

# Computer algebra independent integration tests

## 8-Special-functions/8.8-Polylogarithm-function

Nasser M. Abbasi

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3.164	$\int \log(1 - cx) \text{PolyLog}(2, cx) dx$	554
3.165	$\int \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{x} dx$	558
3.166	$\int \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{x^2} dx$	560
3.167	$\int \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{x^3} dx$	564
3.168	$\int \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{x^4} dx$	569
3.169	$\int \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{x^5} dx$	574
3.170	$\int x^2 (g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$	579
3.171	$\int x (g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$	584
3.172	$\int (g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$	589
3.173	$\int \frac{(g + h \log(1 - cx)) \text{PolyLog}(2, cx)}{x} dx$	593
3.174	$\int \frac{(g + h \log(1 - cx)) \text{PolyLog}(2, cx)}{x^2} dx$	596
3.175	$\int \frac{(g + h \log(1 - cx)) \text{PolyLog}(2, cx)}{x^3} dx$	601
3.176	$\int \frac{(g + h \log(1 - cx)) \text{PolyLog}(2, cx)}{x^4} dx$	606
3.177	$\int x^2 (g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx)) dx$	612
3.178	$\int x (g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx)) dx$	621
3.179	$\int (g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx)) dx$	629
3.180	$\int \frac{(g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx))}{x} dx$	636
3.181	$\int \frac{(g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx))}{x^2} dx$	638
3.182	$\int \frac{(g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx))}{x^3} dx$	644
3.183	$\int \frac{(g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a + bx))}{x^4} dx$	651
3.184	$\int x^2 (a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$	659
3.185	$\int x (a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$	664

3.186	$\int (a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$	670
3.187	$\int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x} dx$	675
3.188	$\int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^2} dx$	680
3.189	$\int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^3} dx$	685
3.190	$\int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^4} dx$	690
3.191	$\int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^5} dx$	695
3.192	$\int x (a + bx + cx^2) \log(1 - dx) \text{PolyLog}(2, dx) dx$	701
3.193	$\int (a + bx + cx^2) \log(1 - dx) \text{PolyLog}(2, dx) dx$	708
3.194	$\int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2, dx)}{x} dx$	714
3.195	$\int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2, dx)}{x^2} dx$	720
3.196	$\int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2, dx)}{x^3} dx$	725
3.197	$\int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2, dx)}{x^4} dx$	730
3.198	$\int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2, dx)}{x^5} dx$	736

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 198 ]. This is test number [ 208 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 198 )	% 0. ( 0 )
Mathematica	% 97.47 ( 193 )	% 2.53 ( 5 )
Maple	% 72.73 ( 144 )	% 27.27 ( 54 )
Maxima	% 44.44 ( 88 )	% 55.56 ( 110 )
Fricas	% 51.52 ( 102 )	% 48.48 ( 96 )
Sympy	% 18.18 ( 36 )	% 81.82 ( 162 )
Giac	% 8.08 ( 16 )	% 91.92 ( 182 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

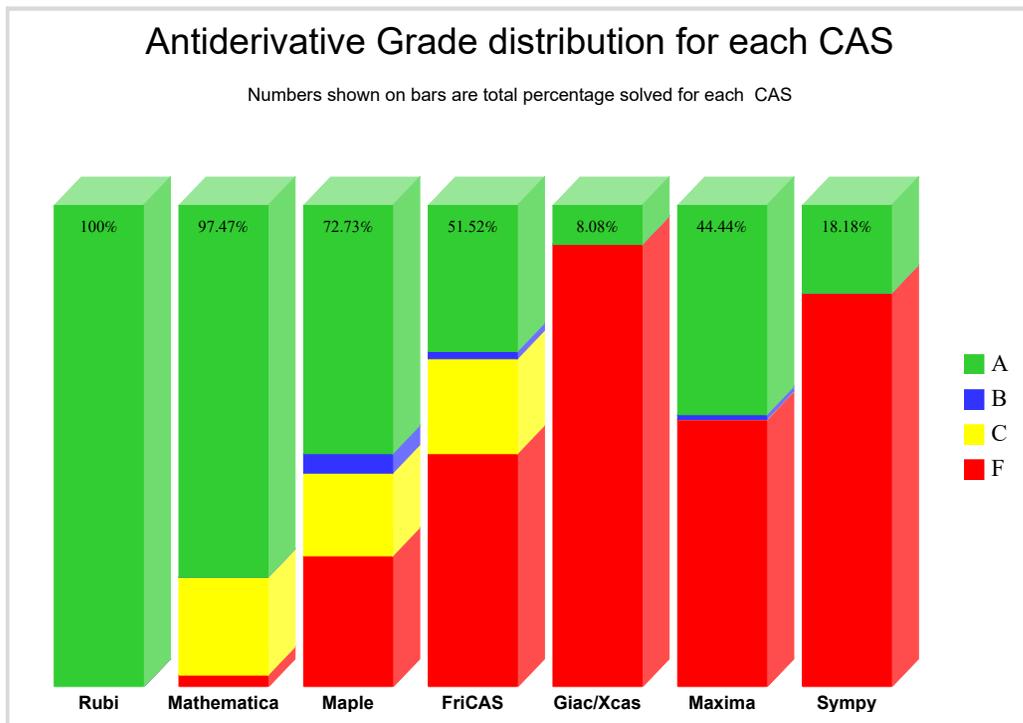
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

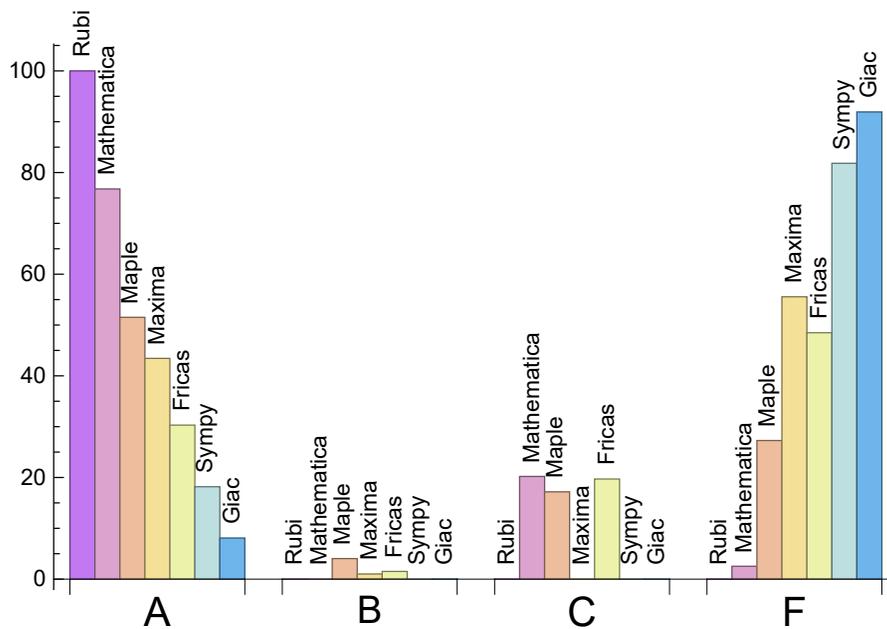
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	76.77	0.	20.2	2.53
Maple	51.52	4.04	17.17	27.27
Maxima	43.43	1.01	0.	55.56
Fricas	30.3	1.52	19.7	48.48
Sympy	18.18	0.	0.	81.82
Giac	8.08	0.	0.	91.92



The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	215.93	0.92	91.	1.
Mathematica	0.3	137.58	0.73	63.	0.8
Maple	0.18	116.35	1.09	101.5	1.11
Maxima	0.86	187.28	1.08	80.	1.09
Fricas	2.27	294.7	3.41	228.	2.78
Sympy	11.27	28.83	0.57	13.5	0.69
Giac	0.	0.	0.	0.	0.

## 1.4 list of integrals that has no closed form antiderivative

{96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {17, 18, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

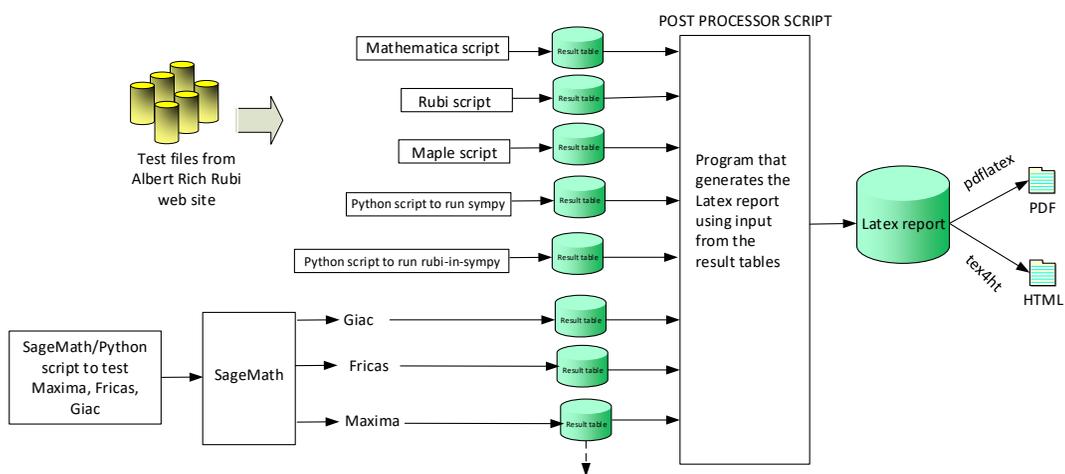
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 87, 96, 97, 98, 99, 100, 102, 105, 108, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198 }

B grade: { }

C grade: { 17, 18, 30, 31, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113 }

F grade: { 101, 181, 182, 183, 196 }

#### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 134, 138, 139, 140, 142, 143, 153, 154, 155, 159, 160, 165, 173, 180 }

B grade: { 39, 40, 41, 42, 43, 44, 137, 144 }

C grade: { 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { 101, 127, 131, 132, 133, 135, 136, 141, 145, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 32, 33, 34, 36, 37, 38, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 153, 154, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 180, 184, 185, 186, 189, 190, 191, 192, 193, 197, 198 }

B grade: { 144, 155 }

C grade: { }

F grade: { 22, 26, 27, 28, 29, 30, 31, 35, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 135, 136, 141, 149, 150, 151, 152, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 187, 188, 194, 195, 196 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 59, 60, 61, 62, 63, 64, 72, 73, 74, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 134, 137, 138, 139, 140, 151, 152, 153, 154, 155, 160, 165, 180 }

B grade: { 75, 76, 77 }

C grade: { 11, 12, 13, 14, 16, 17, 18, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 55, 65, 66, 67, 68, 69, 70, 71, 78, 79, 80, 81, 82, 83, 84, 85, 131, 132, 133, 150 }

F grade: { 6, 15, 22, 35, 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 128, 129, 130, 135, 136, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 19, 20, 21, 23, 24, 25, 27, 28, 29, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 122, 123, 134, 160 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 16, 17, 18, 22, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }



## 2.1.7 Giac

A grade: { 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	73	76	97	177	66	0
normalized size	1	1.	0.85	0.88	1.13	2.06	0.77	0.
time (sec)	N/A	0.053	0.042	0.046	0.964	2.689	15.264	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	65	68	86	153	58	0
normalized size	1	1.	0.86	0.89	1.13	2.01	0.76	0.
time (sec)	N/A	0.045	0.033	0.046	0.983	2.662	8.741	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	57	60	76	132	49	0
normalized size	1	1.	0.86	0.91	1.15	2.	0.74	0.
time (sec)	N/A	0.04	0.029	0.047	0.973	2.599	4.428	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	52	65	111	41	0
normalized size	1	1.	0.86	0.93	1.16	1.98	0.73	0.
time (sec)	N/A	0.028	0.024	0.046	0.965	2.582	2.437	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	36	39	70	22	0
normalized size	1	1.	0.9	1.24	1.34	2.41	0.76	0.
time (sec)	N/A	0.009	0.012	0.045	0.981	2.635	1.209	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	0	3	0
normalized size	1	1.	1.	1.2	1.4	0.	0.6	0.
time (sec)	N/A	0.009	0.001	0.043	0.975	0.	1.712	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	40	38	88	24	0
normalized size	1	1.	1.	1.11	1.06	2.44	0.67	0.
time (sec)	N/A	0.022	0.01	0.119	1.009	2.782	1.563	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	52	54	117	42	0
normalized size	1	1.	0.86	0.9	0.93	2.02	0.72	0.
time (sec)	N/A	0.033	0.022	0.118	0.981	2.629	2.965	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	60	66	143	51	0
normalized size	1	1.	0.76	0.88	0.97	2.1	0.75	0.
time (sec)	N/A	0.036	0.037	0.119	0.99	2.688	5.569	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	60	68	78	163	60	0
normalized size	1	1.	0.77	0.87	1.	2.09	0.77	0.
time (sec)	N/A	0.039	0.035	0.128	0.986	2.786	11.28	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	86	78	104	244	0	0
normalized size	1	1.	0.98	0.89	1.18	2.77	0.	0.
time (sec)	N/A	0.057	0.012	0.161	0.999	2.549	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	69	93	224	0	0
normalized size	1	1.	1.	0.88	1.19	2.87	0.	0.
time (sec)	N/A	0.051	0.011	0.164	0.996	2.676	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	62	82	201	0	0
normalized size	1	1.	1.01	0.91	1.21	2.96	0.	0.
time (sec)	N/A	0.035	0.009	0.155	1.014	2.616	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	41	53	151	0	0
normalized size	1	1.	1.15	1.21	1.56	4.44	0.	0.
time (sec)	N/A	0.011	0.012	0.092	0.991	2.598	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	0	3	0
normalized size	1	1.	1.	1.2	1.4	0.	0.6	0.
time (sec)	N/A	0.009	0.001	0.04	0.984	0.	0.552	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	57	45	162	0	0
normalized size	1	1.	0.96	1.24	0.98	3.52	0.	0.
time (sec)	N/A	0.03	0.032	0.08	0.995	2.662	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	25	90	63	194	0	0
normalized size	1	1.	0.36	1.29	0.9	2.77	0.	0.
time (sec)	N/A	0.043	0.009	0.165	0.993	2.718	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	25	106	76	221	0	0
normalized size	1	1.	0.31	1.32	0.95	2.76	0.	0.
time (sec)	N/A	0.048	0.009	0.168	0.988	2.814	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	62	84	142	56	0
normalized size	1	1.	0.88	0.84	1.14	1.92	0.76	0.
time (sec)	N/A	0.061	0.022	0.044	1.008	2.527	42.518	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	54	73	120	48	0
normalized size	1	1.	0.88	0.84	1.14	1.88	0.75	0.
time (sec)	N/A	0.05	0.017	0.045	0.969	2.584	13.184	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	52	54	89	39	0
normalized size	1	1.	0.93	1.13	1.17	1.93	0.85	0.
time (sec)	N/A	0.025	0.009	0.045	0.985	2.57	4.027	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.009	0.001	0.115	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	46	112	37	0
normalized size	1	1.	1.	0.88	0.94	2.29	0.76	0.
time (sec)	N/A	0.041	0.012	0.05	0.96	2.68	4.88	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	54	62	131	49	0
normalized size	1	1.	0.8	0.84	0.97	2.05	0.77	0.
time (sec)	N/A	0.05	0.028	0.05	0.988	2.649	14.997	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	62	74	154	58	0
normalized size	1	1.	0.81	0.84	1.	2.08	0.78	0.
time (sec)	N/A	0.054	0.031	0.05	0.981	2.657	43.062	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	58	0	398	0	0
normalized size	1	1.	0.89	0.79	0.	5.45	0.	0.
time (sec)	N/A	0.045	0.072	0.046	0.	2.698	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	50	0	351	83	0
normalized size	1	1.	0.9	0.79	0.	5.57	1.32	0.
time (sec)	N/A	0.039	0.051	0.046	0.	2.639	119.376	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	39	37	0	269	60	0
normalized size	1	1.	0.98	0.92	0.	6.72	1.5	0.
time (sec)	N/A	0.017	0.028	0.046	0.	2.631	23.224	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	39	0	232	184	0
normalized size	1	1.	0.98	0.93	0.	5.52	4.38	0.
time (sec)	N/A	0.026	0.018	0.049	0.	2.792	84.144	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	45	0	289	0	0
normalized size	1	1.	0.84	0.8	0.	5.16	0.	0.
time (sec)	N/A	0.032	0.014	0.053	0.	2.746	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	47	53	0	333	0	0
normalized size	1	1.	0.71	0.8	0.	5.05	0.	0.
time (sec)	N/A	0.038	0.015	0.052	0.	2.661	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	80	104	240	0	0
normalized size	1	1.	1.	0.91	1.18	2.73	0.	0.
time (sec)	N/A	0.075	0.018	0.058	0.98	2.717	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	79	72	93	217	0	0
normalized size	1	1.	1.01	0.92	1.19	2.78	0.	0.
time (sec)	N/A	0.061	0.014	0.054	0.968	2.706	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	56	72	181	0	0
normalized size	1	1.	0.87	0.93	1.2	3.02	0.	0.
time (sec)	N/A	0.03	0.011	0.055	1.001	2.664	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.009	0.001	0.154	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	68	55	194	0	0
normalized size	1	1.	0.95	1.08	0.87	3.08	0.	0.
time (sec)	N/A	0.047	0.027	0.061	0.99	2.719	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	30	98	74	217	0	0
normalized size	1	1.	0.38	1.26	0.95	2.78	0.	0.
time (sec)	N/A	0.064	0.012	0.065	1.003	2.718	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	30	115	86	242	0	0
normalized size	1	1.	0.34	1.31	0.98	2.75	0.	0.
time (sec)	N/A	0.065	0.012	0.066	0.998	2.797	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	144	0	608	0	0
normalized size	1	1.	0.89	1.66	0.	6.99	0.	0.
time (sec)	N/A	0.053	0.161	0.178	0.	2.777	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	136	0	555	0	0
normalized size	1	1.	0.9	1.77	0.	7.21	0.	0.
time (sec)	N/A	0.049	0.138	0.175	0.	2.743	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	119	0	452	0	0
normalized size	1	1.	1.	2.38	0.	9.04	0.	0.
time (sec)	N/A	0.023	0.088	0.173	0.	2.694	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	112	0	402	0	0
normalized size	1	1.	0.93	2.07	0.	7.44	0.	0.
time (sec)	N/A	0.036	0.081	0.172	0.	2.871	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	125	0	462	0	0
normalized size	1	1.	0.87	1.79	0.	6.6	0.	0.
time (sec)	N/A	0.041	0.089	0.177	0.	3.227	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	138	0	513	0	0
normalized size	1	1.	0.86	1.72	0.	6.41	0.	0.
time (sec)	N/A	0.047	0.101	0.18	0.	3.241	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0
normalized size	1	1.	0.97	1.52	0.	0.	0.	0.
time (sec)	N/A	0.037	0.044	0.26	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0
normalized size	1	1.	0.97	1.52	0.	0.	0.	0.
time (sec)	N/A	0.032	0.036	0.207	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	88	0	0	0	0
normalized size	1	1.	0.94	1.63	0.	0.	0.	0.
time (sec)	N/A	0.023	0.045	0.207	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	28	0	0
normalized size	1	1.	1.	1.09	0.	2.55	0.	0.
time (sec)	N/A	0.01	0.002	0.044	0.	2.839	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	106	0	0	0	0
normalized size	1	1.	0.87	1.54	0.	0.	0.	0.
time (sec)	N/A	0.039	0.053	0.208	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	108	0	0	0	0
normalized size	1	1.	0.78	1.38	0.	0.	0.	0.
time (sec)	N/A	0.042	0.052	0.209	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	108	0	0	0	0
normalized size	1	1.	0.8	1.42	0.	0.	0.	0.
time (sec)	N/A	0.04	0.052	0.205	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0
normalized size	1	1.	0.47	1.5	0.	0.	0.	0.
time (sec)	N/A	0.052	0.009	0.345	0.	0.	0.	0.



Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0
normalized size	1	1.	0.47	1.5	0.	0.	0.	0.
time (sec)	N/A	0.043	0.008	0.338	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	39	105	0	0	0	0
normalized size	1	1.	0.57	1.52	0.	0.	0.	0.
time (sec)	N/A	0.028	0.006	0.334	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	209	0	0
normalized size	1	1.	1.	1.09	0.	19.	0.	0.
time (sec)	N/A	0.01	0.001	0.041	0.	2.724	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	37	129	0	0	0	0
normalized size	1	1.	0.44	1.54	0.	0.	0.	0.
time (sec)	N/A	0.05	0.009	0.333	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	41	132	0	0	0	0
normalized size	1	1.	0.43	1.39	0.	0.	0.	0.
time (sec)	N/A	0.05	0.009	0.343	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	41	132	0	0	0	0
normalized size	1	1.	0.44	1.42	0.	0.	0.	0.
time (sec)	N/A	0.051	0.01	0.34	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	90	96	0	470	0	0
normalized size	1	1.	0.77	0.82	0.	4.02	0.	0.
time (sec)	N/A	0.072	0.102	0.194	0.	2.82	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	75	83	0	362	0	0
normalized size	1	1.	0.74	0.81	0.	3.55	0.	0.
time (sec)	N/A	0.053	0.082	0.05	0.	2.569	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	69	0	331	0	0
normalized size	1	1.	0.79	0.86	0.	4.14	0.	0.
time (sec)	N/A	0.046	0.079	0.05	0.	2.615	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	63	0	316	0	0
normalized size	1	1.	0.75	0.93	0.	4.65	0.	0.
time (sec)	N/A	0.043	0.071	0.053	0.	2.712	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	76	0	365	0	0
normalized size	1	1.	0.64	0.85	0.	4.1	0.	0.
time (sec)	N/A	0.05	0.074	0.055	0.	2.665	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	65	89	0	412	0	0
normalized size	1	1.	0.61	0.84	0.	3.89	0.	0.
time (sec)	N/A	0.057	0.095	0.054	0.	2.593	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	149	0	822	0	0
normalized size	1	1.	0.64	0.97	0.	5.37	0.	0.
time (sec)	N/A	0.098	0.26	0.182	0.	2.975	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	88	141	0	721	0	0
normalized size	1	1.	0.65	1.04	0.	5.3	0.	0.
time (sec)	N/A	0.083	0.218	0.061	0.	2.874	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	73	133	0	593	0	0
normalized size	1	1.	0.6	1.1	0.	4.9	0.	0.
time (sec)	N/A	0.067	0.188	0.058	0.	2.848	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	57	127	0	547	0	0
normalized size	1	1.	0.59	1.31	0.	5.64	0.	0.
time (sec)	N/A	0.062	0.139	0.057	0.	2.726	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	111	0	518	0	0
normalized size	1	1.	0.68	1.31	0.	6.09	0.	0.
time (sec)	N/A	0.059	0.102	0.056	0.	2.589	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	64	122	0	575	0	0
normalized size	1	1.	0.59	1.13	0.	5.32	0.	0.
time (sec)	N/A	0.066	0.096	0.06	0.	2.789	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	72	135	0	629	0	0
normalized size	1	1.	0.58	1.08	0.	5.03	0.	0.
time (sec)	N/A	0.076	0.13	0.062	0.	2.86	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	150	0	471	0	0
normalized size	1	1.	0.72	1.07	0.	3.36	0.	0.
time (sec)	N/A	0.107	0.104	0.214	0.	2.846	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	139	0	440	0	0
normalized size	1	1.	0.73	1.11	0.	3.52	0.	0.
time (sec)	N/A	0.086	0.07	0.052	0.	2.703	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	57	137	0	408	0	0
normalized size	1	1.	0.5	1.19	0.	3.55	0.	0.
time (sec)	N/A	0.08	0.074	0.053	0.	2.882	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	62	127	0	425	0	0
normalized size	1	1.	0.6	1.23	0.	4.13	0.	0.
time (sec)	N/A	0.074	0.07	0.052	0.	2.664	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	62	129	0	470	0	0
normalized size	1	1.	0.56	1.16	0.	4.23	0.	0.
time (sec)	N/A	0.072	0.067	0.054	0.	2.735	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	70	140	0	512	0	0
normalized size	1	1.	0.56	1.11	0.	4.06	0.	0.
time (sec)	N/A	0.086	0.077	0.056	0.	2.755	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	0	682	0	0
normalized size	1	1.	0.55	0.96	0.	4.24	0.	0.
time (sec)	N/A	0.127	0.103	0.193	0.	3.106	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	0	606	0	0
normalized size	1	1.	0.55	0.96	0.	3.76	0.	0.
time (sec)	N/A	0.116	0.1	0.208	0.	2.915	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	68	147	0	579	0	0
normalized size	1	1.	0.47	1.01	0.	3.97	0.	0.
time (sec)	N/A	0.097	0.092	0.224	0.	2.852	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	68	147	0	521	0	0
normalized size	1	1.	0.51	1.1	0.	3.89	0.	0.
time (sec)	N/A	0.101	0.086	0.191	0.	2.862	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	71	131	0	549	0	0
normalized size	1	1.	0.58	1.07	0.	4.5	0.	0.
time (sec)	N/A	0.093	0.09	0.191	0.	3.007	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	71	131	0	591	0	0
normalized size	1	1.	0.54	0.99	0.	4.48	0.	0.
time (sec)	N/A	0.091	0.094	0.194	0.	2.818	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	79	142	0	641	0	0
normalized size	1	1.	0.54	0.97	0.	4.36	0.	0.
time (sec)	N/A	0.107	0.099	0.206	0.	3.003	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	84	142	0	629	0	0
normalized size	1	1.	0.57	0.97	0.	4.28	0.	0.
time (sec)	N/A	0.106	0.096	0.191	0.	3.057	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	82	121	0	0	0	0
normalized size	1	1.	0.81	1.2	0.	0.	0.	0.
time (sec)	N/A	0.059	0.124	0.255	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	121	0	0	0	0
normalized size	1	1.	0.82	1.21	0.	0.	0.	0.
time (sec)	N/A	0.055	0.106	0.229	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	48	109	0	0	0	0
normalized size	1	1.	0.52	1.17	0.	0.	0.	0.
time (sec)	N/A	0.054	0.021	0.232	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	48	121	0	0	0	0
normalized size	1	1.	0.49	1.25	0.	0.	0.	0.
time (sec)	N/A	0.06	0.029	0.224	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	48	121	0	0	0	0
normalized size	1	1.	0.46	1.15	0.	0.	0.	0.
time (sec)	N/A	0.061	0.026	0.23	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	50	145	0	0	0	0
normalized size	1	1.	0.4	1.16	0.	0.	0.	0.
time (sec)	N/A	0.075	0.03	0.379	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	50	145	0	0	0	0
normalized size	1	1.	0.4	1.17	0.	0.	0.	0.
time (sec)	N/A	0.07	0.026	0.381	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	50	133	0	0	0	0
normalized size	1	1.	0.43	1.16	0.	0.	0.	0.
time (sec)	N/A	0.069	0.02	0.372	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	50	145	0	0	0	0
normalized size	1	1.	0.42	1.22	0.	0.	0.	0.
time (sec)	N/A	0.078	0.025	0.372	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	50	145	0	0	0	0
normalized size	1	1.	0.39	1.12	0.	0.	0.	0.
time (sec)	N/A	0.075	0.024	0.381	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.043	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.006	0.046	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.002	0.007	0.048	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.006	0.062	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.047	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.056	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	144	0	0	0	0
normalized size	1	1.	0.68	1.85	0.	0.	0.	0.
time (sec)	N/A	0.049	0.039	0.227	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	88	173	0	0	0	0
normalized size	1	1.	0.86	1.7	0.	0.	0.	0.
time (sec)	N/A	0.064	0.06	0.388	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	119	198	0	0	0	0
normalized size	1	1.	0.98	1.64	0.	0.	0.	0.
time (sec)	N/A	0.086	0.073	0.794	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0
normalized size	1	1.	0.77	1.88	0.	0.	0.	0.
time (sec)	N/A	0.055	0.045	0.236	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0
normalized size	1	1.	1.07	1.85	0.	0.	0.	0.
time (sec)	N/A	0.071	0.079	0.41	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0
normalized size	1	1.	1.17	1.82	0.	0.	0.	0.
time (sec)	N/A	0.093	0.1	0.792	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0
normalized size	1	1.	0.77	1.88	0.	0.	0.	0.
time (sec)	N/A	0.055	0.045	0.277	0.	0.	0.	0.



Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0
normalized size	1	1.	1.07	1.85	0.	0.	0.	0.
time (sec)	N/A	0.071	0.08	0.414	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0
normalized size	1	1.	1.17	1.82	0.	0.	0.	0.
time (sec)	N/A	0.094	0.097	0.856	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	80	148	0	0	0	0
normalized size	1	1.	0.79	1.47	0.	0.	0.	0.
time (sec)	N/A	0.06	0.064	0.293	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	50	180	0	0	0	0
normalized size	1	1.	0.38	1.38	0.	0.	0.	0.
time (sec)	N/A	0.076	0.025	0.529	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	52	217	0	0	0	0
normalized size	1	1.	0.34	1.41	0.	0.	0.	0.
time (sec)	N/A	0.102	0.02	3.379	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.022	0.05	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.002	0.001	0.047	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	0	0	5	0
normalized size	1	1.	1.	1.14	0.	0.	0.71	0.
time (sec)	N/A	0.009	0.001	0.044	0.	0.	0.528	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.05	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.049	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.023	0.046	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	0.004	0.049	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	0	0	0
normalized size	1	1.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.01	0.001	0.046	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.046	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.022	0.046	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	144	269	270	366	0	0
normalized size	1	1.	0.55	1.03	1.04	1.41	0.	0.
time (sec)	N/A	0.321	0.208	0.008	0.995	2.393	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	96	177	196	242	0	0
normalized size	1	1.	0.63	1.16	1.29	1.59	0.	0.
time (sec)	N/A	0.169	0.101	0.004	0.993	2.348	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	96	122	126	0	0
normalized size	1	1.	0.88	1.6	2.03	2.1	0.	0.
time (sec)	N/A	0.049	0.018	0.003	0.983	2.416	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	422	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.146	0.01	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	85	154	0	0	0
normalized size	1	1.	0.87	1.01	1.83	0.	0.	0.
time (sec)	N/A	0.117	0.052	0.13	0.982	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	195	261	0	0	0
normalized size	1	1.	0.76	1.13	1.51	0.	0.	0.
time (sec)	N/A	0.178	0.18	0.227	0.994	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	210	376	408	0	0	0
normalized size	1	1.	0.76	1.36	1.48	0.	0.	0.
time (sec)	N/A	0.273	0.304	0.214	1.027	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	296	0	356	675	0	0
normalized size	1	1.	0.85	0.	1.03	1.95	0.	0.
time (sec)	N/A	0.638	0.068	0.005	1.03	2.345	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	198	0	261	510	0	0
normalized size	1	1.	1.	0.	1.32	2.58	0.	0.
time (sec)	N/A	0.287	0.047	0.006	1.011	2.511	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	0	162	355	0	0
normalized size	1	1.	0.79	0.	1.93	4.23	0.	0.
time (sec)	N/A	0.069	0.023	0.005	1.03	2.355	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.037	0.004	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	477	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.742	0.005	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	573	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.664	1.931	0.004	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	605	485	1177	919	1339	0	0
normalized size	1	1.	0.8	1.95	1.52	2.21	0.	0.
time (sec)	N/A	0.587	0.562	0.064	1.043	2.378	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	274	687	548	779	0	0
normalized size	1	1.	0.71	1.78	1.42	2.02	0.	0.
time (sec)	N/A	0.339	0.183	0.058	1.034	2.321	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	161	292	286	377	0	0
normalized size	1	1.	0.77	1.39	1.36	1.8	0.	0.
time (sec)	N/A	0.197	0.09	0.049	1.019	2.491	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	96	122	126	0	0
normalized size	1	1.	0.88	1.6	2.03	2.1	0.	0.
time (sec)	N/A	0.052	0.016	0.006	1.003	2.436	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	622	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	0.308	0.401	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	108	189	224	0	0	0
normalized size	1	1.	0.78	1.37	1.62	0.	0.	0.
time (sec)	N/A	0.184	0.141	0.38	1.031	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	190	437	512	0	0	0
normalized size	1	1.	0.68	1.57	1.84	0.	0.	0.
time (sec)	N/A	0.27	0.421	0.462	1.008	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	313	1075	1928	0	0	0
normalized size	1	1.	0.7	2.4	4.3	0.	0.	0.
time (sec)	N/A	0.426	0.632	0.394	1.224	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	59	0	0	0
normalized size	1	1.	1.	0.	1.28	0.	0.	0.
time (sec)	N/A	0.066	0.039	0.322	1.022	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	59	0	0	0
normalized size	1	1.	1.	0.	1.28	0.	0.	0.
time (sec)	N/A	0.067	0.007	0.281	0.957	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	66	0	0	0
normalized size	1	1.	1.	0.	1.29	0.	0.	0.
time (sec)	N/A	0.133	0.039	0.302	0.965	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	66	0	0	0
normalized size	1	1.	1.	0.	1.29	0.	0.	0.
time (sec)	N/A	0.134	0.015	0.282	0.978	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.02	0.891	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	782	0	0
normalized size	1	1.	0.97	0.	0.	23.7	0.	0.
time (sec)	N/A	0.061	0.007	0.703	0.	3.456	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	74	0	0
normalized size	1	1.	0.97	0.	0.	2.24	0.	0.
time (sec)	N/A	0.061	0.007	0.805	0.	2.71	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	40	0	0	68	0	0
normalized size	1	1.	1.21	0.	0.	2.06	0.	0.
time (sec)	N/A	0.058	1.823	0.702	0.	2.445	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	37	78	72	0	0
normalized size	1	1.	1.06	1.03	2.17	2.	0.	0.
time (sec)	N/A	0.317	0.091	0.121	1.014	2.34	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	56	70	84	0	0
normalized size	1	1.	0.97	1.56	1.94	2.33	0.	0.
time (sec)	N/A	0.367	0.103	0.171	1.045	2.478	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	57	285	182	0	0
normalized size	1	1.	1.	1.1	5.48	3.5	0.	0.
time (sec)	N/A	2.025	0.265	0.276	1.179	2.276	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.012	0.185	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.006	0.053	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.005	0.055	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	0	0	0	0
normalized size	1	1.	1.	1.03	0.	0.	0.	0.
time (sec)	N/A	0.018	0.005	0.061	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.051	0.088	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	223	0	508	0	0	0
normalized size	1	1.	0.74	0.	1.69	0.	0.	0.
time (sec)	N/A	0.523	0.573	0.052	1.129	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	192	0	400	0	0	0
normalized size	1	1.	0.74	0.	1.55	0.	0.	0.
time (sec)	N/A	0.405	0.357	0.051	1.119	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	160	0	300	0	0	0
normalized size	1	1.	0.61	0.	1.15	0.	0.	0.
time (sec)	N/A	0.255	0.3	0.049	1.152	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	0	190	0	0	0
normalized size	1	1.	0.9	0.	1.44	0.	0.	0.
time (sec)	N/A	0.212	0.023	0.049	1.102	0.	0.	0.



Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	26	0	0
normalized size	1	1.	1.	0.91	1.	2.36	0.	0.
time (sec)	N/A	0.026	0.009	0.046	0.966	2.534	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	115	0	153	0	0	0
normalized size	1	1.	1.04	0.	1.38	0.	0.	0.
time (sec)	N/A	0.164	0.136	0.049	1.166	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	185	0	219	0	0	0
normalized size	1	1.	0.97	0.	1.15	0.	0.	0.
time (sec)	N/A	0.282	0.275	0.05	1.664	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	246	0	254	0	0	0
normalized size	1	1.	1.	0.	1.04	0.	0.	0.
time (sec)	N/A	0.355	0.235	0.05	1.653	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	277	0	289	0	0	0
normalized size	1	1.	0.97	0.	1.01	0.	0.	0.
time (sec)	N/A	0.449	0.236	0.05	1.74	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	366	252	0	0	0	0	0
normalized size	1	0.87	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	0.461	0.23	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	287	211	0	0	0	0	0
normalized size	1	0.87	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	0.339	0.227	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	149	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.068	0.199	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	0	0
normalized size	1	1.	1.	0.95	0.	0.	0.	0.
time (sec)	N/A	0.058	0.011	0.155	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	165	150	0	0	0	0	0
normalized size	1	1.06	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.172	0.276	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	278	238	0	0	0	0	0
normalized size	1	1.05	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.496	0.256	0.333	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	351	301	0	0	0	0	0
normalized size	1	1.03	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.651	0.201	0.349	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2995	2995	2610	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	4.6	11.481	0.493	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2252	2252	1996	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	2.904	8.832	0.289	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1653	1653	1546	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	3.186	4.922	0.285	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.553	0.302	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2498	2498	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.667	10.192	0.428	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3119	3119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	3.238	10.412	0.521	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3733	3733	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.308	10.519	0.537	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	425	0	560	0	0	0
normalized size	1	1.	0.64	0.	0.85	0.	0.	0.
time (sec)	N/A	0.982	0.755	0.007	1.04	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	362	0	466	0	0	0
normalized size	1	1.	0.66	0.	0.85	0.	0.	0.
time (sec)	N/A	0.718	0.611	0.007	1.038	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	285	0	348	0	0	0
normalized size	1	1.	0.73	0.	0.89	0.	0.	0.
time (sec)	N/A	0.449	0.49	0.005	1.007	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	137	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.235	0.005	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	0.714	0.008	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	285	0	288	0	0	0
normalized size	1	1.	0.86	0.	0.87	0.	0.	0.
time (sec)	N/A	0.541	1.188	0.007	1.164	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	389	0	387	0	0	0
normalized size	1	1.	0.85	0.	0.84	0.	0.	0.
time (sec)	N/A	0.672	1.304	0.006	1.201	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	584	505	0	460	0	0	0
normalized size	1	1.	0.86	0.	0.79	0.	0.	0.
time (sec)	N/A	0.845	1.518	0.007	1.189	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	900	900	583	0	699	0	0	0
normalized size	1	1.	0.65	0.	0.78	0.	0.	0.
time (sec)	N/A	1.185	1.28	0.047	1.054	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	645	472	0	556	0	0	0
normalized size	1	1.	0.73	0.	0.86	0.	0.	0.
time (sec)	N/A	0.818	1.033	0.05	1.05	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	298	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.619	0.356	0.049	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	280	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	0.834	0.049	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	343	343	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.74	1.986	0.049	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	488	0	431	0	0	0
normalized size	1	1.	0.95	0.	0.84	0.	0.	0.
time (sec)	N/A	0.823	1.632	0.048	1.218	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	767	767	621	0	544	0	0	0
normalized size	1	1.	0.81	0.	0.71	0.	0.	0.
time (sec)	N/A	1.125	1.953	0.052	1.234	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder

the integral was to solve. In this test, problem number [163] had the largest ratio of [ 1.214 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	9	0.333
2	A	4	3	1.	9	0.333
3	A	4	3	1.	9	0.333
4	A	4	3	1.	7	0.429
5	A	3	3	1.	5	0.6
6	A	1	1	1.	9	0.111
7	A	5	5	1.	9	0.556
8	A	4	3	1.	9	0.333
9	A	4	3	1.	9	0.333
10	A	4	3	1.	9	0.333
11	A	5	3	1.	9	0.333
12	A	5	3	1.	9	0.333
13	A	5	3	1.	7	0.429
14	A	4	3	1.	5	0.6
15	A	1	1	1.	9	0.111
16	A	6	5	1.	9	0.556
17	A	5	3	1.	9	0.333
18	A	5	3	1.	9	0.333
19	A	5	4	1.	11	0.364
20	A	5	4	1.	11	0.364
21	A	4	4	1.	9	0.444
22	A	1	1	1.	11	0.091
23	A	6	6	1.	11	0.546
24	A	5	4	1.	11	0.364
25	A	5	4	1.	11	0.364
26	A	5	4	1.	11	0.364
27	A	5	4	1.	11	0.364
28	A	4	4	1.	7	0.571
29	A	3	3	1.	11	0.273
30	A	4	4	1.	11	0.364
31	A	5	4	1.	11	0.364
32	A	6	4	1.	11	0.364
33	A	6	4	1.	11	0.364
34	A	5	4	1.	9	0.444
35	A	1	1	1.	11	0.091
36	A	7	6	1.	11	0.546
37	A	6	4	1.	11	0.364
38	A	6	4	1.	11	0.364
39	A	6	4	1.	11	0.364
40	A	6	4	1.	11	0.364
41	A	5	4	1.	7	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	4	3	1.	11	0.273
43	A	5	4	1.	11	0.364
44	A	6	4	1.	11	0.364
45	A	3	3	1.	11	0.273
46	A	3	3	1.	9	0.333
47	A	3	3	1.	7	0.429
48	A	1	1	1.	11	0.091
49	A	3	3	1.	11	0.273
50	A	3	3	1.	11	0.273
51	A	3	3	1.	11	0.273
52	A	4	3	1.	11	0.273
53	A	4	3	1.	9	0.333
54	A	4	3	1.	7	0.429
55	A	1	1	1.	11	0.091
56	A	4	3	1.	11	0.273
57	A	4	3	1.	11	0.273
58	A	4	3	1.	11	0.273
59	A	7	5	1.	13	0.385
60	A	6	5	1.	13	0.385
61	A	5	5	1.	13	0.385
62	A	4	4	1.	13	0.308
63	A	5	5	1.	13	0.385
64	A	6	5	1.	13	0.385
65	A	9	5	1.	13	0.385
66	A	8	5	1.	13	0.385
67	A	7	5	1.	13	0.385
68	A	6	5	1.	13	0.385
69	A	5	4	1.	13	0.308
70	A	6	5	1.	13	0.385
71	A	7	5	1.	13	0.385
72	A	9	8	1.	15	0.533
73	A	8	8	1.	15	0.533
74	A	8	8	1.	15	0.533
75	A	7	7	1.	15	0.467
76	A	7	7	1.	15	0.467
77	A	8	8	1.	15	0.533
78	A	10	8	1.	15	0.533
79	A	10	8	1.	15	0.533
80	A	9	8	1.	15	0.533
81	A	9	8	1.	15	0.533
82	A	8	7	1.	15	0.467
83	A	8	7	1.	15	0.467
84	A	9	8	1.	15	0.533
85	A	9	8	1.	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.	15	0.267
87	A	4	4	1.	15	0.267
88	A	4	4	1.	15	0.267
89	A	4	4	1.	15	0.267
90	A	4	4	1.	15	0.267
91	A	5	4	1.	15	0.267
92	A	5	4	1.	15	0.267
93	A	5	4	1.	15	0.267
94	A	5	4	1.	15	0.267
95	A	5	4	1.	15	0.267
96	A	0	0	0.	0	0.
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	0	0	0.	0	0.
100	A	0	0	0.	0	0.
101	A	2	1	1.	15	0.067
102	A	3	3	1.	11	0.273
103	A	4	3	1.	11	0.273
104	A	5	3	1.	11	0.273
105	A	4	4	1.	13	0.308
106	A	5	4	1.	13	0.308
107	A	6	4	1.	13	0.308
108	A	4	4	1.	13	0.308
109	A	5	4	1.	13	0.308
110	A	6	4	1.	13	0.308
111	A	4	4	1.	13	0.308
112	A	5	4	1.	13	0.308
113	A	6	4	1.	13	0.308
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.
116	A	1	1	1.	9	0.111
117	A	0	0	0.	0	0.
118	A	0	0	0.	0	0.
119	A	0	0	0.	0	0.
120	A	0	0	0.	0	0.
121	A	1	1	1.	11	0.091
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	13	8	1.	13	0.615
125	A	10	8	1.	11	0.727
126	A	7	7	1.	9	0.778
127	A	3	3	1.	13	0.231
128	A	7	9	1.	13	0.692
129	A	11	11	1.	13	0.846

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	14	11	1.	13	0.846
131	A	33	13	1.	13	1.
132	A	19	12	1.	11	1.091
133	A	9	8	1.	9	0.889
134	A	0	0	0.	0	0.
135	A	6	5	1.	13	0.385
136	A	12	13	1.	13	1.
137	A	16	8	1.	17	0.471
138	A	13	8	1.	17	0.471
139	A	10	8	1.	15	0.533
140	A	7	7	1.	9	0.778
141	A	3	3	1.	17	0.176
142	A	8	5	1.	17	0.294
143	A	12	8	1.	17	0.471
144	A	15	9	1.	17	0.529
145	A	5	5	1.	9	0.556
146	A	5	5	1.	12	0.417
147	A	8	6	1.	12	0.5
148	A	8	6	1.	15	0.4
149	A	1	1	1.	34	0.029
150	A	1	1	1.	34	0.029
151	A	1	1	1.	34	0.029
152	A	1	1	1.	37	0.027
153	A	2	2	1.	53	0.038
154	A	2	2	1.	53	0.038
155	A	4	4	1.	76	0.053
156	A	5	3	1.	19	0.158
157	A	4	3	1.	19	0.158
158	A	3	3	1.	17	0.176
159	A	2	2	1.	15	0.133
160	A	0	0	0.	0	0.
161	A	38	16	1.	16	1.
162	A	31	16	1.	16	1.
163	A	22	17	1.	14	1.214
164	A	15	12	1.	13	0.923
165	A	1	2	1.	16	0.125
166	A	10	13	1.	16	0.812
167	A	23	17	1.	16	1.062
168	A	30	17	1.	16	1.062
169	A	37	17	1.	16	1.062
170	A	37	20	0.87	20	1.
171	A	30	20	0.87	18	1.111
172	A	18	14	1.	17	0.824
173	A	3	3	1.	20	0.15

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
174	A	19	15	1.06	20	0.75
175	A	31	22	1.05	20	1.1
176	A	42	24	1.03	20	1.2
177	A	108	20	1.	27	0.741
178	A	67	20	1.	25	0.8
179	A	42	17	1.	24	0.708
180	A	0	0	0.	0	0.
181	A	22	9	1.	27	0.333
182	A	44	16	1.	27	0.593
183	A	78	18	1.	27	0.667
184	A	52	17	1.	21	0.81
185	A	40	21	1.	19	1.105
186	A	26	20	1.	18	1.111
187	A	18	15	1.	21	0.714
188	A	13	17	1.	21	0.81
189	A	30	20	1.	21	0.952
190	A	41	19	1.	21	0.905
191	A	51	19	1.	21	0.905
192	A	60	22	1.	24	0.917
193	A	43	21	1.	23	0.913
194	A	29	24	1.	26	0.923
195	A	19	21	1.	26	0.808
196	A	32	22	1.	26	0.846
197	A	43	20	1.	26	0.769
198	A	61	19	1.	26	0.731

# Chapter 3

## Listing of integrals

### 3.1 $\int x^4 \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=86

$$\frac{1}{5}x^5 \text{PolyLog}(2, ax) - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} - \frac{\log(1-ax)}{25a^5} - \frac{x^4}{100a} + \frac{1}{25}x^5 \log(1-ax) - \frac{x^5}{125}$$

[Out]  $-x/(25*a^4) - x^2/(50*a^3) - x^3/(75*a^2) - x^4/(100*a) - x^5/125 - \text{Log}[1 - a*x]/(25*a^5) + (x^5*\text{Log}[1 - a*x])/25 + (x^5*\text{PolyLog}[2, a*x])/5$

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**Rubi [A]** time = 0.0533019, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 43}

$$\frac{1}{5}x^5 \text{PolyLog}(2, ax) - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} - \frac{\log(1-ax)}{25a^5} - \frac{x^4}{100a} + \frac{1}{25}x^5 \log(1-ax) - \frac{x^5}{125}$$

Antiderivative was successfully verified.

[In] Int[x^4\*PolyLog[2, a\*x], x]

[Out]  $-x/(25*a^4) - x^2/(50*a^3) - x^3/(75*a^2) - x^4/(100*a) - x^5/125 - \text{Log}[1 - a*x]/(25*a^5) + (x^5*\text{Log}[1 - a*x])/25 + (x^5*\text{PolyLog}[2, a*x])/5$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n},

$x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

### Rubi steps

$$\begin{aligned} \int x^4 \text{Li}_2(ax) dx &= \frac{1}{5} x^5 \text{Li}_2(ax) + \frac{1}{5} \int x^4 \log(1 - ax) dx \\ &= \frac{1}{25} x^5 \log(1 - ax) + \frac{1}{5} x^5 \text{Li}_2(ax) + \frac{1}{25} a \int \frac{x^5}{1 - ax} dx \\ &= \frac{1}{25} x^5 \log(1 - ax) + \frac{1}{5} x^5 \text{Li}_2(ax) + \frac{1}{25} a \int \left( -\frac{1}{a^5} - \frac{x}{a^4} - \frac{x^2}{a^3} - \frac{x^3}{a^2} - \frac{x^4}{a} - \frac{1}{a^5(-1 + ax)} \right) dx \\ &= -\frac{x}{25a^4} - \frac{x^2}{50a^3} - \frac{x^3}{75a^2} - \frac{x^4}{100a} - \frac{x^5}{125} - \frac{\log(1 - ax)}{25a^5} + \frac{1}{25} x^5 \log(1 - ax) + \frac{1}{5} x^5 \text{Li}_2(ax) \end{aligned}$$

**Mathematica [A]** time = 0.04193, size = 73, normalized size = 0.85

$$\frac{300a^5x^5\text{PolyLog}(2, ax) - ax(12a^4x^4 + 15a^3x^3 + 20a^2x^2 + 30ax + 60) + 60(a^5x^5 - 1)\log(1 - ax)}{1500a^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*PolyLog[2, a\*x], x]

[Out]  $(-(a*x*(60 + 30*a*x + 20*a^2*x^2 + 15*a^3*x^3 + 12*a^4*x^4)) + 60*(-1 + a^5*x^5)*\text{Log}[1 - a*x] + 300*a^5*x^5*\text{PolyLog}[2, a*x])/(1500*a^5)$

**Maple [A]** time = 0.046, size = 76, normalized size = 0.9

$$-\frac{x}{25a^4} - \frac{\ln(-ax + 1)}{25a^5} - \frac{x^2}{50a^3} + \frac{x^5 \text{polylog}(2, ax)}{5} + \frac{x^5 \ln(-ax + 1)}{25} + \frac{137}{1500a^5} - \frac{x^4}{100a} - \frac{x^3}{75a^2} - \frac{x^5}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*polylog(2,a\*x), x)

[Out]  $-1/25*x/a^4 - 1/25*\ln(-a*x+1)/a^5 - 1/50*x^2/a^3 + 1/5*x^5*\text{polylog}(2, a*x) + 1/25*x^5*\ln(-a*x+1) + 137/1500/a^5 - 1/100*x^4/a - 1/75*x^3/a^2 - 1/125*x^5$

**Maxima [A]** time = 0.964112, size = 97, normalized size = 1.13

$$\frac{300a^5x^5\text{Li}_2(ax) - 12a^5x^5 - 15a^4x^4 - 20a^3x^3 - 30a^2x^2 - 60ax + 60(a^5x^5 - 1)\log(-ax + 1)}{1500a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x), x, algorithm="maxima")

[Out]  $1/1500*(300*a^5*x^5*\text{dilog}(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30*a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*\log(-a*x + 1))/a^5$

**Fricas [A]** time = 2.68893, size = 177, normalized size = 2.06

$$\frac{300 a^5 x^5 \operatorname{Li}_2(ax) - 12 a^5 x^5 - 15 a^4 x^4 - 20 a^3 x^3 - 30 a^2 x^2 - 60 a x + 60 (a^5 x^5 - 1) \log(-a x + 1)}{1500 a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x),x, algorithm="fricas")

[Out] 1/1500\*(300\*a^5\*x^5\*dilog(a\*x) - 12\*a^5\*x^5 - 15\*a^4\*x^4 - 20\*a^3\*x^3 - 30\*a^2\*x^2 - 60\*a\*x + 60\*(a^5\*x^5 - 1)\*log(-a\*x + 1))/a^5

**Sympy [A]** time = 15.264, size = 66, normalized size = 0.77

$$\begin{cases} -\frac{x^5 \operatorname{Li}_1(ax)}{25} + \frac{x^5 \operatorname{Li}_2(ax)}{5} - \frac{x^5}{125} - \frac{x^4}{100a} - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} + \frac{\operatorname{Li}_1(ax)}{25a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*polylog(2,a\*x),x)

[Out] Piecewise((-x\*\*5\*polylog(1, a\*x)/25 + x\*\*5\*polylog(2, a\*x)/5 - x\*\*5/125 - x\*\*4/(100\*a) - x\*\*3/(75\*a\*\*2) - x\*\*2/(50\*a\*\*3) - x/(25\*a\*\*4) + polylog(1, a\*x)/(25\*a\*\*5), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate(x^4\*dilog(a\*x), x)

## 3.2 $\int x^3 \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=76

$$\frac{1}{4}x^4 \text{PolyLog}(2, ax) - \frac{x^2}{32a^2} - \frac{x}{16a^3} - \frac{\log(1-ax)}{16a^4} - \frac{x^3}{48a} + \frac{1}{16}x^4 \log(1-ax) - \frac{x^4}{64}$$

[Out]  $-x/(16*a^3) - x^2/(32*a^2) - x^3/(48*a) - x^4/64 - \text{Log}[1 - a*x]/(16*a^4) + (x^4*\text{Log}[1 - a*x])/16 + (x^4*\text{PolyLog}[2, a*x])/4$

**Rubi [A]** time = 0.0447573, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 43}

$$\frac{1}{4}x^4 \text{PolyLog}(2, ax) - \frac{x^2}{32a^2} - \frac{x}{16a^3} - \frac{\log(1-ax)}{16a^4} - \frac{x^3}{48a} + \frac{1}{16}x^4 \log(1-ax) - \frac{x^4}{64}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{PolyLog}[2, a*x], x]$

[Out]  $-x/(16*a^3) - x^2/(32*a^2) - x^3/(48*a) - x^4/64 - \text{Log}[1 - a*x]/(16*a^4) + (x^4*\text{Log}[1 - a*x])/16 + (x^4*\text{PolyLog}[2, a*x])/4$

### Rule 6591

$\text{Int}[\text{((d_.)*(x_.))^m_*PolyLog}[n, (a_.)*\text{((b_.)*(x_.)^p)^q], x\_Symbol] \text{ :> } \text{Simp}[\text{((d*x)^{m+1}*PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[\text{((p*q)/(m+1)), \text{Int}[\text{((d*x)^m*PolyLog}[n-1, a*(b*x^p)^q], x], x] /; \text{FreeQ}\{a, b, d, m, p, q\}, x] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0]$

### Rule 2395

$\text{Int}[\text{((a_.) + Log}[\text{(c_.)*((d_.) + (e_.)*(x_.))^n] \text{)*(b_.)*((f_.) + (g_.)*(x_.))^q}, x\_Symbol] \text{ :> } \text{Simp}[\text{((f + g*x)^{q+1}*(a + b*Log[c*(d + e*x)^n])/(g*(q+1)), x] - \text{Dist}[\text{(b*e*n)/(g*(q+1))}, \text{Int}[\text{(f + g*x)^{q+1}/(d + e*x)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[q, -1]$

### Rule 43

$\text{Int}[\text{((a_.) + (b_.)*(x_.))^m_*\text{((c_.) + (d_.)*(x_.))^n}, x\_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[\text{(a + b*x)^m*(c + d*x)^n}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\text{!IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n+1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

### Rubi steps

$$\begin{aligned} \int x^3 \text{Li}_2(ax) dx &= \frac{1}{4}x^4 \text{Li}_2(ax) + \frac{1}{4} \int x^3 \log(1-ax) dx \\ &= \frac{1}{16}x^4 \log(1-ax) + \frac{1}{4}x^4 \text{Li}_2(ax) + \frac{1}{16}a \int \frac{x^4}{1-ax} dx \\ &= \frac{1}{16}x^4 \log(1-ax) + \frac{1}{4}x^4 \text{Li}_2(ax) + \frac{1}{16}a \int \left( -\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)} \right) dx \\ &= -\frac{x}{16a^3} - \frac{x^2}{32a^2} - \frac{x^3}{48a} - \frac{x^4}{64} - \frac{\log(1-ax)}{16a^4} + \frac{1}{16}x^4 \log(1-ax) + \frac{1}{4}x^4 \text{Li}_2(ax) \end{aligned}$$

**Mathematica [A]** time = 0.0327425, size = 65, normalized size = 0.86

$$\frac{48a^4x^4\text{PolyLog}(2,ax) - ax(3a^3x^3 + 4a^2x^2 + 6ax + 12) + 12(a^4x^4 - 1)\log(1 - ax)}{192a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*PolyLog[2, a\*x], x]

[Out]  $(-(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3)) + 12*(-1 + a^4*x^4)*\text{Log}[1 - a*x] + 48*a^4*x^4*\text{PolyLog}[2, a*x])/(192*a^4)$

**Maple [A]** time = 0.046, size = 68, normalized size = 0.9

$$\frac{x^4\text{polylog}(2,ax)}{4} + \frac{x^4\ln(-ax+1)}{16} - \frac{\ln(-ax+1)}{16a^4} - \frac{x^4}{64} - \frac{x^3}{48a} - \frac{x^2}{32a^2} - \frac{x}{16a^3} + \frac{25}{192a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*polylog(2,a\*x), x)

[Out]  $1/4*x^4*\text{polylog}(2,a*x) + 1/16*x^4*\ln(-a*x+1) - 1/16*\ln(-a*x+1)/a^4 - 1/64*x^4 - 1/4*8*x^3/a - 1/32*x^2/a^2 - 1/16*x/a^3 + 25/192/a^4$

**Maxima [A]** time = 0.98341, size = 86, normalized size = 1.13

$$\frac{48a^4x^4\text{Li}_2(ax) - 3a^4x^4 - 4a^3x^3 - 6a^2x^2 - 12ax + 12(a^4x^4 - 1)\log(-ax + 1)}{192a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2,a\*x), x, algorithm="maxima")

[Out]  $1/192*(48*a^4*x^4*\text{dilog}(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*\log(-a*x + 1))/a^4$

**Fricas [A]** time = 2.66203, size = 153, normalized size = 2.01

$$\frac{48a^4x^4\text{Li}_2(ax) - 3a^4x^4 - 4a^3x^3 - 6a^2x^2 - 12ax + 12(a^4x^4 - 1)\log(-ax + 1)}{192a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2,a\*x), x, algorithm="fricas")

[Out]  $1/192*(48*a^4*x^4*\text{dilog}(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*\log(-a*x + 1))/a^4$

**Sympy [A]** time = 8.74118, size = 58, normalized size = 0.76

$$\begin{cases} -\frac{x^4 \operatorname{Li}_1(ax)}{16} + \frac{x^4 \operatorname{Li}_2(ax)}{4} - \frac{x^4}{64} - \frac{x^3}{48a} - \frac{x^2}{32a^2} - \frac{x}{16a^3} + \frac{\operatorname{Li}_1(ax)}{16a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*polylog(2,a\*x),x)

[Out] Piecewise((-x\*\*4\*polylog(1, a\*x)/16 + x\*\*4\*polylog(2, a\*x)/4 - x\*\*4/64 - x\*\*3/(48\*a) - x\*\*2/(32\*a\*\*2) - x/(16\*a\*\*3) + polylog(1, a\*x)/(16\*a\*\*4), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate(x^3\*dilog(a\*x), x)



### 3.3 $\int x^2 \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=66

$$\frac{1}{3}x^3 \text{PolyLog}(2, ax) - \frac{x}{9a^2} - \frac{\log(1-ax)}{9a^3} - \frac{x^2}{18a} + \frac{1}{9}x^3 \log(1-ax) - \frac{x^3}{27}$$

[Out]  $-x/(9*a^2) - x^2/(18*a) - x^3/27 - \text{Log}[1 - a*x]/(9*a^3) + (x^3*\text{Log}[1 - a*x])/9 + (x^3*\text{PolyLog}[2, a*x])/3$

**Rubi [A]** time = 0.0396548, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 43}

$$\frac{1}{3}x^3 \text{PolyLog}(2, ax) - \frac{x}{9a^2} - \frac{\log(1-ax)}{9a^3} - \frac{x^2}{18a} + \frac{1}{9}x^3 \log(1-ax) - \frac{x^3}{27}$$

Antiderivative was successfully verified.

