.29	1.12	2.41	0.82	1.41	1.29
time (sec)	N/A	0.04	0.008	0.019	1.336	0.265	0.119	0.198	3.339

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	11	26	15	8	5
normalized size	1	1.	3.4	1.8	2.2	5.2	3.	1.6	1.
time (sec)	N/A	0.005	0.004	0.001	1.486	0.269	0.098	0.213	0.03

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	47	18	36	59	27	73	17
normalized size	1	1.	2.94	1.12	2.25	3.69	1.69	4.56	1.06
time (sec)	N/A	0.012	0.007	0.047	1.42	0.265	0.13	0.205	0.516

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	23	28	19	26	7
normalized size	1	1.	2.38	1.5	2.88	3.5	2.38	3.25	0.88
time (sec)	N/A	0.022	0.007	0.013	1.342	0.272	0.097	0.207	2.797

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	12	19	34	20	19	20
normalized size	1	1.	1.31	0.92	1.46	2.62	1.54	1.46	1.54
time (sec)	N/A	0.012	0.004	0.045	1.347	0.22	0.043	0.201	0.537

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	32	26	18	12
normalized size	1	1.	1.	0.82	1.06	1.88	1.53	1.06	0.71
time (sec)	N/A	0.015	0.011	0.059	1.394	0.226	0.717	0.199	1.02

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	22	18	14
normalized size	1	1.	1.	0.82	1.06	1.06	1.29	1.06	0.82
time (sec)	N/A	0.015	0.009	0.046	1.363	0.229	0.728	0.2	0.996

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	32	26	15	10
normalized size	1	1.	1.	0.8	1.	2.13	1.73	1.	0.67
time (sec)	N/A	0.015	0.01	0.078	1.408	0.222	0.717	0.204	1.042

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	18	19	24	18	12
normalized size	1	1.	1.	0.53	1.06	1.12	1.41	1.06	0.71
time (sec)	N/A	0.016	0.011	0.018	1.342	0.215	0.772	0.202	1.021

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.019	0.002	0.004	1.347	0.219	0.038	0.198	1.122

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	34	114	30	27
normalized size	1	1.	1.	0.77	1.	1.13	3.8	1.	0.9
time (sec)	N/A	0.048	0.014	0.003	1.389	0.26	22.067	0.204	2.271

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	15	7	7	12	0
normalized size	1	1.	1.6	1.2	3.	1.4	1.4	2.4	0.
time (sec)	N/A	0.031	0.003	0.023	1.323	0.252	0.768	0.199	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	93	47	32	39	0
normalized size	1	1.	4.07	1.33	6.2	3.13	2.13	2.6	0.
time (sec)	N/A	0.073	0.015	0.063	1.654	0.232	0.876	0.222	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	19	10	24	0
normalized size	1	1.	1.	0.93	1.57	1.36	0.71	1.71	0.
time (sec)	N/A	0.022	0.007	0.014	1.335	0.265	0.071	0.201	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	29	27	50	20	49	0
normalized size	1	1.	1.	1.32	1.23	2.27	0.91	2.23	0.
time (sec)	N/A	0.054	0.009	0.02	1.356	0.232	0.099	0.206	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.019	0.002	0.	1.341	0.305	0.052	0.2	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	19	15	19	0
normalized size	1	1.	1.	2.47	1.12	1.12	0.88	1.12	0.
time (sec)	N/A	0.038	0.012	0.	1.354	0.214	0.109	0.201	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	90	15	53	15
normalized size	1	1.	1.	1.36	1.12	3.6	0.6	2.12	0.6
time (sec)	N/A	0.016	0.014	0.007	1.548	0.244	0.338	0.208	1.513

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	22	12	26	12
normalized size	1	1.	1.	0.81	1.	1.38	0.75	1.62	0.75
time (sec)	N/A	0.012	0.009	0.005	1.495	0.207	1.23	0.203	1.274

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	41	7	9	7
normalized size	1	1.	1.	0.89	1.	4.56	0.78	1.	0.78
time (sec)	N/A	0.005	0.001	0.004	1.346	0.209	0.152	0.201	0.762

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	24	24	19	26	12
normalized size	1	1.	2.88	0.94	1.5	1.5	1.19	1.62	0.75
time (sec)	N/A	0.007	0.005	0.004	1.345	0.207	1.764	0.232	0.927

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	85	27	31	24
normalized size	1	1.	0.71	0.61	1.13	2.74	0.87	1.	0.77
time (sec)	N/A	0.036	0.01	0.007	1.513	0.209	1.016	0.2	2.57

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	28	46	0	31	36
normalized size	1	1.	1.	0.89	1.04	1.7	0.	1.15	1.33
time (sec)	N/A	0.026	0.017	0.007	1.545	0.216	0.	0.206	1.915

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	19	43	27	45	10
normalized size	1	1.	1.	1.25	1.19	2.69	1.69	2.81	0.62
time (sec)	N/A	0.014	0.009	0.007	1.484	0.21	1.258	0.208	1.394

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	35	115	39	41	20
normalized size	1	1.	0.71	0.81	1.13	3.71	1.26	1.32	0.65
time (sec)	N/A	0.035	0.012	0.005	1.505	0.211	0.878	0.2	2.507

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	23	8	15	8
normalized size	1	1.	1.	1.31	1.15	1.77	0.62	1.15	0.62
time (sec)	N/A	0.006	0.002	0.004	1.357	0.211	0.151	0.2	0.888

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	82	24	15	10
normalized size	1	1.	1.	1.2	1.	5.47	1.6	1.	0.67
time (sec)	N/A	0.006	0.003	0.004	1.336	0.201	0.231	0.2	0.898

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	112	19	26	19
normalized size	1	1.	1.	0.8	1.04	4.48	0.76	1.04	0.76
time (sec)	N/A	0.01	0.011	0.004	1.492	0.208	0.244	0.207	0.619

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	18	15	30	82	24	26	19
normalized size	1	1.	0.72	0.6	1.2	3.28	0.96	1.04	0.76
time (sec)	N/A	0.027	0.006	0.004	1.498	0.223	0.489	0.198	1.877

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	19	3	19	3
normalized size	1	1.	1.	0.83	0.83	3.17	0.5	3.17	0.5
time (sec)	N/A	0.004	0.006	0.004	1.49	0.2	0.158	0.211	0.497

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	20	105	15	34	15
normalized size	1	1.	0.86	0.76	0.95	5.	0.71	1.62	0.71
time (sec)	N/A	0.006	0.008	0.003	1.519	0.198	0.242	0.206	0.558

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	26	30	115	68	34	26
normalized size	1	1.	0.94	0.74	0.86	3.29	1.94	0.97	0.74
time (sec)	N/A	0.034	0.021	0.01	1.501	0.204	4.004	0.208	2.166

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	0	115	76	65	17
normalized size	1	1.	1.	1.22	0.	5.	3.3	2.83	0.74
time (sec)	N/A	0.016	0.012	0.007	0.	0.2	1.034	0.207	1.562

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	25	26	96	94	32	24
normalized size	1	1.	0.93	0.83	0.87	3.2	3.13	1.07	0.8
time (sec)	N/A	0.039	0.013	0.01	1.498	0.207	2.246	0.205	2.579

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	27	39	31	12
normalized size	1	1.	1.	1.39	1.06	1.5	2.17	1.72	0.67
time (sec)	N/A	0.014	0.01	0.007	1.499	0.202	1.279	0.209	1.41

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	32	123	49	32	24
normalized size	1	1.	1.	0.91	0.94	3.62	1.44	0.94	0.71
time (sec)	N/A	0.022	0.048	0.011	1.502	0.203	2.647	0.221	2.111

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	30	39	24	30	24
normalized size	1	1.	1.	0.79	1.03	1.34	0.83	1.03	0.83
time (sec)	N/A	0.021	0.019	0.01	1.492	0.206	0.262	0.211	1.735

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	29	18	19	70	15	50	24
normalized size	1	1.	1.26	0.78	0.83	3.04	0.65	2.17	1.04
time (sec)	N/A	0.029	0.014	0.005	1.48	0.207	1.772	0.206	1.822

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	82	26	12	12
normalized size	1	1.	1.	0.77	0.92	6.31	2.	0.92	0.92
time (sec)	N/A	0.005	0.003	0.004	1.327	0.2	4.084	0.201	0.763

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	117	44	43	24
normalized size	1	1.	0.71	0.94	1.13	3.77	1.42	1.39	0.77
time (sec)	N/A	0.035	0.013	0.006	1.519	0.202	0.905	0.204	2.568

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	32	42	166	112	35	34
normalized size	1	1.	0.73	0.71	0.93	3.69	2.49	0.78	0.76
time (sec)	N/A	0.032	0.024	0.007	1.507	0.21	4.457	0.216	2.771

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	93	26	12	10
normalized size	1	1.	1.	0.77	0.92	7.15	2.	0.92	0.77
time (sec)	N/A	0.007	0.003	0.003	1.768	0.198	0.223	0.209	0.838

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	23	16	31	36	16	14
normalized size	1	1.	1.	1.44	1.	1.94	2.25	1.	0.88
time (sec)	N/A	0.006	0.009	0.003	1.387	0.202	1.222	0.205	0.517

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	26	49	47	0	31	22
normalized size	1	1.	1.36	0.79	1.48	1.42	0.	0.94	0.67
time (sec)	N/A	0.017	0.041	0.006	1.538	0.204	0.	0.213	0.799

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	24	0	24	19
normalized size	1	1.	1.	0.88	1.	3.	0.	3.	2.38
time (sec)	N/A	0.013	0.008	0.006	1.638	0.201	0.	0.213	0.665

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	27	0	28	22
normalized size	1	1.	0.96	1.2	1.2	1.08	0.	1.12	0.88
time (sec)	N/A	0.014	0.009	0.004	1.562	0.197	0.	0.208	0.702

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	37	49	47	0	34	32
normalized size	1	1.	1.2	0.84	1.11	1.07	0.	0.77	0.73
time (sec)	N/A	0.045	0.027	0.005	1.699	0.204	0.	0.215	2.683

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	25	30	38	19	30	20
normalized size	1	1.	0.88	0.96	1.15	1.46	0.73	1.15	0.77
time (sec)	N/A	0.013	0.012	0.006	1.624	0.196	0.131	0.209	0.73

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	80	66	0	49	36
normalized size	1	1.	0.72	0.84	1.86	1.53	0.	1.14	0.84
time (sec)	N/A	0.018	0.026	0.004	1.438	0.206	0.	0.214	0.803

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	23	30	124	29	30	24
normalized size	1	1.	1.	0.7	0.91	3.76	0.88	0.91	0.73
time (sec)	N/A	0.046	0.024	0.014	1.593	0.212	1.632	0.212	3.101

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	30	22	30	24
normalized size	1	1.	1.	0.77	1.	1.	0.73	1.	0.8
time (sec)	N/A	0.033	0.013	0.009	1.537	0.212	1.361	0.209	2.093

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	11	22	3	22	12
normalized size	1	1.	3.	0.93	0.79	1.57	0.21	1.57	0.86
time (sec)	N/A	0.007	0.004	0.004	1.431	0.217	1.701	0.209	0.705

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	10	20	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.67	1.33	0.67
time (sec)	N/A	0.013	0.006	0.007	1.473	0.199	0.091	0.207	1.676

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	27	27	19	28	0
normalized size	1	1.	0.96	0.81	1.04	1.04	0.73	1.08	0.
time (sec)	N/A	0.031	0.007	0.003	1.358	0.192	0.059	0.211	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	30	19
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.2	0.76
time (sec)	N/A	0.055	0.01	0.01	1.336	0.203	0.156	0.208	6.002

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	32	49	20	35	0
normalized size	1	1.	0.97	0.83	1.07	1.63	0.67	1.17	0.
time (sec)	N/A	0.053	0.026	0.013	1.346	0.211	0.098	0.212	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	17	24	17
normalized size	1	1.	1.	0.78	1.	1.	0.74	1.04	0.74
time (sec)	N/A	0.05	0.007	0.009	1.523	0.212	0.149	0.213	4.332

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	32	42	54	34	42	32
normalized size	1	1.	1.	0.84	1.11	1.42	0.89	1.11	0.84
time (sec)	N/A	0.059	0.017	0.009	1.514	0.233	0.12	0.206	5.675

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	104	211	88	100	0
normalized size	1	1.	0.9	0.71	1.01	2.05	0.85	0.97	0.
time (sec)	N/A	0.829	0.074	0.016	1.521	0.245	0.823	0.208	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	39	59	27	41	29
normalized size	1	1.	1.	0.85	1.18	1.79	0.82	1.24	0.88
time (sec)	N/A	0.074	0.035	0.011	1.547	0.209	0.18	0.206	7.376

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	20	26	12	20	12
normalized size	1	1.	0.84	0.84	1.05	1.37	0.63	1.05	0.63
time (sec)	N/A	0.007	0.007	0.	1.502	0.227	0.106	0.202	0.552

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	18	12	20	12
normalized size	1	1.	1.	0.74	0.95	0.95	0.63	1.05	0.63
time (sec)	N/A	0.011	0.004	0.009	1.376	0.232	0.089	0.21	1.119

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	20	20	17	23	17
normalized size	1	1.	1.11	0.84	1.05	1.05	0.89	1.21	0.89
time (sec)	N/A	0.014	0.005	0.009	1.348	0.2	0.091	0.205	1.529

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	50	26	58	26
normalized size	1	1.	1.	0.84	1.09	1.56	0.81	1.81	0.81
time (sec)	N/A	0.059	0.024	0.013	1.377	0.211	0.146	0.217	3.796

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	46	72	32	42	34
normalized size	1	1.	0.77	0.79	1.07	1.67	0.74	0.98	0.79
time (sec)	N/A	0.062	0.034	0.013	1.385	0.23	0.183	0.219	3.971

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	26	35	17	28	17
normalized size	1	1.	1.	0.86	1.24	1.67	0.81	1.33	0.81
time (sec)	N/A	0.019	0.003	0.01	1.361	0.203	0.105	0.222	1.509

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	26	17	35	0
normalized size	1	1.	1.	0.96	1.24	1.04	0.68	1.4	0.
time (sec)	N/A	0.047	0.009	0.01	1.325	0.2	0.094	0.218	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	15	8	18	12
normalized size	1	1.	1.	1.09	1.64	1.36	0.73	1.64	1.09
time (sec)	N/A	0.036	0.005	0.006	1.365	0.2	0.091	0.217	3.199

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	42	42	36	42	36
normalized size	1	1.	1.	0.89	1.17	1.17	1.	1.17	1.
time (sec)	N/A	0.197	0.026	0.007	1.548	0.212	0.276	0.22	14.579

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	45	22	28	22
normalized size	1	1.	1.	0.76	0.97	1.55	0.76	0.97	0.76
time (sec)	N/A	0.174	0.026	0.013	1.517	0.225	0.253	0.213	13.11

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	63	99	63	81	60
normalized size	1	1.	0.9	0.8	1.05	1.65	1.05	1.35	1.
time (sec)	N/A	0.402	0.088	0.015	1.527	0.205	0.365	0.223	65.307

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	25	41	53	26	34	27
normalized size	1	1.	0.76	0.68	1.11	1.43	0.7	0.92	0.73
time (sec)	N/A	0.024	0.02	0.009	1.512	0.198	0.156	0.214	2.39

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	101	197	88	96	83
normalized size	1	1.	0.8	0.7	1.04	2.03	0.91	0.99	0.86
time (sec)	N/A	0.159	0.063	0.019	1.498	0.238	0.293	0.214	9.737

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	47	66	46	49	42
normalized size	1	1.	1.	0.78	1.02	1.43	1.	1.07	0.91
time (sec)	N/A	0.085	0.043	0.013	1.524	0.206	0.198	0.214	9.327

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	50	76	48	51	44
normalized size	1	1.	1.	0.79	1.04	1.58	1.	1.06	0.92
time (sec)	N/A	0.049	0.019	0.01	1.517	0.202	0.214	0.212	3.472

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	18	18	10	19	0
normalized size	1	1.	1.27	0.93	1.2	1.2	0.67	1.27	0.
time (sec)	N/A	0.015	0.004	0.001	1.36	0.195	0.056	0.208	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	8	9	11	11	7	12	7
normalized size	1	1.	0.8	0.9	1.1	1.1	0.7	1.2	0.7
time (sec)	N/A	0.012	0.002	0.003	1.361	0.195	0.057	0.208	1.066

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	10	18	10
normalized size	1	1.	1.	0.92	1.15	1.15	0.77	1.38	0.77
time (sec)	N/A	0.021	0.006	0.009	1.406	0.196	0.1	0.204	1.707

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	8	18	8
normalized size	1	1.	1.	1.09	1.36	1.36	0.73	1.64	0.73
time (sec)	N/A	0.008	0.004	0.007	1.396	0.196	0.09	0.206	1.051

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	13	16	16	10	18	10
normalized size	1	1.	1.25	1.08	1.33	1.33	0.83	1.5	0.83
time (sec)	N/A	0.016	0.004	0.003	1.411	0.19	0.06	0.207	1.617

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	19	27	35	27	80	38	15
normalized size	1	1.	0.73	1.04	1.35	1.04	3.08	1.46	0.58
time (sec)	N/A	0.021	0.01	0.009	1.361	0.221	0.287	0.212	1.792

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	10	19	10
normalized size	1	1.	1.	0.93	1.14	1.14	0.71	1.36	0.71
time (sec)	N/A	0.032	0.005	0.009	1.352	0.196	0.096	0.206	2.18

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	24	24	17	27	0
normalized size	1	1.	1.	0.73	0.92	0.92	0.65	1.04	0.
time (sec)	N/A	0.025	0.008	0.009	1.428	0.203	0.09	0.213	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	23	10	20	10
normalized size	1	1.	1.	1.07	1.36	1.64	0.71	1.43	0.71
time (sec)	N/A	0.015	0.006	0.007	1.457	0.202	0.074	0.203	1.451

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	26	19	30	19
normalized size	1	1.	1.	0.87	1.13	1.13	0.83	1.3	0.83
time (sec)	N/A	0.027	0.007	0.01	1.338	0.211	0.147	0.215	1.898

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	15	24	15
normalized size	1	1.	1.	0.94	1.18	1.18	0.88	1.41	0.88
time (sec)	N/A	0.049	0.009	0.01	1.353	0.209	0.143	0.202	5.725

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	22	8	18	8
normalized size	1	1.	1.	1.08	1.33	1.83	0.67	1.5	0.67
time (sec)	N/A	0.014	0.005	0.009	1.329	0.202	0.07	0.209	2.289

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	21	27	35	19	28	19
normalized size	1	1.	0.73	0.7	0.9	1.17	0.63	0.93	0.63
time (sec)	N/A	0.024	0.012	0.01	1.364	0.204	0.118	0.204	1.781

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	27	36	22	35	22
normalized size	1	1.	0.93	0.75	0.96	1.29	0.79	1.25	0.79
time (sec)	N/A	0.033	0.024	0.01	1.364	0.204	0.129	0.206	2.05

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	24	14	22	14
normalized size	1	1.	1.	1.07	1.36	1.71	1.	1.57	1.
time (sec)	N/A	0.042	0.006	0.01	1.375	0.206	0.115	0.211	3.162

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	23	23	17	27	0
normalized size	1	1.	1.32	0.95	1.21	1.21	0.89	1.42	0.
time (sec)	N/A	0.048	0.011	0.012	1.357	0.217	0.138	0.203	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	19	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.27	0.8
time (sec)	N/A	0.007	0.007	0.002	1.341	0.196	0.081	0.21	1.83

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	36	54	20	41	19
normalized size	1	1.	1.	0.96	1.44	2.16	0.8	1.64	0.76
time (sec)	N/A	0.02	0.019	0.01	1.381	0.198	0.117	0.208	1.53

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	30	42	19	24	17
normalized size	1	1.	1.	0.95	1.43	2.	0.9	1.14	0.81
time (sec)	N/A	0.019	0.014	0.007	1.366	0.189	0.093	0.209	1.507

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	22	17	22	27	15	24	5
normalized size	1	1.	2.75	2.12	2.75	3.38	1.88	3.	0.62
time (sec)	N/A	0.012	0.004	0.008	1.361	0.196	0.1	0.205	1.704

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	10	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.67	1.2	0.67
time (sec)	N/A	0.008	0.006	0.003	1.336	0.194	0.09	0.207	2.166

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	12	19	12
normalized size	1	1.	1.	0.83	1.06	1.06	0.67	1.06	0.67
time (sec)	N/A	0.02	0.003	0.001	1.415	0.194	0.063	0.205	1.897

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	24	17	24	17
normalized size	1	1.	1.	0.95	1.2	1.2	0.85	1.2	0.85
time (sec)	N/A	0.019	0.006	0.003	1.495	0.199	0.104	0.207	2.421

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	35	41	34	35	31
normalized size	1	1.	1.	0.87	1.13	1.32	1.1	1.13	1.
time (sec)	N/A	0.031	0.012	0.002	1.516	0.195	0.099	0.205	1.964

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	28	28	22	28	24
normalized size	1	1.	1.15	0.81	1.04	1.04	0.81	1.04	0.89
time (sec)	N/A	0.04	0.008	0.006	1.504	0.197	0.118	0.206	5.851

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	26	26	19	27	19
normalized size	1	1.	1.22	0.87	1.13	1.13	0.83	1.17	0.83
time (sec)	N/A	0.056	0.011	0.009	1.493	0.202	0.149	0.208	3.863

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	31	43	29	32	29
normalized size	1	1.	1.	0.86	1.11	1.54	1.04	1.14	1.04
time (sec)	N/A	0.049	0.013	0.009	1.511	0.205	0.153	0.204	3.157

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	53	41	45	37
normalized size	1	1.	1.	0.8	1.05	1.29	1.	1.1	0.9
time (sec)	N/A	0.04	0.009	0.003	1.514	0.204	0.172	0.205	2.5

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	36	47	63	42	49	39
normalized size	1	1.	1.02	0.88	1.15	1.54	1.02	1.2	0.95
time (sec)	N/A	0.05	0.013	0.009	1.487	0.202	0.175	0.207	4.227

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	49	20	63	20
normalized size	1	1.	0.92	0.88	1.12	2.04	0.83	2.62	0.83
time (sec)	N/A	0.055	0.022	0.01	1.504	0.199	0.158	0.21	3.947

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	26	19	24	24	19	27	10
normalized size	1	1.	1.86	1.36	1.71	1.71	1.36	1.93	0.71
time (sec)	N/A	0.015	0.007	0.006	1.49	0.203	0.17	0.21	1.445

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	32	32	29	32	29
normalized size	1	1.	1.	0.86	1.1	1.1	1.	1.1	1.
time (sec)	N/A	0.191	0.024	0.007	1.478	0.204	0.263	0.213	17.249

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	19	23	19
normalized size	1	1.	1.	0.78	1.	1.	0.83	1.	0.83
time (sec)	N/A	0.048	0.014	0.008	1.511	0.204	0.25	0.211	8.109

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	35	43	59	41	43	37
normalized size	1	1.	1.	0.9	1.1	1.51	1.05	1.1	0.95
time (sec)	N/A	0.032	0.037	0.005	1.503	0.197	0.142	0.211	2.01

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	19	24	8	19	12
normalized size	1	1.	1.	1.1	1.9	2.4	0.8	1.9	1.2
time (sec)	N/A	0.048	0.008	0.	1.387	0.193	0.104	0.212	3.577

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	15	20	12	20	12
normalized size	1	1.	2.36	1.09	1.36	1.82	1.09	1.82	1.09
time (sec)	N/A	0.067	0.063	0.043	1.341	0.229	0.284	0.214	6.965

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	23	23	19	23	19
normalized size	1	1.	4.1	0.9	1.15	1.15	0.95	1.15	0.95
time (sec)	N/A	0.08	0.227	0.02	1.495	0.259	1.046	0.21	8.512

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	18	12	20	12
normalized size	1	1.	1.	0.74	0.95	0.95	0.63	1.05	0.63
time (sec)	N/A	0.01	0.004	0.008	1.365	0.199	0.089	0.208	0.708

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	15	10	18	10
normalized size	1	1.	1.	0.71	0.88	0.88	0.59	1.06	0.59
time (sec)	N/A	0.009	0.003	0.007	1.341	0.195	0.087	0.205	1.109

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	17	26	17
normalized size	1	1.	1.	0.86	1.1	1.1	0.81	1.24	0.81
time (sec)	N/A	0.018	0.007	0.009	1.336	0.197	0.102	0.207	2.204

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	27	50	62	46	54	56
normalized size	1	1.	0.9	0.55	1.02	1.27	0.94	1.1	1.14
time (sec)	N/A	0.036	0.03	0.003	1.532	0.199	0.098	0.21	1.683

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	68	86	68	72	0
normalized size	1	1.	0.9	0.81	1.08	1.37	1.08	1.14	0.
time (sec)	N/A	0.15	0.041	0.017	1.468	0.208	0.539	0.212	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	54	80	155	63	80	0
normalized size	1	1.	0.78	0.63	0.93	1.8	0.73	0.93	0.
time (sec)	N/A	0.174	0.073	0.02	1.477	0.223	0.278	0.211	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	36	29	38	38	44	39	19
normalized size	1	1.	1.5	1.21	1.58	1.58	1.83	1.62	0.79
time (sec)	N/A	0.018	0.011	0.013	1.327	0.204	1.176	0.209	1.401

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	180	175	367	0	0	188	0
normalized size	1	1.	0.9	0.88	1.84	0.	0.	0.94	0.
time (sec)	N/A	0.669	0.236	0.053	1.542	0.	0.	0.311	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	41	22	41	36	20	31	17
normalized size	1	1.	2.28	1.22	2.28	2.	1.11	1.72	0.94
time (sec)	N/A	0.021	0.014	0.053	1.479	0.255	0.348	0.228	0.579

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	20	19	15	19	15
normalized size	1	1.	1.	1.5	1.11	1.06	0.83	1.06	0.83
time (sec)	N/A	0.015	0.006	0.003	1.357	0.2	0.125	0.208	1.097

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	21	38	27	26	27	0
normalized size	1	1.	0.88	0.66	1.19	0.84	0.81	0.84	0.
time (sec)	N/A	0.028	0.009	0.004	1.344	0.202	0.145	0.209	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	51	16	14
normalized size	1	1.	1.	0.81	1.	1.	3.19	1.	0.88
time (sec)	N/A	0.011	0.006	0.005	1.493	0.206	2.03	0.21	1.16

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	25	9	26	26	26	27	10
normalized size	1	1.	2.5	0.9	2.6	2.6	2.6	2.7	1.
time (sec)	N/A	0.011	0.004	0.006	1.35	0.206	0.787	0.215	0.976

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	50	23	11	0	24	10
normalized size	1	1.	1.	3.57	1.64	0.79	0.	1.71	0.71
time (sec)	N/A	0.01	0.006	0.008	1.361	0.199	0.	0.211	0.96

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	28	28	0	30	26
normalized size	1	1.	1.	1.74	0.9	0.9	0.	0.97	0.84
time (sec)	N/A	0.042	0.008	0.02	1.325	0.197	0.	0.211	2.19

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	18	30	23	76	30	27
normalized size	1	1.	0.66	0.56	0.94	0.72	2.38	0.94	0.84
time (sec)	N/A	0.018	0.008	0.006	1.355	0.226	1.954	0.207	1.288

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	32	32	76	32	29
normalized size	1	1.	1.	0.81	1.03	1.03	2.45	1.03	0.94
time (sec)	N/A	0.026	0.014	0.009	1.498	0.205	2.088	0.21	1.629

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	20	26	19	117	26	24
normalized size	1	1.	0.76	0.69	0.9	0.66	4.03	0.9	0.83
time (sec)	N/A	0.018	0.008	0.007	1.338	0.205	1.44	0.207	1.052

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.01	0.004	0.006	1.525	0.206	0.971	0.209	1.236

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	20	20	17	22	0
normalized size	1	1.	0.95	0.76	0.95	0.95	0.81	1.05	0.
time (sec)	N/A	0.028	0.007	0.005	1.361	0.202	0.163	0.208	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	30	26	31	0
normalized size	1	1.	1.	0.77	1.	1.	0.87	1.03	0.
time (sec)	N/A	0.044	0.012	0.006	1.366	0.2	0.194	0.214	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	26	22	26	26	22
normalized size	1	1.	0.74	0.63	0.96	0.81	0.96	0.96	0.81
time (sec)	N/A	0.028	0.007	0.006	1.339	0.575	1.456	0.206	1.805

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	183	242	396	0	0	189	267
normalized size	1	1.	0.91	1.2	1.97	0.	0.	0.94	1.33
time (sec)	N/A	0.497	0.236	0.022	1.562	0.	0.	0.316	100.799

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	61	77	68	61	61
normalized size	1	1.	1.	0.74	0.98	1.24	1.1	0.98	0.98
time (sec)	N/A	0.076	0.025	0.006	1.53	0.221	1.334	0.214	4.799

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	111	103	0	111	0
normalized size	1	1.	1.	0.64	0.85	0.79	0.	0.85	0.
time (sec)	N/A	0.088	0.031	0.006	1.347	0.205	0.	0.209	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	40	40	50	35	0	38	19
normalized size	1	1.	1.67	1.67	2.08	1.46	0.	1.58	0.79
time (sec)	N/A	0.026	0.022	0.011	1.517	0.211	0.	0.217	1.465

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	15	18	10	16	10
normalized size	1	1.	1.82	1.09	1.36	1.64	0.91	1.45	0.91
time (sec)	N/A	0.035	0.013	0.044	1.396	0.23	0.244	0.21	3.537

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	20	14	20	0
normalized size	1	1.	1.	0.94	1.18	1.18	0.82	1.18	0.
time (sec)	N/A	0.056	0.01	0.01	1.378	0.213	0.129	0.204	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	28	28	26	28	12
normalized size	1	1.	1.	0.83	2.33	2.33	2.17	2.33	1.
time (sec)	N/A	0.018	0.007	0.006	1.385	0.211	0.574	0.204	1.359

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	36	47	47	32	50	20
normalized size	1	1.	1.	1.29	1.68	1.68	1.14	1.79	0.71
time (sec)	N/A	0.031	0.016	0.008	1.362	0.211	0.659	0.205	2.033

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	41	36	20	31	20
normalized size	1	1.	1.	0.51	0.95	0.84	0.47	0.72	0.47
time (sec)	N/A	0.034	0.009	0.023	1.336	0.231	0.283	0.216	0.914

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	54	51	0	50	22
normalized size	1	1.	1.14	0.9	2.57	2.43	0.	2.38	1.05
time (sec)	N/A	0.02	0.025	0.001	1.586	0.225	15.109	0.239	0.567

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	24	16	34	23	14	23	14
normalized size	1	1.	2.18	1.45	3.09	2.09	1.27	2.09	1.27
time (sec)	N/A	0.018	0.014	0.045	1.34	0.238	0.316	0.214	1.43

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	43	22	41	36	20	31	17
normalized size	1	1.	2.39	1.22	2.28	2.	1.11	1.72	0.94
time (sec)	N/A	0.021	0.014	0.046	1.368	0.222	0.328	0.212	0.585

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	34	47	0	38	27
normalized size	1	1.	1.46	1.	1.42	1.96	0.	1.58	1.12
time (sec)	N/A	0.079	0.023	0.046	1.391	0.233	0.	0.204	6.871

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	297	47	0	38	0
normalized size	1	1.	1.62	1.	12.38	1.96	0.	1.58	0.
time (sec)	N/A	0.046	0.04	0.076	1.356	0.225	0.	0.202	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	24	31	45	0	34	14
normalized size	1	1.	3.	1.33	1.72	2.5	0.	1.89	0.78
time (sec)	N/A	0.055	0.038	0.039	1.396	0.218	0.	0.201	3.576

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	154	0	82	32
normalized size	1	1.	1.06	0.97	0.	4.28	0.	2.28	0.89
time (sec)	N/A	0.041	0.056	0.071	0.	0.235	0.	0.225	2.184

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	58	0	35	10
normalized size	1	1.	1.	1.07	1.33	3.87	0.	2.33	0.67
time (sec)	N/A	0.043	0.052	0.003	1.537	0.233	0.	0.204	26.905

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	11	11	7	12	7
normalized size	1	1.	0.83	1.17	0.92	0.92	0.58	1.	0.58
time (sec)	N/A	0.006	0.002	0.003	1.338	0.191	0.06	0.2	0.773

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	35	15	12	15	0
normalized size	1	1.	1.	0.71	2.06	0.88	0.71	0.88	0.
time (sec)	N/A	0.008	0.004	0.001	1.344	0.195	0.143	0.198	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	14	7	11	8
normalized size	1	1.	0.67	0.75	1.17	1.17	0.58	0.92	0.67
time (sec)	N/A	0.015	0.007	0.	1.333	0.205	0.688	0.2	0.502

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	42	24	36	46	27	39	14
normalized size	1	1.	2.62	1.5	2.25	2.88	1.69	2.44	0.88
time (sec)	N/A	0.024	0.069	0.002	1.337	0.225	0.128	0.204	1.629

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	19	15	19	0
normalized size	1	1.	1.	2.47	1.12	1.12	0.88	1.12	0.
time (sec)	N/A	0.039	0.012	0.	1.381	0.209	0.134	0.198	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	15	20	15	20
normalized size	1	1.	0.67	0.71	0.62	0.62	0.83	0.62	0.83
time (sec)	N/A	0.015	0.004	0.003	1.351	0.205	0.22	0.198	1.136

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	41	41	36	45	0
normalized size	1	1.	1.	0.74	0.98	0.98	0.86	1.07	0.
time (sec)	N/A	0.054	0.011	0.012	1.368	0.202	0.163	0.201	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.88
time (sec)	N/A	0.021	0.002	0.006	1.366	0.198	0.546	0.215	1.496

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	15	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.25	0.67
time (sec)	N/A	0.014	0.004	0.002	1.45	0.192	0.059	0.2	1.441

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	4	4	3	4	3
normalized size	1	1.	1.	0.8	0.8	0.8	0.6	0.8	0.6
time (sec)	N/A	0.01	0.003	0.003	1.33	0.205	0.597	0.207	1.976

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	26	14	14	20
normalized size	1	1.	0.58	0.79	0.58	1.08	0.58	0.58	0.83
time (sec)	N/A	0.042	0.007	0.	1.359	0.215	0.056	0.198	1.611

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	15	8	24	8
normalized size	1	1.	1.	1.12	1.38	1.88	1.	3.	1.
time (sec)	N/A	0.035	0.012	0.032	1.346	0.222	0.175	0.211	3.258

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	23	8	15	8
normalized size	1	1.	1.	1.31	1.15	1.77	0.62	1.15	0.62
time (sec)	N/A	0.006	0.003	0.	1.351	0.204	0.274	0.198	0.885

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	12
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.71
time (sec)	N/A	0.012	0.002	0.001	1.342	0.207	0.08	0.199	1.102

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	24	24	109	24	19
normalized size	1	1.	1.	0.79	1.	1.	4.54	1.	0.79
time (sec)	N/A	0.02	0.011	0.009	1.493	0.207	2.541	0.2	1.61

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	22	8	18	8
normalized size	1	1.	1.	1.08	1.33	1.83	0.67	1.5	0.67
time (sec)	N/A	0.013	0.005	0.005	1.323	0.197	0.078	0.199	1.111

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	22	15	22	15
normalized size	1	1.	1.	1.06	1.38	1.38	0.94	1.38	0.94
time (sec)	N/A	0.014	0.004	0.002	1.516	0.209	0.127	0.202	1.611

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	30	39	39	32	0	20
normalized size	1	1.	1.68	1.36	1.77	1.77	1.45	0.	0.91
time (sec)	N/A	0.091	0.016	0.004	1.356	0.238	0.95	0.	5.202

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	60	43	26	62	54	57	20
normalized size	1	1.	2.22	1.59	0.96	2.3	2.	2.11	0.74
time (sec)	N/A	0.019	0.01	0.003	1.335	0.194	1.209	0.236	2.076

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	41	19	15	19	0
normalized size	1	1.	1.	1.29	2.41	1.12	0.88	1.12	0.
time (sec)	N/A	0.037	0.008	0.015	1.33	0.218	0.118	0.223	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	15	22	15
normalized size	1	1.	1.	0.77	1.	1.	0.68	1.	0.68
time (sec)	N/A	0.018	0.005	0.006	1.514	0.197	0.093	0.223	2.016

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	32	32	24	35	24
normalized size	1	1.	0.88	0.78	1.	1.	0.75	1.09	0.75
time (sec)	N/A	0.027	0.015	0.003	1.481	0.22	0.248	0.227	2.033

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	25	47	65	68	54	20
normalized size	1	1.	1.07	0.83	1.57	2.17	2.27	1.8	0.67
time (sec)	N/A	0.04	0.012	0.006	1.509	0.201	2.255	0.223	2.565

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	18	10	20	10
normalized size	1	1.	1.	1.08	1.38	1.38	0.77	1.54	0.77
time (sec)	N/A	0.011	0.005	0.002	1.346	0.196	0.093	0.237	1.592

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	59	19	17	36	0
normalized size	1	1.	0.88	1.06	3.69	1.19	1.06	2.25	0.
time (sec)	N/A	0.041	0.009	0.008	1.378	0.205	0.387	0.224	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	20	20	14	24	12
normalized size	1	1.	1.18	0.82	1.18	1.18	0.82	1.41	0.71
time (sec)	N/A	0.007	0.009	0.003	1.324	0.195	0.1	0.22	6.535

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	3	4	3
normalized size	1	1.	1.	1.33	1.33	1.33	1.	1.33	1.
time (sec)	N/A	0.028	0.008	0.015	1.489	0.22	0.279	0.226	2.316

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	20	22	48
normalized size	1	1.	1.	0.77	1.	1.	0.91	1.	2.18
time (sec)	N/A	0.018	0.011	0.	1.353	0.223	0.424	0.22	2.639

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	7	12	7
normalized size	1	1.	1.	1.11	1.33	1.33	0.78	1.33	0.78
time (sec)	N/A	0.008	0.005	0.004	1.335	0.212	0.036	0.217	0.467

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.67
time (sec)	N/A	0.034	0.005	0.	1.333	0.211	0.079	0.229	3.341

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	26	26	26
normalized size	1	1.	0.81	0.81	0.96	1.04	0.96	0.96	0.96
time (sec)	N/A	0.02	0.019	0.01	1.357	0.303	0.37	0.224	1.49

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	30	26	18	12
normalized size	1	1.	1.	0.82	1.06	1.76	1.53	1.06	0.71
time (sec)	N/A	0.016	0.012	0.049	1.348	0.221	0.736	0.226	1.039

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	27	31
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.08	1.24
time (sec)	N/A	0.03	0.008	0.008	1.497	0.205	0.135	0.215	57.362

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	46	39	34	29	66	0
normalized size	1	1.	1.	1.18	1.	0.87	0.74	1.69	0.
time (sec)	N/A	0.031	0.003	0.004	1.342	0.206	0.11	0.227	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	18	14	18	18	12	18	19
normalized size	1	1.	0.69	0.54	0.69	0.69	0.46	0.69	0.73
time (sec)	N/A	0.046	0.005	0.003	1.348	0.208	0.079	0.223	2.518

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	12	14	7
normalized size	1	1.	1.	0.92	1.17	1.17	1.	1.17	0.58
time (sec)	N/A	0.011	0.01	0.006	1.479	0.214	0.051	0.234	0.487

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	27	0	28	22
normalized size	1	1.	0.96	1.2	1.2	1.08	0.	1.12	0.88
time (sec)	N/A	0.014	0.009	0.003	1.501	0.209	0.	0.221	0.697

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	28	20	28	20
normalized size	1	1.	1.	0.81	1.04	1.04	0.74	1.04	0.74
time (sec)	N/A	0.032	0.003	0.004	1.346	0.218	0.496	0.227	2.624

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	15	15	15	15	15
normalized size	1	1.	1.	0.63	0.79	0.79	0.79	0.79	0.79
time (sec)	N/A	0.009	0.006	0.001	1.345	0.204	0.681	0.219	1.186

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	6	26	20	15	22	7
normalized size	1	1.	3.83	1.	4.33	3.33	2.5	3.67	1.17
time (sec)	N/A	0.018	0.006	0.004	1.344	0.22	0.113	0.227	3.551

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.028	0.007	0.003	1.474	0.228	0.102	0.228	2.281

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	10	43	10
normalized size	1	1.	1.	0.75	0.92	0.92	0.83	3.58	0.83
time (sec)	N/A	0.009	0.005	0.003	1.5	0.208	0.606	0.228	0.896

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	24	34	31	30	31
normalized size	1	1.	0.88	0.76	0.71	1.	0.91	0.88	0.91
time (sec)	N/A	0.052	0.013	0.003	1.343	0.238	0.055	0.227	1.706

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	23	0	8	22
normalized size	1	1.	1.	0.58	0.92	1.92	0.	0.67	1.83
time (sec)	N/A	0.014	0.009	0.004	1.501	0.213	0.	0.23	0.673

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	59	19	28	17	19	12
normalized size	1	1.	1.	3.69	1.19	1.75	1.06	1.19	0.75
time (sec)	N/A	0.079	0.013	0.023	1.342	0.234	1.918	0.227	3.252

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	7	9	12	9	5
normalized size	1	1.	2.86	0.86	1.	1.29	1.71	1.29	0.71
time (sec)	N/A	0.022	0.009	0.019	1.346	0.228	2.264	0.226	1.377

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	6	20	20	15	22	5
normalized size	1	1.	3.83	1.	3.33	3.33	2.5	3.67	0.83
time (sec)	N/A	0.029	0.005	0.003	1.349	0.217	0.099	0.225	2.876

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	43	53	41	45	37
normalized size	1	1.	1.	0.81	1.	1.23	0.95	1.05	0.86
time (sec)	N/A	0.044	0.013	0.009	1.483	0.214	0.188	0.219	2.864

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	100	41	42	77	0
normalized size	1	1.	0.78	1.03	2.7	1.11	1.14	2.08	0.
time (sec)	N/A	0.116	0.015	0.01	1.34	0.21	3.091	0.21	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	16	0	9	48
normalized size	1	1.	1.	0.89	1.	1.78	0.	1.	5.33
time (sec)	N/A	0.033	0.006	0.024	1.342	0.224	0.	0.205	9.289

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	1	12	22	0
normalized size	1	1.	1.	0.85	1.1	0.05	0.6	1.1	0.
time (sec)	N/A	0.008	0.	0.	1.342	0.177	0.025	0.214	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	12	12	8	14	15
normalized size	1	1.	1.	1.	1.	1.	0.67	1.17	1.25
time (sec)	N/A	0.043	0.006	0.009	1.349	0.223	0.073	0.209	5.306

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	15	23	15
normalized size	1	1.	1.	0.86	1.1	1.1	0.71	1.1	0.71
time (sec)	N/A	0.034	0.007	0.009	1.346	0.203	0.104	0.216	3.037

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	41	36	20	31	20
normalized size	1	1.	1.	0.51	0.95	0.84	0.47	0.72	0.47
time (sec)	N/A	0.033	0.009	0.021	1.357	0.232	0.288	0.221	0.927

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	15	22	30	144	22	20
normalized size	1	1.	1.04	0.62	0.92	1.25	6.	0.92	0.83
time (sec)	N/A	0.015	0.008	0.004	1.363	0.227	1.683	0.213	1.176

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	26	22	22	34	22	34
normalized size	1	1.	0.63	0.68	0.58	0.58	0.89	0.58	0.89
time (sec)	N/A	0.033	0.006	0.003	1.374	0.231	0.438	0.214	1.95

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	41	51	0	41	39
normalized size	1	1.	1.	2.51	1.11	1.38	0.	1.11	1.05
time (sec)	N/A	0.065	0.02	0.022	1.58	0.242	0.	0.214	2.646

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	28	14	23	14
normalized size	1	1.	1.	0.94	1.18	1.65	0.82	1.35	0.82
time (sec)	N/A	0.033	0.006	0.011	1.374	0.247	0.107	0.209	3.494

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	51	35	39	35	0
normalized size	1	1.	0.72	0.88	1.27	0.88	0.98	0.88	0.
time (sec)	N/A	0.101	0.038	0.012	1.344	0.237	0.434	0.215	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	27	5	7	5
normalized size	1	1.	1.	1.2	1.4	5.4	1.	1.4	1.
time (sec)	N/A	0.015	2.119	0.012	1.327	0.235	0.735	0.207	29.472

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	24	3	5	3
normalized size	1	1.	1.	0.83	0.83	4.	0.5	0.83	0.5
time (sec)	N/A	0.004	0.007	0.005	1.499	0.221	0.155	0.219	0.529

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	30	84	84	61	30	32
normalized size	1	1.	0.65	0.81	2.27	2.27	1.65	0.81	0.86
time (sec)	N/A	0.027	0.01	0.008	1.355	0.228	0.198	0.209	2.156

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	53	58	24	49	17	24	0
normalized size	1	1.	2.52	2.76	1.14	2.33	0.81	1.14	0.
time (sec)	N/A	0.046	0.035	0.036	1.377	0.228	0.116	0.23	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	30	41	32	0
normalized size	1	1.	1.	0.83	1.07	1.	1.37	1.07	0.
time (sec)	N/A	0.053	0.063	0.014	1.449	0.22	0.236	0.212	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	4	3	4	3
normalized size	1	1.	1.	1.	1.	1.	0.75	1.	0.75
time (sec)	N/A	0.031	0.003	0.003	1.348	0.218	1.574	0.224	2.801

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	20	15	10	20	14
normalized size	1	1.	1.	0.92	1.54	1.15	0.77	1.54	1.08
time (sec)	N/A	0.015	0.004	0.007	1.36	0.195	0.096	0.215	1.475

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	28	28	22	28	41
normalized size	1	1.	0.55	0.55	0.64	0.64	0.5	0.64	0.93
time (sec)	N/A	0.059	0.006	0.005	1.33	0.201	0.073	0.208	2.823

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	34	37	36	37
normalized size	1	1.	1.	0.68	0.88	0.83	0.9	0.88	0.9
time (sec)	N/A	0.033	0.014	0.004	1.493	0.215	4.81	0.213	2.712

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	23	114	18	0
normalized size	1	1.	1.	0.8	1.04	0.92	4.56	0.72	0.
time (sec)	N/A	0.049	0.015	0.001	1.329	0.229	22.198	0.211	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	7	14	7
normalized size	1	1.	1.	0.92	1.17	1.17	0.58	1.17	0.58
time (sec)	N/A	0.005	0.002	0.003	1.329	0.237	0.07	0.213	0.465

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	60	41	58	43	0	41	32
normalized size	1	1.	1.46	1.	1.41	1.05	0.	1.	0.78
time (sec)	N/A	0.03	0.042	0.009	1.53	0.208	0.	0.211	1.69

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	36	93	29	38	29
normalized size	1	1.	0.79	3.5	1.06	2.74	0.85	1.12	0.85
time (sec)	N/A	0.07	0.022	0.	1.552	0.233	4.293	0.217	4.216

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	42	23	14
normalized size	1	1.	1.	0.95	1.21	1.21	2.21	1.21	0.74
time (sec)	N/A	0.017	0.006	0.004	1.51	0.202	0.111	0.213	1.956

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	30	53	159	0	42	39
normalized size	1	1.	1.03	0.79	1.39	4.18	0.	1.11	1.03
time (sec)	N/A	0.027	0.023	0.005	1.555	0.209	0.	0.211	0.814

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	7	11	7
normalized size	1	1.	1.	0.75	0.92	0.92	0.58	0.92	0.58
time (sec)	N/A	0.014	0.005	0.002	1.525	0.229	0.142	0.214	1.236

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	35	41	36	35	32
normalized size	1	1.	1.	0.87	1.13	1.32	1.16	1.13	1.03
time (sec)	N/A	0.033	0.017	0.005	1.485	0.255	0.117	0.214	2.211

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	37	16	163	39	0	203	8
normalized size	1	1.	3.7	1.6	16.3	3.9	0.	20.3	0.8
time (sec)	N/A	0.017	0.009	0.009	1.511	0.245	0.	0.248	1.373

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	39	35	22	41	14
normalized size	1	1.	1.	2.	2.6	2.33	1.47	2.73	0.93
time (sec)	N/A	0.018	0.006	0.007	1.341	0.221	0.17	0.211	1.764

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	0	18	63	18	17
normalized size	1	1.	0.9	0.67	0.	0.86	3.	0.86	0.81
time (sec)	N/A	0.012	0.021	0.003	0.	0.251	0.702	0.214	0.637

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	18	26	23	17	24	17
normalized size	1	1.	0.91	0.78	1.13	1.	0.74	1.04	0.74
time (sec)	N/A	0.03	0.015	0.01	1.348	0.226	0.134	0.212	2.504

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	17	22	17
normalized size	1	1.	1.	0.85	1.1	1.1	0.85	1.1	0.85
time (sec)	N/A	0.015	0.006	0.007	1.336	0.225	0.708	0.214	1.764

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	24	26	14	27	14
normalized size	1	1.	1.	0.89	1.33	1.44	0.78	1.5	0.78
time (sec)	N/A	0.018	0.004	0.01	1.349	0.216	0.127	0.21	1.505

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	32	20	26	34	20	27	10
normalized size	1	1.	2.67	1.67	2.17	2.83	1.67	2.25	0.83
time (sec)	N/A	0.025	0.022	0.013	1.34	0.217	0.115	0.22	4.734

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	14	20	12	22	0
normalized size	1	1.	1.	1.38	1.75	2.5	1.5	2.75	0.
time (sec)	N/A	0.058	0.013	0.062	1.511	0.218	2.373	0.225	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	16	46	16	14
normalized size	1	1.	1.	0.72	0.89	0.89	2.56	0.89	0.78
time (sec)	N/A	0.012	0.006	0.008	1.518	0.21	1.675	0.217	1.124

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	10	61	24	0	0	0
normalized size	1	1.	2.36	0.91	5.55	2.18	0.	0.	0.
time (sec)	N/A	0.083	0.12	0.045	6.504	0.272	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	41	27	30	27
normalized size	1	1.	1.	0.77	1.	1.37	0.9	1.	0.9
time (sec)	N/A	0.021	0.019	0.008	1.526	0.212	0.269	0.221	1.784

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	28	28	26	28	26
normalized size	1	1.	0.83	1.04	1.17	1.17	1.08	1.17	1.08
time (sec)	N/A	0.054	0.013	0.002	1.342	0.244	0.844	0.21	2.033

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	38	42	66	220	0	54	54
normalized size	1	1.	0.76	0.84	1.32	4.4	0.	1.08	1.08
time (sec)	N/A	0.034	0.032	0.007	1.521	0.207	0.	0.213	1.925

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	47	12	1	51	12	7
normalized size	1	1.	1.	4.27	1.09	0.09	4.64	1.09	0.64
time (sec)	N/A	0.007	0.003	0.003	1.351	0.168	0.038	0.209	0.778

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	28	26	26	20	26	20
normalized size	1	1.	1.24	1.12	1.04	1.04	0.8	1.04	0.8
time (sec)	N/A	0.047	0.017	0.016	1.369	0.227	0.051	0.205	3.105

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	26	26	26
normalized size	1	1.	0.81	0.81	0.96	1.04	0.96	0.96	0.96
time (sec)	N/A	0.02	0.021	0.012	1.338	0.211	0.785	0.21	1.558

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	41	24	46	76	36	95	19
normalized size	1	1.	1.71	1.	1.92	3.17	1.5	3.96	0.79
time (sec)	N/A	0.022	0.008	0.016	1.454	0.26	0.132	0.223	0.561

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	47	45	78	17	59	27
normalized size	1	1.	1.03	1.38	1.32	2.29	0.5	1.74	0.79
time (sec)	N/A	0.022	0.016	0.01	1.487	0.212	0.343	0.218	1.684

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	25	47	68	75	51	24
normalized size	1	1.	1.07	0.83	1.57	2.27	2.5	1.7	0.8
time (sec)	N/A	0.042	0.014	0.006	1.502	0.204	2.551	0.215	2.68

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	17	22	22	15	22	27
normalized size	1	1.	0.59	0.53	0.69	0.69	0.47	0.69	0.84
time (sec)	N/A	0.032	0.004	0.005	1.35	0.199	0.085	0.208	2.153

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	23	16	26	23	20
normalized size	1	1.	1.35	0.78	1.	0.7	1.13	1.	0.87
time (sec)	N/A	0.059	0.037	0.011	1.368	0.21	0.471	0.21	2.55

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	28	28	19	28	0
normalized size	1	1.	0.89	0.81	1.04	1.04	0.7	1.04	0.
time (sec)	N/A	0.026	0.009	0.003	1.513	0.223	0.222	0.217	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	72	38	29	43	0
normalized size	1	1.	0.84	0.82	1.89	1.	0.76	1.13	0.
time (sec)	N/A	0.055	0.017	0.004	1.607	0.227	0.897	0.205	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	11	5	5	22	0	8	0
normalized size	1	1.	2.2	1.	1.	4.4	0.	1.6	0.
time (sec)	N/A	0.015	0.022	0.017	1.598	0.206	0.	0.203	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	28	27	28	27
normalized size	1	1.	0.86	0.83	0.97	0.97	0.93	0.97	0.93
time (sec)	N/A	0.042	0.025	0.001	1.483	0.213	0.403	0.203	1.609

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	49	50	0	35	44
normalized size	1	1.	0.92	0.81	1.36	1.39	0.	0.97	1.22
time (sec)	N/A	0.022	0.028	0.004	1.518	0.204	0.	0.206	0.919

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	23	30	0	24	30	0
normalized size	1	1.	1.11	0.82	1.07	0.	0.86	1.07	0.
time (sec)	N/A	0.046	0.014	0.01	1.485	0.	0.143	0.206	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	58	25	57	58	46	51	29
normalized size	1	1.	2.23	0.96	2.19	2.23	1.77	1.96	1.12
time (sec)	N/A	0.025	0.187	0.052	1.322	0.223	0.169	0.207	0.607

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	32	32	45	46	30	46
normalized size	1	1.	0.65	0.7	0.7	0.98	1.	0.65	1.
time (sec)	N/A	0.035	0.004	0.007	1.34	0.3	0.046	0.201	0.777

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	22	32	17	22	17
normalized size	1	1.	0.75	0.85	1.1	1.6	0.85	1.1	0.85
time (sec)	N/A	0.054	0.019	0.009	1.328	0.224	10.023	0.207	2.334

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	22	22	32	14	26	20
normalized size	1	1.	0.86	1.05	1.05	1.52	0.67	1.24	0.95
time (sec)	N/A	0.044	0.012	0.017	1.337	0.23	0.088	0.2	4.164

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	51	31	66	78	0	68	0
normalized size	1	1.	1.38	0.84	1.78	2.11	0.	1.84	0.
time (sec)	N/A	0.071	0.125	0.021	1.488	0.313	0.	0.211	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	26	39	96	0	38	24
normalized size	1	1.	1.57	0.93	1.39	3.43	0.	1.36	0.86
time (sec)	N/A	0.021	0.024	0.009	1.335	0.229	0.	0.203	1.496

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	20	24	17	20	17
normalized size	1	1.	1.21	0.89	1.05	1.26	0.89	1.05	0.89
time (sec)	N/A	0.015	0.003	0.041	1.33	0.234	0.043	0.199	0.719

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	26	25	32	32	22	32	36
normalized size	1	1.	0.57	0.54	0.7	0.7	0.48	0.7	0.78
time (sec)	N/A	0.067	0.006	0.004	1.326	0.231	0.075	0.199	4.026

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	42	34	45	22	34	23	14
normalized size	1	1.	2.33	1.89	2.5	1.22	1.89	1.28	0.78
time (sec)	N/A	0.019	0.007	0.068	1.392	0.238	1.762	0.227	1.287

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	28	24	26	24
normalized size	1	1.	0.81	0.81	0.96	1.04	0.89	0.96	0.89
time (sec)	N/A	0.018	0.026	0.008	1.357	0.23	0.381	0.212	1.419

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	20	0	22	19
normalized size	1	1.	1.	0.94	1.22	1.11	0.	1.22	1.06
time (sec)	N/A	0.021	0.006	0.003	1.328	0.223	0.	0.205	4.759

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	28	20	28	20
normalized size	1	1.	1.	0.81	1.04	1.04	0.74	1.04	0.74
time (sec)	N/A	0.032	0.003	0.	1.337	0.231	0.477	0.206	2.72

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	27	19	27	20
normalized size	1	1.	1.	0.81	1.04	1.04	0.73	1.04	0.77
time (sec)	N/A	0.03	0.013	0.009	1.64	0.238	1.338	0.205	2.038

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	14	12	12	15	721	0
normalized size	1	1.	0.73	0.93	0.8	0.8	1.	48.07	0.
time (sec)	N/A	0.031	0.017	0.013	1.429	0.25	3.673	0.218	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	35	46	50	122	39	39
normalized size	1	1.	0.74	0.74	0.98	1.06	2.6	0.83	0.83
time (sec)	N/A	0.034	0.03	0.01	1.518	0.233	4.473	0.209	2.798

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	36	27	12	1	27	12	7
normalized size	1	1.	3.27	2.45	1.09	0.09	2.45	1.09	0.64
time (sec)	N/A	0.007	0.002	0.001	1.337	0.19	0.032	0.204	0.794

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	18	18	12	18	12
normalized size	1	1.	1.	1.06	1.06	1.06	0.71	1.06	0.71
time (sec)	N/A	0.035	0.01	0.01	1.347	0.218	0.044	0.201	2.426

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	18	27	29	18	0
normalized size	1	1.	1.59	1.29	1.06	1.59	1.71	1.06	0.
time (sec)	N/A	0.035	0.012	0.002	1.392	0.209	0.052	0.202	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	23	36	26	19
normalized size	1	1.	0.67	0.56	0.96	0.85	1.33	0.96	0.7
time (sec)	N/A	0.014	0.006	0.003	1.495	0.212	1.542	0.204	1.505

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	26	24	22	24
normalized size	1	1.	0.92	0.75	0.92	1.08	1.	0.92	1.
time (sec)	N/A	0.018	0.003	0.003	1.376	0.257	0.04	0.201	0.567

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	27	24	12	22	10
normalized size	1	1.	1.	1.42	2.25	2.	1.	1.83	0.83
time (sec)	N/A	0.011	0.005	0.	1.357	0.239	0.088	0.201	0.493

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	22	46	174	53	38	31
normalized size	1	1.	0.62	0.55	1.15	4.35	1.32	0.95	0.78
time (sec)	N/A	0.034	0.011	0.007	1.498	0.205	3.177	0.199	2.162

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [19] had the largest ratio of [1.]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	3	0.333
2	A	1	1	1.	3	0.333
3	A	1	1	1.	3	0.333
4	A	1	1	1.	3	0.333
5	A	1	1	1.	2	0.5
6	A	1	1	1.	2	0.5
7	A	2	2	1.	4	0.5
8	A	2	2	1.	4	0.5
9	A	2	2	1.	5	0.4
10	A	2	2	1.	5	0.4
11	A	1	1	1.	2	0.5
12	A	1	1	1.	2	0.5
13	A	1	1	1.	2	0.5
14	A	1	1	1.	2	0.5
15	A	2	2	1.	4	0.5
16	A	1	1	1.	2	0.5
17	A	3	2	1.	7	0.286
18	A	1	1	1.	6	0.167
19	A	2	2	1.	2	1.
20	A	2	2	1.	7	0.286
21	A	2	2	1.	4	0.5
22	A	2	2	1.	6	0.333
23	A	1	1	1.	4	0.25
24	A	3	2	1.	8	0.25
25	A	3	2	1.	8	0.25
26	A	2	2	1.	4	0.5
27	A	2	2	1.	2	1.
28	A	3	3	1.	6	0.5
29	A	2	2	1.	6	0.333
30	A	1	1	1.	6	0.167
31	A	4	2	1.	7	0.286
32	A	1	1	1.	10	0.1
33	A	1	1	1.	10	0.1
34	A	2	2	1.	4	0.5
35	A	2	2	1.	6	0.333
36	A	2	2	1.	7	0.286
37	A	1	1	1.	8	0.125
38	A	2	2	1.	6	0.333
39	A	3	2	1.	9	0.222
40	A	2	2	1.	2	1.
41	A	2	2	1.	6	0.333
42	A	1	1	1.	9	0.111
43	A	1	1	1.	9	0.111
44	A	2	2	1.	6	0.333
45	A	2	2	1.	9	0.222
46	A	2	2	1.	9	0.222
47	A	2	2	1.	5	0.4
48	A	1	1	1.	3	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	3	3	1.	7	0.429
50	A	1	1	1.	6	0.167
51	A	1	1	1.	3	0.333
52	A	3	3	1.	6	0.5
53	A	3	3	1.	8	0.375
54	A	3	2	1.	9	0.222
55	A	3	3	1.	4	0.75
56	A	2	2	1.	6	0.333
57	A	1	1	1.	8	0.125
58	A	2	2	1.	6	0.333
59	A	2	2	1.	4	0.5
60	A	3	2	1.	4	0.5
61	A	2	1	1.	4	0.25
62	A	3	2	1.	9	0.222
63	A	3	2	1.	9	0.222
64	A	4	3	1.	9	0.333
65	A	3	3	1.	9	0.333
66	A	1	1	1.	10	0.1
67	A	3	2	1.	11	0.182
68	A	4	3	1.	9	0.333
69	A	4	2	1.	4	0.5
70	A	4	2	1.	4	0.5
71	A	4	3	1.	13	0.231
72	A	2	1	1.	4	0.25
73	A	5	3	1.	9	0.333
74	A	3	2	1.	11	0.182
75	A	3	2	1.	11	0.182
76	A	3	3	1.	14	0.214
77	A	3	2	1.	8	0.25
78	A	3	2	1.	7	0.286
79	A	4	3	1.	9	0.333
80	A	2	2	1.	9	0.222
81	A	1	1	1.	8	0.125
82	A	2	2	1.	4	0.5
83	A	3	2	1.	4	0.5
84	A	2	1	1.	4	0.25
85	A	2	1	1.	4	0.25
86	A	2	2	1.	9	0.222
87	A	3	2	1.	9	0.222
88	A	2	2	1.	7	0.286
89	A	3	2	1.	9	0.222
90	A	3	2	1.	4	0.5
91	A	4	2	1.	4	0.5
92	A	3	2	1.	7	0.286
93	A	3	2	1.	9	0.222
94	A	2	2	1.	7	0.286
95	A	3	2	1.	9	0.222
96	A	2	2	1.	7	0.286
97	A	2	2	1.	7	0.286

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
98	A	2	2	1.	4	0.5
99	A	2	2	1.	4	0.5
100	A	3	2	1.	9	0.222
101	A	3	2	1.	9	0.222
102	A	1	1	1.	2	0.5
103	A	2	2	1.	4	0.5
104	A	3	3	1.	5	0.6
105	A	2	1	1.	4	0.25
106	A	1	1	1.	9	0.111
107	A	1	1	1.	7	0.143
108	A	1	1	1.	9	0.111
109	A	1	1	1.	9	0.111
110	A	2	2	1.	7	0.286
111	A	5	2	1.	11	0.182
112	A	2	2	1.	13	0.154
113	A	6	4	1.	10	0.4
114	A	3	2	1.	7	0.286
115	A	4	3	1.	9	0.333
116	A	2	2	1.	7	0.286
117	A	3	2	1.	9	0.222
118	A	2	2	1.	15	0.133
119	A	1	1	1.	13	0.077
120	A	1	1	1.	11	0.091
121	A	2	2	1.	13	0.154
122	A	3	2	1.	15	0.133
123	A	3	3	1.	16	0.188
124	A	1	1	1.	15	0.067
125	A	3	2	1.	15	0.133
126	A	1	1	1.	13	0.077
127	A	1	1	1.	13	0.077
128	A	2	2	1.	11	0.182
129	A	3	2	1.	13	0.154
130	A	1	1	1.	9	0.111
131	A	2	2	1.	9	0.222
132	A	4	4	1.	13	0.308
133	A	1	1	1.	17	0.059
134	A	4	4	1.	15	0.267
135	A	1	1	1.	15	0.067
136	A	3	3	1.	17	0.176
137	A	2	2	1.	15	0.133
138	A	3	3	1.	13	0.231
139	A	1	1	1.	11	0.091
140	A	3	2	1.	15	0.133
141	A	3	3	1.	15	0.2
142	A	2	2	1.	12	0.167
143	A	1	1	1.	11	0.091
144	A	3	3	1.	13	0.231
145	A	2	2	1.	12	0.167
146	A	2	2	1.	14	0.143

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	4	4	1.	17	0.235
148	A	3	3	1.	10	0.3
149	A	2	2	1.	14	0.143
150	A	3	3	1.	17	0.176
151	A	4	4	1.	11	0.364
152	A	2	2	1.	11	0.182
153	A	3	2	1.	12	0.167
154	A	3	2	1.	11	0.182
155	A	3	2	1.	25	0.08
156	A	2	1	1.	29	0.034
157	A	6	5	1.	20	0.25
158	A	6	5	1.	23	0.217
159	A	14	9	1.	32	0.281
160	A	6	5	1.	26	0.192
161	A	2	2	1.	7	0.286
162	A	3	2	1.	11	0.182
163	A	4	3	1.	14	0.214
164	A	2	1	1.	23	0.043
165	A	3	2	1.	22	0.091
166	A	3	2	1.	11	0.182
167	A	4	3	1.	26	0.115
168	A	3	2	1.	16	0.125
169	A	8	4	1.	25	0.16
170	A	6	3	1.	23	0.13
171	A	6	5	1.	26	0.192
172	A	3	2	1.	11	0.182
173	A	8	7	1.	20	0.35
174	A	7	6	1.	23	0.261
175	A	8	8	1.	11	0.727
176	A	2	1	1.	9	0.111
177	A	2	1	1.	7	0.143
178	A	2	1	1.	16	0.062
179	A	3	2	1.	11	0.182
180	A	2	1	1.	13	0.077
181	A	3	2	1.	11	0.182
182	A	3	2	1.	15	0.133
183	A	5	3	1.	20	0.15
184	A	2	1	1.	11	0.091
185	A	2	1	1.	16	0.062
186	A	3	2	1.	25	0.08
187	A	3	2	1.	12	0.167
188	A	2	1	1.	11	0.091
189	A	2	1	1.	14	0.071
190	A	3	2	1.	22	0.091
191	A	2	1	1.	24	0.042
192	A	1	1	1.	20	0.05
193	A	2	1	1.	9	0.111
194	A	2	1	1.	9	0.111
195	A	3	3	1.	11	0.273

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
196	A	1	1	1.	22	0.045
197	A	3	2	1.	11	0.182
198	A	4	4	1.	14	0.286
199	A	4	4	1.	10	0.4
200	A	6	5	1.	23	0.217
201	A	5	4	1.	23	0.174
202	A	6	5	1.	15	0.333
203	A	6	6	1.	7	0.857
204	A	7	7	1.	11	0.636
205	A	5	4	1.	21	0.19
206	A	4	4	1.	11	0.364
207	A	6	4	1.	30	0.133
208	A	6	5	1.	24	0.208
209	A	3	3	1.	14	0.214
210	A	3	2	1.	16	0.125
211	A	2	2	1.	21	0.095
212	A	3	3	1.	15	0.2
213	A	3	2	1.	10	0.2
214	A	4	4	1.	9	0.444
215	A	3	2	1.	18	0.111
216	A	3	2	1.	10	0.2
217	A	6	5	1.	43	0.116
218	A	7	5	1.	50	0.1
219	A	3	3	1.	11	0.273
220	A	9	9	1.	15	0.6
221	A	2	2	1.	11	0.182
222	A	3	2	1.	9	0.222
223	A	3	2	1.	9	0.222
224	A	3	3	1.	11	0.273
225	A	2	2	1.	11	0.182
226	A	2	2	1.	11	0.182
227	A	4	2	1.	13	0.154
228	A	2	1	1.	11	0.091
229	A	3	3	1.	13	0.231
230	A	3	2	1.	11	0.182
231	A	3	3	1.	13	0.231
232	A	3	2	1.	17	0.118
233	A	4	3	1.	17	0.176
234	A	3	2	1.	13	0.154
235	A	10	9	1.	21	0.429
236	A	9	9	1.	13	0.692
237	A	4	3	1.	13	0.231
238	A	5	5	1.	13	0.385
239	A	5	5	1.	12	0.417
240	A	4	3	1.	20	0.15
241	A	3	3	1.	9	0.333
242	A	4	4	1.	11	0.364
243	A	4	3	1.	8	0.375
244	A	2	2	1.	7	0.286

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	2	2	1.	10	0.2
246	A	2	2	1.	11	0.182
247	A	6	6	1.	7	0.857
248	A	4	2	1.	11	0.182
249	A	4	3	1.	9	0.333
250	A	2	2	1.	11	0.182
251	A	2	1	1.	19	0.053
252	A	1	1	1.	9	0.111
253	A	2	1	1.	13	0.077
254	A	1	1	1.	8	0.125
255	A	2	2	1.	7	0.286
256	A	3	2	1.	9	0.222
257	A	3	3	1.	7	0.429
258	A	3	2	1.	20	0.1
259	A	2	2	1.	10	0.2
260	A	2	1	1.	11	0.091
261	A	2	2	1.	7	0.286
262	A	3	3	1.	9	0.333
263	A	1	1	1.	15	0.067
264	A	1	1	1.	13	0.077
265	A	1	1	1.	6	0.167
266	A	3	3	1.	13	0.231
267	A	2	1	1.	7	0.143
268	A	3	3	1.	6	0.5
269	A	4	4	1.	16	0.25
270	A	3	2	1.	9	0.222
271	A	3	2	1.	9	0.222
272	A	4	4	1.	12	0.333
273	A	3	3	1.	4	0.75
274	A	4	4	1.	15	0.267
275	A	3	2	1.	12	0.167
276	A	3	2	1.	6	0.333
277	A	1	1	1.	21	0.048
278	A	2	2	1.	11	0.182
279	A	3	3	1.	6	0.5
280	A	1	1	1.	4	0.25
281	A	3	2	1.	13	0.154
282	A	1	1	1.	10	0.1
283	A	1	1	1.	9	0.111
284	A	5	4	1.	11	0.364
285	A	3	2	1.	8	0.25
286	A	2	2	1.	11	0.182
287	A	2	2	1.	6	0.333
288	A	2	2	1.	14	0.143
289	A	4	3	1.	6	0.5
290	A	2	1	1.	15	0.067
291	A	2	2	1.	13	0.154
292	A	3	3	1.	14	0.214
293	A	2	2	1.	9	0.222

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	4	3	1.	9	0.333
295	A	2	2	1.	14	0.143
296	A	3	2	1.	22	0.091
297	A	2	2	1.	7	0.286
298	A	2	2	1.	13	0.154
299	A	6	6	1.	7	0.857
300	A	6	2	1.	6	0.333
301	A	1	3	1.	8	0.375
302	A	1	0	1.	10	0.
303	A	3	2	1.	15	0.133
304	A	4	3	1.	16	0.188
305	A	4	3	1.	8	0.375
306	A	2	1	1.	9	0.111
307	A	4	3	1.	7	0.429
308	A	5	4	1.	12	0.333
309	A	3	2	1.	17	0.118
310	A	8	4	1.	13	0.308
311	A	2	2	1.	6	0.333
312	A	1	1	1.	11	0.091
313	A	2	1	1.	9	0.111
314	A	3	2	1.	13	0.154
315	A	6	5	1.	6	0.833
316	A	1	1	1.	12	0.083
317	A	4	4	1.	11	0.364
318	A	4	2	1.	9	0.222
319	A	7	4	1.	15	0.267
320	A	5	2	1.	11	0.182
321	A	1	1	1.	6	0.167
322	A	3	3	1.	15	0.2
323	A	5	5	1.	13	0.385
324	A	3	3	1.	13	0.231
325	A	3	3	1.	12	0.25
326	A	2	2	1.	11	0.182
327	A	4	4	1.	12	0.333
328	A	2	2	1.	6	0.333
329	A	2	2	1.	13	0.154
330	A	3	3	1.	15	0.2
331	A	4	3	1.	16	0.188
332	A	2	2	1.	12	0.167
333	A	4	4	1.	8	0.5
334	A	3	3	1.	13	0.231
335	A	4	4	1.	17	0.235
336	A	2	2	1.	13	0.154
337	A	5	4	1.	17	0.235
338	A	2	2	1.	15	0.133
339	A	4	2	1.	6	0.333
340	A	4	4	1.	14	0.286
341	A	1	1	1.	9	0.111
342	A	3	2	1.	9	0.222

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
343	A	1	1	1.	10	0.1
344	A	2	2	1.	8	0.25
345	A	3	3	1.	15	0.2
346	A	4	4	1.	15	0.267
347	A	3	2	1.	9	0.222
348	A	3	2	1.	13	0.154
349	A	3	3	1.	6	0.5
350	A	5	5	1.	8	0.625
351	A	2	2	1.	8	0.25
352	A	3	2	1.	8	0.25
353	A	3	3	1.	14	0.214
354	A	3	2	1.	15	0.133
355	A	3	2	1.	4	0.5
356	A	4	2	1.	6	0.333
357	A	4	4	1.	10	0.4
358	A	3	2	1.	15	0.133
359	A	4	4	1.	13	0.308
360	A	3	3	1.	13	0.231
361	A	2	1	1.	4	0.25
362	A	5	2	1.	9	0.222
363	A	3	3	1.	13	0.231
364	A	1	1	1.	10	0.1
365	A	3	3	1.	10	0.3
366	A	4	3	1.	6	0.5
367	A	4	4	1.	11	0.364
368	A	4	3	1.	9	0.333
369	A	3	3	1.	15	0.2
370	A	1	1	1.	11	0.091
371	A	3	2	1.	9	0.222
372	A	3	2	1.	9	0.222
373	A	2	1	1.	11	0.091
374	A	3	2	1.	4	0.5
375	A	2	2	1.	4	0.5
376	A	3	2	1.	13	0.154

3 Listing of integrals

3.1 $\int x^n\,dx$

Optimal. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

```
[Out] x^(1 + n)/(1 + n)
```

Rubi [A] time = 0.00693595, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rubi in Sympy [A] time = 0.613254, size = 7, normalized size = 0.64

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**n,x)
```

```
[Out] x**(n + 1)/(n + 1)
```

Mathematica [A] time = 0.00182262, size = 11, normalized size = 1.

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$\frac{x^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^n,x)
```

```
[Out] x^(1+n)/(1+n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.213276, size = 14, normalized size = 1.27

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n,x, algorithm="fricas")
```

```
[Out] x*x^n/(n + 1)
```

Sympy [A] time = 0.030976, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

GIAC/XCAS [A] time = 0.205615, size = 15, normalized size = 1.36

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

3.2 $\int e^x\,dx$

Optimal. Leaf size=3

$$e^x$$

```
[Out] E^x
```

Rubi [A] time = 0.00253907, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x,x]
```

```
[Out] E^x
```

Rubi in Sympy [A] time = 0.479961, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x),x)
```

```
[Out] exp(x)
```

Mathematica [A] time = 0.000169911, size = 3, normalized size = 1.

$$e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x,x]
```

```
[Out] E^x
```

Maple [A] time = 0., size = 3, normalized size = 1.

$$\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x),x)
```

```
[Out] exp(x)
```

Maxima [A] time = 1.36485, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="maxima")
```

```
[Out] e^x
```

Fricas [A] time = 0.201549, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="fricas")
```

```
[Out] e^x
```

Sympy [A] time = 0.045966, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x),x)
```

```
[Out] exp(x)
```

GIAC/XCAS [A] time = 0.204118, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x,x, algorithm="giac")
```

```
[Out] e^x
```

3.3 $\int \frac{1}{x}\,dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.00182742, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rubi in Sympy [A] time = 0.023074, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x,x)
```

```
[Out] log(x)
```

Mathematica [A] time = 0.00019487, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x,x)
```

```
[Out] ln(x)
```

Maxima [A] time = 1.36357, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 0.192177, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.026122, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x)
```

```
[Out] log(x)
```

GIAC/XCAS [A] time = 0.202451, size = 4, normalized size = 2.

$$\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="giac")
```

```
[Out] ln(abs(x))
```

3.4 $\int a^x\,dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/Log[a]
```

Rubi [A] time = 0.00507109, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rubi in Sympy [A] time = 0.576181, size = 5, normalized size = 0.62

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(a**x,x)
```

```
[Out] a**x/log(a)
```

Mathematica [A] time = 0.000869394, size = 8, normalized size = 1.

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(a^x,x)
```

```
[Out] a^x/ln(a)
```

Maxima [A] time = 1.30066, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

Fricas [A] time = 0.207634, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

Sympy [A] time = 0.068792, size = 8, normalized size = 1.

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

GIAC/XCAS [A] time = 0.20003, size = 11, normalized size = 1.38

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/ln(a)
```

3.5 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -Cos[x]
```

Rubi [A] time = 0.00383948, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x],x]
```

```
[Out] -Cos[x]
```

Rubi in Sympy [A] time = 0.02618, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Mathematica [A] time = 0.00153816, size = 4, normalized size = 1.

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x],x]
```

```
[Out] -Cos[x]
```

Maple [A] time = 0., size = 5, normalized size = 1.3

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x),x)
```

```
[Out] -cos(x)
```

Maxima [A] time = 1.32517, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A] time = 0.220138, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [A] time = 0.032019, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x)
```

```
[Out] -cos(x)
```

GIAC/XCAS [A] time = 0.201676, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

3.6 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.00299504, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rubi in Sympy [A] time = 0.023028, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x),x)
```

```
[Out] sin(x)
```

Mathematica [A] time = 0.00148056, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x),x)
```

```
[Out] sin(x)
```

Maxima [A] time = 1.36831, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 0.247253, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.030372, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x)
```

```
[Out] sin(x)
```

GIAC/XCAS [A] time = 0.207464, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.7 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

```
[Out] Tan[x]
```

Rubi [A] time = 0.00792822, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Rubi in Sympy [A] time = 0.471343, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

Mathematica [A] time = 0.00265266, size = 2, normalized size = 1.

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Maple [A] time = 0.005, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2,x)
```

```
[Out] tan(x)
```

Maxima [A] time = 1.36381, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

Fricas [A] time = 0.206787, size = 9, normalized size = 4.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

Sympy [A] time = 0.044717, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.208382, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

3.8 $\int \csc^2(x)\,dx$

Optimal. Leaf size=4

$$-\cot(x)$$

```
[Out] -Cot[x]
```

Rubi [A] time = 0.00735385, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Rubi in Sympy [A] time = 0.473958, size = 7, normalized size = 1.75

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csc(x)**2,x)
```

```
[Out] -cos(x)/sin(x)
```

Mathematica [A] time = 0.00273265, size = 4, normalized size = 1.

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Maple [A] time = 0.006, size = 5, normalized size = 1.3

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csc(x)^2,x)
```

```
[Out] -cot(x)
```

Maxima [A] time = 1.42099, size = 8, normalized size = 2.

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x)
```

Fricas [A] time = 0.206007, size = 11, normalized size = 2.75

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/sin(x)
```

Sympy [A] time = 0.043857, size = 7, normalized size = 1.75

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)**2,x)
```

```
[Out] -cos(x)/sin(x)
```

GIAC/XCAS [A] time = 0.210021, size = 8, normalized size = 2.

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x)
```

3.9 $\int \sec(x)\tan(x)\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] Sec[x]
```

Rubi [A] time = 0.00938926, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rubi in Sympy [A] time = 0.723498, size = 3, normalized size = 1.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

Mathematica [A] time = 0.00202805, size = 2, normalized size = 1.

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Maple [A] time = 0.006, size = 3, normalized size = 1.5

$$\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)*tan(x),x)
```

```
[Out] sec(x)
```

Maxima [A] time = 1.38172, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

Fricas [A] time = 0.21349, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [A] time = 0.047054, size = 3, normalized size = 1.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

GIAC/XCAS [A] time = 0.202731, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

3.10 $\int \cot(x)\csc(x)\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -Csc[x]
```

Rubi [A] time = 0.0108525, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\sin(x)}{\tan^2(x)} - \int \frac{\sin(x)}{\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)*csc(x),x)
```

```
[Out] -sin(x)/tan(x)**2 - Integral(sin(x)/tan(x), x)
```

Mathematica [A] time = 0.00243347, size = 4, normalized size = 1.

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Maple [A] time = 0.009, size = 5, normalized size = 1.3

$$-\csc(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)*csc(x),x)
```

```
[Out] -csc(x)
```

Maxima [A] time = 1.37498, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A] time = 0.20786, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [A] time = 0.049476, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x)
```

```
[Out] -1/sin(x)
```

GIAC/XCAS [A] time = 0.203036, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)*csc(x),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.11 $\int \sinh(x)\,dx$

Optimal. Leaf size=2

$$\cosh(x)$$

```
[Out] Cosh[x]
```

Rubi [A] time = 0.00535652, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sinh(x),x)
```

```
[Out] Integral(sinh(x), x)
```

Mathematica [A] time = 0.00316687, size = 2, normalized size = 1.

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sinh(x),x)
```

```
[Out] cosh(x)
```

Maxima [A] time = 1.47955, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

Fricas [A] time = 0.201718, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="fricas")
```

```
[Out] cosh(x)
```

Sympy [A] time = 0.106007, size = 2, normalized size = 1.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x)
```

```
[Out] cosh(x)
```

GIAC/XCAS [A] time = 0.202882, size = 15, normalized size = 7.5

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x
```

3.12 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

```
[Out] Sinh[x]
```

Rubi [A] time = 0.0038318, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cosh(x),x)
```

```
[Out] Integral(cosh(x), x)
```

Mathematica [A] time = 0.00153208, size = 2, normalized size = 1.

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Maple [A] time = 0.003, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cosh(x),x)
```

```
[Out] sinh(x)
```

Maxima [A] time = 1.54598, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="maxima")
```

```
[Out] sinh(x)
```

Fricas [A] time = 0.207141, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [A] time = 0.101384, size = 2, normalized size = 1.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x)
```

```
[Out] sinh(x)
```

GIAC/XCAS [A] time = 0.201525, size = 15, normalized size = 7.5

$$-\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.13 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -Log[Cos[x]]
```

Rubi [A] time = 0.00331054, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Rubi in Sympy [A] time = 0.032367, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

Mathematica [A] time = 0.00322319, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Maple [A] time = 0.001, size = 6, normalized size = 1.2

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x),x)
```

```
[Out] -ln(cos(x))
```

Maxima [A] time = 1.47927, size = 4, normalized size = 0.8

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x))
```

Fricas [A] time = 0.250653, size = 15, normalized size = 3.

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.039097, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

GIAC/XCAS [A] time = 0.201731, size = 8, normalized size = 1.6

$$-\ln(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x),x, algorithm="giac")
```

```
[Out] -ln(abs(cos(x)))
```

3.14 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.00330414, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.031111, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Mathematica [A] time = 0.00322735, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 1.44271, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [A] time = 0.221127, size = 15, normalized size = 5.

$$\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.043343, size = 3, normalized size = 1.

