Computer algebra independent integration tests
`0_Independent_test_suites/Apostol_Problems`

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November 25, 2018 Compiled on November 25, 2018 at 10:10pm

Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

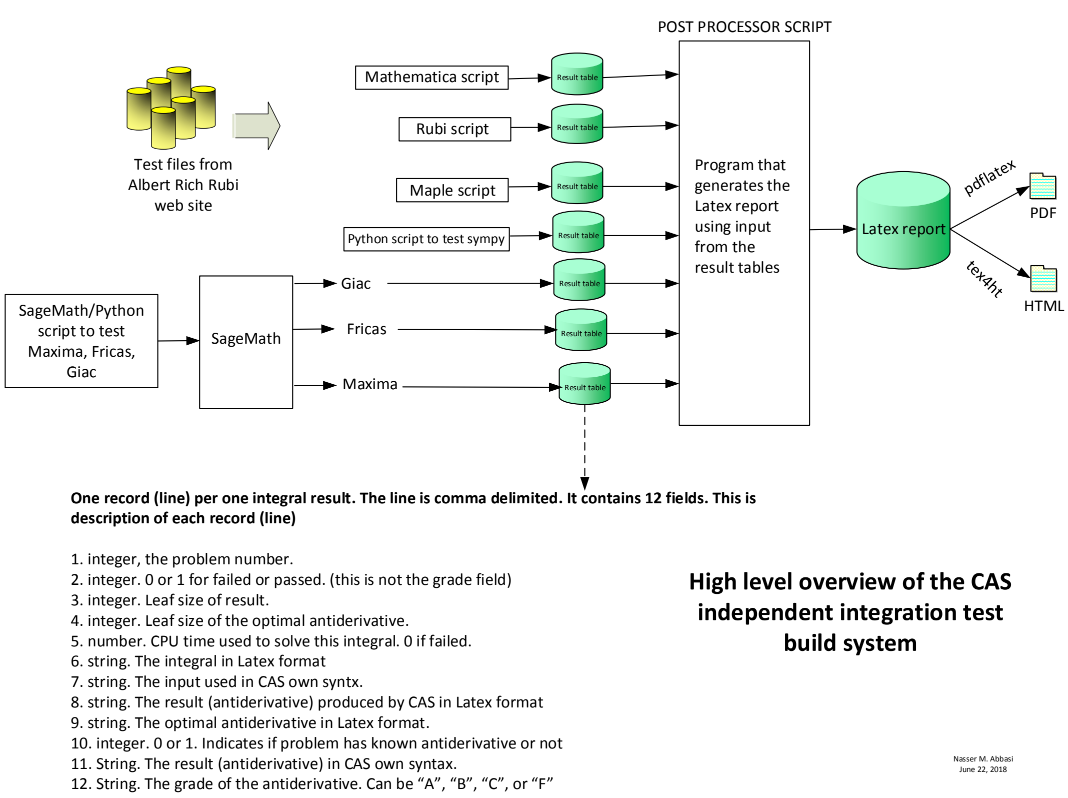
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (175)	% 0. (0)
Rubi in Sympy	% 94.29 (165)	% 5.71 (10)
Mathematica	% 100. (175)	% 0. (0)
Maple	% 98.86 (173)	% 1.14 (2)
Maxima	% 94.29 (165)	% 5.71 (10)
Fricas	% 97.14 (170)	% 2.86 (5)
Sympy	% 85.14 (149)	% 14.86 (26)
Giac	% 94.29 (165)	% 5.71 (10)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

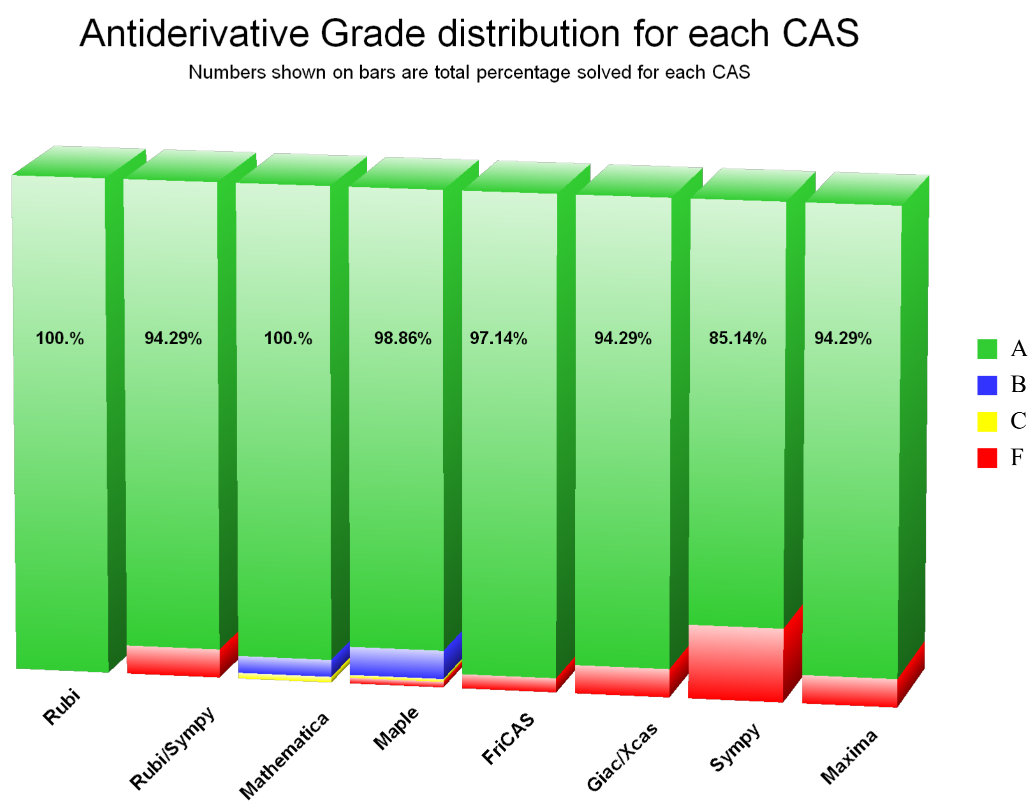
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

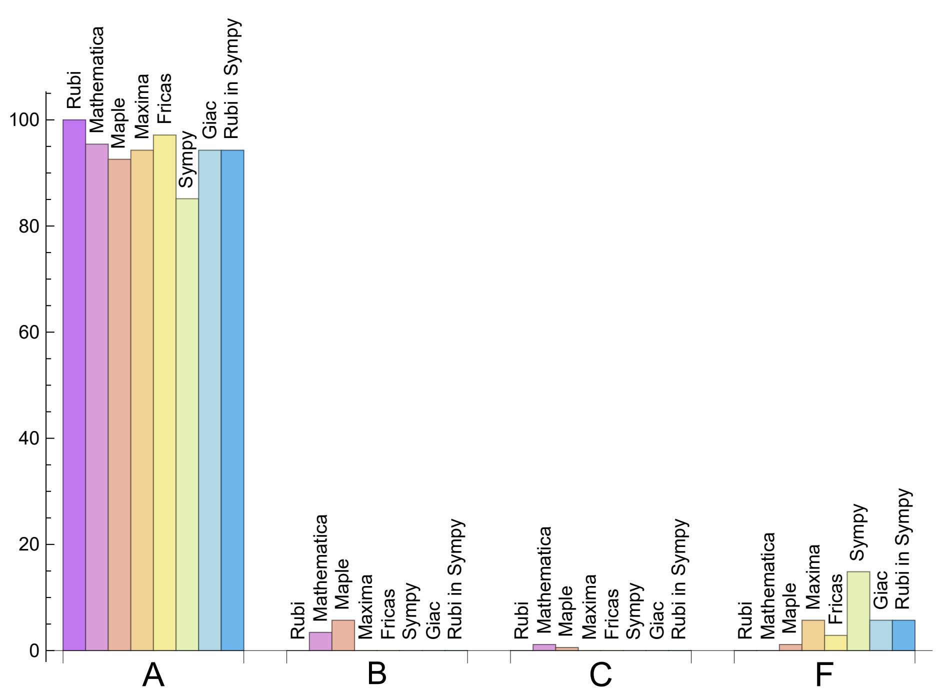
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	94.29	0.	0.	5.71
Mathematica	95.43	3.43	1.14	0.
Maple	92.57	5.71	0.57	1.14
Maxima	94.29	0.	0.	5.71
Fricas	97.14	0.	0.	2.86
Sympy	85.14	0.	0.	14.86
Giac	94.29	0.	0.	5.71

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.03	23.09	1.	19.	1.
Rubi in Sympy	2.04	19.16	0.82	15.	0.81
Mathematica	0.02	22.53	1.03	18.	1.
Maple	0.03	23.19	1.11	16.	0.88
Maxima	1.43	24.67	1.16	19.	1.1
Fricas	0.21	31.38	1.45	22.	1.15
Sympy	1.92	28.72	1.17	17.	0.83
Giac	0.22	28.28	1.34	20.	1.13

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {26, 33, 58, 59, 96, 108, 112, 118, 119, 126}

Not solved by Mathematica {}

Not solved by Maple {19, 172}

Not solved by Maxima {41, 62, 90, 98, 99, 104, 105, 141, 174, 175}

Not solved by Fricas {41, 156, 170, 171, 175}

Not solved by Sympy {13, 19, 62, 83, 84, 88, 98, 99, 103, 104, 105, 145, 146, 151, 153, 154, 155, 162, 163, 164, 165, 170, 171, 172, 173, 174}

Not solved by Giac {21, 41, 62, 98, 99, 156, 161, 164, 172, 175}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {41, 175}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	12	8	12	8
normalized size	1	1.	1.	0.77	0.92	0.92	0.62	0.92	0.62
time (sec)	N/A	0.005	0.003	0.264	1.325	0.209	0.031	0.231	0.518

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	26	39	26	22
normalized size	1	1.	0.67	0.56	0.96	0.96	1.44	0.96	0.81
time (sec)	N/A	0.015	0.006	0.004	1.354	0.218	1.489	0.214	1.479

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	30	30	48	30	29
normalized size	1	1.	0.62	0.53	0.88	0.88	1.41	0.88	0.85
time (sec)	N/A	0.017	0.008	0.006	1.347	0.238	2.128	0.216	1.371

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	19	61	26	22
normalized size	1	1.	0.67	0.56	0.96	0.7	2.26	0.96	0.81
time (sec)	N/A	0.015	0.006	0.004	1.343	0.211	1.589	0.221	1.56

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	30	22	16	14
normalized size	1	1.	1.	0.93	1.14	2.14	1.57	1.14	1.
time (sec)	N/A	0.006	0.007	0.172	1.343	0.219	0.132	0.222	1.235

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	15	8	15	8
normalized size	1	1.	1.15	0.85	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.01	0.003	0.797	1.347	0.217	0.043	0.212	0.712

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	23	97	20	19
normalized size	1	1.	0.7	0.57	0.87	1.	4.22	0.87	0.83
time (sec)	N/A	0.012	0.005	0.004	1.348	0.207	1.447	0.217	0.988

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	14	8	14	12
normalized size	1	1.	1.	0.88	1.	1.75	1.	1.75	1.5
time (sec)	N/A	0.021	0.003	0.021	1.355	0.206	0.04	0.218	1.154

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	24	29	16	12
normalized size	1	1.	1.	0.81	1.	1.5	1.81	1.	0.75
time (sec)	N/A	0.036	0.018	0.021	1.355	0.234	0.399	0.22	1.885

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	5	8	5
normalized size	1	1.	1.	1.17	1.33	1.33	0.83	1.33	0.83
time (sec)	N/A	0.027	0.005	0.325	1.353	0.223	1.018	0.22	1.521

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	16	12	8	14
normalized size	1	1.	1.	0.92	1.17	1.33	1.	0.67	1.17
time (sec)	N/A	0.04	0.013	0.21	1.353	0.234	1.138	0.214	2.428

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	10	11	10
normalized size	1	1.	1.	0.9	1.1	1.1	1.	1.1	1.
time (sec)	N/A	0.027	0.005	0.015	1.371	0.218	0.434	0.213	2.317

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	12	0	12	7
normalized size	1	1.	1.	1.11	1.33	1.33	0.	1.33	0.78
time (sec)	N/A	0.019	0.009	0.031	1.337	0.269	0.	0.215	1.38

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	32	15	15	10	15	10
normalized size	1	1.	1.	2.13	1.	1.	0.67	1.	0.67
time (sec)	N/A	0.007	0.005	0.013	1.343	0.21	0.342	0.215	0.977

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	20	34	20	19
normalized size	1	1.	0.7	0.57	0.87	0.87	1.48	0.87	0.83
time (sec)	N/A	0.012	0.005	0.004	1.324	0.208	1.536	0.217	1.02

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	23	8	12	8
normalized size	1	1.	1.	0.91	1.09	2.09	0.73	1.09	0.73
time (sec)	N/A	0.005	0.006	0.003	1.351	0.207	1.181	0.222	0.496

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	15	15	27	15	10
normalized size	1	1.	1.	1.8	1.	1.	1.8	1.	0.67
time (sec)	N/A	0.007	0.006	0.006	1.367	0.205	0.452	0.216	0.931

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	15	12	15	12
normalized size	1	1.	1.	0.8	1.	1.	0.8	1.	0.8
time (sec)	N/A	0.043	0.029	0.017	1.347	0.223	0.482	0.229	2.863

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	37	0	18	31	0	20	26
normalized size	1	1.	1.16	0.	0.56	0.97	0.	0.62	0.81
time (sec)	N/A	0.194	0.027	0.308	1.374	0.242	0.	0.223	5.816

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	14	18	14
normalized size	1	1.	1.	0.82	1.06	1.06	0.82	1.06	0.82
time (sec)	N/A	0.191	0.014	0.007	1.36	0.24	0.479	0.217	4.909

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	13	9	16	15	0	15
normalized size	1	1.	0.81	0.81	0.56	1.	0.94	0.	0.94
time (sec)	N/A	0.021	0.009	0.004	1.352	0.234	1.164	0.	2.341

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	11	7	11	7
normalized size	1	1.	1.	1.12	1.38	1.38	0.88	1.38	0.88
time (sec)	N/A	0.015	0.003	0.093	1.364	0.242	0.181	0.214	0.754

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	20	20	17	20	17
normalized size	1	1.	0.88	1.06	1.18	1.18	1.	1.18	1.
time (sec)	N/A	0.034	0.01	0.007	1.341	0.239	0.394	0.219	1.391

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	24	27	27	26	27	26
normalized size	1	1.	0.83	1.04	1.17	1.17	1.13	1.17	1.13
time (sec)	N/A	0.053	0.012	0.01	1.335	0.24	0.87	0.216	1.922

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	28	28	26	28	26
normalized size	1	1.	0.83	1.04	1.17	1.17	1.08	1.17	1.08
time (sec)	N/A	0.052	0.012	0.007	1.35	0.243	0.924	0.222	1.926

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	5	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.62	1.	0.
time (sec)	N/A	0.011	0.002	0.003	1.34	0.235	0.036	0.218	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	19	23	24	19	19
normalized size	1	1.	0.78	0.78	0.83	1.	1.04	0.83	0.83
time (sec)	N/A	0.02	0.004	0.004	1.396	0.226	0.413	0.214	1.04

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.01	0.003	0.009	1.351	0.22	0.035	0.214	0.483

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	15	8	15	8
normalized size	1	1.	1.15	0.85	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.011	0.003	0.001	1.361	0.212	0.039	0.227	0.633

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	26	24	22	24
normalized size	1	1.	0.92	0.75	0.92	1.08	1.	0.92	1.
time (sec)	N/A	0.017	0.003	0.105	1.33	0.225	0.038	0.217	0.559

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	23	23	17	23	17
normalized size	1	1.	1.1	0.81	1.1	1.1	0.81	1.1	0.81
time (sec)	N/A	0.013	0.003	0.053	1.481	0.216	0.046	0.223	0.744

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	34	36	30	36
normalized size	1	1.	0.88	0.71	0.94	1.	1.06	0.88	1.06
time (sec)	N/A	0.025	0.003	0.047	1.352	0.256	0.039	0.225	0.665

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	26	36	26	0
normalized size	1	1.	1.	1.	1.04	1.04	1.44	1.04	0.
time (sec)	N/A	0.023	0.005	0.007	1.348	0.237	0.41	0.23	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	31	39	31	32
normalized size	1	1.	0.94	0.7	0.94	0.94	1.18	0.94	0.97
time (sec)	N/A	0.033	0.006	0.078	1.357	0.227	0.819	0.222	1.326

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	39	56	35	36
normalized size	1	1.	0.71	0.9	0.85	0.95	1.37	0.85	0.88
time (sec)	N/A	0.046	0.034	0.036	1.354	0.217	0.85	0.216	1.568

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.01	0.003	0.01	1.362	0.223	0.034	0.216	0.509

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	14	8	12	8
normalized size	1	1.	1.36	1.	1.09	1.27	0.73	1.09	0.73
time (sec)	N/A	0.011	0.003	0.04	1.355	0.206	0.04	0.235	0.671

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	26	24	22	24
normalized size	1	1.	0.92	0.75	0.92	1.08	1.	0.92	1.
time (sec)	N/A	0.019	0.003	0.062	1.358	0.246	0.039	0.237	0.591

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	60	69	84	343	180	68	70
normalized size	1	1.	0.71	0.82	1.	4.08	2.14	0.81	0.83
time (sec)	N/A	0.036	0.061	0.028	1.507	0.214	6.101	0.249	2.687

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	22	46	134	39	38	31
normalized size	1	1.	0.66	0.58	1.21	3.53	1.03	1.	0.82
time (sec)	N/A	0.035	0.01	0.007	1.511	0.195	1.686	0.235	2.363

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	122	168	0	0	31	0	162
normalized size	1	1.	0.71	0.98	0.	0.	0.18	0.	0.94
time (sec)	N/A	0.126	0.266	0.658	0.	0.	0.9	0.	2.248

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	8	7	8	3
normalized size	1	1.	1.	1.17	1.33	1.33	1.17	1.33	0.5
time (sec)	N/A	0.009	0.004	0.019	1.505	0.213	0.045	0.221	0.046

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	16	19	16	10
normalized size	1	1.	1.29	0.93	1.14	1.14	1.36	1.14	0.71
time (sec)	N/A	0.016	0.005	0.004	1.499	0.243	0.05	0.233	0.473

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	14	27	8	24	7
normalized size	1	1.	1.	1.5	1.75	3.38	1.	3.	0.88
time (sec)	N/A	0.01	0.004	0.011	1.502	0.215	0.045	0.226	0.457

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	65	19	46	14
normalized size	1	1.	1.5	1.17	1.83	5.42	1.58	3.83	1.17
time (sec)	N/A	0.017	0.005	0.004	1.5	0.206	0.053	0.22	0.491

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	27	24	26	24	20
normalized size	1	1.	1.18	0.95	1.23	1.09	1.18	1.09	0.91
time (sec)	N/A	0.021	0.008	0.019	1.325	0.215	0.212	0.218	1.16

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	93	22	12	8
normalized size	1	1.	1.	0.77	0.92	7.15	1.69	0.92	0.62
time (sec)	N/A	0.005	0.004	0.005	1.355	0.203	0.215	0.22	0.739

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	52	12	1	58	12	7
normalized size	1	1.	0.85	4.	0.92	0.08	4.46	0.92	0.54
time (sec)	N/A	0.007	0.003	0.026	1.369	0.194	0.038	0.228	0.719

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	20	26	26	15	19	15
normalized size	1	1.	0.89	1.11	1.44	1.44	0.83	1.06	0.83
time (sec)	N/A	0.009	0.006	0.01	1.36	0.19	0.103	0.215	1.139

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	43	32	12	1	31	12	7
normalized size	1	1.	3.91	2.91	1.09	0.09	2.82	1.09	0.64
time (sec)	N/A	0.007	0.002	0.003	1.413	0.21	0.033	0.219	0.761

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	140	107	143	1	131	143	36
normalized size	1	1.	2.5	1.91	2.55	0.02	2.34	2.55	0.64
time (sec)	N/A	0.093	0.003	0.003	1.378	0.178	0.067	0.219	7.446

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	3	5	3
normalized size	1	1.	1.	1.25	1.25	1.25	0.75	1.25	0.75
time (sec)	N/A	0.015	0.006	0.005	1.34	0.216	1.073	0.216	1.049

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	49	50	50	60	50	60
normalized size	1	1.	0.74	0.79	0.81	0.81	0.97	0.81	0.97
time (sec)	N/A	0.07	0.028	0.007	1.386	0.215	3.389	0.218	2.242

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	7	11	7
normalized size	1	1.	1.	0.9	1.1	1.1	0.7	1.1	0.7
time (sec)	N/A	0.013	0.004	0.003	1.353	0.25	0.36	0.225	0.994

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	49	13	16	16	15	248	15
normalized size	1	1.	3.06	0.81	1.	1.	0.94	15.5	0.94
time (sec)	N/A	0.055	0.107	0.043	1.414	0.228	3.298	0.234	2.741

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	7	12	7
normalized size	1	1.	1.	0.9	1.1	1.1	0.7	1.2	0.7
time (sec)	N/A	0.005	0.001	0.	1.445	0.204	0.031	0.215	0.51

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	20	15	20	15
normalized size	1	1.	1.	1.07	1.07	1.33	1.	1.33	1.
time (sec)	N/A	0.008	0.002	0.025	1.385	0.259	0.079	0.22	0.514

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.008	0.001	0.003	1.544	0.212	0.069	0.223	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	30	22	30	0
normalized size	1	1.	1.	0.82	0.82	1.07	0.79	1.07	0.
time (sec)	N/A	0.017	0.002	0.003	1.656	0.203	0.088	0.219	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	5	3	7	3
normalized size	1	1.	1.	1.25	1.25	1.25	0.75	1.75	0.75
time (sec)	N/A	0.003	0.001	0.002	1.624	0.191	0.026	0.216	0.462

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	15	3	15	3
normalized size	1	1.	1.	1.33	1.33	5.	1.	5.	1.
time (sec)	N/A	0.005	0.003	0.001	1.62	0.219	0.044	0.217	0.03

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	0	43	0	0	22
normalized size	1	1.	0.75	1.29	0.	1.54	0.	0.	0.79
time (sec)	N/A	0.02	0.014	0.083	0.	0.219	0.	0.	1.616

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	30	26	30	26
normalized size	1	1.	1.	0.82	0.82	1.07	0.93	1.07	0.93
time (sec)	N/A	0.031	0.004	0.001	1.416	0.206	0.091	0.235	1.928

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	4	3	5	3
normalized size	1	1.	1.	1.33	1.33	1.33	1.	1.67	1.
time (sec)	N/A	0.018	0.001	0.	1.43	0.218	0.079	0.238	1.118

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	8
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.67
time (sec)	N/A	0.026	0.003	0.002	1.448	0.207	0.08	0.24	1.463

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	23	16	20	23	24
normalized size	1	1.	0.7	0.78	1.	0.7	0.87	1.	1.04
time (sec)	N/A	0.065	0.008	0.009	1.502	0.203	1.435	0.235	5.066

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	31	42	37	42	37
normalized size	1	1.	1.	0.82	0.79	1.08	0.95	1.08	0.95
time (sec)	N/A	0.049	0.004	0.003	1.564	0.205	0.113	0.215	2.806

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	8	5	8	5
normalized size	1	1.	1.	0.78	0.89	0.89	0.56	0.89	0.56
time (sec)	N/A	0.02	0.003	0.004	1.548	0.208	0.063	0.222	1.384

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	16	15	10	15	10
normalized size	1	1.	1.	0.86	1.14	1.07	0.71	1.07	0.71
time (sec)	N/A	0.02	0.005	0.006	1.483	0.207	0.153	0.216	1.321

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	9	7	9	7
normalized size	1	1.	1.	0.8	0.9	0.9	0.7	0.9	0.7
time (sec)	N/A	0.016	0.008	0.01	1.419	0.214	0.355	0.223	1.671

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	18	15	15	15
normalized size	1	1.	0.74	0.74	0.79	0.95	0.79	0.79	0.79
time (sec)	N/A	0.014	0.016	0.029	1.377	0.21	0.349	0.218	1.173

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	14	12	18	15	12	15
normalized size	1	1.	0.63	0.74	0.63	0.95	0.79	0.63	0.79
time (sec)	N/A	0.014	0.007	0.007	1.364	0.214	0.341	0.217	1.172

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	12	12	7	12	10
normalized size	1	1.	1.	1.2	1.2	1.2	0.7	1.2	1.
time (sec)	N/A	0.013	0.003	0.008	1.349	0.218	0.056	0.219	1.188

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	8	8	5	8	7
normalized size	1	1.	0.64	0.64	0.73	0.73	0.45	0.73	0.64
time (sec)	N/A	0.01	0.002	0.002	1.374	0.198	0.057	0.22	0.926

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	12	7	12	10
normalized size	1	1.	0.69	0.62	0.75	0.75	0.44	0.75	0.62
time (sec)	N/A	0.013	0.002	0.003	1.418	0.193	0.07	0.238	1.039

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	15	15	10	15	17
normalized size	1	1.	0.63	0.63	0.79	0.79	0.53	0.79	0.89
time (sec)	N/A	0.026	0.002	0.004	1.361	0.206	0.062	0.216	1.717

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	19	22	22	17	22	27
normalized size	1	1.	0.59	0.59	0.69	0.69	0.53	0.69	0.84
time (sec)	N/A	0.03	0.004	0.003	1.416	0.208	0.079	0.237	1.892

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	15	20	15	20
normalized size	1	1.	0.67	0.71	0.62	0.62	0.83	0.62	0.83
time (sec)	N/A	0.014	0.004	0.004	1.469	0.21	0.211	0.234	1.024

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	18	14	18	18	12	18	19
normalized size	1	1.	0.69	0.54	0.69	0.69	0.46	0.69	0.73
time (sec)	N/A	0.034	0.004	0.004	1.407	0.212	0.08	0.237	1.914

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	40	36	42	139	49	36
normalized size	1	1.	0.68	0.98	0.88	1.02	3.39	1.2	0.88
time (sec)	N/A	0.028	0.029	0.015	1.347	0.22	2.095	0.236	1.907

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	29	41	39	45	136	51	36
normalized size	1	1.	0.69	0.98	0.93	1.07	3.24	1.21	0.86
time (sec)	N/A	0.026	0.029	0.006	1.385	0.224	2.135	0.236	1.884

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	20	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.33	0.8
time (sec)	N/A	0.007	0.002	0.004	1.387	0.224	0.234	0.218	0.843

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	47	45	0	34	15
normalized size	1	1.	3.37	1.16	2.47	2.37	0.	1.79	0.79
time (sec)	N/A	0.028	0.112	0.004	1.373	0.242	0.	0.231	1.765

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	47	47	0	35	15
normalized size	1	1.	3.76	1.18	2.76	2.76	0.	2.06	0.88
time (sec)	N/A	0.028	0.071	0.004	1.496	0.23	0.	0.229	1.74

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	31	22	31	22
normalized size	1	1.	1.	0.96	1.24	1.24	0.88	1.24	0.88
time (sec)	N/A	0.054	0.012	0.158	1.574	0.219	0.21	0.235	2.871

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	45	53	22	51	15
normalized size	1	1.	1.18	0.95	2.05	2.41	1.	2.32	0.68
time (sec)	N/A	0.035	0.008	0.013	1.529	0.238	2.239	0.237	2.698

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	11	31	19	12	12
normalized size	1	1.	1.	0.94	0.69	1.94	1.19	0.75	0.75
time (sec)	N/A	0.007	0.005	0.005	1.572	0.2	1.632	0.239	0.883

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	10	15	28	0	12	22
normalized size	1	1.	1.4	1.	1.5	2.8	0.	1.2	2.2
time (sec)	N/A	0.022	0.009	0.026	1.676	0.201	0.	0.223	0.689

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	14	20	14	5
normalized size	1	1.	1.	1.1	1.4	1.4	2.	1.4	0.5
time (sec)	N/A	0.007	0.003	0.013	1.572	0.194	0.112	0.22	0.731

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	53	20	22
normalized size	1	1.	1.	0.67	0.	0.04	2.21	0.83	0.92
time (sec)	N/A	0.017	0.008	0.005	0.	0.195	0.135	0.224	1.071

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	22	26	22	22
normalized size	1	1.	1.	0.89	1.16	1.16	1.37	1.16	1.16
time (sec)	N/A	0.026	0.009	0.004	1.532	0.194	0.094	0.225	0.663

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	18	15	20	15
normalized size	1	1.	1.	0.76	0.95	0.86	0.71	0.95	0.71
time (sec)	N/A	0.018	0.003	0.005	1.525	0.208	0.32	0.218	1.829

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	45	32	32	45	27
normalized size	1	1.	0.75	0.85	1.12	0.8	0.8	1.12	0.68
time (sec)	N/A	0.048	0.014	0.019	1.523	0.22	0.495	0.212	3.055

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	46	34	29	42	29
normalized size	1	1.	0.74	0.86	1.31	0.97	0.83	1.2	0.83
time (sec)	N/A	0.083	0.007	0.01	1.562	0.216	0.467	0.223	6.056

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	22	19	19	22	19
normalized size	1	1.	0.82	0.77	1.	0.86	0.86	1.	0.86
time (sec)	N/A	0.013	0.011	0.003	1.54	0.217	1.135	0.215	1.368

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	0
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.	0.
time (sec)	N/A	0.055	0.004	0.006	1.376	0.231	3.551	0.218	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	23	109	15	23	15
normalized size	1	1.	0.87	0.78	1.	4.74	0.65	1.	0.65
time (sec)	N/A	0.009	0.009	0.004	1.537	0.204	0.22	0.217	0.565

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	16	0	20	0	0	19
normalized size	1	1.	0.91	0.73	0.	0.91	0.	0.	0.86
time (sec)	N/A	0.062	0.082	0.006	0.	0.222	0.	0.	3.298

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	20	0	0	17
normalized size	1	1.	1.	0.8	0.	1.	0.	0.	0.85
time (sec)	N/A	0.035	0.064	0.006	0.	0.228	0.	0.	2.392

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	28	12	20	12
normalized size	1	1.	1.	0.84	1.05	1.47	0.63	1.05	0.63
time (sec)	N/A	0.013	0.012	0.019	1.516	0.19	0.108	0.208	1.358

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	4	15	4	3
normalized size	1	1.	1.	1.	1.	1.	3.75	1.	0.75
time (sec)	N/A	0.028	0.006	0.004	1.551	0.203	0.115	0.207	2.555

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	25	26	38	19	28	26
normalized size	1	1.	0.89	0.93	0.96	1.41	0.7	1.04	0.96
time (sec)	N/A	0.039	0.016	0.01	1.384	0.228	22.768	0.208	2.949

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	67	62	66	51	0	49	36
normalized size	1	1.	1.6	1.48	1.57	1.21	0.	1.17	0.86
time (sec)	N/A	0.034	0.082	0.027	1.501	0.209	0.	0.218	1.858

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	122	0	88	0	82	56
normalized size	1	1.	1.18	1.72	0.	1.24	0.	1.15	0.79
time (sec)	N/A	0.052	0.185	0.02	0.	0.223	0.	0.215	1.931

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	64	28	0	35	0	30	26
normalized size	1	1.	2.	0.88	0.	1.09	0.	0.94	0.81
time (sec)	N/A	0.026	0.039	0.006	0.	0.208	0.	0.242	1.191

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	12	20	12
normalized size	1	1.	1.	0.93	1.2	1.2	0.8	1.33	0.8
time (sec)	N/A	0.015	0.006	0.007	1.378	0.198	0.114	0.222	2.017

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	18	14	20	14
normalized size	1	1.	1.	0.74	0.95	0.95	0.74	1.05	0.74
time (sec)	N/A	0.015	0.006	0.007	1.576	0.195	0.094	0.227	2.574

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	26	19	28	0
normalized size	1	1.	1.	0.87	1.13	1.13	0.83	1.22	0.
time (sec)	N/A	0.04	0.008	0.01	1.422	0.196	0.096	0.221	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	17	27	17
normalized size	1	1.	1.	0.78	1.	1.	0.74	1.17	0.74
time (sec)	N/A	0.049	0.009	0.012	1.448	0.201	0.142	0.219	5.961

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	24	35	19	32	19
normalized size	1	1.	0.92	0.79	1.	1.46	0.79	1.33	0.79
time (sec)	N/A	0.043	0.016	0.012	1.384	0.196	0.121	0.211	2.945

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	51	3	39	29
normalized size	1	1.	1.	1.04	1.36	1.82	0.11	1.39	1.04
time (sec)	N/A	0.049	0.017	0.01	1.503	0.199	0.141	0.211	4.142

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	49	84	14	50	0
normalized size	1	1.	1.24	0.76	1.	1.71	0.29	1.02	0.
time (sec)	N/A	0.136	0.051	0.014	1.577	0.204	0.196	0.21	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	54	51	0	50	22
normalized size	1	1.	1.14	0.9	2.57	2.43	0.	2.38	1.05
time (sec)	N/A	0.019	0.024	0.033	1.616	0.216	14.501	0.238	0.541

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	266	19	74	17	19	12
normalized size	1	1.	1.	16.62	1.19	4.62	1.06	1.19	0.75
time (sec)	N/A	0.081	0.013	0.089	1.348	0.212	2.161	0.209	3.026

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	12	12	8	15	8
normalized size	1	1.	0.82	0.82	1.09	1.09	0.73	1.36	0.73
time (sec)	N/A	0.018	0.006	0.002	1.343	0.19	0.084	0.225	1.477

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	26	20	30	20
normalized size	1	1.	1.	0.87	1.13	1.13	0.87	1.3	0.87
time (sec)	N/A	0.037	0.009	0.01	1.366	0.202	0.131	0.23	2.214

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	27	36	22	30	22
normalized size	1	1.	0.93	0.7	0.9	1.2	0.73	1.	0.73
time (sec)	N/A	0.036	0.014	0.011	1.396	0.195	0.094	0.225	4.234

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	31	20	35	0
normalized size	1	1.	1.	0.89	1.15	1.15	0.74	1.3	0.
time (sec)	N/A	0.047	0.009	0.012	1.431	0.201	0.13	0.232	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	24	27	43	17	22	0
normalized size	1	1.	1.04	1.04	1.17	1.87	0.74	0.96	0.
time (sec)	N/A	0.039	0.017	0.011	1.553	0.194	0.117	0.224	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	19	14	20	14
normalized size	1	1.	1.	0.94	1.19	1.19	0.88	1.25	0.88
time (sec)	N/A	0.028	0.007	0.008	1.622	0.192	0.085	0.217	3.872

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	16	14	16	14
normalized size	1	1.	1.	0.72	0.89	0.89	0.78	0.89	0.78
time (sec)	N/A	0.033	0.011	0.011	1.519	0.195	0.192	0.215	5.02

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	15	8	18	8
normalized size	1	1.	1.	1.09	1.36	1.36	0.73	1.64	0.73
time (sec)	N/A	0.02	0.003	0.009	1.337	0.232	0.089	0.211	1.571

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	32	43	19	39	22
normalized size	1	1.	1.	0.88	1.33	1.79	0.79	1.62	0.92
time (sec)	N/A	0.026	0.013	0.016	1.348	0.23	0.11	0.218	1.954

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	62	112	46	70	41
normalized size	1	1.	0.96	0.85	1.35	2.43	1.	1.52	0.89
time (sec)	N/A	0.052	0.025	0.016	1.355	0.206	0.231	0.213	3.032

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	8	15	8
normalized size	1	1.	1.	1.1	1.4	2.2	0.8	1.5	0.8
time (sec)	N/A	0.011	0.004	0.007	1.342	0.191	0.064	0.21	1.019

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	23	18	10	22	0
normalized size	1	1.	1.	1.06	1.35	1.06	0.59	1.29	0.
time (sec)	N/A	0.019	0.004	0.01	1.339	0.194	0.081	0.21	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	19	17	22	17
normalized size	1	1.	1.	0.75	0.95	0.95	0.85	1.1	0.85
time (sec)	N/A	0.022	0.005	0.007	1.35	0.194	0.096	0.215	2.591

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	22	8	18	8
normalized size	1	1.	0.75	0.81	1.	1.38	0.5	1.12	0.5
time (sec)	N/A	0.016	0.005	0.01	1.363	0.194	0.071	0.212	2.189

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	23	10	19	10
normalized size	1	1.	1.	1.07	1.36	1.64	0.71	1.36	0.71
time (sec)	N/A	0.024	0.012	0.009	1.487	0.197	0.141	0.221	3.553

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	20	27	20
normalized size	1	1.	1.	0.86	1.1	1.1	0.95	1.29	0.95
time (sec)	N/A	0.041	0.009	0.01	1.342	0.201	0.133	0.218	4.992

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	46	20	34	12
normalized size	1	1.	1.29	1.33	1.48	2.19	0.95	1.62	0.57
time (sec)	N/A	0.008	0.012	0.014	1.345	0.193	0.101	0.212	0.566

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	22	17	23	17
normalized size	1	1.	1.	0.77	1.	1.	0.77	1.05	0.77
time (sec)	N/A	0.018	0.005	0.009	1.516	0.188	0.078	0.213	2.68

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	19	24	8	19	12
normalized size	1	1.	1.	1.1	1.9	2.4	0.8	1.9	1.2
time (sec)	N/A	0.046	0.008	0.009	1.343	0.196	0.102	0.223	3.272

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	26	34	19	28	22
normalized size	1	1.	1.	0.71	0.84	1.1	0.61	0.9	0.71
time (sec)	N/A	0.022	0.003	0.01	1.342	0.196	0.105	0.23	1.66

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	22	15	23	15
normalized size	1	1.	1.	0.94	1.22	1.22	0.83	1.28	0.83
time (sec)	N/A	0.043	0.008	0.007	1.503	0.202	0.129	0.227	4.772

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	23	17	26	10
normalized size	1	1.	1.92	0.77	1.77	1.77	1.31	2.	0.77
time (sec)	N/A	0.008	0.005	0.001	1.506	0.2	0.163	0.228	0.585

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	131	73	97	73
normalized size	1	1.	0.75	0.68	1.14	1.54	0.86	1.14	0.86
time (sec)	N/A	0.081	0.03	0.023	1.523	0.202	0.19	0.215	5.485

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	20	35	14	20	20
normalized size	1	1.	0.65	0.7	0.87	1.52	0.61	0.87	0.87
time (sec)	N/A	0.02	0.014	0.007	1.532	0.195	0.127	0.214	1.805

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	15	15	8	15	8
normalized size	1	1.	1.	3.73	1.36	1.36	0.73	1.36	0.73
time (sec)	N/A	0.006	0.01	0.013	1.386	0.187	0.161	0.214	2.756

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	23	20	31	49	39	63	24
normalized size	1	1.	0.51	0.44	0.69	1.09	0.87	1.4	0.53
time (sec)	N/A	0.084	0.035	0.053	1.607	0.221	0.835	0.216	0.802

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	1	110	72	34
normalized size	1	1.	0.84	0.81	0.	0.03	2.97	1.95	0.92
time (sec)	N/A	0.064	0.03	0.02	0.	0.227	11.177	0.215	2.368

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	20	16	50	70	36	47	20
normalized size	1	1.	0.36	0.29	0.89	1.25	0.64	0.84	0.36
time (sec)	N/A	0.038	0.013	0.013	1.484	0.222	0.537	0.249	0.58

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	26	31	32	54	20
normalized size	1	1.	0.65	0.52	0.84	1.	1.03	1.74	0.65
time (sec)	N/A	0.05	0.011	0.023	1.556	0.211	0.406	0.226	0.589

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	18	15	19	51	416	65	24
normalized size	1	1.	0.5	0.42	0.53	1.42	11.56	1.81	0.67
time (sec)	N/A	0.067	0.021	0.029	1.522	0.243	155.717	0.228	4.666

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	58	0	35	10
normalized size	1	1.	1.	1.07	1.33	3.87	0.	2.33	0.67
time (sec)	N/A	0.042	0.053	0.083	1.502	0.239	0.	0.234	25.182

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	19	53	0	18	14
normalized size	1	1.	1.	0.82	1.12	3.12	0.	1.06	0.82
time (sec)	N/A	0.022	0.038	0.234	1.344	0.218	0.	0.216	0.675