[In] Int[x^2\*PolyLog[2, a\*x], x]

[Out]  $-x/(9*a^2) - x^2/(18*a) - x^3/27 - \text{Log}[1 - a*x]/(9*a^3) + (x^3*\text{Log}[1 - a*x])/9 + (x^3*\text{PolyLog}[2, a*x])/3$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m+n+2, 0])

#### Rubi steps

$$\begin{aligned} \int x^2 \text{Li}_2(ax) dx &= \frac{1}{3}x^3 \text{Li}_2(ax) + \frac{1}{3} \int x^2 \log(1-ax) dx \\ &= \frac{1}{9}x^3 \log(1-ax) + \frac{1}{3}x^3 \text{Li}_2(ax) + \frac{1}{9}a \int \frac{x^3}{1-ax} dx \\ &= \frac{1}{9}x^3 \log(1-ax) + \frac{1}{3}x^3 \text{Li}_2(ax) + \frac{1}{9}a \int \left( -\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)} \right) dx \\ &= -\frac{x}{9a^2} - \frac{x^2}{18a} - \frac{x^3}{27} - \frac{\log(1-ax)}{9a^3} + \frac{1}{9}x^3 \log(1-ax) + \frac{1}{3}x^3 \text{Li}_2(ax) \end{aligned}$$

**Mathematica [A]** time = 0.0292432, size = 57, normalized size = 0.86

$$\frac{18a^3x^3\text{PolyLog}(2,ax) - ax(2a^2x^2 + 3ax + 6) + 6(a^3x^3 - 1)\log(1 - ax)}{54a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[2, a\*x], x]

[Out]  $(-(a*x*(6 + 3*a*x + 2*a^2*x^2)) + 6*(-1 + a^3*x^3)*\text{Log}[1 - a*x] + 18*a^3*x^3*\text{PolyLog}[2, a*x])/(54*a^3)$

**Maple [A]** time = 0.047, size = 60, normalized size = 0.9

$$\frac{x^3\text{polylog}(2,ax)}{3} + \frac{x^3\ln(-ax+1)}{9} - \frac{\ln(-ax+1)}{9a^3} - \frac{x^3}{27} - \frac{x^2}{18a} - \frac{x}{9a^2} + \frac{11}{54a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(2,a\*x), x)

[Out]  $1/3*x^3*\text{polylog}(2,a*x)+1/9*x^3*\ln(-a*x+1)-1/9*\ln(-a*x+1)/a^3-1/27*x^3-1/18*x^2/a-1/9*x/a^2+11/54/a^3$

**Maxima [A]** time = 0.973299, size = 76, normalized size = 1.15

$$\frac{18a^3x^3\text{Li}_2(ax) - 2a^3x^3 - 3a^2x^2 - 6ax + 6(a^3x^3 - 1)\log(-ax + 1)}{54a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x), x, algorithm="maxima")

[Out]  $1/54*(18*a^3*x^3*\text{dilog}(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*\log(-a*x + 1))/a^3$

**Fricas [A]** time = 2.59888, size = 132, normalized size = 2.

$$\frac{18a^3x^3\text{Li}_2(ax) - 2a^3x^3 - 3a^2x^2 - 6ax + 6(a^3x^3 - 1)\log(-ax + 1)}{54a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x), x, algorithm="fricas")

[Out]  $1/54*(18*a^3*x^3*\text{dilog}(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*\log(-a*x + 1))/a^3$

**Sympy [A]** time = 4.42809, size = 49, normalized size = 0.74

$$\begin{cases} -\frac{x^3 \operatorname{Li}_1(ax)}{9} + \frac{x^3 \operatorname{Li}_2(ax)}{3} - \frac{x^3}{27} - \frac{x^2}{18a} - \frac{x}{9a^2} + \frac{\operatorname{Li}_1(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(2,a\*x),x)

[Out] Piecewise((-x\*\*3\*polylog(1, a\*x)/9 + x\*\*3\*polylog(2, a\*x)/3 - x\*\*3/27 - x\*\*2/(18\*a) - x/(9\*a\*\*2) + polylog(1, a\*x)/(9\*a\*\*3), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate(x^2\*dilog(a\*x), x)

### 3.4 $\int x \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=56

$$\frac{1}{2}x^2 \text{PolyLog}(2, ax) - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2 \log(1-ax) - \frac{x}{4a} - \frac{x^2}{8}$$

[Out]  $-x/(4*a) - x^2/8 - \text{Log}[1 - a*x]/(4*a^2) + (x^2*\text{Log}[1 - a*x])/4 + (x^2*\text{PolyLog}[2, a*x])/2$

**Rubi [A]** time = 0.0282725, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6591, 2395, 43}

$$\frac{1}{2}x^2 \text{PolyLog}(2, ax) - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2 \log(1-ax) - \frac{x}{4a} - \frac{x^2}{8}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[2, a\*x], x]

[Out]  $-x/(4*a) - x^2/8 - \text{Log}[1 - a*x]/(4*a^2) + (x^2*\text{Log}[1 - a*x])/4 + (x^2*\text{PolyLog}[2, a*x])/2$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int x \text{Li}_2(ax) dx &= \frac{1}{2}x^2 \text{Li}_2(ax) + \frac{1}{2} \int x \log(1-ax) dx \\ &= \frac{1}{4}x^2 \log(1-ax) + \frac{1}{2}x^2 \text{Li}_2(ax) + \frac{1}{4}a \int \frac{x^2}{1-ax} dx \\ &= \frac{1}{4}x^2 \log(1-ax) + \frac{1}{2}x^2 \text{Li}_2(ax) + \frac{1}{4}a \int \left( -\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)} \right) dx \\ &= -\frac{x}{4a} - \frac{x^2}{8} - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2 \log(1-ax) + \frac{1}{2}x^2 \text{Li}_2(ax) \end{aligned}$$

**Mathematica [A]** time = 0.0238254, size = 48, normalized size = 0.86

$$\frac{4a^2x^2\text{PolyLog}(2, ax) + 2(a^2x^2 - 1)\log(1 - ax) - ax(ax + 2)}{8a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[2, a\*x], x]

[Out]  $(-(a*x*(2 + a*x)) + 2*(-1 + a^2*x^2)*\text{Log}[1 - a*x] + 4*a^2*x^2*\text{PolyLog}[2, a*x])/(8*a^2)$

**Maple [A]** time = 0.046, size = 52, normalized size = 0.9

$$\frac{x^2\text{polylog}(2, ax)}{2} + \frac{x^2 \ln(-ax + 1)}{4} - \frac{\ln(-ax + 1)}{4a^2} - \frac{x^2}{8} - \frac{x}{4a} + \frac{3}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(2, a\*x), x)

[Out]  $1/2*x^2*\text{polylog}(2, a*x) + 1/4*x^2*\ln(-a*x+1) - 1/4*\ln(-a*x+1)/a^2 - 1/8*x^2 - 1/4*x/a + 3/8/a^2$

**Maxima [A]** time = 0.965084, size = 65, normalized size = 1.16

$$\frac{4a^2x^2\text{Li}_2(ax) - a^2x^2 - 2ax + 2(a^2x^2 - 1)\log(-ax + 1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2, a\*x), x, algorithm="maxima")

[Out]  $1/8*(4*a^2*x^2*\text{dilog}(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*\log(-a*x + 1))/a^2$

**Fricas [A]** time = 2.58187, size = 111, normalized size = 1.98

$$\frac{4a^2x^2\text{Li}_2(ax) - a^2x^2 - 2ax + 2(a^2x^2 - 1)\log(-ax + 1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2, a\*x), x, algorithm="fricas")

[Out]  $1/8*(4*a^2*x^2*\text{dilog}(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*\log(-a*x + 1))/a^2$

**Sympy [A]** time = 2.43658, size = 41, normalized size = 0.73

$$\begin{cases} -\frac{x^2 \operatorname{Li}_1(ax)}{4} + \frac{x^2 \operatorname{Li}_2(ax)}{2} - \frac{x^2}{8} - \frac{x}{4a} + \frac{\operatorname{Li}_1(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,a\*x),x)

[Out] Piecewise((-x\*\*2\*polylog(1, a\*x)/4 + x\*\*2\*polylog(2, a\*x)/2 - x\*\*2/8 - x/(4\*a) + polylog(1, a\*x)/(4\*a\*\*2), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate(x\*dilog(a\*x), x)

### 3.5 $\int \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=29

$$x\text{PolyLog}(2, ax) - \frac{(1 - ax)\log(1 - ax)}{a} - x$$

[Out]  $-x - ((1 - a*x)*\text{Log}[1 - a*x])/a + x*\text{PolyLog}[2, a*x]$

**Rubi [A]** time = 0.0085078, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {6586, 2389, 2295}

$$x\text{PolyLog}(2, ax) - \frac{(1 - ax)\log(1 - ax)}{a} - x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[2, a*x], x]$

[Out]  $-x - ((1 - a*x)*\text{Log}[1 - a*x])/a + x*\text{PolyLog}[2, a*x]$

#### Rule 6586

$\text{Int}[\text{PolyLog}[n, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x\_Symbol] \rightarrow \text{Simp}[x*\text{PolyLog}[n, a*(b*x^p)^q], x] - \text{Dist}[p*q, \text{Int}[\text{PolyLog}[n - 1, a*(b*x^p)^q], x], x] / ; \text{FreeQ}\{a, b, p, q\}, x \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 2389

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^(n_.))* (b_.)]^(p_.), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] / ; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$

#### Rule 2295

$\text{Int}[\text{Log}[(c_.)*(x_)^(n_.)], x\_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] / ; \text{FreeQ}\{c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int \text{Li}_2(ax) dx &= x\text{Li}_2(ax) + \int \log(1 - ax) dx \\ &= x\text{Li}_2(ax) - \frac{\text{Subst}(\int \log(x) dx, x, 1 - ax)}{a} \\ &= -x - \frac{(1 - ax)\log(1 - ax)}{a} + x\text{Li}_2(ax) \end{aligned}$$

**Mathematica [A]** time = 0.011601, size = 26, normalized size = 0.9

$$x\text{PolyLog}(2, ax) + \left(x - \frac{1}{a}\right)\log(1 - ax) - x$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x], x]

[Out]  $-x + (-a^{-1} + x) \cdot \text{Log}[1 - a*x] + x \cdot \text{PolyLog}[2, a*x]$

**Maple [A]** time = 0.045, size = 36, normalized size = 1.2

$$x \text{polylog}(2, ax) + \ln(-ax + 1)x - x - \frac{\ln(-ax + 1)}{a} + a^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x), x)

[Out]  $x \cdot \text{polylog}(2, a*x) + \ln(-a*x+1) * x - x - 1/a * \ln(-a*x+1) + 1/a$

**Maxima [A]** time = 0.980908, size = 39, normalized size = 1.34

$$\frac{ax \text{Li}_2(ax) - ax + (ax - 1) \log(-ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x), x, algorithm="maxima")

[Out]  $(a*x*dilog(a*x) - a*x + (a*x - 1)*\log(-a*x + 1))/a$

**Fricas [A]** time = 2.63468, size = 70, normalized size = 2.41

$$\frac{ax \text{Li}_2(ax) - ax + (ax - 1) \log(-ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x), x, algorithm="fricas")

[Out]  $(a*x*dilog(a*x) - a*x + (a*x - 1)*\log(-a*x + 1))/a$

**Sympy [A]** time = 1.20944, size = 22, normalized size = 0.76

$$\begin{cases} -x \text{Li}_1(ax) + x \text{Li}_2(ax) - x + \frac{\text{Li}_1(ax)}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x), x)

[Out] Piecewise((-x\*polylog(1, a\*x) + x\*polylog(2, a\*x) - x + polylog(1, a\*x)/a, Ne(a, 0)), (0, True))



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x), x)
```

### 3.6 $\int \frac{\text{PolyLog}(2,ax)}{x} dx$

**Optimal.** Leaf size=5

PolyLog(3, ax)

[Out] PolyLog[3, a\*x]

**Rubi [A]** time = 0.0090331, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {6589}

PolyLog(3, ax)

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/x, x]

[Out] PolyLog[3, a\*x]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\int \frac{\text{Li}_2(ax)}{x} dx = \text{Li}_3(ax)$$

**Mathematica [A]** time = 0.0012581, size = 5, normalized size = 1.

PolyLog(3, ax)

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/x, x]

[Out] PolyLog[3, a\*x]

**Maple [A]** time = 0.043, size = 6, normalized size = 1.2

polylog(3, ax)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/x, x)

[Out] polylog(3, a\*x)

---

**Maxima [A]** time = 0.975433, size = 7, normalized size = 1.4

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x,x, algorithm="maxima")

[Out] polylog(3, a\*x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x,x, algorithm="fricas")

[Out] integral(dilog(a\*x)/x, x)

---

**Sympy [A]** time = 1.71222, size = 3, normalized size = 0.6

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x,x)

[Out] polylog(3, a\*x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x,x, algorithm="giac")

[Out] integrate(dilog(a\*x)/x, x)

### 3.7 $\int \frac{\text{PolyLog}(2, ax)}{x^2} dx$

**Optimal.** Leaf size=36

$$-\frac{\text{PolyLog}(2, ax)}{x} + a \log(x) - a \log(1 - ax) + \frac{\log(1 - ax)}{x}$$

[Out] a\*Log[x] - a\*Log[1 - a\*x] + Log[1 - a\*x]/x - PolyLog[2, a\*x]/x

**Rubi [A]** time = 0.0218984, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6591, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2, ax)}{x} + a \log(x) - a \log(1 - ax) + \frac{\log(1 - ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/x^2, x]

[Out] a\*Log[x] - a\*Log[1 - a\*x] + Log[1 - a\*x]/x - PolyLog[2, a\*x]/x

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e^n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(ax)}{x^2} dx &= -\frac{\operatorname{Li}_2(ax)}{x} - \int \frac{\log(1-ax)}{x^2} dx \\
&= \frac{\log(1-ax)}{x} - \frac{\operatorname{Li}_2(ax)}{x} + a \int \frac{1}{x(1-ax)} dx \\
&= \frac{\log(1-ax)}{x} - \frac{\operatorname{Li}_2(ax)}{x} + a \int \frac{1}{x} dx + a^2 \int \frac{1}{1-ax} dx \\
&= a \log(x) - a \log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\operatorname{Li}_2(ax)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0097221, size = 36, normalized size = 1.

$$-\frac{\operatorname{PolyLog}(2, ax)}{x} + a \log(x) - a \log(1-ax) + \frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/x^2, x]

[Out] a\*Log[x] - a\*Log[1 - a\*x] + Log[1 - a\*x]/x - PolyLog[2, a\*x]/x

**Maple [A]** time = 0.119, size = 40, normalized size = 1.1

$$-\frac{\operatorname{polylog}(2, ax)}{x} + a \ln(-ax) - a \ln(-ax + 1) + \frac{\ln(-ax + 1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/x^2, x)

[Out] -polylog(2, a\*x)/x + a\*ln(-a\*x) - a\*ln(-a\*x+1) + ln(-a\*x+1)/x

**Maxima [A]** time = 1.00914, size = 38, normalized size = 1.06

$$a \log(x) - \frac{(ax - 1) \log(-ax + 1) + \operatorname{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x)/x^2, x, algorithm="maxima")

[Out] a\*log(x) - ((a\*x - 1)\*log(-a\*x + 1) + dilog(a\*x))/x

**Fricas [A]** time = 2.78236, size = 88, normalized size = 2.44

$$-\frac{ax \log(ax - 1) - ax \log(x) + \operatorname{Li}_2(ax) - \log(-ax + 1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x)/x^2, x, algorithm="fricas")

[Out]  $-(a*x*\log(a*x - 1) - a*x*\log(x) + \operatorname{dilog}(a*x) - \log(-a*x + 1))/x$

**Sympy [A]** time = 1.56346, size = 24, normalized size = 0.67

$$a \log(x) + a \operatorname{Li}_1(ax) - \frac{\operatorname{Li}_1(ax)}{x} - \frac{\operatorname{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x)/x**2,x)`

[Out]  $a*\log(x) + a*\operatorname{polylog}(1, a*x) - \operatorname{polylog}(1, a*x)/x - \operatorname{polylog}(2, a*x)/x$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x)/x^2,x, algorithm="giac")`

[Out] `integrate(dilog(a*x)/x^2, x)`

### 3.8 $\int \frac{\text{PolyLog}(2,ax)}{x^3} dx$

**Optimal.** Leaf size=58

$$-\frac{\text{PolyLog}(2,ax)}{2x^2} + \frac{1}{4}a^2 \log(x) - \frac{1}{4}a^2 \log(1-ax) + \frac{\log(1-ax)}{4x^2} - \frac{a}{4x}$$

[Out] -a/(4\*x) + (a^2\*Log[x])/4 - (a^2\*Log[1 - a\*x])/4 + Log[1 - a\*x]/(4\*x^2) - PolyLog[2, a\*x]/(2\*x^2)

**Rubi [A]** time = 0.0332573, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{2x^2} + \frac{1}{4}a^2 \log(x) - \frac{1}{4}a^2 \log(1-ax) + \frac{\log(1-ax)}{4x^2} - \frac{a}{4x}$$

Antiderivative was successfully verified.

[In] Int [PolyLog [2, a\*x] / x^3, x]

[Out] -a/(4\*x) + (a^2\*Log[x])/4 - (a^2\*Log[1 - a\*x])/4 + Log[1 - a\*x]/(4\*x^2) - PolyLog[2, a\*x]/(2\*x^2)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax)}{x^3} dx &= -\frac{\text{Li}_2(ax)}{2x^2} - \frac{1}{2} \int \frac{\log(1-ax)}{x^3} dx \\ &= \frac{\log(1-ax)}{4x^2} - \frac{\text{Li}_2(ax)}{2x^2} + \frac{1}{4}a \int \frac{1}{x^2(1-ax)} dx \\ &= \frac{\log(1-ax)}{4x^2} - \frac{\text{Li}_2(ax)}{2x^2} + \frac{1}{4}a \int \left( \frac{1}{x^2} + \frac{a}{x} - \frac{a^2}{-1+ax} \right) dx \\ &= -\frac{a}{4x} + \frac{1}{4}a^2 \log(x) - \frac{1}{4}a^2 \log(1-ax) + \frac{\log(1-ax)}{4x^2} - \frac{\text{Li}_2(ax)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0221772, size = 50, normalized size = 0.86

$$\frac{-2\text{PolyLog}(2, ax) + a^2x^2 \log(x) - a^2x^2 \log(1 - ax) - ax + \log(1 - ax)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/x^3, x]

[Out]  $(-(a*x) + a^2*x^2*\text{Log}[x] + \text{Log}[1 - a*x] - a^2*x^2*\text{Log}[1 - a*x] - 2*\text{PolyLog}[2, a*x])/(4*x^2)$

**Maple [A]** time = 0.118, size = 52, normalized size = 0.9

$$-\frac{\text{polylog}(2, ax)}{2x^2} - \frac{a}{4x} + \frac{a^2 \ln(-ax)}{4} - \frac{a^2 \ln(-ax + 1)}{4} + \frac{\ln(-ax + 1)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x)/x^3,x)

[Out]  $-1/2*\text{polylog}(2, a*x)/x^2 - 1/4*a/x + 1/4*a^2*\ln(-a*x) - 1/4*a^2*\ln(-a*x+1) + 1/4*\ln(-a*x+1)/x^2$

**Maxima [A]** time = 0.981041, size = 54, normalized size = 0.93

$$\frac{1}{4}a^2 \log(x) - \frac{ax + (a^2x^2 - 1) \log(-ax + 1) + 2 \text{Li}_2(ax)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x^3,x, algorithm="maxima")

[Out]  $1/4*a^2*\log(x) - 1/4*(a*x + (a^2*x^2 - 1)*\log(-a*x + 1) + 2*\text{dilog}(a*x))/x^2$

**Fricas [A]** time = 2.62905, size = 117, normalized size = 2.02

$$\frac{a^2x^2 \log(ax - 1) - a^2x^2 \log(x) + ax + 2 \text{Li}_2(ax) - \log(-ax + 1)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x^3,x, algorithm="fricas")

[Out]  $-1/4*(a^2*x^2*\log(a*x - 1) - a^2*x^2*\log(x) + a*x + 2*\text{dilog}(a*x) - \log(-a*x + 1))/x^2$

**Sympy [A]** time = 2.96484, size = 42, normalized size = 0.72

$$\frac{a^2 \log(x)}{4} + \frac{a^2 \text{Li}_1(ax)}{4} - \frac{a}{4x} - \frac{\text{Li}_1(ax)}{4x^2} - \frac{\text{Li}_2(ax)}{2x^2}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**3,x)
```

```
[Out] a**2*log(x)/4 + a**2*polylog(1, a*x)/4 - a/(4*x) - polylog(1, a*x)/(4*x**2)
- polylog(2, a*x)/(2*x**2)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^3, x)
```

### 3.9 $\int \frac{\text{PolyLog}(2, ax)}{x^4} dx$

**Optimal.** Leaf size=68

$$-\frac{\text{PolyLog}(2, ax)}{3x^3} - \frac{a^2}{9x} + \frac{1}{9}a^3 \log(x) - \frac{1}{9}a^3 \log(1 - ax) - \frac{a}{18x^2} + \frac{\log(1 - ax)}{9x^3}$$

[Out]  $-a/(18*x^2) - a^2/(9*x) + (a^3*\text{Log}[x])/9 - (a^3*\text{Log}[1 - a*x])/9 + \text{Log}[1 - a*x]/(9*x^3) - \text{PolyLog}[2, a*x]/(3*x^3)$

**Rubi [A]** time = 0.0363225, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax)}{3x^3} - \frac{a^2}{9x} + \frac{1}{9}a^3 \log(x) - \frac{1}{9}a^3 \log(1 - ax) - \frac{a}{18x^2} + \frac{\log(1 - ax)}{9x^3}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/x^4, x]

[Out]  $-a/(18*x^2) - a^2/(9*x) + (a^3*\text{Log}[x])/9 - (a^3*\text{Log}[1 - a*x])/9 + \text{Log}[1 - a*x]/(9*x^3) - \text{PolyLog}[2, a*x]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^4} dx &= -\frac{\text{Li}_2(ax)}{3x^3} - \frac{1}{3} \int \frac{\log(1-ax)}{x^4} dx \\
&= \frac{\log(1-ax)}{9x^3} - \frac{\text{Li}_2(ax)}{3x^3} + \frac{1}{9} a \int \frac{1}{x^3(1-ax)} dx \\
&= \frac{\log(1-ax)}{9x^3} - \frac{\text{Li}_2(ax)}{3x^3} + \frac{1}{9} a \int \left( \frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax} \right) dx \\
&= -\frac{a}{18x^2} - \frac{a^2}{9x} + \frac{1}{9} a^3 \log(x) - \frac{1}{9} a^3 \log(1-ax) + \frac{\log(1-ax)}{9x^3} - \frac{\text{Li}_2(ax)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0371128, size = 52, normalized size = 0.76

$$-\frac{6\text{PolyLog}[2, ax] - 2a^3x^3 \log(x) + 2(a^3x^3 - 1) \log(1-ax) + ax(2ax + 1)}{18x^3}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/x^4, x]

[Out]  $-(a*x*(1 + 2*a*x) - 2*a^3*x^3*\text{Log}[x] + 2*(-1 + a^3*x^3)*\text{Log}[1 - a*x] + 6*\text{PolyLog}[2, a*x])/(18*x^3)$

**Maple [A]** time = 0.119, size = 60, normalized size = 0.9

$$-\frac{\text{polylog}(2, ax)}{3x^3} - \frac{a^2}{9x} + \frac{a^3 \ln(-ax)}{9} - \frac{a}{18x^2} - \frac{a^3 \ln(-ax + 1)}{9} + \frac{\ln(-ax + 1)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/x^4, x)

[Out]  $-1/3*\text{polylog}(2, a*x)/x^3 - 1/9*a^2/x + 1/9*a^3*\ln(-a*x) - 1/18*a/x^2 - 1/9*a^3*\ln(-a*x+1) + 1/9*\ln(-a*x+1)/x^3$

**Maxima [A]** time = 0.989735, size = 66, normalized size = 0.97

$$\frac{1}{9} a^3 \log(x) - \frac{2a^2x^2 + ax + 2(a^3x^3 - 1) \log(-ax + 1) + 6 \text{Li}_2(ax)}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x)/x^4, x, algorithm="maxima")

[Out]  $1/9*a^3*\log(x) - 1/18*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*\log(-a*x + 1) + 6*\text{dilog}(a*x))/x^3$

**Fricas [A]** time = 2.68791, size = 143, normalized size = 2.1

$$-\frac{2a^3x^3 \log(ax - 1) - 2a^3x^3 \log(x) + 2a^2x^2 + ax + 6 \text{Li}_2(ax) - 2 \log(-ax + 1)}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dilog(a*x) - 2*log(-a*x + 1))/x^3
```

**Sympy [A]** time = 5.56873, size = 51, normalized size = 0.75

$$\frac{a^3 \log(x)}{9} + \frac{a^3 \operatorname{Li}_1(ax)}{9} - \frac{a^2}{9x} - \frac{a}{18x^2} - \frac{\operatorname{Li}_1(ax)}{9x^3} - \frac{\operatorname{Li}_2(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**4,x)
```

```
[Out] a**3*log(x)/9 + a**3*polylog(1, a*x)/9 - a**2/(9*x) - a/(18*x**2) - polylog(1, a*x)/(9*x**3) - polylog(2, a*x)/(3*x**3)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^4, x)
```

### 3.10 $\int \frac{\text{PolyLog}(2, ax)}{x^5} dx$

**Optimal.** Leaf size=78

$$-\frac{\text{PolyLog}(2, ax)}{4x^4} - \frac{a^2}{32x^2} - \frac{a^3}{16x} + \frac{1}{16}a^4 \log(x) - \frac{1}{16}a^4 \log(1 - ax) - \frac{a}{48x^3} + \frac{\log(1 - ax)}{16x^4}$$

[Out]  $-a/(48*x^3) - a^2/(32*x^2) - a^3/(16*x) + (a^4*\text{Log}[x])/16 - (a^4*\text{Log}[1 - a*x])/16 + \text{Log}[1 - a*x]/(16*x^4) - \text{PolyLog}[2, a*x]/(4*x^4)$

**Rubi [A]** time = 0.0392817, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax)}{4x^4} - \frac{a^2}{32x^2} - \frac{a^3}{16x} + \frac{1}{16}a^4 \log(x) - \frac{1}{16}a^4 \log(1 - ax) - \frac{a}{48x^3} + \frac{\log(1 - ax)}{16x^4}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/x^5, x]

[Out]  $-a/(48*x^3) - a^2/(32*x^2) - a^3/(16*x) + (a^4*\text{Log}[x])/16 - (a^4*\text{Log}[1 - a*x])/16 + \text{Log}[1 - a*x]/(16*x^4) - \text{PolyLog}[2, a*x]/(4*x^4)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^5} dx &= -\frac{\text{Li}_2(ax)}{4x^4} - \frac{1}{4} \int \frac{\log(1-ax)}{x^5} dx \\
&= \frac{\log(1-ax)}{16x^4} - \frac{\text{Li}_2(ax)}{4x^4} + \frac{1}{16} a \int \frac{1}{x^4(1-ax)} dx \\
&= \frac{\log(1-ax)}{16x^4} - \frac{\text{Li}_2(ax)}{4x^4} + \frac{1}{16} a \int \left( \frac{1}{x^4} + \frac{a}{x^3} + \frac{a^2}{x^2} + \frac{a^3}{x} - \frac{a^4}{-1+ax} \right) dx \\
&= -\frac{a}{48x^3} - \frac{a^2}{32x^2} - \frac{a^3}{16x} + \frac{1}{16} a^4 \log(x) - \frac{1}{16} a^4 \log(1-ax) + \frac{\log(1-ax)}{16x^4} - \frac{\text{Li}_2(ax)}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.034528, size = 60, normalized size = 0.77

$$-\frac{24\text{PolyLog}(2, ax) + ax(6a^2x^2 + 3ax + 2) - 6a^4x^4 \log(x) + 6(a^4x^4 - 1) \log(1-ax)}{96x^4}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/x^5, x]

[Out] -(a\*x\*(2 + 3\*a\*x + 6\*a^2\*x^2) - 6\*a^4\*x^4\*Log[x] + 6\*(-1 + a^4\*x^4)\*Log[1 - a\*x] + 24\*PolyLog[2, a\*x])/(96\*x^4)

**Maple [A]** time = 0.128, size = 68, normalized size = 0.9

$$-\frac{\text{polylog}(2, ax)}{4x^4} - \frac{a^3}{16x} - \frac{a}{48x^3} - \frac{a^2}{32x^2} + \frac{a^4 \ln(-ax)}{16} - \frac{a^4 \ln(-ax + 1)}{16} + \frac{\ln(-ax + 1)}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/x^5, x)

[Out] -1/4\*polylog(2, a\*x)/x^4 - 1/16\*a^3/x - 1/48\*a/x^3 - 1/32\*a^2/x^2 + 1/16\*a^4\*ln(-a\*x) - 1/16\*a^4\*ln(-a\*x+1) + 1/16\*ln(-a\*x+1)/x^4

**Maxima [A]** time = 0.986238, size = 78, normalized size = 1.

$$\frac{1}{16} a^4 \log(x) - \frac{6a^3x^3 + 3a^2x^2 + 2ax + 6(a^4x^4 - 1) \log(-ax + 1) + 24 \text{Li}_2(ax)}{96x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x)/x^5, x, algorithm="maxima")

[Out] 1/16\*a^4\*log(x) - 1/96\*(6\*a^3\*x^3 + 3\*a^2\*x^2 + 2\*a\*x + 6\*(a^4\*x^4 - 1)\*log(-a\*x + 1) + 24\*dilog(a\*x))/x^4

**Fricas [A]** time = 2.78575, size = 163, normalized size = 2.09

$$-\frac{6a^4x^4 \log(ax - 1) - 6a^4x^4 \log(x) + 6a^3x^3 + 3a^2x^2 + 2ax + 24 \text{Li}_2(ax) - 6 \log(-ax + 1)}{96x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x^5,x, algorithm="fricas")

[Out]  $-1/96*(6*a^4*x^4*\log(ax - 1) - 6*a^4*x^4*\log(x) + 6*a^3*x^3 + 3*a^2*x^2 + 2*a*x + 24*\operatorname{dilog}(ax) - 6*\log(-ax + 1))/x^4$

**Sympy [A]** time = 11.2801, size = 60, normalized size = 0.77

$$\frac{a^4 \log(x)}{16} + \frac{a^4 \operatorname{Li}_1(ax)}{16} - \frac{a^3}{16x} - \frac{a^2}{32x^2} - \frac{a}{48x^3} - \frac{\operatorname{Li}_1(ax)}{16x^4} - \frac{\operatorname{Li}_2(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x\*\*5,x)

[Out]  $a**4*\log(x)/16 + a**4*\operatorname{polylog}(1, a*x)/16 - a**3/(16*x) - a**2/(32*x**2) - a/(48*x**3) - \operatorname{polylog}(1, a*x)/(16*x**4) - \operatorname{polylog}(2, a*x)/(4*x**4)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/x^5,x, algorithm="giac")

[Out] integrate(dilog(a\*x)/x^5, x)

### 3.11 $\int x^3 \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=88

$$-\frac{1}{16}x^4 \text{PolyLog}(2, ax) + \frac{1}{4}x^4 \text{PolyLog}(3, ax) + \frac{x^2}{128a^2} + \frac{x}{64a^3} + \frac{\log(1-ax)}{64a^4} + \frac{x^3}{192a} - \frac{1}{64}x^4 \log(1-ax) + \frac{x^4}{256}$$

[Out]  $x/(64*a^3) + x^2/(128*a^2) + x^3/(192*a) + x^4/256 + \text{Log}[1 - a*x]/(64*a^4) - (x^4*\text{Log}[1 - a*x])/64 - (x^4*\text{PolyLog}[2, a*x])/16 + (x^4*\text{PolyLog}[3, a*x])/4$

**Rubi [A]** time = 0.0573359, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 43}

$$-\frac{1}{16}x^4 \text{PolyLog}(2, ax) + \frac{1}{4}x^4 \text{PolyLog}(3, ax) + \frac{x^2}{128a^2} + \frac{x}{64a^3} + \frac{\log(1-ax)}{64a^4} + \frac{x^3}{192a} - \frac{1}{64}x^4 \log(1-ax) + \frac{x^4}{256}$$

Antiderivative was successfully verified.

[In] Int[x^3\*PolyLog[3, a\*x], x]

[Out]  $x/(64*a^3) + x^2/(128*a^2) + x^3/(192*a) + x^4/256 + \text{Log}[1 - a*x]/(64*a^4) - (x^4*\text{Log}[1 - a*x])/64 - (x^4*\text{PolyLog}[2, a*x])/16 + (x^4*\text{PolyLog}[3, a*x])/4$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int x^3 \text{Li}_3(ax) dx &= \frac{1}{4} x^4 \text{Li}_3(ax) - \frac{1}{4} \int x^3 \text{Li}_2(ax) dx \\
&= -\frac{1}{16} x^4 \text{Li}_2(ax) + \frac{1}{4} x^4 \text{Li}_3(ax) - \frac{1}{16} \int x^3 \log(1-ax) dx \\
&= -\frac{1}{64} x^4 \log(1-ax) - \frac{1}{16} x^4 \text{Li}_2(ax) + \frac{1}{4} x^4 \text{Li}_3(ax) - \frac{1}{64} a \int \frac{x^4}{1-ax} dx \\
&= -\frac{1}{64} x^4 \log(1-ax) - \frac{1}{16} x^4 \text{Li}_2(ax) + \frac{1}{4} x^4 \text{Li}_3(ax) - \frac{1}{64} a \int \left( -\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)} \right) dx \\
&= \frac{x}{64a^3} + \frac{x^2}{128a^2} + \frac{x^3}{192a} + \frac{x^4}{256} + \frac{\log(1-ax)}{64a^4} - \frac{1}{64} x^4 \log(1-ax) - \frac{1}{16} x^4 \text{Li}_2(ax) + \frac{1}{4} x^4 \text{Li}_3(ax)
\end{aligned}$$

**Mathematica [A]** time = 0.0118425, size = 86, normalized size = 0.98

$$\frac{-48a^4x^4\text{PolyLog}(2,ax) + 192a^4x^4\text{PolyLog}(3,ax) + 3a^4x^4 + 4a^3x^3 + 6a^2x^2 - 12a^4x^4\log(1-ax) + 12ax + 12\log(1-ax)}{768a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*PolyLog[3, a\*x], x]

[Out] (12\*a\*x + 6\*a^2\*x^2 + 4\*a^3\*x^3 + 3\*a^4\*x^4 + 12\*Log[1 - a\*x] - 12\*a^4\*x^4\*Log[1 - a\*x] - 48\*a^4\*x^4\*PolyLog[2, a\*x] + 192\*a^4\*x^4\*PolyLog[3, a\*x])/(768\*a^4)

**Maple [A]** time = 0.161, size = 78, normalized size = 0.9

$$\frac{1}{a^4} \left( -\frac{xa(15x^3a^3 + 20a^2x^2 + 30ax + 60)}{3840} - \frac{(-5x^4a^4 + 5)\ln(-ax + 1)}{320} + \frac{x^4a^4\text{polylog}(2,ax)}{16} - \frac{x^4a^4\text{polylog}(3,ax)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*polylog(3,a\*x), x)

[Out] -1/a^4\*(-1/3840\*x\*a\*(15\*a^3\*x^3+20\*a^2\*x^2+30\*a\*x+60)-1/320\*(-5\*a^4\*x^4+5)\*ln(-a\*x+1)+1/16\*x^4\*a^4\*polylog(2,a\*x)-1/4\*x^4\*a^4\*polylog(3,a\*x))

**Maxima [A]** time = 0.998854, size = 104, normalized size = 1.18

$$\frac{48a^4x^4\text{Li}_2(ax) - 192a^4x^4\text{Li}_3(ax) - 3a^4x^4 - 4a^3x^3 - 6a^2x^2 - 12ax + 12(a^4x^4 - 1)\log(-ax + 1)}{768a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x), x, algorithm="maxima")

[Out] -1/768\*(48\*a^4\*x^4\*dilog(a\*x) - 192\*a^4\*x^4\*polylog(3, a\*x) - 3\*a^4\*x^4 - 4\*a^3\*x^3 - 6\*a^2\*x^2 - 12\*a\*x + 12\*(a^4\*x^4 - 1)\*log(-a\*x + 1))/a^4

**Fricas [C]** time = 2.54882, size = 244, normalized size = 2.77

$$\frac{48 a^4 x^4 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 192 a^4 x^4 \operatorname{polylog}(3, ax) - 3 a^4 x^4 - 4 a^3 x^3 - 6 a^2 x^2 - 12 ax + 12 (a^4 x^4 - 1)}{768 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x),x, algorithm="fricas")

[Out] -1/768\*(48\*a^4\*x^4\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 192\*a^4\*x^4\*polylog(3, a\*x) - 3\*a^4\*x^4 - 4\*a^3\*x^3 - 6\*a^2\*x^2 - 12\*a\*x + 12\*(a^4\*x^4 - 1)\*log(-a\*x + 1))/a^4

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*polylog(3,a\*x),x)

[Out] Integral(x\*\*3\*polylog(3, a\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x),x, algorithm="giac")

[Out] integrate(x^3\*polylog(3, a\*x), x)

## 3.12 $\int x^2 \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=78

$$-\frac{1}{9}x^3 \text{PolyLog}(2, ax) + \frac{1}{3}x^3 \text{PolyLog}(3, ax) + \frac{x}{27a^2} + \frac{\log(1-ax)}{27a^3} + \frac{x^2}{54a} - \frac{1}{27}x^3 \log(1-ax) + \frac{x^3}{81}$$

[Out]  $x/(27*a^2) + x^2/(54*a) + x^3/81 + \text{Log}[1 - a*x]/(27*a^3) - (x^3*\text{Log}[1 - a*x])/27 - (x^3*\text{PolyLog}[2, a*x])/9 + (x^3*\text{PolyLog}[3, a*x])/3$

**Rubi [A]** time = 0.0506273, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 43}

$$-\frac{1}{9}x^3 \text{PolyLog}(2, ax) + \frac{1}{3}x^3 \text{PolyLog}(3, ax) + \frac{x}{27a^2} + \frac{\log(1-ax)}{27a^3} + \frac{x^2}{54a} - \frac{1}{27}x^3 \log(1-ax) + \frac{x^3}{81}$$

Antiderivative was successfully verified.

[In] Int[x^2\*PolyLog[3, a\*x], x]

[Out]  $x/(27*a^2) + x^2/(54*a) + x^3/81 + \text{Log}[1 - a*x]/(27*a^3) - (x^3*\text{Log}[1 - a*x])/27 - (x^3*\text{PolyLog}[2, a*x])/9 + (x^3*\text{PolyLog}[3, a*x])/3$

### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int x^2 \text{Li}_3(ax) dx &= \frac{1}{3} x^3 \text{Li}_3(ax) - \frac{1}{3} \int x^2 \text{Li}_2(ax) dx \\
&= -\frac{1}{9} x^3 \text{Li}_2(ax) + \frac{1}{3} x^3 \text{Li}_3(ax) - \frac{1}{9} \int x^2 \log(1-ax) dx \\
&= -\frac{1}{27} x^3 \log(1-ax) - \frac{1}{9} x^3 \text{Li}_2(ax) + \frac{1}{3} x^3 \text{Li}_3(ax) - \frac{1}{27} a \int \frac{x^3}{1-ax} dx \\
&= -\frac{1}{27} x^3 \log(1-ax) - \frac{1}{9} x^3 \text{Li}_2(ax) + \frac{1}{3} x^3 \text{Li}_3(ax) - \frac{1}{27} a \int \left( -\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)} \right) dx \\
&= \frac{x}{27a^2} + \frac{x^2}{54a} + \frac{x^3}{81} + \frac{\log(1-ax)}{27a^3} - \frac{1}{27} x^3 \log(1-ax) - \frac{1}{9} x^3 \text{Li}_2(ax) + \frac{1}{3} x^3 \text{Li}_3(ax)
\end{aligned}$$

**Mathematica [A]** time = 0.0113884, size = 78, normalized size = 1.

$$\frac{-18a^3x^3\text{PolyLog}(2,ax) + 54a^3x^3\text{PolyLog}(3,ax) + 2a^3x^3 + 3a^2x^2 - 6a^3x^3\log(1-ax) + 6ax + 6\log(1-ax)}{162a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[3, a\*x], x]

[Out] (6\*a\*x + 3\*a^2\*x^2 + 2\*a^3\*x^3 + 6\*Log[1 - a\*x] - 6\*a^3\*x^3\*Log[1 - a\*x] - 18\*a^3\*x^3\*PolyLog[2, a\*x] + 54\*a^3\*x^3\*PolyLog[3, a\*x])/(162\*a^3)

**Maple [A]** time = 0.164, size = 69, normalized size = 0.9

$$\frac{1}{a^3} \left( \frac{xa(4a^2x^2 + 6ax + 12)}{324} + \frac{(-4x^3a^3 + 4)\ln(-ax + 1)}{108} - \frac{x^3a^3\text{polylog}(2,ax)}{9} + \frac{x^3a^3\text{polylog}(3,ax)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(3,a\*x), x)

[Out] 1/a^3\*(1/324\*x\*a\*(4\*a^2\*x^2+6\*a\*x+12)+1/108\*(-4\*a^3\*x^3+4)\*ln(-a\*x+1)-1/9\*x^3\*a^3\*polylog(2,a\*x)+1/3\*x^3\*a^3\*polylog(3,a\*x))

**Maxima [A]** time = 0.995622, size = 93, normalized size = 1.19

$$\frac{18a^3x^3\text{Li}_2(ax) - 54a^3x^3\text{Li}_3(ax) - 2a^3x^3 - 3a^2x^2 - 6ax + 6(a^3x^3 - 1)\log(-ax + 1)}{162a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x), x, algorithm="maxima")

[Out] -1/162\*(18\*a^3\*x^3\*dilog(a\*x) - 54\*a^3\*x^3\*polylog(3, a\*x) - 2\*a^3\*x^3 - 3\*a^2\*x^2 - 6\*a\*x + 6\*(a^3\*x^3 - 1)\*log(-a\*x + 1))/a^3

**Fricas [C]** time = 2.67641, size = 224, normalized size = 2.87

$$\frac{18 a^3 x^3 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 54 a^3 x^3 \operatorname{polylog}(3, ax) - 2 a^3 x^3 - 3 a^2 x^2 - 6 ax + 6 (a^3 x^3 - 1) \log(-ax)}{162 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x),x, algorithm="fricas")

[Out] -1/162\*(18\*a^3\*x^3\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 54\*a^3\*x^3\*polylog(3, a\*x) - 2\*a^3\*x^3 - 3\*a^2\*x^2 - 6\*a\*x + 6\*(a^3\*x^3 - 1)\*log(-a\*x + 1))/a^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(3,a\*x),x)

[Out] Integral(x\*\*2\*polylog(3, a\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x),x, algorithm="giac")

[Out] integrate(x^2\*polylog(3, a\*x), x)

### 3.13 $\int x \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=68

$$-\frac{1}{4}x^2 \text{PolyLog}(2, ax) + \frac{1}{2}x^2 \text{PolyLog}(3, ax) + \frac{\log(1-ax)}{8a^2} - \frac{1}{8}x^2 \log(1-ax) + \frac{x}{8a} + \frac{x^2}{16}$$

[Out]  $x/(8*a) + x^2/16 + \text{Log}[1 - a*x]/(8*a^2) - (x^2*\text{Log}[1 - a*x])/8 - (x^2*\text{PolyLog}[2, a*x])/4 + (x^2*\text{PolyLog}[3, a*x])/2$

**Rubi [A]** time = 0.0351582, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6591, 2395, 43}

$$-\frac{1}{4}x^2 \text{PolyLog}(2, ax) + \frac{1}{2}x^2 \text{PolyLog}(3, ax) + \frac{\log(1-ax)}{8a^2} - \frac{1}{8}x^2 \log(1-ax) + \frac{x}{8a} + \frac{x^2}{16}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{PolyLog}[3, a*x], x]$

[Out]  $x/(8*a) + x^2/16 + \text{Log}[1 - a*x]/(8*a^2) - (x^2*\text{Log}[1 - a*x])/8 - (x^2*\text{PolyLog}[2, a*x])/4 + (x^2*\text{PolyLog}[3, a*x])/2$

#### Rule 6591

$\text{Int}[\{(d \cdot x)^m \cdot \text{PolyLog}[n, (a \cdot x^p)^q]\}, x\_Symbol] \rightarrow \text{Simp}[\{(d \cdot x)^{m+1} \cdot \text{PolyLog}[n, a \cdot (b \cdot x^p)^q]\}/(d \cdot (m+1)), x] - \text{Dist}[\{(p \cdot q)/(m+1)\}, \text{Int}[\{(d \cdot x)^m \cdot \text{PolyLog}[n-1, a \cdot (b \cdot x^p)^q]\}, x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

$\text{Int}[\{(a \cdot x + \text{Log}[c \cdot (d + e \cdot x)^n] \cdot (b \cdot x)^m \cdot ((f \cdot x + g \cdot x)^{q+1})^n)\}, x\_Symbol] \rightarrow \text{Simp}[\{(f + g \cdot x)^{q+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])\}/(g \cdot (q+1)), x] - \text{Dist}[\{(b \cdot e \cdot n)/(g \cdot (q+1))\}, \text{Int}[\{(f + g \cdot x)^{q+1}/(d + e \cdot x)\}, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e \cdot f - d \cdot g, 0] && NeQ[q, -1]

#### Rule 43

$\text{Int}[\{(a \cdot x + (b \cdot x)^m \cdot ((c \cdot x + d \cdot x)^n)\}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7 \cdot m + 4 \cdot n + 4, 0]) || LtQ[9 \cdot m + 5 \cdot (n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int x\text{Li}_3(ax) dx &= \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{2} \int x\text{Li}_2(ax) dx \\
&= -\frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{4} \int x \log(1-ax) dx \\
&= -\frac{1}{8}x^2 \log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{8}a \int \frac{x^2}{1-ax} dx \\
&= -\frac{1}{8}x^2 \log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax) - \frac{1}{8}a \int \left( -\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)} \right) dx \\
&= \frac{x}{8a} + \frac{x^2}{16} + \frac{\log(1-ax)}{8a^2} - \frac{1}{8}x^2 \log(1-ax) - \frac{1}{4}x^2\text{Li}_2(ax) + \frac{1}{2}x^2\text{Li}_3(ax)
\end{aligned}$$

**Mathematica [A]** time = 0.0090438, size = 69, normalized size = 1.01

$$\frac{-4a^2x^2\text{PolyLog}(2, ax) + 8a^2x^2\text{PolyLog}(3, ax) + a^2x^2 - 2a^2x^2 \log(1-ax) + 2ax + 2 \log(1-ax)}{16a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[3, a\*x], x]

[Out] (2\*a\*x + a^2\*x^2 + 2\*Log[1 - a\*x] - 2\*a^2\*x^2\*Log[1 - a\*x] - 4\*a^2\*x^2\*PolyLog[2, a\*x] + 8\*a^2\*x^2\*PolyLog[3, a\*x])/(16\*a^2)

**Maple [A]** time = 0.155, size = 62, normalized size = 0.9

$$-\frac{1}{a^2} \left( -\frac{xa(3ax+6)}{48} - \frac{(-3a^2x^2+3)\ln(-ax+1)}{24} + \frac{x^2a^2\text{polylog}(2, ax)}{4} - \frac{x^2a^2\text{polylog}(3, ax)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(3,a\*x), x)

[Out] -1/a^2\*(-1/48\*x\*a\*(3\*a\*x+6)-1/24\*(-3\*a^2\*x^2+3)\*ln(-a\*x+1)+1/4\*x^2\*a^2\*polylog(2,a\*x)-1/2\*x^2\*a^2\*polylog(3,a\*x))

**Maxima [A]** time = 1.01375, size = 82, normalized size = 1.21

$$\frac{4a^2x^2\text{Li}_2(ax) - 8a^2x^2\text{Li}_3(ax) - a^2x^2 - 2ax + 2(a^2x^2 - 1)\log(-ax + 1)}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x), x, algorithm="maxima")

[Out] -1/16\*(4\*a^2\*x^2\*dilog(a\*x) - 8\*a^2\*x^2\*polylog(3, a\*x) - a^2\*x^2 - 2\*a\*x + 2\*(a^2\*x^2 - 1)\*log(-a\*x + 1))/a^2

**Fricas [C]** time = 2.61559, size = 201, normalized size = 2.96

$$\frac{4 a^2 x^2 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 8 a^2 x^2 \operatorname{polylog}(3, ax) - a^2 x^2 - 2 ax + 2 (a^2 x^2 - 1) \log(-ax + 1)}{16 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x),x, algorithm="fricas")

[Out] -1/16\*(4\*a^2\*x^2\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 8\*a^2\*x^2\*polylog(3, a\*x) - a^2\*x^2 - 2\*a\*x + 2\*(a^2\*x^2 - 1)\*log(-a\*x + 1))/a^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x),x)

[Out] Integral(x\*polylog(3, a\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x),x, algorithm="giac")

[Out] integrate(x\*polylog(3, a\*x), x)



### 3.14 $\int \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=34

$$x(-\text{PolyLog}(2, ax)) + x\text{PolyLog}(3, ax) + \frac{(1 - ax)\log(1 - ax)}{a} + x$$

[Out]  $x + ((1 - a*x)*\text{Log}[1 - a*x])/a - x*\text{PolyLog}[2, a*x] + x*\text{PolyLog}[3, a*x]$

**Rubi [A]** time = 0.0108174, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {6586, 2389, 2295}

$$x(-\text{PolyLog}(2, ax)) + x\text{PolyLog}(3, ax) + \frac{(1 - ax)\log(1 - ax)}{a} + x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[3, a*x], x]$

[Out]  $x + ((1 - a*x)*\text{Log}[1 - a*x])/a - x*\text{PolyLog}[2, a*x] + x*\text{PolyLog}[3, a*x]$

#### Rule 6586

$\text{Int}[\text{PolyLog}[n, (a_*)*((b_*)*(x_)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[x*\text{PolyLog}[n, a*(b*x^p)^q], x] - \text{Dist}[p*q, \text{Int}[\text{PolyLog}[n - 1, a*(b*x^p)^q], x], x] / ; \text{FreeQ}\{a, b, p, q\}, x \ \&\& \ \text{GtQ}[n, 0]$

#### Rule 2389

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_))^{(n_*)}]]*(b_*)^{(p_*)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p], x], x, d + e*x], x] / ; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$

#### Rule 2295

$\text{Int}[\text{Log}[(c_*)*(x_)^{(n_*)}], x\_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] / ; \text{FreeQ}\{c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int \text{Li}_3(ax) dx &= x\text{Li}_3(ax) - \int \text{Li}_2(ax) dx \\ &= -x\text{Li}_2(ax) + x\text{Li}_3(ax) - \int \log(1 - ax) dx \\ &= -x\text{Li}_2(ax) + x\text{Li}_3(ax) + \frac{\text{Subst}(\int \log(x) dx, x, 1 - ax)}{a} \\ &= x + \frac{(1 - ax)\log(1 - ax)}{a} - x\text{Li}_2(ax) + x\text{Li}_3(ax) \end{aligned}$$

**Mathematica [A]** time = 0.0116823, size = 39, normalized size = 1.15

$$x \left( -\text{PolyLog}(2, ax) + \text{PolyLog}(3, ax) + \frac{\log(1 - ax)}{ax} - \log(1 - ax) + 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x], x]

[Out] x\*(1 - Log[1 - a\*x] + Log[1 - a\*x]/(a\*x) - PolyLog[2, a\*x] + PolyLog[3, a\*x])

**Maple [A]** time = 0.092, size = 41, normalized size = 1.2

$$\frac{1}{a} \left( ax + \frac{(-2ax + 2) \ln(-ax + 1)}{2} - ax \operatorname{polylog}(2, ax) + ax \operatorname{polylog}(3, ax) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x), x)

[Out] 1/a\*(a\*x+1/2\*(-2\*a\*x+2)\*ln(-a\*x+1)-a\*x\*polylog(2,a\*x)+a\*x\*polylog(3,a\*x))

**Maxima [A]** time = 0.990846, size = 53, normalized size = 1.56

$$\frac{ax \operatorname{Li}_2(ax) - ax \operatorname{Li}_3(ax) - ax + (ax - 1) \log(-ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x), x, algorithm="maxima")

[Out] -(a\*x\*dilog(a\*x) - a\*x\*polylog(3, a\*x) - a\*x + (a\*x - 1)\*log(-a\*x + 1))/a

**Fricas [C]** time = 2.5983, size = 151, normalized size = 4.44

$$\frac{ax \int \int \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - ax \operatorname{polylog}(3, ax) - ax + (ax - 1) \log(-ax + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x), x, algorithm="fricas")

[Out] -(a\*x\*\int\int(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - a\*x\*polylog(3, a\*x) - a\*x + (a\*x - 1)\*log(-a\*x + 1))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x), x)