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

GIAC/XCAS [A] time = 0.201969, size = 15, normalized size = 5.

$$\frac{1}{2}\,\ln\left(-\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*ln(-cos(x)^2 + 1)
```

3.15 $\int x \sin(x)\, dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0119021, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi in Sympy [A] time = 0.782283, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Mathematica [A] time = 0.00329263, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.1

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 1.36075, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 0.223055, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.182351, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

GIAC/XCAS [A] time = 0.199181, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

3.16 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x) - x$$

```
[Out] -x + x*Log[x]
```

Rubi [A] time = 0.00288465, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rubi in Sympy [A] time = 0.453226, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Mathematica [A] time = 0.000831316, size = 8, normalized size = 1.

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$-x + x\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x),x)
```

```
[Out] -x+x*ln(x)
```

Maxima [A] time = 1.34864, size = 11, normalized size = 1.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A] time = 0.215977, size = 11, normalized size = 1.38

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A] time = 0.064165, size = 5, normalized size = 0.62

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

GIAC/XCAS [A] time = 0.200761, size = 11, normalized size = 1.38

$$x \ln(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x),x, algorithm="giac")
```

```
[Out] x*ln(x) - x
```

3.17 $\int e^x x^2\,dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi [A] time = 0.0227575, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi in Sympy [A] time = 1.77955, size = 17, normalized size = 0.89

$$x^2 e^x - 2xe^x + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x**2,x)
```

```
[Out] x**2*exp(x) - 2*x*exp(x) + 2*exp(x)
```

Mathematica [A] time = 0.00237395, size = 12, normalized size = 0.63

$$e^x \left(x^2 - 2x + 2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] time = 0., size = 12, normalized size = 0.6

$$\left(x^2 - 2\,x + 2\right) \mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x^2,x)
```

```
[Out] (x^2-2*x+2)*exp(x)
```

Maxima [A] time = 1.33839, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="maxima")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 0.200032, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] time = 0.065619, size = 10, normalized size = 0.53

$$\left(x^2 - 2x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x**2,x)
```

```
[Out] (x**2 - 2*x + 2)*exp(x)
```

GIAC/XCAS [A] time = 0.199207, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="giac")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

3.18 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0131769, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi in Sympy [A] time = 1.17853, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Mathematica [A] time = 0.0161079, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.36702, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x)-\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.216431, size = 18, normalized size = 0.95

$$-\frac{1}{2}\cos(x)\,e^x+\frac{1}{2}\,e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.369177, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.200891, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x)-\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.19 $\int \tan^{-1}(x)\,dx$

Optimal. Leaf size=15

$$x\tan^{-1}(x) - \frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0067478, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$

$$x\tan^{-1}(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 0.908175, size = 12, normalized size = 0.8

$$x\operatorname{atan}(x) - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2
```

Mathematica [A] time = 0.00229204, size = 15, normalized size = 1.

$$x\tan^{-1}(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$x\arctan(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x),x)
```

```
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.35691, size = 18, normalized size = 1.2

$$x \arctan(x) - \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x),x, algorithm="maxima")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.232247, size = 18, normalized size = 1.2

$$x \arctan(x) - \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.235287, size = 12, normalized size = 0.8

$$x \operatorname{atan}(x) - \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.199097, size = 18, normalized size = 1.2

$$x \arctan(x) - \frac{1}{2} \ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(x),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*ln(x^2 + 1)
```

3.20 $\int e^{2x} x \, dx$

Optimal. Leaf size=20

$$\frac{1}{2} e^{2x} x - \frac{e^{2x}}{4}$$

```
[Out] -E^(2*x)/4 + (E^(2*x)*x)/2
```

Rubi [A] time = 0.0144578, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{2} e^{2x} x - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)*x,x]
```

```
[Out] -E^(2*x)/4 + (E^(2*x)*x)/2
```

Rubi in Sympy [A] time = 1.13688, size = 14, normalized size = 0.7

$$\frac{x e^{2x}}{2} - \frac{e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)*x,x)
```

```
[Out] x*exp(2*x)/2 - exp(2*x)/4
```

Mathematica [A] time = 0.00214549, size = 15, normalized size = 0.75

$$e^{2x} \left(\frac{x}{2} - \frac{1}{4} \right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)*x,x]
```

```
[Out] E^(2*x)*(-1/4 + x/2)
```

Maple [A] time = 0.003, size = 12, normalized size = 0.6

$$\frac{(2x - 1) e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)*x,x)
```

```
[Out] 1/4*(2*x-1)*exp(2*x)
```

Maxima [A] time = 1.35264, size = 15, normalized size = 0.75

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 0.2061, size = 15, normalized size = 0.75

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A] time = 0.064042, size = 10, normalized size = 0.5

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)*x,x)
```

```
[Out] (2*x - 1)*exp(2*x)/4
```

GIAC/XCAS [A] time = 0.20213, size = 15, normalized size = 0.75

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(2*x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

3.21 $\int x \cos(x)\, dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0135401, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rubi in Sympy [A] time = 0.779238, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Mathematica [A] time = 0.00356333, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 1.34618, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 0.228681, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.178842, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

GIAC/XCAS [A] time = 0.198707, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.22 $\int x \sin(4x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{16}\sin(4x) - \frac{1}{4}x\cos(4x)$$

```
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Rubi [A] time = 0.0175443, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{16}\sin(4x) - \frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[4*x],x]
```

```
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Rubi in Sympy [A] time = 0.82888, size = 14, normalized size = 0.78

$$-\frac{x\cos(4x)}{4} + \frac{\sin(4x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(4*x),x)
```

```
[Out] -x*cos(4*x)/4 + sin(4*x)/16
```

Mathematica [A] time = 0.00524932, size = 18, normalized size = 1.

$$\frac{1}{16}\sin(4x) - \frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[4*x],x]
```

```
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$-\frac{x\cos(4x)}{4} + \frac{\sin(4x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(4*x),x)
```

```
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

Maxima [A] time = 1.37156, size = 19, normalized size = 1.06

$$-\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Fricas [A] time = 0.227059, size = 19, normalized size = 1.06

$$-\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Sympy [A] time = 0.19319, size = 14, normalized size = 0.78

$$-\frac{x\cos\left(4x\right)}{4}+\frac{\sin\left(4x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(4*x),x)
```

```
[Out] -x*cos(4*x)/4 + sin(4*x)/16
```

GIAC/XCAS [A] time = 0.199063, size = 19, normalized size = 1.06

$$-\frac{1}{4}\,x\cos\left(4\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

3.23 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.00772183, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 \log(x)}{2} - \frac{\int x\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.00118458, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 1.33885, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.216551, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.069775, size = 12, normalized size = 0.71

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

GIAC/XCAS [A] time = 0.201276, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x) - 1/4*x^2
```

3.24 $\int x^2 \cos(3x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2 \sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi [A] time = 0.0391707, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{3}x^2 \sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi in Sympy [A] time = 1.53755, size = 27, normalized size = 0.93

$$\frac{x^2 \sin(3x)}{3} + \frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Mathematica [A] time = 0.0275825, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2 - 2\right)\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] time = 0.008, size = 24, normalized size = 0.8

$$\frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27} + \frac{x^2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(3*x),x)
```

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Maxima [A] time = 1.37338, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="maxima")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Fricas [A] time = 0.227633, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="fricas")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] time = 0.397508, size = 27, normalized size = 0.93

$$\frac{x^2\sin(3x)}{3}+\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

GIAC/XCAS [A] time = 0.200987, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="giac")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

3.25 $\int x^2 \sin(2x)\,dx$

Optimal. Leaf size=29

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rubi [A] time = 0.0352794, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[2*x],x]
```

```
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rubi in Sympy [A] time = 1.46451, size = 24, normalized size = 0.83

$$-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 + cos(2*x)/4
```

Mathematica [A] time = 0.0253343, size = 25, normalized size = 0.86

$$\frac{1}{2}x\sin(2x)-\frac{1}{4}\left(2x^2-1\right)\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[2*x],x]
```

```
[Out] -((-1 + 2*x^2)*Cos[2*x])/4 + (x*Sin[2*x])/2
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$\frac{\cos(2x)}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(2*x),x)
```

```
[Out] 1/4*cos(2*x)-1/2*x^2*cos(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 1.32902, size = 28, normalized size = 0.97

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Fricas [A] time = 0.223277, size = 28, normalized size = 0.97

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Sympy [A] time = 0.405627, size = 24, normalized size = 0.83

$$-\frac{x^2\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 + cos(2*x)/4
```

GIAC/XCAS [A] time = 0.201599, size = 28, normalized size = 0.97

$$-\frac{1}{4}\left(2\,x^2-1\right)\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

3.26 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.00775479, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi in Sympy [A] time = 0.562101, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Mathematica [A] time = 0.00155224, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0., size = 16, normalized size = 1.1

$$2\,x - 2\,x\ln(x) + x\,(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 1.45953, size = 16, normalized size = 1.07

$$\left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 0.217105, size = 20, normalized size = 1.33

$$x \log(x)^2 - 2\,x \log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.082173, size = 15, normalized size = 1.

$$x \log(x)^2 - 2x \log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

GIAC/XCAS [A] time = 0.199812, size = 20, normalized size = 1.33

$$x \ln(x)^2 - 2\,x \ln(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*ln(x)^2 - 2*x*ln(x) + 2*x
```

3.27 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rubi [A] time = 0.00831988, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1$.

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rubi in Sympy [A] time = 1.062, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.00451368, size = 16, normalized size = 1.

$$\sqrt{1-x^2} + x \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$\arcsin(x)\, x + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x),x)
```

```
[Out] arcsin(x)*x+(-x^2+1)^(1/2)
```

Maxima [A] time = 1.50196, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="maxima")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Fricas [A] time = 0.23218, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="fricas")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A] time = 0.132064, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.201503, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

3.28 $\int t\cos(t)\sin(t)\,dt$

Optimal. Leaf size=23

$$-\frac{t}{4}+\frac{1}{2}t\sin^2(t)+\frac{1}{4}\sin(t)\cos(t)$$

```
[Out] -t/4 + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rubi [A] time = 0.0215704, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{t}{4}+\frac{1}{2}t\sin^2(t)+\frac{1}{4}\sin(t)\cos(t)$$

Antiderivative was successfully verified.

```
[In]  Int[t*Cos[t]*Sin[t],t]
```

```
[Out] -t/4 + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rubi in Sympy [A] time = 0.98802, size = 19, normalized size = 0.83

$$\frac{t\sin^2(t)}{2}-\frac{t}{4}+\frac{\sin(t)\cos(t)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t*cos(t)*sin(t),t)
```

```
[Out] t*sin(t)**2/2 - t/4 + sin(t)*cos(t)/4
```

Mathematica [A] time = 0.0045764, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2t)-\frac{1}{4}t\cos(2t)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t*Cos[t]*Sin[t],t]
```

```
[Out] -(t*Cos[2*t])/4 + Sin[2*t]/8
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$-\frac{t(\cos(t))^2}{2}+\frac{\cos(t)\sin(t)}{4}+\frac{t}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t*cos(t)*sin(t),t)
```

```
[Out] -1/2*t*cos(t)^2+1/4*cos(t)*sin(t)+1/4*t
```

Maxima [A] time = 1.34905, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,t\cos\left(2\,t\right)+\frac{1}{8}\,\sin\left(2\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*cos(t)*sin(t),t, algorithm="maxima")
```

```
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

Fricas [A] time = 0.229342, size = 23, normalized size = 1.

$$-\frac{1}{2}\,t\cos\left(t\right)^2+\frac{1}{4}\,\cos\left(t\right)\sin\left(t\right)+\frac{1}{4}\,t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*cos(t)*sin(t),t, algorithm="fricas")
```

```
[Out] -1/2*t*cos(t)^2 + 1/4*cos(t)*sin(t) + 1/4*t
```

Sympy [A] time = 0.410469, size = 24, normalized size = 1.04

$$\frac{t\sin^2\left(t\right)}{4}-\frac{t\cos^2\left(t\right)}{4}+\frac{\sin\left(t\right)\cos\left(t\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*cos(t)*sin(t),t)
```

```
[Out] t*sin(t)**2/4 - t*cos(t)**2/4 + sin(t)*cos(t)/4
```

GIAC/XCAS [A] time = 0.200692, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,t\cos\left(2\,t\right)+\frac{1}{8}\,\sin\left(2\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*cos(t)*sin(t),t, algorithm="giac")
```

```
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

3.29 $\int t \sec^2(t)\, dt$

Optimal. Leaf size=8

$$t \tan(t) + \log(\cos(t))$$

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rubi [A] time = 0.0275211, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$t \tan(t) + \log(\cos(t))$$

Antiderivative was successfully verified.

```
[In]  Int[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{t}{\cos^2(t)}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t*sec(t)**2,t)
```

```
[Out] Integral(t/cos(t)**2, t)
```

Mathematica [A] time = 0.0114151, size = 8, normalized size = 1.

$$t \tan(t) + \log(\cos(t))$$

Antiderivative was successfully verified.

```
[In]  Integrate[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Maple [A] time = 0.007, size = 9, normalized size = 1.1

$$\ln(\cos(t)) + t \tan(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t*sec(t)^2,t)
```

```
[Out] ln(cos(t))+t*tan(t)
```

Maxima [A] time = 1.52218, size = 100, normalized size = 12.5

$$\frac{\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right)\log\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right) + 4t\sin(2t)}{2\left(\cos(2t)^2 + \sin(2t)^2 + 2\cos(2t) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*sec(t)^2,t, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) + 1)*log(cos(2*t)^2 +
sin(2*t)^2 + 2*cos(2*t) + 1) + 4*t*sin(2*t))/(cos(2*t)^2 + sin(2*
t)^2 + 2*cos(2*t) + 1)
```

Fricas [A] time = 0.243203, size = 24, normalized size = 3.

$$\frac{\cos(t)\log(-\cos(t)) + t\sin(t)}{\cos(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*sec(t)^2,t, algorithm="fricas")
```

```
[Out] (cos(t)*log(-cos(t)) + t*sin(t))/cos(t)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int t\sec^2(t)\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*sec(t)**2,t)
```

```
[Out] Integral(t*sec(t)**2, t)
```

GIAC/XCAS [A] time = 0.22179, size = 139, normalized size = 17.38

$$\frac{\ln\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4-2\tan\left(\frac{1}{2}t\right)^2+1\right)}{\tan\left(\frac{1}{2}t\right)^4+2\tan\left(\frac{1}{2}t\right)^2+1}\right)\tan\left(\frac{1}{2}t\right)^2 - 4t\tan\left(\frac{1}{2}t\right) - \ln\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4-2\tan\left(\frac{1}{2}t\right)^2+1\right)}{\tan\left(\frac{1}{2}t\right)^4+2\tan\left(\frac{1}{2}t\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}t\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*sec(t)^2,t, algorithm="giac")
```

```
[Out] 1/2*(ln(4*(tan(1/2*t)^4 - 2*tan(1/2*t)^2 + 1)/(tan(1/2*t)^4 + 2*t
an(1/2*t)^2 + 1))*tan(1/2*t)^2 - 4*t*tan(1/2*t) - ln(4*(tan(1/2*t
)^4 - 2*tan(1/2*t)^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)^2 + 1)))/(
tan(1/2*t)^2 - 1)
```

3.30 $\int t^2 \log(t)\, dt$

Optimal. Leaf size=17

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Rubi [A] time = 0.0132054, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[t^2*Log[t],t]
```

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Rubi in Sympy [A] time = 1.12878, size = 12, normalized size = 0.71

$$\frac{t^3 \log(t)}{3} - \frac{t^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**2*ln(t),t)
```

```
[Out] t**3*log(t)/3 - t**3/9
```

Mathematica [A] time = 0.00172183, size = 17, normalized size = 1.

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[t^2*Log[t],t]
```

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^2*ln(t),t)
```

```
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

Maxima [A] time = 1.41644, size = 18, normalized size = 1.06

$$\frac{1}{3}\,t^3\log(t) - \frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*log(t),t, algorithm="maxima")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Fricas [A] time = 0.220029, size = 18, normalized size = 1.06

$$\frac{1}{3}\,t^3\log(t) - \frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*log(t),t, algorithm="fricas")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Sympy [A] time = 0.082239, size = 12, normalized size = 0.71

$$\frac{t^3\log(t)}{3} - \frac{t^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**2*ln(t),t)
```

```
[Out] t**3*log(t)/3 - t**3/9
```

GIAC/XCAS [A] time = 0.200718, size = 18, normalized size = 1.06

$$\frac{1}{3}\,t^3\ln(t) - \frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*log(t),t, algorithm="giac")
```

```
[Out] 1/3*t^3*ln(t) - 1/9*t^3
```

3.31 $\int e^t t^3\,dt$

Optimal. Leaf size=27

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rubi [A] time = 0.047074, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

Antiderivative was successfully verified.

```
[In]  Int[E^t*t^3,t]
```

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rubi in Sympy [A] time = 2.64576, size = 26, normalized size = 0.96

$$t^3 e^t - 3t^2 e^t + 6te^t - 6e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)*t**3,t)
```

```
[Out] t**3*exp(t) - 3*t**2*exp(t) + 6*t*exp(t) - 6*exp(t)
```

Mathematica [A] time = 0.00284049, size = 17, normalized size = 0.63

$$e^t \left(t^3 - 3t^2 + 6t - 6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t*t^3,t]
```

```
[Out] E^t*(-6 + 6*t - 3*t^2 + t^3)
```

Maple [A] time = 0.004, size = 17, normalized size = 0.6

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right)\,\mathrm{e}^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)*t^3,t)
```

```
[Out] (t^3-3*t^2+6*t-6)*exp(t)
```

Maxima [A] time = 1.40972, size = 22, normalized size = 0.81

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right)e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^t,t, algorithm="maxima")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Fricas [A] time = 0.205059, size = 22, normalized size = 0.81

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^t,t, algorithm="fricas")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Sympy [A] time = 0.06951, size = 15, normalized size = 0.56

$$\left(t^3 - 3t^2 + 6t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)*t**3,t)
```

```
[Out] (t**3 - 3*t**2 + 6*t - 6)*exp(t)
```

GIAC/XCAS [A] time = 0.198196, size = 22, normalized size = 0.81

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^t,t, algorithm="giac")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

3.32 $\int e^{2t} \sin(3t)\, dt$

Optimal. Leaf size=27

$$\frac{2}{13} e^{2t} \sin(3t) - \frac{3}{13} e^{2t} \cos(3t)$$

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rubi [A] time = 0.0206872, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2}{13} e^{2t} \sin(3t) - \frac{3}{13} e^{2t} \cos(3t)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*t)*Sin[3*t],t]
```

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rubi in Sympy [A] time = 1.44696, size = 26, normalized size = 0.96

$$\frac{2e^{2t} \sin(3t)}{13} - \frac{3e^{2t} \cos(3t)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*t)*sin(3*t),t)
```

```
[Out] 2*exp(2*t)*sin(3*t)/13 - 3*exp(2*t)*cos(3*t)/13
```

Mathematica [A] time = 0.0288039, size = 22, normalized size = 0.81

$$\frac{1}{13} e^{2t} (2 \sin(3t) - 3 \cos(3t))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*t)*Sin[3*t],t]
```

```
[Out] (E^(2*t)*(-3*Cos[3*t] + 2*Sin[3*t]))/13
```

Maple [A] time = 0.009, size = 22, normalized size = 0.8

$$-\frac{3\, e^{2t} \cos(3t)}{13} + \frac{2\, e^{2t} \sin(3t)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*t)*sin(3*t),t)
```

```
[Out] -3/13*exp(2*t)*cos(3*t)+2/13*exp(2*t)*sin(3*t)
```

Maxima [A] time = 1.40284, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\,\cos\left(3\,t\right)-2\,\sin\left(3\,t\right)\right)e^{\left(2\,t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)*sin(3*t),t, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

Fricas [A] time = 0.217567, size = 28, normalized size = 1.04

$$-\frac{3}{13}\,\cos\left(3\,t\right)e^{\left(2\,t\right)}+\frac{2}{13}\,e^{\left(2\,t\right)}\sin\left(3\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)*sin(3*t),t, algorithm="fricas")
```

```
[Out] -3/13*cos(3*t)*e^(2*t) + 2/13*e^(2*t)*sin(3*t)
```

Sympy [A] time = 0.373493, size = 26, normalized size = 0.96

$$\frac{2e^{2t}\sin\left(3t\right)}{13}-\frac{3e^{2t}\cos\left(3t\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*t)*sin(3*t),t)
```

```
[Out] 2*exp(2*t)*sin(3*t)/13 - 3*exp(2*t)*cos(3*t)/13
```

GIAC/XCAS [A] time = 0.201499, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\,\cos\left(3\,t\right)-2\,\sin\left(3\,t\right)\right)e^{\left(2\,t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)*sin(3*t),t, algorithm="giac")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

3.33 $\int e^{-t}\cos(3t)\,dt$

Optimal. Leaf size=27

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

```
[Out] -Cos[3*t]/(10*E^t) + (3*Sin[3*t])/(10*E^t)
```

Rubi [A] time = 0.0219816, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3*t]/E^t,t]
```

```
[Out] -Cos[3*t]/(10*E^t) + (3*Sin[3*t])/(10*E^t)
```

Rubi in Sympy [A] time = 1.44892, size = 20, normalized size = 0.74

$$\frac{3e^{-t}\sin(3t)}{10}-\frac{e^{-t}\cos(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3*t)/exp(t),t)
```

```
[Out] 3*exp(-t)*sin(3*t)/10 - exp(-t)*cos(3*t)/10
```

Mathematica [A] time = 0.0189033, size = 20, normalized size = 0.74

$$-\frac{1}{10}e^{-t}(\cos(3t)-3\sin(3t))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3*t]/E^t,t]
```

```
[Out] -(Cos[3*t] - 3*Sin[3*t])/(10*E^t)
```

Maple [A] time = 0.01, size = 22, normalized size = 0.8

$$-\frac{\mathrm{e}^{-t}\cos(3t)}{10}+\frac{3\,\mathrm{e}^{-t}\sin(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3*t)/exp(t),t)
```

```
[Out] -1/10*exp(-t)*cos(3*t)+3/10*exp(-t)*sin(3*t)
```

Maxima [A] time = 1.35266, size = 23, normalized size = 0.85

$$-\frac{1}{10}\left(\cos\left(3\,t\right)-3\,\sin\left(3\,t\right)\right)e^{\left(-t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*t)*e^(-t),t, algorithm="maxima")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

Fricas [A] time = 0.223178, size = 28, normalized size = 1.04

$$-\frac{1}{10}\,\cos\left(3\,t\right)e^{\left(-t\right)}+\frac{3}{10}\,e^{\left(-t\right)}\sin\left(3\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*t)*e^(-t),t, algorithm="fricas")
```

```
[Out] -1/10*cos(3*t)*e^(-t) + 3/10*e^(-t)*sin(3*t)
```

Sympy [A] time = 0.793812, size = 20, normalized size = 0.74

$$\frac{3e^{-t}\sin\left(3t\right)}{10}-\frac{e^{-t}\cos\left(3t\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*t)/exp(t),t)
```

```
[Out] 3*exp(-t)*sin(3*t)/10 - exp(-t)*cos(3*t)/10
```

GIAC/XCAS [A] time = 0.199597, size = 23, normalized size = 0.85

$$-\frac{1}{10}\left(\cos\left(3\,t\right)-3\,\sin\left(3\,t\right)\right)e^{\left(-t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*t)*e^(-t),t, algorithm="giac")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

3.34 $\int y \sinh(y)\, dy$

Optimal. Leaf size=9

$$y \cosh(y) - \sinh(y)$$

```
[Out] y*Cosh[y] - Sinh[y]
```

Rubi [A] time = 0.0191523, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$y \cosh(y) - \sinh(y)$$

Antiderivative was successfully verified.

```
[In]  Int[y*Sinh[y],y]
```

```
[Out] y*Cosh[y] - Sinh[y]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$y \cosh(y) - \int \cosh(y)\, dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(y*sinh(y),y)
```

```
[Out] y*cosh(y) - Integral(cosh(y), y)
```

Mathematica [A] time = 0.00513669, size = 9, normalized size = 1.

$$y \cosh(y) - \sinh(y)$$

Antiderivative was successfully verified.

```
[In]  Integrate[y*Sinh[y],y]
```

```
[Out] y*Cosh[y] - Sinh[y]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$y \cosh(y) - \sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(y*sinh(y),y)
```

```
[Out] y*cosh(y)-sinh(y)
```

Maxima [A] time = 1.35613, size = 46, normalized size = 5.11

$$\frac{1}{2} y^2 \sinh(y) + \frac{1}{4} \left(y^2 + 2y + 2\right) e^{(-y)} - \frac{1}{4} \left(y^2 - 2y + 2\right) e^{y}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*sinh(y),y, algorithm="maxima")
```

```
[Out] 1/2*y^2*sinh(y) + 1/4*(y^2 + 2*y + 2)*e^(-y) - 1/4*(y^2 - 2*y + 2
)*e^y
```

Fricas [A] time = 0.210783, size = 12, normalized size = 1.33

$$y\cosh(y) - \sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*sinh(y),y, algorithm="fricas")
```

```
[Out] y*cosh(y) - sinh(y)
```

Sympy [A] time = 0.177784, size = 7, normalized size = 0.78

$$y\cosh(y) - \sinh(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*sinh(y),y)
```

```
[Out] y*cosh(y) - sinh(y)
```

GIAC/XCAS [A] time = 0.198044, size = 23, normalized size = 2.56

$$\frac{1}{2}(y+1)e^{(-y)} + \frac{1}{2}(y-1)e^{y}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*sinh(y),y, algorithm="giac")
```

```
[Out] 1/2*(y + 1)*e^(-y) + 1/2*(y - 1)*e^y
```

3.35 $\int y \cosh(ay)\, dy$

Optimal. Leaf size=19

$$\frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rubi [A] time = 0.0289063, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[y*Cosh[a*y],y]
```

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{y \sinh(ay)}{a} - \frac{\int \sinh(ay)\, dy}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(y*cosh(a*y),y)
```

```
[Out] y*sinh(a*y)/a - Integral(sinh(a*y), y)/a
```

Mathematica [A] time = 0.00822932, size = 19, normalized size = 1.

$$\frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[y*Cosh[a*y],y]
```

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Maple [A] time = 0.007, size = 19, normalized size = 1.

$$\frac{ya \sinh(ay) - \cosh(ay)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(y*cosh(a*y),y)
```

```
[Out] 1/a^2*(y*a*sinh(a*y)-cosh(a*y))
```

Maxima [A] time = 1.39774, size = 77, normalized size = 4.05

$$\frac{1}{2}\,y^2\cosh(ay) - \frac{1}{4}\,a\left(\frac{\left(a^2y^2 - 2\,ay + 2\right)e^{(ay)}}{a^3} + \frac{\left(a^2y^2 + 2\,ay + 2\right)e^{(-ay)}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*cosh(a*y),y, algorithm="maxima")
```

```
[Out] 1/2*y^2*cosh(a*y) - 1/4*a*((a^2*y^2 - 2*a*y + 2)*e^(a*y)/a^3 + (a
^2*y^2 + 2*a*y + 2)*e^(-a*y)/a^3)
```

Fricas [A] time = 0.207366, size = 24, normalized size = 1.26

$$\frac{ay\sinh(ay) - \cosh(ay)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*cosh(a*y),y, algorithm="fricas")
```

```
[Out] (a*y*sinh(a*y) - cosh(a*y))/a^2
```

Sympy [A] time = 0.253345, size = 20, normalized size = 1.05

$$\begin{cases} \frac{y\sinh(ay)}{a} - \frac{\cosh(ay)}{a^2} & \text{for } a \neq 0 \\ \frac{y^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*cosh(a*y),y)
```

```
[Out] Piecewise((y*sinh(a*y)/a - cosh(a*y)/a**2, Ne(a, 0)), (y**2/2, Tr
ue))
```

GIAC/XCAS [A] time = 0.197812, size = 41, normalized size = 2.16

$$\frac{(ay-1)e^{(ay)}}{2\,a^2} - \frac{(ay+1)e^{(-ay)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(y*cosh(a*y),y, algorithm="giac")
```

```
[Out] 1/2*(a*y - 1)*e^(a*y)/a^2 - 1/2*(a*y + 1)*e^(-a*y)/a^2
```

3.36 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t} t - e^{-t}$$

```
[Out] -E^(-t) - t/E^t
```

Rubi [A] time = 0.0165281, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-e^{-t} t - e^{-t}$$

Antiderivative was successfully verified.

```
[In]  Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rubi in Sympy [A] time = 1.07455, size = 10, normalized size = 0.62

$$-te^{-t} - e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t/exp(t),t)
```

```
[Out] -t*exp(-t) - exp(-t)
```

Mathematica [A] time = 0.00209173, size = 11, normalized size = 0.69

$$e^{-t}(-t - 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t/E^t,t]
```

```
[Out] (-1 - t)/E^t
```

Maple [A] time = 0., size = 10, normalized size = 0.6

$$-\frac{1+t}{e^t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t/exp(t),t)
```

```
[Out] -(1+t)/exp(t)
```

Maxima [A] time = 1.35348, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="maxima")
```

```
[Out] -(t + 1)*e^(-t)
```

Fricas [A] time = 0.213559, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

Sympy [A] time = 0.061141, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

GIAC/XCAS [A] time = 0.200484, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

3.37 $\int \sqrt{t}\log(t)\,dt$

Optimal. Leaf size=21

$$\frac{2}{3}t^{3/2}\log(t)-\frac{4t^{3/2}}{9}$$

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rubi [A] time = 0.0140761, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2}{3}t^{3/2}\log(t)-\frac{4t^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[t]*Log[t],t]
```

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rubi in Sympy [A] time = 1.10487, size = 19, normalized size = 0.9

$$\frac{2t^{\frac{3}{2}}\log(t)}{3}-\frac{4t^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(t)*t**(1/2),t)
```

```
[Out] 2*t**(3/2)*log(t)/3 - 4*t**(3/2)/9
```

Mathematica [A] time = 0.00441865, size = 15, normalized size = 0.71

$$\frac{2}{9}t^{3/2}(3\log(t)-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[t]*Log[t],t]
```

```
[Out] (2*t^(3/2)*(-2 + 3*Log[t]))/9
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$-\frac{4}{9}t^{\frac{3}{2}}+\frac{2\ln(t)}{3}t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(t)*t^(1/2),t)
```

```
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

Maxima [A] time = 1.34677, size = 18, normalized size = 0.86

$$\frac{2}{3}\,t^{\frac{3}{2}}\log(t)-\frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)*log(t),t, algorithm="maxima")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

Fricas [A] time = 0.248596, size = 19, normalized size = 0.9

$$\frac{2}{9}\,(3\,t\log(t)-2\,t)\sqrt{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)*log(t),t, algorithm="fricas")
```

```
[Out] 2/9*(3*t*log(t) - 2*t)*sqrt(t)
```

Sympy [A] time = 3.08115, size = 66, normalized size = 3.14

$$\begin{cases} \frac{2t^{\frac{3}{2}}\log(t)}{3}-\frac{4t^{\frac{3}{2}}}{9} & \text{for } |t|<1 \\ -\frac{2t^{\frac{3}{2}}\log\left(\frac{1}{t}\right)}{3}-\frac{4t^{\frac{3}{2}}}{9} & \text{for } \left|\frac{1}{t}\right|<1 \\ -G^{2,1}_{3,3}\left(\begin{matrix}1 & \frac{5}{2},\frac{5}{2}\\ \frac{3}{2},\frac{3}{2} & 0\end{matrix}\middle|t\right)+G^{0,3}_{3,3}\left(\begin{matrix}\frac{5}{2},\frac{5}{2},1 & \\ & \frac{3}{2},\frac{3}{2},0\end{matrix}\middle|t\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(t)*t**(1/2),t)
```

```
[Out] Piecewise((2*t**(3/2)*log(t)/3 - 4*t**(3/2)/9, Abs(t) < 1), (-2*t
**(3/2)*log(1/t)/3 - 4*t**(3/2)/9, Abs(1/t) < 1), (-meijerg(((1,)
, (5/2, 5/2)), ((3/2, 3/2), (0,)), t) + meijerg(((5/2, 5/2, 1), (
)), ((), (3/2, 3/2, 0)), t), True))
```

GIAC/XCAS [A] time = 0.218856, size = 18, normalized size = 0.86

$$\frac{2}{3}\,t^{\frac{3}{2}}\ln(t)-\frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)*log(t),t, algorithm="giac")
```

```
[Out] 2/3*t^(3/2)*ln(t) - 4/9*t^(3/2)
```

3.38 $\int x \cos(2x)\, dx$

Optimal. Leaf size=18

$$\frac{1}{2}x \sin(2x) + \frac{1}{4}\cos(2x)$$

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rubi [A] time = 0.0191007, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}x \sin(2x) + \frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[2*x],x]
```

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rubi in Sympy [A] time = 0.850406, size = 14, normalized size = 0.78

$$\frac{x \sin(2x)}{2} + \frac{\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(2*x),x)
```

```
[Out] x*sin(2*x)/2 + cos(2*x)/4
```

Mathematica [A] time = 0.00491014, size = 18, normalized size = 1.

$$\frac{1}{2}x \sin(2x) + \frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[2*x],x]
```

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$\frac{\cos(2x)}{4} + \frac{x \sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(2*x),x)
```

```
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 1.34922, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Fricas [A] time = 0.22197, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Sympy [A] time = 0.188105, size = 14, normalized size = 0.78

$$\frac{x\sin\left(2x\right)}{2}+\frac{\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(2*x),x)
```

```
[Out] x*sin(2*x)/2 + cos(2*x)/4
```

GIAC/XCAS [A] time = 0.232345, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{1}{4}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

3.39 $\int e^{-x} x^2 \, dx$

Optimal. Leaf size=26

$$-e^{-x} x^2 - 2e^{-x} x - 2e^{-x}$$

```
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rubi [A] time = 0.0321554, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-e^{-x} x^2 - 2e^{-x} x - 2e^{-x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/E^x,x]
```

```
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rubi in Sympy [A] time = 1.95296, size = 19, normalized size = 0.73

$$-x^2 e^{-x} - 2xe^{-x} - 2e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/exp(x),x)
```

```
[Out] -x**2*exp(-x) - 2*x*exp(-x) - 2*exp(-x)
```

Mathematica [A] time = 0.00363885, size = 16, normalized size = 0.62

$$e^{-x} \left(-x^2 - 2x - 2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/E^x,x]
```

```
[Out] (-2 - 2*x - x^2)/E^x
```

Maple [A] time = 0.003, size = 15, normalized size = 0.6

$$-\frac{x^2 + 2x + 2}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/exp(x),x)
```

```
[Out] -(x^2+2*x+2)/exp(x)
```

Maxima [A] time = 1.35939, size = 19, normalized size = 0.73

$$-\left(x^2 + 2\,x + 2\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-x),x, algorithm="maxima")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Fricas [A] time = 0.242986, size = 19, normalized size = 0.73

$$-\left(x^2 + 2\,x + 2\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-x),x, algorithm="fricas")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Sympy [A] time = 0.069028, size = 12, normalized size = 0.46

$$\left(-x^2 - 2x - 2\right) e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/exp(x),x)
```

```
[Out] (-x**2 - 2*x - 2)*exp(-x)
```

GIAC/XCAS [A] time = 0.218072, size = 19, normalized size = 0.73

$$-\left(x^2 + 2\,x + 2\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-x),x, algorithm="giac")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

3.40 $\int \cos^{-1}(x)\,dx$

Optimal. Leaf size=18

$$x\cos^{-1}(x) - \sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rubi [A] time = 0.00999595, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 1$.

$$x\cos^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rubi in Sympy [A] time = 1.02105, size = 12, normalized size = 0.67

$$x\operatorname{acos}(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acos(x),x)
```

```
[Out] x*acos(x) - sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.00474919, size = 18, normalized size = 1.

$$x\cos^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$x\arccos(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccos(x),x)
```

```
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.48087, size = 22, normalized size = 1.22

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x),x, algorithm="maxima")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Fricas [A] time = 0.225051, size = 22, normalized size = 1.22

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x),x, algorithm="fricas")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Sympy [A] time = 0.137691, size = 12, normalized size = 0.67

$$x \operatorname{acos}(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acos(x),x)
```

```
[Out] x*acos(x) - sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.215181, size = 22, normalized size = 1.22

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccos(x),x, algorithm="giac")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

3.41 $\int x \csc^2(x)\, dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x\cot(x)$$

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rubi [A] time = 0.022737, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sin^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x/sin(x)**2, x)
```

Mathematica [A] time = 0.00653949, size = 9, normalized size = 1.

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Maple [A] time = 0.008, size = 10, normalized size = 1.1

$$-x\cot(x) + \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*csc(x)^2,x)
```

```
[Out] -x*cot(x)+ln(sin(x))
```

Maxima [A] time = 1.38577, size = 140, normalized size = 15.56

$$\frac{\left(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\right)\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\left(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\right)\log$$
$$2\left(\cos(2x)^2+\sin(2x)^2-2\cos(2x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 + 2*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) +
1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2
*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)
```

Fricas [A] time = 0.230848, size = 27, normalized size = 3.

$$-\frac{x\cos(x)-\log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

GIAC/XCAS [A] time = 0.221, size = 70, normalized size = 7.78

$$\frac{x\tan\left(\frac{1}{2}x\right)^2+\ln\left(\frac{16\tan\left(\frac{1}{2}x\right)^2}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)-x}{2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + ln(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/
2*x)^2 + 1))*tan(1/2*x) - x)/tan(1/2*x)
```

3.42 $\int \cos(5x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x)$$

```
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rubi [A] time = 0.0150587, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[5*x]*Sin[3*x],x]
```

```
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rubi in Sympy [A] time = 1.04645, size = 12, normalized size = 0.71

$$\frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(5*x)*sin(3*x),x)
```

```
[Out] cos(2*x)/4 - cos(8*x)/16
```

Mathematica [A] time = 0.0122521, size = 17, normalized size = 1.

$$\frac{\cos^2(x)}{2} - \frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[5*x]*Sin[3*x],x]
```

```
[Out] Cos[x]^2/2 - Cos[8*x]/16
```

Maple [A] time = 0.104, size = 14, normalized size = 0.8

$$\frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(5*x)*sin(3*x),x)
```

```
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

Maxima [A] time = 1.35434, size = 18, normalized size = 1.06

$$-\frac{1}{16}\cos(8x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

Fricas [A] time = 0.291506, size = 34, normalized size = 2.

$$-8\cos(x)^8+16\cos(x)^6-10\cos(x)^4+\frac{5}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -8*cos(x)^8 + 16*cos(x)^6 - 10*cos(x)^4 + 5/2*cos(x)^2
```

Sympy [A] time = 0.716914, size = 26, normalized size = 1.53

$$\frac{5\sin(3x)\sin(5x)}{16}+\frac{3\cos(3x)\cos(5x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*sin(3*x),x)
```

```
[Out] 5*sin(3*x)*sin(5*x)/16 + 3*cos(3*x)*cos(5*x)/16
```

GIAC/XCAS [A] time = 0.198772, size = 18, normalized size = 1.06

$$-\frac{1}{16}\cos(8x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

3.43 $\int \sin(2x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rubi [A] time = 0.0153777, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rubi in Sympy [A] time = 1.01537, size = 12, normalized size = 0.71

$$\frac{\sin(2x)}{4} - \frac{\sin(6x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)*sin(4*x),x)
```

```
[Out] sin(2*x)/4 - sin(6*x)/12
```

Mathematica [A] time = 0.0104433, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Maple [A] time = 0.014, size = 9, normalized size = 0.5

$$\frac{(\sin(2x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)*sin(4*x),x)
```

```
[Out] 1/3*sin(2*x)^3
```

Maxima [A] time = 1.33073, size = 18, normalized size = 1.06

$$-\frac{1}{12}\sin(6x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.237042, size = 19, normalized size = 1.12

$$-\frac{1}{3}\left(\cos(2x)^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(2*x)^2 - 1)*sin(2*x)
```

Sympy [A] time = 0.777018, size = 22, normalized size = 1.29

$$-\frac{\sin(2x)\cos(4x)}{3}+\frac{\sin(4x)\cos(2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*sin(4*x),x)
```

```
[Out] -sin(2*x)*cos(4*x)/3 + sin(4*x)*cos(2*x)/6
```

GIAC/XCAS [A] time = 0.201293, size = 18, normalized size = 1.06

$$-\frac{1}{12}\sin(6x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(4*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

3.44 $\int \cos(x) \log(\sin(x))\, dx$

Optimal. Leaf size=11

$$\sin(x) \log(\sin(x)) - \sin(x)$$

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rubi [A] time = 0.0160724, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\sin(x) \log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rubi in Sympy [A] time = 1.06105, size = 10, normalized size = 0.91

$$\log(\sin(x)) \sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Mathematica [A] time = 0.00306192, size = 11, normalized size = 1.

$$\sin(x) \log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.1

$$-\sin(x) + \ln(\sin(x)) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*ln(sin(x)),x)
```

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Maxima [A] time = 1.33769, size = 15, normalized size = 1.36

$$\log(\sin(x)) \sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Fricas [A] time = 0.240417, size = 15, normalized size = 1.36

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A] time = 1.37889, size = 10, normalized size = 0.91

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

GIAC/XCAS [A] time = 0.198805, size = 15, normalized size = 1.36

$$\ln(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] ln(sin(x))*sin(x) - sin(x)
```

3.45 $\int e^{x^2} x^3 \, dx$

Optimal. Leaf size=22

$$\frac{1}{2} e^{x^2} x^2 - \frac{e^{x^2}}{2}$$

```
[Out] -E^x^2/2 + (E^x^2*x^2)/2
```

Rubi [A] time = 0.0316217, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{2} e^{x^2} x^2 - \frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*x^3,x]
```

```
[Out] -E^x^2/2 + (E^x^2*x^2)/2
```

Rubi in Sympy [A] time = 1.89002, size = 15, normalized size = 0.68

$$\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*x**3,x)
```

```
[Out] x**2*exp(x**2)/2 - exp(x**2)/2
```

Mathematica [A] time = 0.00330734, size = 14, normalized size = 0.64

$$\frac{1}{2} e^{x^2} \left(x^2 - 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*x^3,x]
```

```
[Out] (E^x^2*(-1 + x^2))/2
```

Maple [A] time = 0.004, size = 12, normalized size = 0.6

$$\frac{\left(x^2 - 1\right) \mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*x^3,x)
```

```
[Out] 1/2*(x^2-1)*exp(x^2)
```

Maxima [A] time = 1.35231, size = 15, normalized size = 0.68

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Fricas [A] time = 0.222875, size = 15, normalized size = 0.68

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] time = 0.067692, size = 10, normalized size = 0.45

$$\frac{\left(x^2-1\right)\,e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x**3,x)
```

```
[Out] (x**2 - 1)*exp(x**2)/2
```

GIAC/XCAS [A] time = 0.198047, size = 15, normalized size = 0.68

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

3.46 $\int e^x(3+2x)\,dx$

Optimal. Leaf size=15

$$e^x(2x+3)-2e^x$$

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rubi [A] time = 0.0151646, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$e^x(2x+3)-2e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*(3 + 2*x),x]
```

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rubi in Sympy [A] time = 1.3574, size = 12, normalized size = 0.8

$$(2x+3)\,e^x-2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*(3+2*x),x)
```

```
[Out] (2*x + 3)*exp(x) - 2*exp(x)
```

Mathematica [A] time = 0.00281233, size = 9, normalized size = 0.6

$$e^x(2x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*(3 + 2*x),x]
```

```
[Out] E^x*(1 + 2*x)
```

Maple [A] time = 0.002, size = 9, normalized size = 0.6

$$(1+2\,x)\,\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*(3+2*x),x)
```

```
[Out] (1+2*x)*exp(x)
```

Maxima [A] time = 1.34867, size = 16, normalized size = 1.07

$$2\,(x-1)e^x+3\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)*e^x,x, algorithm="maxima")
```

```
[Out] 2*(x - 1)*e^x + 3*e^x
```

Fricas [A] time = 0.21919, size = 11, normalized size = 0.73

$$(2\,x + 1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)*e^x,x, algorithm="fricas")
```

```
[Out] (2*x + 1)*e^x
```

Sympy [A] time = 0.064496, size = 7, normalized size = 0.47

$$(2x + 1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*(3+2*x),x)
```

```
[Out] (2*x + 1)*exp(x)
```

GIAC/XCAS [A] time = 0.201512, size = 11, normalized size = 0.73

$$(2\,x + 1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)*e^x,x, algorithm="giac")
```

```
[Out] (2*x + 1)*e^x
```

3.47 $\int 5^x x\, dx$

Optimal. Leaf size=19

$$\frac{5^x x}{\log(5)} - \frac{5^x}{\log^2(5)}$$

```
[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rubi [A] time = 0.017137, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{5^x x}{\log(5)} - \frac{5^x}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In]  Int[5^x*x,x]
```

```
[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rubi in Sympy [A] time = 1.03444, size = 15, normalized size = 0.79

$$\frac{5^x x}{\log(5)} - \frac{5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(5**x*x,x)
```

```
[Out] 5**x*x/log(5) - 5**x/log(5)**2
```

Mathematica [A] time = 0.00446792, size = 14, normalized size = 0.74

$$\frac{5^x(x\log(5)-1)}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[5^x*x,x]
```

```
[Out] (5^x*(-1 + x*Log[5]))/Log[5]^2
```

Maple [A] time = 0.01, size = 15, normalized size = 0.8

$$\frac{(\ln(5)x-1)\,5^x}{(\ln(5))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(5^x*x,x)
```

```
[Out] (ln(5)*x-1)*5^x/ln(5)^2
```

Maxima [A] time = 1.50922, size = 19, normalized size = 1.

$$\frac{(x\log(5)-1)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5^x*x,x, algorithm="maxima")
```

```
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Fricas [A] time = 0.249188, size = 19, normalized size = 1.

$$\frac{(x\log(5)-1)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5^x*x,x, algorithm="fricas")
```

```
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Sympy [A] time = 0.087708, size = 14, normalized size = 0.74

$$\frac{5^x\,(x\log(5)-1)}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5**x*x,x)
```

```
[Out] 5**x*(x*log(5) - 1)/log(5)**2
```

GIAC/XCAS [A] time = 0.198895, size = 22, normalized size = 1.16

$$\frac{(x\ln(5)-1)e^{(x\ln(5))}}{\ln(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5^x*x,x, algorithm="giac")
```

```
[Out] (x*ln(5) - 1)*e^(x*ln(5))/ln(5)^2
```

3.48 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.00728089, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi in Sympy [A] time = 0.481551, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} + \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Mathematica [A] time = 0.00444584, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$\frac{x\cos(\ln(x))}{2} + \frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 1.34606, size = 14, normalized size = 0.82

$$\frac{1}{2}\,x(\cos(\log(x)) + \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 0.224918, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.555811, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} + \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

GIAC/XCAS [A] time = 0.203847, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos(\ln(x)) + \frac{1}{2}\,x\sin(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(ln(x)) + 1/2*x*sin(ln(x))
```

3.49 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0153121, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi in Sympy [A] time = 1.08443, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Mathematica [A] time = 0.00415274, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x} - 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0.001, size = 17, normalized size = 0.7

$$-2\,\mathrm{e}^{\sqrt{x}} + 2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^(1/2)),x)
```

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.35016, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 0.211357, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.218524, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

GIAC/XCAS [A] time = 0.200558, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.50 $\int \log\left(\sqrt{x}\right)\, dx$

Optimal. Leaf size=14

$$x \log\left(\sqrt{x}\right) - \frac{x}{2}$$

```
[Out] -x/2 + x*Log[Sqrt[x]]
```

Rubi [A] time = 0.00450824, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$x \log\left(\sqrt{x}\right) - \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[Sqrt[x]],x]
```

```
[Out] -x/2 + x*Log[Sqrt[x]]
```

Rubi in Sympy [A] time = 0.559459, size = 8, normalized size = 0.57

$$\frac{x \log(x)}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/2*ln(x),x)
```

```
[Out] x*log(x)/2 - x/2
```

Mathematica [A] time = 0.00109658, size = 12, normalized size = 0.86

$$\frac{1}{2}(x \log(x) - x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[Sqrt[x]],x]
```

```
[Out] (-x + x*Log[x])/2
```

Maple [A] time = 0.001, size = 10, normalized size = 0.7

$$-\frac{x}{2} + \frac{x \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/2*ln(x),x)
```

```
[Out] -1/2*x+1/2*x*ln(x)
```

Maxima [A] time = 1.3634, size = 12, normalized size = 0.86

$$\frac{1}{2}x\log(x) - \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Fricas [A] time = 0.212711, size = 12, normalized size = 0.86

$$\frac{1}{2}x\log(x) - \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Sympy [A] time = 0.068229, size = 8, normalized size = 0.57

$$\frac{x\log(x)}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*ln(x),x)
```

```
[Out] x*log(x)/2 - x/2
```

GIAC/XCAS [A] time = 0.199026, size = 12, normalized size = 0.86

$$\frac{1}{2}x\ln(x) - \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/2*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x*ln(x) - 1/2*x
```

3.51 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.00635582, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi in Sympy [A] time = 0.4971, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Mathematica [A] time = 0.00427433, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x\cos(\ln(x))}{2} + \frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(ln(x)),x)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 1.40398, size = 16, normalized size = 0.94

$$-\frac{1}{2}\,x(\cos(\log(x)) - \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A] time = 0.220473, size = 18, normalized size = 1.06

$$-\frac{1}{2}\,x\cos(\log(x)) + \frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.572867, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

GIAC/XCAS [A] time = 0.199993, size = 18, normalized size = 1.06

$$-\frac{1}{2}\,x\cos(\ln(x)) + \frac{1}{2}\,x\sin(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(ln(x)) + 1/2*x*sin(ln(x))
```

3.52 $\int \sin\left(\sqrt{x}\right)\, dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0175559, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.83802, size = 46, normalized size = 2.09

$$-\sqrt{x}e^{i\sqrt{x}} - \sqrt{x}e^{-i\sqrt{x}} - ie^{i\sqrt{x}} + ie^{-i\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x**(1/2)),x)
```

```
[Out] -sqrt(x)*exp(I*sqrt(x)) - sqrt(x)*exp(-I*sqrt(x)) - I*exp(I*sqrt(
x)) + I*exp(-I*sqrt(x))
```

Mathematica [A] time = 0.00935726, size = 22, normalized size = 1.

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\,\sin\left(\sqrt{x}\right) - 2\,\cos\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x^(1/2)),x)
```

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.37321, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A] time = 0.232913, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] time = 0.405953, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x**(1/2)),x)
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

GIAC/XCAS [A] time = 0.201899, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.53 $\int x^5 \cos\left(x^3\right)\, dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{\cos\left(x^3\right)}{3}$$

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rubi [A] time = 0.0255628, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rubi in Sympy [A] time = 1.44231, size = 15, normalized size = 0.75

$$\frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

Mathematica [A] time = 0.00490662, size = 20, normalized size = 1.

$$\frac{1}{3}x^3 \sin\left(x^3\right) + \frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Maple [A] time = 0.009, size = 17, normalized size = 0.9

$$\frac{\cos\left(x^3\right)}{3} + \frac{x^3 \sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*cos(x^3),x)
```

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Maxima [A] time = 1.34878, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cos(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Fricas [A] time = 0.21938, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cos(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Sympy [A] time = 2.95039, size = 15, normalized size = 0.75

$$\frac{x^3\sin\left(x^3\right)}{3}+\frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

GIAC/XCAS [A] time = 0.199178, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cos(x^3),x, algorithm="giac")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.54 $\int e^{x^2} x^5 \, dx$

Optimal. Leaf size=28

$$-e^{x^2} x^2 + e^{x^2} + \frac{1}{2} e^{x^2} x^4$$

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rubi [A] time = 0.0503848, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-e^{x^2} x^2 + e^{x^2} + \frac{1}{2} e^{x^2} x^4$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*x^5,x]
```

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rubi in Sympy [A] time = 2.87378, size = 22, normalized size = 0.79

$$\frac{x^4 e^{x^2}}{2} - x^2 e^{x^2} + e^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*x**5,x)
```

```
[Out] x**4*exp(x**2)/2 - x**2*exp(x**2) + exp(x**2)
```

Mathematica [A] time = 0.00367308, size = 19, normalized size = 0.68

$$\frac{1}{2} e^{x^2} \left(x^4 - 2x^2 + 2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*x^5,x]
```

```
[Out] (E^x^2*(2 - 2*x^2 + x^4))/2
```

Maple [A] time = 0.004, size = 17, normalized size = 0.6

$$\frac{\left(x^4 - 2x^2 + 2\right) e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*x^5,x)
```

```
[Out] 1/2*(x^4-2*x^2+2)*exp(x^2)
```

Maxima [A] time = 1.45211, size = 22, normalized size = 0.79

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Fricas [A] time = 0.213719, size = 22, normalized size = 0.79

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Sympy [A] time = 0.081417, size = 15, normalized size = 0.54

$$\frac{\left(x^4 - 2x^2 + 2\right)\,e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x**5,x)
```

```
[Out] (x**4 - 2*x**2 + 2)*exp(x**2)/2
```

GIAC/XCAS [A] time = 0.204191, size = 22, normalized size = 0.79

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

3.55 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.0182701, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi in Sympy [A] time = 1.89198, size = 15, normalized size = 0.71

$$\frac{x^2 \operatorname{atan}(x)}{2} - \frac{x}{2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Mathematica [A] time = 0.00337518, size = 21, normalized size = 1.

$$\frac{1}{2}x^2 \tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0., size = 16, normalized size = 0.8

$$-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2 \arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.54811, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 0.215169, size = 18, normalized size = 0.86

$$\frac{1}{2}\,\left(x^2+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.328835, size = 15, normalized size = 0.71

$$\frac{x^2\,\mathrm{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

GIAC/XCAS [A] time = 0.203573, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.56 $\int x \cos(\pi x)\, dx$

Optimal. Leaf size=18

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rubi [A] time = 0.0228439, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[Pi*x],x]
```

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rubi in Sympy [A] time = 0.874022, size = 15, normalized size = 0.83

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(pi*x),x)
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi**2
```

Mathematica [A] time = 0.00968013, size = 18, normalized size = 1.

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[Pi*x],x]
```

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Maple [A] time = 0.009, size = 17, normalized size = 0.9

$$\frac{\cos(\pi x) + x\pi \sin(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(Pi*x),x)
```

```
[Out] 1/Pi^2*(cos(Pi*x)+x*Pi*sin(Pi*x))
```

Maxima [A] time = 1.4244, size = 22, normalized size = 1.22

$$\frac{\pi x \sin(\pi x) + \cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(pi*x),x, algorithm="maxima")
```

```
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Fricas [A] time = 0.221687, size = 22, normalized size = 1.22

$$\frac{\pi x \sin(\pi x) + \cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(pi*x),x, algorithm="fricas")
```

```
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Sympy [A] time = 0.207466, size = 15, normalized size = 0.83

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(pi*x),x)
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi**2
```

GIAC/XCAS [A] time = 0.200375, size = 24, normalized size = 1.33

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(pi*x),x, algorithm="giac")
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi^2
```

3.57 $\int \sqrt{x}\log(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}x^{3/2}\log(x)-\frac{4x^{3/2}}{9}$$

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rubi [A] time = 0.0114925, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2}{3}x^{3/2}\log(x)-\frac{4x^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*Log[x],x]
```

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rubi in Sympy [A] time = 1.07153, size = 19, normalized size = 0.9

$$\frac{2x^{\frac{3}{2}}\log(x)}{3}-\frac{4x^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*log(x)/3 - 4*x**(3/2)/9
```

Mathematica [A] time = 0.0041041, size = 15, normalized size = 0.71

$$\frac{2}{9}x^{3/2}(3\log(x)-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*Log[x],x]
```

```
[Out] (2*x^(3/2)*(-2 + 3*Log[x]))/9
```

Maple [A] time = 0.001, size = 14, normalized size = 0.7

$$-\frac{4}{9}x^{\frac{3}{2}}+\frac{2\ln(x)}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)*x^(1/2),x)
```

```
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

Maxima [A] time = 1.54764, size = 18, normalized size = 0.86

$$\frac{2}{3}x^{\frac{3}{2}}\log(x) - \frac{4}{9}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*log(x),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

Fricas [A] time = 0.211946, size = 19, normalized size = 0.9

$$\frac{2}{9}(3\,x\log(x) - 2\,x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*log(x),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x*log(x) - 2*x)*sqrt(x)
```

Sympy [A] time = 3.09091, size = 66, normalized size = 3.14

$$\begin{cases} \frac{2x^{\frac{3}{2}}\log(x)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for } |x| < 1 \\ -\frac{2x^{\frac{3}{2}}\log\left(\frac{1}{x}\right)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for } \left|\frac{1}{x}\right| < 1 \\ -G^{2,1}_{3,3}\left(\begin{matrix}1 & \frac{5}{2}, \frac{5}{2} \\ \frac{3}{2}, \frac{3}{2} & 0\end{matrix}\middle| x\right) + G^{0,3}_{3,3}\left(\begin{matrix}\frac{5}{2}, \frac{5}{2}, 1 & \\ & \frac{3}{2}, \frac{3}{2}, 0\end{matrix}\middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)*x**(1/2),x)
```

```
[Out] Piecewise((2*x**(3/2)*log(x)/3 - 4*x**(3/2)/9, Abs(x) < 1), (-2*x
**(3/2)*log(1/x)/3 - 4*x**(3/2)/9, Abs(1/x) < 1), (-meijerg(((1,)
, (5/2, 5/2)), ((3/2, 3/2), (0,)), x) + meijerg(((5/2, 5/2, 1), (
)), ((), (3/2, 3/2, 0)), x), True))
```

GIAC/XCAS [A] time = 0.204378, size = 18, normalized size = 0.86

$$\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*log(x),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*ln(x) - 4/9*x^(3/2)
```

3.58 $\int \sin^2(3x)\,dx$

Optimal. Leaf size=18

$$\frac{x}{2} - \frac{1}{6}\sin(3x)\cos(3x)$$

```
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rubi [A] time = 0.0123712, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{2} - \frac{1}{6}\sin(3x)\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[3*x]^2,x]
```

```
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rubi in Sympy [A] time = 0.507694, size = 14, normalized size = 0.78

$$\frac{x}{2} - \frac{\sin(3x)\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(3*x)**2,x)
```

```
[Out] x/2 - sin(3*x)*cos(3*x)/6
```

Mathematica [A] time = 0.00436073, size = 14, normalized size = 0.78

$$\frac{x}{2} - \frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[3*x]^2,x]
```

```
[Out] x/2 - Sin[6*x]/12
```

Maple [A] time = 0.008, size = 15, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(3x)\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(3*x)^2,x)
```

```
[Out] 1/2*x-1/6*cos(3*x)*sin(3*x)
```

Maxima [A] time = 1.41452, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x - \frac{1}{12}\,\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/12*sin(6*x)
```

Fricas [A] time = 0.233194, size = 19, normalized size = 1.06

$$-\frac{1}{6}\,\cos\left(3\,x\right)\sin\left(3\,x\right) + \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/6*cos(3*x)*sin(3*x) + 1/2*x
```

Sympy [A] time = 0.040998, size = 14, normalized size = 0.78

$$\frac{x}{2} - \frac{\sin\left(3x\right)\cos\left(3x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)**2,x)
```

```
[Out] x/2 - sin(3*x)*cos(3*x)/6
```

GIAC/XCAS [A] time = 0.20009, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x - \frac{1}{12}\,\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/12*sin(6*x)
```

3.59 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0105966, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.486551, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00281425, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.40424, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.216031, size = 14, normalized size = 1.

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.035631, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.199655, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.60 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8} + \frac{1}{4}\sin(x)\cos^3(x) + \frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0192441, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{3x}{8} + \frac{1}{4}\sin(x)\cos^3(x) + \frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 0.570009, size = 24, normalized size = 1.

$$\frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00292656, size = 22, normalized size = 0.92

$$\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$\frac{\sin(x)}{4}\left((\cos(x))^3 + \frac{3\cos(x)}{2}\right) + \frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4,x)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

Maxima [A] time = 1.43496, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.220437, size = 26, normalized size = 1.08

$$\frac{1}{8}\left(2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.036894, size = 24, normalized size = 1.

$$\frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

GIAC/XCAS [A] time = 0.20372, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.61 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3} - \cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0107098, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi in Sympy [A] time = 0.63178, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.00294256, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x) - \frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$-\frac{\left(2 + (\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.33341, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.222937, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.042351, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A] time = 0.200353, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

3.62 $\int \cos^4(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

```
[Out] -Cos[x]^5/5 + Cos[x]^7/7
```

Rubi [A] time = 0.0360282, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] -Cos[x]^5/5 + Cos[x]^7/7
```

Rubi in Sympy [A] time = 2.33001, size = 12, normalized size = 0.71

$$\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*sin(x)**3,x)
```

```
[Out] cos(x)**7/7 - cos(x)**5/5
```

Mathematica [A] time = 0.0157803, size = 31, normalized size = 1.82

$$-\frac{3\cos(x)}{64} - \frac{1}{64}\cos(3x) + \frac{1}{320}\cos(5x) + \frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/64 - Cos[3*x]/64 + Cos[5*x]/320 + Cos[7*x]/448
```

Maple [A] time = 0.01, size = 18, normalized size = 1.1

$$-\frac{(\cos(x))^5(\sin(x))^2}{7} - \frac{2(\cos(x))^5}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*sin(x)^3,x)
```

```
[Out] -1/7*cos(x)^5*sin(x)^2-2/35*cos(x)^5
```

Maxima [A] time = 1.32518, size = 18, normalized size = 1.06

$$\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Fricas [A] time = 0.221402, size = 18, normalized size = 1.06

$$\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Sympy [A] time = 0.05273, size = 12, normalized size = 0.71

$$\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*sin(x)**3,x)
```

```
[Out] cos(x)**7/7 - cos(x)**5/5
```

GIAC/XCAS [A] time = 0.201719, size = 18, normalized size = 1.06

$$\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

3.63 $\int \cos^3(x)\sin^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rubi [A] time = 0.0366809, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rubi in Sympy [A] time = 2.34185, size = 12, normalized size = 0.71

$$-\frac{\sin^7(x)}{7}+\frac{\sin^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3*sin(x)**4,x)
```

```
[Out] -sin(x)**7/7 + sin(x)**5/5
```

Mathematica [A] time = 0.0159662, size = 31, normalized size = 1.82

$$\frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] (3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448
```

Maple [B] time = 0.01, size = 30, normalized size = 1.8

$$-\frac{(\cos(x))^4(\sin(x))^3}{7}-\frac{3\sin(x)(\cos(x))^4}{35}+\frac{\left(2+(\cos(x))^2\right)\sin(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3*sin(x)^4,x)
```

```
[Out] -1/7*cos(x)^4*sin(x)^3-3/35*sin(x)*cos(x)^4+1/35*(2+cos(x)^2)*sin
(x)
```

Maxima [A] time = 1.33705, size = 18, normalized size = 1.06

$$-\frac{1}{7}\,\sin(x)^7 + \frac{1}{5}\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

Fricas [A] time = 0.231487, size = 30, normalized size = 1.76

$$\frac{1}{35}\left(5\,\cos(x)^6 - 8\,\cos(x)^4 + \cos(x)^2 + 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.044802, size = 12, normalized size = 0.71

$$-\frac{\sin^7(x)}{7} + \frac{\sin^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3*sin(x)**4,x)
```

```
[Out] -sin(x)**7/7 + sin(x)**5/5
```

GIAC/XCAS [A] time = 0.198859, size = 18, normalized size = 1.06

$$-\frac{1}{7}\,\sin(x)^7 + \frac{1}{5}\,\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^4,x, algorithm="giac")
```

```
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

3.64 $\int \cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=36

$$\frac{x}{16} - \frac{1}{6}\sin^3(x)\cos^3(x) - \frac{1}{8}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x
]^3)/6
```

Rubi [A] time = 0.0698417, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{16} - \frac{1}{6}\sin^3(x)\cos^3(x) - \frac{1}{8}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x
]^3)/6
```

Rubi in Sympy [A] time = 2.65918, size = 32, normalized size = 0.89

$$\frac{x}{16} - \frac{\sin^3(x)\cos^3(x)}{6} - \frac{\sin(x)\cos^3(x)}{8} + \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**4,x)
```

```
[Out] x/16 - sin(x)**3*cos(x)**3/6 - sin(x)*cos(x)**3/8 + sin(x)*cos(x)
/16
```

Mathematica [A] time = 0.0125056, size = 30, normalized size = 0.83

$$\frac{x}{16} - \frac{1}{64}\sin(2x) - \frac{1}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192
```

Maple [A] time = 0.01, size = 29, normalized size = 0.8

$$\frac{x}{16} + \frac{\cos(x)\sin(x)}{16} - \frac{(\cos(x))^3\sin(x)}{8} - \frac{(\cos(x))^3(\sin(x))^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^4,x)
```

```
[Out] 1/16*x+1/16*cos(x)*sin(x)-1/8*cos(x)^3*sin(x)-1/6*cos(x)^3*sin(x)
^3
```

Maxima [A] time = 1.34965, size = 24, normalized size = 0.67

$$-\frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 0.259057, size = 34, normalized size = 0.94

$$\frac{1}{48}\left(8\cos(x)^5 - 14\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.041135, size = 31, normalized size = 0.86

$$\frac{x}{16} + \frac{\sin^5(x)\cos(x)}{6} - \frac{\sin^3(x)\cos(x)}{24} - \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**4,x)
```

```
[Out] x/16 + sin(x)**5*cos(x)/6 - sin(x)**3*cos(x)/24 - sin(x)*cos(x)/1
6
```

GIAC/XCAS [A] time = 0.20482, size = 30, normalized size = 0.83

$$\frac{1}{16}x + \frac{1}{192}\sin(6x) - \frac{1}{64}\sin(4x) - \frac{1}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*x) - 1/64*sin(4*x) - 1/64*sin(2*x)
```

3.65 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8} - \frac{1}{4}\sin(x)\cos^3(x) + \frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0387615, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{8} - \frac{1}{4}\sin(x)\cos^3(x) + \frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 1.62506, size = 20, normalized size = 0.83

$$\frac{x}{8} - \frac{\sin(x)\cos^3(x)}{4} + \frac{\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(x)*cos(x)**3/4 + sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00647966, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0.002, size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 1.34772, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 0.221972, size = 26, normalized size = 1.08

$$-\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.052749, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

GIAC/XCAS [A] time = 0.219669, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.66 $\int (1-\sin(2x))^2\,dx$

Optimal. Leaf size=22

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

```
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rubi [A] time = 0.0164481, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rubi in Sympy [A] time = 0.600005, size = 20, normalized size = 0.91

$$\frac{3x}{2}-\frac{\sin(2x)\cos(2x)}{4}+\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-sin(2*x))**2,x)
```

```
[Out] 3*x/2 - sin(2*x)*cos(2*x)/4 + cos(2*x)
```

Mathematica [A] time = 0.00696155, size = 18, normalized size = 0.82

$$\frac{3x}{2}-\frac{1}{8}\sin(4x)+\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - Sin[4*x]/8
```

Maple [A] time = 0.039, size = 19, normalized size = 0.9

$$\frac{3x}{2}+\cos(2x)-\frac{\cos(2x)\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-sin(2*x))^2,x)
```

```
[Out] 3/2*x+cos(2*x)-1/4*cos(2*x)*sin(2*x)
```

Maxima [A] time = 1.33529, size = 19, normalized size = 0.86

$$\frac{3}{2}\,x + \cos(2\,x) - \frac{1}{8}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(2*x) - 1)^2,x, algorithm="maxima")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

Fricas [A] time = 0.219756, size = 24, normalized size = 1.09

$$-\frac{1}{4}\,\cos(2\,x)\sin(2\,x) + \frac{3}{2}\,x + \cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(2*x) - 1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x)*sin(2*x) + 3/2*x + cos(2*x)
```

Sympy [A] time = 0.210323, size = 37, normalized size = 1.68

$$\frac{x\sin^2(2x)}{2} + \frac{x\cos^2(2x)}{2} + x - \frac{\sin(2x)\cos(2x)}{4} + \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-sin(2*x))**2,x)
```

```
[Out] x*sin(2*x)**2/2 + x*cos(2*x)**2/2 + x - sin(2*x)*cos(2*x)/4 + cos
(2*x)
```

GIAC/XCAS [A] time = 0.212307, size = 19, normalized size = 0.86

$$\frac{3}{2}\,x + \cos(2\,x) - \frac{1}{8}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(2*x) - 1)^2,x, algorithm="giac")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

3.67 $\int \cos(x) \sin\left(\frac{\pi}{6} + x\right)\, dx$

Optimal. Leaf size=20

$$\frac{x}{4} - \frac{1}{4}\cos\left(2x + \frac{\pi}{6}\right)$$

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rubi [A] time = 0.026581, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{x}{4} - \frac{1}{4}\cos\left(2x + \frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\cos\left(2x + \frac{\pi}{6}\right)}{4} + \int \frac{1}{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -cos(2*x + pi/6)/4 + Integral(1/4, x)
```

Mathematica [A] time = 0.0118138, size = 20, normalized size = 1.

$$\frac{x}{4} - \frac{1}{4}\cos\left(2x + \frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Maple [A] time = 0.014, size = 15, normalized size = 0.8

$$\frac{x}{4} - \frac{1}{4}\cos\left(\frac{\pi}{6} + 2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*sin(1/6*Pi+x),x)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Maxima [A] time = 1.37012, size = 19, normalized size = 0.95

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(1/6*pi + x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Fricas [A] time = 0.245362, size = 77, normalized size = 3.85

$$-\frac{\left(362\sqrt{3}+627\right)\cos\left(\frac{1}{6}\pi+x\right)^2+\left(209\sqrt{3}+362\right)\cos\left(\frac{1}{6}\pi+x\right)\sin\left(\frac{1}{6}\pi+x\right)-209\sqrt{3}x-362x}{4\left(209\sqrt{3}+362\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(1/6*pi + x),x, algorithm="fricas")
```

```
[Out] -1/4*((362*sqrt(3) + 627)*cos(1/6*pi + x)^2 + (209*sqrt(3) + 362)
*cos(1/6*pi + x)*sin(1/6*pi + x) - 209*sqrt(3)*x - 362*x)/(209*sq
rt(3) + 362)
```

Sympy [A] time = 0.760257, size = 37, normalized size = 1.85

$$-\frac{x\sin(x)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos(x)}{2}-\frac{\cos(x)\cos\left(x+\frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 - cos(x)*cos
(x + pi/6)/2
```

GIAC/XCAS [A] time = 0.20282, size = 19, normalized size = 0.95

$$\frac{1}{4}x - \frac{1}{4}\cos\left(\frac{1}{6}\pi + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(1/6*pi + x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

3.68 $\int \cos^5(x)\sin^5(x)\,dx$

Optimal. Leaf size=25

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rubi [A] time = 0.0600106, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rubi in Sympy [A] time = 3.44483, size = 19, normalized size = 0.76

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**5*sin(x)**5,x)
```

```
[Out] sin(x)**10/10 - sin(x)**8/4 + sin(x)**6/6
```

Mathematica [A] time = 0.0156388, size = 25, normalized size = 1.

$$-\frac{5}{512}\cos(2x) + \frac{5\cos(6x)}{3072} - \frac{\cos(10x)}{5120}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] (-5*Cos[2*x])/512 + (5*Cos[6*x])/3072 - Cos[10*x]/5120
```

Maple [A] time = 0.01, size = 28, normalized size = 1.1

$$-\frac{(\cos(x))^6(\sin(x))^4}{10} - \frac{(\sin(x))^2(\cos(x))^6}{20} - \frac{(\cos(x))^6}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^5*sin(x)^5,x)
```

```
[Out] -1/10*cos(x)^6*sin(x)^4-1/20*sin(x)^2*cos(x)^6-1/60*cos(x)^6
```

Maxima [A] time = 1.33287, size = 26, normalized size = 1.04

$$\frac{1}{10}\sin(x)^{10} - \frac{1}{4}\sin(x)^{8} + \frac{1}{6}\sin(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/10*sin(x)^10 - 1/4*sin(x)^8 + 1/6*sin(x)^6
```

Fricas [A] time = 0.230845, size = 26, normalized size = 1.04

$$-\frac{1}{10}\cos(x)^{10} + \frac{1}{4}\cos(x)^{8} - \frac{1}{6}\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

Sympy [A] time = 0.053033, size = 19, normalized size = 0.76

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^{8}(x)}{4} + \frac{\sin^{6}(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**5*sin(x)**5,x)
```

```
[Out] sin(x)**10/10 - sin(x)**8/4 + sin(x)**6/6
```

GIAC/XCAS [A] time = 0.212994, size = 26, normalized size = 1.04

$$-\frac{1}{10}\cos(x)^{10} + \frac{1}{4}\cos(x)^{8} - \frac{1}{6}\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

3.69 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16} - \frac{1}{6}\sin^5(x)\cos(x) - \frac{5}{24}\sin^3(x)\cos(x) - \frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x
]*Sin[x]^5)/6
```

Rubi [A] time = 0.0260975, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{5x}{16} - \frac{1}{6}\sin^5(x)\cos(x) - \frac{5}{24}\sin^3(x)\cos(x) - \frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x
]*Sin[x]^5)/6
```

Rubi in Sympy [A] time = 0.654927, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*co
s(x)/16
```

Mathematica [A] time = 0.00323663, size = 30, normalized size = 0.88

$$\frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] time = 0., size = 24, normalized size = 0.7

$$-\frac{\cos(x)}{6}\left((\sin(x))^5 + \frac{5\,(\sin(x))^3}{4} + \frac{15\,\sin(x)}{8}\right) + \frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^6,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

Maxima [A] time = 1.32798, size = 32, normalized size = 0.94

$$\frac{1}{48}\sin(2x)^3 + \frac{5}{16}x + \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.227009, size = 34, normalized size = 1.

$$-\frac{1}{48}\left(8\cos(x)^5 - 26\cos(x)^3 + 33\cos(x)\right)\sin(x) + \frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.040056, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*co
s(x)/16
```

GIAC/XCAS [A] time = 0.213831, size = 30, normalized size = 0.88

$$\frac{5}{16}x - \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) - \frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

3.70 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16} + \frac{1}{6}\sin(x)\cos^5(x) + \frac{5}{24}\sin(x)\cos^3(x) + \frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x
]^5*Sin[x])/6
```

Rubi [A] time = 0.0285844, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{5x}{16} + \frac{1}{6}\sin(x)\cos^5(x) + \frac{5}{24}\sin(x)\cos^3(x) + \frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x
]^5*Sin[x])/6
```

Rubi in Sympy [A] time = 0.67649, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*co
s(x)/16
```

Mathematica [A] time = 0.0034475, size = 30, normalized size = 0.88

$$\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] time = 0.056, size = 24, normalized size = 0.7

$$\frac{\sin(x)}{6}\left((\cos(x))^5 + \frac{5(\cos(x))^3}{4} + \frac{15\cos(x)}{8}\right) + \frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^6,x)
```

```
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

Maxima [A] time = 1.5101, size = 32, normalized size = 0.94

$$-\frac{1}{48}\sin(2x)^3 + \frac{5}{16}x + \frac{3}{64}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.246802, size = 34, normalized size = 1.

$$\frac{1}{48}\left(8\cos(x)^5 + 10\cos(x)^3 + 15\cos(x)\right)\sin(x) + \frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.038221, size = 36, normalized size = 1.06

$$\frac{5x}{16} + \frac{\sin(x)\cos^5(x)}{6} + \frac{5\sin(x)\cos^3(x)}{24} + \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*co
s(x)/16
```

GIAC/XCAS [A] time = 0.200061, size = 30, normalized size = 0.88

$$\frac{5}{16}x + \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) + \frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

3.71 $\int \cos^4(2x)\sin^2(2x)\,dx$

Optimal. Leaf size=46

$$\frac{x}{16} - \frac{1}{12}\sin(2x)\cos^5(2x) + \frac{1}{48}\sin(2x)\cos^3(2x) + \frac{1}{32}\sin(2x)\cos(2x)$$

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2
*x]^5*Sin[2*x])/12
```

Rubi [A] time = 0.0575614, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{x}{16} - \frac{1}{12}\sin(2x)\cos^5(2x) + \frac{1}{48}\sin(2x)\cos^3(2x) + \frac{1}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2
*x]^5*Sin[2*x])/12
```

Rubi in Sympy [A] time = 1.8985, size = 41, normalized size = 0.89

$$\frac{x}{16} - \frac{\sin(2x)\cos^5(2x)}{12} + \frac{\sin(2x)\cos^3(2x)}{48} + \frac{\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(2*x)**4*sin(2*x)**2,x)
```

```
[Out] x/16 - sin(2*x)*cos(2*x)**5/12 + sin(2*x)*cos(2*x)**3/48 + sin(2*
x)*cos(2*x)/32
```

Mathematica [A] time = 0.0206472, size = 30, normalized size = 0.65

$$\frac{x}{16} + \frac{1}{128}\sin(4x) - \frac{1}{128}\sin(8x) - \frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + Sin[4*x]/128 - Sin[8*x]/128 - Sin[12*x]/384
```

Maple [A] time = 0.019, size = 36, normalized size = 0.8

$$-\frac{(\cos(2x))^5\sin(2x)}{12} + \frac{\sin(2x)}{48}\left((\cos(2x))^3 + \frac{3\cos(2x)}{2}\right) + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(2*x)^4*sin(2*x)^2,x)
```

```
[Out] -1/12*cos(2*x)^5*sin(2*x)+1/48*(cos(2*x)^3+3/2*cos(2*x))*sin(2*x)
+1/16*x
```

Maxima [A] time = 1.33205, size = 24, normalized size = 0.52

$$\frac{1}{96}\sin(4x)^3 + \frac{1}{16}x - \frac{1}{128}\sin(8x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 1/16*x - 1/128*sin(8*x)
```

Fricas [A] time = 0.234865, size = 45, normalized size = 0.98

$$-\frac{1}{96}\left(8\cos(2x)^5 - 2\cos(2x)^3 - 3\cos(2x)\right)\sin(2x) + \frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 2*cos(2*x)^3 - 3*cos(2*x))*sin(2*x) + 1/16*
x
```

Sympy [A] time = 0.046342, size = 41, normalized size = 0.89

$$\frac{x}{16} - \frac{\sin(2x)\cos^5(2x)}{12} + \frac{\sin(2x)\cos^3(2x)}{48} + \frac{\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)**4*sin(2*x)**2,x)
```

```
[Out] x/16 - sin(2*x)*cos(2*x)**5/12 + sin(2*x)*cos(2*x)**3/48 + sin(2*
x)*cos(2*x)/32
```

GIAC/XCAS [A] time = 0.199566, size = 30, normalized size = 0.65

$$\frac{1}{16}x - \frac{1}{384}\sin(12x) - \frac{1}{128}\sin(8x) + \frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/384*sin(12*x) - 1/128*sin(8*x) + 1/128*sin(4*x)
```

3.72 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi [A] time = 0.0131811, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi in Sympy [A] time = 0.724471, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**5,x)
```

```
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.003076, size = 23, normalized size = 1.1

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$-\frac{\cos(x)}{5}\left(\frac{8}{3}+(\sin(x))^4+\frac{4\,(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^5,x)
```

```
[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 1.32601, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="maxima")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.222993, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.047376, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**5,x)
```

```
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A] time = 0.205165, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.73 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128} - \frac{1}{8}\sin^3(x)\cos^5(x) - \frac{1}{16}\sin(x)\cos^5(x) + \frac{1}{64}\sin(x)\cos^3(x) + \frac{3}{128}\sin(x)\cos(x)$$

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x
]^5*Sin[x])/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi [A] time = 0.0794012, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3x}{128} - \frac{1}{8}\sin^3(x)\cos^5(x) - \frac{1}{16}\sin(x)\cos^5(x) + \frac{1}{64}\sin(x)\cos^3(x) + \frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x
]^5*Sin[x])/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi in Sympy [A] time = 2.72374, size = 46, normalized size = 1.

$$\frac{3x}{128} - \frac{\sin^3(x)\cos^5(x)}{8} - \frac{\sin(x)\cos^5(x)}{16} + \frac{\sin(x)\cos^3(x)}{64} + \frac{3\sin(x)\cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(x)**3*cos(x)**5/8 - sin(x)*cos(x)**5/16 + sin(x)*co
s(x)**3/64 + 3*sin(x)*cos(x)/128
```

Mathematica [A] time = 0.0094539, size = 22, normalized size = 0.48

$$\frac{3x}{128} - \frac{1}{128}\sin(4x) + \frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] time = 0.011, size = 36, normalized size = 0.8

$$-\frac{(\cos(x))^5(\sin(x))^3}{8} - \frac{(\cos(x))^5\sin(x)}{16} + \frac{\sin(x)}{64}\left((\cos(x))^3 + \frac{3\cos(x)}{2}\right) + \frac{3x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*co
s(x))*sin(x)+3/128*x
```

Maxima [A] time = 1.36931, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Fricas [A] time = 0.230924, size = 42, normalized size = 0.91

$$\frac{1}{128}\left(16\,\cos\left(x\right)^{7} - 24\,\cos\left(x\right)^{5} + 2\,\cos\left(x\right)^{3} + 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{128}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x)
+ 3/128*x
```

Sympy [A] time = 0.055402, size = 31, normalized size = 0.67

$$\frac{3x}{128} - \frac{\sin^{3}\left(2x\right)\cos\left(2x\right)}{128} - \frac{3\sin\left(2x\right)\cos\left(2x\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

GIAC/XCAS [A] time = 0.205748, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.74 $\int \sqrt{\cos(x)}\sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rubi [A] time = 0.0356935, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rubi in Sympy [A] time = 2.2582, size = 19, normalized size = 0.9

$$\frac{2\cos^{\frac{7}{2}}(x)}{7}-\frac{2\cos^{\frac{3}{2}}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3*cos(x)**(1/2),x)
```

```
[Out] 2*cos(x)**(7/2)/7 - 2*cos(x)**(3/2)/3
```

Mathematica [A] time = 0.0343083, size = 22, normalized size = 1.05

$$\sqrt{\cos(x)}\left(\frac{1}{14}\cos(3x)-\frac{19\cos(x)}{42}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] Sqrt[Cos[x]]*((-19*Cos[x])/42 + Cos[3*x]/14)
```

Maple [B] time = 0.091, size = 39, normalized size = 1.9

$$-\frac{8}{21}\sqrt{-2\left(\sin(x/2)\right)^2+1}\left(6\left(\sin(x/2)\right)^6-9\left(\sin(x/2)\right)^4+\left(\sin\left(\frac{x}{2}\right)\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3*cos(x)^(1/2),x)
```

```
[Out] -8/21*(-2*sin(1/2*x)^2+1)^(1/2)*(6*sin(1/2*x)^6-9*sin(1/2*x)^4+si
n(1/2*x)^2+1)
```

Maxima [A] time = 1.36288, size = 18, normalized size = 0.86

$$\frac{2}{7}\cos(x)^{\frac{7}{2}} - \frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x))*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

Fricas [A] time = 0.237176, size = 23, normalized size = 1.1

$$\frac{2}{21}\left(3\cos(x)^3 - 7\cos(x)\right)\sqrt{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x))*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^3 - 7*cos(x))*sqrt(cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3*cos(x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.213291, size = 18, normalized size = 0.86

$$\frac{2}{7}\cos(x)^{\frac{7}{2}} - \frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(cos(x))*sin(x)^3,x, algorithm="giac")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

3.75 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rubi [A] time = 0.0357328, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rubi in Sympy [A] time = 2.25573, size = 19, normalized size = 0.9

$$-\frac{2\sin^{\frac{7}{2}}(x)}{7} + \frac{2\sin^{\frac{3}{2}}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3*sin(x)**(1/2),x)
```

```
[Out] -2*sin(x)**(7/2)/7 + 2*sin(x)**(3/2)/3
```

Mathematica [A] time = 0.0276347, size = 22, normalized size = 1.05

$$\sqrt{\sin(x)}\left(\frac{19\sin(x)}{42} + \frac{1}{14}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] Sqrt[Sin[x]]*((19*Sin[x])/42 + Sin[3*x]/14)
```

Maple [A] time = 0.037, size = 14, normalized size = 0.7

$$\frac{2}{3}(\sin(x))^{\frac{3}{2}} - \frac{2}{7}(\sin(x))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3*sin(x)^(1/2),x)
```

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Maxima [A] time = 1.49179, size = 18, normalized size = 0.86

$$-\frac{2}{7}\,\sin(x)^{\frac{7}{2}}+\frac{2}{3}\,\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sqrt(sin(x)),x, algorithm="maxima")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Fricas [A] time = 0.226532, size = 19, normalized size = 0.9

$$\frac{2}{21}\left(3\,\cos(x)^2+4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sqrt(sin(x)),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [A] time = 103.477, size = 167, normalized size = 7.95

$$\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\,\tan^{\frac{11}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{8\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\,\tan^{\frac{7}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\,\tan^{\frac{3}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3*sin(x)**(1/2),x)
```

```
[Out] 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(11/2)/(21*tan(x/2
)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(1/(
tan(x/2)**2 + 1))*tan(x/2)**(7/2)/(21*tan(x/2)**6 + 63*tan(x/2)**
4 + 63*tan(x/2)**2 + 21) + 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*t
an(x/2)**(3/2)/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2
 + 21)
```

GIAC/XCAS [A] time = 0.201983, size = 18, normalized size = 0.86

$$-\frac{2}{7}\,\sin(x)^{\frac{7}{2}}+\frac{2}{3}\,\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sqrt(sin(x)),x, algorithm="giac")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

3.76 $\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$\sqrt{x} + \sin(\sqrt{x})\cos(\sqrt{x})$$

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.03187, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\sqrt{x} + \sin(\sqrt{x})\cos(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.6876, size = 17, normalized size = 0.89

$$\sqrt{x} + \sin(\sqrt{x})\cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x**(1/2))**2/x**(1/2),x)
```

```
[Out] sqrt(x) + sin(sqrt(x))*cos(sqrt(x))
```

Mathematica [A] time = 0.0100187, size = 18, normalized size = 0.95

$$\sqrt{x} + \frac{1}{2}\sin(2\sqrt{x})$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

Maple [A] time = 0.017, size = 14, normalized size = 0.7

$$\cos(\sqrt{x})\sin(\sqrt{x}) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x^(1/2))^2/x^(1/2),x)
```

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Maxima [A] time = 1.33552, size = 16, normalized size = 0.84

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x))^2/sqrt(x),x, algorithm="maxima")
```

```
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Fricas [A] time = 0.226285, size = 18, normalized size = 0.95

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x))^2/sqrt(x),x, algorithm="fricas")
```

```
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

Sympy [A] time = 0.455867, size = 39, normalized size = 2.05

$$\sqrt{x}\sin^2\left(\sqrt{x}\right) + \sqrt{x}\cos^2\left(\sqrt{x}\right) + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x**(1/2))**2/x**(1/2),x)
```

```
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*
cos(sqrt(x))
```

GIAC/XCAS [A] time = 0.198724, size = 16, normalized size = 0.84

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x))^2/sqrt(x),x, algorithm="giac")
```

```
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

3.77 $\int x \sin^3\left(x^2\right)\, dx$

Optimal. Leaf size=19

$$\frac{1}{6}\cos^3\left(x^2\right) - \frac{\cos\left(x^2\right)}{2}$$

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rubi [A] time = 0.0222967, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{6}\cos^3\left(x^2\right) - \frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x^2]^3,x]
```

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rubi in Sympy [A] time = 1.35506, size = 14, normalized size = 0.74

$$\frac{\cos^3\left(x^2\right)}{6} - \frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x**2)**3,x)
```

```
[Out] cos(x**2)**3/6 - cos(x**2)/2
```

Mathematica [A] time = 0.00472583, size = 19, normalized size = 1.

$$\frac{1}{24}\cos\left(3x^2\right) - \frac{3\cos\left(x^2\right)}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x^2]^3,x]
```

```
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$-\frac{\left(2 + \left(\sin\left(x^2\right)\right)^2\right)\cos\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x^2)^3,x)
```

```
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A] time = 1.35989, size = 20, normalized size = 1.05

$$\frac{1}{24}\cos\left(3\,x^2\right)-\frac{3}{8}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

Fricas [A] time = 0.227814, size = 20, normalized size = 1.05

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Sympy [A] time = 0.822632, size = 22, normalized size = 1.16

$$-\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}-\frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

GIAC/XCAS [A] time = 0.200703, size = 20, normalized size = 1.05

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

3.78 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi [A] time = 0.0222209, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos^2(x)\tan^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*tan(x)**3,x)
```

```
[Out] Integral(cos(x)**2*tan(x)**3, x)
```

Mathematica [A] time = 0.00658397, size = 14, normalized size = 1.

$$\frac{1}{4}\cos(2x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[2*x]/4 - Log[Cos[x]]
```

Maple [A] time = 0.017, size = 13, normalized size = 0.9

$$-\frac{(\sin(x))^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*tan(x)^3,x)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A] time = 1.48407, size = 22, normalized size = 1.57

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 0.235024, size = 19, normalized size = 1.36

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.078063, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*tan(x)**3,x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

GIAC/XCAS [A] time = 0.203484, size = 19, normalized size = 1.36

$$\frac{1}{2}\cos(x)^2 - \frac{1}{2}\ln\left(\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*cos(x)^2 - 1/2*ln(cos(x)^2)
```

3.79 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi [A] time = 0.05361, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi in Sympy [A] time = 3.6298, size = 26, normalized size = 1.18

$$-\log\left(-\cos^2(x)+1\right) - \frac{\cos^2(x)}{2} - \frac{1}{2\left(-\cos^2(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**5*sin(x)**2,x)
```

```
[Out] -log(-cos(x)**2 + 1) - cos(x)**2/2 - 1/(2*(-cos(x)**2 + 1))
```

Mathematica [A] time = 0.00890417, size = 22, normalized size = 1.

$$-\frac{1}{4}\cos(2x) - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Cos[2*x]/4 - Csc[x]^2/2 - 2*Log[Sin[x]]
```

Maple [A] time = 0.02, size = 29, normalized size = 1.3

$$-\frac{(\cos(x))^6}{2\,(\sin(x))^2} - \frac{(\cos(x))^4}{2} - (\cos(x))^2 - 2\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^5*sin(x)^2,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A] time = 1.33848, size = 27, normalized size = 1.23

$$\frac{1}{2}\,\sin(x)^2 - \frac{1}{2\,\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^5*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [A] time = 0.232614, size = 50, normalized size = 2.27

$$-\frac{2\,\cos(x)^4 - 3\,\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\,\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^5*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x))
- 1)/(cos(x)^2 - 1)
```

Sympy [A] time = 0.098456, size = 20, normalized size = 0.91

$$-2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**5*sin(x)**2,x)
```

```
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

GIAC/XCAS [A] time = 0.209236, size = 49, normalized size = 2.23

$$-\frac{1}{2}\,\cos(x)^2 + \frac{2\,\cos(x)^2 - 1}{2\left(\cos(x)^2 - 1\right)} - \ln\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^5*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2*(2*cos(x)^2 - 1)/(cos(x)^2 - 1) - ln(-cos(x)^
2 + 1)
```

3.80 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] Log[1 + Sin[x]]
```

Rubi [A] time = 0.0220001, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rubi in Sympy [A] time = 1.55506, size = 5, normalized size = 1.

