Computer algebra independent integration tests
`0_Independent_test_suites/Moses_Problems`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

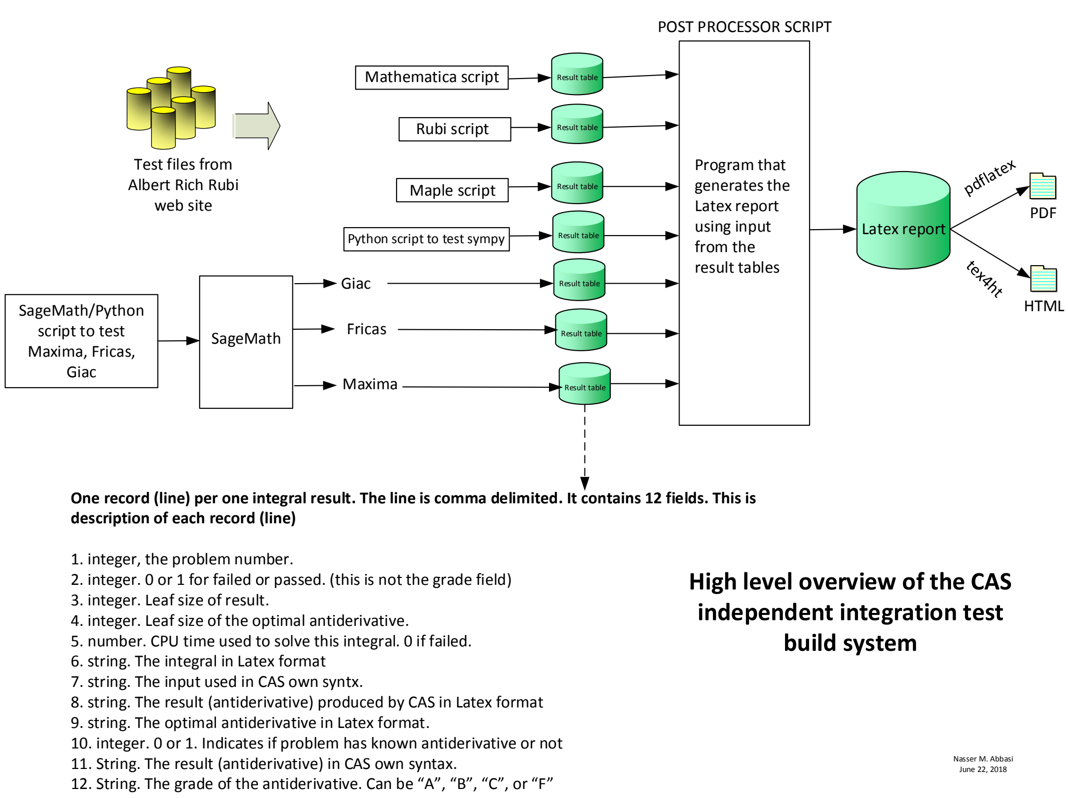
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (113)	% 0. (0)
Rubi in Sympy	% 75.22 (85)	% 24.78 (28)
Mathematica	% 100. (113)	% 0. (0)
Maple	% 100. (113)	% 0. (0)
Maxima	% 94.69 (107)	% 5.31 (6)
Fricas	% 96.46 (109)	% 3.54 (4)
Sympy	% 89.38 (101)	% 10.62 (12)
Giac	% 95.58 (108)	% 4.42 (5)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

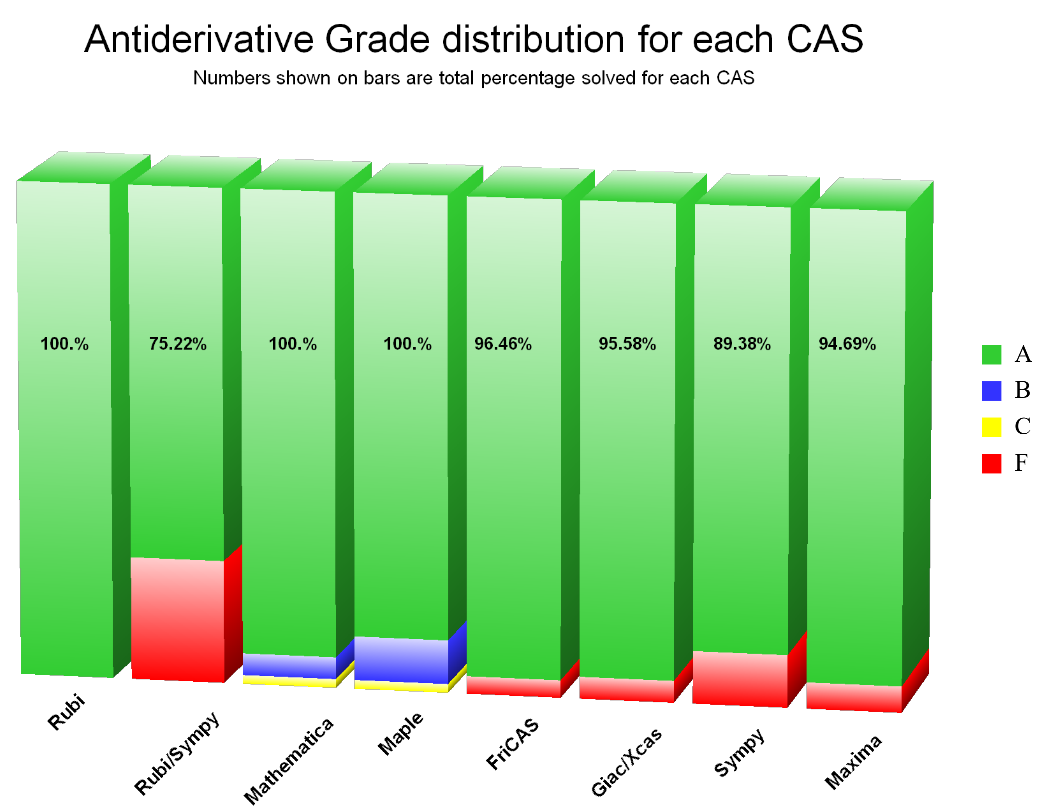
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

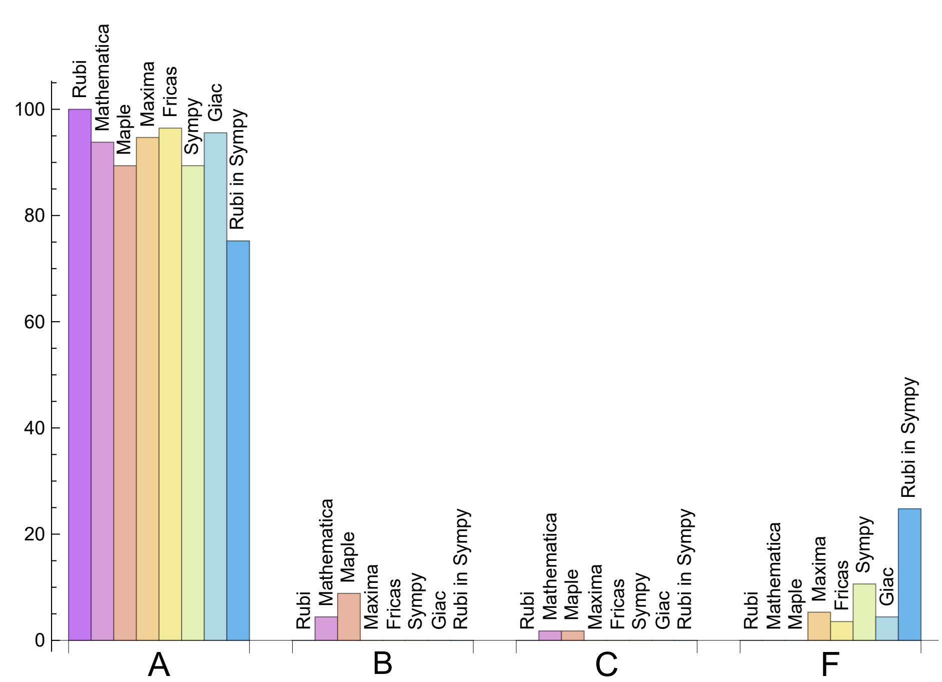
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	75.22	0.	0.	24.78
Mathematica	93.81	4.42	1.77	0.
Maple	89.38	8.85	1.77	0.
Maxima	94.69	0.	0.	5.31
Fricas	96.46	0.	0.	3.54
Sympy	89.38	0.	0.	10.62
Giac	95.58	0.	0.	4.42

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	20.14	1.01	16.	1.
Rubi in Sympy	2.65	16.76	0.87	10.	0.79
Mathematica	0.02	24.1	1.11	16.	1.
Maple	0.01	25.83	1.28	14.	0.92
Maxima	1.51	28.51	1.54	18.	1.17
Fricas	0.22	41.18	1.81	20.	1.25
Sympy	0.86	29.66	1.61	15.	0.82
Giac	0.21	24.74	1.4	18.	1.13

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {6, 14, 16, 35, 52, 55, 58, 59, 60, 64, 71, 77, 79, 87, 95, 96, 97, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113}

Not solved by Mathematica {}

Not solved by Maple {}

Not solved by Maxima {27, 32, 40, 42, 67, 69}

Not solved by Fricas {10, 11, 32, 57}

Not solved by Sympy {10, 32, 35, 36, 40, 42, 57, 69, 70, 87, 90, 98}

Not solved by Giac {32, 40, 42, 45, 69}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	65	19	46	14
normalized size	1	1.	1.5	1.17	1.83	5.42	1.58	3.83	1.17
time (sec)	N/A	0.017	0.005	0.	1.49	0.221	0.052	0.21	0.502

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	20	26	14	20	12
normalized size	1	1.	1.	0.92	1.54	2.	1.08	1.54	0.92
time (sec)	N/A	0.017	0.006	0.007	1.484	0.21	0.117	0.202	2.248

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	15	19	15	15	15
normalized size	1	1.	0.74	0.58	0.79	1.	0.79	0.79	0.79
time (sec)	N/A	0.008	0.004	0.003	1.331	0.211	0.272	0.198	1.025

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	3	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.005	0.002	0.001	1.372	0.208	0.03	0.2	0.023

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	8	5	8	5
normalized size	1	1.	1.	0.78	0.89	0.89	0.56	0.89	0.56
time (sec)	N/A	0.01	0.002	0.	1.37	0.205	0.06	0.2	1.036

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.017	0.003	0.009	1.326	0.224	0.051	0.198	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	93	22	12	8
normalized size	1	1.	1.	0.77	0.92	7.15	1.69	0.92	0.62
time (sec)	N/A	0.005	0.003	0.003	1.34	0.234	0.216	0.204	0.763

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	18	15	15	15
normalized size	1	1.	0.74	0.74	0.79	0.95	0.79	0.79	0.79
time (sec)	N/A	0.013	0.016	0.003	1.353	0.226	0.358	0.2	1.193

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	8	8	8
normalized size	1	1.	1.	0.88	1.	2.38	1.	1.	1.
time (sec)	N/A	0.021	0.004	0.018	1.357	0.22	0.051	0.201	1.111

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	20	0	0	4	3
normalized size	1	1.	1.	1.	5.	0.	0.	1.	0.75
time (sec)	N/A	0.019	0.009	0.004	1.52	0.	0.	0.202	1.36

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	0	2	3	2
normalized size	1	1.	1.	1.5	9.	0.	1.	1.5	1.
time (sec)	N/A	0.018	0.008	0.001	1.487	0.	0.829	0.199	0.697

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	9	5	9	5
normalized size	1	1.	1.	1.	1.12	1.12	0.62	1.12	0.62
time (sec)	N/A	0.008	0.005	0.001	1.355	0.236	0.059	0.198	0.476

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	8	5	8	5
normalized size	1	1.	1.	1.	1.14	1.14	0.71	1.14	0.71
time (sec)	N/A	0.032	0.004	0.003	1.349	0.202	0.069	0.209	1.617

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	26	26	20	26	0
normalized size	1	1.	0.93	0.79	0.93	0.93	0.71	0.93	0.
time (sec)	N/A	0.037	0.009	0.003	1.377	0.203	0.094	0.214	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	15	22	15
normalized size	1	1.	1.	0.77	1.	1.	0.68	1.	0.68
time (sec)	N/A	0.011	0.004	0.004	1.392	0.214	0.083	0.211	0.637

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.
time (sec)	N/A	0.012	0.002	0.	1.344	0.226	0.034	0.215	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	8	5	8	5
normalized size	1	1.	1.	0.78	0.89	0.89	0.56	0.89	0.56
time (sec)	N/A	0.012	0.001	0.	1.352	0.212	0.059	0.224	1.061

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	93	22	12	8
normalized size	1	1.	1.	0.77	0.92	7.15	1.69	0.92	0.62
time (sec)	N/A	0.005	0.001	0.	1.333	0.244	0.23	0.222	0.759

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	7	5	7	5
normalized size	1	1.	1.	1.	1.17	1.17	0.83	1.17	0.83
time (sec)	N/A	0.026	0.002	0.003	1.328	0.198	0.058	0.212	2.742

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	7	7	7	7
normalized size	1	1.	1.	0.67	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.003	0.001	0.003	1.335	0.203	0.028	0.201	0.454

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	7	11	7
normalized size	1	1.	1.	0.9	1.1	1.1	0.7	1.1	0.7
time (sec)	N/A	0.007	0.003	0.007	1.366	0.215	0.122	0.198	0.515

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	9	7	9	7
normalized size	1	1.	1.	1.	1.12	1.12	0.88	1.12	0.88
time (sec)	N/A	0.007	0.001	0.003	1.347	0.234	0.072	0.2	0.839

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	9	9	8	9	8
normalized size	1	1.	1.9	0.8	0.9	0.9	0.8	0.9	0.8
time (sec)	N/A	0.039	0.017	0.007	1.372	0.231	2.943	0.198	6.09

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	20	34	20	19
normalized size	1	1.	0.7	0.57	0.87	0.87	1.48	0.87	0.83
time (sec)	N/A	0.012	0.005	0.004	1.34	0.205	1.449	0.198	0.968

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	23	17	26	10
normalized size	1	1.	1.92	0.77	1.77	1.77	1.31	2.	0.77
time (sec)	N/A	0.008	0.005	0.001	1.528	0.205	0.162	0.199	0.62

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	18	18	15	18	17
normalized size	1	1.	1.	0.78	1.	1.	0.83	1.	0.94
time (sec)	N/A	0.036	0.013	0.005	1.55	0.207	0.113	0.207	3.154

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	0	1	22	26	27
normalized size	1	1.	1.	0.65	0.	0.03	0.71	0.84	0.87
time (sec)	N/A	0.059	0.012	0.01	0.	0.215	0.171	0.203	4.474

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	15	7	11	7
normalized size	1	1.	1.	1.12	1.38	1.88	0.88	1.38	0.88
time (sec)	N/A	0.049	0.002	0.003	1.355	0.222	0.08	0.2	2.739

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	16	8	16	8
normalized size	1	1.	1.	1.08	1.33	1.33	0.67	1.33	0.67
time (sec)	N/A	0.011	0.005	0.02	1.345	0.222	0.074	0.199	0.487

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	22	15
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.1	0.75
time (sec)	N/A	0.027	0.005	0.	1.335	0.229	0.776	0.198	1.413

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	54	63	46	54	42
normalized size	1	1.	5.31	0.84	1.1	1.29	0.94	1.1	0.86
time (sec)	N/A	0.076	0.198	0.001	1.535	0.21	0.258	0.208	3.955

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	0	0	87
normalized size	1	1.	1.23	0.36	0.	0.	0.	0.	0.76
time (sec)	N/A	0.099	0.457	0.149	0.	0.	0.	0.	5.274

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	20	22	48
normalized size	1	1.	1.	0.77	1.	1.	0.91	1.	2.18
time (sec)	N/A	0.018	0.01	0.012	1.339	0.24	0.405	0.2	2.596

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	20	34	20	19
normalized size	1	1.	0.7	0.57	0.87	0.87	1.48	0.87	0.83
time (sec)	N/A	0.011	0.005	0.	1.34	0.209	1.445	0.199	0.981

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	32	0	32	0
normalized size	1	1.	1.	2.88	1.	1.	0.	1.	0.
time (sec)	N/A	0.027	0.013	0.001	1.352	0.207	0.	0.206	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	66	75	108	82	0	82	48
normalized size	1	1.	1.5	1.7	2.45	1.86	0.	1.86	1.09
time (sec)	N/A	0.033	0.063	0.014	1.519	0.21	0.	0.215	1.928

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	182	105	39	26
normalized size	1	1.	0.74	0.86	1.69	5.2	3.	1.11	0.74
time (sec)	N/A	0.028	0.045	0.009	1.5	0.392	3.934	0.207	2.5

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	41	70	153	266	190	111	66
normalized size	1	1.	0.55	0.93	2.04	3.55	2.53	1.48	0.88
time (sec)	N/A	0.037	0.04	0.006	1.348	0.214	39.165	0.29	2.669

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	182	105	39	26
normalized size	1	1.	0.74	0.86	1.69	5.2	3.	1.11	0.74
time (sec)	N/A	0.028	0.011	0.	1.493	0.214	4.139	0.207	2.478

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	134	262	0	174	0	0	46
normalized size	1	1.	2.63	5.14	0.	3.41	0.	0.	0.9
time (sec)	N/A	0.077	0.032	0.035	0.	0.25	0.	0.	12.011

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.01	0.003	0.	1.339	0.221	0.036	0.236	0.488

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	C	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	0	378	0	0	54
normalized size	1	1.16	2.02	3.04	0.	7.71	0.	0.	1.1
time (sec)	N/A	0.159	0.132	0.156	0.	0.33	0.	0.	15.247

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	12	3	41	7
normalized size	1	1.	0.67	0.56	1.33	1.33	0.33	4.56	0.78
time (sec)	N/A	0.015	0.006	0.	1.425	0.211	0.212	0.206	0.483

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	8	8	5	8	7
normalized size	1	1.	0.64	0.64	0.73	0.73	0.45	0.73	0.64
time (sec)	N/A	0.011	0.002	0.	1.4	0.228	0.058	0.226	1.018

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	11	5	0	5
normalized size	1	1.	1.	1.	1.22	1.22	0.56	0.	0.56
time (sec)	N/A	0.038	0.005	0.005	1.349	0.202	0.076	0.	2.003

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	8	5	8	5
normalized size	1	1.	1.	1.	1.14	1.14	0.71	1.14	0.71
time (sec)	N/A	0.047	0.004	0.003	1.358	0.199	0.068	0.209	2.778

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	12	9	8	12	8
normalized size	1	1.	1.	0.73	1.09	0.82	0.73	1.09	0.73
time (sec)	N/A	0.005	0.002	0.002	1.36	0.219	0.301	0.215	0.495

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	3	3	2	3	2
normalized size	1	1.	1.	4.	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.014	0.002	0.003	1.421	0.212	1.271	0.226	1.32

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	46	55	41	47	37
normalized size	1	1.	0.98	0.85	1.12	1.34	1.	1.15	0.9
time (sec)	N/A	0.044	0.013	0.	1.486	0.208	0.173	0.224	3.185

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	88	101	83	90	68
normalized size	1	1.55	1.6	1.4	1.87	2.15	1.77	1.91	1.45
time (sec)	N/A	0.189	0.021	0.	1.527	0.203	0.358	0.225	16.996

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	44	53	36	70	55	12
normalized size	1	1.	1.	2.1	2.52	1.71	3.33	2.62	0.57
time (sec)	N/A	0.02	0.006	0.009	1.349	0.198	0.4	0.209	1.311

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.008	0.001	0.002	12.108	0.202	0.07	0.233	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	45	32	32	51	27
normalized size	1	1.	0.72	0.85	1.12	0.8	0.8	1.27	0.68
time (sec)	N/A	0.049	0.02	0.	1.498	0.221	0.461	0.212	3.12

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	5	9	15
normalized size	1	1.	1.	1.14	1.29	1.29	0.71	1.29	2.14
time (sec)	N/A	0.005	0.001	0.001	1.327	0.208	0.062	0.227	0.561

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	5	11	0
normalized size	1	1.	1.	1.12	1.38	1.38	0.62	1.38	0.
time (sec)	N/A	0.063	0.005	0.051	1.451	0.217	0.077	0.227	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	15	4	3
normalized size	1	1.	1.	1.33	1.33	1.33	5.	1.33	1.
time (sec)	N/A	0.028	0.006	0.005	1.509	0.221	0.142	0.232	3.264

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	4	0	0	4	2
normalized size	1	1.	1.	4.5	2.	0.	0.	2.	1.
time (sec)	N/A	0.004	0.003	0.	1.415	0.	0.	0.214	0.026

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	19	15	19	0
normalized size	1	1.	1.	1.07	1.36	1.36	1.07	1.36	0.
time (sec)	N/A	0.032	0.008	0.016	1.358	0.209	0.191	0.222	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	16	15	19	8	15	0
normalized size	1	1.	0.76	0.94	0.88	1.12	0.47	0.88	0.
time (sec)	N/A	0.05	0.01	0.003	1.352	0.22	0.07	0.229	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	29	32	32	26	32	0
normalized size	1	1.	0.76	0.76	0.84	0.84	0.68	0.84	0.
time (sec)	N/A	0.049	0.016	0.002	1.357	0.213	0.084	0.214	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	7	9	7
normalized size	1	1.	1.	1.14	1.29	1.29	1.	1.29	1.
time (sec)	N/A	0.015	0.004	0.	1.341	0.219	0.181	0.206	0.767

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	20	22	48
normalized size	1	1.	1.	0.77	1.	1.	0.91	1.	2.18
time (sec)	N/A	0.017	0.01	0.	1.345	0.23	0.401	0.229	2.612

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	9	7	9	7
normalized size	1	1.	1.	1.14	1.29	1.29	1.	1.29	1.
time (sec)	N/A	0.014	0.003	0.	1.313	0.233	0.181	0.226	0.784

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	30	22	30	0
normalized size	1	1.	1.	0.82	0.82	1.07	0.79	1.07	0.
time (sec)	N/A	0.016	0.002	0.	1.346	0.225	0.093	0.21	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	20	8	12	8
normalized size	1	1.	1.	0.91	1.09	1.82	0.73	1.09	0.73
time (sec)	N/A	0.019	0.005	0.023	1.344	0.238	0.816	0.207	1.17

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	15	4	3
normalized size	1	1.	1.	1.33	1.33	1.33	5.	1.33	1.
time (sec)	N/A	0.029	0.005	0.002	1.497	0.223	0.141	0.206	3.182

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	4	3	4	3
normalized size	1	1.	1.	1.33	0.	1.33	1.	1.33	1.
time (sec)	N/A	0.075	0.007	0.009	0.	0.23	0.714	0.212	4.847