[Out] Integral(polylog(3, a\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x), x)

$$3.15 \quad \int \frac{\text{PolyLog}(3, ax)}{x} dx$$

**Optimal.** Leaf size=5

PolyLog(4, ax)

[Out] PolyLog[4, a\*x]

**Rubi [A]** time = 0.0085138, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {6589}

PolyLog(4, ax)

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/x, x]

[Out] PolyLog[4, a\*x]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{x} dx = \text{Li}_4(ax)$$

**Mathematica [A]** time = 0.0012345, size = 5, normalized size = 1.

PolyLog(4, ax)

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x]/x, x]

[Out] PolyLog[4, a\*x]

**Maple [A]** time = 0.04, size = 6, normalized size = 1.2

polylog(4, ax)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x)/x, x)

[Out] polylog(4, a\*x)

---

**Maxima [A]** time = 0.983743, size = 7, normalized size = 1.4

$$\text{Li}_4(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x,x, algorithm="maxima")

[Out] polylog(4, a\*x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x,x, algorithm="fricas")

[Out] integral(polylog(3, a\*x)/x, x)

---

**Sympy [A]** time = 0.551546, size = 3, normalized size = 0.6

$$\text{Li}_4(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x,x)

[Out] polylog(4, a\*x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x)/x, x)

### 3.16 $\int \frac{\text{PolyLog}(3, ax)}{x^2} dx$

**Optimal.** Leaf size=46

$$-\frac{\text{PolyLog}(2, ax)}{x} - \frac{\text{PolyLog}(3, ax)}{x} + a \log(x) - a \log(1 - ax) + \frac{\log(1 - ax)}{x}$$

[Out] a\*Log[x] - a\*Log[1 - a\*x] + Log[1 - a\*x]/x - PolyLog[2, a\*x]/x - PolyLog[3, a\*x]/x

**Rubi [A]** time = 0.0304426, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6591, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2, ax)}{x} - \frac{\text{PolyLog}(3, ax)}{x} + a \log(x) - a \log(1 - ax) + \frac{\log(1 - ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/x^2, x]

[Out] a\*Log[x] - a\*Log[1 - a\*x] + Log[1 - a\*x]/x - PolyLog[2, a\*x]/x - PolyLog[3, a\*x]/x

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol]
:> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol]
:> Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_.), x_Symbol]
:> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^2} dx &= -\frac{\text{Li}_3(ax)}{x} + \int \frac{\text{Li}_2(ax)}{x^2} dx \\
&= -\frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} - \int \frac{\log(1-ax)}{x^2} dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} + a \int \frac{1}{x(1-ax)} dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} + a \int \frac{1}{x} dx + a^2 \int \frac{1}{1-ax} dx \\
&= a \log(x) - a \log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0324637, size = 44, normalized size = 0.96

$$\frac{\text{PolyLog}(2, ax) + \text{PolyLog}(3, ax) - ax \log(-ax) + ax \log(1-ax) - \log(1-ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x]/x^2, x]

[Out] -((- (a\*x\*Log[-(a\*x)]) - Log[1 - a\*x] + a\*x\*Log[1 - a\*x] + PolyLog[2, a\*x] + PolyLog[3, a\*x])/x)

**Maple [A]** time = 0.08, size = 57, normalized size = 1.2

$$a \left( \ln(x) + \ln(-a) + \frac{(-8ax + 8) \ln(-ax + 1)}{8ax} - \frac{\text{polylog}(2, ax)}{ax} - \frac{\text{polylog}(3, ax)}{ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x)/x^2, x)

[Out] a\*(ln(x)+ln(-a)+1/8/a/x\*(-8\*a\*x+8)\*ln(-a\*x+1)-polylog(2, a\*x)/a/x-1/a/x\*polylog(3, a\*x))

**Maxima [A]** time = 0.995112, size = 45, normalized size = 0.98

$$a \log(x) - \frac{(ax - 1) \log(-ax + 1) + \text{Li}_2(ax) + \text{Li}_3(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x)/x^2, x, algorithm="maxima")

[Out] a\*log(x) - ((a\*x - 1)\*log(-a\*x + 1) + dilog(a\*x) + polylog(3, a\*x))/x

**Fricas [C]** time = 2.66231, size = 162, normalized size = 3.52

$$\frac{ax \log(ax - 1) - ax \log(x) + \%iint \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - \log(-ax + 1) + \text{polylog}(3, ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + \int(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - log(-a*x + 1) + polylog(3, a*x))/x
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**2,x)
```

```
[Out] Integral(polylog(3, a*x)/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^2, x)
```



### 3.17 $\int \frac{\text{PolyLog}(3,ax)}{x^3} dx$

**Optimal.** Leaf size=70

$$-\frac{\text{PolyLog}(2,ax)}{4x^2} - \frac{\text{PolyLog}(3,ax)}{2x^2} + \frac{1}{8}a^2 \log(x) - \frac{1}{8}a^2 \log(1-ax) + \frac{\log(1-ax)}{8x^2} - \frac{a}{8x}$$

[Out]  $-a/(8*x) + (a^2*\text{Log}[x])/8 - (a^2*\text{Log}[1 - a*x])/8 + \text{Log}[1 - a*x]/(8*x^2) - \text{PolyLog}[2, a*x]/(4*x^2) - \text{PolyLog}[3, a*x]/(2*x^2)$

**Rubi [A]** time = 0.0431429, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{4x^2} - \frac{\text{PolyLog}(3,ax)}{2x^2} + \frac{1}{8}a^2 \log(x) - \frac{1}{8}a^2 \log(1-ax) + \frac{\log(1-ax)}{8x^2} - \frac{a}{8x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/x^3,x]

[Out]  $-a/(8*x) + (a^2*\text{Log}[x])/8 - (a^2*\text{Log}[1 - a*x])/8 + \text{Log}[1 - a*x]/(8*x^2) - \text{PolyLog}[2, a*x]/(4*x^2) - \text{PolyLog}[3, a*x]/(2*x^2)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))\*(b\_.)))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^3} dx &= -\frac{\text{Li}_3(ax)}{2x^2} + \frac{1}{2} \int \frac{\text{Li}_2(ax)}{x^3} dx \\
&= -\frac{\text{Li}_2(ax)}{4x^2} - \frac{\text{Li}_3(ax)}{2x^2} - \frac{1}{4} \int \frac{\log(1-ax)}{x^3} dx \\
&= \frac{\log(1-ax)}{8x^2} - \frac{\text{Li}_2(ax)}{4x^2} - \frac{\text{Li}_3(ax)}{2x^2} + \frac{1}{8} a \int \frac{1}{x^2(1-ax)} dx \\
&= \frac{\log(1-ax)}{8x^2} - \frac{\text{Li}_2(ax)}{4x^2} - \frac{\text{Li}_3(ax)}{2x^2} + \frac{1}{8} a \int \left( \frac{1}{x^2} + \frac{a}{x} - \frac{a^2}{-1+ax} \right) dx \\
&= -\frac{a}{8x} + \frac{1}{8} a^2 \log(x) - \frac{1}{8} a^2 \log(1-ax) + \frac{\log(1-ax)}{8x^2} - \frac{\text{Li}_2(ax)}{4x^2} - \frac{\text{Li}_3(ax)}{2x^2}
\end{aligned}$$

**Mathematica [C]** time = 0.0090975, size = 25, normalized size = 0.36

$$\frac{G_{5,5}^{2,4} \left( -ax \left| \begin{matrix} 1, 1, 1, 1, 3 \\ 1, 2, 0, 0, 0 \end{matrix} \right. \right)}{x^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x]/x^3,x]

[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a\*x)]/x^2

**Maple [A]** time = 0.165, size = 90, normalized size = 1.3

$$-a^2 \left( \frac{1}{ax} + \frac{3}{16} - \frac{\ln(x)}{8} - \frac{\ln(-a)}{8} - \frac{81ax + 378}{432ax} - \frac{(-27a^2x^2 + 27) \ln(-ax + 1)}{216a^2x^2} + \frac{\text{polylog}(2, ax)}{4a^2x^2} + \frac{\text{polylog}(3, ax)}{2a^2x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x)/x^3,x)

[Out] -a^2\*(1/a/x+3/16-1/8\*ln(x)-1/8\*ln(-a)-1/432/a/x\*(81\*a\*x+378)-1/216/a^2/x^2\*(-27\*a^2\*x^2+27)\*ln(-a\*x+1)+1/4/a^2/x^2\*polylog(2,a\*x)+1/2/a^2/x^2\*polylog(3,a\*x))

**Maxima [A]** time = 0.992882, size = 63, normalized size = 0.9

$$\frac{1}{8} a^2 \log(x) - \frac{ax + (a^2x^2 - 1) \log(-ax + 1) + 2 \text{Li}_2(ax) + 4 \text{Li}_3(ax)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x^3,x, algorithm="maxima")

[Out] 1/8\*a^2\*log(x) - 1/8\*(a\*x + (a^2\*x^2 - 1)\*log(-a\*x + 1) + 2\*dilog(a\*x) + 4\*polylog(3, a\*x))/x^2

**Fricas [C]** time = 2.71799, size = 194, normalized size = 2.77

$$\frac{a^2 x^2 \log(ax - 1) - a^2 x^2 \log(x) + ax + 2 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - \log(-ax + 1) + 4 \operatorname{polylog}(3, ax)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x^3,x, algorithm="fricas")

[Out] -1/8\*(a^2\*x^2\*log(a\*x - 1) - a^2\*x^2\*log(x) + a\*x + 2\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - log(-a\*x + 1) + 4\*polylog(3, a\*x))/x^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x\*\*3,x)

[Out] Integral(polylog(3, a\*x)/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x^3,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x)/x^3, x)

### 3.18 $\int \frac{\text{PolyLog}(3, ax)}{x^4} dx$

**Optimal.** Leaf size=80

$$-\frac{\text{PolyLog}(2, ax)}{9x^3} - \frac{\text{PolyLog}(3, ax)}{3x^3} - \frac{a^2}{27x} + \frac{1}{27}a^3 \log(x) - \frac{1}{27}a^3 \log(1 - ax) - \frac{a}{54x^2} + \frac{\log(1 - ax)}{27x^3}$$

[Out]  $-a/(54*x^2) - a^2/(27*x) + (a^3*\text{Log}[x])/27 - (a^3*\text{Log}[1 - a*x])/27 + \text{Log}[1 - a*x]/(27*x^3) - \text{PolyLog}[2, a*x]/(9*x^3) - \text{PolyLog}[3, a*x]/(3*x^3)$

**Rubi [A]** time = 0.047769, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax)}{9x^3} - \frac{\text{PolyLog}(3, ax)}{3x^3} - \frac{a^2}{27x} + \frac{1}{27}a^3 \log(x) - \frac{1}{27}a^3 \log(1 - ax) - \frac{a}{54x^2} + \frac{\log(1 - ax)}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/x^4, x]

[Out]  $-a/(54*x^2) - a^2/(27*x) + (a^3*\text{Log}[x])/27 - (a^3*\text{Log}[1 - a*x])/27 + \text{Log}[1 - a*x]/(27*x^3) - \text{PolyLog}[2, a*x]/(9*x^3) - \text{PolyLog}[3, a*x]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^4} dx &= -\frac{\text{Li}_3(ax)}{3x^3} + \frac{1}{3} \int \frac{\text{Li}_2(ax)}{x^4} dx \\
&= -\frac{\text{Li}_2(ax)}{9x^3} - \frac{\text{Li}_3(ax)}{3x^3} - \frac{1}{9} \int \frac{\log(1-ax)}{x^4} dx \\
&= \frac{\log(1-ax)}{27x^3} - \frac{\text{Li}_2(ax)}{9x^3} - \frac{\text{Li}_3(ax)}{3x^3} + \frac{1}{27} a \int \frac{1}{x^3(1-ax)} dx \\
&= \frac{\log(1-ax)}{27x^3} - \frac{\text{Li}_2(ax)}{9x^3} - \frac{\text{Li}_3(ax)}{3x^3} + \frac{1}{27} a \int \left( \frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax} \right) dx \\
&= -\frac{a}{54x^2} - \frac{a^2}{27x} + \frac{1}{27} a^3 \log(x) - \frac{1}{27} a^3 \log(1-ax) + \frac{\log(1-ax)}{27x^3} - \frac{\text{Li}_2(ax)}{9x^3} - \frac{\text{Li}_3(ax)}{3x^3}
\end{aligned}$$

**Mathematica [C]** time = 0.00906, size = 25, normalized size = 0.31

$$\frac{G_{5,5}^{2,4} \left( -ax \left| \begin{array}{l} 1, 1, 1, 1, 4 \\ 1, 3, 0, 0, 0 \end{array} \right. \right)}{x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x]/x^4, x]

[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a\*x)]/x^3

**Maple [A]** time = 0.168, size = 106, normalized size = 1.3

$$a^3 \left( -\frac{1}{2a^2x^2} - \frac{1}{8ax} - \frac{1}{27} + \frac{\ln(x)}{27} + \frac{\ln(-a)}{27} + \frac{64a^2x^2 + 152ax + 832}{1728a^2x^2} + \frac{(-64x^3a^3 + 64)\ln(-ax + 1)}{1728x^3a^3} - \frac{\text{polylog}(2, a^2x^2 + 152ax + 832)}{9x^3a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x)/x^4, x)

[Out] a^3\*(-1/2/a^2/x^2-1/8/a/x-1/27+1/27\*ln(x)+1/27\*ln(-a)+1/1728/a^2/x^2\*(64\*a^2\*x^2+152\*a\*x+832)+1/1728/a^3/x^3\*(-64\*a^3\*x^3+64)\*ln(-a\*x+1)-1/9/a^3/x^3\*polylog(2, a\*x)-1/3/a^3/x^3\*polylog(3, a\*x))

**Maxima [A]** time = 0.988239, size = 76, normalized size = 0.95

$$\frac{1}{27} a^3 \log(x) - \frac{2a^2x^2 + ax + 2(a^3x^3 - 1)\log(-ax + 1) + 6\text{Li}_2(ax) + 18\text{Li}_3(ax)}{54x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x)/x^4, x, algorithm="maxima")

[Out] 1/27\*a^3\*log(x) - 1/54\*(2\*a^2\*x^2 + a\*x + 2\*(a^3\*x^3 - 1)\*log(-a\*x + 1) + 6\*dilog(a\*x) + 18\*polylog(3, a\*x))/x^3

**Fricas [C]** time = 2.81394, size = 221, normalized size = 2.76

$$\frac{2a^3x^3 \log(ax-1) - 2a^3x^3 \log(x) + 2a^2x^2 + ax + 6 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 2 \log(-ax+1) + 18 \operatorname{polylog}(3, ax)}{54x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x^4,x, algorithm="fricas")

[Out] -1/54\*(2\*a^3\*x^3\*log(a\*x - 1) - 2\*a^3\*x^3\*log(x) + 2\*a^2\*x^2 + a\*x + 6\*Li3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 2\*log(-a\*x + 1) + 18\*polylog(3, a\*x))/x^3

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x\*\*4,x)

[Out] Integral(polylog(3, a\*x)/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/x^4,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x)/x^4, x)

### 3.19 $\int x^5 \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=74

$$\frac{1}{6}x^6 \text{PolyLog}(2, ax^2) - \frac{x^2}{18a^2} - \frac{\log(1-ax^2)}{18a^3} - \frac{x^4}{36a} + \frac{1}{18}x^6 \log(1-ax^2) - \frac{x^6}{54}$$

[Out]  $-x^2/(18*a^2) - x^4/(36*a) - x^6/54 - \text{Log}[1 - a*x^2]/(18*a^3) + (x^6*\text{Log}[1 - a*x^2])/18 + (x^6*\text{PolyLog}[2, a*x^2])/6$

**Rubi [A]** time = 0.0614215, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 43}

$$\frac{1}{6}x^6 \text{PolyLog}(2, ax^2) - \frac{x^2}{18a^2} - \frac{\log(1-ax^2)}{18a^3} - \frac{x^4}{36a} + \frac{1}{18}x^6 \log(1-ax^2) - \frac{x^6}{54}$$

Antiderivative was successfully verified.

[In] Int[x^5\*PolyLog[2, a\*x^2], x]

[Out]  $-x^2/(18*a^2) - x^4/(36*a) - x^6/54 - \text{Log}[1 - a*x^2]/(18*a^3) + (x^6*\text{Log}[1 - a*x^2])/18 + (x^6*\text{PolyLog}[2, a*x^2])/6$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int x^5 \text{Li}_2(ax^2) dx &= \frac{1}{6} x^6 \text{Li}_2(ax^2) + \frac{1}{3} \int x^5 \log(1 - ax^2) dx \\
&= \frac{1}{6} x^6 \text{Li}_2(ax^2) + \frac{1}{6} \text{Subst} \left( \int x^2 \log(1 - ax) dx, x, x^2 \right) \\
&= \frac{1}{18} x^6 \log(1 - ax^2) + \frac{1}{6} x^6 \text{Li}_2(ax^2) + \frac{1}{18} a \text{Subst} \left( \int \frac{x^3}{1 - ax} dx, x, x^2 \right) \\
&= \frac{1}{18} x^6 \log(1 - ax^2) + \frac{1}{6} x^6 \text{Li}_2(ax^2) + \frac{1}{18} a \text{Subst} \left( \int \left( -\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1 + ax)} \right) dx, x, x^2 \right) \\
&= -\frac{x^2}{18a^2} - \frac{x^4}{36a} - \frac{x^6}{54} - \frac{\log(1 - ax^2)}{18a^3} + \frac{1}{18} x^6 \log(1 - ax^2) + \frac{1}{6} x^6 \text{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0220162, size = 65, normalized size = 0.88

$$\frac{18a^3x^6\text{PolyLog}(2,ax^2) - ax^2(2a^2x^4 + 3ax^2 + 6) + 6(a^3x^6 - 1)\log(1 - ax^2)}{108a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*PolyLog[2, a\*x^2], x]

[Out]  $(-(a*x^2*(6 + 3*a*x^2 + 2*a^2*x^4)) + 6*(-1 + a^3*x^6)*\text{Log}[1 - a*x^2] + 18*a^3*x^6*\text{PolyLog}[2, a*x^2])/(108*a^3)$

**Maple [A]** time = 0.044, size = 62, normalized size = 0.8

$$\frac{x^6 \text{polylog}(2, ax^2)}{6} + \frac{x^6 \ln(-ax^2 + 1)}{18} - \frac{x^6}{54} - \frac{x^4}{36a} - \frac{x^2}{18a^2} - \frac{\ln(ax^2 - 1)}{18a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*polylog(2,a\*x^2), x)

[Out]  $1/6*x^6*\text{polylog}(2,a*x^2)+1/18*x^6*\ln(-a*x^2+1)-1/54*x^6-1/36*x^4/a-1/18*x^2/a^2-1/18/a^3*\ln(a*x^2-1)$

**Maxima [A]** time = 1.00801, size = 84, normalized size = 1.14

$$\frac{18a^3x^6\text{Li}_2(ax^2) - 2a^3x^6 - 3a^2x^4 - 6ax^2 + 6(a^3x^6 - 1)\log(-ax^2 + 1)}{108a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(2,a\*x^2), x, algorithm="maxima")

[Out]  $1/108*(18*a^3*x^6*\text{dilog}(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*\log(-a*x^2 + 1))/a^3$



**Fricas [A]** time = 2.5273, size = 142, normalized size = 1.92

$$\frac{18 a^3 x^6 \operatorname{Li}_2(ax^2) - 2 a^3 x^6 - 3 a^2 x^4 - 6 a x^2 + 6 (a^3 x^6 - 1) \log(-ax^2 + 1)}{108 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(2,a\*x^2),x, algorithm="fricas")

[Out] 1/108\*(18\*a^3\*x^6\*dilog(a\*x^2) - 2\*a^3\*x^6 - 3\*a^2\*x^4 - 6\*a\*x^2 + 6\*(a^3\*x^6 - 1)\*log(-a\*x^2 + 1))/a^3

**Sympy [A]** time = 42.5184, size = 56, normalized size = 0.76

$$\begin{cases} -\frac{x^6 \operatorname{Li}_1(ax^2)}{18} + \frac{x^6 \operatorname{Li}_2(ax^2)}{6} - \frac{x^6}{54} - \frac{x^4}{36a} - \frac{x^2}{18a^2} + \frac{\operatorname{Li}_1(ax^2)}{18a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*polylog(2,a\*x\*\*2),x)

[Out] Piecewise((-x\*\*6\*polylog(1, a\*x\*\*2)/18 + x\*\*6\*polylog(2, a\*x\*\*2)/6 - x\*\*6/54 - x\*\*4/(36\*a) - x\*\*2/(18\*a\*\*2) + polylog(1, a\*x\*\*2)/(18\*a\*\*3), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate(x^5\*dilog(a\*x^2), x)

## 3.20 $\int x^3 \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=64

$$\frac{1}{4}x^4 \text{PolyLog}(2, ax^2) - \frac{\log(1 - ax^2)}{8a^2} - \frac{x^2}{8a} + \frac{1}{8}x^4 \log(1 - ax^2) - \frac{x^4}{16}$$

[Out]  $-x^2/(8*a) - x^4/16 - \text{Log}[1 - a*x^2]/(8*a^2) + (x^4*\text{Log}[1 - a*x^2])/8 + (x^4*\text{PolyLog}[2, a*x^2])/4$

**Rubi [A]** time = 0.0495752, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 43}

$$\frac{1}{4}x^4 \text{PolyLog}(2, ax^2) - \frac{\log(1 - ax^2)}{8a^2} - \frac{x^2}{8a} + \frac{1}{8}x^4 \log(1 - ax^2) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

[In] Int[x^3\*PolyLog[2, a\*x^2], x]

[Out]  $-x^2/(8*a) - x^4/16 - \text{Log}[1 - a*x^2]/(8*a^2) + (x^4*\text{Log}[1 - a*x^2])/8 + (x^4*\text{PolyLog}[2, a*x^2])/4$

### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*((b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*((b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int x^3 \text{Li}_2(ax^2) dx &= \frac{1}{4} x^4 \text{Li}_2(ax^2) + \frac{1}{2} \int x^3 \log(1 - ax^2) dx \\
&= \frac{1}{4} x^4 \text{Li}_2(ax^2) + \frac{1}{4} \text{Subst} \left( \int x \log(1 - ax) dx, x, x^2 \right) \\
&= \frac{1}{8} x^4 \log(1 - ax^2) + \frac{1}{4} x^4 \text{Li}_2(ax^2) + \frac{1}{8} a \text{Subst} \left( \int \frac{x^2}{1 - ax} dx, x, x^2 \right) \\
&= \frac{1}{8} x^4 \log(1 - ax^2) + \frac{1}{4} x^4 \text{Li}_2(ax^2) + \frac{1}{8} a \text{Subst} \left( \int \left( -\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1 + ax)} \right) dx, x, x^2 \right) \\
&= -\frac{x^2}{8a} - \frac{x^4}{16} - \frac{\log(1 - ax^2)}{8a^2} + \frac{1}{8} x^4 \log(1 - ax^2) + \frac{1}{4} x^4 \text{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0172903, size = 56, normalized size = 0.88

$$\frac{4a^2x^4\text{PolyLog}(2, ax^2) + 2(a^2x^4 - 1)\log(1 - ax^2) - ax^2(ax^2 + 2)}{16a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*PolyLog[2, a\*x^2], x]

[Out]  $(-(a*x^2*(2 + a*x^2)) + 2*(-1 + a^2*x^4)*\text{Log}[1 - a*x^2] + 4*a^2*x^4*\text{PolyLog}[2, a*x^2])/(16*a^2)$

**Maple [A]** time = 0.045, size = 54, normalized size = 0.8

$$\frac{x^4 \text{polylog}(2, ax^2)}{4} + \frac{x^4 \ln(-ax^2 + 1)}{8} - \frac{x^4}{16} - \frac{x^2}{8a} - \frac{\ln(ax^2 - 1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*polylog(2, a\*x^2), x)

[Out]  $1/4*x^4*\text{polylog}(2, a*x^2) + 1/8*x^4*\ln(-a*x^2+1) - 1/16*x^4 - 1/8*x^2/a - 1/8/a^2*\ln(a*x^2-1)$

**Maxima [A]** time = 0.96931, size = 73, normalized size = 1.14

$$\frac{4a^2x^4\text{Li}_2(ax^2) - a^2x^4 - 2ax^2 + 2(a^2x^4 - 1)\log(-ax^2 + 1)}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2, a\*x^2), x, algorithm="maxima")

[Out]  $1/16*(4*a^2*x^4*\text{dilog}(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*\log(-a*x^2 + 1))/a^2$

**Fricas [A]** time = 2.58406, size = 120, normalized size = 1.88

$$\frac{4a^2x^4\text{Li}_2(ax^2) - a^2x^4 - 2ax^2 + 2(a^2x^4 - 1)\log(-ax^2 + 1)}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2,a\*x^2),x, algorithm="fricas")

[Out] 1/16\*(4\*a^2\*x^4\*dilog(a\*x^2) - a^2\*x^4 - 2\*a\*x^2 + 2\*(a^2\*x^4 - 1)\*log(-a\*x^2 + 1))/a^2

**Sympy [A]** time = 13.1842, size = 48, normalized size = 0.75

$$\begin{cases} -\frac{x^4\text{Li}_1(ax^2)}{8} + \frac{x^4\text{Li}_2(ax^2)}{4} - \frac{x^4}{16} - \frac{x^2}{8a} + \frac{\text{Li}_1(ax^2)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*polylog(2,a\*x\*\*2),x)

[Out] Piecewise((-x\*\*4\*polylog(1, a\*x\*\*2)/8 + x\*\*4\*polylog(2, a\*x\*\*2)/4 - x\*\*4/16 - x\*\*2/(8\*a) + polylog(1, a\*x\*\*2)/(8\*a\*\*2), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3\text{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate(x^3\*dilog(a\*x^2), x)

### 3.21 $\int x \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=46

$$\frac{1}{2}x^2 \text{PolyLog}(2, ax^2) - \frac{(1 - ax^2) \log(1 - ax^2)}{2a} - \frac{x^2}{2}$$

[Out]  $-x^2/2 - ((1 - a*x^2)*\text{Log}[1 - a*x^2])/(2*a) + (x^2*\text{PolyLog}[2, a*x^2])/2$

**Rubi [A]** time = 0.0252799, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {6591, 2454, 2389, 2295}

$$\frac{1}{2}x^2 \text{PolyLog}(2, ax^2) - \frac{(1 - ax^2) \log(1 - ax^2)}{2a} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[2, a\*x^2], x]

[Out]  $-x^2/2 - ((1 - a*x^2)*\text{Log}[1 - a*x^2])/(2*a) + (x^2*\text{PolyLog}[2, a*x^2])/2$

#### Rule 6591

Int[((d\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_.)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rubi steps

$$\begin{aligned}
\int x \operatorname{Li}_2(ax^2) dx &= \frac{1}{2} x^2 \operatorname{Li}_2(ax^2) + \int x \log(1 - ax^2) dx \\
&= \frac{1}{2} x^2 \operatorname{Li}_2(ax^2) + \frac{1}{2} \operatorname{Subst}\left(\int \log(1 - ax) dx, x, x^2\right) \\
&= \frac{1}{2} x^2 \operatorname{Li}_2(ax^2) - \frac{\operatorname{Subst}\left(\int \log(x) dx, x, 1 - ax^2\right)}{2a} \\
&= -\frac{x^2}{2} - \frac{(1 - ax^2) \log(1 - ax^2)}{2a} + \frac{1}{2} x^2 \operatorname{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0091547, size = 43, normalized size = 0.93

$$\frac{ax^2 \operatorname{PolyLog}(2, ax^2) - ax^2 + (ax^2 - 1) \log(1 - ax^2)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[2, a\*x^2], x]

[Out]  $(-(a*x^2) + (-1 + a*x^2)*\operatorname{Log}[1 - a*x^2] + a*x^2*\operatorname{PolyLog}[2, a*x^2])/(2*a)$

**Maple [A]** time = 0.045, size = 52, normalized size = 1.1

$$\frac{x^2 \operatorname{polylog}(2, ax^2)}{2} + \frac{\ln(-ax^2 + 1)x^2}{2} - \frac{x^2}{2} - \frac{\ln(-ax^2 + 1)}{2a} + \frac{1}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(2,a\*x^2), x)

[Out]  $1/2*x^2*\operatorname{polylog}(2, a*x^2) + 1/2*\ln(-a*x^2+1)*x^2 - 1/2*x^2 - 1/2/a*\ln(-a*x^2+1) + 1/2/a$

**Maxima [A]** time = 0.985418, size = 54, normalized size = 1.17

$$\frac{ax^2 \operatorname{Li}_2(ax^2) - ax^2 + (ax^2 - 1) \log(-ax^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,a\*x^2), x, algorithm="maxima")

[Out]  $1/2*(a*x^2*\operatorname{dilog}(a*x^2) - a*x^2 + (a*x^2 - 1)*\log(-a*x^2 + 1))/a$

**Fricas [A]** time = 2.57043, size = 89, normalized size = 1.93

$$\frac{ax^2 \operatorname{Li}_2(ax^2) - ax^2 + (ax^2 - 1) \log(-ax^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

**Sympy [A]** time = 4.02697, size = 39, normalized size = 0.85

$$\begin{cases} -\frac{x^2 \operatorname{Li}_1(ax^2)}{2} + \frac{x^2 \operatorname{Li}_2(ax^2)}{2} - \frac{x^2}{2} + \frac{\operatorname{Li}_1(ax^2)}{2a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**2*polylog(1, a*x**2)/2 + x**2*polylog(2, a*x**2)/2 - x**2/2 + polylog(1, a*x**2)/(2*a), Ne(a, 0)), (0, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x^2), x)
```

$$3.22 \quad \int \frac{\text{PolyLog}(2, ax^2)}{x} dx$$

**Optimal.** Leaf size=11

$$\frac{1}{2} \text{PolyLog}(3, ax^2)$$

[Out] PolyLog[3, a\*x^2]/2

**Rubi [A]** time = 0.0094764, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6589}

$$\frac{1}{2} \text{PolyLog}(3, ax^2)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x, x]

[Out] PolyLog[3, a\*x^2]/2

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{x} dx = \frac{\text{Li}_3(ax^2)}{2}$$

**Mathematica [A]** time = 0.0012499, size = 11, normalized size = 1.

$$\frac{1}{2} \text{PolyLog}(3, ax^2)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x, x]

[Out] PolyLog[3, a\*x^2]/2

**Maple [A]** time = 0.115, size = 10, normalized size = 0.9

$$\frac{\text{polylog}(3, ax^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(polylog(2,a*x^2)/x,x)`

[Out] `1/2*polylog(3,a*x^2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/x,x, algorithm="maxima")`

[Out] `integrate(dilog(a*x^2)/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax^2)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/x,x, algorithm="fricas")`

[Out] `integral(dilog(a*x^2)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x**2)/x,x)`

[Out] `Integral(polylog(2, a*x**2)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/x,x, algorithm="giac")`

[Out] `integrate(dilog(a*x^2)/x, x)`

### 3.23 $\int \frac{\text{PolyLog}(2, ax^2)}{x^3} dx$

**Optimal.** Leaf size=49

$$-\frac{\text{PolyLog}(2, ax^2)}{2x^2} - \frac{1}{2}a \log(1 - ax^2) + \frac{\log(1 - ax^2)}{2x^2} + a \log(x)$$

[Out] a\*Log[x] - (a\*Log[1 - a\*x^2])/2 + Log[1 - a\*x^2]/(2\*x^2) - PolyLog[2, a\*x^2]/(2\*x^2)

**Rubi [A]** time = 0.040811, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$ , Rules used = {6591, 2454, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2, ax^2)}{2x^2} - \frac{1}{2}a \log(1 - ax^2) + \frac{\log(1 - ax^2)}{2x^2} + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^3, x]

[Out] a\*Log[x] - (a\*Log[1 - a\*x^2])/2 + Log[1 - a\*x^2]/(2\*x^2) - PolyLog[2, a\*x^2]/(2\*x^2)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

### Rule 31

`Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(ax^2)}{x^3} dx &= -\frac{\text{Li}_2(ax^2)}{2x^2} - \int \frac{\log(1-ax^2)}{x^3} dx \\
 &= -\frac{\text{Li}_2(ax^2)}{2x^2} - \frac{1}{2} \text{Subst}\left(\int \frac{\log(1-ax)}{x^2} dx, x, x^2\right) \\
 &= \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2} + \frac{1}{2}a \text{Subst}\left(\int \frac{1}{x(1-ax)} dx, x, x^2\right) \\
 &= \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2} + \frac{1}{2}a \text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) + \frac{1}{2}a^2 \text{Subst}\left(\int \frac{1}{1-ax} dx, x, x^2\right) \\
 &= a \log(x) - \frac{1}{2}a \log(1-ax^2) + \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0123956, size = 49, normalized size = 1.

$$-\frac{\text{PolyLog}(2, ax^2)}{2x^2} - \frac{1}{2}a \log(1-ax^2) + \frac{\log(1-ax^2)}{2x^2} + a \log(x)$$

Antiderivative was successfully verified.

`[In] Integrate[PolyLog[2, a*x^2]/x^3, x]`

`[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2]/(2*x^2)`

**Maple [A]** time = 0.05, size = 43, normalized size = 0.9

$$-\frac{\text{polylog}(2, ax^2)}{2x^2} + \frac{\ln(-ax^2 + 1)}{2x^2} + a \ln(x) - \frac{a \ln(ax^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(polylog(2, a*x^2)/x^3, x)`

`[Out] -1/2*polylog(2, a*x^2)/x^2+1/2*ln(-a*x^2+1)/x^2+a*ln(x)-1/2*a*ln(a*x^2-1)`

**Maxima [A]** time = 0.960105, size = 46, normalized size = 0.94

$$a \log(x) - \frac{(ax^2 - 1) \log(-ax^2 + 1) + \text{Li}_2(ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^3,x, algorithm="maxima")

[Out] a\*log(x) - 1/2\*((a\*x^2 - 1)\*log(-a\*x^2 + 1) + dilog(a\*x^2))/x^2

**Fricas [A]** time = 2.68007, size = 112, normalized size = 2.29

$$\frac{ax^2 \log(ax^2 - 1) - 2ax^2 \log(x) + \text{Li}_2(ax^2) - \log(-ax^2 + 1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^3,x, algorithm="fricas")

[Out] -1/2\*(a\*x^2\*log(a\*x^2 - 1) - 2\*a\*x^2\*log(x) + dilog(a\*x^2) - log(-a\*x^2 + 1))/x^2

**Sympy [A]** time = 4.87995, size = 37, normalized size = 0.76

$$a \log(x) + \frac{a \text{Li}_1(ax^2)}{2} - \frac{\text{Li}_1(ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/x\*\*3,x)

[Out] a\*log(x) + a\*polylog(1, a\*x\*\*2)/2 - polylog(1, a\*x\*\*2)/(2\*x\*\*2) - polylog(2, a\*x\*\*2)/(2\*x\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^3,x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/x^3, x)

### 3.24 $\int \frac{\text{PolyLog}(2, ax^2)}{x^5} dx$

**Optimal.** Leaf size=64

$$-\frac{\text{PolyLog}(2, ax^2)}{4x^4} - \frac{1}{8}a^2 \log(1 - ax^2) + \frac{1}{4}a^2 \log(x) - \frac{a}{8x^2} + \frac{\log(1 - ax^2)}{8x^4}$$

[Out]  $-a/(8*x^2) + (a^2*\text{Log}[x])/4 - (a^2*\text{Log}[1 - a*x^2])/8 + \text{Log}[1 - a*x^2]/(8*x^4) - \text{PolyLog}[2, a*x^2]/(4*x^4)$

**Rubi [A]** time = 0.0500673, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax^2)}{4x^4} - \frac{1}{8}a^2 \log(1 - ax^2) + \frac{1}{4}a^2 \log(x) - \frac{a}{8x^2} + \frac{\log(1 - ax^2)}{8x^4}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^5, x]

[Out]  $-a/(8*x^2) + (a^2*\text{Log}[x])/4 - (a^2*\text{Log}[1 - a*x^2])/8 + \text{Log}[1 - a*x^2]/(8*x^4) - \text{PolyLog}[2, a*x^2]/(4*x^4)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(ax^2)}{x^5} dx &= -\frac{\operatorname{Li}_2(ax^2)}{4x^4} - \frac{1}{2} \int \frac{\log(1-ax^2)}{x^5} dx \\
&= -\frac{\operatorname{Li}_2(ax^2)}{4x^4} - \frac{1}{4} \operatorname{Subst}\left(\int \frac{\log(1-ax)}{x^3} dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{8x^4} - \frac{\operatorname{Li}_2(ax^2)}{4x^4} + \frac{1}{8}a \operatorname{Subst}\left(\int \frac{1}{x^2(1-ax)} dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{8x^4} - \frac{\operatorname{Li}_2(ax^2)}{4x^4} + \frac{1}{8}a \operatorname{Subst}\left(\int \left(\frac{1}{x^2} + \frac{a}{x} - \frac{a^2}{-1+ax}\right) dx, x, x^2\right) \\
&= -\frac{a}{8x^2} + \frac{1}{4}a^2 \log(x) - \frac{1}{8}a^2 \log(1-ax^2) + \frac{\log(1-ax^2)}{8x^4} - \frac{\operatorname{Li}_2(ax^2)}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0283499, size = 51, normalized size = 0.8

$$-\frac{2\operatorname{PolyLog}(2, ax^2) - 2a^2x^4 \log(x) + (a^2x^4 - 1) \log(1 - ax^2) + ax^2}{8x^4}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x^5, x]

[Out] -(a\*x^2 - 2\*a^2\*x^4\*Log[x] + (-1 + a^2\*x^4)\*Log[1 - a\*x^2] + 2\*PolyLog[2, a\*x^2])/(8\*x^4)

**Maple [A]** time = 0.05, size = 54, normalized size = 0.8

$$-\frac{\operatorname{polylog}(2, ax^2)}{4x^4} + \frac{\ln(-ax^2 + 1)}{8x^4} - \frac{a}{8x^2} + \frac{a^2 \ln(x)}{4} - \frac{a^2 \ln(ax^2 - 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^2)/x^5, x)

[Out] -1/4\*polylog(2, a\*x^2)/x^4+1/8\*ln(-a\*x^2+1)/x^4-1/8\*a/x^2+1/4\*a^2\*ln(x)-1/8\*a^2\*ln(a\*x^2-1)

**Maxima [A]** time = 0.987978, size = 62, normalized size = 0.97

$$\frac{1}{4}a^2 \log(x) - \frac{ax^2 + (a^2x^4 - 1) \log(-ax^2 + 1) + 2 \operatorname{Li}_2(ax^2)}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^2)/x^5, x, algorithm="maxima")

[Out] 1/4\*a^2\*log(x) - 1/8\*(a\*x^2 + (a^2\*x^4 - 1)\*log(-a\*x^2 + 1) + 2\*dilog(a\*x^2))/x^4

**Fricas [A]** time = 2.64939, size = 131, normalized size = 2.05

$$\frac{a^2 x^4 \log(ax^2 - 1) - 2 a^2 x^4 \log(x) + ax^2 + 2 \operatorname{Li}_2(ax^2) - \log(-ax^2 + 1)}{8 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^5,x, algorithm="fricas")

[Out] -1/8\*(a^2\*x^4\*log(a\*x^2 - 1) - 2\*a^2\*x^4\*log(x) + a\*x^2 + 2\*dilog(a\*x^2) - log(-a\*x^2 + 1))/x^4

**Sympy [A]** time = 14.9968, size = 49, normalized size = 0.77

$$\frac{a^2 \log(x)}{4} + \frac{a^2 \operatorname{Li}_1(ax^2)}{8} - \frac{a}{8x^2} - \frac{\operatorname{Li}_1(ax^2)}{8x^4} - \frac{\operatorname{Li}_2(ax^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/x\*\*5,x)

[Out] a\*\*2\*log(x)/4 + a\*\*2\*polylog(1, a\*x\*\*2)/8 - a/(8\*x\*\*2) - polylog(1, a\*x\*\*2)/(8\*x\*\*4) - polylog(2, a\*x\*\*2)/(4\*x\*\*4)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^5,x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/x^5, x)

### 3.25 $\int \frac{\text{PolyLog}(2, ax^2)}{x^7} dx$

**Optimal.** Leaf size=74

$$-\frac{\text{PolyLog}(2, ax^2)}{6x^6} - \frac{a^2}{18x^2} - \frac{1}{18}a^3 \log(1 - ax^2) + \frac{1}{9}a^3 \log(x) - \frac{a}{36x^4} + \frac{\log(1 - ax^2)}{18x^6}$$

[Out]  $-a/(36*x^4) - a^2/(18*x^2) + (a^3*\text{Log}[x])/9 - (a^3*\text{Log}[1 - a*x^2])/18 + \text{Log}[1 - a*x^2]/(18*x^6) - \text{PolyLog}[2, a*x^2]/(6*x^6)$

**Rubi [A]** time = 0.0541201, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax^2)}{6x^6} - \frac{a^2}{18x^2} - \frac{1}{18}a^3 \log(1 - ax^2) + \frac{1}{9}a^3 \log(x) - \frac{a}{36x^4} + \frac{\log(1 - ax^2)}{18x^6}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^7, x]

[Out]  $-a/(36*x^4) - a^2/(18*x^2) + (a^3*\text{Log}[x])/9 - (a^3*\text{Log}[1 - a*x^2])/18 + \text{Log}[1 - a*x^2]/(18*x^6) - \text{PolyLog}[2, a*x^2]/(6*x^6)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{x^7} dx &= -\frac{\text{Li}_2(ax^2)}{6x^6} - \frac{1}{3} \int \frac{\log(1-ax^2)}{x^7} dx \\
&= -\frac{\text{Li}_2(ax^2)}{6x^6} - \frac{1}{6} \text{Subst}\left(\int \frac{\log(1-ax)}{x^4} dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6} + \frac{1}{18} a \text{Subst}\left(\int \frac{1}{x^3(1-ax)} dx, x, x^2\right) \\
&= \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6} + \frac{1}{18} a \text{Subst}\left(\int \left(\frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax}\right) dx, x, x^2\right) \\
&= -\frac{a}{36x^4} - \frac{a^2}{18x^2} + \frac{1}{9} a^3 \log(x) - \frac{1}{18} a^3 \log(1-ax^2) + \frac{\log(1-ax^2)}{18x^6} - \frac{\text{Li}_2(ax^2)}{6x^6}
\end{aligned}$$

**Mathematica [A]** time = 0.0314649, size = 60, normalized size = 0.81

$$-\frac{6\text{PolyLog}\left(2, ax^2\right) - 4a^3x^6 \log(x) + 2\left(a^3x^6 - 1\right) \log\left(1 - ax^2\right) + ax^2\left(2ax^2 + 1\right)}{36x^6}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x^7, x]

[Out] -(a\*x^2\*(1 + 2\*a\*x^2) - 4\*a^3\*x^6\*Log[x] + 2\*(-1 + a^3\*x^6)\*Log[1 - a\*x^2] + 6\*PolyLog[2, a\*x^2])/(36\*x^6)

**Maple [A]** time = 0.05, size = 62, normalized size = 0.8

$$-\frac{\text{polylog}\left(2, ax^2\right)}{6x^6} + \frac{\ln\left(-ax^2 + 1\right)}{18x^6} - \frac{a}{36x^4} - \frac{a^2}{18x^2} + \frac{a^3 \ln(x)}{9} - \frac{a^3 \ln(ax^2 - 1)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^2)/x^7, x)

[Out] -1/6\*polylog(2, a\*x^2)/x^6+1/18\*ln(-a\*x^2+1)/x^6-1/36\*a/x^4-1/18\*a^2/x^2+1/9\*a^3\*ln(x)-1/18\*a^3\*ln(a\*x^2-1)

**Maxima [A]** time = 0.981148, size = 74, normalized size = 1.

$$\frac{1}{9} a^3 \log(x) - \frac{2a^2x^4 + ax^2 + 2(a^3x^6 - 1) \log(-ax^2 + 1) + 6 \text{Li}_2(ax^2)}{36x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^2)/x^7, x, algorithm="maxima")

[Out] 1/9\*a^3\*log(x) - 1/36\*(2\*a^2\*x^4 + a\*x^2 + 2\*(a^3\*x^6 - 1)\*log(-a\*x^2 + 1) + 6\*dilog(a\*x^2))/x^6

**Fricas [A]** time = 2.65693, size = 154, normalized size = 2.08

$$\frac{2 a^3 x^6 \log(ax^2 - 1) - 4 a^3 x^6 \log(x) + 2 a^2 x^4 + ax^2 + 6 \operatorname{Li}_2(ax^2) - 2 \log(-ax^2 + 1)}{36 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^7,x, algorithm="fricas")

[Out] -1/36\*(2\*a^3\*x^6\*log(a\*x^2 - 1) - 4\*a^3\*x^6\*log(x) + 2\*a^2\*x^4 + a\*x^2 + 6\*dilog(a\*x^2) - 2\*log(-a\*x^2 + 1))/x^6

**Sympy [A]** time = 43.0619, size = 58, normalized size = 0.78

$$\frac{a^3 \log(x)}{9} + \frac{a^3 \operatorname{Li}_1(ax^2)}{18} - \frac{a^2}{18x^2} - \frac{a}{36x^4} - \frac{\operatorname{Li}_1(ax^2)}{18x^6} - \frac{\operatorname{Li}_2(ax^2)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/x\*\*7,x)

[Out] a\*\*3\*log(x)/9 + a\*\*3\*polylog(1, a\*x\*\*2)/18 - a\*\*2/(18\*x\*\*2) - a/(36\*x\*\*4) - polylog(1, a\*x\*\*2)/(18\*x\*\*6) - polylog(2, a\*x\*\*2)/(6\*x\*\*6)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^7,x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/x^7, x)

## 3.26 $\int x^4 \mathbf{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=73

$$\frac{1}{5}x^5 \mathbf{PolyLog}(2, ax^2) - \frac{4x}{25a^2} + \frac{4 \tanh^{-1}(\sqrt{ax})}{25a^{5/2}} - \frac{4x^3}{75a} + \frac{2}{25}x^5 \log(1 - ax^2) - \frac{4x^5}{125}$$

[Out]  $(-4*x)/(25*a^2) - (4*x^3)/(75*a) - (4*x^5)/125 + (4*ArcTanh[Sqrt[a]*x])/(25*a^{(5/2)}) + (2*x^5*Log[1 - a*x^2])/25 + (x^5*PolyLog[2, a*x^2])/5$

**Rubi [A]** time = 0.0450937, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 302, 206}

$$\frac{1}{5}x^5 \mathbf{PolyLog}(2, ax^2) - \frac{4x}{25a^2} + \frac{4 \tanh^{-1}(\sqrt{ax})}{25a^{5/2}} - \frac{4x^3}{75a} + \frac{2}{25}x^5 \log(1 - ax^2) - \frac{4x^5}{125}$$

Antiderivative was successfully verified.

[In] Int[x^4\*PolyLog[2, a\*x^2], x]

[Out]  $(-4*x)/(25*a^2) - (4*x^3)/(75*a) - (4*x^5)/125 + (4*ArcTanh[Sqrt[a]*x])/(25*a^{(5/2)}) + (2*x^5*Log[1 - a*x^2])/25 + (x^5*PolyLog[2, a*x^2])/5$

### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int x^4 \text{Li}_2(ax^2) dx &= \frac{1}{5} x^5 \text{Li}_2(ax^2) + \frac{2}{5} \int x^4 \log(1 - ax^2) dx \\
&= \frac{2}{25} x^5 \log(1 - ax^2) + \frac{1}{5} x^5 \text{Li}_2(ax^2) + \frac{1}{25} (4a) \int \frac{x^6}{1 - ax^2} dx \\
&= \frac{2}{25} x^5 \log(1 - ax^2) + \frac{1}{5} x^5 \text{Li}_2(ax^2) + \frac{1}{25} (4a) \int \left( -\frac{1}{a^3} - \frac{x^2}{a^2} - \frac{x^4}{a} + \frac{1}{a^3(1 - ax^2)} \right) dx \\
&= -\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{2}{25} x^5 \log(1 - ax^2) + \frac{1}{5} x^5 \text{Li}_2(ax^2) + \frac{4 \int \frac{1}{1 - ax^2} dx}{25a^2} \\
&= -\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{4 \tanh^{-1}(\sqrt{ax})}{25a^{5/2}} + \frac{2}{25} x^5 \log(1 - ax^2) + \frac{1}{5} x^5 \text{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0723123, size = 65, normalized size = 0.89

$$\frac{1}{375} \left( 75x^5 \text{PolyLog}(2, ax^2) - \frac{60x}{a^2} + \frac{60 \tanh^{-1}(\sqrt{ax})}{a^{5/2}} - \frac{20x^3}{a} + 30x^5 \log(1 - ax^2) - 12x^5 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*PolyLog[2, a\*x^2], x]

[Out] ((-60\*x)/a^2 - (20\*x^3)/a - 12\*x^5 + (60\*ArcTanh[Sqrt[a]\*x])/a^(5/2) + 30\*x^5\*Log[1 - a\*x^2] + 75\*x^5\*PolyLog[2, a\*x^2])/375

**Maple [A]** time = 0.046, size = 58, normalized size = 0.8

$$-\frac{4x}{25a^2} - \frac{4x^3}{75a} - \frac{4x^5}{125} + \frac{4}{25} \text{Artanh}(x\sqrt{a}) a^{-\frac{5}{2}} + \frac{2x^5 \ln(-ax^2 + 1)}{25} + \frac{x^5 \text{polylog}(2, ax^2)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*polylog(2,a\*x^2), x)

[Out] -4/25\*x/a^2-4/75\*x^3/a-4/125\*x^5+4/25\*arctanh(x\*a^(1/2))/a^(5/2)+2/25\*x^5\*ln(-a\*x^2+1)+1/5\*x^5\*polylog(2,a\*x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.69839, size = 398, normalized size = 5.45

$$\left[ \frac{75 a^3 x^5 \operatorname{Li}_2(ax^2) + 30 a^3 x^5 \log(-ax^2 + 1) - 12 a^3 x^5 - 20 a^2 x^3 - 60 ax + 30 \sqrt{a} \log\left(\frac{ax^2 + 2\sqrt{a}x + 1}{ax^2 - 1}\right)}{375 a^3}, \frac{75 a^3 x^5 \operatorname{Li}_2(ax^2) + \dots}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x^2),x, algorithm="fricas")

[Out] [1/375\*(75\*a^3\*x^5\*dilog(a\*x^2) + 30\*a^3\*x^5\*log(-a\*x^2 + 1) - 12\*a^3\*x^5 - 20\*a^2\*x^3 - 60\*a\*x + 30\*sqrt(a)\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)))/a^3, 1/375\*(75\*a^3\*x^5\*dilog(a\*x^2) + 30\*a^3\*x^5\*log(-a\*x^2 + 1) - 12\*a^3\*x^5 - 20\*a^2\*x^3 - 60\*a\*x - 60\*sqrt(-a)\*arctan(sqrt(-a)\*x))/a^3]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*polylog(2,a\*x\*\*2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate(x^4\*dilog(a\*x^2), x)

### 3.27 $\int x^2 \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=63

$$\frac{1}{3}x^3 \text{PolyLog}(2, ax^2) + \frac{4 \tanh^{-1}(\sqrt{ax})}{9a^{3/2}} + \frac{2}{9}x^3 \log(1 - ax^2) - \frac{4x}{9a} - \frac{4x^3}{27}$$

[Out]  $(-4*x)/(9*a) - (4*x^3)/27 + (4*\text{ArcTanh}[\text{Sqrt}[a]*x])/(9*a^{(3/2)}) + (2*x^3*\text{Log}[1 - a*x^2])/9 + (x^3*\text{PolyLog}[2, a*x^2])/3$

**Rubi [A]** time = 0.0387699, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 302, 206}

$$\frac{1}{3}x^3 \text{PolyLog}(2, ax^2) + \frac{4 \tanh^{-1}(\sqrt{ax})}{9a^{3/2}} + \frac{2}{9}x^3 \log(1 - ax^2) - \frac{4x}{9a} - \frac{4x^3}{27}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{PolyLog}[2, a*x^2], x]$

[Out]  $(-4*x)/(9*a) - (4*x^3)/27 + (4*\text{ArcTanh}[\text{Sqrt}[a]*x])/(9*a^{(3/2)}) + (2*x^3*\text{Log}[1 - a*x^2])/9 + (x^3*\text{PolyLog}[2, a*x^2])/3$

#### Rule 6591

$\text{Int}[(d_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_*, (a_*)*((b_*)*(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})^{(p_*)}]* (b_*)*((f_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])/ (f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^{(n-1)}*(f*x)^{(m+1)})/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 302

$\text{Int}[(x_*)^{(m_*)}/((a_*) + (b_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Int}[\text{PolynomialDivide}[x^m, a + b*x^n, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 206

$\text{Int}[(a_*) + (b_*)*(x_*)^2)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int x^2 \text{Li}_2(ax^2) dx &= \frac{1}{3} x^3 \text{Li}_2(ax^2) + \frac{2}{3} \int x^2 \log(1 - ax^2) dx \\
&= \frac{2}{9} x^3 \log(1 - ax^2) + \frac{1}{3} x^3 \text{Li}_2(ax^2) + \frac{1}{9} (4a) \int \frac{x^4}{1 - ax^2} dx \\
&= \frac{2}{9} x^3 \log(1 - ax^2) + \frac{1}{3} x^3 \text{Li}_2(ax^2) + \frac{1}{9} (4a) \int \left( -\frac{1}{a^2} - \frac{x^2}{a} + \frac{1}{a^2(1 - ax^2)} \right) dx \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{2}{9} x^3 \log(1 - ax^2) + \frac{1}{3} x^3 \text{Li}_2(ax^2) + \frac{4 \int \frac{1}{1 - ax^2} dx}{9a} \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4 \tanh^{-1}(\sqrt{ax})}{9a^{3/2}} + \frac{2}{9} x^3 \log(1 - ax^2) + \frac{1}{3} x^3 \text{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0514052, size = 57, normalized size = 0.9

$$\frac{1}{27} \left( 9x^3 \text{PolyLog}(2, ax^2) + \frac{12 \tanh^{-1}(\sqrt{ax})}{a^{3/2}} + 6x^3 \log(1 - ax^2) - \frac{12x}{a} - 4x^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[2, a\*x^2], x]

[Out] ((-12\*x)/a - 4\*x^3 + (12\*ArcTanh[Sqrt[a]\*x])/a^(3/2) + 6\*x^3\*Log[1 - a\*x^2] + 9\*x^3\*PolyLog[2, a\*x^2])/27

**Maple [A]** time = 0.046, size = 50, normalized size = 0.8

$$-\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4}{9} \text{Artanh}(x\sqrt{a}) a^{-\frac{3}{2}} + \frac{2x^3 \ln(-ax^2 + 1)}{9} + \frac{x^3 \text{polylog}(2, ax^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(2,a\*x^2), x)

[Out] -4/9\*x/a-4/27\*x^3+4/9\*arctanh(x\*a^(1/2))/a^(3/2)+2/9\*x^3\*ln(-a\*x^2+1)+1/3\*x^3\*polylog(2,a\*x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.63943, size = 351, normalized size = 5.57

$$\left[ \frac{9 a^2 x^3 \operatorname{Li}_2(ax^2) + 6 a^2 x^3 \log(-ax^2 + 1) - 4 a^2 x^3 - 12 ax + 6 \sqrt{a} \log\left(\frac{ax^2 + 2\sqrt{ax} + 1}{ax^2 - 1}\right)}{27 a^2}, \frac{9 a^2 x^3 \operatorname{Li}_2(ax^2) + 6 a^2 x^3 \log(-ax^2 + 1)}{27 a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^2),x, algorithm="fricas")

[Out] [1/27\*(9\*a^2\*x^3\*dilog(a\*x^2) + 6\*a^2\*x^3\*log(-a\*x^2 + 1) - 4\*a^2\*x^3 - 12\*a\*x + 6\*sqrt(a)\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)))/a^2, 1/27\*(9\*a^2\*x^3\*dilog(a\*x^2) + 6\*a^2\*x^3\*log(-a\*x^2 + 1) - 4\*a^2\*x^3 - 12\*a\*x - 12\*sqrt(-a)\*arctan(sqrt(-a)\*x))/a^2]

**Sympy [A]** time = 119.376, size = 83, normalized size = 1.32

$$\begin{cases} -\frac{2x^3 \operatorname{Li}_1(ax^2)}{9} + \frac{x^3 \operatorname{Li}_2(ax^2)}{3} - \frac{4x^3}{27} - \frac{4x}{9a} - \frac{4 \log\left(x - \sqrt{\frac{1}{a}}\right)}{9a^2 \sqrt{\frac{1}{a}}} - \frac{2 \operatorname{Li}_1(ax^2)}{9a^2 \sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(2,a\*x\*\*2),x)