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-sin(x))/cos(x),x)
```

```
[Out] log(sin(x) + 1)
```

Mathematica [B] time = 0.0111511, size = 36, normalized size = 7.2

$$\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.036, size = 6, normalized size = 1.2

$$\ln(1+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-sin(x))/cos(x),x)
```

```
[Out] ln(1+sin(x))
```

Maxima [A] time = 1.35132, size = 7, normalized size = 1.4

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)/cos(x),x, algorithm="maxima")
```

```
[Out] log(sin(x) + 1)
```

Fricas [A] time = 0.223727, size = 7, normalized size = 1.4

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)/cos(x),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [A] time = 0.481488, size = 19, normalized size = 3.8

$$2\log\left(\tan\left(\frac{x}{2}\right) + 1\right) - \log\left(\tan^2\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-sin(x))/cos(x),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)**2 + 1)
```

GIAC/XCAS [A] time = 0.214291, size = 7, normalized size = 1.4

$$\ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sin(x) - 1)/cos(x),x, algorithm="giac")
```

```
[Out] ln(sin(x) + 1)
```

3.81 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.015708, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi in Sympy [A] time = 0.51639, size = 7, normalized size = 0.64

$$\frac{\cos(x)}{-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-sin(x)),x)
```

```
[Out] cos(x)/(-sin(x) + 1)
```

Mathematica [B] time = 0.0104177, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.003, size = 11, normalized size = 1.

$$-2\,(-1+\tan(x/2))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-sin(x)),x)
```

```
[Out] -2/(-1+tan(1/2*x))
```

Maxima [A] time = 1.35804, size = 20, normalized size = 1.82

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 0.211194, size = 23, normalized size = 2.09

$$\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 0.714072, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

GIAC/XCAS [A] time = 0.215994, size = 14, normalized size = 1.27

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(sin(x) - 1),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

3.82 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x + Tan[x]
```

Rubi [A] time = 0.00858802, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rubi in Sympy [A] time = 0.046925, size = 3, normalized size = 0.5

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**2,x)
```

```
[Out] -x + tan(x)
```

Mathematica [A] time = 0.00364365, size = 6, normalized size = 1.

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Maple [A] time = 0., size = 7, normalized size = 1.2

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^2,x)
```

```
[Out] -x+tan(x)
```

Maxima [A] time = 1.59995, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="maxima")
```

```
[Out] -x + tan(x)
```

Fricas [A] time = 0.217639, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

Sympy [A] time = 0.04436, size = 7, normalized size = 1.17

$$-x + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.204827, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

3.83 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x + \frac{\tan^3(x)}{3} - \tan(x)$$

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.0146248, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x + \frac{\tan^3(x)}{3} - \tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi in Sympy [A] time = 0.48958, size = 10, normalized size = 0.71

$$x + \frac{\tan^3(x)}{3} - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**4,x)
```

```
[Out] x + tan(x)**3/3 - tan(x)
```

Mathematica [A] time = 0.00526244, size = 18, normalized size = 1.29

$$x - \frac{4\tan(x)}{3} + \frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^4,x]
```

```
[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$x - \tan(x) + \frac{(\tan(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^4,x)
```

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Maxima [A] time = 1.52597, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A] time = 0.224233, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] time = 0.052392, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.200462, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

3.84 $\int \sec^4(x)\,dx$

Optimal. Leaf size=11

$$\frac{\tan^3(x)}{3} + \tan(x)$$

```
[Out] Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.012423, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\tan^3(x)}{3} + \tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^4,x]
```

```
[Out] Tan[x] + Tan[x]^3/3
```

Rubi in Sympy [A] time = 0.52396, size = 19, normalized size = 1.73

$$\frac{2\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**4,x)
```

```
[Out] 2*sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

Mathematica [A] time = 0.00391115, size = 17, normalized size = 1.55

$$\frac{2\tan(x)}{3} + \frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^4,x]
```

```
[Out] (2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.043, size = 13, normalized size = 1.2

$$-\left(-\frac{2}{3} - \frac{(\sec(x))^2}{3}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^4,x)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)
```

Maxima [A] time = 1.34197, size = 12, normalized size = 1.09

$$\frac{1}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + tan(x)
```

Fricas [A] time = 0.206108, size = 22, normalized size = 2.

$$\frac{\left(2\cos(x)^2 + 1\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/cos(x)^3
```

Sympy [A] time = 0.044559, size = 19, normalized size = 1.73

$$\frac{2\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**4,x)
```

```
[Out] 2*sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

GIAC/XCAS [A] time = 0.208536, size = 12, normalized size = 1.09

$$\frac{1}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + tan(x)
```

3.85 $\int \sec^6(x)\,dx$

Optimal. Leaf size=19

$$\frac{\tan^5(x)}{5} + \frac{2\tan^3(x)}{3} + \tan(x)$$

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0156942, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\tan^5(x)}{5} + \frac{2\tan^3(x)}{3} + \tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^6,x]
```

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rubi in Sympy [A] time = 0.60728, size = 31, normalized size = 1.63

$$\frac{8\sin(x)}{15\cos(x)} + \frac{4\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**6,x)
```

```
[Out] 8*sin(x)/(15*cos(x)) + 4*sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)
**5)
```

Mathematica [A] time = 0.00471719, size = 27, normalized size = 1.42

$$\frac{8\tan(x)}{15} + \frac{1}{5}\tan(x)\sec^4(x) + \frac{4}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^6,x]
```

```
[Out] (8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.043, size = 19, normalized size = 1.

$$-\left(-\frac{8}{15} - \frac{(\sec(x))^4}{5} - \frac{4(\sec(x))^2}{15}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^6,x)
```

```
[Out] -(-8/15-1/5*sec(x)^4-4/15*sec(x)^2)*tan(x)
```

Maxima [A] time = 1.35219, size = 20, normalized size = 1.05

$$\frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

Fricas [A] time = 0.208285, size = 30, normalized size = 1.58

$$\frac{\left(8\cos(x)^4 + 4\cos(x)^2 + 3\right)\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/cos(x)^5
```

Sympy [A] time = 0.047546, size = 31, normalized size = 1.63

$$\frac{8\sin(x)}{15\cos(x)} + \frac{4\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**6,x)
```

```
[Out] 8*sin(x)/(15*cos(x)) + 4*sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)
**5)
```

GIAC/XCAS [A] time = 0.20162, size = 20, normalized size = 1.05

$$\frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

3.86 $\int \sec^2(x)\tan^4(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^5(x)}{5}$$

```
[Out] Tan[x]^5/5
```

Rubi [A] time = 0.0316572, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Rubi in Sympy [A] time = 1.78333, size = 5, normalized size = 0.62

$$\frac{\tan^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2*tan(x)**4,x)
```

```
[Out] tan(x)**5/5
```

Mathematica [A] time = 0.00265554, size = 8, normalized size = 1.

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Maple [A] time = 0.021, size = 11, normalized size = 1.4

$$\frac{(\sin(x))^5}{5\,(\cos(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2*tan(x)^4,x)
```

```
[Out] 1/5*sin(x)^5/cos(x)^5
```

Maxima [A] time = 1.34337, size = 8, normalized size = 1.

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5
```

Fricas [A] time = 0.221578, size = 27, normalized size = 3.38

$$\frac{\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\sin(x)}{5\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)^5
```

Sympy [A] time = 0.051708, size = 29, normalized size = 3.62

$$\frac{\sin(x)}{5\cos(x)} - \frac{2\sin(x)}{5\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2*tan(x)**4,x)
```

```
[Out] sin(x)/(5*cos(x)) - 2*sin(x)/(5*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

GIAC/XCAS [A] time = 0.199173, size = 8, normalized size = 1.

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5
```

3.87 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0363302, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(x)}{\cos^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] Integral(tan(x)**2/cos(x)**4, x)
```

Mathematica [A] time = 0.0172234, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{1}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.022, size = 22, normalized size = 1.3

$$\frac{(\sin(x))^3}{5\,(\cos(x))^5}+\frac{2\,(\sin(x))^3}{15\,(\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 1/5*sin(x)^3/cos(x)^5+2/15*sin(x)^3/cos(x)^3
```

Maxima [A] time = 1.34295, size = 18, normalized size = 1.06

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Fricas [A] time = 0.211663, size = 27, normalized size = 1.59

$$-\frac{\left(2\cos(x)^4 + \cos(x)^2 - 3\right)\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [A] time = 0.053007, size = 29, normalized size = 1.71

$$-\frac{2\sin(x)}{15\cos(x)} - \frac{\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)*
*5)
```

GIAC/XCAS [A] time = 0.199124, size = 18, normalized size = 1.06

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

3.88 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.0191417, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x),x)
```

```
[Out] Integral(tan(x)/cos(x)**3, x)
```

Mathematica [A] time = 0.00303344, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0.01, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 1.3978, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 0.213743, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.053887, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

GIAC/XCAS [A] time = 0.201988, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.89 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0355875, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] Integral(tan(x)**3/cos(x)**3, x)
```

Mathematica [A] time = 0.0171287, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [B] time = 0.02, size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5\,(\cos(x))^5} + \frac{(\sin(x))^4}{15\,(\cos(x))^3} - \frac{(\sin(x))^4}{15\,\cos(x)} - \frac{\left(2 + (\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)
-1/15*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.34905, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 0.21406, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.106965, size = 15, normalized size = 0.88

$$-\frac{5\cos^2(x)-3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

GIAC/XCAS [A] time = 0.202015, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.90 $\int \tan^5(x)\,dx$

Optimal. Leaf size=22

$$\frac{\tan^4(x)}{4} - \frac{\tan^2(x)}{2} - \log(\cos(x))$$

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rubi [A] time = 0.017887, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\tan^4(x)}{4} - \frac{\tan^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rubi in Sympy [A] time = 0.517309, size = 17, normalized size = 0.77

$$-\log(\cos(x)) + \frac{\tan^4(x)}{4} - \frac{\tan^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**5,x)
```

```
[Out] -log(cos(x)) + tan(x)**4/4 - tan(x)**2/2
```

Mathematica [A] time = 0.0052138, size = 20, normalized size = 0.91

$$\frac{\sec^4(x)}{4} - \sec^2(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Sec[x]^2 + Sec[x]^4/4
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$-\frac{(\tan(x))^2}{2} + \frac{(\tan(x))^4}{4} + \frac{\ln\left(1+(\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^5,x)
```

```
[Out] -1/2*tan(x)^2+1/4*tan(x)^4+1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.34596, size = 46, normalized size = 2.09

$$\frac{4\,\sin(x)^2-3}{4\left(\sin(x)^4-2\,\sin(x)^2+1\right)}-\frac{1}{2}\,\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)
^2 - 1)
```

Fricas [A] time = 0.223933, size = 32, normalized size = 1.45

$$\frac{1}{4}\,\tan(x)^4-\frac{1}{2}\,\tan(x)^2-\frac{1}{2}\,\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.121459, size = 20, normalized size = 0.91

$$-\frac{4\cos^2(x)-1}{4\cos^4(x)}-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

GIAC/XCAS [A] time = 0.205359, size = 30, normalized size = 1.36

$$\frac{1}{4}\,\tan(x)^4-\frac{1}{2}\,\tan(x)^2+\frac{1}{2}\,\ln\left(\tan(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*ln(tan(x)^2 + 1)
```

3.91 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0198949, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi in Sympy [A] time = 0.526201, size = 17, normalized size = 0.77

$$-x + \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**6,x)
```

```
[Out] -x + tan(x)**5/5 - tan(x)**3/3 + tan(x)
```

Mathematica [A] time = 0.00622399, size = 30, normalized size = 1.36

$$-x + \frac{23\tan(x)}{15} + \frac{1}{5}\tan(x)\sec^4(x) - \frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])
/5
```

Maple [A] time = 0.005, size = 19, normalized size = 0.9

$$-x + \tan(x) - \frac{(\tan(x))^3}{3} + \frac{(\tan(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^6,x)
```

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Maxima [A] time = 1.49675, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Fricas [A] time = 0.21848, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] time = 0.063295, size = 31, normalized size = 1.41

$$-x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**6,x)
```

```
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/c
os(x)
```

GIAC/XCAS [A] time = 0.205459, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.92 $\int \sec(x)\tan^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sec^5(x)}{5} - \frac{2\sec^3(x)}{3} + \sec(x)$$

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0254294, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^5(x)}{5} - \frac{2\sec^3(x)}{3} + \sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rubi in Sympy [A] time = 1.82537, size = 20, normalized size = 1.05

$$\frac{1}{\cos(x)} - \frac{2}{3\cos^3(x)} + \frac{1}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)*tan(x)**5,x)
```

```
[Out] 1/cos(x) - 2/(3*cos(x)**3) + 1/(5*cos(x)**5)
```

Mathematica [A] time = 0.00588609, size = 19, normalized size = 1.

$$\frac{\sec^5(x)}{5} - \frac{2\sec^3(x)}{3} + \sec(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Maple [B] time = 0.016, size = 48, normalized size = 2.5

$$\frac{(\sin(x))^6}{5\,(\cos(x))^5} - \frac{(\sin(x))^6}{15\,(\cos(x))^3} + \frac{(\sin(x))^6}{5\,\cos(x)} + \frac{\cos(x)}{5}\left(\frac{8}{3} + (\sin(x))^4 + \frac{4\,(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)*tan(x)^5,x)
```

```
[Out] 1/5*sin(x)^6/cos(x)^5-1/15*sin(x)^6/cos(x)^3+1/5*sin(x)^6/cos(x)+
1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 1.355, size = 27, normalized size = 1.42

$$\frac{15\,\cos(x)^4 - 10\,\cos(x)^2 + 3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Fricas [A] time = 0.229705, size = 27, normalized size = 1.42

$$\frac{15\,\cos(x)^4 - 10\,\cos(x)^2 + 3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Sympy [A] time = 0.126915, size = 20, normalized size = 1.05

$$\frac{15\cos^4(x) - 10\cos^2(x) + 3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)**5,x)
```

```
[Out] (15*cos(x)**4 - 10*cos(x)**2 + 3)/(15*cos(x)**5)
```

GIAC/XCAS [A] time = 0.200672, size = 27, normalized size = 1.42

$$\frac{15\,\cos(x)^4 - 10\,\cos(x)^2 + 3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

3.93 $\int \sec^3(x)\tan^5(x)\,dx$

Optimal. Leaf size=25

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rubi [A] time = 0.0472868, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x]^5,x]
```

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x)**5,x)
```

```
[Out] Integral(tan(x)**5/cos(x)**3, x)
```

Mathematica [A] time = 0.00809365, size = 25, normalized size = 1.

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x]^5,x]
```

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Maple [B] time = 0.019, size = 58, normalized size = 2.3

$$\frac{(\sin(x))^6}{7(\cos(x))^7} + \frac{(\sin(x))^6}{35(\cos(x))^5} - \frac{(\sin(x))^6}{105(\cos(x))^3} + \frac{(\sin(x))^6}{35\cos(x)} + \frac{\cos(x)}{35}\left(\frac{8}{3} + (\sin(x))^4 + \frac{4(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x)^5,x)
```

```
[Out] 1/7*sin(x)^6/cos(x)^7+1/35*sin(x)^6/cos(x)^5-1/105*sin(x)^6/cos(x
)^3+1/35*sin(x)^6/cos(x)+1/35*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 1.36036, size = 27, normalized size = 1.08

$$\frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Fricas [A] time = 0.2299, size = 27, normalized size = 1.08

$$\frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Sympy [A] time = 0.152236, size = 20, normalized size = 0.8

$$\frac{35\cos^4(x) - 42\cos^2(x) + 15}{105\cos^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x)**5,x)
```

```
[Out] (35*cos(x)**4 - 42*cos(x)**2 + 15)/(105*cos(x)**7)
```

GIAC/XCAS [A] time = 0.20515, size = 27, normalized size = 1.08

$$\frac{35\,\cos(x)^4 - 42\,\cos(x)^2 + 15}{105\,\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

3.94 $\int \sec^6(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^6(x)}{6}$$

```
[Out] Sec[x]^6/6
```

Rubi [A] time = 0.0189001, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**6*tan(x),x)
```

```
[Out] Integral(tan(x)/cos(x)**6, x)
```

Mathematica [A] time = 0.0030136, size = 8, normalized size = 1.

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Maple [A] time = 0.01, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^6*tan(x),x)
```

```
[Out] 1/6*sec(x)^6
```

Maxima [A] time = 1.36296, size = 14, normalized size = 1.75

$$-\frac{1}{6\left(\sin(x)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x),x, algorithm="maxima")
```

```
[Out] -1/6/(sin(x)^2 - 1)^3
```

Fricas [A] time = 0.216379, size = 8, normalized size = 1.

$$\frac{1}{6\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x),x, algorithm="fricas")
```

```
[Out] 1/6/cos(x)^6
```

Sympy [A] time = 0.050575, size = 7, normalized size = 0.88

$$\frac{1}{6\cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**6*tan(x),x)
```

```
[Out] 1/(6*cos(x)**6)
```

GIAC/XCAS [A] time = 0.198959, size = 8, normalized size = 1.

$$\frac{1}{6\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x),x, algorithm="giac")
```

```
[Out] 1/6/cos(x)^6
```

3.95 $\int \sec^6(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{6}$$

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Rubi [A] time = 0.0368012, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\cos^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**6*tan(x)**3,x)
```

```
[Out] Integral(tan(x)**3/cos(x)**6, x)
```

Mathematica [A] time = 0.0104471, size = 17, normalized size = 1.

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Maple [B] time = 0.02, size = 32, normalized size = 1.9

$$\frac{(\sin(x))^4}{8\,(\cos(x))^8} + \frac{(\sin(x))^4}{12\,(\cos(x))^6} + \frac{(\sin(x))^4}{24\,(\cos(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^6*tan(x)^3,x)
```

```
[Out] 1/8*sin(x)^4/cos(x)^8+1/12*sin(x)^4/cos(x)^6+1/24*sin(x)^4/cos(x)
^4
```

Maxima [A] time = 1.37368, size = 49, normalized size = 2.88

$$\frac{4\,\sin(x)^2 - 1}{24\left(\sin(x)^8 - 4\,\sin(x)^6 + 6\,\sin(x)^4 - 4\,\sin(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(4*sin(x)^2 - 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*sin
(x)^2 + 1)
```

Fricas [A] time = 0.25274, size = 19, normalized size = 1.12

$$-\frac{4\,\cos(x)^2 - 3}{24\,\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

Sympy [A] time = 0.132363, size = 15, normalized size = 0.88

$$-\frac{4\cos^2(x) - 3}{24\cos^8(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**6*tan(x)**3,x)
```

```
[Out] -(4*cos(x)**2 - 3)/(24*cos(x)**8)
```

GIAC/XCAS [A] time = 0.204358, size = 19, normalized size = 1.12

$$-\frac{4\,\cos(x)^2 - 3}{24\,\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^6*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

3.96 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.0197046, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\tan(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2/cot(x),x)
```

```
[Out] Integral(x, (x, tan(x)))
```

Mathematica [A] time = 0.00268946, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A] time = 0.018, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2/cot(x),x)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A] time = 1.37236, size = 14, normalized size = 1.75

$$-\frac{1}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/cot(x),x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1)
```

Fricas [A] time = 0.212081, size = 8, normalized size = 1.

$$\frac{1}{2\ \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/cot(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.057229, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2/cot(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

GIAC/XCAS [A] time = 0.20019, size = 8, normalized size = 1.

$$\frac{1}{2\ \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2/cot(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.97 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.0223515, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi in Sympy [A] time = 1.65538, size = 14, normalized size = 0.88

$$-\frac{\operatorname{atanh}(\sin(x))}{2} + \frac{\tan(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)*tan(x)**2,x)
```

```
[Out] -atanh(sin(x))/2 + tan(x)/(2*cos(x))
```

Mathematica [B] time = 0.0726269, size = 42, normalized size = 2.62

$$\frac{1}{2}\left(\tan(x)\sec(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan
[x])/2
```

Maple [A] time = 0.017, size = 24, normalized size = 1.5

$$\frac{(\sin(x))^3}{2\,(\cos(x))^2} + \frac{\sin(x)}{2} - \frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [A] time = 1.37254, size = 36, normalized size = 2.25

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x)
 - 1)
```

Fricas [A] time = 0.26328, size = 46, normalized size = 2.88

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*si
n(x))/cos(x)^2
```

Sympy [A] time = 0.126013, size = 27, normalized size = 1.69

$$\frac{\log(\sin(x)-1)}{4}-\frac{\log(\sin(x)+1)}{4}-\frac{\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

GIAC/XCAS [A] time = 0.206006, size = 39, normalized size = 2.44

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\ln(\sin(x)+1)+\frac{1}{4}\ln(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*ln(sin(x) + 1) + 1/4*ln(-sin(x)
+ 1)
```

3.98 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x - \cot(x)$$

```
[Out] -x - Cot[x]
```

Rubi [A] time = 0.00993355, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-x - \cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rubi in Sympy [A] time = 0.47489, size = 7, normalized size = 0.88

$$-x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**2,x)
```

```
[Out] -x - 1/tan(x)
```

Mathematica [A] time = 0.00365997, size = 8, normalized size = 1.

$$-x - \cot(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.5

$$-\cot(x) + \frac{\pi}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^2,x)
```

```
[Out] -cot(x)+1/2*Pi-x
```

Maxima [A] time = 1.5032, size = 14, normalized size = 1.75

$$-x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

Fricas [A] time = 0.223829, size = 27, normalized size = 3.38

$$-\frac{x\sin(2\,x) + \cos(2\,x) + 1}{\sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] time = 0.046527, size = 8, normalized size = 1.

$$-x - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

GIAC/XCAS [A] time = 0.205638, size = 24, normalized size = 3.

$$-x - \frac{1}{2\,\tan\left(\frac{1}{2}\,x\right)} + \frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.99 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x) - \log(\sin(x))$$

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi [A] time = 0.0138111, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{2}\cot^2(x) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^3,x]
```

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.497006, size = 14, normalized size = 1.

$$-\log(\sin(x)) - \frac{1}{2\tan^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*tan(x)**2)
```

Mathematica [A] time = 0.00494694, size = 14, normalized size = 1.

$$-\frac{1}{2}\csc^2(x) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - Log[Sin[x]]
```

Maple [A] time = 0.005, size = 17, normalized size = 1.2

$$-\frac{(\cot(x))^2}{2} + \frac{\ln\left((\cot(x))^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^3,x)
```

```
[Out] -1/2*cot(x)^2+1/2*ln(cot(x)^2+1)
```

Maxima [A] time = 1.34688, size = 19, normalized size = 1.36

$$-\frac{1}{2\,\sin(x)^2}-\frac{1}{2}\,\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Fricas [A] time = 0.24413, size = 38, normalized size = 2.71

$$-\frac{(\cos(2\,x)-1)\log\left(-\frac{1}{2}\,\cos(2\,x)+\frac{1}{2}\right)-2}{2\,(\cos(2\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2) - 2)/(cos(2*x) - 1)
```

Sympy [A] time = 0.091527, size = 14, normalized size = 1.

$$-\log(\sin(x))-\frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

GIAC/XCAS [A] time = 0.20577, size = 30, normalized size = 2.14

$$\frac{1}{2\left(\cos(x)^2-1\right)}-\frac{1}{2}\ln\left(-\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3,x, algorithm="giac")
```

```
[Out] 1/2/(cos(x)^2 - 1) - 1/2*ln(-cos(x)^2 + 1)
```

3.100 $\int \cot^4(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi [A] time = 0.0395665, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi in Sympy [A] time = 2.84001, size = 44, normalized size = 2.59

$$-\frac{2\cos(x)}{35\sin(x)}-\frac{\cos(x)}{35\sin^3(x)}+\frac{3}{35\sin^4(x)\tan(x)}-\frac{1}{7\sin^4(x)\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**4*csc(x)**4,x)
```

```
[Out] -2*cos(x)/(35*sin(x)) - cos(x)/(35*sin(x)**3) + 3/(35*sin(x)**4*t
an(x)) - 1/(7*sin(x)**4*tan(x)**3)
```

Mathematica [B] time = 0.0199545, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35}-\frac{1}{7}\cot(x)\csc^6(x)+\frac{8}{35}\cot(x)\csc^4(x)-\frac{1}{35}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 -
(Cot[x]*Csc[x]^6)/7
```

Maple [A] time = 0.019, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^5}{7\,(\sin(x))^7}-\frac{2\,(\cos(x))^5}{35\,(\sin(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^4*csc(x)^4,x)
```

```
[Out] -1/7/sin(x)^7*cos(x)^5-2/35/sin(x)^5*cos(x)^5
```

Maxima [A] time = 1.45815, size = 19, normalized size = 1.12

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Fricas [A] time = 0.234645, size = 53, normalized size = 3.12

$$-\frac{2\,\cos(x)^7-7\,\cos(x)^5}{35\left(\cos(x)^6-3\,\cos(x)^4+3\,\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x
)^2 - 1)*sin(x))
```

Sympy [A] time = 0.057872, size = 41, normalized size = 2.41

$$-\frac{2\cos(x)}{35\sin(x)}-\frac{\cos(x)}{35\sin^3(x)}+\frac{8\cos(x)}{35\sin^5(x)}-\frac{\cos(x)}{7\sin^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**4*csc(x)**4,x)
```

```
[Out] -2*cos(x)/(35*sin(x)) - cos(x)/(35*sin(x)**3) + 8*cos(x)/(35*sin(
x)**5) - cos(x)/(7*sin(x)**7)
```

GIAC/XCAS [A] time = 0.213308, size = 19, normalized size = 1.12

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4*csc(x)^4,x, algorithm="giac")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

3.101 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] time = 0.0403828, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^3*Csc[x]^4,x]

[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi in Sympy [A] time = 3.33918, size = 22, normalized size = 1.29

$$\frac{1}{4\left(-\cos^2(x)+1\right)^2} - \frac{1}{6\left(-\cos^2(x)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**3*csc(x)**4,x)

[Out] 1/(4*(-cos(x)**2 + 1)**2) - 1/(6*(-cos(x)**2 + 1)**3)
```

Mathematica [A] time = 0.008245, size = 17, normalized size = 1.

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^3*Csc[x]^4,x]

[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] time = 0.019, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^4}{6\,(\sin(x))^6} - \frac{(\cos(x))^4}{12\,(\sin(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^3*csc(x)^4,x)

[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Maxima [A] time = 1.33638, size = 19, normalized size = 1.12

$$\frac{3\ \sin(x)^2 - 2}{12\ \sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3*csc(x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Fricas [A] time = 0.265129, size = 41, normalized size = 2.41

$$\frac{3\ \cos(x)^2 - 1}{12\left(\cos(x)^6 - 3\ \cos(x)^4 + 3\ \cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3*csc(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.119313, size = 14, normalized size = 0.82

$$\frac{3\sin^2(x) - 2}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**3*csc(x)**4,x)
```

```
[Out] (3*sin(x)**2 - 2)/(12*sin(x)**6)
```

GIAC/XCAS [A] time = 0.197925, size = 24, normalized size = 1.41

$$\frac{3\ \cos(x)^2 - 1}{12\left(\cos(x)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^3*csc(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^2 - 1)^3
```

3.102 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]
```

Rubi [A] time = 0.00521444, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rubi in Sympy [A] time = 0.029621, size = 5, normalized size = 1.

$$-\operatorname{atanh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csc(x),x)
```

```
[Out] -atanh(cos(x))
```

Mathematica [B] time = 0.0041537, size = 17, normalized size = 3.4

$$\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.8

$$-\ln(\csc(x) + \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csc(x),x)
```

```
[Out] -ln(csc(x)+cot(x))
```

Maxima [A] time = 1.48608, size = 11, normalized size = 2.2

$$-\log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x),x, algorithm="maxima")
```

```
[Out] -log(cot(x) + csc(x))
```

Fricas [A] time = 0.268854, size = 26, normalized size = 5.2

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.097842, size = 15, normalized size = 3.

$$\frac{\log(\cos(x)-1)}{2}-\frac{\log(\cos(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

GIAC/XCAS [A] time = 0.213039, size = 8, normalized size = 1.6

$$\ln\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x),x, algorithm="giac")
```

```
[Out] ln(abs(tan(1/2*x)))
```

3.103 $\int \csc^3(x)\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2}\tanh^{-1}(\cos(x)) - \frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/2 - (Cot[x]*Csc[x])/2
```

Rubi [A] time = 0.0121642, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{1}{2}\tanh^{-1}(\cos(x)) - \frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^3,x]
```

```
[Out] -ArcTanh[Cos[x]]/2 - (Cot[x]*Csc[x])/2
```

Rubi in Sympy [A] time = 0.515742, size = 17, normalized size = 1.06

$$-\frac{\operatorname{atanh}(\cos(x))}{2} - \frac{\cos(x)}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csc(x)**3,x)
```

```
[Out] -atanh(cos(x))/2 - cos(x)/(2*sin(x)**2)
```

Mathematica [B] time = 0.00674204, size = 47, normalized size = 2.94

$$-\frac{1}{8}\csc^2\left(\frac{x}{2}\right) + \frac{1}{8}\sec^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^3,x]
```

```
[Out] -Csc[x/2]^2/8 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8
```

Maple [A] time = 0.047, size = 18, normalized size = 1.1

$$-\frac{\cot(x)\csc(x)}{2} + \frac{\ln(\csc(x) - \cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csc(x)^3,x)
```

```
[Out] -1/2*cot(x)*csc(x)+1/2*ln(csc(x)-cot(x))
```

Maxima [A] time = 1.41989, size = 36, normalized size = 2.25

$$\frac{\cos(x)}{2\left(\cos(x)^2-1\right)}-\frac{1}{4}\log(\cos(x)+1)+\frac{1}{4}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*cos(x)/(cos(x)^2 - 1) - 1/4*log(cos(x) + 1) + 1/4*log(cos(x)
- 1)
```

Fricas [A] time = 0.264926, size = 59, normalized size = 3.69

$$-\frac{\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-
1/2*cos(x) + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

Sympy [A] time = 0.129703, size = 27, normalized size = 1.69

$$\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}+\frac{\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)**3,x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)/(2*cos(x)**2 - 2)
```

GIAC/XCAS [A] time = 0.204533, size = 73, normalized size = 4.56

$$-\frac{\left(\frac{2(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{8(\cos(x)-1)}-\frac{\cos(x)-1}{8(\cos(x)+1)}+\frac{1}{4}\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1)
- 1/8*(cos(x) - 1)/(cos(x) + 1) + 1/4*ln(-(cos(x) - 1)/(cos(x) +
1))
```

3.104 $\int \cos(x)\cot(x)\,dx$

Optimal. Leaf size=8

$$\cos(x) - \tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rubi [A] time = 0.0217969, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\cos(x) - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cot[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rubi in Sympy [A] time = 2.79744, size = 7, normalized size = 0.88

$$\cos(x) - \operatorname{atanh}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2/sin(x),x)
```

```
[Out] cos(x) - atanh(cos(x))
```

Mathematica [B] time = 0.00678012, size = 19, normalized size = 2.38

$$\cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cot[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.013, size = 12, normalized size = 1.5

$$\cos(x) + \ln(\csc(x) - \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2/sin(x),x)
```

```
[Out] cos(x)+ln(csc(x)-cot(x))
```

Maxima [A] time = 1.34157, size = 23, normalized size = 2.88

$$\cos(x) - \frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/sin(x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [A] time = 0.271954, size = 28, normalized size = 3.5

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/sin(x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.097092, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2/sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

GIAC/XCAS [A] time = 0.206512, size = 26, normalized size = 3.25

$$\cos(x) - \frac{1}{2}\ln(\cos(x) + 1) + \frac{1}{2}\ln(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2/sin(x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/2*ln(cos(x) + 1) + 1/2*ln(-cos(x) + 1)
```

3.105 $\int \csc^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3}\cot^3(x) - \cot(x)$$

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0115975, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{1}{3}\cot^3(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]^4,x]
```

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rubi in Sympy [A] time = 0.537123, size = 20, normalized size = 1.54

$$-\frac{2\cos(x)}{3\sin(x)} - \frac{\cos(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sin(x)**4,x)
```

```
[Out] -2*cos(x)/(3*sin(x)) - cos(x)/(3*sin(x)**3)
```

Mathematica [A] time = 0.00387051, size = 17, normalized size = 1.31

$$-\frac{2\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]^4,x]
```

```
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.045, size = 12, normalized size = 0.9

$$\left(-\frac{2}{3} - \frac{(\csc(x))^2}{3}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sin(x)^4,x)
```

```
[Out] (-2/3-1/3*csc(x)^2)*cot(x)
```

Maxima [A] time = 1.34744, size = 19, normalized size = 1.46

$$-\frac{3\,\tan(x)^2+1}{3\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-4),x, algorithm="maxima")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Fricas [A] time = 0.219669, size = 34, normalized size = 2.62

$$-\frac{2\,\cos(x)^3-3\,\cos(x)}{3\left(\cos(x)^2-1\right)\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-4),x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.043178, size = 20, normalized size = 1.54

$$-\frac{2\cos(x)}{3\sin(x)}-\frac{\cos(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sin(x)**4,x)
```

```
[Out] -2*cos(x)/(3*sin(x)) - cos(x)/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.201392, size = 19, normalized size = 1.46

$$-\frac{3\,\tan(x)^2+1}{3\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^(-4),x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

3.106 $\int \sin(2x)\sin(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rubi [A] time = 0.0149131, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]*Sin[5*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rubi in Sympy [A] time = 1.01965, size = 12, normalized size = 0.71

$$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)*sin(5*x),x)
```

```
[Out] sin(3*x)/6 - sin(7*x)/14
```

Mathematica [A] time = 0.0112471, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x) - \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]*Sin[5*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Maple [A] time = 0.059, size = 14, normalized size = 0.8

$$\frac{\sin(3\,x)}{6} - \frac{\sin(7\,x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)*sin(5*x),x)
```

```
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

Maxima [A] time = 1.39357, size = 18, normalized size = 1.06

$$-\frac{1}{14}\sin(7x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(5*x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

Fricas [A] time = 0.226308, size = 32, normalized size = 1.88

$$-\frac{2}{21}\left(48\cos(x)^6-60\cos(x)^4+11\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(5*x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/21*(48*cos(x)^6 - 60*cos(x)^4 + 11*cos(x)^2 + 1)*sin(x)
```

Sympy [A] time = 0.717103, size = 26, normalized size = 1.53

$$-\frac{5\sin(2x)\cos(5x)}{21}+\frac{2\sin(5x)\cos(2x)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*sin(5*x),x)
```

```
[Out] -5*sin(2*x)*cos(5*x)/21 + 2*sin(5*x)*cos(2*x)/21
```

GIAC/XCAS [A] time = 0.19943, size = 18, normalized size = 1.06

$$-\frac{1}{14}\sin(7x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(5*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

3.107 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rubi [A] time = 0.0153035, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rubi in Sympy [A] time = 0.996206, size = 14, normalized size = 0.82

$$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*sin(3*x),x)
```

```
[Out] -cos(2*x)/4 - cos(4*x)/8
```

Mathematica [A] time = 0.00944942, size = 17, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[x]^2/2 - Cos[4*x]/8
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$-(\cos(x))^4+\frac{(\cos(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*sin(3*x),x)
```

```
[Out] -cos(x)^4+1/2*cos(x)^2
```

Maxima [A] time = 1.36308, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

Fricas [A] time = 0.228532, size = 18, normalized size = 1.06

$$-\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Sympy [A] time = 0.727638, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(3x)}{8} - \frac{3\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

GIAC/XCAS [A] time = 0.199836, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

3.108 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi [A] time = 0.0149115, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi in Sympy [A] time = 1.04208, size = 10, normalized size = 0.67

$$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] sin(x)/2 + sin(7*x)/14
```

Mathematica [A] time = 0.00982412, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2} + \frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] time = 0.078, size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3*x)*cos(4*x),x)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Maxima [A] time = 1.40762, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Fricas [A] time = 0.22198, size = 32, normalized size = 2.13

$$\frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [A] time = 0.716822, size = 26, normalized size = 1.73

$$-\frac{3\sin(3x)\cos(4x)}{7} + \frac{4\sin(4x)\cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

GIAC/XCAS [A] time = 0.204454, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

3.109 $\int \sin(3x)\sin(6x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rubi [A] time = 0.0160628, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[3*x]*Sin[6*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rubi in Sympy [A] time = 1.02061, size = 12, normalized size = 0.71

$$\frac{\sin(3x)}{6} - \frac{\sin(9x)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(3*x)*sin(6*x),x)
```

```
[Out] sin(3*x)/6 - sin(9*x)/18
```

Mathematica [A] time = 0.0112272, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x) - \frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[3*x]*Sin[6*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Maple [A] time = 0.018, size = 9, normalized size = 0.5

$$\frac{2\,(\sin(3x))^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(3*x)*sin(6*x),x)
```

```
[Out] 2/9*sin(3*x)^3
```

Maxima [A] time = 1.34152, size = 18, normalized size = 1.06

$$-\frac{1}{18}\sin(9x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(6*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

Fricas [A] time = 0.215392, size = 19, normalized size = 1.12

$$-\frac{2}{9}\left(\cos(3x)^2-1\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(6*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -2/9*(cos(3*x)^2 - 1)*sin(3*x)
```

Sympy [A] time = 0.772494, size = 24, normalized size = 1.41

$$-\frac{2\sin(3x)\cos(6x)}{9}+\frac{\sin(6x)\cos(3x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(6*x),x)
```

```
[Out] -2*sin(3*x)*cos(6*x)/9 + sin(6*x)*cos(3*x)/9
```

GIAC/XCAS [A] time = 0.202366, size = 18, normalized size = 1.06

$$-\frac{1}{18}\sin(9x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(6*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

3.110 $\int \cos^5(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{6}\cos^6(x)$$

```
[Out] -Cos[x]^6/6
```

Rubi [A] time = 0.0186179, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^5*Sin[x],x]
```

```
[Out] -Cos[x]^6/6
```

Rubi in Sympy [A] time = 1.12206, size = 7, normalized size = 0.88

$$-\frac{\cos^6(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**5*sin(x),x)
```

```
[Out] -cos(x)**6/6
```

Mathematica [A] time = 0.00192406, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^5*Sin[x],x]
```

```
[Out] -Cos[x]^6/6
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$-\frac{(\cos(x))^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^5*sin(x),x)
```

```
[Out] -1/6*cos(x)^6
```

Maxima [A] time = 1.34717, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(x)^6
```

Fricas [A] time = 0.219202, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x),x, algorithm="fricas")
```

```
[Out] -1/6*cos(x)^6
```

Sympy [A] time = 0.038085, size = 7, normalized size = 0.88

$$-\frac{\cos^6(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**5*sin(x),x)
```

```
[Out] -cos(x)**6/6
```

GIAC/XCAS [A] time = 0.197663, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*cos(x)^6
```

3.111 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] time = 0.0482573, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi in Sympy [A] time = 2.2711, size = 27, normalized size = 0.9

$$\frac{x}{4}+\frac{\sin(2x)}{4}+\frac{\sin(3x)\cos(3x)}{12}+\frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] x/4 + sin(2*x)/4 + sin(3*x)*cos(3*x)/12 + sin(4*x)/8
```

Mathematica [A] time = 0.0137906, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A] time = 1.38854, size = 30, normalized size = 1.

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 0.25978, size = 34, normalized size = 1.13

$$\frac{1}{12}\left(16\cos(x)^5 - 10\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [A] time = 22.0666, size = 114, normalized size = 3.8

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} - \frac{\sin(x)\cos(2x)\cos(3x)}{24} - \frac{\sin(2x)\cos(x)\cos(3x)}{6} + \frac{3\sin(3x)\cos(x)\cos(2x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*
sin(2*x)*sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 - sin(x
)*cos(2*x)*cos(3*x)/24 - sin(2*x)*cos(x)*cos(3*x)/6 + 3*sin(3*x)*
cos(x)*cos(2*x)/8
```

GIAC/XCAS [A] time = 0.204311, size = 30, normalized size = 1.

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(2*x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.112 $\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx$

Optimal. Leaf size=5

$$\sin(x)\cos(x)$$

```
[Out] Cos[x]*Sin[x]
```

Rubi [A] time = 0.0314083, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Cos[x]*Sin[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\tan^2(x)+1\right)\cos^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-tan(x)**2)/sec(x)**2,x)
```

```
[Out] Integral((-tan(x)**2 + 1)*cos(x)**2, x)
```

Mathematica [A] time = 0.00306608, size = 8, normalized size = 1.6

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Sin[2*x]/2
```

Maple [A] time = 0.023, size = 6, normalized size = 1.2

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-tan(x)^2)/sec(x)^2,x)
```

```
[Out] cos(x)*sin(x)
```

Maxima [A] time = 1.32288, size = 15, normalized size = 3.

$$\frac{\tan(x)}{\tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 - 1)/sec(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 0.252341, size = 7, normalized size = 1.4

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 - 1)/sec(x)^2,x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

Sympy [A] time = 0.768139, size = 7, normalized size = 1.4

$$\frac{\tan(x)}{\sec^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-tan(x)**2)/sec(x)**2,x)
```

```
[Out] tan(x)/sec(x)**2
```

GIAC/XCAS [A] time = 0.19876, size = 12, normalized size = 2.4

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(tan(x)^2 - 1)/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/(1/tan(x) + tan(x))
```

3.113 $\int \csc(2x)(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rubi [A] time = 0.0733967, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)+\cos(x)}{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] Integral((sin(x) + cos(x))/sin(2*x), x)
```

Mathematica [B] time = 0.014988, size = 61, normalized size = 4.07

$$\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x/2]]/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 +
 Log[Cos[x/2] + Sin[x/2]]/2
```

Maple [A] time = 0.063, size = 20, normalized size = 1.3

$$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(csc(x)-cot(x))
```

Maxima [A] time = 1.65356, size = 93, normalized size = 6.2

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$
$$+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + sin(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 1/4*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A] time = 0.232271, size = 47, normalized size = 3.13

$$-\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + sin(x))/sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-
1/2*(cos(x) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

Sympy [A] time = 0.875583, size = 32, normalized size = 2.13

$$-\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4}+\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(
cos(x) + 1)/4
```

GIAC/XCAS [A] time = 0.222145, size = 39, normalized size = 2.6

$$\frac{1}{2}\ln\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{2}\ln\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\ln\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + sin(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(tan(1/2*x) + 1)) - 1/2*ln(abs(tan(1/2*x) - 1)) + 1/2*l
n(abs(tan(1/2*x)))
```

3.114 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi [A] time = 0.0219364, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\log(\cos(x)) + \int^{\cos(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2*tan(x),x)
```

```
[Out] -log(cos(x)) + Integral(x, (x, cos(x)))
```

Mathematica [A] time = 0.00692507, size = 14, normalized size = 1.

$$\frac{1}{4}\cos(2x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[2*x]/4 - Log[Cos[x]]
```

Maple [A] time = 0.014, size = 13, normalized size = 0.9

$$-\frac{(\sin(x))^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2*tan(x),x)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A] time = 1.33534, size = 22, normalized size = 1.57

$$-\frac{1}{2}\,\sin(x)^2-\frac{1}{2}\,\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 0.264833, size = 19, normalized size = 1.36

$$\frac{1}{2}\,\cos(x)^2-\log\left(-\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.07138, size = 10, normalized size = 0.71

$$-\log\left(\cos(x)\right)+\frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2*tan(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

GIAC/XCAS [A] time = 0.201228, size = 24, normalized size = 1.71

$$-\frac{1}{2}\,\sin(x)^2-\frac{1}{2}\,\ln\left(-\sin(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*sin(x)^2 - 1/2*ln(-sin(x)^2 + 1)
```

3.115 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi [A] time = 0.0539437, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(x)}{\tan^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(cos(x)**2/tan(x)**3, x)
```

Mathematica [A] time = 0.00876849, size = 22, normalized size = 1.

$$-\frac{1}{4}\cos(2x) - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Cos[2*x]/4 - Csc[x]^2/2 - 2*Log[Sin[x]]
```

Maple [A] time = 0.02, size = 29, normalized size = 1.3

$$-\frac{(\cos(x))^6}{2\,(\sin(x))^2} - \frac{(\cos(x))^4}{2} - (\cos(x))^2 - 2\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*cot(x)^3,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A] time = 1.3562, size = 27, normalized size = 1.23

$$\frac{1}{2}\,\sin(x)^2 - \frac{1}{2\,\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [A] time = 0.23225, size = 50, normalized size = 2.27

$$-\frac{2\,\cos(x)^4 - 3\,\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\,\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x))
- 1)/(cos(x)^2 - 1)
```

Sympy [A] time = 0.099241, size = 20, normalized size = 0.91

$$-2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*cot(x)**3,x)
```

```
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

GIAC/XCAS [A] time = 0.205748, size = 49, normalized size = 2.23

$$-\frac{1}{2}\,\cos(x)^2 + \frac{2\,\cos(x)^2 - 1}{2\left(\cos(x)^2 - 1\right)} - \ln\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2*(2*cos(x)^2 - 1)/(cos(x)^2 - 1) - ln(-cos(x)^
2 + 1)
```

3.116 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.0188348, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x),x)
```

```
[Out] Integral(tan(x)/cos(x)**3, x)
```

Mathematica [A] time = 0.00183606, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 1.34065, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 0.304967, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.051899, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

GIAC/XCAS [A] time = 0.20016, size = 8, normalized size = 1.

$$\frac{1}{3\ \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.117 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0376297, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] Integral(tan(x)**3/cos(x)**3, x)
```

Mathematica [A] time = 0.0115383, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [B] time = 0., size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5\,(\cos(x))^5}+\frac{(\sin(x))^4}{15\,(\cos(x))^3}-\frac{(\sin(x))^4}{15\,\cos(x)}-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)
-1/15*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.35367, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 0.214097, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.108544, size = 15, normalized size = 0.88

$$-\frac{5\cos^2(x)-3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

GIAC/XCAS [A] time = 0.200934, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.118 $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rubi [A] time = 0.0164001, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rubi in Sympy [A] time = 1.51271, size = 15, normalized size = 0.6

$$-\operatorname{asin}\left(\frac{x}{3}\right)-\frac{\sqrt{-x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+9)**(1/2)/x**2,x)
```

```
[Out] -asin(x/3) - sqrt(-x**2 + 9)/x
```

Mathematica [A] time = 0.0141205, size = 25, normalized size = 1.

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Maple [A] time = 0.007, size = 34, normalized size = 1.4

$$-\frac{1}{9x}\left(-x^2+9\right)^{\frac{3}{2}}-\frac{x}{9}\sqrt{-x^2+9}-\arcsin\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(-x^2+9)^(3/2)-1/9*x*(-x^2+9)^(1/2)-arcsin(1/3*x)
```

Maxima [A] time = 1.54754, size = 28, normalized size = 1.12

$$-\frac{\sqrt{-x^2+9}}{x} - \arcsin\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 9)/x - arcsin(1/3*x)
```

Fricas [A] time = 0.243756, size = 90, normalized size = 3.6

$$\frac{x^2 + 2\left(\sqrt{-x^2+9}x - 3\,x\right)\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right) + 3\sqrt{-x^2+9} - 9}{\sqrt{-x^2+9}x - 3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x^2,x, algorithm="fricas")
```

```
[Out] (x^2 + 2*(sqrt(-x^2 + 9)*x - 3*x)*arctan((sqrt(-x^2 + 9) - 3)/x)
+ 3*sqrt(-x^2 + 9) - 9)/(sqrt(-x^2 + 9)*x - 3*x)
```

Sympy [A] time = 0.338443, size = 15, normalized size = 0.6

$$-\operatorname{asin}\left(\frac{x}{3}\right) - \frac{\sqrt{-x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+9)**(1/2)/x**2,x)
```

```
[Out] -asin(x/3) - sqrt(-x**2 + 9)/x
```

GIAC/XCAS [A] time = 0.207845, size = 53, normalized size = 2.12

$$\frac{x}{2\left(\sqrt{-x^2+9}-3\right)} - \frac{\sqrt{-x^2+9}-3}{2\,x} - \arcsin\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 9) - 3) - 1/2*(sqrt(-x^2 + 9) - 3)/x - arcsin(
1/3*x)
```

3.119 $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^2+4}}{4x}$$

```
[Out] -Sqrt[4 + x^2]/(4*x)
```

Rubi [A] time = 0.0115661, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[4 + x^2]),x]
```

```
[Out] -Sqrt[4 + x^2]/(4*x)
```

Rubi in Sympy [A] time = 1.27352, size = 12, normalized size = 0.75

$$-\frac{\sqrt{x^2+4}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(x**2+4)**(1/2),x)
```

```
[Out] -sqrt(x**2 + 4)/(4*x)
```

Mathematica [A] time = 0.00861714, size = 16, normalized size = 1.

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[4 + x^2]),x]
```

```
[Out] -Sqrt[4 + x^2]/(4*x)
```

Maple [A] time = 0.005, size = 13, normalized size = 0.8

$$-\frac{1}{4\,x}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(x^2+4)^(1/2),x)
```

```
[Out] -1/4*(x^2+4)^(1/2)/x
```

Maxima [A] time = 1.49468, size = 16, normalized size = 1.

$$-\frac{\sqrt{x^2+4}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 4)*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^2 + 4)/x
```

Fricas [A] time = 0.20664, size = 22, normalized size = 1.38

$$\frac{1}{x^2-\sqrt{x^2+4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 4)*x^2),x, algorithm="fricas")
```

```
[Out] 1/(x^2 - sqrt(x^2 + 4)*x)
```

Sympy [A] time = 1.23042, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{4}{x^2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(x**2+4)**(1/2),x)
```

```
[Out] -sqrt(1 + 4/x**2)/4
```

GIAC/XCAS [A] time = 0.203059, size = 26, normalized size = 1.62

$$\frac{2}{\left(x-\sqrt{x^2+4}\right)^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 4)*x^2),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 4))^2 - 4)
```

3.120 $\int \frac{x}{\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=9

$$\sqrt{x^2+4}$$

```
[Out] Sqrt[4 + x^2]
```

Rubi [A] time = 0.00466439, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Rubi in Sympy [A] time = 0.761721, size = 7, normalized size = 0.78

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+4)**(1/2),x)
```

```
[Out] sqrt(x**2 + 4)
```

Mathematica [A] time = 0.00133305, size = 9, normalized size = 1.

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Maple [A] time = 0.004, size = 8, normalized size = 0.9

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+4)^(1/2),x)
```

```
[Out] (x^2+4)^(1/2)
```

Maxima [A] time = 1.34601, size = 9, normalized size = 1.

$$\sqrt{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 4),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 4)
```

Fricas [A] time = 0.20864, size = 41, normalized size = 4.56

$$-\frac{x^2 - \sqrt{x^2 + 4}x + 4}{x - \sqrt{x^2 + 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 4),x, algorithm="fricas")
```

```
[Out] -(x^2 - sqrt(x^2 + 4)*x + 4)/(x - sqrt(x^2 + 4))
```

Sympy [A] time = 0.152024, size = 7, normalized size = 0.78

$$\sqrt{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+4)**(1/2),x)
```

```
[Out] sqrt(x**2 + 4)
```

GIAC/XCAS [A] time = 0.200531, size = 9, normalized size = 1.

$$\sqrt{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 4),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 4)
```

3.121 $\int \frac{1}{\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rubi [A] time = 0.00715674, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rubi in Sympy [A] time = 0.927429, size = 12, normalized size = 0.75

$$\operatorname{atanh}\left(\frac{x}{\sqrt{-a^2+x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-a**2+x**2)**(1/2),x)
```

```
[Out] atanh(x/sqrt(-a**2 + x**2))
```

Mathematica [B] time = 0.00492518, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[-a^2 + x^2]]/2 + Log[1 + x/Sqrt[-a^2 + x^2]]/2
```

Maple [A] time = 0.004, size = 15, normalized size = 0.9

$$\ln\left(x+\sqrt{-a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-a^2+x^2)^(1/2),x)
```

```
[Out] ln(x+(-a^2+x^2)^(1/2))
```

Maxima [A] time = 1.34544, size = 24, normalized size = 1.5

$$\log\left(2\,x + 2\,\sqrt{-a^2 + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a^2 + x^2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(-a^2 + x^2))
```

Fricas [A] time = 0.207408, size = 24, normalized size = 1.5

$$-\log\left(-x + \sqrt{-a^2 + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a^2 + x^2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(-a^2 + x^2))
```

Sympy [A] time = 1.76401, size = 19, normalized size = 1.19

$$\begin{cases} \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ -i\,\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-a**2+x**2)**(1/2),x)
```

```
[Out] Piecewise((acosh(x/a), Abs(x**2/a**2) > 1), (-I*asin(x/a), True))
```

GIAC/XCAS [A] time = 0.231935, size = 26, normalized size = 1.62

$$-\ln\left(\left|-x + \sqrt{-a^2 + x^2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-a^2 + x^2),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(-a^2 + x^2)))
```

3.122 $\int \frac{x^3}{(9+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{16}\sqrt{4x^2+9}+\frac{9}{16\sqrt{4x^2+9}}$$

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rubi [A] time = 0.0356915, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{16}\sqrt{4x^2+9}+\frac{9}{16\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rubi in Sympy [A] time = 2.57013, size = 24, normalized size = 0.77

$$\frac{\sqrt{4x^2+9}}{16}+\frac{9}{16\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(4*x**2+9)**(3/2),x)
```

```
[Out] sqrt(4*x**2 + 9)/16 + 9/(16*sqrt(4*x**2 + 9))
```

Mathematica [A] time = 0.00993003, size = 22, normalized size = 0.71

$$\frac{2x^2+9}{8\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] (9 + 2*x^2)/(8*Sqrt[9 + 4*x^2])
```

Maple [A] time = 0.007, size = 19, normalized size = 0.6

$$\frac{2\,x^2+9}{8}\frac{1}{\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(4*x^2+9)^(3/2),x)
```

```
[Out] 1/8*(2*x^2+9)/(4*x^2+9)^(1/2)
```

Maxima [A] time = 1.51263, size = 35, normalized size = 1.13

$$\frac{x^2}{4\sqrt{4x^2+9}} + \frac{9}{8\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(4*x^2 + 9)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/sqrt(4*x^2 + 9) + 9/8/sqrt(4*x^2 + 9)
```

Fricas [A] time = 0.208885, size = 85, normalized size = 2.74

$$-\frac{16x^4 + 90x^2 - 4\left(2x^3 + 9x\right)\sqrt{4x^2+9} + 81}{8\left(16x^3 - (8x^2+9)\sqrt{4x^2+9} + 36x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(4*x^2 + 9)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(16*x^4 + 90*x^2 - 4*(2*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81)/(16
*x^3 - (8*x^2 + 9)*sqrt(4*x^2 + 9) + 36*x)
```

Sympy [A] time = 1.01573, size = 27, normalized size = 0.87

$$\frac{x^2}{4\sqrt{4x^2+9}} + \frac{9}{8\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(4*x**2+9)**(3/2),x)
```

```
[Out] x**2/(4*sqrt(4*x**2 + 9)) + 9/(8*sqrt(4*x**2 + 9))
```

GIAC/XCAS [A] time = 0.199521, size = 31, normalized size = 1.

$$\frac{1}{16}\sqrt{4x^2+9} + \frac{9}{16\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(4*x^2 + 9)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(4*x^2 + 9) + 9/16/sqrt(4*x^2 + 9)
```

3.123 $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$

Optimal. Leaf size=27

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rubi [A] time = 0.0264479, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rubi in Sympy [A] time = 1.91482, size = 36, normalized size = 1.33

$$-\sqrt{-x^2-2x+3}-\operatorname{atan}\left(-\frac{-2x-2}{2\sqrt{-x^2-2x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2-2*x+3)**(1/2),x)
```

```
[Out] -sqrt(-x**2 - 2*x + 3) - atan(-(-2*x - 2)/(2*sqrt(-x**2 - 2*x + 3
)))
```

Mathematica [A] time = 0.0166762, size = 27, normalized size = 1.

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Maple [A] time = 0.007, size = 24, normalized size = 0.9

$$-\arcsin\left(\frac{1}{2}+\frac{x}{2}\right)-\sqrt{-x^2-2\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2-2*x+3)^(1/2),x)
```

```
[Out] -arcsin(1/2+1/2*x)-(-x^2-2*x+3)^(1/2)
```

Maxima [A] time = 1.54533, size = 28, normalized size = 1.04

$$-\sqrt{-x^2-2\,x+3}+\arcsin\left(-\frac{1}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 - 2*x + 3),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) + arcsin(-1/2*x - 1/2)
```

Fricas [A] time = 0.216014, size = 46, normalized size = 1.7

$$-\sqrt{-x^2-2\,x+3}-\arctan\left(\frac{x+1}{\sqrt{-x^2-2\,x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 - 2*x + 3),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) - arctan((x + 1)/sqrt(-x^2 - 2*x + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x-1)\,(x+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2-2*x+3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(x - 1)*(x + 3)), x)
```

GIAC/XCAS [A] time = 0.206193, size = 31, normalized size = 1.15

$$-\sqrt{-x^2-2\,x+3}-\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 - 2*x + 3),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) - arcsin(1/2*x + 1/2)
```

3.124 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{1-x^2}}{x}$$

```
[Out] -(Sqrt[1 - x^2]/x)
```

Rubi [A] time = 0.0141113, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

Rubi in Sympy [A] time = 1.39447, size = 10, normalized size = 0.62

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 1)/x
```

Mathematica [A] time = 0.00914447, size = 16, normalized size = 1.

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

Maple [A] time = 0.007, size = 20, normalized size = 1.3

$$\frac{(-1+x)(1+x)}{x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/x*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.48379, size = 19, normalized size = 1.19

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*x^2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/x
```

Fricas [A] time = 0.210296, size = 43, normalized size = 2.69

$$\frac{x^2+\sqrt{-x^2+1}-1}{\sqrt{-x^2+1}x-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + sqrt(-x^2 + 1) - 1)/(sqrt(-x^2 + 1)*x - x)
```

Sympy [A] time = 1.25761, size = 27, normalized size = 1.69

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } \left|x^2\right| > 1 \\ -\frac{\sqrt{-x^2+1}}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(-x**2 + 1)
/x, True))
```

GIAC/XCAS [A] time = 0.208064, size = 45, normalized size = 2.81

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\sqrt{-x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x
```

3.125 $\int x^3\sqrt{4-x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rubi [A] time = 0.0348676, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[4 - x^2],x]
```

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rubi in Sympy [A] time = 2.50655, size = 20, normalized size = 0.65

$$\frac{\left(-x^2+4\right)^{\frac{5}{2}}}{5}-\frac{4\left(-x^2+4\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-x**2+4)**(1/2),x)
```

```
[Out] (-x**2 + 4)**(5/2)/5 - 4*(-x**2 + 4)**(3/2)/3
```

Mathematica [A] time = 0.0120726, size = 22, normalized size = 0.71

$$-\frac{1}{15}\left(4-x^2\right)^{3/2}\left(3x^2+8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[4 - x^2],x]
```

```
[Out] -((4 - x^2)^(3/2)*(8 + 3*x^2))/15
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$\frac{(-2+x)(2+x)\left(3x^2+8\right)}{15}\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-x^2+4)^(1/2),x)
```

```
[Out] 1/15*(-2+x)*(2+x)*(3*x^2+8)*(-x^2+4)^(1/2)
```

Maxima [A] time = 1.50529, size = 35, normalized size = 1.13

$$-\frac{1}{5}\left(-x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{15}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x^3,x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 4)^(3/2)*x^2 - 8/15*(-x^2 + 4)^(3/2)
```

Fricas [A] time = 0.210511, size = 115, normalized size = 3.71

$$\frac{3\,x^{10}-160\,x^8+1520\,x^6-3840\,x^4+10\left(3\,x^8-52\,x^6+192\,x^4\right)\sqrt{-x^2+4}}{15\left(10\,x^4-160\,x^2-\left(x^4-48\,x^2+256\right)\sqrt{-x^2+4}+512\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^10 - 160*x^8 + 1520*x^6 - 3840*x^4 + 10*(3*x^8 - 52*x^6
 + 192*x^4)*sqrt(-x^2 + 4))/(10*x^4 - 160*x^2 - (x^4 - 48*x^2 + 2
56)*sqrt(-x^2 + 4) + 512)
```

Sympy [A] time = 0.877572, size = 39, normalized size = 1.26

$$\frac{x^4\sqrt{-x^2+4}}{5}-\frac{4x^2\sqrt{-x^2+4}}{15}-\frac{32\sqrt{-x^2+4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-x**2+4)**(1/2),x)
```

```
[Out] x**4*sqrt(-x**2 + 4)/5 - 4*x**2*sqrt(-x**2 + 4)/15 - 32*sqrt(-x**
2 + 4)/15
```

GIAC/XCAS [A] time = 0.19978, size = 41, normalized size = 1.32

$$\frac{1}{5}\left(x^2-4\right)^2\sqrt{-x^2+4}-\frac{4}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x^3,x, algorithm="giac")
```

```
[Out] 1/5*(x^2 - 4)^2*sqrt(-x^2 + 4) - 4/3*(-x^2 + 4)^(3/2)
```

3.126 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2]
```

Rubi [A] time = 0.00643198, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2]
```

Rubi in Sympy [A] time = 0.888419, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**(1/2),x)

[Out] -sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.0019103, size = 13, normalized size = 1.

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2]
```

Maple [A] time = 0.004, size = 17, normalized size = 1.3

$$(-1+x)(1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^(1/2),x)

[Out] (-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.35661, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

Fricas [A] time = 0.210514, size = 23, normalized size = 1.77

$$\frac{x^2}{\sqrt{-x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] x^2/(sqrt(-x^2 + 1) - 1)
```

Sympy [A] time = 0.151225, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.200192, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

3.127 $\int x\sqrt{4-x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] -(4 - x^2)^(3/2)/3
```

Rubi [A] time = 0.00602848, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[4 - x^2],x]
```

```
[Out] -(4 - x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 0.898091, size = 10, normalized size = 0.67

$$-\frac{\left(-x^2+4\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(-x**2+4)**(1/2),x)
```

```
[Out] -(-x**2 + 4)**(3/2)/3
```

Mathematica [A] time = 0.00255922, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[4 - x^2],x]
```

```
[Out] -(4 - x^2)^(3/2)/3
```

Maple [A] time = 0.004, size = 18, normalized size = 1.2

$$\frac{(-2+x)(2+x)}{3}\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(-x^2+4)^(1/2),x)
```

```
[Out] 1/3*(-2+x)*(2+x)*(-x^2+4)^(1/2)
```

Maxima [A] time = 1.33639, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x,x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 4)^(3/2)
```

Fricas [A] time = 0.20129, size = 82, normalized size = 5.47

$$\frac{x^6-24\,x^4+96\,x^2+6\left(x^4-8\,x^2\right)\sqrt{-x^2+4}}{3\left(6\,x^2-(x^2-16)\sqrt{-x^2+4}-32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x,x, algorithm="fricas")
```

```
[Out] 1/3*(x^6 - 24*x^4 + 96*x^2 + 6*(x^4 - 8*x^2)*sqrt(-x^2 + 4))/(6*x
^2 - (x^2 - 16)*sqrt(-x^2 + 4) - 32)
```

Sympy [A] time = 0.231288, size = 24, normalized size = 1.6

$$\frac{x^2\sqrt{-x^2+4}}{3}-\frac{4\sqrt{-x^2+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(-x**2+4)**(1/2),x)
```

```
[Out] x**2*sqrt(-x**2 + 4)/3 - 4*sqrt(-x**2 + 4)/3
```

GIAC/XCAS [A] time = 0.200366, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)*x,x, algorithm="giac")
```

```
[Out] -1/3*(-x^2 + 4)^(3/2)
```

3.128 $\int \sqrt{1-4x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sqrt{1-4x^2}x+\frac{1}{4}\sin^{-1}(2x)$$

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rubi [A] time = 0.00996907, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2}\sqrt{1-4x^2}x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rubi in Sympy [A] time = 0.618543, size = 19, normalized size = 0.76

$$\frac{x\sqrt{-4x^2+1}}{2}+\frac{\operatorname{asin}(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 + 1)/2 + asin(2*x)/4
```

Mathematica [A] time = 0.0108913, size = 25, normalized size = 1.

$$\frac{1}{2}\sqrt{1-4x^2}x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{\arcsin(2x)}{4}+\frac{x}{2}\sqrt{-4x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+1)^(1/2),x)
```

```
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

Maxima [A] time = 1.49218, size = 26, normalized size = 1.04

$$\frac{1}{2}\sqrt{-4x^2+1}x+\frac{1}{4}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

Fricas [A] time = 0.208477, size = 112, normalized size = 4.48

$$-\frac{4x^3+\left(2x^2+\sqrt{-4x^2+1}-1\right)\arctan\left(\frac{\sqrt{-4x^2+1}-1}{2x}\right)-\left(2x^3-x\right)\sqrt{-4x^2+1}-x}{2\left(2x^2+\sqrt{-4x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x^3 + (2*x^2 + sqrt(-4*x^2 + 1) - 1)*arctan(1/2*(sqrt(-4*
x^2 + 1) - 1)/x) - (2*x^3 - x)*sqrt(-4*x^2 + 1) - x)/(2*x^2 + sqr
t(-4*x^2 + 1) - 1)
```

Sympy [A] time = 0.243862, size = 19, normalized size = 0.76

$$\frac{x\sqrt{-4x^2+1}}{2}+\frac{\operatorname{asin}(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-4*x**2 + 1)/2 + asin(2*x)/4
```

GIAC/XCAS [A] time = 0.206941, size = 26, normalized size = 1.04

$$\frac{1}{2}\sqrt{-4x^2+1}x+\frac{1}{4}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

3.129 $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

```
[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0270034, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[4 + x^2],x]
```

```
[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 1.87661, size = 19, normalized size = 0.76

$$\frac{\left(x^2+4\right)^{\frac{3}{2}}}{3}-4\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**2+4)**(1/2),x)
```

```
[Out] (x**2 + 4)**(3/2)/3 - 4*sqrt(x**2 + 4)
```

Mathematica [A] time = 0.00602464, size = 18, normalized size = 0.72

$$\frac{1}{3}\left(x^2-8\right)\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[4 + x^2],x]
```

```
[Out] ((-8 + x^2)*Sqrt[4 + x^2])/3
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$\frac{x^2-8}{3}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^2+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+4)^(1/2)*(x^2-8)
```

Maxima [A] time = 1.49785, size = 30, normalized size = 1.2

$$\frac{1}{3}\sqrt{x^2+4}x^2 - \frac{8}{3}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + 4)*x^2 - 8/3*sqrt(x^2 + 4)
```

Fricas [A] time = 0.222778, size = 82, normalized size = 3.28

$$-\frac{x^6 - 3\,x^4 - 36\,x^2 - \left(x^5 - 5\,x^3 - 24\,x\right)\sqrt{x^2+4} - 32}{3\left(x^3 - \sqrt{x^2+4}(x^2+1) + 3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + 4),x, algorithm="fricas")
```

```
[Out] -1/3*(x^6 - 3*x^4 - 36*x^2 - (x^5 - 5*x^3 - 24*x)*sqrt(x^2 + 4) -
 32)/(x^3 - sqrt(x^2 + 4)*(x^2 + 1) + 3*x)
```

Sympy [A] time = 0.489138, size = 24, normalized size = 0.96

$$\frac{x^2\sqrt{x^2+4}}{3} - \frac{8\sqrt{x^2+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**2+4)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 4)/3 - 8*sqrt(x**2 + 4)/3
```

GIAC/XCAS [A] time = 0.197753, size = 26, normalized size = 1.04

$$\frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}} - 4\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 4)^(3/2) - 4*sqrt(x^2 + 4)
```

3.130 $\int \frac{1}{\sqrt{9+x^2}}\,dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcSinh[x/3]
```

Rubi [A] time = 0.00408074, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

Rubi in Sympy [A] time = 0.497259, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+9)**(1/2),x)
```

```
[Out] asinh(x/3)
```

Mathematica [A] time = 0.00556258, size = 6, normalized size = 1.

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

Maple [A] time = 0.004, size = 5, normalized size = 0.8

$$Arcsinh\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+9)^(1/2),x)
```

```
[Out] arcsinh(1/3*x)
```

Maxima [A] time = 1.48957, size = 5, normalized size = 0.83

$$\operatorname{arsinh}\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 9),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*x)
```

Fricas [A] time = 0.199918, size = 19, normalized size = 3.17

$$-\log\left(-x + \sqrt{x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 9),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 9))
```

Sympy [A] time = 0.158328, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+9)**(1/2),x)
```

```
[Out] asinh(x/3)
```

GIAC/XCAS [A] time = 0.210534, size = 19, normalized size = 3.17

$$-\ln\left(-x + \sqrt{x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 9),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 9))
```

3.131 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.00627967, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi in Sympy [A] time = 0.558377, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Mathematica [A] time = 0.00793974, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{Arcsinh(x)}{2}+\frac{x}{2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Maxima [A] time = 1.51876, size = 20, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A] time = 0.198284, size = 105, normalized size = 5.

$$-\frac{2x^4+2x^2+\left(2x^2-2\sqrt{x^2+1}x+1\right)\log\left(-x+\sqrt{x^2+1}\right)-\left(2x^3+x\right)\sqrt{x^2+1}}{2\left(2x^2-2\sqrt{x^2+1}x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 2*x^2 + (2*x^2 - 2*sqrt(x^2 + 1)*x + 1)*log(-x + sq
rt(x^2 + 1)) - (2*x^3 + x)*sqrt(x^2 + 1))/(2*x^2 - 2*sqrt(x^2 + 1
)*x + 1)
```

Sympy [A] time = 0.242164, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

GIAC/XCAS [A] time = 0.20612, size = 34, normalized size = 1.62

$$\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*ln(-x + sqrt(x^2 + 1))
```

3.132 $\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rubi [A] time = 0.0339582, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rubi in Sympy [A] time = 2.16624, size = 26, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{x^2-16}}{4}\right)}{128}+\frac{\sqrt{x^2-16}}{32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(x**2-16)**(1/2),x)
```

```
[Out] atan(sqrt(x**2 - 16)/4)/128 + sqrt(x**2 - 16)/(32*x**2)
```

Mathematica [A] time = 0.0206181, size = 33, normalized size = 0.94

$$\frac{\sqrt{x^2-16}}{32x^2}-\frac{1}{128}\tan^{-1}\left(\frac{4}{\sqrt{x^2-16}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] Sqrt[-16 + x^2]/(32*x^2) - ArcTan[4/Sqrt[-16 + x^2]]/128
```

Maple [A] time = 0.01, size = 26, normalized size = 0.7

$$\frac{1}{32\,x^2}\sqrt{x^2-16}-\frac{1}{128}\arctan\left(4\,\frac{1}{\sqrt{x^2-16}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(x^2-16)^(1/2),x)
```

```
[Out] 1/32*(x^2-16)^(1/2)/x^2-1/128*arctan(4/(x^2-16)^(1/2))
```

Maxima [A] time = 1.50128, size = 30, normalized size = 0.86

$$\frac{\sqrt{x^2-16}}{32\,x^2}-\frac{1}{128}\,\arcsin\left(\frac{4}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 16)*x^3),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 - 1/128*arcsin(4/abs(x))
```

Fricas [A] time = 0.20376, size = 115, normalized size = 3.29

$$-\frac{2\,x^3-\left(x^4-\sqrt{x^2-16}x^3-8\,x^2\right)\arctan\left(-\frac{1}{4}\,x+\frac{1}{4}\,\sqrt{x^2-16}\right)-2\left(x^2-8\right)\sqrt{x^2-16}-32\,x}{64\left(x^4-\sqrt{x^2-16}x^3-8\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 16)*x^3),x, algorithm="fricas")
```

```
[Out] -1/64*(2*x^3 - (x^4 - sqrt(x^2 - 16)*x^3 - 8*x^2)*arctan(-1/4*x +
 1/4*sqrt(x^2 - 16)) - 2*(x^2 - 8)*sqrt(x^2 - 16) - 32*x)/(x^4 -
sqrt(x^2 - 16)*x^3 - 8*x^2)
```

Sympy [A] time = 4.0036, size = 68, normalized size = 1.94

$$\begin{cases}\frac{i\,\operatorname{acosh}\left(\frac{4}{x}\right)}{128}+\frac{i\sqrt{-1+\frac{16}{x^2}}}{32x} & \text{for } 16\left|\frac{1}{x^2}\right|>1\\ -\frac{\operatorname{asin}\left(\frac{4}{x}\right)}{128}+\frac{1}{32x\sqrt{1-\frac{16}{x^2}}}-\frac{1}{2x^3\sqrt{1-\frac{16}{x^2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(x**2-16)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(4/x)/128 + I*sqrt(-1 + 16/x**2)/(32*x), 16*Abs
(x**(-2)) > 1), (-asin(4/x)/128 + 1/(32*x*sqrt(1 - 16/x**2)) - 1/
(2*x**3*sqrt(1 - 16/x**2)), True))
```

GIAC/XCAS [A] time = 0.207506, size = 34, normalized size = 0.97

$$\frac{\sqrt{x^2-16}}{32\,x^2}+\frac{1}{128}\,\arctan\left(\frac{1}{4}\,\sqrt{x^2-16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 16)*x^3),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 + 1/128*arctan(1/4*sqrt(x^2 - 16))
```

3.133 $\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rubi [A] time = 0.0160103, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rubi in Sympy [A] time = 1.56211, size = 17, normalized size = 0.74

$$\frac{\left(-a^2+x^2\right)^{\frac{3}{2}}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-a**2+x**2)**(1/2)/x**4,x)
```

```
[Out] (-a**2 + x**2)**(3/2)/(3*a**2*x**3)
```

Mathematica [A] time = 0.0118861, size = 23, normalized size = 1.

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Maple [A] time = 0.007, size = 28, normalized size = 1.2

$$-\frac{(a+x)(a-x)}{3\,a^2x^3}\sqrt{-a^2+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-a^2+x^2)^(1/2)/x^4,x)
```

```
[Out] -1/3/x^3*(a+x)*(a-x)/a^2*(-a^2+x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-a^2 + x^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.200202, size = 115, normalized size = 5.

$$-\frac{a^4 - 6\,a^2x^2 + 6\,x^4 + 3\left(a^2x - 2\,x^3\right)\sqrt{-a^2 + x^2}}{3\left(3\,a^2x^4 - 4\,x^6 - (a^2x^3 - 4\,x^5)\sqrt{-a^2 + x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-a^2 + x^2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(a^4 - 6*a^2*x^2 + 6*x^4 + 3*(a^2*x - 2*x^3)*sqrt(-a^2 + x^2
))/(3*a^2*x^4 - 4*x^6 - (a^2*x^3 - 4*x^5)*sqrt(-a^2 + x^2))
```

Sympy [A] time = 1.03356, size = 76, normalized size = 3.3

$$\begin{cases} -\frac{i\sqrt{\frac{a^2}{x^2}-1}}{3x^2} + \frac{i\sqrt{\frac{a^2}{x^2}-1}}{3a^2} & \text{for } \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{\sqrt{-\frac{a^2}{x^2}+1}}{3x^2} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{3a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-a**2+x**2)**(1/2)/x**4,x)
```

```
[Out] Piecewise((-I*sqrt(a**2/x**2 - 1)/(3*x**2) + I*sqrt(a**2/x**2 - 1
)/(3*a**2), Abs(a**2/x**2) > 1), (-sqrt(-a**2/x**2 + 1)/(3*x**2)
+ sqrt(-a**2/x**2 + 1)/(3*a**2), True))
```

GIAC/XCAS [A] time = 0.207051, size = 65, normalized size = 2.83

$$\frac{2\left(a^4 + 3\left(x - \sqrt{-a^2 + x^2}\right)^4\right)}{3\left(a^2 + \left(x - \sqrt{-a^2 + x^2}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-a^2 + x^2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(a^4 + 3*(x - sqrt(-a^2 + x^2))^4)/(a^2 + (x - sqrt(-a^2 + x^
2))^2)^3
```

3.134 $\int \frac{\sqrt{-4+9x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Rubi [A] time = 0.0388239, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-4 + 9*x^2]/x,x]
```

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Rubi in Sympy [A] time = 2.57937, size = 24, normalized size = 0.8

$$\sqrt{9x^2-4}-2\operatorname{atan}\left(\frac{\sqrt{9x^2-4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9*x**2-4)**(1/2)/x,x)
```

```
[Out] sqrt(9*x**2 - 4) - 2*atan(sqrt(9*x**2 - 4)/2)
```

Mathematica [A] time = 0.0132835, size = 28, normalized size = 0.93

$$\sqrt{9x^2-4}+2\tan^{-1}\left(\frac{2}{\sqrt{9x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-4 + 9*x^2]/x,x]
```

```
[Out] Sqrt[-4 + 9*x^2] + 2*ArcTan[2/Sqrt[-4 + 9*x^2]]
```

Maple [A] time = 0.01, size = 25, normalized size = 0.8

$$\sqrt{9\,x^2-4}+2\,\arctan\left(2\,\frac{1}{\sqrt{9\,x^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2-4)^(1/2)/x,x)
```

```
[Out] (9*x^2-4)^(1/2)+2*arctan(2/(9*x^2-4)^(1/2))
```

Maxima [A] time = 1.4975, size = 26, normalized size = 0.87

$$\sqrt{9\,x^2-4}+2\,\arcsin\left(\frac{2}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 4)/x,x, algorithm="maxima")
```

```
[Out] sqrt(9*x^2 - 4) + 2*arcsin(2/3/abs(x))
```

Fricas [A] time = 0.206904, size = 96, normalized size = 3.2

$$-\frac{9\,x^2+4\left(3\,x-\sqrt{9\,x^2-4}\right)\arctan\left(-\frac{3}{2}\,x+\frac{1}{2}\,\sqrt{9\,x^2-4}\right)-3\,\sqrt{9\,x^2-4}x-4}{3\,x-\sqrt{9\,x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 4)/x,x, algorithm="fricas")
```

```
[Out] -(9*x^2 + 4*(3*x - sqrt(9*x^2 - 4))*arctan(-3/2*x + 1/2*sqrt(9*x^
2 - 4)) - 3*sqrt(9*x^2 - 4)*x - 4)/(3*x - sqrt(9*x^2 - 4))
```

Sympy [A] time = 2.24578, size = 94, normalized size = 3.13

$$\begin{cases} -\frac{3ix}{\sqrt{-1+\frac{4}{9x^2}}}-2i\,\operatorname{acosh}\left(\frac{2}{3x}\right)+\frac{4i}{3x\sqrt{-1+\frac{4}{9x^2}}} & \text{for } \frac{4\left|\frac{1}{x^2}\right|}{9}>1 \\ \frac{3x}{\sqrt{1-\frac{4}{9x^2}}}+2\,\operatorname{asin}\left(\frac{2}{3x}\right)-\frac{4}{3x\sqrt{1-\frac{4}{9x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2-4)**(1/2)/x,x)
```

```
[Out] Piecewise((-3*I*x/sqrt(-1 + 4/(9*x**2)) - 2*I*acosh(2/(3*x)) + 4*
I/(3*x*sqrt(-1 + 4/(9*x**2))), 4*Abs(x**(-2))/9 > 1), (3*x/sqrt(1
 - 4/(9*x**2)) + 2*asin(2/(3*x)) - 4/(3*x*sqrt(1 - 4/(9*x**2))),
True))
```

GIAC/XCAS [A] time = 0.205206, size = 32, normalized size = 1.07

$$\sqrt{9\,x^2-4}-2\,\arctan\left(\frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 4)/x,x, algorithm="giac")
```

```
[Out] sqrt(9*x^2 - 4) - 2*arctan(1/2*sqrt(9*x^2 - 4))
```

3.135 $\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{16x^2-9}}{9x}$$

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rubi [A] time = 0.0136649, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rubi in Sympy [A] time = 1.40988, size = 12, normalized size = 0.67

$$\frac{\sqrt{16x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(16*x**2-9)**(1/2),x)
```

```
[Out] sqrt(16*x**2 - 9)/(9*x)
```

Mathematica [A] time = 0.00983276, size = 18, normalized size = 1.

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Maple [A] time = 0.007, size = 25, normalized size = 1.4

$$\frac{(4x-3)(3+4x)}{9x}\frac{1}{\sqrt{16x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(16*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(4*x-3)*(3+4*x)/(16*x^2-9)^(1/2)
```

Maxima [A] time = 1.49915, size = 19, normalized size = 1.06

$$\frac{\sqrt{16\,x^2-9}}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(16*x^2 - 9)*x^2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(16*x^2 - 9)/x
```

Fricas [A] time = 0.202311, size = 27, normalized size = 1.5

$$\frac{1}{4\,x^2-\sqrt{16\,x^2-9}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(16*x^2 - 9)*x^2),x, algorithm="fricas")
```

```
[Out] 1/(4*x^2 - sqrt(16*x^2 - 9)*x)
```

Sympy [A] time = 1.27853, size = 39, normalized size = 2.17

$$\begin{cases} \frac{4i\sqrt{-1+\frac{9}{16x^2}}}{9} & \text{for } \frac{9\left|\frac{1}{x^2}\right|}{16} > 1 \\ \frac{4\sqrt{1-\frac{9}{16x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(16*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 9/(16*x**2))/9, 9*Abs(x**(-2))/16 > 1),
(4*sqrt(1 - 9/(16*x**2))/9, True))
```

GIAC/XCAS [A] time = 0.208524, size = 31, normalized size = 1.72

$$\frac{8}{\left(4\,x-\sqrt{16\,x^2-9}\right)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(16*x^2 - 9)*x^2),x, algorithm="giac")
```

```
[Out] 8/((4*x - sqrt(16*x^2 - 9))^2 + 9)
```

3.136 $\int \frac{x^2}{(a^2-x^2)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{a^2-x^2}} - \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] time = 0.0219736, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{x}{\sqrt{a^2-x^2}} - \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi in Sympy [A] time = 2.11096, size = 24, normalized size = 0.71

$$\frac{x}{\sqrt{a^2-x^2}} - \operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a**2-x**2)**(3/2),x)
```

```
[Out] x/sqrt(a**2 - x**2) - atan(x/sqrt(a**2 - x**2))
```

Mathematica [A] time = 0.0481261, size = 34, normalized size = 1.

$$\frac{x}{\sqrt{a^2-x^2}} - \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] time = 0.011, size = 31, normalized size = 0.9

$$-\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right) + x\frac{1}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a^2-x^2)^(3/2),x)
```

```
[Out] -arctan(x/(a^2-x^2)^(1/2))+x/(a^2-x^2)^(1/2)
```

Maxima [A] time = 1.50236, size = 32, normalized size = 0.94

$$\frac{x}{\sqrt{a^2-x^2}} - \arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^2 - x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a^2 - x^2) - arcsin(x/sqrt(a^2))
```

Fricas [A] time = 0.203253, size = 123, normalized size = 3.62

$$-\frac{ax - 2\left(a^2 - x^2 - \sqrt{a^2-x^2}a\right)\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right) - \sqrt{a^2-x^2}x}{a^2 - x^2 - \sqrt{a^2-x^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^2 - x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(a*x - 2*(a^2 - x^2 - sqrt(a^2 - x^2)*a)*arctan(-(a - sqrt(a^2 -
 x^2))/x) - sqrt(a^2 - x^2)*x)/(a^2 - x^2 - sqrt(a^2 - x^2)*a)
```

Sympy [A] time = 2.6471, size = 49, normalized size = 1.44

$$\begin{cases} i\,\operatorname{acosh}\left(\frac{x}{a}\right) - \frac{ix}{a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ -\operatorname{asin}\left(\frac{x}{a}\right) + \frac{x}{a\sqrt{1-\frac{x^2}{a^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a**2-x**2)**(3/2),x)
```

```
[Out] Piecewise((I*acosh(x/a) - I*x/(a*sqrt(-1 + x**2/a**2)), Abs(x**2/
a**2) > 1), (-asin(x/a) + x/(a*sqrt(1 - x**2/a**2)), True))
```

GIAC/XCAS [A] time = 0.221116, size = 32, normalized size = 0.94

$$-\arcsin\left(\frac{x}{a}\right)\operatorname{sign}(a) + \frac{x}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a^2 - x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(x/a)*sign(a) + x/sqrt(a^2 - x^2)
```

3.137 $\int \frac{x^2}{\sqrt{5-x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

```
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Rubi [A] time = 0.0208859, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[5 - x^2],x]
```

```
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 1.73471, size = 24, normalized size = 0.83

$$-\frac{x\sqrt{-x^2+5}}{2}+\frac{5\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(-x**2 + 5)/2 + 5*asin(sqrt(5)*x/5)/2
```

Mathematica [A] time = 0.0193436, size = 29, normalized size = 1.