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	55	15	22	34	24
normalized size	1	1.	0.73	0.83	1.83	0.5	0.73	1.13	0.8
time (sec)	N/A	0.045	0.011	0.066	1.489	0.231	0.419	0.225	2.118

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	30	39	24	30	24
normalized size	1	1.	1.	0.79	1.03	1.34	0.83	1.03	0.83
time (sec)	N/A	0.011	0.014	0.005	1.516	0.212	0.227	0.217	0.597

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	15	8	15	8
normalized size	1	1.	1.	0.92	1.15	1.15	0.62	1.15	0.62
time (sec)	N/A	0.006	0.003	0.004	1.338	0.208	0.151	0.215	0.876

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	30	55	54	88	63	29
normalized size	1	1.	1.11	0.81	1.49	1.46	2.38	1.7	0.78
time (sec)	N/A	0.047	0.021	0.006	1.514	0.217	2.227	0.226	2.372

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	31	22	34	99	0	35	19
normalized size	1	1.	1.41	1.	1.55	4.5	0.	1.59	0.86
time (sec)	N/A	0.02	0.025	0.006	1.337	0.21	0.	0.228	1.326

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	27	109	24	34	24
normalized size	1	1.	1.	0.78	1.	4.04	0.89	1.26	0.89
time (sec)	N/A	0.01	0.013	0.004	1.504	0.21	0.271	0.225	0.557

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	30	104	0	36	29
normalized size	1	1.	1.	0.78	1.11	3.85	0.	1.33	1.07
time (sec)	N/A	0.028	0.013	0.008	1.496	0.217	0.	0.222	1.503

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	29	12	20	23	0	24	12
normalized size	1	1.	2.07	0.86	1.43	1.64	0.	1.71	0.86
time (sec)	N/A	0.008	0.011	0.004	1.345	0.198	0.	0.216	0.554

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	77	88	80	126	0	227	61
normalized size	1	1.	1.13	1.29	1.18	1.85	0.	3.34	0.9
time (sec)	N/A	0.1	0.065	0.007	1.597	0.209	0.	0.233	5.768

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	16	0	58	0	12
normalized size	1	1.	1.	1.	1.23	0.	4.46	0.	0.92
time (sec)	N/A	0.025	0.005	0.016	1.345	0.	2.177	0.	1.88

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	3	3	2	3	15
normalized size	1	1.	1.	1.	0.2	0.2	0.13	0.2	1.
time (sec)	N/A	0.018	0.016	0.016	1.349	0.212	0.22	0.211	1.066

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	3	3	2	3	2
normalized size	1	1.	1.	4.	1.5	1.5	1.	1.5	1.
time (sec)	N/A	0.017	0.002	0.004	1.414	0.2	1.241	0.228	1.278

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	5	5	3	5	3
normalized size	1	1.	1.	2.25	1.25	1.25	0.75	1.25	0.75
time (sec)	N/A	0.021	0.002	0.004	1.416	0.198	1.343	0.21	1.366

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	7	18	7	18	7
normalized size	1	1.	1.	1.45	0.64	1.64	0.64	1.64	0.64
time (sec)	N/A	0.033	0.005	0.004	1.43	0.198	1.604	0.226	1.896

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	18	18	10	0	10
normalized size	1	1.	1.	1.07	1.29	1.29	0.71	0.	0.71
time (sec)	N/A	0.023	0.003	0.004	1.406	0.202	1.703	0.	1.363

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	22	19	0	19	12
normalized size	1	1.	1.	1.13	1.47	1.27	0.	1.27	0.8
time (sec)	N/A	0.034	0.007	0.023	1.442	0.199	0.	0.232	2.028

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	14	0	14	8
normalized size	1	1.	1.	1.08	1.38	1.08	0.	1.08	0.62
time (sec)	N/A	0.135	0.006	0.007	1.403	0.198	0.	0.225	5.998

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	22	31	0	0	12
normalized size	1	1.	1.	1.16	1.16	1.63	0.	0.	0.63
time (sec)	N/A	0.04	0.011	0.008	1.439	0.201	0.	0.	2.506

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	26	0	22	14
normalized size	1	1.	1.	1.06	1.33	1.44	0.	1.22	0.78
time (sec)	N/A	0.038	0.009	0.175	1.413	0.211	0.	0.211	2.456

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	12	7	12	10
normalized size	1	1.	0.69	0.62	0.75	0.75	0.44	0.75	0.62
time (sec)	N/A	0.016	0.002	0.001	1.351	0.209	0.09	0.211	1.102

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	19	19	12	19	19
normalized size	1	1.	0.62	0.58	0.73	0.73	0.46	0.73	0.73
time (sec)	N/A	0.035	0.003	0.004	1.347	0.212	0.089	0.216	1.755

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	26	26	17	26	27
normalized size	1	1.	0.58	0.56	0.72	0.72	0.47	0.72	0.75
time (sec)	N/A	0.054	0.004	0.004	1.357	0.198	0.125	0.228	2.577

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	39	111	244	81	360	104	39
normalized size	1	1.	0.81	2.31	5.08	1.69	7.5	2.17	0.81
time (sec)	N/A	0.069	0.111	0.119	1.529	0.233	4.261	0.246	4.609

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	4	0	0	4	2
normalized size	1	1.	1.	4.5	2.	0.	0.	2.	1.
time (sec)	N/A	0.004	0.003	0.007	1.422	0.	0.	0.229	0.025

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	8	0	0	15	7
normalized size	1	1.	1.	1.7	0.8	0.	0.	1.5	0.7
time (sec)	N/A	0.007	0.002	0.003	1.412	0.	0.	0.227	0.471

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	30	20	0	0	24
normalized size	1	1.	1.	0.	1.36	0.91	0.	0.	1.09
time (sec)	N/A	0.029	0.033	0.135	1.501	0.226	0.	0.	0.584

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	15	12	0	12	8
normalized size	1	1.	0.83	1.	1.25	1.	0.	1.	0.67
time (sec)	N/A	0.027	0.004	0.006	1.43	0.199	0.	0.224	1.598

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	27	0	27	19
normalized size	1	1.	1.	1.05	0.	1.23	0.	1.23	0.86
time (sec)	N/A	0.095	0.007	0.014	0.	0.201	0.	0.226	6.847

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	88	116	0	0	27	0	95
normalized size	1	1.	0.85	1.13	0.	0.	0.26	0.	0.92
time (sec)	N/A	0.04	0.069	0.096	0.	0.	0.853	0.	0.809

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [2.]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	9	0.111
2	A	2	1	1.	11	0.091
3	A	2	1	1.	11	0.091
4	A	2	1	1.	11	0.091
5	A	1	1	1.	14	0.071
6	A	2	1	1.	4	0.25
7	A	2	1	1.	9	0.111
8	A	2	2	1.	7	0.286
9	A	2	2	1.	17	0.118
10	A	2	2	1.	9	0.222
11	A	3	3	1.	11	0.273
12	A	3	3	1.	16	0.188
13	A	2	2	1.	10	0.2
14	A	1	1	1.	15	0.067
15	A	2	1	1.	9	0.111
16	A	1	1	1.	9	0.111
17	A	1	1	1.	15	0.067
18	A	1	1	1.	17	0.059
19	A	3	2	1.	20	0.1
20	A	1	1	1.	26	0.038
21	A	2	2	1.	20	0.1
22	A	2	2	1.	4	0.5
23	A	3	2	1.	6	0.333
24	A	4	2	1.	6	0.333
25	A	4	2	1.	6	0.333
26	A	2	2	1.	5	0.4
27	A	3	3	1.	6	0.5
28	A	2	2	1.	4	0.5
29	A	2	1	1.	4	0.25
30	A	3	2	1.	4	0.5
31	A	2	1	1.	4	0.25
32	A	4	2	1.	4	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
33	A	2	2	1.	6	0.333
34	A	3	3	1.	6	0.5
35	A	4	4	1.	8	0.5
36	A	2	2	1.	4	0.5
37	A	2	1	1.	4	0.25
38	A	3	2	1.	4	0.5
39	A	5	3	1.	13	0.231
40	A	3	2	1.	13	0.154
41	A	2	2	1.	13	0.154
42	A	2	2	1.	4	0.5
43	A	3	2	1.	4	0.5
44	A	2	2	1.	4	0.5
45	A	3	2	1.	4	0.5
46	A	2	2	1.	10	0.2
47	A	1	1	1.	11	0.091
48	A	1	1	1.	9	0.111
49	A	1	1	1.	13	0.077
50	A	1	1	1.	11	0.091
51	A	2	1	1.	11	0.091
52	A	2	2	1.	8	0.25
53	A	5	3	1.	8	0.375
54	A	1	1	1.	10	0.1
55	A	3	2	1.	17	0.118
56	A	1	1	1.	7	0.143
57	A	2	2	1.	4	0.5
58	A	1	1	1.	4	0.25
59	A	2	2	1.	6	0.333
60	A	1	1	1.	5	0.2
61	A	1	1	1.	2	0.5
62	A	1	1	1.	8	0.125
63	A	2	2	1.	8	0.25
64	A	2	2	1.	8	0.25
65	A	2	2	1.	14	0.143
66	A	3	2	1.	14	0.143
67	A	3	2	1.	8	0.25
68	A	1	1	1.	9	0.111
69	A	1	1	1.	13	0.077
70	A	2	2	1.	9	0.222
71	A	1	1	1.	6	0.167
72	A	1	1	1.	6	0.167
73	A	4	4	1.	7	0.571
74	A	2	2	1.	5	0.4
75	A	2	2	1.	7	0.286
76	A	3	2	1.	7	0.286
77	A	3	2	1.	9	0.222
78	A	3	3	1.	7	0.429
79	A	2	2	1.	11	0.182
80	A	1	1	1.	10	0.1
81	A	1	1	1.	10	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
82	A	2	2	1.	2	1.
83	A	4	4	1.	2	2.
84	A	4	4	1.	2	2.
85	A	3	3	1.	4	0.75
86	A	4	4	1.	6	0.667
87	A	2	2	1.	13	0.154
88	A	2	2	1.	14	0.143
89	A	1	1	1.	9	0.111
90	A	1	1	1.	9	0.111
91	A	2	2	1.	10	0.2
92	A	3	3	1.	4	0.75
93	A	4	3	1.	6	0.5
94	A	5	5	1.	6	0.833
95	A	4	4	1.	6	0.667
96	A	1	3	1.	17	0.176
97	A	2	2	1.	11	0.182
98	A	1	1	1.	15	0.067
99	A	1	1	1.	14	0.071
100	A	2	2	1.	11	0.182
101	A	2	2	1.	13	0.154
102	A	6	6	1.	10	0.6
103	A	3	3	1.	15	0.2
104	A	4	4	1.	15	0.267
105	A	3	3	1.	15	0.2
106	A	3	2	1.	16	0.125
107	A	3	2	1.	16	0.125
108	A	6	4	1.	18	0.222
109	A	3	2	1.	23	0.087
110	A	2	1	1.	19	0.053
111	A	6	6	1.	18	0.333
112	A	6	5	1.	31	0.161
113	A	2	2	1.	7	0.286
114	A	3	2	1.	22	0.091
115	A	2	1	1.	16	0.062
116	A	2	1	1.	17	0.059
117	A	2	1	1.	12	0.083
118	A	3	2	1.	21	0.095
119	A	2	1	1.	20	0.05
120	A	3	3	1.	16	0.188
121	A	4	3	1.	16	0.188
122	A	3	2	1.	11	0.182
123	A	3	2	1.	11	0.182
124	A	2	1	1.	16	0.062
125	A	2	1	1.	7	0.143
126	A	5	5	1.	9	0.556
127	A	4	3	1.	12	0.25
128	A	3	2	1.	14	0.143
129	A	4	4	1.	21	0.19
130	A	3	2	1.	18	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.	7	0.286
132	A	3	3	1.	11	0.273
133	A	3	2	1.	16	0.125
134	A	3	2	1.	11	0.182
135	A	5	4	1.	18	0.222
136	A	3	3	1.	7	0.429
137	A	9	6	1.	7	0.857
138	A	3	3	1.	14	0.214
139	A	1	1	1.	16	0.062
140	A	3	3	1.	12	0.25
141	A	2	2	1.	8	0.25
142	A	2	2	1.	8	0.25
143	A	1	1	1.	10	0.1
144	A	3	3	1.	13	0.231
145	A	2	1	1.	19	0.053
146	A	1	1	1.	11	0.091
147	A	3	3	1.	11	0.273
148	A	2	2	1.	11	0.182
149	A	1	1	1.	13	0.077
150	A	4	4	1.	15	0.267
151	A	3	3	1.	13	0.231
152	A	2	2	1.	9	0.222
153	A	3	3	1.	12	0.25
154	A	2	2	1.	9	0.222
155	A	6	6	1.	18	0.333
156	A	2	2	1.	8	0.25
157	A	3	3	1.	5	0.6
158	A	1	1	1.	7	0.143
159	A	1	1	1.	9	0.111
160	A	2	2	1.	7	0.286
161	A	2	2	1.	5	0.4
162	A	1	1	1.	14	0.071
163	A	2	2	1.	14	0.143
164	A	2	2	1.	9	0.222
165	A	2	3	1.	8	0.375
166	A	2	2	1.	7	0.286
167	A	3	2	1.	9	0.222
168	A	4	2	1.	9	0.222
169	A	1	1	1.	21	0.048
170	A	1	1	1.	4	0.25
171	A	2	2	1.	4	0.5
172	A	2	2	1.	8	0.25
173	A	1	1	1.	11	0.091
174	A	4	2	1.	16	0.125
175	A	1	1	1.	9	0.111

3 Listing of integrals

3.1 $\int \sqrt{1+2x}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}(2x+1)^{3/2}$$

```
[Out] (1 + 2*x)^(3/2)/3
```

Rubi [A] time = 0.00507333, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 2*x],x]

[Out] (1 + 2*x)^(3/2)/3
```

Rubi in Sympy [A] time = 0.517797, size = 8, normalized size = 0.62

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+2*x)**(1/2),x)

[Out] (2*x + 1)**(3/2)/3
```

Mathematica [A] time = 0.00289457, size = 13, normalized size = 1.

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 2*x],x]

[Out] (1 + 2*x)^(3/2)/3
```

Maple [A] time = 0.264, size = 10, normalized size = 0.8

$$\frac{1}{3}(1+2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+2*x)^(1/2),x)
```

```
[Out] 1/3*(1+2*x)^(3/2)
```

Maxima [A] time = 1.3247, size = 12, normalized size = 0.92

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Fricas [A] time = 0.208579, size = 12, normalized size = 0.92

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Sympy [A] time = 0.031499, size = 8, normalized size = 0.62

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+2*x)**(1/2),x)
```

```
[Out] (2*x + 1)**(3/2)/3
```

GIAC/XCAS [A] time = 0.231308, size = 12, normalized size = 0.92

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(2*x + 1),x, algorithm="giac")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

3.2 $\int x\sqrt{1+3x}\,dx$

Optimal. Leaf size=27

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

```
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rubi [A] time = 0.0145727, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 3*x],x]
```

```
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rubi in Sympy [A] time = 1.47911, size = 22, normalized size = 0.81

$$\frac{2\,(3x+1)^{\frac{5}{2}}}{45}-\frac{2\,(3x+1)^{\frac{3}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(1+3*x)**(1/2),x)
```

```
[Out] 2*(3*x + 1)**(5/2)/45 - 2*(3*x + 1)**(3/2)/27
```

Mathematica [A] time = 0.00604768, size = 18, normalized size = 0.67

$$\frac{2}{135}(3x+1)^{3/2}(9x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 3*x],x]
```

```
[Out] (2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$\frac{18\,x-4}{135}\,(1+3\,x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+3*x)^(1/2),x)
```

```
[Out] 2/135*(1+3*x)^(3/2)*(9*x-2)
```

Maxima [A] time = 1.35391, size = 26, normalized size = 0.96

$$\frac{2}{45}(3x+1)^{\frac{5}{2}} - \frac{2}{27}(3x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 1)*x,x, algorithm="maxima")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Fricas [A] time = 0.218311, size = 26, normalized size = 0.96

$$\frac{2}{135}\left(27x^2 + 3x - 2\right)\sqrt{3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 1)*x,x, algorithm="fricas")
```

```
[Out] 2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

Sympy [A] time = 1.48857, size = 39, normalized size = 1.44

$$\frac{2x^2\sqrt{3x+1}}{5} + \frac{2x\sqrt{3x+1}}{45} - \frac{4\sqrt{3x+1}}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(1+3*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/1
35
```

GIAC/XCAS [A] time = 0.213925, size = 26, normalized size = 0.96

$$\frac{2}{45}(3x+1)^{\frac{5}{2}} - \frac{2}{27}(3x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x + 1)*x,x, algorithm="giac")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

3.3 $\int x^2\sqrt{1+x}\,dx$

Optimal. Leaf size=34

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rubi [A] time = 0.016929, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rubi in Sympy [A] time = 1.37069, size = 29, normalized size = 0.85

$$\frac{2\,(x+1)^{\frac{7}{2}}}{7}-\frac{4\,(x+1)^{\frac{5}{2}}}{5}+\frac{2\,(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(1+x)**(1/2),x)

[Out] 2*(x + 1)**(7/2)/7 - 4*(x + 1)**(5/2)/5 + 2*(x + 1)**(3/2)/3
```

Mathematica [A] time = 0.0083522, size = 21, normalized size = 0.62

$$\frac{2}{105}(x+1)^{3/2}\left(15x^2-12x+8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

Maple [A] time = 0.006, size = 18, normalized size = 0.5

$$\frac{30\,x^2-24\,x+16}{105}\,(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(1+x)^(1/2),x)

[Out] 2/105*(1+x)^(3/2)*(15*x^2-12*x+8)
```

Maxima [A] time = 1.34744, size = 30, normalized size = 0.88

$$\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x^2,x, algorithm="maxima")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.237783, size = 30, normalized size = 0.88

$$\frac{2}{105}\left(15x^3 + 3x^2 - 4x + 8\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x^2,x, algorithm="fricas")
```

```
[Out] 2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

Sympy [A] time = 2.1282, size = 48, normalized size = 1.41

$$\frac{2x^3\sqrt{x+1}}{7} + \frac{2x^2\sqrt{x+1}}{35} - \frac{8x\sqrt{x+1}}{105} + \frac{16\sqrt{x+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(1+x)**(1/2),x)
```

```
[Out] 2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/10
5 + 16*sqrt(x + 1)/105
```

GIAC/XCAS [A] time = 0.21592, size = 30, normalized size = 0.88

$$\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x + 1)*x^2,x, algorithm="giac")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

3.4 $\int \frac{x}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

```
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rubi [A] time = 0.0154312, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[2 - 3*x],x]

[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rubi in Sympy [A] time = 1.55981, size = 22, normalized size = 0.81

$$\frac{2\left(-3x+2\right)^{\frac{3}{2}}}{27}-\frac{4\sqrt{-3x+2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(2-3*x)**(1/2),x)

[Out] 2*(-3*x + 2)**(3/2)/27 - 4*sqrt(-3*x + 2)/9
```

Mathematica [A] time = 0.00579681, size = 18, normalized size = 0.67

$$-\frac{2}{27}\sqrt{2-3x}(3x+4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[2 - 3*x],x]

[Out] (-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$-\frac{6x+8}{27}\sqrt{2-3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(2-3*x)^(1/2),x)

[Out] -2/27*(3*x+4)*(2-3*x)^(1/2)
```

Maxima [A] time = 1.34264, size = 26, normalized size = 0.96

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-3*x + 2),x, algorithm="maxima")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Fricas [A] time = 0.2114, size = 19, normalized size = 0.7

$$-\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-3*x + 2),x, algorithm="fricas")
```

```
[Out] -2/27*(3*x + 4)*sqrt(-3*x + 2)
```

Sympy [A] time = 1.5892, size = 61, normalized size = 2.26

$$\begin{cases} -\frac{2ix\sqrt{3x-2}}{9}-\frac{8i\sqrt{3x-2}}{27} & \text{for}\ \frac{3|x|}{2}>1 \\ -\frac{2x\sqrt{-3x+2}}{9}-\frac{8\sqrt{-3x+2}}{27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(2-3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, 3*Abs(x
)/2 > 1), (-2*x*sqrt(-3*x + 2)/9 - 8*sqrt(-3*x + 2)/27, True))
```

GIAC/XCAS [A] time = 0.221197, size = 26, normalized size = 0.96

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-3*x + 2),x, algorithm="giac")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

3.5 $\int \frac{1+x}{(2+2x+x^2)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

```
[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Rubi [A] time = 0.00632382, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(2 + 2*x + x^2)^3,x]

[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Rubi in Sympy [A] time = 1.23509, size = 14, normalized size = 1.

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**2+2*x+2)**3,x)

[Out] -1/(4*(x**2 + 2*x + 2)**2)
```

Mathematica [A] time = 0.00674588, size = 14, normalized size = 1.

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]

[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Maple [A] time = 0.172, size = 13, normalized size = 0.9

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^2+2*x+2)^3,x)

[Out] -1/4/(x^2+2*x+2)^2
```

Maxima [A] time = 1.34274, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^2 + 2*x + 2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Fricas [A] time = 0.218837, size = 30, normalized size = 2.14

$$-\frac{1}{4\left(x^4+4x^3+8x^2+8x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^2 + 2*x + 2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

Sympy [A] time = 0.132066, size = 22, normalized size = 1.57

$$-\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**2+2*x+2)**3,x)
```

```
[Out] -1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

GIAC/XCAS [A] time = 0.221687, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^2 + 2*x + 2)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^2 + 2*x + 2)^2
```

3.6 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0100641, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^3,x]

[Out] -Cos[x] + Cos[x]^3/3
```

Rubi in Sympy [A] time = 0.712416, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.00284177, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^3,x]

[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0.797, size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^3,x)

[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.34692, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="maxima")

[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.217004, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="fricas")

[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.04262, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A] time = 0.212124, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="giac")

[Out] 1/3*cos(x)^3 - cos(x)
```

3.7 $\int \sqrt[3]{-1+z}z\,dz$

Optimal. Leaf size=23

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rubi [A] time = 0.0115898, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rubi in Sympy [A] time = 0.987883, size = 19, normalized size = 0.83

$$\frac{3\,(z-1)^{\frac{7}{3}}}{7}+\frac{3\,(z-1)^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-1+z)**(1/3)*z,z)
```

```
[Out] 3*(z - 1)**(7/3)/7 + 3*(z - 1)**(4/3)/4
```

Mathematica [A] time = 0.00535108, size = 16, normalized size = 0.7

$$\frac{3}{28}(z-1)^{4/3}(4z+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3)*(3 + 4*z))/28
```

Maple [A] time = 0.004, size = 13, normalized size = 0.6

$$\frac{12\,z+9}{28}\,(-1+z)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-1+z)^(1/3)*z,z)
```

```
[Out] 3/28*(-1+z)^(4/3)*(4*z+3)
```

Maxima [A] time = 1.34828, size = 20, normalized size = 0.87

$$\frac{3}{7}(z-1)^{\frac{7}{3}}+\frac{3}{4}(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((z - 1)^(1/3)*z,z, algorithm="maxima")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Fricas [A] time = 0.207213, size = 23, normalized size = 1.

$$\frac{3}{28}\left(4z^2-z-3\right)(z-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((z - 1)^(1/3)*z,z, algorithm="fricas")
```

```
[Out] 3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

Sympy [A] time = 1.44727, size = 97, normalized size = 4.22

$$\begin{cases} \frac{3z^2\sqrt[3]{z-1}}{7}-\frac{3z\sqrt[3]{z-1}}{28}-\frac{9\sqrt[3]{z-1}}{28} & \text{for } |z|>1 \\ \frac{3z^2\sqrt[3]{-z+1}e^{\frac{13i\pi}{3}}}{7}-\frac{3z\sqrt[3]{-z+1}e^{\frac{13i\pi}{3}}}{28}-\frac{9\sqrt[3]{-z+1}e^{\frac{13i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-1+z)**(1/3)*z,z)
```

```
[Out] Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z
 - 1)**(1/3)/28, Abs(z) > 1), (3*z**2*(-z + 1)**(1/3)*exp(13*I*pi
/3)/7 - 3*z*(-z + 1)**(1/3)*exp(13*I*pi/3)/28 - 9*(-z + 1)**(1/3)
*exp(13*I*pi/3)/28, True))
```

GIAC/XCAS [A] time = 0.21711, size = 20, normalized size = 0.87

$$\frac{3}{7}(z-1)^{\frac{7}{3}}+\frac{3}{4}(z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((z - 1)^(1/3)*z,z, algorithm="giac")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\csc^2(x)$$

```
[Out] -Csc[x]^2/2
```

Rubi [A] time = 0.0205464, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]*Csc[x]^2,x]

[Out] -Csc[x]^2/2
```

Rubi in Sympy [A] time = 1.15365, size = 12, normalized size = 1.5

$$-\frac{\cos^2(x)}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)/sin(x)**3,x)

[Out] -cos(x)**2/(2*sin(x)**2)
```

Mathematica [A] time = 0.00307728, size = 8, normalized size = 1.

$$-\frac{1}{2}\cot^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]*Csc[x]^2,x]

[Out] -Cot[x]^2/2
```

Maple [A] time = 0.021, size = 7, normalized size = 0.9

$$-\frac{1}{2\,(\sin(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)/sin(x)^3,x)

[Out] -1/2/sin(x)^2
```

Maxima [A] time = 1.35531, size = 8, normalized size = 1.

$$-\frac{1}{2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2
```

Fricas [A] time = 0.206378, size = 14, normalized size = 1.75

$$\frac{1}{2\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

Sympy [A] time = 0.04039, size = 8, normalized size = 1.

$$-\frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)**3,x)
```

```
[Out] -1/(2*sin(x)**2)
```

GIAC/XCAS [A] time = 0.218373, size = 14, normalized size = 1.75

$$\frac{1}{2\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)/sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[Out] -(4 - Sin[2*x])^(3/2)/3
```

Rubi [A] time = 0.0363133, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]

[Out] -(4 - Sin[2*x])^(3/2)/3
```

Rubi in Sympy [A] time = 1.88455, size = 12, normalized size = 0.75

$$-\frac{(-\sin(2x)+4)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)

[Out] -(-sin(2*x) + 4)**(3/2)/3
```

Mathematica [A] time = 0.0184198, size = 16, normalized size = 1.

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]

[Out] -(4 - Sin[2*x])^(3/2)/3
```

Maple [A] time = 0.021, size = 13, normalized size = 0.8

$$-\frac{1}{3}(4-\sin(2x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(2*x)*(4-sin(2*x))^(1/2),x)

[Out] -1/3*(4-sin(2*x))^(3/2)
```

Maxima [A] time = 1.35489, size = 16, normalized size = 1.

$$-\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 4)*cos(2*x),x, algorithm="maxima")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Fricas [A] time = 0.233611, size = 24, normalized size = 1.5

$$\frac{1}{3}\left(\sin\left(2\,x\right)-4\right)\sqrt{-\sin\left(2\,x\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 4)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

Sympy [A] time = 0.399012, size = 29, normalized size = 1.81

$$\frac{\sqrt{-\sin\left(2x\right)+4}\sin\left(2x\right)}{3}-\frac{4\sqrt{-\sin\left(2x\right)+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
```

```
[Out] sqrt(-sin(2*x) + 4)*sin(2*x)/3 - 4*sqrt(-sin(2*x) + 4)/3
```

GIAC/XCAS [A] time = 0.219949, size = 16, normalized size = 1.

$$-\frac{1}{3}\left(-\sin\left(2\,x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-sin(2*x) + 4)*cos(2*x),x, algorithm="giac")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2}\,dx$

Optimal. Leaf size=6

$$\frac{1}{\cos(x)+3}$$

```
[Out] (3 + Cos[x])^(-1)
```

Rubi [A] time = 0.0270149, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(3 + Cos[x])^2,x]

[Out] (3 + Cos[x])^(-1)
```

Rubi in Sympy [A] time = 1.5209, size = 5, normalized size = 0.83

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)/(3+cos(x))**2,x)

[Out] 1/(cos(x) + 3)
```

Mathematica [A] time = 0.00475847, size = 6, normalized size = 1.

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(3 + Cos[x])^2,x]

[Out] (3 + Cos[x])^(-1)
```

Maple [A] time = 0.325, size = 7, normalized size = 1.2

$$(3+\cos(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(3+cos(x))^2,x)

[Out] 1/(3+cos(x))
```

Maxima [A] time = 1.3528, size = 8, normalized size = 1.33

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + 3)^2,x, algorithm="maxima")
```

```
[Out] 1/(cos(x) + 3)
```

Fricas [A] time = 0.222832, size = 8, normalized size = 1.33

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + 3)^2,x, algorithm="fricas")
```

```
[Out] 1/(cos(x) + 3)
```

Sympy [A] time = 1.01798, size = 5, normalized size = 0.83

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(3+cos(x))**2,x)
```

```
[Out] 1/(cos(x) + 3)
```

GIAC/XCAS [A] time = 0.219896, size = 8, normalized size = 1.33

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + 3)^2,x, algorithm="giac")
```

```
[Out] 1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

Optimal. Leaf size=12

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rubi [A] time = 0.0400302, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/Sqrt[Cos[x]^3],x]

[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rubi in Sympy [A] time = 2.42827, size = 14, normalized size = 1.17

$$\frac{2\sqrt{\cos^3(x)}}{\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/(cos(x)**3)**(1/2),x)

[Out] 2*sqrt(cos(x)**3)/cos(x)**2
```

Mathematica [A] time = 0.0127974, size = 12, normalized size = 1.

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/Sqrt[Cos[x]^3],x]

[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Maple [A] time = 0.21, size = 11, normalized size = 0.9

$$2\,\frac{\cos(x)}{\sqrt{(\cos(x))^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/(cos(x)^3)^(1/2),x)
```

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Maxima [A] time = 1.35309, size = 14, normalized size = 1.17

$$\frac{2\cos(x)}{\sqrt{\cos(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(cos(x)^3),x, algorithm="maxima")
```

```
[Out] 2*cos(x)/sqrt(cos(x)^3)
```

Fricas [A] time = 0.233897, size = 16, normalized size = 1.33

$$\frac{2\sqrt{\cos(x)^3}}{\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(cos(x)^3),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^3)/cos(x)^2
```

Sympy [A] time = 1.13783, size = 12, normalized size = 1.

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x)**3)**(1/2),x)
```

```
[Out] 2*cos(x)/sqrt(cos(x)**3)
```

GIAC/XCAS [A] time = 0.214051, size = 8, normalized size = 0.67

$$\frac{2}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/sqrt(cos(x)^3),x, algorithm="giac")
```

```
[Out] 2/sqrt(cos(x))
```

3.12 $\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{x+1}\right)$$

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rubi [A] time = 0.0267323, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]

[Out] -2*Cos[Sqrt[1 + x]]
```

Rubi in Sympy [A] time = 2.31727, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)

[Out] -2*cos(sqrt(x + 1))
```

Mathematica [A] time = 0.00520036, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]

[Out] -2*Cos[Sqrt[1 + x]]
```

Maple [A] time = 0.015, size = 9, normalized size = 0.9

$$-2\cos\left(\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((1+x)^(1/2))/(1+x)^(1/2),x)

[Out] -2*cos((1+x)^(1/2))
```

Maxima [A] time = 1.37061, size = 11, normalized size = 1.1

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x + 1))/sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] -2*cos(sqrt(x + 1))
```

Fricas [A] time = 0.21849, size = 11, normalized size = 1.1

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x + 1))/sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] -2*cos(sqrt(x + 1))
```

Sympy [A] time = 0.433712, size = 10, normalized size = 1.

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
```

```
[Out] -2*cos(sqrt(x + 1))
```

GIAC/XCAS [A] time = 0.213129, size = 11, normalized size = 1.1

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(sqrt(x + 1))/sqrt(x + 1),x, algorithm="giac")
```

```
[Out] -2*cos(sqrt(x + 1))
```

3.13 $\int x^{-1+n} \sin(x^n)\, dx$

Optimal. Leaf size=9

$$-\frac{\cos(x^n)}{n}$$

```
[Out] -(Cos[x^n]/n)
```

Rubi [A] time = 0.0190793, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Rubi in Sympy [A] time = 1.38042, size = 7, normalized size = 0.78

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(-1+n)*sin(x**n),x)
```

```
[Out] -cos(x**n)/n
```

Mathematica [A] time = 0.00858962, size = 9, normalized size = 1.

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(-1 + n)*Sin[x^n],x]
```

```
[Out] -(Cos[x^n]/n)
```

Maple [A] time = 0.031, size = 10, normalized size = 1.1

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+n)*sin(x^n),x)
```

```
[Out] -cos(x^n)/n
```

Maxima [A] time = 1.33749, size = 12, normalized size = 1.33

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)*sin(x^n),x, algorithm="maxima")
```

```
[Out] -cos(x^n)/n
```

Fricas [A] time = 0.269024, size = 12, normalized size = 1.33

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)*sin(x^n),x, algorithm="fricas")
```

```
[Out] -cos(x^n)/n
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(-1+n)*sin(x**n),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.214675, size = 12, normalized size = 1.33

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(n - 1)*sin(x^n),x, algorithm="giac")
```

```
[Out] -cos(x^n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sqrt{1-x^6}$$

```
[Out] -Sqrt[1 - x^6]/3
```

Rubi [A] time = 0.00652509, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[1 - x^6],x]
```

```
[Out] -Sqrt[1 - x^6]/3
```

Rubi in Sympy [A] time = 0.977442, size = 10, normalized size = 0.67

$$-\frac{\sqrt{-x^6+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(-x**6+1)**(1/2),x)
```

```
[Out] -sqrt(-x**6 + 1)/3
```

Mathematica [A] time = 0.00535236, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[1 - x^6],x]
```

```
[Out] -Sqrt[1 - x^6]/3
```

Maple [B] time = 0.013, size = 32, normalized size = 2.1

$$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3}\frac{1}{\sqrt{-x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(-x^6+1)^(1/2),x)
```

```
[Out] 1/3*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(-x^6+1)^(1/2)
```

Maxima [A] time = 1.3434, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-x^6 + 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Fricas [A] time = 0.209514, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-x^6 + 1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Sympy [A] time = 0.341842, size = 10, normalized size = 0.67

$$-\frac{\sqrt{-x^6+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(-x**6+1)**(1/2),x)
```

```
[Out] -sqrt(-x**6 + 1)/3
```

GIAC/XCAS [A] time = 0.214874, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-x^6 + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

3.15 $\int t\sqrt[4]{1+t}\,dt$

Optimal. Leaf size=23

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

```
[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rubi [A] time = 0.0118474, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

Antiderivative was successfully verified.

```
[In]  Int[t*(1 + t)^(1/4),t]

[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rubi in Sympy [A] time = 1.01989, size = 19, normalized size = 0.83

$$\frac{4\,(t+1)^{\frac{9}{4}}}{9}-\frac{4\,(t+1)^{\frac{5}{4}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t*(1+t)**(1/4),t)

[Out] 4*(t + 1)**(9/4)/9 - 4*(t + 1)**(5/4)/5
```

Mathematica [A] time = 0.00483334, size = 16, normalized size = 0.7

$$\frac{4}{45}(t+1)^{5/4}(5t-4)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t*(1 + t)^(1/4),t]

[Out] (4*(1 + t)^(5/4)*(-4 + 5*t))/45
```

Maple [A] time = 0.004, size = 13, normalized size = 0.6

$$\frac{20\,t-16}{45}\,(1+t)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t*(1+t)^(1/4),t)

[Out] 4/45*(1+t)^(5/4)*(5*t-4)
```

Maxima [A] time = 1.32413, size = 20, normalized size = 0.87

$$\frac{4}{9}(t+1)^{\frac{9}{4}} - \frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((t + 1)^(1/4)*t,t, algorithm="maxima")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Fricas [A] time = 0.207628, size = 20, normalized size = 0.87

$$\frac{4}{45}\left(5t^2+t-4\right)(t+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((t + 1)^(1/4)*t,t, algorithm="fricas")
```

```
[Out] 4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

Sympy [A] time = 1.53629, size = 34, normalized size = 1.48

$$\frac{4t^2\sqrt[4]{t+1}}{9} + \frac{4t\sqrt[4]{t+1}}{45} - \frac{16\sqrt[4]{t+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*(1+t)**(1/4),t)
```

```
[Out] 4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/
4)/45
```

GIAC/XCAS [A] time = 0.217156, size = 20, normalized size = 0.87

$$\frac{4}{9}(t+1)^{\frac{9}{4}} - \frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((t + 1)^(1/4)*t,t, algorithm="giac")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

3.16 $\int \frac{1}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=11

$$\frac{x}{\sqrt{x^2+1}}$$

```
[Out] x/Sqrt[1 + x^2]
```

Rubi [A] time = 0.00470503, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2)^(-3/2),x]

[Out] x/Sqrt[1 + x^2]
```

Rubi in Sympy [A] time = 0.496187, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(3/2),x)

[Out] x/sqrt(x**2 + 1)
```

Mathematica [A] time = 0.00605728, size = 11, normalized size = 1.

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2)^(-3/2),x]

[Out] x/Sqrt[1 + x^2]
```

Maple [A] time = 0.003, size = 10, normalized size = 0.9

$$x\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(3/2),x)

[Out] x/(x^2+1)^(1/2)
```

Maxima [A] time = 1.35077, size = 12, normalized size = 1.09

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 1)
```

Fricas [A] time = 0.206744, size = 23, normalized size = 2.09

$$\frac{1}{x^2 - \sqrt{x^2 + 1}x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/(x^2 - sqrt(x^2 + 1)*x + 1)
```

Sympy [A] time = 1.18098, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(3/2),x)
```

```
[Out] x/sqrt(x**2 + 1)
```

GIAC/XCAS [A] time = 0.221558, size = 12, normalized size = 1.09

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 1)^(-3/2),x, algorithm="giac")
```

```
[Out] x/sqrt(x^2 + 1)
```

3.17 $\int x^2 \left(27+8x^3\right)^{2/3} dx$

Optimal. Leaf size=15

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

```
[Out] (27 + 8*x^3)^(5/3)/40
```

Rubi [A] time = 0.00691163, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(27 + 8*x^3)^(2/3),x]

[Out] (27 + 8*x^3)^(5/3)/40
```

Rubi in Sympy [A] time = 0.931484, size = 10, normalized size = 0.67

$$\frac{\left(8x^3+27\right)^{\frac{5}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(8*x**3+27)**(2/3),x)

[Out] (8*x**3 + 27)**(5/3)/40
```

Mathematica [A] time = 0.00629887, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(27 + 8*x^3)^(2/3),x]

[Out] (27 + 8*x^3)^(5/3)/40
```

Maple [B] time = 0.006, size = 27, normalized size = 1.8

$$\frac{(3+2x)\left(4x^2-6x+9\right)}{40}\left(8x^3+27\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3+27)^(2/3),x)

[Out] 1/40*(3+2*x)*(4*x^2-6*x+9)*(8*x^3+27)^(2/3)
```

Maxima [A] time = 1.36691, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 27)^(2/3)*x^2,x, algorithm="maxima")

[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Fricas [A] time = 0.20508, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 27)^(2/3)*x^2,x, algorithm="fricas")

[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Sympy [A] time = 0.45183, size = 27, normalized size = 1.8

$$\frac{x^3\left(8x^3+27\right)^{\frac{2}{3}}}{5}+\frac{27\left(8x^3+27\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(8*x**3+27)**(2/3),x)

[Out] x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

GIAC/XCAS [A] time = 0.216462, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8\,x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 27)^(2/3)*x^2,x, algorithm="giac")

[Out] 1/40*(8*x^3 + 27)^(5/3)
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rubi [A] time = 0.0430681, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rubi in Sympy [A] time = 2.8627, size = 12, normalized size = 0.8

$$\frac{3\left(\sin(x)-\cos(x)\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
```

```
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

Mathematica [A] time = 0.0289661, size = 15, normalized size = 1.

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Maple [A] time = 0.017, size = 12, normalized size = 0.8

$$\frac{3}{2}\left(-\cos(x)+\sin(x)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x)
```

```
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Maxima [A] time = 1.34672, size = 15, normalized size = 1.

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x) + sin(x))/(-cos(x) + sin(x))^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Fricas [A] time = 0.223118, size = 15, normalized size = 1.

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x) + sin(x))/(-cos(x) + sin(x))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Sympy [A] time = 0.482085, size = 12, normalized size = 0.8

$$\frac{3\left(\sin\left(x\right)-\cos\left(x\right)\right)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
```

```
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

GIAC/XCAS [A] time = 0.22928, size = 15, normalized size = 1.

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x) + sin(x))/(-cos(x) + sin(x))^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

3.19 $\int \frac{x}{\sqrt{1+x^2+(1+x^2)^{3/2}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.194228, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]

[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rubi in Sympy [A] time = 5.81592, size = 26, normalized size = 0.81

$$\frac{2\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)

[Out] 2*sqrt(x**2 + (x**2 + 1)**(3/2) + 1)/sqrt(x**2 + 1)
```

Mathematica [A] time = 0.0274833, size = 37, normalized size = 1.16

$$\frac{2\left(x^2+\sqrt{x^2+1}+1\right)}{\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]

[Out] (2*(1 + x^2 + Sqrt[1 + x^2]))/Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])]
```

Maple [F] time = 0.308, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{1+x^2+(x^2+1)^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

```
[Out] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

Maxima [A] time = 1.37432, size = 18, normalized size = 0.56

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Fricas [A] time = 0.241932, size = 31, normalized size = 0.97

$$\frac{2\sqrt{x^2+(x^2+1)^{\frac{3}{2}}+1}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(x^2+1)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

GIAC/XCAS [A] time = 0.222985, size = 20, normalized size = 0.62

$$2\sqrt{\sqrt{x^2+1}+1}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=17

$$2\sqrt{\sqrt{x^2+1}+1}$$

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.191203, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]

[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rubi in Sympy [A] time = 4.9089, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+1)**(1/2)/((x**2+1)**(1/2)+1)**(1/2),x)

[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

Mathematica [A] time = 0.0139193, size = 17, normalized size = 1.