[Out] Piecewise((-2\*x\*\*3\*polylog(1, a\*x\*\*2)/9 + x\*\*3\*polylog(2, a\*x\*\*2)/3 - 4\*x\*\*3/27 - 4\*x/(9\*a) - 4\*log(x - sqrt(1/a))/(9\*a\*\*2\*sqrt(1/a)) - 2\*polylog(1, a\*x\*\*2)/(9\*a\*\*2\*sqrt(1/a)), Ne(a, 0)), (0, True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate(x^2\*dilog(a\*x^2), x)



## 3.28 $\int \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=40

$$x \text{PolyLog}(2, ax^2) + 2x \log(1 - ax^2) + \frac{4 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} - 4x$$

[Out] -4\*x + (4\*ArcTanh[Sqrt[a]\*x])/Sqrt[a] + 2\*x\*Log[1 - a\*x^2] + x\*PolyLog[2, a\*x^2]

**Rubi [A]** time = 0.0174073, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6586, 2448, 321, 206}

$$x \text{PolyLog}(2, ax^2) + 2x \log(1 - ax^2) + \frac{4 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} - 4x$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2], x]

[Out] -4\*x + (4\*ArcTanh[Sqrt[a]\*x])/Sqrt[a] + 2\*x\*Log[1 - a\*x^2] + x\*PolyLog[2, a\*x^2]

### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] :> Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \operatorname{Li}_2(ax^2) dx &= x\operatorname{Li}_2(ax^2) + 2 \int \log(1 - ax^2) dx \\
&= 2x \log(1 - ax^2) + x\operatorname{Li}_2(ax^2) + (4a) \int \frac{x^2}{1 - ax^2} dx \\
&= -4x + 2x \log(1 - ax^2) + x\operatorname{Li}_2(ax^2) + 4 \int \frac{1}{1 - ax^2} dx \\
&= -4x + \frac{4 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} + 2x \log(1 - ax^2) + x\operatorname{Li}_2(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0278312, size = 39, normalized size = 0.98

$$x\operatorname{PolyLog}(2, ax^2) + 2x(\log(1 - ax^2) - 2) + \frac{4 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2], x]

[Out] (4\*ArcTanh[Sqrt[a]\*x])/Sqrt[a] + 2\*x\*(-2 + Log[1 - a\*x^2]) + x\*PolyLog[2, a\*x^2]

**Maple [A]** time = 0.046, size = 37, normalized size = 0.9

$$-4x + 2x \ln(-ax^2 + 1) + x\operatorname{polylog}(2, ax^2) + 4 \frac{\operatorname{Arctanh}(x\sqrt{a})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^2), x)

[Out] -4\*x+2\*x\*ln(-a\*x^2+1)+x\*polylog(2, a\*x^2)+4\*arctanh(x\*a^(1/2))/a^(1/2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.63136, size = 269, normalized size = 6.72

$$\left[ \frac{ax\operatorname{Li}_2(ax^2) + 2ax \log(-ax^2 + 1) - 4ax + 2\sqrt{a} \log\left(\frac{ax^2 + 2\sqrt{ax} + 1}{ax^2 - 1}\right)}{a}, \frac{ax\operatorname{Li}_2(ax^2) + 2ax \log(-ax^2 + 1) - 4ax - 4\sqrt{-a} \operatorname{arccot}\left(\frac{\sqrt{-a}}{ax}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [(a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1) - 4*a*x + 2*sqrt(a)*log((a*x^2 +
2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, (a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1
) - 4*a*x - 4*sqrt(-a)*arctan(sqrt(-a)*x))/a]
```

**Sympy [A]** time = 23.2238, size = 60, normalized size = 1.5

$$\begin{cases} -2x \operatorname{Li}_1(ax^2) + x \operatorname{Li}_2(ax^2) - 4x - \frac{4 \log\left(x - \sqrt{\frac{1}{a}}\right)}{a \sqrt{\frac{1}{a}}} - \frac{2 \operatorname{Li}_1(ax^2)}{a \sqrt{\frac{1}{a}}} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x*polylog(1, a*x**2) + x*polylog(2, a*x**2) - 4*x - 4*log(x -
sqrt(1/a))/(a*sqrt(1/a)) - 2*polylog(1, a*x**2)/(a*sqrt(1/a)), Ne(a, 0)),
(0, True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2), x)
```

$$3.29 \quad \int \frac{\text{PolyLog}(2, ax^2)}{x^2} dx$$

**Optimal.** Leaf size=42

$$-\frac{\text{PolyLog}(2, ax^2)}{x} + \frac{2 \log(1 - ax^2)}{x} + 4\sqrt{a} \tanh^{-1}(\sqrt{ax})$$

[Out] 4\*Sqrt[a]\*ArcTanh[Sqrt[a]\*x] + (2\*Log[1 - a\*x^2])/x - PolyLog[2, a\*x^2]/x

**Rubi [A]** time = 0.0261791, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 206}

$$-\frac{\text{PolyLog}(2, ax^2)}{x} + \frac{2 \log(1 - ax^2)}{x} + 4\sqrt{a} \tanh^{-1}(\sqrt{ax})$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^2, x]

[Out] 4\*Sqrt[a]\*ArcTanh[Sqrt[a]\*x] + (2\*Log[1 - a\*x^2])/x - PolyLog[2, a\*x^2]/x

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax^2)}{x^2} dx &= -\frac{\text{Li}_2(ax^2)}{x} - 2 \int \frac{\log(1 - ax^2)}{x^2} dx \\ &= \frac{2 \log(1 - ax^2)}{x} - \frac{\text{Li}_2(ax^2)}{x} + (4a) \int \frac{1}{1 - ax^2} dx \\ &= 4\sqrt{a} \tanh^{-1}(\sqrt{ax}) + \frac{2 \log(1 - ax^2)}{x} - \frac{\text{Li}_2(ax^2)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0177403, size = 41, normalized size = 0.98

$$\frac{-\text{PolyLog}\left(2, ax^2\right) + 2 \log\left(1 - ax^2\right) + 4\sqrt{ax} \tanh^{-1}\left(\sqrt{ax}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x^2,x]

[Out] (4\*Sqrt[a]\*x\*ArcTanh[Sqrt[a]\*x] + 2\*Log[1 - a\*x^2] - PolyLog[2, a\*x^2])/x

**Maple [A]** time = 0.049, size = 39, normalized size = 0.9

$$2 \frac{\ln(-ax^2 + 1)}{x} - \frac{\text{polylog}(2, ax^2)}{x} + 4 \text{Artanh}(x\sqrt{a})\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^2)/x^2,x)

[Out] 2\*ln(-a\*x^2+1)/x-polylog(2,a\*x^2)/x+4\*arctanh(x\*a^(1/2))\*a^(1/2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.79204, size = 232, normalized size = 5.52

$$\left[ \frac{2\sqrt{ax} \log\left(\frac{ax^2+2\sqrt{ax}+1}{ax^2-1}\right) - \text{Li}_2(ax^2) + 2 \log(-ax^2 + 1)}{x}, -\frac{4\sqrt{-ax} \arctan(\sqrt{-ax}) + \text{Li}_2(ax^2) - 2 \log(-ax^2 + 1)}{x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^2,x, algorithm="fricas")

[Out] [(2\*sqrt(a)\*x\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)) - dilog(a\*x^2) + 2\*log(-a\*x^2 + 1))/x, -(4\*sqrt(-a)\*x\*arctan(sqrt(-a)\*x) + dilog(a\*x^2) - 2\*log(-a\*x^2 + 1))/x]

**Sympy [A]** time = 84.144, size = 184, normalized size = 4.38

$$\begin{cases} 0 \\ -\frac{\pi^2}{6x} \\ -\frac{4ax^3\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}} - \frac{2ax^3\sqrt{\frac{1}{a}}\operatorname{Li}_1(ax^2)}{x^3-\frac{x}{a}} - \frac{2x^2\operatorname{Li}_1(ax^2)}{x^3-\frac{x}{a}} - \frac{x^2\operatorname{Li}_2(ax^2)}{x^3-\frac{x}{a}} + \frac{4x\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}} + \frac{2x\sqrt{\frac{1}{a}}\operatorname{Li}_1(ax^2)}{x^3-\frac{x}{a}} + \frac{2\operatorname{Li}_1(ax^2)}{ax^3-x} + \frac{\operatorname{Li}_2(ax^2)}{ax^3-x} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/x\*\*2,x)

[Out] Piecewise((0, Eq(a, 0)), (-pi\*\*2/(6\*x), Eq(a, x\*\*(-2))), (-4\*a\*x\*\*3\*sqrt(1/a)\*log(x - sqrt(1/a))/(x\*\*3 - x/a) - 2\*a\*x\*\*3\*sqrt(1/a)\*polylog(1, a\*x\*\*2)/(x\*\*3 - x/a) - 2\*x\*\*2\*polylog(1, a\*x\*\*2)/(x\*\*3 - x/a) - x\*\*2\*polylog(2, a\*x\*\*2)/(x\*\*3 - x/a) + 4\*x\*sqrt(1/a)\*log(x - sqrt(1/a))/(x\*\*3 - x/a) + 2\*x\*sqrt(1/a)\*polylog(1, a\*x\*\*2)/(x\*\*3 - x/a) + 2\*polylog(1, a\*x\*\*2)/(a\*x\*\*3 - x) + polylog(2, a\*x\*\*2)/(a\*x\*\*3 - x), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^2,x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/x^2, x)

### 3.30 $\int \frac{\text{PolyLog}(2, ax^2)}{x^4} dx$

**Optimal.** Leaf size=56

$$-\frac{\text{PolyLog}(2, ax^2)}{3x^3} + \frac{4}{9}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{2 \log(1 - ax^2)}{9x^3} - \frac{4a}{9x}$$

[Out]  $(-4*a)/(9*x) + (4*a^{(3/2)}*ArcTanh[Sqrt[a]*x])/9 + (2*Log[1 - a*x^2])/(9*x^3) - \text{PolyLog}[2, a*x^2]/(3*x^3)$

**Rubi [A]** time = 0.0321837, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 325, 206}

$$-\frac{\text{PolyLog}(2, ax^2)}{3x^3} + \frac{4}{9}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{2 \log(1 - ax^2)}{9x^3} - \frac{4a}{9x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^4, x]

[Out]  $(-4*a)/(9*x) + (4*a^{(3/2)}*ArcTanh[Sqrt[a]*x])/9 + (2*Log[1 - a*x^2])/(9*x^3) - \text{PolyLog}[2, a*x^2]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(ax^2)}{x^4} dx &= -\frac{\operatorname{Li}_2(ax^2)}{3x^3} - \frac{2}{3} \int \frac{\log(1-ax^2)}{x^4} dx \\
&= \frac{2 \log(1-ax^2)}{9x^3} - \frac{\operatorname{Li}_2(ax^2)}{3x^3} + \frac{1}{9}(4a) \int \frac{1}{x^2(1-ax^2)} dx \\
&= -\frac{4a}{9x} + \frac{2 \log(1-ax^2)}{9x^3} - \frac{\operatorname{Li}_2(ax^2)}{3x^3} + \frac{1}{9}(4a^2) \int \frac{1}{1-ax^2} dx \\
&= -\frac{4a}{9x} + \frac{4}{9}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{2 \log(1-ax^2)}{9x^3} - \frac{\operatorname{Li}_2(ax^2)}{3x^3}
\end{aligned}$$

**Mathematica [C]** time = 0.0136452, size = 47, normalized size = 0.84

$$\frac{4ax^2 \operatorname{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, ax^2\right) + 3 \operatorname{PolyLog}(2, ax^2) - 2 \log(1-ax^2)}{9x^3}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x^4, x]

[Out] -(4\*a\*x^2\*Hypergeometric2F1[-1/2, 1, 1/2, a\*x^2] - 2\*Log[1 - a\*x^2] + 3\*PolyLog[2, a\*x^2])/(9\*x^3)

**Maple [A]** time = 0.053, size = 45, normalized size = 0.8

$$-\frac{4a}{9x} + \frac{4}{9}a^{\frac{3}{2}} \operatorname{Artanh}(x\sqrt{a}) + \frac{2 \ln(-ax^2 + 1)}{9x^3} - \frac{\operatorname{polylog}(2, ax^2)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^2)/x^4,x)

[Out] -4/9\*a/x+4/9\*a^(3/2)\*arctanh(x\*a^(1/2))+2/9\*ln(-a\*x^2+1)/x^3-1/3\*polylog(2, a\*x^2)/x^3

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.74604, size = 289, normalized size = 5.16

$$\left[ \frac{2a^{\frac{3}{2}}x^3 \log\left(\frac{ax^2+2\sqrt{ax}+1}{ax^2-1}\right) - 4ax^2 - 3\operatorname{Li}_2(ax^2) + 2 \log(-ax^2 + 1)}{9x^3}, -\frac{4\sqrt{-a}ax^3 \arctan(\sqrt{-ax}) + 4ax^2 + 3\operatorname{Li}_2(ax^2) - 2 \log(1-ax^2)}{9x^3} \right]$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(2*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 4*a*x^2 -
3*dilog(a*x^2) + 2*log(-a*x^2 + 1))/x^3, -1/9*(4*sqrt(-a)*a*x^3*arctan(sqrt
(-a)*x) + 4*a*x^2 + 3*dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^3]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^4, x)
```

### 3.31 $\int \frac{\text{PolyLog}(2, ax^2)}{x^6} dx$

**Optimal.** Leaf size=66

$$-\frac{\text{PolyLog}(2, ax^2)}{5x^5} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2} \tanh^{-1}(\sqrt{ax}) - \frac{4a}{75x^3} + \frac{2 \log(1 - ax^2)}{25x^5}$$

[Out]  $(-4*a)/(75*x^3) - (4*a^2)/(25*x) + (4*a^{(5/2)}*ArcTanh[Sqrt[a]*x])/25 + (2*Log[1 - a*x^2])/(25*x^5) - PolyLog[2, a*x^2]/(5*x^5)$

**Rubi [A]** time = 0.0376341, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 325, 206}

$$-\frac{\text{PolyLog}(2, ax^2)}{5x^5} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2} \tanh^{-1}(\sqrt{ax}) - \frac{4a}{75x^3} + \frac{2 \log(1 - ax^2)}{25x^5}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/x^6, x]

[Out]  $(-4*a)/(75*x^3) - (4*a^2)/(25*x) + (4*a^{(5/2)}*ArcTanh[Sqrt[a]*x])/25 + (2*Log[1 - a*x^2])/(25*x^5) - PolyLog[2, a*x^2]/(5*x^5)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(ax^2)}{x^6} dx &= -\frac{\operatorname{Li}_2(ax^2)}{5x^5} - \frac{2}{5} \int \frac{\log(1-ax^2)}{x^6} dx \\
&= \frac{2 \log(1-ax^2)}{25x^5} - \frac{\operatorname{Li}_2(ax^2)}{5x^5} + \frac{1}{25}(4a) \int \frac{1}{x^4(1-ax^2)} dx \\
&= -\frac{4a}{75x^3} + \frac{2 \log(1-ax^2)}{25x^5} - \frac{\operatorname{Li}_2(ax^2)}{5x^5} + \frac{1}{25}(4a^2) \int \frac{1}{x^2(1-ax^2)} dx \\
&= -\frac{4a}{75x^3} - \frac{4a^2}{25x} + \frac{2 \log(1-ax^2)}{25x^5} - \frac{\operatorname{Li}_2(ax^2)}{5x^5} + \frac{1}{25}(4a^3) \int \frac{1}{1-ax^2} dx \\
&= -\frac{4a}{75x^3} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2} \tanh^{-1}(\sqrt{ax}) + \frac{2 \log(1-ax^2)}{25x^5} - \frac{\operatorname{Li}_2(ax^2)}{5x^5}
\end{aligned}$$

**Mathematica [C]** time = 0.0151209, size = 47, normalized size = 0.71

$$\frac{4ax^2 \operatorname{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, ax^2\right) + 15 \operatorname{PolyLog}(2, ax^2) - 6 \log(1-ax^2)}{75x^5}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^2]/x^6, x]

[Out]  $-(4ax^2 \operatorname{Hypergeometric2F1}[-3/2, 1, -1/2, ax^2] - 6 \operatorname{Log}[1 - ax^2] + 15 \operatorname{PolyLog}[2, ax^2]) / (75x^5)$

**Maple [A]** time = 0.052, size = 53, normalized size = 0.8

$$-\frac{4a}{75x^3} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2} \operatorname{Artanh}(x\sqrt{a}) + \frac{2 \ln(-ax^2 + 1)}{25x^5} - \frac{\operatorname{polylog}(2, ax^2)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^2)/x^6, x)

[Out]  $-4/75*a/x^3 - 4/25*a^2/x + 4/25*a^{5/2}*\operatorname{arctanh}(x*a^{1/2}) + 2/25*\ln(-a*x^2+1)/x^5 - 1/5*\operatorname{polylog}(2, a*x^2)/x^5$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^2)/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.66061, size = 333, normalized size = 5.05

$$\left[ \frac{6a^{\frac{5}{2}}x^5 \log\left(\frac{ax^2+2\sqrt{ax}+1}{ax^2-1}\right) - 12a^2x^4 - 4ax^2 - 15\operatorname{Li}_2(ax^2) + 6\log(-ax^2+1)}{75x^5}, -\frac{12\sqrt{-a}a^2x^5 \arctan(\sqrt{-ax}) + 12a^2x^4 + 4a^2x^2 - 15\operatorname{dilog}(ax^2) - 6\log(-ax^2+1)}{75x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^6,x, algorithm="fricas")

[Out] [1/75\*(6\*a^(5/2)\*x^5\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)) - 12\*a^2\*x^4 - 4\*a\*x^2 - 15\*dilog(a\*x^2) + 6\*log(-a\*x^2 + 1))/x^5, -1/75\*(12\*sqrt(-a)\*a^2\*x^5\*arctan(sqrt(-a)\*x) + 12\*a^2\*x^4 + 4\*a\*x^2 + 15\*dilog(a\*x^2) - 6\*log(-a\*x^2 + 1))/x^5]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/x\*\*6,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/x^6,x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/x^6, x)

### 3.32 $\int x^5 \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=88

$$-\frac{1}{18}x^6 \text{PolyLog}(2, ax^2) + \frac{1}{6}x^6 \text{PolyLog}(3, ax^2) + \frac{x^2}{54a^2} + \frac{\log(1-ax^2)}{54a^3} + \frac{x^4}{108a} - \frac{1}{54}x^6 \log(1-ax^2) + \frac{x^6}{162}$$

[Out]  $x^2/(54*a^2) + x^4/(108*a) + x^6/162 + \text{Log}[1 - a*x^2]/(54*a^3) - (x^6*\text{Log}[1 - a*x^2])/54 - (x^6*\text{PolyLog}[2, a*x^2])/18 + (x^6*\text{PolyLog}[3, a*x^2])/6$

**Rubi [A]** time = 0.075215, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 43}

$$-\frac{1}{18}x^6 \text{PolyLog}(2, ax^2) + \frac{1}{6}x^6 \text{PolyLog}(3, ax^2) + \frac{x^2}{54a^2} + \frac{\log(1-ax^2)}{54a^3} + \frac{x^4}{108a} - \frac{1}{54}x^6 \log(1-ax^2) + \frac{x^6}{162}$$

Antiderivative was successfully verified.

[In] Int[x^5\*PolyLog[3, a\*x^2], x]

[Out]  $x^2/(54*a^2) + x^4/(108*a) + x^6/162 + \text{Log}[1 - a*x^2]/(54*a^3) - (x^6*\text{Log}[1 - a*x^2])/54 - (x^6*\text{PolyLog}[2, a*x^2])/18 + (x^6*\text{PolyLog}[3, a*x^2])/6$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int x^5 \text{Li}_3(ax^2) dx &= \frac{1}{6} x^6 \text{Li}_3(ax^2) - \frac{1}{3} \int x^5 \text{Li}_2(ax^2) dx \\
&= -\frac{1}{18} x^6 \text{Li}_2(ax^2) + \frac{1}{6} x^6 \text{Li}_3(ax^2) - \frac{1}{9} \int x^5 \log(1-ax^2) dx \\
&= -\frac{1}{18} x^6 \text{Li}_2(ax^2) + \frac{1}{6} x^6 \text{Li}_3(ax^2) - \frac{1}{18} \text{Subst} \left( \int x^2 \log(1-ax) dx, x, x^2 \right) \\
&= -\frac{1}{54} x^6 \log(1-ax^2) - \frac{1}{18} x^6 \text{Li}_2(ax^2) + \frac{1}{6} x^6 \text{Li}_3(ax^2) - \frac{1}{54} a \text{Subst} \left( \int \frac{x^3}{1-ax} dx, x, x^2 \right) \\
&= -\frac{1}{54} x^6 \log(1-ax^2) - \frac{1}{18} x^6 \text{Li}_2(ax^2) + \frac{1}{6} x^6 \text{Li}_3(ax^2) - \frac{1}{54} a \text{Subst} \left( \int \left( -\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)} \right) dx, x, x^2 \right) \\
&= \frac{x^2}{54a^2} + \frac{x^4}{108a} + \frac{x^6}{162} + \frac{\log(1-ax^2)}{54a^3} - \frac{1}{54} x^6 \log(1-ax^2) - \frac{1}{18} x^6 \text{Li}_2(ax^2) + \frac{1}{6} x^6 \text{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0180466, size = 88, normalized size = 1.

$$\frac{-18a^3x^6\text{PolyLog}(2,ax^2) + 54a^3x^6\text{PolyLog}(3,ax^2) + 2a^3x^6 + 3a^2x^4 - 6a^3x^6\log(1-ax^2) + 6ax^2 + 6\log(1-ax^2)}{324a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*PolyLog[3, a\*x^2], x]

[Out] (6\*a\*x^2 + 3\*a^2\*x^4 + 2\*a^3\*x^6 + 6\*Log[1 - a\*x^2] - 6\*a^3\*x^6\*Log[1 - a\*x^2] - 18\*a^3\*x^6\*PolyLog[2, a\*x^2] + 54\*a^3\*x^6\*PolyLog[3, a\*x^2])/(324\*a^3)

**Maple [A]** time = 0.058, size = 80, normalized size = 0.9

$$\frac{1}{2a^3} \left( \frac{x^2a(4a^2x^4 + 6ax^2 + 12)}{324} + \frac{(-4x^6a^3 + 4)\ln(-ax^2 + 1)}{108} - \frac{x^6a^3\text{polylog}(2, ax^2)}{9} + \frac{x^6a^3\text{polylog}(3, ax^2)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*polylog(3,a\*x^2), x)

[Out] 1/2/a^3\*(1/324\*x^2\*a\*(4\*a^2\*x^4+6\*a\*x^2+12)+1/108\*(-4\*a^3\*x^6+4)\*ln(-a\*x^2+1)-1/9\*x^6\*a^3\*polylog(2,a\*x^2)+1/3\*x^6\*a^3\*polylog(3,a\*x^2))

**Maxima [A]** time = 0.97977, size = 104, normalized size = 1.18

$$\frac{18a^3x^6\text{Li}_2(ax^2) - 54a^3x^6\text{Li}_3(ax^2) - 2a^3x^6 - 3a^2x^4 - 6ax^2 + 6(a^3x^6 - 1)\log(-ax^2 + 1)}{324a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] -1/324\*(18\*a^3\*x^6\*dilog(a\*x^2) - 54\*a^3\*x^6\*polylog(3, a\*x^2) - 2\*a^3\*x^6 - 3\*a^2\*x^4 - 6\*a\*x^2 + 6\*(a^3\*x^6 - 1)\*log(-a\*x^2 + 1))/a^3

---

**Fricas [C]** time = 2.71681, size = 240, normalized size = 2.73

$$\frac{18 a^3 x^6 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) - 54 a^3 x^6 \operatorname{polylog}\left(3, ax^2\right) - 2 a^3 x^6 - 3 a^2 x^4 - 6 a x^2 + 6 (a^3 x^6 - 1) \log(-ax^2+1)}{324 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] -1/324\*(18\*a^3\*x^6\*Li3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) - 54\*a^3\*x^6\*polylog(3, a\*x^2) - 2\*a^3\*x^6 - 3\*a^2\*x^4 - 6\*a\*x^2 + 6\*(a^3\*x^6 - 1)\*log(-a\*x^2 + 1))/a^3

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*polylog(3,a\*x\*\*2),x)

[Out] Integral(x\*\*5\*polylog(3, a\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(x^5\*polylog(3, a\*x^2), x)

### 3.33 $\int x^3 \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=78

$$-\frac{1}{8}x^4 \text{PolyLog}(2, ax^2) + \frac{1}{4}x^4 \text{PolyLog}(3, ax^2) + \frac{\log(1-ax^2)}{16a^2} + \frac{x^2}{16a} - \frac{1}{16}x^4 \log(1-ax^2) + \frac{x^4}{32}$$

[Out]  $x^2/(16*a) + x^4/32 + \text{Log}[1 - a*x^2]/(16*a^2) - (x^4*\text{Log}[1 - a*x^2])/16 - (x^4*\text{PolyLog}[2, a*x^2])/8 + (x^4*\text{PolyLog}[3, a*x^2])/4$

**Rubi [A]** time = 0.0606042, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 43}

$$-\frac{1}{8}x^4 \text{PolyLog}(2, ax^2) + \frac{1}{4}x^4 \text{PolyLog}(3, ax^2) + \frac{\log(1-ax^2)}{16a^2} + \frac{x^2}{16a} - \frac{1}{16}x^4 \log(1-ax^2) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Int[x^3\*PolyLog[3, a\*x^2], x]

[Out]  $x^2/(16*a) + x^4/32 + \text{Log}[1 - a*x^2]/(16*a^2) - (x^4*\text{Log}[1 - a*x^2])/16 - (x^4*\text{PolyLog}[2, a*x^2])/8 + (x^4*\text{PolyLog}[3, a*x^2])/4$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n+1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int x^3 \text{Li}_3(ax^2) dx &= \frac{1}{4} x^4 \text{Li}_3(ax^2) - \frac{1}{2} \int x^3 \text{Li}_2(ax^2) dx \\
&= -\frac{1}{8} x^4 \text{Li}_2(ax^2) + \frac{1}{4} x^4 \text{Li}_3(ax^2) - \frac{1}{4} \int x^3 \log(1-ax^2) dx \\
&= -\frac{1}{8} x^4 \text{Li}_2(ax^2) + \frac{1}{4} x^4 \text{Li}_3(ax^2) - \frac{1}{8} \text{Subst} \left( \int x \log(1-ax) dx, x, x^2 \right) \\
&= -\frac{1}{16} x^4 \log(1-ax^2) - \frac{1}{8} x^4 \text{Li}_2(ax^2) + \frac{1}{4} x^4 \text{Li}_3(ax^2) - \frac{1}{16} a \text{Subst} \left( \int \frac{x^2}{1-ax} dx, x, x^2 \right) \\
&= -\frac{1}{16} x^4 \log(1-ax^2) - \frac{1}{8} x^4 \text{Li}_2(ax^2) + \frac{1}{4} x^4 \text{Li}_3(ax^2) - \frac{1}{16} a \text{Subst} \left( \int \left( -\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)} \right) dx, x, x^2 \right) \\
&= \frac{x^2}{16a} + \frac{x^4}{32} + \frac{\log(1-ax^2)}{16a^2} - \frac{1}{16} x^4 \log(1-ax^2) - \frac{1}{8} x^4 \text{Li}_2(ax^2) + \frac{1}{4} x^4 \text{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0144214, size = 79, normalized size = 1.01

$$\frac{-4a^2x^4\text{PolyLog}(2, ax^2) + 8a^2x^4\text{PolyLog}(3, ax^2) + a^2x^4 - 2a^2x^4 \log(1-ax^2) + 2ax^2 + 2 \log(1-ax^2)}{32a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*PolyLog[3, a\*x^2], x]

[Out] (2\*a\*x^2 + a^2\*x^4 + 2\*Log[1 - a\*x^2] - 2\*a^2\*x^4\*Log[1 - a\*x^2] - 4\*a^2\*x^4\*PolyLog[2, a\*x^2] + 8\*a^2\*x^4\*PolyLog[3, a\*x^2])/(32\*a^2)

**Maple [A]** time = 0.054, size = 72, normalized size = 0.9

$$-\frac{1}{2a^2} \left( -\frac{x^2a(3ax^2+6)}{48} - \frac{(-3a^2x^4+3)\ln(-ax^2+1)}{24} + \frac{x^4a^2\text{polylog}(2, ax^2)}{4} - \frac{x^4a^2\text{polylog}(3, ax^2)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*polylog(3,a\*x^2), x)

[Out] -1/2/a^2\*(-1/48\*x^2\*a\*(3\*a\*x^2+6)-1/24\*(-3\*a^2\*x^4+3)\*ln(-a\*x^2+1)+1/4\*x^4\*a^2\*polylog(2,a\*x^2)-1/2\*x^4\*a^2\*polylog(3,a\*x^2))

**Maxima [A]** time = 0.968293, size = 93, normalized size = 1.19

$$-\frac{4a^2x^4\text{Li}_2(ax^2) - 8a^2x^4\text{Li}_3(ax^2) - a^2x^4 - 2ax^2 + 2(a^2x^4 - 1)\log(-ax^2 + 1)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] -1/32\*(4\*a^2\*x^4\*dilog(a\*x^2) - 8\*a^2\*x^4\*polylog(3, a\*x^2) - a^2\*x^4 - 2\*a\*x^2 + 2\*(a^2\*x^4 - 1)\*log(-a\*x^2 + 1))/a^2

**Fricas [C]** time = 2.70568, size = 217, normalized size = 2.78

$$\frac{4a^2x^4 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2\log(-ax^2+1)}{x}\right) - 8a^2x^4 \operatorname{polylog}(3, ax^2) - a^2x^4 - 2ax^2 + 2(a^2x^4 - 1)\log(-ax^2 + 1)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] -1/32\*(4\*a^2\*x^4\*\operatorname{Li}\_3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) - 8\*a^2\*x^4\*polylog(3, a\*x^2) - a^2\*x^4 - 2\*a\*x^2 + 2\*(a^2\*x^4 - 1)\*log(-a\*x^2 + 1))/a^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*polylog(3,a\*x\*\*2),x)

[Out] Integral(x\*\*3\*polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(x^3\*polylog(3, a\*x^2), x)

### 3.34 $\int x \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=60

$$-\frac{1}{2}x^2 \text{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \text{PolyLog}(3, ax^2) + \frac{(1-ax^2) \log(1-ax^2)}{2a} + \frac{x^2}{2}$$

[Out]  $x^2/2 + ((1 - a*x^2)*\text{Log}[1 - a*x^2])/(2*a) - (x^2*\text{PolyLog}[2, a*x^2])/2 + (x^2*\text{PolyLog}[3, a*x^2])/2$

**Rubi [A]** time = 0.029869, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {6591, 2454, 2389, 2295}

$$-\frac{1}{2}x^2 \text{PolyLog}(2, ax^2) + \frac{1}{2}x^2 \text{PolyLog}(3, ax^2) + \frac{(1-ax^2) \log(1-ax^2)}{2a} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[3, a\*x^2], x]

[Out]  $x^2/2 + ((1 - a*x^2)*\text{Log}[1 - a*x^2])/(2*a) - (x^2*\text{PolyLog}[2, a*x^2])/2 + (x^2*\text{PolyLog}[3, a*x^2])/2$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(b\_.)^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(b\_.)^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rubi steps

$$\begin{aligned}
\int x\text{Li}_3(ax^2) dx &= \frac{1}{2}x^2\text{Li}_3(ax^2) - \int x\text{Li}_2(ax^2) dx \\
&= -\frac{1}{2}x^2\text{Li}_2(ax^2) + \frac{1}{2}x^2\text{Li}_3(ax^2) - \int x \log(1-ax^2) dx \\
&= -\frac{1}{2}x^2\text{Li}_2(ax^2) + \frac{1}{2}x^2\text{Li}_3(ax^2) - \frac{1}{2} \text{Subst}\left(\int \log(1-ax) dx, x, x^2\right) \\
&= -\frac{1}{2}x^2\text{Li}_2(ax^2) + \frac{1}{2}x^2\text{Li}_3(ax^2) + \frac{\text{Subst}\left(\int \log(x) dx, x, 1-ax^2\right)}{2a} \\
&= \frac{x^2}{2} + \frac{(1-ax^2)\log(1-ax^2)}{2a} - \frac{1}{2}x^2\text{Li}_2(ax^2) + \frac{1}{2}x^2\text{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0112108, size = 52, normalized size = 0.87

$$\frac{1}{2}x^2 \left( -\text{PolyLog}(2, ax^2) + \text{PolyLog}(3, ax^2) + \frac{\log(1-ax^2)}{ax^2} - \log(1-ax^2) + 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[3, a\*x^2], x]

[Out] (x^2\*(1 - Log[1 - a\*x^2] + Log[1 - a\*x^2]/(a\*x^2) - PolyLog[2, a\*x^2] + PolyLog[3, a\*x^2]))/2

**Maple [A]** time = 0.055, size = 56, normalized size = 0.9

$$\frac{1}{2a} \left( ax^2 + \frac{(-2ax^2 + 2)\ln(-ax^2 + 1)}{2} - ax^2 \text{polylog}(2, ax^2) + ax^2 \text{polylog}(3, ax^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(3,a\*x^2), x)

[Out] 1/2/a\*(a\*x^2+1/2\*(-2\*a\*x^2+2)\*ln(-a\*x^2+1)-a\*x^2\*polylog(2,a\*x^2)+a\*x^2\*polylog(3,a\*x^2))

**Maxima [A]** time = 1.00071, size = 72, normalized size = 1.2

$$-\frac{ax^2\text{Li}_2(ax^2) - ax^2\text{Li}_3(ax^2) - ax^2 + (ax^2 - 1)\log(-ax^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] -1/2\*(a\*x^2\*dilog(a\*x^2) - a\*x^2\*polylog(3, a\*x^2) - a\*x^2 + (a\*x^2 - 1)\*log(-a\*x^2 + 1))/a

**Fricas [C]** time = 2.66414, size = 181, normalized size = 3.02

$$\frac{ax^2 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) - ax^2 \operatorname{polylog}(3, ax^2) - ax^2 + (ax^2 - 1) \log(-ax^2 + 1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] -1/2\*(a\*x^2\*\operatorname{Li}\_3(a, x, -\log(-a\*x^2 + 1)/a, -2\*\log(-a\*x^2 + 1)/x) - a\*x^2\*polylog(3, a\*x^2) - a\*x^2 + (a\*x^2 - 1)\*\log(-a\*x^2 + 1))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x\*\*2),x)

[Out] Integral(x\*polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(x\*polylog(3, a\*x^2), x)

$$3.35 \quad \int \frac{\text{PolyLog}(3, ax^2)}{x} dx$$

**Optimal.** Leaf size=11

$$\frac{1}{2} \text{PolyLog}(4, ax^2)$$

[Out] PolyLog[4, a\*x^2]/2

**Rubi [A]** time = 0.0090189, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6589}

$$\frac{1}{2} \text{PolyLog}(4, ax^2)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x, x]

[Out] PolyLog[4, a\*x^2]/2

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{x} dx = \frac{\text{Li}_4(ax^2)}{2}$$

**Mathematica [A]** time = 0.0014221, size = 11, normalized size = 1.

$$\frac{1}{2} \text{PolyLog}(4, ax^2)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2]/x, x]

[Out] PolyLog[4, a\*x^2]/2

**Maple [A]** time = 0.154, size = 10, normalized size = 0.9

$$\frac{\text{polylog}(4, ax^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x,x)
```

```
[Out] 1/2*polylog(4,a*x^2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, ax^2)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^2)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

$$3.36 \quad \int \frac{\text{PolyLog}(3, ax^2)}{x^3} dx$$

**Optimal.** Leaf size=63

$$-\frac{\text{PolyLog}(2, ax^2)}{2x^2} - \frac{\text{PolyLog}(3, ax^2)}{2x^2} - \frac{1}{2}a \log(1 - ax^2) + \frac{\log(1 - ax^2)}{2x^2} + a \log(x)$$

[Out] a\*Log[x] - (a\*Log[1 - a\*x^2])/2 + Log[1 - a\*x^2]/(2\*x^2) - PolyLog[2, a\*x^2]/(2\*x^2) - PolyLog[3, a\*x^2]/(2\*x^2)

**Rubi [A]** time = 0.047025, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$ , Rules used = {6591, 2454, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2, ax^2)}{2x^2} - \frac{\text{PolyLog}(3, ax^2)}{2x^2} - \frac{1}{2}a \log(1 - ax^2) + \frac{\log(1 - ax^2)}{2x^2} + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^3, x]

[Out] a\*Log[x] - (a\*Log[1 - a\*x^2])/2 + Log[1 - a\*x^2]/(2\*x^2) - PolyLog[2, a\*x^2]/(2\*x^2) - PolyLog[3, a\*x^2]/(2\*x^2)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29



Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_3(ax^2)}{x^3} dx &= -\frac{\text{Li}_3(ax^2)}{2x^2} + \int \frac{\text{Li}_2(ax^2)}{x^3} dx \\
 &= -\frac{\text{Li}_2(ax^2)}{2x^2} - \frac{\text{Li}_3(ax^2)}{2x^2} - \int \frac{\log(1-ax^2)}{x^3} dx \\
 &= -\frac{\text{Li}_2(ax^2)}{2x^2} - \frac{\text{Li}_3(ax^2)}{2x^2} - \frac{1}{2} \text{Subst}\left(\int \frac{\log(1-ax)}{x^2} dx, x, x^2\right) \\
 &= \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2} - \frac{\text{Li}_3(ax^2)}{2x^2} + \frac{1}{2}a \text{Subst}\left(\int \frac{1}{x(1-ax)} dx, x, x^2\right) \\
 &= \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2} - \frac{\text{Li}_3(ax^2)}{2x^2} + \frac{1}{2}a \text{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) + \frac{1}{2}a^2 \text{Subst}\left(\int \frac{1}{1-ax} dx, x, x^2\right) \\
 &= a \log(x) - \frac{1}{2}a \log(1-ax^2) + \frac{\log(1-ax^2)}{2x^2} - \frac{\text{Li}_2(ax^2)}{2x^2} - \frac{\text{Li}_3(ax^2)}{2x^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0269275, size = 60, normalized size = 0.95

$$-\frac{\text{PolyLog}(2, ax^2) + \text{PolyLog}(3, ax^2) - ax^2 \log(-ax^2) + ax^2 \log(1-ax^2) - \log(1-ax^2)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2]/x^3, x]

[Out] -(-(a\*x^2\*Log[-(a\*x^2)]) - Log[1 - a\*x^2] + a\*x^2\*Log[1 - a\*x^2] + PolyLog[2, a\*x^2] + PolyLog[3, a\*x^2])/(2\*x^2)

**Maple [A]** time = 0.061, size = 68, normalized size = 1.1

$$\frac{a}{2} \left( 2 \ln(x) + \ln(-a) + \frac{(-8ax^2 + 8) \ln(-ax^2 + 1)}{8ax^2} - \frac{\text{polylog}(2, ax^2)}{ax^2} - \frac{\text{polylog}(3, ax^2)}{ax^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^2)/x^3, x)

[Out] 1/2\*a\*(2\*ln(x)+ln(-a)+1/8/a/x^2\*(-8\*a\*x^2+8)\*ln(-a\*x^2+1)-polylog(2, a\*x^2)/a/x^2-1/a/x^2\*polylog(3, a\*x^2))

**Maxima [A]** time = 0.989681, size = 55, normalized size = 0.87

$$a \log(x) - \frac{(ax^2 - 1) \log(-ax^2 + 1) + \text{Li}_2(ax^2) + \text{Li}_3(ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^3,x, algorithm="maxima")

[Out]  $a \log(x) - \frac{1}{2}((a x^2 - 1) \log(-a x^2 + 1) + \operatorname{dilog}(a x^2) + \operatorname{polylog}(3, a x^2)) / x^2$

**Fricas [C]** time = 2.71896, size = 194, normalized size = 3.08

$$\frac{ax^2 \log(ax^2 - 1) - 2ax^2 \log(x) + \%iint\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) - \log(-ax^2 + 1) + \operatorname{polylog}(3, ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^3,x, algorithm="fricas")

[Out]  $-\frac{1}{2}(ax^2 \log(ax^2 - 1) - 2ax^2 \log(x) + \%iint(a, x, -\log(-ax^2 + 1)/a, -2 \log(-ax^2 + 1)/x) - \log(-ax^2 + 1) + \operatorname{polylog}(3, ax^2)) / x^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/x\*\*3,x)

[Out] Integral(polylog(3, a\*x\*\*2)/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^3,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2)/x^3, x)

### 3.37 $\int \frac{\text{PolyLog}(3, ax^2)}{x^5} dx$

**Optimal.** Leaf size=78

$$-\frac{\text{PolyLog}(2, ax^2)}{8x^4} - \frac{\text{PolyLog}(3, ax^2)}{4x^4} - \frac{1}{16}a^2 \log(1 - ax^2) + \frac{1}{8}a^2 \log(x) - \frac{a}{16x^2} + \frac{\log(1 - ax^2)}{16x^4}$$

[Out]  $-a/(16*x^2) + (a^2*\text{Log}[x])/8 - (a^2*\text{Log}[1 - a*x^2])/16 + \text{Log}[1 - a*x^2]/(16*x^4) - \text{PolyLog}[2, a*x^2]/(8*x^4) - \text{PolyLog}[3, a*x^2]/(4*x^4)$

**Rubi [A]** time = 0.0637554, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax^2)}{8x^4} - \frac{\text{PolyLog}(3, ax^2)}{4x^4} - \frac{1}{16}a^2 \log(1 - ax^2) + \frac{1}{8}a^2 \log(x) - \frac{a}{16x^2} + \frac{\log(1 - ax^2)}{16x^4}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^5, x]

[Out]  $-a/(16*x^2) + (a^2*\text{Log}[x])/8 - (a^2*\text{Log}[1 - a*x^2])/16 + \text{Log}[1 - a*x^2]/(16*x^4) - \text{PolyLog}[2, a*x^2]/(8*x^4) - \text{PolyLog}[3, a*x^2]/(4*x^4)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^5} dx &= -\frac{\text{Li}_3(ax^2)}{4x^4} + \frac{1}{2} \int \frac{\text{Li}_2(ax^2)}{x^5} dx \\
&= -\frac{\text{Li}_2(ax^2)}{8x^4} - \frac{\text{Li}_3(ax^2)}{4x^4} - \frac{1}{4} \int \frac{\log(1-ax^2)}{x^5} dx \\
&= -\frac{\text{Li}_2(ax^2)}{8x^4} - \frac{\text{Li}_3(ax^2)}{4x^4} - \frac{1}{8} \text{Subst} \left( \int \frac{\log(1-ax)}{x^3} dx, x, x^2 \right) \\
&= \frac{\log(1-ax^2)}{16x^4} - \frac{\text{Li}_2(ax^2)}{8x^4} - \frac{\text{Li}_3(ax^2)}{4x^4} + \frac{1}{16} a \text{Subst} \left( \int \frac{1}{x^2(1-ax)} dx, x, x^2 \right) \\
&= \frac{\log(1-ax^2)}{16x^4} - \frac{\text{Li}_2(ax^2)}{8x^4} - \frac{\text{Li}_3(ax^2)}{4x^4} + \frac{1}{16} a \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{a}{x} - \frac{a^2}{-1+ax} \right) dx, x, x^2 \right) \\
&= -\frac{a}{16x^2} + \frac{1}{8} a^2 \log(x) - \frac{1}{16} a^2 \log(1-ax^2) + \frac{\log(1-ax^2)}{16x^4} - \frac{\text{Li}_2(ax^2)}{8x^4} - \frac{\text{Li}_3(ax^2)}{4x^4}
\end{aligned}$$

**Mathematica [C]** time = 0.0118626, size = 30, normalized size = 0.38

$$\frac{G_{5,5}^{2,4} \left( -ax^2 \mid \begin{matrix} 1, 1, 1, 1, 3 \\ 1, 2, 0, 0, 0 \end{matrix} \right)}{2x^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/x^5,x]

[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a\*x^2)]/(2\*x^4)

**Maple [A]** time = 0.065, size = 98, normalized size = 1.3

$$-\frac{a^2}{2} \left( \frac{1}{ax^2} + \frac{3}{16} - \frac{\ln(x)}{4} - \frac{\ln(-a)}{8} - \frac{81ax^2 + 378}{432ax^2} - \frac{(-27a^2x^4 + 27)\ln(-ax^2 + 1)}{216a^2x^4} \right) + \frac{\text{polylog}(2, ax^2)}{4a^2x^4} + \frac{\text{polylog}(3, ax^2)}{2a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/x^5,x)

[Out] -1/2\*a^2\*(1/a/x^2+3/16-1/4\*ln(x)-1/8\*ln(-a)-1/432/a/x^2\*(81\*a\*x^2+378)-1/216/a^2/x^4\*(-27\*a^2\*x^4+27)\*ln(-a\*x^2+1)+1/4/a^2/x^4\*polylog(2,a\*x^2)+1/2/a^2/x^4\*polylog(3,a\*x^2))

**Maxima [A]** time = 1.0028, size = 74, normalized size = 0.95

$$\frac{1}{8} a^2 \log(x) - \frac{ax^2 + (a^2x^4 - 1)\log(-ax^2 + 1) + 2\text{Li}_2(ax^2) + 4\text{Li}_3(ax^2)}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{8}a^2\log(x) - \frac{1}{16}(ax^2 + (a^2x^4 - 1)\log(-ax^2 + 1) + 2\operatorname{dilog}(ax^2) + 4\operatorname{polylog}(3, ax^2))/x^4$

**Fricas [C]** time = 2.71783, size = 217, normalized size = 2.78

$$\frac{a^2x^4\log(ax^2 - 1) - 2a^2x^4\log(x) + ax^2 + 2\%iint\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2\log(-ax^2+1)}{x}\right) - \log(-ax^2 + 1) + 4\operatorname{polylog}(3, ax^2)}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax^2)/x^5,x, algorithm="fricas")`

[Out]  $-\frac{1}{16}(a^2x^4\log(ax^2 - 1) - 2a^2x^4\log(x) + ax^2 + 2\%iint(a, x, -\log(-ax^2 + 1)/a, -2\log(-ax^2 + 1)/x) - \log(-ax^2 + 1) + 4\operatorname{polylog}(3, ax^2))/x^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax**2)/x**5,x)`

[Out] `Integral(polylog(3, ax**2)/x**5, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax^2)/x^5,x, algorithm="giac")`

[Out] `integrate(polylog(3, ax^2)/x^5, x)`

### 3.38 $\int \frac{\text{PolyLog}(3, ax^2)}{x^7} dx$

**Optimal.** Leaf size=88

$$-\frac{\text{PolyLog}(2, ax^2)}{18x^6} - \frac{\text{PolyLog}(3, ax^2)}{6x^6} - \frac{a^2}{54x^2} - \frac{1}{54}a^3 \log(1 - ax^2) + \frac{1}{27}a^3 \log(x) - \frac{a}{108x^4} + \frac{\log(1 - ax^2)}{54x^6}$$

[Out] -a/(108\*x^4) - a^2/(54\*x^2) + (a^3\*Log[x])/27 - (a^3\*Log[1 - a\*x^2])/54 + Log[1 - a\*x^2]/(54\*x^6) - PolyLog[2, a\*x^2]/(18\*x^6) - PolyLog[3, a\*x^2]/(6\*x^6)

**Rubi [A]** time = 0.0654848, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2, ax^2)}{18x^6} - \frac{\text{PolyLog}(3, ax^2)}{6x^6} - \frac{a^2}{54x^2} - \frac{1}{54}a^3 \log(1 - ax^2) + \frac{1}{27}a^3 \log(x) - \frac{a}{108x^4} + \frac{\log(1 - ax^2)}{54x^6}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^7, x]

[Out] -a/(108\*x^4) - a^2/(54\*x^2) + (a^3\*Log[x])/27 - (a^3\*Log[1 - a\*x^2])/54 + Log[1 - a\*x^2]/(54\*x^6) - PolyLog[2, a\*x^2]/(18\*x^6) - PolyLog[3, a\*x^2]/(6\*x^6)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m+1)/n]] && (GtQ[(m+1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{x^7} dx &= -\frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{3} \int \frac{\text{Li}_2(ax^2)}{x^7} dx \\
&= -\frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} - \frac{1}{9} \int \frac{\log(1-ax^2)}{x^7} dx \\
&= -\frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} - \frac{1}{18} \text{Subst} \left( \int \frac{\log(1-ax)}{x^4} dx, x, x^2 \right) \\
&= \frac{\log(1-ax^2)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{54} a \text{Subst} \left( \int \frac{1}{x^3(1-ax)} dx, x, x^2 \right) \\
&= \frac{\log(1-ax^2)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6} + \frac{1}{54} a \text{Subst} \left( \int \left( \frac{1}{x^3} + \frac{a}{x^2} + \frac{a^2}{x} - \frac{a^3}{-1+ax} \right) dx, x, x^2 \right) \\
&= -\frac{a}{108x^4} - \frac{a^2}{54x^2} + \frac{1}{27} a^3 \log(x) - \frac{1}{54} a^3 \log(1-ax^2) + \frac{\log(1-ax^2)}{54x^6} - \frac{\text{Li}_2(ax^2)}{18x^6} - \frac{\text{Li}_3(ax^2)}{6x^6}
\end{aligned}$$

**Mathematica [C]** time = 0.0119825, size = 30, normalized size = 0.34

$$\frac{G_{5,5}^{2,4} \left( -ax^2 \mid \begin{matrix} 1, 1, 1, 1, 4 \\ 1, 3, 0, 0, 0 \end{matrix} \right)}{2x^6}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/x^7, x]

[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a\*x^2)]/(2\*x^6)

**Maple [A]** time = 0.066, size = 115, normalized size = 1.3

$$\frac{a^3}{2} \left( -\frac{1}{2a^2x^4} - \frac{1}{8ax^2} - \frac{1}{27} + \frac{2 \ln(x)}{27} + \frac{\ln(-a)}{27} + \frac{64a^2x^4 + 152ax^2 + 832}{1728a^2x^4} + \frac{(-64x^6a^3 + 64) \ln(-ax^2 + 1)}{1728x^6a^3} - \frac{\text{polylog}(3, ax^2)}{9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^2)/x^7, x)

[Out] 1/2\*a^3\*(-1/2/a^2/x^4-1/8/a/x^2-1/27+2/27\*ln(x)+1/27\*ln(-a)+1/1728/a^2/x^4\*(64\*a^2\*x^4+152\*a\*x^2+832)+1/1728/a^3/x^6\*(-64\*a^3\*x^6+64)\*ln(-a\*x^2+1)-1/9/a^3/x^6\*polylog(2, a\*x^2)-1/3/a^3/x^6\*polylog(3, a\*x^2))

**Maxima [A]** time = 0.997547, size = 86, normalized size = 0.98

$$\frac{1}{27} a^3 \log(x) - \frac{2a^2x^4 + ax^2 + 2(a^3x^6 - 1) \log(-ax^2 + 1) + 6 \text{Li}_2(ax^2) + 18 \text{Li}_3(ax^2)}{108x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x^2)/x^7, x, algorithm="maxima")

[Out]  $\frac{1}{27}a^3\log(x) - \frac{1}{108}(2a^2x^4 + ax^2 + 2(a^3x^6 - 1)\log(-ax^2 + 1) + 6\operatorname{dilog}(ax^2) + 18\operatorname{polylog}(3, ax^2))/x^6$

**Fricas [C]** time = 2.79682, size = 242, normalized size = 2.75

$$\frac{2a^3x^6\log(ax^2 - 1) - 4a^3x^6\log(x) + 2a^2x^4 + ax^2 + 6\%iint\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2\log(-ax^2+1)}{x}\right) - 2\log(-ax^2 + 1) + 18\operatorname{polylog}(3, ax^2)}{108x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax^2)/x^7,x, algorithm="fricas")`

[Out]  $-\frac{1}{108}(2a^3x^6\log(ax^2 - 1) - 4a^3x^6\log(x) + 2a^2x^4 + ax^2 + 6\%iint(a, x, -\log(-ax^2 + 1)/a, -2\log(-ax^2 + 1)/x) - 2\log(-ax^2 + 1) + 18\operatorname{polylog}(3, ax^2))/x^6$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax**2)/x**7,x)`

[Out] `Integral(polylog(3, ax**2)/x**7, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,ax^2)/x^7,x, algorithm="giac")`

[Out] `integrate(polylog(3, ax^2)/x^7, x)`



### 3.39 $\int x^4 \mathbf{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=87

$$-\frac{2}{25}x^5 \mathbf{PolyLog}(2, ax^2) + \frac{1}{5}x^5 \mathbf{PolyLog}(3, ax^2) + \frac{8x}{125a^2} - \frac{8 \tanh^{-1}(\sqrt{ax})}{125a^{5/2}} + \frac{8x^3}{375a} - \frac{4}{125}x^5 \log(1 - ax^2) + \frac{8x^5}{625}$$

[Out] (8\*x)/(125\*a^2) + (8\*x^3)/(375\*a) + (8\*x^5)/625 - (8\*ArcTanh[Sqrt[a]\*x])/(125\*a^(5/2)) - (4\*x^5\*Log[1 - a\*x^2])/125 - (2\*x^5\*PolyLog[2, a\*x^2])/25 + (x^5\*PolyLog[3, a\*x^2])/5

**Rubi [A]** time = 0.0530276, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 302, 206}

$$-\frac{2}{25}x^5 \mathbf{PolyLog}(2, ax^2) + \frac{1}{5}x^5 \mathbf{PolyLog}(3, ax^2) + \frac{8x}{125a^2} - \frac{8 \tanh^{-1}(\sqrt{ax})}{125a^{5/2}} + \frac{8x^3}{375a} - \frac{4}{125}x^5 \log(1 - ax^2) + \frac{8x^5}{625}$$

Antiderivative was successfully verified.

[In] Int[x^4\*PolyLog[3, a\*x^2], x]

[Out] (8\*x)/(125\*a^2) + (8\*x^3)/(375\*a) + (8\*x^5)/625 - (8\*ArcTanh[Sqrt[a]\*x])/(125\*a^(5/2)) - (4\*x^5\*Log[1 - a\*x^2])/125 - (2\*x^5\*PolyLog[2, a\*x^2])/25 + (x^5\*PolyLog[3, a\*x^2])/5

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int x^4 \text{Li}_3(ax^2) dx &= \frac{1}{5} x^5 \text{Li}_3(ax^2) - \frac{2}{5} \int x^4 \text{Li}_2(ax^2) dx \\
&= -\frac{2}{25} x^5 \text{Li}_2(ax^2) + \frac{1}{5} x^5 \text{Li}_3(ax^2) - \frac{4}{25} \int x^4 \log(1-ax^2) dx \\
&= -\frac{4}{125} x^5 \log(1-ax^2) - \frac{2}{25} x^5 \text{Li}_2(ax^2) + \frac{1}{5} x^5 \text{Li}_3(ax^2) - \frac{1}{125} (8a) \int \frac{x^6}{1-ax^2} dx \\
&= -\frac{4}{125} x^5 \log(1-ax^2) - \frac{2}{25} x^5 \text{Li}_2(ax^2) + \frac{1}{5} x^5 \text{Li}_3(ax^2) - \frac{1}{125} (8a) \int \left( -\frac{1}{a^3} - \frac{x^2}{a^2} - \frac{x^4}{a} + \frac{1}{a^3(1-ax^2)} \right) dx \\
&= \frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{4}{125} x^5 \log(1-ax^2) - \frac{2}{25} x^5 \text{Li}_2(ax^2) + \frac{1}{5} x^5 \text{Li}_3(ax^2) - \frac{8 \int \frac{1}{1-ax^2} dx}{125a^2} \\
&= \frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{8 \tanh^{-1}(\sqrt{ax})}{125a^{5/2}} - \frac{4}{125} x^5 \log(1-ax^2) - \frac{2}{25} x^5 \text{Li}_2(ax^2) + \frac{1}{5} x^5 \text{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.161049, size = 77, normalized size = 0.89

$$\frac{-150x^5 \text{PolyLog}(2, ax^2) + 375x^5 \text{PolyLog}(3, ax^2) + \frac{120x}{a^2} - \frac{120 \tanh^{-1}(\sqrt{ax})}{a^{5/2}} + \frac{40x^3}{a} - 60x^5 \log(1-ax^2) + 24x^5}{1875}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*PolyLog[3, a\*x^2], x]

[Out] ((120\*x)/a^2 + (40\*x^3)/a + 24\*x^5 - (120\*ArcTanh[Sqrt[a]\*x])/a^(5/2) - 60\*x^5\*Log[1 - a\*x^2] - 150\*x^5\*PolyLog[2, a\*x^2] + 375\*x^5\*PolyLog[3, a\*x^2])/1875

**Maple [B]** time = 0.178, size = 144, normalized size = 1.7

$$-\frac{1}{2a^2} \left( \frac{2x(168a^2x^4 + 280ax^2 + 840)}{13125a^3} (-a)^{\frac{7}{2}} + \frac{8x}{125a^3} (-a)^{\frac{7}{2}} \left( \ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}) \right) \right) \frac{1}{\sqrt{ax^2}} - \frac{8x^5 \ln(-ax^2 + 1)}{125a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*polylog(3,a\*x^2),x)

[Out] -1/2/a^2/(-a)^(1/2)\*(2/13125\*x\*(-a)^(7/2)\*(168\*a^2\*x^4+280\*a\*x^2+840)/a^3+8/125\*x\*(-a)^(7/2)/a^3/(a\*x^2)^(1/2)\*(ln(1-(a\*x^2)^(1/2))-ln(1+(a\*x^2)^(1/2))))-8/125\*x^5\*(-a)^(7/2)/a\*ln(-a\*x^2+1)-4/25\*x^5\*(-a)^(7/2)\*polylog(2,a\*x^2)/a+2/5\*x^5\*(-a)^(7/2)/a\*polylog(3,a\*x^2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(3,a\*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 2.77746, size = 608, normalized size = 6.99

$$\left[ \frac{150 a^3 x^5 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) + 60 a^3 x^5 \log(-ax^2+1) - 375 a^3 x^5 \operatorname{polylog}(3, ax^2) - 24 a^3 x^5 - 40 a^2 x^3 - 120 a x - 60 \sqrt{a} \log\left(\frac{ax^2 - 2\sqrt{a}x + 1}{ax^2 - 1}\right)}{1875 a^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] [-1/1875\*(150\*a^3\*x^5\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 60\*a^3\*x^5\*log(-a\*x^2 + 1) - 375\*a^3\*x^5\*polylog(3, a\*x^2) - 24\*a^3\*x^5 - 40\*a^2\*x^3 - 120\*a\*x - 60\*sqrt(a)\*log((a\*x^2 - 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)))/a^3, -1/1875\*(150\*a^3\*x^5\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 60\*a^3\*x^5\*log(-a\*x^2 + 1) - 375\*a^3\*x^5\*polylog(3, a\*x^2) - 24\*a^3\*x^5 - 40\*a^2\*x^3 - 120\*a\*x - 120\*sqrt(-a)\*arctan(sqrt(-a)\*x))/a^3]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*polylog(3,a\*x\*\*2),x)

[Out] Integral(x\*\*4\*polylog(3, a\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(x^4\*polylog(3, a\*x^2), x)

### 3.40 $\int x^2 \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=77

$$-\frac{2}{9}x^3 \text{PolyLog}(2, ax^2) + \frac{1}{3}x^3 \text{PolyLog}(3, ax^2) - \frac{8 \tanh^{-1}(\sqrt{ax})}{27a^{3/2}} - \frac{4}{27}x^3 \log(1 - ax^2) + \frac{8x}{27a} + \frac{8x^3}{81}$$

[Out] (8\*x)/(27\*a) + (8\*x^3)/81 - (8\*ArcTanh[Sqrt[a]\*x])/(27\*a^(3/2)) - (4\*x^3\*Log[1 - a\*x^2])/27 - (2\*x^3\*PolyLog[2, a\*x^2])/9 + (x^3\*PolyLog[3, a\*x^2])/3

**Rubi [A]** time = 0.0489431, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 302, 206}

$$-\frac{2}{9}x^3 \text{PolyLog}(2, ax^2) + \frac{1}{3}x^3 \text{PolyLog}(3, ax^2) - \frac{8 \tanh^{-1}(\sqrt{ax})}{27a^{3/2}} - \frac{4}{27}x^3 \log(1 - ax^2) + \frac{8x}{27a} + \frac{8x^3}{81}$$

Antiderivative was successfully verified.

[In] Int[x^2\*PolyLog[3, a\*x^2], x]

[Out] (8\*x)/(27\*a) + (8\*x^3)/81 - (8\*ArcTanh[Sqrt[a]\*x])/(27\*a^(3/2)) - (4\*x^3\*Log[1 - a\*x^2])/27 - (2\*x^3\*PolyLog[2, a\*x^2])/9 + (x^3\*PolyLog[3, a\*x^2])/3

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int x^2 \operatorname{Li}_3(ax^2) dx &= \frac{1}{3} x^3 \operatorname{Li}_3(ax^2) - \frac{2}{3} \int x^2 \operatorname{Li}_2(ax^2) dx \\
&= -\frac{2}{9} x^3 \operatorname{Li}_2(ax^2) + \frac{1}{3} x^3 \operatorname{Li}_3(ax^2) - \frac{4}{9} \int x^2 \log(1-ax^2) dx \\
&= -\frac{4}{27} x^3 \log(1-ax^2) - \frac{2}{9} x^3 \operatorname{Li}_2(ax^2) + \frac{1}{3} x^3 \operatorname{Li}_3(ax^2) - \frac{1}{27} (8a) \int \frac{x^4}{1-ax^2} dx \\
&= -\frac{4}{27} x^3 \log(1-ax^2) - \frac{2}{9} x^3 \operatorname{Li}_2(ax^2) + \frac{1}{3} x^3 \operatorname{Li}_3(ax^2) - \frac{1}{27} (8a) \int \left( -\frac{1}{a^2} - \frac{x^2}{a} + \frac{1}{a^2(1-ax^2)} \right) dx \\
&= \frac{8x}{27a} + \frac{8x^3}{81} - \frac{4}{27} x^3 \log(1-ax^2) - \frac{2}{9} x^3 \operatorname{Li}_2(ax^2) + \frac{1}{3} x^3 \operatorname{Li}_3(ax^2) - \frac{8 \int \frac{1}{1-ax^2} dx}{27a} \\
&= \frac{8x}{27a} + \frac{8x^3}{81} - \frac{8 \tanh^{-1}(\sqrt{ax})}{27a^{3/2}} - \frac{4}{27} x^3 \log(1-ax^2) - \frac{2}{9} x^3 \operatorname{Li}_2(ax^2) + \frac{1}{3} x^3 \operatorname{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.138001, size = 69, normalized size = 0.9

$$\frac{1}{81} \left( -18x^3 \operatorname{PolyLog}(2, ax^2) + 27x^3 \operatorname{PolyLog}(3, ax^2) - \frac{24 \tanh^{-1}(\sqrt{ax})}{a^{3/2}} - 12x^3 \log(1-ax^2) + \frac{24x}{a} + 8x^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[3, a\*x^2], x]

[Out] ((24\*x)/a + 8\*x^3 - (24\*ArcTanh[Sqrt[a]\*x])/a^(3/2) - 12\*x^3\*Log[1 - a\*x^2] - 18\*x^3\*PolyLog[2, a\*x^2] + 27\*x^3\*PolyLog[3, a\*x^2])/81

**Maple [B]** time = 0.175, size = 136, normalized size = 1.8

$$\frac{1}{2a} \left( \frac{2x(40ax^2 + 120)}{405a^2} (-a)^{5/2} + \frac{8x}{27a^2} (-a)^{5/2} \left( \ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}) \right) \frac{1}{\sqrt{ax^2}} - \frac{8x^3 \ln(-ax^2 + 1)}{27a} (-a)^{5/2} - \frac{4x^3}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(3,a\*x^2), x)

[Out] 1/2/a/(-a)^(1/2)\*(2/405\*x\*(-a)^(5/2)\*(40\*a\*x^2+120)/a^2+8/27\*x\*(-a)^(5/2)/a^(1/2)/(a\*x^2)^(1/2)\*(ln(1-(a\*x^2)^(1/2))-ln(1+(a\*x^2)^(1/2))))-8/27\*x^3\*(-a)^(5/2)/a\*ln(-a\*x^2+1)-4/9\*x^3\*(-a)^(5/2)\*polylog(2,a\*x^2)/a+2/3\*x^3\*(-a)^(5/2)/a\*polylog(3,a\*x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 2.74317, size = 555, normalized size = 7.21

$$\left[ \frac{18 a^2 x^3 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) + 12 a^2 x^3 \log(-ax^2+1) - 27 a^2 x^3 \operatorname{polylog}(3, ax^2) - 8 a^2 x^3 - 24 ax - 12 \sqrt{a} \log\left(\frac{ax^2 - 2\sqrt{a}x + 1}{ax^2 - 1}\right)}{81 a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] [-1/81\*(18\*a^2\*x^3\*\operatorname{Li}\_3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 12\*a^2\*x^3\*log(-a\*x^2 + 1) - 27\*a^2\*x^3\*polylog(3, a\*x^2) - 8\*a^2\*x^3 - 24\*a\*x - 12\*sqrt(a)\*log((a\*x^2 - 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)))/a^2, -1/81\*(18\*a^2\*x^3\*\operatorname{Li}\_3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 12\*a^2\*x^3\*log(-a\*x^2 + 1) - 27\*a^2\*x^3\*polylog(3, a\*x^2) - 8\*a^2\*x^3 - 24\*a\*x - 24\*sqrt(-a)\*arctan(sqrt(-a)\*x))/a^2]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(3,a\*x\*\*2),x)

[Out] Integral(x\*\*2\*polylog(3, a\*x\*\*2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(x^2\*polylog(3, a\*x^2), x)

### 3.41 $\int \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=50

$$-2x\text{PolyLog}(2, ax^2) + x\text{PolyLog}(3, ax^2) - 4x \log(1 - ax^2) - \frac{8 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} + 8x$$

[Out]  $8*x - (8*\text{ArcTanh}[\text{Sqrt}[a]*x])/\text{Sqrt}[a] - 4*x*\text{Log}[1 - a*x^2] - 2*x*\text{PolyLog}[2, a*x^2] + x*\text{PolyLog}[3, a*x^2]$

**Rubi [A]** time = 0.0233193, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6586, 2448, 321, 206}

$$-2x\text{PolyLog}(2, ax^2) + x\text{PolyLog}(3, ax^2) - 4x \log(1 - ax^2) - \frac{8 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} + 8x$$

Antiderivative was successfully verified.

[In] `Int[PolyLog[3, a*x^2], x]`

[Out]  $8*x - (8*\text{ArcTanh}[\text{Sqrt}[a]*x])/\text{Sqrt}[a] - 4*x*\text{Log}[1 - a*x^2] - 2*x*\text{PolyLog}[2, a*x^2] + x*\text{PolyLog}[3, a*x^2]$

#### Rule 6586

`Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] / ; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]`

#### Rule 2448

`Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] / ; FreeQ[{c, d, e, n, p}, x]`

#### Rule 321

`Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] / ; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]`

#### Rule 206

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] / ; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rubi steps

$$\begin{aligned}
\int \operatorname{Li}_3(ax^2) dx &= x\operatorname{Li}_3(ax^2) - 2 \int \operatorname{Li}_2(ax^2) dx \\
&= -2x\operatorname{Li}_2(ax^2) + x\operatorname{Li}_3(ax^2) - 4 \int \log(1 - ax^2) dx \\
&= -4x \log(1 - ax^2) - 2x\operatorname{Li}_2(ax^2) + x\operatorname{Li}_3(ax^2) - (8a) \int \frac{x^2}{1 - ax^2} dx \\
&= 8x - 4x \log(1 - ax^2) - 2x\operatorname{Li}_2(ax^2) + x\operatorname{Li}_3(ax^2) - 8 \int \frac{1}{1 - ax^2} dx \\
&= 8x - \frac{8 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} - 4x \log(1 - ax^2) - 2x\operatorname{Li}_2(ax^2) + x\operatorname{Li}_3(ax^2)
\end{aligned}$$

**Mathematica [A]** time = 0.087684, size = 50, normalized size = 1.

$$-2x\operatorname{PolyLog}(2, ax^2) + x\operatorname{PolyLog}(3, ax^2) - 4x \log(1 - ax^2) - \frac{8 \tanh^{-1}(\sqrt{ax})}{\sqrt{a}} + 8x$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2], x]

[Out] 8\*x - (8\*ArcTanh[Sqrt[a]\*x])/Sqrt[a] - 4\*x\*Log[1 - a\*x^2] - 2\*x\*PolyLog[2, a\*x^2] + x\*PolyLog[3, a\*x^2]

**Maple [B]** time = 0.173, size = 119, normalized size = 2.4

$$-\frac{1}{2} \left( 16 \frac{x(-a)^{3/2}}{a} + 8 \frac{x(-a)^{3/2} (\ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}))}{a\sqrt{ax^2}} - 8 \frac{x(-a)^{3/2} \ln(-ax^2 + 1)}{a} - 4 \frac{x(-a)^{3/2} \operatorname{polylog}(2, ax^2)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^2), x)

[Out] -1/2/(-a)^(1/2)\*(16\*x\*(-a)^(3/2)/a+8\*x\*(-a)^(3/2)/a/(a\*x^2)^(1/2)\*(ln(1-(a\*x^2)^(1/2))-ln(1+(a\*x^2)^(1/2))))-8\*x\*(-a)^(3/2)/a\*ln(-a\*x^2+1)-4\*x\*(-a)^(3/2)\*polylog(2, a\*x^2)/a+2\*x\*(-a)^(3/2)/a\*polylog(3, a\*x^2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [C]** time = 2.69418, size = 452, normalized size = 9.04

$$\left[ \frac{2ax \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2\log(-ax^2+1)}{x}\right) + 4ax \log(-ax^2+1) - ax \operatorname{polylog}(3, ax^2) - 8ax - 4\sqrt{a} \log\left(\frac{ax^2-2\sqrt{ax}}{ax^2-1}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2),x, algorithm="fricas")

[Out] [-(2\*a\*x\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 4\*a\*x\*log(-a\*x^2 + 1) - a\*x\*polylog(3, a\*x^2) - 8\*a\*x - 4\*sqrt(a)\*log((a\*x^2 - 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)))/a, -(2\*a\*x\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 4\*a\*x\*log(-a\*x^2 + 1) - a\*x\*polylog(3, a\*x^2) - 8\*a\*x - 8\*sqrt(-a)\*arctan(sqrt(-a)\*x))/a]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2),x)

[Out] Integral(polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2), x)

### 3.42 $\int \frac{\text{PolyLog}(3, ax^2)}{x^2} dx$

**Optimal.** Leaf size=54

$$-\frac{2\text{PolyLog}(2, ax^2)}{x} - \frac{\text{PolyLog}(3, ax^2)}{x} + \frac{4\log(1 - ax^2)}{x} + 8\sqrt{a} \tanh^{-1}(\sqrt{ax})$$

[Out] 8\*Sqrt[a]\*ArcTanh[Sqrt[a]\*x] + (4\*Log[1 - a\*x^2])/x - (2\*PolyLog[2, a\*x^2])/x - PolyLog[3, a\*x^2]/x

**Rubi [A]** time = 0.0357649, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 206}

$$-\frac{2\text{PolyLog}(2, ax^2)}{x} - \frac{\text{PolyLog}(3, ax^2)}{x} + \frac{4\log(1 - ax^2)}{x} + 8\sqrt{a} \tanh^{-1}(\sqrt{ax})$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^2, x]

[Out] 8\*Sqrt[a]\*ArcTanh[Sqrt[a]\*x] + (4\*Log[1 - a\*x^2])/x - (2\*PolyLog[2, a\*x^2])/x - PolyLog[3, a\*x^2]/x

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_3(ax^2)}{x^2} dx &= -\frac{\text{Li}_3(ax^2)}{x} + 2 \int \frac{\text{Li}_2(ax^2)}{x^2} dx \\ &= -\frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x} - 4 \int \frac{\log(1 - ax^2)}{x^2} dx \\ &= \frac{4\log(1 - ax^2)}{x} - \frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x} + (8a) \int \frac{1}{1 - ax^2} dx \\ &= 8\sqrt{a} \tanh^{-1}(\sqrt{ax}) + \frac{4\log(1 - ax^2)}{x} - \frac{2\text{Li}_2(ax^2)}{x} - \frac{\text{Li}_3(ax^2)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0806658, size = 50, normalized size = 0.93

$$\frac{-2\text{PolyLog}(2, ax^2) - \text{PolyLog}(3, ax^2) + 4\log(1 - ax^2) + 8\sqrt{ax} \tanh^{-1}(\sqrt{ax})}{x}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2]/x^2,x]

[Out] (8\*sqrt[a]\*x\*ArcTanh[Sqrt[a]\*x] + 4\*Log[1 - a\*x^2] - 2\*PolyLog[2, a\*x^2] - PolyLog[3, a\*x^2])/x

**Maple [B]** time = 0.172, size = 112, normalized size = 2.1

$$\frac{a}{2} \left( -8 \frac{x\sqrt{-a} \left( \ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}) \right)}{\sqrt{ax^2}} + 8 \frac{\sqrt{-a} \ln(-ax^2 + 1)}{ax} - 4 \frac{\sqrt{-a} \text{polylog}(2, ax^2)}{ax} - 2 \frac{\sqrt{-a} \text{polylog}(3, ax^2)}{ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/x^2,x)

[Out] 1/2\*a/(-a)^(1/2)\*(-8\*x\*(-a)^(1/2)/(a\*x^2)^(1/2)\*(ln(1-(a\*x^2)^(1/2))-ln(1+(a\*x^2)^(1/2)))+8/x\*(-a)^(1/2)/a\*ln(-a\*x^2+1)-4/x\*(-a)^(1/2)\*polylog(2,a\*x^2)/a-2/x\*(-a)^(1/2)/a\*polylog(3,a\*x^2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.87069, size = 402, normalized size = 7.44

$$\frac{4\sqrt{ax} \log\left(\frac{ax^2+2\sqrt{ax}+1}{ax^2-1}\right) - 2\%iint\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2\log(-ax^2+1)}{x}\right) + 4\log(-ax^2+1) - \text{polylog}(3, ax^2) - 8\sqrt{-ax}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^2,x, algorithm="fricas")

[Out] [(4\*sqrt(a)\*x\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)) - 2\*%iint(a, x, -1\*log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 4\*log(-a\*x^2 + 1) - polylog(3, a\*x^2))/x, -(8\*sqrt(-a)\*x\*arctan(sqrt(-a)\*x) + 2\*%iint(a, x, -log(-a\*x^2 + 1)

`/a, -2*log(-a*x^2 + 1)/x) - 4*log(-a*x^2 + 1) + polylog(3, a*x^2))/x]`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,a*x**2)/x**2,x)`

[Out] `Integral(polylog(3, a*x**2)/x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,a*x^2)/x^2,x, algorithm="giac")`

[Out] `integrate(polylog(3, a*x^2)/x^2, x)`

### 3.43 $\int \frac{\text{PolyLog}(3, ax^2)}{x^4} dx$

**Optimal.** Leaf size=70

$$-\frac{2\text{PolyLog}(2, ax^2)}{9x^3} - \frac{\text{PolyLog}(3, ax^2)}{3x^3} + \frac{8}{27}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{4\log(1 - ax^2)}{27x^3} - \frac{8a}{27x}$$

[Out]  $(-8*a)/(27*x) + (8*a^{(3/2)}*ArcTanh[Sqrt[a]*x])/27 + (4*Log[1 - a*x^2])/(27*x^3) - (2*PolyLog[2, a*x^2])/(9*x^3) - PolyLog[3, a*x^2]/(3*x^3)$

**Rubi [A]** time = 0.0409967, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 325, 206}

$$-\frac{2\text{PolyLog}(2, ax^2)}{9x^3} - \frac{\text{PolyLog}(3, ax^2)}{3x^3} + \frac{8}{27}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{4\log(1 - ax^2)}{27x^3} - \frac{8a}{27x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^4, x]

[Out]  $(-8*a)/(27*x) + (8*a^{(3/2)}*ArcTanh[Sqrt[a]*x])/27 + (4*Log[1 - a*x^2])/(27*x^3) - (2*PolyLog[2, a*x^2])/(9*x^3) - PolyLog[3, a*x^2]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_3(ax^2)}{x^4} dx &= -\frac{\operatorname{Li}_3(ax^2)}{3x^3} + \frac{2}{3} \int \frac{\operatorname{Li}_2(ax^2)}{x^4} dx \\
&= -\frac{2\operatorname{Li}_2(ax^2)}{9x^3} - \frac{\operatorname{Li}_3(ax^2)}{3x^3} - \frac{4}{9} \int \frac{\log(1-ax^2)}{x^4} dx \\
&= \frac{4\log(1-ax^2)}{27x^3} - \frac{2\operatorname{Li}_2(ax^2)}{9x^3} - \frac{\operatorname{Li}_3(ax^2)}{3x^3} + \frac{1}{27}(8a) \int \frac{1}{x^2(1-ax^2)} dx \\
&= -\frac{8a}{27x} + \frac{4\log(1-ax^2)}{27x^3} - \frac{2\operatorname{Li}_2(ax^2)}{9x^3} - \frac{\operatorname{Li}_3(ax^2)}{3x^3} + \frac{1}{27}(8a^2) \int \frac{1}{1-ax^2} dx \\
&= -\frac{8a}{27x} + \frac{8}{27}a^{3/2} \tanh^{-1}(\sqrt{ax}) + \frac{4\log(1-ax^2)}{27x^3} - \frac{2\operatorname{Li}_2(ax^2)}{9x^3} - \frac{\operatorname{Li}_3(ax^2)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.088813, size = 61, normalized size = 0.87

$$-\frac{6\operatorname{PolyLog}(2, ax^2) + 9\operatorname{PolyLog}(3, ax^2) - 8a^{3/2}x^3 \tanh^{-1}(\sqrt{ax}) + 8ax^2 - 4\log(1-ax^2)}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2]/x^4, x]

[Out]  $-(8*a*x^2 - 8*a^{(3/2)}*x^3*\operatorname{ArcTanh}[\operatorname{Sqrt}[a]*x] - 4*\operatorname{Log}[1 - a*x^2] + 6*\operatorname{PolyLog}[2, a*x^2] + 9*\operatorname{PolyLog}[3, a*x^2])/(27*x^3)$

**Maple [B]** time = 0.177, size = 125, normalized size = 1.8

$$-\frac{a^2}{2} \left( -\frac{16}{27x} \frac{1}{\sqrt{-a}} - \frac{8ax}{27} \left( \ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}) \right) \frac{1}{\sqrt{-a}} \frac{1}{\sqrt{ax^2}} + \frac{8 \ln(-ax^2 + 1)}{27x^3a} \frac{1}{\sqrt{-a}} - \frac{4 \operatorname{polylog}(2, ax^2)}{9x^3a} \frac{1}{\sqrt{-a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/x^4,x)

[Out]  $-1/2*a^2/(-a)^{(1/2)}*(-16/27/x/(-a)^{(1/2)}-8/27*x/(-a)^{(1/2)}*a/(a*x^2)^{(1/2)}*(\ln(1-(a*x^2)^{(1/2)})-\ln(1+(a*x^2)^{(1/2)}))+8/27/x^3/(-a)^{(1/2)}/a*\ln(-a*x^2+1))-4/9/x^3/(-a)^{(1/2)}*polylog(2,a*x^2)/a-2/3/x^3/(-a)^{(1/2)}/a*polylog(3,a*x^2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.22669, size = 462, normalized size = 6.6

$$\left[ \frac{4 a^{\frac{3}{2}} x^3 \log\left(\frac{ax^2+2\sqrt{ax}+1}{ax^2-1}\right) - 8ax^2 - 6 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) + 4 \log(-ax^2+1) - 9 \operatorname{polylog}(3, ax^2)}{27 x^3}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^4,x, algorithm="fricas")

[Out] [1/27\*(4\*a^(3/2)\*x^3\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)) - 8\*a\*x^2 - 6\*\operatorname{Li}\_3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 4\*log(-a\*x^2 + 1) - 9\*polylog(3, a\*x^2))/x^3, -1/27\*(8\*sqrt(-a)\*a\*x^3\*arctan(sqrt(-a)\*x) + 8\*a\*x^2 + 6\*\operatorname{Li}\_3(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) - 4\*log(-a\*x^2 + 1) + 9\*polylog(3, a\*x^2))/x^3]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/x\*\*4,x)

[Out] Integral(polylog(3, a\*x\*\*2)/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^4,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2)/x^4, x)

### 3.44 $\int \frac{\text{PolyLog}(3, ax^2)}{x^6} dx$

**Optimal.** Leaf size=80

$$-\frac{2\text{PolyLog}(2, ax^2)}{25x^5} - \frac{\text{PolyLog}(3, ax^2)}{5x^5} - \frac{8a^2}{125x} + \frac{8}{125}a^{5/2} \tanh^{-1}(\sqrt{ax}) - \frac{8a}{375x^3} + \frac{4 \log(1 - ax^2)}{125x^5}$$

[Out]  $(-8*a)/(375*x^3) - (8*a^2)/(125*x) + (8*a^{(5/2)}*ArcTanh[Sqrt[a]*x])/125 + (4*\text{Log}[1 - a*x^2])/(125*x^5) - (2*\text{PolyLog}[2, a*x^2])/(25*x^5) - \text{PolyLog}[3, a*x^2]/(5*x^5)$

**Rubi [A]** time = 0.0469064, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {6591, 2455, 325, 206}

$$-\frac{2\text{PolyLog}(2, ax^2)}{25x^5} - \frac{\text{PolyLog}(3, ax^2)}{5x^5} - \frac{8a^2}{125x} + \frac{8}{125}a^{5/2} \tanh^{-1}(\sqrt{ax}) - \frac{8a}{375x^3} + \frac{4 \log(1 - ax^2)}{125x^5}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/x^6, x]

[Out]  $(-8*a)/(375*x^3) - (8*a^2)/(125*x) + (8*a^{(5/2)}*ArcTanh[Sqrt[a]*x])/125 + (4*\text{Log}[1 - a*x^2])/(125*x^5) - (2*\text{PolyLog}[2, a*x^2])/(25*x^5) - \text{PolyLog}[3, a*x^2]/(5*x^5)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^p)/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{\operatorname{Li}_3(ax^2)}{x^6} dx &= -\frac{\operatorname{Li}_3(ax^2)}{5x^5} + \frac{2}{5} \int \frac{\operatorname{Li}_2(ax^2)}{x^6} dx \\
&= -\frac{2\operatorname{Li}_2(ax^2)}{25x^5} - \frac{\operatorname{Li}_3(ax^2)}{5x^5} - \frac{4}{25} \int \frac{\log(1-ax^2)}{x^6} dx \\
&= \frac{4\log(1-ax^2)}{125x^5} - \frac{2\operatorname{Li}_2(ax^2)}{25x^5} - \frac{\operatorname{Li}_3(ax^2)}{5x^5} + \frac{1}{125}(8a) \int \frac{1}{x^4(1-ax^2)} dx \\
&= -\frac{8a}{375x^3} + \frac{4\log(1-ax^2)}{125x^5} - \frac{2\operatorname{Li}_2(ax^2)}{25x^5} - \frac{\operatorname{Li}_3(ax^2)}{5x^5} + \frac{1}{125}(8a^2) \int \frac{1}{x^2(1-ax^2)} dx \\
&= -\frac{8a}{375x^3} - \frac{8a^2}{125x} + \frac{4\log(1-ax^2)}{125x^5} - \frac{2\operatorname{Li}_2(ax^2)}{25x^5} - \frac{\operatorname{Li}_3(ax^2)}{5x^5} + \frac{1}{125}(8a^3) \int \frac{1}{1-ax^2} dx \\
&= -\frac{8a}{375x^3} - \frac{8a^2}{125x} + \frac{8}{125}a^{5/2} \tanh^{-1}(\sqrt{ax}) + \frac{4\log(1-ax^2)}{125x^5} - \frac{2\operatorname{Li}_2(ax^2)}{25x^5} - \frac{\operatorname{Li}_3(ax^2)}{5x^5}
\end{aligned}$$

**Mathematica [A]** time = 0.100693, size = 69, normalized size = 0.86

$$\frac{30\operatorname{PolyLog}(2, ax^2) + 75\operatorname{PolyLog}(3, ax^2) + 24a^2x^4 - 24a^{5/2}x^5 \tanh^{-1}(\sqrt{ax}) + 8ax^2 - 12\log(1-ax^2)}{375x^5}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^2]/x^6, x]

[Out]  $-(8ax^2 + 24a^2x^4 - 24a^{5/2}x^5 \operatorname{ArcTanh}[\operatorname{Sqrt}[a]x] - 12\operatorname{Log}[1 - ax^2] + 30\operatorname{PolyLog}[2, ax^2] + 75\operatorname{PolyLog}[3, ax^2]) / (375x^5)$

**Maple [B]** time = 0.18, size = 138, normalized size = 1.7

$$\frac{a^3}{2} \left( -\frac{16}{375x^3} (-a)^{-\frac{3}{2}} - \frac{16a}{125x} (-a)^{-\frac{3}{2}} - \frac{8a^2x}{125} \left( \ln(1 - \sqrt{ax^2}) - \ln(1 + \sqrt{ax^2}) \right) (-a)^{-\frac{3}{2}} \frac{1}{\sqrt{ax^2}} + \frac{8 \ln(-ax^2 + 1)}{125ax^5} (-a)^{-\frac{3}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^2)/x^6, x)

[Out]  $\frac{1}{2}a^3/(-a)^{(1/2)}*(-16/375/x^3/(-a)^{(3/2)}-16/125/x/(-a)^{(3/2)}*a-8/125*x/(-a)^{(3/2)}*a^2/(a*x^2)^{(1/2)}*(\ln(1-(a*x^2)^{(1/2)})-\ln(1+(a*x^2)^{(1/2)}))+8/125/x^5/(-a)^{(3/2)}/a*\ln(-a*x^2+1)-4/25/x^5/(-a)^{(3/2)}*\operatorname{polylog}(2, a*x^2)/a-2/5/x^5/(-a)^{(3/2)}/a*\operatorname{polylog}(3, a*x^2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x^2)/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 3.24127, size = 513, normalized size = 6.41

$$\left[ \frac{12 a^{\frac{5}{2}} x^5 \log\left(\frac{ax^2+2\sqrt{ax+1}}{ax^2-1}\right) - 24 a^2 x^4 - 8 a x^2 - 30 \operatorname{Li}_3\left(a, x, -\frac{\log(-ax^2+1)}{a}, -\frac{2 \log(-ax^2+1)}{x}\right) + 12 \log(-ax^2+1) - 75 \operatorname{polylog}(3, ax^2)}{375 x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^6,x, algorithm="fricas")

[Out] [1/375\*(12\*a^(5/2)\*x^5\*log((a\*x^2 + 2\*sqrt(a)\*x + 1)/(a\*x^2 - 1)) - 24\*a^2\*x^4 - 8\*a\*x^2 - 30\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 12\*log(-a\*x^2 + 1) - 75\*polylog(3, a\*x^2))/x^5, -1/375\*(24\*sqrt(-a)\*a^2\*x^5\*arctan(sqrt(-a)\*x) + 24\*a^2\*x^4 + 8\*a\*x^2 + 30\*\%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) - 12\*log(-a\*x^2 + 1) + 75\*polylog(3, a\*x^2))/x^5 ]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/x\*\*6,x)

[Out] Integral(polylog(3, a\*x\*\*2)/x\*\*6, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/x^6,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2)/x^6, x)

### 3.45 $\int x^2 \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=71

$$\frac{aq^2 x^{q+3} \text{Hypergeometric2F1}\left(1, \frac{q+3}{q}, \frac{3}{q} + 2, ax^q\right)}{9(q+3)} + \frac{1}{3} x^3 \text{PolyLog}(2, ax^q) + \frac{1}{9} q x^3 \log(1 - ax^q)$$

[Out] (a\*q^2\*x^(3 + q)\*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a\*x^q])/(9\*(3 + q)) + (q\*x^3\*Log[1 - a\*x^q])/9 + (x^3\*PolyLog[2, a\*x^q])/3

**Rubi [A]** time = 0.0373625, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$\frac{1}{3} x^3 \text{PolyLog}(2, ax^q) + \frac{aq^2 x^{q+3} {}_2F_1\left(1, \frac{q+3}{q}; 2 + \frac{3}{q}; ax^q\right)}{9(q+3)} + \frac{1}{9} q x^3 \log(1 - ax^q)$$

Antiderivative was successfully verified.

[In] Int[x^2\*PolyLog[2, a\*x^q], x]

[Out] (a\*q^2\*x^(3 + q)\*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a\*x^q])/(9\*(3 + q)) + (q\*x^3\*Log[1 - a\*x^q])/9 + (x^3\*PolyLog[2, a\*x^q])/3

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x^2 \text{Li}_2(ax^q) dx &= \frac{1}{3} x^3 \text{Li}_2(ax^q) + \frac{1}{3} q \int x^2 \log(1 - ax^q) dx \\ &= \frac{1}{9} q x^3 \log(1 - ax^q) + \frac{1}{3} x^3 \text{Li}_2(ax^q) + \frac{1}{9} (aq^2) \int \frac{x^{2+q}}{1 - ax^q} dx \\ &= \frac{aq^2 x^{3+q} {}_2F_1\left(1, \frac{3+q}{q}; 2 + \frac{3}{q}; ax^q\right)}{9(3+q)} + \frac{1}{9} q x^3 \log(1 - ax^q) + \frac{1}{3} x^3 \text{Li}_2(ax^q) \end{aligned}$$

**Mathematica [A]** time = 0.0439455, size = 69, normalized size = 0.97

$$\frac{qx^3 \left( aqx^q \text{Hypergeometric2F1} \left( 1, \frac{q+3}{q}, \frac{3}{q} + 2, ax^q \right) + (q+3) \log(1 - ax^q) \right)}{9(q+3)} + \frac{1}{3} x^3 \text{PolyLog}(2, ax^q)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[2, a\*x^q], x]

[Out] (q\*x^3\*(a\*q\*x^q\*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a\*x^q] + (3 + q)\*Log[1 - a\*x^q]))/(9\*(3 + q)) + (x^3\*PolyLog[2, a\*x^q])/3

**Maple [C]** time = 0.26, size = 108, normalized size = 1.5

$$-\frac{1}{q} (-a)^{-3q-1} \left( -\frac{q^2 x^3 \ln(1 - ax^q)}{9} (-a)^{3q-1} - \frac{qx^3 \text{polylog}(2, ax^q)}{3+q} (-a)^{3q-1} \left( 1 + \frac{q}{3} \right) - \frac{q^2 x^{3+q} a}{9} (-a)^{3q-1} \text{LerchPhi} \left( ax^q, 1, \frac{3}{q} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(2,a\*x^q), x)

[Out] -(-a)^(-3/q)/q\*(-1/9\*q^2\*x^3\*(-a)^(3/q)\*ln(1-a\*x^q)-q/(3+q)\*x^3\*(-a)^(3/q)\*(1+1/3\*q)\*polylog(2,a\*x^q)-1/9\*q^2\*x^(3+q)\*a\*(-a)^(3/q)\*LerchPhi(a\*x^q,1,(3+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{27} q^2 x^3 + \frac{1}{9} q x^3 \log(-ax^q + 1) + \frac{1}{3} x^3 \text{Li}_2(ax^q) - q^2 \int \frac{x^2}{9(ax^q - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^q), x, algorithm="maxima")

[Out] -1/27\*q^2\*x^3 + 1/9\*q\*x^3\*log(-a\*x^q + 1) + 1/3\*x^3\*dilog(a\*x^q) - q^2\*integrate(1/9\*x^2/(a\*x^q - 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{Li}_2(ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^q), x, algorithm="fricas")

[Out] integral(x^2\*dilog(a\*x^q), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(2,a\*x\*\*q),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,a\*x^q),x, algorithm="giac")

[Out] integrate(x^2\*dilog(a\*x^q), x)

### 3.46 $\int x \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=71

$$\frac{aq^2 x^{q+2} \text{Hypergeometric2F1}\left(1, \frac{q+2}{q}, 2\left(\frac{1}{q} + 1\right), ax^q\right)}{4(q+2)} + \frac{1}{2} x^2 \text{PolyLog}(2, ax^q) + \frac{1}{4} qx^2 \log(1 - ax^q)$$

[Out] (a\*q^2\*x^(2 + q)\*Hypergeometric2F1[1, (2 + q)/q, 2\*(1 + q^(-1)), a\*x^q])/(4\*(2 + q)) + (q\*x^2\*Log[1 - a\*x^q])/4 + (x^2\*PolyLog[2, a\*x^q])/2

**Rubi [A]** time = 0.0317548, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2455, 364}

$$\frac{1}{2} x^2 \text{PolyLog}(2, ax^q) + \frac{aq^2 x^{q+2} {}_2F_1\left(1, \frac{q+2}{q}; 2\left(1 + \frac{1}{q}\right); ax^q\right)}{4(q+2)} + \frac{1}{4} qx^2 \log(1 - ax^q)$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[2, a\*x^q], x]

[Out] (a\*q^2\*x^(2 + q)\*Hypergeometric2F1[1, (2 + q)/q, 2\*(1 + q^(-1)), a\*x^q])/(4\*(2 + q)) + (q\*x^2\*Log[1 - a\*x^q])/4 + (x^2\*PolyLog[2, a\*x^q])/2

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int x \text{Li}_2(ax^q) dx &= \frac{1}{2} x^2 \text{Li}_2(ax^q) + \frac{1}{2} q \int x \log(1 - ax^q) dx \\ &= \frac{1}{4} qx^2 \log(1 - ax^q) + \frac{1}{2} x^2 \text{Li}_2(ax^q) + \frac{1}{4} (aq^2) \int \frac{x^{1+q}}{1 - ax^q} dx \\ &= \frac{aq^2 x^{2+q} {}_2F_1\left(1, \frac{2+q}{q}; 2\left(1 + \frac{1}{q}\right); ax^q\right)}{4(2+q)} + \frac{1}{4} qx^2 \log(1 - ax^q) + \frac{1}{2} x^2 \text{Li}_2(ax^q) \end{aligned}$$

**Mathematica [A]** time = 0.036441, size = 69, normalized size = 0.97

$$\frac{qx^2 \left( aqx^q \text{Hypergeometric2F1} \left( 1, \frac{q+2}{q}, \frac{2}{q} + 2, ax^q \right) + (q+2) \log(1 - ax^q) \right)}{4(q+2)} + \frac{1}{2} x^2 \text{PolyLog}(2, ax^q)$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[2, a\*x^q], x]

[Out] (q\*x^2\*(a\*q\*x^q\*Hypergeometric2F1[1, (2 + q)/q, 2 + 2/q, a\*x^q] + (2 + q)\*Log[1 - a\*x^q]))/(4\*(2 + q)) + (x^2\*PolyLog[2, a\*x^q])/2

**Maple [C]** time = 0.207, size = 108, normalized size = 1.5

$$-\frac{1}{q} (-a)^{-2q-1} \left( -\frac{q^2 x^2 \ln(1 - ax^q)}{4} (-a)^{2q-1} - \frac{qx^2 \text{polylog}(2, ax^q)}{2+q} (-a)^{2q-1} \left( 1 + \frac{q}{2} \right) - \frac{q^2 x^{2+q} a}{4} (-a)^{2q-1} \text{LerchPhi}(ax^q, 1, (2+q)/q) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(2,a\*x^q), x)

[Out] -(-a)^(-2/q)/q\*(-1/4\*q^2\*x^2\*(-a)^(2/q)\*ln(1-a\*x^q)-q/(2+q)\*x^2\*(-a)^(2/q)\*(1+1/2\*q)\*polylog(2,a\*x^q)-1/4\*q^2\*x^(2+q)\*a\*(-a)^(2/q)\*LerchPhi(a\*x^q,1,(2+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8} q^2 x^2 + \frac{1}{4} qx^2 \log(-ax^q + 1) + \frac{1}{2} x^2 \text{Li}_2(ax^q) - q^2 \int \frac{x}{4(ax^q - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,a\*x^q), x, algorithm="maxima")

[Out] -1/8\*q^2\*x^2 + 1/4\*q\*x^2\*log(-a\*x^q + 1) + 1/2\*x^2\*dilog(a\*x^q) - q^2\*integrate(1/4\*x/(a\*x^q - 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \text{Li}_2(ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,a\*x^q), x, algorithm="fricas")

[Out] integral(x\*dilog(a\*x^q), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*polylog(2,a*x**q),x)`

[Out] `Integral(x*polylog(2, a*x**q), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*polylog(2,a*x^q),x, algorithm="giac")`

[Out] `integrate(x*dilog(a*x^q), x)`



### 3.47 $\int \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=54

$$\frac{aq^2x^{q+1}\text{Hypergeometric2F1}\left(1, \frac{1}{q} + 1, \frac{1}{q} + 2, ax^q\right)}{q+1} + x\text{PolyLog}(2, ax^q) + qx \log(1 - ax^q)$$

[Out] (a\*q^2\*x^(1 + q)\*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a\*x^q])/(1 + q) + q\*x\*Log[1 - a\*x^q] + x\*PolyLog[2, a\*x^q]

**Rubi [A]** time = 0.023219, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6586, 2448, 364}

$$x\text{PolyLog}(2, ax^q) + \frac{aq^2x^{q+1} {}_2F_1\left(1, 1 + \frac{1}{q}; 2 + \frac{1}{q}; ax^q\right)}{q+1} + qx \log(1 - ax^q)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q], x]

[Out] (a\*q^2\*x^(1 + q)\*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a\*x^q])/(1 + q) + q\*x\*Log[1 - a\*x^q] + x\*PolyLog[2, a\*x^q]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] :> Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])]/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \text{Li}_2(ax^q) dx &= x\text{Li}_2(ax^q) + q \int \log(1 - ax^q) dx \\ &= qx \log(1 - ax^q) + x\text{Li}_2(ax^q) + (aq^2) \int \frac{x^q}{1 - ax^q} dx \\ &= \frac{aq^2x^{1+q} {}_2F_1\left(1, 1 + \frac{1}{q}; 2 + \frac{1}{q}; ax^q\right)}{1+q} + qx \log(1 - ax^q) + x\text{Li}_2(ax^q) \end{aligned}$$

**Mathematica [A]** time = 0.0450111, size = 51, normalized size = 0.94

$$qx \left( \frac{aqx^q \text{Hypergeometric2F1} \left( 1, \frac{1}{q} + 1, \frac{1}{q} + 2, ax^q \right)}{q + 1} + \log(1 - ax^q) \right) + x \text{PolyLog}(2, ax^q)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^q], x]

[Out] q\*x\*((a\*q\*x^q\*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a\*x^q])/(1 + q) + Log[1 - a\*x^q]) + x\*PolyLog[2, a\*x^q]

**Maple [C]** time = 0.207, size = 88, normalized size = 1.6

$$-\frac{1}{q} (-a)^{-q-1} \left( -q^2 x \sqrt[q]{-a} \ln(1 - ax^q) - qx \sqrt[q]{-a} \text{polylog}(2, ax^q) - q^2 x^{1+q} a \sqrt[q]{-a} \text{LerchPhi} \left( ax^q, 1, \frac{1+q}{q} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^q), x)

[Out] -1/q\*(-a)^(-1/q)\*(-q^2\*x\*(-a)^(1/q)\*ln(1-a\*x^q)-q\*x\*(-a)^(1/q)\*polylog(2, a\*x^q)-q^2\*x^(1+q)\*a\*(-a)^(1/q)\*LerchPhi(a\*x^q, 1, (1+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^2 x - q^2 \int \frac{1}{ax^q - 1} dx + qx \log(-ax^q + 1) + x \text{Li}_2(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^q), x, algorithm="maxima")

[Out] -q^2\*x - q^2\*integrate(1/(a\*x^q - 1), x) + q\*x\*log(-a\*x^q + 1) + x\*dilog(a\*x^q)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{Li}_2(ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^q), x, algorithm="fricas")

[Out] integral(dilog(a\*x^q), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q),x)
```

```
[Out] Integral(polylog(2, a*x**q), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q), x)
```