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[5 - x^2],x]
```

```
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Maple [A] time = 0.01, size = 23, normalized size = 0.8

$$\frac{5}{2}\arcsin\left(\frac{x\sqrt{5}}{5}\right)-\frac{x}{2}\sqrt{-x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+5)^(1/2),x)
```

```
[Out] 5/2*arcsin(1/5*x*5^(1/2))-1/2*x*(-x^2+5)^(1/2)
```

Maxima [A] time = 1.49167, size = 30, normalized size = 1.03

$$-\frac{1}{2}\sqrt{-x^2+5}x+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 5),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

Fricas [A] time = 0.205964, size = 39, normalized size = 1.34

$$-\frac{1}{2}\sqrt{-x^2+5}x-\frac{5}{2}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 5),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x - 5/2*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [A] time = 0.261825, size = 24, normalized size = 0.83

$$-\frac{x\sqrt{-x^2+5}}{2}+\frac{5\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(-x**2 + 5)/2 + 5*asin(sqrt(5)*x/5)/2
```

GIAC/XCAS [A] time = 0.211, size = 30, normalized size = 1.03

$$-\frac{1}{2}\sqrt{-x^2+5}x+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 5),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

3.138 $\int \frac{1}{x\sqrt{3+x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rubi [A] time = 0.0290964, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rubi in Sympy [A] time = 1.82175, size = 24, normalized size = 1.04

$$-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\sqrt{x^2+3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*atanh(sqrt(3)*sqrt(x**2 + 3)/3)/3
```

Mathematica [A] time = 0.0140943, size = 29, normalized size = 1.26

$$\frac{\log(x)-\log\left(\sqrt{3}\sqrt{x^2+3}+3\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] (Log[x] - Log[3 + Sqrt[3]*Sqrt[3 + x^2]])/Sqrt[3]
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$-\frac{\sqrt{3}}{3}\mathit{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2+3)^(1/2),x)
```

```
[Out] -1/3*3^(1/2)*arctanh(3^(1/2)/(x^2+3)^(1/2))
```

Maxima [A] time = 1.48, size = 19, normalized size = 0.83

$$-\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{\sqrt{3}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 3)*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arcsinh(sqrt(3)/abs(x))
```

Fricas [A] time = 0.207199, size = 70, normalized size = 3.04

$$\frac{1}{3}\sqrt{3}\log\left(\frac{\sqrt{3}\left(x^2+3\right)-\sqrt{x^2+3}\left(\sqrt{3}x+3\right)+3\,x}{x^2-\sqrt{x^2+3}x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 3)*x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((sqrt(3)*(x^2 + 3) - sqrt(x^2 + 3)*(sqrt(3)*x + 3
) + 3*x)/(x^2 - sqrt(x^2 + 3)*x))
```

Sympy [A] time = 1.77199, size = 15, normalized size = 0.65

$$-\frac{\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{3}}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*asinh(sqrt(3)/x)/3
```

GIAC/XCAS [A] time = 0.206418, size = 50, normalized size = 2.17

$$-\frac{1}{6}\sqrt{3}\ln\left(\sqrt{3}+\sqrt{x^2+3}\right)+\frac{1}{6}\sqrt{3}\ln\left(-\sqrt{3}+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 3)*x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*ln(sqrt(3) + sqrt(x^2 + 3)) + 1/6*sqrt(3)*ln(-sqrt(3
) + sqrt(x^2 + 3))
```

3.139 $\int \frac{x}{(4+x^2)^{5/2}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Rubi [A] time = 0.00546531, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(4 + x^2)^(5/2),x]
```

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Rubi in Sympy [A] time = 0.763352, size = 12, normalized size = 0.92

$$-\frac{1}{3\left(x^2+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+4)**(5/2),x)
```

```
[Out] -1/(3*(x**2 + 4)**(3/2))
```

Mathematica [A] time = 0.00290577, size = 13, normalized size = 1.

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(4 + x^2)^(5/2),x]
```

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Maple [A] time = 0.004, size = 10, normalized size = 0.8

$$-\frac{1}{3}\left(x^2+4\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+4)^(5/2),x)
```

```
[Out] -1/3/(x^2+4)^(3/2)
```

Maxima [A] time = 1.32695, size = 12, normalized size = 0.92

$$-\frac{1}{3\left(x^2+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

Fricas [A] time = 0.200361, size = 82, normalized size = 6.31

$$\frac{x^3-\sqrt{x^2+4}\left(x^2+1\right)+3\,x}{3\left(x^6+9\,x^4+24\,x^2-\left(x^5+7\,x^3+12\,x\right)\sqrt{x^2+4}+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - sqrt(x^2 + 4)*(x^2 + 1) + 3*x)/(x^6 + 9*x^4 + 24*x^2 -
 (x^5 + 7*x^3 + 12*x)*sqrt(x^2 + 4) + 16)
```

Sympy [A] time = 4.08395, size = 26, normalized size = 2.

$$-\frac{1}{3x^2\sqrt{x^2+4}+12\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+4)**(5/2),x)
```

```
[Out] -1/(3*x**2*sqrt(x**2 + 4) + 12*sqrt(x**2 + 4))
```

GIAC/XCAS [A] time = 0.200548, size = 12, normalized size = 0.92

$$-\frac{1}{3\left(x^2+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

3.140 $\int x^3\sqrt{4-9x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rubi [A] time = 0.0345316, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rubi in Sympy [A] time = 2.56827, size = 24, normalized size = 0.77

$$\frac{\left(-9x^2+4\right)^{\frac{5}{2}}}{405}-\frac{4\left(-9x^2+4\right)^{\frac{3}{2}}}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(-9*x**2+4)**(1/2),x)
```

```
[Out] (-9*x**2 + 4)**(5/2)/405 - 4*(-9*x**2 + 4)**(3/2)/243
```

Mathematica [A] time = 0.012912, size = 22, normalized size = 0.71

$$-\frac{\left(4-9x^2\right)^{3/2}\left(27x^2+8\right)}{1215}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] -((4 - 9*x^2)^(3/2)*(8 + 27*x^2))/1215
```

Maple [A] time = 0.006, size = 29, normalized size = 0.9

$$\frac{(-2+3x)(2+3x)\left(27x^2+8\right)}{1215}\sqrt{-9x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/1215*(-2+3*x)*(2+3*x)*(27*x^2+8)*(-9*x^2+4)^(1/2)
```

Maxima [A] time = 1.51905, size = 35, normalized size = 1.13

$$-\frac{1}{45}\left(-9x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{1215}\left(-9x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 4)*x^3,x, algorithm="maxima")
```

```
[Out] -1/45*(-9*x^2 + 4)^(3/2)*x^2 - 8/1215*(-9*x^2 + 4)^(3/2)
```

Fricas [A] time = 0.201533, size = 117, normalized size = 3.77

$$\frac{729x^{10}-4320x^8+4560x^6-1280x^4+10\left(81x^8-156x^6+64x^4\right)\sqrt{-9x^2+4}}{5\left(810x^4-1440x^2-(81x^4-432x^2+256)\sqrt{-9x^2+4}+512\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 4)*x^3,x, algorithm="fricas")
```

```
[Out] 1/5*(729*x^10 - 4320*x^8 + 4560*x^6 - 1280*x^4 + 10*(81*x^8 - 156
*x^6 + 64*x^4)*sqrt(-9*x^2 + 4))/(810*x^4 - 1440*x^2 - (81*x^4 -
432*x^2 + 256)*sqrt(-9*x^2 + 4) + 512)
```

Sympy [A] time = 0.905343, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{-9x^2+4}}{5}-\frac{4x^2\sqrt{-9x^2+4}}{135}-\frac{32\sqrt{-9x^2+4}}{1215}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(-9*x**2+4)**(1/2),x)
```

```
[Out] x**4*sqrt(-9*x**2 + 4)/5 - 4*x**2*sqrt(-9*x**2 + 4)/135 - 32*sqrt
(-9*x**2 + 4)/1215
```

GIAC/XCAS [A] time = 0.204293, size = 43, normalized size = 1.39

$$\frac{1}{405}\left(9x^2-4\right)^2\sqrt{-9x^2+4}-\frac{4}{243}\left(-9x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 4)*x^3,x, algorithm="giac")
```

```
[Out] 1/405*(9*x^2 - 4)^2*sqrt(-9*x^2 + 4) - 4/243*(-9*x^2 + 4)^(3/2)
```

3.141 $\int x^2\sqrt{9-x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{9}{8}\sqrt{9-x^2}x+\frac{1}{4}\sqrt{9-x^2}x^3+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])
/8
```

Rubi [A] time = 0.0323906, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{9}{8}\sqrt{9-x^2}x+\frac{1}{4}\sqrt{9-x^2}x^3+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[9 - x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])
/8
```

Rubi in Sympy [A] time = 2.77129, size = 34, normalized size = 0.76

$$\frac{x^3\sqrt{-x^2+9}}{4}-\frac{9x\sqrt{-x^2+9}}{8}+\frac{81\operatorname{asin}\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**2+9)**(1/2),x)
```

```
[Out] x**3*sqrt(-x**2 + 9)/4 - 9*x*sqrt(-x**2 + 9)/8 + 81*asin(x/3)/8
```

Mathematica [A] time = 0.0241363, size = 33, normalized size = 0.73

$$\frac{1}{8}\left(x\sqrt{9-x^2}\left(2x^2-9\right)+81\sin^{-1}\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[9 - x^2],x]
```

```
[Out] (x*Sqrt[9 - x^2]*(-9 + 2*x^2) + 81*ArcSin[x/3])/8
```

Maple [A] time = 0.007, size = 32, normalized size = 0.7

$$-\frac{x}{4}\left(-x^2+9\right)^{\frac{3}{2}}+\frac{9x}{8}\sqrt{-x^2+9}+\frac{81}{8}\arcsin\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^2+9)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2+9)^(3/2)+9/8*x*(-x^2+9)^(1/2)+81/8*arcsin(1/3*x)
```

Maxima [A] time = 1.50664, size = 42, normalized size = 0.93

$$-\frac{1}{4}\left(-x^2+9\right)^{\frac{3}{2}}x+\frac{9}{8}\sqrt{-x^2+9}x+\frac{81}{8}\arcsin\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)*x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 9)^(3/2)*x + 9/8*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*
x)
```

Fricas [A] time = 0.209553, size = 166, normalized size = 3.69

$$-\frac{24\,x^7-756\,x^5+6804\,x^3+162\left(x^4-72\,x^2+12\left(x^2-18\right)\sqrt{-x^2+9}+648\right)\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)-\left(2\,x^7-153\,x^5+1944\,x^3-\right.}{8\left(x^4-72\,x^2+12\left(x^2-18\right)\sqrt{-x^2+9}+648\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)*x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(24*x^7 - 756*x^5 + 6804*x^3 + 162*(x^4 - 72*x^2 + 12*(x^2 -
 18)*sqrt(-x^2 + 9) + 648)*arctan((sqrt(-x^2 + 9) - 3)/x) - (2*x^
7 - 153*x^5 + 1944*x^3 - 5832*x)*sqrt(-x^2 + 9) - 17496*x)/(x^4 -
 72*x^2 + 12*(x^2 - 18)*sqrt(-x^2 + 9) + 648)
```

Sympy [A] time = 4.45745, size = 112, normalized size = 2.49

$$\begin{cases}\frac{ix^5}{4\sqrt{x^2-9}}-\frac{27ix^3}{8\sqrt{x^2-9}}+\frac{81ix}{8\sqrt{x^2-9}}-\frac{81i\,\operatorname{acosh}\left(\frac{x}{3}\right)}{8} & \text{for } \frac{\left|x^2\right|}{9}>1\\ -\frac{x^5}{4\sqrt{-x^2+9}}+\frac{27x^3}{8\sqrt{-x^2+9}}-\frac{81x}{8\sqrt{-x^2+9}}+\frac{81\,\operatorname{asin}\left(\frac{x}{3}\right)}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 9)) - 27*I*x**3/(8*sqrt(x**2 - 9
)) + 81*I*x/(8*sqrt(x**2 - 9)) - 81*I*acosh(x/3)/8, Abs(x**2)/9 >
 1), (-x**5/(4*sqrt(-x**2 + 9)) + 27*x**3/(8*sqrt(-x**2 + 9)) - 8
1*x/(8*sqrt(-x**2 + 9)) + 81*asin(x/3)/8, True))
```

GIAC/XCAS [A] time = 0.21574, size = 35, normalized size = 0.78

$$\frac{1}{8}\left(2\,x^2-9\right)\sqrt{-x^2+9}x+\frac{81}{8}\arcsin\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2 - 9)*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

3.142 $\int 5x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Rubi [A] time = 0.00650621, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Rubi in Sympy [A] time = 0.838377, size = 10, normalized size = 0.77

$$\frac{5\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(5*x*(x**2+1)**(1/2),x)
```

```
[Out] 5*(x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.00325007, size = 13, normalized size = 1.

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Maple [A] time = 0.003, size = 10, normalized size = 0.8

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(5*x*(x^2+1)^(1/2),x)
```

```
[Out] 5/3*(x^2+1)^(3/2)
```

Maxima [A] time = 1.76761, size = 12, normalized size = 0.92

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5*sqrt(x^2 + 1)*x,x, algorithm="maxima")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.198133, size = 93, normalized size = 7.15

$$-\frac{5\left(4\,x^6+9\,x^4+6\,x^2-\left(4\,x^5+7\,x^3+3\,x\right)\sqrt{x^2+1}+1\right)}{3\left(4\,x^3-\left(4\,x^2+1\right)\sqrt{x^2+1}+3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5*sqrt(x^2 + 1)*x,x, algorithm="fricas")
```

```
[Out] -5/3*(4*x^6 + 9*x^4 + 6*x^2 - (4*x^5 + 7*x^3 + 3*x)*sqrt(x^2 + 1)
 + 1)/(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [A] time = 0.222907, size = 26, normalized size = 2.

$$\frac{5x^2\sqrt{x^2+1}}{3}+\frac{5\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5*x*(x**2+1)**(1/2),x)
```

```
[Out] 5*x**2*sqrt(x**2 + 1)/3 + 5*sqrt(x**2 + 1)/3
```

GIAC/XCAS [A] time = 0.209215, size = 12, normalized size = 0.92

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(5*sqrt(x^2 + 1)*x,x, algorithm="giac")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

3.143 $\int \frac{1}{(-25+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{x}{25\sqrt{4x^2-25}}$$

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Rubi [A] time = 0.0056477, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-25 + 4*x^2)^(-3/2),x]
```

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Rubi in Sympy [A] time = 0.516835, size = 14, normalized size = 0.88

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**2-25)**(3/2),x)
```

```
[Out] -x/(25*sqrt(4*x**2 - 25))
```

Mathematica [A] time = 0.00889073, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-25 + 4*x^2)^(-3/2),x]
```

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Maple [A] time = 0.003, size = 23, normalized size = 1.4

$$-\frac{(2x-5)(5+2x)x}{25}\left(4x^2-25\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^2-25)^(3/2),x)
```

```
[Out] -1/25*(2*x-5)*(5+2*x)*x/(4*x^2-25)^(3/2)
```

Maxima [A] time = 1.38729, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 25)^(-3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

Fricas [A] time = 0.202002, size = 31, normalized size = 1.94

$$\frac{1}{2\left(4x^2-2\sqrt{4x^2-25}x-25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 25)^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/2/(4*x^2 - 2*sqrt(4*x^2 - 25)*x - 25)
```

Sympy [A] time = 1.22245, size = 36, normalized size = 2.25

$$\begin{cases} -\frac{x}{25\sqrt{4x^2-25}} & \text{for } \frac{4\left|x^2\right|}{25} > 1 \\ \frac{ix}{25\sqrt{-4x^2+25}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**2-25)**(3/2),x)
```

```
[Out] Piecewise((-x/(25*sqrt(4*x**2 - 25)), 4*Abs(x**2)/25 > 1), (I*x/(
25*sqrt(-4*x**2 + 25)), True))
```

GIAC/XCAS [A] time = 0.20469, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 25)^(-3/2),x, algorithm="giac")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

3.144 $\int \sqrt{2x-x^2}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{2x-x^2}(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - ArcSin[1 - x]/2
```

Rubi [A] time = 0.017401, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{1}{2}\sqrt{2x-x^2}(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2*x - x^2],x]
```

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - ArcSin[1 - x]/2
```

Rubi in Sympy [A] time = 0.798937, size = 22, normalized size = 0.67

$$-\frac{(-2x+2)\sqrt{-x^2+2x}}{4}+\frac{\operatorname{asin}(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+2*x)**(1/2),x)
```

```
[Out] -(-2*x + 2)*sqrt(-x**2 + 2*x)/4 + asin(x - 1)/2
```

Mathematica [A] time = 0.0408164, size = 45, normalized size = 1.36

$$\frac{1}{2}\sqrt{-(x-2)x}\left(x-\frac{2\log\left(\sqrt{x-2}+\sqrt{x}\right)}{\sqrt{x-2}\sqrt{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2*x - x^2],x]
```

```
[Out] (Sqrt[-((-2 + x)*x)]*(-1 + x - (2*Log[Sqrt[-2 + x] + Sqrt[x]])/(S
qrt[-2 + x]*Sqrt[x])))/2
```

Maple [A] time = 0.006, size = 26, normalized size = 0.8

$$-\frac{-2x+2}{4}\sqrt{-x^2+2x}+\frac{\arcsin(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+2*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+2)*(-x^2+2*x)^(1/2)+1/2*arcsin(-1+x)
```

Maxima **[A]** time = 1.53843, size = 49, normalized size = 1.48

$$\frac{1}{2}\sqrt{-x^2+2\,x}x-\frac{1}{2}\sqrt{-x^2+2\,x}-\frac{1}{2}\arcsin\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*arcsin(-x + 1
)
```

Fricas **[A]** time = 0.203513, size = 47, normalized size = 1.42

$$\frac{1}{2}\sqrt{-x^2+2\,x}(x-1)-\arctan\left(\frac{\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) - arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy **[F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^2+2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + 2*x), x)
```

GIAC/XCAS **[A]** time = 0.213038, size = 31, normalized size = 0.94

$$\frac{1}{2}\sqrt{-x^2+2\,x}(x-1)+\frac{1}{2}\arcsin\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 2*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) + 1/2*arcsin(x - 1)
```

3.145 $\int \frac{1}{\sqrt{8+4x+x^2}}\,dx$

Optimal. Leaf size=8

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

```
[Out] ArcSinh[(2 + x)/2]
```

Rubi [A] time = 0.0129142, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Rubi in Sympy [A] time = 0.665103, size = 19, normalized size = 2.38

$$\operatorname{atanh}\left(\frac{2x+4}{2\sqrt{x^2+4x+8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4*x+8)**(1/2),x)
```

```
[Out] atanh((2*x + 4)/(2*sqrt(x**2 + 4*x + 8)))
```

Mathematica [A] time = 0.00797654, size = 8, normalized size = 1.

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$Arcsinh\left(1+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4*x+8)^(1/2),x)
```

```
[Out] arcsinh(1+1/2*x)
```

Maxima [A] time = 1.63778, size = 8, normalized size = 1.

$$\operatorname{arsinh}\left(\frac{1}{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x + 8),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*x + 1)
```

Fricas [A] time = 0.201239, size = 24, normalized size = 3.

$$-\log\left(-x + \sqrt{x^2 + 4x + 8} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x + 8),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + 4x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x + 8), x)
```

GIAC/XCAS [A] time = 0.212927, size = 24, normalized size = 3.

$$-\ln\left(-x + \sqrt{x^2 + 4x + 8} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x + 8),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(x^2 + 4*x + 8) - 2)
```

3.146 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi [A] time = 0.0137868, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi in Sympy [A] time = 0.701501, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{18x+6}{6\sqrt{9x^2+6x-8}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] atanh((18*x + 6)/(6*sqrt(9*x**2 + 6*x - 8)))/3
```

Mathematica [A] time = 0.00895472, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3
```

Maple [A] time = 0.004, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(3+9x)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/9*ln(1/9*(3+9*x)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A] time = 1.56198, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+6\,x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 0.196927, size = 27, normalized size = 1.08

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

GIAC/XCAS [A] time = 0.207921, size = 28, normalized size = 1.12

$$-\frac{1}{3}\ln\left(\left|-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

3.147 $\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{4x-x^2}x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0448581, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$-\frac{1}{2}\sqrt{4x-x^2}x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[4*x - x^2],x]
```

```
[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rubi in Sympy [A] time = 2.68261, size = 32, normalized size = 0.73

$$-\frac{x\sqrt{-x^2+4x}}{2}-3\sqrt{-x^2+4x}+6\operatorname{asin}\left(\frac{x}{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-x**2+4*x)**(1/2),x)
```

```
[Out] -x*sqrt(-x**2 + 4*x)/2 - 3*sqrt(-x**2 + 4*x) + 6*asin(x/2 - 1)
```

Mathematica [A] time = 0.0271179, size = 53, normalized size = 1.2

$$\frac{x\left(x^2+2x-24\right)+24\sqrt{x-4}\sqrt{x}\log\left(\sqrt{x-4}+\sqrt{x}\right)}{2\sqrt{-(x-4)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[4*x - x^2],x]
```

```
[Out] (x*(-24 + 2*x + x^2) + 24*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] +
 Sqrt[x]])/(2*Sqrt[-((-4 + x)*x)])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.8

$$6\arcsin\left(-1+x/2\right)-3\sqrt{-x^2+4x}-\frac{x}{2}\sqrt{-x^2+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-x^2+4*x)^(1/2),x)
```

```
[Out] 6*arcsin(-1+1/2*x)-3*(-x^2+4*x)^(1/2)-1/2*x*(-x^2+4*x)^(1/2)
```

Maxima [A] time = 1.69884, size = 49, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2+4\,x}x-3\sqrt{-x^2+4\,x}-6\arcsin\left(-\frac{1}{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*x - 3*sqrt(-x^2 + 4*x) - 6*arcsin(-1/2*x +
1)
```

Fricas [A] time = 0.20351, size = 47, normalized size = 1.07

$$-\frac{1}{2}\sqrt{-x^2+4\,x}(x+6)-12\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) - 12*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x - 4)), x)
```

GIAC/XCAS [A] time = 0.214958, size = 34, normalized size = 0.77

$$-\frac{1}{2}\sqrt{-x^2+4\,x}(x+6)+6\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-x^2 + 4*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) + 6*arcsin(1/2*x - 1)
```

3.148 $\int \frac{1}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=26

$$\frac{x+1}{2\,(x^2+2x+2)}+\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rubi [A] time = 0.0131628, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x+1}{2\,(x^2+2x+2)}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rubi in Sympy [A] time = 0.729881, size = 20, normalized size = 0.77

$$\frac{2x+2}{4\,(x^2+2x+2)}+\frac{\operatorname{atan}(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+2*x+2)**2,x)
```

```
[Out] (2*x + 2)/(4*(x**2 + 2*x + 2)) + atan(x + 1)/2
```

Mathematica [A] time = 0.0124541, size = 23, normalized size = 0.88

$$\frac{1}{2}\left(\frac{x+1}{x^2+2x+2}+\tan^{-1}(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] ((1 + x)/(2 + 2*x + x^2) + ArcTan[1 + x])/2
```

Maple [A] time = 0.006, size = 25, normalized size = 1.

$$\frac{2\,x+2}{4\,x^2+8\,x+8}+\frac{\arctan(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+2*x+2)^2,x)
```

```
[Out] 1/4*(2*x+2)/(x^2+2*x+2)+1/2*arctan(1+x)
```

Maxima [A] time = 1.62446, size = 30, normalized size = 1.15

$$\frac{x+1}{2\left(x^2+2x+2\right)}+\frac{1}{2}\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 2)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

Fricas [A] time = 0.195729, size = 38, normalized size = 1.46

$$\frac{\left(x^2+2x+2\right)\arctan\left(x+1\right)+x+1}{2\left(x^2+2x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 2)^(-2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.130854, size = 19, normalized size = 0.73

$$\frac{x+1}{2x^2+4x+4}+\frac{\operatorname{atan}\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+2*x+2)**2,x)
```

```
[Out] (x + 1)/(2*x**2 + 4*x + 4) + atan(x + 1)/2
```

GIAC/XCAS [A] time = 0.208532, size = 30, normalized size = 1.15

$$\frac{x+1}{2\left(x^2+2x+2\right)}+\frac{1}{2}\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 2)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

3.149 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*
x - x^2])
```

Rubi [A] time = 0.0175299, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*
x - x^2])
```

Rubi in Sympy [A] time = 0.802721, size = 36, normalized size = 0.84

$$\frac{2x+4}{54\left(-x^2-4x+5\right)^{\frac{3}{2}}}+\frac{4x+8}{486\sqrt{-x^2-4x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] (2*x + 4)/(54*(-x**2 - 4*x + 5)**(3/2)) + (4*x + 8)/(486*sqrt(-x*
*2 - 4*x + 5))
```

Mathematica [A] time = 0.0258725, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] -((2 + x)*(-19 + 8*x + 2*x^2))/(243*(5 - 4*x - x^2)^(3/2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{(5+x)(-1+x)\left(2x^3+12x^2-3x-38\right)}{243}\left(-x^2-4x+5\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(5+x)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Maxima [A] time = 1.43765, size = 80, normalized size = 1.86

$$\frac{2\,x}{243\,\sqrt{-x^2-4\,x+5}}+\frac{4}{243\,\sqrt{-x^2-4\,x+5}}+\frac{x}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}+\frac{2}{27\,(-x^2-4\,x+5)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*
x/(-x^2 - 4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Fricas [A] time = 0.206224, size = 66, normalized size = 1.53

$$-\frac{\left(2\,x^3+12\,x^2-3\,x-38\right)\sqrt{-x^2-4\,x+5}}{243\,\left(x^4+8\,x^3+6\,x^2-40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*
x^3 + 6*x^2 - 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^2-4x+5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.214396, size = 49, normalized size = 1.14

$$-\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\,\left(x^2+4\,x-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x
 - 5)^2
```

3.150 $\int e^t \sqrt{9 - e^{2t}}\, dt$

Optimal. Leaf size=33

$$\frac{1}{2} e^t \sqrt{9 - e^{2t}} + \frac{9}{2} \sin^{-1}\left(\frac{e^t}{3}\right)$$

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rubi [A] time = 0.0459188, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{1}{2} e^t \sqrt{9 - e^{2t}} + \frac{9}{2} \sin^{-1}\left(\frac{e^t}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rubi in Sympy [A] time = 3.10135, size = 24, normalized size = 0.73

$$\frac{\sqrt{-e^{2t} + 9} e^t}{2} + \frac{9 \operatorname{asin}\left(\frac{e^t}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)*(9-exp(2*t))**(1/2),t)
```

```
[Out] sqrt(-exp(2*t) + 9)*exp(t)/2 + 9*asin(exp(t)/3)/2
```

Mathematica [A] time = 0.0242512, size = 33, normalized size = 1.

$$\frac{1}{2} e^t \sqrt{9 - e^{2t}} + \frac{9}{2} \sin^{-1}\left(\frac{e^t}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Maple [A] time = 0.014, size = 23, normalized size = 0.7

$$\frac{\mathrm{e}^t}{2} \sqrt{9 - (\mathrm{e}^t)^2} + \frac{9}{2} \arcsin\left(\frac{\mathrm{e}^t}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)*(9-exp(2*t))^(1/2),t)
```

```
[Out] 1/2*exp(t)*(9-exp(t)^2)^(1/2)+9/2*arcsin(1/3*exp(t))
```

Maxima [A] time = 1.59341, size = 30, normalized size = 0.91

$$\frac{1}{2}\sqrt{-e^{(2\,t)}+9}e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^(2*t) + 9)*e^t,t, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

Fricas [A] time = 0.211551, size = 124, normalized size = 3.76

$$-\frac{18\left(6\sqrt{-e^{(2\,t)}+9}+e^{(2\,t)}-18\right)\arctan\left(\left(\sqrt{-e^{(2\,t)}+9}-3\right)e^{(-t)}\right)-\left(e^{(3\,t)}-18\,e^{t}\right)\sqrt{-e^{(2\,t)}+9}+6\,e^{(3\,t)}-54\,e^{t}}{2\left(6\sqrt{-e^{(2\,t)}+9}+e^{(2\,t)}-18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^(2*t) + 9)*e^t,t, algorithm="fricas")
```

```
[Out] -1/2*(18*(6*sqrt(-e^(2*t) + 9) + e^(2*t) - 18)*arctan((sqrt(-e^(2
*t) + 9) - 3)*e^(-t)) - (e^(3*t) - 18*e^t)*sqrt(-e^(2*t) + 9) + 6
*e^(3*t) - 54*e^t)/(6*sqrt(-e^(2*t) + 9) + e^(2*t) - 18)
```

Sympy [A] time = 1.63177, size = 29, normalized size = 0.88

$$\begin{cases}\frac{\sqrt{-e^{2t}+9}e^{t}}{2}+\frac{9\operatorname{asin}\left(\frac{e^{t}}{3}\right)}{2} & \text{for } e^{t}<\log(3)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)*(9-exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(-exp(2*t) + 9)*exp(t)/2 + 9*asin(exp(t)/3)/2, exp
(t) < log(3)))
```

GIAC/XCAS [A] time = 0.211744, size = 30, normalized size = 0.91

$$\frac{1}{2}\sqrt{-e^{(2\,t)}+9}e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^(2*t) + 9)*e^t,t, algorithm="giac")
```

```
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

3.151 $\int \sqrt{-9+e^{2t}}\,dt$

Optimal. Leaf size=30

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rubi [A] time = 0.0333224, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rubi in Sympy [A] time = 2.09252, size = 24, normalized size = 0.8

$$\sqrt{e^{2t}-9}-3\operatorname{atan}\left(\frac{\sqrt{e^{2t}-9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-9+exp(2*t))**(1/2),t)
```

```
[Out] sqrt(exp(2*t) - 9) - 3*atan(sqrt(exp(2*t) - 9)/3)
```

Mathematica [A] time = 0.0134914, size = 30, normalized size = 1.

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Maple [A] time = 0.009, size = 23, normalized size = 0.8

$$-3\arctan\left(1/3\sqrt{-9+\mathrm{e}^{2t}}\right)+\sqrt{-9+\mathrm{e}^{2t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-9+exp(2*t))^(1/2),t)
```

```
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

Maxima [A] time = 1.53706, size = 30, normalized size = 1.

$$\sqrt{e^{(2\,t)}-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*t) - 9),t, algorithm="maxima")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Fricas [A] time = 0.212037, size = 30, normalized size = 1.

$$\sqrt{e^{(2\,t)}-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*t) - 9),t, algorithm="fricas")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Sympy [A] time = 1.361, size = 22, normalized size = 0.73

$$\begin{cases}\sqrt{e^{2t}-9}-3\operatorname{acos}\left(3e^{-t}\right) & \text{for } e^{t}<\log(3)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-9+exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(exp(2*t) - 9) - 3*acos(3*exp(-t)), exp(t) < log(3
)))
```

GIAC/XCAS [A] time = 0.208623, size = 30, normalized size = 1.

$$\sqrt{e^{(2\,t)}-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)}-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*t) - 9),t, algorithm="giac")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

3.152 $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rubi [A] time = 0.00705851, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rubi in Sympy [A] time = 0.704878, size = 12, normalized size = 0.86

$$\operatorname{atanh}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+x**2)**(1/2),x)
```

```
[Out] atanh(x/sqrt(a**2 + x**2))
```

Mathematica [B] time = 0.00394987, size = 42, normalized size = 3.

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[a^2 + x^2]]/2 + Log[1 + x/Sqrt[a^2 + x^2]]/2
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$\ln\left(x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+x^2)^(1/2),x)
```

```
[Out] ln(x+(a^2+x^2)^(1/2))
```

Maxima [A] time = 1.43065, size = 11, normalized size = 0.79

$$\operatorname{arsinh}\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] arcsinh(x/sqrt(a^2))
```

Fricas [A] time = 0.216675, size = 22, normalized size = 1.57

$$-\log\left(-x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(a^2 + x^2))
```

Sympy [A] time = 1.70107, size = 3, normalized size = 0.21

$$\operatorname{asinh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+x**2)**(1/2),x)
```

```
[Out] asinh(x/a)
```

GIAC/XCAS [A] time = 0.208962, size = 22, normalized size = 1.57

$$-\ln\left(-x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 + x^2),x, algorithm="giac")
```

```
[Out] -ln(-x + sqrt(a^2 + x^2))
```

3.153 $\int \frac{5+x}{-2+x+x^2}\,dx$

Optimal. Leaf size=15

$$2\log(1-x)-\log(x+2)$$

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Rubi [A] time = 0.0126646, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + x)/(-2 + x + x^2),x]
```

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Rubi in Sympy [A] time = 1.67623, size = 10, normalized size = 0.67

$$2\log(-x+1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+x)/(x**2+x-2),x)
```

```
[Out] 2*log(-x + 1) - log(x + 2)
```

Mathematica [A] time = 0.00568866, size = 15, normalized size = 1.

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + x)/(-2 + x + x^2),x]
```

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$-\ln(2+x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+x)/(x^2+x-2),x)
```

```
[Out] -ln(2+x)+2*ln(-1+x)
```

Maxima [A] time = 1.47329, size = 18, normalized size = 1.2

$$-\log(x+2)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 5)/(x^2 + x - 2),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + 2*log(x - 1)
```

Fricas [A] time = 0.19868, size = 18, normalized size = 1.2

$$-\log(x+2)+2\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 5)/(x^2 + x - 2),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + 2*log(x - 1)
```

Sympy [A] time = 0.091234, size = 10, normalized size = 0.67

$$2\log(x-1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+x)/(x**2+x-2),x)
```

```
[Out] 2*log(x - 1) - log(x + 2)
```

GIAC/XCAS [A] time = 0.207374, size = 20, normalized size = 1.33

$$-\ln(|x+2|)+2\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 5)/(x^2 + x - 2),x, algorithm="giac")
```

```
[Out] -ln(abs(x + 2)) + 2*ln(abs(x - 1))
```

3.154 $\int \frac{x+x^3}{-1+x}\,dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi [A] time = 0.0306489, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+2x+2\log(-x+1)+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + 2*x + 2*log(-x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.00714938, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}+2\,x+2\,\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x)/(-1+x),x)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(-1+x)
```

Maxima [A] time = 1.35759, size = 27, normalized size = 1.04

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Fricas [A] time = 0.192253, size = 27, normalized size = 1.04

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] time = 0.059164, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

GIAC/XCAS [A] time = 0.211187, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x)/(x - 1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*ln(abs(x - 1))
```

3.155 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi [A] time = 0.0551782, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi in Sympy [A] time = 6.00211, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2}+\frac{\log(-2x+1)}{10}-\frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(-2*x + 1)/10 - log(x + 2)/10
```

Mathematica [A] time = 0.0097486, size = 25, normalized size = 1.

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] time = 0.01, size = 20, normalized size = 0.8

$$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
```

```
[Out] -1/10*ln(2+x)+1/2*ln(x)+1/10*ln(2*x-1)
```

Maxima [A] time = 1.33618, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Fricas [A] time = 0.202873, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.1562, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log(x+2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

GIAC/XCAS [A] time = 0.208091, size = 30, normalized size = 1.2

$$\frac{1}{10}\ln(|2x-1|)-\frac{1}{10}\ln(|x+2|)+\frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x - 1)/(2*x^3 + 3*x^2 - 2*x),x, algorithm="giac")
```

```
[Out] 1/10*ln(abs(2*x - 1)) - 1/10*ln(abs(x + 2)) + 1/2*ln(abs(x))
```

3.156 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi [A] time = 0.0527774, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] Integral((x**4 - 2*x**2 + 4*x + 1)/(x**3 - x**2 - x + 1), x)
```

Mathematica [A] time = 0.0261084, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] time = 0.013, size = 25, normalized size = 0.8

$$x+\frac{x^2}{2}-\ln(1+x)+\ln(-1+x)-2\,(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)
```

```
[Out] x+1/2*x^2-ln(1+x)+ln(-1+x)-2/(-1+x)
```

Maxima [A] time = 1.34602, size = 32, normalized size = 1.07

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Fricas [A] time = 0.210583, size = 49, normalized size = 1.63

$$\frac{x^3 + x^2 - 2(x-1)\log(x+1) + 2(x-1)\log(x-1) - 2x - 4}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*
x - 4)/(x - 1)
```

Sympy [A] time = 0.097528, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

GIAC/XCAS [A] time = 0.211804, size = 35, normalized size = 1.17

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \ln(|x+1|) + \ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - 2*x^2 + 4*x + 1)/(x^3 - x^2 - x + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - ln(abs(x + 1)) + ln(abs(x - 1))
```

3.157 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.050405, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi in Sympy [A] time = 4.33239, size = 17, normalized size = 0.74

$$\log(x)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Mathematica [A] time = 0.00730777, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.009, size = 18, normalized size = 0.8

$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2-x+4)/(x^3+4*x),x)
```

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.52335, size = 23, normalized size = 1.

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 0.212299, size = 23, normalized size = 1.

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.14869, size = 17, normalized size = 0.74

$$\log(x)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

GIAC/XCAS [A] time = 0.212899, size = 24, normalized size = 1.04

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\ln\left(x^2+4\right)+\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 - x + 4)/(x^3 + 4*x),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*ln(x^2 + 4) + ln(abs(x))
```

3.158 $\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx$

Optimal. Leaf size=38

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/
8
```

Rubi [A] time = 0.0592986, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/
8
```

Rubi in Sympy [A] time = 5.67476, size = 32, normalized size = 0.84

$$x+\frac{\log\left(4x^2-4x+3\right)}{8}-\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\left(x-\frac{1}{2}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2-3*x+2)/(4*x**2-4*x+3),x)
```

```
[Out] x + log(4*x**2 - 4*x + 3)/8 - sqrt(2)*atan(sqrt(2)*(x - 1/2))/8
```

Mathematica [A] time = 0.0168122, size = 38, normalized size = 1.

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]
/8
```

Maple [A] time = 0.009, size = 32, normalized size = 0.8

$$x+\frac{\ln\left(4x^2-4x+3\right)}{8}-\frac{\sqrt{2}}{8}\arctan\left(\frac{(8x-4)\sqrt{2}}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2-3*x+2)/(4*x^2-4*x+3),x)
```

```
[Out] x+1/8*ln(4*x^2-4*x+3)-1/8*2^(1/2)*arctan(1/8*(8*x-4)*2^(1/2))
```

Maxima [A] time = 1.51412, size = 42, normalized size = 1.11

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)+x+\frac{1}{8}\log\left(4x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 3*x + 2)/(4*x^2 - 4*x + 3),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 -
4*x + 3)
```

Fricas [A] time = 0.233421, size = 54, normalized size = 1.42

$$\frac{1}{16}\sqrt{2}\left(8\sqrt{2}x+\sqrt{2}\log\left(4x^2-4x+3\right)-2\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 3*x + 2)/(4*x^2 - 4*x + 3),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*(8*sqrt(2)*x + sqrt(2)*log(4*x^2 - 4*x + 3) - 2*arct
an(1/2*sqrt(2)*(2*x - 1)))
```

Sympy [A] time = 0.119971, size = 34, normalized size = 0.89

$$x+\frac{\log\left(x^2-x+\frac{3}{4}\right)}{8}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2-3*x+2)/(4*x**2-4*x+3),x)
```

```
[Out] x + log(x**2 - x + 3/4)/8 - sqrt(2)*atan(sqrt(2)*x - sqrt(2)/2)/8
```

GIAC/XCAS [A] time = 0.20602, size = 42, normalized size = 1.11

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)+x+\frac{1}{8}\ln\left(4x^2-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 3*x + 2)/(4*x^2 - 4*x + 3),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*ln(4*x^2 - 4
*x + 3)
```

3.159 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)$$
$$+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(
1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
+ Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]
/2
```

Rubi [A] time = 0.829011, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)$$
$$+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(
1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
+ Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]
/2
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0741753, size = 93, normalized size = 0.9

$$\frac{1}{48}\left(-14\log\left(1-x^3\right)+\frac{6(x+1)}{\left(x^2+1\right)^2}+\frac{9(3x-2)}{x^2+1}+45\log\left(x^2+1\right)-10\log\left(x^2+x+1\right)\right.$$
$$\left.+20\log(1-x)-48\log(x)+21\tan^{-1}(x)-16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x
] - 16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log
```

```
[x] + 45*Log[1 + x^2] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] time = 0.016, size = 73, normalized size = 0.7

$$\frac{1}{8\,(x^2+1)^2}\left(\frac{9\,x^3}{2}-3\,x^2+\frac{11\,x}{2}-2\right)+\frac{15\,\ln\left(x^2+1\right)}{16}+\frac{7\,\arctan\left(x\right)}{16}$$
$$-\ln\left(x\right)+\frac{\ln\left(-1+x\right)}{8}-\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arcta
n(x)-ln(x)+1/8*ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^
(1/2))*3^(1/2)
```

Maxima [A] time = 1.52115, size = 104, normalized size = 1.01

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{9\,x^3-6\,x^2+11\,x-4}{16\,(x^4+2\,x^2+1)}+\frac{7}{16}\,\arctan\left(x\right)$$
$$-\frac{1}{2}\,\log\left(x^2+x+1\right)+\frac{15}{16}\,\log\left(x^2+1\right)+\frac{1}{8}\,\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="maxi
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2
+ 11*x - 4)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x
+ 1) + 15/16*log(x^2 + 1) + 1/8*log(x - 1) - log(x)
```

Fricas [A] time = 0.245389, size = 211, normalized size = 2.05

$$\frac{\sqrt{3}\Big(7\,\sqrt{3}\left(x^4+2\,x^2+1\right)\arctan\left(x\right)-8\,\sqrt{3}\left(x^4+2\,x^2+1\right)\log\left(x^2+x+1\right)+15\,\sqrt{3}\left(x^4+2\,x^2+1\right)\log\left(x^2+1\right)+2\,\sqrt{3}\left(x^4+2\right.}{48\,(x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="frica
```

```
[Out] 1/48*sqrt(3)*(7*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(x) - 8*sqrt(3)*(
x^4 + 2*x^2 + 1)*log(x^2 + x + 1) + 15*sqrt(3)*(x^4 + 2*x^2 + 1)*
log(x^2 + 1) + 2*sqrt(3)*(x^4 + 2*x^2 + 1)*log(x - 1) - 16*sqrt(3
)*(x^4 + 2*x^2 + 1)*log(x) - 16*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt
(3)*(2*x + 1)) + sqrt(3)*(9*x^3 - 6*x^2 + 11*x - 4))/(x^4 + 2*x^2
 + 1)
```

Sympy [A] time = 0.822652, size = 88, normalized size = 0.85

$$-\log\left(x\right)+\frac{\log\left(x-1\right)}{8}+\frac{15\log\left(x^2+1\right)}{16}-\frac{\log\left(x^2+x+1\right)}{2}$$
$$+\frac{7\,\mathrm{atan}\left(x\right)}{16}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/
2 + 7*atan(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9
*x**3 - 6*x**2 + 11*x - 4)/(16*x**4 + 32*x**2 + 16)
```

GIAC/XCAS [A] time = 0.207955, size = 100, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^3-6x^2+11x-4}{16\left(x^2+1\right)^2}+\frac{7}{16}\arctan(x)$$
$$-\frac{1}{2}\ln\left(x^2+x+1\right)+\frac{15}{16}\ln\left(x^2+1\right)+\frac{1}{8}\ln(|x-1|)-\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/((x^2 + x + 1)*(x^2 + 1)^3*(x - 1)*x),x, algorithm="giac
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2
+ 11*x - 4)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*ln(x^2 + x + 1) +
15/16*ln(x^2 + 1) + 1/8*ln(abs(x - 1)) - ln(abs(x))
```

3.160 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0743529, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 7.3762, size = 29, normalized size = 0.88

$$-\frac{x\left(2+\frac{1}{x}\right)}{2\left(x^2+1\right)}+2\log(x)-\log\left(x^2+1\right)-2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -x*(2 + 1/x)/(2*(x**2 + 1)) + 2*log(x) - log(x**2 + 1) - 2*atan(x
)
```

Mathematica [A] time = 0.0352772, size = 33, normalized size = 1.

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.011, size = 28, normalized size = 0.9

$$-\frac{1}{x^2+1}\left(x+\frac{1}{2}\right)-\frac{\ln\left(x^2+1\right)}{2}-2\arctan(x)+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

Maxima [A] time = 1.54702, size = 39, normalized size = 1.18

$$-\frac{2x+1}{2\left(x^2+1\right)}-2\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x
)
```

Fricas [A] time = 0.209482, size = 59, normalized size = 1.79

$$-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1
)*log(x) + 2*x + 1)/(x^2 + 1)
```

Sympy [A] time = 0.180143, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log(x)-\frac{\log\left(x^2+1\right)}{2}-2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

GIAC/XCAS [A] time = 0.205696, size = 41, normalized size = 1.24

$$-\frac{2x+1}{2\left(x^2+1\right)}-2\arctan(x)-\frac{1}{2}\ln\left(x^2+1\right)+\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(x^3 - 2*x^2 + 3*x - 1)/((x^2 + 1)^2*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*ln(x^2 + 1) + ln(abs
(x))
```

3.161 $\int \frac{1}{(1+x^2)^2}\,dx$

Optimal. Leaf size=19

$$\frac{x}{2\,(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.00722714, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{x}{2\,(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 0.5522, size = 12, normalized size = 0.63

$$\frac{x}{2\,(x^2+1)} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**2,x)
```

```
[Out] x/(2*(x**2 + 1)) + atan(x)/2
```

Mathematica [A] time = 0.0070905, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1} + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^(-2),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A] time = 0., size = 16, normalized size = 0.8

$$\frac{x}{2\,x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^2,x)
```

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.50214, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.226824, size = 26, normalized size = 1.37

$$\frac{\left(x^2+1\right)\arctan\left(x\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.105739, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

GIAC/XCAS [A] time = 0.202456, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.162 $\int \frac{1}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rubi [A] time = 0.0111303, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rubi in Sympy [A] time = 1.11906, size = 12, normalized size = 0.63

$$\frac{\log(-x+1)}{3}-\frac{\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)/(2+x),x)
```

```
[Out] log(-x + 1)/3 - log(x + 2)/3
```

Mathematica [A] time = 0.00400331, size = 19, normalized size = 1.

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Maple [A] time = 0.009, size = 14, normalized size = 0.7

$$-\frac{\ln(2+x)}{3}+\frac{\ln(-1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)/(2+x),x)
```

```
[Out] -1/3*ln(2+x)+1/3*ln(-1+x)
```

Maxima [A] time = 1.37572, size = 18, normalized size = 0.95

$$-\frac{1}{3}\log(x+2)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x - 1)),x, algorithm="maxima")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Fricas [A] time = 0.232421, size = 18, normalized size = 0.95

$$-\frac{1}{3}\log(x+2)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x - 1)),x, algorithm="fricas")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Sympy [A] time = 0.089443, size = 12, normalized size = 0.63

$$\frac{\log(x-1)}{3}-\frac{\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)/(2+x),x)
```

```
[Out] log(x - 1)/3 - log(x + 2)/3
```

GIAC/XCAS [A] time = 0.210328, size = 20, normalized size = 1.05

$$-\frac{1}{3}\ln(|x+2|)+\frac{1}{3}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x - 1)),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(x + 2)) + 1/3*ln(abs(x - 1))
```

3.163 $\int \frac{7}{-12+5x+2x^2}\,dx$

Optimal. Leaf size=19

$$\frac{7}{11}\log(3-2x)-\frac{7}{11}\log(x+4)$$

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rubi [A] time = 0.0137621, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{7}{11}\log(3-2x)-\frac{7}{11}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rubi in Sympy [A] time = 1.52941, size = 17, normalized size = 0.89

$$\frac{7\log(-2x+3)}{11}-\frac{7\log(x+4)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(7/(2*x**2+5*x-12),x)
```

```
[Out] 7*log(-2*x + 3)/11 - 7*log(x + 4)/11
```

Mathematica [A] time = 0.00472871, size = 21, normalized size = 1.11

$$7\left(\frac{1}{11}\log(3-2x)-\frac{1}{11}\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] 7*(Log[3 - 2*x]/11 - Log[4 + x]/11)
```

Maple [A] time = 0.009, size = 16, normalized size = 0.8

$$-\frac{7\ln(4+x)}{11}+\frac{7\ln(-3+2x)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(7/(2*x^2+5*x-12),x)
```

```
[Out] -7/11*ln(4+x)+7/11*ln(-3+2*x)
```

Maxima [A] time = 1.34806, size = 20, normalized size = 1.05

$$\frac{7}{11}\log(2x-3)-\frac{7}{11}\log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(7/(2*x^2 + 5*x - 12),x, algorithm="maxima")
```

```
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Fricas [A] time = 0.199643, size = 20, normalized size = 1.05

$$\frac{7}{11}\log(2x-3)-\frac{7}{11}\log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(7/(2*x^2 + 5*x - 12),x, algorithm="fricas")
```

```
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Sympy [A] time = 0.091085, size = 17, normalized size = 0.89

$$\frac{7\log\left(x-\frac{3}{2}\right)}{11}-\frac{7\log(x+4)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(7/(2*x**2+5*x-12),x)
```

```
[Out] 7*log(x - 3/2)/11 - 7*log(x + 4)/11
```

GIAC/XCAS [A] time = 0.204714, size = 23, normalized size = 1.21

$$\frac{7}{11}\ln(|2x-3|)-\frac{7}{11}\ln(|x+4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(7/(2*x^2 + 5*x - 12),x, algorithm="giac")
```

```
[Out] 7/11*ln(abs(2*x - 3)) - 7/11*ln(abs(x + 4))
```

3.164 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi [A] time = 0.0588609, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi in Sympy [A] time = 3.79581, size = 26, normalized size = 0.81

$$\frac{41\log(-2x+1)}{128}-\frac{25\log(2x+3)}{128}-\frac{9}{32(-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+3*x-4)/(2*x-1)**2/(3+2*x),x)
```

```
[Out] 41*log(-2*x + 1)/128 - 25*log(2*x + 3)/128 - 9/(32*(-2*x + 1))
```

Mathematica [A] time = 0.0241322, size = 32, normalized size = 1.

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] time = 0.013, size = 27, normalized size = 0.8

$$-\frac{25\,\ln(3+2x)}{128}+\frac{9}{64x-32}+\frac{41\,\ln(2x-1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] -25/128*ln(3+2*x)+9/32/(2*x-1)+41/128*ln(2*x-1)
```

Maxima [A] time = 1.37685, size = 35, normalized size = 1.09

$$\frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log\left(2\,x+3\right)+\frac{41}{128}\,\log\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Fricas [A] time = 0.211184, size = 50, normalized size = 1.56

$$-\frac{25\,(2\,x-1)\log\left(2\,x+3\right)-41\,(2\,x-1)\log\left(2\,x-1\right)-36}{128\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 3
6)/(2*x - 1)
```

Sympy [A] time = 0.146379, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+3*x-4)/(2*x-1)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

GIAC/XCAS [A] time = 0.217145, size = 58, normalized size = 1.81

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\ln\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right)-\frac{25}{128}\ln\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 + 3*x - 4)/((2*x + 3)*(2*x - 1)^2),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*ln(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*ln
(abs(-4/(2*x - 1) - 1))
```

3.165 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)} - \frac{12}{1375(5x+3)^2} + \frac{20\log(6-x)}{3993} + \frac{1493\log(5x+3)}{499125}$$

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/
3993 + (1493*Log[3 + 5*x])/499125
```

Rubi [A] time = 0.0615206, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{201}{15125(5x+3)} - \frac{12}{1375(5x+3)^2} + \frac{20\log(6-x)}{3993} + \frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/
3993 + (1493*Log[3 + 5*x])/499125
```

Rubi in Sympy [A] time = 3.97143, size = 34, normalized size = 0.79

$$\frac{20\log(-x+6)}{3993} + \frac{1493\log(5x+3)}{499125} + \frac{201}{15125(5x+3)} - \frac{12}{1375(5x+3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] 20*log(-x + 6)/3993 + 1493*log(5*x + 3)/499125 + 201/(15125*(5*x
+ 3)) - 12/(1375*(5*x + 3)**2)
```

Mathematica [A] time = 0.0339592, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2} + 2500\log(x-6) + 1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 +
 5*x])/499125
```

Maple [A] time = 0.013, size = 34, normalized size = 0.8

$$-\frac{12}{1375\,(3+5\,x)^2} + \frac{201}{45375+75625\,x} + \frac{1493\,\ln(3+5\,x)}{499125} + \frac{20\,\ln(-6+x)}{3993}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-x^2)/(-6+x)/(3+5*x)^3,x)
```

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+1493/499125*ln(3+5*x)+20/399
3*ln(-6+x)
```

Maxima [A] time = 1.38515, size = 46, normalized size = 1.07

$$\frac{3\,(335\,x+157)}{15125\,(25\,x^2+30\,x+9)}+\frac{1493}{499125}\,\log(5\,x+3)+\frac{20}{3993}\,\log(x-6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2)/((5*x + 3)^3*(x - 6)),x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x +
 3) + 20/3993*log(x - 6)
```

Fricas [A] time = 0.230355, size = 72, normalized size = 1.67

$$\frac{1493\left(25\,x^2+30\,x+9\right)\log(5\,x+3)+2500\left(25\,x^2+30\,x+9\right)\log(x-6)+33165\,x+15543}{499125\,(25\,x^2+30\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2)/((5*x + 3)^3*(x - 6)),x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 +
30*x + 9)*log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] time = 0.183053, size = 32, normalized size = 0.74

$$\frac{1005x+471}{378125x^2+453750x+136125}+\frac{20\log(x-6)}{3993}+\frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/
3993 + 1493*log(x + 3/5)/499125
```

GIAC/XCAS [A] time = 0.218564, size = 42, normalized size = 0.98

$$\frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2}+\frac{1493}{499125}\ln(|5\,x+3|)+\frac{20}{3993}\ln(|x-6|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 - x^2)/((5*x + 3)^3*(x - 6)),x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*ln(abs(5*x + 3))
+ 20/3993*ln(abs(x - 6))
```

3.166 $\int \frac{1}{-x^3+x^4}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0185005, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 1.50932, size = 17, normalized size = 0.81

$$-\log(x)+\log(-x+1)+\frac{1}{x}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-x**3),x)
```

```
[Out] -log(x) + log(-x + 1) + 1/x + 1/(2*x**2)
```

Mathematica [A] time = 0.00311023, size = 21, normalized size = 1.

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Maple [A] time = 0.01, size = 18, normalized size = 0.9

$$\frac{1}{2\,x^2}+x^{-1}-\ln(x)+\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-x^3),x)
```

```
[Out] 1/2/x^2+1/x-ln(x)+ln(-1+x)
```

Maxima [A] time = 1.36104, size = 26, normalized size = 1.24

$$\frac{2\,x+1}{2\,x^2}+\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x + 1)/x^2 + log(x - 1) - log(x)
```

Fricas [A] time = 0.203377, size = 35, normalized size = 1.67

$$\frac{2\,x^2\log\left(x-1\right)-2\,x^2\log\left(x\right)+2\,x+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x - 1) - 2*x^2*log(x) + 2*x + 1)/x^2
```

Sympy [A] time = 0.104677, size = 17, normalized size = 0.81

$$-\log\left(x\right)+\log\left(x-1\right)+\frac{2x+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-x**3),x)
```

```
[Out] -log(x) + log(x - 1) + (2*x + 1)/(2*x**2)
```

GIAC/XCAS [A] time = 0.221906, size = 28, normalized size = 1.33

$$\frac{2\,x+1}{2\,x^2}+\ln\left(\left|x-1\right|\right)-\ln\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*x + 1)/x^2 + ln(abs(x - 1)) - ln(abs(x))
```

3.167 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.046602, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x-\log(x)+\frac{\log\left(-x^2+1\right)}{2}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x - log(x) + log(-x**2 + 1)/2 + Integral(x, x)
```

Mathematica [A] time = 0.00890225, size = 25, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.01, size = 24, normalized size = 1.

$$x+\frac{x^2}{2}+\frac{\ln(1+x)}{2}-\ln(x)+\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^3-x^2-x+1)/(x^3-x),x)
```

```
[Out] x+1/2*x^2+1/2*ln(1+x)-ln(x)+1/2*ln(-1+x)
```

Maxima [A] time = 1.32485, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 0.199789, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.094391, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.217555, size = 35, normalized size = 1.4

$$\frac{1}{2}x^2 + x + \frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + x^3 - x^2 - x + 1)/(x^3 - x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*ln(abs(x + 1)) + 1/2*ln(abs(x - 1)) - ln(abs(x)
)
```

3.168 $\int \frac{-2+x^2}{x(2+x^2)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

```
[Out] -Log[x] + Log[2 + x^2]
```

Rubi [A] time = 0.0357837, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rubi in Sympy [A] time = 3.19894, size = 12, normalized size = 1.09

$$-\frac{\log\left(x^2\right)}{2}+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x**2)/2 + log(x**2 + 2)
```

Mathematica [A] time = 0.00504965, size = 11, normalized size = 1.

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.1

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-2)/x/(x^2+2),x)
```

```
[Out] -ln(x)+ln(x^2+2)
```

Maxima [A] time = 1.36547, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="maxima")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Fricas [A] time = 0.200052, size = 15, normalized size = 1.36

$$\log\left(x^2+2\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] time = 0.091153, size = 8, normalized size = 0.73

$$-\log\left(x\right)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x) + log(x**2 + 2)
```

GIAC/XCAS [A] time = 0.217447, size = 18, normalized size = 1.64

$$\ln\left(x^2+2\right)-\frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 2)/((x^2 + 2)*x),x, algorithm="giac")
```

```
[Out] ln(x^2 + 2) - 1/2*ln(x^2)
```

3.169 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Rubi [A] time = 0.196964, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Rubi in Sympy [A] time = 14.5789, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\operatorname{atan}(x)-5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqr
t(2)*x/2)
```

Mathematica [A] time = 0.0264357, size = 36, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[
2 + x^2]
```

Maple [A] time = 0.007, size = 32, normalized size = 0.9

$$6\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/
2)
```

Maxima [A] time = 1.54831, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6\arctan(x) + \log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1
/2*log(x^2 + 1)
```

Fricas [A] time = 0.211656, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6\arctan(x) + \log\left(x^2+2\right) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1
/2*log(x^2 + 1)
```

Sympy [A] time = 0.275928, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2} + \log\left(x^2+2\right) + 6\operatorname{atan}(x) - 5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqr
t(2)*x/2)
```

GIAC/XCAS [A] time = 0.220002, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6\arctan(x) + \ln\left(x^2+2\right) - \frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 4*x^2 + 2)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + ln(x^2 + 2) - 1/
2*ln(x^2 + 1)
```

3.170 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{13x}{24\,(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi [A] time = 0.173995, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$-\frac{13x}{24\,(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi in Sympy [A] time = 13.1103, size = 22, normalized size = 0.76

$$-\frac{13x}{24\,(x^2+4)}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\,(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*(x**2 + 4)) + 25*atan(x/2)/144 + atan(x)/9
```

Mathematica [A] time = 0.0262965, size = 29, normalized size = 1.

$$-\frac{13x}{24\,(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] time = 0.013, size = 22, normalized size = 0.8

$$-\frac{13\,x}{24\,x^2+96}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan\,(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Maxima [A] time = 1.51722, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Fricas [A] time = 0.225003, size = 45, normalized size = 1.55

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)+16\left(x^2+4\right)\arctan\left(x\right)-78\,x}{144\,(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x
)/(x^2 + 4)
```

Sympy [A] time = 0.252808, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

GIAC/XCAS [A] time = 0.213176, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,(x^2+4)}+\frac{25}{144}\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 + x^2 + 1)/((x^2 + 4)^2*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.171 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]
) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[
1 + x + x^2])/5586
```

Rubi [A] time = 0.401882, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]
) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[
1 + x + x^2])/5586
```

Rubi in Sympy [A] time = 65.3073, size = 60, normalized size = 1.

$$\frac{200\log(-2x+3)}{3211}+\frac{2731\log(x+5)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{8379}-\frac{79}{273(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(-2*x + 3)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x
 + 1)/5586 + 451*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/8379 - 79/(2
73*(x + 5))
```

Mathematica [A] time = 0.0878075, size = 54, normalized size = 0.9

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176
400*Log[3 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2
832102
```

Maple [A] time = 0.015, size = 48, normalized size = 0.8

$$-\frac{481\,\ln\left(x^2+x+1\right)}{5586}+\frac{451\,\sqrt{3}}{8379}\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)-\frac{79}{1365+273\,x}+\frac{2731\,\ln(5+x)}{24843}+\frac{200\,\ln(-3+2\,x)}{3211}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2
)-79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)
```

Maxima [A] time = 1.52683, size = 63, normalized size = 1.05

$$\frac{451}{8379}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{79}{273\,(x+5)}$$
$$-\frac{481}{5586}\,\log\left(x^2+x+1\right)+\frac{200}{3211}\,\log(2\,x-3)+\frac{2731}{24843}\,\log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) -
 481/5586*log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*l
og(x + 5)
```

Fricas [A] time = 0.204922, size = 99, normalized size = 1.65

$$-\frac{\sqrt{3}\left(81289\,\sqrt{3}(x+5)\log\left(x^2+x+1\right)-58800\,\sqrt{3}(x+5)\log(2\,x-3)-103778\,\sqrt{3}(x+5)\log(x+5)-152438\,(x+5)\arctan\right.}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="fricas")
```

```
[Out] -1/2832102*sqrt(3)*(81289*sqrt(3)*(x + 5)*log(x^2 + x + 1) - 5880
0*sqrt(3)*(x + 5)*log(2*x - 3) - 103778*sqrt(3)*(x + 5)*log(x + 5
) - 152438*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) + 273182*sqrt(3)
)/(x + 5)
```

Sympy [A] time = 0.364901, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log(x+5)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x
+ 1)/5586 + 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79
/(273*x + 1365)
```

GIAC/XCAS [A] time = 0.222988, size = 81, normalized size = 1.35

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\ln\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\ln\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((16*x + 1)/((x^2 + x + 1)*(2*x - 3)*(x + 5)^2),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x +
5) - 481/5586*ln(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*ln(abs
(-13/(x + 5) + 2))
```

3.172 $\int \frac{x^4}{(9+x^2)^3}\,dx$

Optimal. Leaf size=37

$$-\frac{3x}{8\left(x^2+9\right)}-\frac{x^3}{4\left(x^2+9\right)^2}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -x^3/(4*(9 + x^2)^2) - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rubi [A] time = 0.0240618, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{3x}{8\left(x^2+9\right)}-\frac{x^3}{4\left(x^2+9\right)^2}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(9 + x^2)^3,x]
```

```
[Out] -x^3/(4*(9 + x^2)^2) - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rubi in Sympy [A] time = 2.38952, size = 27, normalized size = 0.73

$$-\frac{x^3}{4\left(x^2+9\right)^2}-\frac{3x}{8\left(x^2+9\right)}+\frac{\operatorname{atan}\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**2+9)**3,x)
```

```
[Out] -x**3/(4*(x**2 + 9)**2) - 3*x/(8*(x**2 + 9)) + atan(x/3)/8
```

Mathematica [A] time = 0.0199029, size = 28, normalized size = 0.76

$$\frac{1}{8}\left(\tan^{-1}\left(\frac{x}{3}\right)-\frac{x\left(5x^2+27\right)}{\left(x^2+9\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(9 + x^2)^3,x]
```

```
[Out] (-((x*(27 + 5*x^2))/(9 + x^2)^2) + ArcTan[x/3])/8
```

Maple [A] time = 0.009, size = 25, normalized size = 0.7

$$\frac{1}{\left(x^2+9\right)^2}\left(-\frac{5x^3}{8}-\frac{27x}{8}\right)+\frac{1}{8}\arctan\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^2+9)^3,x)
```

```
[Out] (-5/8*x^3-27/8*x)/(x^2+9)^2+1/8*arctan(1/3*x)
```

Maxima [A] time = 1.51219, size = 41, normalized size = 1.11

$$-\frac{5\,x^3+27\,x}{8\,(x^4+18\,x^2+81)}+\frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^2 + 9)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^3 + 27*x)/(x^4 + 18*x^2 + 81) + 1/8*arctan(1/3*x)
```

Fricas [A] time = 0.198462, size = 53, normalized size = 1.43

$$-\frac{5\,x^3-\left(x^4+18\,x^2+81\right)\,\arctan\left(\frac{1}{3}\,x\right)+27\,x}{8\,(x^4+18\,x^2+81)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^2 + 9)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(5*x^3 - (x^4 + 18*x^2 + 81)*arctan(1/3*x) + 27*x)/(x^4 + 18
*x^2 + 81)
```

Sympy [A] time = 0.156236, size = 26, normalized size = 0.7

$$-\frac{5x^3+27x}{8x^4+144x^2+648}+\frac{\operatorname{atan}\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**2+9)**3,x)
```

```
[Out] -(5*x**3 + 27*x)/(8*x**4 + 144*x**2 + 648) + atan(x/3)/8
```

GIAC/XCAS [A] time = 0.213696, size = 34, normalized size = 0.92

$$-\frac{5\,x^3+27\,x}{8\,\left(x^2+9\right)^2}+\frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^2 + 9)^3,x, algorithm="giac")
```

```
[Out] -1/8*(5*x^3 + 27*x)/(x^2 + 9)^2 + 1/8*arctan(1/3*x)
```

3.173 $\int \frac{19x}{(-1+x)^3(3+5x+4x^2)^2}\,dx$

Optimal. Leaf size=97

$$\frac{19(44x+39)}{276(1-x)^2(4x^2+5x+3)} - \frac{209\log\left(4x^2+5x+3\right)}{4608} - \frac{1843}{4416(1-x)} - \frac{399}{736(1-x)^2} + \frac{209\log(1-x)}{2304} + \frac{114437\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(27
6*(1 - x)^2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23
]])/(52992*Sqrt[23]) + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x +
 4*x^2])/4608
```

Rubi [A] time = 0.158717, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{19(44x+39)}{276(1-x)^2(4x^2+5x+3)} - \frac{209\log\left(4x^2+5x+3\right)}{4608} - \frac{1843}{4416(1-x)} - \frac{399}{736(1-x)^2} + \frac{209\log(1-x)}{2304} + \frac{114437\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In]  Int[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(27
6*(1 - x)^2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23
]])/(52992*Sqrt[23]) + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x +
 4*x^2])/4608
```

Rubi in Sympy [A] time = 9.73698, size = 83, normalized size = 0.86

$$\frac{209\log(-x+1)}{2304} - \frac{209\log\left(4x^2+5x+3\right)}{4608} + \frac{114437\sqrt{23}\operatorname{atan}\left(\sqrt{23}\left(\frac{8x}{23}+\frac{5}{23}\right)\right)}{1218816} - \frac{1843}{4416(-x+1)} + \frac{19(44x+39)}{276(-x+1)^2(4x^2+5x+3)} - \frac{399}{736(-x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(19*x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] 209*log(-x + 1)/2304 - 209*log(4*x**2 + 5*x + 3)/4608 + 114437*sq
rt(23)*atan(sqrt(23)*(8*x/23 + 5/23))/1218816 - 1843/(4416*(-x +
1)) + 19*(44*x + 39)/(276*(-x + 1)**2*(4*x**2 + 5*x + 3)) - 399/(
736*(-x + 1)**2)
```

Mathematica [A] time = 0.0633816, size = 78, normalized size = 0.8

$$\frac{19\left(\frac{184(2204x+975)}{4x^2+5x+3} - 17457\log\left(4x^2+5x+3\right) + \frac{59248}{x-1} - \frac{25392}{(x-1)^2} + 34914\log(1-x) + 36138\sqrt{23}\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (19*(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3
 + 5*x + 4*x^2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 349
14*Log[1 - x] - 17457*Log[3 + 5*x + 4*x^2]))/7312896
```

Maple [A] time = 0.019, size = 68, normalized size = 0.7

$$-\frac{19}{288\,(-1+x)^2}+\frac{133}{-864+864\,x}+\frac{209\,\ln(-1+x)}{2304}-\frac{19}{6912}\left(-\frac{2204\,x}{23}-\frac{975}{23}\right)\left(x^2+\frac{5\,x}{4}+\frac{3}{4}\right)^{-1}-\frac{209\,\ln\left(16\,x^2+20\,x+12\right)}{4608}+\frac{114437\,\sqrt{23}}{1218816}\arctan\left(\frac{(32\,x+20)\,\sqrt{23}}{92}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -19/288/(-1+x)^2+133/864/(-1+x)+209/2304*ln(-1+x)-19/6912*(-2204/
23*x-975/23)/(x^2+5/4*x+3/4)-209/4608*ln(16*x^2+20*x+12)+114437/1
218816*23^(1/2)*arctan(1/92*(32*x+20)*23^(1/2))
```

Maxima [A] time = 1.49752, size = 101, normalized size = 1.04

$$\frac{114437}{1218816}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(8\,x+5)\right)+\frac{19\left(388\,x^3-407\,x^2-120\,x-45\right)}{4416\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)}-\frac{209}{4608}\,\log\left(4\,x^2+5\,x+3\right)+\frac{209}{2304}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(19*x/((4*x^2 + 5*x + 3)^2*(x - 1)^3),x, algorithm="maxima")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416
*(388*x^3 - 407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
 - 209/4608*log(4*x^2 + 5*x + 3) + 209/2304*log(x - 1)
```

Fricas [A] time = 0.237594, size = 197, normalized size = 2.03

$$-\frac{19\,\sqrt{23}\left(253\,\sqrt{23}\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\log\left(4\,x^2+5\,x+3\right)-506\,\sqrt{23}\left(4\,x^4-3\,x^3-3\,x^2-x+3\right)\log(x-1)-12046\right.}{2437632\,(4\,x^4-3\,x^3-3\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(19*x/((4*x^2 + 5*x + 3)^2*(x - 1)^3),x, algorithm="fricas")
```

```
[Out] -19/2437632*sqrt(23)*(253*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3
)*log(4*x^2 + 5*x + 3) - 506*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x
 + 3)*log(x - 1) - 12046*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arctan(1/
23*sqrt(23)*(8*x + 5)) - 24*sqrt(23)*(388*x^3 - 407*x^2 - 120*x -
 45))/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] time = 0.293401, size = 88, normalized size = 0.91

$$\frac{19\left(388x^3-407x^2-120x-45\right)}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{209\log\left(x-1\right)}{2304}$$
$$-\frac{209\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{114437\sqrt{23}\,\operatorname{atan}\left(\frac{8\sqrt{23}x}{23}+\frac{5\sqrt{23}}{23}\right)}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(19*x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] 19*(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 -
13248*x**2 - 4416*x + 13248) + 209*log(x - 1)/2304 - 209*log(x**2
 + 5*x/4 + 3/4)/4608 + 114437*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*s
qrt(23)/23)/1218816
```

GIAC/XCAS [A] time = 0.214362, size = 96, normalized size = 0.99

$$\frac{114437}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8\,x+5)\right)+\frac{19\left(388\,x^3-407\,x^2-120\,x-45\right)}{4416\left(4\,x^2+5\,x+3\right)(x-1)^2}$$
$$-\frac{209}{4608}\ln\left(4\,x^2+5\,x+3\right)+\frac{209}{2304}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(19*x/((4*x^2 + 5*x + 3)^2*(x - 1)^3),x, algorithm="giac")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416
*(388*x^3 - 407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) -
 209/4608*ln(4*x^2 + 5*x + 3) + 209/2304*ln(abs(x - 1))
```

3.174 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Rubi [A] time = 0.0854745, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Rubi in Sympy [A] time = 9.32745, size = 42, normalized size = 0.91

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\sqrt{7}\left(\frac{2x}{7}+\frac{1}{7}\right)\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(sqrt(7)*(2*x/7 +
 1/7))/28 - 1/(2*x)
```

Mathematica [A] time = 0.0430825, size = 46, normalized size = 1.

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*
Log[2 + x + x^2])/8
```

Maple [A] time = 0.013, size = 36, normalized size = 0.8

$$-\frac{1}{2x}-\frac{\ln(x)}{4}+\frac{5\ln\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}}{28}\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))
*7^(1/2)
```

Maxima [A] time = 1.52415, size = 47, normalized size = 1.02

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2
+ x + 2) - 1/4*log(x)
```

Fricas [A] time = 0.205981, size = 66, normalized size = 1.43

$$\frac{\sqrt{7}\left(5\sqrt{7}x\log\left(x^2+x+2\right)-2\sqrt{7}x\log(x)+2x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-4\sqrt{7}\right)}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*(5*sqrt(7)*x*log(x^2 + x + 2) - 2*sqrt(7)*x*log(x) +
 2*x*arctan(1/7*sqrt(7)*(2*x + 1)) - 4*sqrt(7))/x
```

Sympy [A] time = 0.197508, size = 46, normalized size = 1.

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 +
sqrt(7)/7)/28 - 1/(2*x)
```

GIAC/XCAS [A] time = 0.214113, size = 49, normalized size = 1.07

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\ln\left(x^2+x+2\right)-\frac{1}{4}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x^2 + 1)/(x^4 + x^3 + 2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*ln(x^2 +
 x + 2) - 1/4*ln(abs(x))
```

3.175 $\int \frac{1}{-x^3+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Rubi [A] time = 0.0490998, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$

$$\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Rubi in Sympy [A] time = 3.47222, size = 44, normalized size = 0.92

$$\frac{\log(-x+1)}{3} - \frac{\log\left(x^2+x+1\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3} + \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-x**3),x)
```

```
[Out] log(-x + 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x/3
 + 1/3))/3 + 1/(2*x**2)
```

Mathematica [A] time = 0.0191526, size = 48, normalized size = 1.

$$\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Lo
g[1 + x + x^2]/6
```

Maple [A] time = 0.01, size = 38, normalized size = 0.8

$$\frac{1}{2x^2} + \frac{\ln(-1+x)}{3} - \frac{\ln\left(x^2+x+1\right)}{6} - \frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-x^3),x)
```

```
[Out] 1/2/x^2+1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)
```

Maxima [A] time = 1.51673, size = 50, normalized size = 1.04

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^
2 + x + 1) + 1/3*log(x - 1)
```

Fricas [A] time = 0.202438, size = 76, normalized size = 1.58

$$-\frac{\sqrt{3}\left(\sqrt{3}x^2\log\left(x^2+x+1\right)-2\sqrt{3}x^2\log(x-1)+6x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-3\sqrt{3}\right)}{18x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*x^2*log(x^2 + x + 1) - 2*sqrt(3)*x^2*log(x
 - 1) + 6*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*sqrt(3))/x^2
```

Sympy [A] time = 0.214274, size = 48, normalized size = 1.