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	21	72	20	12	28	12
normalized size	1	1.	1.	1.31	4.5	1.25	0.75	1.75	0.75
time (sec)	N/A	0.058	0.011	0.036	1.502	0.226	0.157	0.232	4.644

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	127	262	0	173	0	0	48
normalized size	1	1.	2.4	4.94	0.	3.26	0.	0.	0.91
time (sec)	N/A	0.129	0.086	0.016	0.	0.243	0.	0.	14.425

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	127	93	90	0	112	17
normalized size	1	1.	2.19	7.94	5.81	5.62	0.	7.	1.06
time (sec)	N/A	0.136	0.09	0.056	1.509	0.271	0.	0.257	14.036

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	92	35	422	111	0
normalized size	1	1.	2.19	7.56	5.75	2.19	26.38	6.94	0.
time (sec)	N/A	0.199	0.028	0.012	1.506	0.214	2.324	0.222	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	88	35	422	107	14
normalized size	1	1.	2.19	7.56	5.5	2.19	26.38	6.69	0.88
time (sec)	N/A	0.041	0.02	0.01	1.505	0.216	2.275	0.209	9.155

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	182	105	39	26
normalized size	1	1.	0.74	0.86	1.69	5.2	3.	1.11	0.74
time (sec)	N/A	0.029	0.044	0.	1.479	0.212	4.076	0.207	2.561

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	35	19	16	19
normalized size	1	1.	1.29	0.93	1.14	2.5	1.36	1.14	1.36
time (sec)	N/A	0.014	0.01	0.011	1.503	0.229	0.047	0.207	2.583

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	8	15	8
normalized size	1	1.	1.	0.92	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.015	0.006	0.002	1.496	0.199	0.075	0.203	1.564

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	8	5	8	5
normalized size	1	1.	1.	1.	1.14	1.14	0.71	1.14	0.71
time (sec)	N/A	0.046	0.004	0.	1.332	0.209	0.068	0.205	2.783

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	24	31	31	20	41	0
normalized size	1	1.	0.79	1.	1.29	1.29	0.83	1.71	0.
time (sec)	N/A	0.493	0.021	0.007	1.631	0.236	0.116	0.204	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	11	7	11	7
normalized size	1	1.	1.	1.	1.22	1.22	0.78	1.22	0.78
time (sec)	N/A	0.006	0.002	0.001	1.339	0.231	0.053	0.203	0.522

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	32	26	19	26	0
normalized size	1	1.	1.	0.8	1.28	1.04	0.76	1.04	0.
time (sec)	N/A	0.048	0.002	0.006	1.33	0.212	0.081	0.205	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	5
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.62
time (sec)	N/A	0.009	0.005	0.003	1.54	0.204	0.088	0.205	1.064

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	10	16	10
normalized size	1	1.	1.	0.81	1.	1.	0.62	1.	0.62
time (sec)	N/A	0.018	0.005	0.003	1.529	0.197	0.09	0.203	2.225

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	22	27	36	22	10
normalized size	1	1.	1.	1.21	1.57	1.93	2.57	1.57	0.71
time (sec)	N/A	0.019	0.004	0.011	1.48	0.217	0.725	0.209	0.564

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	182	105	39	26
normalized size	1	1.	0.74	0.86	1.69	5.2	3.	1.11	0.74
time (sec)	N/A	0.03	0.012	0.	1.49	0.224	4.099	0.209	2.543

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	86	34	28	12
normalized size	1	1.	1.	0.94	1.18	5.06	2.	1.65	0.71
time (sec)	N/A	0.017	0.024	0.007	1.485	0.243	0.97	0.209	1.668

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	18	15	15	15
normalized size	1	1.	0.74	0.74	0.79	0.95	0.79	0.79	0.79
time (sec)	N/A	0.014	0.016	0.	1.341	0.257	0.365	0.203	1.207

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	3	2	4	2
normalized size	1	1.	1.	1.5	1.5	1.5	1.	2.	1.
time (sec)	N/A	0.002	0.	0.	1.332	0.227	0.025	0.201	0.022

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	346	96	0	45	0
normalized size	1	1.	1.62	0.69	7.69	2.13	0.	1.	0.
time (sec)	N/A	0.198	0.254	0.194	1.695	0.272	0.	0.22	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	22	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.57	0.71
time (sec)	N/A	0.011	0.003	0.	1.359	0.247	0.044	0.209	0.493

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	15	14	14	15	14
normalized size	1	1.	0.82	0.65	0.88	0.82	0.82	0.88	0.82
time (sec)	N/A	0.007	0.004	0.006	1.324	0.232	0.27	0.203	0.977

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	107	0	36	31
normalized size	1	1.	1.	0.87	1.13	4.65	0.	1.57	1.35
time (sec)	N/A	0.023	0.013	0.008	1.496	0.233	0.	0.213	1.722

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	14	5	8	5
normalized size	1	1.	1.	0.88	1.	1.75	0.62	1.	0.62
time (sec)	N/A	0.019	0.002	0.003	1.339	0.273	0.039	0.201	1.093

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	7	5	7	5
normalized size	1	1.	1.	1.	1.17	1.17	0.83	1.17	0.83
time (sec)	N/A	0.026	0.002	0.	1.363	0.22	0.06	0.204	2.757

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.67
time (sec)	N/A	0.036	0.005	0.001	1.369	0.234	0.076	0.204	3.397

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	14	7	11	8
normalized size	1	1.	0.67	0.75	1.17	1.17	0.58	0.92	0.67
time (sec)	N/A	0.015	0.008	0.003	1.33	0.223	0.656	0.212	0.511

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.019	0.003	0.	1.373	0.256	0.052	0.202	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.008	0.001	0.001	1.339	0.232	0.069	0.201	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.
time (sec)	N/A	0.012	0.002	0.	1.337	0.221	0.033	0.201	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	45	21	30	43	0	27	15
normalized size	1	1.	1.88	0.88	1.25	1.79	0.	1.12	0.62
time (sec)	N/A	0.024	0.027	0.008	1.503	0.248	0.	0.212	1.901

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	18	26	15	18	17
normalized size	1	1.	1.	0.7	0.9	1.3	0.75	0.9	0.85
time (sec)	N/A	0.038	0.014	0.003	1.485	0.219	0.115	0.21	3.228

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	182	105	39	26
normalized size	1	1.	0.74	0.86	1.69	5.2	3.	1.11	0.74
time (sec)	N/A	0.029	0.041	0.	1.495	0.246	4.03	0.209	2.551

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	19	24	19	14
normalized size	1	1.	1.	0.75	0.95	0.95	1.2	0.95	0.7
time (sec)	N/A	0.04	0.009	0.006	1.484	0.222	0.12	0.201	4.071

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	54	31	35	31
normalized size	1	1.	1.	0.82	1.06	1.64	0.94	1.06	0.94
time (sec)	N/A	0.024	0.019	0.	1.486	0.222	0.14	0.206	1.897

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	26	26	15	26	0
normalized size	1	1.	1.	0.95	1.24	1.24	0.71	1.24	0.
time (sec)	N/A	0.03	0.	0.002	1.344	0.227	0.034	0.203	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	32	32	19	31	0
normalized size	1	1.	1.	0.96	1.23	1.23	0.73	1.19	0.
time (sec)	N/A	0.036	0.	0.001	1.343	0.202	0.037	0.201	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	34	22	34	0
normalized size	1	1.	1.	0.96	1.26	1.26	0.81	1.26	0.
time (sec)	N/A	0.04	0.	0.001	1.337	0.226	0.037	0.211	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	41	41	26	39	0
normalized size	1	1.	1.	0.97	1.28	1.28	0.81	1.22	0.
time (sec)	N/A	0.047	0.	0.001	1.348	0.201	0.04	0.2	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	31	31	20	31	0
normalized size	1	1.	1.04	1.	1.29	1.29	0.83	1.29	0.
time (sec)	N/A	0.052	0.	0.002	1.368	0.203	0.037	0.206	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	38	38	24	36	0
normalized size	1	1.	1.03	1.	1.31	1.31	0.83	1.24	0.
time (sec)	N/A	0.059	0.	0.002	1.364	0.203	0.04	0.208	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	23	15	23	0
normalized size	1	1.	1.	0.95	1.21	1.21	0.79	1.21	0.
time (sec)	N/A	0.019	0.	0.002	1.439	0.199	0.033	0.203	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	30	19	28	0
normalized size	1	1.	1.	0.96	1.25	1.25	0.79	1.17	0.
time (sec)	N/A	0.023	0.	0.002	1.345	0.207	0.036	0.203	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	31	22	31	0
normalized size	1	1.	1.	0.96	1.24	1.24	0.88	1.24	0.
time (sec)	N/A	0.025	0.	0.001	1.361	0.206	0.036	0.207	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	38	38	26	36	0
normalized size	1	1.	1.	0.97	1.27	1.27	0.87	1.2	0.
time (sec)	N/A	0.03	0.	0.002	1.351	0.202	0.04	0.207	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	35	35	24	34	0
normalized size	1	1.	1.04	1.	1.3	1.3	0.89	1.26	0.
time (sec)	N/A	0.018	0.	0.001	1.422	0.198	0.039	0.208	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [0.8571]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	4	0.5
2	A	3	2	1.	11	0.182
3	A	2	1	1.	11	0.091
4	A	1	1	1.	2	0.5
5	A	1	1	1.	7	0.143
6	A	2	2	1.	7	0.286
7	A	1	1	1.	11	0.091
8	A	1	1	1.	6	0.167
9	A	2	2	1.	7	0.286
10	A	2	2	1.	4	0.5
11	A	1	1	1.	6	0.167
12	A	3	2	1.	6	0.333
13	A	4	2	1.	16	0.125
14	A	5	3	1.	7	0.429
15	A	3	1	1.	14	0.071
16	A	2	2	1.	5	0.4
17	A	1	1	1.	7	0.143
18	A	1	1	1.	11	0.091
19	A	2	2	1.	11	0.182
20	A	1	1	1.	5	0.2
21	A	1	1	1.	6	0.167
22	A	2	2	1.	9	0.222
23	A	3	3	1.	14	0.214
24	A	2	1	1.	9	0.111
25	A	3	3	1.	7	0.429
26	A	2	2	1.	15	0.133

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	2	2	1.	17	0.118
28	A	3	3	1.	13	0.231
29	A	1	1	1.	5	0.2
30	A	3	3	1.	8	0.375
31	A	7	7	1.	11	0.636
32	A	3	2	1.	12	0.167
33	A	3	3	1.	6	0.5
34	A	2	1	1.	9	0.111
35	A	4	3	1.	13	0.231
36	A	4	4	1.	15	0.267
37	A	3	2	1.	15	0.133
38	A	6	3	1.	13	0.231
39	A	3	2	1.	15	0.133
40	A	5	5	1.	29	0.172
41	A	2	2	1.	4	0.5
42	A	6	6	1.16	19	0.316
43	A	1	1	1.	6	0.167
44	A	2	2	1.	5	0.4
45	A	1	1	1.	10	0.1
46	A	5	3	1.	13	0.231
47	A	1	1	1.	5	0.2
48	A	1	1	1.	7	0.143
49	A	6	6	1.	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.	27	0.037
52	A	1	1	1.	4	0.25
53	A	4	3	1.	6	0.5
54	A	2	2	1.	10	0.2
55	A	7	4	1.	9	0.444
56	A	2	1	1.	12	0.083
57	A	1	1	1.	4	0.25
58	A	6	4	1.	7	0.571
59	A	4	3	1.	11	0.273
60	A	6	3	1.	9	0.333
61	A	2	2	1.	4	0.5
62	A	3	3	1.	6	0.5
63	A	2	2	1.	4	0.5
64	A	2	2	1.	6	0.333
65	A	2	1	1.	9	0.111
66	A	2	1	1.	12	0.083
67	A	2	2	1.	20	0.1
68	A	2	2	1.	10	0.2
69	A	6	6	1.	30	0.2
70	A	5	5	1.	39	0.128
71	A	6	5	1.	45	0.111
72	A	4	4	1.	31	0.129
73	A	3	2	1.	15	0.133
74	A	3	2	1.	4	0.5
75	A	3	2	1.	11	0.182

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
76	A	5	3	1.	13	0.231
77	A	10	6	1.	33	0.182
78	A	1	1	1.	7	0.143
79	A	5	5	1.	8	0.625
80	A	2	2	1.	9	0.222
81	A	3	3	1.	11	0.273
82	A	3	3	1.	8	0.375
83	A	3	2	1.	15	0.133
84	A	2	2	1.	16	0.125
85	A	1	1	1.	6	0.167
86	A	1	1	1.	3	0.333
87	A	4	2	1.	17	0.118
88	A	2	2	1.	4	0.5
89	A	2	1	1.	11	0.091
90	A	3	3	1.	14	0.214
91	A	2	2	1.	7	0.286
92	A	2	2	1.	11	0.182
93	A	3	2	1.	13	0.154
94	A	1	1	1.	8	0.125
95	A	2	2	1.	7	0.286
96	A	1	1	1.	4	0.25
97	A	2	2	1.	5	0.4
98	A	3	3	1.	17	0.176
99	A	3	3	1.	16	0.188
100	A	3	2	1.	15	0.133
101	A	3	3	1.	15	0.2
102	A	3	3	1.	8	0.375
103	A	1	1	1.	20	0.05
104	A	1	1	1.	25	0.04
105	A	1	1	1.	26	0.038
106	A	1	1	1.	31	0.032
107	A	1	1	1.	24	0.042
108	A	1	1	1.	29	0.034
109	A	1	1	1.	18	0.056
110	A	1	1	1.	23	0.043
111	A	1	1	1.	24	0.042
112	A	1	1	1.	29	0.034
113	A	1	1	1.	27	0.037

3 Listing of integrals

3.1 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0165754, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^4,x]

[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi in Sympy [A] time = 0.501687, size = 14, normalized size = 1.17

$$x+\frac{1}{\tan(x)}-\frac{1}{3\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**4,x)

[Out] x + 1/tan(x) - 1/(3*tan(x)**3)
```

Mathematica [A] time = 0.00541795, size = 18, normalized size = 1.5

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^4,x]

[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0., size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3}+\cot(x)-\frac{\pi}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.48997, size = 22, normalized size = 1.83

$$x + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [A] time = 0.221089, size = 65, normalized size = 5.42

$$\frac{4 \cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2 \cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)
/((cos(2*x) - 1)*sin(2*x))
```

Sympy [A] time = 0.051641, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.210257, size = 46, normalized size = 3.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15 \tan\left(\frac{1}{2}x\right)^2 - 1}{24 \tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
 5/8*tan(1/2*x)
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

```
[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Rubi [A] time = 0.0172013, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(1 + x^2)),x]

[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Rubi in Sympy [A] time = 2.24829, size = 12, normalized size = 0.92

$$\operatorname{atan}(x)+\frac{1}{x}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(x**2+1),x)

[Out] atan(x) + 1/x - 1/(3*x**3)
```

Mathematica [A] time = 0.00598656, size = 13, normalized size = 1.

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(1 + x^2)),x]

[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Maple [A] time = 0.007, size = 12, normalized size = 0.9

$$-\frac{1}{3\,x^3}+x^{-1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(x^2+1),x)

[Out] -1/3/x^3+1/x+arctan(x)
```

Maxima [A] time = 1.484, size = 20, normalized size = 1.54

$$\frac{3\,x^2-1}{3\,x^3}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x^4),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Fricas [A] time = 0.210309, size = 26, normalized size = 2.

$$\frac{3\,x^3\arctan\left(x\right)+3\,x^2-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

Sympy [A] time = 0.116823, size = 14, normalized size = 1.08

$$\operatorname{atan}\left(x\right)+\frac{3x^2-1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(x**2+1),x)
```

```
[Out] atan(x) + (3*x**2 - 1)/(3*x**3)
```

GIAC/XCAS [A] time = 0.202067, size = 20, normalized size = 1.54

$$\frac{3\,x^2-1}{3\,x^3}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

3.3 $\int \frac{x+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rubi [A] time = 0.0078335, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rubi in Sympy [A] time = 1.02458, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

Mathematica [A] time = 0.00382188, size = 14, normalized size = 0.74

$$\frac{2}{15}x^{3/2}(3x+5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(5 + 3*x))/15
```

Maple [A] time = 0.003, size = 11, normalized size = 0.6

$$\frac{10+6\,x}{15}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x)/x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)*(5+3*x)
```

Maxima [A] time = 1.33123, size = 15, normalized size = 0.79

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Fricas [A] time = 0.211054, size = 19, normalized size = 1.

$$\frac{2}{15}\left(3\,x^2 + 5\,x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 5*x)*sqrt(x)
```

Sympy [A] time = 0.27182, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

GIAC/XCAS [A] time = 0.198108, size = 15, normalized size = 0.79

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + x)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

3.4 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.00461319, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x],x]

[Out] Sin[x]
```

Rubi in Sympy [A] time = 0.023007, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x),x)

[Out] sin(x)
```

Mathematica [A] time = 0.00154168, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x],x]

[Out] Sin[x]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x),x)

[Out] sin(x)
```

Maxima [A] time = 1.37232, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 0.207523, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.029884, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x)
```

```
[Out] sin(x)
```

GIAC/XCAS [A] time = 0.199812, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.5 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] E^x^2/2
```

Rubi [A] time = 0.0104122, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rubi in Sympy [A] time = 1.03643, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Mathematica [A] time = 0.00214517, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 1.37013, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 0.204732, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.059663, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

GIAC/XCAS [A] time = 0.200049, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.017114, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]

[Out] Sec[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\tan(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sec(x)**2*tan(x),x)

[Out] Integral(x, (x, tan(x)))
```

Mathematica [A] time = 0.0030232, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]

[Out] Sec[x]^2/2
```

Maple [A] time = 0.009, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x)

[Out] 1/2*sec(x)^2
```

Maxima [A] time = 1.32574, size = 8, normalized size = 1.

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A] time = 0.224094, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.050914, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

GIAC/XCAS [A] time = 0.198484, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.7 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.00497958, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 0.76266, size = 8, normalized size = 0.62

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.00328751, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0.003, size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 1.34017, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.234433, size = 93, normalized size = 7.15

$$-\frac{4x^6+9x^4+6x^2-\left(4x^5+7x^3+3x\right)\sqrt{x^2+1}+1}{3\left(4x^3-\left(4x^2+1\right)\sqrt{x^2+1}+3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^6 + 9*x^4 + 6*x^2 - (4*x^5 + 7*x^3 + 3*x)*sqrt(x^2 + 1)
 + 1)/(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [A] time = 0.216007, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

GIAC/XCAS [A] time = 0.203724, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

3.8 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0127187, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Sin[x],x]

[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi in Sympy [A] time = 1.19299, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*sin(x),x)

[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Mathematica [A] time = 0.0161374, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Sin[x],x]

[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*sin(x),x)

[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.35255, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="maxima")

[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.225591, size = 18, normalized size = 0.95

$$-\frac{1}{2}\cos\left(x\right)e^{x} + \frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.357772, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2} - \frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*sin(x),x)

[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.199609, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="giac")

[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\csc^3(x)$$

```
[Out] -Csc[x]^3/3
```

Rubi [A] time = 0.0210645, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x]^3,x]

[Out] -Csc[x]^3/3
```

Rubi in Sympy [A] time = 1.11114, size = 8, normalized size = 1.

$$-\frac{1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*csc(x)**2/sin(x)**2,x)

[Out] -1/(3*sin(x)**3)
```

Mathematica [A] time = 0.00417226, size = 8, normalized size = 1.

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^3,x]

[Out] -Csc[x]^3/3
```

Maple [A] time = 0.018, size = 7, normalized size = 0.9

$$-\frac{1}{3\,(\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^2/sin(x)^2,x)

[Out] -1/3/sin(x)^3
```

Maxima [A] time = 1.35713, size = 8, normalized size = 1.

$$-\frac{1}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/3/sin(x)^3
```

Fricas [A] time = 0.219539, size = 19, normalized size = 2.38

$$\frac{1}{3\,\left(\cos\left(x\right)^{2}-1\right)\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.050865, size = 8, normalized size = 1.

$$-\frac{1}{3\sin^{3}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*csc(x)**2/sin(x)**2,x)
```

```
[Out] -1/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.20089, size = 8, normalized size = 1.

$$-\frac{1}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/3/sin(x)^3
```

3.10 $\int \sin(e^x)\,dx$

Optimal. Leaf size=4

$$\text{Si}(e^x)$$

```
[Out] SinIntegral[E^x]
```

Rubi [A] time = 0.0189059, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\text{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[E^x],x]

[Out] SinIntegral[E^x]
```

Rubi in Sympy [A] time = 1.35991, size = 3, normalized size = 0.75

$$\text{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(exp(x)),x)

[Out] Si(exp(x))
```

Mathematica [A] time = 0.00881329, size = 4, normalized size = 1.

$$\text{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[E^x],x]

[Out] SinIntegral[E^x]
```

Maple [A] time = 0.004, size = 4, normalized size = 1.

$$Si(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x)

[Out] Si(exp(x))
```

Maxima [A] time = 1.51957, size = 20, normalized size = 5.

$$-\frac{1}{2} i \,\text{Ei}(i e^x) + \frac{1}{2} i \,\text{Ei}(-i e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(e^x),x, algorithm="maxima")

[Out] -1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(e^x),x, algorithm="fricas")

[Out] sin_integral(e^x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin\left(e^{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(exp(x)),x)

[Out] Integral(sin(exp(x)), x)
```

GIAC/XCAS [A] time = 0.201689, size = 4, normalized size = 1.