$$3.48 \quad \int \frac{\text{PolyLog}(2, ax^q)}{x} dx$$

**Optimal.** Leaf size=11

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

[Out] PolyLog[3, a\*x^q]/q

**Rubi [A]** time = 0.0102243, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6589}

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/x, x]

[Out] PolyLog[3, a\*x^q]/q

**Rule 6589**

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

**Rubi steps**

$$\int \frac{\text{Li}_2(ax^q)}{x} dx = \frac{\text{Li}_3(ax^q)}{q}$$

**Mathematica [A]** time = 0.0017468, size = 11, normalized size = 1.

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^q]/x, x]

[Out] PolyLog[3, a\*x^q]/q

**Maple [A]** time = 0.044, size = 12, normalized size = 1.1

$$\frac{\text{polylog}(3, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/x,x)

[Out] polylog(3,a\*x^q)/q

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}q^2 \log(x)^3 + \frac{1}{2}q \log(-ax^q + 1) \log(x)^2 - q^2 \int \frac{\log(x)^2}{2(axx^q - x)} dx + \text{Li}_2(ax^q) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x,x, algorithm="maxima")

[Out] -1/6\*q^2\*log(x)^3 + 1/2\*q\*log(-a\*x^q + 1)\*log(x)^2 - q^2\*integrate(1/2\*log(x)^2/(a\*x\*x^q - x), x) + dilog(a\*x^q)\*log(x)

**Fricas [A]** time = 2.83852, size = 28, normalized size = 2.55

$$\frac{\text{polylog}(3, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x,x, algorithm="fricas")

[Out] polylog(3, a\*x^q)/q

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*q)/x,x)

[Out] Integral(polylog(2, a\*x\*\*q)/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x,x, algorithm="giac")

[Out] integrate(dilog(a\*x^q)/x, x)

### 3.49 $\int \frac{\text{PolyLog}(2, ax^q)}{x^2} dx$

**Optimal.** Leaf size=69

$$-\frac{aq^2x^{q-1}\text{Hypergeometric2F1}\left(1, -\frac{1-q}{q}, 2 - \frac{1}{q}, ax^q\right)}{1-q} - \frac{\text{PolyLog}(2, ax^q)}{x} + \frac{q \log(1 - ax^q)}{x}$$

[Out] -((a\*q^2\*x^(-1 + q)\*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a\*x^q])/(1 - q)) + (q\*Log[1 - a\*x^q])/x - PolyLog[2, a\*x^q]/x

**Rubi [A]** time = 0.0393505, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}(2, ax^q)}{x} - \frac{aq^2x^{q-1} {}_2F_1\left(1, -\frac{1-q}{q}; 2 - \frac{1}{q}; ax^q\right)}{1-q} + \frac{q \log(1 - ax^q)}{x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/x^2, x]

[Out] -((a\*q^2\*x^(-1 + q)\*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a\*x^q])/(1 - q)) + (q\*Log[1 - a\*x^q])/x - PolyLog[2, a\*x^q]/x

#### Rule 6591

Int[((d\_)\*(x\_))^(m\_)\*PolyLog[n\_, (a\_)\*((b\_)\*(x\_)^(p\_))^(q\_)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax^q)}{x^2} dx &= -\frac{\text{Li}_2(ax^q)}{x} - q \int \frac{\log(1 - ax^q)}{x^2} dx \\ &= \frac{q \log(1 - ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x} + (aq^2) \int \frac{x^{-2+q}}{1 - ax^q} dx \\ &= -\frac{aq^2x^{-1+q} {}_2F_1\left(1, -\frac{1-q}{q}; 2 - \frac{1}{q}; ax^q\right)}{1-q} + \frac{q \log(1 - ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.052905, size = 60, normalized size = 0.87

$$\frac{q \left( \frac{{}_2F_1\left(1, \frac{q-1}{q}, 2 - \frac{1}{q}, ax^q\right)}{q-1} + \log(1 - ax^q) \right)}{x} - \frac{\text{PolyLog}(2, ax^q)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^q]/x^2,x]

[Out] (q\*((a\*q\*x^q\*Hypergeometric2F1[1, (-1 + q)/q, 2 - q^(-1), a\*x^q])/(-1 + q) + Log[1 - a\*x^q]))/x - PolyLog[2, a\*x^q]/x

**Maple [C]** time = 0.208, size = 106, normalized size = 1.5

$$-\frac{\sqrt[q]{-a}}{q} \left( -\frac{q^2 \ln(1 - ax^q)}{x} (-a)^{-q-1} - \frac{(1 - q) q \text{polylog}(2, ax^q)}{(-1 + q)x} (-a)^{-q-1} - q^2 x^{-1+q} a (-a)^{-q-1} \text{LerchPhi}\left(ax^q, 1, \frac{-1 + q}{q}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/x^2,x)

[Out] -(-a)^(1/q)/q\*(-q^2/x\*(-a)^(-1/q)\*ln(1-a\*x^q)-q/(-1+q)/x\*(-a)^(-1/q)\*(1-q)\*polylog(2,a\*x^q)-q^2\*x^(-1+q)\*a\*(-a)^(-1/q)\*LerchPhi(a\*x^q,1,(-1+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{ax^2x^q - x^2} dx + \frac{q^2 + q \log(-ax^q + 1) - \text{Li}_2(ax^q)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^2,x, algorithm="maxima")

[Out] -q^2\*integrate(1/(a\*x^2\*x^q - x^2), x) + (q^2 + q\*log(-a\*x^q + 1) - dilog(a\*x^q))/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax^q)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^2,x, algorithm="fricas")

[Out] integral(dilog(a\*x^q)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*q)/x\*\*2,x)

[Out] Integral(polylog(2, a\*x\*\*q)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^2,x, algorithm="giac")

[Out] integrate(dilog(a\*x^q)/x^2, x)



### 3.50 $\int \frac{\text{PolyLog}(2, ax^q)}{x^3} dx$

**Optimal.** Leaf size=78

$$-\frac{aq^2x^{q-2}\text{Hypergeometric2F1}\left(1, -\frac{2-q}{q}, 2\left(1 - \frac{1}{q}\right), ax^q\right)}{4(2-q)} - \frac{\text{PolyLog}(2, ax^q)}{2x^2} + \frac{q \log(1 - ax^q)}{4x^2}$$

[Out]  $-(a*q^2*x^{(-2+q)}*Hypergeometric2F1[1, -((2-q)/q), 2*(1-q^{-1}), a*x^q])/ (4*(2-q)) + (q*Log[1-a*x^q])/ (4*x^2) - PolyLog[2, a*x^q]/ (2*x^2)$

**Rubi [A]** time = 0.0418889, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}(2, ax^q)}{2x^2} - \frac{aq^2x^{q-2} {}_2F_1\left(1, -\frac{2-q}{q}; 2\left(1 - \frac{1}{q}\right); ax^q\right)}{4(2-q)} + \frac{q \log(1 - ax^q)}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/x^3, x]

[Out]  $-(a*q^2*x^{(-2+q)}*Hypergeometric2F1[1, -((2-q)/q), 2*(1-q^{-1}), a*x^q])/ (4*(2-q)) + (q*Log[1-a*x^q])/ (4*x^2) - PolyLog[2, a*x^q]/ (2*x^2)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/ (d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/ (f\*(m+1)), x] - Dist[(b\*e\*n\*p)/ (f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/ (d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/ (c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax^q)}{x^3} dx &= -\frac{\text{Li}_2(ax^q)}{2x^2} - \frac{1}{2}q \int \frac{\log(1 - ax^q)}{x^3} dx \\ &= \frac{q \log(1 - ax^q)}{4x^2} - \frac{\text{Li}_2(ax^q)}{2x^2} + \frac{1}{4}(aq^2) \int \frac{x^{-3+q}}{1 - ax^q} dx \\ &= -\frac{aq^2x^{-2+q} {}_2F_1\left(1, -\frac{2-q}{q}; 2\left(1 - \frac{1}{q}\right); ax^q\right)}{4(2-q)} + \frac{q \log(1 - ax^q)}{4x^2} - \frac{\text{Li}_2(ax^q)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0521594, size = 61, normalized size = 0.78

$$\frac{q \left( \frac{{}_2F_1\left(1, \frac{q-2}{q}, 2-\frac{2}{q}, ax^q\right)}{q-2} + \log(1-ax^q) \right) - 2\text{PolyLog}(2, ax^q)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^q]/x^3,x]

[Out] (q\*((a\*q\*x^q\*Hypergeometric2F1[1, (-2 + q)/q, 2 - 2/q, a\*x^q])/(-2 + q) + Log[1 - a\*x^q]) - 2\*PolyLog[2, a\*x^q])/(4\*x^2)

**Maple [C]** time = 0.209, size = 108, normalized size = 1.4

$$-\frac{1}{q}(-a)^{2q-1} \left( -\frac{q^2 \ln(1-ax^q)}{4x^2} (-a)^{-2q-1} - \frac{q \text{polylog}(2, ax^q)}{(-2+q)x^2} (-a)^{-2q-1} \left(1 - \frac{q}{2}\right) - \frac{q^2 x^{-2+q} a}{4} (-a)^{-2q-1} \text{LerchPhi}\left(ax^q, 1, \frac{-2}{q}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/x^3,x)

[Out] -(-a)^(2/q)/q\*(-1/4\*q^2/x^2\*(-a)^(-2/q)\*ln(1-a\*x^q)-q/(-2+q)/x^2\*(-a)^(-2/q))\*(1-1/2\*q)\*polylog(2,a\*x^q)-1/4\*q^2\*x^(-2+q)\*a\*(-a)^(-2/q)\*LerchPhi(a\*x^q, 1, (-2+q)/q)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{4(ax^3x^q - x^3)} dx + \frac{q^2 + 2q \log(-ax^q + 1) - 4 \text{Li}_2(ax^q)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^3,x, algorithm="maxima")

[Out] -q^2\*integrate(1/4/(a\*x^3\*x^q - x^3), x) + 1/8\*(q^2 + 2\*q\*log(-a\*x^q + 1) - 4\*dilog(a\*x^q))/x^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax^q)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^3,x, algorithm="fricas")

[Out] integral(dilog(a\*x^q)/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*q)/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^3,x, algorithm="giac")

[Out] integrate(dilog(a\*x^q)/x^3, x)

### 3.51 $\int \frac{\text{PolyLog}(2, ax^q)}{x^4} dx$

**Optimal.** Leaf size=76

$$-\frac{aq^2x^{q-3}\text{Hypergeometric2F1}\left(1, -\frac{3-q}{q}, 2 - \frac{3}{q}, ax^q\right)}{9(3-q)} - \frac{\text{PolyLog}(2, ax^q)}{3x^3} + \frac{q \log(1 - ax^q)}{9x^3}$$

[Out]  $-(a*q^2*x^{(-3 + q)}*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(9*(3 - q)) + (q*Log[1 - a*x^q])/(9*x^3) - PolyLog[2, a*x^q]/(3*x^3)$

**Rubi [A]** time = 0.040045, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}(2, ax^q)}{3x^3} - \frac{aq^2x^{q-3} {}_2F_1\left(1, -\frac{3-q}{q}; 2 - \frac{3}{q}; ax^q\right)}{9(3-q)} + \frac{q \log(1 - ax^q)}{9x^3}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/x^4, x]

[Out]  $-(a*q^2*x^{(-3 + q)}*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(9*(3 - q)) + (q*Log[1 - a*x^q])/(9*x^3) - PolyLog[2, a*x^q]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax^q)}{x^4} dx &= -\frac{\text{Li}_2(ax^q)}{3x^3} - \frac{1}{3}q \int \frac{\log(1 - ax^q)}{x^4} dx \\ &= \frac{q \log(1 - ax^q)}{9x^3} - \frac{\text{Li}_2(ax^q)}{3x^3} + \frac{1}{9}(aq^2) \int \frac{x^{-4+q}}{1 - ax^q} dx \\ &= -\frac{aq^2x^{-3+q} {}_2F_1\left(1, -\frac{3-q}{q}; 2 - \frac{3}{q}; ax^q\right)}{9(3-q)} + \frac{q \log(1 - ax^q)}{9x^3} - \frac{\text{Li}_2(ax^q)}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.0518445, size = 61, normalized size = 0.8

$$\frac{q \left( \frac{{}_2F_1\left(1, \frac{q-3}{q}, 2-\frac{3}{q}, ax^q\right)}{q-3} + \log(1-ax^q) \right) - 3 \text{PolyLog}(2, ax^q)}{9x^3}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x^q]/x^4, x]

[Out] (q\*((a\*q\*x^q\*Hypergeometric2F1[1, (-3 + q)/q, 2 - 3/q, a\*x^q])/(-3 + q) + Log[1 - a\*x^q]) - 3\*PolyLog[2, a\*x^q])/(9\*x^3)