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]

[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Maple [A] time = 0.007, size = 14, normalized size = 0.8

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x)

[Out] 2*((x^2+1)^(1/2)+1)^(1/2)
```

Maxima [A] time = 1.35976, size = 18, normalized size = 1.06

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(x^2 + 1)*sqrt(sqrt(x^2 + 1) + 1)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Fricas [A] time = 0.240288, size = 18, normalized size = 1.06

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(x^2 + 1)*sqrt(sqrt(x^2 + 1) + 1)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Sympy [A] time = 0.479281, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+1)**(1/2)/((x**2+1)**(1/2)+1)**(1/2),x)
```

```
[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

GIAC/XCAS [A] time = 0.216606, size = 18, normalized size = 1.06

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(sqrt(x^2 + 1)*sqrt(sqrt(x^2 + 1) + 1)),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx$

Optimal. Leaf size=16

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rubi [A] time = 0.0211461, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rubi in Sympy [A] time = 2.34145, size = 15, normalized size = 0.94

$$-\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((x**2-2*x+1)**(1/5)/(1-x),x)
```

```
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

Mathematica [A] time = 0.00881329, size = 13, normalized size = 0.81

$$-\frac{5}{2}\sqrt[5]{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*((-1 + x)^2)^(1/5))/2
```

Maple [A] time = 0.004, size = 13, normalized size = 0.8

$$-\frac{5}{2}\sqrt[5]{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)^(1/5)/(1-x),x)
```

```
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

Maxima [A] time = 1.35173, size = 9, normalized size = 0.56

$$-\frac{5}{2}(x-1)^{\frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x, algorithm="maxima")
```

```
[Out] -5/2*(x - 1)^(2/5)
```

Fricas [A] time = 0.233747, size = 16, normalized size = 1.

$$-\frac{5}{2}\left(x^{2}-2x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x, algorithm="fricas")
```

```
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Sympy [A] time = 1.16359, size = 15, normalized size = 0.94

$$-\frac{5\sqrt[5]{x^{2}-2x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-2*x+1)**(1/5)/(1-x),x)
```

```
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(x^{2}-2x+1\right)^{\frac{1}{5}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^2 - 2*x + 1)^(1/5)/(x - 1),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x + 1)^(1/5)/(x - 1), x)
```

3.22 $\int x \sin(x)\, dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0146978, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]
```

Rubi in Sympy [A] time = 0.754444, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*sin(x),x)

[Out] -x*cos(x) + sin(x)
```

Mathematica [A] time = 0.00333774, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]

[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0.093, size = 9, normalized size = 1.1

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)

[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 1.36352, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="maxima")

[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 0.241875, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="fricas")

[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.181218, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x)

[Out] -x*cos(x) + sin(x)
```

GIAC/XCAS [A] time = 0.214402, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x),x, algorithm="giac")

[Out] -x*cos(x) + sin(x)
```

3.23 $\int x^2 \sin(x)\, dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi [A] time = 0.0335054, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[x],x]

[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi in Sympy [A] time = 1.39113, size = 17, normalized size = 1.

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x),x)

[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Mathematica [A] time = 0.0102955, size = 15, normalized size = 0.88

$$2x\sin(x) - (x^2 - 2)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[x],x]

[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A] time = 0.007, size = 18, normalized size = 1.1

$$2\cos(x) - x^2\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x),x)

[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A] time = 1.34057, size = 20, normalized size = 1.18

$$-(x^2 - 2)\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A] time = 0.238809, size = 20, normalized size = 1.18

$$-\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A] time = 0.393845, size = 17, normalized size = 1.

$$-x^2\cos\left(x\right)+2x\sin\left(x\right)+2\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

GIAC/XCAS [A] time = 0.218543, size = 20, normalized size = 1.18

$$-\left(x^2-2\right)\cos\left(x\right)+2\,x\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.24 $\int x^3 \cos(x)\,dx$

Optimal. Leaf size=23

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

```
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rubi [A] time = 0.0532141, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Cos[x],x]

[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rubi in Sympy [A] time = 1.92231, size = 26, normalized size = 1.13

$$x^3 \sin{(x)} + 3x^2 \cos{(x)} - 6x \sin{(x)} - 6 \cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*cos(x),x)

[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

Mathematica [A] time = 0.0118285, size = 19, normalized size = 0.83

$$x\left(x^2-6\right)\sin(x)+3\left(x^2-2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Cos[x],x]

[Out] 3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

Maple [A] time = 0.01, size = 24, normalized size = 1.

$$-6\,\cos(x)+3\,x^2\cos(x)-6\,x\sin(x)+x^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*cos(x),x)

[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Maxima [A] time = 1.33521, size = 27, normalized size = 1.17

$$3\left(x^2-2\right)\cos(x)+\left(x^3-6\,x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*cos(x),x, algorithm="maxima")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Fricas [A] time = 0.239757, size = 27, normalized size = 1.17

$$3\left(x^2-2\right)\cos\left(x\right)+\left(x^3-6\,x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*cos(x),x, algorithm="fricas")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Sympy [A] time = 0.870053, size = 26, normalized size = 1.13

$$x^3\sin\left(x\right)+3x^2\cos\left(x\right)-6x\sin\left(x\right)-6\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*cos(x),x)
```

```
[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

GIAC/XCAS [A] time = 0.215839, size = 27, normalized size = 1.17

$$3\left(x^2-2\right)\cos\left(x\right)+\left(x^3-6\,x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*cos(x),x, algorithm="giac")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

3.25 $\int x^3 \sin(x)\,dx$

Optimal. Leaf size=24

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi [A] time = 0.0522071, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$x^3(-\cos(x)) + 3x^2\sin(x) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sin[x],x]

[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi in Sympy [A] time = 1.92561, size = 26, normalized size = 1.08

$$-x^3\cos(x) + 3x^2\sin(x) + 6x\cos(x) - 6\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*sin(x),x)

[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Mathematica [A] time = 0.012055, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x) - x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sin[x],x]

[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] time = 0.007, size = 25, normalized size = 1.

$$6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*sin(x),x)

[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Maxima [A] time = 1.35021, size = 28, normalized size = 1.17

$$-\left(x^3-6x\right)\cos(x) + 3\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Fricas [A] time = 0.242894, size = 28, normalized size = 1.17

$$-\left(x^{3} - 6\,x\right)\,\cos\left(x\right) + 3\,\left(x^{2} - 2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] time = 0.924283, size = 26, normalized size = 1.08

$$-x^{3}\cos\left(x\right) + 3x^{2}\sin\left(x\right) + 6x\cos\left(x\right) - 6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

GIAC/XCAS [A] time = 0.221625, size = 28, normalized size = 1.17

$$-\left(x^{3} - 6\,x\right)\,\cos\left(x\right) + 3\,\left(x^{2} - 2\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*sin(x),x, algorithm="giac")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.26 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.0114538, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int^{\sin(x)} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cos(x)*sin(x),x)
```

```
[Out] Integral(x, (x, sin(x)))
```

Mathematica [A] time = 0.00207701, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -Cos[x]^2/2
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A] time = 1.34008, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 0.235346, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.03586, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

GIAC/XCAS [A] time = 0.218294, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.27 $\int x\cos(x)\sin(x)\,dx$

Optimal. Leaf size=23

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rubi [A] time = 0.0204367, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]*Sin[x],x]
```

```
[Out] -x/4 + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rubi in Sympy [A] time = 1.03985, size = 19, normalized size = 0.83

$$\frac{x\sin^2(x)}{2}-\frac{x}{4}+\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*cos(x)*sin(x),x)
```

```
[Out] x*sin(x)**2/2 - x/4 + sin(x)*cos(x)/4
```

Mathematica [A] time = 0.00437193, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2x)-\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]*Sin[x],x]
```

```
[Out] -(x*Cos[2*x])/4 + Sin[2*x]/8
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{x(\cos(x))^2}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x)
```

```
[Out] -1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x
```

Maxima [A] time = 1.39561, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*sin(x),x, algorithm="maxima")

[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Fricas [A] time = 0.225672, size = 23, normalized size = 1.

$$-\frac{1}{2}\,x\cos\left(x\right)^{2}+\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.412793, size = 24, normalized size = 1.04

$$\frac{x\sin^{2}\left(x\right)}{4}-\frac{x\cos^{2}\left(x\right)}{4}+\frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*sin(x),x)

[Out] x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

GIAC/XCAS [A] time = 0.213684, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x)*sin(x),x, algorithm="giac")

[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

3.28 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0102308, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.482715, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00277329, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0.009, size = 11, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2,x)

[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.35077, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 0.219675, size = 14, normalized size = 1.

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.034959, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.214403, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.29 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0105249, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]

[Out] -Cos[x] + Cos[x]^3/3
```

Rubi in Sympy [A] time = 0.632831, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.00274161, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]

[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)

[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 1.36099, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos{(x)}^3-\cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="maxima")

[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.212313, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos{(x)}^3-\cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="fricas")

[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.038895, size = 8, normalized size = 0.62

$$\frac{\cos^3{(x)}}{3}-\cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**3,x)

[Out] cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A] time = 0.227016, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos{(x)}^3-\cos{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^3,x, algorithm="giac")

[Out] 1/3*cos(x)^3 - cos(x)
```

3.30 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0167588, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi in Sympy [A] time = 0.558954, size = 24, normalized size = 1.

$$\frac{3x}{8}-\frac{\sin^3(x)\cos(x)}{4}-\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00284657, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.105, size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 1.33013, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.224621, size = 26, normalized size = 1.08

$$\frac{1}{8}\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.037607, size = 24, normalized size = 1.

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

GIAC/XCAS [A] time = 0.216976, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.31 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi [A] time = 0.0126419, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^5,x]

[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi in Sympy [A] time = 0.744482, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**5,x)

[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Mathematica [A] time = 0.00320367, size = 23, normalized size = 1.1

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^5,x]

[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

Maple [A] time = 0.053, size = 17, normalized size = 0.8

$$-\frac{\cos(x)}{5}\left(\frac{8}{3}+(\sin(x))^4+\frac{4(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^5,x)

[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 1.48142, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="maxima")

[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.216444, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="fricas")

[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.046102, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5}+\frac{2\cos^3(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**5,x)

[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

GIAC/XCAS [A] time = 0.223361, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^5,x, algorithm="giac")

[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.32 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x
]*Sin[x]^5)/6
```

Rubi [A] time = 0.0252156, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x
]*Sin[x]^5)/6
```

Rubi in Sympy [A] time = 0.665223, size = 36, normalized size = 1.06

$$\frac{5x}{16}-\frac{\sin^5(x)\cos(x)}{6}-\frac{5\sin^3(x)\cos(x)}{24}-\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*co
s(x)/16
```

Mathematica [A] time = 0.00301104, size = 30, normalized size = 0.88

$$\frac{5x}{16}-\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] time = 0.047, size = 24, normalized size = 0.7

$$-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5\,(\sin(x))^3}{4}+\frac{15\,\sin(x)}{8}\right)+\frac{5\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^6,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

Maxima [A] time = 1.35154, size = 32, normalized size = 0.94

$$\frac{1}{48}\sin(2x)^3 + \frac{5}{16}x + \frac{3}{64}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.255924, size = 34, normalized size = 1.

$$-\frac{1}{48}\left(8\cos(x)^5 - 26\cos(x)^3 + 33\cos(x)\right)\sin(x) + \frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.039202, size = 36, normalized size = 1.06

$$\frac{5x}{16} - \frac{\sin^5(x)\cos(x)}{6} - \frac{5\sin^3(x)\cos(x)}{24} - \frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*co
s(x)/16
```

GIAC/XCAS [A] time = 0.224505, size = 30, normalized size = 0.88

$$\frac{5}{16}x - \frac{1}{192}\sin(6x) + \frac{3}{64}\sin(4x) - \frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

3.33 $\int x\sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi [A] time = 0.0225089, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x\sin(x)\cos(x)}{2}+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*sin(x)**2,x)
```

```
[Out] -x*sin(x)*cos(x)/2 + sin(x)**2/4 + Integral(x, x)/2
```

Mathematica [A] time = 0.00520388, size = 25, normalized size = 1.

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A] time = 0.007, size = 25, normalized size = 1.

$$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(x)^2,x)
```

```
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A] time = 1.34804, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A] time = 0.236728, size = 26, normalized size = 1.04

$$-\frac{1}{2}x\cos(x)\sin(x) + \frac{1}{4}x^2 - \frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A] time = 0.409773, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4} + \frac{x^2\cos^2(x)}{4} - \frac{x\sin(x)\cos(x)}{2} + \frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)*
*2/4
```

GIAC/XCAS [A] time = 0.229944, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.34 $\int x \sin^3(x)\, dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3
/9
```

Rubi [A] time = 0.0334462, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3
/9
```

Rubi in Sympy [A] time = 1.32645, size = 32, normalized size = 0.97

$$-\frac{x\sin^2(x)\cos(x)}{3}-\frac{2x\cos(x)}{3}+\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x)/3 - 2*x*cos(x)/3 + sin(x)**3/9 + 2*sin(x)/3
```

Mathematica [A] time = 0.00627071, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A] time = 0.078, size = 23, normalized size = 0.7

$$-\frac{x\left(2+(\sin(x))^2\right)\cos(x)}{3}+\frac{(\sin(x))^3}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
```

```
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A] time = 1.35688, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A] time = 0.226867, size = 31, normalized size = 0.94

$$\frac{1}{3}\,x\cos\left(x\right)^{3}-x\cos\left(x\right)-\frac{1}{9}\,\left(\cos\left(x\right)^{2}-7\right)\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A] time = 0.818743, size = 39, normalized size = 1.18

$$-x\sin^{2}\left(x\right)\cos\left(x\right)-\frac{2x\cos^{3}\left(x\right)}{3}+\frac{7\sin^{3}\left(x\right)}{9}+\frac{2\sin\left(x\right)\cos^{2}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*
cos(x)**2/3
```

GIAC/XCAS [A] time = 0.221893, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\cos\left(3\,x\right)-\frac{3}{4}\,x\cos\left(x\right)-\frac{1}{36}\,\sin\left(3\,x\right)+\frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.35 $\int x^2 \sin^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin
[x]^2)/2
```

Rubi [A] time = 0.0463262, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin
[x]^2)/2
```

Rubi in Sympy [A] time = 1.56759, size = 36, normalized size = 0.88

$$\frac{x^3}{6}-\frac{x^2\sin(x)\cos(x)}{2}+\frac{x\sin^2(x)}{2}-\frac{x}{4}+\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/2 - x/4 + sin(x)*cos(
x)/4
```

Mathematica [A] time = 0.034314, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0.036, size = 37, normalized size = 0.9

$$x^2\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x(\cos(x))^2}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*sin(x)^2,x)
```

```
[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/
4*x-1/3*x^3
```

Maxima [A] time = 1.35386, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 0.217224, size = 39, normalized size = 0.95

$$\frac{1}{6}x^3 - \frac{1}{2}x\cos(x)^2 - \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.850384, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} - \frac{x^2\sin(x)\cos(x)}{2} + \frac{x\sin^2(x)}{4} - \frac{x\cos^2(x)}{4} + \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*si
n(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

GIAC/XCAS [A] time = 0.216416, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.36 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0103652, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi in Sympy [A] time = 0.5092, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Mathematica [A] time = 0.00282993, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.01, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.36247, size = 14, normalized size = 1.

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.223295, size = 14, normalized size = 1.

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right) + \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.033515, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.216073, size = 14, normalized size = 1.

$$\frac{1}{2}\,x + \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.37 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.0107204, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi in Sympy [A] time = 0.671478, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

Mathematica [A] time = 0.00293136, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0.04, size = 11, normalized size = 1.

$$\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 1.35495, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="maxima")

[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.206491, size = 14, normalized size = 1.27

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="fricas")

[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.040248, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**3,x)

[Out] -sin(x)**3/3 + sin(x)
```

GIAC/XCAS [A] time = 0.234835, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^3,x, algorithm="giac")

[Out] -1/3*sin(x)^3 + sin(x)
```

3.38 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0185558, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi in Sympy [A] time = 0.59117, size = 24, normalized size = 1.

$$\frac{3x}{8}+\frac{\sin(x)\cos^3(x)}{4}+\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Mathematica [A] time = 0.00288593, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.062, size = 18, normalized size = 0.8

$$\frac{\sin(x)}{4}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cos(x)^4,x)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

Maxima [A] time = 1.35768, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.246385, size = 26, normalized size = 1.08

$$\frac{1}{8}\left(2\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.03856, size = 24, normalized size = 1.

$$\frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

GIAC/XCAS [A] time = 0.236757, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.39 $\int \left(a^2-x^2\right)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16}a^4x\sqrt{a^2-x^2}$$

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (
x*(a^2 - x^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rubi [A] time = 0.036184, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16}a^4x\sqrt{a^2-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 - x^2)^(5/2),x]
```

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (
x*(a^2 - x^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rubi in Sympy [A] time = 2.68722, size = 70, normalized size = 0.83

$$\frac{5a^6\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)}{16}+\frac{5a^4x\sqrt{a^2-x^2}}{16}+\frac{5a^2x\left(a^2-x^2\right)^{\frac{3}{2}}}{24}+\frac{x\left(a^2-x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((a**2-x**2)**(5/2),x)
```

```
[Out] 5*a**6*atan(x/sqrt(a**2 - x**2))/16 + 5*a**4*x*sqrt(a**2 - x**2)/
16 + 5*a**2*x*(a**2 - x**2)**(3/2)/24 + x*(a**2 - x**2)**(5/2)/6
```

Mathematica [A] time = 0.0612947, size = 60, normalized size = 0.71

$$\frac{1}{48}\left(15a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+x\sqrt{a^2-x^2}\left(33a^4-26a^2x^2+8x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 - x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a^2 - x^2]*(33*a^4 - 26*a^2*x^2 + 8*x^4) + 15*a^6*ArcTan[
x/Sqrt[a^2 - x^2]])/48
```

Maple [A] time = 0.028, size = 69, normalized size = 0.8

$$\frac{5\,a^2x}{24}\left(a^2-x^2\right)^{\frac{3}{2}}+\frac{x}{6}\left(a^2-x^2\right)^{\frac{5}{2}}+\frac{5\,a^6}{16}\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)+\frac{5\,a^4x}{16}\sqrt{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((a^2-x^2)^(5/2),x)
```

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(
x/(a^2-x^2)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

Maxima [A] time = 1.50729, size = 84, normalized size = 1.

$$\frac{5}{16}\,a^6\arcsin\left(\frac{x}{\sqrt{a^2}}\right)+\frac{5}{16}\,\sqrt{a^2-x^2}a^4x+\frac{5}{24}\,\left(a^2-x^2\right)^{\frac{3}{2}}a^2x+\frac{1}{6}\,\left(a^2-x^2\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*arcsin(x/sqrt(a^2)) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*
(a^2 - x^2)^(3/2)*a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

Fricas [A] time = 0.214446, size = 343, normalized size = 4.08

$$-\frac{1056\,a^{11}x-2944\,a^9x^3+3174\,a^7x^5-1698\,a^5x^7+460\,a^3x^9-48\,ax^{11}+30\left(32\,a^{12}-48\,a^{10}x^2+18\,a^8x^4-a^6x^6-2\left(16\,a^{11}\cdots\right.\right.}{48\left(32\,a^6-48\,a^4x^2+18\,a^2x^4-x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/48*(1056*a^11*x - 2944*a^9*x^3 + 3174*a^7*x^5 - 1698*a^5*x^7 +
 460*a^3*x^9 - 48*a*x^11 + 30*(32*a^12 - 48*a^10*x^2 + 18*a^8*x^4
 - a^6*x^6 - 2*(16*a^11 - 16*a^9*x^2 + 3*a^7*x^4)*sqrt(a^2 - x^2)
)*arctan(-(a - sqrt(a^2 - x^2))/x) - (1056*a^10*x - 2416*a^8*x^3
+ 2098*a^6*x^5 - 885*a^4*x^7 + 170*a^2*x^9 - 8*x^11)*sqrt(a^2 - x
^2))/(32*a^6 - 48*a^4*x^2 + 18*a^2*x^4 - x^6 - 2*(16*a^5 - 16*a^3
*x^2 + 3*a*x^4)*sqrt(a^2 - x^2))
```

Sympy [A] time = 6.10086, size = 180, normalized size = 2.14

$$\begin{cases} -\frac{5ia^6\operatorname{acosh}\left(\frac{x}{a}\right)}{16}-\frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}}+\frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}}-\frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}}+\frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for}\ \left|\frac{x^2}{a^2}\right|>1 \\ \frac{5a^6\operatorname{asin}\left(\frac{x}{a}\right)}{16}+\frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16}-\frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24}+\frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a**2-x**2)**(5/2),x)
```

```
[Out] Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x*
*2/a**2)) + 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**
5/(24*sqrt(-1 + x**2/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)),
Abs(x**2/a**2) > 1), (5*a**6*asin(x/a)/16 + 11*a**5*x*sqrt(1 - x*
*2/a**2)/16 - 13*a**3*x**3*sqrt(1 - x**2/a**2)/24 + a*x**5*sqrt(1
 - x**2/a**2)/6, True))
```

GIAC/XCAS [A] time = 0.249435, size = 68, normalized size = 0.81

$$\frac{5}{16}\,a^6\arcsin\left(\frac{x}{a}\right)\operatorname{sign}(a)+\frac{1}{48}\,\left(33\,a^4-2\left(13\,a^2-4\,x^2\right)x^2\right)\sqrt{a^2-x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((a^2 - x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/16*a^6*arcsin(x/a)*sign(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*
x^2)*sqrt(a^2 - x^2)*x
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rubi [A] time = 0.034993, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[5 + x^2],x]
```

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rubi in Sympy [A] time = 2.36343, size = 31, normalized size = 0.82

$$\frac{\left(x^2+5\right)^{\frac{5}{2}}}{5}-\frac{10\left(x^2+5\right)^{\frac{3}{2}}}{3}+25\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(x**2+5)**(1/2),x)
```

```
[Out] (x**2 + 5)**(5/2)/5 - 10*(x**2 + 5)**(3/2)/3 + 25*sqrt(x**2 + 5)
```

Mathematica [A] time = 0.0101233, size = 25, normalized size = 0.66

$$\frac{1}{15}\sqrt{x^2+5}\left(3x^4-20x^2+200\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[5 + x^2],x]
```

```
[Out] (Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

Maple [A] time = 0.007, size = 22, normalized size = 0.6

$$\frac{3\,x^4-20\,x^2+200}{15}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(x^2+5)^(1/2),x)
```

```
[Out] 1/15*(x^2+5)^(1/2)*(3*x^4-20*x^2+200)
```

Maxima [A] time = 1.51067, size = 46, normalized size = 1.21

$$\frac{1}{5}\sqrt{x^2+5}x^4 - \frac{4}{3}\sqrt{x^2+5}x^2 + \frac{40}{3}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(x^2 + 5),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5
)
```

Fricas [A] time = 0.195129, size = 134, normalized size = 3.53

$$-\frac{48\,x^{10} + 100\,x^{8} + 1375\,x^{6} + 21875\,x^{4} + 62500\,x^{2} - \left(48\,x^{9} - 20\,x^{7} + 1575\,x^{5} + 17500\,x^{3} + 25000\,x\right)\sqrt{x^2+5} + 25000}{15\left(16\,x^{5} + 100\,x^{3} - (16\,x^{4} + 60\,x^{2} + 25)\sqrt{x^2+5} + 125\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(x^2 + 5),x, algorithm="fricas")
```

```
[Out] -1/15*(48*x^10 + 100*x^8 + 1375*x^6 + 21875*x^4 + 62500*x^2 - (48
*x^9 - 20*x^7 + 1575*x^5 + 17500*x^3 + 25000*x)*sqrt(x^2 + 5) + 2
5000)/(16*x^5 + 100*x^3 - (16*x^4 + 60*x^2 + 25)*sqrt(x^2 + 5) +
125*x)
```

Sympy [A] time = 1.68638, size = 39, normalized size = 1.03

$$\frac{x^4\sqrt{x^2+5}}{5} - \frac{4x^2\sqrt{x^2+5}}{3} + \frac{40\sqrt{x^2+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(x**2+5)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 +
5)/3
```

GIAC/XCAS [A] time = 0.235436, size = 38, normalized size = 1.

$$\frac{1}{5}\left(x^2+5\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2+5\right)^{\frac{3}{2}} + 25\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(x^2 + 5),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}}\,dt$

Optimal. Leaf size=172

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t
)*Sqrt[(2*2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^
2]*EllipticF[ArcSin[(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqr
t[3]) + t)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2
/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rubi [A] time = 0.125541, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\ 2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}F\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right)\middle|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t
)*Sqrt[(2*2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^
2]*EllipticF[ArcSin[(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqr
t[3]) + t)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2
/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rubi in Sympy [A] time = 2.24772, size = 162, normalized size = 0.94

$$\frac{2t\sqrt{t^3+4}}{5}-\frac{8\cdot 3^{\frac{3}{4}}\sqrt{\frac{2^{\frac{2}{3}}t^2-2\sqrt[3]{2}t+4}{\left(\sqrt[3]{2}t+2+2\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\left(2t+2\cdot 2^{\frac{2}{3}}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{2}t-2\sqrt{3}+2}{\sqrt[3]{2}t+2+2\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{15\sqrt{\frac{2\sqrt[3]{2}t+4}{\left(\sqrt[3]{2}t+2+2\sqrt{3}\right)^2}}\sqrt{t^3+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**3/(t**3+4)**(1/2),t)
```

```
[Out] 2*t*sqrt(t**3 + 4)/5 - 8*3**(3/4)*sqrt((2**(2/3)*t**2 - 2*2**(1/3
)*t + 4)/(2**(1/3)*t + 2 + 2*sqrt(3))**2)*sqrt(sqrt(3) + 2)*(2*t
+ 2*2**(2/3))*elliptic_f(asin((2**(1/3)*t - 2*sqrt(3) + 2)/(2**(1
/3)*t + 2 + 2*sqrt(3))), -7 - 4*sqrt(3))/(15*sqrt((2*2**(1/3)*t +
 4)/(2**(1/3)*t + 2 + 2*sqrt(3))**2)*sqrt(t**3 + 4))
```

Mathematica [C] time = 0.266495, size = 122, normalized size = 0.71

$$\frac{6t\left(t^3+4\right)-8\sqrt[6]{-2}3^{3/4}\sqrt{-\sqrt[6]{-1}\left(\sqrt[3]{2}t+2(-1)^{2/3}\right)}\sqrt{(-2)^{2/3}t^2+2\sqrt[3]{-2}t+4}F\left(\sin^{-1}\left(\frac{\sqrt{\left(-i+\sqrt{3}\right)\left(\sqrt[3]{2}t+2\right)}}{2\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)}{15\sqrt{t^3+4}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (6*t*(4 + t^3) - 8*(-2)^(1/6)*3^(3/4)*Sqrt[-((-1)^(1/6)*(2*(-1)^(
2/3) + 2^(1/3)*t))]*Sqrt[4 + 2*(-2)^(1/3)*t + (-2)^(2/3)*t^2]*Ell
ipticF[ArcSin[Sqrt[(-I + Sqrt[3])*(2 + 2^(1/3)*t)]/(2*3^(1/4))],
(-1)^(1/3)])/(15*Sqrt[4 + t^3])
```

Maple [A] time = 0.658, size = 168, normalized size = 1.

$$\frac{2\,t}{5}\sqrt{t^3+4}$$
$$+\frac{8\,i}{15}\sqrt{3}2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}\right)\sqrt{3}\sqrt[3]{2}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{2/3}}{2}+\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}\right)\sqrt{3}\sqrt[3]{2}}\mathit{EllipticF}\left(\frac{\sqrt{6}}{6}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^3/(t^3+4)^(1/2),t)
```

```
[Out] 2/5*t*(t^3+4)^(1/2)+8/15*I*3^(1/2)*2^(2/3)*(I*(t-1/2*2^(2/3)-1/2*
I*3^(1/2)*2^(2/3))*3^(1/2)*2^(1/3))^(1/2)*((2^(2/3)+t)/(3/2*2^(2/
3)+1/2*I*3^(1/2)*2^(2/3)))^(1/2)*(-I*(t-1/2*2^(2/3)+1/2*I*3^(1/2)
*2^(2/3))*3^(1/2)*2^(1/3))^(1/2)/(t^3+4)^(1/2)*EllipticF(1/6*6^(1
/2)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*2^(2/3))*3^(1/2)*2^(1/3))^(1/
2),(I*3^(1/2)*2^(2/3)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2^(2/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3/sqrt(t^3 + 4),t, algorithm="maxima")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{t^3}{\sqrt{t^3+4}}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3/sqrt(t^3 + 4),t, algorithm="fricas")
```

```
[Out] integral(t^3/sqrt(t^3 + 4), t)
```

Sympy [A] time = 0.899525, size = 31, normalized size = 0.18

$$\frac{t^4\,\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{t^3e^{i\pi}}{4}\right)}{6\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**3/(t**3+4)**(1/2),t)
```

```
[Out] t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)
/(6*gamma(7/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3/sqrt(t^3 + 4),t, algorithm="giac")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

3.42 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x + Tan[x]
```

Rubi [A] time = 0.0085973, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]

[Out] -x + Tan[x]
```

Rubi in Sympy [A] time = 0.045619, size = 3, normalized size = 0.5

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(tan(x)**2,x)

[Out] -x + tan(x)
```

Mathematica [A] time = 0.00369132, size = 6, normalized size = 1.

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]

[Out] -x + Tan[x]
```

Maple [A] time = 0.019, size = 7, normalized size = 1.2

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)

[Out] -x+tan(x)
```

Maxima [A] time = 1.50498, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="maxima")
```

```
[Out] -x + tan(x)
```

Fricas [A] time = 0.213338, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

Sympy [A] time = 0.044702, size = 7, normalized size = 1.17

$$-x + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.220536, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

3.43 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.0159044, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Tan[x]^4,x]

[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi in Sympy [A] time = 0.473357, size = 10, normalized size = 0.71

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(tan(x)**4,x)

[Out] x + tan(x)**3/3 - tan(x)
```

Mathematica [A] time = 0.0051962, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Tan[x]^4,x]

[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$x-\tan(x)+\frac{(\tan(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(tan(x)^4,x)

[Out] x-tan(x)+1/3*tan(x)^3
```

Maxima [A] time = 1.49914, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A] time = 0.242919, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] time = 0.049686, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

GIAC/XCAS [A] time = 0.232812, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

3.44 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x - Cot[x]
```

Rubi [A] time = 0.00979404, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]

[Out] -x - Cot[x]
```

Rubi in Sympy [A] time = 0.45654, size = 7, normalized size = 0.88

$$-x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(cot(x)**2,x)

[Out] -x - 1/tan(x)
```

Mathematica [A] time = 0.00359821, size = 8, normalized size = 1.

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]

[Out] -x - Cot[x]
```

Maple [A] time = 0.011, size = 12, normalized size = 1.5

$$-\cot(x)+\frac{\pi}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)

[Out] -cot(x)+1/2*Pi-x
```

Maxima [A] time = 1.50198, size = 14, normalized size = 1.75

$$-x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

Fricas [A] time = 0.215025, size = 27, normalized size = 3.38

$$-\frac{x\sin(2x) + \cos(2x) + 1}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] time = 0.044884, size = 8, normalized size = 1.

$$-x - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

GIAC/XCAS [A] time = 0.226111, size = 24, normalized size = 3.

$$-x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.45 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0174922, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x]^4,x]

[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi in Sympy [A] time = 0.491307, size = 14, normalized size = 1.17

$$x+\frac{1}{\tan(x)}-\frac{1}{3\tan^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x)**4,x)

[Out] x + 1/tan(x) - 1/(3*tan(x)**3)
```

Mathematica [A] time = 0.00532964, size = 18, normalized size = 1.5

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x]^4,x]

[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.004, size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3}+\cot(x)-\frac{\pi}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x)^4,x)

[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.49967, size = 22, normalized size = 1.83

$$x + \frac{3\,\tan\left(x\right)^{2} - 1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [A] time = 0.206218, size = 65, normalized size = 5.42

$$\frac{4\,\cos\left(2\,x\right)^{2} + 3\,(x\cos\left(2\,x\right) - x)\sin\left(2\,x\right) + 2\,\cos\left(2\,x\right) - 2}{3\,(\cos\left(2\,x\right) - 1)\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)
/((cos(2*x) - 1)*sin(2*x))
```

Sympy [A] time = 0.053439, size = 19, normalized size = 1.58

$$x + \frac{\cos\left(x\right)}{\sin\left(x\right)} - \frac{\cos^{3}\left(x\right)}{3\sin^{3}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

GIAC/XCAS [A] time = 0.220438, size = 46, normalized size = 3.83

$$\frac{1}{24}\,\tan\left(\frac{1}{2}\,x\right)^{3} + x + \frac{15\,\tan\left(\frac{1}{2}\,x\right)^{2} - 1}{24\,\tan\left(\frac{1}{2}\,x\right)^{3}} - \frac{5}{8}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
 5/8*tan(1/2*x)
```

3.46 $\int (2+3x)\sin(5x)\,dx$

Optimal. Leaf size=22

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

```
[Out] -((2 + 3*x)*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Rubi [A] time = 0.0213374, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*Sin[5*x],x]

[Out] -((2 + 3*x)*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Rubi in Sympy [A] time = 1.15993, size = 20, normalized size = 0.91

$$-\left(\frac{3x}{5}+\frac{2}{5}\right)\cos(5x)+\frac{3\sin(5x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((2+3*x)*sin(5*x),x)

[Out] -(3*x/5 + 2/5)*cos(5*x) + 3*sin(5*x)/25
```

Mathematica [A] time = 0.00817653, size = 26, normalized size = 1.18

$$\frac{3}{25}\sin(5x)-\frac{3}{5}x\cos(5x)-\frac{2}{5}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*Sin[5*x],x]

[Out] (-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Maple [A] time = 0.019, size = 21, normalized size = 1.

$$-\frac{2\cos(5x)}{5}+\frac{3\sin(5x)}{25}-\frac{3\cos(5x)x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*sin(5*x),x)

[Out] -2/5*cos(5*x)+3/25*sin(5*x)-3/5*cos(5*x)*x
```

Maxima [A] time = 1.3251, size = 27, normalized size = 1.23

$$-\frac{3}{5}\,x\cos\left(5\,x\right)-\frac{2}{5}\,\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)*sin(5*x),x, algorithm="maxima")
```

```
[Out] -3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

Fricas [A] time = 0.215428, size = 24, normalized size = 1.09

$$-\frac{1}{5}\left(3\,x+2\right)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)*sin(5*x),x, algorithm="fricas")
```

```
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Sympy [A] time = 0.212422, size = 26, normalized size = 1.18

$$-\frac{3x\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}-\frac{2\cos\left(5x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+3*x)*sin(5*x),x)
```

```
[Out] -3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

GIAC/XCAS [A] time = 0.217808, size = 24, normalized size = 1.09

$$-\frac{1}{5}\left(3\,x+2\right)\cos\left(5\,x\right)+\frac{3}{25}\,\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x + 2)*sin(5*x),x, algorithm="giac")
```

```
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

3.47 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.00530596, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Rubi in Sympy [A] time = 0.738792, size = 8, normalized size = 0.62

$$\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(x**2+1)**(1/2),x)

[Out] (x**2 + 1)**(3/2)/3
```

Mathematica [A] time = 0.00351149, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)

[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 1.35494, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.203395, size = 93, normalized size = 7.15

$$-\frac{4\,x^6+9\,x^4+6\,x^2-\left(4\,x^5+7\,x^3+3\,x\right)\sqrt{x^2+1}+1}{3\left(4\,x^3-\left(4\,x^2+1\right)\sqrt{x^2+1}+3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^6 + 9*x^4 + 6*x^2 - (4*x^5 + 7*x^3 + 3*x)*sqrt(x^2 + 1)
 + 1)/(4*x^3 - (4*x^2 + 1)*sqrt(x^2 + 1) + 3*x)
```

Sympy [A] time = 0.215418, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

GIAC/XCAS [A] time = 0.219743, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)*x,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

3.48 $\int x\left(-1+x^2\right)^9\,dx$

Optimal. Leaf size=13

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

```
[Out] (1 - x^2)^10/20
```

Rubi [A] time = 0.00727161, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(-1 + x^2)^9,x]

[Out] (1 - x^2)^10/20
```

Rubi in Sympy [A] time = 0.718554, size = 7, normalized size = 0.54

$$\frac{\left(-x^2+1\right)^{10}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(x**2-1)**9,x)

[Out] (-x**2 + 1)**10/20
```

Mathematica [A] time = 0.00273841, size = 11, normalized size = 0.85

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(-1 + x^2)^9,x]

[Out] (-1 + x^2)^10/20
```

Maple [B] time = 0.026, size = 52, normalized size = 4.

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9\,x^{16}}{4}-6\,x^{14}+\frac{21\,x^{12}}{2}-\frac{63\,x^{10}}{5}+\frac{21\,x^{8}}{2}-6\,x^{6}+\frac{9\,x^{4}}{4}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(x^2-1)^9,x)

[Out] 1/20*x^20-1/2*x^18+9/4*x^16-6*x^14+21/2*x^12-63/5*x^10+21/2*x^8-6
*x^6+9/4*x^4-1/2*x^2
```

Maxima [A] time = 1.36909, size = 12, normalized size = 0.92

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^9*x,x, algorithm="maxima")
```

```
[Out] 1/20*(x^2 - 1)^10
```

Fricas [A] time = 0.194271, size = 1, normalized size = 0.08

$$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^{8}-6x^{6}+\frac{9}{4}x^{4}-\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^9*x,x, algorithm="fricas")
```

```
[Out] 1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10
+ 21/2*x^8 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

Sympy [A] time = 0.038316, size = 58, normalized size = 4.46

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9x^{16}}{4}-6x^{14}+\frac{21x^{12}}{2}-\frac{63x^{10}}{5}+\frac{21x^{8}}{2}-6x^{6}+\frac{9x^{4}}{4}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(x**2-1)**9,x)
```

```
[Out] x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/
5 + 21*x**8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

GIAC/XCAS [A] time = 0.228086, size = 12, normalized size = 0.92

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^9*x,x, algorithm="giac")
```

```
[Out] 1/20*(x^2 - 1)^10
```

3.49 $\int \frac{3+2x}{(7+6x)^3}\,dx$

Optimal. Leaf size=18

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

```
[Out] -(3 + 2*x)^2/(8*(7 + 6*x)^2)
```