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(e^x),x, algorithm="giac")

[Out] Si(e^x)
```

3.11 $\int \frac{\sin(y)}{y}\,dy$

Optimal. Leaf size=2

$$\mathrm{Si}(y)$$

```
[Out] SinIntegral[y]
```

Rubi [A] time = 0.018495, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[y]/y,y]

[Out] SinIntegral[y]
```

Rubi in Sympy [A] time = 0.697378, size = 2, normalized size = 1.

$$\mathrm{Si}\,(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(y)/y,y)

[Out] Si(y)
```

Mathematica [A] time = 0.00816405, size = 2, normalized size = 1.

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[y]/y,y]

[Out] SinIntegral[y]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$Si\,(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(y)/y,y)

[Out] Si(y)
```

Maxima [A] time = 1.48743, size = 18, normalized size = 9.

$$-\frac{1}{2}i\,\mathrm{Ei}\,(i\,y) + \frac{1}{2}i\,\mathrm{Ei}\,(-i\,y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)/y,y, algorithm="maxima")

[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)/y,y, algorithm="fricas")

[Out] sin_integral(y)
```

Sympy [A] time = 0.828821, size = 2, normalized size = 1.

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)/y,y)

[Out] Si(y)
```

GIAC/XCAS [A] time = 0.198507, size = 3, normalized size = 1.5

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)/y,y, algorithm="giac")

[Out] Si(y)
```

3.12 $\int (e^x + \sin(x))\, dx$

Optimal. Leaf size=8

$$e^x - \cos(x)$$

```
[Out] E^x - Cos[x]
```

Rubi [A] time = 0.00751288, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x + Sin[x],x]

[Out] E^x - Cos[x]
```

Rubi in Sympy [A] time = 0.475919, size = 5, normalized size = 0.62

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)+sin(x),x)

[Out] exp(x) - cos(x)
```

Mathematica [A] time = 0.00532548, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x + Sin[x],x]

[Out] E^x - Cos[x]
```

Maple [A] time = 0.001, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)+sin(x),x)

[Out] exp(x)-cos(x)
```

Maxima [A] time = 1.35502, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x + sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x) + e^x
```

Fricas [A] time = 0.236425, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x + sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x) + e^x
```

Sympy [A] time = 0.059039, size = 5, normalized size = 0.62

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)+sin(x),x)
```

```
[Out] exp(x) - cos(x)
```

GIAC/XCAS [A] time = 0.197748, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x + sin(x),x, algorithm="giac")
```

```
[Out] -cos(x) + e^x
```

3.13 $\int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.0317724, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Rubi in Sympy [A] time = 1.61684, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)+2*exp(x**2)*x**2,x)
```

```
[Out] x*exp(x**2)
```

Mathematica [A] time = 0.00391435, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0.003, size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)+2*exp(x^2)*x^2,x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 1.34934, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x^2*e^(x^2) + e^(x^2),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 0.202078, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x^2*e^(x^2) + e^(x^2),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.068657, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)+2*exp(x**2)*x**2,x)
```

```
[Out] x*exp(x**2)
```

GIAC/XCAS [A] time = 0.209303, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*x^2*e^(x^2) + e^(x^2),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.14 $\int (e^x + x)^2 \, dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rubi [A] time = 0.0370384, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)^2,x]
```

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x + e^x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x+exp(x))**2,x)
```

```
[Out] Integral((x + exp(x))**2, x)
```

Mathematica [A] time = 0.00920591, size = 26, normalized size = 0.93

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + e^x(2x - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)^2,x]
```

```
[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

Maple [A] time = 0.003, size = 22, normalized size = 0.8

$$\frac{x^3}{3} + \frac{(e^x)^2}{2} + 2\,e^x x - 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+x)^2,x)
```

```
[Out] 1/3*x^3+1/2*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A] time = 1.37696, size = 26, normalized size = 0.93

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Fricas [A] time = 0.202844, size = 26, normalized size = 0.93

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Sympy [A] time = 0.093709, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)\,e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+exp(x))**2,x)
```

```
[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

GIAC/XCAS [A] time = 0.214104, size = 26, normalized size = 0.93

$$\frac{1}{3}\,x^3 + 2\,(x-1)e^x + \frac{1}{2}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

3.15 $\int \left(2e^x + e^{2x} + x^2\right) \, dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Rubi [A] time = 0.0113693, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[2*E^x + E^(2*x) + x^2,x]

[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Rubi in Sympy [A] time = 0.637238, size = 15, normalized size = 0.68

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2*exp(x)+exp(2*x)+x**2,x)

[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

Mathematica [A] time = 0.00388875, size = 22, normalized size = 1.

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[2*E^x + E^(2*x) + x^2,x]

[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$2\,e^x + \frac{e^{2\,x}}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2*exp(x)+exp(2*x)+x^2,x)

[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Maxima [A] time = 1.39223, size = 22, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + e^(2*x) + 2*e^x,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Fricas [A] time = 0.213708, size = 22, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + e^(2*x) + 2*e^x,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Sympy [A] time = 0.083403, size = 15, normalized size = 0.68

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*exp(x)+exp(2*x)+x**2,x)
```

```
[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

GIAC/XCAS [A] time = 0.210618, size = 22, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2 + e^(2*x) + 2*e^x,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

3.16 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.0120211, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\sin(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*sin(x),x)

[Out] Integral(x, (x, sin(x)))
```

Mathematica [A] time = 0.00210773, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]

[Out] -Cos[x]^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)

[Out] 1/2*sin(x)^2
```

Maxima [A] time = 1.34442, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 0.225704, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.034036, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

GIAC/XCAS [A] time = 0.21465, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.17 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] E^x^2/2
```

Rubi [A] time = 0.0118742, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*x,x]

[Out] E^x^2/2
```

Rubi in Sympy [A] time = 1.06089, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*x,x)

[Out] exp(x**2)/2
```

Mathematica [A] time = 0.00116154, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*x,x]

[Out] E^x^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*x,x)

[Out] 1/2*exp(x^2)
```

Maxima [A] time = 1.35177, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="maxima")

[Out] 1/2*e^(x^2)
```

Fricas [A] time = 0.212342, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="fricas")

[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.05864, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*x,x)

[Out] exp(x**2)/2
```

GIAC/XCAS [A] time = 0.223895, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(x^2),x, algorithm="giac")

[Out] 1/2*e^(x^2)
```

3.18 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.00477223, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 0.759376, size = 8, normalized size = 0.62

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2+1)**(1/2),x)

[Out] (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.000587809, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0., size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2+1)^(1/2),x)

[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 1.33344, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.243513, size = 93, normalized size = 7.15

$$-\frac{4x^6+9x^4+6x^2-\left(4x^5+7x^3+3x\right)\sqrt{x^2+1}+1}{3\left(4x^3-\left(4x^2+1\right)\sqrt{x^2+1}+3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^6 + 9*x^4 + 6*x^2 - (4*x^5 + 7*x^3 + 3*x)*sqrt(x^2 + 1)
 + 1)/(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [A] time = 0.229841, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

GIAC/XCAS [A] time = 0.222474, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

3.19 $\int \frac{e^x}{1+e^x}\,dx$

Optimal. Leaf size=6

$$\log(e^x + 1)$$

```
[Out] Log[1 + E^x]
```

Rubi [A] time = 0.0258402, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 + E^x),x]

[Out] Log[1 + E^x]
```

Rubi in Sympy [A] time = 2.74219, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1+exp(x)),x)

[Out] log(exp(x) + 1)
```

Mathematica [A] time = 0.00232404, size = 6, normalized size = 1.

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 + E^x),x]

[Out] Log[1 + E^x]
```

Maple [A] time = 0.003, size = 6, normalized size = 1.

$$\ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1+exp(x)),x)

[Out] ln(1+exp(x))
```

Maxima [A] time = 1.32803, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="maxima")
```

```
[Out] log(e^x + 1)
```

Fricas [A] time = 0.197977, size = 7, normalized size = 1.17

$$\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

Sympy [A] time = 0.057866, size = 5, normalized size = 0.83

$$\log\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1+exp(x)),x)
```

```
[Out] log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.212002, size = 7, normalized size = 1.17

$$\ln\left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="giac")
```

```
[Out] ln(e^x + 1)
```

3.20 $\int x^{3/2} \, dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

```
[Out] (2*x^(5/2))/5
```

Rubi [A] time = 0.00306384, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5
```

Rubi in Sympy [A] time = 0.454371, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(3/2),x)

[Out] 2*x**(5/2)/5
```

Mathematica [A] time = 0.00075388, size = 9, normalized size = 1.

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5
```

Maple [A] time = 0.003, size = 6, normalized size = 0.7

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)

[Out] 2/5*x^(5/2)
```

Maxima [A] time = 1.3355, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2),x, algorithm="maxima")

[Out] 2/5*x^(5/2)
```

Fricas [A] time = 0.203467, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2),x, algorithm="fricas")

[Out] 2/5*x^(5/2)
```

Sympy [A] time = 0.027784, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2),x)

[Out] 2*x**(5/2)/5
```

GIAC/XCAS [A] time = 0.201266, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2),x, algorithm="giac")

[Out] 2/5*x^(5/2)
```

3.21 $\int \cos(3+2x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin(2x+3)$$

```
[Out] Sin[3 + 2*x]/2
```

Rubi [A] time = 0.00661853, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Rubi in Sympy [A] time = 0.515343, size = 7, normalized size = 0.7

$$\frac{\sin(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(3+2*x),x)
```

```
[Out] sin(2*x + 3)/2
```

Mathematica [A] time = 0.0027125, size = 10, normalized size = 1.

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Maple [A] time = 0.007, size = 9, normalized size = 0.9

$$\frac{\sin(3+2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(3+2*x),x)
```

```
[Out] 1/2*sin(3+2*x)
```

Maxima [A] time = 1.3664, size = 11, normalized size = 1.1

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x + 3),x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*x + 3)
```

Fricas [A] time = 0.214726, size = 11, normalized size = 1.1

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x + 3),x, algorithm="fricas")
```

```
[Out] 1/2*sin(2*x + 3)
```

Sympy [A] time = 0.122377, size = 7, normalized size = 0.7

$$\frac{\sin\left(2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(3+2*x),x)
```

```
[Out] sin(2*x + 3)/2
```

GIAC/XCAS [A] time = 0.198078, size = 11, normalized size = 1.1

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x + 3),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*x + 3)
```

3.22 $\int 2e^{2x}yz\,dx$

Optimal. Leaf size=8

$$e^{2x}yz$$

```
[Out] E^(2*x)*y*z
```

Rubi [A] time = 0.00743992, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In]  Int[2*E^(2*x)*y*z,x]

[Out] E^(2*x)*y*z
```

Rubi in Sympy [A] time = 0.839445, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2*exp(2*x)*y*z,x)

[Out] y*z*exp(2*x)
```

Mathematica [A] time = 0.000889233, size = 8, normalized size = 1.

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In]  Integrate[2*E^(2*x)*y*z,x]

[Out] E^(2*x)*y*z
```

Maple [A] time = 0.003, size = 8, normalized size = 1.

$$e^{2x}yz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2*exp(2*x)*y*z,x)

[Out] exp(2*x)*y*z
```

Maxima [A] time = 1.34695, size = 9, normalized size = 1.12

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*y*z*e^(2*x),x, algorithm="maxima")
```

```
[Out] y*z*e^(2*x)
```

Fricas [A] time = 0.234294, size = 9, normalized size = 1.12

$$yze^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*y*z*e^(2*x),x, algorithm="fricas")
```

```
[Out] y*z*e^(2*x)
```

Sympy [A] time = 0.072371, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*exp(2*x)*y*z,x)
```

```
[Out] y*z*exp(2*x)
```

GIAC/XCAS [A] time = 0.200352, size = 9, normalized size = 1.12

$$yze^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*y*z*e^(2*x),x, algorithm="giac")
```

```
[Out] y*z*e^(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x) \, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos^3(e^x)$$

```
[Out] -Cos[E^x]^3/3
```

Rubi [A] time = 0.0392648, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{1}{3}\cos^3(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]

[Out] -Cos[E^x]^3/3
```

Rubi in Sympy [A] time = 6.08972, size = 8, normalized size = 0.8

$$-\frac{\cos^3(e^x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)

[Out] -cos(exp(x))**3/3
```

Mathematica [A] time = 0.0169012, size = 19, normalized size = 1.9

$$-\frac{1}{4}\cos(e^x) - \frac{1}{12}\cos(3e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]

[Out] -Cos[E^x]/4 - Cos[3*E^x]/12
```

Maple [A] time = 0.007, size = 8, normalized size = 0.8

$$-\frac{(\cos(e^x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x)

[Out] -1/3*cos(exp(x))^3
```

Maxima [A] time = 1.37219, size = 9, normalized size = 0.9

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(e^x)^2*e^x*sin(e^x),x, algorithm="maxima")
```

```
[Out] -1/3*cos(e^x)^3
```

Fricas [A] time = 0.231044, size = 9, normalized size = 0.9

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(e^x)^2*e^x*sin(e^x),x, algorithm="fricas")
```

```
[Out] -1/3*cos(e^x)^3
```

Sympy [A] time = 2.94266, size = 8, normalized size = 0.8

$$-\frac{\cos^{3}\left(e^{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))**3/3
```

GIAC/XCAS [A] time = 0.197629, size = 9, normalized size = 0.9

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(e^x)^2*e^x*sin(e^x),x, algorithm="giac")
```

```
[Out] -1/3*cos(e^x)^3
```

3.24 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.0118266, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi in Sympy [A] time = 0.967715, size = 19, normalized size = 0.83

$$\frac{2\,(x+1)^{\frac{5}{2}}}{5}-\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.00503877, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

Maple [A] time = 0.004, size = 13, normalized size = 0.6

$$\frac{-4+6\,x}{15}\,(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+x)^(1/2),x)
```

```
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Maxima [A] time = 1.33996, size = 20, normalized size = 0.87

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.205207, size = 20, normalized size = 0.87

$$\frac{2}{15}\,\left(3\,x^{2}+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] time = 1.44905, size = 34, normalized size = 1.48

$$\frac{2x^{2}\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

GIAC/XCAS [A] time = 0.198081, size = 20, normalized size = 0.87

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.25 $\int \frac{1}{-1+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.00794742, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^4)^(-1),x]

[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi in Sympy [A] time = 0.619817, size = 10, normalized size = 0.77

$$-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-1),x)

[Out] -atan(x)/2 - atanh(x)/2
```

Mathematica [A] time = 0.00537219, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^4)^(-1),x]

[Out] -ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$-\frac{\arctan(x)}{2}-\frac{\mathit{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-1),x)

[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A] time = 1.52759, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="maxima")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 0.20465, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="fricas")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.16239, size = 17, normalized size = 1.31

$$\frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-1),x)

[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

GIAC/XCAS [A] time = 0.19935, size = 26, normalized size = 2.

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\ln(|x+1|) + \frac{1}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="giac")

[Out] -1/2*arctan(x) - 1/4*ln(abs(x + 1)) + 1/4*ln(abs(x - 1))
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Rubi [A] time = 0.0357168, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(2 + 3*E^(2*x)),x]

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Rubi in Sympy [A] time = 3.15427, size = 17, normalized size = 0.94

$$\frac{\sqrt{6}\,\text{atan}\left(\frac{\sqrt{6}e^x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(2+3*exp(2*x)),x)

[Out] sqrt(6)*atan(sqrt(6)*exp(x)/2)/6
```

Mathematica [A] time = 0.0131084, size = 18, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(2 + 3*E^(2*x)),x]

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{e^x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(2+3*exp(2*x)),x)

[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.54985, size = 18, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(3*e^(2*x) + 2),x, algorithm="maxima")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A] time = 0.207156, size = 18, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(3*e^(2*x) + 2),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Sympy [A] time = 0.113463, size = 15, normalized size = 0.83

$$\operatorname{RootSum}\left(24z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(2+3*exp(2*x)),x)

[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

GIAC/XCAS [A] time = 0.207173, size = 18, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(3*e^(2*x) + 2),x, algorithm="giac")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rubi [A] time = 0.0590941, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(A + B*E^(4*x)),x]

[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rubi in Sympy [A] time = 4.47356, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(A+B*exp(4*x)),x)

[Out] atan(sqrt(B)*exp(2*x)/sqrt(A))/(2*sqrt(A)*sqrt(B))
```

Mathematica [A] time = 0.0122646, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(A + B*E^(4*x)),x]

[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Maple [A] time = 0.01, size = 20, normalized size = 0.7

$$\frac{1}{2}\arctan\left(B\left(e^{x}\right)^{2}\frac{1}{\sqrt{AB}}\right)\frac{1}{\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(B*e^(4*x) + A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.214702, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,ABe^{(2\,x)}+\sqrt{-AB}\left(Be^{(4\,x)}-A\right)}{Be^{(4\,x)}+A}\right)}{4\,\sqrt{-AB}}, -\frac{\arctan\left(\frac{Ae^{(-2\,x)}}{\sqrt{AB}}\right)}{2\,\sqrt{AB}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(B*e^(4*x) + A),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*A*B*e^(2*x) + sqrt(-A*B)*(B*e^(4*x) - A))/(B*e^(4*x)
+ A))/sqrt(-A*B), -1/2*arctan(A*e^(-2*x)/sqrt(A*B))/sqrt(A*B)]
```

Sympy [A] time = 0.170636, size = 22, normalized size = 0.71

$$\operatorname{RootSum}\left(16z^2AB + 1, \left(i \mapsto i\log\left(4iA + e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

GIAC/XCAS [A] time = 0.202735, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{Be^{(2\,x)}}{\sqrt{AB}}\right)}{2\,\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(B*e^(4*x) + A),x, algorithm="giac")
```

```
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

3.28 $\int \frac{e^{1+x}}{1+e^x} dx$

Optimal. Leaf size=8

$$e \log (e^x + 1)$$

```
[Out] E*Log[1 + E^x]
```

Rubi [A] time = 0.0492838, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$e \log (e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

Rubi in Sympy [A] time = 2.7392, size = 7, normalized size = 0.88

$$e \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1+x)/(1+exp(x)),x)
```

```
[Out] E*log(exp(x) + 1)
```

Mathematica [A] time = 0.00240883, size = 8, normalized size = 1.

$$e \log (e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

Maple [A] time = 0.003, size = 9, normalized size = 1.1

$$\mathrm{e} \ln (1 + \mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1+x)/(1+exp(x)),x)
```

```
[Out] exp(1)*ln(1+exp(x))
```

Maxima [A] time = 1.35457, size = 11, normalized size = 1.38

$$e \log (e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + 1)/(e^x + 1),x, algorithm="maxima")
```

```
[Out] e*log(e^x + 1)
```

Fricas [A] time = 0.221606, size = 15, normalized size = 1.88

$$e \log \left(e + e^{(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + 1)/(e^x + 1),x, algorithm="fricas")
```

```
[Out] e*log(e + e^(x + 1))
```

Sympy [A] time = 0.080139, size = 7, normalized size = 0.88

$$e \log \left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1+x)/(1+exp(x)),x)
```

```
[Out] E*log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.20004, size = 11, normalized size = 1.38

$$e \ln \left(e^{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x + 1)/(e^x + 1),x, algorithm="giac")
```

```
[Out] e*ln(e^x + 1)
```

3.29 $\int (10e)^x \, dx$

Optimal. Leaf size=12

$$\frac{(10e)^x}{1 + \log(10)}$$

```
[Out] (10*E)^x/(1 + Log[10])
```

Rubi [A] time = 0.0114692, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{(10e)^x}{1 + \log(10)}$$

Antiderivative was successfully verified.

```
[In]  Int[(10*E)^x,x]
```

```
[Out] (10*E)^x/(1 + Log[10])
```

Rubi in Sympy [A] time = 0.48664, size = 8, normalized size = 0.67

$$\frac{(10e)^x}{\log(10e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((10*E)**x,x)
```

```
[Out] (10*E)**x/log(10*E)
```

Mathematica [A] time = 0.0053498, size = 12, normalized size = 1.

$$\frac{(10e)^x}{\log(10e)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(10*E)^x,x]
```

```
[Out] (10*E)^x/Log[10*E]
```

Maple [A] time = 0.02, size = 13, normalized size = 1.1

$$\frac{(10\,E)^x}{\ln(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((10*E)^x,x)
```

```
[Out] 1/ln(10*E)*(10*E)^x
```

Maxima [A] time = 1.34536, size = 16, normalized size = 1.33

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((10*E)^x,x, algorithm="maxima")
```

```
[Out] (10*E)^x/log(10*E)
```

Fricas [A] time = 0.221903, size = 16, normalized size = 1.33

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((10*E)^x,x, algorithm="fricas")
```

```
[Out] (10*E)^x/log(10*E)
```

Sympy [A] time = 0.074312, size = 8, normalized size = 0.67

$$\frac{(10e)^x}{1+\log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((10*E)**x,x)
```

```
[Out] (10*E)**x/(1 + log(10))
```

GIAC/XCAS [A] time = 0.198523, size = 16, normalized size = 1.33

$$\frac{(10\,E)^x}{\ln(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((10*E)^x,x, algorithm="giac")
```

```
[Out] (10*E)^x/ln(10*E)
```

3.30 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi [A] time = 0.0267647, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi in Sympy [A] time = 1.41254, size = 15, normalized size = 0.75

$$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Mathematica [A] time = 0.00476135, size = 20, normalized size = 1.

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2\cos(x^2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-\frac{x^2\cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A] time = 1.33502, size = 22, normalized size = 1.1

$$-\frac{1}{2}\,x^{2}\cos\left(x^{2}\right)+\frac{1}{2}\,\sin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A] time = 0.229118, size = 22, normalized size = 1.1

$$-\frac{1}{2}\,x^{2}\cos\left(x^{2}\right)+\frac{1}{2}\,\sin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] time = 0.776329, size = 15, normalized size = 0.75

$$-\frac{x^{2}\cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

GIAC/XCAS [A] time = 0.198359, size = 22, normalized size = 1.1

$$-\frac{1}{2}\,x^{2}\cos\left(x^{2}\right)+\frac{1}{2}\,\sin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.31 $\int \frac{x^7}{1+x^{12}}\, dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[
1 - x^4 + x^8]/24
```

Rubi [A] time = 0.0761665, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$

$$-\frac{1}{12}\log\left(x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{24}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[
1 - x^4 + x^8]/24
```

Rubi in Sympy [A] time = 3.95518, size = 42, normalized size = 0.86

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^4}{3}-\frac{1}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(sqrt(3
)*(2*x**4/3 - 1/3))/12
```

Mathematica [B] time = 0.198422, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}x+1\right)-2\log\left(x^2+\sqrt{2}x+1\right)+\log\left(2x^2-\sqrt{6}x+\sqrt{2}x+2\right)\right.$$
$$+\log\left(2x^2+\sqrt{2}\left(\sqrt{3}-1\right)x+2\right)+\log\left(2x^2-\left(\sqrt{2}+\sqrt{6}\right)x+2\right)$$
$$+\log\left(2x^2+\left(\sqrt{2}+\sqrt{6}\right)x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{-2\sqrt{2}x+\sqrt{3}+1}{1-\sqrt{3}}\right)$$
$$\left.-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x-\sqrt{3}+1}{1+\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x+\sqrt{3}-1}{1+\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt{2}x+\sqrt{3}+1}{\sqrt{3}-1}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*
Sqrt[3]*ArcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqr
t[3]*ArcTan[(-1 + Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[
```

```
3]*ArcTan[(1 + Sqrt[3] + 2*Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 -
 Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x + x^2] + Log[2 + Sqrt[2]*
x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqrt[3])*x + 2*x^2
] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2] + S
qrt[6])*x + 2*x^2])/24
```

Maple [A] time = 0.001, size = 41, normalized size = 0.8

$$-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2\,x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(x^12+1),x)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4
-1)*3^(1/2))
```

Maxima [A] time = 1.53504, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4
 + 1) - 1/12*log(x^4 + 1)
```

Fricas [A] time = 0.209945, size = 63, normalized size = 1.29

$$\frac{1}{72}\sqrt{3}\left(\sqrt{3}\log\left(x^8-x^4+1\right)-2\sqrt{3}\log\left(x^4+1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(sqrt(3)*log(x^8 - x^4 + 1) - 2*sqrt(3)*log(x^4 + 1)
 + 6*arctan(1/3*sqrt(3)*(2*x^4 - 1)))
```