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3 + 1/(2*x**2)
```

GIAC/XCAS [A] time = 0.211882, size = 51, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{3}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*ln(x^2
 + x + 1) + 1/3*ln(abs(x - 1))
```

3.176 $\int \frac{x^2}{1+x}\,dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - x + \log(x+1)$$

```
[Out] -x + x^2/2 + Log[1 + x]
```

Rubi [A] time = 0.0154363, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x^2}{2} - x + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x),x]
```

```
[Out] -x + x^2/2 + Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x + \log(x+1) + \int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+x),x)
```

```
[Out] -x + log(x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.00422826, size = 19, normalized size = 1.27

$$\frac{1}{2}(x+1)^2 - 2(x+1) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + x),x]
```

```
[Out] -2*(1 + x) + (1 + x)^2/2 + Log[1 + x]
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$-x + \frac{x^2}{2} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+x),x)
```

```
[Out] -x+1/2*x^2+ln(1+x)
```

Maxima [A] time = 1.35955, size = 18, normalized size = 1.2

$$\frac{1}{2}x^2 - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 1)
```

Fricas [A] time = 0.194772, size = 18, normalized size = 1.2

$$\frac{1}{2}x^2 - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + log(x + 1)
```

Sympy [A] time = 0.055608, size = 10, normalized size = 0.67

$$\frac{x^2}{2} - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+x),x)
```

```
[Out] x**2/2 - x + log(x + 1)
```

GIAC/XCAS [A] time = 0.208186, size = 19, normalized size = 1.27

$$\frac{1}{2}x^2 - x + \ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + ln(abs(x + 1))
```

3.177 $\int \frac{x}{-5+x}\,dx$

Optimal. Leaf size=10

$$x + 5\log(5 - x)$$

```
[Out] x + 5*Log[5 - x]
```

Rubi [A] time = 0.0118842, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$x + 5\log(5 - x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-5 + x),x]
```

```
[Out] x + 5*Log[5 - x]
```

Rubi in Sympy [A] time = 1.06624, size = 7, normalized size = 0.7

$$x + 5\log(-x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-5+x),x)
```

```
[Out] x + 5*log(-x + 5)
```

Mathematica [A] time = 0.00193654, size = 8, normalized size = 0.8

$$x + 5\log(x - 5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-5 + x),x]
```

```
[Out] x + 5*Log[-5 + x]
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$x + 5\ln(-5 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-5+x),x)
```

```
[Out] x+5*ln(-5+x)
```

Maxima [A] time = 1.3611, size = 11, normalized size = 1.1

$$x + 5\log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - 5),x, algorithm="maxima")
```

```
[Out] x + 5*log(x - 5)
```

Fricas [A] time = 0.195011, size = 11, normalized size = 1.1

$$x + 5 \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - 5),x, algorithm="fricas")
```

```
[Out] x + 5*log(x - 5)
```

Sympy [A] time = 0.0566, size = 7, normalized size = 0.7

$$x + 5 \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-5+x),x)
```

```
[Out] x + 5*log(x - 5)
```

GIAC/XCAS [A] time = 0.208451, size = 12, normalized size = 1.2

$$x + 5 \ln(|x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x - 5),x, algorithm="giac")
```

```
[Out] x + 5*ln(abs(x - 5))
```

3.178 $\int \frac{-1+4x}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=13

$$\log(1-x)+3\log(x+2)$$

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0206523, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Rubi in Sympy [A] time = 1.70712, size = 10, normalized size = 0.77

$$\log(-x+1)+3\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+4*x)/(-1+x)/(2+x),x)
```

```
[Out] log(-x + 1) + 3*log(x + 2)
```

Mathematica [A] time = 0.00609344, size = 13, normalized size = 1.

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Maple [A] time = 0.009, size = 12, normalized size = 0.9

$$3\ln(2+x)+\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+4*x)/(-1+x)/(2+x),x)
```

```
[Out] 3*ln(2+x)+ln(-1+x)
```

Maxima [A] time = 1.4057, size = 15, normalized size = 1.15

$$3\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x - 1)/((x + 2)*(x - 1)),x, algorithm="maxima")
```

```
[Out] 3*log(x + 2) + log(x - 1)
```

Fricas [A] time = 0.195854, size = 15, normalized size = 1.15

$$3 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x - 1)/((x + 2)*(x - 1)),x, algorithm="fricas")
```

```
[Out] 3*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.100406, size = 10, normalized size = 0.77

$$\log(x - 1) + 3 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+4*x)/(-1+x)/(2+x),x)
```

```
[Out] log(x - 1) + 3*log(x + 2)
```

GIAC/XCAS [A] time = 0.204396, size = 18, normalized size = 1.38

$$3 \ln(|x + 2|) + \ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x - 1)/((x + 2)*(x - 1)),x, algorithm="giac")
```

```
[Out] 3*ln(abs(x + 2)) + ln(abs(x - 1))
```

3.179 $\int \frac{1}{(1+x)(2+x)}\,dx$

Optimal. Leaf size=11

$$\log(x+1)-\log(x+2)$$

```
[Out] Log[1 + x] - Log[2 + x]
```

Rubi [A] time = 0.00801813, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x)*(2 + x)),x]

[Out] Log[1 + x] - Log[2 + x]
```

Rubi in Sympy [A] time = 1.05143, size = 8, normalized size = 0.73

$$\log(x+1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x)/(2+x),x)

[Out] log(x + 1) - log(x + 2)
```

Mathematica [A] time = 0.00433449, size = 11, normalized size = 1.

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((1 + x)*(2 + x)),x]

[Out] Log[1 + x] - Log[2 + x]
```

Maple [A] time = 0.007, size = 12, normalized size = 1.1

$$\ln(1+x)-\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(2+x),x)

[Out] ln(1+x)-ln(2+x)
```

Maxima [A] time = 1.39602, size = 15, normalized size = 1.36

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x + 1)),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Fricas [A] time = 0.195656, size = 15, normalized size = 1.36

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x + 1)),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A] time = 0.089869, size = 8, normalized size = 0.73

$$\log(x+1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(2+x),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

GIAC/XCAS [A] time = 0.206493, size = 18, normalized size = 1.64

$$-\ln(|x+2|)+\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 2)*(x + 1)),x, algorithm="giac")
```

```
[Out] -ln(abs(x + 2)) + ln(abs(x + 1))
```

3.180 $\int \frac{-5+6x}{3+2x}\,dx$

Optimal. Leaf size=12

$$3x - 7\log(2x+3)$$

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rubi [A] time = 0.0157124, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$3x - 7\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rubi in Sympy [A] time = 1.61684, size = 10, normalized size = 0.83

$$3x - 7\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Mathematica [A] time = 0.00435113, size = 15, normalized size = 1.25

$$3x - 7\log(2x+3) + \frac{9}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 9/2 + 3*x - 7*Log[3 + 2*x]
```

Maple [A] time = 0.003, size = 13, normalized size = 1.1

$$3x - 7\ln(3+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x-7*ln(3+2*x)
```

Maxima [A] time = 1.41085, size = 16, normalized size = 1.33

$$3x - 7\log(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x - 5)/(2*x + 3),x, algorithm="maxima")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Fricas [A] time = 0.189837, size = 16, normalized size = 1.33

$$3\,x - 7\,\log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x - 5)/(2*x + 3),x, algorithm="fricas")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Sympy [A] time = 0.060313, size = 10, normalized size = 0.83

$$3x - 7\log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x - 7*log(2*x + 3)
```

GIAC/XCAS [A] time = 0.207369, size = 18, normalized size = 1.5

$$3\,x - 7\ln\left(\left|2\,x + 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x - 5)/(2*x + 3),x, algorithm="giac")
```

```
[Out] 3*x - 7*ln(abs(2*x + 3))
```

3.181 $\int \frac{1}{(a+x)(b+x)}\,dx$

Optimal. Leaf size=26

$$\frac{\log(b+x)}{a-b} - \frac{\log(a+x)}{a-b}$$

```
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rubi [A] time = 0.0205634, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\log(b+x)}{a-b} - \frac{\log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + x)*(b + x)),x]
```

```
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rubi in Sympy [A] time = 1.79153, size = 15, normalized size = 0.58

$$-\frac{\log(a+x)}{a-b} + \frac{\log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a+x)/(b+x),x)
```

```
[Out] -log(a + x)/(a - b) + log(b + x)/(a - b)
```

Mathematica [A] time = 0.00975916, size = 19, normalized size = 0.73

$$\frac{\log(b+x) - \log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + x)*(b + x)),x]
```

```
[Out] (-Log[a + x] + Log[b + x])/(a - b)
```

Maple [A] time = 0.009, size = 27, normalized size = 1.

$$-\frac{\ln(a+x)}{a-b} + \frac{\ln(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a+x)/(b+x),x)
```

```
[Out] -ln(a+x)/(a-b)+ln(b+x)/(a-b)
```

Maxima [A] time = 1.36114, size = 35, normalized size = 1.35

$$-\frac{\log(a+x)}{a-b}+\frac{\log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + x)*(b + x)),x, algorithm="maxima")
```

```
[Out] -log(a + x)/(a - b) + log(b + x)/(a - b)
```

Fricas [A] time = 0.221043, size = 27, normalized size = 1.04

$$-\frac{\log(a+x)-\log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + x)*(b + x)),x, algorithm="fricas")
```

```
[Out] -(log(a + x) - log(b + x))/(a - b)
```

Sympy [A] time = 0.28664, size = 80, normalized size = 3.08

$$\frac{\log\left(-\frac{a^2}{2(a-b)}+\frac{ab}{a-b}+\frac{a}{2}-\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}-\frac{\log\left(\frac{a^2}{2(a-b)}-\frac{ab}{a-b}+\frac{a}{2}+\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a+x)/(b+x),x)
```

```
[Out] log(-a**2/(2*(a - b)) + a*b/(a - b) + a/2 - b**2/(2*(a - b)) + b/
2 + x)/(a - b) - log(a**2/(2*(a - b)) - a*b/(a - b) + a/2 + b**2/
(2*(a - b)) + b/2 + x)/(a - b)
```

GIAC/XCAS [A] time = 0.21192, size = 38, normalized size = 1.46

$$-\frac{\ln(|a+x|)}{a-b}+\frac{\ln(|b+x|)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a + x)*(b + x)),x, algorithm="giac")
```

```
[Out] -ln(abs(a + x))/(a - b) + ln(abs(b + x))/(a - b)
```

3.182 $\int \frac{1+x^2}{-x+x^2}\,dx$

Optimal. Leaf size=14

$$x + 2\log(1-x) - \log(x)$$

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0319346, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 2.18017, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/(x**2-x),x)
```

```
[Out] x - log(x) + 2*log(-x + 1)
```

Mathematica [A] time = 0.00492774, size = 14, normalized size = 1.

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.009, size = 13, normalized size = 0.9

$$x - \ln(x) + 2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/(x^2-x),x)
```

```
[Out] x-ln(x)+2*ln(-1+x)
```

Maxima [A] time = 1.35165, size = 16, normalized size = 1.14

$$x + 2\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 0.195945, size = 16, normalized size = 1.14

$$x + 2\,\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.095633, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/(x**2-x),x)
```

```
[Out] x - log(x) + 2*log(x - 1)
```

GIAC/XCAS [A] time = 0.205721, size = 19, normalized size = 1.36

$$x + 2\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/(x^2 - x),x, algorithm="giac")
```

```
[Out] x + 2*ln(abs(x - 1)) - ln(abs(x))
```

3.183 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx$

Optimal. Leaf size=26

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rubi [A] time = 0.0249968, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(-x+3)}{7}-\frac{\log(x+4)}{7}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] log(-x + 3)/7 - log(x + 4)/7 + Integral(x, x)
```

Mathematica [A] time = 0.00847475, size = 26, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] time = 0.009, size = 19, normalized size = 0.7

$$\frac{x^2}{2}+\frac{\ln(-3+x)}{7}-\frac{\ln(4+x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x^2-12*x+1)/(x^2+x-12),x)
```

```
[Out] 1/2*x^2+1/7*ln(-3+x)-1/7*ln(4+x)
```

Maxima [A] time = 1.42766, size = 24, normalized size = 0.92

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Fricas [A] time = 0.202811, size = 24, normalized size = 0.92

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] time = 0.089927, size = 17, normalized size = 0.65

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

GIAC/XCAS [A] time = 0.213389, size = 27, normalized size = 1.04

$$\frac{1}{2}x^2 - \frac{1}{7}\ln(|x+4|) + \frac{1}{7}\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 + x^2 - 12*x + 1)/(x^2 + x - 12),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/7*ln(abs(x + 4)) + 1/7*ln(abs(x - 3))
```

3.184 $\int \frac{3+2x}{(1+x)^2}\,dx$

Optimal. Leaf size=14

$$2\log(x+1)-\frac{1}{x+1}$$

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rubi [A] time = 0.014629, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rubi in Sympy [A] time = 1.45106, size = 10, normalized size = 0.71

$$2\log(x+1)-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(1+x)**2,x)
```

```
[Out] 2*log(x + 1) - 1/(x + 1)
```

Mathematica [A] time = 0.00560706, size = 14, normalized size = 1.

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Maple [A] time = 0.007, size = 15, normalized size = 1.1

$$-(1+x)^{-1}+2\,\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(1+x)^2,x)
```

```
[Out] -1/(1+x)+2*ln(1+x)
```

Maxima [A] time = 1.45698, size = 19, normalized size = 1.36

$$-\frac{1}{x+1}+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x + 1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x + 1) + 2*log(x + 1)
```

Fricas [A] time = 0.202169, size = 23, normalized size = 1.64

$$\frac{2(x+1)\log(x+1)-1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x + 1)^2,x, algorithm="fricas")
```

```
[Out] (2*(x + 1)*log(x + 1) - 1)/(x + 1)
```

Sympy [A] time = 0.074011, size = 10, normalized size = 0.71

$$2\log(x+1)-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(1+x)**2,x)
```

```
[Out] 2*log(x + 1) - 1/(x + 1)
```

GIAC/XCAS [A] time = 0.203052, size = 20, normalized size = 1.43

$$-\frac{1}{x+1}+2\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x + 1)^2,x, algorithm="giac")
```

```
[Out] -1/(x + 1) + 2*ln(abs(x + 1))
```

3.185 $\int \frac{1}{x(1+x)(3+2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rubi [A] time = 0.0272626, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rubi in Sympy [A] time = 1.89823, size = 19, normalized size = 0.83

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2\log(2x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+x)/(3+2*x),x)
```

```
[Out] log(x)/3 - log(x + 1) + 2*log(2*x + 3)/3
```

Mathematica [A] time = 0.00727353, size = 23, normalized size = 1.

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Maple [A] time = 0.01, size = 20, normalized size = 0.9

$$\frac{\ln(x)}{3} - \ln(1+x) + \frac{2\ln(3+2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+x)/(3+2*x),x)
```

```
[Out] 1/3*ln(x)-ln(1+x)+2/3*ln(3+2*x)
```

Maxima [A] time = 1.33825, size = 26, normalized size = 1.13

$$\frac{2}{3}\log(2x+3) - \log(x+1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x + 3)*(x + 1)*x),x, algorithm="maxima")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Fricas [A] time = 0.210755, size = 26, normalized size = 1.13

$$\frac{2}{3}\log(2x+3) - \log(x+1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x + 3)*(x + 1)*x),x, algorithm="fricas")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Sympy [A] time = 0.14694, size = 19, normalized size = 0.83

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2\log\left(x+\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+x)/(3+2*x),x)
```

```
[Out] log(x)/3 - log(x + 1) + 2*log(x + 3/2)/3
```

GIAC/XCAS [A] time = 0.21507, size = 30, normalized size = 1.3

$$\frac{2}{3}\ln(|2x+3|) - \ln(|x+1|) + \frac{1}{3}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((2*x + 3)*(x + 1)*x),x, algorithm="giac")
```

```
[Out] 2/3*ln(abs(2*x + 3)) - ln(abs(x + 1)) + 1/3*ln(abs(x))
```

3.186 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi [A] time = 0.0486953, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi in Sympy [A] time = 5.72484, size = 15, normalized size = 0.88

$$\log(x)+2\log(-x+1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(-x + 1) + 3*log(x + 3)
```

Mathematica [A] time = 0.00926639, size = 17, normalized size = 1.

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$\ln(x)+2\ln(-1+x)+3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
```

```
[Out] ln(x)+2*ln(-1+x)+3*ln(3+x)
```

Maxima [A] time = 1.35308, size = 20, normalized size = 1.18

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Fricas [A] time = 0.209134, size = 20, normalized size = 1.18

$$3\,\log\left(x+3\right)+2\,\log\left(x-1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] time = 0.142826, size = 15, normalized size = 0.88

$$\log\left(x\right)+2\log\left(x-1\right)+3\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

GIAC/XCAS [A] time = 0.202057, size = 24, normalized size = 1.41

$$3\ln\left(|x+3|\right)+2\ln\left(|x-1|\right)+\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((6*x^2 + 5*x - 3)/(x^3 + 2*x^2 - 3*x),x, algorithm="giac")
```

```
[Out] 3*ln(abs(x + 3)) + 2*ln(abs(x - 1)) + ln(abs(x))
```

3.187 $\int \frac{x}{4+4x+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{x+2} + \log(x+2)$$

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi [A] time = 0.0138489, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi in Sympy [A] time = 2.28869, size = 8, normalized size = 0.67

$$\log(x+2) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+4*x+4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

Mathematica [A] time = 0.0049511, size = 12, normalized size = 1.

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] time = 0.009, size = 13, normalized size = 1.1

$$2\,(2+x)^{-1} + \ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+4*x+4),x)
```

```
[Out] 2/(2+x)+ln(2+x)
```

Maxima [A] time = 1.32862, size = 16, normalized size = 1.33

$$\frac{2}{x+2} + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4*x + 4),x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

Fricas [A] time = 0.202383, size = 22, normalized size = 1.83

$$\frac{(x+2)\log(x+2)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4*x + 4),x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] time = 0.069793, size = 8, normalized size = 0.67

$$\log(x+2) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+4*x+4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

GIAC/XCAS [A] time = 0.209304, size = 18, normalized size = 1.5

$$\frac{2}{x+2} + \ln(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 4*x + 4),x, algorithm="giac")
```

```
[Out] 2/(x + 2) + ln(abs(x + 2))
```

3.188 $\int \frac{1}{(-1+x)^2(4+x)}\,dx$

Optimal. Leaf size=30

$$\frac{1}{5(1-x)} - \frac{1}{25}\log(1-x) + \frac{1}{25}\log(x+4)$$

```
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rubi [A] time = 0.0243878, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{5(1-x)} - \frac{1}{25}\log(1-x) + \frac{1}{25}\log(x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rubi in Sympy [A] time = 1.78079, size = 19, normalized size = 0.63

$$-\frac{\log(-x+1)}{25} + \frac{\log(x+4)}{25} + \frac{1}{5(-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**2/(4+x),x)
```

```
[Out] -log(-x + 1)/25 + log(x + 4)/25 + 1/(5*(-x + 1))
```

Mathematica [A] time = 0.0120141, size = 22, normalized size = 0.73

$$\frac{1}{25}\left(-\frac{5}{x-1} - \log(x-1) + \log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] (-5/(-1 + x) - Log[-1 + x] + Log[4 + x])/25
```

Maple [A] time = 0.01, size = 21, normalized size = 0.7

$$\frac{\ln(4+x)}{25} - \frac{1}{-5+5x} - \frac{\ln(-1+x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^2/(4+x),x)
```

```
[Out] 1/25*ln(4+x)-1/5/(-1+x)-1/25*ln(-1+x)
```

Maxima [A] time = 1.36408, size = 27, normalized size = 0.9

$$-\frac{1}{5\,(x-1)}+\frac{1}{25}\,\log\left(x+4\right)-\frac{1}{25}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 4)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] -1/5/(x - 1) + 1/25*log(x + 4) - 1/25*log(x - 1)
```

Fricas [A] time = 0.204318, size = 35, normalized size = 1.17

$$\frac{(x-1)\log\left(x+4\right)-(x-1)\log\left(x-1\right)-5}{25\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 4)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] 1/25*((x - 1)*log(x + 4) - (x - 1)*log(x - 1) - 5)/(x - 1)
```

Sympy [A] time = 0.118058, size = 19, normalized size = 0.63

$$-\frac{\log\left(x-1\right)}{25}+\frac{\log\left(x+4\right)}{25}-\frac{1}{5x-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**2/(4+x),x)
```

```
[Out] -log(x - 1)/25 + log(x + 4)/25 - 1/(5*x - 5)
```

GIAC/XCAS [A] time = 0.20437, size = 28, normalized size = 0.93

$$-\frac{1}{5\,(x-1)}+\frac{1}{25}\ln\left(\left|-\frac{5}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 4)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] -1/5/(x - 1) + 1/25*ln(abs(-5/(x - 1) - 1))
```

3.189 $\int \frac{x^2}{(-3+x)(2+x)^2}\,dx$

Optimal. Leaf size=28

$$\frac{4}{5(x+2)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(x+2)$$

```
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rubi [A] time = 0.0325676, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{4}{5(x+2)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rubi in Sympy [A] time = 2.05015, size = 22, normalized size = 0.79

$$\frac{9\log(-x+3)}{25} + \frac{16\log(x+2)}{25} + \frac{4}{5(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-3+x)/(2+x)**2,x)
```

```
[Out] 9*log(-x + 3)/25 + 16*log(x + 2)/25 + 4/(5*(x + 2))
```

Mathematica [A] time = 0.0235232, size = 26, normalized size = 0.93

$$\frac{4}{5(x+2)} + \frac{9}{25}\log(x-3) + \frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[-3 + x])/25 + (16*Log[2 + x])/25
```

Maple [A] time = 0.01, size = 21, normalized size = 0.8

$$\frac{9\,\ln(-3+x)}{25} + \frac{4}{10+5\,x} + \frac{16\,\ln(2+x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-3+x)/(2+x)^2,x)
```

```
[Out] 9/25*ln(-3+x)+4/5/(2+x)+16/25*ln(2+x)
```

Maxima [A] time = 1.36354, size = 27, normalized size = 0.96

$$\frac{4}{5\,(x+2)} + \frac{16}{25}\,\log(x+2) + \frac{9}{25}\,\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x + 2)^2*(x - 3)),x, algorithm="maxima")
```

```
[Out] 4/5/(x + 2) + 16/25*log(x + 2) + 9/25*log(x - 3)
```

Fricas [A] time = 0.203826, size = 36, normalized size = 1.29

$$\frac{16\,(x+2)\log(x+2) + 9\,(x+2)\log(x-3) + 20}{25\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x + 2)^2*(x - 3)),x, algorithm="fricas")
```

```
[Out] 1/25*(16*(x + 2)*log(x + 2) + 9*(x + 2)*log(x - 3) + 20)/(x + 2)
```

Sympy [A] time = 0.129302, size = 22, normalized size = 0.79

$$\frac{9\log(x-3)}{25} + \frac{16\log(x+2)}{25} + \frac{4}{5x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-3+x)/(2+x)**2,x)
```

```
[Out] 9*log(x - 3)/25 + 16*log(x + 2)/25 + 4/(5*x + 10)
```

GIAC/XCAS [A] time = 0.206013, size = 35, normalized size = 1.25

$$\frac{4}{5\,(x+2)} + \ln(|x+2|) + \frac{9}{25}\ln\left(\left|-\frac{5}{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/((x + 2)^2*(x - 3)),x, algorithm="giac")
```

```
[Out] 4/5/(x + 2) + ln(abs(x + 2)) + 9/25*ln(abs(-5/(x + 2) + 1))
```

3.190 $\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx$

Optimal. Leaf size=14

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0420218, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi in Sympy [A] time = 3.16199, size = 14, normalized size = 1.

$$2\log(x) + 3\log(x+2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Mathematica [A] time = 0.00573666, size = 14, normalized size = 1.

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] time = 0.01, size = 15, normalized size = 1.1

$$x^{-1} + 2\ln(x) + 3\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Maxima [A] time = 1.37504, size = 19, normalized size = 1.36

$$\frac{1}{x} + 3\,\log(x+2) + 2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Fricas [A] time = 0.205905, size = 24, normalized size = 1.71

$$\frac{3\,x\log(x+2) + 2\,x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.115328, size = 14, normalized size = 1.

$$2\log(x) + 3\log(x+2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

GIAC/XCAS [A] time = 0.211322, size = 22, normalized size = 1.57

$$\frac{1}{x} + 3\ln(|x+2|) + 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 3*x - 2)/(x^3 + 2*x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*ln(abs(x + 2)) + 2*ln(abs(x))
```

3.191 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi [A] time = 0.0477299, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-4x^2-2x+18}{x^3+4x^2+x-6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

```
[Out] Integral((-4*x**2 - 2*x + 18)/(x**3 + 4*x**2 + x - 6), x)
```

Mathematica [A] time = 0.0108794, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-Log[1 - x]/2 + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] time = 0.012, size = 18, normalized size = 1.

$$-2\,\ln(2+x)+\ln(-1+x)-3\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
```

```
[Out] -2*ln(2+x)+ln(-1+x)-3*ln(3+x)
```

Maxima [A] time = 1.35688, size = 23, normalized size = 1.21

$$-3\log(x+3) - 2\log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="maxima")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Fricas [A] time = 0.217169, size = 23, normalized size = 1.21

$$-3\log(x+3) - 2\log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.138196, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

```
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

GIAC/XCAS [A] time = 0.203251, size = 27, normalized size = 1.42

$$-3\ln(|x+3|) - 2\ln(|x+2|) + \ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*(2*x^2 + x - 9)/(x^3 + 4*x^2 + x - 6),x, algorithm="giac")
```

```
[Out] -3*ln(abs(x + 3)) - 2*ln(abs(x + 2)) + ln(abs(x - 1))
```

3.192 $\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi [A] time = 0.00740857, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi in Sympy [A] time = 1.83027, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

Mathematica [A] time = 0.00699195, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^3+3\,x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x)/(x^3+3*x^2+4),x)
```

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Maxima [A] time = 1.34131, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+3\,x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Fricas [A] time = 0.195751, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^3+3\,x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] time = 0.080698, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

GIAC/XCAS [A] time = 0.209547, size = 19, normalized size = 1.27

$$\frac{1}{3}\ln\left(\left|x^3+3\,x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x)/(x^3 + 3*x^2 + 4),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(x^3 + 3*x^2 + 4))
```

3.193 $\int \frac{1}{(-1+x)^2 x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{1-x} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

```
[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Rubi [A] time = 0.0204274, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{1-x} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-1 + x)^2*x^2),x]
```

```
[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Rubi in Sympy [A] time = 1.53029, size = 19, normalized size = 0.76

$$2\log(x) - 2\log(-x+1) + \frac{1}{-x+1} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1+x)**2/x**2,x)
```

```
[Out] 2*log(x) - 2*log(-x + 1) + 1/(-x + 1) - 1/x
```

Mathematica [A] time = 0.0187926, size = 25, normalized size = 1.

$$-\frac{1}{x-1} - \frac{1}{x} - 2\log(1-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((-1 + x)^2*x^2),x]
```

```
[Out] -(-1 + x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] time = 0.01, size = 24, normalized size = 1.

$$-x^{-1} + 2\ln(x) - (-1+x)^{-1} - 2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1+x)^2/x^2,x)
```

```
[Out] -1/x+2*ln(x)-1/(-1+x)-2*ln(-1+x)
```

Maxima [A] time = 1.38095, size = 36, normalized size = 1.44

$$-\frac{2\,x-1}{x^2-x}-2\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^2*x^2),x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - x) - 2*log(x - 1) + 2*log(x)
```

Fricas [A] time = 0.198302, size = 54, normalized size = 2.16

$$-\frac{2\left(x^2-x\right)\log\left(x-1\right)-2\left(x^2-x\right)\log\left(x\right)+2\,x-1}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^2*x^2),x, algorithm="fricas")
```

```
[Out] -(2*(x^2 - x)*log(x - 1) - 2*(x^2 - x)*log(x) + 2*x - 1)/(x^2 - x
)
```

Sympy [A] time = 0.11697, size = 20, normalized size = 0.8

$$-\frac{2x-1}{x^2-x}+2\log\left(x\right)-2\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1+x)**2/x**2,x)
```

```
[Out] -(2*x - 1)/(x**2 - x) + 2*log(x) - 2*log(x - 1)
```

GIAC/XCAS [A] time = 0.207611, size = 41, normalized size = 1.64

$$-\frac{1}{x-1}+\frac{1}{\frac{1}{x-1}+1}+2\ln\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x - 1)^2*x^2),x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/(1/(x - 1) + 1) + 2*ln(abs(-1/(x - 1) - 1))
```

3.194 $\int \frac{x^2}{(1+x)^3}\,dx$

Optimal. Leaf size=21

$$\frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Rubi [A] time = 0.0186345, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Rubi in Sympy [A] time = 1.5072, size = 17, normalized size = 0.81

$$\log(x+1) + \frac{2}{x+1} - \frac{1}{2(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(1+x)**3,x)
```

```
[Out] log(x + 1) + 2/(x + 1) - 1/(2*(x + 1)**2)
```

Mathematica [A] time = 0.0139596, size = 21, normalized size = 1.

$$\frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Maple [A] time = 0.007, size = 20, normalized size = 1.

$$-\frac{1}{2(1+x)^2} + 2(1+x)^{-1} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(1+x)^3,x)
```

```
[Out] -1/2/(1+x)^2+2/(1+x)+ln(1+x)
```

Maxima [A] time = 1.36583, size = 30, normalized size = 1.43

$$\frac{4\,x+3}{2\,(x^2+2\,x+1)}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*x + 3)/(x^2 + 2*x + 1) + log(x + 1)
```

Fricas [A] time = 0.189447, size = 42, normalized size = 2.

$$\frac{2\left(x^2+2\,x+1\right)\log(x+1)+4\,x+3}{2\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 2*x + 1)*log(x + 1) + 4*x + 3)/(x^2 + 2*x + 1)
```

Sympy [A] time = 0.092839, size = 19, normalized size = 0.9

$$\frac{4x+3}{2x^2+4x+2}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(1+x)**3,x)
```

```
[Out] (4*x + 3)/(2*x**2 + 4*x + 2) + log(x + 1)
```

GIAC/XCAS [A] time = 0.208517, size = 24, normalized size = 1.14

$$\frac{4\,x+3}{2\,(x+1)^2}+\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x + 1)^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*x + 3)/(x + 1)^2 + ln(abs(x + 1))
```

3.195 $\int \frac{1}{-x^2+x^4}\,dx$

Optimal. Leaf size=8

$$\frac{1}{x} - \tanh^{-1}(x)$$

```
[Out] x^(-1) - ArcTanh[x]
```

Rubi [A] time = 0.0119814, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{1}{x} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) - ArcTanh[x]
```

Rubi in Sympy [A] time = 1.70363, size = 5, normalized size = 0.62

$$-\operatorname{atanh}(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-x**2),x)
```

```
[Out] -atanh(x) + 1/x
```

Mathematica [B] time = 0.00399979, size = 22, normalized size = 2.75

$$\frac{1}{x} + \frac{1}{2}\log(1-x) - \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) + Log[1 - x]/2 - Log[1 + x]/2
```

Maple [A] time = 0.008, size = 17, normalized size = 2.1

$$-\frac{\ln(1+x)}{2} + x^{-1} + \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-x^2),x)
```

```
[Out] -1/2*ln(1+x)+1/x+1/2*ln(-1+x)
```

Maxima [A] time = 1.36111, size = 22, normalized size = 2.75

$$\frac{1}{x} - \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2),x, algorithm="maxima")
```

```
[Out] 1/x - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 0.195611, size = 27, normalized size = 3.38

$$-\frac{x\log(x+1) - x\log(x-1) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [A] time = 0.099833, size = 15, normalized size = 1.88

$$\frac{\log(x-1)}{2} - \frac{\log(x+1)}{2} + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-x**2),x)
```

```
[Out] log(x - 1)/2 - log(x + 1)/2 + 1/x
```

GIAC/XCAS [A] time = 0.204851, size = 24, normalized size = 3.

$$\frac{1}{x} - \frac{1}{2}\ln(|x+1|) + \frac{1}{2}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2),x, algorithm="giac")
```

```
[Out] 1/x - 1/2*ln(abs(x + 1)) + 1/2*ln(abs(x - 1))
```

3.196 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[Out] Log[1 - x^2 + x^4]/2
```

Rubi [A] time = 0.00774903, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rubi in Sympy [A] time = 2.16588, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

Mathematica [A] time = 0.00636606, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^4-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^3-x)/(x^4-x^2+1),x)
```

```
[Out] 1/2*ln(x^4-x^2+1)
```

Maxima [A] time = 1.33633, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Fricas [A] time = 0.194308, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] time = 0.089786, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.207107, size = 18, normalized size = 1.2

$$\frac{1}{2}\ln\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^3 - x)/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^4 - x^2 + 1)
```

3.197 $\int \frac{x^3}{1+x^2}\,dx$

Optimal. Leaf size=18

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0201464, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 1.89675, size = 12, normalized size = 0.67

$$\frac{x^2}{2} - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**2+1),x)
```

```
[Out] x**2/2 - log(x**2 + 1)/2
```

Mathematica [A] time = 0.00331598, size = 18, normalized size = 1.

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0.001, size = 15, normalized size = 0.8

$$\frac{x^2}{2} - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^2+1),x)
```

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Maxima [A] time = 1.41546, size = 19, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.193889, size = 19, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.063491, size = 12, normalized size = 0.67

$$\frac{x^2}{2} - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**2+1),x)
```

```
[Out] x**2/2 - log(x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.205198, size = 19, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*ln(x^2 + 1)
```

3.198 $\int \frac{-1+x}{2+2x+x^2}\,dx$

Optimal. Leaf size=20

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Rubi [A] time = 0.019214, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x)/(2 + 2*x + x^2),x]
```

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Rubi in Sympy [A] time = 2.42106, size = 17, normalized size = 0.85

$$\frac{\log\left(x^2+2x+2\right)}{2}-2\operatorname{atan}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)/(x**2+2*x+2),x)
```

```
[Out] log(x**2 + 2*x + 2)/2 - 2*atan(x + 1)
```

Mathematica [A] time = 0.00644126, size = 20, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x)/(2 + 2*x + x^2),x]
```

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Maple [A] time = 0.003, size = 19, normalized size = 1.

$$-2\arctan(1+x)+\frac{\ln\left(x^2+2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)/(x^2+2*x+2),x)
```

```
[Out] -2*arctan(1+x)+1/2*ln(x^2+2*x+2)
```

Maxima [A] time = 1.49476, size = 24, normalized size = 1.2

$$-2\arctan(x+1)+\frac{1}{2}\log\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 2*x + 2),x, algorithm="maxima")
```

```
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Fricas [A] time = 0.198663, size = 24, normalized size = 1.2

$$-2\arctan(x+1)+\frac{1}{2}\log\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 2*x + 2),x, algorithm="fricas")
```

```
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Sympy [A] time = 0.104379, size = 17, normalized size = 0.85

$$\frac{\log\left(x^2+2x+2\right)}{2}-2\operatorname{atan}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)/(x**2+2*x+2),x)
```

```
[Out] log(x**2 + 2*x + 2)/2 - 2*atan(x + 1)
```

GIAC/XCAS [A] time = 0.206936, size = 24, normalized size = 1.2

$$-2\arctan(x+1)+\frac{1}{2}\ln\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)/(x^2 + 2*x + 2),x, algorithm="giac")
```

```
[Out] -2*arctan(x + 1) + 1/2*ln(x^2 + 2*x + 2)
```

3.199 $\int \frac{x}{1+x+x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0308665, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rubi in Sympy [A] time = 1.96419, size = 31, normalized size = 1.

$$\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+x+1),x)
```

```
[Out] log(x**2 + x + 1)/2 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/3
```

Mathematica [A] time = 0.0123744, size = 31, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Maple [A] time = 0.002, size = 27, normalized size = 0.9

$$\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+x+1),x)
```

```
[Out] 1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.51573, size = 35, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Fricas [A] time = 0.19494, size = 41, normalized size = 1.32

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - 2*arctan(1/3*sqrt(3)*(2*x
 + 1)))
```

Sympy [A] time = 0.099489, size = 34, normalized size = 1.1

$$\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x+1),x)
```

```
[Out] log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.20532, size = 35, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*ln(x^2 + x + 1)
```

3.200 $\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rubi [A] time = 0.0402132, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rubi in Sympy [A] time = 5.85108, size = 24, normalized size = 0.89

$$x+\frac{\log\left(4x^2+4x+5\right)}{8}+\frac{3\operatorname{atan}\left(x+\frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(4*x**2 + 4*x + 5)/8 + 3*atan(x + 1/2)/8
```

Mathematica [A] time = 0.00769655, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{\ln\left(4x^2+4x+5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
```

```
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Maxima [A] time = 1.50444, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\log\left(4x^2 + 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 5*x + 7)/(4*x^2 + 4*x + 5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Fricas [A] time = 0.197281, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\log\left(4x^2 + 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 5*x + 7)/(4*x^2 + 4*x + 5),x, algorithm="fricas")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A] time = 0.117667, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

GIAC/XCAS [A] time = 0.206186, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\ln\left(4x^2 + 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 5*x + 7)/(4*x^2 + 4*x + 5),x, algorithm="giac")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*ln(4*x^2 + 4*x + 5)
```

3.201 $\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0555433, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 3.86313, size = 19, normalized size = 0.83

$$2\log(-x+1)+\frac{\log\left(x^2+1\right)}{2}-3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(-x + 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Mathematica [A] time = 0.0111344, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] time = 0.009, size = 20, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{2}-3\arctan(x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-4*x+5)/(-1+x)/(x^2+1),x)
```

```
[Out] 1/2*ln(x^2+1)-3*arctan(x)+2*ln(-1+x)
```

Maxima [A] time = 1.49311, size = 26, normalized size = 1.13

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="maxima")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.202086, size = 26, normalized size = 1.13

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2 + 1\right) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="fricas")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.148641, size = 19, normalized size = 0.83

$$2 \log(x - 1) + \frac{\log\left(x^2 + 1\right)}{2} - 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

GIAC/XCAS [A] time = 0.207591, size = 27, normalized size = 1.17

$$-3 \arctan(x) + \frac{1}{2} \ln\left(x^2 + 1\right) + 2 \ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x, algorithm="giac")
```

```
[Out] -3*arctan(x) + 1/2*ln(x^2 + 1) + 2*ln(abs(x - 1))
```

3.202 $\int \frac{3+2x}{3x+x^3}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rubi [A] time = 0.0492847, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rubi in Sympy [A] time = 3.15747, size = 29, normalized size = 1.04

$$\log(x)-\frac{\log\left(x^2+3\right)}{2}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(x**3+3*x),x)
```

```
[Out] log(x) - log(x**2 + 3)/2 + 2*sqrt(3)*atan(sqrt(3)*x/3)/3
```

Mathematica [A] time = 0.0125286, size = 28, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^2+3\right)}{2}+\frac{2\sqrt{3}}{3}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(x^3+3*x),x)
```

```
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.5114, size = 31, normalized size = 1.11

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{2}\log\left(x^2+3\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x^3 + 3*x),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

Fricas [A] time = 0.204753, size = 43, normalized size = 1.54

$$-\frac{1}{6}\sqrt{3}\left(\sqrt{3}\log\left(x^2+3\right) - 2\sqrt{3}\log\left(x\right) - 4\arctan\left(\frac{1}{3}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x^3 + 3*x),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*(sqrt(3)*log(x^2 + 3) - 2*sqrt(3)*log(x) - 4*arctan(
1/3*sqrt(3)*x))
```

Sympy [A] time = 0.153047, size = 29, normalized size = 1.04

$$\log\left(x\right) - \frac{\log\left(x^2+3\right)}{2} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(x**3+3*x),x)
```

```
[Out] log(x) - log(x**2 + 3)/2 + 2*sqrt(3)*atan(sqrt(3)*x/3)/3
```

GIAC/XCAS [A] time = 0.204067, size = 32, normalized size = 1.14

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{2}\ln\left(x^2+3\right) + \ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(x^3 + 3*x),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*ln(x^2 + 3) + ln(abs(x))
```

3.203 $\int \frac{1}{-1+x^3}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x +
 x^2]/6
```

Rubi [A] time = 0.0395499, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x +
 x^2]/6
```

Rubi in Sympy [A] time = 2.50005, size = 37, normalized size = 0.9

$$\frac{\log(-x+1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-1),x)
```

```
[Out] log(-x + 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*x/3
 + 1/3))/3
```

Mathematica [A] time = 0.00903536, size = 41, normalized size = 1.

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x +
 x^2]/6
```

Maple [A] time = 0.003, size = 33, normalized size = 0.8

$$\frac{\ln(-1+x)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-1),x)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1
/2)
```

Maxima [A] time = 1.51389, size = 43, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

Fricas [A] time = 0.203758, size = 53, normalized size = 1.29

$$-\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-2\sqrt{3}\log\left(x-1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - 2*sqrt(3)*log(x - 1) +
6*arctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.171504, size = 41, normalized size = 1.

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.205249, size = 45, normalized size = 1.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*ln(x^2 + x + 1)
+ 1/3*ln(abs(x - 1))
```

3.204 $\int \frac{x^3}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x
+ x^2]/6
```

Rubi [A] time = 0.0496655, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x
+ x^2]/6
```

Rubi in Sympy [A] time = 4.22711, size = 39, normalized size = 0.95

$$x-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**3+1),x)
```

```
[Out] x - log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(sqrt(3)*(2*
x/3 - 1/3))/3
```

Mathematica [A] time = 0.013335, size = 42, normalized size = 1.02

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x^3),x]
```

```
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x
 + x^2]/6
```

Maple [A] time = 0.009, size = 36, normalized size = 0.9

$$x-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^3+1),x)
```

```
[Out] x-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

Maxima [A] time = 1.48685, size = 47, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x
+ 1) - 1/3*log(x + 1)
```

Fricas [A] time = 0.201715, size = 63, normalized size = 1.54

$$\frac{1}{18}\sqrt{3}\left(6\sqrt{3}x+\sqrt{3}\log\left(x^2-x+1\right)-2\sqrt{3}\log\left(x+1\right)-6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(6*sqrt(3)*x + sqrt(3)*log(x^2 - x + 1) - 2*sqrt(3)*
log(x + 1) - 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.175274, size = 42, normalized size = 1.02

$$x-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**3+1),x)
```

```
[Out] x - log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x
/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.206819, size = 49, normalized size = 1.2

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{1}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^3 + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*ln(x^2 - x +
 1) - 1/3*ln(abs(x + 1))
```

3.205 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0551849, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 3.94718, size = 20, normalized size = 0.83

$$\log(-x+1)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)-\frac{1}{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(-x + 1) - log(x**2 + 1)/2 + atan(x) - 1/(-x + 1)
```

Mathematica [A] time = 0.0224286, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] time = 0.01, size = 21, normalized size = 0.9

$$-\frac{\ln\left(x^2+1\right)}{2}+\arctan(x)+\ln(-1+x)+(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-2*x-1)/(-1+x)^2/(x^2+1),x)
```

```
[Out] -1/2*ln(x^2+1)+arctan(x)+ln(-1+x)+1/(-1+x)
```

Maxima [A] time = 1.50396, size = 27, normalized size = 1.12

$$\frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 0.199105, size = 49, normalized size = 2.04

$$\frac{2(x-1)\arctan(x) - (x-1)\log\left(x^2+1\right) + 2(x-1)\log(x-1) + 2}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) + 2)/(x - 1)
```

Sympy [A] time = 0.157644, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

GIAC/XCAS [A] time = 0.210148, size = 63, normalized size = 2.62

$$\frac{1}{4}\pi - \pi\left\lfloor \frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\ln\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - 2*x - 1)/((x^2 + 1)*(x - 1)^2),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) +
arctan(x) - 1/2*ln(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.206 $\int \frac{x^4}{-1+x^4}\,dx$

Optimal. Leaf size=14

$$x - \frac{1}{2}\tan^{-1}(x) - \frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x - ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.0149922, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$x - \frac{1}{2}\tan^{-1}(x) - \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(-1 + x^4),x]
```

```
[Out] x - ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi in Sympy [A] time = 1.44532, size = 10, normalized size = 0.71

$$x - \frac{\operatorname{atan}(x)}{2} - \frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**4-1),x)
```

```
[Out] x - atan(x)/2 - atanh(x)/2
```

Mathematica [A] time = 0.00656541, size = 26, normalized size = 1.86

$$x + \frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+1) - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(-1 + x^4),x]
```

```
[Out] x - ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0.006, size = 19, normalized size = 1.4

$$x + \frac{\ln(-1+x)}{4} - \frac{\ln(1+x)}{4} - \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^4-1),x)
```

```
[Out] x+1/4*ln(-1+x)-1/4*ln(1+x)-1/2*arctan(x)
```

Maxima [A] time = 1.48974, size = 24, normalized size = 1.71

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 - 1),x, algorithm="maxima")
```

```
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 0.202815, size = 24, normalized size = 1.71

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 - 1),x, algorithm="fricas")
```

```
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.169575, size = 19, normalized size = 1.36

$$x + \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**4-1),x)
```

```
[Out] x + log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

GIAC/XCAS [A] time = 0.210215, size = 27, normalized size = 1.93

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\ln(|x+1|) + \frac{1}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 - 1),x, algorithm="giac")
```

```
[Out] x - 1/2*arctan(x) - 1/4*ln(abs(x + 1)) + 1/4*ln(abs(x - 1))
```

3.207 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi [A] time = 0.190829, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi in Sympy [A] time = 17.2487, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2+1\right)}{2}-3\operatorname{atan}(x)+\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Mathematica [A] time = 0.0241885, size = 29, normalized size = 1.

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] time = 0.007, size = 25, normalized size = 0.9

$$-3\arctan(x)+\frac{3\ln\left(x^2+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
```

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.47778, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Fricas [A] time = 0.204457, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] time = 0.262575, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2+1\right)}{2} - 3\operatorname{atan}(x) + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

GIAC/XCAS [A] time = 0.213332, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3 - x^2 + 6*x - 4)/((x^2 + 2)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*ln(x^2 + 1)
```

3.208 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0478941, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi in Sympy [A] time = 8.10866, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\operatorname{atan}\left(\frac{x}{2}\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Mathematica [A] time = 0.0144226, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.008, size = 18, normalized size = 0.8

$$-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
```

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.5113, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Fricas [A] time = 0.203852, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] time = 0.249894, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.211046, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\ln\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3 - 2*x^2 + x + 1)/(x^4 + 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*ln(x^2 + 4)
```

3.209 $\int \frac{-3+x}{(4+2x+x^2)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{4x+7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*S
qrt[3])
```

Rubi [A] time = 0.0321465, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{4x+7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] -(7 + 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*S
qrt[3])
```

Rubi in Sympy [A] time = 2.00999, size = 37, normalized size = 0.95

$$-\frac{8x+14}{12\left(x^2+2x+4\right)}-\frac{2\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{x}{3}+\frac{1}{3}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+x)/(x**2+2*x+4)**2,x)
```

```
[Out] -(8*x + 14)/(12*(x**2 + 2*x + 4)) - 2*sqrt(3)*atan(sqrt(3)*(x/3 +
 1/3))/9
```

Mathematica [A] time = 0.0370192, size = 39, normalized size = 1.

$$\frac{-4x-7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] (-7 - 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*S
qrt[3])
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$\frac{-8\,x-14}{12\,x^2+24\,x+48}-\frac{2\,\sqrt{3}}{9}\arctan\left(\frac{(2\,x+2)\,\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+x)/(x^2+2*x+4)^2,x)
```

```
[Out] 1/12*(-8*x-14)/(x^2+2*x+4)-2/9*3^(1/2)*arctan(1/6*(2*x+2)*3^(1/2)
)
```

Maxima [A] time = 1.50289, size = 43, normalized size = 1.1

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{4x+7}{6\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^2 + 2*x + 4)^2,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2
*x + 4)
```

Fricas [A] time = 0.197225, size = 59, normalized size = 1.51

$$-\frac{\sqrt{3}\left(4\left(x^2+2x+4\right)\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)+\sqrt{3}(4x+7)\right)}{18\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^2 + 2*x + 4)^2,x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*(4*(x^2 + 2*x + 4)*arctan(1/3*sqrt(3)*(x + 1)) + sq
rt(3)*(4*x + 7))/(x^2 + 2*x + 4)
```

Sympy [A] time = 0.142352, size = 41, normalized size = 1.05

$$-\frac{4x+7}{6x^2+12x+24}-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+x)/(x**2+2*x+4)**2,x)
```

```
[Out] -(4*x + 7)/(6*x**2 + 12*x + 24) - 2*sqrt(3)*atan(sqrt(3)*x/3 + sq
rt(3)/3)/9
```

GIAC/XCAS [A] time = 0.210942, size = 43, normalized size = 1.1

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{4x+7}{6\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^2 + 2*x + 4)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2
*x + 4)
```

3.210 $\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1}+\log(x)$$

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi [A] time = 0.0477536, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi in Sympy [A] time = 3.57674, size = 12, normalized size = 1.2

$$\frac{\log\left(x^2\right)}{2}+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x**2)/2 + 1/(x**2 + 1)
```

Mathematica [A] time = 0.00844627, size = 10, normalized size = 1.

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] time = 0., size = 11, normalized size = 1.1

$$\left(x^2+1\right)^{-1}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/x/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)+ln(x)
```

Maxima [A] time = 1.38651, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1} + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 0.193307, size = 24, normalized size = 2.4

$$\frac{\left(x^2+1\right)\log\left(x\right)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.104472, size = 8, normalized size = 0.8

$$\log\left(x\right)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x) + 1/(x**2 + 1)
```

GIAC/XCAS [A] time = 0.211513, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1} + \frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="giac")
```

```
[Out] 1/(x^2 + 1) + 1/2*ln(x^2)
```

3.211 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log\left(\sin^2(x) - 3\sin(x) + 2\right)$$

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rubi [A] time = 0.0673052, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\log\left(\sin^2(x) - 3\sin(x) + 2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rubi in Sympy [A] time = 6.96482, size = 12, normalized size = 1.09

$$\log\left(\sin^2(x) - 3\sin(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x)**2 - 3*sin(x) + 2)
```

Mathematica [B] time = 0.0625948, size = 26, normalized size = 2.36

$$\log(2 - \sin(x)) + 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

Maple [A] time = 0.043, size = 12, normalized size = 1.1

$$\ln\left(2 - 3\,\sin(x) + (\sin(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)
```

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Maxima [A] time = 1.3412, size = 15, normalized size = 1.36

$$\log\left(\sin(x)^2 - 3\,\sin(x) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sin(x) - 3)*cos(x)/(sin(x)^2 - 3*sin(x) + 2),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

Fricas [A] time = 0.228665, size = 20, normalized size = 1.82

$$\log\left(-\frac{1}{2}\,\sin(x) + 1\right) + \log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sin(x) - 3)*cos(x)/(sin(x)^2 - 3*sin(x) + 2),x, algorithm="fricas")
```

```
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

Sympy [A] time = 0.284128, size = 12, normalized size = 1.09

$$\log(\sin(x) - 2) + \log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

GIAC/XCAS [A] time = 0.213584, size = 20, normalized size = 1.82

$$\ln(-\sin(x) + 2) + \ln(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*sin(x) - 3)*cos(x)/(sin(x)^2 - 3*sin(x) + 2),x, algorithm="giac")
```

```
[Out] ln(-sin(x) + 2) + ln(-sin(x) + 1)
```

3.212 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rubi [A] time = 0.079717, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rubi in Sympy [A] time = 8.51223, size = 19, normalized size = 0.95

$$-\cos(x)+\sqrt{5}\,\text{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

```
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

Mathematica [B] time = 0.226915, size = 82, normalized size = 4.1

$$\frac{1}{20}\left(-20\cos(x)+21\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right)+21\sqrt{5}\tan^{-1}\left(\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)+\frac{1}{\sqrt{5}}\right)-\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5]
- Sqrt[6/5]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*T
an[x/2]] - 20*Cos[x])/20
```

Maple [A] time = 0.02, size = 18, normalized size = 0.9

$$-\cos(x)+\arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)/(5+cos(x)^2),x)
```

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.49503, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)/(cos(x)^2 + 5),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Fricas [A] time = 0.25907, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)/(cos(x)^2 + 5),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Sympy [A] time = 1.04585, size = 19, normalized size = 0.95

$$-\cos(x) + \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

```
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

GIAC/XCAS [A] time = 0.210227, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)/(cos(x)^2 + 5),x, algorithm="giac")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.213 $\int \frac{1}{-3+2x+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Rubi [A] time = 0.009731, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 2*x + x^2)^(-1),x]
```

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Rubi in Sympy [A] time = 0.707519, size = 12, normalized size = 0.63

$$\frac{\log(-x+1)}{4}-\frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+2*x-3),x)
```

```
[Out] log(-x + 1)/4 - log(x + 3)/4
```

Mathematica [A] time = 0.00414442, size = 19, normalized size = 1.

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 2*x + x^2)^(-1),x]
```

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Maple [A] time = 0.008, size = 14, normalized size = 0.7

$$\frac{\ln(-1+x)}{4}-\frac{\ln(3+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+2*x-3),x)
```

```
[Out] 1/4*ln(-1+x)-1/4*ln(3+x)
```

Maxima [A] time = 1.36491, size = 18, normalized size = 0.95

$$-\frac{1}{4}\log(x+3)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x - 3),x, algorithm="maxima")
```

```
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Fricas [A] time = 0.198622, size = 18, normalized size = 0.95

$$-\frac{1}{4}\log(x+3)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x - 3),x, algorithm="fricas")
```

```
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Sympy [A] time = 0.088778, size = 12, normalized size = 0.63

$$\frac{\log(x-1)}{4}-\frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+2*x-3),x)
```

```
[Out] log(x - 1)/4 - log(x + 3)/4
```

GIAC/XCAS [A] time = 0.208498, size = 20, normalized size = 1.05

$$-\frac{1}{4}\ln(|x+3|)+\frac{1}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x - 3),x, algorithm="giac")
```

```
[Out] -1/4*ln(abs(x + 3)) + 1/4*ln(abs(x - 1))
```

3.214 $\int \frac{1}{-2x+x^2}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rubi [A] time = 0.00907664, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rubi in Sympy [A] time = 1.10927, size = 10, normalized size = 0.59

$$-\frac{\log(x)}{2}+\frac{\log(-x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x),x)
```

```
[Out] -log(x)/2 + log(-x + 2)/2
```

Mathematica [A] time = 0.0025605, size = 17, normalized size = 1.

$$\frac{1}{2}\log(2-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Maple [A] time = 0.007, size = 12, normalized size = 0.7

$$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x),x)
```

```
[Out] -1/2*ln(x)+1/2*ln(-2+x)
```

Maxima [A] time = 1.34144, size = 15, normalized size = 0.88

$$\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Fricas [A] time = 0.194603, size = 15, normalized size = 0.88

$$\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Sympy [A] time = 0.087498, size = 10, normalized size = 0.59

$$-\frac{\log(x)}{2}+\frac{\log(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x),x)
```

```
[Out] -log(x)/2 + log(x - 2)/2
```

GIAC/XCAS [A] time = 0.205118, size = 18, normalized size = 1.06

$$\frac{1}{2}\ln(|x-2|)-\frac{1}{2}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x - 2)) - 1/2*ln(abs(x))
```

3.215 $\int \frac{1+2x}{-7+12x+4x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rubi [A] time = 0.0177255, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rubi in Sympy [A] time = 2.2045, size = 17, normalized size = 0.81

$$\frac{\log(-2x+1)}{8}+\frac{3\log(2x+7)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)/(4*x**2+12*x-7),x)
```

```
[Out] log(-2*x + 1)/8 + 3*log(2*x + 7)/8
```

Mathematica [A] time = 0.00724762, size = 21, normalized size = 1.

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$\frac{3\,\ln(7+2\,x)}{8}+\frac{\ln(2\,x-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)/(4*x^2+12*x-7),x)
```

```
[Out] 3/8*ln(7+2*x)+1/8*ln(2*x-1)
```

Maxima [A] time = 1.33617, size = 23, normalized size = 1.1

$$\frac{3}{8}\log(2x+7)+\frac{1}{8}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(4*x^2 + 12*x - 7),x, algorithm="maxima")
```

```
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Fricas [A] time = 0.19669, size = 23, normalized size = 1.1

$$\frac{3}{8}\log(2x+7)+\frac{1}{8}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(4*x^2 + 12*x - 7),x, algorithm="fricas")
```

```
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Sympy [A] time = 0.101859, size = 17, normalized size = 0.81

$$\frac{\log\left(x-\frac{1}{2}\right)}{8}+\frac{3\log\left(x+\frac{7}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)/(4*x**2+12*x-7),x)
```

```
[Out] log(x - 1/2)/8 + 3*log(x + 7/2)/8
```

GIAC/XCAS [A] time = 0.206786, size = 26, normalized size = 1.24

$$\frac{3}{8}\ln(|2x+7|)+\frac{1}{8}\ln(|2x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 1)/(4*x^2 + 12*x - 7),x, algorithm="giac")
```

```
[Out] 3/8*ln(abs(2*x + 7)) + 1/8*ln(abs(2*x - 1))
```

3.216 $\int \frac{x}{-1+x+x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*x])/10
```

Rubi [A] time = 0.0359463, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-1 + x + x^2),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*x])/10
```

Rubi in Sympy [A] time = 1.68311, size = 56, normalized size = 1.14

$$\frac{\sqrt{5}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(2x+1+\sqrt{5}\right)}{5}-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\log\left(2x-\sqrt{5}+1\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+x-1),x)
```

```
[Out] sqrt(5)*(1/2 + sqrt(5)/2)*log(2*x + 1 + sqrt(5))/5 - sqrt(5)*(-sq
rt(5)/2 + 1/2)*log(2*x - sqrt(5) + 1)/5
```

Mathematica [A] time = 0.0304941, size = 44, normalized size = 0.9

$$\frac{1}{10}\left(\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-1 + x + x^2),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) + (5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*x])/10
```

Maple [A] time = 0.003, size = 27, normalized size = 0.6

$$\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\sqrt{5}}{5}\mathit{Artanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+x-1),x)

[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Maxima [A] time = 1.53222, size = 50, normalized size = 1.02

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x - 1),x, algorithm="maxima")

[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) + 1/2*
log(x^2 + x - 1)
```

Fricas [A] time = 0.198717, size = 62, normalized size = 1.27

$$\frac{1}{10}\sqrt{5}\left(\sqrt{5}\log\left(x^2+x-1\right)+\log\left(\frac{\sqrt{5}\left(2x^2+2x+3\right)+10x+5}{x^2+x-1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x - 1),x, algorithm="fricas")

[Out] 1/10*sqrt(5)*(sqrt(5)*log(x^2 + x - 1) + log((sqrt(5)*(2*x^2 + 2*
x + 3) + 10*x + 5)/(x^2 + x - 1)))
```

Sympy [A] time = 0.09795, size = 46, normalized size = 0.94

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\left(-\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x-1),x)

[Out] (sqrt(5)/10 + 1/2)*log(x + 1/2 + sqrt(5)/2) + (-sqrt(5)/10 + 1/2)
*log(x - sqrt(5)/2 + 1/2)
```

GIAC/XCAS [A] time = 0.209999, size = 54, normalized size = 1.1

$$-\frac{1}{10}\sqrt{5}\ln\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)+\frac{1}{2}\ln\left(\left|x^2+x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + x - 1),x, algorithm="giac")

[Out] -1/10*sqrt(5)*ln(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) +
 1/2*ln(abs(x^2 + x - 1))
```

3.217 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$

Optimal. Leaf size=63

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7
 - 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879
 + (11049*Log[5 + x + x^2])/260015
```

Rubi [A] time = 0.149534, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5)
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7
 - 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879
 + (11049*Log[5 + x + x^2])/260015
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x^3-27x^2+5x-32}{30x^5-13x^4+50x^3-286x^2-299x-70}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   rubi_integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-
286*x**2-299*x-70),x)
```

```
[Out] Integral((4*x**3 - 27*x**2 + 5*x - 32)/(30*x**5 - 13*x**4 + 50*x*
*3 - 286*x**2 - 299*x - 70), x)
```

Mathematica [A] time = 0.0407242, size = 57, normalized size = 0.9

$$\frac{453009\log\left(x^2+x+5\right)-418418\log(7-3x)-11023670\log(2x+1)+10536070\log(5x+2)+163508\sqrt{19}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{10660615}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 3
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x]
 - 11023670*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 +
 x + x^2])/10660615
```

Maple [A] time = 0.017, size = 51, normalized size = 0.8

$$\frac{4822\,\ln(2+5\,x)}{4879}+\frac{11049\,\ln\left(x^2+x+5\right)}{260015}+\frac{3988\,\sqrt{19}}{260015}\arctan\left(\frac{(1+2\,x)\,\sqrt{19}}{19}\right)-\frac{334\,\ln(1+2\,x)}{323}-\frac{3146\,\ln(3\,x-7)}{80155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)+11049/260015*ln(x^2+x+5)+3988/260015*arctan(1
/19*(1+2*x)*19^(1/2))*19^(1/2)-334/323*ln(1+2*x)-3146/80155*ln(3*
x-7)
```

Maxima [A] time = 1.46791, size = 68, normalized size = 1.08

$$\frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right)+\frac{4822}{4879}\,\log(5\,x+2)-\frac{3146}{80155}\,\log(3\,x-7)-\frac{334}{323}\,\log(2\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/2600
15*log(x^2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x
 - 7) - 334/323*log(2*x + 1)
```

Fricas [A] time = 0.207955, size = 86, normalized size = 1.37

$$\frac{1}{202551685}\,\sqrt{19}\Big(453009\,\sqrt{19}\log\left(x^2+x+5\right)+10536070\,\sqrt{19}\log(5\,x+2)-418418\,\sqrt{19}\log(3\,x-7)-11023670\,\sqrt{19}\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 1/202551685*sqrt(19)*(453009*sqrt(19)*log(x^2 + x + 5) + 10536070
*sqrt(19)*log(5*x + 2) - 418418*sqrt(19)*log(3*x - 7) - 11023670*
sqrt(19)*log(2*x + 1) + 3106652*arctan(1/19*sqrt(19)*(2*x + 1)))
```

Sympy [A] time = 0.538577, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\,\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-
299*x-70),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1
/2)/323 + 11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*s
```

```
qrt(19)*x/19 + sqrt(19)/19)/260015
```

GIAC/XCAS [A] time = 0.211537, size = 72, normalized size = 1.14

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\ln\left(x^2+x+5\right)+\frac{4822}{4879}\ln\left(|5x+2|\right)-\frac{3146}{80155}\ln\left(|3x-7|\right)-\frac{334}{323}\ln\left(|2x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3 - 27*x^2 + 5*x - 32)/(30*x^5 - 13*x^4 + 50*x^3 - 286*x^2 - 299*x -
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/2600
15*ln(x^2 + x + 5) + 4822/4879*ln(abs(5*x + 2)) - 3146/80155*ln(a
bs(3*x - 7)) - 334/323*ln(abs(2*x + 1))
```

3.218 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}$$
$$-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*A
rcTan[Sqrt[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])
/1331 - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rubi [A] time = 0.173691, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}$$
$$-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 1
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*A
rcTan[Sqrt[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])
/1331 - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-
80*x**3+41*x**2-20*x+4),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0730768, size = 67, normalized size = 0.78

$$\frac{142150\log\left(2x^2+1\right)-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}-236384\log(2-5x)+12575\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) +
12575*Sqrt[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Lo
g[1 + 2*x^2])/399300
```

Maple [A] time = 0.02, size = 54, normalized size = 0.6

$$-\frac{5828}{45375\,x-18150}-\frac{59096\,\ln(5\,x-2)}{99825}+\frac{1}{3993}\left(-\frac{2761\,x}{4}-\frac{3443}{8}\right)\left(x^2+\frac{1}{2}\right)^{-1}+\frac{2843\,\ln\left(4\,x^2+2\right)}{7986}+\frac{503\,\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-
20*x+4),x)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8
)/(x^2+1/2)+2843/7986*ln(4*x^2+2)+503/15972*arctan(x*2^(1/2))*2^(
1/2)
```

Maxima [A] time = 1.47718, size = 80, normalized size = 0.93

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^2+4675\,x+2554}{12100\,(10\,x^3-4\,x^2+5\,x-2)}+\frac{2843}{7986}\log\left(2\,x^2+1\right)-\frac{59096}{99825}\log(5\,x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x
 + 2554)/(10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) -
59096/99825*log(5*x - 2)
```

Fricas [A] time = 0.223186, size = 155, normalized size = 1.8

$$\frac{\sqrt{2}\left(142150\,\sqrt{2}\left(10\,x^3-4\,x^2+5\,x-2\right)\log\left(2\,x^2+1\right)-236384\,\sqrt{2}\left(10\,x^3-4\,x^2+5\,x-2\right)\log(5\,x-2)+25150\left(10\,x^3-4\,x\right.\right.}{798600\,(10\,x^3-4\,x^2+5\,x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 1/798600*sqrt(2)*(142150*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*log(2
*x^2 + 1) - 236384*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*log(5*x - 2
) + 25150*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 33*sqrt(
2)*(36458*x^2 + 4675*x + 2554))/(10*x^3 - 4*x^2 + 5*x - 2)
```

Sympy [A] time = 0.278232, size = 63, normalized size = 0.73

$$-\frac{36458x^2+4675x+2554}{121000x^3-48400x^2+60500x-24200}-\frac{59096\log\left(x-\frac{2}{5}\right)}{99825}+\frac{2843\log\left(x^2+\frac{1}{2}\right)}{7986}+\frac{503\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-
80*x**3+41*x**2-20*x+4),x)
```

```
[Out] -(36458*x**2 + 4675*x + 2554)/(121000*x**3 - 48400*x**2 + 60500*x
 - 24200) - 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986
```

```
+ 503*sqrt(2)*atan(sqrt(2)*x)/15972
```

GIAC/XCAS [A] time = 0.211198, size = 80, normalized size = 0.93

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2 + 4675\,x + 2554}{12100\,(2\,x^2+1)(5\,x-2)} + \frac{2843}{7986}\ln\left(2\,x^2+1\right) - \frac{59096}{99825}\ln\left(|5\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5 - 7*x^3 - 13*x^2 + 8)/(100*x^6 - 80*x^5 + 116*x^4 - 80*x^3 + 41*x
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x
 + 2554)/((2*x^2 + 1)*(5*x - 2)) + 2843/7986*ln(2*x^2 + 1) - 5909
6/99825*ln(abs(5*x - 2))
```

3.219 $\int \frac{\sqrt{4+x}}{x}\,dx$

Optimal. Leaf size=24

$$2\sqrt{x+4}-4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rubi [A] time = 0.0178314, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$2\sqrt{x+4}-4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rubi in Sympy [A] time = 1.40066, size = 19, normalized size = 0.79

$$2\sqrt{x+4}-4\operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4+x)**(1/2)/x,x)
```

```
[Out] 2*sqrt(x + 4) - 4*atanh(sqrt(x + 4)/2)
```

Mathematica [A] time = 0.0110996, size = 36, normalized size = 1.5

$$2\sqrt{x+4}+2\log\left(2-\sqrt{x+4}\right)-2\log\left(\sqrt{x+4}+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] + 2*Log[2 - Sqrt[4 + x]] - 2*Log[2 + Sqrt[4 + x]]
```

Maple [A] time = 0.013, size = 29, normalized size = 1.2

$$2\sqrt{4+x}-2\ln\left(\sqrt{4+x}+2\right)+2\ln\left(\sqrt{4+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4+x)^(1/2)/x,x)
```

```
[Out] 2*(4+x)^(1/2)-2*ln((4+x)^(1/2)+2)+2*ln((4+x)^(1/2)-2)
```

Maxima [A] time = 1.32665, size = 38, normalized size = 1.58

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 4)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Fricas [A] time = 0.20379, size = 38, normalized size = 1.58

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 4)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Sympy [A] time = 1.17596, size = 44, normalized size = 1.83

$$\begin{cases} 2\sqrt{x+4} - 4\operatorname{acoth}\left(\frac{\sqrt{x+4}}{2}\right) & \text{for } \frac{|x+4|}{4} > 1 \\ 2\sqrt{x+4} - 4\operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4+x)**(1/2)/x,x)
```

```
[Out] Piecewise((2*sqrt(x + 4) - 4*acoth(sqrt(x + 4)/2), Abs(x + 4)/4 >
 1), (2*sqrt(x + 4) - 4*atanh(sqrt(x + 4)/2), True))
```

GIAC/XCAS [A] time = 0.20871, size = 39, normalized size = 1.62

$$2\sqrt{x+4} - 2\ln\left(\sqrt{x+4}+2\right) + 2\ln\left(\left|\sqrt{x+4}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 4)/x,x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 4) - 2*ln(sqrt(x + 4) + 2) + 2*ln(abs(sqrt(x + 4) - 2)
)
```

3.220 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$
$$-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[
(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log
[1 - x^(1/6)])/5 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(
1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) + Sqrt[5
]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.669162, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$
$$-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[
(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log
[1 - x^(1/6)])/5 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(
1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) + Sqrt[5
]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.236188, size = 180, normalized size = 0.9

$$\frac{1}{10}\left(20\sqrt{x}+12\log\left(1-\sqrt[6]{x}\right)-3\left(1+\sqrt{5}\right)\log\left(\sqrt[3]{x}-\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt[6]{x}+1\right)\right.$$
$$+3\left(\sqrt{5}-1\right)\log\left(\sqrt[3]{x}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[6]{x}+1\right)$$
$$\left.+6\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-6\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] (20*Sqrt[x] + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x^(1
/6))/Sqrt[2*(5 + Sqrt[5])]] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 +
 Sqrt[5] + 4*x^(1/6))/Sqrt[10 - 2*Sqrt[5]]] + 12*Log[1 - x^(1/6)]
 - 3*(1 + Sqrt[5])*Log[1 - ((-1 + Sqrt[5])*x^(1/6))/2 + x^(1/3)]
+ 3*(-1 + Sqrt[5])*Log[1 + ((1 + Sqrt[5])*x^(1/6))/2 + x^(1/3)])/
10
```

Maple [A] time = 0.053, size = 175, normalized size = 0.9

$$2\sqrt{x}-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)$$
$$+\frac{12\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10+2\sqrt{5}}}\left(1+4\sqrt[6]{x}-\sqrt{5}\right)\right)$$
$$-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)+\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)$$
$$-\frac{12\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10-2\sqrt{5}}}\left(1+4\sqrt[6]{x}+\sqrt{5}\right)\right)+\frac{6}{5}\ln\left(-1+\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-3/
10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+12/5/(10+2*5^(1/2))^(1
/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)-3/
10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))+3/10*ln(2+x^(1/6)+2*x^
(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-12/5/(10-2*5^(1/2))^(1/2)*arctan((
1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)+6/5*ln(-1+x^(1
/6))
```

Maxima [A] time = 1.54237, size = 367, normalized size = 1.84

$$-\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}$$
$$+\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}+2\sqrt{x}$$
$$+\frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)}$$
$$-\frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5
)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1
)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt
(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*s
qrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(-2*sq
rt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sq
rt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) +
(-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1
)^(2/5)) - 6/5*log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*
(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3)
 + sqrt(x) + 1)), x)
```

GIAC/XCAS [A] time = 0.310757, size = 188, normalized size = 0.94

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$
$$+\frac{3}{10}\sqrt{5}\ln\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)-\frac{3}{10}\sqrt{5}\ln\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)$$
$$+2\sqrt{x}-\frac{3}{10}\ln\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\ln\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) - 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(
2*sqrt(5) + 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x
^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) + 3/10*sqrt(5)*ln(1/2*x^(1/6)*
(sqrt(5) + 1) + x^(1/3) + 1) - 3/10*sqrt(5)*ln(-1/2*x^(1/6)*(sqrt
(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) - 3/10*ln(x^(2/3) + sqrt(x) +
 x^(1/3) + x^(1/6) + 1) + 6/5*ln(abs(x^(1/6) - 1))
```

3.221 $\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

```
[Out] -ArcTanh[(3*Cos[x] + 4*Sin[x])/5]/5
```

Rubi [A] time = 0.0212366, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] + 4*Sin[x])/5]/5
```

Rubi in Sympy [A] time = 0.579014, size = 17, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{4\sin(x)}{5}+\frac{3\cos(x)}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] -atanh(4*sin(x)/5 + 3*cos(x)/5)/5
```

Mathematica [B] time = 0.0144412, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] Log[Cos[x/2] - 2*Sin[x/2]]/5 - Log[2*Cos[x/2] + Sin[x/2]]/5
```

Maple [A] time = 0.053, size = 22, normalized size = 1.2

$$-\frac{1}{5}\ln\left(\tan\left(\frac{x}{2}\right)+2\right)+\frac{1}{5}\ln(2\tan(x/2)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] -1/5*ln(tan(1/2*x)+2)+1/5*ln(2*tan(1/2*x)-1)
```

Maxima [A] time = 1.47864, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}-1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(4*cos(x) - 3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) - 1) - 1/5*log(sin(x)/(cos(x) + 1)
+ 2)
```

Fricas [A] time = 0.254983, size = 36, normalized size = 2.

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(4*cos(x) - 3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) + 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) - 2
*sin(x) + 5/2)
```

Sympy [A] time = 0.348493, size = 20, normalized size = 1.11

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{2}\right)}{5}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] log(tan(x/2) - 1/2)/5 - log(tan(x/2) + 2)/5
```

GIAC/XCAS [A] time = 0.228226, size = 31, normalized size = 1.72

$$\frac{1}{5}\ln\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)-\frac{1}{5}\ln\left(\left|\tan\left(\frac{1}{2}x\right)+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(4*cos(x) - 3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*ln(abs(2*tan(1/2*x) - 1)) - 1/5*ln(abs(tan(1/2*x) + 2))
```

3.222 $\int \frac{1}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=18

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rubi [A] time = 0.0146088, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rubi in Sympy [A] time = 1.09657, size = 15, normalized size = 0.83

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Mathematica [A] time = 0.00604224, size = 18, normalized size = 1.

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Maple [A] time = 0.003, size = 27, normalized size = 1.5

$$2\sqrt{x} + \ln\left(\sqrt{x}-1\right) - \ln\left(1+\sqrt{x}\right) - \ln\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))-ln(-1+x)
```

Maxima [A] time = 1.35673, size = 20, normalized size = 1.11

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

Fricas [A] time = 0.19982, size = 19, normalized size = 1.06

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Sympy [A] time = 0.125218, size = 15, normalized size = 0.83

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

GIAC/XCAS [A] time = 0.207748, size = 19, normalized size = 1.06

$$2\sqrt{x} - 2\ln\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*ln(sqrt(x) + 1)
```

3.223 $\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=32

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\frac{1}{\sqrt[3]{x}}+1\right)-\log(x)$$

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rubi [A] time = 0.0275681, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\frac{1}{\sqrt[3]{x}}+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$3\sqrt[3]{x}+x-3\log\left(\sqrt[3]{x}+1\right)-3\int^{\sqrt[3]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+1/x**(1/3)),x)
```

```
[Out] 3*x**(1/3) + x - 3*log(x**(1/3) + 1) - 3*Integral(x, (x, x**(1/3)
))
```

Mathematica [A] time = 0.00853203, size = 28, normalized size = 0.88

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(1/3)]
```

Maple [A] time = 0.004, size = 21, normalized size = 0.7

$$x-\frac{3}{2}x^{\frac{2}{3}}+3\sqrt[3]{x}-3\ln\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+1/x^(1/3)),x)
```

```
[Out] x-3/2*x^(2/3)+3*x^(1/3)-3*ln(x^(1/3)+1)
```

Maxima [A] time = 1.34434, size = 38, normalized size = 1.19

$$-\frac{1}{2}x\left(\frac{3}{x^{\frac{1}{3}}}-\frac{6}{x^{\frac{2}{3}}}-2\right)-\log(x)-3\log\left(\frac{1}{x^{\frac{1}{3}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/3) + 1),x, algorithm="maxima")
```

```
[Out] -1/2*x*(3/x^(1/3) - 6/x^(2/3) - 2) - log(x) - 3*log(1/x^(1/3) + 1
)
```

Fricas [A] time = 0.20167, size = 27, normalized size = 0.84

$$x-\frac{3}{2}x^{\frac{2}{3}}+3x^{\frac{1}{3}}-3\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/3) + 1),x, algorithm="fricas")
```

```
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

Sympy [A] time = 0.145261, size = 26, normalized size = 0.81

$$-\frac{3x^{\frac{2}{3}}}{2}+3\sqrt[3]{x}+x-3\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3)/2 + 3*x**(1/3) + x - 3*log(x**(1/3) + 1)
```

GIAC/XCAS [A] time = 0.208532, size = 27, normalized size = 0.84

$$x-\frac{3}{2}x^{\frac{2}{3}}+3x^{\frac{1}{3}}-3\ln\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/3) + 1),x, algorithm="giac")
```

```
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*ln(x^(1/3) + 1)
```

3.224 $\int \frac{\sqrt{x}}{1+x}\,dx$

Optimal. Leaf size=16

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0111232, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.16002, size = 14, normalized size = 0.88

$$2\sqrt{x} - 2\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(1+x),x)
```

```
[Out] 2*sqrt(x) - 2*atan(sqrt(x))
```

Mathematica [A] time = 0.00647774, size = 16, normalized size = 1.

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.005, size = 13, normalized size = 0.8

$$-2\,\arctan\left(\sqrt{x}\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(1+x),x)
```

```
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

Maxima [A] time = 1.49315, size = 16, normalized size = 1.

$$2\sqrt{x} - 2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Fricas [A] time = 0.205837, size = 16, normalized size = 1.

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Sympy [A] time = 2.03, size = 51, normalized size = 3.19

$$\begin{cases} 2\sqrt{x} + 2\operatorname{asin}\left(\frac{1}{\sqrt{x+1}}\right) & \text{for } |x+1| > 1 \\ 2i\sqrt{-x} + i\log(x+1) - 2i\log\left(\sqrt{-x}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(1+x),x)
```

```
[Out] Piecewise((2*sqrt(x) + 2*asin(1/sqrt(x + 1)), Abs(x + 1) > 1), (2
*I*sqrt(-x) + I*log(x + 1) - 2*I*log(sqrt(-x) + 1), True))
```

GIAC/XCAS [A] time = 0.209568, size = 16, normalized size = 1.

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

3.225 $\int \frac{1}{x\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rubi [A] time = 0.010595, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[1 + x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 0.975793, size = 10, normalized size = 1.

$$-2\operatorname{atanh}\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+x)**(1/2),x)
```

```
[Out] -2*atanh(sqrt(x + 1))
```

Mathematica [B] time = 0.00410762, size = 25, normalized size = 2.5

$$\log\left(1-\sqrt{x+1}\right)-\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[1 + x]),x]
```

```
[Out] Log[1 - Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]]
```

Maple [A] time = 0.006, size = 9, normalized size = 0.9

$$-2\,\mathit{Artanh}\left(\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+x)^(1/2),x)
```

```
[Out] -2*arctanh((1+x)^(1/2))
```

Maxima [A] time = 1.34975, size = 26, normalized size = 2.6

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*x),x, algorithm="maxima")
```

```
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Fricas [A] time = 0.206026, size = 26, normalized size = 2.6

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*x),x, algorithm="fricas")
```

```
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.787056, size = 26, normalized size = 2.6

$$\begin{cases} -2\operatorname{acoth}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2\operatorname{atanh}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*acoth(sqrt(x + 1)), Abs(x + 1) > 1), (-2*atanh(sqrt
(x + 1)), True))
```

GIAC/XCAS [A] time = 0.214613, size = 27, normalized size = 2.7

$$-\ln\left(\sqrt{x+1}+1\right)+\ln\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1)*x),x, algorithm="giac")
```

```
[Out] -ln(sqrt(x + 1) + 1) + ln(abs(sqrt(x + 1) - 1))
```

3.226 $\int \frac{1}{-\sqrt[3]{x}+x}\,dx$

Optimal. Leaf size=14

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Rubi [A] time = 0.010194, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Rubi in Sympy [A] time = 0.959532, size = 10, normalized size = 0.71

$$\frac{3\log\left(-x^{\frac{2}{3}}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**(1/3)+x),x)
```

```
[Out] 3*log(-x**(2/3) + 1)/2
```

Mathematica [A] time = 0.00567106, size = 14, normalized size = 1.

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Maple [B] time = 0.008, size = 50, normalized size = 3.6

$$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}+\ln\left(-1+\sqrt[3]{x}\right)-\frac{1}{2}\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)+\ln\left(\sqrt[3]{x}+1\right)-\frac{1}{2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^(1/3)+x),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)+ln(-1+x^(1/3))-1/2*ln(x^(2/3)+x^(1/3)+1)
+ln(x^(1/3)+1)-1/2*ln(x^(2/3)-x^(1/3)+1)
```

Maxima [A] time = 1.36053, size = 23, normalized size = 1.64

$$\frac{3}{2}\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(x^(1/3) - 1)
```

Fricas [A] time = 0.198534, size = 11, normalized size = 0.79

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*log(x^(2/3) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-\sqrt[3]{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**(1/3)+x),x)
```

```
[Out] Integral(1/(-x**(1/3) + x), x)
```

GIAC/XCAS [A] time = 0.210512, size = 24, normalized size = 1.71

$$\frac{3}{2}\ln\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\ln\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*ln(x^(1/3) + 1) + 3/2*ln(abs(x^(1/3) - 1))
```

3.227 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi [A] time = 0.0424525, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi in Sympy [A] time = 2.18986, size = 26, normalized size = 0.84

$$\frac{4\log\left(-\sqrt{x+2}+2\right)}{3}+\frac{2\log\left(\sqrt{x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] 4*log(-sqrt(x + 2) + 2)/3 + 2*log(sqrt(x + 2) + 1)/3
```

Mathematica [A] time = 0.00837652, size = 31, normalized size = 1.

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [B] time = 0.02, size = 54, normalized size = 1.7

$$\frac{\ln(1+x)}{3}+\frac{2\ln(-2+x)}{3}-\frac{2}{3}\ln\left(\sqrt{2+x}+2\right)+\frac{1}{3}\ln\left(1+\sqrt{2+x}\right)-\frac{1}{3}\ln\left(-1+\sqrt{2+x}\right)+\frac{2}{3}\ln\left(\sqrt{2+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x-(2+x)^(1/2)),x)
```

```
[Out] 1/3*ln(1+x)+2/3*ln(-2+x)-2/3*ln((2+x)^(1/2)+2)+1/3*ln(1+(2+x)^(1/
2))-1/3*ln(-1+(2+x)^(1/2))+2/3*ln((2+x)^(1/2)-2)
```

Maxima [A] time = 1.32545, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Fricas [A] time = 0.196754, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x-\sqrt{x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(x + 2)), x)
```

GIAC/XCAS [A] time = 0.210727, size = 30, normalized size = 0.97

$$\frac{2}{3}\ln\left(\sqrt{x+2}+1\right)+\frac{4}{3}\ln\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x - sqrt(x + 2)),x, algorithm="giac")
```

```
[Out] 2/3*ln(sqrt(x + 2) + 1) + 4/3*ln(abs(sqrt(x + 2) - 2))
```

3.228 $\int \frac{x^2}{\sqrt{-1+x}}\,dx$

Optimal. Leaf size=32

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rubi [A] time = 0.0180979, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[-1 + x],x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rubi in Sympy [A] time = 1.28766, size = 27, normalized size = 0.84

$$\frac{2(x-1)^{\frac{5}{2}}}{5}+\frac{4(x-1)^{\frac{3}{2}}}{3}+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-1+x)**(1/2),x)
```

```
[Out] 2*(x - 1)**(5/2)/5 + 4*(x - 1)**(3/2)/3 + 2*sqrt(x - 1)
```

Mathematica [A] time = 0.00798838, size = 21, normalized size = 0.66

$$\frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[-1 + x],x]
```

```
[Out] (2*Sqrt[-1 + x]*(8 + 4*x + 3*x^2))/15
```

Maple [A] time = 0.006, size = 18, normalized size = 0.6

$$\frac{6x^2+8x+16}{15}\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-1+x)^(1/2),x)
```

```
[Out] 2/15*(-1+x)^(1/2)*(3*x^2+4*x+8)
```

Maxima [A] time = 1.35459, size = 30, normalized size = 0.94

$$\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(x - 1),x, algorithm="maxima")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

Fricas [A] time = 0.226445, size = 23, normalized size = 0.72

$$\frac{2}{15}\left(3x^2 + 4x + 8\right)\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(x - 1),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

Sympy [A] time = 1.95417, size = 76, normalized size = 2.38

$$\begin{cases} \frac{2x^2\sqrt{x-1}}{5} + \frac{8x\sqrt{x-1}}{15} + \frac{16\sqrt{x-1}}{15} & \text{for } |x| > 1 \\ \frac{2ix^2\sqrt{-x+1}}{5} + \frac{8ix\sqrt{-x+1}}{15} + \frac{16i\sqrt{-x+1}}{15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15 + 16*sqrt(x
- 1)/15, Abs(x) > 1), (2*I*x**2*sqrt(-x + 1)/5 + 8*I*x*sqrt(-x +
1)/15 + 16*I*sqrt(-x + 1)/15, True))
```

GIAC/XCAS [A] time = 0.207487, size = 30, normalized size = 0.94

$$\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(x - 1),x, algorithm="giac")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

3.229 $\int \frac{\sqrt{-1+x}}{1+x}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rubi [A] time = 0.026014, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rubi in Sympy [A] time = 1.62914, size = 29, normalized size = 0.94

$$2\sqrt{x-1}-2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+x)**(1/2)/(1+x),x)
```

```
[Out] 2*sqrt(x - 1) - 2*sqrt(2)*atan(sqrt(2)*sqrt(x - 1)/2)
```

Mathematica [A] time = 0.0137113, size = 31, normalized size = 1.

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Maple [A] time = 0.009, size = 25, normalized size = 0.8

$$-2\arctan\left(1/2\sqrt{-1+x}\sqrt{2}\right)\sqrt{2}+2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+x)^(1/2)/(1+x),x)
```

```
[Out] -2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(-1+x)^(1/2)
```

Maxima [A] time = 1.49829, size = 32, normalized size = 1.03

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(x + 1),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Fricas [A] time = 0.204941, size = 32, normalized size = 1.03

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(x + 1),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Sympy [A] time = 2.08826, size = 76, normalized size = 2.45

$$\begin{cases} 2\sqrt{x-1}+2\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for } \frac{|x+1|}{2}>1 \\ 2i\sqrt{-x+1}+\sqrt{2}i\log(x+1)-2\sqrt{2}i\log\left(\sqrt{-\frac{x}{2}+\frac{1}{2}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+x)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((2*sqrt(x - 1) + 2*sqrt(2)*asin(sqrt(2)/sqrt(x + 1)), A
bs(x + 1)/2 > 1), (2*I*sqrt(-x + 1) + sqrt(2)*I*log(x + 1) - 2*sq
rt(2)*I*log(sqrt(-x/2 + 1/2) + 1), True))
```

GIAC/XCAS [A] time = 0.209863, size = 32, normalized size = 1.03

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 1)/(x + 1),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

3.230 $\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=29

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

```
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rubi [A] time = 0.0177053, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + Sqrt[x]],x]
```

```
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rubi in Sympy [A] time = 1.05162, size = 24, normalized size = 0.83

$$\frac{4\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{3}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x**(1/2))**(1/2),x)
```

```
[Out] 4*(sqrt(x) + 1)**(3/2)/3 - 4*sqrt(sqrt(x) + 1)
```

Mathematica [A] time = 0.00751384, size = 22, normalized size = 0.76

$$\frac{4}{3}\left(\sqrt{x}-2\right)\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + Sqrt[x]],x]
```

```
[Out] (4*(-2 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/3
```

Maple [A] time = 0.007, size = 20, normalized size = 0.7

$$\frac{4}{3}\left(1+\sqrt{x}\right)^{\frac{3}{2}}-4\sqrt{1+\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x^(1/2))^(1/2),x)
```

```
[Out] 4/3*(1+x^(1/2))^(3/2)-4*(1+x^(1/2))^(1/2)
```

Maxima [A] time = 1.33752, size = 26, normalized size = 0.9

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

Fricas [A] time = 0.204602, size = 19, normalized size = 0.66

$$\frac{4}{3}\sqrt{\sqrt{x}+1}\left(\sqrt{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(x) + 1)*(sqrt(x) - 2)
```

Sympy [A] time = 1.43966, size = 117, normalized size = 4.03

$$-\frac{4x^{\frac{5}{2}}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}+\frac{8x^{\frac{5}{2}}}{3x^{\frac{5}{2}}+3x^2}+\frac{4x^3\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}-\frac{8x^2\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^2}+\frac{8x^2}{3x^{\frac{5}{2}}+3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x**(1/2))**(1/2),x)
```

```
[Out] -4*x**(5/2)*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**(5/2)/
(3*x**(5/2) + 3*x**2) + 4*x**3*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*
x**2) - 8*x**2*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**2/(
3*x**(5/2) + 3*x**2)
```

GIAC/XCAS [A] time = 0.206516, size = 26, normalized size = 0.9

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

3.231 $\int \frac{\sqrt{x}}{x+x^2}\,dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0100113, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.23645, size = 7, normalized size = 0.88

$$2\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**2+x),x)

[Out] 2*atan(sqrt(x))
```

Mathematica [A] time = 0.00403947, size = 8, normalized size = 1.