Rubi [A] time = 0.00943694, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -(3 + 2*x)^2/(8*(7 + 6*x)^2)
```

Rubi in Sympy [A] time = 1.13854, size = 15, normalized size = 0.83

$$-\frac{(2x+3)^2}{8\,(6x+7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(7+6*x)**3,x)
```

```
[Out] -(2*x + 3)**2/(8*(6*x + 7)**2)
```

Mathematica [A] time = 0.00576289, size = 16, normalized size = 0.89

$$-\frac{3x+4}{9(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -(4 + 3*x)/(9*(7 + 6*x)^2)
```

Maple [A] time = 0.01, size = 20, normalized size = 1.1

$$-\frac{1}{126+108\,x}-\frac{1}{18\,(7+6\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(7+6*x)^3,x)
```

```
[Out] -1/18/(7+6*x)-1/18/(7+6*x)^2
```

Maxima [A] time = 1.35996, size = 26, normalized size = 1.44

$$-\frac{3\,x + 4}{9\,(36\,x^2 + 84\,x + 49)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(6*x + 7)^3,x, algorithm="maxima")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Fricas [A] time = 0.189883, size = 26, normalized size = 1.44

$$-\frac{3\,x + 4}{9\,(36\,x^2 + 84\,x + 49)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(6*x + 7)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Sympy [A] time = 0.102592, size = 15, normalized size = 0.83

$$-\frac{3x + 4}{324x^2 + 756x + 441}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(7+6*x)**3,x)
```

```
[Out] -(3*x + 4)/(324*x**2 + 756*x + 441)
```

GIAC/XCAS [A] time = 0.214813, size = 19, normalized size = 1.06

$$-\frac{3\,x + 4}{9\,(6\,x + 7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/(6*x + 7)^3,x, algorithm="giac")
```

```
[Out] -1/9*(3*x + 4)/(6*x + 7)^2
```

3.50 $\int x^4 \left(1+x^5\right)^5 \, dx$

Optimal. Leaf size=11

$$\frac{1}{30} \left(x^5+1\right)^6$$

```
[Out] (1 + x^5)^6/30
```

Rubi [A] time = 0.00684124, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{1}{30} \left(x^5+1\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^5)^5,x]
```

```
[Out] (1 + x^5)^6/30
```

Rubi in Sympy [A] time = 0.76069, size = 7, normalized size = 0.64

$$\frac{\left(x^5+1\right)^6}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**4*(x**5+1)**5,x)
```

```
[Out] (x**5 + 1)**6/30
```

Mathematica [B] time = 0.0022178, size = 43, normalized size = 3.91

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^5)^5,x]
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

Maple [B] time = 0.003, size = 32, normalized size = 2.9

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2\,x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+1)^5,x)
```

```
[Out] 1/30*x^30+1/5*x^25+1/2*x^20+2/3*x^15+1/2*x^10+1/5*x^5
```

Maxima [A] time = 1.41261, size = 12, normalized size = 1.09

$$\frac{1}{30}\left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)^5*x^4,x, algorithm="maxima")
```

```
[Out] 1/30*(x^5 + 1)^6
```

Fricas [A] time = 0.210004, size = 1, normalized size = 0.09

$$\frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)^5*x^4,x, algorithm="fricas")
```

```
[Out] 1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

Sympy [A] time = 0.033167, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(x**5+1)**5,x)
```

```
[Out] x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

GIAC/XCAS [A] time = 0.219424, size = 12, normalized size = 1.09

$$\frac{1}{30}\left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^5 + 1)^5*x^4,x, algorithm="giac")
```

```
[Out] 1/30*(x^5 + 1)^6
```

3.51 $\int(1-x)^{20}x^4\,dx$

Optimal. Leaf size=56

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

```
[Out] -(1 - x)^21/21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^
24/6 - (1 - x)^25/25
```

Rubi [A] time = 0.0931179, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x)^20*x^4,x]
```

```
[Out] -(1 - x)^21/21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^
24/6 - (1 - x)^25/25
```

Rubi in Sympy [A] time = 7.44586, size = 36, normalized size = 0.64

$$-\frac{(-x+1)^{25}}{25}+\frac{(-x+1)^{24}}{6}-\frac{6(-x+1)^{23}}{23}+\frac{2(-x+1)^{22}}{11}-\frac{(-x+1)^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-x)**20*x**4,x)
```

```
[Out] -(-x + 1)**25/25 + (-x + 1)**24/6 - 6*(-x + 1)**23/23 + 2*(-x + 1
)**22/11 - (-x + 1)**21/21
```

Mathematica [B] time = 0.00280465, size = 140, normalized size = 2.5

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-\frac{83980x^{14}}{7}+9690x^{13}-6460x^{12}+\frac{38760x^{11}}{11}-\frac{7752x^{10}}{5}+\frac{1615x^9}{3}-\frac{285x^8}{2}+\frac{190x^7}{7}-\frac{10x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x)^20*x^4,x]
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (
7752*x^10)/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x
^14)/7 + (184756*x^15)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x
^18)/3 + 2040*x^19 - (3876*x^20)/5 + (1615*x^21)/7 - (570*x^22)/1
1 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

Maple [B] time = 0.003, size = 107, normalized size = 1.9

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-\frac{83980x^{14}}{7}+9690x^{13}-6460x^{12}+\frac{38760x^{11}}{11}-\frac{7752x^{10}}{5}+\frac{1615x^{9}}{3}-\frac{285x^{8}}{2}+\frac{190x^{7}}{7}-\frac{10x^{6}}{3}+\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((1-x)^20*x^4,x)
```

```
[Out] 1/25*x^25-5/6*x^24+190/23*x^23-570/11*x^22+1615/7*x^21-3876/5*x^2
0+2040*x^19-12920/3*x^18+7410*x^17-20995/2*x^16+184756/15*x^15-83
980/7*x^14+9690*x^13-6460*x^12+38760/11*x^11-7752/5*x^10+1615/3*x
^9-285/2*x^8+190/7*x^7-10/3*x^6+1/5*x^5
```

Maxima [A] time = 1.37836, size = 143, normalized size = 2.55

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{83980}{7}x^{14}+9690x^{13}-6460x^{12}+\frac{38760}{11}x^{11}-\frac{7752}{5}x^{10}+\frac{1615}{3}x^{9}-\frac{285}{2}x^{8}+\frac{190}{7}x^{7}-\frac{10}{3}x^{6}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x - 1)^20*x^4,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 -
3876/5*x^20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16
 + 184756/15*x^15 - 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/
11*x^11 - 7752/5*x^10 + 1615/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3
*x^6 + 1/5*x^5
```

Fricas [A] time = 0.178299, size = 1, normalized size = 0.02

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}-\frac{83980}{7}x^{14}+9690x^{13}-6460x^{12}+\frac{38760}{11}x^{11}-\frac{7752}{5}x^{10}+\frac{1615}{3}x^{9}-\frac{285}{2}x^{8}+\frac{190}{7}x^{7}-\frac{10}{3}x^{6}+\frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate((x - 1)^20*x^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 -
3876/5*x^20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16
 + 184756/15*x^15 - 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/
11*x^11 - 7752/5*x^10 + 1615/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3
*x^6 + 1/5*x^5
```

Sympy [A] time = 0.066715, size = 131, normalized size = 2.34

$$\frac{x^{25}}{25} - \frac{5x^{24}}{6} + \frac{190x^{23}}{23} - \frac{570x^{22}}{11} + \frac{1615x^{21}}{7} - \frac{3876x^{20}}{5} + 2040x^{19} - \frac{12920x^{18}}{3} + 7410x^{17} - \frac{20995x^{16}}{2} + \frac{184756x^{15}}{15} - \frac{83980x^{14}}{7} + 9690x^{13} - 6460x^{12} + \frac{38760x^{11}}{11} - \frac{7752x^{10}}{5} + \frac{1615x^{9}}{3} - \frac{285x^{8}}{2} + \frac{190x^{7}}{7} - \frac{10x^{6}}{3} + \frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-x)**20*x**4,x)
```

```
[Out] x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7
 - 3876*x**20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995
*x**16/2 + 184756*x**15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x*
*12 + 38760*x**11/11 - 7752*x**10/5 + 1615*x**9/3 - 285*x**8/2 +
190*x**7/7 - 10*x**6/3 + x**5/5
```

GIAC/XCAS [A] time = 0.218893, size = 143, normalized size = 2.55

$$\frac{1}{25}\,x^{25} - \frac{5}{6}\,x^{24} + \frac{190}{23}\,x^{23} - \frac{570}{11}\,x^{22} + \frac{1615}{7}\,x^{21} - \frac{3876}{5}\,x^{20} + 2040\,x^{19} - \frac{12920}{3}\,x^{18} + 7410\,x^{17} - \frac{20995}{2}\,x^{16} + \frac{184756}{15}\,x^{15} - \frac{83980}{7}\,x^{14} + 9690\,x^{13} - 6460\,x^{12} + \frac{38760}{11}\,x^{11} - \frac{7752}{5}\,x^{10} + \frac{1615}{3}\,x^{9} - \frac{285}{2}\,x^{8} + \frac{190}{7}\,x^{7} - \frac{10}{3}\,x^{6} + \frac{1}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 1)^20*x^4,x, algorithm="giac")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 -
3876/5*x^20 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16
 + 184756/15*x^15 - 83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/
11*x^11 - 7752/5*x^10 + 1615/3*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3
*x^6 + 1/5*x^5
```

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=4

$$\cos\left(\frac{1}{x}\right)$$

```
[Out] Cos[x^(-1)]
```

Rubi [A] time = 0.0145071, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x^(-1)]/x^2,x]

[Out] Cos[x^(-1)]
```

Rubi in Sympy [A] time = 1.04911, size = 3, normalized size = 0.75

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(1/x)/x**2,x)

[Out] cos(1/x)
```

Mathematica [A] time = 0.00632318, size = 4, normalized size = 1.

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x^(-1)]/x^2,x]

[Out] Cos[x^(-1)]
```

Maple [A] time = 0.005, size = 5, normalized size = 1.3

$$\cos\left(x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(1/x)/x^2,x)

[Out] cos(1/x)
```

Maxima [A] time = 1.34044, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] cos(1/x)
```

Fricas [A] time = 0.216484, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] cos(1/x)
```

Sympy [A] time = 1.07347, size = 3, normalized size = 0.75

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/x)/x**2,x)
```

```
[Out] cos(1/x)
```

GIAC/XCAS [A] time = 0.215654, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(1/x)/x^2,x, algorithm="giac")
```

```
[Out] cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right) dx$

Optimal. Leaf size=62

$$12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1
+ x)^(1/4)] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 +
x)^(1/4)]
```

Rubi [A] time = 0.06974, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375

$$12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1
+ x)^(1/4)] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 +
x)^(1/4)]
```

Rubi in Sympy [A] time = 2.24162, size = 60, normalized size = 0.97

$$-4\left(x-1\right)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x
- 1)**(1/4)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1
)**(1/4))
```

Mathematica [A] time = 0.0282817, size = 46, normalized size = 0.74

$$12\left(\sqrt{x-1}-2\right)\sin\left(\sqrt[4]{x-1}\right)-4\left(\sqrt{x-1}-6\right)\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] -4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-
2 + Sqrt[-1 + x])*Sin[(-1 + x)^(1/4)]
```

Maple [A] time = 0.007, size = 49, normalized size = 0.8

$$24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4\left(-1+x\right)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\sin\left(\sqrt[4]{-1+x}\right)+12\sin\left(\sqrt[4]{-1+x}\right)\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin((-1+x)^(1/4)),x)
```

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4)
)-24*sin((-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Maxima [A] time = 1.38576, size = 50, normalized size = 0.81

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin((x - 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqr
t(x - 1) - 2)*sin((x - 1)^(1/4))
```

Fricas [A] time = 0.214825, size = 50, normalized size = 0.81

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin((x - 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqr
t(x - 1) - 2)*sin((x - 1)^(1/4))
```

Sympy [A] time = 3.38887, size = 60, normalized size = 0.97

$$-4\,(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x
- 1)**(1/4)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1
)**(1/4))
```

GIAC/XCAS [A] time = 0.217693, size = 50, normalized size = 0.81

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin((x - 1)^(1/4)),x, algorithm="giac")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqr
t(x - 1) - 2)*sin((x - 1)^(1/4))
```

3.54 $\int x \cos(x^2) \sin(x^2)\, dx$

Optimal. Leaf size=10

$$\frac{1}{4} \sin^2(x^2)$$

```
[Out] Sin[x^2]^2/4
```

Rubi [A] time = 0.0129177, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{1}{4} \sin^2(x^2)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Cos[x^2]*Sin[x^2],x]
```

```
[Out] Sin[x^2]^2/4
```

Rubi in Sympy [A] time = 0.993824, size = 7, normalized size = 0.7

$$\frac{\sin^2(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*cos(x**2)*sin(x**2),x)
```

```
[Out] sin(x**2)**2/4
```

Mathematica [A] time = 0.00431081, size = 10, normalized size = 1.

$$-\frac{1}{4} \cos^2(x^2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Cos[x^2]*Sin[x^2],x]
```

```
[Out] -Cos[x^2]^2/4
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\frac{(\cos(x^2))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*cos(x^2)*sin(x^2),x)
```

```
[Out] -1/4*cos(x^2)^2
```

Maxima [A] time = 1.35312, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x^2)^2
```

Fricas [A] time = 0.249941, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(x^2)^2
```

Sympy [A] time = 0.360121, size = 7, normalized size = 0.7

$$\frac{\sin^2\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x**2)*sin(x**2),x)
```

```
[Out] sin(x**2)**2/4
```

GIAC/XCAS [A] time = 0.224803, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(x^2)^2
```

3.55 $\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rubi [A] time = 0.0545069, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]

[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rubi in Sympy [A] time = 2.74074, size = 15, normalized size = 0.94

$$-\frac{2\left(3\cos^2(x)+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)

[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

Mathematica [B] time = 0.107486, size = 49, normalized size = 3.06

$$\frac{-3\sqrt{3\cos(2x)+5}\cos(2x)-5\sqrt{3\cos(2x)+5}+5\sqrt{5}}{9\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]

[Out] (5*Sqrt[5] - 5*Sqrt[5 + 3*Cos[2*x]] - 3*Cos[2*x]*Sqrt[5 + 3*Cos[2
*x]])/(9*Sqrt[2])
```

Maple [A] time = 0.043, size = 13, normalized size = 0.8

$$-\frac{2}{9}\left(1+3\left(\cos(x)\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x)

[Out] -2/9*(1+3*cos(x)^2)^(3/2)
```

Maxima [A] time = 1.414, size = 16, normalized size = 1.

$$-\frac{2}{9}\left(3\,\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x)^2 + 1)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Fricas [A] time = 0.227562, size = 16, normalized size = 1.

$$-\frac{2}{9}\left(3\,\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x)^2 + 1)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Sympy [A] time = 3.29817, size = 15, normalized size = 0.94

$$-\frac{2\left(3\cos^{2}\left(x\right)+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
```

```
[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

GIAC/XCAS [A] time = 0.234006, size = 248, normalized size = 15.5

$$-\frac{16\left(\left(\tan\left(\frac{1}{2}\,x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}\,x\right)^{4}-\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{5}-\left(\tan\left(\frac{1}{2}\,x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}\,x\right)^{4}-\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{3}-2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-\sqrt{\mathrm{ta}\cdots}\right.\right.}{\left(\left(\tan\left(\frac{1}{2}\,x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}\,x\right)^{4}-\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{2}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}-2\sqrt{\mathrm{tan}\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*cos(x)^2 + 1)*sin(2*x),x, algorithm="giac")
```

```
[Out] -16*((tan(1/2*x)^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^5 - (
tan(1/2*x)^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^3 - 2*(tan(
1/2*x)^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^2 + 3*tan(1/2*x
)^2 - 3*sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1) - 1)/((tan(1/2*x)^2
 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^2 + 2*tan(1/2*x)^2 - 2*
sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1) - 2)^3
```

3.56 $\int \frac{1}{2+3x}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

```
[Out] Log[2 + 3*x]/3
```

Rubi [A] time = 0.00471719, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rubi in Sympy [A] time = 0.509688, size = 7, normalized size = 0.7

$$\frac{\log{(3x+2)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Mathematica [A] time = 0.00111162, size = 10, normalized size = 1.

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Maple [A] time = 0., size = 9, normalized size = 0.9

$$\frac{\ln(2+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2+3*x),x)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A] time = 1.44508, size = 11, normalized size = 1.1

$$\frac{1}{3}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="maxima")

[Out] 1/3*log(3*x + 2)
```

Fricas [A] time = 0.203973, size = 11, normalized size = 1.1

$$\frac{1}{3}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="fricas")

[Out] 1/3*log(3*x + 2)
```

Sympy [A] time = 0.031041, size = 7, normalized size = 0.7

$$\frac{\log\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2+3*x),x)

[Out] log(3*x + 2)/3
```

GIAC/XCAS [A] time = 0.21534, size = 12, normalized size = 1.2

$$\frac{1}{3}\ln\left(\left|3\,x+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x + 2),x, algorithm="giac")

[Out] 1/3*ln(abs(3*x + 2))
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.00791702, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi in Sympy [A] time = 0.51416, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)**2,x)

[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Mathematica [A] time = 0.0015164, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]^2,x]

[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0.025, size = 16, normalized size = 1.1

$$2x - 2x\ln(x) + x(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)^2,x)

[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 1.38508, size = 16, normalized size = 1.07

$$\left(\log(x)^2 - 2\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 0.258505, size = 20, normalized size = 1.33

$$x \log (x)^2 - 2 x \log (x) + 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.079342, size = 15, normalized size = 1.

$$x \log (x)^2 - 2x \log (x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

GIAC/XCAS [A] time = 0.220463, size = 20, normalized size = 1.33

$$x\ln (x)^2 - 2 x\ln (x) + 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*ln(x)^2 - 2*x*ln(x) + 2*x
```

3.58 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.00825204, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)}{2}-\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - Integral(x, x)/2
```

Mathematica [A] time = 0.00112954, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x)-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x),x)

[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 1.54391, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="maxima")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.211709, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="fricas")

[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.069315, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x),x)

[Out] x**2*log(x)/2 - x**2/4
```

GIAC/XCAS [A] time = 0.223215, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\ln\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x),x, algorithm="giac")

[Out] 1/2*x^2*ln(x) - 1/4*x^2
```

3.59 $\int x \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.0168769, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2\log(x)^2}{2}-\frac{x^2\log(x)}{2}+\frac{\int x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + Integral(x, x)/2
```

Mathematica [A] time = 0.00188918, size = 28, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 1.65583, size = 23, normalized size = 0.82

$$\frac{1}{4}\left(2\log\left(x\right)^{2}-2\log\left(x\right)+1\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="maxima")

[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 0.203275, size = 30, normalized size = 1.07

$$\frac{1}{2}x^{2}\log\left(x\right)^{2}-\frac{1}{2}x^{2}\log\left(x\right)+\frac{1}{4}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="fricas")

[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.087873, size = 22, normalized size = 0.79

$$\frac{x^{2}\log\left(x\right)^{2}}{2}-\frac{x^{2}\log\left(x\right)}{2}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*ln(x)**2,x)

[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

GIAC/XCAS [A] time = 0.218519, size = 30, normalized size = 1.07

$$\frac{1}{2}x^{2}\ln\left(x\right)^{2}-\frac{1}{2}x^{2}\ln\left(x\right)+\frac{1}{4}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*log(x)^2,x, algorithm="giac")

[Out] 1/2*x^2*ln(x)^2 - 1/2*x^2*ln(x) + 1/4*x^2
```

3.60 $\int \frac{1}{1+t}\,dt$

Optimal. Leaf size=4

$$\log(t+1)$$

```
[Out] Log[1 + t]
```

Rubi [A] time = 0.00291505, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Rubi in Sympy [A] time = 0.461584, size = 3, normalized size = 0.75

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+t),t)
```

```
[Out] log(t + 1)
```

Mathematica [A] time = 0.000867474, size = 4, normalized size = 1.

$$\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$\ln(1+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+t),t)
```

```
[Out] ln(1+t)
```

Maxima [A] time = 1.62359, size = 5, normalized size = 1.25

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(t + 1),t, algorithm="maxima")
```

```
[Out] log(t + 1)
```

Fricas [A] time = 0.191291, size = 5, normalized size = 1.25

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(t + 1),t, algorithm="fricas")
```

```
[Out] log(t + 1)
```

Sympy [A] time = 0.026006, size = 3, normalized size = 0.75

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+t),t)
```

```
[Out] log(t + 1)
```

GIAC/XCAS [A] time = 0.216236, size = 7, normalized size = 1.75

$$\ln(|t+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(t + 1),t, algorithm="giac")
```

```
[Out] ln(abs(t + 1))
```

3.61 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.00456008, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Int[Cot[x],x]

[Out] Log[Sin[x]]
```

Rubi in Sympy [A] time = 0.030328, size = 3, normalized size = 1.

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(cot(x),x)

[Out] log(sin(x))
```

Mathematica [A] time = 0.00338638, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Cot[x],x]

[Out] Log[Sin[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(cot(x),x)

[Out] ln(sin(x))
```

Maxima [A] time = 1.61971, size = 4, normalized size = 1.33

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [A] time = 0.219355, size = 15, normalized size = 5.

$$\frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(2\,x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.043976, size = 3, normalized size = 1.

$$\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

GIAC/XCAS [A] time = 0.217179, size = 15, normalized size = 5.

$$\frac{1}{2}\ln\left(-\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*ln(-cos(x)^2 + 1)
```

3.62 $\int x^n \log(ax)\,dx$

Optimal. Leaf size=28

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

```
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rubi [A] time = 0.0199525, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^n*Log[a*x],x]

[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rubi in Sympy [A] time = 1.61621, size = 22, normalized size = 0.79

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**n*ln(a*x),x)

[Out] x**(n + 1)*log(a*x)/(n + 1) - x**(n + 1)/(n + 1)**2
```

Mathematica [A] time = 0.0141084, size = 21, normalized size = 0.75

$$\frac{x^{n+1}((n+1)\log(ax)-1)}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^n*Log[a*x],x]

[Out] (x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

Maple [A] time = 0.083, size = 36, normalized size = 1.3

$$\frac{x\ln(ax)\,e^{n\ln(x)}}{1+n}-\frac{xe^{n\ln(x)}}{n^2+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^n*ln(a*x),x)

[Out] 1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n*log(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218856, size = 43, normalized size = 1.54

$$\frac{\left((n+1)x\log\left(a\right)+(n+1)x\log\left(x\right)-x\right)x^{n}}{n^{2}+2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n*log(a*x),x, algorithm="fricas")
```

```
[Out] ((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**n*ln(a*x),x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int x^{n}\log\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^n*log(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^n*log(a*x), x)
```

3.63 $\int x^2 \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rubi [A] time = 0.0305475, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Log[x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rubi in Sympy [A] time = 1.92764, size = 26, normalized size = 0.93

$$\frac{x^3\log{(x)}^2}{3}-\frac{2x^3\log{(x)}}{9}+\frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*ln(x)**2,x)
```

```
[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

Mathematica [A] time = 0.00370572, size = 28, normalized size = 1.

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Log[x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Maple [A] time = 0.001, size = 23, normalized size = 0.8

$$\frac{2\,x^3}{27}-\frac{2\,x^3\ln{(x)}}{9}+\frac{x^3\,(\ln{(x)})^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*ln(x)^2,x)
```

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Maxima [A] time = 1.41614, size = 23, normalized size = 0.82

$$\frac{1}{27}\left(9\log\left(x\right)^{2} - 6\log\left(x\right) + 2\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x)^2,x, algorithm="maxima")

[Out] 1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

Fricas [A] time = 0.20615, size = 30, normalized size = 1.07

$$\frac{1}{3}\,x^{3}\log\left(x\right)^{2} - \frac{2}{9}\,x^{3}\log\left(x\right) + \frac{2}{27}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x)^2,x, algorithm="fricas")

[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Sympy [A] time = 0.091174, size = 26, normalized size = 0.93

$$\frac{x^{3}\log\left(x\right)^{2}}{3} - \frac{2x^{3}\log\left(x\right)}{9} + \frac{2x^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*ln(x)**2,x)

[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

GIAC/XCAS [A] time = 0.23451, size = 30, normalized size = 1.07

$$\frac{1}{3}\,x^{3}\ln\left(x\right)^{2} - \frac{2}{9}\,x^{3}\ln\left(x\right) + \frac{2}{27}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*log(x)^2,x, algorithm="giac")

[Out] 1/3*x^3*ln(x)^2 - 2/9*x^3*ln(x) + 2/27*x^3
```

3.64 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] Log[Log[x]]
```

Rubi [A] time = 0.0183933, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rubi in Sympy [A] time = 1.11765, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Mathematica [A] time = 0.00100283, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 1.43033, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 0.218183, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.078941, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

GIAC/XCAS [A] time = 0.238495, size = 5, normalized size = 1.67

$$\ln(|\ln(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x*log(x)),x, algorithm="giac")
```

```
[Out] ln(abs(ln(x)))
```

3.65 $\int \frac{\log(1-t)}{1-t}\,dt$

Optimal. Leaf size=12

$$-\frac{1}{2}\log^2(1-t)$$

```
[Out] -Log[1 - t]^2/2
```

Rubi [A] time = 0.0255394, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - t]/(1 - t),t]

[Out] -Log[1 - t]^2/2
```

Rubi in Sympy [A] time = 1.46301, size = 8, normalized size = 0.67

$$-\frac{\log(-t+1)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(ln(1-t)/(1-t),t)

[Out] -log(-t + 1)**2/2
```

Mathematica [A] time = 0.00309904, size = 12, normalized size = 1.

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - t]/(1 - t),t]

[Out] -Log[1 - t]^2/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$-\frac{(\ln(1-t))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-t)/(1-t),t)

[Out] -1/2*ln(1-t)^2
```

Maxima [A] time = 1.44768, size = 14, normalized size = 1.17

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-log(-t + 1)/(t - 1),t, algorithm="maxima")
```

```
[Out] -1/2*log(-t + 1)^2
```

Fricas [A] time = 0.206628, size = 14, normalized size = 1.17

$$-\frac{1}{2}\log\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-log(-t + 1)/(t - 1),t, algorithm="fricas")
```

```
[Out] -1/2*log(-t + 1)^2
```

Sympy [A] time = 0.079763, size = 8, normalized size = 0.67

$$-\frac{\log\left(-t+1\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(1-t)/(1-t),t)
```

```
[Out] -log(-t + 1)**2/2
```

GIAC/XCAS [A] time = 0.240403, size = 14, normalized size = 1.17

$$-\frac{1}{2}\ln\left(-t+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-log(-t + 1)/(t - 1),t, algorithm="giac")
```

```
[Out] -1/2*ln(-t + 1)^2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rubi [A] time = 0.065073, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]

[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rubi in Sympy [A] time = 5.06574, size = 24, normalized size = 1.04

$$-\frac{4\left(\log\left(x\right)+1\right)^{\frac{3}{2}}}{3}+2\sqrt{\log\left(x\right)+1}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(x)/x/(1+ln(x))**(1/2),x)

[Out] -4*(log(x) + 1)**(3/2)/3 + 2*sqrt(log(x) + 1)*log(x)
```

Mathematica [A] time = 0.00760408, size = 16, normalized size = 0.7

$$\frac{2}{3}(\log(x)-2)\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]

[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A] time = 0.009, size = 18, normalized size = 0.8

$$\frac{2}{3}\left(1+\ln\left(x\right)\right)^{\frac{3}{2}}-2\sqrt{1+\ln\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(x)/x/(1+ln(x))^(1/2),x)

[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Maxima [A] time = 1.50231, size = 23, normalized size = 1.

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}-2\sqrt{\log\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(x*sqrt(log(x) + 1)),x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Fricas [A] time = 0.203332, size = 16, normalized size = 0.7

$$\frac{2}{3}\sqrt{\log\left(x\right)+1}\left(\log\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(x*sqrt(log(x) + 1)),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] time = 1.43454, size = 20, normalized size = 0.87

$$\frac{2\left(\log\left(x\right)+1\right)^{\frac{3}{2}}}{3}-2\sqrt{\log\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

GIAC/XCAS [A] time = 0.2355, size = 23, normalized size = 1.

$$\frac{2}{3}\left(\ln\left(x\right)+1\right)^{\frac{3}{2}}-2\sqrt{\ln\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(x)/(x*sqrt(log(x) + 1)),x, algorithm="giac")
```

```
[Out] 2/3*(ln(x) + 1)^(3/2) - 2*sqrt(ln(x) + 1)
```

3.67 $\int x^3 \log^3(x)\, dx$

Optimal. Leaf size=39

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log
[x]^3)/4
```

Rubi [A] time = 0.0485654, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log
[x]^3)/4
```

Rubi in Sympy [A] time = 2.80623, size = 37, normalized size = 0.95

$$\frac{x^4\log(x)^3}{4}-\frac{3x^4\log(x)^2}{16}+\frac{3x^4\log(x)}{32}-\frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*ln(x)**3,x)
```

```
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**
4/128
```

Mathematica [A] time = 0.00378348, size = 39, normalized size = 1.

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log
[x]^3)/4
```

Maple [A] time = 0.003, size = 32, normalized size = 0.8

$$-\frac{3\,x^4}{128}+\frac{3\,x^4\ln(x)}{32}-\frac{3\,x^4(\ln(x))^2}{16}+\frac{x^4(\ln(x))^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*ln(x)^3,x)
```

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Maxima [A] time = 1.56424, size = 31, normalized size = 0.79

$$\frac{1}{128}\left(32\,\log\left(x\right)^{3}-24\,\log\left(x\right)^{2}+12\,\log\left(x\right)-3\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

Fricas [A] time = 0.204995, size = 42, normalized size = 1.08

$$\frac{1}{4}\,x^{4}\log\left(x\right)^{3}-\frac{3}{16}\,x^{4}\log\left(x\right)^{2}+\frac{3}{32}\,x^{4}\log\left(x\right)-\frac{3}{128}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^
4
```

Sympy [A] time = 0.113336, size = 37, normalized size = 0.95

$$\frac{x^{4}\log\left(x\right)^{3}}{4}-\frac{3x^{4}\log\left(x\right)^{2}}{16}+\frac{3x^{4}\log\left(x\right)}{32}-\frac{3x^{4}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*ln(x)**3,x)
```

```
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**
4/128
```

GIAC/XCAS [A] time = 0.215188, size = 42, normalized size = 1.08

$$\frac{1}{4}\,x^{4}\ln\left(x\right)^{3}-\frac{3}{16}\,x^{4}\ln\left(x\right)^{2}+\frac{3}{32}\,x^{4}\ln\left(x\right)-\frac{3}{128}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*log(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*ln(x)^3 - 3/16*x^4*ln(x)^2 + 3/32*x^4*ln(x) - 3/128*x^4
```

3.68 $\int e^{x^3} x^2 \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^3}}{3}$$

```
[Out] E^x^3/3
```

Rubi [A] time = 0.0200876, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[E^x^3*x^2,x]

[Out] E^x^3/3
```

Rubi in Sympy [A] time = 1.3841, size = 5, normalized size = 0.56

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**3)*x**2,x)

[Out] exp(x**3)/3
```

Mathematica [A] time = 0.00274353, size = 9, normalized size = 1.

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x^3*x^2,x]

[Out] E^x^3/3
```

Maple [A] time = 0.004, size = 7, normalized size = 0.8

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^3)*x^2,x)

[Out] 1/3*exp(x^3)
```

Maxima [A] time = 1.54756, size = 8, normalized size = 0.89

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*e^(x^3)
```

Fricas [A] time = 0.208143, size = 8, normalized size = 0.89

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*e^(x^3)
```

Sympy [A] time = 0.062543, size = 5, normalized size = 0.56

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x**3)*x**2,x)
```

```
[Out] exp(x**3)/3
```

GIAC/XCAS [A] time = 0.221627, size = 8, normalized size = 0.89

$$\frac{1}{3}\,e^{\left(x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(x^3),x, algorithm="giac")
```

```
[Out] 1/3*e^(x^3)
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$

Optimal. Leaf size=14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

```
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rubi [A] time = 0.0196678, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In]  Int[2^Sqrt[x]/Sqrt[x],x]

[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rubi in Sympy [A] time = 1.32137, size = 10, normalized size = 0.71

$$\frac{2 \cdot 2^{\sqrt{x}}}{\log{(2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2**(x**(1/2))/x**(1/2),x)

[Out] 2*2**(sqrt(x))/log(2)
```

Mathematica [A] time = 0.00503077, size = 14, normalized size = 1.

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[2^Sqrt[x]/Sqrt[x],x]

[Out] 2^(1 + Sqrt[x])/Log[2]
```

Maple [A] time = 0.006, size = 12, normalized size = 0.9

$$2\,{\frac{{2}^{\sqrt{x}}}{\ln \left(2 \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2^(x^(1/2))/x^(1/2),x)
```

```
[Out] 2/ln(2)*2^(x^(1/2))
```

Maxima [A] time = 1.48324, size = 16, normalized size = 1.14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2^sqrt(x)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2^(sqrt(x) + 1)/log(2)
```

Fricas [A] time = 0.207196, size = 15, normalized size = 1.07

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2^sqrt(x)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2*2^sqrt(x)/log(2)
```

Sympy [A] time = 0.152574, size = 10, normalized size = 0.71

$$\frac{2 \cdot 2^{\sqrt{x}}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2**(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*2**(sqrt(x))/log(2)
```

GIAC/XCAS [A] time = 0.215751, size = 15, normalized size = 1.07

$$\frac{2 \cdot 2^{\left(\sqrt{x}\right)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2^sqrt(x)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2*2^sqrt(x)/ln(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}e^{2\sin(x)}$$

```
[Out] E^(2*Sin[x])/2
```

Rubi [A] time = 0.0157524, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*Sin[x])*Cos[x],x]

[Out] E^(2*Sin[x])/2
```

Rubi in Sympy [A] time = 1.67072, size = 7, normalized size = 0.7

$$\frac{e^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*sin(x))*cos(x),x)

[Out] exp(2*sin(x))/2
```

Mathematica [A] time = 0.00846611, size = 10, normalized size = 1.

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*Sin[x])*Cos[x],x]

[Out] E^(2*Sin[x])/2
```

Maple [A] time = 0.01, size = 8, normalized size = 0.8

$$\frac{e^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*sin(x))*cos(x),x)

[Out] 1/2*exp(2*sin(x))
```

Maxima [A] time = 1.419, size = 9, normalized size = 0.9

$$\frac{1}{2} e^{(2 \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^(2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*sin(x))
```

Fricas [A] time = 0.214045, size = 9, normalized size = 0.9

$$\frac{1}{2} e^{(2 \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^(2*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*e^(2*sin(x))
```

Sympy [A] time = 0.354896, size = 7, normalized size = 0.7

$$\frac{e^{2 \sin (x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*sin(x))*cos(x),x)
```

```
[Out] exp(2*sin(x))/2
```

GIAC/XCAS [A] time = 0.223314, size = 9, normalized size = 0.9

$$\frac{1}{2} e^{(2 \sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^(2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*sin(x))
```

3.71 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0135615, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x*Sin[x],x]

[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi in Sympy [A] time = 1.17316, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)*sin(x),x)

[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Mathematica [A] time = 0.0160791, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x)-\cos(x))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x*Sin[x],x]

[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0.029, size = 14, normalized size = 0.7

$$-\frac{e^x\cos(x)}{2}+\frac{e^x\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)*sin(x),x)

[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.37677, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.210022, size = 18, normalized size = 0.95

$$-\frac{1}{2}\cos\left(x\right)e^{x}+\frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.3489, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}-\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.217649, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos\left(x\right)-\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.72 $\int e^x \cos(x)\, dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x \cos(x)$$

```
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0135202, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x],x]

[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi in Sympy [A] time = 1.1717, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} + \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*cos(x),x)

[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

Mathematica [A] time = 0.0071641, size = 12, normalized size = 0.63

$$\frac{1}{2}e^x(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x],x]

[Out] (E^x*(Cos[x] + Sin[x]))/2
```

Maple [A] time = 0.007, size = 14, normalized size = 0.7

$$\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x),x)

[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.36374, size = 12, normalized size = 0.63

$$\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^x,x, algorithm="maxima")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Fricas [A] time = 0.213841, size = 18, normalized size = 0.95

$$\frac{1}{2}\cos\left(x\right)e^{x}+\frac{1}{2}e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^x,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.341191, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin\left(x\right)}{2}+\frac{e^{x}\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*cos(x),x)
```

```
[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

GIAC/XCAS [A] time = 0.216733, size = 12, normalized size = 0.63

$$\frac{1}{2}\left(\cos\left(x\right)+\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(x)*e^x,x, algorithm="giac")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

3.73 $\int \frac{1}{1+e^x}\,dx$

Optimal. Leaf size=10

$$x-\log\left(e^x+1\right)$$

```
[Out] x - Log[1 + E^x]
```

Rubi [A] time = 0.0134802, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571

$$x-\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + E^x)^(-1),x]

[Out] x - Log[1 + E^x]
```

Rubi in Sympy [A] time = 1.18841, size = 10, normalized size = 1.

$$-\log\left(e^x+1\right)+\log\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+exp(x)),x)

[Out] -log(exp(x) + 1) + log(exp(x))
```

Mathematica [A] time = 0.00310863, size = 10, normalized size = 1.

$$x-\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + E^x)^(-1),x]

[Out] x - Log[1 + E^x]
```

Maple [A] time = 0.008, size = 12, normalized size = 1.2

$$-\ln\left(1+e^x\right)+\ln\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+exp(x)),x)

[Out] -ln(1+exp(x))+ln(exp(x))
```

Maxima [A] time = 1.34933, size = 12, normalized size = 1.2

$$x-\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^x + 1),x, algorithm="maxima")
```

```
[Out] x - log(e^x + 1)
```

Fricas [A] time = 0.218086, size = 12, normalized size = 1.2

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^x + 1),x, algorithm="fricas")
```

```
[Out] x - log(e^x + 1)
```

Sympy [A] time = 0.055553, size = 7, normalized size = 0.7

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+exp(x)),x)
```

```
[Out] x - log(exp(x) + 1)
```

GIAC/XCAS [A] time = 0.219186, size = 12, normalized size = 1.2

$$x - \ln(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(e^x + 1),x, algorithm="giac")
```

```
[Out] x - ln(e^x + 1)
```

3.74 $\int e^x x \, dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

```
[Out] -E^x + E^x*x
```

Rubi [A] time = 0.0104852, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$e^x x - e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x,x]
```

```
[Out] -E^x + E^x*x
```

Rubi in Sympy [A] time = 0.926222, size = 7, normalized size = 0.64

$$xe^x - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*x,x)
```

```
[Out] x*exp(x) - exp(x)
```

Mathematica [A] time = 0.00151928, size = 7, normalized size = 0.64

$$e^x(x - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x,x]
```

```
[Out] E^x*(-1 + x)
```

Maple [A] time = 0.002, size = 7, normalized size = 0.6

$$(-1 + x)\, \mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x)
```

```
[Out] (-1+x)*exp(x)
```

Maxima [A] time = 1.37359, size = 8, normalized size = 0.73

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A] time = 0.198197, size = 8, normalized size = 0.73

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A] time = 0.057101, size = 5, normalized size = 0.45

$$(x-1)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

GIAC/XCAS [A] time = 0.220094, size = 8, normalized size = 0.73

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

3.75 $\int e^{-x} x \, dx$

Optimal. Leaf size=16

$$-e^{-x}x - e^{-x}$$

```
[Out] -E^(-x) - x/E^x
```

Rubi [A] time = 0.0134441, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-e^{-x}x - e^{-x}$$

Antiderivative was successfully verified.

```
[In]  Int[x/E^x,x]

[Out] -E^(-x) - x/E^x
```

Rubi in Sympy [A] time = 1.0391, size = 10, normalized size = 0.62

$$-xe^{-x} - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/exp(x),x)

[Out] -x*exp(-x) - exp(-x)
```

Mathematica [A] time = 0.00248595, size = 11, normalized size = 0.69

$$e^{-x}(-x-1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/E^x,x]

[Out] (-1 - x)/E^x
```

Maple [A] time = 0.003, size = 10, normalized size = 0.6

$$-\frac{1+x}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/exp(x),x)