Sympy [A] time = 0.258389, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt
(3)*x**4/3 - sqrt(3)/3)/12
```

GIAC/XCAS [A] time = 0.208241, size = 54, normalized size = 1.1

$$\frac{1}{12}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^{4}-1\right)\right)+\frac{1}{24}\ln\left(x^{8}-x^{4}+1\right)-\frac{1}{12}\ln\left(x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(x^12 + 1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*ln(x^8 - x^4
+ 1) - 1/12*ln(x^4 + 1)
```

3.32 $\int x^{3a}\sin\left(x^{2a}\right)\,dx$

Optimal. Leaf size=115

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*
x^(2*a))^((1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1
))/2, I*x^(2*a)])/(a*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rubi [A] time = 0.0987692, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\text{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*
x^(2*a))^((1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1
))/2, I*x^(2*a)])/(a*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rubi in Sympy [A] time = 5.27377, size = 87, normalized size = 0.76

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\left(\frac{3a+1}{2a},-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\left(\frac{3a+1}{2a},ix^{2a}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] I*x**(3*a + 1)*(-I*x**(2*a))**(-(3*a + 1)/(2*a))*Gamma((3*a + 1)/
(2*a), -I*x**(2*a))/(4*a) - I*x**(3*a + 1)*(I*x**(2*a))**(-(3*a +
 1)/(2*a))*Gamma((3*a + 1)/(2*a), I*x**(2*a))/(4*a)
```

Mathematica [A] time = 0.457046, size = 142, normalized size = 1.23

$$-\frac{x^{a+1}\left(x^{4a}\right)^{-\frac{a+1}{2a}}\left((a+1)\left(-ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},ix^{2a}\right)+(a+1)\left(ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},-ix^{2a}\right)+4a\left(x^{4a}\right)^{\frac{a+1}{2a}}\cos\left(x^{2a}\right)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] -(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)
*(I*x^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] +
 (1 + a)*((-I)*x^(2*a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^
(2*a)]))/(8*a^2*(x^(4*a))^((1 + a)/(2*a)))
```

Maple [C] time = 0.149, size = 41, normalized size = 0.4

$$\frac{x^{5a+1}}{5a+1}\,{}_1F_2\left(\frac{5}{4}+\frac{1}{4a};\frac{3}{2},\frac{9}{4}+\frac{1}{4a};-\frac{x^{4a}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3*a)*sin(x^(2*a)),x)
```

```
[Out] 1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^
(4*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{xx^{a}\cos\left(x^{2\,a}\right)-(a+1)\int x^{a}\cos\left(x^{2\,a}\right)\,dx}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
```

```
[Out] -1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x)
)/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{3\,a}\sin\left(x^{2\,a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
```

```
[Out] integral(x^(3*a)*sin(x^(2*a)), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3\,a}\sin\left(x^{2\,a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0177472, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.59603, size = 48, normalized size = 2.18

$$-i\sqrt{x}e^{i\sqrt{x}}+i\sqrt{x}e^{-i\sqrt{x}}+e^{i\sqrt{x}}+e^{-i\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x**(1/2)),x)
```

```
[Out] -I*sqrt(x)*exp(I*sqrt(x)) + I*sqrt(x)*exp(-I*sqrt(x)) + exp(I*sqr
t(x)) + exp(-I*sqrt(x))
```

Mathematica [A] time = 0.0101979, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0.012, size = 17, normalized size = 0.8

$$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.33931, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="maxima")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 0.239963, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="fricas")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.405013, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x**(1/2)),x)

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

GIAC/XCAS [A] time = 0.199515, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="giac")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.34 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.0114164, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi in Sympy [A] time = 0.980611, size = 19, normalized size = 0.83

$$\frac{2\,(x+1)^{\frac{5}{2}}}{5}-\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.00503717, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

Maple [A] time = 0., size = 13, normalized size = 0.6

$$\frac{-4+6\,x}{15}\,(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(1+x)^(1/2),x)
```

```
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Maxima [A] time = 1.34049, size = 20, normalized size = 0.87

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.208882, size = 20, normalized size = 0.87

$$\frac{2}{15}\,\left(3\,x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] time = 1.44474, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5}+\frac{2x\sqrt{x+1}}{15}-\frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

GIAC/XCAS [A] time = 0.198518, size = 20, normalized size = 0.87

$$\frac{2}{5}\,(x+1)^{\frac{5}{2}}-\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x,x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.0267247, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$6\sqrt[6]{x}+2\sqrt{x}-6\log\left(\sqrt[6]{x}+1\right)-6\int^{\sqrt[6]{x}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] 6*x**(1/6) + 2*sqrt(x) - 6*log(x**(1/6) + 1) - 6*Integral(x, (x,
x**(1/6)))
```

Mathematica [A] time = 0.0130262, size = 32, normalized size = 1.

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0.001, size = 92, normalized size = 2.9

$$2\ln\left(-1+\sqrt[6]{x}\right)-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)-2\ln\left(1+\sqrt[6]{x}\right)+\ln\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)+2\sqrt{x}+\ln\left(\sqrt{x}-1\right)\\-\ln\left(1+\sqrt{x}\right)+6\sqrt[6]{x}-\ln(-1+x)-2\ln\left(-1+\sqrt[3]{x}\right)+\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)-3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)
-x^(1/6)+1)+2*x^(1/2)+ln(x^(1/2)-1)-ln(1+x^(1/2))+6*x^(1/6)-ln(-1
```

```
+x)-2*ln(-1+x^(1/3))+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A] time = 1.35231, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="maxima")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 0.207079, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="fricas")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**(1/3)+x**(1/2)),x)

[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

GIAC/XCAS [A] time = 0.206078, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x) + x^(1/3)),x, algorithm="giac")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*ln(x^(1/6) + 1)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\, dx$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*S
qrt[2])
```

Rubi [A] time = 0.0331115, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*S
qrt[2])
```

Rubi in Sympy [A] time = 1.92834, size = 48, normalized size = 1.09

$$\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(-\frac{2(x+1)}{2x+3}+1\right)}-\frac{\sqrt{2}\,\text{atanh}\left(\sqrt{2}\sqrt{\frac{x+1}{2x+3}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((1+x)/(3+2*x))**(1/2),x)
```

```
[Out] sqrt((x + 1)/(2*x + 3))/(2*(-2*(x + 1)/(2*x + 3) + 1)) - sqrt(2)*
atanh(sqrt(2)*sqrt((x + 1)/(2*x + 3)))/4
```

Mathematica [A] time = 0.0633515, size = 66, normalized size = 1.5

$$\frac{\sqrt{\frac{x+1}{2x+3}}\left(2\sqrt{x+1}(2x+3)-\sqrt{4x+6}\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)\right)}{4\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[(1 + x)/(3 + 2*x)]*(2*Sqrt[1 + x]*(3 + 2*x) - Sqrt[6 + 4*x]
*ArcSinh[Sqrt[2]*Sqrt[1 + x]]))/(4*Sqrt[1 + x])
```

Maple [B] time = 0.014, size = 75, normalized size = 1.7

$$-\frac{3+2\,x}{8}\sqrt{\frac{1+x}{3+2\,x}}\left(\ln\left(\frac{5\,\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2\,x^2+5\,x+3}\right)\sqrt{2}-4\,\sqrt{2\,x^2+5\,x+3}\right)\frac{1}{\sqrt{(3+2\,x)\,(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((1+x)/(3+2*x))^(1/2),x)
```

```
[Out] -1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x
^2+5*x+3)^(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(
1/2)
```

Maxima [A] time = 1.51942, size = 108, normalized size = 2.45

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-2\sqrt{\frac{x+1}{2x+3}}\right)}{2\sqrt{2}+4\sqrt{\frac{x+1}{2x+3}}}\right)-\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2(x+1)}{2x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/(2*x + 3)),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-2*(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/((2*sqrt
(2)) + 4*sqrt((x + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/
(2*(x + 1)/(2*x + 3) - 1)
```

Fricas [A] time = 0.209959, size = 82, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\left(2\sqrt{2}(2x+3)\sqrt{\frac{x+1}{2x+3}}+\log\left(-\sqrt{2}(4x+5)+4(2x+3)\sqrt{\frac{x+1}{2x+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/(2*x + 3)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2*sqrt(2)*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + log(-s
qrt(2)*(4*x + 5) + 4*(2*x + 3)*sqrt((x + 1)/(2*x + 3))))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x+1}{2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((1+x)/(3+2*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)
```

GIAC/XCAS [A] time = 0.214876, size = 82, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\ln\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+5x+3}\right)-5\right|\right)\mathrm{sign}(2x+3)+\frac{1}{2}\sqrt{2x^2+5x+3}\,\mathrm{sign}(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x + 1)/(2*x + 3)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*ln(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3))
 - 5))*sign(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sign(2*x + 3)
```

3.37 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0283531, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^2)^(5/2),x]

[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [A] time = 2.50033, size = 26, normalized size = 0.74

$$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(5/2),x)

[Out] x**3/(3*(-x**2 + 1)**(3/2)) - x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0448373, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0.009, size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}}+\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.50027, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(
-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.391992, size = 182, normalized size = 5.2

$$-\frac{12x^5 - 25x^3 + 6\left(x^6 - 6x^4 + 9x^2 + \left(3x^4 - 7x^2 + 4\right)\sqrt{-x^2+1} - 4\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \left(4x^5 - 19x^3 + 12x\right)\sqrt{-x^2+1}}{3\left(x^6 - 6x^4 + 9x^2 + (3x^4 - 7x^2 + 4)\sqrt{-x^2+1} - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 - 25*x^3 + 6*(x^6 - 6*x^4 + 9*x^2 + (3*x^4 - 7*x^2 +
 4)*sqrt(-x^2 + 1) - 4)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^5 -
 19*x^3 + 12*x)*sqrt(-x^2 + 1) + 12*x)/(x^6 - 6*x^4 + 9*x^2 + (3*
x^4 - 7*x^2 + 4)*sqrt(-x^2 + 1) - 4)
```

Sympy [A] time = 3.93425, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} - \frac{6x^2\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} + \frac{3\operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(-x**2 + 1)/(3*
x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*s
qrt(-x**2 + 1)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2
 + 3)
```

GIAC/XCAS [A] time = 0.207422, size = 39, normalized size = 1.11

$$\frac{\left(4x^2 - 3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.38 $\int \sqrt{x}(1+x)^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3
/2)*(1 + x)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sq
rt[x]])/64
```

Rubi [A] time = 0.0365901, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3
/2)*(1 + x)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sq
rt[x]])/64
```

Rubi in Sympy [A] time = 2.66932, size = 66, normalized size = 0.88

$$\frac{\sqrt{x}\,(x+1)^{\frac{7}{2}}}{4}-\frac{\sqrt{x}\,(x+1)^{\frac{5}{2}}}{24}-\frac{5\sqrt{x}\,(x+1)^{\frac{3}{2}}}{96}-\frac{5\sqrt{x}\sqrt{x+1}}{64}-\frac{5\,\mathrm{asinh}\left(\sqrt{x}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] sqrt(x)*(x + 1)**(7/2)/4 - sqrt(x)*(x + 1)**(5/2)/24 - 5*sqrt(x)*
(x + 1)**(3/2)/96 - 5*sqrt(x)*sqrt(x + 1)/64 - 5*asinh(sqrt(x))/6
4
```

Mathematica [A] time = 0.0395441, size = 41, normalized size = 0.55

$$\frac{1}{192}\left(\sqrt{x}\sqrt{x+1}\left(48x^3+136x^2+118x+15\right)-15\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3) - 15*ArcSinh
[Sqrt[x]])/192
```

Maple [A] time = 0.006, size = 70, normalized size = 0.9

$$\frac{1}{4}\sqrt{x}\,(1+x)^{\frac{7}{2}}-\frac{1}{24}\sqrt{x}\,(1+x)^{\frac{5}{2}}-\frac{5}{96}\sqrt{x}\,(1+x)^{\frac{3}{2}}-\frac{5}{64}\sqrt{x}\sqrt{1+x}$$
$$-\frac{5}{128}\sqrt{x\,(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(1+x)^(5/2),x)
```

```
[Out] 1/4*x^(1/2)*(1+x)^(7/2)-1/24*x^(1/2)*(1+x)^(5/2)-5/96*x^(1/2)*(1+
x)^(3/2)-5/64*x^(1/2)*(1+x)^(1/2)-5/128*(x*(1+x))^(1/2)/(1+x)^(1/
2)/x^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 1.34759, size = 153, normalized size = 2.04

$$\frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}}+\frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}}-\frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{15\,\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^{4}}{x^{4}}-\frac{4\,(x+1)^{3}}{x^{3}}+\frac{6\,(x+1)^{2}}{x^{2}}-\frac{4\,(x+1)}{x}+1\right)}-\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(5/2)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(
x + 1)^(3/2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4
*(x + 1)^3/x^3 + 6*(x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(s
qrt(x + 1)/sqrt(x) + 1) + 5/128*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 0.214251, size = 266, normalized size = 3.55

$$-\frac{98304\,x^{8}+524288\,x^{7}+1146880\,x^{6}+1294336\,x^{5}+788608\,x^{4}+250112\,x^{3}-8\left(12288\,x^{7}+59392\,x^{6}+115200\,x^{5}+110848\right.}{3072\left(12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(5/2)*sqrt(x),x, algorithm="fricas")
```

```
[Out] -1/3072*(98304*x^8 + 524288*x^7 + 1146880*x^6 + 1294336*x^5 + 788
608*x^4 + 250112*x^3 - 8*(12288*x^7 + 59392*x^6 + 115200*x^5 + 11
0848*x^4 + 54320*x^3 + 12744*x^2 + 1246*x + 35)*sqrt(x + 1)*sqrt(
x) + 37024*x^2 - 120*(128*x^4 + 256*x^3 - 8*(16*x^3 + 24*x^2 + 10
*x + 1)*sqrt(x + 1)*sqrt(x) + 160*x^2 + 32*x + 1)*log(2*sqrt(x +
1)*sqrt(x) - 2*x - 1) + 2080*x + 5)/(128*x^4 + 256*x^3 - 8*(16*x^
3 + 24*x^2 + 10*x + 1)*sqrt(x + 1)*sqrt(x) + 160*x^2 + 32*x + 1)
```

Sympy [A] time = 39.1646, size = 190, normalized size = 2.53

$$\begin{cases} -\frac{5\,\operatorname{acosh}\left(\sqrt{x+1}\right)}{64}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}}-\frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}}-\frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}}-\frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}}+\frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1|>1 \\ \frac{5i\,\operatorname{asin}\left(\sqrt{x+1}\right)}{64}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}}+\frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}}+\frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}}+\frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}}-\frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x))
- 7*(x + 1)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5
*(x + 1)**(3/2)/(192*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x
 + 1) > 1), (5*I*asin(sqrt(x + 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(
-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x)) + I*(x + 1)**(5/2)/(96*sq
rt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*sqrt(x + 1)/(64
```

```
*sqrt(-x)), True))
```

GIAC/XCAS [A] time = 0.28991, size = 111, normalized size = 1.48

$$\frac{1}{192}\,(2\,(4\,(6\,x-11)(x+1)+59)(x+1)-15)\sqrt{x+1}\sqrt{x}+\frac{1}{12}\,(2\,(4\,x-3)(x+1)+3)\sqrt{x+1}\sqrt{x}+\frac{1}{4}\,(2\,x+1)\sqrt{x+1}\sqrt{x}+\frac{5}{64}\,\ln\left(\left|-\sqrt{x+1}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)^(5/2)*sqrt(x),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 11)*(x + 1) + 59)*(x + 1) - 15)*sqrt(x + 1)*sq
rt(x) + 1/12*(2*(4*x - 3)*(x + 1) + 3)*sqrt(x + 1)*sqrt(x) + 1/4*
(2*x + 1)*sqrt(x + 1)*sqrt(x) + 5/64*ln(abs(-sqrt(x + 1) + sqrt(x
)))
```

3.39 $\int \frac{x^4}{(1-x^2)^{5/2}}\, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0283467, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [A] time = 2.47774, size = 26, normalized size = 0.74

$$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] x**3/(3*(-x**2 + 1)**(3/2)) - x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0105914, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}}+\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49307, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(
-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.213741, size = 182, normalized size = 5.2

$$-\frac{12x^5 - 25x^3 + 6\left(x^6 - 6x^4 + 9x^2 + \left(3x^4 - 7x^2 + 4\right)\sqrt{-x^2+1} - 4\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \left(4x^5 - 19x^3 + 12x\right)\sqrt{-x^2+1}}{3\left(x^6 - 6x^4 + 9x^2 + (3x^4 - 7x^2 + 4)\sqrt{-x^2+1} - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 - 25*x^3 + 6*(x^6 - 6*x^4 + 9*x^2 + (3*x^4 - 7*x^2 +
 4)*sqrt(-x^2 + 1) - 4)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^5 -
 19*x^3 + 12*x)*sqrt(-x^2 + 1) + 12*x)/(x^6 - 6*x^4 + 9*x^2 + (3*
x^4 - 7*x^2 + 4)*sqrt(-x^2 + 1) - 4)
```

Sympy [A] time = 4.1388, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} - \frac{6x^2\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} + \frac{3\operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(-x**2 + 1)/(3*
x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*s
qrt(-x**2 + 1)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2
 + 3)
```

GIAC/XCAS [A] time = 0.207361, size = 39, normalized size = 1.11

$$\frac{\left(4x^2 - 3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$

Optimal. Leaf size=51

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[
A^2 + B^2 - B^2*y^2]]
```

Rubi [A] time = 0.0773143, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[
A^2 + B^2 - B^2*y^2]]
```

Rubi in Sympy [A] time = 12.0107, size = 46, normalized size = 0.9

$$A\,\mathrm{atanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)+B\,\mathrm{atan}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

```
[Out] A*atanh(A*y/sqrt(A**2 - B**2*y**2 + B**2)) + B*atan(B*y/sqrt(A**2
 - B**2*y**2 + B**2))
```

Mathematica [C] time = 0.0324764, size = 134, normalized size = 2.63

$$iB\log\left(2\sqrt{A^2-B^2y^2+B^2}-2iBy\right)+\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right)$$
$$-\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2+B^2y+B^2\right)-\frac{1}{2}A\log(1-y)+\frac{1}{2}A\log(y+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] -(A*Log[1 - y])/2 + (A*Log[1 + y])/2 + I*B*Log[(-2*I)*B*y + 2*Sqr
t[A^2 + B^2 - B^2*y^2]] + (A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 +
 B^2 - B^2*y^2]])/2 - (A*Log[A^2 + B^2 + B^2*y + A*Sqrt[A^2 + B^2
 - B^2*y^2]])/2
```

Maple [B] time = 0.035, size = 262, normalized size = 5.1

$$-\frac{1}{2}\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}+\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}}\right)\frac{1}{\sqrt{B^2}}$$
$$+\frac{A^2}{2}\ln\left(\frac{1}{y-1}\left(2A^2-2B^2(y-1)+2\sqrt{A^2}\sqrt{-B^2(y-1)^2-2B^2(y-1)+A^2}\right)\right)\frac{1}{\sqrt{A^2}}$$
$$+\frac{1}{2}\sqrt{-B^2(1+y)^2+2B^2(1+y)+A^2}+\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2(1+y)^2+2B^2(1+y)+A^2}}\right)\frac{1}{\sqrt{B^2}}$$
$$-\frac{A^2}{2}\ln\left(\frac{1}{1+y}\left(2A^2+2B^2(1+y)+2\sqrt{A^2}\sqrt{-B^2(1+y)^2+2B^2(1+y)+A^2}\right)\right)\frac{1}{\sqrt{A^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y)
```

```
[Out] -1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arc
tan((B^2)^(1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))+1/2*A^2/(
A^2)^(1/2)*ln((2*A^2-2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^
2*(y-1)+A^2)^(1/2))/(y-1))+1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/
2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(1+y)^2+2*B^2*(
1+y)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B^2*(1+y)+2*(A^2
)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-B^2*y^2 + A^2 + B^2)/(y^2 - 1),y, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250368, size = 174, normalized size = 3.41

$$-B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)+\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2+2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)$$
$$-\frac{1}{4}A\log\left(-\frac{(A^2-B^2)y^2-2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-sqrt(-B^2*y^2 + A^2 + B^2)/(y^2 - 1),y, algorithm="fricas")
```

```
[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 -
B^2)*y^2 + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1
/4*A*log(-((A^2 - B^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A
^2 + B^2)/y^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sqrt{A^2-B^2y^2+B^2}}{y^2-1}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

```
[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sqrt(-B^2*y^2 + A^2 + B^2)/(y^2 - 1),y, algorithm="giac")
```

```
[Out] Timed out
```

3.41 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.00987212, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.487638, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00289777, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2,x)

[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.33867, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 0.221373, size = 14, normalized size = 1.