**Maple [C]** time = 0.205, size = 108, normalized size = 1.4

$$-\frac{1}{q}(-a)^{3q-1} \left( -\frac{q^2 \ln(1-ax^q)}{9x^3} (-a)^{-3q-1} - \frac{q \text{polylog}(2, ax^q)}{(-3+q)x^3} (-a)^{-3q-1} \left(1 - \frac{q}{3}\right) - \frac{q^2 x^{-3+q} a}{9} (-a)^{-3q-1} \text{LerchPhi}(ax^q, 1, (-3+q)/q) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x^q)/x^4, x)

[Out] -(-a)^(3/q)/q\*(-1/9\*q^2/x^3\*(-a)^(-3/q)\*ln(1-a\*x^q)-q/(-3+q)/x^3\*(-a)^(-3/q))\*(1-1/3\*q)\*polylog(2, a\*x^q)-1/9\*q^2\*x^(-3+q)\*a\*(-a)^(-3/q)\*LerchPhi(a\*x^q, 1, (-3+q)/q)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{9(ax^4x^q - x^4)} dx + \frac{q^2 + 3q \log(-ax^q + 1) - 9 \text{Li}_2(ax^q)}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^q)/x^4, x, algorithm="maxima")

[Out] -q^2\*integrate(1/9/(a\*x^4\*x^q - x^4), x) + 1/27\*(q^2 + 3\*q\*log(-a\*x^q + 1) - 9\*dilog(a\*x^q))/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax^q)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x^q)/x^4, x, algorithm="fricas")

[Out] integral(dilog(a\*x^q)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*q)/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/x^4,x, algorithm="giac")

[Out] integrate(dilog(a\*x^q)/x^4, x)

## 3.52 $\int x^2 \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=88

$$-\frac{aq^3x^{q+3}\text{Hypergeometric2F1}\left(1, \frac{q+3}{q}, \frac{3}{q} + 2, ax^q\right)}{27(q+3)} - \frac{1}{9}qx^3\text{PolyLog}(2, ax^q) + \frac{1}{3}x^3\text{PolyLog}(3, ax^q) - \frac{1}{27}q^2x^3 \log(1 - ax^q)$$

[Out]  $-(a*q^3*x^(3 + q)*\text{Hypergeometric2F1}[1, (3 + q)/q, 2 + 3/q, a*x^q])/(27*(3 + q)) - (q^2*x^3*\text{Log}[1 - a*x^q])/27 - (q*x^3*\text{PolyLog}[2, a*x^q])/9 + (x^3*\text{PolyLog}[3, a*x^q])/3$

**Rubi [A]** time = 0.0515477, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{1}{9}qx^3\text{PolyLog}(2, ax^q) + \frac{1}{3}x^3\text{PolyLog}(3, ax^q) - \frac{aq^3x^{q+3} {}_2F_1\left(1, \frac{q+3}{q}; 2 + \frac{3}{q}; ax^q\right)}{27(q+3)} - \frac{1}{27}q^2x^3 \log(1 - ax^q)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{PolyLog}[3, a*x^q], x]$

[Out]  $-(a*q^3*x^(3 + q)*\text{Hypergeometric2F1}[1, (3 + q)/q, 2 + 3/q, a*x^q])/(27*(3 + q)) - (q^2*x^3*\text{Log}[1 - a*x^q])/27 - (q*x^3*\text{PolyLog}[2, a*x^q])/9 + (x^3*\text{PolyLog}[3, a*x^q])/3$

### Rule 6591

$\text{Int}[(d*x)^m*\text{PolyLog}[n, (a + b*x^p)^q], x] \text{Symbol} \rightarrow \text{Simp}[(d*x)^{m+1}*\text{PolyLog}[n, (a + b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, (a + b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

### Rule 2455

$\text{Int}[(a + \text{Log}[c*(d + e*x^n)^p])*b*(f*x)^m, x] \text{Symbol} \rightarrow \text{Simp}[(f*x)^{m+1}*(a + b*\text{Log}[c*(d + e*x^n)^p])]/(f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^{n-1}*(f*x)^{m+1})/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 364

$\text{Int}[(c*(x)^m*(a + b*(x)^n)^p, x] \text{Symbol} \rightarrow \text{Simp}[(a + b*(x)^n)^p*(c*x)^{m+1}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])/c*(m+1), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !GtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
\int x^2 \text{Li}_3(ax^q) dx &= \frac{1}{3} x^3 \text{Li}_3(ax^q) - \frac{1}{3} q \int x^2 \text{Li}_2(ax^q) dx \\
&= -\frac{1}{9} q x^3 \text{Li}_2(ax^q) + \frac{1}{3} x^3 \text{Li}_3(ax^q) - \frac{1}{9} q^2 \int x^2 \log(1 - ax^q) dx \\
&= -\frac{1}{27} q^2 x^3 \log(1 - ax^q) - \frac{1}{9} q x^3 \text{Li}_2(ax^q) + \frac{1}{3} x^3 \text{Li}_3(ax^q) - \frac{1}{27} (aq^3) \int \frac{x^{2+q}}{1 - ax^q} dx \\
&= -\frac{aq^3 x^{3+q} {}_2F_1\left(1, \frac{3+q}{q}; 2 + \frac{3}{q}; ax^q\right)}{27(3+q)} - \frac{1}{27} q^2 x^3 \log(1 - ax^q) - \frac{1}{9} q x^3 \text{Li}_2(ax^q) + \frac{1}{3} x^3 \text{Li}_3(ax^q)
\end{aligned}$$

**Mathematica [C]** time = 0.0089413, size = 41, normalized size = 0.47

$$\frac{x^3 G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, \frac{q-3}{q} \\ 1, 0, 0, 0, -\frac{3}{q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*PolyLog[3, a\*x^q], x]

[Out] -((x^3\*MeijerG[{{1, 1, 1, 1, (-3 + q)/q}, {}}, {{1}}, {0, 0, 0, -3/q}], -(a\*x^q)))/q)

**Maple [C]** time = 0.345, size = 132, normalized size = 1.5

$$-\frac{1}{q} (-a)^{-3q^{-1}} \left( \frac{q^3 x^3 \ln(1 - ax^q)}{27} (-a)^{3q^{-1}} + \frac{q^2 x^3 \text{polylog}(2, ax^q)}{9} (-a)^{3q^{-1}} - \frac{q x^3 \text{polylog}(3, ax^q)}{3+q} (-a)^{3q^{-1}} \left(1 + \frac{q}{3}\right) + \frac{q^3 x^{3+q}}{27} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(3,a\*x^q), x)

[Out] -(-a)^(-3/q)/q\*(1/27\*q^3\*x^3\*(-a)^(3/q)\*ln(1-a\*x^q)+1/9\*q^2\*x^3\*(-a)^(3/q)\*polylog(2,a\*x^q)-q/(3+q)\*x^3\*(-a)^(3/q)\*(1+1/3\*q)\*polylog(3,a\*x^q)+1/27\*q^3\*x^(3+q)\*a\*(-a)^(3/q)\*LerchPhi(a\*x^q, 1, (3+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{81} q^3 x^3 - \frac{1}{27} q^2 x^3 \log(-ax^q + 1) - \frac{1}{9} q x^3 \text{Li}_2(ax^q) + q^3 \int \frac{x^2}{27(ax^q - 1)} dx + \frac{1}{3} x^3 \text{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^q), x, algorithm="maxima")

[Out] 1/81\*q^3\*x^3 - 1/27\*q^2\*x^3\*log(-a\*x^q + 1) - 1/9\*q\*x^3\*dilog(a\*x^q) + q^3\*integrate(1/27\*x^2/(a\*x^q - 1), x) + 1/3\*x^3\*polylog(3, a\*x^q)



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \text{polylog}(3, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^q),x, algorithm="fricas")

[Out] integral(x^2\*polylog(3, a\*x^q), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(3,a\*x\*\*q),x)

[Out] Integral(x\*\*2\*polylog(3, a\*x\*\*q), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(3,a\*x^q),x, algorithm="giac")

[Out] integrate(x^2\*polylog(3, a\*x^q), x)

### 3.53 $\int x \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=88

$$-\frac{aq^3x^{q+2}\text{Hypergeometric2F1}\left(1, \frac{q+2}{q}, 2\left(\frac{1}{q}+1\right), ax^q\right)}{8(q+2)} - \frac{1}{4}qx^2\text{PolyLog}(2, ax^q) + \frac{1}{2}x^2\text{PolyLog}(3, ax^q) - \frac{1}{8}q^2x^2\log(1 - ax^q)$$

[Out]  $-(a*q^3*x^{(2+q)}*Hypergeometric2F1[1, (2+q)/q, 2*(1+q^{-1}), a*x^q])/ (8*(2+q)) - (q^2*x^2*Log[1 - a*x^q])/8 - (q*x^2*PolyLog[2, a*x^q])/4 + (x^2*PolyLog[3, a*x^q])/2$

**Rubi [A]** time = 0.0425662, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6591, 2455, 364}

$$-\frac{1}{4}qx^2\text{PolyLog}(2, ax^q) + \frac{1}{2}x^2\text{PolyLog}(3, ax^q) - \frac{aq^3x^{q+2} {}_2F_1\left(1, \frac{q+2}{q}; 2\left(1 + \frac{1}{q}\right); ax^q\right)}{8(q+2)} - \frac{1}{8}q^2x^2\log(1 - ax^q)$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[3, a\*x^q], x]

[Out]  $-(a*q^3*x^{(2+q)}*Hypergeometric2F1[1, (2+q)/q, 2*(1+q^{-1}), a*x^q])/ (8*(2+q)) - (q^2*x^2*Log[1 - a*x^q])/8 - (q*x^2*PolyLog[2, a*x^q])/4 + (x^2*PolyLog[3, a*x^q])/2$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int x \operatorname{Li}_3(ax^q) dx &= \frac{1}{2} x^2 \operatorname{Li}_3(ax^q) - \frac{1}{2} q \int x \operatorname{Li}_2(ax^q) dx \\
&= -\frac{1}{4} q x^2 \operatorname{Li}_2(ax^q) + \frac{1}{2} x^2 \operatorname{Li}_3(ax^q) - \frac{1}{4} q^2 \int x \log(1 - ax^q) dx \\
&= -\frac{1}{8} q^2 x^2 \log(1 - ax^q) - \frac{1}{4} q x^2 \operatorname{Li}_2(ax^q) + \frac{1}{2} x^2 \operatorname{Li}_3(ax^q) - \frac{1}{8} (aq^3) \int \frac{x^{1+q}}{1 - ax^q} dx \\
&= -\frac{aq^3 x^{2+q} {}_2F_1\left(1, \frac{2+q}{q}; 2\left(1 + \frac{1}{q}\right); ax^q\right)}{8(2+q)} - \frac{1}{8} q^2 x^2 \log(1 - ax^q) - \frac{1}{4} q x^2 \operatorname{Li}_2(ax^q) + \frac{1}{2} x^2 \operatorname{Li}_3(ax^q)
\end{aligned}$$

**Mathematica [C]** time = 0.0076607, size = 41, normalized size = 0.47

$$-\frac{x^2 G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, \frac{q-2}{q} \\ 1, 0, 0, 0, -\frac{2}{q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*PolyLog[3, a\*x^q], x]

[Out] -((x^2\*MeijerG[{{1, 1, 1, 1, (-2 + q)/q}, {}}, {{1}}, {0, 0, 0, -2/q}}, -(a\*x^q)])/q)

**Maple [C]** time = 0.338, size = 132, normalized size = 1.5

$$-\frac{1}{q} (-a)^{-2q-1} \left( \frac{q^3 x^2 \ln(1 - ax^q)}{8} (-a)^{2q-1} + \frac{q^2 x^2 \operatorname{polylog}(2, ax^q)}{4} (-a)^{2q-1} - \frac{q x^2 \operatorname{polylog}(3, ax^q)}{2+q} (-a)^{2q-1} \left(1 + \frac{q}{2}\right) + \frac{q^3 x^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(3,a\*x^q),x)

[Out] -(-a)^(-2/q)/q\*(1/8\*q^3\*x^2\*(-a)^(2/q)\*ln(1-a\*x^q)+1/4\*q^2\*x^2\*(-a)^(2/q)\*polylog(2,a\*x^q)-q/(2+q)\*x^2\*(-a)^(2/q)\*(1+1/2\*q)\*polylog(3,a\*x^q)+1/8\*q^3\*x^2\*(-a)^(2/q)\*LerchPhi(a\*x^q,1,(2+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{16} q^3 x^2 - \frac{1}{8} q^2 x^2 \log(-ax^q + 1) - \frac{1}{4} q x^2 \operatorname{Li}_2(ax^q) + q^3 \int \frac{x}{8(ax^q - 1)} dx + \frac{1}{2} x^2 \operatorname{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^q),x, algorithm="maxima")

[Out] 1/16\*q^3\*x^2 - 1/8\*q^2\*x^2\*log(-a\*x^q + 1) - 1/4\*q\*x^2\*dilog(a\*x^q) + q^3\*integrate(1/8\*x/(a\*x^q - 1), x) + 1/2\*x^2\*polylog(3, a\*x^q)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \text{polylog}(3, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^q),x, algorithm="fricas")

[Out] integral(x\*polylog(3, a\*x^q), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x\*\*q),x)

[Out] Integral(x\*polylog(3, a\*x\*\*q), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,a\*x^q),x, algorithm="giac")

[Out] integrate(x\*polylog(3, a\*x^q), x)

### 3.54 $\int \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=69

$$\frac{aq^3x^{q+1}\text{Hypergeometric2F1}\left(1, \frac{1}{q} + 1, \frac{1}{q} + 2, ax^q\right)}{q + 1} - qx\text{PolyLog}(2, ax^q) + x\text{PolyLog}(3, ax^q) - q^2x \log(1 - ax^q)$$

[Out] -((a\*q^3\*x^(1 + q)\*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a\*x^q])/(1 + q)) - q^2\*x\*Log[1 - a\*x^q] - q\*x\*PolyLog[2, a\*x^q] + x\*PolyLog[3, a\*x^q]

**Rubi [A]** time = 0.0280803, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6586, 2448, 364}

$$-qx\text{PolyLog}(2, ax^q) + x\text{PolyLog}(3, ax^q) - \frac{aq^3x^{q+1} {}_2F_1\left(1, 1 + \frac{1}{q}; 2 + \frac{1}{q}; ax^q\right)}{q + 1} - q^2x \log(1 - ax^q)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^q], x]

[Out] -((a\*q^3\*x^(1 + q)\*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a\*x^q])/(1 + q)) - q^2\*x\*Log[1 - a\*x^q] - q\*x\*PolyLog[2, a\*x^q] + x\*PolyLog[3, a\*x^q]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \text{Li}_3(ax^q) dx &= x\text{Li}_3(ax^q) - q \int \text{Li}_2(ax^q) dx \\ &= -qx\text{Li}_2(ax^q) + x\text{Li}_3(ax^q) - q^2 \int \log(1 - ax^q) dx \\ &= -q^2x \log(1 - ax^q) - qx\text{Li}_2(ax^q) + x\text{Li}_3(ax^q) - (aq^3) \int \frac{x^q}{1 - ax^q} dx \\ &= -\frac{aq^3x^{1+q} {}_2F_1\left(1, 1 + \frac{1}{q}; 2 + \frac{1}{q}; ax^q\right)}{1 + q} - q^2x \log(1 - ax^q) - qx\text{Li}_2(ax^q) + x\text{Li}_3(ax^q) \end{aligned}$$

**Mathematica [C]** time = 0.0060684, size = 39, normalized size = 0.57

$$\frac{x G_{5,5}^{1,5} \left( -ax^q \begin{matrix} 1, 1, 1, 1, \frac{q-1}{q} \\ 1, 0, 0, 0, -\frac{1}{q} \end{matrix} \right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q], x]

[Out] -((x\*MeijerG[{{1, 1, 1, 1, (-1 + q)/q}, {}}, {{1}}, {0, 0, 0, -q^(-1)}], -(a\*x^q)))/q

**Maple [C]** time = 0.334, size = 105, normalized size = 1.5

$$-\frac{1}{q} (-a)^{-q-1} \left( q^3 x \sqrt[q]{-a} \ln(1 - ax^q) + q^2 x \sqrt[q]{-a} \operatorname{polylog}(2, ax^q) - qx \sqrt[q]{-a} \operatorname{polylog}(3, ax^q) + q^3 x^{1+q} a \sqrt[q]{-a} \operatorname{LerchPhi} \left( ax^q, 1, \frac{1}{q} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^q), x)

[Out] -1/q\*(-a)^(-1/q)\*(q^3\*x\*(-a)^(1/q)\*ln(1-a\*x^q)+q^2\*x\*(-a)^(1/q)\*polylog(2,a\*x^q)-q\*x\*(-a)^(1/q)\*polylog(3,a\*x^q)+q^3\*x^(1+q)\*a\*(-a)^(1/q)\*LerchPhi(a\*x^q,1,(1+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$q^3 x + q^3 \int \frac{1}{ax^q - 1} dx - q^2 x \log(-ax^q + 1) - qx \operatorname{Li}_2(ax^q) + x \operatorname{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q), x, algorithm="maxima")

[Out] q^3\*x + q^3\*integrate(1/(a\*x^q - 1), x) - q^2\*x\*log(-a\*x^q + 1) - q\*x\*dilog(a\*x^q) + x\*polylog(3, a\*x^q)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{polylog}(3, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q), x, algorithm="fricas")

[Out] integral(polylog(3, a\*x^q), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q),x)

[Out] Integral(polylog(3, a\*x\*\*q), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q), x)

$$3.55 \quad \int \frac{\text{PolyLog}(3, ax^q)}{x} dx$$

**Optimal.** Leaf size=11

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

[Out] PolyLog[4, a\*x^q]/q

**Rubi [A]** time = 0.0097364, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6589}

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^q]/x, x]

[Out] PolyLog[4, a\*x^q]/q

**Rule 6589**

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

**Rubi steps**

$$\int \frac{\text{Li}_3(ax^q)}{x} dx = \frac{\text{Li}_4(ax^q)}{q}$$

**Mathematica [A]** time = 0.0014392, size = 11, normalized size = 1.

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x^q]/x, x]

[Out] PolyLog[4, a\*x^q]/q

**Maple [A]** time = 0.041, size = 12, normalized size = 1.1

$$\frac{\text{polylog}(4, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int(polylog(3,a\*x^q)/x,x)

[Out] polylog(4,a\*x^q)/q

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{24} q^3 \log(x)^4 - \frac{1}{6} q^2 \log(-ax^q + 1) \log(x)^3 + q^3 \int \frac{\log(x)^3}{6(axx^q - x)} dx - \frac{1}{2} q \text{Li}_2(ax^q) \log(x)^2 + \log(x) \text{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x,x, algorithm="maxima")

[Out] 1/24\*q^3\*log(x)^4 - 1/6\*q^2\*log(-a\*x^q + 1)\*log(x)^3 + q^3\*integrate(1/6\*log(x)^3/(a\*x\*x^q - x), x) - 1/2\*q\*dilog(a\*x^q)\*log(x)^2 + log(x)\*polylog(3, a\*x^q)

**Fricas [C]** time = 2.72366, size = 209, normalized size = 19.

$$\frac{q^2 \int \left( a, q, x, -\frac{\log(-ax^q+1)}{a}, -\log(-ax^q+1) \log(x), -\frac{q \log(-ax^q+1)}{x} \right) \log(x)^2 - q^2 \text{Li}_2(ax^q) \log(x)^2 - 2 \text{polylog}(4, ax^q)}{2q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x,x, algorithm="fricas")

[Out] -1/2\*(q^2\*\int(a, q, x, -log(-a\*x^q + 1)/a, -log(-a\*x^q + 1)\*log(x), -q\*log(-a\*x^q + 1)/x)\*log(x)^2 - q^2\*dilog(a\*x^q)\*log(x)^2 - 2\*polylog(4, a\*x^q))/q

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q)/x,x)

[Out] Integral(polylog(3, a\*x\*\*q)/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q)/x, x)

### 3.56 $\int \frac{\text{PolyLog}(3, ax^q)}{x^2} dx$

**Optimal.** Leaf size=84

$$\frac{aq^3x^{q-1}\text{Hypergeometric2F1}\left(1, -\frac{1-q}{q}, 2 - \frac{1}{q}, ax^q\right)}{1-q} - \frac{q\text{PolyLog}(2, ax^q)}{x} - \frac{\text{PolyLog}(3, ax^q)}{x} + \frac{q^2 \log(1 - ax^q)}{x}$$

[Out] -((a\*q^3\*x^(-1 + q)\*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a\*x^q])/(1 - q)) + (q^2\*Log[1 - a\*x^q])/x - (q\*PolyLog[2, a\*x^q])/x - PolyLog[3, a\*x^q]/x

**Rubi [A]** time = 0.0501811, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}(2, ax^q)}{x} - \frac{\text{PolyLog}(3, ax^q)}{x} - \frac{aq^3x^{q-1} {}_2F_1\left(1, -\frac{1-q}{q}; 2 - \frac{1}{q}; ax^q\right)}{1-q} + \frac{q^2 \log(1 - ax^q)}{x}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^q]/x^2, x]

[Out] -((a\*q^3\*x^(-1 + q)\*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a\*x^q])/(1 - q)) + (q^2\*Log[1 - a\*x^q])/x - (q\*PolyLog[2, a\*x^q])/x - PolyLog[3, a\*x^q]/x

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b\*x^n)/a)])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^2} dx &= -\frac{\text{Li}_3(ax^q)}{x} + q \int \frac{\text{Li}_2(ax^q)}{x^2} dx \\
&= -\frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x} - q^2 \int \frac{\log(1-ax^q)}{x^2} dx \\
&= \frac{q^2 \log(1-ax^q)}{x} - \frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x} + (aq^3) \int \frac{x^{-2+q}}{1-ax^q} dx \\
&= -\frac{aq^3 x^{-1+q} {}_2F_1\left(1, -\frac{1-q}{q}; 2 - \frac{1}{q}; ax^q\right)}{1-q} + \frac{q^2 \log(1-ax^q)}{x} - \frac{q\text{Li}_2(ax^q)}{x} - \frac{\text{Li}_3(ax^q)}{x}
\end{aligned}$$

**Mathematica [C]** time = 0.0088089, size = 37, normalized size = 0.44

$$\frac{G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 + \frac{1}{q} \\ 1, 0, 0, 0, \frac{1}{q} \end{matrix}\right)}{qx}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/x^2, x]

[Out] -(MeijerG[{{1, 1, 1, 1, 1 + q^(-1)}}, {}], {{1}, {0, 0, 0, q^(-1)}}}, -(a\*x^q)/(q\*x))

**Maple [C]** time = 0.333, size = 129, normalized size = 1.5

$$-\frac{\sqrt[q]{-a} \left( -\frac{q^3 \ln(1-ax^q)}{x} (-a)^{-q^{-1}} + \frac{q^2 \text{polylog}(2, ax^q)}{x} (-a)^{-q^{-1}} - \frac{(1-q) q \text{polylog}(3, ax^q)}{(-1+q)x} (-a)^{-q^{-1}} - q^3 x^{-1+q} a (-a)^{-q} \right)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^q)/x^2, x)

[Out] -(-a)^(1/q)/q\*(-q^3/x\*(-a)^(-1/q)\*ln(1-a\*x^q)+q^2/x\*(-a)^(-1/q)\*polylog(2, a\*x^q)-q/(-1+q)/x\*(-a)^(-1/q)\*(1-q)\*polylog(3, a\*x^q)-q^3\*x^(-1+q)\*a\*(-a)^(-1/q)\*LerchPhi(a\*x^q, 1, (-1+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{ax^2x^q - x^2} dx + \frac{q^3 + q^2 \log(-ax^q + 1) - q\text{Li}_2(ax^q) - \text{Li}_3(ax^q)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x^q)/x^2, x, algorithm="maxima")

[Out] -q^3\*integrate(1/(a\*x^2\*x^q - x^2), x) + (q^3 + q^2\*log(-a\*x^q + 1) - q\*dilog(a\*x^q) - polylog(3, a\*x^q))/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, ax^q)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^2,x, algorithm="fricas")

[Out] integral(polylog(3, a\*x^q)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q)/x\*\*2,x)

[Out] Integral(polylog(3, a\*x\*\*q)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^2,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q)/x^2, x)

$$3.57 \quad \int \frac{\text{PolyLog}(3, ax^q)}{x^3} dx$$

**Optimal.** Leaf size=95

$$\frac{aq^3x^{q-2}\text{Hypergeometric2F1}\left(1, -\frac{2-q}{q}, 2\left(1-\frac{1}{q}\right), ax^q\right)}{8(2-q)} - \frac{q\text{PolyLog}(2, ax^q)}{4x^2} - \frac{\text{PolyLog}(3, ax^q)}{2x^2} + \frac{q^2 \log(1-ax^q)}{8x^2}$$

[Out]  $-(a*q^3*x^{(-2+q)}*Hypergeometric2F1[1, -((2-q)/q), 2*(1-q^{-1}), a*x^q])/((8*(2-q)) + (q^2*Log[1-a*x^q]))/(8*x^2) - (q*PolyLog[2, a*x^q])/(4*x^2) - PolyLog[3, a*x^q]/(2*x^2)$

**Rubi [A]** time = 0.0502811, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}(2, ax^q)}{4x^2} - \frac{\text{PolyLog}(3, ax^q)}{2x^2} - \frac{aq^3x^{q-2} {}_2F_1\left(1, -\frac{2-q}{q}; 2\left(1-\frac{1}{q}\right); ax^q\right)}{8(2-q)} + \frac{q^2 \log(1-ax^q)}{8x^2}$$

Antiderivative was successfully verified.

[In] Int [PolyLog [3, a\*x^q] / x^3, x]

[Out]  $-(a*q^3*x^{(-2+q)}*Hypergeometric2F1[1, -((2-q)/q), 2*(1-q^{-1}), a*x^q])/((8*(2-q)) + (q^2*Log[1-a*x^q]))/(8*x^2) - (q*PolyLog[2, a*x^q])/(4*x^2) - PolyLog[3, a*x^q]/(2*x^2)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^3} dx &= -\frac{\text{Li}_3(ax^q)}{2x^2} + \frac{1}{2}q \int \frac{\text{Li}_2(ax^q)}{x^3} dx \\
&= -\frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2} - \frac{1}{4}q^2 \int \frac{\log(1-ax^q)}{x^3} dx \\
&= \frac{q^2 \log(1-ax^q)}{8x^2} - \frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2} + \frac{1}{8}(aq^3) \int \frac{x^{-3+q}}{1-ax^q} dx \\
&= -\frac{aq^3 x^{-2+q} {}_2F_1\left(1, -\frac{2-q}{q}; 2\left(1 - \frac{1}{q}\right); ax^q\right)}{8(2-q)} + \frac{q^2 \log(1-ax^q)}{8x^2} - \frac{q\text{Li}_2(ax^q)}{4x^2} - \frac{\text{Li}_3(ax^q)}{2x^2}
\end{aligned}$$

**Mathematica [C]** time = 0.0085129, size = 41, normalized size = 0.43

$$\frac{G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, \frac{q+2}{q} \\ 1, 0, 0, 0, \frac{2}{q} \end{matrix}\right)}{qx^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/x^3, x]

[Out] -(MeijerG[{{1, 1, 1, 1, (2 + q)/q}, {}}, {{1}}, {0, 0, 0, 2/q}}, -(a\*x^q)]/(q\*x^2))

**Maple [C]** time = 0.343, size = 132, normalized size = 1.4

$$-\frac{1}{q}(-a)^{2q-1} \left( -\frac{q^3 \ln(1-ax^q)}{8x^2} (-a)^{-2q-1} + \frac{q^2 \text{polylog}(2, ax^q)}{4x^2} (-a)^{-2q-1} - \frac{q \text{polylog}(3, ax^q)}{(-2+q)x^2} (-a)^{-2q-1} \left(1 - \frac{q}{2}\right) - \frac{q^3 x^{-2+q} a}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^q)/x^3,x)

[Out] -(-a)^(2/q)/q\*(-1/8\*q^3/x^2\*(-a)^(-2/q)\*ln(1-a\*x^q)+1/4\*q^2/x^2\*(-a)^(-2/q)\*polylog(2,a\*x^q)-q/(-2+q)/x^2\*(-a)^(-2/q)\*(1-1/2\*q)\*polylog(3,a\*x^q)-1/8\*q^3\*x^(-2+q)\*a\*(-a)^(-2/q)\*LerchPhi(a\*x^q,1,(-2+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{8(ax^3x^q - x^3)} dx + \frac{q^3 + 2q^2 \log(-ax^q + 1) - 4q\text{Li}_2(ax^q) - 8\text{Li}_3(ax^q)}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^3,x, algorithm="maxima")

[Out] -q^3\*integrate(1/8/(a\*x^3\*x^q - x^3), x) + 1/16\*(q^3 + 2\*q^2\*log(-a\*x^q + 1) - 4\*q\*dilog(a\*x^q) - 8\*polylog(3, a\*x^q))/x^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, ax^q)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^3,x, algorithm="fricas")

[Out] integral(polylog(3, a\*x^q)/x^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q)/x\*\*3,x)

[Out] Integral(polylog(3, a\*x\*\*q)/x\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^3,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q)/x^3, x)

### 3.58 $\int \frac{\text{PolyLog}(3, ax^q)}{x^4} dx$

**Optimal.** Leaf size=93

$$\frac{aq^3x^{q-3}\text{Hypergeometric2F1}\left(1, -\frac{3-q}{q}, 2 - \frac{3}{q}, ax^q\right)}{27(3-q)} - \frac{q\text{PolyLog}(2, ax^q)}{9x^3} - \frac{\text{PolyLog}(3, ax^q)}{3x^3} + \frac{q^2 \log(1 - ax^q)}{27x^3}$$

[Out]  $-(a*q^3*x^{(-3 + q)}*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(27*(3 - q)) + (q^2*Log[1 - a*x^q])/(27*x^3) - (q*PolyLog[2, a*x^q])/(9*x^3) - PolyLog[3, a*x^q]/(3*x^3)$

**Rubi [A]** time = 0.0508152, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}(2, ax^q)}{9x^3} - \frac{\text{PolyLog}(3, ax^q)}{3x^3} - \frac{aq^3x^{q-3} {}_2F_1\left(1, -\frac{3-q}{q}; 2 - \frac{3}{q}; ax^q\right)}{27(3-q)} + \frac{q^2 \log(1 - ax^q)}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^q]/x^4, x]

[Out]  $-(a*q^3*x^{(-3 + q)}*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(27*(3 - q)) + (q^2*Log[1 - a*x^q])/(27*x^3) - (q*PolyLog[2, a*x^q])/(9*x^3) - PolyLog[3, a*x^q]/(3*x^3)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^4} dx &= -\frac{\text{Li}_3(ax^q)}{3x^3} + \frac{1}{3}q \int \frac{\text{Li}_2(ax^q)}{x^4} dx \\
&= -\frac{q\text{Li}_2(ax^q)}{9x^3} - \frac{\text{Li}_3(ax^q)}{3x^3} - \frac{1}{9}q^2 \int \frac{\log(1-ax^q)}{x^4} dx \\
&= \frac{q^2 \log(1-ax^q)}{27x^3} - \frac{q\text{Li}_2(ax^q)}{9x^3} - \frac{\text{Li}_3(ax^q)}{3x^3} + \frac{1}{27}(aq^3) \int \frac{x^{-4+q}}{1-ax^q} dx \\
&= -\frac{aq^3 x^{-3+q} {}_2F_1\left(1, -\frac{3-q}{q}; 2 - \frac{3}{q}; ax^q\right)}{27(3-q)} + \frac{q^2 \log(1-ax^q)}{27x^3} - \frac{q\text{Li}_2(ax^q)}{9x^3} - \frac{\text{Li}_3(ax^q)}{3x^3}
\end{aligned}$$

**Mathematica [C]** time = 0.0104082, size = 41, normalized size = 0.44

$$-\frac{G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, \frac{q+3}{q} \\ 1, 0, 0, 0, \frac{3}{q} \end{matrix}\right)}{qx^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/x^4, x]

[Out] -(MeijerG[{{1, 1, 1, 1, (3 + q)/q}, {}}, {{1}, {0, 0, 0, 3/q}}, -(a\*x^q)]/(q\*x^3))

**Maple [C]** time = 0.34, size = 132, normalized size = 1.4

$$-\frac{1}{q}(-a)^{3q-1} \left( -\frac{q^3 \ln(1-ax^q)}{27x^3} (-a)^{-3q-1} + \frac{q^2 \text{polylog}(2, ax^q)}{9x^3} (-a)^{-3q-1} - \frac{q \text{polylog}(3, ax^q)}{(-3+q)x^3} (-a)^{-3q-1} \left(1 - \frac{q}{3}\right) - \frac{q^3 x^{-3+q}}{27} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^q)/x^4, x)

[Out] -(-a)^(3/q)/q\*(-1/27\*q^3/x^3\*(-a)^(-3/q)\*ln(1-a\*x^q)+1/9\*q^2/x^3\*(-a)^(-3/q)\*polylog(2, a\*x^q)-q/(-3+q)/x^3\*(-a)^(-3/q)\*(1-1/3\*q)\*polylog(3, a\*x^q)-1/27\*q^3\*x^(-3+q)\*a\*(-a)^(-3/q)\*LerchPhi(a\*x^q, 1, (-3+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{27(ax^4x^q - x^4)} dx + \frac{q^3 + 3q^2 \log(-ax^q + 1) - 9q\text{Li}_2(ax^q) - 27\text{Li}_3(ax^q)}{81x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3, a\*x^q)/x^4, x, algorithm="maxima")

[Out] -q^3\*integrate(1/27/(a\*x^4\*x^q - x^4), x) + 1/81\*(q^3 + 3\*q^2\*log(-a\*x^q + 1) - 9\*q\*dilog(a\*x^q) - 27\*polylog(3, a\*x^q))/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, ax^q)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^4,x, algorithm="fricas")

[Out] integral(polylog(3, a\*x^q)/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q)/x\*\*4,x)

[Out] Integral(polylog(3, a\*x\*\*q)/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/x^4,x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q)/x^4, x)

### 3.59 $\int (dx)^{3/2} \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=117

$$\frac{2(dx)^{5/2} \text{PolyLog}(2, ax)}{5d} + \frac{8d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}} - \frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} - \frac{8(dx)^{5/2}}{125d}$$

[Out]  $(-8*d*\text{Sqrt}[d*x])/(25*a^2) - (8*(d*x)^{(3/2)})/(75*a) - (8*(d*x)^{(5/2)})/(125*d) + (8*d^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(25*a^{(5/2)}) + (4*(d*x)^{(5/2)}*\text{Log}[1 - a*x])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x])/(5*d)$

**Rubi [A]** time = 0.0721405, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2(dx)^{5/2} \text{PolyLog}(2, ax)}{5d} + \frac{8d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}} - \frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} - \frac{8(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*\text{PolyLog}[2, a*x], x]$

[Out]  $(-8*d*\text{Sqrt}[d*x])/(25*a^2) - (8*(d*x)^{(3/2)})/(75*a) - (8*(d*x)^{(5/2)})/(125*d) + (8*d^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(25*a^{(5/2)}) + (4*(d*x)^{(5/2)}*\text{Log}[1 - a*x])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x])/(5*d)$

#### Rule 6591

$\text{Int}[(d_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_*, (a_*)*((b_*)*(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})*(b_*)]*((f_*) + (g_*)*(x_*)^{(q_*)}), x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])]/(g*(q+1)), x] - \text{Dist}[(b*e*n)/(g*(q+1)), \text{Int}[(f + g*x)^{(q+1)}/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 50

$\text{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*(c + d*x)^n/(b*(m+n+1)), x] + \text{Dist}[(n*(b*c - a*d))/(b*(m+n+1)), \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && (!IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

$\text{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int (dx)^{3/2} \text{Li}_2(ax) dx &= \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{2}{5} \int (dx)^{3/2} \log(1-ax) dx \\
 &= \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{(4a) \int \frac{(dx)^{5/2}}{1-ax} dx}{25d} \\
 &= -\frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{4}{25} \int \frac{(dx)^{3/2}}{1-ax} dx \\
 &= -\frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{(4d) \int \frac{\sqrt{dx}}{1-ax} dx}{25a} \\
 &= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{(4d^2) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{25a^2} \\
 &= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d} + \frac{(8d) \text{Subst} \left( \int \frac{1}{1-\frac{ax^2}{d}} dx \right)}{25a^2} \\
 &= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{8d^{3/2} \tanh^{-1} \left( \frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}} \right)}{25a^{5/2}} + \frac{4(dx)^{5/2} \log(1-ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax)}{5d}
 \end{aligned}$$

**Mathematica [A]** time = 0.102173, size = 90, normalized size = 0.77

$$\frac{2(dx)^{3/2} \left( x^{5/2} \text{PolyLog}(2, ax) + \frac{2}{75} \sqrt{x} \left( 15x^2 \log(1-ax) - \frac{2(3a^2x^2+5ax+15)}{a^2} \right) + \frac{4 \tanh^{-1}(\sqrt{a}\sqrt{x})}{5a^{5/2}} \right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*PolyLog[2, a\*x], x]

[Out] (2\*(d\*x)^(3/2)\*((4\*ArcTanh[Sqrt[a]\*Sqrt[x]])/(5\*a^(5/2)) + (2\*Sqrt[x]\*((-2\*(15 + 5\*a\*x + 3\*a^2\*x^2))/a^2 + 15\*x^2\*Log[1 - a\*x]))/75 + x^(5/2)\*PolyLog[2, a\*x]))/(5\*x^(3/2))

**Maple [A]** time = 0.194, size = 96, normalized size = 0.8

$$\frac{2 \text{polylog}(2, ax)}{5d} (dx)^{5/2} + \frac{4}{25d} (dx)^{5/2} \ln \left( \frac{-adx+d}{d} \right) - \frac{8}{125d} (dx)^{5/2} - \frac{8}{75a} (dx)^{3/2} - \frac{8d}{25a^2} \sqrt{dx} + \frac{8d^2}{25a^2} \text{Artanh} \left( a\sqrt{dx} \frac{1}{\sqrt{ad}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*polylog(2,a\*x), x)

[Out] 2/5\*(d\*x)^(5/2)\*polylog(2,a\*x)/d+4/25/d\*(d\*x)^(5/2)\*ln((-a\*d\*x+d)/d)-8/125\*(d\*x)^(5/2)/d-8/75\*(d\*x)^(3/2)/a-8/25\*d\*(d\*x)^(1/2)/a^2+8/25\*d^2/a^2/(a\*d)^

$(1/2)*\operatorname{arctanh}(a*(d*x)^{(1/2)/(a*d)^{(1/2)})}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(2,a\*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.82002, size = 470, normalized size = 4.02

$$\left[ \frac{2 \left( 30 d \sqrt{\frac{d}{a}} \log \left( \frac{adx + 2 \sqrt{dxa} \sqrt{\frac{d}{a}} + d}{ax-1} \right) + (75 a^2 dx^2 \operatorname{Li}_2(ax) + 30 a^2 dx^2 \log(-ax + 1) - 12 a^2 dx^2 - 20 adx - 60 d) \sqrt{dx} \right)}{375 a^2}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(2,a\*x),x, algorithm="fricas")

[Out] [2/375\*(30\*d\*sqrt(d/a)\*log((a\*d\*x + 2\*sqrt(d\*x)\*a\*sqrt(d/a) + d)/(a\*x - 1)) + (75\*a^2\*d\*x^2\*dilog(a\*x) + 30\*a^2\*d\*x^2\*log(-a\*x + 1) - 12\*a^2\*d\*x^2 - 20\*a\*d\*x - 60\*d)\*sqrt(d\*x))/a^2, -2/375\*(60\*d\*sqrt(-d/a)\*arctan(sqrt(d\*x)\*a\*sqrt(-d/a)/d) - (75\*a^2\*d\*x^2\*dilog(a\*x) + 30\*a^2\*d\*x^2\*log(-a\*x + 1) - 12\*a^2\*d\*x^2 - 20\*a\*d\*x - 60\*d)\*sqrt(d\*x))/a^2]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*polylog(2,a\*x),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)\*dilog(a\*x), x)

### 3.60 $\int \sqrt{dx} \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=102

$$\frac{2(dx)^{3/2} \text{PolyLog}(2, ax)}{3d} + \frac{8\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}} - \frac{8\sqrt{dx}}{9a} + \frac{4(dx)^{3/2} \log(1 - ax)}{9d} - \frac{8(dx)^{3/2}}{27d}$$

[Out]  $(-8\sqrt{d*x})/(9*a) - (8*(d*x)^{(3/2)})/(27*d) + (8\sqrt{d}*ArcTanh[(\sqrt{a}*\sqrt{d*x})/\sqrt{d}])/(9*a^{(3/2)}) + (4*(d*x)^{(3/2)}*Log[1 - a*x])/(9*d) + (2*(d*x)^{(3/2)}*PolyLog[2, a*x])/(3*d)$

**Rubi [A]** time = 0.0530667, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2(dx)^{3/2} \text{PolyLog}(2, ax)}{3d} + \frac{8\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}} - \frac{8\sqrt{dx}}{9a} + \frac{4(dx)^{3/2} \log(1 - ax)}{9d} - \frac{8(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*PolyLog[2, a\*x], x]

[Out]  $(-8\sqrt{d*x})/(9*a) - (8*(d*x)^{(3/2)})/(27*d) + (8\sqrt{d}*ArcTanh[(\sqrt{a}*\sqrt{d*x})/\sqrt{d}])/(9*a^{(3/2)}) + (4*(d*x)^{(3/2)}*Log[1 - a*x])/(9*d) + (2*(d*x)^{(3/2)}*PolyLog[2, a*x])/(3*d)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \sqrt{dx} \operatorname{Li}_2(ax) dx &= \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d} + \frac{2}{3} \int \sqrt{dx} \log(1-ax) dx \\
 &= \frac{4(dx)^{3/2} \log(1-ax)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d} + \frac{(4a) \int \frac{(dx)^{3/2}}{1-ax} dx}{9d} \\
 &= -\frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2} \log(1-ax)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d} + \frac{4}{9} \int \frac{\sqrt{dx}}{1-ax} dx \\
 &= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2} \log(1-ax)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d} + \frac{(4d) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{9a} \\
 &= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2} \log(1-ax)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d} + \frac{8 \operatorname{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{9a} \\
 &= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{8\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}} + \frac{4(dx)^{3/2} \log(1-ax)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax)}{3d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0816363, size = 75, normalized size = 0.74

$$\frac{2\sqrt{dx} \left( 9x^{3/2} \operatorname{PolyLog}(2, ax) + \frac{12 \tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{3/2}} + \frac{2\sqrt{x}(-2ax+3ax \log(1-ax)-6)}{a} \right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*PolyLog[2, a\*x], x]

[Out] (2\*Sqrt[d\*x]\*((12\*ArcTanh[Sqrt[a]\*Sqrt[x]])/a^(3/2) + (2\*Sqrt[x]\*(-6 - 2\*a\*x + 3\*a\*x\*Log[1 - a\*x]))/a + 9\*x^(3/2)\*PolyLog[2, a\*x]))/(27\*Sqrt[x])

**Maple [A]** time = 0.05, size = 83, normalized size = 0.8

$$\frac{2 \operatorname{polylog}(2, ax)}{3d} (dx)^{\frac{3}{2}} + \frac{4}{9d} (dx)^{\frac{3}{2}} \ln\left(\frac{-adx+d}{d}\right) - \frac{8}{27d} (dx)^{\frac{3}{2}} - \frac{8}{9a} \sqrt{dx} + \frac{8d}{9a} \operatorname{Arctanh}\left(a\sqrt{dx} \frac{1}{\sqrt{ad}}\right) \frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*polylog(2, a\*x), x)

[Out] 2/3\*(d\*x)^(3/2)\*polylog(2, a\*x)/d+4/9/d\*(d\*x)^(3/2)\*ln((-a\*d\*x+d)/d)-8/27\*(d\*x)^(3/2)/d-8/9\*(d\*x)^(1/2)/a+8/9\*d/a/(a\*d)^(1/2)\*arctanh(a\*(d\*x)^(1/2)/(a\*d)^(1/2))

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(2,a\*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.56879, size = 362, normalized size = 3.55

$$\left[ \frac{2 \left( (9 ax \operatorname{Li}_2(ax) + 6 ax \log(-ax + 1) - 4 ax - 12) \sqrt{dx} + 6 \sqrt{\frac{d}{a}} \log\left(\frac{adx + 2 \sqrt{dxa} \sqrt{\frac{d}{a} + d}}{ax - 1}\right) \right)}{27 a}, \frac{2 \left( (9 ax \operatorname{Li}_2(ax) + 6 ax \log(-ax + 1) - 4 ax - 12) \sqrt{dx} + 6 \sqrt{\frac{d}{a}} \log\left(\frac{adx + 2 \sqrt{dxa} \sqrt{\frac{d}{a} + d}}{ax - 1}\right) \right)}{27 a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(2,a\*x),x, algorithm="fricas")

[Out] [2/27\*((9\*a\*x\*dilog(a\*x) + 6\*a\*x\*log(-a\*x + 1) - 4\*a\*x - 12)\*sqrt(d\*x) + 6\*sqrt(d/a)\*log((a\*d\*x + 2\*sqrt(d\*x)\*a\*sqrt(d/a) + d)/(a\*x - 1)))/a, 2/27\*((9\*a\*x\*dilog(a\*x) + 6\*a\*x\*log(-a\*x + 1) - 4\*a\*x - 12)\*sqrt(d\*x) - 12\*sqrt(-d/a)\*arctan(sqrt(d\*x)\*a\*sqrt(-d/a)/d))/a]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*polylog(2,a\*x),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(2,a\*x),x, algorithm="giac")

[Out] integrate(sqrt(d\*x)\*dilog(a\*x), x)



### 3.61 $\int \frac{\text{PolyLog}(2, ax)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=80

$$\frac{2\sqrt{dx}\text{PolyLog}(2, ax)}{d} + \frac{4\sqrt{dx}\log(1 - ax)}{d} + \frac{8 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} - \frac{8\sqrt{dx}}{d}$$

[Out]  $(-8*\text{Sqrt}[d*x])/d + (8*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])]/(\text{Sqrt}[a]*\text{Sqrt}[d])) + (4*\text{Sqrt}[d*x]*\text{Log}[1 - a*x])/d + (2*\text{Sqrt}[d*x]*\text{PolyLog}[2, a*x])/d$

**Rubi [A]** time = 0.0458829, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2\sqrt{dx}\text{PolyLog}(2, ax)}{d} + \frac{4\sqrt{dx}\log(1 - ax)}{d} + \frac{8 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} - \frac{8\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[2, a*x]/\text{Sqrt}[d*x], x]$

[Out]  $(-8*\text{Sqrt}[d*x])/d + (8*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])]/(\text{Sqrt}[a]*\text{Sqrt}[d])) + (4*\text{Sqrt}[d*x]*\text{Log}[1 - a*x])/d + (2*\text{Sqrt}[d*x]*\text{PolyLog}[2, a*x])/d$

#### Rule 6591

$\text{Int}[(d_*)(x_*)^{(m_*)} \text{PolyLog}[n_*, (a_*)((b_*)(x_*)^{(p_*)})^{(q_*)}], x_{\text{Symbol}}] \rightarrow \text{Simp}[(d*x)^{(m+1)} \text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m \text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

$\text{Int}[(a_*) + \text{Log}[(c_*)(d_*) + (e_*)(x_*)^{(n_*)}] * (b_*) * ((f_*) + (g_*)(x_*)^{(q_*)}), x_{\text{Symbol}}] \rightarrow \text{Simp}[(f + g*x)^{(q+1)} * (a + b*\text{Log}[c*(d + e*x)^n])]/(g*(q+1)), x] - \text{Dist}[(b*e*n)/(g*(q+1)), \text{Int}[(f + g*x)^{(q+1)}/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 50

$\text{Int}[(a_*) + (b_*)(x_*)^{(m_*)} * ((c_*) + (d_*)(x_*)^{(n_*)}), x_{\text{Symbol}}] \rightarrow \text{Simp}[(a + b*x)^{(m+1)} * (c + d*x)^n / (b*(m+n+1)), x] + \text{Dist}[(n*(b*c - a*d))/(b*(m+n+1)), \text{Int}[(a + b*x)^m * (c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m+n+1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m-n, 0]))) && !ILtQ[m+n+2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

$\text{Int}[(a_*) + (b_*)(x_*)^{(m_*)} * ((c_*) + (d_*)(x_*)^{(n_*)}), x_{\text{Symbol}}] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_2(ax)}{d} + 2 \int \frac{\log(1-ax)}{\sqrt{dx}} dx \\ &= \frac{4\sqrt{dx}\log(1-ax)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{(4a) \int \frac{\sqrt{dx}}{1-ax} dx}{d} \\ &= -\frac{8\sqrt{dx}}{d} + \frac{4\sqrt{dx}\log(1-ax)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax)}{d} + 4 \int \frac{1}{\sqrt{dx}(1-ax)} dx \\ &= -\frac{8\sqrt{dx}}{d} + \frac{4\sqrt{dx}\log(1-ax)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{8 \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{d} \\ &= -\frac{8\sqrt{dx}}{d} + \frac{8 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} + \frac{4\sqrt{dx}\log(1-ax)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.078857, size = 63, normalized size = 0.79

$$\frac{2\sqrt{ax}\text{PolyLog}[2, ax] + 4\sqrt{ax}(\log(1-ax) - 2) + 8\sqrt{x} \tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/Sqrt[d\*x], x]

[Out] (8\*Sqrt[x]\*ArcTanh[Sqrt[a]\*Sqrt[x]] + 4\*Sqrt[a]\*x\*(-2 + Log[1 - a\*x]) + 2\*Sqrt[a]\*x\*PolyLog[2, a\*x])/(Sqrt[a]\*Sqrt[d\*x])

**Maple [A]** time = 0.05, size = 69, normalized size = 0.9

$$2 \frac{\sqrt{dx}\text{polylog}(2, ax)}{d} + 4 \frac{\sqrt{dx}}{d} \ln\left(\frac{-adx+d}{d}\right) - 8 \frac{\sqrt{dx}}{d} + 8 \frac{1}{\sqrt{ad}} \text{Arctanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x)/(d\*x)^(1/2), x)