[Out] -(1+x)/exp(x)
```

Maxima [A] time = 1.41769, size = 12, normalized size = 0.75

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(-x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x)
```

Fricas [A] time = 0.193265, size = 12, normalized size = 0.75

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(-x),x, algorithm="fricas")
```

```
[Out] -(x + 1)*e^(-x)
```

Sympy [A] time = 0.070098, size = 7, normalized size = 0.44

$$(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/exp(x),x)
```

```
[Out] (-x - 1)*exp(-x)
```

GIAC/XCAS [A] time = 0.237688, size = 12, normalized size = 0.75

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^(-x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x)
```

3.76 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi [A] time = 0.0255442, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi in Sympy [A] time = 1.71701, size = 17, normalized size = 0.89

$$x^2 e^x - 2xe^x + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(x)*x**2,x)
```

```
[Out] x**2*exp(x) - 2*x*exp(x) + 2*exp(x)
```

Mathematica [A] time = 0.0023394, size = 12, normalized size = 0.63

$$e^x \left(x^2 - 2x + 2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] time = 0.004, size = 12, normalized size = 0.6

$$\left(x^2 - 2\,x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x)
```

```
[Out] (x^2-2*x+2)*exp(x)
```

Maxima [A] time = 1.36132, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right) e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="maxima")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 0.206407, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] time = 0.062245, size = 10, normalized size = 0.53

$$\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)*x**2,x)
```

```
[Out] (x**2 - 2*x + 2)*exp(x)
```

GIAC/XCAS [A] time = 0.216404, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^x,x, algorithm="giac")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

3.77 $\int e^{-2x}x^2\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

```
[Out] -1/(4*E^(2*x)) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rubi [A] time = 0.0300944, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/E^(2*x),x]

[Out] -1/(4*E^(2*x)) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rubi in Sympy [A] time = 1.89209, size = 27, normalized size = 0.84

$$-\frac{x^2e^{-2x}}{2}-\frac{xe^{-2x}}{2}-\frac{e^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/exp(2*x),x)

[Out] -x**2*exp(-2*x)/2 - x*exp(-2*x)/2 - exp(-2*x)/4
```

Mathematica [A] time = 0.00410474, size = 19, normalized size = 0.59

$$-\frac{1}{4}e^{-2x}\left(2x^2+2x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/E^(2*x),x]

[Out] -(1 + 2*x + 2*x^2)/(4*E^(2*x))
```

Maple [A] time = 0.003, size = 19, normalized size = 0.6

$$-\frac{2\,x^2+2\,x+1}{4\,e^{2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/exp(2*x),x)

[Out] -1/4*(2*x^2+2*x+1)/exp(2*x)
```

Maxima [A] time = 1.41579, size = 22, normalized size = 0.69

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Fricas [A] time = 0.208442, size = 22, normalized size = 0.69

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Sympy [A] time = 0.078853, size = 17, normalized size = 0.53

$$\frac{\left(-2x^2-2x-1\right)e^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/exp(2*x),x)
```

```
[Out] (-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

GIAC/XCAS [A] time = 0.237221, size = 22, normalized size = 0.69

$$-\frac{1}{4}\left(2\,x^2+2\,x+1\right)e^{(-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*e^(-2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

3.78 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0138079, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^Sqrt[x],x]

[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi in Sympy [A] time = 1.02438, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x**(1/2)),x)

[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Mathematica [A] time = 0.00384076, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^Sqrt[x],x]

[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0.004, size = 17, normalized size = 0.7

$$-2\,e^{\sqrt{x}}+2\,e^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x^(1/2)),x)

[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 1.4689, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^sqrt(x),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 0.210481, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^sqrt(x),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.211322, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

GIAC/XCAS [A] time = 0.233823, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e^sqrt(x),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.79 $\int e^{-x^2} x^3 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

```
[Out] -1/(2*E^x^2) - x^2/(2*E^x^2)
```

Rubi [A] time = 0.0340855, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/E^x^2,x]

[Out] -1/(2*E^x^2) - x^2/(2*E^x^2)
```

Rubi in Sympy [A] time = 1.91424, size = 19, normalized size = 0.73

$$-\frac{x^2e^{-x^2}}{2} - \frac{e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/exp(x**2),x)

[Out] -x**2*exp(-x**2)/2 - exp(-x**2)/2
```

Mathematica [A] time = 0.00428297, size = 18, normalized size = 0.69

$$\frac{1}{2}e^{-x^2}\left(-x^2-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/E^x^2,x]

[Out] (-1 - x^2)/(2*E^x^2)
```

Maple [A] time = 0.004, size = 14, normalized size = 0.5

$$-\frac{x^2+1}{2\,e^{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/exp(x^2),x)

[Out] -1/2*(x^2+1)/exp(x^2)
```

Maxima [A] time = 1.40734, size = 18, normalized size = 0.69

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Fricas [A] time = 0.212363, size = 18, normalized size = 0.69

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A] time = 0.079511, size = 12, normalized size = 0.46

$$\frac{\left(-x^2-1\right)e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/exp(x**2),x)
```

```
[Out] (-x**2 - 1)*exp(-x**2)/2
```

GIAC/XCAS [A] time = 0.236536, size = 18, normalized size = 0.69

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3*e^(-x^2),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

3.80 $\int e^{ax}\cos(bx)\,dx$

Optimal. Leaf size=41

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^
2)
```

Rubi [A] time = 0.0282219, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(a*x)*Cos[b*x],x]
```

```
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^
2)
```

Rubi in Sympy [A] time = 1.90678, size = 36, normalized size = 0.88

$$\frac{ae^{ax}\cos{(bx)}}{a^2+b^2}+\frac{be^{ax}\sin{(bx)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 + b
**2)
```

Mathematica [A] time = 0.0293159, size = 28, normalized size = 0.68

$$\frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(a*x)*Cos[b*x],x]
```

```
[Out] (E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] time = 0.015, size = 40, normalized size = 1.

$$\frac{ae^{ax}\cos{(bx)}}{a^2+b^2}+\frac{e^{ax}b\sin{(bx)}}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x)*cos(b*x),x)
```

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A] time = 1.3467, size = 36, normalized size = 0.88

$$\frac{(a\cos(bx) + b\sin(bx))e^{(ax)}}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x)*e^(a*x),x, algorithm="maxima")
```

```
[Out] (a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A] time = 0.219554, size = 42, normalized size = 1.02

$$\frac{a\cos(bx)\,e^{(ax)} + be^{(ax)}\sin(bx)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x)*e^(a*x),x, algorithm="fricas")
```

```
[Out] (a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [A] time = 2.09457, size = 139, normalized size = 3.39

$$\begin{cases} x & \text{for } a = 0 \wedge b = 0 \\ \frac{ixe^{-ibx}\sin(bx)}{2} + \frac{xe^{-ibx}\cos(bx)}{2} + \frac{ie^{-ibx}\cos(bx)}{2b} & \text{for } a = -ib \\ -\frac{ixe^{ibx}\sin(bx)}{2} + \frac{xe^{ibx}\cos(bx)}{2} - \frac{ie^{ibx}\cos(bx)}{2b} & \text{for } a = ib \\ \frac{ae^{ax}\cos(bx)}{a^2+b^2} + \frac{be^{ax}\sin(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 +
 x*exp(-I*b*x)*cos(b*x)/2 + I*exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -
I*b)), (-I*x*exp(I*b*x)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 - I*
exp(I*b*x)*cos(b*x)/(2*b), Eq(a, I*b)), (a*exp(a*x)*cos(b*x)/(a**
2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 + b**2), True))
```

GIAC/XCAS [A] time = 0.236036, size = 49, normalized size = 1.2

$$\left(\frac{a\cos(bx)}{a^2 + b^2} + \frac{b\sin(bx)}{a^2 + b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(cos(b*x)*e^(a*x),x, algorithm="giac")
```

```
[Out] (a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.81 $\int e^{ax} \sin(bx)\,dx$

Optimal. Leaf size=42

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 +
 b^2)
```

Rubi [A] time = 0.0259106, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[E^(a*x)*Sin[b*x],x]
```

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 +
 b^2)
```

Rubi in Sympy [A] time = 1.8836, size = 36, normalized size = 0.86

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(a*x)*sin(b*x),x)
```

```
[Out] a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b
**2)
```

Mathematica [A] time = 0.0292896, size = 29, normalized size = 0.69

$$\frac{e^{ax}(a\sin(bx)-b\cos(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(a*x)*Sin[b*x],x]
```

```
[Out] (E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] time = 0.006, size = 41, normalized size = 1.

$$-\frac{e^{ax}b\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(a*x)*sin(b*x),x)
```

```
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A] time = 1.38498, size = 39, normalized size = 0.93

$$-\frac{(b\cos(bx) - a\sin(bx))e^{(ax)}}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)*sin(b*x),x, algorithm="maxima")
```

```
[Out] -(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A] time = 0.223701, size = 45, normalized size = 1.07

$$-\frac{b\cos(bx)\,e^{(ax)} - ae^{(ax)}\sin(bx)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)*sin(b*x),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [A] time = 2.13496, size = 136, normalized size = 3.24

$$\begin{cases} 0 & \text{for } a = 0 \wedge b = 0 \\ \frac{xe^{-ibx}\sin(bx)}{2} - \frac{ixe^{-ibx}\cos(bx)}{2} - \frac{e^{-ibx}\cos(bx)}{2b} & \text{for } a = -ib \\ \frac{xe^{ibx}\sin(bx)}{2} + \frac{ixe^{ibx}\cos(bx)}{2} - \frac{e^{ibx}\cos(bx)}{2b} & \text{for } a = ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2} - \frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*x)*sin(b*x),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I
*x*exp(-I*b*x)*cos(b*x)/2 - exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*
b)), (x*exp(I*b*x)*sin(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - exp(I
*b*x)*cos(b*x)/(2*b), Eq(a, I*b)), (a*exp(a*x)*sin(b*x)/(a**2 + b
**2) - b*exp(a*x)*cos(b*x)/(a**2 + b**2), True))
```

GIAC/XCAS [A] time = 0.235554, size = 51, normalized size = 1.21

$$-\left(\frac{b\cos(bx)}{a^2 + b^2} - \frac{a\sin(bx)}{a^2 + b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*x)*sin(b*x),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.82 $\int \cot^{-1}(x)\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.00722682, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 0.842846, size = 12, normalized size = 0.8

$$x\,\mathrm{acot}\,(x)+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acot(x),x)
```

```
[Out] x*acot(x) + log(x**2 + 1)/2
```

Mathematica [A] time = 0.00247219, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$x\mathrm{arccot}\,(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccot(x),x)
```

```
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 1.38675, size = 18, normalized size = 1.2

$$x \operatorname{arccot}(x) + \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(x),x, algorithm="maxima")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.22433, size = 18, normalized size = 1.2

$$x \operatorname{arccot}(x) + \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(x),x, algorithm="fricas")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.233644, size = 12, normalized size = 0.8

$$x \operatorname{acot}(x) + \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acot(x),x)
```

```
[Out] x*acot(x) + log(x**2 + 1)/2
```

GIAC/XCAS [A] time = 0.218393, size = 20, normalized size = 1.33

$$x \arctan\left(\frac{1}{x}\right) + \frac{1}{2} \ln\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(x),x, algorithm="giac")
```

```
[Out] x*arctan(1/x) + 1/2*ln(x^2 + 1)
```

3.83 $\int \sec^{-1}(x)\,dx$

Optimal. Leaf size=19

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.0280833, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.

$$x\sec^{-1}(x)-\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSec[x],x]

[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi in Sympy [A] time = 1.76474, size = 15, normalized size = 0.79

$$x\operatorname{asec}(x)-\operatorname{atanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asec(x),x)

[Out] x*asec(x) - atanh(sqrt(1 - 1/x**2))
```

Mathematica [B] time = 0.112068, size = 64, normalized size = 3.37

$$x\sec^{-1}(x)-\frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSec[x],x]

[Out] x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1
 + x/Sqrt[-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] time = 0.004, size = 22, normalized size = 1.2

$$x\operatorname{arcsec}(x)-\ln\left(x+x\sqrt{1-x^{-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsec(x),x)
```

```
[Out] x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [A] time = 1.37341, size = 47, normalized size = 2.47

$$x \operatorname{arcsec}(x) - \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) + \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x),x, algorithm="maxima")
```

```
[Out] x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^
2 + 1) + 1)
```

Fricas [A] time = 0.241886, size = 45, normalized size = 2.37

$$(x - 2) \operatorname{arcsec}(x) + 4 \arctan\left(-x + \sqrt{x^2 - 1}\right) + \log\left(-x + \sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(
x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asec}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asec(x),x)
```

```
[Out] Integral(asec(x), x)
```

GIAC/XCAS [A] time = 0.231164, size = 34, normalized size = 1.79

$$x \arccos\left(\frac{1}{x}\right) + \frac{\ln\left(\left|-x + \sqrt{x^2 - 1}\right|\right)}{\operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsec(x),x, algorithm="giac")
```

```
[Out] x*arccos(1/x) + ln(abs(-x + sqrt(x^2 - 1)))/sign(x)
```

3.84 $\int \csc^{-1}(x)\,dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.0275982, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi in Sympy [A] time = 1.74025, size = 15, normalized size = 0.88

$$x\,\mathrm{acsc}\,(x)+\mathrm{atanh}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acsc(x),x)
```

```
[Out] x*acsc(x) + atanh(sqrt(1 - 1/x**2))
```

Mathematica [B] time = 0.0713409, size = 64, normalized size = 3.76

$$\frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}x}+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1
 + x/Sqrt[-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] time = 0.004, size = 20, normalized size = 1.2

$$x\mathrm{arccsc}\,(x)+\ln\left(x+x\sqrt{1-x^{-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccsc(x),x)
```

```
[Out] x*arccsc(x)+ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [A] time = 1.49562, size = 47, normalized size = 2.76

$$x \operatorname{arccsc}(x) + \frac{1}{2} \log\left(\sqrt{-\frac{1}{x^2} + 1} + 1\right) - \frac{1}{2} \log\left(-\sqrt{-\frac{1}{x^2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x),x, algorithm="maxima")
```

```
[Out] x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^
2 + 1) + 1)
```

Fricas [A] time = 0.229933, size = 47, normalized size = 2.76

$$(x - 2) \operatorname{arccsc}(x) - 4 \arctan\left(-x + \sqrt{x^2 - 1}\right) - \log\left(-x + \sqrt{x^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(
x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acsc}(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acsc(x),x)
```

```
[Out] Integral(acsc(x), x)
```

GIAC/XCAS [A] time = 0.229477, size = 35, normalized size = 2.06

$$x \arcsin\left(\frac{1}{x}\right) - \frac{\ln\left(\left|-x + \sqrt{x^2 - 1}\right|\right)}{\operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccsc(x),x, algorithm="giac")
```

```
[Out] x*arcsin(1/x) - ln(abs(-x + sqrt(x^2 - 1)))/sign(x)
```

3.85 $\int \sin^{-1}(x)^2\,dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi [A] time = 0.0539379, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Int[ArcSin[x]^2,x]

[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi in Sympy [A] time = 2.87102, size = 22, normalized size = 0.88

$$x\,\mathrm{asin}^2(x)-2x+2\sqrt{-x^2+1}\,\mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(asin(x)**2,x)

[Out] x*asin(x)**2 - 2*x + 2*sqrt(-x**2 + 1)*asin(x)
```

Mathematica [A] time = 0.0117959, size = 25, normalized size = 1.

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcSin[x]^2,x]

[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] time = 0.158, size = 24, normalized size = 1.

$$-2\,x+x\,(\arcsin(x))^2+2\,\arcsin(x)\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arcsin(x)^2,x)

[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.57379, size = 31, normalized size = 1.24

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A] time = 0.219299, size = 31, normalized size = 1.24

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] time = 0.210235, size = 22, normalized size = 0.88

$$x \operatorname{asin}^2(x) - 2x + 2\sqrt{-x^2+1}\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(-x**2 + 1)*asin(x)
```

GIAC/XCAS [A] time = 0.235074, size = 31, normalized size = 1.24

$$x \arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.86 $\int \frac{\sin^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0350145, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 2.69826, size = 15, normalized size = 0.68

$$-\operatorname{atanh}\left(\sqrt{-x^2+1}\right)-\frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(asin(x)/x**2,x)

[Out] -atanh(sqrt(-x**2 + 1)) - asin(x)/x
```

Mathematica [A] time = 0.00754904, size = 26, normalized size = 1.18

$$-\log\left(\sqrt{1-x^2}+1\right)+\log(x)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) + Log[x] - Log[1 + Sqrt[1 - x^2]]
```

Maple [A] time = 0.013, size = 21, normalized size = 1.

$$-\frac{\arcsin(x)}{x}-\mathit{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x)

[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.5286, size = 45, normalized size = 2.05

$$-\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.237668, size = 53, normalized size = 2.41

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right) - x\log\left(\sqrt{-x^2+1}-1\right) + 2\arcsin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*a
rcsin(x))/x
```

Sympy [A] time = 2.2394, size = 22, normalized size = 1.

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \left|\frac{1}{x^2}\right| > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(asin(x)/x**2,x)
```

```
[Out] Piecewise((-acosh(1/x), Abs(x**(-2)) > 1), (I*asin(1/x), True)) -
 asin(x)/x
```

GIAC/XCAS [A] time = 0.23681, size = 51, normalized size = 2.32

$$-\frac{\arcsin(x)}{x} - \frac{1}{2}\ln\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\ln\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arcsin(x)/x^2,x, algorithm="giac")
```

```
[Out] -arcsin(x)/x - 1/2*ln(sqrt(-x^2 + 1) + 1) + 1/2*ln(-sqrt(-x^2 + 1
) + 1)
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] time = 0.00725337, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 - x^2],x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi in Sympy [A] time = 0.883175, size = 12, normalized size = 0.75

$$\operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2-x**2)**(1/2),x)

[Out] atan(x/sqrt(a**2 - x**2))
```

Mathematica [A] time = 0.00463751, size = 16, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 - x^2],x]

[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] time = 0.005, size = 15, normalized size = 0.9

$$\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2-x^2)^(1/2),x)

[Out] arctan(x/(a^2-x^2)^(1/2))
```

Maxima [A] time = 1.57245, size = 11, normalized size = 0.69

$$\arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 - x^2),x, algorithm="maxima")
```

```
[Out] arcsin(x/sqrt(a^2))
```

Fricas [A] time = 0.200098, size = 31, normalized size = 1.94

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 - x^2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [A] time = 1.63174, size = 19, normalized size = 1.19

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

GIAC/XCAS [A] time = 0.238503, size = 12, normalized size = 0.75

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sign}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a^2 - x^2),x, algorithm="giac")
```

```
[Out] arcsin(x/a)*sign(a)
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

Optimal. Leaf size=10

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rubi [A] time = 0.0223867, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - 2*x - x^2],x]

[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rubi in Sympy [A] time = 0.689149, size = 22, normalized size = 2.2

$$\operatorname{atan}\left(-\frac{-2x-2}{2\sqrt{-x^2-2x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-2*x+1)**(1/2),x)

[Out] atan(-(-2*x - 2)/(2*sqrt(-x**2 - 2*x + 1)))
```

Mathematica [A] time = 0.00936366, size = 14, normalized size = 1.4

$$-\sin^{-1}\left(\frac{-x-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - 2*x - x^2],x]

[Out] -ArcSin[(-1 - x)/Sqrt[2]]
```

Maple [A] time = 0.026, size = 10, normalized size = 1.

$$\arcsin\left(\frac{(1+x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-2*x+1)^(1/2),x)
```

```
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Maxima [A] time = 1.67626, size = 15, normalized size = 1.5

$$-\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 1),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/2*sqrt(2)*(x + 1))
```

Fricas [A] time = 0.201066, size = 28, normalized size = 2.8

$$-2\arctan\left(\frac{\sqrt{-x^2-2x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 1),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-2*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 1), x)
```

GIAC/XCAS [A] time = 0.223129, size = 12, normalized size = 1.2

$$\arcsin\left(\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 - 2*x + 1),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sqrt(2)*(x + 1))
```

3.89 $\int \frac{1}{a^2+x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] ArcTan[x/a]/a
```

Rubi [A] time = 0.00697499, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + x^2)^(-1),x]

[Out] ArcTan[x/a]/a
```

Rubi in Sympy [A] time = 0.730526, size = 5, normalized size = 0.5

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a**2+x**2),x)

[Out] atan(x/a)/a
```

Mathematica [A] time = 0.00315407, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + x^2)^(-1),x]

[Out] ArcTan[x/a]/a
```

Maple [A] time = 0.013, size = 11, normalized size = 1.1

$$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a^2+x^2),x)

[Out] arctan(x/a)/a
```

Maxima [A] time = 1.57168, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A] time = 0.19404, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [A] time = 0.11201, size = 20, normalized size = 2.

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

GIAC/XCAS [A] time = 0.22019, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a^2 + x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

3.90 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0165668, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 1.07101, size = 22, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a),x)
```

```
[Out] atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0079423, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.005, size = 16, normalized size = 0.7

$$1\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.194793, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)}{2\,\sqrt{-ab}}, \frac{\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a))/sqrt(-a*
b), arctan(sqrt(a*b)*x/a)/sqrt(a*b)]
```

Sympy [A] time = 0.134841, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log
(a*sqrt(-1/(a*b)) + x)/2
```

GIAC/XCAS [A] time = 0.223779, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.91 $\int \frac{1}{2-x+x^2} \, dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi [A] time = 0.0257762, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi in Sympy [A] time = 0.663014, size = 22, normalized size = 1.16

$$\frac{2\sqrt{7}\operatorname{atan}\left(\sqrt{7}\left(\frac{2x}{7}-\frac{1}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(sqrt(7)*(2*x/7 - 1/7))/7
```

Mathematica [A] time = 0.00933582, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x + x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$\frac{2\sqrt{7}}{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-x+2),x)
```

```
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A] time = 1.5319, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A] time = 0.194369, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] time = 0.093559, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

GIAC/XCAS [A] time = 0.224673, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - x + 2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.92 $\int x \tan^{-1}(x)\, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.0181306, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi in Sympy [A] time = 1.82926, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}(x)}{2} - \frac{x}{2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Mathematica [A] time = 0.00348781, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.52478, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^{2}\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 0.207887, size = 18, normalized size = 0.86

$$\frac{1}{2}\,\left(x^{2}+1\right)\,\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.319526, size = 15, normalized size = 0.71

$$\frac{x^{2}\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

GIAC/XCAS [A] time = 0.217736, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^{2}\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.93 $\int x^2 \cos^{-1}(x)\, dx$

Optimal. Leaf size=40

$$\frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

```
[Out] -Sqrt[1 - x^2]/3 + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rubi [A] time = 0.0476349, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*ArcCos[x],x]

[Out] -Sqrt[1 - x^2]/3 + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rubi in Sympy [A] time = 3.05471, size = 27, normalized size = 0.68

$$\frac{x^3\operatorname{acos}(x)}{3}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{9}-\frac{\sqrt{-x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*acos(x),x)

[Out] x**3*acos(x)/3 + (-x**2 + 1)**(3/2)/9 - sqrt(-x**2 + 1)/3
```

Mathematica [A] time = 0.0140873, size = 30, normalized size = 0.75

$$\frac{1}{3}x^3\cos^{-1}(x)-\frac{1}{9}\sqrt{1-x^2}\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*ArcCos[x],x]

[Out] -(Sqrt[1 - x^2]*(2 + x^2))/9 + (x^3*ArcCos[x])/3
```

Maple [A] time = 0.019, size = 34, normalized size = 0.9

$$\frac{x^3\arccos(x)}{3}-\frac{x^2}{9}\sqrt{-x^2+1}-\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*arccos(x),x)

[Out] 1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.52309, size = 45, normalized size = 1.12

$$\frac{1}{3}\,x^3\arccos\left(x\right)-\frac{1}{9}\,\sqrt{-x^2+1}x^2-\frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arccos(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Fricas [A] time = 0.219983, size = 32, normalized size = 0.8

$$\frac{1}{3}\,x^3\arccos\left(x\right)-\frac{1}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arccos(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.49457, size = 32, normalized size = 0.8

$$\frac{x^3\operatorname{acos}\left(x\right)}{3}-\frac{x^2\sqrt{-x^2+1}}{9}-\frac{2\sqrt{-x^2+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*acos(x),x)
```

```
[Out] x**3*acos(x)/3 - x**2*sqrt(-x**2 + 1)/9 - 2*sqrt(-x**2 + 1)/9
```

GIAC/XCAS [A] time = 0.21215, size = 45, normalized size = 1.12

$$\frac{1}{3}\,x^3\arccos\left(x\right)-\frac{1}{9}\,\sqrt{-x^2+1}x^2-\frac{2}{9}\,\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*arccos(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

3.94 $\int x \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=35

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^
2]/2
```

Rubi [A] time = 0.0831479, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]^2,x]
```

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^
2]/2
```

Rubi in Sympy [A] time = 6.05555, size = 29, normalized size = 0.83

$$\frac{x^2\operatorname{atan}^2(x)}{2}-x\operatorname{atan}(x)+\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*atan(x)**2,x)
```

```
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

Mathematica [A] time = 0.00722106, size = 26, normalized size = 0.74

$$\frac{1}{2}\left(\log\left(x^2+1\right)+\left(x^2+1\right)\tan^{-1}(x)^2-2x\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]^2,x]
```

```
[Out] (-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

Maple [A] time = 0.01, size = 30, normalized size = 0.9

$$-x\arctan(x)+\frac{(\arctan(x))^2}{2}+\frac{x^2(\arctan(x))^2}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)^2,x)
```

```
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Maxima [A] time = 1.56241, size = 46, normalized size = 1.31

$$\frac{1}{2}\,x^2\arctan\left(x\right)^2-\left(x-\arctan\left(x\right)\right)\arctan\left(x\right)-\frac{1}{2}\,\arctan\left(x\right)^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2
 + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.2159, size = 34, normalized size = 0.97

$$\frac{1}{2}\,\left(x^2+1\right)\,\arctan\left(x\right)^2-x\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.46711, size = 29, normalized size = 0.83

$$\frac{x^2\operatorname{atan}^2\left(x\right)}{2}-x\operatorname{atan}\left(x\right)+\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}^2\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*atan(x)**2,x)
```

```
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

GIAC/XCAS [A] time = 0.222593, size = 42, normalized size = 1.2

$$\frac{1}{2}\,x^2\arctan\left(x\right)^2-x\arctan\left(x\right)+\frac{1}{2}\,\arctan\left(x\right)^2+\frac{1}{2}\,\ln\left(-i\,x^2-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*ln(-I*x
^2 - I)
```

3.95 $\int \tan^{-1}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0125779, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rubi in Sympy [A] time = 1.3685, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(atan(x**(1/2)),x)
```

```
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Mathematica [A] time = 0.0106311, size = 18, normalized size = 0.82

$$(x+1)\tan^{-1}\left(\sqrt{x}\right)-\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x)
```

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Maxima [A] time = 1.53953, size = 22, normalized size = 1.

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Fricas [A] time = 0.217494, size = 19, normalized size = 0.86

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] time = 1.13534, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x**(1/2)),x)
```

```
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

GIAC/XCAS [A] time = 0.214596, size = 22, normalized size = 1.

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.96 $\int \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}(1+x)}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

```
[Out] ArcTan[Sqrt[x]]^2
```

Rubi [A] time = 0.0553465, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\,\text{atan}^2\left(\sqrt{x}\right) - \int \frac{\text{atan}\left(\sqrt{x}\right)}{\sqrt{x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

```
[Out] 2*atan(sqrt(x))**2 - Integral(atan(sqrt(x))/(sqrt(x)*(x + 1)), x)
```

Mathematica [A] time = 0.00404746, size = 8, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\left(\arctan\left(\sqrt{x}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/(1+x)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))^2
```

Maxima [A] time = 1.37559, size = 8, normalized size = 1.

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/((x + 1)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))^2
```

Fricas [A] time = 0.231464, size = 8, normalized size = 1.

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/((x + 1)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))^2
```

Sympy [A] time = 3.55055, size = 7, normalized size = 0.88

$$\operatorname{atan}^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

```
[Out] atan(sqrt(x))**2
```

GIAC/XCAS [A] time = 0.218283, size = 8, normalized size = 1.

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arctan(sqrt(x))/((x + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))^2
```

3.97 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0086933, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi in Sympy [A] time = 0.564831, size = 15, normalized size = 0.65

$$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2),x)

[Out] x*sqrt(-x**2 + 1)/2 + asin(x)/2
```

Mathematica [A] time = 0.00871474, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$\frac{\arcsin(x)}{2}+\frac{x}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2),x)

[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.53721, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 0.204438, size = 109, normalized size = 4.74

$$-\frac{2x^3+2\left(x^2+2\sqrt{-x^2+1}-2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(x^3-2x\right)\sqrt{-x^2+1}-2x}{2\left(x^2+2\sqrt{-x^2+1}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 + 2*(x^2 + 2*sqrt(-x^2 + 1) - 2)*arctan((sqrt(-x^2 +
1) - 1)/x) - (x^3 - 2*x)*sqrt(-x^2 + 1) - 2*x)/(x^2 + 2*sqrt(-x^2
 + 1) - 2)
```

Sympy [A] time = 0.220365, size = 15, normalized size = 0.65

$$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 1)/2 + asin(x)/2
```

GIAC/XCAS [A] time = 0.217321, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.98 $\int \frac{e^{\tan^{-1}(x)}x}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] -(E^ArcTan[x]*(1 - x))/(2*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0622105, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]

[Out] -(E^ArcTan[x]*(1 - x))/(2*Sqrt[1 + x^2])
```

Rubi in Sympy [A] time = 3.29783, size = 19, normalized size = 0.86

$$-\frac{(-x+1)\,e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)

[Out] -(-x + 1)*exp(atan(x))/(2*sqrt(x**2 + 1))
```

Mathematica [A] time = 0.0821633, size = 20, normalized size = 0.91

$$\frac{(x-1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]

[Out] (E^ArcTan[x]*(-1 + x))/(2*Sqrt[1 + x^2])
```

Maple [A] time = 0.006, size = 16, normalized size = 0.7

$$\frac{(-1+x)\,e^{\arctan(x)}}{2}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(arctan(x))*x/(x^2+1)^(3/2),x)
```

```
[Out] 1/2*(-1+x)*exp(arctan(x))/(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0.22213, size = 20, normalized size = 0.91

$$\frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.99 $\int \frac{e^{\tan^{-1}(x)}}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0350961, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]

[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rubi in Sympy [A] time = 2.39228, size = 17, normalized size = 0.85

$$\frac{(x+1)\,e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(atan(x))/(x**2+1)**(3/2),x)

[Out] (x + 1)*exp(atan(x))/(2*sqrt(x**2 + 1))
```

Mathematica [A] time = 0.064379, size = 20, normalized size = 1.

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]

[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Maple [A] time = 0.006, size = 16, normalized size = 0.8

$$\frac{e^{\arctan(x)}(1+x)}{2}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(arctan(x))/(x^2+1)^(3/2),x)
```

```
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0.227675, size = 20, normalized size = 1.

$$\frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(atan(x))/(x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^arctan(x)/(x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.100 $\int \frac{x^2}{(1+x^2)^2}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.01292, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 1.35815, size = 12, normalized size = 0.63

$$-\frac{x}{2(x^2+1)} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+1)**2,x)

[Out] -x/(2*(x**2 + 1)) + atan(x)/2
```

Mathematica [A] time = 0.0116371, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.019, size = 16, normalized size = 0.8

$$-\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+1)^2,x)

[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.51569, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.190376, size = 28, normalized size = 1.47

$$\frac{\left(x^{2}+1\right)\arctan\left(x\right)-x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.108322, size = 12, normalized size = 0.63

$$-\frac{x}{2x^{2}+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

GIAC/XCAS [A] time = 0.20756, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.101 $\int \frac{e^x}{1+e^{2x}}\,dx$

Optimal. Leaf size=4

$$\tan^{-1}\left(e^x\right)$$

```
[Out] ArcTan[E^x]
```

Rubi [A] time = 0.0280923, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^x/(1 + E^(2*x)),x]

[Out] ArcTan[E^x]
```

Rubi in Sympy [A] time = 2.55456, size = 3, normalized size = 0.75

$$\operatorname{atan}\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(x)/(1+exp(2*x)),x)

[Out] atan(exp(x))
```

Mathematica [A] time = 0.00597664, size = 4, normalized size = 1.

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^x/(1 + E^(2*x)),x]

[Out] ArcTan[E^x]
```

Maple [A] time = 0.004, size = 4, normalized size = 1.

$$\arctan\left(\mathrm{e}^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(x)/(1+exp(2*x)),x)

[Out] arctan(exp(x))
```

Maxima [A] time = 1.55077, size = 4, normalized size = 1.

$$\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) + 1),x, algorithm="maxima")
```

```
[Out] arctan(e^x)
```

Fricas [A] time = 0.202516, size = 4, normalized size = 1.

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) + 1),x, algorithm="fricas")
```

```
[Out] arctan(e^x)
```

Sympy [A] time = 0.114726, size = 15, normalized size = 3.75

$$\operatorname{RootSum}\left(4z^{2} + 1, \left(i \mapsto i\log\left(2i + e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(x)/(1+exp(2*x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

GIAC/XCAS [A] time = 0.206595, size = 4, normalized size = 1.

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^x/(e^(2*x) + 1),x, algorithm="giac")
```

```
[Out] arctan(e^x)
```

3.102 $\int e^{-x}\cot^{-1}(e^x)\,dx$

Optimal. Leaf size=27

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rubi [A] time = 0.0389215, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[ArcCot[E^x]/E^x,x]

[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rubi in Sympy [A] time = 2.94892, size = 26, normalized size = 0.96

$$\frac{\log\left(e^{2x}+1\right)}{2}-\frac{\log\left(e^{2x}\right)}{2}-e^{-x}\operatorname{acot}\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(acot(exp(x))/exp(x),x)

[Out] log(exp(2*x) + 1)/2 - log(exp(2*x))/2 - exp(-x)*acot(exp(x))
```

Mathematica [A] time = 0.0164087, size = 24, normalized size = 0.89

$$\frac{1}{2}\log\left(e^{-2x}+1\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[ArcCot[E^x]/E^x,x]

[Out] -(ArcCot[E^x]/E^x) + Log[1 + E^(-2*x)]/2
```

Maple [A] time = 0.01, size = 25, normalized size = 0.9

$$-\frac{\operatorname{arccot}\left(e^x\right)}{e^x}+\frac{\ln\left(\left(e^x\right)^2+1\right)}{2}-\ln\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(arccot(exp(x))/exp(x),x)

[Out] -arccot(exp(x))/exp(x)+1/2*ln(exp(x)^2+1)-ln(exp(x))
```

Maxima [A] time = 1.38409, size = 26, normalized size = 0.96

$$-\operatorname{arccot}\left(e^{x}\right)e^{(-x)} + \frac{1}{2}\log\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(e^x)*e^(-x),x, algorithm="maxima")
```

```
[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Fricas [A] time = 0.228226, size = 38, normalized size = 1.41

$$-\frac{1}{2}\left(2\,xe^{x} - e^{x}\log\left(e^{(2x)} + 1\right) + 2\,\operatorname{arccot}\left(e^{x}\right)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(e^x)*e^(-x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A] time = 22.7683, size = 19, normalized size = 0.7

$$-x + \frac{\log\left(e^{2x} + 1\right)}{2} - e^{-x}\operatorname{acot}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(acot(exp(x))/exp(x),x)
```

```
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

GIAC/XCAS [A] time = 0.208363, size = 28, normalized size = 1.04

$$-\arctan\left(e^{(-x)}\right)e^{(-x)} + \frac{1}{2}\ln\left(e^{(-2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(arccot(e^x)*e^(-x),x, algorithm="giac")
```

```
[Out] -arctan(e^(-x))*e^(-x) + 1/2*ln(e^(-2*x) + 1)
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x
)]]
```

Rubi [A] time = 0.033987, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x
)]]
```

Rubi in Sympy [A] time = 1.85796, size = 36, normalized size = 0.86

$$-\frac{2a\sqrt{\frac{a+x}{a-x}}}{1+\frac{a+x}{a-x}}+2a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] -2*a*sqrt((a + x)/(a - x))/(1 + (a + x)/(a - x)) + 2*a*atan(sqrt(
(a + x)/(a - x)))
```

Mathematica [A] time = 0.0823051, size = 67, normalized size = 1.6

$$\frac{\sqrt{\frac{a+x}{a-x}}\left(\sqrt{a+x}(x-a)+a\sqrt{a-x}\tan^{-1}\left(\frac{x}{\sqrt{a-x}\sqrt{a+x}}\right)\right)}{\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + a*Sqrt[a - x]*ArcT
an[x/(Sqrt[a - x]*Sqrt[a + x])]))/Sqrt[a + x]
```

Maple [A] time = 0.027, size = 62, normalized size = 1.5

$$(-a+x)\sqrt{-\frac{a+x}{-a+x}}\left(\sqrt{a^2-x^2}-a\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)\right)\frac{1}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] (-(a+x)/(-a+x))^(1/2)*(-a+x)/(-(a+x)*(-a+x))^(1/2)*((a^2-x^2)^(1/
2)-a*arctan(x/(a^2-x^2)^(1/2)))
```

Maxima [A] time = 1.50089, size = 66, normalized size = 1.57

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((
a + x)/(a - x))))
```

Fricas [A] time = 0.209414, size = 51, normalized size = 1.21

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

GIAC/XCAS [A] time = 0.217732, size = 49, normalized size = 1.17

$$a\arcsin\left(\frac{x}{a}\right)\operatorname{sign}(a-x)\operatorname{sign}(a)-\sqrt{a^2-x^2}\operatorname{sign}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((a + x)/(a - x)),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sign(a - x)*sign(a) - sqrt(a^2 - x^2)*sign(a - x)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*Ar
cTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi [A] time = 0.0523851, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*Ar
cTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi in Sympy [A] time = 1.93067, size = 56, normalized size = 0.79

$$-\frac{(a-b)^2\operatorname{atan}\left(\frac{a+b-2x}{2\sqrt{-ab-x^2+x(a+b)}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^2+x(a+b)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] -(a - b)**2*atan((a + b - 2*x)/(2*sqrt(-a*b - x**2 + x*(a + b))))
/8 - (a + b - 2*x)*sqrt(-a*b - x**2 + x*(a + b))/4
```

Mathematica [A] time = 0.184948, size = 84, normalized size = 1.18

$$\frac{1}{8}\sqrt{(a-x)(x-b)}\left(-2(a+b-2x)-\frac{(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x-a}\sqrt{b-x}}\right)}{\sqrt{x-a}\sqrt{b-x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (Sqrt[(a - x)*(-b + x)]*(-2*(a + b - 2*x) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[b - x]*Sqrt[-a + x])])/(Sqrt[b - x]*Sqrt[-a +
x])))/8
```

Maple [A] time = 0.02, size = 122, normalized size = 1.7

$$-\frac{a+b-2x}{4}\sqrt{-ab+(a+b)x-x^2}-\frac{ab}{4}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)x-x^2}}\right)$$
$$+\frac{a^2}{8}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)x-x^2}}\right)$$
$$+\frac{b^2}{8}\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*a-1/2*b
)/(-a*b+(a+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*a-1/2*b)/(-a*b+
(a+b)*x-x^2)^(1/2))*a^2+1/8*arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-
x^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223167, size = 88, normalized size = 1.24

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{a+b-2\,x}{2\,\sqrt{-ab+(a+b)x-x^2}}\right)-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="fricas")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*(a + b - 2*x)/sqrt(-a*b + (a
+ b)*x - x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(-a+x)\,(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

GIAC/XCAS [A] time = 0.214958, size = 82, normalized size = 1.15

$$\frac{1}{8}\left(a^2 - 2\,ab + b^2\right)\arcsin\left(\frac{a + b - 2\,x}{a - b}\right)\operatorname{sign}\left(-a + b\right) - \frac{1}{4}\sqrt{-ab + ax + bx - x^2}(a + b - 2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(a - x)*(b - x)),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sign(-a + b
) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi [A] time = 0.0263628, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi in Sympy [A] time = 1.19132, size = 26, normalized size = 0.81

$$-\operatorname{atan}\left(\frac{a+b-2x}{2\sqrt{-ab-x^2+x\,(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] -atan((a + b - 2*x)/(2*sqrt(-a*b - x**2 + x*(a + b))))
```

Mathematica [A] time = 0.0385634, size = 64, normalized size = 2.

$$-\frac{\sqrt{x-a}\sqrt{b-x}\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x-a}\sqrt{b-x}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((Sqrt[b - x]*Sqrt[-a + x]*ArcTan[(a + b - 2*x)/(2*Sqrt[b - x]*S
qrt[-a + x])])/Sqrt[(a - x)*(-b + x)])
```

Maple [A] time = 0.006, size = 28, normalized size = 0.9

$$\arctan\left(1\left(x-\frac{a}{2}-\frac{b}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b-x)*(-a+x))^(1/2),x)

[Out] arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.207562, size = 35, normalized size = 1.09

$$\arctan\left(-\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="fricas")

[Out] arctan(-1/2*(a + b - 2*x)/sqrt(-a*b + (a + b)*x - x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(-a+x)(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b-x)*(-a+x))**(1/2),x)

[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

GIAC/XCAS [A] time = 0.24245, size = 30, normalized size = 0.94

$$\arcsin\left(\frac{a+b-2x}{a-b}\right)\mathrm{sign}\left(-a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(a - x)*(b - x)),x, algorithm="giac")

[Out] arcsin((a + b - 2*x)/(a - b))*sign(-a + b)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2}\,dx$

Optimal. Leaf size=15

$$2\log(1-x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Rubi [A] time = 0.0148709, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Rubi in Sympy [A] time = 2.01697, size = 12, normalized size = 0.8

$$2\log(-x+1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+5*x)/(x**2+2*x-3),x)
```

```
[Out] 2*log(-x + 1) + 3*log(x + 3)
```

Mathematica [A] time = 0.00618239, size = 15, normalized size = 1.