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.035787, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.236203, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.42 $\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$-B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)$$

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2*Sin[x]^2]]) - A*ArcTanh[(A*C
os[x])/Sqrt[A^2 + B^2*Sin[x]^2]]
```

Rubi [A] time = 0.1588, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTan
h[(A*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

Rubi in Sympy [A] time = 15.2472, size = 54, normalized size = 1.1

$$-A\,\text{atanh}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)-B\,\text{atan}\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

```
[Out] -A*atanh(A*cos(x)/sqrt(A**2 - B**2*cos(x)**2 + B**2)) - B*atan(B*
cos(x)/sqrt(A**2 - B**2*cos(x)**2 + B**2))
```

Mathematica [B] time = 0.132149, size = 99, normalized size = 2.02

$$\sqrt{-B^2}\log\left(\sqrt{2A^2-B^2\cos(2x)+B^2}+\sqrt{2}\sqrt{-B^2}\cos(x)\right)-\sqrt{A^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2-B^2\cos(2x)+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 -
 B^2*Cos[2*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqr
t[2*A^2 + B^2 - B^2*Cos[2*x]]]
```

Maple [C] time = 0.156, size = 149, normalized size = 3.

$$-\frac{1}{2\cos(x)}\sqrt{\left(A^2+B^2(\sin(x))^2\right)(\cos(x))^2}\left(Acsgn(A)\ln\left(-\frac{1}{(\sin(x))^2}\left(A^2(\sin(x))^2-B^2(\sin(x))^2-2\,csgn(A)A\sqrt{\left(A^2+B^2(s\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x)
```

```
[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(
x)^2-B^2*sin(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)
-2*A^2)/sin(x)^2)-B*csgn(B)*arctan(1/2*(2*B^2*sin(x)^2+A^2-B^2)/B
*csgn(B)/((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)))/cos(x)/(A^2+B^2*si
n(x)^2)^(1/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.330358, size = 378, normalized size = 7.71

$$-\frac{1}{2}B\arctan\left(-\frac{\left(4B^3\cos(x)^3-3\left(A^2B+B^3\right)\cos(x)\right)\sin(x)+\left(4B^2\cos(x)^3-\left(A^2+3B^2\right)\cos(x)\right)\sqrt{-B^2\cos(x)^2+A^2+B^2}}{4B^3\cos(x)^4+A^2B+B^3-\left(3A^2B+5B^3\right)\cos(x)^2-\left(4B^2\cos(x)^2-A^2-B^2\right)\sqrt{-B^2\cos(x)^2+A^2+B^2}\sin(x)}\right)$$
$$-\frac{1}{2}B\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$
$$-\frac{1}{2}A\log\left(-B^2\cos(x)^2+AB\cos(x)\sin(x)+A^2+B^2+\sqrt{-B^2\cos(x)^2+A^2+B^2}(A\cos(x)+B\sin(x))\right)$$
$$+\frac{1}{2}A\log\left(-B^2\cos(x)^2-AB\cos(x)\sin(x)+A^2+B^2-\sqrt{-B^2\cos(x)^2+A^2+B^2}(A\cos(x)-B\sin(x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*B*arctan(-((4*B^3*cos(x)^3 - 3*(A^2*B + B^3)*cos(x))*sin(x)
+ (4*B^2*cos(x)^3 - (A^2 + 3*B^2)*cos(x))*sqrt(-B^2*cos(x)^2 + A^
2 + B^2))/(4*B^3*cos(x)^4 + A^2*B + B^3 - (3*A^2*B + 5*B^3)*cos(x
)^2 - (4*B^2*cos(x)^2 - A^2 - B^2)*sqrt(-B^2*cos(x)^2 + A^2 + B^2
)*sin(x))) - 1/2*B*arctan(sin(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^
2 + A*B*cos(x)*sin(x) + A^2 + B^2 + sqrt(-B^2*cos(x)^2 + A^2 + B^
2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(x)^2 - A*B*cos(x)*
sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) -
B*sin(x)))
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{B^2 \sin(x)^2 + A^2}}{\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x),x, algorithm="giac")

[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0154872, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x])^(-1),x]

[Out] Sin[x]/(1 + Cos[x])
```

Rubi in Sympy [A] time = 0.482682, size = 7, normalized size = 0.78

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+cos(x)),x)

[Out] sin(x)/(cos(x) + 1)
```

Mathematica [A] time = 0.00588993, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x])^(-1),x]

[Out] Tan[x/2]
```

Maple [A] time = 0., size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+cos(x)),x)

[Out] tan(1/2*x)
```

Maxima [A] time = 1.42482, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 0.21069, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.211935, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

GIAC/XCAS [A] time = 0.206499, size = 41, normalized size = 4.56

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{(x^2+1)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + 1),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.44 $\int e^x x \, dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

```
[Out] -E^x + E^x*x
```

Rubi [A] time = 0.0110612, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$e^x x - e^x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*x,x]

[Out] -E^x + E^x*x
```

Rubi in Sympy [A] time = 1.01764, size = 7, normalized size = 0.64

$$xe^x - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x,x)

[Out] x*exp(x) - exp(x)
```

Mathematica [A] time = 0.00170775, size = 7, normalized size = 0.64

$$e^x(x - 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*x,x]

[Out] E^x*(-1 + x)
```

Maple [A] time = 0., size = 7, normalized size = 0.6

$$(-1 + x)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x,x)

[Out] (-1+x)*exp(x)
```

Maxima [A] time = 1.39961, size = 8, normalized size = 0.73

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A] time = 0.228127, size = 8, normalized size = 0.73

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A] time = 0.058299, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

GIAC/XCAS [A] time = 0.226422, size = 8, normalized size = 0.73

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

3.45 $\int \frac{e^x x}{(1+x)^2} \, dx$

Optimal. Leaf size=9

$$\frac{e^x}{x+1}$$

```
[Out] E^x/(1 + x)
```

Rubi [A] time = 0.0382296, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x*x)/(1 + x)^2,x]

[Out] E^x/(1 + x)
```

Rubi in Sympy [A] time = 2.0026, size = 5, normalized size = 0.56

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*x/(1+x)**2,x)

[Out] exp(x)/(x + 1)
```

Mathematica [A] time = 0.00537475, size = 9, normalized size = 1.

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x*x)/(1 + x)^2,x]

[Out] E^x/(1 + x)
```

Maple [A] time = 0.005, size = 9, normalized size = 1.

$$\frac{e^x}{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*x/(1+x)^2,x)

[Out] exp(x)/(1+x)
```

Maxima [A] time = 1.34865, size = 11, normalized size = 1.22

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x/(x + 1)^2,x, algorithm="maxima")
```

```
[Out] e^x/(x + 1)
```

Fricas [A] time = 0.202348, size = 11, normalized size = 1.22

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x/(x + 1)^2,x, algorithm="fricas")
```

```
[Out] e^x/(x + 1)
```

Sympy [A] time = 0.076076, size = 5, normalized size = 0.56

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x/(1+x)**2,x)
```

```
[Out] exp(x)/(x + 1)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x/(x + 1)^2,x, algorithm="giac")
```

```
[Out] undef
```

3.46 $\int e^{x^2}\left(1+2x^2\right)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.0473044, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rubi in Sympy [A] time = 2.77753, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Mathematica [A] time = 0.00378284, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0.003, size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*(2*x^2+1),x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 1.3579, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 0.19915, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.067613, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

GIAC/XCAS [A] time = 0.208539, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.47 $\int e^{x^2}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi [A] time = 0.0052378, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi in Sympy [A] time = 0.495381, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\text{erfi}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

Mathematica [A] time = 0.00205717, size = 11, normalized size = 1.

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A] time = 0.002, size = 8, normalized size = 0.7

$$\frac{\mathit{erfi}\,(x)\,\sqrt{\pi}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2),x)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [A] time = 1.36038, size = 12, normalized size = 1.09

$$-\frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A] time = 0.218831, size = 9, normalized size = 0.82

$$\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A] time = 0.300879, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\mathrm{erfi}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

GIAC/XCAS [A] time = 0.215141, size = 12, normalized size = 1.09

$$\frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(-ix\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

3.48 $\int \frac{e^x}{x}\,dx$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(x)$$

```
[Out] ExpIntegralEi[x]
```

Rubi [A] time = 0.0144882, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Rubi in Sympy [A] time = 1.32033, size = 2, normalized size = 1.

$$\text{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/x,x)
```

```
[Out] Ei(x)
```

Mathematica [A] time = 0.00164759, size = 2, normalized size = 1.

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Maple [B] time = 0.003, size = 8, normalized size = 4.

$$-Ei\left(1,-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x*exp(x),x)
```

```
[Out] -Ei(1,-x)
```

Maxima [A] time = 1.42112, size = 3, normalized size = 1.5

$$\text{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/x,x, algorithm="maxima")
```

```
[Out] Ei(x)
```

Fricas [A] time = 0.211563, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/x,x, algorithm="fricas")
```

```
[Out] Ei(x)
```

Sympy [A] time = 1.2707, size = 2, normalized size = 1.

$$\mathrm{Ei}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/x,x)
```

```
[Out] Ei(x)
```

GIAC/XCAS [A] time = 0.22617, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/x,x, algorithm="giac")
```

```
[Out] Ei(x)
```

3.49 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x +
 x^2]/6
```

Rubi [A] time = 0.0439775, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x +
 x^2]/6
```

Rubi in Sympy [A] time = 3.18486, size = 37, normalized size = 0.9

$$-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(sqrt(3)*(2*x/3
 - 1/3))/3
```

Mathematica [A] time = 0.0128659, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x
^2]/6
```

Maple [A] time = 0., size = 35, normalized size = 0.9

$$-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x/(x^3+1),x)
```

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))
```

Maxima [A] time = 1.48578, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/(x^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1)
- 1/3*log(x + 1)
```

Fricas [A] time = 0.208438, size = 55, normalized size = 1.34

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\log\left(x^2-x+1\right)-2\sqrt{3}\log\left(x+1\right)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/(x^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*log(x^2 - x + 1) - 2*sqrt(3)*log(x + 1) + 6
*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.173209, size = 41, normalized size = 1.

$$-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.224054, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\ln\left(x^2-x+1\right)-\frac{1}{3}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(x/(x^3 + 1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*ln(x^2 - x + 1) -
 1/3*ln(abs(x + 1))
```

3.50 $\int \frac{1}{-1+x^6}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}}-\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -ArcTan[(Sqrt[3]*x)/(1 - x^2)]/(2*Sqrt[3]) - ArcTanh[x]/3 - ArcTa
nh[x/(1 + x^2)]/6
```

Rubi [A] time = 0.188862, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]
/(2*Sqrt[3]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x
^2]/12
```

Rubi in Sympy [A] time = 16.9956, size = 68, normalized size = 1.45

$$\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}-\frac{1}{3}\right)\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{6}-\frac{\mathrm{atanh}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-1),x)
```

```
[Out] log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 - sqrt(3)*atan(sqrt(3
)*(2*x/3 - 1/3))/6 - sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/6 - atan
h(x)/3
```

Mathematica [A] time = 0.0207816, size = 75, normalized size = 1.6

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)\right.$$
$$\left.-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*
x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Lo
g[1 + x + x^2])/12
```

Maple [A] time = 0., size = 66, normalized size = 1.4

$$\frac{\ln(-1+x)}{6}-\frac{\ln(x^2+x+1)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-1),x)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(
1/2)-1/6*ln(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))
```

Maxima [A] time = 1.527, size = 88, normalized size = 1.87

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x
+ 1) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Fricas [A] time = 0.203179, size = 101, normalized size = 2.15

$$-\frac{1}{36}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)-\sqrt{3}\log\left(x^2-x+1\right)+2\sqrt{3}\log(x+1)-2\sqrt{3}\log(x-1)+6\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+6\text{ arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) - sqrt(3)*log(x^2 - x + 1
) + 2*sqrt(3)*log(x + 1) - 2*sqrt(3)*log(x - 1) + 6*arctan(1/3*sq
rt(3)*(2*x + 1)) + 6*arctan(1/3*sqrt(3)*(2*x - 1)))
```

Sympy [A] time = 0.358155, size = 83, normalized size = 1.77

$$\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}+\frac{\log\left(x^2-x+1\right)}{12}-\frac{\log\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x
 + 1)/12 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*at
```

```
an(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

GIAC/XCAS [A] time = 0.225219, size = 90, normalized size = 1.91

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\ln\left(x^2+x+1\right)+\frac{1}{12}\ln\left(x^2-x+1\right)-\frac{1}{6}\ln\left(|x+1|\right)+\frac{1}{6}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^6 - 1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1
/3*sqrt(3)*(2*x - 1)) - 1/12*ln(x^2 + x + 1) + 1/12*ln(x^2 - x +
1) - 1/6*ln(abs(x + 1)) + 1/6*ln(abs(x - 1))
```

3.51 $\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)x^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Rubi [A] time = 0.0202066, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Rubi in Sympy [A] time = 1.3105, size = 12, normalized size = 0.57

$$\frac{\operatorname{atanh}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)

[Out] atanh(x/A)/(A*(A**2 - B**2))
```

Mathematica [A] time = 0.00642334, size = 21, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Maple [B] time = 0.009, size = 44, normalized size = 2.1

$$-\frac{\ln\left(A-x\right)}{\left(2A^2-2B^2\right)A}+\frac{\ln\left(A+x\right)}{\left(2A^2-2B^2\right)A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x)

[Out] -1/2/(A^2-B^2)/A*ln(A-x)+1/2/(A^2-B^2)/A*ln(A+x)
```

Maxima [A] time = 1.34928, size = 53, normalized size = 2.52

$$\frac{\log(A+x)}{2\,(A^3-AB^2)}-\frac{\log(-A+x)}{2\,(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(A^4 - A^2*B^2 - (A^2 - B^2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

Fricas [A] time = 0.198048, size = 36, normalized size = 1.71

$$\frac{\log(A+x)-\log(-A+x)}{2\,(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(A^4 - A^2*B^2 - (A^2 - B^2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

Sympy [A] time = 0.399846, size = 70, normalized size = 3.33

$$-\frac{\log\left(-\frac{A^3}{(A-B)(A+B)}+\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A\,(A-B)\,(A+B)}+\frac{\log\left(\frac{A^3}{(A-B)(A+B)}-\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A\,(A-B)\,(A+B)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

```
[Out] -log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A
*(A - B)*(A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*
(A + B)) + x)/(2*A*(A - B)*(A + B))
```

GIAC/XCAS [A] time = 0.209492, size = 55, normalized size = 2.62

$$\frac{\ln(|A+x|)}{2\,(A^3-AB^2)}-\frac{\ln(|-A+x|)}{2\,(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(A^4 - A^2*B^2 - (A^2 - B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(A + x))/(A^3 - A*B^2) - 1/2*ln(abs(-A + x))/(A^3 - A*B
^2)
```

3.52 $\int x \log(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.00835924, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)}{2}-\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.00116538, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x),x)

[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 12.1083, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="maxima")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.201786, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="fricas")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.070298, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - x**2/4
```

GIAC/XCAS [A] time = 0.233025, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\ln\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="giac")

[Out] 1/2*x^2*ln(x) - 1/4*x^2
```

3.53 $\int x^2 \sin^{-1}(x)\,dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi [A] time = 0.0485389, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcSin[x],x]

[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi in Sympy [A] time = 3.11959, size = 27, normalized size = 0.68

$$\frac{x^3\operatorname{asin}(x)}{3}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{9}+\frac{\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*asin(x),x)

[Out] x**3*asin(x)/3 - (-x**2 + 1)**(3/2)/9 + sqrt(-x**2 + 1)/3
```

Mathematica [A] time = 0.020374, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(3x^3\sin^{-1}(x)+\sqrt{1-x^2}\left(x^2+2\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcSin[x],x]

[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A] time = 0., size = 34, normalized size = 0.9

$$\frac{x^3\arcsin(x)}{3}+\frac{x^2}{9}\sqrt{-x^2+1}+\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arcsin(x),x)

[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49809, size = 45, normalized size = 1.12

$$\frac{1}{3}\,x^3\arcsin\left(x\right) + \frac{1}{9}\,\sqrt{-x^2+1}x^2 + \frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Fricas [A] time = 0.220537, size = 32, normalized size = 0.8

$$\frac{1}{3}\,x^3\arcsin\left(x\right) + \frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.461047, size = 32, normalized size = 0.8

$$\frac{x^3\operatorname{asin}\left(x\right)}{3} + \frac{x^2\sqrt{-x^2+1}}{9} + \frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(-x**2 + 1)/9 + 2*sqrt(-x**2 + 1)/9
```

GIAC/XCAS [A] time = 0.211564, size = 51, normalized size = 1.27

$$\frac{1}{3}\,\left(x^2-1\right)x\arcsin\left(x\right) + \frac{1}{3}\,x\arcsin\left(x\right) - \frac{1}{9}\,\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{3}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2
) + 1/3*sqrt(-x^2 + 1)
```

3.54 $\int \frac{1}{1+2x+x^2}\,dx$

Optimal. Leaf size=7

$$-\frac{1}{x+1}$$

```
[Out] -(1 + x)^(-1)
```

Rubi [A] time = 0.00452872, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2)^(-1),x]

[Out] -(1 + x)^(-1)
```

Rubi in Sympy [A] time = 0.561463, size = 15, normalized size = 2.14

$$-\frac{2x+2}{2\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**2+2*x+1),x)

[Out] -(2*x + 2)/(2*(x**2 + 2*x + 1))
```

Mathematica [A] time = 0.00118298, size = 7, normalized size = 1.

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)^(-1),x]

[Out] -(1 + x)^(-1)
```

Maple [A] time = 0.001, size = 8, normalized size = 1.1

$$-\left(1+x\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+1),x)

[Out] -1/(1+x)
```

Maxima [A] time = 1.32656, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x + 1),x, algorithm="maxima")
```

```
[Out] -1/(x + 1)
```

Fricas [A] time = 0.208182, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x + 1),x, algorithm="fricas")
```

```
[Out] -1/(x + 1)
```

Sympy [A] time = 0.061913, size = 5, normalized size = 0.71

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+2*x+1),x)
```

```
[Out] -1/(x + 1)
```

GIAC/XCAS [A] time = 0.226792, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x + 1),x, algorithm="giac")
```

```
[Out] -1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2} \, dx$

Optimal. Leaf size=8

$$\frac{x}{\log(x)+1}$$

```
[Out] x/(1 + Log[x])
```

Rubi [A] time = 0.0632274, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]/(1 + Log[x])^2,x]

[Out] x/(1 + Log[x])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{(\log(x)+1)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)/(1+ln(x))**2,x)

[Out] Integral(log(x)/(log(x) + 1)**2, x)
```

Mathematica [A] time = 0.00512997, size = 8, normalized size = 1.

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]/(1 + Log[x])^2,x]

[Out] x/(1 + Log[x])
```

Maple [A] time = 0.051, size = 9, normalized size = 1.1

$$\frac{x}{1+\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/(1+ln(x))^2,x)

[Out] x/(1+ln(x))
```

Maxima [A] time = 1.45072, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(log(x) + 1)^2,x, algorithm="maxima")
```

```
[Out] x/(log(x) + 1)
```

Fricas [A] time = 0.21709, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(log(x) + 1)^2,x, algorithm="fricas")
```

```
[Out] x/(log(x) + 1)
```

Sympy [A] time = 0.077185, size = 5, normalized size = 0.62

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/(1+ln(x))**2,x)
```

```
[Out] x/(log(x) + 1)
```

GIAC/XCAS [A] time = 0.227057, size = 11, normalized size = 1.38

$$\frac{x}{\ln(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(log(x) + 1)^2,x, algorithm="giac")
```

```
[Out] x/(ln(x) + 1)
```

3.56 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0281508, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rubi in Sympy [A] time = 3.26357, size = 3, normalized size = 1.

$$\operatorname{atan}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] atan(log(x))
```

Mathematica [A] time = 0.00575297, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.005, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.50924, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 0.220967, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [A] time = 0.141512, size = 15, normalized size = 5.

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log(2i+\log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

GIAC/XCAS [A] time = 0.231733, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="giac")
```

```
[Out] arctan(ln(x))
```

3.57 $\int \frac{1}{\log(x)} \, dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] LogIntegral[x]
```

Rubi [A] time = 0.00402891, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rubi in Sympy [A] time = 0.025707, size = 2, normalized size = 1.

$$\text{li}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/ln(x),x)
```

```
[Out] li(x)
```

Mathematica [A] time = 0.00289905, size = 2, normalized size = 1.

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Maple [B] time = 0., size = 9, normalized size = 4.5

$$-Ei\,(1, -\ln\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A] time = 1.41474, size = 4, normalized size = 2.

$$\text{Ei}\,(\log\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$log_i ntegral\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/ln(x),x)
```

```
[Out] Integral(1/log(x), x)
```

GIAC/XCAS [A] time = 0.214497, size = 4, normalized size = 2.

$$\operatorname{Ei}\left(\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(ln(x))
```

3.58 $\int x(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=14

$$x\sin(x) + \sin(x) - x\cos(x) + \cos(x)$$

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rubi [A] time = 0.0318898, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$

$$x\sin(x) + \sin(x) - x\cos(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x] + Sin[x]),x]

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\sin\left(x\right) + \cos\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(cos(x)+sin(x)),x)

[Out] Integral(x*(sin(x) + cos(x)), x)
```

Mathematica [A] time = 0.00771415, size = 14, normalized size = 1.

$$x\sin(x) + \sin(x) - x\cos(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x] + Sin[x]),x]

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Maple [A] time = 0.016, size = 15, normalized size = 1.1

$$\cos\left(x\right) - x\cos\left(x\right) + \sin\left(x\right) + x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x)+sin(x)),x)

[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Maxima [A] time = 1.35785, size = 19, normalized size = 1.36

$$-x\cos\left(x\right) + x\sin\left(x\right) + \cos\left(x\right) + \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(cos(x) + sin(x)),x, algorithm="maxima")

[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Fricas [A] time = 0.208738, size = 19, normalized size = 1.36

$$-(x - 1)\cos(x) + (x + 1)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(cos(x) + sin(x)),x, algorithm="fricas")

[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)
```

Sympy [A] time = 0.191208, size = 15, normalized size = 1.07

$$x\sin(x) - x\cos(x) + \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(cos(x)+sin(x)),x)

[Out] x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

GIAC/XCAS [A] time = 0.222319, size = 19, normalized size = 1.36

$$-x\cos(x) + x\sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(cos(x) + sin(x)),x, algorithm="giac")

[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

3.59 $\int e^{-x} (e^x + x) \, dx$

Optimal. Leaf size=17

$$-e^{-x}x + x - e^{-x}$$

```
[Out] -E^(-x) + x - x/E^x
```

Rubi [A] time = 0.050053, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-e^{-x}x + x - e^{-x}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x + x)/E^x,x]

[Out] -E^(-x) + x - x/E^x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (x + e^x) e^{-x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x+exp(x))/exp(x),x)

[Out] Integral((x + exp(x))*exp(-x), x)
```

Mathematica [A] time = 0.00978476, size = 13, normalized size = 0.76

$$e^{-x}(-x - 1) + x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x + x)/E^x,x]

[Out] (-1 - x)/E^x + x
```

Maple [A] time = 0.003, size = 16, normalized size = 0.9

$$-\left(e^x\right)^{-1} + x - \frac{x}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((exp(x)+x)/exp(x),x)

[Out] -1/exp(x)+x-x/exp(x)
```

Maxima [A] time = 1.35165, size = 15, normalized size = 0.88

$$-(x+1)e^{(-x)}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)*e^(-x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + x
```

Fricas [A] time = 0.220337, size = 19, normalized size = 1.12

$$(xe^x-x-1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)*e^(-x),x, algorithm="fricas")
```

```
[Out] (x*e^x - x - 1)*e^(-x)
```

Sympy [A] time = 0.070081, size = 8, normalized size = 0.47

$$x+(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x+exp(x))/exp(x),x)
```

```
[Out] x + (-x - 1)*exp(-x)
```

GIAC/XCAS [A] time = 0.228953, size = 15, normalized size = 0.88

$$-(x+1)e^{(-x)}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + e^x)*e^(-x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + x
```

3.60 $\int (1+e^x)^2 x \, dx$

Optimal. Leaf size=38

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rubi [A] time = 0.049431, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + E^x)^2*x,x]
```

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{2x}}{2}+2xe^x-\frac{e^{2x}}{4}-2e^x+\int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+exp(x))**2*x,x)
```