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^2+x),x)

[Out] 2*arctan(x^(1/2))
```

Maxima [A] time = 1.52463, size = 8, normalized size = 1.

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 0.205759, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [A] time = 0.971066, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**2+x),x)
```

```
[Out] 2*atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.208505, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(x^2 + x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.232 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$x + 4\sqrt{x} + 4\log\left(1-\sqrt{x}\right)$$

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi [A] time = 0.0281364, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$x + 4\sqrt{x} + 4\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$4\sqrt{x} + 4\log\left(-\sqrt{x}+1\right) + 2\int^{\sqrt{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + 4*log(-sqrt(x) + 1) + 2*Integral(x, (x, sqrt(x)))
```

Mathematica [A] time = 0.00737561, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x}-1\right) - 5$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] -5 + 4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$x + 4\sqrt{x} + 4\ln\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/2))/(x^(1/2)-1),x)
```

```
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

Maxima [A] time = 1.36087, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Fricas [A] time = 0.20237, size = 20, normalized size = 0.95

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] time = 0.162879, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

GIAC/XCAS [A] time = 0.208148, size = 22, normalized size = 1.05

$$x + 4\sqrt{x} + 4\ln\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)/(sqrt(x) - 1),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 4*ln(abs(sqrt(x) - 1))
```

3.233 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rubi [A] time = 0.0440313, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-6\sqrt[3]{x} - x - 6\log\left(-\sqrt[3]{x}+1\right) - 6\int^{\sqrt[3]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -6*x**(1/3) - x - 6*log(-x**(1/3) + 1) - 6*Integral(x, (x, x**(1/
3)))
```

Mathematica [A] time = 0.0116087, size = 30, normalized size = 1.

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Maple [A] time = 0.006, size = 23, normalized size = 0.8

$$-x - 3\,x^{2/3} - 6\sqrt[3]{x} - 6\,\ln\left(-1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
```

```
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(-1+x^(1/3))
```

Maxima [A] time = 1.36593, size = 30, normalized size = 1.

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="maxima")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Fricas [A] time = 0.199908, size = 30, normalized size = 1.

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="fricas")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] time = 0.19399, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

GIAC/XCAS [A] time = 0.213506, size = 31, normalized size = 1.03

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\ln\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x, algorithm="giac")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*ln(abs(x^(1/3) - 1))
```

3.234 $\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

```
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rubi [A] time = 0.0281291, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x^2)^(1/3),x]
```

```
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rubi in Sympy [A] time = 1.80511, size = 22, normalized size = 0.81

$$\frac{3\left(x^2+1\right)^{\frac{5}{3}}}{10}-\frac{3\left(x^2+1\right)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**2+1)**(1/3),x)
```

```
[Out] 3*(x**2 + 1)**(5/3)/10 - 3*(x**2 + 1)**(2/3)/4
```

Mathematica [A] time = 0.00718938, size = 20, normalized size = 0.74

$$\frac{3}{20}\left(x^2+1\right)^{2/3}\left(2x^2-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x^2)^(1/3),x]
```

```
[Out] (3*(1 + x^2)^(2/3)*(-3 + 2*x^2))/20
```

Maple [A] time = 0.006, size = 17, normalized size = 0.6

$$\frac{6x^2-9}{20}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^2+1)^(1/3),x)
```

```
[Out] 3/20*(x^2+1)^(2/3)*(2*x^2-3)
```

Maxima [A] time = 1.33868, size = 26, normalized size = 0.96

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

Fricas [A] time = 0.574585, size = 22, normalized size = 0.81

$$\frac{3}{20}\left(2\,x^2-3\right)\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/20*(2*x^2 - 3)*(x^2 + 1)^(2/3)
```

Sympy [A] time = 1.45604, size = 26, normalized size = 0.96

$$\frac{3x^2\left(x^2+1\right)^{\frac{2}{3}}}{10}-\frac{9\left(x^2+1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**2+1)**(1/3),x)
```

```
[Out] 3*x**2*(x**2 + 1)**(2/3)/10 - 9*(x**2 + 1)**(2/3)/20
```

GIAC/XCAS [A] time = 0.205704, size = 26, normalized size = 0.96

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^2 + 1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

3.235 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$
$$-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*
x^(1/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*Arc
Tan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6
*Log[1 - x^(1/6)])/5 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]
*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) + Sq
rt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.497266, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$
$$-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)$$
$$-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*
x^(1/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*Arc
Tan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6
*Log[1 - x^(1/6)])/5 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]
*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) + Sq
rt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi in Sympy [A] time = 100.799, size = 267, normalized size = 1.33

$$6\sqrt[6]{x}+x+\frac{6\log\left(-\sqrt[6]{x}+1\right)}{5}-\left(\frac{3}{10}+\frac{3\sqrt{5}}{10}\right)\log\left(\sqrt[6]{x}\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\sqrt[3]{x}+1\right)$$
$$-\left(-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)\log\left(\sqrt[6]{x}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+\sqrt[3]{x}+1\right)$$
$$-\frac{12\left(-\left(\frac{1}{4}+\frac{\sqrt{5}}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{\sqrt[6]{x}+\frac{1}{4}+\frac{\sqrt{5}}{4}}{\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}}\right)}{5\sqrt{-\frac{\sqrt{5}}{4}+\frac{3}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{5}{4}}}-\frac{12\left(-\left(-\frac{\sqrt{5}}{4}+\frac{1}{4}\right)^2+1\right)\operatorname{atan}\left(\frac{\sqrt[6]{x}-\frac{\sqrt{5}}{4}+\frac{1}{4}}{\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}\right)}{5\sqrt{-\frac{\sqrt{5}}{4}+\frac{5}{4}}\sqrt{\frac{\sqrt{5}}{4}+\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + x + 6*log(-x**(1/6) + 1)/5 - (3/10 + 3*sqrt(5)/10)*l
og(x**(1/6)*(1/2 + sqrt(5)/2) + x**(1/3) + 1) - (-3*sqrt(5)/10 +
3/10)*log(x**(1/6)*(-sqrt(5)/2 + 1/2) + x**(1/3) + 1) - 12*(-(1/4
 + sqrt(5)/4)**2 + 1)*atan((x**(1/6) + 1/4 + sqrt(5)/4)/(sqrt(-sq
rt(5)/4 + 3/4)*sqrt(sqrt(5)/4 + 5/4)))/(5*sqrt(-sqrt(5)/4 + 3/4)*
sqrt(sqrt(5)/4 + 5/4)) - 12*(-(-sqrt(5)/4 + 1/4)**2 + 1)*atan((x*
*(1/6) - sqrt(5)/4 + 1/4)/(sqrt(-sqrt(5)/4 + 5/4)*sqrt(sqrt(5)/4
+ 3/4)))/(5*sqrt(-sqrt(5)/4 + 5/4)*sqrt(sqrt(5)/4 + 3/4))
```

Mathematica [A] time = 0.235666, size = 183, normalized size = 0.91

$$\frac{1}{10}\left(10x+60\sqrt[6]{x}+12\log\left(1-\sqrt[6]{x}\right)+3\left(\sqrt{5}-1\right)\log\left(\sqrt[3]{x}-\frac{1}{2}\left(\sqrt{5}-1\right)\sqrt[6]{x}+1\right)\right.$$
$$-3\left(1+\sqrt{5}\right)\log\left(\sqrt[3]{x}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[6]{x}+1\right)$$
$$\left.-6\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-6\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{4\sqrt[6]{x}+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] (60*x^(1/6) + 10*x - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5]
+ 4*x^(1/6))/Sqrt[2*(5 + Sqrt[5])]] - 6*Sqrt[10 - 2*Sqrt[5]]*ArcT
an[(1 + Sqrt[5] + 4*x^(1/6))/Sqrt[10 - 2*Sqrt[5]]] + 12*Log[1 - x
^(1/6)] + 3*(-1 + Sqrt[5])*Log[1 - ((-1 + Sqrt[5])*x^(1/6))/2 + x
^(1/3)] - 3*(1 + Sqrt[5])*Log[1 + ((1 + Sqrt[5])*x^(1/6))/2 + x^(
1/3)])/10
```

Maple [A] time = 0.022, size = 242, normalized size = 1.2

$$x+6\sqrt[6]{x}-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)$$
$$+\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}-\sqrt[6]{x}\sqrt{5}\right)-6\frac{1}{\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1+4\sqrt[6]{x}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)$$
$$-\frac{6\sqrt{5}}{5\sqrt{10+2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10+2\sqrt{5}}}\left(1+4\sqrt[6]{x}-\sqrt{5}\right)\right)-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)$$
$$-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-6\frac{1}{\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1+4\sqrt[6]{x}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)$$
$$+\frac{6\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10-2\sqrt{5}}}\left(1+4\sqrt[6]{x}+\sqrt{5}\right)\right)+\frac{6}{5}\ln\left(-1+\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+3/10*ln(
2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-6/(10+2*5^(1/2))^(1/
2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))-6/5/(10+2*5
^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*
5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-3/10
*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-6/(10-2*5^(1/2))^(1/2)*a
```

```
rctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*5^(1/
2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1
/2)+6/5*ln(-1+x^(1/6))
```

Maxima [A] time = 1.56231, size = 396, normalized size = 1.97

$$-\frac{3\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}$$
$$-\frac{3\sqrt{5}\,(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}-\frac{6}{5}\,(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)$$
$$+x-\frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)}$$
$$-\frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}\,(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2\,(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}\,(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (
-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)
*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^
(1/6)))/sqrt(2*sqrt(5) - 10) - 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) +
1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (
-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*s
qrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10) - 6
/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*l
og(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*
x^(1/3))/(sqrt(5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*lo
g(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^
(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(4/5)) + 6*x^(1/6)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3)
 + sqrt(x) + 1)), x)
```

GIAC/XCAS [A] time = 0.316483, size = 189, normalized size = 0.94

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)$$
$$-\frac{3}{10}\sqrt{5}\ln\left(\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)+\frac{3}{10}\sqrt{5}\ln\left(-\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)$$
$$+x+6\,x^{\frac{1}{6}}-\frac{3}{10}\ln\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\ln\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(sqrt(x) - 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(
2*sqrt(5) + 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*
x^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) - 3/10*sqrt(5)*ln(1/2*x^(1/6)
*(sqrt(5) + 1) + x^(1/3) + 1) + 3/10*sqrt(5)*ln(-1/2*x^(1/6)*(sqr
t(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/6) - 3/10*ln(x^(2/3) + sqrt
(x) + x^(1/3) + x^(1/6) + 1) + 6/5*ln(abs(x^(1/6) - 1))
```

3.236 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[
1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi [A] time = 0.0755541, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[
1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi in Sympy [A] time = 4.799, size = 61, normalized size = 0.98

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-\sqrt[4]{x}+\sqrt{x}+1\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\sqrt{3}\left(\frac{2\sqrt[4]{x}}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-x**(1/4) + sqrt(x) + 1
)/3 - 4*sqrt(3)*atan(sqrt(3)*(2*x**(1/4)/3 - 1/3))/3
```

Mathematica [A] time = 0.02472, size = 62, normalized size = 1.

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)-\frac{4\tan^{-1}\left(\frac{2\sqrt[4]{x}-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - (4*ArcTan[(-1 + 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log
[1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Maple [A] time = 0.006, size = 46, normalized size = 0.7

$$2\sqrt{x}+\frac{4}{3}\ln\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\ln\left(1-\sqrt[4]{x}+\sqrt{x}\right)-\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[4]{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)
*arctan(1/3*(2*x^(1/4)-1)*3^(1/2))
```

Maxima [A] time = 1.53031, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/
3*log(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Fricas [A] time = 0.221387, size = 77, normalized size = 1.24

$$-\frac{2}{9}\sqrt{3}\left(\sqrt{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)-2\sqrt{3}\log\left(x^{\frac{1}{4}}+1\right)-3\sqrt{3}\sqrt{x}+6\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] -2/9*sqrt(3)*(sqrt(3)*log(sqrt(x) - x^(1/4) + 1) - 2*sqrt(3)*log(
x^(1/4) + 1) - 3*sqrt(3)*sqrt(x) + 6*arctan(2/3*sqrt(3)*x^(1/4) -
 1/3*sqrt(3)))
```

Sympy [A] time = 1.33415, size = 68, normalized size = 1.1

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x)
 + 4)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.213663, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\ln\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\ln\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + 1/x^(1/4)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/
3*ln(sqrt(x) - x^(1/4) + 1) + 4/3*ln(x^(1/4) + 1)
```

3.237 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Rubi [A] time = 0.0878171, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)-12\int\sqrt[12]{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/1
2)/5 + 12*x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 + 4*x**(5/4)/5
+ 4*x**(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(
x) - x - 12*log(x**(1/12) + 1) - 12*Integral(x, (x, x**(1/12)))
```

Mathematica [A] time = 0.0312092, size = 130, normalized size = 1.

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5
 - 2*Sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 -
(6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^
(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]
```

Maple [A] time = 0.006, size = 83, normalized size = 0.6

$$12x^{1/12} - 6\sqrt[6]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} + \frac{12}{5}x^{\frac{5}{12}} + \frac{12}{7}x^{\frac{7}{12}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{12}{11}x^{\frac{11}{12}} - x + \frac{12}{13}x^{\frac{13}{12}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{4}{5}x^{\frac{5}{4}} - 12\ln\left(1 + x^{1/12}\right) - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7
/12)-3/2*x^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*
x^(13/12)-6/7*x^(7/6)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Maxima [A] time = 1.34691, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sq
rt(x) + 12/5*x^(5/12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^
(1/12) - 12*log(x^(1/12) + 1)
```

Fricas [A] time = 0.204537, size = 103, normalized size = 0.79

$$\frac{4}{5}(x+5)x^{\frac{1}{4}} - \frac{6}{7}(x+7)x^{\frac{1}{6}} + \frac{12}{13}(x+13)x^{\frac{1}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} - 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/1
2) - x + 12/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3
) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*lo
g(x^(1/12) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{12}}}{\sqrt[4]{x} + \sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] Integral(x**(7/12)/(x**(1/4) + x**(1/3)), x)
```

GIAC/XCAS [A] time = 0.208707, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12\ln\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x^(1/4) + 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12)
 - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sq
rt(x) + 12/5*x^(5/12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^
(1/12) - 12*ln(x^(1/12) + 1)
```

3.238 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi [A] time = 0.0263871, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi in Sympy [A] time = 1.46503, size = 19, normalized size = 0.79

$$x\sqrt{-1+\frac{1}{x}}-\operatorname{atan}\left(\sqrt{-1+\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1-x)/x)**(1/2),x)
```

```
[Out] x*sqrt(-1 + 1/x) - atan(sqrt(-1 + 1/x))
```

Mathematica [A] time = 0.0217444, size = 40, normalized size = 1.67

$$\sqrt{\frac{1}{x}-1}x-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x}-1}(2x-1)}{2(x-1)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[(Sqrt[-1 + x^(-1)]*(-1 + 2*x))/(2*(-
1 + x))]/2
```

Maple [A] time = 0.011, size = 40, normalized size = 1.7

$$\frac{x}{2}\sqrt{-\frac{-1+x}{x}}\left(2\sqrt{-x^2+x}+\arcsin\left(2x-1\right)\right)\frac{1}{\sqrt{-x\left(-1+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(-(-1+x)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(-1+
x))^(1/2)
```

Maxima [A] time = 1.51653, size = 50, normalized size = 2.08

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Fricas [A] time = 0.211082, size = 35, normalized size = 1.46

$$x\sqrt{-\frac{x-1}{x}}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="fricas")
```

```
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{-x+1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1-x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((-x + 1)/x), x)
```

GIAC/XCAS [A] time = 0.216564, size = 38, normalized size = 1.58

$$\frac{1}{4}\,\pi\operatorname{sign}(x)+\frac{1}{2}\,\arcsin(2\,x-1)\operatorname{sign}(x)+\sqrt{-x^2+x}\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x - 1)/x),x, algorithm="giac")
```

```
[Out] 1/4*pi*sign(x) + 1/2*arcsin(2*x - 1)*sign(x) + sqrt(-x^2 + x)*sig
n(x)
```

3.239 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rubi [A] time = 0.0352887, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rubi in Sympy [A] time = 3.53724, size = 10, normalized size = 0.91

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Mathematica [A] time = 0.0125043, size = 20, normalized size = 1.82

$$\log(\sin(x)) - 2\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.044, size = 12, normalized size = 1.1

$$\ln(\sin(x)) - \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/(sin(x)+sin(x)^2),x)
```

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Maxima [A] time = 1.39643, size = 15, normalized size = 1.36

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + sin(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Fricas [A] time = 0.229957, size = 18, normalized size = 1.64

$$\log\left(\frac{1}{2}\,\sin\left(x\right)\right) - \log\left(\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + sin(x)),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [A] time = 0.243918, size = 10, normalized size = 0.91

$$-\log\left(\sin\left(x\right) + 1\right) + \log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

GIAC/XCAS [A] time = 0.210021, size = 16, normalized size = 1.45

$$-\ln\left(\sin\left(x\right) + 1\right) + \ln\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + sin(x)),x, algorithm="giac")
```

```
[Out] -ln(sin(x) + 1) + ln(abs(sin(x)))
```

3.240 $\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx$

Optimal. Leaf size=17

$$2\log(e^x+2)-\log(e^x+1)$$

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rubi [A] time = 0.0555049, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$2\log(e^x+2)-\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] Integral(exp(2*x)/(exp(2*x) + 3*exp(x) + 2), x)
```

Mathematica [A] time = 0.00993707, size = 17, normalized size = 1.

$$2\log(e^x+2)-\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$-\ln(1+e^x)+2\ln(2+e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

Maxima [A] time = 1.37825, size = 20, normalized size = 1.18

$$2\log(e^x+2)-\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^(2*x) + 3*e^x + 2),x, algorithm="maxima")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Fricas [A] time = 0.212622, size = 20, normalized size = 1.18

$$2 \log (e^x + 2) - \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^(2*x) + 3*e^x + 2),x, algorithm="fricas")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Sympy [A] time = 0.128722, size = 14, normalized size = 0.82

$$-\log (e^x + 1) + 2 \log (e^x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -log(exp(x) + 1) + 2*log(exp(x) + 2)
```

GIAC/XCAS [A] time = 0.204425, size = 20, normalized size = 1.18

$$2 \ln (e^x + 2) - \ln (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^(2*x) + 3*e^x + 2),x, algorithm="giac")
```

```
[Out] 2*ln(e^x + 2) - ln(e^x + 1)
```

3.241 $\int \frac{1}{\sqrt{1+e^x}}\,dx$

Optimal. Leaf size=12

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rubi [A] time = 0.0177997, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rubi in Sympy [A] time = 1.35893, size = 12, normalized size = 1.

$$-2\operatorname{atanh}\left(\sqrt{e^x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+exp(x))**(1/2),x)
```

```
[Out] -2*atanh(sqrt(exp(x) + 1))
```

Mathematica [A] time = 0.00726841, size = 12, normalized size = 1.

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Maple [A] time = 0.006, size = 10, normalized size = 0.8

$$-2\,Artanh\left(\sqrt{1+e^x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+exp(x))^(1/2),x)
```

```
[Out] -2*arctanh((1+exp(x))^(1/2))
```

Maxima [A] time = 1.38464, size = 28, normalized size = 2.33

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e^x + 1),x, algorithm="maxima")
```

```
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Fricas [A] time = 0.211383, size = 28, normalized size = 2.33

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e^x + 1),x, algorithm="fricas")
```

```
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Sympy [A] time = 0.573748, size = 26, normalized size = 2.17

$$\log\left(-1+\frac{1}{\sqrt{e^x+1}}\right)-\log\left(1+\frac{1}{\sqrt{e^x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+exp(x))**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(exp(x) + 1)) - log(1 + 1/sqrt(exp(x) + 1))
```

GIAC/XCAS [A] time = 0.204356, size = 28, normalized size = 2.33

$$-\ln\left(\sqrt{e^x+1}+1\right)+\ln\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(e^x + 1),x, algorithm="giac")
```

```
[Out] -ln(sqrt(e^x + 1) + 1) + ln(sqrt(e^x + 1) - 1)
```

3.242 $\int \sqrt{1-e^x}\,dx$

Optimal. Leaf size=28

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rubi [A] time = 0.031239, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rubi in Sympy [A] time = 2.0329, size = 20, normalized size = 0.71

$$2\sqrt{-e^x+1}-2\operatorname{atanh}\left(\sqrt{-e^x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-exp(x))**(1/2),x)
```

```
[Out] 2*sqrt(-exp(x) + 1) - 2*atanh(sqrt(-exp(x) + 1))
```

Mathematica [A] time = 0.0162804, size = 28, normalized size = 1.

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Maple [A] time = 0.008, size = 36, normalized size = 1.3

$$2\sqrt{1-\mathrm{e}^x}+\ln\left(-1+\sqrt{1-\mathrm{e}^x}\right)-\ln\left(1+\sqrt{1-\mathrm{e}^x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-exp(x))^(1/2),x)
```

```
[Out] 2*(1-exp(x))^(1/2)+ln(-1+(1-exp(x))^(1/2))-ln(1+(1-exp(x))^(1/2))
```

Maxima [A] time = 1.36187, size = 47, normalized size = 1.68

$$2\sqrt{-e^x+1}-\log\left(\sqrt{-e^x+1}+1\right)+\log\left(\sqrt{-e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^x + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) -
 1)
```

Fricas [A] time = 0.211196, size = 47, normalized size = 1.68

$$2\sqrt{-e^x+1}-\log\left(\sqrt{-e^x+1}+1\right)+\log\left(\sqrt{-e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^x + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) -
 1)
```

Sympy [A] time = 0.658805, size = 32, normalized size = 1.14

$$2\sqrt{-e^x+1}+\log\left(\sqrt{-e^x+1}-1\right)-\log\left(\sqrt{-e^x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-exp(x))**(1/2),x)
```

```
[Out] 2*sqrt(-exp(x) + 1) + log(sqrt(-exp(x) + 1) - 1) - log(sqrt(-exp(
x) + 1) + 1)
```

GIAC/XCAS [A] time = 0.205491, size = 50, normalized size = 1.79

$$2\sqrt{-e^x+1}-\ln\left(\sqrt{-e^x+1}+1\right)+\ln\left(-\sqrt{-e^x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-e^x + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(-e^x + 1) - ln(sqrt(-e^x + 1) + 1) + ln(-sqrt(-e^x + 1) +
1)
```

3.243 $\int \frac{1}{3-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rubi [A] time = 0.0339761, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rubi in Sympy [A] time = 0.913506, size = 20, normalized size = 0.47

$$-\frac{\log\left(-3\tan\left(\frac{x}{2}\right)+1\right)}{4}+\frac{\log\left(-\tan\left(\frac{x}{2}\right)+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3-5*sin(x)),x)
```

```
[Out] -log(-3*tan(x/2) + 1)/4 + log(-tan(x/2) + 3)/4
```

Mathematica [A] time = 0.00889649, size = 43, normalized size = 1.

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Maple [A] time = 0.023, size = 22, normalized size = 0.5

$$-\frac{1}{4}\ln\left(-1+3\,\tan\left(x/2\right)\right)+\frac{1}{4}\ln\left(\tan\left(\frac{x}{2}\right)-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3-5*sin(x)),x)
```

```
[Out] -1/4*ln(-1+3*tan(1/2*x))+1/4*ln(tan(1/2*x)-3)
```

Maxima [A] time = 1.33605, size = 41, normalized size = 0.95

$$-\frac{1}{4}\log\left(\frac{3\sin(x)}{\cos(x)+1}-1\right)+\frac{1}{4}\log\left(\frac{\sin(x)}{\cos(x)+1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 3),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*sin(x)/(cos(x) + 1) - 1) + 1/4*log(sin(x)/(cos(x) + 1)
 - 3)
```

Fricas [A] time = 0.231099, size = 36, normalized size = 0.84

$$\frac{1}{8}\log(4\cos(x)-3\sin(x)+5)-\frac{1}{8}\log(-4\cos(x)-3\sin(x)+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 3),x, algorithm="fricas")
```

```
[Out] 1/8*log(4*cos(x) - 3*sin(x) + 5) - 1/8*log(-4*cos(x) - 3*sin(x) +
 5)
```

Sympy [A] time = 0.283378, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-3\right)}{4}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3-5*sin(x)),x)
```

```
[Out] log(tan(x/2) - 3)/4 - log(tan(x/2) - 1/3)/4
```

GIAC/XCAS [A] time = 0.215824, size = 31, normalized size = 0.72

$$-\frac{1}{4}\ln\left(\left|3\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{4}\ln\left(\left|\tan\left(\frac{1}{2}x\right)-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 3),x, algorithm="giac")
```

```
[Out] -1/4*ln(abs(3*tan(1/2*x) - 1)) + 1/4*ln(abs(tan(1/2*x) - 3))
```

3.244 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0197602, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi in Sympy [A] time = 0.567327, size = 22, normalized size = 1.05

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}(-\sin(x)+\cos(x))}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] -sqrt(2)*atanh(sqrt(2)*(-sin(x) + cos(x))/2)/2
```

Mathematica [C] time = 0.024974, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] time = 0.001, size = 19, normalized size = 0.9

$$\sqrt{2}\,Artanh\left(\frac{\sqrt{2}}{4}\left(2\,\tan(x/2)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Maxima [A] time = 1.58619, size = 54, normalized size = 2.57

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1\right)}{2\sqrt{2}+\frac{2\sin(x)}{\cos(x)+1}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-2*(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/((2*sqrt(
2)) + 2*sin(x)/(cos(x) + 1) - 2))
```

Fricas [A] time = 0.224751, size = 51, normalized size = 2.43

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) +
 3)/(2*cos(x)*sin(x) + 1))
```

Sympy [A] time = 15.1089, size = 0, normalized size = 0.

NaN

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.238795, size = 50, normalized size = 2.38

$$-\frac{1}{2}\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2)
+ 2*tan(1/2*x) - 2))
```

3.245 $\int \frac{1}{1-\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=11

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

```
[Out] -Log[1 + Cot[x/2]]
```

Rubi [A] time = 0.0184646, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -Log[1 + Cot[x/2]]
```

Rubi in Sympy [A] time = 1.42978, size = 14, normalized size = 1.27

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(x)+sin(x)),x)
```

```
[Out] -log(tan(x/2) + 1) + log(tan(x/2))
```

Mathematica [B] time = 0.0136735, size = 24, normalized size = 2.18

$$\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] Log[Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.045, size = 16, normalized size = 1.5

$$-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(x)+sin(x)),x)
```

```
[Out] -ln(1+tan(1/2*x))+ln(tan(1/2*x))
```

Maxima [A] time = 1.33959, size = 34, normalized size = 3.09

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - sin(x) - 1),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.237937, size = 23, normalized size = 2.09

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\frac{1}{2}\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - sin(x) - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(x) + 1/2) - 1/2*log(sin(x) + 1)
```

Sympy [A] time = 0.315917, size = 14, normalized size = 1.27

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(x)+sin(x)),x)
```

```
[Out] -log(tan(x/2) + 1) + log(tan(x/2))
```

GIAC/XCAS [A] time = 0.213801, size = 23, normalized size = 2.09

$$-\ln\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\ln\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - sin(x) - 1),x, algorithm="giac")
```

```
[Out] -ln(abs(tan(1/2*x) + 1)) + ln(abs(tan(1/2*x)))
```

3.246 $\int \frac{1}{4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

```
[Out] -ArcTanh[(3*Cos[x] - 4*Sin[x])/5]/5
```

Rubi [A] time = 0.0207096, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] - 4*Sin[x])/5]/5
```

Rubi in Sympy [A] time = 0.585207, size = 17, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(-\frac{4\sin(x)}{5}+\frac{3\cos(x)}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -atanh(-4*sin(x)/5 + 3*cos(x)/5)/5
```

Mathematica [B] time = 0.0143775, size = 43, normalized size = 2.39

$$\frac{1}{5}\log\left(2\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -Log[2*Cos[x/2] - Sin[x/2]]/5 + Log[Cos[x/2] + 2*Sin[x/2]]/5
```

Maple [A] time = 0.046, size = 22, normalized size = 1.2

$$-\frac{1}{5}\ln\left(\tan\left(\frac{x}{2}\right)-2\right)+\frac{1}{5}\ln\left(2\tan(x/2)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -1/5*ln(tan(1/2*x)-2)+1/5*ln(2*tan(1/2*x)+1)
```

Maxima [A] time = 1.36818, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) + 1) - 1/5*log(sin(x)/(cos(x) + 1)
- 2)
```

Fricas [A] time = 0.221554, size = 36, normalized size = 2.

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) - 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) + 2
*sin(x) + 5/2)
```

Sympy [A] time = 0.32849, size = 20, normalized size = 1.11

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{5}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{1}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -log(tan(x/2) - 2)/5 + log(tan(x/2) + 1/2)/5
```

GIAC/XCAS [A] time = 0.212357, size = 31, normalized size = 1.72

$$\frac{1}{5}\ln\left(\left|2\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{5}\ln\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*cos(x) + 3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*ln(abs(2*tan(1/2*x) + 1)) - 1/5*ln(abs(tan(1/2*x) - 2))
```

3.247 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\csc^2(x) - \frac{1}{2}\tanh^{-1}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rubi [A] time = 0.0788112, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$

$$-\frac{1}{2}\csc^2(x) - \frac{1}{2}\tanh^{-1}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rubi in Sympy [A] time = 6.87073, size = 27, normalized size = 1.12

$$-\frac{\operatorname{atanh}(\cos(x))}{2} + \frac{\cos(x)}{2(-\cos^2(x)+1)} - \frac{1}{2(-\cos^2(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] -atanh(cos(x))/2 + cos(x)/(2*(-cos(x)**2 + 1)) - 1/(2*(-cos(x)**2
 + 1))
```

Mathematica [A] time = 0.0225975, size = 35, normalized size = 1.46

$$-\frac{1}{4}\sec^2\left(\frac{x}{2}\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

Maple [A] time = 0.046, size = 24, normalized size = 1.

$$-\frac{1}{2+2\cos(x)} - \frac{\ln(1+\cos(x))}{4} + \frac{\ln(\cos(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(sin(x)+tan(x)),x)
```

```
[Out] -1/2/(1+cos(x))-1/4*ln(1+cos(x))+1/4*ln(cos(x)-1)
```

Maxima [A] time = 1.39106, size = 34, normalized size = 1.42

$$-\frac{\sin(x)^2}{4(\cos(x)+1)^2}+\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + tan(x)),x, algorithm="maxima")
```

```
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.23255, size = 47, normalized size = 1.96

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*
cos(x) + 1/2) + 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] Integral(1/(sin(x) + tan(x)), x)
```

GIAC/XCAS [A] time = 0.204009, size = 38, normalized size = 1.58

$$\frac{\cos(x)-1}{4(\cos(x)+1)}+\frac{1}{4}\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(x) + tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*ln(-(cos(x) - 1)/(cos(x) + 1)
)
```

3.248 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rubi [A] time = 0.0457707, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

Mathematica [A] time = 0.0401553, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]
))
```

Maple [A] time = 0.076, size = 24, normalized size = 1.

$$\frac{1}{4+4\cos(x)}-\frac{\ln(1+\cos(x))}{8}+\frac{\ln(\cos(x)-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] 1/4/(1+cos(x))-1/8*ln(1+cos(x))+1/8*ln(cos(x)-1)
```

Maxima [A] time = 1.35636, size = 297, normalized size = 12.38

$$\frac{4\cos(2x)\cos(x) + 8\cos(x)^2 - \left(2(2\cos(x)+1)\cos(2x) + \cos(2x)^2 + 4\cos(x)^2 + \sin(2x)^2 + 4\sin(2x)\sin(x) + 4\sin(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x) + 2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x)
+ cos(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*si
n(x)^2 + 4*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) +
(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2
 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*sin(x)^2 + 4*cos
(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 + sin(2
*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

Fricas [A] time = 0.224978, size = 47, normalized size = 1.96

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) - (\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right) - 2}{8(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x) + 2*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*
cos(x) + 1/2) - 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{2\sin(x) + \sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

GIAC/XCAS [A] time = 0.202144, size = 38, normalized size = 1.58

$$-\frac{\cos(x)-1}{8(\cos(x)+1)} + \frac{1}{8}\ln\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sin(2*x) + 2*sin(x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*ln(-(cos(x) - 1)/(cos(x) + 1
))
```

3.249 $\int \frac{\sec(x)}{1+\sin(x)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

```
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Rubi [A] time = 0.0546269, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]/(1 + Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Rubi in Sympy [A] time = 3.57574, size = 14, normalized size = 0.78

$$\frac{\operatorname{atanh}(\sin(x))}{2} - \frac{1}{2(\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)/(1+sin(x)),x)
```

```
[Out] atanh(sin(x))/2 - 1/(2*(sin(x) + 1))
```

Mathematica [B] time = 0.0383487, size = 54, normalized size = 3.

$$\frac{1}{2}\left(-\frac{1}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]/(1 + Sin[x]),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - (Cos[x/2]
 + Sin[x/2])^(-2))/2
```

Maple [A] time = 0.039, size = 24, normalized size = 1.3

$$-\frac{1}{2+2\sin(x)} + \frac{\ln(1+\sin(x))}{4} - \frac{\ln(-1+\sin(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)/(1+sin(x)),x)
```

```
[Out] -1/2/(1+sin(x))+1/4*ln(1+sin(x))-1/4*ln(-1+sin(x))
```

Maxima [A] time = 1.39578, size = 31, normalized size = 1.72

$$-\frac{1}{2\left(\sin\left(x\right)+1\right)}+\frac{1}{4}\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(\sin\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)/(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(sin(x) - 1)
```

Fricas [A] time = 0.21788, size = 45, normalized size = 2.5

$$\frac{\left(\sin\left(x\right)+1\right)\log\left(\sin\left(x\right)+1\right)-\left(\sin\left(x\right)+1\right)\log\left(-\sin\left(x\right)+1\right)-2}{4\left(\sin\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)/(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(x) + 1)*log(sin(x) + 1) - (sin(x) + 1)*log(-sin(x) + 1)
 - 2)/(sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(x\right)}{\sin\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)/(1+sin(x)),x)
```

```
[Out] Integral(sec(x)/(sin(x) + 1), x)
```

GIAC/XCAS [A] time = 0.201098, size = 34, normalized size = 1.89

$$-\frac{1}{2\left(\sin\left(x\right)+1\right)}+\frac{1}{4}\ln\left(\sin\left(x\right)+1\right)-\frac{1}{4}\ln\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)/(sin(x) + 1),x, algorithm="giac")
```

```
[Out] -1/2/(sin(x) + 1) + 1/4*ln(sin(x) + 1) - 1/4*ln(-sin(x) + 1)
```

3.250 $\int \frac{1}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0412535, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi in Sympy [A] time = 2.18447, size = 32, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] -atanh((a*cos(x) - b*sin(x))/sqrt(a**2 + b**2))/sqrt(a**2 + b**2)
```

Mathematica [A] time = 0.0564802, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Maple [A] time = 0.071, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2+b^2}}\,Artanh\left(1/2\,\frac{2\,b\tan(x/2)-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a)/(a^2+b^2)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cos(x) + a*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.234516, size = 154, normalized size = 4.28

$$\frac{\log\left(-\frac{2\left(a^3+ab^2\right)\cos(x)-2\left(a^2b+b^3\right)\sin(x)+\left(2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2\,b^2\right)\sqrt{a^2+b^2}}{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2+a^2}\right)}{2\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cos(x) + a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*(a^3 + a*b^2)*cos(x) - 2*(a^2*b + b^3)*sin(x) + (2*a*
b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^2 - 2*b^2)*sqrt(a^2 +
b^2))/(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2))/sqrt(a^
2 + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

GIAC/XCAS [A] time = 0.224888, size = 82, normalized size = 2.28

$$-\frac{\ln\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cos(x) + a*sin(x)),x, algorithm="giac")
```

```
[Out] -ln(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2
*x) - 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.251 $\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi [A] time = 0.0427766, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi in Sympy [A] time = 26.9054, size = 10, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] atan(a*tan(x)/b)/(a*b)
```

Mathematica [A] time = 0.0517988, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] time = 0.003, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{a\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
```

```
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A] time = 1.5373, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [A] time = 0.233105, size = 58, normalized size = 3.87

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))
/(a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.204, size = 35, normalized size = 2.33

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.252 $\int \frac{x}{-1+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] Log[1 - x^2]/2
```

Rubi [A] time = 0.00576833, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rubi in Sympy [A] time = 0.772527, size = 7, normalized size = 0.58

$$\frac{\log\left(-x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-1),x)
```

```
[Out] log(-x**2 + 1)/2
```

Mathematica [A] time = 0.002261, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-1 + x^2),x]
```

```
[Out] Log[-1 + x^2]/2
```

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-1),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A] time = 1.33841, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

Fricas [A] time = 0.191185, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.060374, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.199584, size = 12, normalized size = 1.

$$\frac{1}{2}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 1),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(x^2 - 1))
```

3.253 $\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi [A] time = 0.00827636, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x^{\frac{3}{2}}}{3}+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + Integral(x, x)
```

Mathematica [A] time = 0.00380652, size = 17, normalized size = 1.

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] time = 0.001, size = 12, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(1+x^(1/2)),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Maxima [A] time = 1.3437, size = 35, normalized size = 2.06

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Fricas [A] time = 0.194566, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A] time = 0.142508, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

GIAC/XCAS [A] time = 0.197753, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)*(sqrt(x) + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

3.254 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0150472, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi in Sympy [A] time = 0.502001, size = 8, normalized size = 0.67

$$-\frac{\sin(x)}{-\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(x)),x)
```

```
[Out] -sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.0072089, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 1.33325, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 0.204506, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.687695, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

GIAC/XCAS [A] time = 0.200409, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.255 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.0236035, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{2}\tan(x)\sec(x) - \frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi in Sympy [A] time = 1.62858, size = 14, normalized size = 0.88

$$-\frac{\operatorname{atanh}(\sin(x))}{2} + \frac{\tan(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)*tan(x)**2,x)
```

```
[Out] -atanh(sin(x))/2 + tan(x)/(2*cos(x))
```

Mathematica [B] time = 0.0687743, size = 42, normalized size = 2.62

$$\frac{1}{2}\left(\tan(x)\sec(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan
[x])/2
```

Maple [A] time = 0.002, size = 24, normalized size = 1.5

$$\frac{(\sin(x))^3}{2\,(\cos(x))^2} + \frac{\sin(x)}{2} - \frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [A] time = 1.33703, size = 36, normalized size = 2.25

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x)
 - 1)
```

Fricas [A] time = 0.225058, size = 46, normalized size = 2.88

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*si
n(x))/cos(x)^2
```

Sympy [A] time = 0.127665, size = 27, normalized size = 1.69

$$\frac{\log(\sin(x)-1)}{4}-\frac{\log(\sin(x)+1)}{4}-\frac{\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

GIAC/XCAS [A] time = 0.203761, size = 39, normalized size = 2.44

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\ln(\sin(x)+1)+\frac{1}{4}\ln(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*ln(sin(x) + 1) + 1/4*ln(-sin(x)
+ 1)
```

3.256 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0389816, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] Integral(tan(x)**3/cos(x)**3, x)
```

Mathematica [A] time = 0.0116403, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [B] time = 0., size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5\,(\cos(x))^5}+\frac{(\sin(x))^4}{15\,(\cos(x))^3}-\frac{(\sin(x))^4}{15\,\cos(x)}-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)
-1/15*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.38086, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 0.208504, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.134136, size = 15, normalized size = 0.88

$$-\frac{5\cos^2(x)-3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

GIAC/XCAS [A] time = 0.197901, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2-3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.257 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0153893, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi in Sympy [A] time = 1.13584, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Mathematica [A] time = 0.00404331, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x} - 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$-2\,\mathrm{e}^{\sqrt{x}} + 2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^(1/2)),x)
```

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.35099, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 0.204909, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.219944, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

GIAC/XCAS [A] time = 0.197836, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sqrt(x),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.258 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Rubi [A] time = 0.0543834, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3}{3}+19x-\frac{\log(x)}{10}+\frac{3126\log(-x+5)}{35}-\frac{31\log(x+2)}{14}+3\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 19*x - log(x)/10 + 3126*log(-x + 5)/35 - 31*log(x + 2)/1
4 + 3*Integral(x, x)
```

Mathematica [A] time = 0.0106007, size = 42, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31
*Log[2 + x])/14
```

Maple [A] time = 0.012, size = 31, normalized size = 0.7

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126\ln(-5+x)}{35}-\frac{31\ln(2+x)}{14}-\frac{\ln(x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x+3126/35*ln(-5+x)-31/14*ln(2+x)-1/10*ln(x)
```

Maxima [A] time = 1.36846, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(x+2)+\frac{3126}{35}\log(x-5)-\frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5)
- 1/10*log(x)
```

Fricas [A] time = 0.201931, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(x+2)+\frac{3126}{35}\log(x-5)-\frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5)
- 1/10*log(x)
```

Sympy [A] time = 0.162548, size = 36, normalized size = 0.86

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\log(x)}{10}+\frac{3126\log(x-5)}{35}-\frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*lo
g(x + 2)/14
```

GIAC/XCAS [A] time = 0.201086, size = 45, normalized size = 1.07

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\ln(|x+2|)+\frac{3126}{35}\ln(|x-5|)-\frac{1}{10}\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)/(x^3 - 3*x^2 - 10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*ln(abs(x + 2)) + 3126/35*ln(abs(
x - 5)) - 1/10*ln(abs(x))
```

3.259 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

```
[Out] 2*Sqrt[Log[x]]
```

Rubi [A] time = 0.0206341, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rubi in Sympy [A] time = 1.49633, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

Mathematica [A] time = 0.00179286, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x)^(1/2),x)
```

```
[Out] 2*ln(x)^(1/2)
```

Maxima [A] time = 1.36554, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*sqrt(log(x))),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x))
```

Fricas [A] time = 0.197919, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*sqrt(log(x))),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x))
```

Sympy [A] time = 0.546356, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

GIAC/XCAS [A] time = 0.215078, size = 8, normalized size = 1.

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*sqrt(log(x))),x, algorithm="giac")
```

```
[Out] 2*sqrt(ln(x))
```

3.260 $\int \frac{5+2x}{-3+x}\,dx$

Optimal. Leaf size=12

$$2x + 11\log(3 - x)$$

```
[Out] 2*x + 11*Log[3 - x]
```

Rubi [A] time = 0.0143679, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$2x + 11\log(3 - x)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*x + 11*Log[3 - x]
```

Rubi in Sympy [A] time = 1.44076, size = 8, normalized size = 0.67

$$2x + 11\log(-x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+2*x)/(-3+x),x)
```

```
[Out] 2*x + 11*log(-x + 3)
```

Mathematica [A] time = 0.00387147, size = 12, normalized size = 1.

$$2(x - 3) + 11\log(x - 3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*(-3 + x) + 11*Log[-3 + x]
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$2\,x + 11\,\ln(-3 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+2*x)/(-3+x),x)
```

```
[Out] 2*x+11*ln(-3+x)
```

Maxima [A] time = 1.45021, size = 14, normalized size = 1.17

$$2\,x + 11\,\log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x - 3),x, algorithm="maxima")
```

```
[Out] 2*x + 11*log(x - 3)
```

Fricas [A] time = 0.191847, size = 14, normalized size = 1.17

$$2\,x + 11\,\log\left(x - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x - 3),x, algorithm="fricas")
```

```
[Out] 2*x + 11*log(x - 3)
```

Sympy [A] time = 0.059295, size = 8, normalized size = 0.67

$$2x + 11\log\left(x - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+2*x)/(-3+x),x)
```

```
[Out] 2*x + 11*log(x - 3)
```

GIAC/XCAS [A] time = 0.199789, size = 15, normalized size = 1.25

$$2\,x + 11\ln\left(|x - 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x - 3),x, algorithm="giac")
```

```
[Out] 2*x + 11*ln(abs(x - 3))
```

3.261 $\int e^{e^x+x}\,dx$

Optimal. Leaf size=5

$$e^{e^x}$$

```
[Out] E^E^x
```

Rubi [A] time = 0.00952141, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Rubi in Sympy [A] time = 1.97641, size = 3, normalized size = 0.6

$$e^{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x+exp(x)),x)
```

```
[Out] exp(exp(x))
```

Mathematica [A] time = 0.00302928, size = 5, normalized size = 1.

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Maple [A] time = 0.003, size = 4, normalized size = 0.8

$$\mathrm{e}^{\mathrm{e}^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(exp(x)+x),x)
```

```
[Out] exp(exp(x))
```

Maxima [A] time = 1.3295, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + e^x),x, algorithm="maxima")
```

```
[Out] e^(e^x)
```

Fricas [A] time = 0.205278, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + e^x),x, algorithm="fricas")
```

```
[Out] e^(e^x)
```

Sympy [A] time = 0.597229, size = 3, normalized size = 0.6

$$e^{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x+exp(x)),x)
```

```
[Out] exp(exp(x))
```

GIAC/XCAS [A] time = 0.206549, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + e^x),x, algorithm="giac")
```

```
[Out] e^(e^x)
```

3.262 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8} - \frac{1}{4}\sin(x)\cos^3(x) + \frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0420618, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{8} - \frac{1}{4}\sin(x)\cos^3(x) + \frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 1.61114, size = 20, normalized size = 0.83

$$\frac{x}{8} - \frac{\sin(x)\cos^3(x)}{4} + \frac{\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(x)*cos(x)**3/4 + sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00709434, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0., size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 1.35859, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 0.215232, size = 26, normalized size = 1.08

$$-\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.055724, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

GIAC/XCAS [A] time = 0.197693, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.263 $\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=8

$$-\log(\sin(x)+\cos(x))$$

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rubi [A] time = 0.0352941, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rubi in Sympy [A] time = 3.25811, size = 8, normalized size = 1.

$$-\log(\sin(x)+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] -log(sin(x) + cos(x))
```

Mathematica [A] time = 0.012175, size = 8, normalized size = 1.

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Maple [A] time = 0.032, size = 9, normalized size = 1.1

$$-\ln(\cos(x)+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] -ln(cos(x)+sin(x))
```

Maxima [A] time = 1.34627, size = 11, normalized size = 1.38

$$-\log(\cos(x)+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cos(x) - sin(x))/(cos(x) + sin(x)),x, algorithm="maxima")
```

```
[Out] -log(cos(x) + sin(x))
```

Fricas [A] time = 0.222238, size = 15, normalized size = 1.88

$$-\frac{1}{2}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cos(x) - sin(x))/(cos(x) + sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.175349, size = 8, normalized size = 1.

$$-\log\left(\sin\left(x\right)+\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] -log(sin(x) + cos(x))
```

GIAC/XCAS [A] time = 0.210854, size = 24, normalized size = 3.

$$\frac{1}{2}\ln\left(\tan\left(x\right)^2+1\right)-\ln\left(\left|\tan\left(x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(cos(x) - sin(x))/(cos(x) + sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*ln(tan(x)^2 + 1) - ln(abs(tan(x) + 1))
```

3.264 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2]
```

Rubi [A] time = 0.0064787, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Rubi in Sympy [A] time = 0.884892, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 1)
```

Mathematica [A] time = 0.00314063, size = 13, normalized size = 1.

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Maple [A] time = 0., size = 17, normalized size = 1.3

$$(-1+x)(1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+1)^(1/2),x)
```

```
[Out] (-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.35058, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

Fricas [A] time = 0.203735, size = 23, normalized size = 1.77

$$\frac{x^2}{\sqrt{-x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] x^2/(sqrt(-x^2 + 1) - 1)
```

Sympy [A] time = 0.274035, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 1)
```

GIAC/XCAS [A] time = 0.198465, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

3.265 $\int x^3 \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

```
[Out] -x^4/16 + (x^4*Log[x])/4
```

Rubi [A] time = 0.0120768, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Log[x],x]
```

```
[Out] -x^4/16 + (x^4*Log[x])/4
```

Rubi in Sympy [A] time = 1.10186, size = 12, normalized size = 0.71

$$\frac{x^4 \log(x)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*ln(x),x)
```

```
[Out] x**4*log(x)/4 - x**4/16
```

Mathematica [A] time = 0.00172183, size = 17, normalized size = 1.

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Log[x],x]
```

```
[Out] -x^4/16 + (x^4*Log[x])/4
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x^4}{16} + \frac{x^4 \ln(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*ln(x),x)
```

```
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

Maxima [A] time = 1.34176, size = 18, normalized size = 1.06

$$\frac{1}{4}x^4\log(x) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Fricas [A] time = 0.207398, size = 18, normalized size = 1.06

$$\frac{1}{4}x^4\log(x) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Sympy [A] time = 0.080395, size = 12, normalized size = 0.71

$$\frac{x^4\log(x)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*ln(x),x)
```

```
[Out] x**4*log(x)/4 - x**4/16
```

GIAC/XCAS [A] time = 0.199077, size = 18, normalized size = 1.06

$$\frac{1}{4}x^4\ln(x) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*ln(x) - 1/16*x^4
```

3.266 $\int \frac{\sqrt{-2+x}}{2+x}\,dx$

Optimal. Leaf size=24

$$2\sqrt{x-2}-4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Rubi [A] time = 0.0196514, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$2\sqrt{x-2}-4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2 + x]/(2 + x),x]
```

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Rubi in Sympy [A] time = 1.60992, size = 19, normalized size = 0.79

$$2\sqrt{x-2}-4\operatorname{atan}\left(\frac{\sqrt{x-2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2+x)**(1/2)/(2+x),x)
```

```
[Out] 2*sqrt(x - 2) - 4*atan(sqrt(x - 2)/2)
```

Mathematica [A] time = 0.0110829, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2 + x]/(2 + x),x]
```

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Maple [A] time = 0.009, size = 19, normalized size = 0.8

$$-4\arctan\left(1/2\sqrt{-2+x}\right)+2\sqrt{-2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2+x)^(1/2)/(2+x),x)
```

```
[Out] -4*arctan(1/2*(-2+x)^(1/2))+2*(-2+x)^(1/2)
```

Maxima [A] time = 1.49346, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 2)/(x + 2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Fricas [A] time = 0.206545, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 2)/(x + 2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Sympy [A] time = 2.5413, size = 109, normalized size = 4.54

$$\begin{cases} -4i\,\operatorname{acosh}\left(\frac{2}{\sqrt{x+2}}\right)-\frac{2i\sqrt{x+2}}{\sqrt{-1+\frac{4}{x+2}}}+\frac{8i}{\sqrt{-1+\frac{4}{x+2}}\sqrt{x+2}} & \text{for } 4\left|\frac{1}{x+2}\right|>1 \\ 4\,\operatorname{asin}\left(\frac{2}{\sqrt{x+2}}\right)+\frac{2\sqrt{x+2}}{\sqrt{1-\frac{4}{x+2}}}-\frac{8}{\sqrt{1-\frac{4}{x+2}}\sqrt{x+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2+x)**(1/2)/(2+x),x)
```

```
[Out] Piecewise((-4*I*acosh(2/sqrt(x + 2)) - 2*I*sqrt(x + 2)/sqrt(-1 +
4/(x + 2)) + 8*I/(sqrt(-1 + 4/(x + 2))*sqrt(x + 2)), 4*Abs(1/(x +
 2)) > 1), (4*asin(2/sqrt(x + 2)) + 2*sqrt(x + 2)/sqrt(1 - 4/(x +
 2)) - 8/(sqrt(1 - 4/(x + 2))*sqrt(x + 2)), True))
```

GIAC/XCAS [A] time = 0.19962, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x - 2)/(x + 2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

3.267 $\int \frac{x}{(2+x)^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{x+2} + \log(x+2)$$

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi [A] time = 0.0129833, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(2 + x)^2,x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi in Sympy [A] time = 1.1107, size = 8, normalized size = 0.67

$$\log(x+2) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2+x)**2,x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

Mathematica [A] time = 0.00479015, size = 12, normalized size = 1.

$$\frac{2}{x+2} + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(2 + x)^2,x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] time = 0.005, size = 13, normalized size = 1.1

$$2\,(2+x)^{-1} + \ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2+x)^2,x)
```

```
[Out] 2/(2+x)+ln(2+x)
```

Maxima [A] time = 1.32288, size = 16, normalized size = 1.33

$$\frac{2}{x+2} + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 2)^2,x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

Fricas [A] time = 0.19678, size = 22, normalized size = 1.83

$$\frac{(x+2)\log(x+2)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 2)^2,x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] time = 0.078056, size = 8, normalized size = 0.67

$$\log(x+2) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2+x)**2,x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

GIAC/XCAS [A] time = 0.19873, size = 18, normalized size = 1.5

$$\frac{2}{x+2} + \ln(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 2)^2,x, algorithm="giac")
```

```
[Out] 2/(x + 2) + ln(abs(x + 2))
```

3.268 $\int \log\left(1+x^2\right)\,dx$

Optimal. Leaf size=16

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rubi [A] time = 0.0143375, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rubi in Sympy [A] time = 1.61095, size = 15, normalized size = 0.94

$$x\log\left(x^2+1\right)-2x+2\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

Mathematica [A] time = 0.0038302, size = 16, normalized size = 1.

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Maple [A] time = 0.002, size = 17, normalized size = 1.1

$$-2x+2\arctan(x)+x\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x^2+1),x)
```

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Maxima [A] time = 1.51649, size = 22, normalized size = 1.38

$$x\log\left(x^2+1\right)-2x+2\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2 + 1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Fricas [A] time = 0.208972, size = 22, normalized size = 1.38

$$x \log\left(x^2 + 1\right) - 2\,x + 2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2 + 1),x, algorithm="fricas")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Sympy [A] time = 0.127364, size = 15, normalized size = 0.94

$$x \log\left(x^2 + 1\right) - 2x + 2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

GIAC/XCAS [A] time = 0.202274, size = 22, normalized size = 1.38

$$x\ln\left(x^2 + 1\right) - 2\,x + 2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x^2 + 1),x, algorithm="giac")
```

```
[Out] x*ln(x^2 + 1) - 2*x + 2*arctan(x)
```

3.269 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rubi [A] time = 0.0912611, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rubi in Sympy [A] time = 5.2017, size = 20, normalized size = 0.91

$$2\sqrt{\log(x)+1}-2\operatorname{atanh}\left(\sqrt{\log(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

```
[Out] 2*sqrt(log(x) + 1) - 2*atanh(sqrt(log(x) + 1))
```

Mathematica [A] time = 0.0155022, size = 37, normalized size = 1.68

$$\log\left(1-\sqrt{\log(x)+1}\right)-\log\left(\sqrt{\log(x)+1}+1\right)+2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] 2*Sqrt[1 + Log[x]] + Log[1 - Sqrt[1 + Log[x]]] - Log[1 + Sqrt[1 +
 Log[x]]]
```

Maple [A] time = 0.004, size = 30, normalized size = 1.4

$$2\sqrt{1+\ln(x)}+\ln\left(-1+\sqrt{1+\ln(x)}\right)-\ln\left(1+\sqrt{1+\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+ln(x))^(1/2)/x/ln(x),x)
```

```
[Out] 2*(1+ln(x))^(1/2)+ln(-1+(1+ln(x))^(1/2))-ln(1+(1+ln(x))^(1/2))
```

Maxima [A] time = 1.3562, size = 39, normalized size = 1.77

$$2\sqrt{\log(x)+1}-\log\left(\sqrt{\log(x)+1}+1\right)+\log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(log(x) + 1)/(x*log(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x)
+ 1) - 1)
```

Fricas [A] time = 0.23841, size = 39, normalized size = 1.77

$$2\sqrt{\log(x)+1}-\log\left(\sqrt{\log(x)+1}+1\right)+\log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(log(x) + 1)/(x*log(x)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x)
+ 1) - 1)
```

Sympy [A] time = 0.950221, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1}+\log\left(\sqrt{\log(x)+1}-1\right)-\log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

```
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x)
+ 1) + 1)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(log(x) + 1)/(x*log(x)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \left(1+\sqrt{x}\right)^8\,dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rubi [A] time = 0.0191267, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Sqrt[x])^8,x]
```

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rubi in Sympy [A] time = 2.07644, size = 20, normalized size = 0.74

$$\frac{\left(\sqrt{x}+1\right)^{10}}{5}-\frac{2\left(\sqrt{x}+1\right)^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x**(1/2))**8,x)
```

```
[Out] (sqrt(x) + 1)**10/5 - 2*(sqrt(x) + 1)**9/9
```

Mathematica [B] time = 0.00992939, size = 60, normalized size = 2.22

$$\frac{16x^{9/2}}{9}+16x^{7/2}+\frac{112x^{5/2}}{5}+\frac{16x^{3/2}}{3}+\frac{x^5}{5}+7x^4+\frac{70x^3}{3}+14x^2+x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Sqrt[x])^8,x]
```

```
[Out] x + (16*x^(3/2))/3 + 14*x^2 + (112*x^(5/2))/5 + (70*x^3)/3 + 16*x
^(7/2) + 7*x^4 + (16*x^(9/2))/9 + x^5/5
```

Maple [B] time = 0.003, size = 43, normalized size = 1.6

$$\frac{x^5}{5}+\frac{16}{9}x^{\frac{9}{2}}+7x^4+16x^{7/2}+\frac{70x^3}{3}+\frac{112}{5}x^{\frac{5}{2}}+14x^2+\frac{16}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x^(1/2))^8,x)
```

```
[Out] 1/5*x^5+16/9*x^(9/2)+7*x^4+16*x^(7/2)+70/3*x^3+112/5*x^(5/2)+14*x
^2+16/3*x^(3/2)+x
```

Maxima [A] time = 1.33507, size = 26, normalized size = 0.96

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^8,x, algorithm="maxima")
```

```
[Out] 1/5*(sqrt(x) + 1)^10 - 2/9*(sqrt(x) + 1)^9
```

Fricas [A] time = 0.19428, size = 62, normalized size = 2.3

$$\frac{1}{5}x^5+7x^4+\frac{70}{3}x^3+14x^2+\frac{16}{45}\left(5x^4+45x^3+63x^2+15x\right)\sqrt{x}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^8,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 + 7*x^4 + 70/3*x^3 + 14*x^2 + 16/45*(5*x^4 + 45*x^3 + 63*
x^2 + 15*x)*sqrt(x) + x
```

Sympy [A] time = 1.20865, size = 54, normalized size = 2.

$$\frac{16x^{\frac{9}{2}}}{9}+16x^{\frac{7}{2}}+\frac{112x^{\frac{5}{2}}}{5}+\frac{16x^{\frac{3}{2}}}{3}+\frac{x^5}{5}+7x^4+\frac{70x^3}{3}+14x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x**(1/2))**8,x)
```

```
[Out] 16*x**(9/2)/9 + 16*x**(7/2) + 112*x**(5/2)/5 + 16*x**(3/2)/3 + x*
*5/5 + 7*x**4 + 70*x**3/3 + 14*x**2 + x
```

GIAC/XCAS [A] time = 0.235854, size = 57, normalized size = 2.11

$$\frac{1}{5}x^5+\frac{16}{9}x^{\frac{9}{2}}+7x^4+16x^{\frac{7}{2}}+\frac{70}{3}x^3+\frac{112}{5}x^{\frac{5}{2}}+14x^2+\frac{16}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sqrt(x) + 1)^8,x, algorithm="giac")
```

```
[Out] 1/5*x^5 + 16/9*x^(9/2) + 7*x^4 + 16*x^(7/2) + 70/3*x^3 + 112/5*x^
(5/2) + 14*x^2 + 16/3*x^(3/2) + x
```

3.271 $\int \sec^4(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^6(x)}{6} - \frac{\sec^4(x)}{4}$$

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Rubi [A] time = 0.0373724, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sec^6(x)}{6} - \frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(x)}{\cos^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**4*tan(x)**3,x)
```

```
[Out] Integral(tan(x)**3/cos(x)**4, x)
```

Mathematica [A] time = 0.00833556, size = 17, normalized size = 1.

$$\frac{\sec^6(x)}{6} - \frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Maple [A] time = 0.015, size = 22, normalized size = 1.3

$$\frac{(\sin(x))^4}{6\,(\cos(x))^6} + \frac{(\sin(x))^4}{12\,(\cos(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^4*tan(x)^3,x)
```

```
[Out] 1/6*sin(x)^4/cos(x)^6+1/12*sin(x)^4/cos(x)^4
```

Maxima [A] time = 1.32976, size = 41, normalized size = 2.41

$$-\frac{3\,\sin(x)^2-1}{12\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(x)^2 - 1)/(sin(x)^6 - 3*sin(x)^4 + 3*sin(x)^2 - 1)
```

Fricas [A] time = 0.21826, size = 19, normalized size = 1.12

$$-\frac{3\,\cos(x)^2-2}{12\,\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

Sympy [A] time = 0.11837, size = 15, normalized size = 0.88

$$-\frac{3\cos^2(x)-2}{12\cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**4*tan(x)**3,x)
```

```
[Out] -(3*cos(x)**2 - 2)/(12*cos(x)**6)
```

GIAC/XCAS [A] time = 0.222739, size = 19, normalized size = 1.12

$$-\frac{3\,\cos(x)^2-2}{12\,\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

3.272 $\int \frac{x}{2-2x+x^2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rubi [A] time = 0.0184598, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rubi in Sympy [A] time = 2.01616, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2-2x+2\right)}{2}+\operatorname{atan}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-2*x+2),x)
```

```
[Out] log(x**2 - 2*x + 2)/2 + atan(x - 1)
```

Mathematica [A] time = 0.00538467, size = 22, normalized size = 1.

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Maple [A] time = 0.006, size = 17, normalized size = 0.8

$$\arctan(-1+x)+\frac{\ln\left(x^2-2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-2*x+2),x)
```

```
[Out] arctan(-1+x)+1/2*ln(x^2-2*x+2)
```

Maxima [A] time = 1.51392, size = 22, normalized size = 1.

$$\arctan(x-1)+\frac{1}{2}\log\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 2*x + 2),x, algorithm="maxima")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Fricas [A] time = 0.197379, size = 22, normalized size = 1.

$$\arctan(x-1)+\frac{1}{2}\log\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 2*x + 2),x, algorithm="fricas")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Sympy [A] time = 0.093142, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2-2x+2\right)}{2}+\operatorname{atan}(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-2*x+2),x)
```

```
[Out] log(x**2 - 2*x + 2)/2 + atan(x - 1)
```

GIAC/XCAS [A] time = 0.222689, size = 22, normalized size = 1.

$$\arctan(x-1)+\frac{1}{2}\ln\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 - 2*x + 2),x, algorithm="giac")
```

```
[Out] arctan(x - 1) + 1/2*ln(x^2 - 2*x + 2)
```

3.273 $\int x \sin^{-1}(x)\,dx$

Optimal. Leaf size=32

$$\frac{1}{4}\sqrt{1-x^2}x + \frac{1}{2}x^2\sin^{-1}(x) - \frac{1}{4}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rubi [A] time = 0.0267451, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$

$$\frac{1}{4}\sqrt{1-x^2}x + \frac{1}{2}x^2\sin^{-1}(x) - \frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rubi in Sympy [A] time = 2.03337, size = 24, normalized size = 0.75

$$\frac{x^2\operatorname{asin}(x)}{2} + \frac{x\sqrt{-x^2+1}}{4} - \frac{\operatorname{asin}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*asin(x),x)
```

```
[Out] x**2*asin(x)/2 + x*sqrt(-x**2 + 1)/4 - asin(x)/4
```

Mathematica [A] time = 0.0152347, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(\sqrt{1-x^2}x + \left(2x^2-1\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2] + (-1 + 2*x^2)*ArcSin[x])/4
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{\arcsin(x)}{4} + \frac{x^2\arcsin(x)}{2} + \frac{x}{4}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)*x,x)
```

```
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.48113, size = 32, normalized size = 1.

$$\frac{1}{2}\,x^2\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x-\frac{1}{4}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Fricas [A] time = 0.220367, size = 32, normalized size = 1.

$$\frac{1}{4}\left(2\,x^2-1\right)\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x
```

Sympy [A] time = 0.247828, size = 24, normalized size = 0.75

$$\frac{x^2\,\mathrm{asin}\left(x\right)}{2}+\frac{x\sqrt{-x^2+1}}{4}-\frac{\mathrm{asin}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*asin(x),x)
```

```
[Out] x**2*asin(x)/2 + x*sqrt(-x**2 + 1)/4 - asin(x)/4
```

GIAC/XCAS [A] time = 0.227115, size = 35, normalized size = 1.09

$$\frac{1}{2}\left(x^2-1\right)\arcsin\left(x\right)+\frac{1}{4}\,\sqrt{-x^2+1}x+\frac{1}{4}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

3.274 $\int \frac{\sqrt{9-x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rubi [A] time = 0.0403956, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rubi in Sympy [A] time = 2.56495, size = 20, normalized size = 0.67

$$\sqrt{-x^2+9}-3\operatorname{atanh}\left(\frac{\sqrt{-x^2+9}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+9)**(1/2)/x,x)
```

```
[Out] sqrt(-x**2 + 9) - 3*atanh(sqrt(-x**2 + 9)/3)
```

Mathematica [A] time = 0.0124659, size = 32, normalized size = 1.07

$$\sqrt{9-x^2}-3\log\left(\sqrt{9-x^2}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] + 3*Log[x] - 3*Log[3 + Sqrt[9 - x^2]]
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$\sqrt{-x^2+9}-3\,Artanh\left(3\frac{1}{\sqrt{-x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+9)^(1/2)/x,x)
```

```
[Out] (-x^2+9)^(1/2)-3*arctanh(3/(-x^2+9)^(1/2))
```

Maxima [A] time = 1.5095, size = 47, normalized size = 1.57

$$\sqrt{-x^2+9} - 3\log\left(\frac{6\sqrt{-x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 9) - 3*log(6*sqrt(-x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 0.200965, size = 65, normalized size = 2.17

$$-\frac{x^2 - 3\left(\sqrt{-x^2+9} - 3\right)\log\left(\frac{\sqrt{-x^2+9}-3}{x}\right)}{\sqrt{-x^2+9} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x,x, algorithm="fricas")
```

```
[Out] -(x^2 - 3*(sqrt(-x^2 + 9) - 3)*log((sqrt(-x^2 + 9) - 3)/x))/(sqrt
(-x^2 + 9) - 3)
```

Sympy [A] time = 2.25545, size = 68, normalized size = 2.27

$$\begin{cases} i\sqrt{x^2-9} - 3\log(x) + \frac{3\log(x^2)}{2} + 3i\,\operatorname{asin}\left(\frac{3}{x}\right) & \text{for } \frac{|x^2|}{9} > 1 \\ \sqrt{-x^2+9} + \frac{3\log(x^2)}{2} - 3\log\left(\sqrt{-\frac{x^2}{9}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin
(3/x), Abs(x**2)/9 > 1), (sqrt(-x**2 + 9) + 3*log(x**2)/2 - 3*log
(sqrt(-x**2/9 + 1) + 1), True))
```

GIAC/XCAS [A] time = 0.223263, size = 54, normalized size = 1.8

$$\sqrt{-x^2+9} - \frac{3}{2}\ln\left(\sqrt{-x^2+9}+3\right) + \frac{3}{2}\ln\left(-\sqrt{-x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 9)/x,x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 9) - 3/2*ln(sqrt(-x^2 + 9) + 3) + 3/2*ln(-sqrt(-x^2 +
 9) + 3)
```

3.275 $\int \frac{x}{2+3x+x^2}\,dx$

Optimal. Leaf size=13

$$2\log(x+2)-\log(x+1)$$

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rubi [A] time = 0.0113645, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$2\log(x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rubi in Sympy [A] time = 1.59229, size = 10, normalized size = 0.77

$$-\log(x+1)+2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+3*x+2),x)
```

```
[Out] -log(x + 1) + 2*log(x + 2)
```

Mathematica [A] time = 0.00458344, size = 13, normalized size = 1.

$$2\log(x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Maple [A] time = 0.002, size = 14, normalized size = 1.1

$$-\ln(1+x)+2\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+3*x+2),x)
```

```
[Out] -ln(1+x)+2*ln(2+x)
```

Maxima [A] time = 1.34599, size = 18, normalized size = 1.38

$$2\log(x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 3*x + 2),x, algorithm="maxima")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

Fricas [A] time = 0.195977, size = 18, normalized size = 1.38

$$2\,\log\left(x+2\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 3*x + 2),x, algorithm="fricas")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

Sympy [A] time = 0.09348, size = 10, normalized size = 0.77

$$-\log\left(x+1\right)+2\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+3*x+2),x)
```

```
[Out] -log(x + 1) + 2*log(x + 2)
```

GIAC/XCAS [A] time = 0.237265, size = 20, normalized size = 1.54

$$2\ln\left(|x+2|\right)-\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^2 + 3*x + 2),x, algorithm="giac")
```

```
[Out] 2*ln(abs(x + 2)) - ln(abs(x + 1))
```

3.276 $\int x^2 \cosh(x)\,dx$

Optimal. Leaf size=16

$$x^2 \sinh(x) + 2\sinh(x) - 2x\cosh(x)$$

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rubi [A] time = 0.0410497, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^2 \sinh(x) + 2\sinh(x) - 2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^2 \sinh(x) - 2x\cosh(x) + 2\int \cosh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cosh(x),x)
```

```
[Out] x**2*sinh(x) - 2*x*cosh(x) + 2*Integral(cosh(x), x)
```

Mathematica [A] time = 0.00940846, size = 14, normalized size = 0.88

$$\left(x^2+2\right)\sinh(x) - 2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + (2 + x^2)*Sinh[x]
```

Maple [A] time = 0.008, size = 17, normalized size = 1.1

$$-2x\cosh(x) + 2\sinh(x) + x^2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cosh(x),x)
```

```
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

Maxima [A] time = 1.3776, size = 59, normalized size = 3.69

$$\frac{1}{3}x^3\cosh(x) - \frac{1}{6}\left(x^3 + 3x^2 + 6x + 6\right)e^{(-x)} - \frac{1}{6}\left(x^3 - 3x^2 + 6x - 6\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*cosh(x) - 1/6*(x^3 + 3*x^2 + 6*x + 6)*e^(-x) - 1/6*(x^3 -
 3*x^2 + 6*x - 6)*e^x
```

Fricas [A] time = 0.204867, size = 19, normalized size = 1.19

$$-2x\cosh(x) + \left(x^2 + 2\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cosh(x),x, algorithm="fricas")
```

```
[Out] -2*x*cosh(x) + (x^2 + 2)*sinh(x)
```

Sympy [A] time = 0.386933, size = 17, normalized size = 1.06

$$x^2\sinh(x) - 2x\cosh(x) + 2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cosh(x),x)
```

```
[Out] x**2*sinh(x) - 2*x*cosh(x) + 2*sinh(x)
```

GIAC/XCAS [A] time = 0.224227, size = 36, normalized size = 2.25

$$-\frac{1}{2}\left(x^2 + 2x + 2\right)e^{(-x)} + \frac{1}{2}\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 2*x + 2)*e^(-x) + 1/2*(x^2 - 2*x + 2)*e^x
```

3.277 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.00728313, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi in Sympy [A] time = 6.53486, size = 12, normalized size = 0.71

$$\frac{\log\left(x\left(x^3+2x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x*(x**3 + 2*x + 4))/4
```

Mathematica [A] time = 0.00861106, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$\frac{\ln\left(x\left(x^3+2\,x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

```
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Maxima [A] time = 1.32374, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^4 + 2\,x^2 + 4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Fricas [A] time = 0.194932, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^4 + 2\,x^2 + 4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] time = 0.099663, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4 + 2x^2 + 4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

GIAC/XCAS [A] time = 0.219571, size = 24, normalized size = 1.41

$$\frac{1}{4}\ln\left(\left|x^3 + 2\,x + 4\right|\right) + \frac{1}{4}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + x + 1)/(x^4 + 2*x^2 + 4*x),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(x^3 + 2*x + 4)) + 1/4*ln(abs(x))
```

3.278 $\int \frac{\cos(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] ArcTan[Sin[x]]
```

Rubi [A] time = 0.0278721, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rubi in Sympy [A] time = 2.31605, size = 3, normalized size = 1.

$$\operatorname{atan}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/(1+sin(x)**2),x)
```

```
[Out] atan(sin(x))
```

Mathematica [A] time = 0.00816533, size = 3, normalized size = 1.

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A] time = 0.015, size = 4, normalized size = 1.3

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/(1+sin(x)^2),x)
```

```
[Out] arctan(sin(x))
```

Maxima [A] time = 1.48868, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + 1),x, algorithm="maxima")
```

```
[Out] arctan(sin(x))
```

Fricas [A] time = 0.219653, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + 1),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [A] time = 0.278579, size = 3, normalized size = 1.

$$\operatorname{atan}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(1+sin(x)**2),x)
```

```
[Out] atan(sin(x))
```

GIAC/XCAS [A] time = 0.226133, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/(sin(x)^2 + 1),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

3.279 $\int \cos\left(\sqrt{x}\right)\, dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0179034, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.63869, size = 48, normalized size = 2.18

$$-i\sqrt{x}e^{i\sqrt{x}} + i\sqrt{x}e^{-i\sqrt{x}} + e^{i\sqrt{x}} + e^{-i\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x**(1/2)),x)
```

```
[Out] -I*sqrt(x)*exp(I*sqrt(x)) + I*sqrt(x)*exp(-I*sqrt(x)) + exp(I*sqr
t(x)) + exp(-I*sqrt(x))
```

Mathematica [A] time = 0.010602, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.35318, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 0.223184, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.423534, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

GIAC/XCAS [A] time = 0.219709, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.280 $\int \sin(\pi x)\,dx$

Optimal. Leaf size=9

$$-\frac{\cos(\pi x)}{\pi}$$

```
[Out] -(Cos[Pi*x]/Pi)
```

Rubi [A] time = 0.00805845, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[Pi*x],x]
```

```
[Out] -(Cos[Pi*x]/Pi)
```

Rubi in Sympy [A] time = 0.466595, size = 7, normalized size = 0.78

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(pi*x),x)
```

```
[Out] -cos(pi*x)/pi
```

Mathematica [A] time = 0.00536931, size = 9, normalized size = 1.

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[Pi*x],x]
```

```
[Out] -(Cos[Pi*x]/Pi)
```

Maple [A] time = 0.004, size = 10, normalized size = 1.1

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(Pi*x),x)
```

```
[Out] -cos(Pi*x)/Pi
```

Maxima [A] time = 1.33547, size = 12, normalized size = 1.33

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(pi*x),x, algorithm="maxima")
```

```
[Out] -cos(pi*x)/pi
```

Fricas [A] time = 0.211506, size = 12, normalized size = 1.33

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(pi*x),x, algorithm="fricas")
```

```
[Out] -cos(pi*x)/pi
```

Sympy [A] time = 0.035801, size = 7, normalized size = 0.78

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(pi*x),x)
```

```
[Out] -cos(pi*x)/pi
```

GIAC/XCAS [A] time = 0.216723, size = 12, normalized size = 1.33

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(pi*x),x, algorithm="giac")
```

```
[Out] -cos(pi*x)/pi
```

3.281 $\int \frac{e^{2x}}{1+e^x}\,dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.0344052, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Rubi in Sympy [A] time = 3.34127, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Mathematica [A] time = 0.00543907, size = 12, normalized size = 1.