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$2\ln(-1+x)+3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+5*x)/(x^2+2*x-3),x)
```

```
[Out] 2*ln(-1+x)+3*ln(3+x)
```

Maxima [A] time = 1.37797, size = 18, normalized size = 1.2

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x + 3)/(x^2 + 2*x - 3),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Fricas [A] time = 0.197893, size = 18, normalized size = 1.2

$$3\,\log\left(x+3\right)+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x + 3)/(x^2 + 2*x - 3),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Sympy [A] time = 0.114409, size = 12, normalized size = 0.8

$$2\log\left(x-1\right)+3\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+5*x)/(x**2+2*x-3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3)
```

GIAC/XCAS [A] time = 0.221742, size = 20, normalized size = 1.33

$$3\ln\left(\left|x+3\right|\right)+2\ln\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x + 3)/(x^2 + 2*x - 3),x, algorithm="giac")
```

```
[Out] 3*ln(abs(x + 3)) + 2*ln(abs(x - 1))
```

3.107 $\int \frac{5+2x}{-3+2x+x^2} \, dx$

Optimal. Leaf size=19

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

```
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rubi [A] time = 0.0150267, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(5 + 2*x)/(-3 + 2*x + x^2),x]

[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rubi in Sympy [A] time = 2.57416, size = 14, normalized size = 0.74

$$\frac{7\log(-x+1)}{4}+\frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5+2*x)/(x**2+2*x-3),x)

[Out] 7*log(-x + 1)/4 + log(x + 3)/4
```

Mathematica [A] time = 0.00599392, size = 19, normalized size = 1.

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]

[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Maple [A] time = 0.007, size = 14, normalized size = 0.7

$$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5+2*x)/(x^2+2*x-3),x)

[Out] 7/4*ln(-1+x)+1/4*ln(3+x)
```

Maxima [A] time = 1.57552, size = 18, normalized size = 0.95

$$\frac{1}{4}\log(x+3)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x^2 + 2*x - 3),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Fricas [A] time = 0.195108, size = 18, normalized size = 0.95

$$\frac{1}{4}\log(x+3)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x^2 + 2*x - 3),x, algorithm="fricas")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Sympy [A] time = 0.094239, size = 14, normalized size = 0.74

$$\frac{7\log(x-1)}{4}+\frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5+2*x)/(x**2+2*x-3),x)
```

```
[Out] 7*log(x - 1)/4 + log(x + 3)/4
```

GIAC/XCAS [A] time = 0.226814, size = 20, normalized size = 1.05

$$\frac{1}{4}\ln(|x+3|)+\frac{7}{4}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 5)/(x^2 + 2*x - 3),x, algorithm="giac")
```

```
[Out] 1/4*ln(abs(x + 3)) + 7/4*ln(abs(x - 1))
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rubi [A] time = 0.0402839, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x + x^3)/(-3 - 2*x + x^2),x]

[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$2x+9\log(-x+3)+\log(x+1)+\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**3+3*x)/(x**2-2*x-3),x)

[Out] 2*x + 9*log(-x + 3) + log(x + 1) + Integral(x, x)
```

Mathematica [A] time = 0.0075788, size = 23, normalized size = 1.

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]

[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Maple [A] time = 0.01, size = 20, normalized size = 0.9

$$\frac{x^2}{2}+2x+9\ln(-3+x)+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^3+3*x)/(x^2-2*x-3),x)

[Out] 1/2*x^2+2*x+9*ln(-3+x)+ln(1+x)
```

Maxima [A] time = 1.42222, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 + 2x + \log(x+1) + 9\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 3*x)/(x^2 - 2*x - 3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Fricas [A] time = 0.195934, size = 26, normalized size = 1.13

$$\frac{1}{2}x^2 + 2x + \log(x+1) + 9\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 3*x)/(x^2 - 2*x - 3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Sympy [A] time = 0.096035, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + 2x + 9\log(x-3) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**3+3*x)/(x**2-2*x-3),x)
```

```
[Out] x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

GIAC/XCAS [A] time = 0.220625, size = 28, normalized size = 1.22

$$\frac{1}{2}x^2 + 2x + \ln(|x+1|) + 9\ln(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^3 + 3*x)/(x^2 - 2*x - 3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2*x + ln(abs(x + 1)) + 9*ln(abs(x - 3))
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3}\,dx$

Optimal. Leaf size=23

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rubi [A] time = 0.0489408, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]

[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rubi in Sympy [A] time = 5.9607, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2}+2\log(-x+1)-\frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)

[Out] log(x)/2 + 2*log(-x + 1) - log(x + 2)/2
```

Mathematica [A] time = 0.00915951, size = 23, normalized size = 1.

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]

[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Maple [A] time = 0.012, size = 18, normalized size = 0.8

$$-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+5*x-1)/(x^3+x^2-2*x),x)

[Out] -1/2*ln(2+x)+1/2*ln(x)+2*ln(-1+x)
```

Maxima [A] time = 1.44809, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x+2\right)+2\log\left(x-1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 5*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Fricas [A] time = 0.20072, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x+2\right)+2\log\left(x-1\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 5*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Sympy [A] time = 0.141812, size = 17, normalized size = 0.74

$$\frac{\log\left(x\right)}{2}+2\log\left(x-1\right)-\frac{\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
```

```
[Out] log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

GIAC/XCAS [A] time = 0.218559, size = 27, normalized size = 1.17

$$-\frac{1}{2}\ln\left(|x+2|\right)+2\ln\left(|x-1|\right)+\frac{1}{2}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2 + 5*x - 1)/(x^3 + x^2 - 2*x),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(x + 2)) + 2*ln(abs(x - 1)) + 1/2*ln(abs(x))
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rubi [A] time = 0.0433183, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]

[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rubi in Sympy [A] time = 2.94529, size = 19, normalized size = 0.79

$$\frac{3\log(-x+1)}{2}-\frac{\log(x+1)}{2}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)

[Out] 3*log(-x + 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

Mathematica [A] time = 0.0164353, size = 22, normalized size = 0.92

$$\frac{1}{x+1}+\frac{3}{2}\log(x-1)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]

[Out] (1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

Maple [A] time = 0.012, size = 19, normalized size = 0.8

$$(1+x)^{-1}-\frac{\ln(1+x)}{2}+\frac{3\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2*x+3)/(-1+x)/(1+x)^2,x)

[Out] 1/(1+x)-1/2*ln(1+x)+3/2*ln(-1+x)
```

Maxima [A] time = 1.38366, size = 24, normalized size = 1.

$$\frac{1}{x+1} - \frac{1}{2}\log(x+1) + \frac{3}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 3)/((x + 1)^2*(x - 1)),x, algorithm="maxima")
```

```
[Out] 1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

Fricas [A] time = 0.196349, size = 35, normalized size = 1.46

$$-\frac{(x+1)\log(x+1) - 3(x+1)\log(x-1) - 2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 3)/((x + 1)^2*(x - 1)),x, algorithm="fricas")
```

```
[Out] -1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

Sympy [A] time = 0.120675, size = 19, normalized size = 0.79

$$\frac{3\log(x-1)}{2} - \frac{\log(x+1)}{2} + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
```

```
[Out] 3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

GIAC/XCAS [A] time = 0.210527, size = 32, normalized size = 1.33

$$\frac{1}{x+1} + \ln(|x+1|) + \frac{3}{2}\ln\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 2*x + 3)/((x + 1)^2*(x - 1)),x, algorithm="giac")
```

```
[Out] 1/(x + 1) + ln(abs(x + 1)) + 3/2*ln(abs(-2/(x + 1) + 1))
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3}\,dx$

Optimal. Leaf size=28

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rubi [A] time = 0.0491363, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rubi in Sympy [A] time = 4.14249, size = 29, normalized size = 1.04

$$\log\left(-x^3+1\right)+\frac{4\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+2*x-2)/(x**3-1),x)
```

```
[Out] log(-x**3 + 1) + 4*sqrt(3)*atan(sqrt(3)*(2*x/3 + 1/3))/3
```

Mathematica [A] time = 0.0167873, size = 28, normalized size = 1.

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Maple [A] time = 0.01, size = 29, normalized size = 1.

$$\ln\left(-1+x\right)+\ln\left(x^2+x+1\right)+\frac{4\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+2*x-2)/(x^3-1),x)
```

```
[Out] ln(-1+x)+ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.50273, size = 38, normalized size = 1.36

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 2)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + lo
g(x - 1)
```

Fricas [A] time = 0.19918, size = 51, normalized size = 1.82

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3}\log\left(x^2+x+1\right)+\sqrt{3}\log\left(x-1\right)+4\,\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 2)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3)*log(x^2 + x + 1) + sqrt(3)*log(x - 1) + 4*ar
ctan(1/3*sqrt(3)*(2*x + 1)))
```

Sympy [A] time = 0.141372, size = 3, normalized size = 0.11

$$\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+2*x-2)/(x**3-1),x)
```

```
[Out] log(x - 1)
```

GIAC/XCAS [A] time = 0.211376, size = 39, normalized size = 1.39

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\ln\left(x^2+x+1\right)+\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 2*x - 2)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + ln(x^2 + x + 1) + ln(
abs(x - 1))
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)(2+x^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 +
Log[2 + x^2]/3
```

Rubi [A] time = 0.136234, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161

$$\frac{1}{2(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 +
Log[2 + x^2]/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4-x^3+2x^2-x+2}{(x-1)\left(x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

```
[Out] Integral((x**4 - x**3 + 2*x**2 - x + 2)/((x - 1)*(x**2 + 2)**2),
x)
```

Mathematica [A] time = 0.0505087, size = 61, normalized size = 1.24

$$\frac{1}{2\left((x-1)^2+2(x-1)+3\right)}+\frac{1}{3}\log\left((x-1)^2+2(x-1)+3\right)+\frac{1}{3}\log(x-1)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2
]) + Log[3 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

Maple [A] time = 0.014, size = 37, normalized size = 0.8

$$\frac{\ln(-1+x)}{3}+\frac{1}{2x^2+4}+\frac{\ln\left(x^2+2\right)}{3}-\frac{\sqrt{2}}{6}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x)
```

```
[Out] 1/3*ln(-1+x)+1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*
2^(1/2)
```

Maxima [A] time = 1.57741, size = 49, normalized size = 1.

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\,(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - x^3 + 2*x^2 - x + 2)/((x^2 + 2)^2*(x - 1)),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2
+ 2) + 1/3*log(x - 1)
```

Fricas [A] time = 0.204356, size = 84, normalized size = 1.71

$$\frac{\sqrt{2}\left(2\sqrt{2}(x^2+2)\log\left(x^2+2\right)+2\sqrt{2}(x^2+2)\log\left(x-1\right)-2\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\sqrt{2}\right)}{12\,(x^2+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - x^3 + 2*x^2 - x + 2)/((x^2 + 2)^2*(x - 1)),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*(2*sqrt(2)*(x^2 + 2)*log(x^2 + 2) + 2*sqrt(2)*(x^2 +
 2)*log(x - 1) - 2*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 3*sqrt(2))/(
x^2 + 2)
```

Sympy [A] time = 0.195626, size = 14, normalized size = 0.29

$$\frac{\log\left(x-1\right)}{3}+\frac{1}{2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

```
[Out] log(x - 1)/3 + 1/(2*x**2 + 4)
```

GIAC/XCAS [A] time = 0.209775, size = 50, normalized size = 1.02

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\,(x^2+2)}+\frac{1}{3}\ln\left(x^2+2\right)+\frac{1}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4 - x^3 + 2*x^2 - x + 2)/((x^2 + 2)^2*(x - 1)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*ln(x^2 +
 2) + 1/3*ln(abs(x - 1))
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0186006, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Cos[x] + Sin[x])^(-1),x]

[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi in Sympy [A] time = 0.541286, size = 22, normalized size = 1.05

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}(-\sin(x)+\cos(x))}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(cos(x)+sin(x)),x)

[Out] -sqrt(2)*atanh(sqrt(2)*(-sin(x) + cos(x))/2)/2
```

Mathematica [C] time = 0.0238186, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Cos[x] + Sin[x])^(-1),x]

[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] time = 0.033, size = 19, normalized size = 0.9

$$\sqrt{2}Artanh\left(\frac{\sqrt{2}}{4}(2\tan(x/2)-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Maxima [A] time = 1.61631, size = 54, normalized size = 2.57

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1\right)}{2\sqrt{2}+\frac{2\sin(x)}{\cos(x)+1}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-2*(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/((2*sqrt(
2)) + 2*sin(x)/(cos(x) + 1) - 2))
```

Fricas [A] time = 0.215725, size = 51, normalized size = 2.43

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) +
 3)/(2*cos(x)*sin(x) + 1))
```

Sympy [A] time = 14.5012, size = 0, normalized size = 0.

$$\mathrm{NaN}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] nan
```

GIAC/XCAS [A] time = 0.237818, size = 50, normalized size = 2.38

$$-\frac{1}{2}\sqrt{2}\ln\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(cos(x) + sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2)
+ 2*tan(1/2*x) - 2))
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{4-x^2}+1\right)$$

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rubi [A] time = 0.0811877, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]

[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rubi in Sympy [A] time = 3.02601, size = 12, normalized size = 0.75

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)

[Out] -log(sqrt(-x**2 + 4) + 1)
```

Mathematica [A] time = 0.0129228, size = 16, normalized size = 1.

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]

[Out] -Log[1 + Sqrt[4 - x^2]]
```

Maple [B] time = 0.089, size = 266, normalized size = 16.6

$$
\begin{aligned}
&-\frac{\ln \left(x^{2}-3\right)}{2}+\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \sqrt{-\left(-2+x\right)^{2}-4 x+8} \\
&+\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \sqrt{-\left(2+x\right)^{2}+4 x+8} \\
&+\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \mathit{Artanh}\left(\frac{2-2 \sqrt{3} \left(x-\sqrt{3}\right)}{2} \frac{1}{\sqrt{-\left(x-\sqrt{3}\right)^{2}-2 \sqrt{3} \left(x-\sqrt{3}\right)+1}}\right) \\
&-\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \sqrt{-\left(x-\sqrt{3}\right)^{2}-2 \sqrt{3} \left(x-\sqrt{3}\right)+1} \\
&+\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \mathit{Artanh}\left(\frac{2+2 \sqrt{3} \left(x+\sqrt{3}\right)}{2} \frac{1}{\sqrt{-\left(x+\sqrt{3}\right)^{2}+2 \sqrt{3} \left(x+\sqrt{3}\right)+1}}\right) \\
&-\frac{1}{\left(4+2 \sqrt{3}\right) \left(-2+\sqrt{3}\right)} \sqrt{-\left(x+\sqrt{3}\right)^{2}+2 \sqrt{3} \left(x+\sqrt{3}\right)+1}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(4-x^2+(-x^2+4)^(1/2)),x)
```

```
[Out] -1/2*ln(x^2-3)+1/2/(2+3^(1/2))/(-2+3^(1/2))*(-(-2+x)^2-4*x+8)^(1/
2)+1/2/(2+3^(1/2))/(-2+3^(1/2))*(-(2+x)^2+4*x+8)^(1/2)+1/2/(2+3^(
1/2))/(-2+3^(1/2))*arctanh(1/2*(2-2*3^(1/2)*(x-3^(1/2)))/(-(x-3^(
1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2))-1/2/(2+3^(1/2))/(-2+3^(1/
2))*(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2)+1/2/(2+3^(1/2)
)/(-2+3^(1/2))*arctanh(1/2*(2+2*3^(1/2)*(x+3^(1/2)))/(-(x+3^(1/2)
)^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2))-1/2/(2+3^(1/2))/(-2+3^(1/2))*
(-(x+3^(1/2))^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2)
```

Maxima [A] time = 1.34782, size = 19, normalized size = 1.19

$$
-\log\left(\sqrt{-x^{2}+4}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 4) - 4),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Fricas [A] time = 0.21199, size = 74, normalized size = 4.62

$$
-\frac{1}{2}\log\left(x^{2}-3\right)+\frac{1}{2}\log\left(-\frac{x^{2}+3\sqrt{-x^{2}+4}-6}{x^{2}}\right)-\frac{1}{2}\log\left(-\frac{x^{2}+\sqrt{-x^{2}+4}-2}{x^{2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 4) - 4),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) -
1/2*log(-(x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

Sympy [A] time = 2.16125, size = 17, normalized size = 1.06

$$-\begin{cases}\log\left(\sqrt{-x^2+4}+1\right) & \text{for } x > -2 \wedge x < 2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
```

```
[Out] -Piecewise((log(sqrt(-x**2 + 4) + 1), (x > -2) & (x < 2)))
```

GIAC/XCAS [A] time = 0.209149, size = 19, normalized size = 1.19

$$-\ln\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-x/(x^2 - sqrt(-x^2 + 4) - 4),x, algorithm="giac")
```

```
[Out] -ln(sqrt(-x^2 + 4) + 1)
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] Log[2 - x] + Log[5 + x]
```

Rubi [A] time = 0.0182272, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[2 - x] + Log[5 + x]
```

Rubi in Sympy [A] time = 1.47666, size = 8, normalized size = 0.73

$$\log(-x+2)+\log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+2*x)/(-2+x)/(5+x),x)

[Out] log(-x + 2) + log(x + 5)
```

Mathematica [A] time = 0.00550531, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] time = 0.002, size = 9, normalized size = 0.8

$$\ln((-2+x)(5+x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+2*x)/(-2+x)/(5+x),x)

[Out] ln((-2+x)*(5+x))
```

Maxima [A] time = 1.34278, size = 12, normalized size = 1.09

$$\log(x+5)+\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="maxima")
```

```
[Out] log(x + 5) + log(x - 2)
```

Fricas [A] time = 0.18985, size = 12, normalized size = 1.09

$$\log\left(x^2 + 3\,x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] time = 0.083926, size = 8, normalized size = 0.73

$$\log\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

GIAC/XCAS [A] time = 0.224585, size = 15, normalized size = 1.36

$$\ln\left(|x + 5|\right) + \ln\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x + 3)/((x + 5)*(x - 2)),x, algorithm="giac")
```

```
[Out] ln(abs(x + 5)) + ln(abs(x - 2))
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rubi [A] time = 0.0367001, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[x/((1 + x)*(2 + x)*(3 + x)),x]

[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rubi in Sympy [A] time = 2.21359, size = 20, normalized size = 0.87

$$-\frac{\log(x+1)}{2}+2\log(x+2)-\frac{3\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)/(2+x)/(3+x),x)

[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Mathematica [A] time = 0.00907984, size = 23, normalized size = 1.

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]

[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A] time = 0.01, size = 20, normalized size = 0.9

$$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)/(2+x)/(3+x),x)

[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Maxima [A] time = 1.36606, size = 26, normalized size = 1.13

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 3)*(x + 2)*(x + 1)),x, algorithm="maxima")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Fricas [A] time = 0.202308, size = 26, normalized size = 1.13

$$-\frac{3}{2}\log(x+3)+2\log(x+2)-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 3)*(x + 2)*(x + 1)),x, algorithm="fricas")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] time = 0.130946, size = 20, normalized size = 0.87

$$-\frac{\log(x+1)}{2}+2\log(x+2)-\frac{3\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

GIAC/XCAS [A] time = 0.229923, size = 30, normalized size = 1.3

$$-\frac{3}{2}\ln(|x+3|)+2\ln(|x+2|)-\frac{1}{2}\ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((x + 3)*(x + 2)*(x + 1)),x, algorithm="giac")
```

```
[Out] -3/2*ln(abs(x + 3)) + 2*ln(abs(x + 2)) - 1/2*ln(abs(x + 1))
```

3.117 $\int \frac{x}{2-3x+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rubi [A] time = 0.0357005, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(2 - 3*x + x^3),x]
```

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rubi in Sympy [A] time = 4.23438, size = 22, normalized size = 0.73

$$\frac{2\log\left(-x+1\right)}{9}-\frac{2\log\left(x+2\right)}{9}+\frac{1}{3\left(-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**3-3*x+2),x)
```

```
[Out] 2*log(-x + 1)/9 - 2*log(x + 2)/9 + 1/(3*(-x + 1))
```

Mathematica [A] time = 0.0139849, size = 28, normalized size = 0.93

$$-\frac{1}{3(x-1)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(2 - 3*x + x^3),x]
```

```
[Out] -1/(3*(-1 + x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Maple [A] time = 0.011, size = 21, normalized size = 0.7

$$-\frac{2\ln\left(2+x\right)}{9}-\frac{1}{-3+3x}+\frac{2\ln\left(-1+x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^3-3*x+2),x)
```

```
[Out] -2/9*ln(2+x)-1/3/(-1+x)+2/9*ln(-1+x)
```

Maxima [A] time = 1.39563, size = 27, normalized size = 0.9

$$-\frac{1}{3(x-1)} - \frac{2}{9}\log(x+2) + \frac{2}{9}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 3*x + 2),x, algorithm="maxima")
```

```
[Out] -1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

Fricas [A] time = 0.194873, size = 36, normalized size = 1.2

$$-\frac{2(x-1)\log(x+2) - 2(x-1)\log(x-1) + 3}{9(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 3*x + 2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

Sympy [A] time = 0.093551, size = 22, normalized size = 0.73

$$\frac{2\log(x-1)}{9} - \frac{2\log(x+2)}{9} - \frac{1}{3x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**3-3*x+2),x)
```

```
[Out] 2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

GIAC/XCAS [A] time = 0.225019, size = 30, normalized size = 1.

$$-\frac{1}{3(x-1)} - \frac{2}{9}\ln(|x+2|) + \frac{2}{9}\ln(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^3 - 3*x + 2),x, algorithm="giac")
```

```
[Out] -1/3/(x - 1) - 2/9*ln(abs(x + 2)) + 2/9*ln(abs(x - 1))
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=27

$$\frac{x^2}{2} - x - \log(1-x) + 3\log(x) + \log(x+2)$$

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rubi [A] time = 0.0467185, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{x^2}{2} - x - \log(1-x) + 3\log(x) + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]

[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-x + 3\log(x) - \log(-x+1) + \log(x+2) + \int x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)

[Out] -x + 3*log(x) - log(-x + 1) + log(x + 2) + Integral(x, x)
```

Mathematica [A] time = 0.00879665, size = 27, normalized size = 1.

$$\frac{x^2}{2} - x - \log(1-x) + 3\log(x) + \log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]

[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Maple [A] time = 0.012, size = 24, normalized size = 0.9

$$-x + \frac{x^2}{2} + \ln(2+x) + 3\ln(x) - \ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+2*x-6)/(x^3+x^2-2*x),x)

[Out] -x+1/2*x^2+ln(2+x)+3*ln(x)-ln(-1+x)
```

Maxima [A] time = 1.4311, size = 31, normalized size = 1.15

$$\frac{1}{2}x^2 - x + \log(x+2) - \log(x-1) + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x - 6)/(x^3 + x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Fricas [A] time = 0.200593, size = 31, normalized size = 1.15

$$\frac{1}{2}x^2 - x + \log(x+2) - \log(x-1) + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x - 6)/(x^3 + x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Sympy [A] time = 0.129858, size = 20, normalized size = 0.74

$$\frac{x^2}{2} - x + 3\log(x) - \log(x-1) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
```

```
[Out] x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

GIAC/XCAS [A] time = 0.231505, size = 35, normalized size = 1.3

$$\frac{1}{2}x^2 - x + \ln(|x+2|) - \ln(|x-1|) + 3\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 2*x - 6)/(x^3 + x^2 - 2*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + ln(abs(x + 2)) - ln(abs(x - 1)) + 3*ln(abs(x))
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx$

Optimal. Leaf size=23

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rubi [A] time = 0.0385672, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]

[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{8x^3+7}{(x+1)\,(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)

[Out] Integral((8*x**3 + 7)/((x + 1)*(2*x + 1)**3), x)
```

Mathematica [A] time = 0.0170653, size = 24, normalized size = 1.04

$$\frac{6x+(2x+1)^2\log(x+1)}{(2x+1)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]

[Out] (6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

Maple [A] time = 0.011, size = 24, normalized size = 1.

$$-3\,(1+2\,x)^{-2}+3\,(1+2\,x)^{-1}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((8*x^3+7)/(1+x)/(1+2*x)^3,x)

[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Maxima [A] time = 1.55279, size = 27, normalized size = 1.17

$$\frac{6\,x}{4\,x^2+4\,x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x, algorithm="maxima")
```

```
[Out] 6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

Fricas [A] time = 0.193705, size = 43, normalized size = 1.87

$$\frac{\left(4\,x^2+4\,x+1\right)\log\left(x+1\right)+6\,x}{4\,x^2+4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x, algorithm="fricas")
```

```
[Out] ((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

Sympy [A] time = 0.117101, size = 17, normalized size = 0.74

$$\frac{6x}{4x^2+4x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
```

```
[Out] 6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

GIAC/XCAS [A] time = 0.223899, size = 22, normalized size = 0.96

$$\frac{6\,x}{\left(2\,x+1\right)^2}+\ln\left(\left|x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((8*x^3 + 7)/((2*x + 1)^3*(x + 1)),x, algorithm="giac")
```

```
[Out] 6*x/(2*x + 1)^2 + ln(abs(x + 1))
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} dx$

Optimal. Leaf size=16

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rubi [A] time = 0.027795, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(-1 + x^3),x]

[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rubi in Sympy [A] time = 3.87229, size = 14, normalized size = 0.88

$$2\log(-x+1)+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**2+x+1)/(x**3-1),x)

[Out] 2*log(-x + 1) + log(x**2 + x + 1)
```

Mathematica [A] time = 0.00724154, size = 16, normalized size = 1.

$$\log\left(x^2+x+1\right)+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]

[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Maple [A] time = 0.008, size = 15, normalized size = 0.9

$$2\ln(-1+x)+\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(x^3-1),x)

[Out] 2*ln(-1+x)+ln(x^2+x+1)
```

Maxima [A] time = 1.62193, size = 19, normalized size = 1.19

$$\log\left(x^2+x+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(x^3 - 1),x, algorithm="maxima")

[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.192281, size = 19, normalized size = 1.19

$$\log\left(x^2 + x + 1\right) + 2\,\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(x^3 - 1),x, algorithm="fricas")

[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.085473, size = 14, normalized size = 0.88

$$2\log\left(x - 1\right) + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+x+1)/(x**3-1),x)

[Out] 2*log(x - 1) + log(x**2 + x + 1)
```

GIAC/XCAS [A] time = 0.21679, size = 20, normalized size = 1.25

$$\ln\left(x^2 + x + 1\right) + 2\ln\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + x + 1)/(x^3 - 1),x, algorithm="giac")

[Out] ln(x^2 + x + 1) + 2*ln(abs(x - 1))
```

3.121 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi [A] time = 0.0332514, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(4 + 5*x^2 + x^4),x]

[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi in Sympy [A] time = 5.02038, size = 14, normalized size = 0.78

$$x-\frac{8\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(x**4+5*x**2+4),x)

[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Mathematica [A] time = 0.0110295, size = 18, normalized size = 1.

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(4 + 5*x^2 + x^4),x]

[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] time = 0.011, size = 13, normalized size = 0.7

$$x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(x^4+5*x^2+4),x)

[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 1.51905, size = 16, normalized size = 0.89

$$x - \frac{8}{3} \arctan\left(\frac{1}{2} x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 0.195418, size = 16, normalized size = 0.89

$$x - \frac{8}{3} \arctan\left(\frac{1}{2} x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.192233, size = 14, normalized size = 0.78

$$x - \frac{8 \operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

GIAC/XCAS [A] time = 0.215385, size = 16, normalized size = 0.89

$$x - \frac{8}{3} \arctan\left(\frac{1}{2} x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(x^4 + 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.122 $\int \frac{2+x}{x+x^2}\,dx$

Optimal. Leaf size=11

$$2\log(x) - \log(x+1)$$

```
[Out] 2*Log[x] - Log[1 + x]
```

Rubi [A] time = 0.0200133, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(x + x^2),x]
```

```
[Out] 2*Log[x] - Log[1 + x]
```

Rubi in Sympy [A] time = 1.57101, size = 8, normalized size = 0.73

$$2\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**2+x),x)
```

```
[Out] 2*log(x) - log(x + 1)
```

Mathematica [A] time = 0.00316559, size = 11, normalized size = 1.

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(x + x^2),x]
```

```
[Out] 2*Log[x] - Log[1 + x]
```

Maple [A] time = 0.009, size = 12, normalized size = 1.1

$$2\ln(x) - \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2+x),x)
```

```
[Out] 2*ln(x)-ln(1+x)
```

Maxima [A] time = 1.33706, size = 15, normalized size = 1.36

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Fricas [A] time = 0.231548, size = 15, normalized size = 1.36

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Sympy [A] time = 0.089173, size = 8, normalized size = 0.73

$$2\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2+x),x)
```

```
[Out] 2*log(x) - log(x + 1)
```

GIAC/XCAS [A] time = 0.211365, size = 18, normalized size = 1.64

$$-\ln(|x+1|) + 2\ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 + x),x, algorithm="giac")
```

```
[Out] -ln(abs(x + 1)) + 2*ln(abs(x))
```

3.123 $\int \frac{1}{x(1+x^2)^2} dx$

Optimal. Leaf size=24

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0255199, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 1.95417, size = 22, normalized size = 0.92

$$\frac{\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2} + \frac{1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**2+1)**2,x)
```

```
[Out] log(x**2)/2 - log(x**2 + 1)/2 + 1/(2*(x**2 + 1))
```

Mathematica [A] time = 0.0125833, size = 24, normalized size = 1.

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.016, size = 21, normalized size = 0.9

$$\frac{1}{2x^2+2} + \ln(x) - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.34761, size = 32, normalized size = 1.33

$$\frac{1}{2\,(x^2+1)} - \frac{1}{2}\,\log\left(x^2+1\right) + \frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 0.22982, size = 43, normalized size = 1.79

$$-\frac{\left(x^2+1\right)\log\left(x^2+1\right) - 2\left(x^2+1\right)\log\left(x\right) - 1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

Sympy [A] time = 0.110324, size = 19, normalized size = 0.79

$$\log\left(x\right) - \frac{\log\left(x^2+1\right)}{2} + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

GIAC/XCAS [A] time = 0.217808, size = 39, normalized size = 1.62

$$\frac{x^2+2}{2\,(x^2+1)} - \frac{1}{2}\ln\left(x^2+1\right) + \frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 + 2)/(x^2 + 1) - 1/2*ln(x^2 + 1) + 1/2*ln(x^2)
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2
*Log[2 + x] - (17*Log[3 + x])/8
```

Rubi [A] time = 0.0517896, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2
*Log[2 + x] - (17*Log[3 + x])/8
```

Rubi in Sympy [A] time = 3.03221, size = 41, normalized size = 0.89

$$\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - 17*log(x + 3)/8 + 5/(4*(x + 3)) + 1
/(4*(x + 3)**2) + 1/(x + 2)
```

Mathematica [A] time = 0.0252764, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 +
x] - 17*Log[3 + x])/8
```

Maple [A] time = 0.016, size = 39, normalized size = 0.9

$$(2+x)^{-1}+\frac{1}{4(3+x)^2}+\frac{5}{12+4x}+\frac{\ln(1+x)}{8}+2\ln(2+x)-\frac{17\ln(3+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x)/(2+x)^2/(3+x)^3,x)
```

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Maxima [A] time = 1.35519, size = 62, normalized size = 1.35

$$\frac{9\,x^2+50\,x+68}{4\,(x^3+8\,x^2+21\,x+18)}-\frac{17}{8}\,\log\left(x+3\right)+2\,\log\left(x+2\right)+\frac{1}{8}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x +
3) + 2*log(x + 2) + 1/8*log(x + 1)
```

Fricas [A] time = 0.205921, size = 112, normalized size = 2.43

$$\frac{18\,x^2-17\,\left(x^3+8\,x^2+21\,x+18\right)\,\log\left(x+3\right)+16\,\left(x^3+8\,x^2+21\,x+18\right)\,\log\left(x+2\right)+\left(x^3+8\,x^2+21\,x+18\right)\,\log\left(x+1\right)+1}{8\,(x^3+8\,x^2+21\,x+18)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 +
 8*x^2 + 21*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x
+ 1) + 100*x + 136)/(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] time = 0.230709, size = 46, normalized size = 1.

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log\left(x+1\right)}{8}+2\log\left(x+2\right)-\frac{17\log\left(x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/
8 + 2*log(x + 2) - 17*log(x + 3)/8
```

GIAC/XCAS [A] time = 0.212988, size = 70, normalized size = 1.52

$$\frac{1}{x+2}-\frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2}+\frac{1}{8}\ln\left(\left|-\frac{1}{x+2}+1\right|\right)-\frac{17}{8}\ln\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x + 3)^3*(x + 2)^2*(x + 1)),x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*ln(abs(-1
/(x + 2) + 1)) - 17/8*ln(abs(-1/(x + 2) - 1))
```

3.125 $\int \frac{x}{(1+x)^2} dx$

Optimal. Leaf size=10

$$\frac{1}{x+1} + \log(x+1)$$

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi [A] time = 0.011224, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi in Sympy [A] time = 1.01869, size = 8, normalized size = 0.8

$$\log(x+1) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(1+x)**2,x)

[Out] log(x + 1) + 1/(x + 1)
```

Mathematica [A] time = 0.00390123, size = 10, normalized size = 1.

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A] time = 0.007, size = 11, normalized size = 1.1

$$(1+x)^{-1} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(1+x)^2,x)

[Out] 1/(1+x)+ln(1+x)
```

Maxima [A] time = 1.34165, size = 14, normalized size = 1.4

$$\frac{1}{x+1} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A] time = 0.191065, size = 22, normalized size = 2.2

$$\frac{(x+1)\log(x+1)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A] time = 0.063842, size = 8, normalized size = 0.8

$$\log(x+1) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

GIAC/XCAS [A] time = 0.210087, size = 15, normalized size = 1.5

$$\frac{1}{x+1} + \ln(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x + 1)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + ln(abs(x + 1))
```

3.126 $\int \frac{1}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.0185629, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-x + x^3)^(-1),x]

[Out] -Log[x] + Log[1 - x^2]/2
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**3-x),x)

[Out] Exception raised: TypeError
```

Mathematica [A] time = 0.00358253, size = 17, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-x + x^3)^(-1),x]

[Out] -Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.01, size = 18, normalized size = 1.1

$$\frac{\ln(1+x)}{2}-\ln(x)+\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^3-x),x)

[Out] 1/2*ln(1+x)-ln(x)+1/2*ln(-1+x)
```

Maxima [A] time = 1.33902, size = 23, normalized size = 1.35

$$\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - x),x, algorithm="maxima")

[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 0.194322, size = 18, normalized size = 1.06

$$\frac{1}{2}\log\left(x^2-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - x),x, algorithm="fricas")

[Out] 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.080906, size = 10, normalized size = 0.59

$$-\log\left(x\right)+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**3-x),x)

[Out] -log(x) + log(x**2 - 1)/2
```

GIAC/XCAS [A] time = 0.2098, size = 22, normalized size = 1.29

$$-\frac{1}{2}\ln\left(x^2\right)+\frac{1}{2}\ln\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^3 - x),x, algorithm="giac")

[Out] -1/2*ln(x^2) + 1/2*ln(abs(x^2 - 1))
```

3.127 $\int \frac{x^2}{-6+x+x^2}\,dx$

Optimal. Leaf size=20

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rubi [A] time = 0.0224065, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(-6 + x + x^2),x]

[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rubi in Sympy [A] time = 2.59053, size = 17, normalized size = 0.85

$$x+\frac{4\log\left(-x+2\right)}{5}-\frac{9\log\left(x+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+x-6),x)

[Out] x + 4*log(-x + 2)/5 - 9*log(x + 3)/5
```

Mathematica [A] time = 0.00508133, size = 20, normalized size = 1.

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(-6 + x + x^2),x]

[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$x+\frac{4\,\ln\left(-2+x\right)}{5}-\frac{9\,\ln\left(3+x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+x-6),x)

[Out] x+4/5*ln(-2+x)-9/5*ln(3+x)
```

Maxima [A] time = 1.34977, size = 19, normalized size = 0.95

$$x - \frac{9}{5}\,\log\left(x + 3\right) + \frac{4}{5}\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + x - 6),x, algorithm="maxima")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Fricas [A] time = 0.194364, size = 19, normalized size = 0.95

$$x - \frac{9}{5}\,\log\left(x + 3\right) + \frac{4}{5}\,\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + x - 6),x, algorithm="fricas")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A] time = 0.096395, size = 17, normalized size = 0.85

$$x + \frac{4\log\left(x - 2\right)}{5} - \frac{9\log\left(x + 3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+x-6),x)
```

```
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

GIAC/XCAS [A] time = 0.215049, size = 22, normalized size = 1.1

$$x - \frac{9}{5}\ln\left(|x + 3|\right) + \frac{4}{5}\ln\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + x - 6),x, algorithm="giac")
```

```
[Out] x - 9/5*ln(abs(x + 3)) + 4/5*ln(abs(x - 2))
```

3.128 $\int \frac{2+x}{4-4x+x^2}\,dx$

Optimal. Leaf size=16

$$\frac{4}{2-x}+\log(2-x)$$

```
[Out] 4/(2 - x) + Log[2 - x]
```

Rubi [A] time = 0.0155304, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{4}{2-x}+\log(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] 4/(2 - x) + Log[2 - x]
```

Rubi in Sympy [A] time = 2.18911, size = 8, normalized size = 0.5

$$\log(-x+2)+\frac{4}{-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2+x)/(x**2-4*x+4),x)
```

```
[Out] log(-x + 2) + 4/(-x + 2)
```

Mathematica [A] time = 0.00527172, size = 12, normalized size = 0.75

$$\log(x-2)-\frac{4}{x-2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] -4/(-2 + x) + Log[-2 + x]
```

Maple [A] time = 0.01, size = 13, normalized size = 0.8

$$\ln(-2+x)-4\,(-2+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2+x)/(x^2-4*x+4),x)
```

```
[Out] ln(-2+x)-4/(-2+x)
```

Maxima [A] time = 1.36311, size = 16, normalized size = 1.

$$-\frac{4}{x-2} + \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x + 4),x, algorithm="maxima")
```

```
[Out] -4/(x - 2) + log(x - 2)
```

Fricas [A] time = 0.194168, size = 22, normalized size = 1.38

$$\frac{(x-2)\log(x-2)-4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x + 4),x, algorithm="fricas")
```

```
[Out] ((x - 2)*log(x - 2) - 4)/(x - 2)
```

Sympy [A] time = 0.07067, size = 8, normalized size = 0.5

$$\log(x-2) - \frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2+x)/(x**2-4*x+4),x)
```

```
[Out] log(x - 2) - 4/(x - 2)
```

GIAC/XCAS [A] time = 0.212257, size = 18, normalized size = 1.12

$$-\frac{4}{x-2} + \ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 2)/(x^2 - 4*x + 4),x, algorithm="giac")
```

```
[Out] -4/(x - 2) + ln(abs(x - 2))
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi [A] time = 0.0239587, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi in Sympy [A] time = 3.55335, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)+\frac{2}{-2x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)

[Out] -atan(x - 2) + 2/(-2*x + 4)
```

Mathematica [A] time = 0.0123414, size = 14, normalized size = 1.

$$\tan^{-1}(2-x)-\frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] time = 0.009, size = 15, normalized size = 1.1

$$-\arctan(-2+x)-(-2+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)

[Out] -arctan(-2+x)-1/(-2+x)
```

Maxima [A] time = 1.48672, size = 19, normalized size = 1.36

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A] time = 0.197063, size = 23, normalized size = 1.64

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] time = 0.140555, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2) - \frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

GIAC/XCAS [A] time = 0.220908, size = 19, normalized size = 1.36

$$-\frac{1}{x-2} - \arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 - 4*x + 5)*(x^2 - 4*x + 4)),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3} \, dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5}{2}\log(x+2)$$

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi [A] time = 0.0405114, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi in Sympy [A] time = 4.99209, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

```
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Mathematica [A] time = 0.00854995, size = 21, normalized size = 1.

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] time = 0.01, size = 18, normalized size = 0.9

$$-\frac{3\ln(x)}{2} + 4\ln(1+x) - \frac{5\ln(2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3+x)/(x^3+3*x^2+2*x),x)
```

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Maxima [A] time = 1.34195, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="maxima")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A] time = 0.201458, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log\left(x+2\right)+4\log\left(x+1\right)-\frac{3}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="fricas")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] time = 0.133264, size = 20, normalized size = 0.95

$$-\frac{3\log\left(x\right)}{2}+4\log\left(x+1\right)-\frac{5\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3+x)/(x**3+3*x**2+2*x),x)

[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

GIAC/XCAS [A] time = 0.218444, size = 27, normalized size = 1.29

$$-\frac{5}{2}\ln\left(|x+2|\right)+4\ln\left(|x+1|\right)-\frac{3}{2}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x - 3)/(x^3 + 3*x^2 + 2*x),x, algorithm="giac")

[Out] -5/2*ln(abs(x + 2)) + 4*ln(abs(x + 1)) - 3/2*ln(abs(x))
```

3.131 $\int \frac{1}{(-1+x^2)^2}\, dx$

Optimal. Leaf size=21

$$\frac{x}{2(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.00830644, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x}{2(1-x^2)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)^(-2),x]

[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi in Sympy [A] time = 0.566464, size = 12, normalized size = 0.57

$$\frac{x}{2(-x^2+1)}+\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**2,x)

[Out] x/(2*(-x**2 + 1)) + atanh(x)/2
```

Mathematica [A] time = 0.0124384, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)^(-2),x]

[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.014, size = 28, normalized size = 1.3

$$-\frac{1}{4+4x}+\frac{\ln(1+x)}{4}-\frac{1}{4x-4}-\frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^2,x)

[Out] -1/4/(1+x)+1/4*ln(1+x)-1/4/(-1+x)-1/4*ln(-1+x)
```

Maxima [A] time = 1.34528, size = 31, normalized size = 1.48

$$-\frac{x}{2\,(x^2-1)}+\frac{1}{4}\,\log\left(x+1\right)-\frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [A] time = 0.193479, size = 46, normalized size = 2.19

$$\frac{\left(x^2-1\right)\,\log\left(x+1\right)-\left(x^2-1\right)\,\log\left(x-1\right)-2\,x}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 0.100562, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

GIAC/XCAS [A] time = 0.211894, size = 34, normalized size = 1.62

$$-\frac{x}{2\,(x^2-1)}+\frac{1}{4}\,\ln\left(|x+1|\right)-\frac{1}{4}\,\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^(-2),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*ln(abs(x + 1)) - 1/4*ln(abs(x - 1))
```

3.132 $\int \frac{1+x}{-1+x^3}\,dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rubi [A] time = 0.0184451, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x)/(-1 + x^3),x]

[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rubi in Sympy [A] time = 2.67986, size = 17, normalized size = 0.77

$$\frac{2\log\left(-x+1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1+x)/(x**3-1),x)

[Out] 2*log(-x + 1)/3 - log(x**2 + x + 1)/3
```

Mathematica [A] time = 0.00457224, size = 22, normalized size = 1.

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x)/(-1 + x^3),x]

[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Maple [A] time = 0.009, size = 17, normalized size = 0.8

$$\frac{2\ln\left(-1+x\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1+x)/(x^3-1),x)

[Out] 2/3*ln(-1+x)-1/3*ln(x^2+x+1)
```

Maxima [A] time = 1.51585, size = 22, normalized size = 1.