```
[Out] x*exp(2*x)/2 + 2*x*exp(x) - exp(2*x)/4 - 2*exp(x) + Integral(x, x
)
```

Mathematica [A] time = 0.0162913, size = 29, normalized size = 0.76

$$\frac{1}{4}\left(2x^2+8e^x(x-1)+e^{2x}(2x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + E^x)^2*x,x]
```

```
[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

Maple [A] time = 0.002, size = 29, normalized size = 0.8

$$\frac{x^2}{2}+\frac{(e^x)^2 x}{2}-\frac{(e^x)^2}{4}+2e^x x-2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+exp(x))^2*x,x)
```

```
[Out] 1/2*x^2+1/2*exp(x)^2*x-1/4*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A] time = 1.35686, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x - 1)e^{(2x)} + 2(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e^x + 1)^2,x, algorithm="maxima")

[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Fricas [A] time = 0.213292, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x - 1)e^{(2x)} + 2(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e^x + 1)^2,x, algorithm="fricas")

[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Sympy [A] time = 0.084472, size = 26, normalized size = 0.68

$$\frac{x^2}{2} + \frac{(2x - 1)e^{2x}}{4} + \frac{(8x - 8)e^x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+exp(x))**2*x,x)

[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

GIAC/XCAS [A] time = 0.214277, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x - 1)e^{(2x)} + 2(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e^x + 1)^2,x, algorithm="giac")

[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.61 $\int x \cos(x) \, dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0146882, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rubi in Sympy [A] time = 0.766561, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Mathematica [A] time = 0.003787, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 1.34075, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 0.218749, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.180562, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

GIAC/XCAS [A] time = 0.205832, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.62 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0174071, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi in Sympy [A] time = 2.61152, size = 48, normalized size = 2.18

$$-i\sqrt{x}e^{i\sqrt{x}}+i\sqrt{x}e^{-i\sqrt{x}}+e^{i\sqrt{x}}+e^{-i\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x**(1/2)),x)
```

```
[Out] -I*sqrt(x)*exp(I*sqrt(x)) + I*sqrt(x)*exp(-I*sqrt(x)) + exp(I*sqr
t(x)) + exp(-I*sqrt(x))
```

Mathematica [A] time = 0.0102961, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\cos\left(\sqrt{x}\right)+2\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.34485, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="maxima")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 0.23025, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="fricas")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.401382, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x**(1/2)),x)

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

GIAC/XCAS [A] time = 0.228569, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(sqrt(x)),x, algorithm="giac")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.63 $\int x \cos(x)\,dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0137842, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]
```

Rubi in Sympy [A] time = 0.783996, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*cos(x),x)

[Out] x*sin(x) + cos(x)
```

Mathematica [A] time = 0.00304976, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)

[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 1.31274, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 0.232934, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.18133, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

GIAC/XCAS [A] time = 0.226113, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.64 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.016315, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)^2}{2}-\frac{x^2\log(x)}{2}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + Integral(x, x)/2
```

Mathematica [A] time = 0.00186486, size = 28, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0., size = 23, normalized size = 0.8

$$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 1.3461, size = 23, normalized size = 0.82

$$\frac{1}{4}\left(2\,\log\left(x\right)^{2}-2\,\log\left(x\right)+1\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 0.225426, size = 30, normalized size = 1.07

$$\frac{1}{2}\,x^{2}\log\left(x\right)^{2}-\frac{1}{2}\,x^{2}\log\left(x\right)+\frac{1}{4}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.092948, size = 22, normalized size = 0.79

$$\frac{x^{2}\log\left(x\right)^{2}}{2}-\frac{x^{2}\log\left(x\right)}{2}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

GIAC/XCAS [A] time = 0.210396, size = 30, normalized size = 1.07

$$\frac{1}{2}\,x^{2}\ln\left(x\right)^{2}-\frac{1}{2}\,x^{2}\ln\left(x\right)+\frac{1}{4}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x)^2 - 1/2*x^2*ln(x) + 1/4*x^2
```

3.65 $\int \cos(x)\left(1+\sin^3(x)\right)\,dx$

Optimal. Leaf size=11

$$\frac{\sin^4(x)}{4}+\sin(x)$$

```
[Out] Sin[x] + Sin[x]^4/4
```

Rubi [A] time = 0.0190195, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]*(1 + Sin[x]^3),x]

[Out] Sin[x] + Sin[x]^4/4
```

Rubi in Sympy [A] time = 1.17017, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)*(1+sin(x)**3),x)

[Out] sin(x)**4/4 + sin(x)
```

Mathematica [A] time = 0.00484902, size = 11, normalized size = 1.

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]*(1 + Sin[x]^3),x]

[Out] Sin[x] + Sin[x]^4/4
```

Maple [A] time = 0.023, size = 10, normalized size = 0.9

$$\sin(x)+\frac{(\sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)*(1+sin(x)^3),x)

[Out] sin(x)+1/4*sin(x)^4
```

Maxima [A] time = 1.34406, size = 12, normalized size = 1.09

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^3 + 1)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

Fricas [A] time = 0.238069, size = 20, normalized size = 1.82

$$\frac{1}{4}\cos(x)^4 - \frac{1}{2}\cos(x)^2 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^3 + 1)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

Sympy [A] time = 0.81619, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*(1+sin(x)**3),x)
```

```
[Out] sin(x)**4/4 + sin(x)
```

GIAC/XCAS [A] time = 0.206661, size = 12, normalized size = 1.09

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((sin(x)^3 + 1)*cos(x),x, algorithm="giac")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

3.66 $\int \frac{1}{x(1+\log^2(x))}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.028675, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rubi in Sympy [A] time = 3.18181, size = 3, normalized size = 1.

$$\operatorname{atan}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] atan(log(x))
```

Mathematica [A] time = 0.00528036, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.002, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.49664, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 0.22253, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [A] time = 0.141465, size = 15, normalized size = 5.

$$\operatorname{RootSum}\left(4z^2 + 1, \left(i \mapsto i\log(2i + \log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

GIAC/XCAS [A] time = 0.20646, size = 4, normalized size = 1.33

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((log(x)^2 + 1)*x),x, algorithm="giac")
```

```
[Out] arctan(ln(x))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}\left(1+\sin^{-1}(x)^2\right)} dx$

Optimal. Leaf size=3

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

```
[Out] ArcTan[ArcSin[x]]
```

Rubi [A] time = 0.0746184, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]

[Out] ArcTan[ArcSin[x]]
```

Rubi in Sympy [A] time = 4.84677, size = 3, normalized size = 1.

$$\operatorname{atan}\left(\operatorname{asin}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)

[Out] atan(asin(x))
```

Mathematica [A] time = 0.00707962, size = 3, normalized size = 1.

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]

[Out] ArcTan[ArcSin[x]]
```

Maple [A] time = 0.009, size = 4, normalized size = 1.3

$$\arctan\left(\arcsin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x)

[Out] arctan(arcsin(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\left(\arcsin\left(x\right)^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

Fricas [A] time = 0.230431, size = 4, normalized size = 1.33

$$\arctan\left(\arcsin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)),x, algorithm="fricas")

[Out] arctan(arcsin(x))
```

Sympy [A] time = 0.714087, size = 3, normalized size = 1.

$$\operatorname{atan}\left(\operatorname{asin}\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)

[Out] atan(asin(x))
```

GIAC/XCAS [A] time = 0.212357, size = 4, normalized size = 1.33

$$\arctan\left(\arcsin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)),x, algorithm="giac")

[Out] arctan(arcsin(x))
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rubi [A] time = 0.0580929, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]

[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rubi in Sympy [A] time = 4.64384, size = 12, normalized size = 0.75

$$\frac{x}{2}-\frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)/(cos(x)+sin(x)),x)

[Out] x/2 - log(sin(x) + cos(x))/2
```

Mathematica [A] time = 0.0108244, size = 16, normalized size = 1.

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]

[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.036, size = 21, normalized size = 1.3

$$\frac{\ln\left(1+(\tan(x))^2\right)}{4}-\frac{\ln\left(1+\tan(x)\right)}{2}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+sin(x)),x)

[Out] 1/4*ln(1+tan(x)^2)-1/2*ln(1+tan(x))+1/2*x
```

Maxima [A] time = 1.50212, size = 72, normalized size = 4.5

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{2}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + si
n(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.226001, size = 20, normalized size = 1.25

$$\frac{1}{2}x - \frac{1}{4}\log(2\cos(x)\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.157405, size = 12, normalized size = 0.75

$$\frac{x}{2} - \frac{\log(\sin(x) + \cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/2
```

GIAC/XCAS [A] time = 0.232057, size = 28, normalized size = 1.75

$$\frac{1}{2}x + \frac{1}{4}\ln\left(\tan(x)^2 + 1\right) - \frac{1}{2}\ln(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*ln(tan(x)^2 + 1) - 1/2*ln(abs(tan(x) + 1))
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2}\,dy$

Optimal. Leaf size=53

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sq
rt[A^2 + B^2 - B^2*y^2]]
```

Rubi [A] time = 0.128817, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sq
rt[A^2 + B^2 - B^2*y^2]]
```

Rubi in Sympy [A] time = 14.4245, size = 48, normalized size = 0.91

$$-A\,\text{atanh}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)-B\,\text{atan}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] -A*atanh(A*y/sqrt(A**2 - B**2*y**2 + B**2)) - B*atan(B*y/sqrt(A**
2 - B**2*y**2 + B**2))
```

Mathematica [C] time = 0.0864869, size = 127, normalized size = 2.4

$$\frac{1}{2}\left(-2iB\log\left(2\left(\sqrt{A^2-B^2y^2+B^2}-iBy\right)\right)-A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right)\right.$$
$$\left.+A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2+B^2(y+1)\right)+A\log(1-y)-A\log(y+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] (A*Log[1 - y] - A*Log[1 + y] - (2*I)*B*Log[2*((-I)*B*y + Sqrt[A^2
 + B^2 - B^2*y^2])] - A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2
- B^2*y^2]] + A*Log[A^2 + B^2*(1 + y) + A*Sqrt[A^2 + B^2 - B^2*y^
2]])/2
```

Maple [B] time = 0.016, size = 262, normalized size = 4.9

$$\frac{1}{2}\sqrt{-B^2\,(y-1)^2-2\,B^2\,(y-1)+A^2}-\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2\,(y-1)^2-2\,B^2\,(y-1)+A^2}}\right)\frac{1}{\sqrt{B^2}}$$
$$-\frac{A^2}{2}\ln\left(\frac{1}{y-1}\left(2\,A^2-2\,B^2\,(y-1)+2\,\sqrt{A^2}\sqrt{-B^2\,(y-1)^2-2\,B^2\,(y-1)+A^2}\right)\right)\frac{1}{\sqrt{A^2}}$$
$$-\frac{1}{2}\sqrt{-B^2\,(1+y)^2+2\,B^2\,(1+y)+A^2}-\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2\,(1+y)^2+2\,B^2\,(1+y)+A^2}}\right)\frac{1}{\sqrt{B^2}}$$
$$+\frac{A^2}{2}\ln\left(\frac{1}{1+y}\left(2\,A^2+2\,B^2\,(1+y)+2\,\sqrt{A^2}\sqrt{-B^2\,(1+y)^2+2\,B^2\,(1+y)+A^2}\right)\right)\frac{1}{\sqrt{A^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y)
```

```
[Out] 1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arct
an((B^2)^(1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))-1/2*A^2/(A
^2)^(1/2)*ln((2*A^2-2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2
*(y-1)+A^2)^(1/2))/(y-1))-1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2
)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(1+y)^2+2*B^2*(1
+y)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B^2*(1+y)+2*(A^2)
^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(y^2 - 1)*B^2 + A^2)/(y^2 - 1),y, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.243198, size = 173, normalized size = 3.26

$$B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)-\frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y^2+2\,\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)$$
$$+\frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y^2-2\,\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(y^2 - 1)*B^2 + A^2)/(y^2 - 1),y, algorithm="fricas")
```

```
[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B
^2)*y^2 + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/
4*A*log(-((A^2 - B^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^
2 + B^2)/y^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{A^2-B^2y^2+B^2}}{(y-1)\,(y+1)}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(y^2 - 1)*B^2 + A^2)/(y^2 - 1),y, algorithm="giac")
```

```
[Out] Timed out
```

3.70 $\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)}\,dz$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)-Bz$$

```
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rubi [A] time = 0.135927, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128

$$-A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)-Bz$$

Antiderivative was successfully verified.

```
[In]  Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rubi in Sympy [A] time = 14.0361, size = 17, normalized size = 1.06

$$-A\,\mathrm{atanh}\left(\frac{A\tan(z)}{B}\right)-B\,\mathrm{atan}(\tan(z))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] -A*atanh(A*tan(z)/B) - B*atan(tan(z))
```

Mathematica [B] time = 0.0901274, size = 35, normalized size = 2.19

$$-\frac{B(A^2+B^2)\left(A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)+Bz\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

Maple [B] time = 0.056, size = 127, normalized size = 7.9

$$-\frac{B\arctan(\tan(z))A^2}{A^2+B^2}-\frac{\arctan(\tan(z))B^3}{A^2+B^2}+\frac{A^3\ln(A\tan(z)-B)}{2A^2+2B^2}+\frac{AB^2\ln(A\tan(z)-B)}{2A^2+2B^2}-\frac{A^3\ln(A\tan(z)+B)}{2A^2+2B^2}-\frac{AB^2\ln(A\tan(z)+B)}{2A^2+2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z)
```

```
[Out] -B/(A^2+B^2)*arctan(tan(z))*A^2-1/(A^2+B^2)*arctan(tan(z))*B^3+1/
2*A^3/(A^2+B^2)*ln(A*tan(z)-B)+1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)-B)
-1/2*A^3/(A^2+B^2)*ln(A*tan(z)+B)-1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)
+B)
```

Maxima [A] time = 1.50919, size = 93, normalized size = 5.81

$$-\frac{\left(A^2+B^2\right)\left(\frac{2\,B^2z}{A^2+B^2}+\frac{AB\log(A\tan(z)+B)}{A^2+B^2}-\frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*cos(z)^2/(B*((A^2 + B^2)*sin(z)^2/B^2 - 1)),z, algorithm="
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^
2 + B^2) - A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

Fricas [A] time = 0.270585, size = 90, normalized size = 5.62

$$-Bz-\frac{1}{4}\,A\log\left(2\,AB\cos\left(z\right)\sin\left(z\right)-\left(A^2-B^2\right)\cos\left(z\right)^2+A^2\right)+\frac{1}{4}\,A\log\left(-2\,AB\cos\left(z\right)\sin\left(z\right)-\left(A^2-B^2\right)\cos\left(z\right)^2+A^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*cos(z)^2/(B*((A^2 + B^2)*sin(z)^2/B^2 - 1)),z, algorithm="
```

```
[Out] -B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2
) + 1/4*A*log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.256508, size = 112, normalized size = 7.

$$-\frac{\left(\frac{A^3B\ln(|A\tan(z)+B|)}{A^4+A^2B^2}-\frac{A^3B\ln(|A\tan(z)-B|)}{A^4+A^2B^2}+\frac{2\,B^2z}{A^2+B^2}\right)\left(A^2+B^2\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*cos(z)^2/(B*((A^2 + B^2)*sin(z)^2/B^2 - 1)),z, algorithm="
```

```
[Out] -1/2*(A^3*B*ln(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*ln(abs(
A*tan(z) - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)
/B
```

3.71 $\int -\frac{A^2+B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi [A] time = 0.199218, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

```
[In] Int[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))),w]

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(A^2+B^2)\int\frac{1}{\left(1-\frac{w^2(A^2+B^2)}{B^2(w^2+1)}\right)(w^2+1)^2}\,dw}{B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w

[Out] -(A**2 + B**2)*Integral(1/((1 - w**2*(A**2 + B**2)/(B**2*(w**2 + 
1)))*(w**2 + 1)**2), w)/B
```

Mathematica [B] time = 0.0277822, size = 35, normalized size = 2.19

$$-\frac{B(A^2+B^2)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))),

[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3
))
```

Maple [B] time = 0.012, size = 121, normalized size = 7.6

$$-\frac{B\arctan(w)A^2}{A^2+B^2}-\frac{\arctan(w)B^3}{A^2+B^2}+\frac{A^3\ln(Aw-B)}{2A^2+2B^2}+\frac{AB^2\ln(Aw-B)}{2A^2+2B^2}-\frac{A^3\ln(Aw+B)}{2A^2+2B^2}-\frac{AB^2\ln(Aw+B)}{2A^2+2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w)
```

```
[Out] -B/(A^2+B^2)*arctan(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3+1/2*A^3/(A^2
+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-1/2*A^3/(A^2+B^2)*l
n(A*w+B)-1/2*A*B^2/(A^2+B^2)*ln(A*w+B)
```

Maxima [A] time = 1.50595, size = 92, normalized size = 5.75

$$-\frac{\left(A^2+B^2\right)\left(\frac{2\,B^2\arctan(w)}{A^2+B^2}+\frac{AB\log(Aw+B)}{A^2+B^2}-\frac{AB\log(Aw-B)}{A^2+B^2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A^2 + B^2)/((w^2 + 1)^2*B*((A^2 + B^2)*w^2/((w^2 + 1)*B^2) - 1)),w, alg
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/
(A^2 + B^2) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

Fricas [A] time = 0.214495, size = 35, normalized size = 2.19

$$-B\arctan\left(w\right)-\frac{1}{2}\,A\log\left(Aw+B\right)+\frac{1}{2}\,A\log\left(Aw-B\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((A^2 + B^2)/((w^2 + 1)^2*B*((A^2 + B^2)*w^2/((w^2 + 1)*B^2) - 1)),w, alg
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [A] time = 2.32441, size = 422, normalized size = 26.38

$$\left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B\left(A^2+B^2\right)^3}-\frac{A^7B}{\left(A^2+B^2\right)^3}+\frac{A^5B^3}{\left(A^2+B^2\right)^3}+\frac{A^5}{B\left(A^2+B^2\right)}+\frac{A^3B^5}{\left(A^2+B^2\right)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{A\log\left(w+\frac{\frac{A^9}{B\left(A^2+B^2\right)^3}+\frac{A^7B}{\left(A^2+B^2\right)^3}-\frac{A^5B^3}{\left(A^2+B^2\right)^3}-\frac{A^5}{B\left(A^2+B^2\right)}-\frac{A^3B^5}{\left(A^2+B^2\right)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{\left(A^2+B^2\right)^3}-\frac{iA^4B^4}{\left(A^2+B^2\right)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{\left(A^2+B^2\right)^3}+\frac{iB^8}{\left(A^2+B^2\right)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)}-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{\left(A^2+B^2\right)^3}+\frac{iA^4B^4}{\left(A^2+B^2\right)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{\left(A^2+B^2\right)^3}-\frac{iB^8}{\left(A^2+B^2\right)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/
(A**2 + B**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B
**2)) + A**3*B**5/(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/
(2*B*(A**2 + B**2)) + A*log(w + (A**9/(B*(A**2 + B**2)**3) + A**7
*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 + B**2)**3 - A**5/(B*(A**2
+ B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A**2 + B**2))/A**
2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**2)**3
 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B
**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B
**2))/A**2)/(2*(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B*
*2)**3 + I*A**4*B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*
A**2*B**6/(A**2 + B**2)**3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A*
*2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

GIAC/XCAS [A] time = 0.222453, size = 111, normalized size = 6.94

$$-\frac{\left(\frac{A^3 B \ln(|Aw+B|)}{A^4+A^2B^2} - \frac{A^3 B \ln(|Aw-B|)}{A^4+A^2B^2} + \frac{2\,B^2 \arctan(w)}{A^2+B^2}\right)\left(A^2 + B^2\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)/((w^2 + 1)^2*B*((A^2 + B^2)*w^2/((w^2 + 1)*B^2) - 1)),w, alg
```

```
[Out] -1/2*(A^3*B*ln(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*ln(abs(A*w -
 B))/(A^4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

3.72 $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi [A] time = 0.0405668, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

```
[In]  Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi in Sympy [A] time = 9.15516, size = 14, normalized size = 0.88

$$-A\operatorname{atanh}\left(\frac{Aw}{B}\right)-B\operatorname{atan}(w)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] -A*atanh(A*w/B) - B*atan(w)
```

Mathematica [B] time = 0.0199897, size = 35, normalized size = 2.19

$$-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3
))
```

Maple [B] time = 0.01, size = 121, normalized size = 7.6

$$-\frac{B\arctan(w)A^2}{A^2+B^2}-\frac{\arctan(w)B^3}{A^2+B^2}+\frac{A^3\ln(Aw-B)}{2A^2+2B^2}+\frac{AB^2\ln(Aw-B)}{2A^2+2B^2}-\frac{A^3\ln(Aw+B)}{2A^2+2B^2}-\frac{AB^2\ln(Aw+B)}{2A^2+2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w)
```

```
[Out] -B/(A^2+B^2)*arctan(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3+1/2*A^3/(A^2
+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-1/2*A^3/(A^2+B^2)*l
n(A*w+B)-1/2*A*B^2/(A^2+B^2)*ln(A*w+B)
```