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$e^x - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 1.33294, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 0.211083, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.078676, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.229322, size = 14, normalized size = 1.17

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="giac")
```

```
[Out] e^x - ln(e^x + 1)
```

3.282 $\int e^{3x}\cos(5x)\,dx$

Optimal. Leaf size=27

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rubi [A] time = 0.0198725, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(3*x)*Cos[5*x],x]
```

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rubi in Sympy [A] time = 1.48971, size = 26, normalized size = 0.96

$$\frac{5e^{3x}\sin(5x)}{34}+\frac{3e^{3x}\cos(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(3*x)*cos(5*x),x)
```

```
[Out] 5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

Mathematica [A] time = 0.0189411, size = 22, normalized size = 0.81

$$\frac{1}{34}e^{3x}(5\sin(5x)+3\cos(5x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(3*x)*Cos[5*x],x]
```

```
[Out] (E^(3*x)*(3*Cos[5*x] + 5*Sin[5*x]))/34
```

Maple [A] time = 0.01, size = 22, normalized size = 0.8

$$\frac{3\,e^{3x}\cos(5x)}{34}+\frac{5\,e^{3x}\sin(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(3*x)*cos(5*x),x)
```

```
[Out] 3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

Maxima [A] time = 1.35685, size = 26, normalized size = 0.96

$$\frac{1}{34}(3\cos(5x) + 5\sin(5x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*e^(3*x),x, algorithm="maxima")
```

```
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Fricas [A] time = 0.302901, size = 28, normalized size = 1.04

$$\frac{3}{34}\cos(5x)\,e^{(3x)} + \frac{5}{34}\,e^{(3x)}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*e^(3*x),x, algorithm="fricas")
```

```
[Out] 3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)
```

Sympy [A] time = 0.369806, size = 26, normalized size = 0.96

$$\frac{5e^{3x}\sin(5x)}{34} + \frac{3e^{3x}\cos(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(3*x)*cos(5*x),x)
```

```
[Out] 5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

GIAC/XCAS [A] time = 0.224464, size = 26, normalized size = 0.96

$$\frac{1}{34}(3\cos(5x) + 5\sin(5x))e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*e^(3*x),x, algorithm="giac")
```

```
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

3.283 $\int \cos(3x)\cos(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rubi [A] time = 0.0155857, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rubi in Sympy [A] time = 1.0389, size = 12, normalized size = 0.71

$$\frac{\sin(2x)}{4} + \frac{\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3*x)*cos(5*x),x)
```

```
[Out] sin(2*x)/4 + sin(8*x)/16
```

Mathematica [A] time = 0.0119763, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Maple [A] time = 0.049, size = 14, normalized size = 0.8

$$\frac{\sin(2\,x)}{4} + \frac{\sin(8\,x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3*x)*cos(5*x),x)
```

```
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

Maxima [A] time = 1.3483, size = 18, normalized size = 1.06

$$\frac{1}{16}\sin(8x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.220826, size = 30, normalized size = 1.76

$$\left(8\cos(x)^7 - 12\cos(x)^5 + 5\cos(x)^3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] (8*cos(x)^7 - 12*cos(x)^5 + 5*cos(x)^3)*sin(x)
```

Sympy [A] time = 0.735587, size = 26, normalized size = 1.53

$$-\frac{3\sin(3x)\cos(5x)}{16} + \frac{5\sin(5x)\cos(3x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x)*cos(5*x),x)
```

```
[Out] -3*sin(3*x)*cos(5*x)/16 + 5*sin(5*x)*cos(3*x)/16
```

GIAC/XCAS [A] time = 0.225624, size = 18, normalized size = 1.06

$$\frac{1}{16}\sin(8x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(5*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

3.284 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0297027, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi in Sympy [A] time = 57.3619, size = 31, normalized size = 1.24

$$3\log(3x+5)-\frac{\log\left(-6x+9\left(x+\frac{1}{3}\right)^2+8\right)}{24}+\frac{23\operatorname{atan}(x)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] 3*log(3*x + 5) - log(-6*x + 9*(x + 1/3)**2 + 8)/24 + 23*atan(x)/3
6
```

Mathematica [A] time = 0.007502, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.008, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.49727, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 0.205121, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.134838, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

GIAC/XCAS [A] time = 0.215406, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\ln\left(x^2+1\right) + \frac{1}{2}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 + x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*ln(x^2 + 1) + 1/2*ln(abs(x + 1))
```

3.285 $\int x^2 \log(1+x)\,dx$

Optimal. Leaf size=39

$$-\frac{x^3}{9}+\frac{1}{3}x^3\log(x+1)+\frac{x^2}{6}-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rubi [A] time = 0.0310959, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{x^3}{9}+\frac{1}{3}x^3\log(x+1)+\frac{x^2}{6}-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[1 + x],x]
```

```
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^3\log(x+1)}{3}-\frac{x^3}{9}-\frac{x}{3}+\frac{\log(x+1)}{3}+\frac{\int x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln(1+x),x)
```

```
[Out] x**3*log(x + 1)/3 - x**3/9 - x/3 + log(x + 1)/3 + Integral(x, x)/
3
```

Mathematica [A] time = 0.00314863, size = 39, normalized size = 1.

$$-\frac{x^3}{9}+\frac{1}{3}x^3\log(x+1)+\frac{x^2}{6}-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Log[1 + x],x]
```

```
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Maple [A] time = 0.004, size = 46, normalized size = 1.2

$$\frac{(1+x)^3\ln(1+x)}{3}-\frac{x^3}{9}+\frac{x^2}{6}-\frac{x}{3}-\frac{11}{18}-(1+x)^2\ln(1+x)+\ln(1+x)(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*ln(1+x),x)
```

```
[Out] 1/3*(1+x)^3*ln(1+x)-1/9*x^3+1/6*x^2-1/3*x-11/18-(1+x)^2*ln(1+x)+l
n(1+x)*(1+x)
```

Maxima [A] time = 1.34187, size = 39, normalized size = 1.

$$\frac{1}{3}x^3\log(x+1)-\frac{1}{9}x^3+\frac{1}{6}x^2-\frac{1}{3}x+\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x + 1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x + 1) - 1/9*x^3 + 1/6*x^2 - 1/3*x + 1/3*log(x + 1)
```

Fricas [A] time = 0.205935, size = 34, normalized size = 0.87

$$-\frac{1}{9}x^3+\frac{1}{6}x^2+\frac{1}{3}\left(x^3+1\right)\log(x+1)-\frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x + 1),x, algorithm="fricas")
```

```
[Out] -1/9*x^3 + 1/6*x^2 + 1/3*(x^3 + 1)*log(x + 1) - 1/3*x
```

Sympy [A] time = 0.109592, size = 29, normalized size = 0.74

$$\frac{x^3\log(x+1)}{3}-\frac{x^3}{9}+\frac{x^2}{6}-\frac{x}{3}+\frac{\log(x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln(1+x),x)
```

```
[Out] x**3*log(x + 1)/3 - x**3/9 + x**2/6 - x/3 + log(x + 1)/3
```

GIAC/XCAS [A] time = 0.22657, size = 66, normalized size = 1.69

$$\frac{1}{3}(x+1)^3\ln(x+1)-\frac{1}{9}(x+1)^3-(x+1)^2\ln(x+1)+\frac{1}{2}(x+1)^2+(x+1)\ln(x+1)-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x + 1),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^3*ln(x + 1) - 1/9*(x + 1)^3 - (x + 1)^2*ln(x + 1) + 1
/2*(x + 1)^2 + (x + 1)*ln(x + 1) - x - 1
```

3.286 $\int e^{-x^3} x^5 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{3} e^{-x^3} x^3 - \frac{e^{-x^3}}{3}$$

```
[Out] -1/(3*E^x^3) - x^3/(3*E^x^3)
```

Rubi [A] time = 0.0457995, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{1}{3} e^{-x^3} x^3 - \frac{e^{-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/E^x^3,x]
```

```
[Out] -1/(3*E^x^3) - x^3/(3*E^x^3)
```

Rubi in Sympy [A] time = 2.51772, size = 19, normalized size = 0.73

$$-\frac{x^3 e^{-x^3}}{3} - \frac{e^{-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/exp(x**3),x)
```

```
[Out] -x**3*exp(-x**3)/3 - exp(-x**3)/3
```

Mathematica [A] time = 0.00461831, size = 18, normalized size = 0.69

$$\frac{1}{3} e^{-x^3} \left(-x^3 - 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/E^x^3,x]
```

```
[Out] (-1 - x^3)/(3*E^x^3)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.5

$$-\frac{x^3 + 1}{3\, e^{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/exp(x^3),x)
```

```
[Out] -1/3*(x^3+1)/exp(x^3)
```

Maxima [A] time = 1.34822, size = 18, normalized size = 0.69

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Fricas [A] time = 0.207926, size = 18, normalized size = 0.69

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(-x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Sympy [A] time = 0.078712, size = 12, normalized size = 0.46

$$\frac{\left(-x^3-1\right)\,e^{-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/exp(x**3),x)
```

```
[Out] (-x**3 - 1)*exp(-x**3)/3
```

GIAC/XCAS [A] time = 0.222865, size = 18, normalized size = 0.69

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*e^(-x^3),x, algorithm="giac")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

3.287 $\int \tan^2(4x)\,dx$

Optimal. Leaf size=12

$$\frac{1}{4}\tan(4x) - x$$

```
[Out] -x + Tan[4*x]/4
```

Rubi [A] time = 0.0105326, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{4}\tan(4x) - x$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[4*x]^2,x]
```

```
[Out] -x + Tan[4*x]/4
```

Rubi in Sympy [A] time = 0.486796, size = 7, normalized size = 0.58

$$-x + \frac{\tan(4x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(4*x)**2,x)
```

```
[Out] -x + tan(4*x)/4
```

Mathematica [A] time = 0.0101595, size = 12, normalized size = 1.

$$\frac{1}{4}\tan(4x) - x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[4*x]^2,x]
```

```
[Out] -x + Tan[4*x]/4
```

Maple [A] time = 0.006, size = 11, normalized size = 0.9

$$-x + \frac{\tan(4\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(4*x)^2,x)
```

```
[Out] -x+1/4*tan(4*x)
```

Maxima [A] time = 1.47927, size = 14, normalized size = 1.17

$$-x + \frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(4*x)^2,x, algorithm="maxima")
```

```
[Out] -x + 1/4*tan(4*x)
```

Fricas [A] time = 0.213655, size = 14, normalized size = 1.17

$$-x + \frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(4*x)^2,x, algorithm="fricas")
```

```
[Out] -x + 1/4*tan(4*x)
```

Sympy [A] time = 0.050685, size = 12, normalized size = 1.

$$-x + \frac{\sin(4x)}{4\cos(4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(4*x)**2,x)
```

```
[Out] -x + sin(4*x)/(4*cos(4*x))
```

GIAC/XCAS [A] time = 0.23406, size = 14, normalized size = 1.17

$$-x + \frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(4*x)^2,x, algorithm="giac")
```

```
[Out] -x + 1/4*tan(4*x)
```

3.288 $\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rubi [A] time = 0.0139653, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rubi in Sympy [A] time = 0.697248, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{18x+12}{6\sqrt{9x^2+12x-5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2+12*x-5)**(1/2),x)
```

```
[Out] atanh((18*x + 12)/(6*sqrt(9*x**2 + 12*x - 5)))/3
```

Mathematica [A] time = 0.00937678, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+12x-5}+3x+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] Log[2 + 3*x + Sqrt[-5 + 12*x + 9*x^2]]/3
```

Maple [A] time = 0.003, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(6+9x)\sqrt{9}}{9}+\sqrt{9x^2+12x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2+12*x-5)^(1/2),x)
```

```
[Out] 1/9*ln(1/9*(6+9*x)*9^(1/2)+(9*x^2+12*x-5)^(1/2))*9^(1/2)
```

Maxima [A] time = 1.50057, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18\,x+6\,\sqrt{9\,x^2+12\,x-5}+12\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x - 5),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 12*x - 5) + 12)
```

Fricas [A] time = 0.20906, size = 27, normalized size = 1.08

$$-\frac{1}{3}\log\left(-3\,x+\sqrt{9\,x^2+12\,x-5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x - 5),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 12*x - 5) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2+12x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2+12*x-5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x - 5), x)
```

GIAC/XCAS [A] time = 0.221044, size = 28, normalized size = 1.12

$$-\frac{1}{3}\ln\left(\left|-3\,x+\sqrt{9\,x^2+12\,x-5}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x - 5),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(-3*x + sqrt(9*x^2 + 12*x - 5) - 2))
```

3.289 $\int x^2 \tan^{-1}(x)\,dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi [A] time = 0.0321509, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi in Sympy [A] time = 2.624, size = 20, normalized size = 0.74

$$\frac{x^3 \operatorname{atan}(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Mathematica [A] time = 0.00310192, size = 27, normalized size = 1.

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-\frac{x^2}{6} + \frac{x^3 \arctan(x)}{3} + \frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arctan(x),x)
```

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Maxima [A] time = 1.34599, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Fricas [A] time = 0.218429, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.49625, size = 20, normalized size = 0.74

$$\frac{x^3\operatorname{atan}(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

GIAC/XCAS [A] time = 0.226861, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*ln(x^2 + 1)
```

3.290 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi [A] time = 0.00914927, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi in Sympy [A] time = 1.18633, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Mathematica [A] time = 0.00589761, size = 19, normalized size = 1.

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] time = 0.001, size = 12, normalized size = 0.6

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{6}{7}x^{\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-x^(1/2))/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Maxima [A] time = 1.3449, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Fricas [A] time = 0.20437, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="fricas")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] time = 0.68099, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

GIAC/XCAS [A] time = 0.218681, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(sqrt(x) - 1)/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

3.291 $\int \frac{1}{-e^{-x}+e^{x}}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}\left(e^{x}\right)$$

```
[Out] -ArcTanh[E^x]
```

Rubi [A] time = 0.0175895, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\tanh^{-1}\left(e^{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

Rubi in Sympy [A] time = 3.55055, size = 7, normalized size = 1.17

$$-\operatorname{atanh}\left(e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-1/exp(x)+exp(x)),x)
```

```
[Out] -atanh(exp(-x))
```

Mathematica [B] time = 0.00579425, size = 23, normalized size = 3.83

$$\frac{1}{2}\log\left(1-e^{x}\right)-\frac{1}{2}\log\left(e^{x}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] Log[1 - E^x]/2 - Log[1 + E^x]/2
```

Maple [A] time = 0.004, size = 6, normalized size = 1.

$$-Artanh\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-1/exp(x)+exp(x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [A] time = 1.34387, size = 26, normalized size = 4.33

$$-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - e^x),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [A] time = 0.219767, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - e^x),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [A] time = 0.112954, size = 15, normalized size = 2.5

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-1/exp(x)+exp(x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

GIAC/XCAS [A] time = 0.226885, size = 22, normalized size = 3.67

$$-\frac{1}{2}\ln\left(e^{x}+1\right)+\frac{1}{2}\ln\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - e^x),x, algorithm="giac")
```

```
[Out] -1/2*ln(e^x + 1) + 1/2*ln(abs(e^x - 1))
```

3.292 $\int \frac{x}{10+2x^2+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi [A] time = 0.0279553, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi in Sympy [A] time = 2.28103, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

Mathematica [A] time = 0.00721722, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A] time = 0.003, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+2*x^2+10),x)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Maxima [A] time = 1.47395, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Fricas [A] time = 0.228458, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] time = 0.10184, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

GIAC/XCAS [A] time = 0.228201, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

3.293 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rubi [A] time = 0.00931854, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rubi in Sympy [A] time = 0.895945, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

Mathematica [A] time = 0.00507557, size = 12, normalized size = 1.

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Maple [A] time = 0.003, size = 9, normalized size = 0.8

$$\frac{3}{4}\ln\left(1+x^{\frac{4}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1/x^(1/3)+x),x)
```

```
[Out] 3/4*ln(1+x^(4/3))
```

Maxima [A] time = 1.49971, size = 11, normalized size = 0.92

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + 1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

Fricas [A] time = 0.208122, size = 11, normalized size = 0.92

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + 1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A] time = 0.606153, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

GIAC/XCAS [A] time = 0.227831, size = 43, normalized size = 3.58

$$\frac{3}{4}\ln\left(\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\ln\left(-\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + 1/x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/4*ln(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*ln(-sqrt(2)*x^(1/3) +
 x^(2/3) + 1)
```

3.294 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16} - \frac{1}{6}\sin(x)\cos^5(x) + \frac{1}{24}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[
x])/6
```

Rubi [A] time = 0.0516699, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{16} - \frac{1}{6}\sin(x)\cos^5(x) + \frac{1}{24}\sin(x)\cos^3(x) + \frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[
x])/6
```

Rubi in Sympy [A] time = 1.70566, size = 31, normalized size = 0.91

$$\frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/1
6
```

Mathematica [A] time = 0.0126713, size = 30, normalized size = 0.88

$$\frac{x}{16} + \frac{1}{64}\sin(2x) - \frac{1}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] time = 0.003, size = 26, normalized size = 0.8

$$-\frac{(\cos(x))^5\sin(x)}{6} + \frac{\sin(x)}{24}\left((\cos(x))^3 + \frac{3\cos(x)}{2}\right) + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4*sin(x)^2,x)
```

```
[Out] -1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x
```

Maxima [A] time = 1.34326, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 0.237579, size = 34, normalized size = 1.

$$-\frac{1}{48}\left(8\cos(x)^5 - 2\cos(x)^3 - 3\cos(x)\right)\sin(x) + \frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.054746, size = 31, normalized size = 0.91

$$\frac{x}{16} - \frac{\sin(x)\cos^5(x)}{6} + \frac{\sin(x)\cos^3(x)}{24} + \frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/1
6
```

GIAC/XCAS [A] time = 0.227424, size = 30, normalized size = 0.88

$$\frac{1}{16}x - \frac{1}{192}\sin(6x) - \frac{1}{64}\sin(4x) + \frac{1}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

3.295 $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] -ArcSin[(-2 - x)/3]
```

Rubi [A] time = 0.0144178, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Rubi in Sympy [A] time = 0.672802, size = 22, normalized size = 1.83

$$\operatorname{atan}\left(-\frac{-2x-4}{2\sqrt{-x^2-4x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-4*x+5)**(1/2),x)
```

```
[Out] atan(-(-2*x - 4)/(2*sqrt(-x**2 - 4*x + 5)))
```

Mathematica [A] time = 0.00947182, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-4*x+5)^(1/2),x)
```

```
[Out] arcsin(2/3+1/3*x)
```

Maxima [A] time = 1.50122, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 4*x + 5),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*x - 2/3)
```

Fricas [A] time = 0.21318, size = 23, normalized size = 1.92

$$\arctan\left(\frac{x+2}{\sqrt{-x^2-4x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 4*x + 5),x, algorithm="fricas")
```

```
[Out] arctan((x + 2)/sqrt(-x^2 - 4*x + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-4x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 4*x + 5), x)
```

GIAC/XCAS [A] time = 0.229988, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{3}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 4*x + 5),x, algorithm="giac")
```

```
[Out] arcsin(1/3*x + 2/3)
```

3.296 $\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{1-x^2}+1\right)$$

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0791616, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 3.25225, size = 12, normalized size = 0.75

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1-x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] -log(sqrt(-x**2 + 1) + 1)
```

Mathematica [A] time = 0.012734, size = 16, normalized size = 1.

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Maple [B] time = 0.023, size = 59, normalized size = 3.7

$$-\ln(x)-\frac{1}{2}\sqrt{-(-1+x)^2+2-2x}-\frac{1}{2}\sqrt{-(1+x)^2+2+2x}+\sqrt{-x^2+1}-\mathit{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1-x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] -ln(x)-1/2*(-(-1+x)^2+2-2*x)^(1/2)-1/2*(-(1+x)^2+2+2*x)^(1/2)+(-x
^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.34201, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 1) - 1),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 1) + 1)
```

Fricas [A] time = 0.234352, size = 28, normalized size = 1.75

$$-\log(x)+\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 1) - 1),x, algorithm="fricas")
```

```
[Out] -log(x) + log((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 1.91802, size = 17, normalized size = 1.06

$$-\begin{cases}\log\left(\sqrt{-x^2+1}+1\right) & \text{for } x>-1\wedge x<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1-x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] -Piecewise((log(sqrt(-x**2 + 1) + 1), (x > -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.227232, size = 19, normalized size = 1.19

$$-\ln\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 1) - 1),x, algorithm="giac")
```

```
[Out] -ln(sqrt(-x^2 + 1) + 1)
```

3.297 $\int (1 + \cos(x)) \csc(x)\, dx$

Optimal. Leaf size=7

$$\log(1 - \cos(x))$$

```
[Out] Log[1 - Cos[x]]
```

Rubi [A] time = 0.021914, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rubi in Sympy [A] time = 1.37704, size = 5, normalized size = 0.71

$$\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+cos(x))*csc(x),x)
```

```
[Out] log(-cos(x) + 1)
```

Mathematica [B] time = 0.0090776, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right) + \log(\sin(x)) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x])*Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.019, size = 6, normalized size = 0.9

$$\ln(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+cos(x))*csc(x),x)
```

```
[Out] ln(cos(x)-1)
```

Maxima [A] time = 1.34635, size = 7, normalized size = 1.

$$\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*csc(x),x, algorithm="maxima")
```

```
[Out] log(cos(x) - 1)
```

Fricas [A] time = 0.227807, size = 9, normalized size = 1.29

$$\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [A] time = 2.26426, size = 12, normalized size = 1.71

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+cos(x))*csc(x),x)
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

GIAC/XCAS [A] time = 0.226328, size = 9, normalized size = 1.29

$$\ln(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x) + 1)*csc(x),x, algorithm="giac")
```

```
[Out] ln(-cos(x) + 1)
```

3.298 $\int \frac{e^x}{-1+e^{2x}}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

```
[Out] -ArcTanh[E^x]
```

Rubi [A] time = 0.0290647, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rubi in Sympy [A] time = 2.87644, size = 5, normalized size = 0.83

$$-\operatorname{atanh}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] -atanh(exp(x))
```

Mathematica [B] time = 0.00452744, size = 23, normalized size = 3.83

$$\frac{1}{2}\log(1-e^x)-\frac{1}{2}\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(-1 + E^(2*x)),x]
```

```
[Out] Log[1 - E^x]/2 - Log[1 + E^x]/2
```

Maple [A] time = 0.003, size = 6, normalized size = 1.

$$-Artanh(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [A] time = 1.34911, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log(e^x+1)+\frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [A] time = 0.216822, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [A] time = 0.099362, size = 15, normalized size = 2.5

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

GIAC/XCAS [A] time = 0.225082, size = 22, normalized size = 3.67

$$-\frac{1}{2}\ln\left(e^{x}+1\right)+\frac{1}{2}\ln\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) - 1),x, algorithm="giac")
```

```
[Out] -1/2*ln(e^x + 1) + 1/2*ln(abs(e^x - 1))
```

3.299 $\int \frac{1}{-8+x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*
x + x^2]/24
```

Rubi [A] time = 0.044413, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-8 + x^3)^(-1),x]
```

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*
x + x^2]/24
```

Rubi in Sympy [A] time = 2.86422, size = 37, normalized size = 0.86

$$\frac{\log(-x+2)}{12}-\frac{\log\left(x^2+2x+4\right)}{24}-\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\left(\frac{x}{3}+\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-8),x)
```

```
[Out] log(-x + 2)/12 - log(x**2 + 2*x + 4)/24 - sqrt(3)*atan(sqrt(3)*(x
/3 + 1/3))/12
```

Mathematica [A] time = 0.0127126, size = 43, normalized size = 1.

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-8 + x^3)^(-1),x]
```

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*
x + x^2]/24
```

Maple [A] time = 0.009, size = 35, normalized size = 0.8

$$\frac{\ln(-2+x)}{12}-\frac{\ln\left(x^2+2\,x+4\right)}{24}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2\,x+2)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-8),x)
```

```
[Out] 1/12*ln(-2+x)-1/24*ln(x^2+2*x+4)-1/12*3^(1/2)*arctan(1/6*(2*x+2)*
3^(1/2))
```

Maxima [A] time = 1.48308, size = 43, normalized size = 1.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 8),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x +
4) + 1/12*log(x - 2)
```

Fricas [A] time = 0.213994, size = 53, normalized size = 1.23

$$-\frac{1}{72}\sqrt{3}\left(\sqrt{3}\log\left(x^2+2x+4\right)-2\sqrt{3}\log(x-2)+6\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 8),x, algorithm="fricas")
```

```
[Out] -1/72*sqrt(3)*(sqrt(3)*log(x^2 + 2*x + 4) - 2*sqrt(3)*log(x - 2)
+ 6*arctan(1/3*sqrt(3)*(x + 1)))
```

Sympy [A] time = 0.188309, size = 41, normalized size = 0.95

$$\frac{\log(x-2)}{12}-\frac{\log\left(x^2+2x+4\right)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-8),x)
```

```
[Out] log(x - 2)/12 - log(x**2 + 2*x + 4)/24 - sqrt(3)*atan(sqrt(3)*x/3
 + sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.219035, size = 45, normalized size = 1.05

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\ln\left(x^2+2x+4\right)+\frac{1}{12}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - 8),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*ln(x^2 + 2*x + 4
) + 1/12*ln(abs(x - 2))
```

3.300 $\int x^5 \cosh(x)\,dx$

Optimal. Leaf size=37

$$x^5 \sinh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 2
0*x^3*Sinh[x] + x^5*Sinh[x]
```

Rubi [A] time = 0.116435, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^5 \sinh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Cosh[x],x]
```

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 2
0*x^3*Sinh[x] + x^5*Sinh[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$x^5 \sinh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \int \sinh(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*cosh(x),x)
```

```
[Out] x**5*sinh(x) - 5*x**4*cosh(x) + 20*x**3*sinh(x) - 60*x**2*cosh(x)
 + 120*x*sinh(x) - 120*Integral(sinh(x), x)
```

Mathematica [A] time = 0.0148117, size = 29, normalized size = 0.78

$$x\left(x^4 + 20x^2 + 120\right)\sinh(x) - 5\left(x^4 + 12x^2 + 24\right)\cosh(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Cosh[x],x]
```

```
[Out] -5*(24 + 12*x^2 + x^4)*Cosh[x] + x*(120 + 20*x^2 + x^4)*Sinh[x]
```

Maple [A] time = 0.01, size = 38, normalized size = 1.

$$-120\cosh(x) - 60x^2\cosh(x) - 5x^4\cosh(x) + 120x\sinh(x) + 20x^3\sinh(x) + x^5\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*cosh(x),x)
```

```
[Out] -120*cosh(x)-60*x^2*cosh(x)-5*x^4*cosh(x)+120*x*sinh(x)+20*x^3*si
nh(x)+x^5*sinh(x)
```

Maxima [A] time = 1.33964, size = 100, normalized size = 2.7

$$\frac{1}{6}x^6\cosh(x) - \frac{1}{12}\left(x^6 + 6x^5 + 30x^4 + 120x^3 + 360x^2 + 720x + 720\right)e^{(-x)} - \frac{1}{12}\left(x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*cosh(x) - 1/12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2
+ 720*x + 720)*e^(-x) - 1/12*(x^6 - 6*x^5 + 30*x^4 - 120*x^3 + 36
0*x^2 - 720*x + 720)*e^x
```

Fricas [A] time = 0.210484, size = 41, normalized size = 1.11

$$-5\left(x^4 + 12x^2 + 24\right)\cosh(x) + \left(x^5 + 20x^3 + 120x\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cosh(x),x, algorithm="fricas")
```

```
[Out] -5*(x^4 + 12*x^2 + 24)*cosh(x) + (x^5 + 20*x^3 + 120*x)*sinh(x)
```

Sympy [A] time = 3.09145, size = 42, normalized size = 1.14

$$x^5\sinh(x) - 5x^4\cosh(x) + 20x^3\sinh(x) - 60x^2\cosh(x) + 120x\sinh(x) - 120\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*cosh(x),x)
```

```
[Out] x**5*sinh(x) - 5*x**4*cosh(x) + 20*x**3*sinh(x) - 60*x**2*cosh(x)
 + 120*x*sinh(x) - 120*cosh(x)
```

GIAC/XCAS [A] time = 0.210449, size = 77, normalized size = 2.08

$$-\frac{1}{2}\left(x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120\right)e^{(-x)} + \frac{1}{2}\left(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x) + 1/2*(
x^5 - 5*x^4 + 20*x^3 - 60*x^2 + 120*x - 120)*e^x
```

3.301 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

```
[Out] Log[Tan[x]]^2/2
```

Rubi [A] time = 0.0329528, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Rubi in Sympy [A] time = 9.2886, size = 48, normalized size = 5.33

$$-\frac{(\log(-\cos(2x)+1)-\log(\cos(2x)+1))^2}{8}-\frac{\log\left(-\sin^2(x)+1\right)\log(\tan(x))}{2}+\frac{\log\left(\sin^2(x)\right)\log(\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(tan(x))/cos(x)/sin(x),x)
```

```
[Out] -(log(-cos(2*x) + 1) - log(cos(2*x) + 1))**2/8 - log(-sin(x)**2 +
 1)*log(tan(x))/2 + log(sin(x)**2)*log(tan(x))/2
```

Mathematica [A] time = 0.00588257, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Maple [A] time = 0.024, size = 8, normalized size = 0.9

$$\frac{(\ln(\tan(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(tan(x))/cos(x)/sin(x),x)
```

```
[Out] 1/2*ln(tan(x))^2
```

Maxima [A] time = 1.34224, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/(cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x))^2
```

Fricas [A] time = 0.223696, size = 16, normalized size = 1.78

$$\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/(cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\tan(x))}{\sin(x)\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(tan(x))/cos(x)/sin(x),x)
```

```
[Out] Integral(log(tan(x))/(sin(x)*cos(x)), x)
```

GIAC/XCAS [A] time = 0.205264, size = 9, normalized size = 1.

$$\frac{1}{2}\ln(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(tan(x))/(cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*ln(tan(x))^2
```

3.302 $\int \left(-2x + x^2 + x^3\right)\, dx$

Optimal. Leaf size=20

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi [A] time = 0.00839539, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0$.

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In]  Int[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^4}{4} + \frac{x^3}{3} - 2\int x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - 2*Integral(x, x)
```

Mathematica [A] time = 0.0000428777, size = 20, normalized size = 1.

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3+x^2-2*x,x)
```

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Maxima [A] time = 1.3418, size = 22, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3 + x^2 - 2*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Fricas [A] time = 0.177175, size = 1, normalized size = 0.05

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3 + x^2 - 2*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A] time = 0.025368, size = 12, normalized size = 0.6

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

GIAC/XCAS [A] time = 0.21377, size = 22, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3 + x^2 - 2*x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

3.303 $\int \frac{1+e^x}{1-e^x}\,dx$

Optimal. Leaf size=12

$$x - 2\log(1 - e^x)$$

```
[Out] x - 2*Log[1 - E^x]
```

Rubi [A] time = 0.0432246, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Rubi in Sympy [A] time = 5.30623, size = 15, normalized size = 1.25

$$-x - 2\log(-e^x + 1) + 2\log(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+exp(x))/(1-exp(x)),x)
```

```
[Out] -x - 2*log(-exp(x) + 1) + 2*log(exp(x))
```

Mathematica [A] time = 0.00594752, size = 12, normalized size = 1.

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Maple [A] time = 0.009, size = 12, normalized size = 1.

$$\ln(e^x) - 2\ln(-1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+exp(x))/(1-exp(x)),x)
```

```
[Out] ln(exp(x))-2*ln(-1+exp(x))
```

Maxima [A] time = 1.34889, size = 12, normalized size = 1.

$$x - 2\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^x + 1)/(e^x - 1),x, algorithm="maxima")
```

```
[Out] x - 2*log(e^x - 1)
```

Fricas [A] time = 0.222899, size = 12, normalized size = 1.

$$x - 2 \log (e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^x + 1)/(e^x - 1),x, algorithm="fricas")
```

```
[Out] x - 2*log(e^x - 1)
```

Sympy [A] time = 0.072978, size = 8, normalized size = 0.67

$$x - 2 \log (e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+exp(x))/(1-exp(x)),x)
```

```
[Out] x - 2*log(exp(x) - 1)
```

GIAC/XCAS [A] time = 0.208635, size = 14, normalized size = 1.17

$$x - 2 \ln (|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(e^x + 1)/(e^x - 1),x, algorithm="giac")
```

```
[Out] x - 2*ln(abs(e^x - 1))
```

3.304 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi [A] time = 0.0342478, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi in Sympy [A] time = 3.03663, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Mathematica [A] time = 0.00711514, size = 21, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] time = 0.009, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+1)/(x^2+4),x)
```

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A] time = 1.34584, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 0.202991, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.104243, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

GIAC/XCAS [A] time = 0.215751, size = 23, normalized size = 1.1

$$-\frac{1}{6}\ln\left(x^2+4\right)+\frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x^2 + 4)*(x^2 + 1)),x, algorithm="giac")
```

```
[Out] -1/6*ln(x^2 + 4) + 1/6*ln(x^2 + 1)
```

3.305 $\int \frac{1}{4-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rubi [A] time = 0.0334494, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rubi in Sympy [A] time = 0.926556, size = 20, normalized size = 0.47

$$-\frac{\log\left(-2\tan\left(\frac{x}{2}\right)+1\right)}{3}+\frac{\log\left(-\tan\left(\frac{x}{2}\right)+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4-5*sin(x)),x)
```

```
[Out] -log(-2*tan(x/2) + 1)/3 + log(-tan(x/2) + 2)/3
```

Mathematica [A] time = 0.0085941, size = 43, normalized size = 1.

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Maple [A] time = 0.021, size = 22, normalized size = 0.5

$$\frac{1}{3}\ln\left(\tan\left(\frac{x}{2}\right)-2\right)-\frac{1}{3}\ln\left(2\,\tan(x/2)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4-5*sin(x)),x)
```

```
[Out] 1/3*ln(tan(1/2*x)-2)-1/3*ln(2*tan(1/2*x)-1)
```

Maxima [A] time = 1.35703, size = 41, normalized size = 0.95

$$-\frac{1}{3}\log\left(\frac{2\sin(x)}{\cos(x)+1}-1\right)+\frac{1}{3}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 4),x, algorithm="maxima")
```

```
[Out] -1/3*log(2*sin(x)/(cos(x) + 1) - 1) + 1/3*log(sin(x)/(cos(x) + 1)
 - 2)
```

Fricas [A] time = 0.231989, size = 36, normalized size = 0.84

$$\frac{1}{6}\log\left(\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)-\frac{1}{6}\log\left(-\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 4),x, algorithm="fricas")
```

```
[Out] 1/6*log(3/2*cos(x) - 2*sin(x) + 5/2) - 1/6*log(-3/2*cos(x) - 2*si
n(x) + 5/2)
```

Sympy [A] time = 0.287528, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{3}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4-5*sin(x)),x)
```

```
[Out] log(tan(x/2) - 2)/3 - log(tan(x/2) - 1/2)/3
```

GIAC/XCAS [A] time = 0.220657, size = 31, normalized size = 0.72

$$-\frac{1}{3}\ln\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{3}\ln\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(5*sin(x) - 4),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(2*tan(1/2*x) - 1)) + 1/3*ln(abs(tan(1/2*x) - 2))
```

3.306 $\int x\sqrt[3]{c+x}\,dx$

Optimal. Leaf size=24

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rubi [A] time = 0.0151333, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(c + x)^(1/3),x]
```

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rubi in Sympy [A] time = 1.17599, size = 20, normalized size = 0.83

$$-\frac{3c\,(c+x)^{\frac{4}{3}}}{4}+\frac{3\,(c+x)^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c+x)**(1/3),x)
```

```
[Out] -3*c*(c + x)**(4/3)/4 + 3*(c + x)**(7/3)/7
```

Mathematica [A] time = 0.00801397, size = 25, normalized size = 1.04

$$\frac{3}{28}\sqrt[3]{c+x}\left(-3c^2+cx+4x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(c + x)^(1/3),x]
```

```
[Out] (3*(c + x)^(1/3)*(-3*c^2 + c*x + 4*x^2))/28
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$-\frac{9\,c-12\,x}{28}\,(c+x)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c+x)^(1/3),x)
```

```
[Out] -3/28*(c+x)^(4/3)*(3*c-4*x)
```

Maxima [A] time = 1.36276, size = 22, normalized size = 0.92

$$\frac{3}{7}(c+x)^{\frac{7}{3}} - \frac{3}{4}(c+x)^{\frac{4}{3}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + x)^(1/3)*x,x, algorithm="maxima")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

Fricas [A] time = 0.227295, size = 30, normalized size = 1.25

$$-\frac{3}{28}\left(3c^2 - cx - 4x^2\right)(c+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + x)^(1/3)*x,x, algorithm="fricas")
```

```
[Out] -3/28*(3*c^2 - c*x - 4*x^2)*(c + x)^(1/3)
```

Sympy [A] time = 1.68275, size = 144, normalized size = 6.

$$-\frac{9c^{\frac{13}{3}}\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{9c^{\frac{13}{3}}}{28c^2+28cx} - \frac{6c^{\frac{10}{3}}x\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{9c^{\frac{10}{3}}x}{28c^2+28cx} + \frac{15c^{\frac{7}{3}}x^2\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx} + \frac{12c^{\frac{4}{3}}x^3\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c+x)**(1/3),x)
```

```
[Out] -9*c**(13/3)*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(13/3)/(2
8*c**2 + 28*c*x) - 6*c**(10/3)*x*(1 + x/c)**(1/3)/(28*c**2 + 28*c
*x) + 9*c**(10/3)*x/(28*c**2 + 28*c*x) + 15*c**(7/3)*x**2*(1 + x/
c)**(1/3)/(28*c**2 + 28*c*x) + 12*c**(4/3)*x**3*(1 + x/c)**(1/3)/
(28*c**2 + 28*c*x)
```

GIAC/XCAS [A] time = 0.212761, size = 22, normalized size = 0.92

$$\frac{3}{7}(c+x)^{\frac{7}{3}} - \frac{3}{4}(c+x)^{\frac{4}{3}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c + x)^(1/3)*x,x, algorithm="giac")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

3.307 $\int e^{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=38

$$3e^{\sqrt[3]{x}}x^{2/3} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 6e^{\sqrt[3]{x}}$$

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rubi [A] time = 0.0325605, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$3e^{\sqrt[3]{x}}x^{2/3} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 6e^{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^(1/3),x]
```

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rubi in Sympy [A] time = 1.94994, size = 34, normalized size = 0.89

$$3x^{\frac{2}{3}}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**(1/3)),x)
```

```
[Out] 3*x**(2/3)*exp(x**(1/3)) - 6*x**(1/3)*exp(x**(1/3)) + 6*exp(x**(1
/3))
```

Mathematica [A] time = 0.00603552, size = 24, normalized size = 0.63

$$e^{\sqrt[3]{x}}\left(3x^{2/3} - 6\sqrt[3]{x} + 6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^(1/3),x]
```

```
[Out] E^x^(1/3)*(6 - 6*x^(1/3) + 3*x^(2/3))
```

Maple [A] time = 0.003, size = 26, normalized size = 0.7

$$6\,\mathrm{e}^{\sqrt[3]{x}} - 6\,\mathrm{e}^{\sqrt[3]{x}}\sqrt[3]{x} + 3\,\mathrm{e}^{\sqrt[3]{x}}x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^(1/3)),x)
```

```
[Out] 6*exp(x^(1/3))-6*exp(x^(1/3))*x^(1/3)+3*exp(x^(1/3))*x^(2/3)
```

Maxima [A] time = 1.37396, size = 22, normalized size = 0.58

$$3\left(x^{\frac{2}{3}}-2x^{\frac{1}{3}}+2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Fricas [A] time = 0.231071, size = 22, normalized size = 0.58

$$3\left(x^{\frac{2}{3}}-2x^{\frac{1}{3}}+2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Sympy [A] time = 0.438435, size = 34, normalized size = 0.89

$$3x^{\frac{2}{3}}e^{\sqrt[3]{x}}-6\sqrt[3]{x}e^{\sqrt[3]{x}}+6e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**(1/3)),x)
```

```
[Out] 3*x**(2/3)*exp(x**(1/3)) - 6*x**(1/3)*exp(x**(1/3)) + 6*exp(x**(1
/3))
```

GIAC/XCAS [A] time = 0.213582, size = 22, normalized size = 0.58

$$3\left(x^{\frac{2}{3}}-2x^{\frac{1}{3}}+2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

3.308 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Rubi [A] time = 0.0648394, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 2.64605, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\operatorname{atan}\left(\sqrt{11}\left(\frac{2\sqrt{x+1}}{11}+\frac{1}{11}\right)\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(sqrt(11)*(2*sqrt(x + 1
)/11 + 1/11))/11
```

Mathematica [A] time = 0.0203436, size = 37, normalized size = 1.

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x +
Sqrt[1 + x]]
```

Maple [B] time = 0.022, size = 93, normalized size = 2.5

$$-\frac{1}{2}\ln\left(x+4-\sqrt{1+x}\right)-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(2\sqrt{1+x}-1\right)\right)+\frac{1}{2}\ln\left(4+x+\sqrt{1+x}\right)$$
$$-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(2\sqrt{1+x}+1\right)\right)+\frac{\sqrt{11}}{11}\arctan\left(\frac{(7+2x)\sqrt{11}}{11}\right)+\frac{\ln\left(x^2+7x+15\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4+x+(1+x)^(1/2)),x)
```

```
[Out] -1/2*ln(x+4-(1+x)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(1+x)^(1/2)
-1)*11^(1/2))+1/2*ln(4+x+(1+x)^(1/2))-1/11*arctan(1/11*(2*(1+x)^(
1/2)+1)*11^(1/2))*11^(1/2)+1/11*11^(1/2)*arctan(1/11*(7+2*x)*11^(
1/2))+1/2*ln(x^2+7*x+15)
```

Maxima [A] time = 1.58043, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x
+ sqrt(x + 1) + 4)
```

Fricas [A] time = 0.241737, size = 51, normalized size = 1.38

$$\frac{1}{11}\sqrt{11}\left(\sqrt{11}\log\left(x+\sqrt{x+1}+4\right)-2\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="fricas")
```

```
[Out] 1/11*sqrt(11)*(sqrt(11)*log(x + sqrt(x + 1) + 4) - 2*arctan(2/11*
sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x+1}+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x + 1) + 4), x)
```

GIAC/XCAS [A] time = 0.214459, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\ln\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x + sqrt(x + 1) + 4),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + ln(x +
 sqrt(x + 1) + 4)
```

3.309 $\int \frac{1+x^3}{-x^2+x^3}\,dx$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0332264, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi in Sympy [A] time = 3.49449, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(-x+1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(-x + 1) + 1/x
```

Mathematica [A] time = 0.0061008, size = 17, normalized size = 1.

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.011, size = 16, normalized size = 0.9

$$x + x^{-1} - \ln(x) + 2\,\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+1)/(x^3-x^2),x)
```

```
[Out] x+1/x-ln(x)+2*ln(-1+x)
```

Maxima [A] time = 1.37391, size = 20, normalized size = 1.18

$$x + \frac{1}{x} + 2\,\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 0.247433, size = 28, normalized size = 1.65

$$\frac{x^2 + 2\,x\log(x-1) - x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] time = 0.106666, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

GIAC/XCAS [A] time = 0.208645, size = 23, normalized size = 1.35

$$x + \frac{1}{x} + 2\ln(|x-1|) - \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)/(x^3 - x^2),x, algorithm="giac")
```

```
[Out] x + 1/x + 2*ln(abs(x - 1)) - ln(abs(x))
```

3.310 $\int \left(-3 + 4x + x^2\right) \sin(2x)\, dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \sin(2x) - 2x\cos(2x) + \frac{7}{4}\cos(2x)$$

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*
Sin[2*x])/2
```

Rubi [A] time = 0.100655, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \sin(2x) - 2x\cos(2x) + \frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*
Sin[2*x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + 4x - 3\right) \sin\left(2x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] Integral((x**2 + 4*x - 3)*sin(2*x), x)
```

Mathematica [A] time = 0.0384604, size = 29, normalized size = 0.72

$$\frac{1}{4}\left(\left(-2x^2 - 8x + 7\right)\cos(2x) + 2(x+2)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

Maple [A] time = 0.012, size = 35, normalized size = 0.9

$$\frac{7\cos\left(2x\right)}{4} - 2x\cos\left(2x\right) - \frac{x^2\cos\left(2x\right)}{2} + \sin\left(2x\right) + \frac{x\sin\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4*x-3)*sin(2*x),x)
```

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x
)
```

Maxima [A] time = 1.34438, size = 51, normalized size = 1.27

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x)-2x\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{3}{2}\cos(2x)+\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x - 3)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*c
os(2*x) + sin(2*x)
```

Fricas [A] time = 0.237115, size = 35, normalized size = 0.88

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos(2x)+\frac{1}{2}(x+2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x - 3)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Sympy [A] time = 0.434492, size = 39, normalized size = 0.98

$$-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}-2x\cos(2x)+\sin(2x)+\frac{7\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos
(2*x)/4
```

GIAC/XCAS [A] time = 0.215175, size = 35, normalized size = 0.88

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos(2x)+\frac{1}{2}(x+2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x - 3)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

3.311 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -Sin[Cos[x]]
```

Rubi [A] time = 0.0147589, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rubi in Sympy [A] time = 29.4721, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Mathematica [A] time = 2.11923, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Maple [A] time = 0.012, size = 6, normalized size = 1.2

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Maxima [A] time = 1.32667, size = 7, normalized size = 1.4

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
```

```
[Out] -sin(cos(x))
```

Fricas [A] time = 0.235119, size = 27, normalized size = 5.4

$$\sin\left(\frac{\tan\left(\frac{1}{2}\,x\right)^2 - 1}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A] time = 0.735445, size = 5, normalized size = 1.

$$-\sin\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

GIAC/XCAS [A] time = 0.207089, size = 7, normalized size = 1.4

$$-\sin\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -sin(cos(x))
```

3.312 $\int \frac{1}{\sqrt{16-x^2}}\,dx$

Optimal. Leaf size=6

$$\sin^{-1}\left(\frac{x}{4}\right)$$

```
[Out] ArcSin[x/4]
```

Rubi [A] time = 0.00402379, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

Rubi in Sympy [A] time = 0.528762, size = 3, normalized size = 0.5

$$\operatorname{asin}\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+16)**(1/2),x)
```

```
[Out] asin(x/4)
```

Mathematica [A] time = 0.00652445, size = 6, normalized size = 1.

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

Maple [A] time = 0.005, size = 5, normalized size = 0.8

$$\arcsin\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+16)^(1/2),x)
```

```
[Out] arcsin(1/4*x)
```

Maxima [A] time = 1.49938, size = 5, normalized size = 0.83

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 16),x, algorithm="maxima")
```

```
[Out] arcsin(1/4*x)
```

Fricas [A] time = 0.22106, size = 24, normalized size = 4.

$$-2\,\arctan\left(\frac{\sqrt{-x^2+16}-4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 16),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 16) - 4)/x)
```

Sympy [A] time = 0.154797, size = 3, normalized size = 0.5

$$\operatorname{asin}\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+16)**(1/2),x)
```

```
[Out] asin(x/4)
```

GIAC/XCAS [A] time = 0.219416, size = 5, normalized size = 0.83

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 16),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x)
```

3.313 $\int \frac{x^3}{(1+x)^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

```
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x
)^6)
```

Rubi [A] time = 0.0267131, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + x)^10,x]
```

```
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x
)^6)
```

Rubi in Sympy [A] time = 2.15609, size = 32, normalized size = 0.86

$$-\frac{1}{6\,(x+1)^6}+\frac{3}{7\,(x+1)^7}-\frac{3}{8\,(x+1)^8}+\frac{1}{9\,(x+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(1+x)**10,x)
```

```
[Out] -1/(6*(x + 1)**6) + 3/(7*(x + 1)**7) - 3/(8*(x + 1)**8) + 1/(9*(x
 + 1)**9)
```

Mathematica [A] time = 0.0098142, size = 24, normalized size = 0.65

$$-\frac{84x^3+36x^2+9x+1}{504(x+1)^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + x)^10,x]
```

```
[Out] -(1 + 9*x + 36*x^2 + 84*x^3)/(504*(1 + x)^9)
```

Maple [A] time = 0.008, size = 30, normalized size = 0.8

$$\frac{1}{9\,(1+x)^9}-\frac{3}{8\,(1+x)^8}+\frac{3}{7\,(1+x)^7}-\frac{1}{6\,(1+x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(1+x)^10,x)
```

```
[Out] 1/9/(1+x)^9-3/8/(1+x)^8+3/7/(1+x)^7-1/6/(1+x)^6
```

Maxima [A] time = 1.35507, size = 84, normalized size = 2.27

$$-\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x^9+9\,x^8+36\,x^7+84\,x^6+126\,x^5+126\,x^4+84\,x^3+36\,x^2+9\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x + 1)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6
 + 126*x^5 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Fricas [A] time = 0.228135, size = 84, normalized size = 2.27

$$-\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x^9+9\,x^8+36\,x^7+84\,x^6+126\,x^5+126\,x^4+84\,x^3+36\,x^2+9\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x + 1)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6
 + 126*x^5 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Sympy [A] time = 0.198364, size = 61, normalized size = 1.65

$$-\frac{84x^3+36x^2+9x+1}{504x^9+4536x^8+18144x^7+42336x^6+63504x^5+63504x^4+42336x^3+18144x^2+4536x+504}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(1+x)**10,x)
```

```
[Out] -(84*x**3 + 36*x**2 + 9*x + 1)/(504*x**9 + 4536*x**8 + 18144*x**7
 + 42336*x**6 + 63504*x**5 + 63504*x**4 + 42336*x**3 + 18144*x**2
 + 4536*x + 504)
```

GIAC/XCAS [A] time = 0.20897, size = 30, normalized size = 0.81

$$-\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x + 1)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x + 1)^9
```

3.314 $\int \cot^3(2x)\csc^3(2x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\csc^3(2x) - \frac{1}{10}\csc^5(2x)$$

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rubi [A] time = 0.0461886, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{6}\csc^3(2x) - \frac{1}{10}\csc^5(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sin(2x)}{15\tan^2(2x)} + \frac{2\int \frac{\sin(2x)}{\tan(2x)}\,dx}{15} + \frac{1}{15\sin(2x)\tan^2(2x)} - \frac{1}{10\sin^3(2x)\tan^2(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(2*x)**3*csc(2*x)**3,x)
```

```
[Out] sin(2*x)/(15*tan(2*x)**2) + 2*Integral(sin(2*x)/tan(2*x), x)/15 +
 1/(15*sin(2*x)*tan(2*x)**2) - 1/(10*sin(2*x)**3*tan(2*x)**2)
```

Mathematica [B] time = 0.0349165, size = 53, normalized size = 2.52

$$\frac{11\tan(x)}{480} + \frac{11\cot(x)}{480} - \frac{1}{320}\cot(x)\csc^4(x) + \frac{11}{960}\cot(x)\csc^2(x) - \frac{1}{320}\tan(x)\sec^4(x) + \frac{11}{960}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] (11*Cot[x])/480 + (11*Cot[x]*Csc[x]^2)/960 - (Cot[x]*Csc[x]^4)/32
0 + (11*Tan[x])/480 + (11*Sec[x]^2*Tan[x])/960 - (Sec[x]^4*Tan[x]
)/320
```

Maple [B] time = 0.036, size = 58, normalized size = 2.8

$$-\frac{(\cos(2x))^4}{10\,(\sin(2x))^5} - \frac{(\cos(2x))^4}{30\,(\sin(2x))^3} + \frac{(\cos(2x))^4}{30\,\sin(2x)} + \frac{\left(2+(\cos(2x))^2\right)\sin(2x)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(2*x)^3*csc(2*x)^3,x)
```

```
[Out] -1/10/sin(2*x)^5*cos(2*x)^4-1/30/sin(2*x)^3*cos(2*x)^4+1/30/sin(2
*x)*cos(2*x)^4+1/30*(2+cos(2*x)^2)*sin(2*x)
```

Maxima [A] time = 1.37688, size = 24, normalized size = 1.14

$$\frac{5\,\sin(2\,x)^2-3}{30\,\sin(2\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

Fricas [A] time = 0.227828, size = 49, normalized size = 2.33

$$-\frac{5\,\cos(2\,x)^2-2}{30\left(\cos(2\,x)^4-2\,\cos(2\,x)^2+1\right)\sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(5*cos(2*x)^2 - 2)/((cos(2*x)^4 - 2*cos(2*x)^2 + 1)*sin(2*x
))
```

Sympy [A] time = 0.115788, size = 17, normalized size = 0.81

$$\frac{5\sin^2(2x)-3}{30\sin^5(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(2*x)**3*csc(2*x)**3,x)
```

```
[Out] (5*sin(2*x)**2 - 3)/(30*sin(2*x)**5)
```

GIAC/XCAS [A] time = 0.230217, size = 24, normalized size = 1.14

$$\frac{5\,\sin(2\,x)^2-3}{30\,\sin(2\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

3.315 $\int (x + \sin(x))^2\, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3} + \frac{x}{2} + 2\sin(x) - 2x\cos(x) - \frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0527617, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$

$$\frac{x^3}{3} + \frac{x}{2} + 2\sin(x) - 2x\cos(x) - \frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x + \sin(x))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+sin(x))**2,x)
```

```
[Out] Integral((x + sin(x))**2, x)
```

Mathematica [A] time = 0.0629458, size = 30, normalized size = 1.

$$\frac{1}{6}x\left(2x^2 + 3\right) + 2\sin(x) - \frac{1}{4}\sin(2x) - 2x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + Sin[x])^2,x]
```

```
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

Maple [A] time = 0.014, size = 25, normalized size = 0.8

$$\frac{x}{2} + \frac{x^3}{3} - 2\,x\cos(x) + 2\,\sin(x) - \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x+sin(x))^2,x)
```

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.44905, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Fricas [A] time = 0.220219, size = 30, normalized size = 1.

$$\frac{1}{3}x^3 - 2x\cos(x) - \frac{1}{2}(\cos(x) - 4)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

Sympy [A] time = 0.235873, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - 2x\cos(x) - \frac{\sin(x)\cos(x)}{2} + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+sin(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(
x)/2 + 2*sin(x)
```

GIAC/XCAS [A] time = 0.211972, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

3.316 $\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx$

Optimal. Leaf size=4

$$e^{\tan^{-1}(x)}$$

```
[Out] E^ArcTan[x]
```

Rubi [A] time = 0.031119, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$e^{\tan^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[E^ArcTan[x]/(1 + x^2),x]
```

```
[Out] E^ArcTan[x]
```

Rubi in Sympy [A] time = 2.801, size = 3, normalized size = 0.75

$$e^{\operatorname{atan}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(atan(x))/(x**2+1),x)
```

```
[Out] exp(atan(x))
```

Mathematica [A] time = 0.0029576, size = 4, normalized size = 1.

$$e^{\tan^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^ArcTan[x]/(1 + x^2),x]
```

```
[Out] E^ArcTan[x]
```

Maple [A] time = 0.003, size = 4, normalized size = 1.

$$\mathrm{e}^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(arctan(x))/(x^2+1),x)
```

```
[Out] exp(arctan(x))
```

Maxima [A] time = 1.34774, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1),x, algorithm="maxima")
```

```
[Out] e^arctan(x)
```

Fricas [A] time = 0.21822, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1),x, algorithm="fricas")
```

```
[Out] e^arctan(x)
```

Sympy [A] time = 1.57426, size = 3, normalized size = 0.75

$$e^{\operatorname{atan}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(atan(x))/(x**2+1),x)
```

```
[Out] exp(atan(x))
```

GIAC/XCAS [A] time = 0.224259, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] e^arctan(x)
```

3.317 $\int \frac{1}{x(1+x^4)}\,dx$

Optimal. Leaf size=13

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rubi [A] time = 0.0151275, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x^4)),x]
```

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rubi in Sympy [A] time = 1.47476, size = 14, normalized size = 1.08

$$\frac{\log\left(x^4\right)}{4} - \frac{\log\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**4+1),x)
```

```
[Out] log(x**4)/4 - log(x**4 + 1)/4
```

Mathematica [A] time = 0.00435817, size = 13, normalized size = 1.

$$\log(x) - \frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x^4)),x]
```

```
[Out] Log[x] - Log[1 + x^4]/4
```

Maple [A] time = 0.007, size = 12, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^4+1),x)
```

```
[Out] ln(x)-1/4*ln(x^4+1)
```

Maxima [A] time = 1.35965, size = 20, normalized size = 1.54

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 0.194678, size = 15, normalized size = 1.15

$$-\frac{1}{4}\log\left(x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^4 + 1) + log(x)
```

Sympy [A] time = 0.0963, size = 10, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4
```

GIAC/XCAS [A] time = 0.215362, size = 20, normalized size = 1.54

$$-\frac{1}{4}\ln\left(x^4+1\right)+\frac{1}{4}\ln\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^4 + 1)*x),x, algorithm="giac")
```

```
[Out] -1/4*ln(x^4 + 1) + 1/4*ln(x^4)
```

3.318 $\int e^{-2t} t^3 \, dt$

Optimal. Leaf size=44

$$-\frac{1}{2}e^{-2t}t^3 - \frac{3}{4}e^{-2t}t^2 - \frac{3}{4}e^{-2t}t - \frac{3e^{-2t}}{8}$$

```
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2
*E^(2*t))
```

Rubi [A] time = 0.0585159, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{1}{2}e^{-2t}t^3 - \frac{3}{4}e^{-2t}t^2 - \frac{3}{4}e^{-2t}t - \frac{3e^{-2t}}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[t^3/E^(2*t),t]
```

```
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2
*E^(2*t))
```

Rubi in Sympy [A] time = 2.8235, size = 41, normalized size = 0.93

$$-\frac{t^3 e^{-2t}}{2} - \frac{3t^2 e^{-2t}}{4} - \frac{3te^{-2t}}{4} - \frac{3e^{-2t}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**3/exp(2*t),t)
```

```
[Out] -t**3*exp(-2*t)/2 - 3*t**2*exp(-2*t)/4 - 3*t*exp(-2*t)/4 - 3*exp(
-2*t)/8
```

Mathematica [A] time = 0.00585729, size = 24, normalized size = 0.55

$$-\frac{1}{8}e^{-2t}\left(4t^3 + 6t^2 + 6t + 3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t^3/E^(2*t),t]
```

```
[Out] -(3 + 6*t + 6*t^2 + 4*t^3)/(8*E^(2*t))
```

Maple [A] time = 0.005, size = 24, normalized size = 0.6

$$-\frac{4t^3 + 6t^2 + 6t + 3}{8e^{2t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^3/exp(2*t),t)
```

```
[Out] -1/8*(4*t^3+6*t^2+6*t+3)/exp(2*t)
```

Maxima [A] time = 1.33013, size = 28, normalized size = 0.64

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-2*t),t, algorithm="maxima")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Fricas [A] time = 0.201162, size = 28, normalized size = 0.64

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-2*t),t, algorithm="fricas")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Sympy [A] time = 0.073492, size = 22, normalized size = 0.5

$$\frac{\left(-4t^3-6t^2-6t-3\right)e^{-2t}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**3/exp(2*t),t)
```

```
[Out] (-4*t**3 - 6*t**2 - 6*t - 3)*exp(-2*t)/8
```

GIAC/XCAS [A] time = 0.207787, size = 28, normalized size = 0.64

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-2*t),t, algorithm="giac")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

3.319 $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$

Optimal. Leaf size=41

$$\frac{6t^{7/6}}{7} - \frac{6t^{5/6}}{5} + 2\sqrt{t} - 6\sqrt[6]{t} + 6\tan^{-1}\left(\sqrt[6]{t}\right)$$

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan
[t^(1/6)]
```

Rubi [A] time = 0.033075, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{6t^{7/6}}{7} - \frac{6t^{5/6}}{5} + 2\sqrt{t} - 6\sqrt[6]{t} + 6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan
[t^(1/6)]
```

Rubi in Sympy [A] time = 2.71242, size = 37, normalized size = 0.9

$$\frac{6t^{\frac{7}{6}}}{7} - \frac{6t^{\frac{5}{6}}}{5} - 6\sqrt[6]{t} + 2\sqrt{t} + 6\operatorname{atan}\left(\sqrt[6]{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**(1/2)/(1+t**(1/3)),t)
```

```
[Out] 6*t**(7/6)/7 - 6*t**(5/6)/5 - 6*t**(1/6) + 2*sqrt(t) + 6*atan(t**
(1/6))
```

Mathematica [A] time = 0.0144812, size = 41, normalized size = 1.

$$\frac{6t^{7/6}}{7} - \frac{6t^{5/6}}{5} + 2\sqrt{t} - 6\sqrt[6]{t} + 6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan
[t^(1/6)]
```

Maple [A] time = 0.004, size = 28, normalized size = 0.7

$$-6\sqrt[6]{t} - \frac{6}{5}t^{\frac{5}{6}} + \frac{6}{7}t^{\frac{7}{6}} + 6\arctan\left(\sqrt[6]{t}\right) + 2\sqrt{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^(1/2)/(1+t^(1/3)),t)

[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)
```

Maxima [A] time = 1.49266, size = 36, normalized size = 0.88

$$\frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)/(t^(1/3) + 1),t, algorithm="maxima")

[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1
/6))
```

Fricas [A] time = 0.215419, size = 34, normalized size = 0.83

$$\frac{6}{7}\,(t-7)t^{\frac{1}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)/(t^(1/3) + 1),t, algorithm="fricas")

[Out] 6/7*(t - 7)*t^(1/6) - 6/5*t^(5/6) + 2*sqrt(t) + 6*arctan(t^(1/6))
```

Sympy [A] time = 4.80965, size = 37, normalized size = 0.9

$$\frac{6t^{\frac{7}{6}}}{7} - \frac{6t^{\frac{5}{6}}}{5} - 6\sqrt[6]{t} + 2\sqrt{t} + 6\,\mathrm{atan}\left(\sqrt[6]{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**(1/2)/(1+t**(1/3)),t)

[Out] 6*t**(7/6)/7 - 6*t**(5/6)/5 - 6*t**(1/6) + 2*sqrt(t) + 6*atan(t**
(1/6))
```

GIAC/XCAS [A] time = 0.213486, size = 36, normalized size = 0.88

$$\frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(t)/(t^(1/3) + 1),t, algorithm="giac")

[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1
/6))
```

3.320 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] time = 0.0485529, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}-\frac{\int^{\sin(3x)} x\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -cos(2*x)/8 - cos(4*x)/16 - Integral(x, (x, sin(3*x)))/3
```

Mathematica [A] time = 0.0151771, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A] time = 1.32944, size = 26, normalized size = 1.04

$$\frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 0.228909, size = 23, normalized size = 0.92

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [A] time = 22.1982, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{3\sin(x)\sin(2x)\cos(3x)}{8} + \frac{\sin(x)\sin(3x)\cos(2x)}{6} + \frac{\sin(2x)\sin(3x)\cos(x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*s
in(2*x)*cos(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 3*sin(
x)*sin(2*x)*cos(3*x)/8 + sin(x)*sin(3*x)*cos(2*x)/6 + sin(2*x)*si
n(3*x)*cos(x)/24
```

GIAC/XCAS [A] time = 0.210815, size = 18, normalized size = 0.72

$$-\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(3*x)*sin(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.321 $\int \log\left(\frac{x}{2}\right)\, dx$

Optimal. Leaf size=12

$$x \log\left(\frac{x}{2}\right) - x$$

```
[Out] -x + x*Log[x/2]
```

Rubi [A] time = 0.00460648, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$x \log\left(\frac{x}{2}\right) - x$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Rubi in Sympy [A] time = 0.464809, size = 7, normalized size = 0.58

$$x \log\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(1/2*x),x)
```

```
[Out] x*log(x/2) - x
```

Mathematica [A] time = 0.00153016, size = 12, normalized size = 1.

$$x \log\left(\frac{x}{2}\right) - x$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Maple [A] time = 0.003, size = 11, normalized size = 0.9

$$-x + x \ln\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(1/2*x),x)
```

```
[Out] -x+x*ln(1/2*x)
```

Maxima [A] time = 1.32875, size = 14, normalized size = 1.17

$$x \log\left(\frac{1}{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(1/2*x),x, algorithm="maxima")
```

```
[Out] x*log(1/2*x) - x
```

Fricas [A] time = 0.23654, size = 14, normalized size = 1.17

$$x \log\left(\frac{1}{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(1/2*x),x, algorithm="fricas")
```

```
[Out] x*log(1/2*x) - x
```

Sympy [A] time = 0.069657, size = 7, normalized size = 0.58

$$x \log\left(\frac{x}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(1/2*x),x)
```

```
[Out] x*log(x/2) - x
```

GIAC/XCAS [A] time = 0.21301, size = 14, normalized size = 1.17

$$x \ln\left(\frac{1}{2}x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(1/2*x),x, algorithm="giac")
```

```
[Out] x*ln(1/2*x) - x
```

3.322 $\int \sqrt{\frac{1+x}{1-x}}\,dx$

Optimal. Leaf size=41

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

```
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]
]
```

Rubi [A] time = 0.0304611, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]
]
```

Rubi in Sympy [A] time = 1.69038, size = 32, normalized size = 0.78

$$-\frac{2\sqrt{\frac{x+1}{-x+1}}}{1+\frac{x+1}{-x+1}}+2\operatorname{atan}\left(\sqrt{\frac{x+1}{-x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1+x)/(1-x))**(1/2),x)
```

```
[Out] -2*sqrt((x + 1)/(-x + 1))/(1 + (x + 1)/(-x + 1)) + 2*atan(sqrt((x
 + 1)/(-x + 1)))
```

Mathematica [A] time = 0.0419962, size = 60, normalized size = 1.46

$$\frac{\sqrt{\frac{x+1}{1-x}}\left(\sqrt{x+1}(x-1)+2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)\right)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] (Sqrt[(1 + x)/(1 - x)]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*ArcS
in[Sqrt[1 + x]/Sqrt[2]]))/Sqrt[1 + x]
```

Maple [A] time = 0.009, size = 41, normalized size = 1.

$$(-1+x)\sqrt{-\frac{1+x}{-1+x}}\left(\sqrt{-x^2+1}-\arcsin(x)\right)\frac{1}{\sqrt{-(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1+x)/(1-x))^(1/2),x)
```

```
[Out] (-(1+x)/(-1+x))^(1/2)*(-1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2
)-arcsin(x))
```

Maxima [A] time = 1.53034, size = 58, normalized size = 1.41

$$\frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x + 1)/(x - 1)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-(x + 1)/(x - 1))/((x + 1)/(x - 1) - 1) + 2*arctan(sqrt(-(
x + 1)/(x - 1)))
```

Fricas [A] time = 0.208365, size = 43, normalized size = 1.05

$$(x-1)\sqrt{-\frac{x+1}{x-1}}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x + 1)/(x - 1)),x, algorithm="fricas")
```

```
[Out] (x - 1)*sqrt(-(x + 1)/(x - 1)) + 2*arctan(sqrt(-(x + 1)/(x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{-x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1+x)/(1-x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(-x + 1)), x)
```

GIAC/XCAS [A] time = 0.210591, size = 41, normalized size = 1.

$$\frac{1}{2}\,\pi\,\mathrm{sign}\,(x-1)-\arcsin(x)\,\mathrm{sign}\,(x-1)+\sqrt{-x^2+1}\,\mathrm{sign}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(x + 1)/(x - 1)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sign(x - 1) - arcsin(x)*sign(x - 1) + sqrt(-x^2 + 1)*sign(
x - 1)
```

3.323 $\int \frac{x \log(x)}{\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rubi [A] time = 0.0695099, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rubi in Sympy [A] time = 4.21559, size = 29, normalized size = 0.85

$$\sqrt{x^2-1}\log(x)-\sqrt{x^2-1}+\operatorname{atan}\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*log(x) - sqrt(x**2 - 1) + atan(sqrt(x**2 - 1))
```

Mathematica [A] time = 0.0220327, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}(\log(x)-1)-\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])
```

Maple [C] time = 0., size = 119, normalized size = 3.5

$$\begin{aligned}
&-\frac{1}{4}\sqrt{-signum\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)\frac{1}{\sqrt{signum\left(x^2-1\right)}}\\
&+\frac{\ln(x)}{2}\sqrt{-signum\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)\frac{1}{\sqrt{signum\left(x^2-1\right)}}\\
&+\frac{1}{32}\sqrt{-signum\left(x^2-1\right)}\left(-16+16\sqrt{-x^2+1}-32\ln\left(1/2+1/2\sqrt{-x^2+1}\right)\right)\frac{1}{\sqrt{signum\left(x^2-1\right)}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x)/(x^2-1)^(1/2),x)
```

```
[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/
2))+1/2/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x
^2+1)^(1/2))+1/32/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16
+16*(-x^2+1)^(1/2)-32*ln(1/2+1/2*(-x^2+1)^(1/2)))
```

Maxima [A] time = 1.55179, size = 36, normalized size = 1.06

$$\sqrt{x^2-1}\log(x) - \sqrt{x^2-1} - \arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)/sqrt(x^2 - 1),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

Fricas [A] time = 0.232523, size = 93, normalized size = 2.74

$$\frac{x^2 + 2\left(x - \sqrt{x^2-1}\right)\arctan\left(-x + \sqrt{x^2-1}\right) - \left(x^2-1\right)\log(x) + \sqrt{x^2-1}(x\log(x) - x) - 1}{x - \sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)/sqrt(x^2 - 1),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*(x - sqrt(x^2 - 1))*arctan(-x + sqrt(x^2 - 1)) - (x^2 -
1)*log(x) + sqrt(x^2 - 1)*(x*log(x) - x) - 1)/(x - sqrt(x^2 - 1))
```

Sympy [A] time = 4.29324, size = 29, normalized size = 0.85

$$\sqrt{x^2-1}\log(x) - \begin{cases} \sqrt{x^2-1} - \operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x
 > -1) & (x < 1)))
```

GIAC/XCAS [A] time = 0.216759, size = 38, normalized size = 1.12

$$\sqrt{x^2-1}\ln(x) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)/sqrt(x^2 - 1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 1)*ln(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

3.324 $\int \frac{a+x}{a^2+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rubi [A] time = 0.0170919, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rubi in Sympy [A] time = 1.95558, size = 14, normalized size = 0.74

$$\frac{\log\left(a^2+x^2\right)}{2}+\operatorname{atan}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a+x)/(a**2+x**2),x)
```

```
[Out] log(a**2 + x**2)/2 + atan(x/a)
```

Mathematica [A] time = 0.00632638, size = 19, normalized size = 1.

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Maple [A] time = 0.004, size = 18, normalized size = 1.

$$\arctan\left(\frac{x}{a}\right)+\frac{\ln\left(a^2+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a+x)/(a^2+x^2),x)
```

```
[Out] arctan(x/a)+1/2*ln(a^2+x^2)
```

Maxima [A] time = 1.51017, size = 23, normalized size = 1.21

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + x)/(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Fricas [A] time = 0.201657, size = 23, normalized size = 1.21

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + x)/(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Sympy [A] time = 0.110728, size = 42, normalized size = 2.21

$$\left(\frac{1}{2} - \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} - \frac{i}{2}\right) + x\right) + \left(\frac{1}{2} + \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} + \frac{i}{2}\right) + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a+x)/(a**2+x**2),x)
```

```
[Out] (1/2 - I/2)*log(-a + 2*a*(1/2 - I/2) + x) + (1/2 + I/2)*log(-a +
2*a*(1/2 + I/2) + x)
```

GIAC/XCAS [A] time = 0.21266, size = 23, normalized size = 1.21

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\ln\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a + x)/(a^2 + x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a) + 1/2*ln(a^2 + x^2)
```

3.325 $\int \sqrt{1+x-x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

```
[Out] -((1 - 2*x)*Sqrt[1 + x - x^2])/4 - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/
8
```

Rubi [A] time = 0.0268322, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x - x^2],x]
```

```
[Out] -((1 - 2*x)*Sqrt[1 + x - x^2])/4 - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/
8
```

Rubi in Sympy [A] time = 0.813921, size = 39, normalized size = 1.03

$$-\frac{(-2x+1)\sqrt{-x^2+x+1}}{4}-\frac{5\operatorname{atan}\left(\frac{-2x+1}{2\sqrt{-x^2+x+1}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+x+1)**(1/2),x)
```

```
[Out] -(-2*x + 1)*sqrt(-x**2 + x + 1)/4 - 5*atan((-2*x + 1)/(2*sqrt(-x*
*2 + x + 1)))/8
```

Mathematica [A] time = 0.0231808, size = 39, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1}-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x - x^2],x]
```

```
[Out] (-1/4 + x/2)*Sqrt[1 + x - x^2] - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Maple [A] time = 0.005, size = 30, normalized size = 0.8

$$-\frac{1-2x}{4}\sqrt{-x^2+x+1}+\frac{5}{8}\arcsin\left(\frac{2\sqrt{5}}{5}\left(x-\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+x+1)^(1/2),x)
```

```
[Out] -1/4*(1-2*x)*(-x^2+x+1)^(1/2)+5/8*arcsin(2/5*5^(1/2)*(x-1/2))
```

Maxima [A] time = 1.55516, size = 53, normalized size = 1.39

$$\frac{1}{2}\sqrt{-x^2+x+1}x-\frac{1}{4}\sqrt{-x^2+x+1}-\frac{5}{8}\arcsin\left(-\frac{1}{5}\sqrt{5}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + x + 1)*x - 1/4*sqrt(-x^2 + x + 1) - 5/8*arcsin(-1
/5*sqrt(5)*(2*x - 1))
```

Fricas [A] time = 0.208774, size = 159, normalized size = 4.18

$$-\frac{8x^4+4x^3-25x^2+5\left(3x^2+4\sqrt{-x^2+x+1}(x+2)-8x-8\right)\arctan\left(\frac{\sqrt{-x^2+x+1}-1}{x}\right)-\left(6x^3-19x^2-12x\right)\sqrt{-x^2+x+1}}{4\left(3x^2+4\sqrt{-x^2+x+1}(x+2)-8x-8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4 + 4*x^3 - 25*x^2 + 5*(3*x^2 + 4*sqrt(-x^2 + x + 1)*(x
 + 2) - 8*x - 8)*arctan((sqrt(-x^2 + x + 1) - 1)/x) - (6*x^3 - 19
*x^2 - 12*x)*sqrt(-x^2 + x + 1) - 12*x)/(3*x^2 + 4*sqrt(-x^2 + x
+ 1)*(x + 2) - 8*x - 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x + 1), x)
```

GIAC/XCAS [A] time = 0.210793, size = 42, normalized size = 1.11

$$\frac{1}{4}\sqrt{-x^2+x+1}(2x-1)+\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + x + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) + 5/8*arcsin(1/5*sqrt(5)*(2*x -
1))
```

3.326 $\int \frac{x^4}{16+x^{10}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

```
[Out] ArcTan[x^5/4]/20
```

Rubi [A] time = 0.0137458, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Rubi in Sympy [A] time = 1.23571, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**10+16),x)
```

```
[Out] atan(x**5/4)/20
```

Mathematica [A] time = 0.00538723, size = 12, normalized size = 1.

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Maple [A] time = 0.002, size = 9, normalized size = 0.8

$$\frac{1}{20}\arctan\left(\frac{x^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^10+16),x)
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Maxima [A] time = 1.52469, size = 11, normalized size = 0.92

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + 16),x, algorithm="maxima")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Fricas [A] time = 0.228837, size = 11, normalized size = 0.92

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + 16),x, algorithm="fricas")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Sympy [A] time = 0.141754, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**10+16),x)
```

```
[Out] atan(x**5/4)/20
```

GIAC/XCAS [A] time = 0.214287, size = 11, normalized size = 0.92

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^10 + 16),x, algorithm="giac")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

3.327 $\int \frac{2+x}{2+x+x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rubi [A] time = 0.0334638, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rubi in Sympy [A] time = 2.21122, size = 32, normalized size = 1.03

$$\frac{\log\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}\,\operatorname{atan}\left(\sqrt{7}\left(\frac{2x}{7}+\frac{1}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**2+x+2),x)
```

```
[Out] log(x**2 + x + 2)/2 + 3*sqrt(7)*atan(sqrt(7)*(2*x/7 + 1/7))/7
```

Mathematica [A] time = 0.0167361, size = 31, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Maple [A] time = 0.005, size = 27, normalized size = 0.9

$$\frac{\ln\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}}{7}\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2+x+2),x)
```

```
[Out] 1/2*ln(x^2+x+2)+3/7*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.48483, size = 35, normalized size = 1.13

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x + 2),x, algorithm="maxima")
```

```
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Fricas [A] time = 0.254523, size = 41, normalized size = 1.32

$$\frac{1}{14}\sqrt{7}\left(\sqrt{7}\log\left(x^2+x+2\right)+6\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x + 2),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*(sqrt(7)*log(x^2 + x + 2) + 6*arctan(1/7*sqrt(7)*(2*
x + 1)))
```

Sympy [A] time = 0.116841, size = 36, normalized size = 1.16

$$\frac{\log\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2+x+2),x)
```

```
[Out] log(x**2 + x + 2)/2 + 3*sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)/7
```

GIAC/XCAS [A] time = 0.214339, size = 35, normalized size = 1.13

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+\frac{1}{2}\ln\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x + 2),x, algorithm="giac")
```

```
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*ln(x^2 + x + 2)
```

3.328 $\int x \sec(x) \tan(x)\, dx$

Optimal. Leaf size=10

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rubi [A] time = 0.0168106, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sec[x]*Tan[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rubi in Sympy [A] time = 1.37318, size = 8, normalized size = 0.8

$$\frac{x}{\cos(x)} - \operatorname{atanh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sec(x)*tan(x),x)
```

```
[Out] x/cos(x) - atanh(sin(x))
```

Mathematica [B] time = 0.0086549, size = 37, normalized size = 3.7

$$x \sec(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sec[x]*Tan[x],x]
```

```
[Out] Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + x*Sec[x]
```

Maple [A] time = 0.009, size = 16, normalized size = 1.6

$$\frac{x}{\cos(x)} - \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sec(x)*tan(x),x)
```

```
[Out] x/cos(x)-ln(sec(x)+tan(x))
```

Maxima [A] time = 1.511, size = 163, normalized size = 16.3

$$\frac{4\,x\cos(2\,x)\cos(x)+4\,x\sin(2\,x)\sin(x)+4\,x\cos(x)-\left(\cos(2\,x)^2+\sin(2\,x)^2+2\,\cos(2\,x)+1\right)\log\left(\cos(x)^2+\sin(x)^2+2\,s}{2\left(\cos(2\,x)^2+\sin(2\,x)^2+2\,\cos(2\,x)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*x*cos(2*x)*cos(x) + 4*x*sin(2*x)*sin(x) + 4*x*cos(x) - (co
s(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 +
 2*sin(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(c
os(x)^2 + sin(x)^2 - 2*sin(x) + 1))/(cos(2*x)^2 + sin(2*x)^2 + 2*
cos(2*x) + 1)
```

Fricas [A] time = 0.244844, size = 39, normalized size = 3.9

$$-\frac{\cos(x)\log(\sin(x)+1)-\cos(x)\log(-\sin(x)+1)-2\,x}{2\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*log(sin(x) + 1) - cos(x)*log(-sin(x) + 1) - 2*x)/cos
(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tan(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sec(x)*tan(x),x)
```

```
[Out] Integral(x*tan(x)*sec(x), x)
```

GIAC/XCAS [A] time = 0.247866, size = 203, normalized size = 20.3

$$-\frac{2\,x\tan\left(\frac{1}{2}\,x\right)^2+\ln\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2-\ln\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+2\,x-\ln\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2+2\,ta}{\tan\left(\frac{1}{2}\,x\right)}\right.\right.}{2\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*(2*x*tan(1/2*x)^2 + ln(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(
tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - ln(2*(tan(1/2*x)^2 - 2*tan(1/2*
x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*x - ln(2*(tan(1/2*x)
^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + ln(2*(tan(1/2*x)^2 -
 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)
```

3.329 $\int \frac{x}{-a^4+x^4}\,dx$

Optimal. Leaf size=15

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.0181331, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(-a^4 + x^4),x]
```

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi in Sympy [A] time = 1.76379, size = 14, normalized size = 0.93

$$-\frac{\operatorname{atanh}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-a**4+x**4),x)
```

```
[Out] -atanh(x**2/a**2)/(2*a**2)
```

Mathematica [A] time = 0.00615551, size = 15, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(-a^4 + x^4),x]
```

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Maple [B] time = 0.007, size = 30, normalized size = 2.

$$-\frac{\ln\left(a^2+x^2\right)}{4\,a^2}+\frac{\ln\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-a^4+x^4),x)
```

```
[Out] -1/4/a^2*ln(a^2+x^2)+1/4/a^2*ln(-a^2+x^2)
```

Maxima [A] time = 1.34083, size = 39, normalized size = 2.6

$$-\frac{\log\left(a^2+x^2\right)}{4\,a^2}+\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(a^4 - x^4),x, algorithm="maxima")
```

```
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(-a^2 + x^2)/a^2
```

Fricas [A] time = 0.221401, size = 35, normalized size = 2.33

$$-\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(a^4 - x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] time = 0.170483, size = 22, normalized size = 1.47

$$\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-a**4+x**4),x)
```

```
[Out] (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

GIAC/XCAS [A] time = 0.21137, size = 41, normalized size = 2.73

$$-\frac{\ln\left(a^2+x^2\right)}{4\,a^2}+\frac{\ln\left(\left|-a^2+x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(a^4 - x^4),x, algorithm="giac")
```

```
[Out] -1/4*ln(a^2 + x^2)/a^2 + 1/4*ln(abs(-a^2 + x^2))/a^2
```

3.330 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0123942, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi in Sympy [A] time = 0.637021, size = 17, normalized size = 0.81

$$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.0207058, size = 19, normalized size = 0.9

$$\frac{2}{3}\left((x+1)^{3/2}-x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (2*(-x^(3/2) + (1 + x)^(3/2)))/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{3}\,(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Fricas [A] time = 0.250854, size = 18, normalized size = 0.86

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [A] time = 0.702186, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x
) + 3*sqrt(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

GIAC/XCAS [A] time = 0.214075, size = 18, normalized size = 0.86

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x + 1) + sqrt(x)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.331 $\int \frac{1}{1-e^{-x}+2e^{x}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}\log(1-2e^{x})-\frac{1}{3}\log(e^{x}+1)$$

```
[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3
```

Rubi [A] time = 0.0304393, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{3}\log(1-2e^{x})-\frac{1}{3}\log(e^{x}+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - E^(-x) + 2*E^x)^(-1),x]
```

```
[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3
```

Rubi in Sympy [A] time = 2.50401, size = 17, normalized size = 0.74

$$\frac{\log(-2e^{x}+1)}{3}-\frac{\log(e^{x}+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-1/exp(x)+2*exp(x)),x)
```

```
[Out] log(-2*exp(x) + 1)/3 - log(exp(x) + 1)/3
```

Mathematica [A] time = 0.0151064, size = 21, normalized size = 0.91

$$\frac{1}{3}\left(\log(1-2e^{x})-\log(e^{x}+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - E^(-x) + 2*E^x)^(-1),x]
```

```
[Out] (Log[1 - 2*E^x] - Log[1 + E^x])/3
```

Maple [A] time = 0.01, size = 18, normalized size = 0.8

$$-\frac{\ln(1+e^{x})}{3}+\frac{\ln(2e^{x}-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-1/exp(x)+2*exp(x)),x)
```

```
[Out] -1/3*ln(1+exp(x))+1/3*ln(2*exp(x)-1)
```

Maxima [A] time = 1.34847, size = 26, normalized size = 1.13

$$-\frac{1}{3}\log\left(e^{(-x)}+1\right)+\frac{1}{3}\log\left(e^{(-x)}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - 2*e^x - 1),x, algorithm="maxima")
```

```
[Out] -1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 2)
```

Fricas [A] time = 0.226399, size = 23, normalized size = 1.

$$\frac{1}{3}\log\left(2\,e^{x}-1\right)-\frac{1}{3}\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - 2*e^x - 1),x, algorithm="fricas")
```

```
[Out] 1/3*log(2*e^x - 1) - 1/3*log(e^x + 1)
```

Sympy [A] time = 0.133984, size = 17, normalized size = 0.74

$$\frac{\log\left(e^{x}-\frac{1}{2}\right)}{3}-\frac{\log\left(e^{x}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-1/exp(x)+2*exp(x)),x)
```

```
[Out] log(exp(x) - 1/2)/3 - log(exp(x) + 1)/3
```

GIAC/XCAS [A] time = 0.212068, size = 24, normalized size = 1.04

$$-\frac{1}{3}\ln\left(e^{x}+1\right)+\frac{1}{3}\ln\left(\left|2\,e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(e^(-x) - 2*e^x - 1),x, algorithm="giac")
```

```
[Out] -1/3*ln(e^x + 1) + 1/3*ln(abs(2*e^x - 1))
```

3.332 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rubi [A] time = 0.0154955, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rubi in Sympy [A] time = 1.76436, size = 17, normalized size = 0.85

$$2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(atan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Mathematica [A] time = 0.00634974, size = 20, normalized size = 1.