[Out] 2\*polylog(2,a\*x)\*(d\*x)^(1/2)/d+4/d\*(d\*x)^(1/2)\*ln((-a\*d\*x+d)/d)-8\*(d\*x)^(1/2)/d+8/(a\*d)^(1/2)\*arctanh(a\*(d\*x)^(1/2)/(a\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.61497, size = 331, normalized size = 4.14

$$\left[ \frac{2 \left( \sqrt{dx} (a \operatorname{Li}_2(ax) + 2a \log(-ax + 1) - 4a) + 2\sqrt{ad} \log\left(\frac{adx + 2\sqrt{ad}\sqrt{dx+d}}{ax-1}\right) \right)}{ad}, \frac{2 \left( \sqrt{dx} (a \operatorname{Li}_2(ax) + 2a \log(-ax + 1) - 4a) \right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x + 1) - 4*a) + 2*sqrt(a*d)*log((a*d*x + 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)))/(a*d), 2*(sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x + 1) - 4*a) - 4*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*x)/(a*d*x)))/(a*d)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x)/sqrt(d*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/sqrt(d*x), x)
```

### 3.62 $\int \frac{\text{PolyLog}(2,ax)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=68

$$-\frac{2\text{PolyLog}(2,ax)}{d\sqrt{dx}} + \frac{8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{4\log(1-ax)}{d\sqrt{dx}}$$

[Out]  $(8\sqrt{a}\text{ArcTanh}[(\sqrt{a}\sqrt{dx})/\sqrt{d}])/d^{(3/2)} + (4\text{Log}[1 - ax])/ (d\sqrt{dx}) - (2\text{PolyLog}[2, ax])/(d\sqrt{dx})$

**Rubi [A]** time = 0.0431147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2395, 63, 206}

$$-\frac{2\text{PolyLog}(2,ax)}{d\sqrt{dx}} + \frac{8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{4\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/(d\*x)^(3/2), x]

[Out]  $(8\sqrt{a}\text{ArcTanh}[(\sqrt{a}\sqrt{dx})/\sqrt{d}])/d^{(3/2)} + (4\text{Log}[1 - ax])/ (d\sqrt{dx}) - (2\text{PolyLog}[2, ax])/(d\sqrt{dx})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1)-1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_2(ax)}{d\sqrt{dx}} - 2 \int \frac{\log(1-ax)}{(dx)^{3/2}} dx \\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}} + \frac{(4a) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{d} \\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}} + \frac{(8a) \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{d^2} \\
&= \frac{8\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax)}{d\sqrt{dx}}
\end{aligned}$$

**Mathematica [A]** time = 0.0714846, size = 51, normalized size = 0.75

$$\frac{2x(-\text{PolyLog}(2, ax) + 2\log(1-ax) + 4\sqrt{a}\sqrt{x} \tanh^{-1}(\sqrt{a}\sqrt{x}))}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/(d\*x)^(3/2), x]

[Out] (2\*x\*(4\*Sqrt[a]\*Sqrt[x]\*ArcTanh[Sqrt[a]\*Sqrt[x]] + 2\*Log[1 - a\*x] - PolyLog[2, a\*x]))/(d\*x)^(3/2)

**Maple [A]** time = 0.053, size = 63, normalized size = 0.9

$$-2 \frac{\text{polylog}(2, ax)}{d\sqrt{dx}} + 4 \frac{1}{d\sqrt{dx}} \ln\left(\frac{-adx + d}{d}\right) + 8 \frac{a}{d\sqrt{ad}} \text{Artanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/(d\*x)^(3/2), x)

[Out] -2\*polylog(2, a\*x)/d/(d\*x)^(1/2)+4/d/(d\*x)^(1/2)\*ln((-a\*d\*x+d)/d)+8/d\*a/(a\*d)^(1/2)\*arctanh(a\*(d\*x)^(1/2)/(a\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, a\*x)/(d\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.71198, size = 316, normalized size = 4.65

$$\left[ \frac{2 \left( 2 dx \sqrt{\frac{a}{d}} \log \left( \frac{ax + 2 \sqrt{dx} \sqrt{\frac{a}{d}} + 1}{ax - 1} \right) - \sqrt{dx} (\text{Li}_2(ax) - 2 \log(-ax + 1)) \right)}{d^2 x}, - \frac{2 \left( 4 dx \sqrt{-\frac{a}{d}} \arctan \left( \frac{\sqrt{dx} \sqrt{-\frac{a}{d}}}{ax} \right) + \sqrt{dx} (\text{Li}_2(ax) - 2 \log(-ax + 1)) \right)}{d^2 x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(3/2),x, algorithm="fricas")

[Out] [2\*(2\*d\*x\*sqrt(a/d)\*log((a\*x + 2\*sqrt(d\*x)\*sqrt(a/d) + 1)/(a\*x - 1)) - sqrt(d\*x)\*(dilog(a\*x) - 2\*log(-a\*x + 1)))/(d^2\*x), -2\*(4\*d\*x\*sqrt(-a/d)\*arctan(sqrt(d\*x)\*sqrt(-a/d)/(a\*x)) + sqrt(d\*x)\*(dilog(a\*x) - 2\*log(-a\*x + 1)))/(d^2\*x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)\*\*(3/2),x)

[Out] Integral(polylog(2, a\*x)/(d\*x)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(3/2),x, algorithm="giac")

[Out] integrate(dilog(a\*x)/(d\*x)^(3/2), x)

### 3.63 $\int \frac{\text{PolyLog}(2, ax)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=89

$$-\frac{2\text{PolyLog}(2, ax)}{3d(dx)^{3/2}} + \frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} - \frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}}$$

[Out]  $(-8*a)/(9*d^2*\text{Sqrt}[d*x]) + (8*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])])/(9*d^{(5/2)}) + (4*\text{Log}[1 - a*x])/(9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[2, a*x])/(3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.0504307, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{2\text{PolyLog}(2, ax)}{3d(dx)^{3/2}} + \frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} - \frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/(d\*x)^(5/2), x]

[Out]  $(-8*a)/(9*d^2*\text{Sqrt}[d*x]) + (8*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])])/(9*d^{(5/2)}) + (4*\text{Log}[1 - a*x])/(9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[2, a*x])/(3*d*(d*x)^{(3/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Simp[((a + b\*x)^(m+1)\*(c + d\*x)^(n+1))/((b\*c - a\*d)\*(m+1)), x] - Dist[(d\*(m+n+2))/((b\*c - a\*d)\*(m+1)), Int[(a + b\*x)^(m+1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} - \frac{2}{3} \int \frac{\log(1-ax)}{(dx)^{5/2}} dx \\ &= \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(4a) \int \frac{1}{(dx)^{3/2}(1-ax)} dx}{9d} \\ &= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(4a^2) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{9d^2} \\ &= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(8a^2) \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{9d^3} \\ &= -\frac{8a}{9d^2\sqrt{dx}} + \frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0741523, size = 57, normalized size = 0.64

$$\frac{2x \left( 3\text{PolyLog}(2, ax) - 4a^{3/2}x^{3/2} \tanh^{-1}\left(\sqrt{a}\sqrt{x}\right) + 4ax - 2\log(1-ax) \right)}{9(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/(d\*x)^(5/2), x]

[Out] (-2\*x\*(4\*a\*x - 4\*a^(3/2)\*x^(3/2)\*ArcTanh[Sqrt[a]\*Sqrt[x]] - 2\*Log[1 - a\*x] + 3\*PolyLog[2, a\*x])/(9\*(d\*x)^(5/2))

**Maple [A]** time = 0.055, size = 76, normalized size = 0.9

$$-\frac{2 \text{polylog}(2, ax)}{3d} (dx)^{-\frac{3}{2}} + \frac{4}{9d} \ln\left(\frac{-adx+d}{d}\right) (dx)^{-\frac{3}{2}} - \frac{8a}{9d^2} \frac{1}{\sqrt{dx}} + \frac{8a^2}{9d^2} \text{Artanh}\left(a\sqrt{dx} \frac{1}{\sqrt{ad}}\right) \frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/(d\*x)^(5/2), x)

[Out] -2/3\*polylog(2, a\*x)/d/(d\*x)^(3/2)+4/9/d/(d\*x)^(3/2)\*ln((-a\*d\*x+d)/d)-8/9\*a/d^2/(d\*x)^(1/2)+8/9/d^2\*a^2/(a\*d)^(1/2)\*arctanh(a\*(d\*x)^(1/2)/(a\*d)^(1/2))



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.66517, size = 365, normalized size = 4.1

$$\left[ \frac{2 \left( 2 a d x^2 \sqrt{\frac{a}{d}} \log \left( \frac{a x + 2 \sqrt{d x} \sqrt{\frac{a}{d}} + 1}{a x - 1} \right) - (4 a x + 3 \operatorname{Li}_2(a x) - 2 \log(-a x + 1)) \sqrt{d x} \right)}{9 d^3 x^2}, - \frac{2 \left( 4 a d x^2 \sqrt{-\frac{a}{d}} \arctan \left( \frac{\sqrt{d x} \sqrt{-\frac{a}{d}}}{a x} \right) + (4 a x + 3 \operatorname{dilog}(a x) - 2 \log(-a x + 1)) \sqrt{d x} \right)}{9 d^3 x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(5/2),x, algorithm="fricas")

[Out] [2/9\*(2\*a\*d\*x^2\*sqrt(a/d)\*log((a\*x + 2\*sqrt(d\*x)\*sqrt(a/d) + 1)/(a\*x - 1)) - (4\*a\*x + 3\*dilog(a\*x) - 2\*log(-a\*x + 1))\*sqrt(d\*x))/(d^3\*x^2), -2/9\*(4\*a\*d\*x^2\*sqrt(-a/d)\*arctan(sqrt(d\*x)\*sqrt(-a/d)/(a\*x)) + (4\*a\*x + 3\*dilog(a\*x) - 2\*log(-a\*x + 1))\*sqrt(d\*x))/(d^3\*x^2)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(5/2),x, algorithm="giac")

[Out] integrate(dilog(a\*x)/(d\*x)^(5/2), x)

### 3.64 $\int \frac{\text{PolyLog}(2, ax)}{(dx)^{7/2}} dx$

**Optimal.** Leaf size=106

$$-\frac{2\text{PolyLog}(2, ax)}{5d(dx)^{5/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{8a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} - \frac{8a}{75d^2(dx)^{3/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}}$$

[Out]  $(-8*a)/(75*d^2*(d*x)^{(3/2)}) - (8*a^2)/(25*d^3*\text{Sqrt}[d*x]) + (8*a^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])]/(25*d^{(7/2)})) + (4*\text{Log}[1 - a*x])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[2, a*x])/(5*d*(d*x)^{(5/2)})$

**Rubi [A]** time = 0.057124, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{2\text{PolyLog}(2, ax)}{5d(dx)^{5/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{8a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} - \frac{8a}{75d^2(dx)^{3/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x]/(d\*x)^(7/2), x]

[Out]  $(-8*a)/(75*d^2*(d*x)^{(3/2)}) - (8*a^2)/(25*d^3*\text{Sqrt}[d*x]) + (8*a^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/(\text{Sqrt}[d])]/(25*d^{(7/2)})) + (4*\text{Log}[1 - a*x])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[2, a*x])/(5*d*(d*x)^{(5/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Simp[((a + b\*x)^(m+1)\*(c + d\*x)^(n+1))/((b\*c - a\*d)\*(m+1)), x] - Dist[(d\*(m+n+2))/((b\*c - a\*d)\*(m+1)), Int[(a + b\*x)^(m+1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(ax)}{(dx)^{7/2}} dx &= -\frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} - \frac{2}{5} \int \frac{\log(1-ax)}{(dx)^{7/2}} dx \\
 &= \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a) \int \frac{1}{(dx)^{5/2}(1-ax)} dx}{25d} \\
 &= -\frac{8a}{75d^2(dx)^{3/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a^2) \int \frac{1}{(dx)^{3/2}(1-ax)} dx}{25d^2} \\
 &= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a^3) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{25d^3} \\
 &= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(8a^3) \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{25d^4} \\
 &= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{8a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0946373, size = 65, normalized size = 0.61

$$\frac{2x(15\text{PolyLog}(2, ax) + 12a^2x^2 - 12a^{5/2}x^{5/2} \tanh^{-1}(\sqrt{a}\sqrt{x}) + 4ax - 6\log(1-ax))}{75(dx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a\*x]/(d\*x)^(7/2), x]

[Out] (-2\*x\*(4\*a\*x + 12\*a^2\*x^2 - 12\*a^(5/2)\*x^(5/2)\*ArcTanh[Sqrt[a]\*Sqrt[x]] - 6\*Log[1 - a\*x] + 15\*PolyLog[2, a\*x])/(75\*(d\*x)^(7/2))

**Maple [A]** time = 0.054, size = 89, normalized size = 0.8

$$-\frac{2 \text{polylog}(2, ax)}{5d} (dx)^{-\frac{5}{2}} + \frac{4}{25d} \ln\left(\frac{-adx+d}{d}\right) (dx)^{-\frac{5}{2}} - \frac{8a}{75d^2} (dx)^{-\frac{3}{2}} - \frac{8a^2}{25d^3} \frac{1}{\sqrt{dx}} + \frac{8a^3}{25d^3} \text{Artanh}\left(a\sqrt{dx} \frac{1}{\sqrt{ad}}\right) \frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, a\*x)/(d\*x)^(7/2), x)

[Out] -2/5\*polylog(2, a\*x)/d/(d\*x)^(5/2)+4/25/d/(d\*x)^(5/2)\*ln((-a\*d\*x+d)/d)-8/75\*a/d^2/(d\*x)^(3/2)-8/25\*a^2/d^3/(d\*x)^(1/2)+8/25/d^3\*a^3/(a\*d)^(1/2)\*arctanh(a\*(d\*x)^(1/2)/(a\*d)^(1/2))

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.59325, size = 412, normalized size = 3.89

$$\left[ \frac{2 \left( 6 a^2 d x^3 \sqrt{\frac{a}{d}} \log \left( \frac{a x + 2 \sqrt{d x} \sqrt{\frac{a}{d}} + 1}{a x - 1} \right) - (12 a^2 x^2 + 4 a x + 15 \operatorname{Li}_2(a x) - 6 \log(-a x + 1)) \sqrt{d x} \right)}{75 d^4 x^3}, - \frac{2 \left( 12 a^2 d x^3 \sqrt{-\frac{a}{d}} \arctan \left( \frac{\sqrt{d x}}{\sqrt{-a x + 1}} \right) + (12 a^2 x^2 + 4 a x + 15 \operatorname{dilog}(a x) - 6 \log(-a x + 1)) \sqrt{d x} \right)}{75 d^4 x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(7/2),x, algorithm="fricas")

[Out] [2/75\*(6\*a^2\*d\*x^3\*sqrt(a/d)\*log((a\*x + 2\*sqrt(d\*x)\*sqrt(a/d) + 1)/(a\*x - 1)) - (12\*a^2\*x^2 + 4\*a\*x + 15\*dilog(a\*x) - 6\*log(-a\*x + 1))\*sqrt(d\*x))/(d^4\*x^3), -2/75\*(12\*a^2\*d\*x^3\*sqrt(-a/d)\*arctan(sqrt(d\*x)\*sqrt(-a/d)/(a\*x)) + (12\*a^2\*x^2 + 4\*a\*x + 15\*dilog(a\*x) - 6\*log(-a\*x + 1))\*sqrt(d\*x))/(d^4\*x^3)]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)\*\*(7/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax)}{(dx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x)/(d\*x)^(7/2),x, algorithm="giac")

[Out] integrate(dilog(a\*x)/(d\*x)^(7/2), x)

### 3.65 $\int (dx)^{5/2} \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=153

$$\frac{4(dx)^{7/2} \text{PolyLog}(2, ax)}{49d} + \frac{2(dx)^{7/2} \text{PolyLog}(3, ax)}{7d} + \frac{16d^2 \sqrt{dx}}{343a^3} - \frac{16d^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a}$$

[Out]  $(16*d^2*\text{Sqrt}[d*x])/(343*a^3) + (16*d*(d*x)^{(3/2)})/(1029*a^2) + (16*(d*x)^{(5/2)})/(1715*a) + (16*(d*x)^{(7/2)})/(2401*d) - (16*d^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(343*a^{(7/2)}) - (8*(d*x)^{(7/2)}*\text{Log}[1 - a*x])/(343*d) - (4*(d*x)^{(7/2)}*\text{PolyLog}[2, a*x])/(49*d) + (2*(d*x)^{(7/2)}*\text{PolyLog}[3, a*x])/(7*d)$

**Rubi [A]** time = 0.0982584, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{4(dx)^{7/2} \text{PolyLog}(2, ax)}{49d} + \frac{2(dx)^{7/2} \text{PolyLog}(3, ax)}{7d} + \frac{16d^2 \sqrt{dx}}{343a^3} - \frac{16d^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(5/2)}*\text{PolyLog}[3, a*x], x]$

[Out]  $(16*d^2*\text{Sqrt}[d*x])/(343*a^3) + (16*d*(d*x)^{(3/2)})/(1029*a^2) + (16*(d*x)^{(5/2)})/(1715*a) + (16*(d*x)^{(7/2)})/(2401*d) - (16*d^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(343*a^{(7/2)}) - (8*(d*x)^{(7/2)}*\text{Log}[1 - a*x])/(343*d) - (4*(d*x)^{(7/2)}*\text{PolyLog}[2, a*x])/(49*d) + (2*(d*x)^{(7/2)}*\text{PolyLog}[3, a*x])/(7*d)$

#### Rule 6591

$\text{Int}[(d_.)*(x_.))^{(m_.)*\text{PolyLog}[n_, (a_.)*((b_.)*(x_.))^{(p_.)}]^{(q_.)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]^{(q_.)}]^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])]/(g*(q+1)), x] - \text{Dist}[(b*e^n)/(g*(q+1)), \text{Int}[(f + g*x)^{(q+1)}/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 50

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}], x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*(c + d*x)^n/(b*(m+n+1)), x] + \text{Dist}[(n*(b*c - a*d))/(b*(m+n+1)), \text{Int}[(a + b*x)^m*(c + d*x)^{(n-1)}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int (dx)^{5/2} \text{Li}_3(ax) dx &= \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} - \frac{2}{7} \int (dx)^{5/2} \text{Li}_2(ax) dx \\
&= -\frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} - \frac{4}{49} \int (dx)^{5/2} \log(1-ax) dx \\
&= -\frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} - \frac{(8a) \int \frac{(dx)^{7/2} dx}{1-ax}}{343d} \\
&= \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} - \frac{8}{343} \int \frac{(dx)^{5/2}}{1-ax} dx \\
&= \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} - \frac{(8d) \int \frac{(dx)^{3/2}}{1-ax} dx}{343a} \\
&= \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} \\
&= \frac{16d^2 \sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} \\
&= \frac{16d^2 \sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2} \log(1-ax)}{343d} - \frac{4(dx)^{7/2} \text{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax)}{7d} \\
&= \frac{16d^2 \sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{16d^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} - \frac{8(dx)^{7/2} \log(1-ax)}{343d}
\end{aligned}$$

**Mathematica [A]** time = 0.260023, size = 98, normalized size = 0.64

$$\frac{2(dx)^{5/2} \left( -1470x^3 \text{PolyLog}(2, ax) + 5145x^3 \text{PolyLog}(3, ax) + \frac{8(15a^3x^3 + 21a^2x^2 + 35ax + 105)}{a^3} - \frac{840 \tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{7/2}\sqrt{x}} - 420x^3 \log(1-ax) \right)}{36015x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*PolyLog[3, a*x], x]
```

```
[Out] (2*(d*x)^(5/2)*((8*(105 + 35*a*x + 21*a^2*x^2 + 15*a^3*x^3))/a^3 - (840*Arc
Tanh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*Sqrt[x]) - 420*x^3*Log[1 - a*x] - 1470*x^3*
PolyLog[2, a*x] + 5145*x^3*PolyLog[3, a*x]))/(36015*x^2)
```

**Maple [A]** time = 0.182, size = 149, normalized size = 1.

$$\frac{1}{a} (dx)^{5/2} \left( \frac{720x^3a^3 + 1008a^2x^2 + 1680ax + 5040}{108045a^4} \sqrt{x}(-a)^{9/2} + \frac{8}{343a^4} \sqrt{x}(-a)^{9/2} (\ln(1-\sqrt{ax}) - \ln(1+\sqrt{ax})) \frac{1}{\sqrt{ax}} - \frac{8 \ln}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3,a*x),x)
```

```
[Out] (d*x)^(5/2)/x^(5/2)/(-a)^(5/2)/a*(2/108045*x^(1/2)*(-a)^(9/2)*(360*a^3*x^3+
504*a^2*x^2+840*a*x+2520)/a^4+8/343*x^(1/2)*(-a)^(9/2)/a^4/(a*x)^(1/2)*(ln(
1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8/343*x^(7/2)*(-a)^(9/2)/a*ln(-a*x+1)-4/4
9*x^(7/2)*(-a)^(9/2)*polylog(2,a*x)/a+2/7*x^(7/2)*(-a)^(9/2)/a*polylog(3,a*
x))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 2.97483, size = 822, normalized size = 5.37

$$\frac{2 \left( 1470 \sqrt{d} x a^3 d^2 x^3 \operatorname{iiint} \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - 5145 \sqrt{d} x a^3 d^2 x^3 \operatorname{polylog}(3, ax) - 420 d^2 \sqrt{\frac{d}{a}} \log \left( \frac{adx-2\sqrt{d}x}{ax-d} \right) \right)}{36015 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [-2/36015*(1470*sqrt(d*x)*a^3*d^2*x^3*%iint(a, x, -log(-a*x + 1)/a, -log(-a
*x + 1)/x) - 5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) - 420*d^2*sqrt(d/a)
*log((a*d*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) + 4*(105*a^3*d^2*x^3*
log(-a*x + 1) - 30*a^3*d^2*x^3 - 42*a^2*d^2*x^2 - 70*a*d^2*x - 210*d^2)*sq
rt(d*x))/a^3, -2/36015*(1470*sqrt(d*x)*a^3*d^2*x^3*%iint(a, x, -log(-a*x + 1
)/a, -log(-a*x + 1)/x) - 5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) - 840*d
^2*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d) + 4*(105*a^3*d^2*x^3*log(-a*
x + 1) - 30*a^3*d^2*x^3 - 42*a^2*d^2*x^2 - 70*a*d^2*x - 210*d^2)*sqrt(d*x)
/a^3]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*polylog(3,a*x),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \text{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*polylog(3, a*x), x)
```



### 3.66 $\int (dx)^{3/2} \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=136

$$\frac{4(dx)^{5/2} \text{PolyLog}(2, ax)}{25d} + \frac{2(dx)^{5/2} \text{PolyLog}(3, ax)}{5d} - \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}} + \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} - \frac{8(dx)^{5/2} \log(1 - ax)}{125d}$$

```
[Out] (16*d*Sqrt[d*x])/(125*a^2) + (16*(d*x)^(3/2))/(375*a) + (16*(d*x)^(5/2))/(6
25*d) - (16*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]]/(125*a^(5/2))) - (
8*(d*x)^(5/2)*Log[1 - a*x])/(125*d) - (4*(d*x)^(5/2)*PolyLog[2, a*x])/(25*d
) + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

**Rubi [A]** time = 0.0834981, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{4(dx)^{5/2} \text{PolyLog}(2, ax)}{25d} + \frac{2(dx)^{5/2} \text{PolyLog}(3, ax)}{5d} - \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}} + \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} - \frac{8(dx)^{5/2} \log(1 - ax)}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x], x]
```

```
[Out] (16*d*Sqrt[d*x])/(125*a^2) + (16*(d*x)^(3/2))/(375*a) + (16*(d*x)^(5/2))/(6
25*d) - (16*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]]/(125*a^(5/2))) - (
8*(d*x)^(5/2)*Log[1 - a*x])/(125*d) - (4*(d*x)^(5/2)*PolyLog[2, a*x])/(25*d
) + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 206

$\text{Int}[(a_ + (b_ )*(x_ )^2)^{-1}, x\_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int (dx)^{3/2} \text{Li}_3(ax) dx &= \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{2}{5} \int (dx)^{3/2} \text{Li}_2(ax) dx \\ &= -\frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{4}{25} \int (dx)^{3/2} \log(1-ax) dx \\ &= -\frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{(8a) \int \frac{(dx)^{5/2}}{1-ax} dx}{125d} \\ &= \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{8}{125} \int \frac{(dx)^{3/2}}{1-ax} dx \\ &= \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{(8d) \int \frac{\sqrt{dx}}{1-ax} dx}{125a} \\ &= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{(8d) \int \frac{\sqrt{dx}}{1-ax} dx}{125a} \\ &= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax)}{5d} - \frac{(8d) \int \frac{\sqrt{dx}}{1-ax} dx}{125a} \\ &= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{a}}\right)}{125a^{5/2}} - \frac{8(dx)^{5/2} \log(1-ax)}{125d} - \frac{4(dx)^{5/2} \text{Li}_2(ax)}{25d} \end{aligned}$$

**Mathematica [A]** time = 0.217824, size = 88, normalized size = 0.65

$$\frac{2d\sqrt{dx} \left( -150x^2 \text{PolyLog}(2, ax) + 375x^2 \text{PolyLog}(3, ax) + 4 \left( -\frac{30 \tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{5/2}\sqrt{x}} + \frac{30}{a^2} - 15x^2 \log(1-ax) + \frac{10x}{a} + 6x^2 \right) \right)}{1875}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*PolyLog[3, a\*x], x]

[Out] (2\*d\*Sqrt[d\*x]\*(4\*(30/a^2 + (10\*x)/a + 6\*x^2 - (30\*ArcTanh[Sqrt[a]\*Sqrt[x]])/(a^(5/2)\*Sqrt[x]) - 15\*x^2\*Log[1 - a\*x]) - 150\*x^2\*PolyLog[2, a\*x] + 375\*x^2\*PolyLog[3, a\*x])/1875

**Maple [A]** time = 0.061, size = 141, normalized size = 1.

$$\frac{1}{a} (dx)^{\frac{3}{2}} \left( \frac{336 a^2 x^2 + 560 a x + 1680}{13125 a^3} \sqrt{x} (-a)^{\frac{7}{2}} + \frac{8}{125 a^3} \sqrt{x} (-a)^{\frac{7}{2}} (\ln(1 - \sqrt{ax}) - \ln(1 + \sqrt{ax})) \frac{1}{\sqrt{ax}} - \frac{8 \ln(-ax + 1)}{125 a} x^{\frac{5}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*polylog(3,a*x),x)`

[Out]  $(d*x)^{(3/2)}/x^{(3/2)}/(-a)^{(3/2)}/a*(2/13125*x^{(1/2)}*(-a)^{(7/2)}*(168*a^2*x^2+280*a*x+840)/a^3+8/125*x^{(1/2)}*(-a)^{(7/2)}/a^3/(a*x)^{(1/2)}*(\ln(1-(a*x)^{(1/2)})-\ln(1+(a*x)^{(1/2)}))-8/125*x^{(5/2)}*(-a)^{(7/2)}/a*\ln(-a*x+1)-4/25*x^{(5/2)}*(-a)^{(7/2)}*polylog(2,a*x)/a+2/5*x^{(5/2)}*(-a)^{(7/2)}/a*polylog(3,a*x)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.87358, size = 721, normalized size = 5.3

$$\frac{2 \left( 150 \sqrt{d} x a^2 dx^2 \int \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - 375 \sqrt{d} x a^2 dx^2 \operatorname{polylog}(3, ax) - 60 d \sqrt{\frac{d}{a}} \log \left( \frac{adx-2\sqrt{d}x\sqrt{\frac{d}{a}}+d}{ax-1} \right) \right)}{1875 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="fricas")`

[Out]  $[-2/1875*(150*\sqrt{d*x}*a^2*d*x^2*\int(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) - 375*\sqrt{d*x}*a^2*d*x^2*\operatorname{polylog}(3, a*x) - 60*d*\sqrt{d/a}*\log((a*d*x - 2*\sqrt{d*x}*a*\sqrt{d/a} + d)/(a*x - 1)) + 4*(15*a^2*d*x^2*\log(-a*x + 1) - 6*a^2*d*x^2 - 10*a*d*x - 30*d)*\sqrt{d*x})/a^2, -2/1875*(150*\sqrt{d*x}*a^2*d*x^2*\int(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) - 375*\sqrt{d*x}*a^2*d*x^2*\operatorname{polylog}(3, a*x) - 120*d*\sqrt{-d/a}*\arctan(\sqrt{d*x}*a*\sqrt{-d/a}/d) + 4*(15*a^2*d*x^2*\log(-a*x + 1) - 6*a^2*d*x^2 - 10*a*d*x - 30*d)*\sqrt{d*x})/a^2]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(3/2)*polylog(3,a*x),x)`

[Out] `Integral((d*x)**(3/2)*polylog(3, a*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \text{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x), x)
```

### 3.67 $\int \sqrt{dx} \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=121

$$\frac{4(dx)^{3/2} \text{PolyLog}(2, ax)}{9d} + \frac{2(dx)^{3/2} \text{PolyLog}(3, ax)}{3d} - \frac{16\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}} + \frac{16\sqrt{dx}}{27a} - \frac{8(dx)^{3/2} \log(1 - ax)}{27d} + \frac{16(dx)^{3/2}}{81d}$$

```
[Out] (16*sqrt[d*x])/(27*a) + (16*(d*x)^(3/2))/(81*d) - (16*sqrt[d]*ArcTanh[(sqrt[a]*sqrt[d*x])/sqrt[d]])/(27*a^(3/2)) - (8*(d*x)^(3/2)*Log[1 - a*x])/(27*d) - (4*(d*x)^(3/2)*PolyLog[2, a*x])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x])/(3*d)
```

**Rubi [A]** time = 0.066747, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$\frac{4(dx)^{3/2} \text{PolyLog}(2, ax)}{9d} + \frac{2(dx)^{3/2} \text{PolyLog}(3, ax)}{3d} - \frac{16\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}} + \frac{16\sqrt{dx}}{27a} - \frac{8(dx)^{3/2} \log(1 - ax)}{27d} + \frac{16(dx)^{3/2}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x], x]
```

```
[Out] (16*sqrt[d*x])/(27*a) + (16*(d*x)^(3/2))/(81*d) - (16*sqrt[d]*ArcTanh[(sqrt[a]*sqrt[d*x])/sqrt[d]])/(27*a^(3/2)) - (8*(d*x)^(3/2)*Log[1 - a*x])/(27*d) - (4*(d*x)^(3/2)*PolyLog[2, a*x])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x])/(3*d)
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 206

$\text{Int}[\{(a\_)+ (b\_)*(x\_)^2\}^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \sqrt{dx} \text{Li}_3(ax) dx &= \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{2}{3} \int \sqrt{dx} \text{Li}_2(ax) dx \\ &= -\frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{4}{9} \int \sqrt{dx} \log(1-ax) dx \\ &= -\frac{8(dx)^{3/2} \log(1-ax)}{27d} - \frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{(8a) \int \frac{(dx)^{3/2}}{1-ax} dx}{27d} \\ &= \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2} \log(1-ax)}{27d} - \frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{8}{27} \int \frac{\sqrt{dx}}{1-ax} dx \\ &= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2} \log(1-ax)}{27d} - \frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{(8d) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{27a} \\ &= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2} \log(1-ax)}{27d} - \frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} - \frac{16 \text{Subst} \left( \int \frac{1}{1-\frac{ax^2}{d}} \right)}{27a} \\ &= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{16\sqrt{d} \tanh^{-1} \left( \frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}} \right)}{27a^{3/2}} - \frac{8(dx)^{3/2} \log(1-ax)}{27d} - \frac{4(dx)^{3/2} \text{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2} \text{Li}_3(ax)}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.188443, size = 73, normalized size = 0.6

$$\frac{2}{81} \sqrt{dx} \left( -18x \text{PolyLog}(2, ax) + 27x \text{PolyLog}(3, ax) + 4 \left( -\frac{6 \tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{3/2}\sqrt{x}} - 3x \log(1-ax) + \frac{6}{a} + 2x \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*PolyLog[3, a\*x], x]

[Out] (2\*Sqrt[d\*x]\*(4\*(6/a + 2\*x - (6\*ArcTanh[Sqrt[a]\*Sqrt[x]]))/(a^(3/2)\*Sqrt[x]) - 3\*x\*Log[1 - a\*x]) - 18\*x\*PolyLog[2, a\*x] + 27\*x\*PolyLog[3, a\*x])/81

**Maple [A]** time = 0.058, size = 133, normalized size = 1.1

$$\frac{1}{a} \sqrt{dx} \left( \frac{80ax + 240}{405a^2} \sqrt{x} (-a)^{5/2} + \frac{8}{27a^2} \sqrt{x} (-a)^{5/2} (\ln(1 - \sqrt{ax}) - \ln(1 + \sqrt{ax})) \frac{1}{\sqrt{ax}} - \frac{8 \ln(-ax + 1)}{27a} x^2 (-a)^{5/2} - \frac{4 \text{polylog}}{9a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*polylog(3,a\*x), x)

[Out] (d\*x)^(1/2)/x^(1/2)/(-a)^(1/2)/a\*(2/405\*x^(1/2)\*(-a)^(5/2)\*(40\*a\*x+120)/a^2 + 8/27\*x^(1/2)\*(-a)^(5/2)/a^2/(a\*x)^(1/2)\*(ln(1-(a\*x)^(1/2))-ln(1+(a\*x)^(1/2)))

))) - 8/27\*x^(3/2)\*(-a)^(5/2)/a\*ln(-a\*x+1) - 4/9\*x^(3/2)\*(-a)^(5/2)\*polylog(2,a\*x)/a + 2/3\*x^(3/2)\*(-a)^(5/2)/a\*polylog(3,a\*x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.8478, size = 593, normalized size = 4.9

$$\frac{2 \left( 18 \sqrt{dx} ax \operatorname{Li}_3 \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - 27 \sqrt{dx} ax \operatorname{polylog}(3, ax) + 4(3ax \log(-ax+1) - 2ax - 6) \sqrt{dx} \right)}{81a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x),x, algorithm="fricas")

[Out] [-2/81\*(18\*sqrt(d\*x)\*a\*x\*\%iint(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 27\*sqrt(d\*x)\*a\*x\*polylog(3, a\*x) + 4\*(3\*a\*x\*log(-a\*x + 1) - 2\*a\*x - 6)\*sqrt(d\*x) - 12\*sqrt(d/a)\*log((a\*d\*x - 2\*sqrt(d\*x)\*a\*sqrt(d/a) + d)/(a\*x - 1)))/a, -2/81\*(18\*sqrt(d\*x)\*a\*x\*\%iint(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 27\*sqrt(d\*x)\*a\*x\*polylog(3, a\*x) + 4\*(3\*a\*x\*log(-a\*x + 1) - 2\*a\*x - 6)\*sqrt(d\*x) - 24\*sqrt(-d/a)\*arctan(sqrt(d\*x)\*a\*sqrt(-d/a)/d))/a]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*polylog(3,a\*x),x)

[Out] Integral(sqrt(d\*x)\*polylog(3, a\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x),x, algorithm="giac")

[Out] integrate(sqrt(d\*x)\*polylog(3, a\*x), x)

### 3.68 $\int \frac{\text{PolyLog}(3, ax)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=97

$$-\frac{4\sqrt{dx}\text{PolyLog}(2, ax)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax)}{d} - \frac{8\sqrt{dx}\log(1 - ax)}{d} - \frac{16 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} + \frac{16\sqrt{dx}}{d}$$

[Out] (16\*Sqrt[d\*x])/d - (16\*ArcTanh[(Sqrt[a]\*Sqrt[d\*x])/Sqrt[d]]/(Sqrt[a]\*Sqrt[d])) - (8\*Sqrt[d\*x]\*Log[1 - a\*x])/d - (4\*Sqrt[d\*x]\*PolyLog[2, a\*x])/d + (2\*Sqrt[d\*x]\*PolyLog[3, a\*x])/d

**Rubi [A]** time = 0.0621491, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 50, 63, 206}

$$-\frac{4\sqrt{dx}\text{PolyLog}(2, ax)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax)}{d} - \frac{8\sqrt{dx}\log(1 - ax)}{d} - \frac{16 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} + \frac{16\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/Sqrt[d\*x], x]

[Out] (16\*Sqrt[d\*x])/d - (16\*ArcTanh[(Sqrt[a]\*Sqrt[d\*x])/Sqrt[d]]/(Sqrt[a]\*Sqrt[d])) - (8\*Sqrt[d\*x]\*Log[1 - a\*x])/d - (4\*Sqrt[d\*x]\*PolyLog[2, a\*x])/d + (2\*Sqrt[d\*x]\*PolyLog[3, a\*x])/d

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +



$(d*x^p/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 206

$\text{Int}[(a_ + (b_ )*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_3(ax)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 2 \int \frac{\text{Li}_2(ax)}{\sqrt{dx}} dx \\ &= -\frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 4 \int \frac{\log(1-ax)}{\sqrt{dx}} dx \\ &= -\frac{8\sqrt{dx} \log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - \frac{(8a) \int \frac{\sqrt{dx}}{1-ax} dx}{d} \\ &= \frac{16\sqrt{dx}}{d} - \frac{8\sqrt{dx} \log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 8 \int \frac{1}{\sqrt{dx}(1-ax)} dx \\ &= \frac{16\sqrt{dx}}{d} - \frac{8\sqrt{dx} \log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - \frac{16 \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{d} \\ &= \frac{16\sqrt{dx}}{d} - \frac{16 \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} - \frac{8\sqrt{dx} \log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.138969, size = 57, normalized size = 0.59

$$\frac{2x \left( -2\text{PolyLog}(2, ax) + \text{PolyLog}(3, ax) - 4 \log(1-ax) - \frac{8 \tanh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}\sqrt{x}} + 8 \right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x]/Sqrt[d\*x], x]

[Out]  $(2*x*(8 - (8*\text{ArcTanh}[\text{Sqrt}[a]*\text{Sqrt}[x]]))/(\text{Sqrt}[a]*\text{Sqrt}[x]) - 4*\text{Log}[1 - a*x] - 2*\text{PolyLog}[2, a*x] + \text{PolyLog}[3, a*x])/ \text{Sqrt}[d*x]$

**Maple [A]** time = 0.057, size = 127, normalized size = 1.3

$$\frac{1}{a} \sqrt{x} \sqrt{-a} \left( 16 \frac{\sqrt{x} (-a)^{3/2}}{a} + 8 \frac{\sqrt{x} (-a)^{3/2} (\ln(1 - \sqrt{ax}) - \ln(1 + \sqrt{ax}))}{a \sqrt{ax}} - 8 \frac{\sqrt{x} (-a)^{3/2} \ln(-ax + 1)}{a} - 4 \frac{\sqrt{x} (-a)^{3/2} \text{poly}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x)/(d\*x)^(1/2), x)

[Out]  $1/(d*x)^{(1/2)}*x^{(1/2)}*(-a)^{(1/2)}/a*(16*x^{(1/2)}*(-a)^{(3/2)}/a+8*x^{(1/2)}*(-a)^{(3/2)}/a/(a*x)^{(1/2)}*(\ln(1-(a*x)^{(1/2)})-\ln(1+(a*x)^{(1/2)}))-8*x^{(1/2)}*(-a)^{(3/2)}/a$

$/2)/a*\ln(-a*x+1)-4*x^{(1/2)}*(-a)^{(3/2)}*polylog(2,a*x)/a+2*x^{(1/2)}*(-a)^{(3/2)}/a*polylog(3,a*x))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.72649, size = 547, normalized size = 5.64

$$\left[ \frac{2 \left( 2 \sqrt{dx} \operatorname{Li}_3 \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) - \sqrt{dx} \operatorname{polylog}(3, ax) + 4 \sqrt{dx} (a \log(-ax+1) - 2a) - 4 \sqrt{ad} \log \left( \frac{adx-2}{ad} \right) \right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(1/2),x, algorithm="fricas")

[Out]  $[-2*(2*\sqrt{d*x}*a*\operatorname{Li}_3(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) - \sqrt{d*x}*a*\operatorname{polylog}(3, a*x) + 4*\sqrt{d*x}*(a*\log(-a*x + 1) - 2*a) - 4*\sqrt{a*d}*\log((a*d*x - 2*\sqrt{a*d}*\sqrt{d*x} + d)/(a*x - 1)))/(a*d), -2*(2*\sqrt{d*x}*a*\operatorname{Li}_3(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) - \sqrt{d*x}*a*\operatorname{polylog}(3, a*x) + 4*\sqrt{d*x}*(a*\log(-a*x + 1) - 2*a) - 8*\sqrt{-a*d}*\arctan(\sqrt{-a*d}*\sqrt{d*x}/(a*d*x)))/(a*d)]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)\*\*(1/2),x)

[Out] Integral(polylog(3, a\*x)/sqrt(d\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(1/2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x)/sqrt(d\*x), x)

### 3.69 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=85

$$-\frac{4\text{PolyLog}(2,ax)}{d\sqrt{dx}} - \frac{2\text{PolyLog}(3,ax)}{d\sqrt{dx}} + \frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax)}{d\sqrt{dx}}$$

[Out] (16\*sqrt[a]\*ArcTanh[(sqrt[a]\*sqrt[d\*x])/sqrt[d]])/d^(3/2) + (8\*Log[1 - a\*x])/(d\*sqrt[d\*x]) - (4\*PolyLog[2, a\*x])/(d\*sqrt[d\*x]) - (2\*PolyLog[3, a\*x])/(d\*sqrt[d\*x])

**Rubi [A]** time = 0.0588706, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2395, 63, 206}

$$-\frac{4\text{PolyLog}(2,ax)}{d\sqrt{dx}} - \frac{2\text{PolyLog}(3,ax)}{d\sqrt{dx}} + \frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/(d\*x)^(3/2), x]

[Out] (16\*sqrt[a]\*ArcTanh[(sqrt[a]\*sqrt[d\*x])/sqrt[d]])/d^(3/2) + (8\*Log[1 - a\*x])/(d\*sqrt[d\*x]) - (4\*PolyLog[2, a\*x])/(d\*sqrt[d\*x]) - (2\*PolyLog[3, a\*x])/(d\*sqrt[d\*x])

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + 2 \int \frac{\text{Li}_2(ax)}{(dx)^{3/2}} dx \\
&= -\frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} - 4 \int \frac{\log(1-ax)}{(dx)^{3/2}} dx \\
&= \frac{8 \log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(8a) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{d} \\
&= \frac{8 \log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(16a) \text{Subst} \left( \int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx} \right)}{d^2} \\
&= \frac{16\sqrt{a} \tanh^{-1} \left( \frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{8 \log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}}
\end{aligned}$$

**Mathematica [A]** time = 0.101918, size = 58, normalized size = 0.68

$$\frac{2x \left( -2\text{PolyLog}(2, ax) - \text{PolyLog}(3, ax) + 4 \log(1-ax) + 8\sqrt{a}\sqrt{x} \tanh^{-1}(\sqrt{a}\sqrt{x}) \right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x]/(d\*x)^(3/2), x]

[Out] (2\*x\*(8\*Sqrt[a]\*Sqrt[x]\*ArcTanh[Sqrt[a]\*Sqrt[x]] + 4\*Log[1 - a\*x] - 2\*PolyLog[2, a\*x] - PolyLog[3, a\*x]))/(d\*x)^(3/2)

**Maple [A]** time = 0.056, size = 111, normalized size = 1.3

$$\frac{1}{a} x^{\frac{3}{2}} (-a)^{\frac{3}{2}} \left( -8 \frac{\sqrt{x}\sqrt{-a} (\ln(1-\sqrt{ax}) - \ln(1+\sqrt{ax}))}{\sqrt{ax}} + 8 \frac{\sqrt{-a} \ln(-ax+1)}{\sqrt{xa}} - 4 \frac{\sqrt{-a} \text{polylog}(2, ax)}{\sqrt{xa}} - 2 \frac{\sqrt{-a} \text{polylog}(3, ax)}{\sqrt{xa}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x)/(d\*x)^(3/2), x)

[Out] 1/(d\*x)^(3/2)\*x^(3/2)\*(-a)^(3/2)/a\*(-8\*x^(1/2)\*(-a)^(1/2)/(a\*x)^(1/2)\*(ln(1-(a\*x)^(1/2))-ln(1+(a\*x)^(1/2)))+8/x^(1/2)\*(-a)^(1/2)/a\*ln(-a\*x+1)-4/x^(1/2)\*(-a)^(1/2)\*polylog(2,a\*x)/a-2/x^(1/2)\*(-a)^(1/2)/a\*polylog(3,a\*x))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.5892, size = 518, normalized size = 6.09

$$\frac{2 \left( 4 dx \sqrt{\frac{a}{d}} \log \left( \frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}+1}}{ax-1} \right) - 2 \sqrt{dx} \operatorname{Li}_3 \left( a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x} \right) + 4 \sqrt{dx} \log(-ax+1) - \sqrt{dx} \operatorname{polylog}(3, a, x) \right)}{d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(3/2),x, algorithm="fricas")

[Out] [2\*(4\*d\*x\*sqrt(a/d)\*log((a\*x + 2\*sqrt(d\*x)\*sqrt(a/d) + 1)/(a\*x - 1)) - 2\*sqrt(d\*x)\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) + 4\*sqrt(d\*x)\*log(-a\*x + 1) - sqrt(d\*x)\*polylog(3, a\*x))/(d^2\*x), -2\*(8\*d\*x\*sqrt(-a/d)\*arctan(sqrt(d\*x)\*sqrt(-a/d)/(a\*x)) + 2\*sqrt(d\*x)\*\operatorname{Li}\_3(a, x, -log(-a\*x + 1)/a, -log(-a\*x + 1)/x) - 4\*sqrt(d\*x)\*log(-a\*x + 1) + sqrt(d\*x)\*polylog(3, a\*x))/(d^2\*x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)\*\*(3/2),x)

[Out] Integral(polylog(3, a\*x)/(d\*x)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(3/2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x)/(d\*x)^(3/2), x)

### 3.70 $\int \frac{\text{PolyLog}(3, ax)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=108

$$-\frac{4\text{PolyLog}(2, ax)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax)}{3d(dx)^{3/2}} + \frac{16a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} - \frac{16a}{27d^2\sqrt{dx}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}}$$

[Out]  $(-16*a)/(27*d^2*\text{Sqrt}[d*x]) + (16*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(27*d^{(5/2)}) + (8*\text{Log}[1 - a*x])/(27*d*(d*x)^{(3/2)}) - (4*\text{PolyLog}[2, a*x])/(9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[3, a*x])/(3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.0655397, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{4\text{PolyLog}(2, ax)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax)}{3d(dx)^{3/2}} + \frac{16a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} - \frac{16a}{27d^2\sqrt{dx}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x]/(d\*x)^(5/2), x]

[Out]  $(-16*a)/(27*d^2*\text{Sqrt}[d*x]) + (16*a^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(27*d^{(5/2)}) + (8*\text{Log}[1 - a*x])/(27*d*(d*x)^{(3/2)}) - (4*\text{PolyLog}[2, a*x])/(9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[3, a*x])/(3*d*(d*x)^{(3/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[((a + b\*x)^(m+1)\*(c + d\*x)^(n+1))/((b\*c - a\*d)\*(m+1)), x] - Dist[(d\*(m+n+2))/((b\*c - a\*d)\*(m+1)), Int[(a + b\*x)^(m+1)\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m+1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ

[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_3(ax)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{2}{3} \int \frac{\text{Li}_2(ax)}{(dx)^{5/2}} dx \\
 &= -\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} - \frac{4}{9} \int \frac{\log(1-ax)}{(dx)^{5/2}} dx \\
 &= \frac{8 \log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{(8a) \int \frac{1}{(dx)^{3/2}(1-ax)} dx}{27d} \\
 &= -\frac{16a}{27d^2\sqrt{dx}} + \frac{8 \log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{(8a^2) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{27d^2} \\
 &= -\frac{16a}{27d^2\sqrt{dx}} + \frac{8 \log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{(16a^2) \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x, \sqrt{dx}\right)}{27d^3} \\
 &= -\frac{16a}{27d^2\sqrt{dx}} + \frac{16a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{8 \log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0961699, size = 64, normalized size = 0.59

$$\frac{2x \left( 6\text{PolyLog}(2, ax) + 9\text{PolyLog}(3, ax) - 8a^{3/2}x^{3/2} \tanh^{-1}(\sqrt{a}\sqrt{x}) + 8ax - 4 \log(1-ax) \right)}{27(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, a\*x]/(d\*x)^(5/2), x]

[Out] (-2\*x\*(8\*a\*x - 8\*a^(3/2)\*x^(3/2)\*ArcTanh[Sqrt[a]\*Sqrt[x]] - 4\*Log[1 - a\*x] + 6\*PolyLog[2, a\*x] + 9\*PolyLog[3, a\*x])/(27\*(d\*x)^(5/2))

**Maple [A]** time = 0.06, size = 122, normalized size = 1.1

$$\frac{1}{a} x^{\frac{5}{2}} (-a)^{\frac{5}{2}} \left( -\frac{16}{27} \frac{1}{\sqrt{x}} \frac{1}{\sqrt{-a}} - \frac{8a}{27} \sqrt{x} (\ln(1-\sqrt{ax}) - \ln(1+\sqrt{ax})) \right) \frac{1}{\sqrt{-a}} \frac{1}{\sqrt{ax}} + \frac{8 \ln(-ax+1)}{27a} x^{-\frac{3}{2}} \frac{1}{\sqrt{-a}} - \frac{4 \text{polylog}(2, -ax)}{9a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x)/(d\*x)^(5/2), x)

[Out] 1/(d\*x)^(5/2)\*x^(5/2)\*(-a)^(5/2)/a\*(-16/27/x^(1/2)/(-a)^(1/2)-8/27\*x^(1/2)/(-a)^(1/2)\*a/(a\*x)^(1/2)\*(ln(1-(a\*x)^(1/2))-ln(1+(a\*x)^(1/2)))+8/27/x^(3/2)/(-a)^(1/2)/a\*ln(-a\*x+1)-4/9/x^(3/2)/(-a)^(1/2)\*polylog(2, a\*x)/a-2/3/x^(3/2)

$\text{polylog}(3, a*x)/(-a)^{(1/2)}/a$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.78928, size = 575, normalized size = 5.32

$$\frac{2 \left( 4 a d x^2 \sqrt{\frac{a}{d}} \log\left(\frac{a x + 2 \sqrt{d x} \sqrt{\frac{a}{d}} + 1}{a x - 1}\right) - 4 (2 a x - \log(-a x + 1)) \sqrt{d x} - 6 \sqrt{d x} \operatorname{Li}_2\left(a, x, -\frac{\log(-a x + 1)}{a}, -\frac{\log(-a x + 1)}{x}\right) - 9 \sqrt{d x} \operatorname{polylog}(3, a x) \right)}{27 d^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(5/2),x, algorithm="fricas")

[Out]  $\left[ \frac{2}{27} (4 a d x^2 \sqrt{a/d} \log((a x + 2 \sqrt{d x} \sqrt{a/d} + 1)/(a x - 1)) - 4 (2 a x - \log(-a x + 1)) \sqrt{d x} - 6 \sqrt{d x} \operatorname{Li}_2(a, x, -\log(-a x + 1)/a, -\log(-a x + 1)/x) - 9 \sqrt{d x} \operatorname{polylog}(3, a x) / (d^3 x^2), -2/27 (8 a d x^2 \sqrt{-a/d} \arctan(\sqrt{d x} \sqrt{-a/d} / (a x)) + 4 (2 a x - \log(-a x + 1)) \sqrt{d x} + 6 \sqrt{d x} \operatorname{Li}_2(a, x, -\log(-a x + 1)/a, -\log(-a x + 1)/x) + 9 \sqrt{d x} \operatorname{polylog}(3, a x) / (d^3 x^2)) \right]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)\*\*(5/2),x)

[Out] Integral(polylog(3, a\*x)/(d\*x)\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(5/2), x)
```

### 3.71 $\int \frac{\text{PolyLog}(3, ax)}{(dx)^{7/2}} dx$

**Optimal.** Leaf size=125

$$-\frac{4\text{PolyLog}(2, ax)}{25d(dx)^{5/2}} - \frac{2\text{PolyLog}(3, ax)}{5d(dx)^{5/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} - \frac{16a}{375d^2(dx)^{3/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}}$$

[Out]  $(-16*a)/(375*d^2*(d*x)^{(3/2)}) - (16*a^2)/(125*d^3*\text{Sqrt}[d*x]) + (16*a^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (8*\text{Log}[1 - a*x])/(125*d*(d*x)^{(5/2)}) - (4*\text{PolyLog}[2, a*x])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[3, a*x])/(5*d*(d*x)^{(5/2)})$

**Rubi [A]** time = 0.0763217, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{4\text{PolyLog}(2, ax)}{25d(dx)^{5/2}} - \frac{2\text{PolyLog}(3, ax)}{5d(dx)^{5/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} - \frac{16a}{375d^2(dx)^{3/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[3, a*x]/(d*x)^{(7/2)}, x]$

[Out]  $(-16*a)/(375*d^2*(d*x)^{(3/2)}) - (16*a^2)/(125*d^3*\text{Sqrt}[d*x]) + (16*a^{(5/2)}*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (8*\text{Log}[1 - a*x])/(125*d*(d*x)^{(5/2)}) - (4*\text{PolyLog}[2, a*x])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[3, a*x])/(5*d*(d*x)^{(5/2)})$

#### Rule 6591

$\text{Int}[(d*x)^m * \text{PolyLog}[n, a*(b*x^p)^q], x] \rightarrow \text{Simp}[(d*x)^{m+1} * \text{PolyLog}[n, a*(b*x^p)^q] / (d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m * \text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

$\text{Int}[(a + \text{Log}[c*(d + e*x)^n]) * (f + g*x)^{q+1}, x] \rightarrow \text{Simp}[(f + g*x)^{q+1} * (a + b*\text{Log}[c*(d + e*x)^n]) / (g*(q+1)), x] - \text{Dist}[(b*e*n)/(g*(q+1)), \text{Int}[(f + g*x)^{q+1} / (d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 51

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x] \rightarrow \text{Simp}[(a + b*x)^{m+1} * (c + d*x)^{n+1} / ((b*c - a*d)*(m+1)), x] - \text{Dist}[(d*(m+n+2)) / ((b*c - a*d)*(m+1)), \text{Int}[(a + b*x)^{m+1} * (c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{(dx)^{7/2}} dx &= -\frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{2}{5} \int \frac{\text{Li}_2(ax)}{(dx)^{7/2}} dx \\
&= -\frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} - \frac{4}{25} \int \frac{\log(1-ax)}{(dx)^{7/2}} dx \\
&= \frac{8 \log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a) \int \frac{1}{(dx)^{5/2}(1-ax)} dx}{125d} \\
&= -\frac{16a}{375d^2(dx)^{3/2}} + \frac{8 \log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^2) \int \frac{1}{(dx)^{3/2}(1-ax)} dx}{125d^2} \\
&= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8 \log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^3) \int \frac{1}{\sqrt{dx}(1-ax)} dx}{125d^3} \\
&= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8 \log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(16a^3) \text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} dx, x\right)}{125d^4} \\
&= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{8 \log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.13028, size = 72, normalized size = 0.58

$$\frac{2x \left( 30 \text{PolyLog}(2, ax) + 75 \text{PolyLog}(3, ax) + 24a^2x^2 - 24a^{5/2}x^{5/2} \tanh^{-1}(\sqrt{a}\sqrt{x}) + 8ax - 12 \log(1-ax) \right)}{375(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(7/2), x]
```

```
[Out] (-2*x*(8*a*x + 24*a^2*x^2 - 24*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 1
2*Log[1 - a*x] + 30*PolyLog[2, a*x] + 75*PolyLog[3, a*x])/(375*(d*x)^(7/2)
)
```

**Maple [A]** time = 0.062, size = 135, normalized size = 1.1

$$\frac{1}{a} x^{\frac{7}{2}} (-a)^{\frac{7}{2}} \left( -\frac{16}{375} x^{-\frac{3}{2}} (-a)^{-\frac{3}{2}} - \frac{16a}{125} \frac{1}{\sqrt{x}} (-a)^{-\frac{3}{2}} - \frac{8a^2}{125} \sqrt{x} (\ln(1-\sqrt{ax}) - \ln(1+\sqrt{ax})) (-a)^{-\frac{3}{2}} \frac{1}{\sqrt{ax}} + \frac{8 \ln(-ax+1)}{125a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x)/(d\*x)^(7/2),x)

[Out]  $\frac{1}{(d*x)^{7/2}} * x^{7/2} * (-a)^{7/2} / a * (-16/375/x^{3/2} / (-a)^{3/2} - 16/125/x^{1/2} / (-a)^{3/2} * a - 8/125 * x^{1/2} / (-a)^{3/2} * a^2 / (a*x)^{1/2} * (\ln(1 - (a*x)^{1/2}) - \ln(1 + (a*x)^{1/2})) + 8/125/x^{5/2} / (-a)^{3/2} / a * \ln(-a*x+1) - 4/25/x^{5/2} / (-a)^{3/2} * \text{polylog}(2, a*x) / a - 2/5/x^{5/2} / (-a)^{3/2} / a * \text{polylog}(3, a*x))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.85994, size = 629, normalized size = 5.03

$$\frac{2 \left( 12 a^2 d x^3 \sqrt{\frac{a}{d}} \log \left( \frac{a x + 2 \sqrt{d x} \sqrt{\frac{a}{d}} + 1}{a x - 1} \right) - 4 (6 a^2 x^2 + 2 a x - 3 \log(-a x + 1)) \sqrt{d x} - 30 \sqrt{d x} \int \int \left( a, x, -\frac{\log(-a x + 1)}{a}, -\frac{\log(-a x + 1)}{x} \right) \right)}{375 d^4 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)^(7/2),x, algorithm="fricas")

[Out]  $\frac{2}{375} * (12 * a^2 * d * x^3 * \text{sqrt}(a/d) * \log((a*x + 2 * \text{sqrt}(d*x) * \text{sqrt}(a/d) + 1) / (a*x - 1)) - 4 * (6 * a^2 * x^2 + 2 * a * x - 3 * \log(-a*x + 1)) * \text{sqrt}(d*x) - 30 * \text{sqrt}(d*x) * \int \int (\text{nt}(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) - 75 * \text{sqrt}(d*x) * \text{polylog}(3, a*x)) / (d^4 * x^3), -2/375 * (24 * a^2 * d * x^3 * \text{sqrt}(-a/d) * \arctan(\text{sqrt}(d*x) * \text{sqrt}(-a/d) / (a*x)) + 4 * (6 * a^2 * x^2 + 2 * a * x - 3 * \log(-a*x + 1)) * \text{sqrt}(d*x) + 30 * \text{sqrt}(d*x) * \int \int (\text{nt}(a, x, -\log(-a*x + 1)/a, -\log(-a*x + 1)/x) + 75 * \text{sqrt}(d*x) * \text{polylog}(3, a*x)) / (d^4 * x^3))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x)/(d\*x)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{(dx)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(7/2), x)
```

### 3.72 $\int (dx)^{3/2} \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=140

$$\frac{2(dx)^{5/2} \text{PolyLog}(2, ax^2)}{5d} + \frac{16d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} - \frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125a}$$

[Out]  $(-32*d*\text{Sqrt}[d*x])/(25*a) - (32*(d*x)^{(5/2)})/(125*d) + (16*d^{(3/2)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*a^{(5/4)}) + (16*d^{(3/2)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*a^{(5/4)}) + (8*(d*x)^{(5/2)}*\text{Log}[1 - a*x^2])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x^2])/(5*d)$

**Rubi [A]** time = 0.10671, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{5/2} \text{PolyLog}(2, ax^2)}{5d} + \frac{16d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} - \frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*\text{PolyLog}[2, a*x^2], x]$

[Out]  $(-32*d*\text{Sqrt}[d*x])/(25*a) - (32*(d*x)^{(5/2)})/(125*d) + (16*d^{(3/2)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*a^{(5/4)}) + (16*d^{(3/2)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*a^{(5/4)}) + (8*(d*x)^{(5/2)}*\text{Log}[1 - a*x^2])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x^2])/(5*d)$

#### Rule 6591

$\text{Int}[(d_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_*, (a_*)*((b_*)*(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})^{(p_*)}]* (b_*)*((f_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])]/(f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^{(n-1)}*(f*x)^{(m+1)})/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$  FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n-1)}*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2} \text{Li}_2(ax^2) dx &= \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{4}{5} \int (dx)^{3/2} \log(1 - ax^2) dx \\
&= \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{(16a) \int \frac{x(dx)^{5/2}}{1-ax^2} dx}{25d} \\
&= \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{(16a) \int \frac{(dx)^{7/2}}{1-ax^2} dx}{25d^2} \\
&= -\frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{16}{25} \int \frac{(dx)^{3/2}}{1 - ax^2} dx \\
&= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{(16d^2) \int \frac{1}{\sqrt{dx}(1-ax^2)} dx}{25a} \\
&= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{(32d) \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx\right)}{25a} \\
&= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^2)}{5d} + \frac{(16d^2) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax}} dx\right)}{25a} \\
&= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{16d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{8(dx)^{5/2} \log(1 - ax^2)}{25d}
\end{aligned}$$