Maxima [A] time = 1.50452, size = 88, normalized size = 5.5

$$-\frac{1}{2}\left(A^2+B^2\right)B\left(\frac{A\log\left(Aw+B\right)}{A^2B+B^3}-\frac{A\log\left(Aw-B\right)}{A^2B+B^3}+\frac{2\arctan\left(w\right)}{A^2+B^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*B/((A^2*w^2 - B^2)*(w^2 + 1)),w, algorithm="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)
/(A^2*B + B^3) + 2*arctan(w)/(A^2 + B^2))
```

Fricas [A] time = 0.215967, size = 35, normalized size = 2.19

$$-B\arctan\left(w\right)-\frac{1}{2}A\log\left(Aw+B\right)+\frac{1}{2}A\log\left(Aw-B\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*B/((A^2*w^2 - B^2)*(w^2 + 1)),w, algorithm="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [A] time = 2.27451, size = 422, normalized size = 26.38

$$\left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B\left(A^2+B^2\right)^3}-\frac{A^7B}{\left(A^2+B^2\right)^3}+\frac{A^5B^3}{\left(A^2+B^2\right)^3}+\frac{A^5}{B\left(A^2+B^2\right)}+\frac{A^3B^5}{\left(A^2+B^2\right)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{A\log\left(w+\frac{\frac{A^9}{B\left(A^2+B^2\right)^3}+\frac{A^7B}{\left(A^2+B^2\right)^3}-\frac{A^5B^3}{\left(A^2+B^2\right)^3}-\frac{A^5}{B\left(A^2+B^2\right)}-\frac{A^3B^5}{\left(A^2+B^2\right)^3}-\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{i\log\left(w+\frac{-\frac{iA^6B^2}{\left(A^2+B^2\right)^3}-\frac{iA^4B^4}{\left(A^2+B^2\right)^3}-\frac{iA^4}{A^2+B^2}+\frac{iA^2B^6}{\left(A^2+B^2\right)^3}+\frac{iB^8}{\left(A^2+B^2\right)^3}-\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)}-\frac{i\log\left(w+\frac{\frac{iA^6B^2}{\left(A^2+B^2\right)^3}+\frac{iA^4B^4}{\left(A^2+B^2\right)^3}+\frac{iA^4}{A^2+B^2}-\frac{iA^2B^6}{\left(A^2+B^2\right)^3}-\frac{iB^8}{\left(A^2+B^2\right)^3}+\frac{iB^4}{A^2+B^2}}{A^2}\right)}{2\left(A^2+B^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/
(A**2 + B**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B
```

```
**2)) + A**3*B**5/(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/
(2*B*(A**2 + B**2)) + A*log(w + (A**9/(B*(A**2 + B**2)**3) + A**7
*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 + B**2)**3 - A**5/(B*(A**2
+ B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A**2 + B**2))/A**
2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**2)**3
 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B
**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B
**2))/A**2)/(2*(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B*
*2)**3 + I*A**4*B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*
A**2*B**6/(A**2 + B**2)**3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A*
*2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

GIAC/XCAS [A] time = 0.208557, size = 107, normalized size = 6.69

$$-\frac{1}{2}\left(\frac{A^{3}\ln\left(|Aw+B|\right)}{A^{4}B+A^{2}B^{3}}-\frac{A^{3}\ln\left(|Aw-B|\right)}{A^{4}B+A^{2}B^{3}}+\frac{2\,\arctan\left(w\right)}{A^{2}+B^{2}}\right)\left(A^{2}+B^{2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2 + B^2)*B/((A^2*w^2 - B^2)*(w^2 + 1)),w, algorithm="giac")
```

```
[Out] -1/2*(A^3*ln(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*ln(abs(A*w - B
))/(A^4*B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

3.73 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0289613, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^2)^(5/2),x]

[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [A] time = 2.56076, size = 26, normalized size = 0.74

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(5/2),x)

[Out] x**3/(3*(-x**2 + 1)**(3/2)) - x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0440217, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}}+\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.47949, size = 59, normalized size = 1.69

$$\frac{1}{3}\,x\left(\frac{3\,x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(
-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.212308, size = 182, normalized size = 5.2

$$-\frac{12\,x^5-25\,x^3+6\left(x^6-6\,x^4+9\,x^2+\left(3\,x^4-7\,x^2+4\right)\sqrt{-x^2+1}-4\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4\,x^5-19\,x^3+12\,x\right)\sqrt{-x^2+1}+}{3\left(x^6-6\,x^4+9\,x^2+(3\,x^4-7\,x^2+4)\sqrt{-x^2+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 - 25*x^3 + 6*(x^6 - 6*x^4 + 9*x^2 + (3*x^4 - 7*x^2 +
 4)*sqrt(-x^2 + 1) - 4)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^5 -
 19*x^3 + 12*x)*sqrt(-x^2 + 1) + 12*x)/(x^6 - 6*x^4 + 9*x^2 + (3*
x^4 - 7*x^2 + 4)*sqrt(-x^2 + 1) - 4)
```

Sympy [A] time = 4.07598, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{-x^2+1}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}-\frac{3x\sqrt{-x^2+1}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(-x**2 + 1)/(3*
x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*s
qrt(-x**2 + 1)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2
 + 3)
```

GIAC/XCAS [A] time = 0.207138, size = 39, normalized size = 1.11

$$\frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.74 $\int \tan^4(y)\,dy$

Optimal. Leaf size=14

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

```
[Out] y - Tan[y] + Tan[y]^3/3
```

Rubi [A] time = 0.014388, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[y]^4,y]

[Out] y - Tan[y] + Tan[y]^3/3
```

Rubi in Sympy [A] time = 2.58295, size = 19, normalized size = 1.36

$$y+\frac{\sin^3(y)}{3\cos^3(y)}-\frac{\sin(y)}{\cos(y)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(y)**4/cos(y)**4,y)

[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

Mathematica [A] time = 0.0104996, size = 18, normalized size = 1.29

$$y-\frac{4\tan(y)}{3}+\frac{1}{3}\tan(y)\sec^2(y)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[y]^4,y]

[Out] y - (4*Tan[y])/3 + (Sec[y]^2*Tan[y])/3
```

Maple [A] time = 0.011, size = 13, normalized size = 0.9

$$y-\tan(y)+\frac{(\tan(y))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(y)^4/cos(y)^4,y)

[Out] y-tan(y)+1/3*tan(y)^3
```

Maxima [A] time = 1.50277, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Fricas [A] time = 0.229481, size = 35, normalized size = 2.5

$$\frac{3\,y\cos(y)^3 - \left(4\,\cos(y)^2 - 1\right)\sin(y)}{3\,\cos(y)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
```

```
[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

Sympy [A] time = 0.047141, size = 19, normalized size = 1.36

$$y + \frac{\sin^3(y)}{3\cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)**4/cos(y)**4,y)
```

```
[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

GIAC/XCAS [A] time = 0.207196, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

3.75 $\int \frac{z^4}{1+z^2}\,dz$

Optimal. Leaf size=13

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

```
[Out] -z + z^3/3 + ArcTan[z]
```

Rubi [A] time = 0.0147685, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In]  Int[z^4/(1 + z^2),z]

[Out] -z + z^3/3 + ArcTan[z]
```

Rubi in Sympy [A] time = 1.56443, size = 8, normalized size = 0.62

$$\frac{z^3}{3} - z + \operatorname{atan}(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(z**4/(z**2+1),z)

[Out] z**3/3 - z + atan(z)
```

Mathematica [A] time = 0.00580449, size = 13, normalized size = 1.

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In]  Integrate[z^4/(1 + z^2),z]

[Out] -z + z^3/3 + ArcTan[z]
```

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$-z + \frac{z^3}{3} + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(z^4/(z^2+1),z)

[Out] -z+1/3*z^3+arctan(z)
```

Maxima [A] time = 1.49647, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(z^4/(z^2 + 1),z, algorithm="maxima")

[Out] 1/3*z^3 - z + arctan(z)
```

Fricas [A] time = 0.19936, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(z^4/(z^2 + 1),z, algorithm="fricas")

[Out] 1/3*z^3 - z + arctan(z)
```

Sympy [A] time = 0.074964, size = 8, normalized size = 0.62

$$\frac{z^3}{3} - z + \operatorname{atan}(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(z**4/(z**2+1),z)

[Out] z**3/3 - z + atan(z)
```

GIAC/XCAS [A] time = 0.203337, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(z^4/(z^2 + 1),z, algorithm="giac")

[Out] 1/3*z^3 - z + arctan(z)
```

3.76 $\int e^{x^2} \left(1+2x^2\right) \, dx$

Optimal. Leaf size=7

$$e^{x^2} x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.0459435, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$e^{x^2} x$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rubi in Sympy [A] time = 2.78317, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Mathematica [A] time = 0.00405578, size = 7, normalized size = 1.

$$e^{x^2} x$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0., size = 7, normalized size = 1.

$$e^{x^2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*(2*x^2+1),x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 1.33247, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 0.209038, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.068189, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

GIAC/XCAS [A] time = 0.204917, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 1)*e^(x^2),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.77 $\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=24

$$e^{x^2}x + \frac{e^{x^2}}{2\left(x^2+1\right)}$$

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rubi [A] time = 0.493149, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$e^{x^2}x + \frac{e^{x^2}}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]

[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: GeneratorsError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)

[Out] Exception raised: GeneratorsError
```

Mathematica [A] time = 0.0211515, size = 19, normalized size = 0.79

$$e^{x^2}\left(\frac{1}{2\left(x^2+1\right)}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]

[Out] E^x^2*(x + 1/(2*(1 + x^2)))
```

Maple [A] time = 0.007, size = 24, normalized size = 1.

$$\frac{\left(2x^3+2x+1\right)e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x)

[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

Maxima [A] time = 1.63135, size = 31, normalized size = 1.29

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 5*x^4 + x^3 + 4*x^2 + 1)*e^(x^2)/(x^2 + 1)^2,x, algorithm="maxi
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Fricas [A] time = 0.23555, size = 31, normalized size = 1.29

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 5*x^4 + x^3 + 4*x^2 + 1)*e^(x^2)/(x^2 + 1)^2,x, algorithm="fric
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Sympy [A] time = 0.116101, size = 20, normalized size = 0.83

$$\frac{\left(2x^3+2x+1\right)e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

```
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

GIAC/XCAS [A] time = 0.20368, size = 41, normalized size = 1.71

$$\frac{2\,x^3e^{\left(x^2\right)}+2\,xe^{\left(x^2\right)}+e^{\left(x^2\right)}}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6 + 5*x^4 + x^3 + 4*x^2 + 1)*e^(x^2)/(x^2 + 1)^2,x, algorithm="giac
```

```
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

3.78 $\int e^{-1-x}\,dx$

Optimal. Leaf size=9

$$-e^{-x-1}$$

```
[Out] -E^(-1 - x)
```

Rubi [A] time = 0.00558114, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(-1 - x),x]

[Out] -E^(-1 - x)
```

Rubi in Sympy [A] time = 0.522101, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(-1-x),x)

[Out] -exp(-x - 1)
```

Mathematica [A] time = 0.00215861, size = 9, normalized size = 1.

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(-1 - x),x]

[Out] -E^(-1 - x)
```

Maple [A] time = 0.001, size = 9, normalized size = 1.

$$-\mathrm{e}^{-1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(-1-x),x)

[Out] -exp(-1-x)
```

Maxima [A] time = 1.33892, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x - 1),x, algorithm="maxima")
```

```
[Out] -e^(-x - 1)
```

Fricas [A] time = 0.231354, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x - 1),x, algorithm="fricas")
```

```
[Out] -e^(-x - 1)
```

Sympy [A] time = 0.052826, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(-1-x),x)
```

```
[Out] -exp(-x - 1)
```

GIAC/XCAS [A] time = 0.202637, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(-x - 1),x, algorithm="giac")
```

```
[Out] -e^(-x - 1)
```

3.79 $\int \left(\frac{1}{x} + x\right) \log(x)\, dx$

Optimal. Leaf size=25

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + \frac{\log^2(x)}{2}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rubi [A] time = 0.0481837, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + \frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(x^(-1) + x)*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2 \log(x)}{2} + \frac{\log(x)^2}{2} - \frac{\int x\, dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/x+x)*ln(x),x)

[Out] x**2*log(x)/2 + log(x)**2/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.00248691, size = 25, normalized size = 1.

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + \frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x^(-1) + x)*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Maple [A] time = 0.006, size = 20, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2} + \frac{(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/x+x)*ln(x),x)

[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Maxima [A] time = 1.33035, size = 32, normalized size = 1.28

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(x^2 + 2\log(x)\right)\log(x) - \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

Fricas [A] time = 0.212302, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Sympy [A] time = 0.081117, size = 19, normalized size = 0.76

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4} + \frac{\log(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/x+x)*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

GIAC/XCAS [A] time = 0.204757, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2 + \frac{1}{2}\ln(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1/x)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*ln(x) - 1/4*x^2 + 1/2*ln(x)^2
```

3.80 $\int \frac{x}{1+x^4} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2} \tan^{-1}\left(x^2\right)$$

```
[Out] ArcTan[x^2]/2
```

Rubi [A] time = 0.00874866, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{2} \tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^4),x]

[Out] ArcTan[x^2]/2
```

Rubi in Sympy [A] time = 1.06419, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+1),x)

[Out] atan(x**2)/2
```

Mathematica [A] time = 0.00453224, size = 8, normalized size = 1.

$$\frac{1}{2} \tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x^4),x]

[Out] ArcTan[x^2]/2
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+1),x)

[Out] 1/2*arctan(x^2)
```

Maxima [A] time = 1.54005, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2)
```

Fricas [A] time = 0.203921, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2)
```

Sympy [A] time = 0.087924, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+1),x)
```

```
[Out] atan(x**2)/2
```

GIAC/XCAS [A] time = 0.205238, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2)
```

3.81 $\int \frac{x^5}{1+x^4}\,dx$

Optimal. Leaf size=16

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rubi [A] time = 0.018264, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(1 + x^4),x]

[Out] x^2/2 - ArcTan[x^2]/2
```

Rubi in Sympy [A] time = 2.22511, size = 10, normalized size = 0.62

$$\frac{x^2}{2}-\frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**4+1),x)

[Out] x**2/2 - atan(x**2)/2
```

Mathematica [A] time = 0.00503429, size = 16, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(1 + x^4),x]

[Out] x^2/2 - ArcTan[x^2]/2
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{x^2}{2}-\frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^4+1),x)

[Out] 1/2*x^2-1/2*arctan(x^2)
```

Maxima [A] time = 1.52933, size = 16, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Fricas [A] time = 0.196898, size = 16, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Sympy [A] time = 0.089683, size = 10, normalized size = 0.62

$$\frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**4+1),x)
```

```
[Out] x**2/2 - atan(x**2)/2
```

GIAC/XCAS [A] time = 0.203236, size = 16, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

3.82 $\int \frac{1}{1+\tan^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0190425, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Tan[x]^2)^(-1),x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.564301, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+tan(x)**2),x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00386635, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Tan[x]^2)^(-1),x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.011, size = 17, normalized size = 1.2

$$\frac{\tan(x)}{2+2(\tan(x))^2}+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+tan(x)^2),x)

[Out] 1/2/(1+tan(x)^2)*tan(x)+1/2*x
```

Maxima [A] time = 1.47975, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(tan(x)^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 0.216806, size = 27, normalized size = 1.93

$$\frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(tan(x)^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [A] time = 0.724572, size = 36, normalized size = 2.57

$$\frac{x\tan^2(x)}{2\tan^2(x) + 2} + \frac{x}{2\tan^2(x) + 2} + \frac{\tan(x)}{2\tan^2(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+tan(x)**2),x)
```

```
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*t
an(x)**2 + 2)
```

GIAC/XCAS [A] time = 0.20936, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(tan(x)^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.83 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0301933, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [A] time = 2.54298, size = 26, normalized size = 0.74

$$\frac{x^3}{3(-x^2+1)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] x**3/(3*(-x**2 + 1)**(3/2)) - x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0115405, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}}+\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49014, size = 59, normalized size = 1.69

$$\frac{1}{3}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{x}{3\,\sqrt{-x^2+1}}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(
-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.223922, size = 182, normalized size = 5.2

$$-\frac{12\,x^5-25\,x^3+6\left(x^6-6\,x^4+9\,x^2+\left(3\,x^4-7\,x^2+4\right)\sqrt{-x^2+1}-4\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4\,x^5-19\,x^3+12\,x\right)\sqrt{-x^2+1}}{3\left(x^6-6\,x^4+9\,x^2+\left(3\,x^4-7\,x^2+4\right)\sqrt{-x^2+1}-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 - 25*x^3 + 6*(x^6 - 6*x^4 + 9*x^2 + (3*x^4 - 7*x^2 +
 4)*sqrt(-x^2 + 1) - 4)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^5 -
 19*x^3 + 12*x)*sqrt(-x^2 + 1) + 12*x)/(x^6 - 6*x^4 + 9*x^2 + (3*
x^4 - 7*x^2 + 4)*sqrt(-x^2 + 1) - 4)
```

Sympy [A] time = 4.09897, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{-x^2+1}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}-\frac{3x\sqrt{-x^2+1}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}\left(x\right)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(-x**2 + 1)/(3*
x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*s
qrt(-x**2 + 1)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2
 + 3)
```

GIAC/XCAS [A] time = 0.209024, size = 39, normalized size = 1.11

$$\frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.84 $\int -\frac{x^2}{(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=17

$$\sin^{-1}(x)-\frac{x}{\sqrt{1-x^2}}$$

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rubi [A] time = 0.0171994, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sin^{-1}(x)-\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rubi in Sympy [A] time = 1.66846, size = 12, normalized size = 0.71

$$-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(-x**2/(-x**2+1)**(3/2),x)
```

```
[Out] -x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0236906, size = 17, normalized size = 1.

$$\sin^{-1}(x)-\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(-x^2+1)^(3/2),x)
```

```
[Out] arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.48503, size = 20, normalized size = 1.18

$$-\frac{x}{\sqrt{-x^2+1}}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(-x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.242648, size = 86, normalized size = 5.06

$$-\frac{2\left(x^2+\sqrt{-x^2+1}-1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}x+x}{x^2+\sqrt{-x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(-x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*(x^2 + sqrt(-x^2 + 1) - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - s
qrt(-x^2 + 1)*x + x)/(x^2 + sqrt(-x^2 + 1) - 1)
```

Sympy [A] time = 0.97049, size = 34, normalized size = 2.

$$\frac{x^2\operatorname{asin}(x)}{x^2-1}+\frac{x\sqrt{-x^2+1}}{x^2-1}-\frac{\operatorname{asin}(x)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x**2/(-x**2+1)**(3/2),x)
```

```
[Out] x**2*asin(x)/(x**2 - 1) + x*sqrt(-x**2 + 1)/(x**2 - 1) - asin(x)/
(x**2 - 1)
```

GIAC/XCAS [A] time = 0.208514, size = 28, normalized size = 1.65

$$\frac{\sqrt{-x^2+1}x}{x^2-1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x^2/(-x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

3.85 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0135913, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Sin[x],x]

[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi in Sympy [A] time = 1.20716, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*sin(x),x)

[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Mathematica [A] time = 0.0158542, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Sin[x],x]

[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*sin(x),x)

[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.34136, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.257184, size = 18, normalized size = 0.95

$$-\frac{1}{2}\cos\left(x\right)e^{x} + \frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.364508, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2} - \frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.202718, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right) - \sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.86 $\int \frac{1}{x} dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.00244339, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rubi in Sympy [A] time = 0.021735, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x,x)
```

```
[Out] log(x)
```

Mathematica [A] time = 0.000141432, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

Maxima [A] time = 1.33208, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 0.226681, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.025056, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x)
```

```
[Out] log(x)
```

GIAC/XCAS [A] time = 0.201494, size = 4, normalized size = 2.

$$\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x,x, algorithm="giac")
```

```
[Out] ln(abs(x))
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

Optimal. Leaf size=45

$$-\frac{1}{2(\tan(t)+1)} - \frac{1}{12}\log(\cos(t)-\sin(t)) - \frac{1}{4}\log(\sin(t)+\cos(t)) + \frac{1}{3}\log(\sin(t)+2\cos(t))$$

```
[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t]
+ Sin[t]]/3 - 1/(2*(1 + Tan[t]))
```

Rubi [A] time = 0.197797, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{1}{2(\tan(t)+1)} - \frac{1}{12}\log(\cos(t)-\sin(t)) - \frac{1}{4}\log(\sin(t)+\cos(t)) + \frac{1}{3}\log(\sin(t)+2\cos(t))$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t]
+ Sin[t]]/3 - 1/(2*(1 + Tan[t]))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.254416, size = 73, normalized size = 1.62

$$-\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\sin(t)+\cos(t))-4\log(\sin(t)+2\cos(t)))+\sin(t)(\log(\cos(t)-\sin(t))+3\log(\sin(t)+\cos}{12(\sin(t)+\cos(t))}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2
*Cos[t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] +
Sin[t]] - 4*Log[2*Cos[t] + Sin[t]])*Sin[t])/(12*(Cos[t] + Sin[t])
)
```

Maple [A] time = 0.194, size = 31, normalized size = 0.7

$$\frac{\ln(\tan(t)+2)}{3} - \frac{1}{2+2\tan(t)} - \frac{\ln(1+\tan(t))}{4} - \frac{\ln(-1+\tan(t))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t)
```

```
[Out] 1/3*ln(tan(t)+2)-1/2/(1+tan(t))-1/4*ln(1+tan(t))-1/12*ln(-1+tan(t
))
```

Maxima [A] time = 1.69538, size = 346, normalized size = 7.69

$$\frac{3\left(\cos(2t)^2+\sin(2t)^2+2\sin(2t)+1\right)\log\left(953674316406250\,(3\cos(2t)+\sin(2t)+4)\cos(4t)+2384185791015625\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*t)/(sec(t)^2 + 3*tan(t) + 1),t, algorithm="maxima")
```

```
[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(9536743164
06250*(3*cos(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos
(4*t)^2 + 953674316406250*cos(2*t)^2 - 953674316406250*(cos(2*t)
- 3*sin(2*t) + 3)*sin(4*t) + 2384185791015625*sin(4*t)^2 + 953674
316406250*sin(2*t)^2 + 2861022949218750*cos(2*t) - 95367431640625
0*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + sin(2*t)^2 + 2*s
in(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) + 5*(c
os(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 +
5*sin(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t
)^2 - 2*sin(2*t) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 +
2*sin(2*t) + 1)
```

Fricas [A] time = 0.271645, size = 96, normalized size = 2.13

$$\frac{4(\cos(t)+\sin(t))\log\left(\frac{3}{4}\cos(t)^2+\cos(t)\sin(t)+\frac{1}{4}\right)-3(\cos(t)+\sin(t))\log(2\cos(t)\sin(t)+1)-(\cos(t)+\sin(t))\log(-}{24(\cos(t)+\sin(t))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*t)/(sec(t)^2 + 3*tan(t) + 1),t, algorithm="fricas")
```

```
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4)
 - 3*(cos(t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t
))*log(-2*cos(t)*sin(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin
(t))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2t)}{3\tan(t)+\sec^2(t)+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

GIAC/XCAS [A] time = 0.220375, size = 45, normalized size = 1.

$$-\frac{1}{2(\tan(t)+1)}+\frac{1}{3}\ln(|\tan(t)+2|)-\frac{1}{4}\ln(|\tan(t)+1|)-\frac{1}{12}\ln(|\tan(t)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(2*t)/(sec(t)^2 + 3*tan(t) + 1),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*ln(abs(tan(t) + 2)) - 1/4*ln(abs(tan(t) +
 1)) - 1/12*ln(abs(tan(t) - 1))
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0110279, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.493025, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/sec(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00283697, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/sec(x)^2,x)

[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.3589, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.246999, size = 14, normalized size = 1.