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$-\ln\left(1+x\right) + 2\,\arctan\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arctan(x^(1/2))/x^(1/2),x)
```

```
[Out] -ln(1+x)+2*arctan(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.33564, size = 22, normalized size = 1.1

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Fricas [A] time = 0.225387, size = 22, normalized size = 1.1

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] time = 0.707571, size = 17, normalized size = 0.85

$$2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

GIAC/XCAS [A] time = 0.213661, size = 22, normalized size = 1.1

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/sqrt(x),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - ln(x + 1)
```

3.333 $\int \frac{\log(1+x)}{x^2}\,dx$

Optimal. Leaf size=18

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rubi [A] time = 0.0177651, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rubi in Sympy [A] time = 1.5048, size = 14, normalized size = 0.78

$$\log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(1+x)/x**2,x)
```

```
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

Mathematica [A] time = 0.00361293, size = 18, normalized size = 1.

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Maple [A] time = 0.01, size = 16, normalized size = 0.9

$$\ln(x) - \frac{\ln(1+x)(1+x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(1+x)/x^2,x)
```

```
[Out] ln(x)-ln(1+x)*(1+x)/x
```

Maxima [A] time = 1.34926, size = 24, normalized size = 1.33

$$-\frac{\log(x+1)}{x} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 1)/x^2,x, algorithm="maxima")
```

```
[Out] -log(x + 1)/x - log(x + 1) + log(x)
```

Fricas [A] time = 0.215748, size = 26, normalized size = 1.44

$$-\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 1)/x^2,x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(x + 1) - x*log(x))/x
```

Sympy [A] time = 0.127161, size = 14, normalized size = 0.78

$$\log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(1+x)/x**2,x)
```

```
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

GIAC/XCAS [A] time = 0.210111, size = 27, normalized size = 1.5

$$-\frac{\ln(x+1)}{x} - \ln(|x+1|) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x + 1)/x^2,x, algorithm="giac")
```

```
[Out] -ln(x + 1)/x - ln(abs(x + 1)) + ln(abs(x))
```

3.334 $\int \frac{1}{-e^x+e^{3x}}\,dx$

Optimal. Leaf size=12

$$e^{-x} - \tanh^{-1}(e^x)$$

```
[Out] E^(-x) - ArcTanh[E^x]
```

Rubi [A] time = 0.024791, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$e^{-x} - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-E^x + E^(3*x))^(-1),x]
```

```
[Out] E^(-x) - ArcTanh[E^x]
```

Rubi in Sympy [A] time = 4.73425, size = 10, normalized size = 0.83

$$-\operatorname{atanh}(e^{-x}) + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-exp(x)+exp(3*x)),x)
```

```
[Out] -atanh(exp(-x)) + exp(-x)
```

Mathematica [B] time = 0.021508, size = 32, normalized size = 2.67

$$e^{-x} + \frac{1}{2}\log(1-e^{-x}) - \frac{1}{2}\log(e^{-x}+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-E^x + E^(3*x))^(-1),x]
```

```
[Out] E^(-x) + Log[1 - E^(-x)]/2 - Log[1 + E^(-x)]/2
```

Maple [A] time = 0.013, size = 20, normalized size = 1.7

$$-\frac{\ln(1+e^x)}{2} + (e^x)^{-1} + \frac{\ln(-1+e^x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-exp(x)+exp(3*x)),x)
```

```
[Out] -1/2*ln(1+exp(x))+1/exp(x)+1/2*ln(-1+exp(x))
```

Maxima [A] time = 1.34042, size = 26, normalized size = 2.17

$$e^{(-x)} - \frac{1}{2}\log(e^x + 1) + \frac{1}{2}\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^(3*x) - e^x),x, algorithm="maxima")
```

```
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [A] time = 0.216942, size = 34, normalized size = 2.83

$$-\frac{1}{2}(e^x \log(e^x + 1) - e^x \log(e^x - 1) - 2)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^(3*x) - e^x),x, algorithm="fricas")
```

```
[Out] -1/2*(e^x*log(e^x + 1) - e^x*log(e^x - 1) - 2)*e^(-x)
```

Sympy [A] time = 0.114666, size = 20, normalized size = 1.67

$$\frac{\log(e^x - 1)}{2} - \frac{\log(e^x + 1)}{2} + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-exp(x)+exp(3*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)
```

GIAC/XCAS [A] time = 0.219618, size = 27, normalized size = 2.25

$$e^{(-x)} - \frac{1}{2}\ln(e^x + 1) + \frac{1}{2}\ln(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^(3*x) - e^x),x, algorithm="giac")
```

```
[Out] e^(-x) - 1/2*ln(e^x + 1) + 1/2*ln(abs(e^x - 1))
```

3.335 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-x-2\cot(x)$$

```
[Out] -x - 2*Cot[x]
```

Rubi [A] time = 0.0578814, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(x)+1}{\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+cos(x)**2)/(-cos(x)**2+1),x)
```

```
[Out] Integral((cos(x)**2 + 1)/sin(x)**2, x)
```

Mathematica [A] time = 0.0130457, size = 8, normalized size = 1.

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Maple [A] time = 0.062, size = 11, normalized size = 1.4

$$-2\,(\tan(x))^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+cos(x)^2)/(-cos(x)^2+1),x)
```

```
[Out] -2/tan(x)-x
```

Maxima [A] time = 1.51114, size = 14, normalized size = 1.75

$$-x - \frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x, algorithm="maxima")
```

```
[Out] -x - 2/tan(x)
```

Fricas [A] time = 0.21793, size = 20, normalized size = 2.5

$$-\frac{x\sin(x) + 2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

Sympy [A] time = 2.37255, size = 12, normalized size = 1.5

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+cos(x)**2)/(-cos(x)**2+1),x)
```

```
[Out] -x + tan(x/2) - 1/tan(x/2)
```

GIAC/XCAS [A] time = 0.225478, size = 22, normalized size = 2.75

$$-x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)} + \tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

3.336 $\int \frac{1}{x\sqrt{-25+2x}}\,dx$

Optimal. Leaf size=18

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rubi [A] time = 0.0120941, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rubi in Sympy [A] time = 1.12354, size = 14, normalized size = 0.78

$$\frac{2\,\text{atan}\left(\frac{\sqrt{2x-25}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(-25+2*x)**(1/2),x)
```

```
[Out] 2*atan(sqrt(2*x - 25)/5)/5
```

Mathematica [A] time = 0.00637822, size = 18, normalized size = 1.

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Maple [A] time = 0.008, size = 13, normalized size = 0.7

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{-25+2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(-25+2*x)^(1/2),x)
```

```
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

Maxima [A] time = 1.51821, size = 16, normalized size = 0.89

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x - 25)*x),x, algorithm="maxima")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Fricas [A] time = 0.210336, size = 16, normalized size = 0.89

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x - 25)*x),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Sympy [A] time = 1.67547, size = 46, normalized size = 2.56

$$\begin{cases} \frac{2i\,\operatorname{acosh}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{for } \frac{25\left|\frac{1}{x}\right|}{2} > 1 \\ -\frac{2\operatorname{asin}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(-25+2*x)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(5*sqrt(2)/(2*sqrt(x)))/5, 25*Abs(1/x)/2 > 1)
, (-2*asin(5*sqrt(2)/(2*sqrt(x)))/5, True))
```

GIAC/XCAS [A] time = 0.216565, size = 16, normalized size = 0.89

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x - 25)*x),x, algorithm="giac")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

3.337 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rubi [A] time = 0.082698, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]

[Out] -ArcSin[Cos[x]^2/3]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)

[Out] Timed out
```

Mathematica [C] time = 0.120085, size = 26, normalized size = 2.36

$$i\log\left(\sqrt{9-\cos^4(x)}+i\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]

[Out] I*Log[I*Cos[x]^2 + Sqrt[9 - Cos[x]^4]]
```

Maple [A] time = 0.045, size = 10, normalized size = 0.9

$$-\arcsin\left(\frac{(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)/(9-cos(x)^4)^(1/2),x)

[Out] -arcsin(1/3*cos(x)^2)
```

Maxima [A] time = 6.50359, size = 61, normalized size = 5.55

$$\log\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\sin(2\,x) + 1\right) - \log\left(\cos(2\,x)^2 + \sin(2\,x)^2 - 2\,\sin(2\,x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/sqrt(-cos(x)^4 + 9),x, algorithm="maxima")
```

```
[Out] log(cos(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - log(cos(2*x)^2 +
sin(2*x)^2 - 2*sin(2*x) + 1)
```

Fricas [A] time = 0.271522, size = 24, normalized size = 2.18

$$-\arctan\left(\frac{\cos(x)^2}{\sqrt{-\cos(x)^4 + 9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/sqrt(-cos(x)^4 + 9),x, algorithm="fricas")
```

```
[Out] -arctan(cos(x)^2/sqrt(-cos(x)^4 + 9))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2\,x)}{\sqrt{-\cos(x)^4 + 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)/sqrt(-cos(x)^4 + 9),x, algorithm="giac")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

3.338 $\int \frac{x^2}{\sqrt{5-4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rubi [A] time = 0.0211762, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[5 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rubi in Sympy [A] time = 1.78426, size = 27, normalized size = 0.9

$$-\frac{x\sqrt{-4x^2+5}}{8}+\frac{5\operatorname{asin}\left(\frac{2\sqrt{5}x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-4*x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 + 5)/8 + 5*asin(2*sqrt(5)*x/5)/16
```

Mathematica [A] time = 0.0185408, size = 30, normalized size = 1.

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[5 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Maple [A] time = 0.008, size = 23, normalized size = 0.8

$$\frac{5}{16}\arcsin\left(\frac{2x\sqrt{5}}{5}\right)-\frac{x}{8}\sqrt{-4x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-4*x^2+5)^(1/2),x)
```

```
[Out] 5/16*arcsin(2/5*x*5^(1/2))-1/8*x*(-4*x^2+5)^(1/2)
```

Maxima [A] time = 1.52561, size = 30, normalized size = 1.

$$-\frac{1}{8}\sqrt{-4\,x^2+5}x+\frac{5}{16}\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-4*x^2 + 5),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

Fricas [A] time = 0.212454, size = 41, normalized size = 1.37

$$-\frac{1}{8}\sqrt{-4\,x^2+5}x-\frac{5}{16}\arctan\left(\frac{\sqrt{-4\,x^2+5}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-4*x^2 + 5),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x - 5/16*arctan(1/2*sqrt(-4*x^2 + 5)/x)
```

Sympy [A] time = 0.268908, size = 27, normalized size = 0.9

$$-\frac{x\sqrt{-4x^2+5}}{8}+\frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-4*x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(-4*x**2 + 5)/8 + 5*asin(2*sqrt(5)*x/5)/16
```

GIAC/XCAS [A] time = 0.220797, size = 30, normalized size = 1.

$$-\frac{1}{8}\sqrt{-4\,x^2+5}x+\frac{5}{16}\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-4*x^2 + 5),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

3.339 $\int x^3 \sin(x)\,dx$

Optimal. Leaf size=24

$$x^3(-\cos(x)) + 3x^2 \sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi [A] time = 0.0543254, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^3(-\cos(x)) + 3x^2 \sin(x) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi in Sympy [A] time = 2.03309, size = 26, normalized size = 1.08

$$-x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Mathematica [A] time = 0.0132284, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x) - x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sin[x],x]
```

```
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] time = 0.002, size = 25, normalized size = 1.

$$6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*sin(x),x)
```

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Maxima [A] time = 1.3422, size = 28, normalized size = 1.17

$$-\left(x^3-6x\right)\cos(x) + 3\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Fricas [A] time = 0.244232, size = 28, normalized size = 1.17

$$-\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\left(x^2 - 2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] time = 0.843795, size = 26, normalized size = 1.08

$$-x^3\cos\left(x\right) + 3x^2\sin\left(x\right) + 6x\cos\left(x\right) - 6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

GIAC/XCAS [A] time = 0.209857, size = 28, normalized size = 1.17

$$-\left(x^3 - 6\,x\right)\cos\left(x\right) + 3\left(x^2 - 2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="giac")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.340 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*A
rcSinh[(1 + x)/Sqrt[3]])/2
```

Rubi [A] time = 0.0335192, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*A
rcSinh[(1 + x)/Sqrt[3]])/2
```

Rubi in Sympy [A] time = 1.92478, size = 54, normalized size = 1.08

$$-\frac{(2x+2)\sqrt{x^2+2x+4}}{4}+\frac{\left(x^2+2x+4\right)^{\frac{3}{2}}}{3}-\frac{3\,\text{atanh}\left(\frac{2x+2}{2\sqrt{x^2+2x+4}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+2*x+4)**(1/2),x)
```

```
[Out] -(2*x + 2)*sqrt(x**2 + 2*x + 4)/4 + (x**2 + 2*x + 4)**(3/2)/3 - 3
*atanh((2*x + 2)/(2*sqrt(x**2 + 2*x + 4)))/2
```

Mathematica [A] time = 0.032022, size = 38, normalized size = 0.76

$$\frac{1}{6}\left(\sqrt{x^2+2x+4}\left(2x^2+x+5\right)-9\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] (Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2) - 9*ArcSinh[(1 + x)/Sqrt[3]]
)/6
```

Maple [A] time = 0.007, size = 42, normalized size = 0.8

$$\frac{1}{3}\left(x^2+2\,x+4\right)^{\frac{3}{2}}-\frac{2\,x+2}{4}\sqrt{x^2+2\,x+4}-\frac{3}{2}Arcsinh\left(\frac{(1+x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-1/4*(2*x+2)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1
/3*(1+x)*3^(1/2))
```

Maxima [A] time = 1.52057, size = 66, normalized size = 1.32

$$\frac{1}{3}\left(x^2+2\,x+4\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^2+2\,x+4}x-\frac{1}{2}\sqrt{x^2+2\,x+4}-\frac{3}{2}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)*x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(
x^2 + 2*x + 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

Fricas [A] time = 0.207473, size = 220, normalized size = 4.4

$$-\frac{32\,x^6+144\,x^5+456\,x^4+816\,x^3+1104\,x^2-36\left(4\,x^3+12\,x^2-\left(4\,x^2+8\,x+7\right)\sqrt{x^2+2\,x+4}+21\,x+13\right)\log\left(-x+\sqrt{x^2+}\right.}{24\left(4\,x^3+12\,x^2-(4\,x^2+8\,x+7)\sqrt{x^2+2\,x+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)*x,x, algorithm="fricas")
```

```
[Out] -1/24*(32*x^6 + 144*x^5 + 456*x^4 + 816*x^3 + 1104*x^2 - 36*(4*x^
3 + 12*x^2 - (4*x^2 + 8*x + 7)*sqrt(x^2 + 2*x + 4) + 21*x + 13)*l
og(-x + sqrt(x^2 + 2*x + 4) - 1) - (32*x^5 + 112*x^4 + 296*x^3 +
400*x^2 + 416*x + 211)*sqrt(x^2 + 2*x + 4) + 885*x + 469)/(4*x^3
+ 12*x^2 - (4*x^2 + 8*x + 7)*sqrt(x^2 + 2*x + 4) + 21*x + 13)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2+2x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2 + 2*x + 4), x)
```

GIAC/XCAS [A] time = 0.213015, size = 54, normalized size = 1.08

$$\frac{1}{6}\left((2\,x+1)x+5\right)\sqrt{x^2+2\,x+4}+\frac{3}{2}\ln\left(-x+\sqrt{x^2+2\,x+4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2*x + 4)*x,x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*ln(-x + sqrt(x^2
+ 2*x + 4) - 1)
```

3.341 $\int x \left(5 + x^2\right)^8 \, dx$

Optimal. Leaf size=11

$$\frac{1}{18} \left(x^2 + 5\right)^9$$

```
[Out] (5 + x^2)^9/18
```

Rubi [A] time = 0.00692123, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{18} \left(x^2 + 5\right)^9$$

Antiderivative was successfully verified.

```
[In]  Int[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Rubi in Sympy [A] time = 0.778112, size = 7, normalized size = 0.64

$$\frac{\left(x^2 + 5\right)^9}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+5)**8,x)
```

```
[Out] (x**2 + 5)**9/18
```

Mathematica [A] time = 0.0026133, size = 11, normalized size = 1.

$$\frac{1}{18} \left(x^2 + 5\right)^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Maple [B] time = 0.003, size = 47, normalized size = 4.3

$$\frac{x^{18}}{18} + \frac{5\,x^{16}}{2} + 50\,x^{14} + \frac{1750\,x^{12}}{3} + 4375\,x^{10} + 21875\,x^8 + \frac{218750\,x^6}{3} + 156250\,x^4 + \frac{390625\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+5)^8,x)
```

```
[Out] 1/18*x^18+5/2*x^16+50*x^14+1750/3*x^12+4375*x^10+21875*x^8+218750
/3*x^6+156250*x^4+390625/2*x^2
```

Maxima [A] time = 1.35063, size = 12, normalized size = 1.09

$$\frac{1}{18}\left(x^2+5\right)^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^8*x,x, algorithm="maxima")
```

```
[Out] 1/18*(x^2 + 5)^9
```

Fricas [A] time = 0.167509, size = 1, normalized size = 0.09

$$\frac{1}{18}x^{18}+\frac{5}{2}x^{16}+50x^{14}+\frac{1750}{3}x^{12}+4375x^{10}+21875x^8+\frac{218750}{3}x^6+156250x^4+\frac{390625}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^8*x,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 + 5/2*x^16 + 50*x^14 + 1750/3*x^12 + 4375*x^10 + 21875*
x^8 + 218750/3*x^6 + 156250*x^4 + 390625/2*x^2
```

Sympy [A] time = 0.038304, size = 51, normalized size = 4.64

$$\frac{x^{18}}{18}+\frac{5x^{16}}{2}+50x^{14}+\frac{1750x^{12}}{3}+4375x^{10}+21875x^8+\frac{218750x^6}{3}+156250x^4+\frac{390625x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+5)**8,x)
```

```
[Out] x**18/18 + 5*x**16/2 + 50*x**14 + 1750*x**12/3 + 4375*x**10 + 218
75*x**8 + 218750*x**6/3 + 156250*x**4 + 390625*x**2/2
```

GIAC/XCAS [A] time = 0.209293, size = 12, normalized size = 1.09

$$\frac{1}{18}\left(x^2+5\right)^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 5)^8*x,x, algorithm="giac")
```

```
[Out] 1/18*(x^2 + 5)^9
```

3.342 $\int \cos^2(x)\sin^5(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

```
[Out] -Cos[x]^3/3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rubi [A] time = 0.0467748, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] -Cos[x]^3/3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rubi in Sympy [A] time = 3.10453, size = 20, normalized size = 0.8

$$-\frac{\cos^7(x)}{7}+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2*sin(x)**5,x)
```

```
[Out] -cos(x)**7/7 + 2*cos(x)**5/5 - cos(x)**3/3
```

Mathematica [A] time = 0.0166215, size = 31, normalized size = 1.24

$$-\frac{5\cos(x)}{64}-\frac{1}{192}\cos(3x)+\frac{3}{320}\cos(5x)-\frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/64 - Cos[3*x]/192 + (3*Cos[5*x])/320 - Cos[7*x]/448
```

Maple [A] time = 0.016, size = 28, normalized size = 1.1

$$-\frac{(\cos(x))^3(\sin(x))^4}{7}-\frac{4(\sin(x))^2(\cos(x))^3}{35}-\frac{8(\cos(x))^3}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2*sin(x)^5,x)
```

```
[Out] -1/7*cos(x)^3*sin(x)^4-4/35*sin(x)^2*cos(x)^3-8/105*cos(x)^3
```

Maxima [A] time = 1.3685, size = 26, normalized size = 1.04

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^5,x, algorithm="maxima")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Fricas [A] time = 0.226975, size = 26, normalized size = 1.04

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Sympy [A] time = 0.050655, size = 20, normalized size = 0.8

$$-\frac{\cos^7(x)}{7}+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2*sin(x)**5,x)
```

```
[Out] -cos(x)**7/7 + 2*cos(x)**5/5 - cos(x)**3/3
```

GIAC/XCAS [A] time = 0.205304, size = 26, normalized size = 1.04

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2*sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

3.343 $\int e^{-3x}\cos(4x)\,dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rubi [A] time = 0.0199228, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rubi in Sympy [A] time = 1.55764, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25}-\frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

Mathematica [A] time = 0.021291, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4\sin(4x)-3\cos(4x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

Maple [A] time = 0.012, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3x}\cos(4x)}{25}+\frac{4\,\mathrm{e}^{-3x}\sin(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(4*x)/exp(3*x),x)
```

```
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

Maxima [A] time = 1.33831, size = 26, normalized size = 0.96

$$-\frac{1}{25}(3\cos(4x) - 4\sin(4x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*e^(-3*x),x, algorithm="maxima")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Fricas [A] time = 0.210979, size = 28, normalized size = 1.04

$$-\frac{3}{25}\cos(4x)\,e^{(-3x)} + \frac{4}{25}\,e^{(-3x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*e^(-3*x),x, algorithm="fricas")
```

```
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

Sympy [A] time = 0.785049, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25} - \frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

GIAC/XCAS [A] time = 0.209619, size = 26, normalized size = 0.96

$$-\frac{1}{25}(3\cos(4x) - 4\sin(4x))e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(4*x)*e^(-3*x),x, algorithm="giac")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

3.344 $\int \csc^3\left(\frac{x}{2}\right)\, dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rubi [A] time = 0.0217313, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x/2]^3,x]
```

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rubi in Sympy [A] time = 0.560826, size = 19, normalized size = 0.79

$$-\operatorname{atanh}\left(\cos\left(\frac{x}{2}\right)\right)-\frac{\cos\left(\frac{x}{2}\right)}{\sin^2\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(csc(1/2*x)**3,x)
```

```
[Out] -atanh(cos(x/2)) - cos(x/2)/sin(x/2)**2
```

Mathematica [A] time = 0.00761112, size = 41, normalized size = 1.71

$$-\frac{1}{4}\csc^2\left(\frac{x}{4}\right)+\frac{1}{4}\sec^2\left(\frac{x}{4}\right)+\log\left(\sin\left(\frac{x}{4}\right)\right)-\log\left(\cos\left(\frac{x}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x/2]^3,x]
```

```
[Out] -Csc[x/4]^2/4 - Log[Cos[x/4]] + Log[Sin[x/4]] + Sec[x/4]^2/4
```

Maple [A] time = 0.016, size = 24, normalized size = 1.

$$-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\ln\left(\csc\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(csc(1/2*x)^3,x)
```

```
[Out] -cot(1/2*x)*csc(1/2*x)+ln(csc(1/2*x)-cot(1/2*x))
```

Maxima [A] time = 1.45378, size = 46, normalized size = 1.92

$$\frac{\cos\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)^2-1}-\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(1/2*x)^3,x, algorithm="maxima")
```

```
[Out] cos(1/2*x)/(cos(1/2*x)^2 - 1) - 1/2*log(cos(1/2*x) + 1) + 1/2*log
(cos(1/2*x) - 1)
```

Fricas [A] time = 0.259571, size = 76, normalized size = 3.17

$$-\frac{\left(\cos\left(\frac{1}{2}x\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(\frac{1}{2}x\right)+\frac{1}{2}\right)-\left(\cos\left(\frac{1}{2}x\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(\frac{1}{2}x\right)+\frac{1}{2}\right)-2\cos\left(\frac{1}{2}x\right)}{2\left(\cos\left(\frac{1}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(1/2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(1/2*x)^2 - 1)*log(1/2*cos(1/2*x) + 1/2) - (cos(1/2*x)^
2 - 1)*log(-1/2*cos(1/2*x) + 1/2) - 2*cos(1/2*x))/(cos(1/2*x)^2 -
 1)
```

Sympy [A] time = 0.131938, size = 36, normalized size = 1.5

$$\frac{\log\left(\cos\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\cos\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(1/2*x)**3,x)
```

```
[Out] log(cos(x/2) - 1)/2 - log(cos(x/2) + 1)/2 + 2*cos(x/2)/(2*cos(x/2
)**2 - 2)
```

GIAC/XCAS [A] time = 0.222743, size = 95, normalized size = 3.96

$$-\frac{\left(\frac{2\left(\cos\left(\frac{1}{2}x\right)-1\right)}{\cos\left(\frac{1}{2}x\right)+1}-1\right)\left(\cos\left(\frac{1}{2}x\right)+1\right)}{4\left(\cos\left(\frac{1}{2}x\right)-1\right)}-\frac{\cos\left(\frac{1}{2}x\right)-1}{4\left(\cos\left(\frac{1}{2}x\right)+1\right)}+\frac{1}{2}\ln\left(-\frac{\cos\left(\frac{1}{2}x\right)-1}{\cos\left(\frac{1}{2}x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(csc(1/2*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) - 1)*(cos(1/2*x) + 1)/(
cos(1/2*x) - 1) - 1/4*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) + 1/2*ln(
-(cos(1/2*x) - 1)/(cos(1/2*x) + 1))
```

3.345 $\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rubi [A] time = 0.0222532, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rubi in Sympy [A] time = 1.68437, size = 27, normalized size = 0.79

$$3\,\text{atanh}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9*x**2-1)**(1/2)/x**2,x)
```

```
[Out] 3*atanh(3*x/sqrt(9*x**2 - 1)) - sqrt(9*x**2 - 1)/x
```

Mathematica [A] time = 0.0155297, size = 35, normalized size = 1.03

$$3\log\left(\sqrt{9x^2-1}+3x\right)-\frac{\sqrt{9x^2-1}}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*Log[3*x + Sqrt[-1 + 9*x^2]]
```

Maple [A] time = 0.01, size = 47, normalized size = 1.4

$$\frac{1}{x}\left(9x^2-1\right)^{\frac{3}{2}}-9x\sqrt{9x^2-1}+\ln\left(x\sqrt{9}+\sqrt{9x^2-1}\right)\sqrt{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2-1)^(1/2)/x^2,x)
```

```
[Out] 1/x*(9*x^2-1)^(3/2)-9*x*(9*x^2-1)^(1/2)+ln(x*9^(1/2)+(9*x^2-1)^(1
/2))*9^(1/2)
```

Maxima [A] time = 1.48653, size = 45, normalized size = 1.32

$$-\frac{\sqrt{9\,x^2-1}}{x}+3\,\log\left(18\,x+6\,\sqrt{9\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 1)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(9*x^2 - 1)/x + 3*log(18*x + 6*sqrt(9*x^2 - 1))
```

Fricas [A] time = 0.211885, size = 78, normalized size = 2.29

$$-\frac{3\left(3\,x^2-\sqrt{9\,x^2-1}x\right)\log\left(-3\,x+\sqrt{9\,x^2-1}\right)+1}{3\,x^2-\sqrt{9\,x^2-1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 1)/x^2,x, algorithm="fricas")
```

```
[Out] -(3*(3*x^2 - sqrt(9*x^2 - 1)*x)*log(-3*x + sqrt(9*x^2 - 1)) + 1)/
(3*x^2 - sqrt(9*x^2 - 1)*x)
```

Sympy [A] time = 0.343018, size = 17, normalized size = 0.5

$$3\,\operatorname{acosh}(3x)-\frac{\sqrt{9x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2-1)**(1/2)/x**2,x)
```

```
[Out] 3*acosh(3*x) - sqrt(9*x**2 - 1)/x
```

GIAC/XCAS [A] time = 0.21771, size = 59, normalized size = 1.74

$$-\frac{6}{\left(3\,x-\sqrt{9\,x^2-1}\right)^2+1}-\frac{3}{2}\ln\left(\left(3\,x-\sqrt{9\,x^2-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 1)/x^2,x, algorithm="giac")
```

```
[Out] -6/((3*x - sqrt(9*x^2 - 1))^2 + 1) - 3/2*ln((3*x - sqrt(9*x^2 - 1
))^2)
```

3.346 $\int \frac{\sqrt{4-3x^2}}{x}\,dx$

Optimal. Leaf size=30

$$\sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Rubi [A] time = 0.0418147, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 - 3*x^2]/x,x]
```

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Rubi in Sympy [A] time = 2.68039, size = 24, normalized size = 0.8

$$\sqrt{-3x^2+4}-2\operatorname{atanh}\left(\frac{\sqrt{-3x^2+4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+4)**(1/2)/x,x)
```

```
[Out] sqrt(-3*x**2 + 4) - 2*atanh(sqrt(-3*x**2 + 4)/2)
```

Mathematica [A] time = 0.0135961, size = 32, normalized size = 1.07

$$\sqrt{4-3x^2}-2\log\left(\sqrt{4-3x^2}+2\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 - 3*x^2]/x,x]
```

```
[Out] Sqrt[4 - 3*x^2] + 2*Log[x] - 2*Log[2 + Sqrt[4 - 3*x^2]]
```

Maple [A] time = 0.006, size = 25, normalized size = 0.8

$$\sqrt{-3\,x^2+4}-2\,Artanh\left(2\,\frac{1}{\sqrt{-3\,x^2+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+4)^(1/2)/x,x)
```

```
[Out] (-3*x^2+4)^(1/2)-2*arctanh(2/(-3*x^2+4)^(1/2))
```

Maxima [A] time = 1.5022, size = 47, normalized size = 1.57

$$\sqrt{-3\,x^2+4}-2\,\log\left(\frac{4\,\sqrt{-3\,x^2+4}}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-3*x^2 + 4) - 2*log(4*sqrt(-3*x^2 + 4)/abs(x) + 8/abs(x))
```

Fricas [A] time = 0.204053, size = 68, normalized size = 2.27

$$-\frac{3\,x^2-2\left(\sqrt{-3\,x^2+4}-2\right)\log\left(\frac{\sqrt{-3\,x^2+4}-2}{x}\right)}{\sqrt{-3\,x^2+4}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4)/x,x, algorithm="fricas")
```

```
[Out] -(3*x^2 - 2*(sqrt(-3*x^2 + 4) - 2)*log((sqrt(-3*x^2 + 4) - 2)/x))
/(sqrt(-3*x^2 + 4) - 2)
```

Sympy [A] time = 2.55135, size = 75, normalized size = 2.5

$$\begin{cases} i\sqrt{3x^2-4}-2\log\left(x\right)+\log\left(x^2\right)+2i\,\operatorname{asin}\left(\frac{2\sqrt{3}}{3x}\right) & \text{for } \frac{3\left|x^2\right|}{4}>1 \\ \sqrt{-3x^2+4}+\log\left(x^2\right)-2\log\left(\sqrt{-\frac{3x^2}{4}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+4)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(3*x**2 - 4) - 2*log(x) + log(x**2) + 2*I*asin(2
*sqrt(3)/(3*x)), 3*Abs(x**2)/4 > 1), (sqrt(-3*x**2 + 4) + log(x**
2) - 2*log(sqrt(-3*x**2/4 + 1) + 1), True))
```

GIAC/XCAS [A] time = 0.215165, size = 51, normalized size = 1.7

$$\sqrt{-3\,x^2+4}-\ln\left(\sqrt{-3\,x^2+4}+2\right)+\ln\left(-\sqrt{-3\,x^2+4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4)/x,x, algorithm="giac")
```

```
[Out] sqrt(-3*x^2 + 4) - ln(sqrt(-3*x^2 + 4) + 2) + ln(-sqrt(-3*x^2 + 4
) + 2)
```

3.347 $\int e^{3x} x^2 \, dx$

Optimal. Leaf size=32

$$\frac{1}{3} e^{3x} x^2 - \frac{2}{9} e^{3x} x + \frac{2e^{3x}}{27}$$

```
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rubi [A] time = 0.0322933, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{3} e^{3x} x^2 - \frac{2}{9} e^{3x} x + \frac{2e^{3x}}{27}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(3*x)*x^2,x]
```

```
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rubi in Sympy [A] time = 2.15312, size = 27, normalized size = 0.84

$$\frac{x^2 e^{3x}}{3} - \frac{2x e^{3x}}{9} + \frac{2e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(3*x)*x**2,x)
```

```
[Out] x**2*exp(3*x)/3 - 2*x*exp(3*x)/9 + 2*exp(3*x)/27
```

Mathematica [A] time = 0.00356269, size = 19, normalized size = 0.59

$$\frac{1}{27} e^{3x} \left(9x^2 - 6x + 2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(3*x)*x^2,x]
```

```
[Out] (E^(3*x)*(2 - 6*x + 9*x^2))/27
```

Maple [A] time = 0.005, size = 17, normalized size = 0.5

$$\frac{\left(9x^2 - 6x + 2\right) e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(3*x)*x^2,x)
```

```
[Out] 1/27*(9*x^2-6*x+2)*exp(3*x)
```

Maxima [A] time = 1.35018, size = 22, normalized size = 0.69

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(3*x),x, algorithm="maxima")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Fricas [A] time = 0.198966, size = 22, normalized size = 0.69

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(3*x),x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Sympy [A] time = 0.085207, size = 15, normalized size = 0.47

$$\frac{\left(9x^2-6x+2\right)\,e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(3*x)*x**2,x)
```

```
[Out] (9*x**2 - 6*x + 2)*exp(3*x)/27
```

GIAC/XCAS [A] time = 0.208055, size = 22, normalized size = 0.69

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(3*x),x, algorithm="giac")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

3.348 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rubi [A] time = 0.0585358, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rubi in Sympy [A] time = 2.5498, size = 20, normalized size = 0.87

$$\frac{2\left(\sin(x)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*(sin(x) + 1)**(3/2)/3 - 2*sqrt(sin(x) + 1)
```

Mathematica [A] time = 0.0370521, size = 31, normalized size = 1.35

$$\frac{2(\sin(x)-2)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{3\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

Maple [A] time = 0.011, size = 18, normalized size = 0.8

$$\frac{2}{3}(1+\sin(x))^{\frac{3}{2}}-2\sqrt{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*sin(x)/(1+sin(x))^(1/2),x)
```

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Maxima [A] time = 1.36845, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}} - 2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)/sqrt(sin(x) + 1),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

Fricas [A] time = 0.210375, size = 16, normalized size = 0.7

$$\frac{2}{3}\sqrt{\sin(x)+1}(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)/sqrt(sin(x) + 1),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

Sympy [A] time = 0.470907, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin(x)+1}\sin(x)}{3} - \frac{4\sqrt{\sin(x)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

GIAC/XCAS [A] time = 0.210148, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}} - 2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)/sqrt(sin(x) + 1),x, algorithm="giac")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

3.349 $\int x \sin^{-1}\left(x^2\right)\, dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2 \sin^{-1}\left(x^2\right)$$

```
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rubi [A] time = 0.0255407, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2 \sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x*ArcSin[x^2],x]
```

```
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*asin(x**2),x)
```

```
[Out] Exception raised: ValueError
```

Mathematica [A] time = 0.00850355, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(\sqrt{1-x^4}+x^2 \sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*ArcSin[x^2],x]
```

```
[Out] (Sqrt[1 - x^4] + x^2*ArcSin[x^2])/2
```

Maple [A] time = 0.003, size = 22, normalized size = 0.8

$$\frac{x^2 \arcsin\left(x^2\right)}{2}+\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*arcsin(x^2),x)
```

```
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

Maxima [A] time = 1.51348, size = 28, normalized size = 1.04

$$\frac{1}{2}\,x^2 \arcsin\left(x^2\right) + \frac{1}{2}\,\sqrt{-x^4 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Fricas [A] time = 0.222901, size = 28, normalized size = 1.04

$$\frac{1}{2}\,x^2 \arcsin\left(x^2\right) + \frac{1}{2}\,\sqrt{-x^4 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Sympy [A] time = 0.222138, size = 19, normalized size = 0.7

$$\frac{x^2 \operatorname{asin}\left(x^2\right)}{2} + \frac{\sqrt{-x^4 + 1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*asin(x**2),x)
```

```
[Out] x**2*asin(x**2)/2 + sqrt(-x**4 + 1)/2
```

GIAC/XCAS [A] time = 0.216991, size = 28, normalized size = 1.04

$$\frac{1}{2}\,x^2 \arcsin\left(x^2\right) + \frac{1}{2}\,\sqrt{-x^4 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arcsin(x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

3.350 $\int x^3 \sin^{-1}\left(x^2\right)\, dx$

Optimal. Leaf size=38

$$-\frac{1}{8}\sin^{-1}\left(x^2\right) + \frac{1}{8}\sqrt{1-x^4}x^2 + \frac{1}{4}x^4\sin^{-1}\left(x^2\right)$$

```
[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4
```

Rubi [A] time = 0.0545805, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$

$$-\frac{1}{8}\sin^{-1}\left(x^2\right) + \frac{1}{8}\sqrt{1-x^4}x^2 + \frac{1}{4}x^4\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*ArcSin[x^2],x]
```

```
[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*asin(x**2),x)
```

```
[Out] Exception raised: ValueError
```

Mathematica [A] time = 0.0167284, size = 32, normalized size = 0.84

$$\frac{1}{8}\left(\sqrt{1-x^4}x^2 + \left(2x^4-1\right)\sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*ArcSin[x^2],x]
```

```
[Out] (x^2*Sqrt[1 - x^4] + (-1 + 2*x^4)*ArcSin[x^2])/8
```

Maple [A] time = 0.004, size = 31, normalized size = 0.8

$$-\frac{\arcsin\left(x^2\right)}{8} + \frac{x^4\arcsin\left(x^2\right)}{4} + \frac{x^2}{8}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*arcsin(x^2),x)
```

```
[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)
```

Maxima [A] time = 1.60733, size = 72, normalized size = 1.89

$$\frac{1}{4}x^4\arcsin\left(x^2\right)-\frac{\sqrt{-x^4+1}}{8\,x^2\left(\frac{x^4-1}{x^4}-1\right)}+\frac{1}{8}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsin(x^2) - 1/8*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)
) + 1/8*arctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [A] time = 0.226749, size = 38, normalized size = 1.

$$\frac{1}{8}\sqrt{-x^4+1}x^2+\frac{1}{8}\left(2\,x^4-1\right)\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/8*(2*x^4 - 1)*arcsin(x^2)
```

Sympy [A] time = 0.896623, size = 29, normalized size = 0.76

$$\frac{x^4\operatorname{asin}\left(x^2\right)}{4}+\frac{x^2\sqrt{-x^4+1}}{8}-\frac{\operatorname{asin}\left(x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*asin(x**2),x)
```

```
[Out] x**4*asin(x**2)/4 + x**2*sqrt(-x**4 + 1)/8 - asin(x**2)/8
```

GIAC/XCAS [A] time = 0.204829, size = 43, normalized size = 1.13

$$\frac{1}{8}\sqrt{-x^4+1}x^2+\frac{1}{4}\left(x^4-1\right)\arcsin\left(x^2\right)+\frac{1}{8}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*arcsin(x^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/4*(x^4 - 1)*arcsin(x^2) + 1/8*arcsin(x
^2)
```

3.351 $\int e^x \mathbf{sech}\left(e^x\right)\, dx$

Optimal. Leaf size=5

$$\tan^{-1}\left(\sinh\left(e^x\right)\right)$$

```
[Out] ArcTan[Sinh[E^x]]
```

Rubi [A] time = 0.015468, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\tan^{-1}\left(\sinh\left(e^x\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Sech[E^x],x]
```

```
[Out] ArcTan[Sinh[E^x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{e^x} \frac{1}{\cosh(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*sech(exp(x)),x)
```

```
[Out] Integral(1/cosh(x), (x, exp(x)))
```

Mathematica [B] time = 0.0223236, size = 11, normalized size = 2.2

$$2\tan^{-1}\left(\tanh\left(\frac{e^x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Sech[E^x],x]
```

```
[Out] 2*ArcTan[Tanh[E^x/2]]
```

Maple [A] time = 0.017, size = 5, normalized size = 1.

$$\arctan\left(\sinh\left(\mathrm{e}^x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*sech(exp(x)),x)
```

```
[Out] arctan(sinh(exp(x)))
```

Maxima [A] time = 1.59778, size = 5, normalized size = 1.

$$\arctan(\sinh(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sech(e^x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(e^x))
```

Fricas [A] time = 0.205811, size = 22, normalized size = 4.4

$$2\arctan(\cosh(\cosh(x)+\sinh(x))+\sinh(\cosh(x)+\sinh(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sech(e^x),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*sech(exp(x)),x)
```

```
[Out] Integral(exp(x)*sech(exp(x)), x)
```

GIAC/XCAS [A] time = 0.203114, size = 8, normalized size = 1.6

$$2\arctan\left(e^{(e^x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sech(e^x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(e^x))
```

3.352 $\int x^2 \cos(3x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2 \sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi [A] time = 0.0423021, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1}{3}x^2 \sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi in Sympy [A] time = 1.60912, size = 27, normalized size = 0.93

$$\frac{x^2 \sin(3x)}{3} + \frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Mathematica [A] time = 0.0253331, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2 - 2\right)\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] time = 0.001, size = 24, normalized size = 0.8

$$\frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27} + \frac{x^2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*cos(3*x),x)
```

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Maxima [A] time = 1.48296, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="maxima")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Fricas [A] time = 0.212951, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="fricas")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] time = 0.40292, size = 27, normalized size = 0.93

$$\frac{x^2\sin(3x)}{3}+\frac{2x\cos(3x)}{9}-\frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

GIAC/XCAS [A] time = 0.20279, size = 28, normalized size = 0.97

$$\frac{2}{9}x\cos(3x)+\frac{1}{27}\left(9x^2-2\right)\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*cos(3*x),x, algorithm="giac")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

3.353 $\int \sqrt{5-4x-x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rubi [A] time = 0.0223694, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[5 - 4*x - x^2],x]
```

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rubi in Sympy [A] time = 0.918535, size = 44, normalized size = 1.22

$$\frac{(2x+4)\sqrt{-x^2-4x+5}}{4}+\frac{9\operatorname{atan}\left(-\frac{-2x-4}{2\sqrt{-x^2-4x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-4*x+5)**(1/2),x)
```

```
[Out] (2*x + 4)*sqrt(-x**2 - 4*x + 5)/4 + 9*atan(-(-2*x - 4)/(2*sqrt(-x
**2 - 4*x + 5)))/2
```

Mathematica [A] time = 0.0275601, size = 33, normalized size = 0.92

$$\frac{1}{2}\left(\sqrt{-x^2-4x+5}(x+2)+9\sin^{-1}\left(\frac{x+2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[5 - 4*x - x^2],x]
```

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2] + 9*ArcSin[(2 + x)/3])/2
```

Maple [A] time = 0.004, size = 29, normalized size = 0.8

$$-\frac{-2x-4}{4}\sqrt{-x^2-4x+5}+\frac{9}{2}\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-4*x+5)^(1/2),x)
```

```
[Out] -1/4*(-2*x-4)*(-x^2-4*x+5)^(1/2)+9/2*arcsin(2/3+1/3*x)
```

Maxima [A] time = 1.51817, size = 49, normalized size = 1.36

$$\frac{1}{2}\sqrt{-x^2-4x+5}x+\sqrt{-x^2-4x+5}-\frac{9}{2}\arcsin\left(-\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - 4*x + 5),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*x + sqrt(-x^2 - 4*x + 5) - 9/2*arcsin(-1
/3*x - 2/3)
```

Fricas [A] time = 0.204392, size = 50, normalized size = 1.39

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)+\frac{9}{2}\arctan\left(\frac{x+2}{\sqrt{-x^2-4x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - 4*x + 5),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arctan((x + 2)/sqrt(-x^2 -
 4*x + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2-4x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 - 4*x + 5), x)
```

GIAC/XCAS [A] time = 0.205528, size = 35, normalized size = 0.97

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)+\frac{9}{2}\arcsin\left(\frac{1}{3}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - 4*x + 5),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arcsin(1/3*x + 2/3)
```

3.354 $\int \frac{x^5}{\sqrt{2}+x^2}\,dx$

Optimal. Leaf size=28

$$\frac{x^4}{4} - \frac{x^2}{\sqrt{2}} + \log\left(x^2 + \sqrt{2}\right)$$

```
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rubi [A] time = 0.0455685, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x^4}{4} - \frac{x^2}{\sqrt{2}} + \log\left(x^2 + \sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(x^2 + \sqrt{2}\right)}{4} - \frac{\int^{x^2} \sqrt{2}\,dx}{2} + \frac{\int^{x^2} x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**2+2**(1/2)),x)
```

```
[Out] log(x**2 + sqrt(2))/4 - Integral(sqrt(2), (x, x**2))/2 + Integral
(x, (x, x**2))/2
```

Mathematica [A] time = 0.0143151, size = 31, normalized size = 1.11

$$\frac{1}{4}\left(x^4 - 2\sqrt{2}x^2 + 4\log\left(x^2 + \sqrt{2}\right) - 6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] (-6 - 2*Sqrt[2]*x^2 + x^4 + 4*Log[Sqrt[2] + x^2])/4
```

Maple [A] time = 0.01, size = 23, normalized size = 0.8

$$\frac{x^4}{4} + \ln\left(x^2 + \sqrt{2}\right) - \frac{x^2\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^2+2^(1/2)),x)
```

```
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*x^2*2^(1/2)
```

Maxima [A] time = 1.48534, size = 30, normalized size = 1.07

$$\frac{1}{4}x^4 - \frac{1}{2}\sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 + sqrt(2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 + sqrt(2)),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.143149, size = 24, normalized size = 0.86

$$\frac{x^4}{4} - \frac{\sqrt{2}x^2}{2} + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**2+2**(1/2)),x)
```

```
[Out] x**4/4 - sqrt(2)*x**2/2 + log(x**2 + sqrt(2))
```

GIAC/XCAS [A] time = 0.205903, size = 30, normalized size = 1.07

$$\frac{1}{4}x^4 - \frac{1}{2}\sqrt{2}x^2 + \ln\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^2 + sqrt(2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + ln(x^2 + sqrt(2))
```

3.355 $\int \sec^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tanh^{-1}(\sin(x)) + \frac{1}{4}\tan(x)\sec^3(x) + \frac{3}{8}\tan(x)\sec(x)$$

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rubi [A] time = 0.0250671, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{3}{8}\tanh^{-1}(\sin(x)) + \frac{1}{4}\tan(x)\sec^3(x) + \frac{3}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rubi in Sympy [A] time = 0.607089, size = 29, normalized size = 1.12

$$\frac{3\sin(x)}{8\cos^2(x)} + \frac{\sin(x)}{4\cos^4(x)} + \frac{3\operatorname{atanh}(\sin(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**5,x)
```

```
[Out] 3*sin(x)/(8*cos(x)**2) + sin(x)/(4*cos(x)**4) + 3*atanh(sin(x))/8
```

Mathematica [B] time = 0.187033, size = 58, normalized size = 2.23

$$\frac{1}{16}\left(\frac{1}{2}(11\sin(x) + 3\sin(3x))\sec^4(x) - 6\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 6\log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^5,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2)/16
```

Maple [A] time = 0.052, size = 25, normalized size = 1.

$$-\left(-\frac{(\sec(x))^3}{4} - \frac{3\sec(x)}{8}\right)\tan(x) + \frac{3\ln(\sec(x) + \tan(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x))
```

Maxima [A] time = 1.32185, size = 57, normalized size = 2.19

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^4-2\,\sin(x)^2+1\right)}+\frac{3}{16}\,\log(\sin(x)+1)-\frac{3}{16}\,\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3/16*l
og(sin(x) + 1) - 3/16*log(sin(x) - 1)
```

Fricas [A] time = 0.223194, size = 58, normalized size = 2.23

$$\frac{3\,\cos(x)^4\log(\sin(x)+1)-3\,\cos(x)^4\log(-\sin(x)+1)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) +
2*(3*cos(x)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [A] time = 0.168673, size = 46, normalized size = 1.77

$$-\frac{3\sin^3(x)-5\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}-\frac{3\log(\sin(x)-1)}{16}+\frac{3\log(\sin(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**5,x)
```

```
[Out] -(3*sin(x)**3 - 5*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 3*lo
g(sin(x) - 1)/16 + 3*log(sin(x) + 1)/16
```

GIAC/XCAS [A] time = 0.20665, size = 51, normalized size = 1.96

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^2-1\right)^2}+\frac{3}{16}\ln(\sin(x)+1)-\frac{3}{16}\ln(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^2 - 1)^2 + 3/16*ln(sin(x) +
1) - 3/16*ln(-sin(x) + 1)
```

3.356 $\int \sin^6(2x)\,dx$

Optimal. Leaf size=46

$$\frac{5x}{16} - \frac{1}{12}\sin^5(2x)\cos(2x) - \frac{5}{48}\sin^3(2x)\cos(2x) - \frac{5}{32}\sin(2x)\cos(2x)$$

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48
- (Cos[2*x]*Sin[2*x]^5)/12
```

Rubi [A] time = 0.035017, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{5x}{16} - \frac{1}{12}\sin^5(2x)\cos(2x) - \frac{5}{48}\sin^3(2x)\cos(2x) - \frac{5}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48
- (Cos[2*x]*Sin[2*x]^5)/12
```

Rubi in Sympy [A] time = 0.77697, size = 46, normalized size = 1.

$$\frac{5x}{16} - \frac{\sin^5(2x)\cos(2x)}{12} - \frac{5\sin^3(2x)\cos(2x)}{48} - \frac{5\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)**6,x)
```

```
[Out] 5*x/16 - sin(2*x)**5*cos(2*x)/12 - 5*sin(2*x)**3*cos(2*x)/48 - 5*
sin(2*x)*cos(2*x)/32
```

Mathematica [A] time = 0.00432553, size = 30, normalized size = 0.65

$$\frac{5x}{16} - \frac{15}{128}\sin(4x) + \frac{3}{128}\sin(8x) - \frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[4*x])/128 + (3*Sin[8*x])/128 - Sin[12*x]/384
```

Maple [A] time = 0.007, size = 32, normalized size = 0.7

$$-\frac{\cos(2x)}{12}\left((\sin(2x))^5 + \frac{5(\sin(2x))^3}{4} + \frac{15\sin(2x)}{8}\right) + \frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)^6,x)
```

```
[Out] -1/12*(sin(2*x)^5+5/4*sin(2*x)^3+15/8*sin(2*x))*cos(2*x)+5/16*x
```

Maxima [A] time = 1.33978, size = 32, normalized size = 0.7

$$\frac{1}{96}\sin(4x)^3 + \frac{5}{16}x + \frac{3}{128}\sin(8x) - \frac{1}{8}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^6,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 5/16*x + 3/128*sin(8*x) - 1/8*sin(4*x)
```

Fricas [A] time = 0.300411, size = 45, normalized size = 0.98

$$-\frac{1}{96}\left(8\cos(2x)^5 - 26\cos(2x)^3 + 33\cos(2x)\right)\sin(2x) + \frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^6,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 26*cos(2*x)^3 + 33*cos(2*x))*sin(2*x) + 5/1
6*x
```

Sympy [A] time = 0.046303, size = 46, normalized size = 1.

$$\frac{5x}{16} - \frac{\sin^5(2x)\cos(2x)}{12} - \frac{5\sin^3(2x)\cos(2x)}{48} - \frac{5\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)**6,x)
```

```
[Out] 5*x/16 - sin(2*x)**5*cos(2*x)/12 - 5*sin(2*x)**3*cos(2*x)/48 - 5*
sin(2*x)*cos(2*x)/32
```

GIAC/XCAS [A] time = 0.201014, size = 30, normalized size = 0.65

$$\frac{5}{16}x - \frac{1}{384}\sin(12x) + \frac{3}{128}\sin(8x) - \frac{15}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/384*sin(12*x) + 3/128*sin(8*x) - 15/128*sin(4*x)
```

3.357 $\int \cos(x) \log(\sin(x)) \sin^2(x)\, dx$

Optimal. Leaf size=20

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rubi [A] time = 0.0539514, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rubi in Sympy [A] time = 2.33397, size = 17, normalized size = 0.85

$$\frac{\log(\sin(x))\sin^3(x)}{3} - \frac{\sin^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Mathematica [A] time = 0.0187452, size = 15, normalized size = 0.75

$$\frac{1}{9}\sin^3(x)(3\log(\sin(x)) - 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A] time = 0.009, size = 17, normalized size = 0.9

$$-\frac{(\sin(x))^3}{9} + \frac{\ln(\sin(x))(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*ln(sin(x))*sin(x)^2,x)
```

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Maxima [A] time = 1.32819, size = 22, normalized size = 1.1

$$\frac{1}{3}\log(\sin(x))\sin(x)^3 - \frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Fricas [A] time = 0.224055, size = 32, normalized size = 1.6

$$-\frac{1}{3}\left(\cos(x)^2 - 1\right)\log(\sin(x))\sin(x) + \frac{1}{9}\left(\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x
)
```

Sympy [A] time = 10.0226, size = 17, normalized size = 0.85

$$\frac{\log(\sin(x))\sin^3(x)}{3} - \frac{\sin^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

GIAC/XCAS [A] time = 0.206954, size = 22, normalized size = 1.1

$$\frac{1}{3}\ln(\sin(x))\sin(x)^3 - \frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*ln(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

3.358 $\int \frac{e^{-x}}{1+2e^x}\,dx$

Optimal. Leaf size=21

$$-2x - e^{-x} + 2\log(2e^x + 1)$$

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rubi [A] time = 0.044486, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-2x - e^{-x} + 2\log(2e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rubi in Sympy [A] time = 4.1642, size = 20, normalized size = 0.95

$$2\log(2e^x + 1) - 2\log(e^x) - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/exp(x)/(1+2*exp(x)),x)
```

```
[Out] 2*log(2*exp(x) + 1) - 2*log(exp(x)) - exp(-x)
```

Mathematica [A] time = 0.0115379, size = 18, normalized size = 0.86

$$2\log(e^{-x} + 2) - e^{-x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) + 2*Log[2 + E^(-x)]
```

Maple [A] time = 0.017, size = 22, normalized size = 1.1

$$-(e^x)^{-1} - 2\ln(e^x) + 2\ln(1 + 2e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/exp(x)/(1+2*exp(x)),x)
```

```
[Out] -1/exp(x)-2*ln(exp(x))+2*ln(1+2*exp(x))
```

Maxima [A] time = 1.33652, size = 22, normalized size = 1.05

$$-e^{(-x)} + 2\log\left(e^{(-x)} + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)/(2*e^x + 1),x, algorithm="maxima")
```

```
[Out] -e^(-x) + 2*log(e^(-x) + 2)
```

Fricas [A] time = 0.229665, size = 32, normalized size = 1.52

$$-(2\,xe^{x} - 2\,e^{x}\log\left(2\,e^{x} + 1\right) + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)/(2*e^x + 1),x, algorithm="fricas")
```

```
[Out] -(2*x*e^x - 2*e^x*log(2*e^x + 1) + 1)*e^(-x)
```

Sympy [A] time = 0.087837, size = 14, normalized size = 0.67

$$2\log\left(2 + e^{-x}\right) - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/exp(x)/(1+2*exp(x)),x)
```

```
[Out] 2*log(2 + exp(-x)) - exp(-x)
```

GIAC/XCAS [A] time = 0.200094, size = 26, normalized size = 1.24

$$-2\,x - e^{(-x)} + 2\ln\left(2\,e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x)/(2*e^x + 1),x, algorithm="giac")
```

```
[Out] -2*x - e^(-x) + 2*ln(2*e^x + 1)
```

3.359 $\int \sqrt{2+3\cos(x)}\tan(x)\,dx$

Optimal. Leaf size=37

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[
x]]
```

Rubi [A] time = 0.0710129, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[
x]]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\cos(x)+2}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+3*cos(x))**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(3*cos(x) + 2)*tan(x), x)
```

Mathematica [A] time = 0.125367, size = 51, normalized size = 1.38

$$2\sqrt{3\cos(x)+2}\left(\frac{\sqrt{2}\sqrt{\sec(x)}\sinh^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{\sec(x)}\right)}{\sqrt{2\sec(x)+3}}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2 + 3*Cos[x]]*(-1 + (Sqrt[2]*ArcSinh[Sqrt[2/3]*Sqrt[Sec[x]
]]*Sqrt[Sec[x]])/Sqrt[3 + 2*Sec[x]])
```

Maple [A] time = 0.021, size = 31, normalized size = 0.8

$$2\,Artanh\left(1/2\sqrt{2+3\cos(x)}\sqrt{2}\right)\sqrt{2}-2\sqrt{2+3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+3*cos(x))^(1/2)*tan(x),x)
```

```
[Out] 2*arctanh(1/2*(2+3*cos(x))^(1/2)*2^(1/2))*2^(1/2)-2*(2+3*cos(x))^
(1/2)
```

Maxima [A] time = 1.48829, size = 66, normalized size = 1.78

$$-\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\sqrt{3\,\cos(x)+2}\right)}{2\sqrt{2}+2\sqrt{3\,\cos(x)+2}}\right)-2\sqrt{3\,\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x) + 2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-2*(sqrt(2) - sqrt(3*cos(x) + 2))/((2*sqrt(2)) + 2*s
qrt(3*cos(x) + 2))) - 2*sqrt(3*cos(x) + 2)
```

Fricas [A] time = 0.312748, size = 78, normalized size = 2.11

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{9\,\cos(x)^2+4\left(3\sqrt{2}\cos(x)+4\sqrt{2}\right)\sqrt{3\,\cos(x)+2}+48\,\cos(x)+32}{\cos(x)^2}\right)-2\sqrt{3\,\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x) + 2)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(9*cos(x)^2 + 4*(3*sqrt(2)*cos(x) + 4*sqrt(2))*s
qrt(3*cos(x) + 2) + 48*cos(x) + 32)/cos(x)^2) - 2*sqrt(3*cos(x) +
 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3\cos(x)+2}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*cos(x))**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(3*cos(x) + 2)*tan(x), x)
```

GIAC/XCAS [A] time = 0.210765, size = 68, normalized size = 1.84

$$-\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+2\sqrt{3\,\cos(x)+2}\right|}{2\left(\sqrt{2}+\sqrt{3\,\cos(x)+2}\right)}\right)-2\sqrt{3\,\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x) + 2)*tan(x),x, algorithm="giac")
```

```
[Out] -sqrt(2)*ln(1/2*abs(-2*sqrt(2) + 2*sqrt(3*cos(x) + 2))/(sqrt(2) +
 sqrt(3*cos(x) + 2))) - 2*sqrt(3*cos(x) + 2)
```

3.360 $\int \frac{x}{\sqrt{-4x+x^2}}\,dx$

Optimal. Leaf size=28

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rubi [A] time = 0.0211784, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[-4*x + x^2],x]
```

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rubi in Sympy [A] time = 1.49621, size = 24, normalized size = 0.86

$$\sqrt{x^2-4x}+4\operatorname{atanh}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2-4*x)**(1/2),x)
```

```
[Out] sqrt(x**2 - 4*x) + 4*atanh(x/sqrt(x**2 - 4*x))
```

Mathematica [A] time = 0.0237507, size = 44, normalized size = 1.57

$$\frac{(x-4)x+4\sqrt{x-4}\sqrt{x}\log\left(\sqrt{x-4}+\sqrt{x}\right)}{\sqrt{(x-4)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[-4*x + x^2],x]
```

```
[Out] ((-4 + x)*x + 4*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] + Sqrt[x]])
/Sqrt[(-4 + x)*x]
```

Maple [A] time = 0.009, size = 26, normalized size = 0.9

$$\sqrt{x^2-4\,x}+2\,\ln\left(x-2+\sqrt{x^2-4\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2-4*x)^(1/2),x)
```

```
[Out] (x^2-4*x)^(1/2)+2*ln(x-2+(x^2-4*x)^(1/2))
```

Maxima [A] time = 1.33531, size = 39, normalized size = 1.39

$$\sqrt{x^2 - 4\,x} + 2\,\log\left(2\,x + 2\,\sqrt{x^2 - 4\,x} - 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 - 4*x),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

Fricas [A] time = 0.228701, size = 96, normalized size = 3.43

$$-\frac{x^2 + 2\left(x - \sqrt{x^2 - 4\,x} - 2\right)\log\left(-x + \sqrt{x^2 - 4\,x} + 2\right) - \sqrt{x^2 - 4\,x}(x - 1) - 3\,x - 2}{x - \sqrt{x^2 - 4\,x} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 - 4*x),x, algorithm="fricas")
```

```
[Out] -(x^2 + 2*(x - sqrt(x^2 - 4*x) - 2)*log(-x + sqrt(x^2 - 4*x) + 2)
 - sqrt(x^2 - 4*x)*(x - 1) - 3*x - 2)/(x - sqrt(x^2 - 4*x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2-4*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(x - 4)), x)
```

GIAC/XCAS [A] time = 0.203245, size = 38, normalized size = 1.36

$$\sqrt{x^2 - 4\,x} - 2\ln\left(\left|-x + \sqrt{x^2 - 4\,x} + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 - 4*x),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 4*x) - 2*ln(abs(-x + sqrt(x^2 - 4*x) + 2))
```

3.361 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.0151186, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi in Sympy [A] time = 0.719394, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.00326959, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8} + \frac{5}{48}\sin(3x) + \frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^5,x]
```

```
[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80
```

Maple [A] time = 0.041, size = 17, normalized size = 0.9

$$\frac{\sin(x)}{5}\left(\frac{8}{3} + (\cos(x))^4 + \frac{4(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^5,x)
```

```
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

Maxima [A] time = 1.33042, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.233617, size = 24, normalized size = 1.26

$$\frac{1}{15}\left(3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 0.042946, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.198709, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.362 $\int e^{-x} x^4 \, dx$

Optimal. Leaf size=46

$$-e^{-x} x^4 - 4e^{-x} x^3 - 12e^{-x} x^2 - 24e^{-x} x - 24e^{-x}$$

```
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rubi [A] time = 0.0671766, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-e^{-x} x^4 - 4e^{-x} x^3 - 12e^{-x} x^2 - 24e^{-x} x - 24e^{-x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/E^x,x]
```

```
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rubi in Sympy [A] time = 4.02553, size = 36, normalized size = 0.78

$$-x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/exp(x),x)
```

```
[Out] -x**4*exp(-x) - 4*x**3*exp(-x) - 12*x**2*exp(-x) - 24*x*exp(-x) -
 24*exp(-x)
```

Mathematica [A] time = 0.00573922, size = 26, normalized size = 0.57

$$e^{-x} \left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/E^x,x]
```

```
[Out] (-24 - 24*x - 12*x^2 - 4*x^3 - x^4)/E^x
```

Maple [A] time = 0.004, size = 25, normalized size = 0.5

$$-\frac{x^4 + 4x^3 + 12x^2 + 24x + 24}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/exp(x),x)
```

```
[Out] -(x^4+4*x^3+12*x^2+24*x+24)/exp(x)
```

Maxima [A] time = 1.32588, size = 32, normalized size = 0.7

$$-\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(-x),x, algorithm="maxima")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Fricas [A] time = 0.23054, size = 32, normalized size = 0.7

$$-\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(-x),x, algorithm="fricas")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Sympy [A] time = 0.074717, size = 22, normalized size = 0.48

$$\left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right) e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/exp(x),x)
```

```
[Out] (-x**4 - 4*x**3 - 12*x**2 - 24*x - 24)*exp(-x)
```

GIAC/XCAS [A] time = 0.199475, size = 32, normalized size = 0.7

$$-\left(x^4 + 4x^3 + 12x^2 + 24x + 24\right) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4*e^(-x),x, algorithm="giac")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

3.363 $\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rubi [A] time = 0.0192185, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rubi in Sympy [A] time = 1.28651, size = 14, normalized size = 0.78

$$\frac{\operatorname{atanh}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**10-2)**(1/2),x)
```

```
[Out] atanh(x**5/sqrt(x**10 - 2))/5
```

Mathematica [B] time = 0.00651485, size = 42, normalized size = 2.33

$$\frac{1}{10}\log\left(\frac{x^5}{\sqrt{x^{10}-2}}+1\right)-\frac{1}{10}\log\left(1-\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] -Log[1 - x^5/Sqrt[-2 + x^10]]/10 + Log[1 + x^5/Sqrt[-2 + x^10]]/1
0
```

Maple [C] time = 0.068, size = 34, normalized size = 1.9

$$\frac{1}{5}\sqrt{-signum\left(-1+\frac{x^{10}}{2}\right)}\arcsin\left(\frac{x^5\sqrt{2}}{2}\right)\frac{1}{\sqrt{signum\left(-1+\frac{x^{10}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^10-2)^(1/2),x)
```

```
[Out] 1/5/signum(-1+1/2*x^10)^(1/2)*(-signum(-1+1/2*x^10))^(1/2)*arcsin
(1/2*x^5*2^(1/2))
```

Maxima [A] time = 1.39161, size = 45, normalized size = 2.5

$$\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}+1\right)-\frac{1}{10}\log\left(\frac{\sqrt{x^{10}-2}}{x^5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(x^10 - 2),x, algorithm="maxima")
```

```
[Out] 1/10*log(sqrt(x^10 - 2)/x^5 + 1) - 1/10*log(sqrt(x^10 - 2)/x^5 -
1)
```

Fricas [A] time = 0.23782, size = 22, normalized size = 1.22

$$-\frac{1}{5}\log\left(-x^5+\sqrt{x^{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(x^10 - 2),x, algorithm="fricas")
```

```
[Out] -1/5*log(-x^5 + sqrt(x^10 - 2))
```

Sympy [A] time = 1.76178, size = 34, normalized size = 1.89

$$\begin{cases}\frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{for } \frac{|x^{10}|}{2}>1\\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**10-2)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(2)*x**5/2)/5, Abs(x**10)/2 > 1), (-I*asin(s
qrt(2)*x**5/2)/5, True))
```

GIAC/XCAS [A] time = 0.22727, size = 23, normalized size = 1.28

$$-\frac{1}{5}\ln\left(\left|-x^5+\sqrt{x^{10}-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(x^10 - 2),x, algorithm="giac")
```

```
[Out] -1/5*ln(abs(-x^5 + sqrt(x^10 - 2)))
```

3.364 $\int e^x \cos(4+3x)\,dx$

Optimal. Leaf size=27

$$\frac{3}{10}e^x \sin(3x+4) + \frac{1}{10}e^x \cos(3x+4)$$

```
[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rubi [A] time = 0.0182429, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3}{10}e^x \sin(3x+4) + \frac{1}{10}e^x \cos(3x+4)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Cos[4 + 3*x],x]
```

```
[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rubi in Sympy [A] time = 1.41905, size = 24, normalized size = 0.89

$$\frac{3e^x \sin(3x+4)}{10} + \frac{e^x \cos(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*cos(4+3*x),x)
```

```
[Out] 3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

Mathematica [A] time = 0.0261657, size = 22, normalized size = 0.81

$$\frac{1}{10}e^x(3\sin(3x+4) + \cos(3x+4))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Cos[4 + 3*x],x]
```

```
[Out] (E^x*(Cos[4 + 3*x] + 3*Sin[4 + 3*x]))/10
```

Maple [A] time = 0.008, size = 22, normalized size = 0.8

$$\frac{e^x \cos(3x+4)}{10} + \frac{3e^x \sin(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*cos(3*x+4),x)
```

```
[Out] 1/10*exp(x)*cos(3*x+4)+3/10*exp(x)*sin(3*x+4)
```

Maxima [A] time = 1.35708, size = 26, normalized size = 0.96

$$\frac{1}{10}(\cos(3x+4)+3\sin(3x+4))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x + 4)*e^x,x, algorithm="maxima")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Fricas [A] time = 0.229661, size = 28, normalized size = 1.04

$$\frac{1}{10}\cos(3x+4)e^x+\frac{3}{10}e^x\sin(3x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x + 4)*e^x,x, algorithm="fricas")
```

```
[Out] 1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)
```

Sympy [A] time = 0.380596, size = 24, normalized size = 0.89

$$\frac{3e^x\sin(3x+4)}{10}+\frac{e^x\cos(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*cos(4+3*x),x)
```

```
[Out] 3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

GIAC/XCAS [A] time = 0.21197, size = 26, normalized size = 0.96

$$\frac{1}{10}(\cos(3x+4)+3\sin(3x+4))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3*x + 4)*e^x,x, algorithm="giac")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

3.365 $\int e^x \log(1+e^x)\,dx$

Optimal. Leaf size=18

$$(e^x+1)\log(e^x+1)-e^x$$

```
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Rubi [A] time = 0.0210974, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$(e^x+1)\log(e^x+1)-e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Rubi in Sympy [A] time = 4.75873, size = 19, normalized size = 1.06

$$e^x \log(e^x+1)-e^x+\log(e^x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*ln(1+exp(x)),x)
```

```
[Out] exp(x)*log(exp(x) + 1) - exp(x) + log(exp(x) + 1)
```

Mathematica [A] time = 0.00613439, size = 18, normalized size = 1.

$$(e^x+1)\log(e^x+1)-e^x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$(1+e^x)\ln(1+e^x)-1-e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*ln(1+exp(x)),x)
```

```
[Out] (1+exp(x))*ln(1+exp(x))-1-exp(x)
```

Maxima [A] time = 1.32755, size = 22, normalized size = 1.22

$$(e^x+1)\log(e^x+1)-e^x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(e^x + 1),x, algorithm="maxima")
```

```
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

Fricas [A] time = 0.222511, size = 20, normalized size = 1.11

$$(e^x + 1)\log(e^x + 1) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(e^x + 1),x, algorithm="fricas")
```

```
[Out] (e^x + 1)*log(e^x + 1) - e^x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*ln(1+exp(x)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.205406, size = 22, normalized size = 1.22

$$(e^x + 1)\ln(e^x + 1) - e^x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*log(e^x + 1),x, algorithm="giac")
```

```
[Out] (e^x + 1)*ln(e^x + 1) - e^x - 1
```

3.366 $\int x^2 \tan^{-1}(x)\,dx$

Optimal. Leaf size=27

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi [A] time = 0.0320069, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi in Sympy [A] time = 2.72001, size = 20, normalized size = 0.74

$$\frac{x^3 \operatorname{atan}(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Mathematica [A] time = 0.00299632, size = 27, normalized size = 1.

$$\frac{1}{3}x^3 \tan^{-1}(x) - \frac{x^2}{6} + \frac{1}{6}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Maple [A] time = 0., size = 22, normalized size = 0.8

$$-\frac{x^2}{6} + \frac{x^3 \arctan(x)}{3} + \frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arctan(x),x)
```

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Maxima [A] time = 1.33688, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Fricas [A] time = 0.230583, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.476771, size = 20, normalized size = 0.74

$$\frac{x^3\operatorname{atan}(x)}{3} - \frac{x^2}{6} + \frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

GIAC/XCAS [A] time = 0.205857, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x) - \frac{1}{6}x^2 + \frac{1}{6}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*ln(x^2 + 1)
```

3.367 $\int \sqrt{-1+e^{2x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rubi [A] time = 0.0298099, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + E^(2*x)],x]
```

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rubi in Sympy [A] time = 2.03819, size = 20, normalized size = 0.77

$$\sqrt{e^{2x}-1}-\operatorname{atan}\left(\sqrt{e^{2x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+exp(2*x))**(1/2),x)
```

```
[Out] sqrt(exp(2*x) - 1) - atan(sqrt(exp(2*x) - 1))
```

Mathematica [A] time = 0.0127561, size = 26, normalized size = 1.

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + E^(2*x)],x]
```

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Maple [A] time = 0.009, size = 21, normalized size = 0.8

$$-\arctan\left(\sqrt{-1+e^{2x}}\right)+\sqrt{-1+e^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+exp(2*x))^(1/2),x)
```

```
[Out] -arctan((-1+exp(2*x))^(1/2))+(-1+exp(2*x))^(1/2)
```

Maxima [A] time = 1.6402, size = 27, normalized size = 1.04

$$\sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*x) - 1),x, algorithm="maxima")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Fricas [A] time = 0.23769, size = 27, normalized size = 1.04

$$\sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*x) - 1),x, algorithm="fricas")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Sympy [A] time = 1.33753, size = 19, normalized size = 0.73

$$\begin{cases}\sqrt{e^{2x}-1}-\operatorname{acos}\left(e^{-x}\right) & \text{for } e^{x}<0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((sqrt(exp(2*x) - 1) - acos(exp(-x)), exp(x) < 0))
```

GIAC/XCAS [A] time = 0.204701, size = 27, normalized size = 1.04

$$\sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(e^(2*x) - 1),x, algorithm="giac")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

3.368 $\int e^{\sin(x)} \sin(2x)\, dx$

Optimal. Leaf size=15

$$2e^{\sin(x)} \sin(x) - 2e^{\sin(x)}$$

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rubi [A] time = 0.0311459, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$2e^{\sin(x)} \sin(x) - 2e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[E^Sin[x]*Sin[2*x],x]
```

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(sin(x))*sin(2*x),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.0170362, size = 11, normalized size = 0.73

$$e^{\sin(x)}(2 \sin(x) - 2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^Sin[x]*Sin[2*x],x]
```

```
[Out] E^Sin[x]*(-2 + 2*Sin[x])
```

Maple [A] time = 0.013, size = 14, normalized size = 0.9

$$-2\, \mathrm{e}^{\sin(x)} + 2\, \mathrm{e}^{\sin(x)} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(sin(x))*sin(2*x),x)
```

```
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

Maxima [A] time = 1.42882, size = 12, normalized size = 0.8

$$2\,(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Fricas [A] time = 0.250407, size = 12, normalized size = 0.8

$$2\,(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Sympy [A] time = 3.6733, size = 15, normalized size = 1.

$$2e^{\sin(x)}\sin(x) - 2e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(sin(x))*sin(2*x),x)
```

```
[Out] 2*exp(sin(x))*sin(x) - 2*exp(sin(x))
```

GIAC/XCAS [A] time = 0.217663, size = 721, normalized size = 48.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] 2*(e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6*tan(x)^2 - 2*
e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^5*tan(x)^2 - e^(2*
tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 + 8*e^(2*tan(1/2*x)/(
tan(1/2*x)^2 + 1))*tan(1/2*x)^5*tan(x) - 5*e^(2*tan(1/2*x)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 + 2*e^(2*tan(1/2*x)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^5 - 16*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*
tan(1/2*x)^4*tan(x) + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(
1/2*x)^3*tan(x)^2 + 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2
*x)^4 - 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)
^2 - 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^3 + 16*e^(
2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x) - 2*e^(2*tan
(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)*tan(x)^2 + 5*e^(2*tan(1/2*
x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 8*e^(2*tan(1/2*x)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)*tan(x) + e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
)*tan(x)^2 + 2*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x) - e
^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)))/(tan(1/2*x)^6*tan(x)^2 + tan(
1/2*x)^6 + 3*tan(1/2*x)^4*tan(x)^2 + 3*tan(1/2*x)^4 + 3*tan(1/2*x
)^2*tan(x)^2 + 3*tan(1/2*x)^2 + tan(x)^2 + 1)
```

3.369 $\int x^2\sqrt{5-x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{5}{8}\sqrt{5-x^2}x+\frac{1}{4}\sqrt{5-x^2}x^3+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqr
t[5]])/8
```

Rubi [A] time = 0.0337825, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{5}{8}\sqrt{5-x^2}x+\frac{1}{4}\sqrt{5-x^2}x^3+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[5 - x^2],x]
```

```
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqr
t[5]])/8
```

Rubi in Sympy [A] time = 2.79844, size = 39, normalized size = 0.83

$$\frac{x^3\sqrt{-x^2+5}}{4}-\frac{5x\sqrt{-x^2+5}}{8}+\frac{25\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(-x**2+5)**(1/2),x)
```

```
[Out] x**3*sqrt(-x**2 + 5)/4 - 5*x*sqrt(-x**2 + 5)/8 + 25*asin(sqrt(5)*
x/5)/8
```

Mathematica [A] time = 0.0303433, size = 35, normalized size = 0.74

$$\frac{1}{8}\left(x\sqrt{5-x^2}\left(2x^2-5\right)+25\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[5 - x^2],x]
```

```
[Out] (x*Sqrt[5 - x^2]*(-5 + 2*x^2) + 25*ArcSin[x/Sqrt[5]])/8
```

Maple [A] time = 0.01, size = 35, normalized size = 0.7

$$-\frac{x}{4}\left(-x^2+5\right)^{\frac{3}{2}}+\frac{5x}{8}\sqrt{-x^2+5}+\frac{25}{8}\arcsin\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(-x^2+5)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2+5)^(3/2)+5/8*x*(-x^2+5)^(1/2)+25/8*arcsin(1/5*x*5^(1
/2))
```

Maxima [A] time = 1.51772, size = 46, normalized size = 0.98

$$-\frac{1}{4}\left(-x^2+5\right)^{\frac{3}{2}}x+\frac{5}{8}\sqrt{-x^2+5}x+\frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 5)^(3/2)*x + 5/8*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*
sqrt(5)*x)
```

Fricas [A] time = 0.233044, size = 50, normalized size = 1.06

$$\frac{1}{8}\left(2\,x^3-5\,x\right)\sqrt{-x^2+5}-\frac{25}{8}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 5) - 25/8*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [A] time = 4.47294, size = 122, normalized size = 2.6

$$\begin{cases}\frac{ix^5}{4\sqrt{x^2-5}}-\frac{15ix^3}{8\sqrt{x^2-5}}+\frac{25ix}{8\sqrt{x^2-5}}-\frac{25i\,\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{for } \frac{\left|x^2\right|}{5}>1\\ -\frac{x^5}{4\sqrt{-x^2+5}}+\frac{15x^3}{8\sqrt{-x^2+5}}-\frac{25x}{8\sqrt{-x^2+5}}+\frac{25\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 5)) - 15*I*x**3/(8*sqrt(x**2 - 5
)) + 25*I*x/(8*sqrt(x**2 - 5)) - 25*I*acosh(sqrt(5)*x/5)/8, Abs(x
**2)/5 > 1), (-x**5/(4*sqrt(-x**2 + 5)) + 15*x**3/(8*sqrt(-x**2 +
 5)) - 25*x/(8*sqrt(-x**2 + 5)) + 25*asin(sqrt(5)*x/5)/8, True))
```

GIAC/XCAS [A] time = 0.208678, size = 39, normalized size = 0.83

$$\frac{1}{8}\left(2\,x^2-5\right)\sqrt{-x^2+5}x+\frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 5)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2 - 5)*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

3.370 $\int x^2 \left(1+x^3\right)^4 \, dx$

Optimal. Leaf size=11

$$\frac{1}{15}\left(x^3+1\right)^5$$

```
[Out] (1 + x^3)^5/15
```

Rubi [A] time = 0.00669756, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{1}{15}\left(x^3+1\right)^5$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(1 + x^3)^4,x]
```

```
[Out] (1 + x^3)^5/15
```

Rubi in Sympy [A] time = 0.794474, size = 7, normalized size = 0.64

$$\frac{\left(x^3+1\right)^5}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(x**3+1)**4,x)
```

```
[Out] (x**3 + 1)**5/15
```

Mathematica [B] time = 0.00202133, size = 36, normalized size = 3.27

$$\frac{x^{15}}{15}+\frac{x^{12}}{3}+\frac{2x^9}{3}+\frac{2x^6}{3}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(1 + x^3)^4,x]
```

```
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

Maple [B] time = 0.001, size = 27, normalized size = 2.5

$$\frac{x^{15}}{15}+\frac{x^{12}}{3}+\frac{2\,x^9}{3}+\frac{2\,x^6}{3}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(x^3+1)^4,x)
```

```
[Out] 1/15*x^15+1/3*x^12+2/3*x^9+2/3*x^6+1/3*x^3
```

Maxima [A] time = 1.33682, size = 12, normalized size = 1.09

$$\frac{1}{15}\left(x^3+1\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)^4*x^2,x, algorithm="maxima")
```

```
[Out] 1/15*(x^3 + 1)^5
```

Fricas [A] time = 0.189837, size = 1, normalized size = 0.09

$$\frac{1}{15}x^{15}+\frac{1}{3}x^{12}+\frac{2}{3}x^9+\frac{2}{3}x^6+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)^4*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15 + 1/3*x^12 + 2/3*x^9 + 2/3*x^6 + 1/3*x^3
```

Sympy [A] time = 0.031505, size = 27, normalized size = 2.45

$$\frac{x^{15}}{15}+\frac{x^{12}}{3}+\frac{2x^9}{3}+\frac{2x^6}{3}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(x**3+1)**4,x)
```

```
[Out] x**15/15 + x**12/3 + 2*x**9/3 + 2*x**6/3 + x**3/3
```

GIAC/XCAS [A] time = 0.203509, size = 12, normalized size = 1.09

$$\frac{1}{15}\left(x^3+1\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 1)^4*x^2,x, algorithm="giac")
```

```
[Out] 1/15*(x^3 + 1)^5
```

3.371 $\int \cos^3(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^4(x)}{4} - \frac{\sin^6(x)}{6}$$

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

Rubi [A] time = 0.0351594, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sin^4(x)}{4} - \frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

Rubi in Sympy [A] time = 2.42613, size = 12, normalized size = 0.71

$$-\frac{\sin^6(x)}{6} + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3*sin(x)**3,x)
```

```
[Out] -sin(x)**6/6 + sin(x)**4/4
```

Mathematica [A] time = 0.00985004, size = 17, normalized size = 1.

$$\frac{1}{192}\cos(6x) - \frac{3}{64}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] (-3*Cos[2*x])/64 + Cos[6*x]/192
```

Maple [A] time = 0.01, size = 18, normalized size = 1.1

$$-\frac{(\cos(x))^4(\sin(x))^2}{6} - \frac{(\cos(x))^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3*sin(x)^3,x)
```

```
[Out] -1/6*cos(x)^4*sin(x)^2-1/12*cos(x)^4
```

Maxima [A] time = 1.3473, size = 18, normalized size = 1.06

$$-\frac{1}{6}\sin(x)^6+\frac{1}{4}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(x)^6 + 1/4*sin(x)^4
```

Fricas [A] time = 0.217586, size = 18, normalized size = 1.06

$$\frac{1}{6}\cos(x)^6-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

Sympy [A] time = 0.044095, size = 12, normalized size = 0.71

$$-\frac{\sin^6(x)}{6}+\frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3*sin(x)**3,x)
```

```
[Out] -sin(x)**6/6 + sin(x)**4/4
```

GIAC/XCAS [A] time = 0.20138, size = 18, normalized size = 1.06

$$\frac{1}{6}\cos(x)^6-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

3.372 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0353396, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(x)}{\cos^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] Integral(tan(x)**2/cos(x)**4, x)
```

Mathematica [A] time = 0.011991, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{1}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.002, size = 22, normalized size = 1.3

$$\frac{(\sin(x))^3}{5\,(\cos(x))^5}+\frac{2\,(\sin(x))^3}{15\,(\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 1/5*sin(x)^3/cos(x)^5+2/15*sin(x)^3/cos(x)^3
```

Maxima [A] time = 1.3918, size = 18, normalized size = 1.06

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Fricas [A] time = 0.209219, size = 27, normalized size = 1.59

$$-\frac{\left(2\cos(x)^4 + \cos(x)^2 - 3\right)\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [A] time = 0.052494, size = 29, normalized size = 1.71

$$-\frac{2\sin(x)}{15\cos(x)} - \frac{\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)*
*5)
```

GIAC/XCAS [A] time = 0.201555, size = 18, normalized size = 1.06

$$\frac{1}{5}\tan(x)^5 + \frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

3.373 $\int x\sqrt{1+2x}\,dx$

Optimal. Leaf size=27

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

```
[Out] -(1 + 2*x)^(3/2)/6 + (1 + 2*x)^(5/2)/10
```

Rubi [A] time = 0.0144904, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[1 + 2*x],x]
```

```
[Out] -(1 + 2*x)^(3/2)/6 + (1 + 2*x)^(5/2)/10
```

Rubi in Sympy [A] time = 1.50529, size = 19, normalized size = 0.7

$$\frac{(2x+1)^{\frac{5}{2}}}{10}-\frac{(2x+1)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+2*x)**(1/2),x)
```

```
[Out] (2*x + 1)**(5/2)/10 - (2*x + 1)**(3/2)/6
```

Mathematica [A] time = 0.00623519, size = 18, normalized size = 0.67

$$\frac{1}{15}(2x+1)^{3/2}(3x-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[1 + 2*x],x]
```

```
[Out] ((1 + 2*x)^(3/2)*(-1 + 3*x))/15
```

Maple [A] time = 0.003, size = 15, normalized size = 0.6

$$\frac{3x-1}{15}(1+2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+2*x)^(1/2),x)
```

```
[Out] 1/15*(1+2*x)^(3/2)*(3*x-1)
```

Maxima [A] time = 1.49534, size = 26, normalized size = 0.96

$$\frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

Fricas [A] time = 0.212294, size = 23, normalized size = 0.85

$$\frac{1}{15}\left(6x^2+x-1\right)\sqrt{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1)*x,x, algorithm="fricas")
```

```
[Out] 1/15*(6*x^2 + x - 1)*sqrt(2*x + 1)
```

Sympy [A] time = 1.54197, size = 36, normalized size = 1.33

$$\frac{2x^2\sqrt{2x+1}}{5} + \frac{x\sqrt{2x+1}}{15} - \frac{\sqrt{2x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+2*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(2*x + 1)/5 + x*sqrt(2*x + 1)/15 - sqrt(2*x + 1)/15
```

GIAC/XCAS [A] time = 0.203585, size = 26, normalized size = 0.96

$$\frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1)*x,x, algorithm="giac")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

3.374 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8} - \frac{1}{4}\sin^3(x)\cos(x) - \frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0178135, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{3x}{8} - \frac{1}{4}\sin^3(x)\cos(x) - \frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi in Sympy [A] time = 0.567128, size = 24, normalized size = 1.

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00307024, size = 22, normalized size = 0.92

$$\frac{3x}{8} - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^3 + \frac{3\sin(x)}{2}\right) + \frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 1.37581, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.256808, size = 26, normalized size = 1.08

$$\frac{1}{8}\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.040371, size = 24, normalized size = 1.

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

GIAC/XCAS [A] time = 0.200716, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.375 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi [A] time = 0.0105905, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{\tan^2(x)}{2} + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi in Sympy [A] time = 0.493162, size = 10, normalized size = 0.83

$$\log(\cos(x)) + \frac{\tan^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + tan(x)**2/2
```

Mathematica [A] time = 0.00503877, size = 12, normalized size = 1.

$$\frac{\sec^2(x)}{2} + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

Maple [A] time = 0., size = 17, normalized size = 1.4

$$\frac{(\tan(x))^2}{2} - \frac{\ln\left(1 + (\tan(x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^3,x)
```

```
[Out] 1/2*tan(x)^2-1/2*ln(1+tan(x)^2)
```

Maxima [A] time = 1.35687, size = 27, normalized size = 2.25

$$-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 0.238815, size = 24, normalized size = 2.

$$\frac{1}{2}\tan(x)^2+\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.087913, size = 12, normalized size = 1.

$$\log(\cos(x))+\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

GIAC/XCAS [A] time = 0.201146, size = 22, normalized size = 1.83

$$\frac{1}{2}\tan(x)^2-\frac{1}{2}\ln\left(\tan(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*tan(x)^2 - 1/2*ln(tan(x)^2 + 1)
```

3.376 $\int x^5\sqrt{1+x^2}\,dx$

Optimal. Leaf size=40

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rubi [A] time = 0.0336456, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rubi in Sympy [A] time = 2.16245, size = 31, normalized size = 0.78

$$\frac{\left(x^2+1\right)^{\frac{7}{2}}}{7}-\frac{2\left(x^2+1\right)^{\frac{5}{2}}}{5}+\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(x**2+1)**(1/2),x)
```

```
[Out] (x**2 + 1)**(7/2)/7 - 2*(x**2 + 1)**(5/2)/5 + (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.0108458, size = 25, normalized size = 0.62

$$\frac{1}{105}\left(x^2+1\right)^{3/2}\left(15x^4-12x^2+8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(8 - 12*x^2 + 15*x^4))/105
```

Maple [A] time = 0.007, size = 22, normalized size = 0.6

$$\frac{15x^4-12x^2+8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(x^2+1)^(1/2),x)
```

```
[Out] 1/105*(x^2+1)^(3/2)*(15*x^4-12*x^2+8)
```

Maxima [A] time = 1.4983, size = 46, normalized size = 1.15

$$\frac{1}{7}\left(x^2+1\right)^{\frac{3}{2}}x^4-\frac{4}{35}\left(x^2+1\right)^{\frac{3}{2}}x^2+\frac{8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^5,x, algorithm="maxima")
```

```
[Out] 1/7*(x^2 + 1)^(3/2)*x^4 - 4/35*(x^2 + 1)^(3/2)*x^2 + 8/105*(x^2 +
 1)^(3/2)
```

Fricas [A] time = 0.205229, size = 174, normalized size = 4.35

$$-\frac{960\,x^{14}+2352\,x^{12}+1736\,x^{10}+623\,x^8+826\,x^6+735\,x^4+196\,x^2-\left(960\,x^{13}+1872\,x^{11}+920\,x^9+337\,x^7+693\,x^5+420\,x\right.}{105\left(64\,x^7+112\,x^5+56\,x^3-\left(64\,x^6+80\,x^4+24\,x^2+1\right)\sqrt{x^2+1}+7\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(960*x^14 + 2352*x^12 + 1736*x^10 + 623*x^8 + 826*x^6 + 73
5*x^4 + 196*x^2 - (960*x^13 + 1872*x^11 + 920*x^9 + 337*x^7 + 693
*x^5 + 420*x^3 + 56*x)*sqrt(x^2 + 1) + 8)/(64*x^7 + 112*x^5 + 56*
x^3 - (64*x^6 + 80*x^4 + 24*x^2 + 1)*sqrt(x^2 + 1) + 7*x)
```

Sympy [A] time = 3.17738, size = 53, normalized size = 1.32

$$\frac{x^6\sqrt{x^2+1}}{7}+\frac{x^4\sqrt{x^2+1}}{35}-\frac{4x^2\sqrt{x^2+1}}{105}+\frac{8\sqrt{x^2+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(x**2+1)**(1/2),x)
```

```
[Out] x**6*sqrt(x**2 + 1)/7 + x**4*sqrt(x**2 + 1)/35 - 4*x**2*sqrt(x**2
 + 1)/105 + 8*sqrt(x**2 + 1)/105
```

GIAC/XCAS [A] time = 0.199367, size = 38, normalized size = 0.95

$$\frac{1}{7}\left(x^2+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^2+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x^5,x, algorithm="giac")
```

```
[Out] 1/7*(x^2 + 1)^(7/2) - 2/5*(x^2 + 1)^(5/2) + 1/3*(x^2 + 1)^(3/2)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,``^``) then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,``+``) or type(expn,``*``) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```