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 - 1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Fricas [A] time = 0.188249, size = 22, normalized size = 1.

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 - 1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Sympy [A] time = 0.078172, size = 17, normalized size = 0.77

$$\frac{2\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1+x)/(x**3-1),x)
```

```
[Out] 2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

GIAC/XCAS [A] time = 0.213335, size = 23, normalized size = 1.05

$$-\frac{1}{3}\ln\left(x^2+x+1\right)+\frac{2}{3}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 1)/(x^3 - 1),x, algorithm="giac")
```

```
[Out] -1/3*ln(x^2 + x + 1) + 2/3*ln(abs(x - 1))
```

3.133 $\int \frac{1+x^4}{x(1+x^2)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1} + \log(x)$$

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi [A] time = 0.046195, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{1}{x^2+1} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi in Sympy [A] time = 3.27187, size = 12, normalized size = 1.2

$$\frac{\log\left(x^2\right)}{2} + \frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**4+1)/x/(x**2+1)**2,x)

[Out] log(x**2)/2 + 1/(x**2 + 1)
```

Mathematica [A] time = 0.00782358, size = 10, normalized size = 1.

$$\frac{1}{x^2+1} + \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] time = 0.009, size = 11, normalized size = 1.1

$$\left(x^2+1\right)^{-1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^4+1)/x/(x^2+1)^2,x)

[Out] 1/(x^2+1)+ln(x)
```

Maxima [A] time = 1.3425, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 0.195865, size = 24, normalized size = 2.4

$$\frac{\left(x^2+1\right)\log\left(x\right)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.102103, size = 8, normalized size = 0.8

$$\log\left(x\right)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x) + 1/(x**2 + 1)
```

GIAC/XCAS [A] time = 0.222716, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1}+\frac{1}{2}\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^4 + 1)/((x^2 + 1)^2*x),x, algorithm="giac")
```

```
[Out] 1/(x^2 + 1) + 1/2*ln(x^2)
```

3.134 $\int \frac{1}{-2x^3+x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rubi [A] time = 0.0224945, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(-2*x^3 + x^4)^(-1),x]

[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rubi in Sympy [A] time = 1.65975, size = 22, normalized size = 0.71

$$-\frac{\log(x)}{8}+\frac{\log(-x+2)}{8}+\frac{1}{4x}+\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-2*x**3),x)

[Out] -log(x)/8 + log(-x + 2)/8 + 1/(4*x) + 1/(4*x**2)
```

Mathematica [A] time = 0.00315375, size = 31, normalized size = 1.

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-2*x^3 + x^4)^(-1),x]

[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Maple [A] time = 0.01, size = 22, normalized size = 0.7

$$\frac{1}{4\,x^2}+\frac{1}{4\,x}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-2*x^3),x)

[Out] 1/4/x^2+1/4/x-1/8*ln(x)+1/8*ln(-2+x)
```

Maxima [A] time = 1.34184, size = 26, normalized size = 0.84

$$\frac{x+1}{4\,x^2} + \frac{1}{8}\,\log\left(x-2\right) - \frac{1}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

Fricas [A] time = 0.196035, size = 34, normalized size = 1.1

$$\frac{x^2\log\left(x-2\right) - x^2\log\left(x\right) + 2\,x + 2}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2*x^3),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

Sympy [A] time = 0.105103, size = 19, normalized size = 0.61

$$-\frac{\log\left(x\right)}{8} + \frac{\log\left(x-2\right)}{8} + \frac{x+1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-2*x**3),x)
```

```
[Out] -log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

GIAC/XCAS [A] time = 0.230007, size = 28, normalized size = 0.9

$$\frac{x+1}{4\,x^2} + \frac{1}{8}\ln\left(|x-2|\right) - \frac{1}{8}\ln\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 2*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(x + 1)/x^2 + 1/8*ln(abs(x - 2)) - 1/8*ln(abs(x))
```

3.135 $\int \frac{1-x^3}{x(1+x^2)} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0430217, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^3)/(x*(1 + x^2)),x]

[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 4.77189, size = 15, normalized size = 0.83

$$-x+\log(x)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**3+1)/x/(x**2+1),x)

[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

Mathematica [A] time = 0.00788118, size = 18, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 - x^3)/(x*(1 + x^2)),x]

[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$-x+\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^3+1)/x/(x^2+1),x)

[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.50263, size = 22, normalized size = 1.22

$$-x + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^2 + 1)*x),x, algorithm="maxima")

[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 0.202171, size = 22, normalized size = 1.22

$$-x + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^2 + 1)*x),x, algorithm="fricas")

[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Sympy [A] time = 0.129338, size = 15, normalized size = 0.83

$$-x + \log(x) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**3+1)/x/(x**2+1),x)

[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

GIAC/XCAS [A] time = 0.227086, size = 23, normalized size = 1.28

$$-x + \arctan(x) - \frac{1}{2}\ln\left(x^2+1\right) + \ln(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x^3 - 1)/((x^2 + 1)*x),x, algorithm="giac")

[Out] -x + arctan(x) - 1/2*ln(x^2 + 1) + ln(abs(x))
```

3.136 $\int \frac{1}{-1+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.00800693, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^4)^(-1),x]

[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi in Sympy [A] time = 0.585474, size = 10, normalized size = 0.77

$$-\frac{\operatorname{atan}(x)}{2}-\frac{\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-1),x)

[Out] -atan(x)/2 - atanh(x)/2
```

Mathematica [A] time = 0.00525476, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^4)^(-1),x]

[Out] -ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$-\frac{\arctan(x)}{2}-\frac{\mathit{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-1),x)

[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A] time = 1.50589, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x+1\right)+\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="maxima")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 0.200016, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\log\left(x+1\right)+\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="fricas")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.16301, size = 17, normalized size = 1.31

$$\frac{\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{4}-\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-1),x)

[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

GIAC/XCAS [A] time = 0.228481, size = 26, normalized size = 2.

$$-\frac{1}{2}\arctan\left(x\right)-\frac{1}{4}\ln\left(|x+1|\right)+\frac{1}{4}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 1),x, algorithm="giac")

[Out] -1/2*arctan(x) - 1/4*ln(abs(x + 1)) + 1/4*ln(abs(x - 1))
```

3.137 $\int \frac{1}{1+x^4} dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0805692, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqr
t[2]) - Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x
+ x^2]/(4*Sqrt[2])
```

Rubi in Sympy [A] time = 5.48517, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
x + 1)/4
```

Mathematica [A] time = 0.0299923, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqr
t[2]*x + x^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.023, size = 58, normalized size = 0.7

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(x\sqrt{2}-1\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+1),x)
```

```
[Out] 1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/4*arctan(x*2^(1/2)-1)*2^(1/2)+1
/8*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))
```

Maxima [A] time = 1.5232, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*
x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 0.2022, size = 131, normalized size = 1.54

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}+1}\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}-1}\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x +
 1) + 1)) - 1/2*sqrt(2)*arctan(1/(sqrt(2)*x + sqrt(2)*sqrt(x^2 -
sqrt(2)*x + 1) - 1)) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8
*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.190151, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)
*x + 1)/8 + sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*
x + 1)/4
```

GIAC/XCAS [A] time = 0.214762, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\ln\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\ln\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*ln(x^2 + sqrt(2)*x
 + 1) - 1/8*sqrt(2)*ln(x^2 - sqrt(2)*x + 1)
```

3.138 $\int \frac{x^2}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

```
[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rubi [A] time = 0.0201448, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 + 2*x + x^2)^2,x]

[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rubi in Sympy [A] time = 1.80503, size = 20, normalized size = 0.87

$$-\frac{x\left(2x+4\right)}{4\left(x^2+2x+2\right)}+\operatorname{atan}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**2+2*x+2)**2,x)

[Out] -x*(2*x + 4)/(4*(x**2 + 2*x + 2)) + atan(x + 1)
```

Mathematica [A] time = 0.0137605, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2x+2}+\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 + 2*x + x^2)^2,x]

[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.7

$$\left(x^2+2\,x+2\right)^{-1}+\arctan\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^2+2*x+2)^2,x)

[Out] 1/(x^2+2*x+2)+arctan(1+x)
```

Maxima [A] time = 1.53231, size = 20, normalized size = 0.87

$$\frac{1}{x^2 + 2x + 2} + \arctan(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 2)^2,x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Fricas [A] time = 0.194914, size = 35, normalized size = 1.52

$$\frac{(x^2 + 2x + 2)\arctan(x + 1) + 1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 2)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.127153, size = 14, normalized size = 0.61

$$\operatorname{atan}(x + 1) + \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**2+2*x+2)**2,x)
```

```
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

GIAC/XCAS [A] time = 0.214209, size = 20, normalized size = 0.87

$$\frac{1}{x^2 + 2x + 2} + \arctan(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^2 + 2*x + 2)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2} \, dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

```
[Out] -(x/(1 + x + x^5))
```

Rubi [A] time = 0.0057325, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Rubi in Sympy [A] time = 2.75604, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

Mathematica [A] time = 0.00971564, size = 11, normalized size = 1.

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Maple [B] time = 0.013, size = 41, normalized size = 3.7

$$-\frac{-3\,x^2+5\,x-1}{7\,x^3-7\,x^2+7}+\frac{-3\,x-1}{7\,x^2+7\,x+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
```

```
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

Maxima [A] time = 1.38568, size = 15, normalized size = 1.36

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

Fricas [A] time = 0.186584, size = 15, normalized size = 1.36

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^5 + x + 1)
```

Sympy [A] time = 0.161351, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

GIAC/XCAS [A] time = 0.214393, size = 15, normalized size = 1.36

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^5 - 1)/(x^5 + x + 1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x
] + 2*Sin[x])]/Sqrt[5]
```

Rubi [A] time = 0.0839133, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x
] + 2*Sin[x])]/Sqrt[5]
```

Rubi in Sympy [A] time = 0.801627, size = 24, normalized size = 0.53

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\sqrt{5}\left(\frac{3\tan\left(\frac{x}{2}\right)}{5}+\frac{1}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5-cos(x)+2*sin(x)),x)
```

```
[Out] sqrt(5)*atan(sqrt(5)*(3*tan(x/2)/5 + 1/5))/5
```

Mathematica [A] time = 0.0346503, size = 23, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{3\tan\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

Maple [A] time = 0.053, size = 20, normalized size = 0.4

$$\frac{\sqrt{5}}{5}\arctan\left(\frac{\sqrt{5}}{10}(6\,\tan(x/2)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(1/(5-cos(x)+2*sin(x)),x)

[Out] 1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

Maxima [A] time = 1.60667, size = 31, normalized size = 0.69

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(\frac{3\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-1/(cos(x) - 2*sin(x) - 5),x, algorithm="maxima")

[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

Fricas [A] time = 0.220963, size = 49, normalized size = 1.09

$$\frac{1}{10}\sqrt{5}\arctan\left(-\frac{\sqrt{5}\cos\left(x\right)-2\sqrt{5}\sin\left(x\right)-\sqrt{5}}{2\left(2\cos\left(x\right)+\sin\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-1/(cos(x) - 2*sin(x) - 5),x, algorithm="fricas")

[Out] 1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqr
t(5))/(2*cos(x) + sin(x)))
```

Sympy [A] time = 0.835013, size = 39, normalized size = 0.87

$$\frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(1/(5-cos(x)+2*sin(x)),x)

[Out] sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 -
 pi/2)/pi))/5
```

GIAC/XCAS [A] time = 0.215703, size = 63, normalized size = 1.4

$$\frac{1}{10}\sqrt{5}\left(x+2\arctan\left(-\frac{\sqrt{5}\sin\left(x\right)-\cos\left(x\right)-3\sin\left(x\right)-1}{\sqrt{5}\cos\left(x\right)+\sqrt{5}-3\cos\left(x\right)+\sin\left(x\right)+3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   integrate(-1/(cos(x) - 2*sin(x) - 5),x, algorithm="giac")

[Out] 1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) -
 1)/(sqrt(5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]

Rubi [A] time = 0.0639, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rubi in Sympy [A] time = 2.36815, size = 34, normalized size = 0.92

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-a+1}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{-a+1}\sqrt{a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+a*cos(x)),x)
```

```
[Out] 2*atan(sqrt(-a + 1)*tan(x/2)/sqrt(a + 1))/(sqrt(-a + 1)*sqrt(a +
1))
```

Mathematica [A] time = 0.0297242, size = 31, normalized size = 0.84

$$\frac{2\tanh^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

Maple [A] time = 0.02, size = 30, normalized size = 0.8

$$2\,\frac{1}{\sqrt{(1+a)(a-1)}}\mathit{Artanh}\left(\frac{(a-1)\tan(x/2)}{\sqrt{(1+a)(a-1)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+a*cos(x)),x)
```

```
[Out] 2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*cos(x) + 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227078, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,\left(a^3+\left(a^2-1\right)\,\cos(x)-a\right)\,\sin(x)-\left(\left(a^2-2\right)\,\cos(x)^2-2\,a^2-2\,a\cos(x)+1\right)\sqrt{a^2-1}}{a^2\cos(x)^2+2\,a\cos(x)+1}\right)}{2\sqrt{a^2-1}},\ \frac{\arctan\left(\frac{\sqrt{-a^2+1}(a+\cos(x))}{(a^2-1)\sin(x)}\right)}{\sqrt{-a^2+1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*cos(x) + 1),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*(a^3 + (a^2 - 1)*cos(x) - a)*sin(x) - ((a^2 - 2)*cos(
x)^2 - 2*a^2 - 2*a*cos(x) + 1)*sqrt(a^2 - 1))/(a^2*cos(x)^2 + 2*a
*cos(x) + 1))/sqrt(a^2 - 1), arctan(sqrt(-a^2 + 1)*(a + cos(x))/(
(a^2 - 1)*sin(x)))/sqrt(-a^2 + 1)]
```

Sympy [A] time = 11.1774, size = 110, normalized size = 2.97

$$\begin{cases} -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for}\ a=-1 \\ \tan\left(\frac{x}{2}\right) & \text{for}\ a=1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}}+\frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+a*cos(x)),x)
```

```
[Out] Piecewise((-1/tan(x/2), Eq(a, -1)), (tan(x/2), Eq(a, 1)), (-log(-
sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a
- 1)) - sqrt(a/(a - 1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a
- 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a - 1)
 + 1/(a - 1))), True))
```

GIAC/XCAS [A] time = 0.215294, size = 72, normalized size = 1.95

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sign}(2\,a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*cos(x) + 1),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sign(2*a - 2) + arctan((a*tan(1/2*x)
 - tan(1/2*x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

Optimal. Leaf size=56

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2
] + Sin[x/2]]/Sqrt[3]
```

Rubi [A] time = 0.0375471, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2
] + Sin[x/2]]/Sqrt[3]
```

Rubi in Sympy [A] time = 0.579725, size = 20, normalized size = 0.36

$$\frac{2\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+2*cos(x)),x)
```

```
[Out] 2*sqrt(3)*atanh(sqrt(3)*tan(x/2)/3)/3
```

Mathematica [A] time = 0.0129929, size = 20, normalized size = 0.36

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.013, size = 16, normalized size = 0.3

$$\frac{2\sqrt{3}}{3}\mathit{Artanh}\left(\frac{\sqrt{3}}{3}\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+2*cos(x)),x)
```

```
[Out] 2/3*3^(1/2)*arctanh(1/3*3^(1/2)*tan(1/2*x))
```

Maxima [A] time = 1.48372, size = 50, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{3}+\frac{\sin(x)}{\cos(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*cos(x) + 1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(
x)/(cos(x) + 1)))
```

Fricas [A] time = 0.22219, size = 70, normalized size = 1.25

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{2\sqrt{3}\cos(x)^2 - 6(\cos(x)+2)\sin(x) - 4\sqrt{3}\cos(x) - 7\sqrt{3}}{4\cos(x)^2 + 4\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*cos(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(2*sqrt(3)*cos(x)^2 - 6*(cos(x) + 2)*sin(x) - 4*
sqrt(3)*cos(x) - 7*sqrt(3))/(4*cos(x)^2 + 4*cos(x) + 1))
```

Sympy [A] time = 0.536907, size = 36, normalized size = 0.64

$$-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+2*cos(x)),x)
```

```
[Out] -sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(
3))/3
```

GIAC/XCAS [A] time = 0.248927, size = 47, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\ln\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*cos(x) + 1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*
tan(1/2*x)))
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx$

Optimal. Leaf size=31

$$\frac{2x}{\sqrt{3}}-\frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

```
[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rubi [A] time = 0.05019, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2x}{\sqrt{3}}-\frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Cos[x]/2)^(-1),x]

[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]
```

Rubi in Sympy [A] time = 0.589181, size = 20, normalized size = 0.65

$$\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+1/2*cos(x)),x)

[Out] 4*sqrt(3)*atan(sqrt(3)*tan(x/2)/3)/3
```

Mathematica [A] time = 0.0107652, size = 20, normalized size = 0.65

$$\frac{4\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Cos[x]/2)^(-1),x]

[Out] (4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.023, size = 16, normalized size = 0.5

$$\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+1/2*cos(x)),x)
```

```
[Out] 4/3*3^(1/2)*arctan(1/3*3^(1/2)*tan(1/2*x))
```

Maxima [A] time = 1.55611, size = 26, normalized size = 0.84

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(x)}{3(\cos(x)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(cos(x) + 2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.211495, size = 31, normalized size = 1.

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(x)+\sqrt{3}}{3\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(cos(x) + 2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A] time = 0.40569, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+1/2*cos(x)),x)
```

```
[Out] 4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/
3
```

GIAC/XCAS [A] time = 0.226214, size = 54, normalized size = 1.74

$$\frac{2}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/(cos(x) + 2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos
(x) + sqrt(3) - cos(x) + 1)))
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/
Sqrt[2]
```

Rubi [A] time = 0.0671648, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/
Sqrt[2]
```

Rubi in Sympy [A] time = 4.66621, size = 24, normalized size = 0.67

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}}{2\tan(x)}\right)}{2}-\mathrm{atan}\left(\frac{1}{\tan(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)**2/(1+sin(x)**2),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)/(2*tan(x)))/2 - atan(1/tan(x))
```

Mathematica [A] time = 0.0209109, size = 18, normalized size = 0.5

$$x-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

Maple [A] time = 0.029, size = 15, normalized size = 0.4

$$-\frac{\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)^2/(1+sin(x)^2),x)
```

```
[Out] -1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+x
```

Maxima [A] time = 1.52201, size = 19, normalized size = 0.53

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/(sin(x)^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

Fricas [A] time = 0.243118, size = 51, normalized size = 1.42

$$\frac{1}{4}\sqrt{2}\left(2\sqrt{2}x+\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-2\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/(sin(x)^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*x + arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqr
t(2))/(cos(x)*sin(x))))
```

Sympy [A] time = 155.717, size = 416, normalized size = 11.56

$$\frac{41\sqrt{2}x\sqrt{-2\sqrt{2}+3}}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}+\frac{58x\sqrt{-2\sqrt{2}+3}}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}-\frac{17\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}-\frac{12\sqrt{2}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{-2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}-\frac{17\sqrt{-2\sqrt{2}+3}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}-\frac{12\sqrt{2}\sqrt{-2\sqrt{2}+3}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{41\sqrt{2}\sqrt{-2\sqrt{2}+3}+58\sqrt{-2\sqrt{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)**2/(1+sin(x)**2),x)
```

```
[Out] 41*sqrt(2)*x*sqrt(-2*sqrt(2) + 3)/(41*sqrt(2)*sqrt(-2*sqrt(2) + 3
) + 58*sqrt(-2*sqrt(2) + 3)) + 58*x*sqrt(-2*sqrt(2) + 3)/(41*sqrt
(2)*sqrt(-2*sqrt(2) + 3) + 58*sqrt(-2*sqrt(2) + 3)) - 17*(atan(ta
n(x/2)/sqrt(-2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqr
t(2)*sqrt(-2*sqrt(2) + 3) + 58*sqrt(-2*sqrt(2) + 3)) - 12*sqrt(2)
*(atan(tan(x/2)/sqrt(-2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi)
)/(41*sqrt(2)*sqrt(-2*sqrt(2) + 3) + 58*sqrt(-2*sqrt(2) + 3)) - 1
7*sqrt(-2*sqrt(2) + 3)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*
sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*sqrt(-2*sq
rt(2) + 3) + 58*sqrt(-2*sqrt(2) + 3)) - 12*sqrt(2)*sqrt(-2*sqrt(2
```

```
) + 3)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) +
pi*floor((x/2 - pi/2)/pi))/(41*sqrt(2)*sqrt(-2*sqrt(2) + 3) + 58*
sqrt(-2*sqrt(2) + 3))
```

GIAC/XCAS [A] time = 0.227674, size = 65, normalized size = 1.81

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-2\cos\left(2x\right)+2}\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)^2/(sin(x)^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2
)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2))) + x
```

3.145 $\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi [A] time = 0.0421322, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi in Sympy [A] time = 25.1825, size = 10, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] atan(a*tan(x)/b)/(a*b)
```

Mathematica [A] time = 0.0533469, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] time = 0.083, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{a\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
```

[Out] arctan(a*tan(x)/b)/a/b

Maxima [A] time = 1.50216, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [A] time = 0.238988, size = 58, normalized size = 3.87

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))
/(a*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.234131, size = 35, normalized size = 2.33

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rubi [A] time = 0.0217313, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*Cos[x] + a*Sin[x])^(-2),x]

[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rubi in Sympy [A] time = 0.675075, size = 14, normalized size = 0.82

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*cos(x)+a*sin(x))**2,x)

[Out] sin(x)/(b*(a*sin(x) + b*cos(x)))
```

Mathematica [A] time = 0.0377791, size = 17, normalized size = 1.

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]

[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Maple [A] time = 0.234, size = 14, normalized size = 0.8

$$-\frac{1}{a(a\tan(x)+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*cos(x)+a*sin(x))^2,x)

[Out] -1/a/(a*tan(x)+b)
```

Maxima [A] time = 1.34446, size = 19, normalized size = 1.12

$$-\frac{1}{a^2\tan(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*cos(x) + a*sin(x))^(-2),x, algorithm="maxima")
```

```
[Out] -1/(a^2*tan(x) + a*b)
```

Fricas [A] time = 0.218437, size = 53, normalized size = 3.12

$$-\frac{a\cos(x) - b\sin(x)}{(a^2b + b^3)\cos(x) + (a^3 + ab^2)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*cos(x) + a*sin(x))^(-2),x, algorithm="fricas")
```

```
[Out] -(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(
x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*cos(x)+a*sin(x))**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.216294, size = 18, normalized size = 1.06

$$-\frac{1}{(a\tan(x) + b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*cos(x) + a*sin(x))^(-2),x, algorithm="giac")
```

```
[Out] -1/((a*tan(x) + b)*a)
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rubi [A] time = 0.0454923, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In]  Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rubi in Sympy [A] time = 2.11813, size = 24, normalized size = 0.8

$$\frac{x}{2}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{2}-\frac{\log\left(\sin\left(x\right)+\cos\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(sin(x)/(1+cos(x)+sin(x)),x)

[Out] x/2 - log(tan(x/2) + 1)/2 - log(sin(x) + cos(x) + 1)/2
```

Mathematica [A] time = 0.0109498, size = 22, normalized size = 0.73

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.066, size = 25, normalized size = 0.8

$$\frac{1}{2}\ln\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(sin(x)/(1+cos(x)+sin(x)),x)

[Out] 1/2*ln(tan(1/2*x)^2+1)-ln(1+tan(1/2*x))+1/2*x
```

Maxima [A] time = 1.48879, size = 55, normalized size = 1.83

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x) + 1),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*
log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.230849, size = 15, normalized size = 0.5

$$\frac{1}{2}x - \frac{1}{2}\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x) + 1),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A] time = 0.419479, size = 22, normalized size = 0.73

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right)+1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

GIAC/XCAS [A] time = 0.225072, size = 34, normalized size = 1.13

$$\frac{1}{2}x + \frac{1}{2}\ln\left(\tan\left(\frac{1}{2}x\right)^2+1\right) - \ln\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(x)/(cos(x) + sin(x) + 1),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*ln(tan(1/2*x)^2 + 1) - ln(abs(tan(1/2*x) + 1))
```

3.148 $\int \sqrt{3-x^2}\, dx$

Optimal. Leaf size=29

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rubi [A] time = 0.0114883, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rubi in Sympy [A] time = 0.597389, size = 24, normalized size = 0.83

$$\frac{x\sqrt{-x^2+3}}{2}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 3)/2 + 3*asin(sqrt(3)*x/3)/2
```

Mathematica [A] time = 0.013566, size = 29, normalized size = 1.

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Maple [A] time = 0.005, size = 23, normalized size = 0.8

$$\frac{3}{2}\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{2}\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+3)^(1/2),x)
```

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Maxima [A] time = 1.51561, size = 30, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+3}x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Fricas [A] time = 0.211993, size = 39, normalized size = 1.34

$$\frac{1}{2}\sqrt{-x^2+3}x-\frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

Sympy [A] time = 0.227111, size = 24, normalized size = 0.83

$$\frac{x\sqrt{-x^2+3}}{2}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(-x**2 + 3)/2 + 3*asin(sqrt(3)*x/3)/2
```

GIAC/XCAS [A] time = 0.216836, size = 30, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+3}x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

3.149 $\int \frac{x}{\sqrt{3-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{3-x^2}$$

```
[Out] -Sqrt[3 - x^2]
```

Rubi [A] time = 0.00621599, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]
```

Rubi in Sympy [A] time = 0.87586, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2+3)**(1/2),x)

[Out] -sqrt(-x**2 + 3)
```

Mathematica [A] time = 0.00259858, size = 13, normalized size = 1.

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2+3)^(1/2),x)

[Out] -(-x^2+3)^(1/2)
```

Maxima [A] time = 1.33796, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 3),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 3)
```

Fricas [A] time = 0.207814, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 3),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 3)
```

Sympy [A] time = 0.151469, size = 8, normalized size = 0.62

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 3)
```

GIAC/XCAS [A] time = 0.214832, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-x^2 + 3),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x}\, dx$

Optimal. Leaf size=37

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rubi [A] time = 0.0473773, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2]/x,x]

[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rubi in Sympy [A] time = 2.37152, size = 29, normalized size = 0.78

$$\sqrt{-x^2+3}-\sqrt{3}\,\text{atanh}\left(\frac{\sqrt{3}\sqrt{-x^2+3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-x**2+3)**(1/2)/x,x)

[Out] sqrt(-x**2 + 3) - sqrt(3)*atanh(sqrt(3)*sqrt(-x**2 + 3)/3)
```

Mathematica [A] time = 0.0214785, size = 41, normalized size = 1.11

$$\sqrt{3-x^2}-\sqrt{3}\log\left(\sqrt{9-3x^2}+3\right)+\sqrt{3}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2]/x,x]

[Out] Sqrt[3 - x^2] + Sqrt[3]*Log[x] - Sqrt[3]*Log[3 + Sqrt[9 - 3*x^2]]
```

Maple [A] time = 0.006, size = 30, normalized size = 0.8

$$\sqrt{-x^2+3}-\sqrt{3}Artanh\left(\sqrt{3}\frac{1}{\sqrt{-x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2)/x,x)
```

```
[Out] (-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

Maxima [A] time = 1.51444, size = 55, normalized size = 1.49

$$-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|}+\frac{6}{|x|}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-
x^2 + 3)
```

Fricas [A] time = 0.216663, size = 54, normalized size = 1.46

$$\frac{1}{2}\sqrt{3}\log\left(-\frac{x^2+2\sqrt{3}\sqrt{-x^2+3}-6}{x^2}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt
(-x^2 + 3)
```

Sympy [A] time = 2.22713, size = 88, normalized size = 2.38

$$\begin{cases} i\sqrt{x^2-3}-\sqrt{3}\log(x)+\frac{\sqrt{3}\log(x^2)}{2}+\sqrt{3}i\operatorname{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for}\ \frac{|x^2|}{3}>1 \\ \sqrt{-x^2+3}+\frac{\sqrt{3}\log(x^2)}{2}-\sqrt{3}\log\left(\sqrt{-\frac{x^2}{3}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+3)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/
2 + sqrt(3)*I*asin(sqrt(3)/x), Abs(x**2)/3 > 1), (sqrt(-x**2 + 3)
 + sqrt(3)*log(x**2)/2 - sqrt(3)*log(sqrt(-x**2/3 + 1) + 1), True
))
```

GIAC/XCAS [A] time = 0.226267, size = 63, normalized size = 1.7

$$\frac{1}{2}\sqrt{3}\ln\left(\frac{\sqrt{3}-\sqrt{-x^2+3}}{\sqrt{3}+\sqrt{-x^2+3}}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 3)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*ln((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 +
3))) + sqrt(-x^2 + 3)
```

3.151 $\int \frac{\sqrt{x+x^2}}{x} \, dx$

Optimal. Leaf size=22

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rubi [A] time = 0.019759, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x + x^2]/x,x]

[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rubi in Sympy [A] time = 1.3261, size = 19, normalized size = 0.86

$$\sqrt{x^2+x}+\operatorname{atanh}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+x)**(1/2)/x,x)

[Out] sqrt(x**2 + x) + atanh(x/sqrt(x**2 + x))
```

Mathematica [A] time = 0.0247859, size = 31, normalized size = 1.41

$$\sqrt{x(x+1)}\left(\frac{\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x + x^2]/x,x]

[Out] Sqrt[x*(1 + x)]*(1 + ArcSinh[Sqrt[x]]/(Sqrt[x]*Sqrt[1 + x]))
```

Maple [A] time = 0.006, size = 22, normalized size = 1.

$$\sqrt{x^2+x}+\frac{1}{2}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+x)^(1/2)/x,x)
```

```
[Out] (x^2+x)^(1/2)+1/2*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 1.33722, size = 34, normalized size = 1.55

$$\sqrt{x^2+x}+\frac{1}{2}\log\left(2x+2\sqrt{x^2+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x)/x,x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A] time = 0.21015, size = 99, normalized size = 4.5

$$-\frac{8x^2+2\left(2x-2\sqrt{x^2+x}+1\right)\log\left(-2x+2\sqrt{x^2+x}-1\right)-2\sqrt{x^2+x}(4x+1)+6x-1}{4\left(2x-2\sqrt{x^2+x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^2 + 2*(2*x - 2*sqrt(x^2 + x) + 1)*log(-2*x + 2*sqrt(x^2
 + x) - 1) - 2*sqrt(x^2 + x)*(4*x + 1) + 6*x - 1)/(2*x - 2*sqrt(x
^2 + x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(x + 1))/x, x)
```

GIAC/XCAS [A] time = 0.228088, size = 35, normalized size = 1.59

$$\sqrt{x^2+x}-\frac{1}{2}\ln\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + x)/x,x, algorithm="giac")
```

```
[Out] sqrt(x^2 + x) - 1/2*ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.152 $\int \sqrt{5+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rubi [A] time = 0.0100395, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[5 + x^2],x]

[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rubi in Sympy [A] time = 0.557144, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+5}}{2}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+5)**(1/2),x)

[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

Mathematica [A] time = 0.0129843, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[5 + x^2],x]

[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$\frac{5}{2}Arcsinh\left(\frac{x\sqrt{5}}{5}\right)+\frac{x}{2}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+5)^(1/2),x)
```

```
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Maxima [A] time = 1.50438, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\,\mathrm{arsinh}\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 5),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

Fricas [A] time = 0.209638, size = 109, normalized size = 4.04

$$-\frac{2\,x^4+10\,x^2+5\left(2\,x^2-2\,\sqrt{x^2+5}x+5\right)\log\left(-x+\sqrt{x^2+5}\right)-\left(2\,x^3+5\,x\right)\sqrt{x^2+5}}{2\left(2\,x^2-2\,\sqrt{x^2+5}x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 5),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 10*x^2 + 5*(2*x^2 - 2*sqrt(x^2 + 5)*x + 5)*log(-x +
 sqrt(x^2 + 5)) - (2*x^3 + 5*x)*sqrt(x^2 + 5))/(2*x^2 - 2*sqrt(x^
2 + 5)*x + 5)
```

Sympy [A] time = 0.271315, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+5}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+5)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

GIAC/XCAS [A] time = 0.224888, size = 34, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^2+5}x-\frac{5}{2}\ln\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 5),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*ln(-x + sqrt(x^2 + 5))
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=27

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rubi [A] time = 0.0280356, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[1 + x + x^2],x]

[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rubi in Sympy [A] time = 1.50343, size = 29, normalized size = 1.07

$$\sqrt{x^2+x+1}-\frac{\operatorname{atanh}\left(\frac{2x+1}{2\sqrt{x^2+x+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**2+x+1)**(1/2),x)

[Out] sqrt(x**2 + x + 1) - atanh((2*x + 1)/(2*sqrt(x**2 + x + 1)))/2
```

Mathematica [A] time = 0.013231, size = 27, normalized size = 1.

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[1 + x + x^2],x]

[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Maple [A] time = 0.008, size = 21, normalized size = 0.8

$$\sqrt{x^2+x+1}-\frac{1}{2}Arcsinh\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^2+x+1)^(1/2),x)
```

```
[Out] (x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.49646, size = 30, normalized size = 1.11

$$\sqrt{x^2+x+1}-\frac{1}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + x + 1),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 0.216753, size = 104, normalized size = 3.85

$$-\frac{8x^2-2\left(2x-2\sqrt{x^2+x+1}+1\right)\log\left(-2x+2\sqrt{x^2+x+1}-1\right)-2\sqrt{x^2+x+1}(4x+1)+6x+7}{4\left(2x-2\sqrt{x^2+x+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + x + 1),x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^2 - 2*(2*x - 2*sqrt(x^2 + x + 1) + 1)*log(-2*x + 2*sqrt
(x^2 + x + 1) - 1) - 2*sqrt(x^2 + x + 1)*(4*x + 1) + 6*x + 7)/(2*
x - 2*sqrt(x^2 + x + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + x + 1), x)
```

GIAC/XCAS [A] time = 0.222296, size = 36, normalized size = 1.33

$$\sqrt{x^2+x+1}+\frac{1}{2}\ln\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(x^2 + x + 1),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + x + 1) + 1/2*ln(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.154 $\int \frac{1}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=14

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rubi [A] time = 0.00838707, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rubi in Sympy [A] time = 0.553924, size = 12, normalized size = 0.86

$$2\,\mathrm{atanh}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+x)**(1/2),x)
```

```
[Out] 2*atanh(x/sqrt(x**2 + x))
```

Mathematica [B] time = 0.010579, size = 29, normalized size = 2.07

$$\frac{2\sqrt{x}\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x*(1 + x)]
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+x)^(1/2),x)
```

```
[Out] ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 1.34523, size = 20, normalized size = 1.43

$$\log\left(2x + 2\sqrt{x^2 + x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A] time = 0.197849, size = 23, normalized size = 1.64

$$-\log\left(-2x + 2\sqrt{x^2 + x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x),x, algorithm="fricas")
```

```
[Out] -log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + x), x)
```