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.043698, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.208573, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.89 $\int \frac{1+x^2}{\sqrt{x}} dx$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

```
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rubi [A] time = 0.00734457, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)/Sqrt[x],x]

[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rubi in Sympy [A] time = 0.976895, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)/x**(1/2),x)

[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

Mathematica [A] time = 0.00374348, size = 14, normalized size = 0.82

$$\frac{2}{5}\sqrt{x}\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)/Sqrt[x],x]

[Out] (2*Sqrt[x]*(5 + x^2))/5
```

Maple [A] time = 0.006, size = 11, normalized size = 0.7

$$\frac{2x^2+10}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)/x^(1/2),x)

[Out] 2/5*x^(1/2)*(x^2+5)
```

Maxima [A] time = 1.32423, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Fricas [A] time = 0.231608, size = 14, normalized size = 0.82

$$\frac{2}{5}\left(x^2 + 5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 5)*sqrt(x)
```

Sympy [A] time = 0.269595, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

GIAC/XCAS [A] time = 0.202884, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=23

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rubi [A] time = 0.0233415, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[5 + 2*x + x^2],x]

[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rubi in Sympy [A] time = 1.72235, size = 31, normalized size = 1.35

$$\sqrt{x^2+2x+5}-\operatorname{atanh}\left(\frac{2x+2}{2\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+2*x+5)**(1/2),x)

[Out] sqrt(x**2 + 2*x + 5) - atanh((2*x + 2)/(2*sqrt(x**2 + 2*x + 5)))
```

Mathematica [A] time = 0.0127632, size = 23, normalized size = 1.

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[5 + 2*x + x^2],x]

[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Maple [A] time = 0.008, size = 20, normalized size = 0.9

$$-Arcsinh\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2\,x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+2*x+5)^(1/2),x)

[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Maxima [A] time = 1.49596, size = 26, normalized size = 1.13

$$\sqrt{x^2 + 2\,x + 5} - \operatorname{arsinh}\left(\frac{1}{2}\,x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 2*x + 5),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

Fricas [A] time = 0.233205, size = 107, normalized size = 4.65

$$-\frac{2\,x^2 - 2\left(x - \sqrt{x^2 + 2\,x + 5} + 1\right)\log\left(-x + \sqrt{x^2 + 2\,x + 5} - 1\right) - \sqrt{x^2 + 2\,x + 5}(2\,x + 1) + 3\,x + 9}{2\left(x - \sqrt{x^2 + 2\,x + 5} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 2*x + 5),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 - 2*(x - sqrt(x^2 + 2*x + 5) + 1)*log(-x + sqrt(x^2 +
 2*x + 5) - 1) - sqrt(x^2 + 2*x + 5)*(2*x + 1) + 3*x + 9)/(x - sq
rt(x^2 + 2*x + 5) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2 + 2x + 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + 2*x + 5), x)
```

GIAC/XCAS [A] time = 0.212673, size = 36, normalized size = 1.57

$$\sqrt{x^2 + 2\,x + 5} + \ln\left(-x + \sqrt{x^2 + 2\,x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + 2*x + 5),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x + 5) + ln(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^3(x)}{3}$$

```
[Out] Sin[x]^3/3
```

Rubi [A] time = 0.0186326, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Rubi in Sympy [A] time = 1.09323, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*sin(x)**2,x)
```

```
[Out] sin(x)**3/3
```

Mathematica [A] time = 0.00176855, size = 8, normalized size = 1.

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] 1/3*sin(x)^3
```

Maxima [A] time = 1.33929, size = 8, normalized size = 1.

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3
```

Fricas [A] time = 0.272542, size = 14, normalized size = 1.75

$$-\frac{1}{3}\left(\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.039305, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)**2,x)
```

```
[Out] sin(x)**3/3
```

GIAC/XCAS [A] time = 0.200659, size = 8, normalized size = 1.

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3
```

3.92 $\int \frac{e^x}{1+e^x} \, dx$

Optimal. Leaf size=6

$$\log \left(e^x+1\right)$$

```
[Out] Log[1 + E^x]
```

Rubi [A] time = 0.0256604, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\log \left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 + E^x),x]

[Out] Log[1 + E^x]
```

Rubi in Sympy [A] time = 2.75674, size = 5, normalized size = 0.83

$$\log \left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1+exp(x)),x)

[Out] log(exp(x) + 1)
```

Mathematica [A] time = 0.00216852, size = 6, normalized size = 1.

$$\log \left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 + E^x),x]

[Out] Log[1 + E^x]
```

Maple [A] time = 0., size = 6, normalized size = 1.

$$\ln \left(1+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1+exp(x)),x)

[Out] ln(1+exp(x))
```

Maxima [A] time = 1.36339, size = 7, normalized size = 1.17

$$\log \left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="maxima")

[Out] log(e^x + 1)
```

Fricas [A] time = 0.219867, size = 7, normalized size = 1.17

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="fricas")

[Out] log(e^x + 1)
```

Sympy [A] time = 0.060429, size = 5, normalized size = 0.83

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1+exp(x)),x)

[Out] log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.203518, size = 7, normalized size = 1.17

$$\ln\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^x + 1),x, algorithm="giac")

[Out] ln(e^x + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.035976, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Rubi in Sympy [A] time = 3.39697, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(1+exp(x)),x)

[Out] exp(x) - log(exp(x) + 1)
```

Mathematica [A] time = 0.00474951, size = 12, normalized size = 1.

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$e^x - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(1+exp(x)),x)

[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 1.36942, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="maxima")

[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 0.234482, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="fricas")

[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.076117, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(1+exp(x)),x)

[Out] exp(x) - log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.203971, size = 14, normalized size = 1.17

$$e^x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(e^x + 1),x, algorithm="giac")

[Out] e^x - ln(e^x + 1)
```

3.94 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.015229, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi in Sympy [A] time = 0.510592, size = 8, normalized size = 0.67

$$-\frac{\sin{(x)}}{-\cos{(x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-cos(x)),x)

[Out] -sin(x)/(-cos(x) + 1)
```

Mathematica [A] time = 0.00803861, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - Cos[x])^(-1),x]

[Out] -Cot[x/2]
```

Maple [A] time = 0.003, size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-cos(x)),x)

[Out] -1/tan(1/2*x)
```

Maxima [A] time = 1.32959, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 0.22252, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.656155, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

GIAC/XCAS [A] time = 0.212029, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(cos(x) - 1),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.0194335, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sec[x]^2*Tan[x],x]

[Out] Sec[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\tan(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sec(x)**2*tan(x),x)

[Out] Integral(x, (x, tan(x)))
```

Mathematica [A] time = 0.00285681, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sec[x]^2*Tan[x],x]

[Out] Sec[x]^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sec(x)^2*tan(x),x)

[Out] 1/2*sec(x)^2
```

Maxima [A] time = 1.37296, size = 8, normalized size = 1.

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A] time = 0.255514, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.051811, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

GIAC/XCAS [A] time = 0.201755, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.96 $\int x \log(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.00814133, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)}{2}-\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.00111834, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x),x)

[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 1.33912, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="maxima")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.231848, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="fricas")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.069253, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - x**2/4
```

GIAC/XCAS [A] time = 0.201056, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\ln\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="giac")

[Out] 1/2*x^2*ln(x) - 1/4*x^2
```

3.97 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.0121936, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\sin(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*sin(x),x)

[Out] Integral(x, (x, sin(x)))
```

Mathematica [A] time = 0.00198805, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]

[Out] -Cos[x]^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)

[Out] 1/2*sin(x)^2
```

Maxima [A] time = 1.33721, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 0.22059, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.033149, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

GIAC/XCAS [A] time = 0.201486, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}} \, dx$

Optimal. Leaf size=24

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rubi [A] time = 0.024367, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rubi in Sympy [A] time = 1.90138, size = 15, normalized size = 0.62

$$-\sqrt{-x^2+2x}+2\,\mathrm{asin}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 2*x) + 2*asin(x - 1)
```

Mathematica [A] time = 0.0270264, size = 45, normalized size = 1.88

$$\frac{(x-2)x+4\sqrt{x-2}\sqrt{x}\log\left(\sqrt{x-2}+\sqrt{x}\right)}{\sqrt{-(x-2)x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] ((-2 + x)*x + 4*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] + Sqrt[x]])
/Sqrt[-((-2 + x)*x)]
```

Maple [A] time = 0.008, size = 21, normalized size = 0.9

$$2\,\arcsin\,(-1+x)-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+2*x)^(1/2),x)
```

```
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Maxima [A] time = 1.50315, size = 30, normalized size = 1.25

$$-\sqrt{-x^2+2x}-2\arcsin(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/sqrt(-x^2 + 2*x),x, algorithm="maxima")

[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

Fricas [A] time = 0.247878, size = 43, normalized size = 1.79

$$-\sqrt{-x^2+2x}-4\arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/sqrt(-x^2 + 2*x),x, algorithm="fricas")

[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+1}{\sqrt{-x(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(-x**2+2*x)**(1/2),x)

[Out] Integral((x + 1)/sqrt(-x*(x - 2)), x)
```

GIAC/XCAS [A] time = 0.212486, size = 27, normalized size = 1.12

$$-\sqrt{-x^2+2x}+2\arcsin(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/sqrt(-x^2 + 2*x),x, algorithm="giac")

[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}} \, dx$

Optimal. Leaf size=20

$$\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}} e^x\right)$$

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rubi [A] time = 0.0381164, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}} e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2*E^x)/(2 + 3*E^(2*x)),x]

[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rubi in Sympy [A] time = 3.22773, size = 17, normalized size = 0.85

$$\frac{\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} e^x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2*exp(x)/(2+3*exp(2*x)),x)

[Out] sqrt(6)*atan(sqrt(6)*exp(x)/2)/3
```

Mathematica [A] time = 0.0137087, size = 20, normalized size = 1.

$$\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}} e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]

[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$\frac{\sqrt{6}}{3} \arctan\left(\frac{e^x \sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.48504, size = 18, normalized size = 0.9

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*e^x/(3*e^(2*x) + 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A] time = 0.219175, size = 26, normalized size = 1.3

$$\frac{1}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*e^x/(3*e^(2*x) + 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

Sympy [A] time = 0.114545, size = 15, normalized size = 0.75

$$\operatorname{RootSum}\left(6z^{2} + 1, \left(i \mapsto i\log\left(2i + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

GIAC/XCAS [A] time = 0.209806, size = 18, normalized size = 0.9

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2*e^x/(3*e^(2*x) + 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.100 $\int \frac{x^4}{(1-x^2)^{5/2}} \, dx$

Optimal. Leaf size=35

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0287015, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x}{\sqrt{1-x^2}}+\frac{x^3}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(1 - x^2)^(5/2),x]

[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi in Sympy [A] time = 2.55067, size = 26, normalized size = 0.74

$$\frac{x^3}{3\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{x}{\sqrt{-x^2+1}}+\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-x**2+1)**(5/2),x)

[Out] x**3/(3*(-x**2 + 1)**(3/2)) - x/sqrt(-x**2 + 1) + asin(x)
```

Mathematica [A] time = 0.0406279, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}}+\arcsin(x)-x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.49527, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(
-x^2 + 1) + arcsin(x)
```

Fricas [A] time = 0.246357, size = 182, normalized size = 5.2

$$-\frac{12x^5 - 25x^3 + 6\left(x^6 - 6x^4 + 9x^2 + (3x^4 - 7x^2 + 4)\sqrt{-x^2+1} - 4\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^5 - 19x^3 + 12x)\sqrt{-x^2+1}}{3\left(x^6 - 6x^4 + 9x^2 + (3x^4 - 7x^2 + 4)\sqrt{-x^2+1} - 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(12*x^5 - 25*x^3 + 6*(x^6 - 6*x^4 + 9*x^2 + (3*x^4 - 7*x^2 +
 4)*sqrt(-x^2 + 1) - 4)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^5 -
 19*x^3 + 12*x)*sqrt(-x^2 + 1) + 12*x)/(x^6 - 6*x^4 + 9*x^2 + (3*
x^4 - 7*x^2 + 4)*sqrt(-x^2 + 1) - 4)
```

Sympy [A] time = 4.03007, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} - \frac{6x^2\operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x\sqrt{-x^2+1}}{3x^4 - 6x^2 + 3} + \frac{3\operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(-x**2 + 1)/(3*
x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*s
qrt(-x**2 + 1)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2
 + 3)
```

GIAC/XCAS [A] time = 0.208983, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2+1}x}{3(x^2-1)^2} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(-x^2 + 1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}} \, dx$

Optimal. Leaf size=20

$$\frac{e^{2x}}{2} - \frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rubi [A] time = 0.0396088, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{e^{2x}}{2} - \frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(6*x)/(1 + E^(4*x)),x]

[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rubi in Sympy [A] time = 4.07059, size = 14, normalized size = 0.7

$$\frac{e^{2x}}{2} - \frac{\operatorname{atan}\left(e^{2x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(6*x)/(1+exp(4*x)),x)

[Out] exp(2*x)/2 - atan(exp(2*x))/2
```

Mathematica [A] time = 0.00885969, size = 20, normalized size = 1.

$$\frac{e^{2x}}{2} - \frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(6*x)/(1 + E^(4*x)),x]

[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Maple [A] time = 0.006, size = 15, normalized size = 0.8

$$\frac{\left(e^{x}\right)^{2}}{2} - \frac{\arctan\left(\left(e^{x}\right)^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(6*x)/(1+exp(4*x)),x)

[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

Maxima [A] time = 1.48415, size = 19, normalized size = 0.95

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(6*x)/(e^(4*x) + 1),x, algorithm="maxima")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Fricas [A] time = 0.222383, size = 19, normalized size = 0.95

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(6*x)/(e^(4*x) + 1),x, algorithm="fricas")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Sympy [A] time = 0.120295, size = 24, normalized size = 1.2

$$\frac{e^{2x}}{2} + \operatorname{RootSum}\left(16z^2 + 1, \left(i \mapsto i\log\left(-4i + e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(6*x)/(1+exp(4*x)),x)

[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(
2*x))))
```

GIAC/XCAS [A] time = 0.201267, size = 19, normalized size = 0.95

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right) + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(6*x)/(e^(4*x) + 1),x, algorithm="giac")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

3.102 $\int \log \left(2+3 x^2\right) \, dx$

Optimal. Leaf size=33

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi [A] time = 0.0243126, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi in Sympy [A] time = 1.89674, size = 31, normalized size = 0.94

$$x \log \left(3 x^2+2\right)-2 x+\frac{2 \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Mathematica [A] time = 0.0190889, size = 33, normalized size = 1.

$$x \log \left(3 x^2+2\right)-2 x+2 \sqrt{\frac{2}{3}} \tan ^{-1}\left(\sqrt{\frac{3}{2}} x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A] time = 0., size = 27, normalized size = 0.8

$$-2 x+x \ln \left(3 x^2+2\right)+\frac{2 \sqrt{6}}{3} \arctan \left(\frac{x \sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(3*x^2+2),x)
```

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.48589, size = 35, normalized size = 1.06

$$x \log\left(3 x^2 + 2\right) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A] time = 0.221527, size = 54, normalized size = 1.64

$$\frac{1}{3} \sqrt{3}\left(\sqrt{3} x \log\left(3 x^2 + 2\right) - 2 \sqrt{3} x + 2 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{3} \sqrt{2} x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*x*log(3*x^2 + 2) - 2*sqrt(3)*x + 2*sqrt(2)*a
rctan(1/2*sqrt(3)*sqrt(2)*x))
```

Sympy [A] time = 0.139802, size = 31, normalized size = 0.94

$$x \log\left(3x^2 + 2\right) - 2x + \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

GIAC/XCAS [A] time = 0.205687, size = 35, normalized size = 1.06

$$x \ln\left(3 x^2 + 2\right) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] x*ln(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rubi [A] time = 0.0302064, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*H*r**2-a**2)**(1/2),x)

[Out] Integral(1/r, x)/sqrt(2*H*r**2 - a**2)
```

Mathematica [A] time = 0.0000547171, size = 21, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Maple [A] time = 0.002, size = 20, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,Hr^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*H*r^2-a^2)^(1/2),x)

[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Maxima [A] time = 1.34383, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Fricas [A] time = 0.227218, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Sympy [A] time = 0.033998, size = 15, normalized size = 0.71

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2))
```

GIAC/XCAS [A] time = 0.202825, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rubi [A] time = 0.0362666, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)

[Out] Integral(1/r, x)/sqrt(2*H*r**2 - a**2 - e**2)
```

Mathematica [A] time = 0.0000502373, size = 26, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Maple [A] time = 0.001, size = 25, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,Hr^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x)

[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 1.34267, size = 32, normalized size = 1.23

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - e^2)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A] time = 0.201551, size = 32, normalized size = 1.23

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - e^2)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Sympy [A] time = 0.036634, size = 19, normalized size = 0.73

$$\frac{x}{r\sqrt{2Hr^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

GIAC/XCAS [A] time = 0.200672, size = 31, normalized size = 1.19

$$\frac{x}{\sqrt{2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - e^2)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi [A] time = 0.0395758, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-2Kr^4-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)

[Out] Integral(1/r, x)/sqrt(2*H*r**2 - 2*K*r**4 - a**2)
```

Mathematica [A] time = 0.0000591969, size = 27, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] time = 0.001, size = 26, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{-2\,K{r}^{4}+2\,H{r}^{2}-{a}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x)

[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Maxima [A] time = 1.33696, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Fricas [A] time = 0.225969, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Sympy [A] time = 0.036526, size = 22, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

GIAC/XCAS [A] time = 0.211069, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=32

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi [A] time = 0.0470756, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-2Kr^4-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] Integral(1/r, x)/sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2)
```

Mathematica [A] time = 0.0000492774, size = 32, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] time = 0.001, size = 31, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x)
```

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 1.34838, size = 41, normalized size = 1.28

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A] time = 0.200655, size = 41, normalized size = 1.28

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Sympy [A] time = 0.040091, size = 26, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

GIAC/XCAS [A] time = 0.200058, size = 39, normalized size = 1.22

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rubi [A] time = 0.0517409, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-2Kr-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)

[Out] Integral(1/r, x)/sqrt(2*H*r**2 - 2*K*r - a**2)
```

Mathematica [A] time = 0.0000499173, size = 25, normalized size = 1.04

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

Maple [A] time = 0.002, size = 24, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,Hr^2-2\,Kr-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x)

[Out] 1/r/(2*H*r^2-2*K*r-a^2)^(1/2)*x
```

Maxima [A] time = 1.36765, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Fricas [A] time = 0.20254, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Sympy [A] time = 0.037186, size = 20, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

GIAC/XCAS [A] time = 0.205796, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=29

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rubi [A] time = 0.059488, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{r}\,dx}{\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)

[Out] Integral(1/r, x)/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Mathematica [A] time = 0.0000575969, size = 30, normalized size = 1.03

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,Hr^2-2\,Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)

[Out] 1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)*x
```

Maxima [A] time = 1.36383, size = 38, normalized size = 1.31

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

Fricas [A] time = 0.202852, size = 38, normalized size = 1.31

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r),x, algorithm="fricas")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

Sympy [A] time = 0.040436, size = 24, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

GIAC/XCAS [A] time = 0.208267, size = 36, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}}\,dx$

Optimal. Leaf size=19

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rubi [A] time = 0.0190505, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-a^2 + 2*E*r^2],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int r\,dx}{\sqrt{-a^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*E*r**2-a**2)**(1/2),x)

[Out] Integral(r, x)/sqrt(-a**2 + 2*E*r**2)
```

Mathematica [A] time = 0.0000489574, size = 19, normalized size = 1.

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Maple [A] time = 0.002, size = 18, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*E*r^2-a^2)^(1/2),x)

[Out] r*x/(2*E*r^2-a^2)^(1/2)
```

Maxima [A] time = 1.43947, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

Fricas [A] time = 0.198869, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

Sympy [A] time = 0.033077, size = 15, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(2*E*r**2-a**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-a**2 + 2*E*r**2)
```

GIAC/XCAS [A] time = 0.202844, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

Optimal. Leaf size=24

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rubi [A] time = 0.0226228, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int r\,dx}{\sqrt{-a^2-e^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*E*r**2-a**2-e**2)**(1/2),x)

[Out] Integral(r, x)/sqrt(-a**2 - e**2 + 2*E*r**2)
```

Mathematica [A] time = 0.0000425577, size = 24, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*E*r^2-a^2-e^2)^(1/2),x)

[Out] r*x/(2*E*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 1.34522, size = 30, normalized size = 1.25

$$\frac{rx}{\sqrt{2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2 - e^2),x, algorithm="maxima")

[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

Fricas [A] time = 0.206751, size = 30, normalized size = 1.25

$$\frac{rx}{\sqrt{2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2 - e^2),x, algorithm="fricas")

[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

Sympy [A] time = 0.035578, size = 19, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(2*E*r**2-a**2-e**2)**(1/2),x)

[Out] r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

GIAC/XCAS [A] time = 0.203355, size = 28, normalized size = 1.17

$$\frac{rx}{\sqrt{2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(2*E*r^2 - a^2 - e^2),x, algorithm="giac")

[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=25

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi [A] time = 0.0254233, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int r\,dx}{\sqrt{-2Kr^4-a^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(-2*K*r**4+2*E*r**2-a**2)**(1/2),x)

[Out] Integral(r, x)/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

Mathematica [A] time = 0.0000537571, size = 25, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] time = 0.001, size = 24, normalized size = 1.

$$rx\frac{1}{\sqrt{-2\,K{r}^{4}+2\,E{r}^{2}-{a}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x)

[Out] r*x/(-2*K*r^4+2*E*r^2-a^2)^(1/2)
```

Maxima [A] time = 1.36125, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2),x, algorithm="maxima")

[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

Fricas [A] time = 0.205501, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2),x, algorithm="fricas")

[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

Sympy [A] time = 0.035778, size = 22, normalized size = 0.88

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/(-2*K*r**4+2*E*r**2-a**2)**(1/2),x)

[Out] r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

GIAC/XCAS [A] time = 0.207357, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2),x, algorithm="giac")

[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=30

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi [A] time = 0.0301219, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int r\,dx}{\sqrt{-2Kr^4-a^2-e^2+2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(-2*K*r**4+2*E*r**2-a**2-e**2)**(1/2),x)

[Out] Integral(r, x)/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

Mathematica [A] time = 0.0000406378, size = 30, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] time = 0.002, size = 29, normalized size = 1.

$$rx\frac{1}{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x)

[Out] r*x/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 1.35122, size = 38, normalized size = 1.27

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

Fricas [A] time = 0.201951, size = 38, normalized size = 1.27

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

Sympy [A] time = 0.039571, size = 26, normalized size = 0.87

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r**4+2*E*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

GIAC/XCAS [A] time = 0.207125, size = 36, normalized size = 1.2

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=27

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rubi [A] time = 0.0176643, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In]  Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int r\,dx}{\sqrt{2Hr^2-2Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)

[Out] Integral(r, x)/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Mathematica [A] time = 0.0000406378, size = 28, normalized size = 1.04

$$\frac{rx}{\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

Maple [A] time = 0.001, size = 27, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,H{r}^{2}-2\,Kr-{a}^{2}-{e}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)

[Out] r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)*x
```

Maxima [A] time = 1.42172, size = 35, normalized size = 1.3

$$\frac{rx}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r),x, algorithm="maxima")

[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Fricas [A] time = 0.197883, size = 35, normalized size = 1.3

$$\frac{rx}{\sqrt{2\,Hr^2 - a^2 - e^2 - 2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r),x, algorithm="fricas")

[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Sympy [A] time = 0.03878, size = 24, normalized size = 0.89

$$\frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)

[Out] r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

GIAC/XCAS [A] time = 0.207787, size = 34, normalized size = 1.26

$$\frac{rx}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r),x, algorithm="giac")

[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```