GIAC/XCAS [A] time = 0.215665, size = 24, normalized size = 1.71

$$-\ln\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + x),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2} dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(
2*Sqrt[2]*Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rubi [A] time = 0.100346, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(
2*Sqrt[2]*Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 5.7675, size = 61, normalized size = 0.9

$$-\operatorname{atan}\left(-\frac{-2x-1}{2\sqrt{-x^2-x+2}}\right)+\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}(-x+4)}{4\sqrt{-x^2-x+2}}\right)}{4}-\frac{\sqrt{-x^2-x+2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] -atan(-(-2*x - 1)/(2*sqrt(-x**2 - x + 2))) + sqrt(2)*atanh(sqrt(2
)*(-x + 4)/(4*sqrt(-x**2 - x + 2)))/4 - sqrt(-x**2 - x + 2)/x
```

Mathematica [A] time = 0.0649716, size = 77, normalized size = 1.13

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\log\left(2\sqrt{2}\sqrt{-x^2-x+2}-x+4\right)}{2\sqrt{2}}-\frac{\log(x)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] - Log[x]/(2*Sqrt[2]
) + Log[4 - x + 2*Sqrt[2]*Sqrt[2 - x - x^2]]/(2*Sqrt[2])
```

Maple [A] time = 0.007, size = 88, normalized size = 1.3

$$-\frac{1}{2x}\left(-x^2-x+2\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{-x^2-x+2}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)$$
$$+\frac{\sqrt{2}}{4}\mathit{Artanh}\left(\frac{(4-x)\sqrt{2}}{4}\frac{1}{\sqrt{-x^2-x+2}}\right)+\frac{-1-2x}{4}\sqrt{-x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2-x+2)^(1/2)/x^2,x)
```

```
[Out] -1/2/x*(-x^2-x+2)^(3/2)-1/4*(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/
4*arctanh(1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)+1/4*(-1-2*x
)*(-x^2-x+2)^(1/2)
```

Maxima [A] time = 1.59728, size = 80, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right)-\frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - x + 2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) -
1) - sqrt(-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Fricas [A] time = 0.209001, size = 126, normalized size = 1.85

$$-\frac{\sqrt{2}\left(4\sqrt{2}x\arctan\left(\frac{2x+1}{2\sqrt{-x^2-x+2}}\right)-x\log\left(-\frac{\sqrt{2}(7x^2+16x-32)+8\sqrt{-x^2-x+2}(x-4)}{x^2}\right)+4\sqrt{2}\sqrt{-x^2-x+2}\right)}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - x + 2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(4*sqrt(2)*x*arctan(1/2*(2*x + 1)/sqrt(-x^2 - x + 2)
) - x*log(-(sqrt(2)*(7*x^2 + 16*x - 32) + 8*sqrt(-x^2 - x + 2)*(x
 - 4))/x^2) + 4*sqrt(2)*sqrt(-x^2 - x + 2))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

GIAC/XCAS [A] time = 0.233224, size = 227, normalized size = 3.34

$$-\frac{1}{4}\sqrt{2}\ln\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right)+\frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1}-\arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 - x + 2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*ln(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*
x + 1) + 6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1
) + 6)) + 6*(3*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sq
rt(-x^2 - x + 2) - 3)/(2*x + 1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2
*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)
```

3.156 $\int \frac{\log(t)}{1+t} \, dt$

Optimal. Leaf size=13

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rubi [A] time = 0.0253977, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[t]/(1 + t),t]

[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rubi in Sympy [A] time = 1.87986, size = 12, normalized size = 0.92

$$\log(t)\log(t+1)+\text{Li}_2(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(t)/(1+t),t)

[Out] log(t)*log(t + 1) + polylog(2, -t)
```

Mathematica [A] time = 0.00516197, size = 13, normalized size = 1.

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[t]/(1 + t),t]

[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Maple [C] time = 0.016, size = 13, normalized size = 1.

$$dilog(1+t)+\ln(t)\ln(1+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(t)/(1+t),t)

[Out] dilog(1+t)+ln(t)*ln(1+t)
```

Maxima [A] time = 1.34458, size = 16, normalized size = 1.23

$$\log(t+1)\log(t)+\text{Li}_2(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)/(t + 1),t, algorithm="maxima")
```

```
[Out] log(t + 1)*log(t) + dilog(-t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(t\right)}{t+1}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)/(t + 1),t, algorithm="fricas")
```

```
[Out] integral(log(t)/(t + 1), t)
```

Sympy [A] time = 2.17734, size = 58, normalized size = 4.46

$$\begin{cases} i\pi\log\left(t+1\right) - \mathrm{Li}_{2}\left(t+1\right) & \text{for}\ \left|t+1\right| < 1 \\ -i\pi\log\left(\frac{1}{t+1}\right) - \mathrm{Li}_{2}\left(t+1\right) & \text{for}\ \left|\frac{1}{t+1}\right| < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1, 1 \\ 0, 0 & \end{matrix} \middle| t+1\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| t+1\right) - \mathrm{Li}_{2}\left(t+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(t)/(1+t),t)
```

```
[Out] Piecewise((I*pi*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1),
(-I*pi*log(1/(t + 1)) - polylog(2, t + 1), Abs(1/(t + 1)) < 1), (
-I*pi*meijerg(((), (1, 1)), ((0, 0), ()), t + 1) + I*pi*meijerg((
(1, 1), ()), ((), (0, 0)), t + 1) - polylog(2, t + 1), True))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(t\right)}{t+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)/(t + 1),t, algorithm="giac")
```

```
[Out] integrate(log(t)/(t + 1), t)
```

3.157 $\int \log\left(e^{\cos(x)}\right)\, dx$

Optimal. Leaf size=15

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

```
[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rubi [A] time = 0.0184765, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[E^Cos[x]],x]

[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rubi in Sympy [A] time = 1.06558, size = 15, normalized size = 1.

$$x\log\left(e^{\cos(x)}\right) - x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(exp(cos(x))),x)

[Out] x*log(exp(cos(x))) - x*cos(x) + sin(x)
```

Mathematica [A] time = 0.0164609, size = 15, normalized size = 1.

$$\sin(x) + x\left(\log\left(e^{\cos(x)}\right) - \cos(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[E^Cos[x]],x]

[Out] x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]
```

Maple [A] time = 0.016, size = 15, normalized size = 1.

$$-x\cos(x) + x\ln\left(\mathrm{e}^{\cos(x)}\right) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(exp(cos(x))),x)

[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Maxima [A] time = 1.34898, size = 3, normalized size = 0.2

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(e^cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 0.211671, size = 3, normalized size = 0.2

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(e^cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.219599, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(exp(cos(x))),x)
```

```
[Out] sin(x)
```

GIAC/XCAS [A] time = 0.211012, size = 3, normalized size = 0.2

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(e^cos(x)),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.158 $\int \frac{e^t}{t} \, dt$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(t)$$

```
[Out] ExpIntegralEi[t]
```

Rubi [A] time = 0.0169777, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In]  Int[E^t/t,t]

[Out] ExpIntegralEi[t]
```

Rubi in Sympy [A] time = 1.27845, size = 2, normalized size = 1.

$$\text{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)/t,t)

[Out] Ei(t)
```

Mathematica [A] time = 0.00191894, size = 2, normalized size = 1.

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t/t,t]

[Out] ExpIntegralEi[t]
```

Maple [B] time = 0.004, size = 8, normalized size = 4.

$$-Ei\,(1, -t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)/t,t)

[Out] -Ei(1,-t)
```

Maxima [A] time = 1.41408, size = 3, normalized size = 1.5

$$\text{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t,t, algorithm="maxima")
```

```
[Out] Ei(t)
```

Fricas [A] time = 0.199619, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t,t, algorithm="fricas")
```

```
[Out] Ei(t)
```

Sympy [A] time = 1.24061, size = 2, normalized size = 1.

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)/t,t)
```

```
[Out] Ei(t)
```

GIAC/XCAS [A] time = 0.228238, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t,t, algorithm="giac")
```

```
[Out] Ei(t)
```

3.159 $\int \frac{e^{at}}{t}\,dt$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(at)$$

```
[Out] ExpIntegralEi[a*t]
```

Rubi [A] time = 0.0208139, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*t)/t,t]
```

```
[Out] ExpIntegralEi[a*t]
```

Rubi in Sympy [A] time = 1.36631, size = 3, normalized size = 0.75

$$\text{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(exp(a*t)/t,t)
```

```
[Out] Ei(a*t)
```

Mathematica [A] time = 0.00196054, size = 4, normalized size = 1.

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*t)/t,t]
```

```
[Out] ExpIntegralEi[a*t]
```

Maple [A] time = 0.004, size = 9, normalized size = 2.3

$$-Ei\,(1,-at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t)
```

```
[Out] -Ei(1,-a*t)
```

Maxima [A] time = 1.41595, size = 5, normalized size = 1.25

$$\text{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*t)/t,t, algorithm="maxima")
```

```
[Out] Ei(a*t)
```

Fricas [A] time = 0.198334, size = 5, normalized size = 1.25

$$\operatorname{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*t)/t,t, algorithm="fricas")
```

```
[Out] Ei(a*t)
```

Sympy [A] time = 1.34288, size = 3, normalized size = 0.75

$$\operatorname{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(a*t)/t,t)
```

```
[Out] Ei(a*t)
```

GIAC/XCAS [A] time = 0.210192, size = 5, normalized size = 1.25

$$\operatorname{Ei}(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(a*t)/t,t, algorithm="giac")
```

```
[Out] Ei(a*t)
```

3.160 $\int \frac{e^t}{t^2} \, dt$

Optimal. Leaf size=11

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rubi [A] time = 0.0333979, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In]  Int[E^t/t^2,t]

[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rubi in Sympy [A] time = 1.89631, size = 7, normalized size = 0.64

$$\text{Ei}(t) - \frac{e^t}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)/t**2,t)

[Out] Ei(t) - exp(t)/t
```

Mathematica [A] time = 0.0048487, size = 11, normalized size = 1.

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t/t^2,t]

[Out] -(E^t/t) + ExpIntegralEi[t]
```

Maple [A] time = 0.004, size = 16, normalized size = 1.5

$$-\frac{e^t}{t} - Ei(1, -t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)/t^2,t)

[Out] -exp(t)/t-Ei(1,-t)
```

Maxima [A] time = 1.42966, size = 7, normalized size = 0.64

$$(-1, -t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t^2,t, algorithm="maxima")

[Out] gamma(-1, -t)
```

Fricas [A] time = 0.197507, size = 18, normalized size = 1.64

$$\frac{t\text{Ei}\left(t\right) - e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t^2,t, algorithm="fricas")

[Out] (t*Ei(t) - e^t)/t
```

Sympy [A] time = 1.60351, size = 7, normalized size = 0.64

$$\text{Ei}\left(t\right) - \frac{e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)/t**2,t)

[Out] Ei(t) - exp(t)/t
```

GIAC/XCAS [A] time = 0.226251, size = 18, normalized size = 1.64

$$\frac{t\text{Ei}\left(t\right) - e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/t^2,t, algorithm="giac")

[Out] (t*Ei(t) - e^t)/t
```

3.161 $\int e^{\frac{1}{t}}\,dt$

Optimal. Leaf size=14

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rubi [A] time = 0.0225486, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rubi in Sympy [A] time = 1.36337, size = 10, normalized size = 0.71

$$te^{\frac{1}{t}} - \text{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(1/t),t)
```

```
[Out] t*exp(1/t) - Ei(1/t)
```

Mathematica [A] time = 0.00308048, size = 14, normalized size = 1.

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Maple [A] time = 0.004, size = 15, normalized size = 1.1

$$e^{t^{-1}}t + Ei\left(1, -t^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(1/t),t)
```

```
[Out] exp(1/t)*t+Ei(1,-1/t)
```

Maxima [A] time = 1.40561, size = 18, normalized size = 1.29

$$te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/t),t, algorithm="maxima")

[Out] t*e^(1/t) - Ei(1/t)
```

Fricas [A] time = 0.202327, size = 18, normalized size = 1.29

$$te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/t),t, algorithm="fricas")

[Out] t*e^(1/t) - Ei(1/t)
```

Sympy [A] time = 1.70326, size = 10, normalized size = 0.71

$$te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(1/t),t)

[Out] t*exp(1/t) - Ei(1/t)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(1/t),t, algorithm="giac")

[Out] undef
```

3.162 $\int \frac{e^{-t}}{-1-a+t}\,dt$

Optimal. Leaf size=15

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rubi [A] time = 0.0338097, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(E^t*(-1 - a + t)),t]

[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rubi in Sympy [A] time = 2.02819, size = 12, normalized size = 0.8

$$e^{-a-1}\,\text{Ei}\,(a-t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/exp(t)/(-1-a+t),t)

[Out] exp(-a - 1)*Ei(a - t + 1)
```

Mathematica [A] time = 0.00724314, size = 15, normalized size = 1.

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(E^t*(-1 - a + t)),t]

[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Maple [A] time = 0.023, size = 17, normalized size = 1.1

$$-e^{-1-a}Ei\,(1,-1-a+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/exp(t)/(-1-a+t),t)

[Out] -exp(-1-a)*Ei(1,-1-a+t)
```

Maxima [A] time = 1.44247, size = 22, normalized size = 1.47

$$-e^{(-a-1)}exp_integral_e\,(1,-a+t-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-t)/(a - t + 1),t, algorithm="maxima")
```

```
[Out] -e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

Fricas [A] time = 0.199377, size = 19, normalized size = 1.27

$$\mathrm{Ei}\left(a - t + 1\right) e^{\left(-a-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-t)/(a - t + 1),t, algorithm="fricas")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-t}}{-a + t - 1}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/exp(t)/(-1-a+t),t)
```

```
[Out] Integral(exp(-t)/(-a + t - 1), t)
```

GIAC/XCAS [A] time = 0.232198, size = 19, normalized size = 1.27

$$\mathrm{Ei}\left(a - t + 1\right) e^{\left(-a-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-e^(-t)/(a - t + 1),t, algorithm="giac")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

3.163 $\int \frac{e^{t^2} t}{1+t^2} \, dt$

Optimal. Leaf size=13

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rubi [A] time = 0.135018, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Int[(E^t^2*t)/(1 + t^2),t]

[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rubi in Sympy [A] time = 5.99774, size = 8, normalized size = 0.62

$$\frac{\text{Ei}\left(t^2+1\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t**2)*t/(t**2+1),t)

[Out] exp(-1)*Ei(t**2 + 1)/2
```

Mathematica [A] time = 0.00575937, size = 13, normalized size = 1.

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(E^t^2*t)/(1 + t^2),t]

[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Maple [A] time = 0.007, size = 14, normalized size = 1.1

$$-\frac{\mathrm{e}^{-1} Ei\left(1,-t^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t^2)*t/(t^2+1),t)

[Out] -1/2*exp(-1)*Ei(1,-t^2-1)
```

Maxima [A] time = 1.40266, size = 18, normalized size = 1.38

$$-\frac{1}{2}\,e^{(-1)}exp_integral_e\left(1,-t^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(t^2)/(t^2 + 1),t, algorithm="maxima")
```

```
[Out] -1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

Fricas [A] time = 0.19801, size = 14, normalized size = 1.08

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(t^2)/(t^2 + 1),t, algorithm="fricas")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{te^{t^2}}{t^2+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t**2)*t/(t**2+1),t)
```

```
[Out] Integral(t*exp(t**2)/(t**2 + 1), t)
```

GIAC/XCAS [A] time = 0.225349, size = 14, normalized size = 1.08

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(t^2)/(t^2 + 1),t, algorithm="giac")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

3.164 $\int \frac{e^t}{(1+t)^2} \, dt$

Optimal. Leaf size=19

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rubi [A] time = 0.0398312, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In]  Int[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rubi in Sympy [A] time = 2.50627, size = 12, normalized size = 0.63

$$\frac{\text{Ei}\,(t+1)}{e} - \frac{e^t}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)/(1+t)**2,t)
```

```
[Out] exp(-1)*Ei(t + 1) - exp(t)/(t + 1)
```

Mathematica [A] time = 0.011402, size = 19, normalized size = 1.

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Maple [A] time = 0.008, size = 22, normalized size = 1.2

$$-\frac{e^t}{1+t} - \mathrm{e}^{-1} Ei\,(1,-1-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)/(1+t)^2,t)
```

```
[Out] -exp(t)/(1+t)-exp(-1)*Ei(1,-1-t)
```

Maxima [A] time = 1.43915, size = 22, normalized size = 1.16

$$-\frac{e^{(-1)}exp_integral_e\left(2,-t-1\right)}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/(t + 1)^2,t, algorithm="maxima")
```

```
[Out] -e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

Fricas [A] time = 0.20097, size = 31, normalized size = 1.63

$$\frac{\left((t+1)\mathrm{Ei}\left(t+1\right)-e^{(t+1)}\right)e^{(-1)}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/(t + 1)^2,t, algorithm="fricas")
```

```
[Out] ((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)/(1+t)**2,t)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$undef$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t/(t + 1)^2,t, algorithm="giac")
```

```
[Out] undef
```

3.165 $\int e^t \log(1+t)\, dt$

Optimal. Leaf size=18

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rubi [A] time = 0.0381986, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In]  Int[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rubi in Sympy [A] time = 2.45597, size = 14, normalized size = 0.78

$$e^t \log(t+1) - \frac{\text{Ei}(t+1)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(t)*ln(1+t),t)
```

```
[Out] exp(t)*log(t + 1) - exp(-1)*Ei(t + 1)
```

Mathematica [A] time = 0.00877777, size = 18, normalized size = 1.

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Maple [A] time = 0.175, size = 19, normalized size = 1.1

$$\mathrm{e}^t \ln(1+t) + \mathrm{e}^{-1} Ei(1, -1-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(t)*ln(1+t),t)
```

```
[Out] exp(t)*ln(1+t)+exp(-1)*Ei(1,-1-t)
```

Maxima [A] time = 1.41295, size = 24, normalized size = 1.33

$$e^{(-1)} exp_i ntegral_e (1, -t - 1) + e^t \log(t + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t*log(t + 1),t, algorithm="maxima")
```

```
[Out] e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

Fricas [A] time = 0.210764, size = 26, normalized size = 1.44

$$\left(e^{(t+1)} \log(t + 1) - \mathrm{Ei}(t + 1)\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t*log(t + 1),t, algorithm="fricas")
```

```
[Out] (e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^t \log(t + 1)\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(t)*ln(1+t),t)
```

```
[Out] Integral(exp(t)*log(t + 1), t)
```

GIAC/XCAS [A] time = 0.211015, size = 22, normalized size = 1.22

$$-\mathrm{Ei}(t + 1) e^{(-1)} + e^t \ln(t + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^t*log(t + 1),t, algorithm="giac")
```

```
[Out] -Ei(t + 1)*e^(-1) + e^t*ln(t + 1)
```

3.166 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t}t - e^{-t}$$

```
[Out] -E^(-t) - t/E^t
```

Rubi [A] time = 0.0162737, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-e^{-t}t - e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]

[Out] -E^(-t) - t/E^t
```

Rubi in Sympy [A] time = 1.10154, size = 10, normalized size = 0.62

$$-te^{-t} - e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(t/exp(t),t)

[Out] -t*exp(-t) - exp(-t)
```

Mathematica [A] time = 0.00213749, size = 11, normalized size = 0.69

$$e^{-t}(-t - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]

[Out] (-1 - t)/E^t
```

Maple [A] time = 0.001, size = 10, normalized size = 0.6

$$-\frac{1 + t}{e^t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t)

[Out] -(1+t)/exp(t)
```

Maxima [A] time = 1.35144, size = 12, normalized size = 0.75

$$-(t + 1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="maxima")
```

```
[Out] -(t + 1)*e^(-t)
```

Fricas [A] time = 0.209306, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

Sympy [A] time = 0.090321, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

GIAC/XCAS [A] time = 0.210625, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t*e^(-t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

3.167 $\int e^{-t}t^2\,dt$

Optimal. Leaf size=26

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

```
[Out] -2/E^t - (2*t)/E^t - t^2/E^t
```

Rubi [A] time = 0.0349204, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

Antiderivative was successfully verified.

```
[In]  Int[t^2/E^t,t]

[Out] -2/E^t - (2*t)/E^t - t^2/E^t
```

Rubi in Sympy [A] time = 1.75476, size = 19, normalized size = 0.73

$$-t^2e^{-t}-2te^{-t}-2e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**2/exp(t),t)

[Out] -t**2*exp(-t) - 2*t*exp(-t) - 2*exp(-t)
```

Mathematica [A] time = 0.0034347, size = 16, normalized size = 0.62

$$e^{-t}\left(-t^2-2t-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t^2/E^t,t]

[Out] (-2 - 2*t - t^2)/E^t
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$-{\frac{t^2+2\,t+2}{{{\rm e}^{t}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^2/exp(t),t)

[Out] -(t^2+2*t+2)/exp(t)
```

Maxima [A] time = 1.34743, size = 19, normalized size = 0.73

$$-\left(t^2 + 2\,t + 2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*e^(-t),t, algorithm="maxima")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Fricas [A] time = 0.212409, size = 19, normalized size = 0.73

$$-\left(t^2 + 2\,t + 2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*e^(-t),t, algorithm="fricas")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Sympy [A] time = 0.088941, size = 12, normalized size = 0.46

$$\left(-t^2 - 2t - 2\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**2/exp(t),t)
```

```
[Out] (-t**2 - 2*t - 2)*exp(-t)
```

GIAC/XCAS [A] time = 0.215646, size = 19, normalized size = 0.73

$$-\left(t^2 + 2\,t + 2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^2*e^(-t),t, algorithm="giac")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

3.168 $\int e^{-t}t^3\,dt$

Optimal. Leaf size=36

$$-e^{-t}t^3-3e^{-t}t^2-6e^{-t}t-6e^{-t}$$

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rubi [A] time = 0.0537962, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-e^{-t}t^3-3e^{-t}t^2-6e^{-t}t-6e^{-t}$$

Antiderivative was successfully verified.

```
[In]  Int[t^3/E^t,t]

[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rubi in Sympy [A] time = 2.57745, size = 27, normalized size = 0.75

$$-t^3e^{-t}-3t^2e^{-t}-6te^{-t}-6e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(t**3/exp(t),t)

[Out] -t**3*exp(-t) - 3*t**2*exp(-t) - 6*t*exp(-t) - 6*exp(-t)
```

Mathematica [A] time = 0.00420394, size = 21, normalized size = 0.58

$$e^{-t}\left(-t^3-3t^2-6t-6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[t^3/E^t,t]

[Out] (-6 - 6*t - 3*t^2 - t^3)/E^t
```

Maple [A] time = 0.004, size = 20, normalized size = 0.6

$$-\frac{t^3+3\,t^2+6\,t+6}{e^t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(t^3/exp(t),t)

[Out] -(t^3+3*t^2+6*t+6)/exp(t)
```

Maxima [A] time = 1.3571, size = 26, normalized size = 0.72

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-t),t, algorithm="maxima")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Fricas [A] time = 0.198283, size = 26, normalized size = 0.72

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-t),t, algorithm="fricas")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Sympy [A] time = 0.125162, size = 17, normalized size = 0.47

$$\left(-t^{3}-3t^{2}-6t-6\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t**3/exp(t),t)
```

```
[Out] (-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

GIAC/XCAS [A] time = 0.227735, size = 26, normalized size = 0.72

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(t^3*e^(-t),t, algorithm="giac")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x(aa1+bb1)}{a^2+b^2}-\frac{(a1b-ab1)\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*S
in[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.0691131, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{x(aa1+bb1)}{a^2+b^2}-\frac{(a1b-ab1)\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*S
in[x]])/(a^2 + b^2)
```

Rubi in Sympy [A] time = 4.60883, size = 39, normalized size = 0.81

$$\frac{x\left(aa_1+bb_1\right)}{a^2+b^2}+\frac{\left(ab_1-a_1b\right)\log\left(a\sin\left(x\right)+b\cos\left(x\right)\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] x*(a*a1 + b*b1)/(a**2 + b**2) + (a*b1 - a1*b)*log(a*sin(x) + b*co
s(x))/(a**2 + b**2)
```

Mathematica [A] time = 0.111168, size = 39, normalized size = 0.81

$$\frac{x(aa1+bb1)+(ab1-a1b)\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^
2 + b^2)
```

Maple [B] time = 0.119, size = 111, normalized size = 2.3

$$-\frac{\ln\left(1+(\tan(x))^2\right)ab1}{2a^2+2b^2}+\frac{\ln\left(1+(\tan(x))^2\right)a1b}{2a^2+2b^2}+\frac{\arctan(\tan(x))aa1}{a^2+b^2}+\frac{\arctan(\tan(x))bb1}{a^2+b^2}+\frac{\ln(a\tan(x)+b)ab1}{a^2+b^2}-\frac{\ln(a\tan(x)+b)a1b}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] -1/2/(a^2+b^2)*ln(1+tan(x)^2)*a*b1+1/2/(a^2+b^2)*ln(1+tan(x)^2)*a
1*b+1/(a^2+b^2)*arctan(tan(x))*a*a1+1/(a^2+b^2)*arctan(tan(x))*b*
b1+1/(a^2+b^2)*ln(a*tan(x)+b)*a*b1-1/(a^2+b^2)*ln(a*tan(x)+b)*a1*
b
```

Maxima [A] time = 1.5294, size = 244, normalized size = 5.08

$$a_1\left(\frac{2\,a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}-\frac{b\log\left(-b-\frac{2\,a\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$
$$+\,b_1\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}+\frac{a\log\left(-b-\frac{2\,a\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}-\frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x, algorithm="maxima")
```

```
[Out] a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*
sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*
log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(si
n(x)/(cos(x) + 1))/(a^2 + b^2) + a*log(-b - 2*a*sin(x)/(cos(x) +
1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) - a*log(sin(x)^2/(cos
(x) + 1)^2 + 1)/(a^2 + b^2))
```

Fricas [A] time = 0.232594, size = 81, normalized size = 1.69

$$\frac{2\,(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2\right)}{2\,(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) -
(a^2 - b^2)*cos(x)^2 + a^2))/(a^2 + b^2)
```

Sympy [A] time = 4.26117, size = 360, normalized size = 7.5

$$\begin{cases}\tilde{\infty}\left(-a_1\log\left(\cos\left(x\right)\right)+b_1x\right) & \text{f}\\ -\frac{a_1x\sin\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)}-\frac{ia_1x\cos\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)}+\frac{ia_1\sin\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)}+\frac{ib_1x\sin\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)}-\frac{b_1x\cos\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)}-\frac{b_1\sin\left(x\right)}{2ib\sin\left(x\right)-2b\cos\left(x\right)} & \text{f}\\ \frac{a_1x\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)}-\frac{ia_1x\cos\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)}+\frac{ia_1\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)}+\frac{ib_1x\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)}+\frac{b_1x\cos\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)}+\frac{b_1\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)} & \text{f}\\ \frac{a_1x+b_1\log\left(\sin\left(x\right)\right)}{a} & \text{f}\\ \frac{aa_1x}{a^2+b^2}+\frac{ab_1\log\left(\frac{a\sin\left(x\right)}{b}+\cos\left(x\right)\right)}{a^2+b^2}-\frac{a_1b\log\left(\frac{a\sin\left(x\right)}{b}+\cos\left(x\right)\right)}{a^2+b^2}+\frac{bb_1x}{a^2+b^2} & \text{o}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), (-
a1*x*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)) - I*a1*x*cos(x)/(2*I*b*si
n(x) - 2*b*cos(x)) + I*a1*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)) + I*
b1*x*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)) - b1*x*cos(x)/(2*I*b*sin(
```

```
x) - 2*b*cos(x)) - b1*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)), Eq(a, -
I*b)), (a1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) - I*a1*x*cos(x)/(
2*I*b*sin(x) + 2*b*cos(x)) + I*a1*sin(x)/(2*I*b*sin(x) + 2*b*cos(
x)) + I*b1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(2*
I*b*sin(x) + 2*b*cos(x)) + b1*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)),
 Eq(a, I*b)), ((a1*x + b1*log(sin(x)))/a, Eq(b, 0)), (a*a1*x/(a**
2 + b**2) + a*b1*log(a*sin(x)/b + cos(x))/(a**2 + b**2) - a1*b*lo
g(a*sin(x)/b + cos(x))/(a**2 + b**2) + b*b1*x/(a**2 + b**2), True
))
```

GIAC/XCAS [A] time = 0.246174, size = 104, normalized size = 2.17

$$\frac{(aa_1 + bb_1)x}{a^2 + b^2} + \frac{(a_1b - ab_1)\ln\left(\tan(x)^2 + 1\right)}{2\left(a^2 + b^2\right)} - \frac{\left(aa_1b - a^2b_1\right)\ln\left(|a\tan(x) + b|\right)}{a^3 + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x) + a1*sin(x))/(b*cos(x) + a*sin(x)),x, algorithm="giac")
```

```
[Out] (a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*ln(tan(x)^2 + 1)/
(a^2 + b^2) - (a*a1*b - a^2*b1)*ln(abs(a*tan(x) + b))/(a^3 + a*b^
2)
```

3.170 $\int \frac{1}{\log(t)}\,dt$

Optimal. Leaf size=2

$$\text{LogIntegral}(t)$$

```
[Out] LogIntegral[t]
```

Rubi [A] time = 0.00441417, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In]  Int[Log[t]^(-1),t]

[Out] LogIntegral[t]
```

Rubi in Sympy [A] time = 0.025008, size = 2, normalized size = 1.

$$\text{li}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/ln(t),t)

[Out] li(t)
```

Mathematica [A] time = 0.00282961, size = 2, normalized size = 1.

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[t]^(-1),t]

[Out] LogIntegral[t]
```

Maple [B] time = 0.007, size = 9, normalized size = 4.5

$$-Ei\,(1,-\ln\,(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/ln(t),t)

[Out] -Ei(1,-ln(t))
```

Maxima [A] time = 1.42204, size = 4, normalized size = 2.

$$\text{Ei}\,(\log\,(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(t),t, algorithm="maxima")
```

```
[Out] Ei(log(t))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$log_integral(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(t),t, algorithm="fricas")
```

```
[Out] log_integral(t)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(t)}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/ln(t),t)
```

```
[Out] Integral(1/log(t), t)
```

GIAC/XCAS [A] time = 0.228751, size = 4, normalized size = 2.

$$\operatorname{Ei}(\ln(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/log(t),t, algorithm="giac")
```

```
[Out] Ei(ln(t))
```

3.171 $\int \frac{1}{\log^2(t)}\,dt$

Optimal. Leaf size=10

$$\text{LogIntegral}(t)-\frac{t}{\log(t)}$$

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Rubi [A] time = 0.00679612, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\text{LogIntegral}(t)-\frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In]  Int[Log[t]^(-2),t]

[Out] -(t/Log[t]) + LogIntegral[t]
```

Rubi in Sympy [A] time = 0.471386, size = 7, normalized size = 0.7

$$-\frac{t}{\log(t)}+\operatorname{li}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/ln(t)**2,t)

[Out] -t/log(t) + li(t)
```

Mathematica [A] time = 0.00186582, size = 10, normalized size = 1.

$$\text{LogIntegral}(t)-\frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[t]^(-2),t]

[Out] -(t/Log[t]) + LogIntegral[t]
```

Maple [A] time = 0.003, size = 17, normalized size = 1.7

$$-\frac{t}{\ln(t)}-Ei(1,-\ln(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/ln(t)^2,t)

[Out] -t/ln(t)-Ei(1,-ln(t))
```

Maxima [A] time = 1.41187, size = 8, normalized size = 0.8

$$(-1, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-2),t, algorithm="maxima")
```

```
[Out] gamma(-1, -log(t))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(t)\, log_i ntegral(t) - t}{\log(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-2),t, algorithm="fricas")
```

```
[Out] (log(t)*log_integral(t) - t)/log(t)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{t}{\log(t)} + \int \frac{1}{\log(t)}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/ln(t)**2,t)
```

```
[Out] -t/log(t) + Integral(1/log(t), t)
```

GIAC/XCAS [A] time = 0.226759, size = 15, normalized size = 1.5

$$-\frac{t}{\ln(t)} + \mathrm{Ei}(\ln(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-2),t, algorithm="giac")
```

```
[Out] -t/ln(t) + Ei(ln(t))
```

3.172 $\int \log^{-1-n}(t)\,dt$

Optimal. Leaf size=22

$$(-\log(t))^n \log^{-n}(t)(-\text{Gamma}(-n, -\log(t)))$$

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rubi [A] time = 0.0290269, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$(-\log(t))^n \log^{-n}(t)(-\text{Gamma}(-n, -\log(t)))$$

Antiderivative was successfully verified.

```
[In]  Int[Log[t]^(-1 - n),t]

[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rubi in Sympy [A] time = 0.584314, size = 24, normalized size = 1.09

$$(-\log(t))^{n+1}(-n, -\log(t))\log(t)^{-n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(ln(t)**(-1-n),t)

[Out] (-log(t))**(n + 1)*Gamma(-n, -log(t))*log(t)**(-n - 1)
```

Mathematica [A] time = 0.0331394, size = 22, normalized size = 1.

$$(-\log(t))^n \log^{-n}(t)(-\text{Gamma}(-n, -\log(t)))$$

Antiderivative was successfully verified.

```
[In]  Integrate[Log[t]^(-1 - n),t]

[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int (\ln(t))^{-1-n}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(ln(t)^(-1-n),t)

[Out] int(ln(t)^(-1-n),t)
```

Maxima [A] time = 1.50065, size = 30, normalized size = 1.36

$$-(-\log(t))^n \log(t)^{-n}\ (-n, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-n - 1),t, algorithm="maxima")
```

```
[Out] -(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

Fricas [A] time = 0.226325, size = 20, normalized size = 0.91

$$\cos(\pi + \pi n)\ (-n, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-n - 1),t, algorithm="fricas")
```

```
[Out] cos(pi + pi*n)*gamma(-n, -log(t))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(t)^{-n-1}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(ln(t)**(-1-n),t)
```

```
[Out] Integral(log(t)**(-n - 1), t)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \log(t)^{-n-1}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(log(t)^(-n - 1),t, algorithm="giac")
```

```
[Out] integrate(log(t)^(-n - 1), t)
```

3.173 $\int \frac{e^{2t}}{-1+t} \, dt$

Optimal. Leaf size=12

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rubi [A] time = 0.0269893, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*t)/(-1 + t),t]

[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rubi in Sympy [A] time = 1.59817, size = 8, normalized size = 0.67

$$e^2 \text{Ei}(2t-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*t)/(-1+t),t)

[Out] exp(2)*Ei(2*t - 2)
```

Mathematica [A] time = 0.00358669, size = 10, normalized size = 0.83

$$e^2\text{ExpIntegralEi}(2(t-1))$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*t)/(-1 + t),t]

[Out] E^2*ExpIntegralEi[2*(-1 + t)]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.

$$-\mathrm{e}^2 Ei(1, -2t+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*t)/(-1+t),t)

[Out] -exp(2)*Ei(1,-2*t+2)
```

Maxima [A] time = 1.43033, size = 15, normalized size = 1.25

$$-e^2 exp_integral_e(1, -2t+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)/(t - 1),t, algorithm="maxima")
```

```
[Out] -e^2*exp_integral_e(1, -2*t + 2)
```

Fricas [A] time = 0.199384, size = 12, normalized size = 1.

$$\mathrm{Ei}\left(2\,t-2\right)e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)/(t - 1),t, algorithm="fricas")
```

```
[Out] Ei(2*t - 2)*e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2t}}{t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*t)/(-1+t),t)
```

```
[Out] Integral(exp(2*t)/(t - 1), t)
```

GIAC/XCAS [A] time = 0.224413, size = 12, normalized size = 1.

$$\mathrm{Ei}\left(2\,t-2\right)e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*t)/(t - 1),t, algorithm="giac")
```

```
[Out] Ei(2*t - 2)*e^2
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2}\,dx$

Optimal. Leaf size=22

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rubi [A] time = 0.095385, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In]  Int[E^(2*x)/(2 - 3*x + x^2),x]

[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rubi in Sympy [A] time = 6.84726, size = 19, normalized size = 0.86

$$e^4\,\text{Ei}\,(2x-4)-e^2\,\text{Ei}\,(2x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(exp(2*x)/(x**2-3*x+2),x)

[Out] exp(4)*Ei(2*x - 4) - exp(2)*Ei(2*x - 2)
```

Mathematica [A] time = 0.00691707, size = 22, normalized size = 1.

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[E^(2*x)/(2 - 3*x + x^2),x]

[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Maple [A] time = 0.014, size = 23, normalized size = 1.1

$$e^2 Ei\,(1,2-2\,x)-e^4 Ei\,(1,4-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(exp(2*x)/(x^2-3*x+2),x)

[Out] exp(2)*Ei(1,2-2*x)-exp(4)*Ei(1,4-2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2\,x)}}{x^2 - 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(x^2 - 3*x + 2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

Fricas [A] time = 0.201225, size = 27, normalized size = 1.23

$$\mathrm{Ei}\left(2\,x - 4\right)e^4 - \mathrm{Ei}\left(2\,x - 2\right)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(x^2 - 3*x + 2),x, algorithm="fricas")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{(x-2)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(exp(2*x)/(x**2-3*x+2),x)
```

```
[Out] Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

GIAC/XCAS [A] time = 0.226295, size = 27, normalized size = 1.23

$$\mathrm{Ei}\left(2\,x - 4\right)e^4 - \mathrm{Ei}\left(2\,x - 2\right)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(e^(2*x)/(x^2 - 3*x + 2),x, algorithm="giac")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

Optimal. Leaf size=103

$$\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)
^2]*EllipticF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4
*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^
3])
```

Rubi [A] time = 0.0399457, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)
^2]*EllipticF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4
*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^
3])
```

Rubi in Sympy [A] time = 0.809349, size = 95, normalized size = 0.92

$$\frac{2\cdot 3^{\frac{3}{4}}\sqrt{\frac{t^2-t+1}{\left(t+1+\sqrt{3}\right)^2}}\sqrt{\sqrt{3}+2}\,(t+1)\,F\left(\operatorname{asin}\left(\frac{t-\sqrt{3}+1}{t+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt{\frac{t+1}{\left(t+1+\sqrt{3}\right)^2}}\sqrt{t^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(t**3+1)**(1/2),t)
```

```
[Out] 2*3**(3/4)*sqrt((t**2 - t + 1)/(t + 1 + sqrt(3))**2)*sqrt(sqrt(3)
 + 2)*(t + 1)*elliptic_f(asin((t - sqrt(3) + 1)/(t + 1 + sqrt(3))
), -7 - 4*sqrt(3))/(3*sqrt((t + 1)/(t + 1 + sqrt(3))**2)*sqrt(t**
3 + 1))
```

Mathematica [A] time = 0.068797, size = 88, normalized size = 0.85

$$\frac{2\sqrt[6]{-1}\sqrt{-\sqrt[6]{-1}\left(t+(-1)^{2/3}\right)}\sqrt{(-1)^{2/3}t^2+\sqrt[3]{-1}t+1}F\left(\sin^{-1}\left(\frac{\sqrt{-(-1)^{5/6}(t+1)}}{\sqrt[4]{3}}\right)|\sqrt[3]{-1}\right)}{\sqrt[4]{3}\sqrt{t^3+1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*(-1)^(1/6)*Sqrt[-((-1)^(1/6)*((-1)^(2/3) + t))]*Sqrt[1 + (-1)^
(1/3)*t + (-1)^(2/3)*t^2]*EllipticF[ArcSin[Sqrt[-((-1)^(5/6)*(1 +
 t))]/3^(1/4)], (-1)^(1/3)])/(3^(1/4)*Sqrt[1 + t^3])
```

Maple [A] time = 0.096, size = 116, normalized size = 1.1

$$2\,\frac{3/2-i/2\sqrt{3}}{\sqrt{t^3+1}}\sqrt{\frac{1+t}{3/2-i/2\sqrt{3}}}\sqrt{\frac{t-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{t-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\,EllipticF\left(\sqrt{\frac{1+t}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3+1)^(1/2),t)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+t)/(3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2-1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2+1/2*I*3^(1/2))/
(-3/2+1/2*I*3^(1/2)))^(1/2)/(t^3+1)^(1/2)*EllipticF(((1+t)/(3/2-1
/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{t^3+1}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(t^3 + 1),t, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{t^3+1}}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(t^3 + 1),t, algorithm="fricas")
```

```
[Out] integral(1/sqrt(t^3 + 1), t)
```

Sympy [A] time = 0.852905, size = 27, normalized size = 0.26

$$\frac{t\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| t^3e^{i\pi}\right)}{3\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t**3+1)**(1/2),t)
```

```
[Out] t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*g
amma(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{t^3 + 1}}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(t^3 + 1),t, algorithm="giac")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```