**Mathematica [A]** time = 0.104193, size = 101, normalized size = 0.72

$$\frac{2(dx)^{3/2} \left( 25x^{5/2} \text{PolyLog}\left(2, ax^2\right) + \frac{4\sqrt[4]{a}\sqrt{x}(-4ax^2 + 5ax^2 \log(1 - ax^2) - 20) + 40 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt{d}}\right) + 40 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{x}}{\sqrt{d}}\right)}{a^{5/4}} \right)}{125x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x^2], x]
```

[Out]  $(2*(d*x)^{(3/2)}*((40*\text{ArcTan}[a^{(1/4)}*\text{Sqrt}[x]] + 40*\text{ArcTanh}[a^{(1/4)}*\text{Sqrt}[x]] + 4*a^{(1/4)}*\text{Sqrt}[x]*(-20 - 4*a*x^2 + 5*a*x^2*\text{Log}[1 - a*x^2])))/a^{(5/4)} + 25*x^{(5/2)}*\text{PolyLog}[2, a*x^2])/(125*x^{(3/2)})$

**Maple [A]** time = 0.214, size = 150, normalized size = 1.1

$$\frac{2 \operatorname{polylog}(2, ax^2)}{5d} (dx)^{\frac{5}{2}} + \frac{8}{25d} (dx)^{\frac{5}{2}} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) - \frac{32}{125d} (dx)^{\frac{5}{2}} - \frac{32d}{25a} \sqrt{dx} + \frac{8d}{25a} \sqrt[4]{\frac{d^2}{a}} \ln\left(\left(\sqrt{dx} + \sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx} - \sqrt[4]{\frac{d^2}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*polylog(2,a*x^2),x)`

[Out]  $2/5*(d*x)^{(5/2)}*\text{polylog}(2,a*x^2)/d+8/25/d*(d*x)^{(5/2)}*\ln((-a*d^2*x^2+d^2)/d^2)-32/125*(d*x)^{(5/2)}/d-32/25*d*(d*x)^{(1/2)}/a+8/25*d/a*(d^2/a)^{(1/4)}*\ln(((d*x)^{(1/2)}+(d^2/a)^{(1/4)})/((d*x)^{(1/2)}-(d^2/a)^{(1/4)}))+16/25*d/a*(d^2/a)^{(1/4)}*\arctan((d*x)^{(1/2)}/(d^2/a)^{(1/4)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.84643, size = 471, normalized size = 3.36

$$2 \left( 80 a \left( \frac{d^6}{a^5} \right)^{\frac{1}{4}} \arctan \left( \frac{\sqrt{d^3 x + a^2} \sqrt{\frac{d^6}{a^5}} a^4 \left( \frac{d^6}{a^5} \right)^{\frac{3}{4}} - \sqrt{d^3 x + a^2} \sqrt{\frac{d^6}{a^5}} a^4 \left( \frac{d^6}{a^5} \right)^{\frac{3}{4}}}{d^6} \right) - 20 a \left( \frac{d^6}{a^5} \right)^{\frac{1}{4}} \log \left( 8 \sqrt{d^3 x + a^2} + 8 a \left( \frac{d^6}{a^5} \right)^{\frac{1}{4}} \right) + 20 a \left( \frac{d^6}{a^5} \right)^{\frac{1}{4}} \log \left( 8 \sqrt{d^3 x + a^2} - 8 a \left( \frac{d^6}{a^5} \right)^{\frac{1}{4}} \right) \right) / 125 a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="fricas")`

[Out]  $-2/125*(80*a*(d^6/a^5)^{(1/4)}*\arctan(-(\text{sqrt}(d*x)*a^4*d*(d^6/a^5)^{(3/4)} - \text{sqrt}(d^3*x + a^2*\text{sqrt}(d^6/a^5))*a^4*(d^6/a^5)^{(3/4)})/d^6) - 20*a*(d^6/a^5)^{(1/4)}*\log(8*\text{sqrt}(d*x)*d + 8*a*(d^6/a^5)^{(1/4)}) + 20*a*(d^6/a^5)^{(1/4)}*\log(8*\text{sqrt}(d*x)*d - 8*a*(d^6/a^5)^{(1/4)}) - (25*a*d*x^2*\text{dilog}(a*x^2) + 20*a*d*x^2*\log(-a*x^2 + 1) - 16*a*d*x^2 - 80*d)*\text{sqrt}(d*x))/a$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*polylog(2,a\*x\*\*2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)\*dilog(a\*x^2), x)

### 3.73 $\int \sqrt{dx} \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=125

$$\frac{2(dx)^{3/2} \text{PolyLog}(2, ax^2)}{3d} - \frac{16\sqrt{d} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{16\sqrt{d} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} - \frac{32(dx)^{3/2}}{27d}$$

```
[Out] (-32*(d*x)^(3/2))/(27*d) - (16*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)) + (16*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)) + (8*(d*x)^(3/2)*Log[1 - a*x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[2, a*x^2])/(3*d)
```

**Rubi [A]** time = 0.086485, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$\frac{2(dx)^{3/2} \text{PolyLog}(2, ax^2)}{3d} - \frac{16\sqrt{d} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{16\sqrt{d} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} - \frac{32(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x^2], x]
```

```
[Out] (-32*(d*x)^(3/2))/(27*d) - (16*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)) + (16*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)) + (8*(d*x)^(3/2)*Log[1 - a*x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[2, a*x^2])/(3*d)
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

#### Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_)), x_Symbol]
:> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b)
, 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} \operatorname{Li}_2(ax^2) dx &= \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{4}{3} \int \sqrt{dx} \log(1 - ax^2) dx \\
&= \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{(16a) \int \frac{x(dx)^{3/2}}{1 - ax^2} dx}{9d} \\
&= \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{(16a) \int \frac{(dx)^{5/2}}{1 - ax^2} dx}{9d^2} \\
&= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{16}{9} \int \frac{\sqrt{dx}}{1 - ax^2} dx \\
&= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{32 \operatorname{Subst}\left(\int \frac{x^2}{1 - \frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{9d} \\
&= -\frac{32(dx)^{3/2}}{27d} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^2)}{3d} + \frac{(16d) \operatorname{Subst}\left(\int \frac{1}{d - \sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{9\sqrt{a}} \\
&= -\frac{32(dx)^{3/2}}{27d} - \frac{16\sqrt{d} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{16\sqrt{d} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}} + \frac{8(dx)^{3/2} \log(1 - ax^2)}{9d} + \frac{2(dx)^{3/2}}{9d}
\end{aligned}$$

**Mathematica [A]** time = 0.0696545, size = 91, normalized size = 0.73

$$\frac{2\sqrt{dx} \left( 9x^{3/2} \operatorname{PolyLog}(2, ax^2) + \frac{4(a^{3/4}x^{3/2}(3\log(1-ax^2)-4)-6\tan^{-1}(\sqrt[4]{a}\sqrt{x})+6\tanh^{-1}(\sqrt[4]{a}\sqrt{x}))}{a^{3/4}} \right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x^2], x]
```

[Out]  $(2\sqrt{d*x}*((4*(-6*\text{ArcTan}[a^{1/4}]*\text{Sqrt}[x]] + 6*\text{ArcTanh}[a^{1/4}]*\text{Sqrt}[x]] + a^{3/4}*x^{3/2}*(-4 + 3*\text{Log}[1 - a*x^2]))) / a^{3/4} + 9*x^{3/2}*\text{PolyLog}[2, a*x^2]) / (27*\text{Sqrt}[x])$

**Maple [A]** time = 0.052, size = 139, normalized size = 1.1

$$\frac{2 \operatorname{polylog}(2, ax^2)}{3d} (dx)^{\frac{3}{2}} + \frac{8}{9d} (dx)^{\frac{3}{2}} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) - \frac{32}{27d} (dx)^{\frac{3}{2}} - \frac{16d}{9a} \arctan\left(\sqrt{dx} \frac{1}{\sqrt[4]{d^2}}\right) \frac{1}{\sqrt[4]{d^2}} + \frac{8d}{9a} \ln\left(\left(\sqrt{dx} + \sqrt[4]{\frac{d^2}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(1/2)*polylog(2,a*x^2),x)`

[Out]  $\frac{2}{3}*(d*x)^{3/2}*polylog(2,a*x^2)/d + 8/9*d*(d*x)^{3/2}*ln((-a*d^2*x^2+d^2)/d^2) - 32/27*(d*x)^{3/2}/d - 16/9*d/a/(d^2/a)^{1/4}*arctan((d*x)^{1/2}/(d^2/a)^{1/4}) + 8/9*d/a/(d^2/a)^{1/4}*ln(((d*x)^{1/2}+(d^2/a)^{1/4})/((d*x)^{1/2}-(d^2/a)^{1/4}))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.70334, size = 440, normalized size = 3.52

$$\frac{2}{27} \sqrt{dx} (9x \operatorname{Li}_2(ax^2) + 12x \log(-ax^2 + 1) - 16x) + \frac{32}{9} \left(\frac{d^2}{a^3}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{dx} ad \left(\frac{d^2}{a^3}\right)^{\frac{1}{4}} - \sqrt{d^3x + ad^2} \sqrt{\frac{d^2}{a^3}} a \left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}}{d^2}\right) + \frac{8}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="fricas")`

[Out]  $\frac{2}{27}*\text{sqrt}(d*x)*(9*x*\text{dilog}(a*x^2) + 12*x*\text{log}(-a*x^2 + 1) - 16*x) + 32/9*(d^2/a^3)^{1/4}*arctan(-(\text{sqrt}(d*x)*a*d*(d^2/a^3)^{1/4} - \text{sqrt}(d^3*x + a*d^2*\text{sqrt}(d^2/a^3))*a*(d^2/a^3)^{1/4})/d^2) + 8/9*(d^2/a^3)^{1/4}*\text{log}(512*a^2*(d^2/a^3)^{3/4} + 512*\text{sqrt}(d*x)*d) - 8/9*(d^2/a^3)^{1/4}*\text{log}(-512*a^2*(d^2/a^3)^{3/4} + 512*\text{sqrt}(d*x)*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x^2), x)
```

### 3.74 $\int \frac{\text{PolyLog}(2, ax^2)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=115

$$\frac{2\sqrt{dx}\text{PolyLog}(2, ax^2)}{d} + \frac{8\sqrt{dx}\log(1 - ax^2)}{d} + \frac{16 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{16 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{32\sqrt{dx}}{d}$$

[Out] (-32\*Sqrt[d\*x])/d + (16\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) + (16\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) + (8\*Sqrt[d\*x]\*Log[1 - a\*x^2])/d + (2\*Sqrt[d\*x]\*PolyLog[2, a\*x^2])/d

**Rubi [A]** time = 0.0803542, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$\frac{2\sqrt{dx}\text{PolyLog}(2, ax^2)}{d} + \frac{8\sqrt{dx}\log(1 - ax^2)}{d} + \frac{16 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{16 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{32\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/Sqrt[d\*x], x]

[Out] (-32\*Sqrt[d\*x])/d + (16\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) + (16\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) + (8\*Sqrt[d\*x]\*Log[1 - a\*x^2])/d + (2\*Sqrt[d\*x]\*PolyLog[2, a\*x^2])/d

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 4 \int \frac{\log(1-ax^2)}{\sqrt{dx}} dx \\
&= \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{(16a) \int \frac{x\sqrt{dx}}{1-ax^2} dx}{d} \\
&= \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{(16a) \int \frac{(dx)^{3/2}}{1-ax^2} dx}{d^2} \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 16 \int \frac{1}{\sqrt{dx}(1-ax^2)} dx \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{32 \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{d} \\
&= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 16 \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right) + 16 \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right) \\
&= -\frac{32\sqrt{dx}}{d} + \frac{16 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{16 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.0738958, size = 57, normalized size = 0.5

$$\frac{5x\text{Gamma}\left(\frac{5}{4}\right)\left(16\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) + \text{PolyLog}\left(2, ax^2\right) + 4\log(1-ax^2) - 16\right)}{2\text{Gamma}\left(\frac{9}{4}\right)\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[2, a\*x^2]/Sqrt[d\*x], x]

[Out]  $(5*x*\text{Gamma}[5/4]*(-16 + 16*\text{Hypergeometric2F1}[1/4, 1, 5/4, a*x^2] + 4*\text{Log}[1 - a*x^2] + \text{PolyLog}[2, a*x^2]))/(2*\text{Sqrt}[d*x]*\text{Gamma}[9/4])$

**Maple [A]** time = 0.053, size = 137, normalized size = 1.2

$$2 \frac{\sqrt{dx} \text{polylog}(2, ax^2)}{d} + 8 \frac{\sqrt{dx}}{d} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) + 16 \frac{1}{d} \sqrt[4]{\frac{d^2}{a}} \arctan\left(\sqrt{dx} \frac{1}{\sqrt[4]{\frac{d^2}{a}}}\right) + 8 \frac{1}{d} \sqrt[4]{\frac{d^2}{a}} \ln\left(\left(\sqrt{dx} + \sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx} - \sqrt[4]{\frac{d^2}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(2,a*x^2)/(d*x)^(1/2),x)`

[Out]  $2*\text{polylog}(2,a*x^2)*(d*x)^{(1/2)}/d+8/d*(d*x)^{(1/2)}*\ln((-a*d^2*x^2+d^2)/d^2)+16/d*(d^2/a)^{(1/4)}*\arctan((d*x)^{(1/2)}/(d^2/a)^{(1/4)})+8/d*(d^2/a)^{(1/4)}*\ln(((d*x)^{(1/2)}+(d^2/a)^{(1/4)})/((d*x)^{(1/2)}-(d^2/a)^{(1/4)}))-32*(d*x)^{(1/2)}/d$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.88189, size = 408, normalized size = 3.55

$$2 \frac{\left(16 d \left(\frac{1}{ad^2}\right)^{\frac{1}{4}} \arctan\left(\sqrt{d^2 \sqrt{\frac{1}{ad^2}} + dx} ad \left(\frac{1}{ad^2}\right)^{\frac{3}{4}} - \sqrt{dx} ad \left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right) - 4 d \left(\frac{1}{ad^2}\right)^{\frac{1}{4}} \log\left(d \left(\frac{1}{ad^2}\right)^{\frac{1}{4}} + \sqrt{dx}\right) + 4 d \left(\frac{1}{ad^2}\right)^{\frac{1}{4}} \log\left(-d \left(\frac{1}{ad^2}\right)^{\frac{1}{4}} + \sqrt{dx}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")`

[Out]  $-2*(16*d*(1/(a*d^2))^{(1/4)}*\arctan(\text{sqrt}(d^2*\text{sqrt}(1/(a*d^2)) + d*x)*a*d*(1/(a*d^2))^{(3/4)} - \text{sqrt}(d*x)*a*d*(1/(a*d^2))^{(3/4)}) - 4*d*(1/(a*d^2))^{(1/4)}*\log(d*(1/(a*d^2))^{(1/4)} + \text{sqrt}(d*x)) + 4*d*(1/(a*d^2))^{(1/4)}*\log(-d*(1/(a*d^2))^{(1/4)} + \text{sqrt}(d*x)) - \text{sqrt}(d*x)*(d*\log(a*x^2) + 4*\log(-a*x^2 + 1) - 16))/d$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(polylog(2,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**2)/sqrt(d*x), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/sqrt(d*x), x)
```

### 3.75 $\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=103

$$-\frac{2\text{PolyLog}(2, ax^2)}{d\sqrt{dx}} - \frac{16\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8 \log(1 - ax^2)}{d\sqrt{dx}}$$

[Out]  $(-16*a^{(1/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (16*a^{(1/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (8*Log[1 - a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])$

**Rubi [A]** time = 0.0738395, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {6591, 2455, 16, 329, 298, 205, 208}

$$-\frac{2\text{PolyLog}(2, ax^2)}{d\sqrt{dx}} - \frac{16\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8 \log(1 - ax^2)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/(d\*x)^(3/2), x]

[Out]  $(-16*a^{(1/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (16*a^{(1/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (8*Log[1 - a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k), x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(ax^2)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} - 4 \int \frac{\log(1-ax^2)}{(dx)^{3/2}} dx \\ &= \frac{8 \log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16a) \int \frac{x}{\sqrt{dx}(1-ax^2)} dx}{d} \\ &= \frac{8 \log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16a) \int \frac{\sqrt{dx}}{1-ax^2} dx}{d^2} \\ &= \frac{8 \log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(32a) \text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{d^3} \\ &= \frac{8 \log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} + \frac{(16\sqrt{a}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{d} - \frac{(16\sqrt{a}) \text{Subst}\left(\int \frac{1}{d+\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{d} \\ &= -\frac{16\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8 \log(1-ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^2)}{d\sqrt{dx}} \end{aligned}$$

**Mathematica [C]** time = 0.0700504, size = 62, normalized size = 0.6

$$\frac{x\text{Gamma}\left(\frac{3}{4}\right)\left(16ax^2\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) - 3\text{PolyLog}\left(2, ax^2\right) + 12 \log(1-ax^2)\right)}{2\text{Gamma}\left(\frac{7}{4}\right)(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(3/2), x]
```

```
[Out] (x*Gamma[3/4]*(16*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 12*Log[1 - a*x^2] - 3*PolyLog[2, a*x^2]))/(2*(d*x)^(3/2)*Gamma[7/4])
```

**Maple [A]** time = 0.052, size = 127, normalized size = 1.2

$$-2 \frac{\text{polylog}(2, ax^2)}{d\sqrt{dx}} + 8 \frac{1}{d\sqrt{dx}} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) - 16 \frac{1}{d} \arctan\left(\sqrt{dx} \frac{1}{\sqrt[4]{d^2/a}}\right) \frac{1}{\sqrt[4]{d^2/a}} + 8 \frac{1}{d} \ln\left(\left(\sqrt{dx} + \sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx} - \sqrt[4]{\frac{d^2}{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(2,a*x^2)/(d*x)^(3/2),x)`

[Out]  $-2*\text{polylog}(2,a*x^2)/d/(d*x)^{(1/2)}+8/d/(d*x)^{(1/2)}*\ln((-a*d^2*x^2+d^2)/d^2)-16/d/(d^2/a)^{(1/4)}*\arctan((d*x)^{(1/2)}/(d^2/a)^{(1/4)})+8/d/(d^2/a)^{(1/4)}*\ln((d*x)^{(1/2)}+(d^2/a)^{(1/4)})/((d*x)^{(1/2)}-(d^2/a)^{(1/4)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.66427, size = 425, normalized size = 4.13

$$2 \left( 16 d^2 x \left( \frac{a}{d^6} \right)^{\frac{1}{4}} \arctan \left( -\frac{\sqrt{d x a} \left( \frac{a}{d^6} \right)^{\frac{1}{4}} - \sqrt{a d^4 \sqrt{\frac{a}{d^6}} + a^2 d x} \left( \frac{a}{d^6} \right)^{\frac{1}{4}}}{a} \right) + 4 d^2 x \left( \frac{a}{d^6} \right)^{\frac{1}{4}} \log \left( 512 d^5 \left( \frac{a}{d^6} \right)^{\frac{3}{4}} + 512 \sqrt{d x a} \right) - 4 d^2 x \left( \frac{a}{d^6} \right)^{\frac{1}{4}} \log \left( \dots \right) \right) / d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")`

[Out]  $2*(16*d^2*x*(a/d^6)^{(1/4)}*\arctan(-(\text{sqrt}(d*x)*a*d*(a/d^6)^{(1/4)} - \text{sqrt}(a*d^4*\text{sqrt}(a/d^6) + a^2*d*x)*d*(a/d^6)^{(1/4)})/a) + 4*d^2*x*(a/d^6)^{(1/4)}*\log(512*d^5*(a/d^6)^{(3/4)} + 512*\text{sqrt}(d*x)*a) - 4*d^2*x*(a/d^6)^{(1/4)}*\log(-512*d^5*(a/d^6)^{(3/4)} + 512*\text{sqrt}(d*x)*a) - \text{sqrt}(d*x)*(d*\log(a*x^2) - 4*\log(-a*x^2 + 1)))/(d^2*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x**2)/(d*x)**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^2)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(3/2), x)
```

### 3.76 $\int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=111

$$-\frac{2\text{PolyLog}(2, ax^2)}{3d(dx)^{3/2}} + \frac{16a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8 \log(1 - ax^2)}{9d(dx)^{3/2}}$$

[Out] (16\*a^(3/4)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(9\*d^(5/2)) + (16\*a^(3/4)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(9\*d^(5/2)) + (8\*Log[1 - a\*x^2])/(9\*d\*(d\*x)^(3/2)) - (2\*PolyLog[2, a\*x^2])/(3\*d\*(d\*x)^(3/2))

**Rubi [A]** time = 0.0720639, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {6591, 2455, 16, 329, 212, 208, 205}

$$-\frac{2\text{PolyLog}(2, ax^2)}{3d(dx)^{3/2}} + \frac{16a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8 \log(1 - ax^2)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/(d\*x)^(5/2), x]

[Out] (16\*a^(3/4)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(9\*d^(5/2)) + (16\*a^(3/4)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(9\*d^(5/2)) + (8\*Log[1 - a\*x^2])/(9\*d\*(d\*x)^(3/2)) - (2\*PolyLog[2, a\*x^2])/(3\*d\*(d\*x)^(3/2))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
  Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
  /b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(ax^2)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} - \frac{4}{3} \int \frac{\log(1-ax^2)}{(dx)^{5/2}} dx \\
 &= \frac{8 \log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a) \int \frac{x}{(dx)^{3/2}(1-ax^2)} dx}{9d} \\
 &= \frac{8 \log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a) \int \frac{1}{\sqrt{dx}(1-ax^2)} dx}{9d^2} \\
 &= \frac{8 \log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(32a) \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{9d^3} \\
 &= \frac{8 \log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}} + \frac{(16a) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{9d^2} + \frac{(16a) \text{Subst}\left(\int \frac{1}{d+\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{9d^2} \\
 &= \frac{16a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8 \log(1-ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^2)}{3d(dx)^{3/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0673089, size = 62, normalized size = 0.56

$$\frac{x\text{Gamma}\left(\frac{1}{4}\right)\left(16ax^2\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) - 3\text{PolyLog}\left(2, ax^2\right) + 4 \log\left(1 - ax^2\right)\right)}{18\text{Gamma}\left(\frac{5}{4}\right)(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(5/2), x]
```

```
[Out] (x*Gamma[1/4]*(16*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 - a
*x^2] - 3*PolyLog[2, a*x^2]))/(18*(d*x)^(5/2)*Gamma[5/4])
```

**Maple [A]** time = 0.054, size = 129, normalized size = 1.2

$$-\frac{2 \text{polylog}\left(2, ax^2\right)}{3d} (dx)^{-\frac{3}{2}} + \frac{8}{9d} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) (dx)^{-\frac{3}{2}} + \frac{8a}{9d^3} \sqrt{\frac{d^2}{a}} \ln\left(\left(\sqrt{dx} + \sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx} - \sqrt[4]{\frac{d^2}{a}}\right)^{-1}\right) + \frac{16a^4}{9d^3} \sqrt{\frac{d^2}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(2,a*x^2)/(d*x)^(5/2),x)`

[Out] 
$$-2/3 \operatorname{polylog}(2, a x^2) / d / (d x)^{3/2} + 8/9 / d / (d x)^{3/2} * \ln((-a d^2 x^2 + d^2) / d^2) + 8/9 / d^3 * a * (d^2 / a)^{1/4} * \ln(((d x)^{1/2} + (d^2 / a)^{1/4}) / ((d x)^{1/2} - (d^2 / a)^{1/4})) + 16/9 / d^3 * a * (d^2 / a)^{1/4} * \arctan((d x)^{1/2} / (d^2 / a)^{1/4})$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.73533, size = 470, normalized size = 4.23

$$2 \left( 16 d^3 x^2 \left( \frac{a^3}{d^{10}} \right)^{\frac{1}{4}} \arctan \left( \frac{\sqrt{d x} a d^7 \left( \frac{a^3}{d^{10}} \right)^{\frac{3}{4}} - \sqrt{d^6 \sqrt{\frac{a^3}{d^{10}} + a^2 d x} d^7 \left( \frac{a^3}{d^{10}} \right)^{\frac{3}{4}}}}{a^3} \right) - 4 d^3 x^2 \left( \frac{a^3}{d^{10}} \right)^{\frac{1}{4}} \log \left( 8 d^3 \left( \frac{a^3}{d^{10}} \right)^{\frac{1}{4}} + 8 \sqrt{d x} a \right) + 4 d^3 x^2 \left( \frac{a^3}{d^{10}} \right)^{\frac{1}{4}} \right) / 9 d^3 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")`

[Out] 
$$-2/9 * (16 * d^3 * x^2 * (a^3 / d^{10})^{1/4} * \arctan(-(\sqrt{d x}) * a * d^7 * (a^3 / d^{10})^{3/4} - \sqrt{d^6 * \sqrt{a^3 / d^{10}} + a^2 * d x} * d^7 * (a^3 / d^{10})^{3/4}) / a^3) - 4 * d^3 * x^2 * (a^3 / d^{10})^{1/4} * \log(8 * d^3 * (a^3 / d^{10})^{1/4} + 8 * \sqrt{d x} * a) + 4 * d^3 * x^2 * (a^3 / d^{10})^{1/4} * \log(-8 * d^3 * (a^3 / d^{10})^{1/4} + 8 * \sqrt{d x} * a) + \sqrt{d x} * (3 * \operatorname{dilog}(a x^2) - 4 * \log(-a x^2 + 1)) / (d^3 x^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,a*x**2)/(d*x)**(5/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{(dx)^{\frac{5}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(5/2), x)
```

$$3.77 \quad \int \frac{\text{PolyLog}(2, ax^2)}{(dx)^{7/2}} dx$$

**Optimal.** Leaf size=126

$$-\frac{2\text{PolyLog}(2, ax^2)}{5d(dx)^{5/2}} - \frac{16a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} - \frac{32a}{25d^3\sqrt{dx}} + \frac{8 \log(1 - ax^2)}{25d(dx)^{5/2}}$$

[Out]  $(-32*a)/(25*d^3*\text{Sqrt}[d*x]) - (16*a^{(5/4)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*d^{(7/2)}) + (16*a^{(5/4)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*d^{(7/2)}) + (8*\text{Log}[1 - a*x^2])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[2, a*x^2])/(5*d*(d*x)^{(5/2)})$

**Rubi [A]** time = 0.0858323, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 325, 329, 298, 205, 208}

$$-\frac{2\text{PolyLog}(2, ax^2)}{5d(dx)^{5/2}} - \frac{16a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} - \frac{32a}{25d^3\sqrt{dx}} + \frac{8 \log(1 - ax^2)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^2]/(d\*x)^(7/2), x]

[Out]  $(-32*a)/(25*d^3*\text{Sqrt}[d*x]) - (16*a^{(5/4)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*d^{(7/2)}) + (16*a^{(5/4)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(25*d^{(7/2)}) + (8*\text{Log}[1 - a*x^2])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[2, a*x^2])/(5*d*(d*x)^{(5/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,

x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}} dx &= -\frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} - \frac{4}{5} \int \frac{\log(1-ax^2)}{(dx)^{7/2}} dx \\
&= \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a) \int \frac{x}{(dx)^{5/2}(1-ax^2)} dx}{25d} \\
&= \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a) \int \frac{1}{(dx)^{3/2}(1-ax^2)} dx}{25d^2} \\
&= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^2) \int \frac{\sqrt{dx}}{1-ax^2} dx}{25d^4} \\
&= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(32a^2) \text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{25d^5} \\
&= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^{3/2}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{25d^3} - \frac{(16a^{3/2}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{25d^3} \\
&= -\frac{32a}{25d^3\sqrt{dx}} - \frac{16a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{8 \log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0765584, size = 70, normalized size = 0.56

$$\frac{x\text{Gamma}\left(-\frac{1}{4}\right)\left(16a^2x^4\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) - 15\text{PolyLog}\left(2, ax^2\right) - 48ax^2 + 12 \log(1-ax^2)\right)}{150\text{Gamma}\left(\frac{3}{4}\right)(dx)^{7/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[2, a\*x^2]/(d\*x)^(7/2),x]

[Out]  $-(x*\Gamma[-1/4]*(-48*a*x^2 + 16*a^2*x^4*\text{Hypergeometric2F1}[3/4, 1, 7/4, a*x^2] + 12*\text{Log}[1 - a*x^2] - 15*\text{PolyLog}[2, a*x^2]))/(150*(d*x)^(7/2)*\Gamma[3/4])$

**Maple [A]** time = 0.056, size = 140, normalized size = 1.1

$$-\frac{2 \operatorname{polylog}(2, ax^2)}{5d} (dx)^{-\frac{5}{2}} + \frac{8}{25d} \ln\left(\frac{-ad^2x^2 + d^2}{d^2}\right) (dx)^{-\frac{5}{2}} - \frac{32a}{25d^3} \frac{1}{\sqrt{dx}} - \frac{16a}{25d^3} \arctan\left(\sqrt{dx} \frac{1}{\sqrt[4]{d^2}}\right) \frac{1}{\sqrt[4]{d^2}} + \frac{8a}{25d^3} \ln\left(\sqrt[4]{\frac{d^2}{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^2)/(d\*x)^(7/2),x)

[Out]  $-2/5*\operatorname{polylog}(2, a*x^2)/d/(d*x)^(5/2)+8/25*d/(d*x)^(5/2)*\ln((-a*d^2*x^2+d^2)/d^2)-32/25*a/d^3/(d*x)^(1/2)-16/25/d^3*a/(d^2/a)^(1/4)*\arctan((d*x)^(1/2)/(d^2/a)^(1/4))+8/25/d^3*a/(d^2/a)^(1/4)*\ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/(d\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.75536, size = 512, normalized size = 4.06

$$2 \left( 16 d^4 x^3 \left( \frac{a^5}{d^{14}} \right)^{\frac{1}{4}} \arctan \left( \frac{\sqrt{d} x a^4 d^3 \left( \frac{a^5}{d^{14}} \right)^{\frac{1}{4}} - \sqrt{a^5 d^8 \sqrt{\frac{a^5}{d^{14}} + a^8 d x d^3 \left( \frac{a^5}{d^{14}} \right)^{\frac{1}{4}}}}}{a^5} \right) + 4 d^4 x^3 \left( \frac{a^5}{d^{14}} \right)^{\frac{1}{4}} \log \left( 512 d^{11} \left( \frac{a^5}{d^{14}} \right)^{\frac{3}{4}} + 512 \sqrt{d} x a^4 \right) - 4 d^4 x^3 \right) / 25 d^4 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/(d\*x)^(7/2),x, algorithm="fricas")

[Out]  $2/25*(16*d^4*x^3*(a^5/d^14)^(1/4)*\arctan(-(\sqrt{d*x}*a^4*d^3*(a^5/d^14)^(1/4) - \sqrt{a^5*d^8*\sqrt{a^5/d^14} + a^8*d*x}*d^3*(a^5/d^14)^(1/4))/a^5) + 4*d^4*x^3*(a^5/d^14)^(1/4)*\log(512*d^11*(a^5/d^14)^(3/4) + 512*\sqrt{d*x}*a^4) - 4*d^4*x^3*(a^5/d^14)^(1/4)*\log(-512*d^11*(a^5/d^14)^(3/4) + 512*\sqrt{d*x}*a^4) - (16*a*x^2 + 5*dilog(a*x^2) - 4*\log(-a*x^2 + 1))*\sqrt{d*x}/(d^4*x^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x\*\*2)/(d\*x)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{(dx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^2)/(d\*x)^(7/2),x, algorithm="giac")

[Out] integrate(dilog(a\*x^2)/(d\*x)^(7/2), x)

### 3.78 $\int (dx)^{5/2} \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=161

$$-\frac{8(dx)^{7/2}\text{PolyLog}(2, ax^2)}{49d} + \frac{2(dx)^{7/2}\text{PolyLog}(3, ax^2)}{7d} + \frac{64d^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{64d^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343a^{7/4}}$$

[Out] (128\*d\*(d\*x)^(3/2))/(1029\*a) + (128\*(d\*x)^(7/2))/(2401\*d) + (64\*d^(5/2)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(343\*a^(7/4)) - (64\*d^(5/2)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(343\*a^(7/4)) - (32\*(d\*x)^(7/2)\*Log[1 - a\*x^2])/(343\*d) - (8\*(d\*x)^(7/2)\*PolyLog[2, a\*x^2])/(49\*d) + (2\*(d\*x)^(7/2)\*PolyLog[3, a\*x^2])/(7\*d)

**Rubi [A]** time = 0.126749, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$-\frac{8(dx)^{7/2}\text{PolyLog}(2, ax^2)}{49d} + \frac{2(dx)^{7/2}\text{PolyLog}(3, ax^2)}{7d} + \frac{64d^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{64d^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343a^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*PolyLog[3, a\*x^2], x]

[Out] (128\*d\*(d\*x)^(3/2))/(1029\*a) + (128\*(d\*x)^(7/2))/(2401\*d) + (64\*d^(5/2)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(343\*a^(7/4)) - (64\*d^(5/2)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(343\*a^(7/4)) - (32\*(d\*x)^(7/2)\*Log[1 - a\*x^2])/(343\*d) - (8\*(d\*x)^(7/2)\*PolyLog[2, a\*x^2])/(49\*d) + (2\*(d\*x)^(7/2)\*PolyLog[3, a\*x^2])/(7\*d)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1)))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p]

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int (dx)^{5/2} \text{Li}_3(ax^2) dx &= \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} - \frac{4}{7} \int (dx)^{5/2} \text{Li}_2(ax^2) dx \\
 &= -\frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} - \frac{16}{49} \int (dx)^{5/2} \log(1 - ax^2) dx \\
 &= -\frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} - \frac{(64a) \int \frac{x(dx)^{7/2}}{1 - ax^2} dx}{343d} \\
 &= -\frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} - \frac{(64a) \int \frac{(dx)^{9/2}}{1 - ax^2} dx}{343d^2} \\
 &= \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} - \frac{64}{343} \int \frac{(dx)^5}{1 - ax^2} dx \\
 &= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} \\
 &= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} \\
 &= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343d} - \frac{8(dx)^{7/2} \text{Li}_2(ax^2)}{49d} + \frac{2(dx)^{7/2} \text{Li}_3(ax^2)}{7d} \\
 &= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} + \frac{64d^{5/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{64d^{5/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{32(dx)^{7/2} \log(1 - ax^2)}{343d}
 \end{aligned}$$

**Mathematica [C]** time = 0.103023, size = 89, normalized size = 0.55

$$\frac{11d \text{Gamma}\left(\frac{11}{4}\right) (dx)^{3/2} \left(448 \text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) + 588ax^2 \text{PolyLog}(2, ax^2) - 1029ax^2 \text{PolyLog}(3, ax^2)\right) + 14406a \text{Gamma}\left(\frac{15}{4}\right) (dx)^{7/2} \text{Li}_3(ax^2)}{1029a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^(5/2)\*PolyLog[3, a\*x^2], x]

[Out]  $(-11*d*(d*x)^{(3/2)}*\Gamma[11/4]*(-448 - 192*a*x^2 + 448*\text{Hypergeometric2F1}[3/4, 1, 7/4, a*x^2] + 336*a*x^2*\text{Log}[1 - a*x^2] + 588*a*x^2*\text{PolyLog}[2, a*x^2] - 1029*a*x^2*\text{PolyLog}[3, a*x^2]))/(14406*a*\Gamma[15/4])$

**Maple [A]** time = 0.193, size = 155, normalized size = 1.

$$-\frac{1}{2}(dx)^{\frac{5}{2}}\left(\frac{8448ax^2+19712}{79233a^2}x^{\frac{3}{2}}(-a)^{\frac{11}{4}}+\frac{64}{343a^2}x^{\frac{3}{2}}(-a)^{\frac{11}{4}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)\right)(ax^2)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)\*polylog(3,a\*x^2), x)

[Out]  $-1/2*(d*x)^{(5/2)}/x^{(5/2)}/(-a)^{(7/4)}*(4/79233*x^{(3/2)}*(-a)^{(11/4)}*(2112*a*x^2+4928)/a^2+64/343*x^{(3/2)}*(-a)^{(11/4)}/a^2/(a*x^2)^{(3/4)}*(\ln(1-(a*x^2)^{(1/4)}))-\ln(1+(a*x^2)^{(1/4)})+2*\arctan((a*x^2)^{(1/4)}))-64/343*x^{(7/2)}*(-a)^{(11/4)}/a*\ln(-a*x^2+1)-16/49*x^{(7/2)}*(-a)^{(11/4)}*\text{polylog}(2, a*x^2)/a+4/7*x^{(7/2)}*(-a)^{(11/4)}/a*\text{polylog}(3, a*x^2)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.10612, size = 682, normalized size = 4.24

$$2\left(588\sqrt{d}xad^2x^3\%iint\left(a,x,-\frac{\log(-ax^2+1)}{a},-\frac{2\log(-ax^2+1)}{x}\right)-1029\sqrt{d}xad^2x^3\text{polylog}(3,ax^2)+1344\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\arctan\left(-\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}\sqrt{d}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*polylog(3,a\*x^2), x, algorithm="fricas")

[Out]  $-2/7203*(588*\text{sqrt}(d*x)*a*d^2*x^3*\%iint(a, x, -\log(-a*x^2 + 1)/a, -2*\log(-a*x^2 + 1)/x) - 1029*\text{sqrt}(d*x)*a*d^2*x^3*\text{polylog}(3, a*x^2) + 1344*(d^{10}/a^7)^{(1/4)}*a*\arctan(-((d^{10}/a^7)^{(1/4)}*\text{sqrt}(d*x)*a^2*d^7 - \text{sqrt}(d^{15}*x + \text{sqrt}(d^{10}/a^7))*a^3*d^{10})*(d^{10}/a^7)^{(1/4)}*a^2)/d^{10} + 336*(d^{10}/a^7)^{(1/4)}*a*\log(32768*\text{sqrt}(d*x)*d^7 + 32768*(d^{10}/a^7)^{(3/4)}*a^5) - 336*(d^{10}/a^7)^{(1/4)}*a*\log(32768*\text{sqrt}(d*x)*d^7 - 32768*(d^{10}/a^7)^{(3/4)}*a^5) + 16*(21*a*d^2*x^3*lo$



$$g(-a*x^2 + 1) - 12*a*d^2*x^3 - 28*d^2*x)*sqrt(d*x))/a$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)\*polylog(3,a\*x\*\*2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \text{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)\*polylog(3, a\*x^2), x)

### 3.79 $\int (dx)^{3/2} \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=161

$$\frac{8(dx)^{5/2}\text{PolyLog}(2, ax^2)}{25d} + \frac{2(dx)^{5/2}\text{PolyLog}(3, ax^2)}{5d} - \frac{64d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d}$$

[Out] (128\*d\*Sqrt[d\*x])/(125\*a) + (128\*(d\*x)^(5/2))/(625\*d) - (64\*d^(3/2)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(125\*a^(5/4)) - (64\*d^(3/2)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(125\*a^(5/4)) - (32\*(d\*x)^(5/2)\*Log[1 - a\*x^2])/(125\*d) - (8\*(d\*x)^(5/2)\*PolyLog[2, a\*x^2])/(25\*d) + (2\*(d\*x)^(5/2)\*PolyLog[3, a\*x^2])/(5\*d)

**Rubi [A]** time = 0.115868, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$\frac{8(dx)^{5/2}\text{PolyLog}(2, ax^2)}{25d} + \frac{2(dx)^{5/2}\text{PolyLog}(3, ax^2)}{5d} - \frac{64d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*PolyLog[3, a\*x^2], x]

[Out] (128\*d\*Sqrt[d\*x])/(125\*a) + (128\*(d\*x)^(5/2))/(625\*d) - (64\*d^(3/2)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(125\*a^(5/4)) - (64\*d^(3/2)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(125\*a^(5/4)) - (32\*(d\*x)^(5/2)\*Log[1 - a\*x^2])/(125\*d) - (8\*(d\*x)^(5/2)\*PolyLog[2, a\*x^2])/(25\*d) + (2\*(d\*x)^(5/2)\*PolyLog[3, a\*x^2])/(5\*d)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1)))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p, 0]

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^(p), x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int (dx)^{3/2} \text{Li}_3(ax^2) dx &= \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} - \frac{4}{5} \int (dx)^{3/2} \text{Li}_2(ax^2) dx \\
 &= -\frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} - \frac{16}{25} \int (dx)^{3/2} \log(1 - ax^2) dx \\
 &= -\frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} - \frac{(64a) \int \frac{x(dx)^{5/2}}{1 - ax^2} dx}{125d} \\
 &= -\frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} - \frac{(64a) \int \frac{(dx)^{7/2}}{1 - ax^2} dx}{125d^2} \\
 &= \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} - \frac{64}{125} \int \frac{(dx)^3}{1 - ax^2} \\
 &= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} \\
 &= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} \\
 &= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d} - \frac{8(dx)^{5/2} \text{Li}_2(ax^2)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^2)}{5d} \\
 &= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{64d^{3/2} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{32(dx)^{5/2} \log(1 - ax^2)}{125d}
 \end{aligned}$$

**Mathematica [C]** time = 0.0996985, size = 89, normalized size = 0.55

$$\frac{9d \text{Gamma}\left(\frac{9}{4}\right) \sqrt{dx} \left(320 \text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) + 100ax^2 \text{PolyLog}(2, ax^2) - 125ax^2 \text{PolyLog}(3, ax^2)\right)}{1250a \text{Gamma}\left(\frac{13}{4}\right)}$$



$$^{(1/4)} + 16*(5*a*d*x^2*\log(-a*x^2 + 1) - 4*a*d*x^2 - 20*d)*\text{sqrt}(d*x))/a$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \text{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*polylog(3,a\*x\*\*2),x)

[Out] Integral((d\*x)\*\*(3/2)\*polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \text{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)\*polylog(3, a\*x^2), x)

### 3.80 $\int \sqrt{dx} \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=146

$$-\frac{8(dx)^{3/2}\text{PolyLog}(2, ax^2)}{9d} + \frac{2(dx)^{3/2}\text{PolyLog}(3, ax^2)}{3d} + \frac{64\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{64\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{32(dx)^{3/2}\log}{27a}$$

[Out] (128\*(d\*x)^(3/2))/(81\*d) + (64\*Sqrt[d]\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*a^(3/4)) - (64\*Sqrt[d]\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*a^(3/4)) - (32\*(d\*x)^(3/2)\*Log[1 - a\*x^2])/(27\*d) - (8\*(d\*x)^(3/2)\*PolyLog[2, a\*x^2])/(9\*d) + (2\*(d\*x)^(3/2)\*PolyLog[3, a\*x^2])/(3\*d)

**Rubi [A]** time = 0.0974576, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$-\frac{8(dx)^{3/2}\text{PolyLog}(2, ax^2)}{9d} + \frac{2(dx)^{3/2}\text{PolyLog}(3, ax^2)}{3d} + \frac{64\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{64\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{32(dx)^{3/2}\log}{27a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*PolyLog[3, a\*x^2], x]

[Out] (128\*(d\*x)^(3/2))/(81\*d) + (64\*Sqrt[d]\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*a^(3/4)) - (64\*Sqrt[d]\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*a^(3/4)) - (32\*(d\*x)^(3/2)\*Log[1 - a\*x^2])/(27\*d) - (8\*(d\*x)^(3/2)\*PolyLog[2, a\*x^2])/(9\*d) + (2\*(d\*x)^(3/2)\*PolyLog[3, a\*x^2])/(3\*d)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} \operatorname{Li}_3(ax^2) dx &= \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{4}{3} \int \sqrt{dx} \operatorname{Li}_2(ax^2) dx \\
&= -\frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{16}{9} \int \sqrt{dx} \log(1 - ax^2) dx \\
&= -\frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{(64a) \int \frac{x(dx)^{3/2}}{1 - ax^2} dx}{27d} \\
&= -\frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{(64a) \int \frac{(dx)^{5/2}}{1 - ax^2} dx}{27d^2} \\
&= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{64}{27} \int \frac{\sqrt{dx}}{1 - ax^2} dx \\
&= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{128 \operatorname{Subst}\left(\int \frac{dx}{1 - ax^2}\right)}{27} \\
&= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^{3/2} \operatorname{Li}_2(ax^2)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^2)}{3d} - \frac{(64d) \operatorname{Subst}\left(\int \frac{dx}{1 - ax^2}\right)}{27} \\
&= \frac{128(dx)^{3/2}}{81d} + \frac{64\sqrt{d} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{64\sqrt{d} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{32(dx)^{3/2} \log(1 - ax^2)}{27d} - \frac{8(dx)^3}{27d}
\end{aligned}$$

**Mathematica [C]** time = 0.0915019, size = 68, normalized size = 0.47

$$\frac{7x \operatorname{Gamma}\left(\frac{7}{4}\right) \sqrt{dx} \left(64 \operatorname{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) + 36 \operatorname{PolyLog}(2, ax^2) - 27 \operatorname{PolyLog}(3, ax^2) + 48 \log\left(\frac{11}{4}\right)\right)}{162 \operatorname{Gamma}\left(\frac{11}{4}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d\*x]\*PolyLog[3, a\*x^2], x]

[Out]  $(-7*x*\sqrt{d*x}*\Gamma[7/4]*(-64 + 64*\text{Hypergeometric2F1}[3/4, 1, 7/4, a*x^2] + 48*\text{Log}[1 - a*x^2] + 36*\text{PolyLog}[2, a*x^2] - 27*\text{PolyLog}[3, a*x^2]))/(162*\Gamma[11/4])$

**Maple [A]** time = 0.224, size = 147, normalized size = 1.

$$-\frac{1}{2}\sqrt{dx}\left(\frac{256}{81a}x^{\frac{3}{2}}(-a)^{\frac{7}{4}} + \frac{64}{27a}x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1 - \sqrt[4]{ax^2}\right) - \ln\left(1 + \sqrt[4]{ax^2}\right) + 2\arctan\left(\sqrt[4]{ax^2}\right)\right)\right)(ax^2)^{-\frac{3}{4}} - \frac{64\ln(-ax^2 + 1)}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*polylog(3,a\*x^2), x)

[Out]  $-1/2*(d*x)^{(1/2)}/x^{(1/2)}/(-a)^{(3/4)}*(256/81*x^{(3/2)}*(-a)^{(7/4)}/a+64/27*x^{(3/2)}*(-a)^{(7/4)}/a/(a*x^2)^{(3/4)}*(\ln(1-(a*x^2)^{(1/4)})-\ln(1+(a*x^2)^{(1/4)})+2*\arctan((a*x^2)^{(1/4)}))-64/27*x^{(3/2)}*(-a)^{(7/4)}/a*\ln(-a*x^2+1)-16/9*x^{(3/2)}*(-a)^{(7/4)}*\text{polylog}(2,a*x^2)/a+4/3*x^{(3/2)}*(-a)^{(7/4)}/a*\text{polylog}(3,a*x^2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.8515, size = 579, normalized size = 3.97

$$-\frac{8}{9}\sqrt{dx}\%iint\left(a, x, -\frac{\log(-ax^2 + 1)}{a}, -\frac{2\log(-ax^2 + 1)}{x}\right) + \frac{2}{3}\sqrt{dx}\text{polylog}(3, ax^2) - \frac{32}{81}\sqrt{dx}(3x\log(-ax^2 + 1) - 4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x^2), x, algorithm="fricas")

[Out]  $-8/9*\text{sqrt}(d*x)*x*\%iint(a, x, -\log(-a*x^2 + 1)/a, -2*\log(-a*x^2 + 1)/x) + 2/3*\text{sqrt}(d*x)*x*\text{polylog}(3, a*x^2) - 32/81*\text{sqrt}(d*x)*(3*x*\log(-a*x^2 + 1) - 4*x) - 128/27*(d^2/a^3)^{(1/4)}*\arctan(-(\text{sqrt}(d*x)*a*d*(d^2/a^3)^{(1/4)} - \text{sqrt}(d^3*x + a*d^2*\text{sqrt}(d^2/a^3)))*a*(d^2/a^3)^{(1/4)})/d^2) - 32/27*(d^2/a^3)^{(1/4)}*\log(32768*a^2*(d^2/a^3)^{(3/4)} + 32768*\text{sqrt}(d*x)*d) + 32/27*(d^2/a^3)^{(1/4)}*\log(-32768*a^2*(d^2/a^3)^{(3/4)} + 32768*\text{sqrt}(d*x)*d)$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*polylog(3,a\*x\*\*2),x)

[Out] Integral(sqrt(d\*x)\*polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate(sqrt(d\*x)\*polylog(3, a\*x^2), x)

### 3.81 $\int \frac{\text{PolyLog}(3, ax^2)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=134

$$-\frac{8\sqrt{dx}\text{PolyLog}(2, ax^2)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax^2)}{d} - \frac{32\sqrt{dx}\log(1 - ax^2)}{d} - \frac{64 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{64 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} +$$

[Out] (128\*Sqrt[d\*x])/d - (64\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) - (64\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) - (32\*Sqrt[d\*x]\*Log[1 - a\*x^2])/d - (8\*Sqrt[d\*x]\*PolyLog[2, a\*x^2])/d + (2\*Sqrt[d\*x]\*PolyLog[3, a\*x^2])/d

**Rubi [A]** time = 0.101164, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$-\frac{8\sqrt{dx}\text{PolyLog}(2, ax^2)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax^2)}{d} - \frac{32\sqrt{dx}\log(1 - ax^2)}{d} - \frac{64 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{64 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} +$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/Sqrt[d\*x], x]

[Out] (128\*Sqrt[d\*x])/d - (64\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) - (64\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]]/(a^(1/4)\*Sqrt[d])) - (32\*Sqrt[d\*x]\*Log[1 - a\*x^2])/d - (8\*Sqrt[d\*x]\*PolyLog[2, a\*x^2])/d + (2\*Sqrt[d\*x]\*PolyLog[3, a\*x^2])/d

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a + b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^(n\*(m-n+1)))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_3(ax^2)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - 4 \int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}} dx \\
 &= -\frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - 16 \int \frac{\log(1-ax^2)}{\sqrt{dx}} dx \\
 &= -\frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - \frac{(64a) \int \frac{x\sqrt{dx}}{1-ax^2} dx}{d} \\
 &= -\frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - \frac{(64a) \int \frac{(dx)^{3/2}}{1-ax^2} dx}{d^2} \\
 &= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - 64 \int \frac{1}{\sqrt{dx}(1-ax^2)} dx \\
 &= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - \frac{128 \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{d} \\
 &= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d} - 64 \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right) \\
 &= \frac{128\sqrt{dx}}{d} - \frac{64 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{64 \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{32\sqrt{dx} \log(1-ax^2)}{d} - \frac{8\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^2)}{d}
 \end{aligned}$$

**Mathematica [C]** time = 0.0861366, size = 68, normalized size = 0.51

$$\frac{5x\text{Gamma}\left(\frac{5}{4}\right)\left(64\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) + 4\text{PolyLog}(2, ax^2) - \text{PolyLog}(3, ax^2) + 16 \log(1-ax^2)\right)}{2\text{Gamma}\left(\frac{9}{4}\right)\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/Sqrt[d\*x],x]

[Out]  $(-5*x*\text{Gamma}[5/4]*(-64 + 64*\text{Hypergeometric2F1}[1/4, 1, 5/4, a*x^2] + 16*\text{Log}[1 - a*x^2] + 4*\text{PolyLog}[2, a*x^2] - \text{PolyLog}[3, a*x^2]))/(2*\text{Sqrt}[d*x]*\text{Gamma}[9/4])$

**Maple [A]** time = 0.191, size = 147, normalized size = 1.1

$$-\frac{1}{2}\sqrt{x}\left(256\frac{\sqrt{x}(-a)^{5/4}}{a} + 64\frac{\sqrt{x}(-a)^{5/4}\left(\ln\left(1 - \sqrt[4]{ax^2}\right) - \ln\left(1 + \sqrt[4]{ax^2}\right) - 2\arctan\left(\sqrt[4]{ax^2}\right)\right)}{a\sqrt[4]{ax^2}} - 64\frac{\sqrt{x}(-a)^{5/4}\ln(-ax^2 + 1)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/(d\*x)^(1/2),x)

[Out]  $-1/2/(d*x)^{(1/2)}*x^{(1/2)/(-a)^{(1/4)}*(256*x^{(1/2)}*(-a)^{(5/4)}/a+64*x^{(1/2)}*(-a)^{(5/4)}/a/(a*x^2)^{(1/4)}*(\ln(1-(a*x^2)^{(1/4)})-\ln(1+(a*x^2)^{(1/4)})-2*\arctan((a*x^2)^{(1/4)}))-64*x^{(1/2)}*(-a)^{(5/4)}/a*\ln(-a*x^2+1)-16*x^{(1/2)}*(-a)^{(5/4)}*polylog(2,a*x^2)/a+4*x^{(1/2)}*(-a)^{(5/4)}/a*polylog(3,a*x^2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.8615, size = 521, normalized size = 3.89

$$2\left(64d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{d^2\sqrt{\frac{1}{ad^2}}+dx}ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}-\sqrt{dx}ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right)-16d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+16d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(-\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(1/2),x, algorithm="fricas")

[Out]  $2*(64*d*(1/(a*d^2))^{(1/4)}*\arctan(\text{sqrt}(d^2*\text{sqrt}(1/(a*d^2)) + d*x)*a*d*(1/(a*d^2))^{(3/4)} - \text{sqrt}(d*x)*a*d*(1/(a*d^2))^{(3/4)}) - 16*d*(1/(a*d^2))^{(1/4)}*\log(d*(1/(a*d^2))^{(1/4)} + \text{sqrt}(d*x)) + 16*d*(1/(a*d^2))^{(1/4)}*\log(-d*(1/(a*d^2))^{(1/4)} + \text{sqrt}(d*x)) - 16*\text{sqrt}(d*x)*(\log(-a*x^2 + 1) - 4) - 4*\text{sqrt}(d*x)*\%i\text{int}(a, x, -\log(-a*x^2 + 1)/a, -2*\log(-a*x^2 + 1)/x) + \text{sqrt}(d*x)*\text{polylog}(3, a*x^2))/d$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/sqrt(d*x), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/sqrt(d*x), x)
```

### 3.82 $\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=122

$$-\frac{8\text{PolyLog}(2, ax^2)}{d\sqrt{dx}} - \frac{2\text{PolyLog}(3, ax^2)}{d\sqrt{dx}} - \frac{64\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{64\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{32 \log(1 - ax^2)}{d\sqrt{dx}}$$

[Out]  $(-64*a^{(1/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (64*a^{(1/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (32*Log[1 - a*x^2])/(d*Sqrt[d*x]) - (8*PolyLog[2, a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^2])/(d*Sqrt[d*x])$

**Rubi [A]** time = 0.0932373, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {6591, 2455, 16, 329, 298, 205, 208}

$$-\frac{8\text{PolyLog}(2, ax^2)}{d\sqrt{dx}} - \frac{2\text{PolyLog}(3, ax^2)}{d\sqrt{dx}} - \frac{64\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{64\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{32 \log(1 - ax^2)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/(d\*x)^(3/2), x]

[Out]  $(-64*a^{(1/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (64*a^{(1/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (32*Log[1 - a*x^2])/(d*Sqrt[d*x]) - (8*PolyLog[2, a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^2])/(d*Sqrt[d*x])$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_3(ax^2)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + 4 \int \frac{\text{Li}_2(ax^2)}{(dx)^{3/2}} dx \\
 &= -\frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} - 16 \int \frac{\log(1-ax^2)}{(dx)^{3/2}} dx \\
 &= \frac{32 \log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a) \int \frac{x}{\sqrt{dx}(1-ax^2)} dx}{d} \\
 &= \frac{32 \log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a) \int \frac{\sqrt{dx}}{1-ax^2} dx}{d^2} \\
 &= \frac{32 \log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(128a) \text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{d^3} \\
 &= \frac{32 \log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64\sqrt{a}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{d} - \frac{(64\sqrt{a}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{d} \\
 &= -\frac{64\sqrt[4]{a} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{64\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{32 \log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0897261, size = 71, normalized size = 0.58

$$\frac{x\text{Gamma}\left(\frac{3}{4}\right)\left(64ax^2\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) - 12\text{PolyLog}(2, ax^2) - 3\text{PolyLog}(3, ax^2) + 48 \log(1-ax^2)\right)}{2\text{Gamma}\left(\frac{7}{4}\right)(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/(d\*x)^(3/2), x]

[Out] (x\*Gamma[3/4]\*(64\*a\*x^2\*Hypergeometric2F1[3/4, 1, 7/4, a\*x^2] + 48\*Log[1 - a\*x^2] - 12\*PolyLog[2, a\*x^2] - 3\*PolyLog[3, a\*x^2]))/(2\*(d\*x)^(3/2)\*Gamma[7/4])

**Maple [A]** time = 0.191, size = 131, normalized size = 1.1

$$-\frac{1}{2}x^2\sqrt[4]{-a}\left(-64\frac{x^{3/2}(-a)^{3/4}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)}{(ax^2)^{3/4}}+64\frac{(-a)^{3/4}\ln(-ax^2+1)}{\sqrt{xa}}-16\frac{(-a)^{3/4}}{\sqrt{xa}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/(d\*x)^(3/2),x)

[Out] -1/2/(d\*x)^(3/2)\*x^(3/2)\*(-a)^(1/4)\*(-64\*x^(3/2)\*(-a)^(3/4)/(a\*x^2)^(3/4)\*(ln(1-(a\*x^2)^(1/4))-ln(1+(a\*x^2)^(1/4))+2\*arctan((a\*x^2)^(1/4)))+64/x^(1/2)\*(-a)^(3/4)/a\*ln(-a\*x^2+1)-16/x^(1/2)\*(-a)^(3/4)\*polylog(2,a\*x^2)/a-4/x^(1/2)\*(-a)^(3/4)/a\*polylog(3,a\*x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.0072, size = 549, normalized size = 4.5

$$2\left(64d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dxa}\left(\frac{a}{d^6}\right)^{\frac{1}{4}}-\sqrt{ad^4\sqrt{\frac{a}{d^6}}+a^2dxd}\left(\frac{a}{d^6}\right)^{\frac{1}{4}}}{a}\right)+16d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(32768d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+32768\sqrt{dxa}\right)-16d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(3/2),x, algorithm="fricas")

[Out] 2\*(64\*d^2\*x\*(a/d^6)^(1/4)\*arctan(-(sqrt(d\*x)\*a\*d\*(a/d^6)^(1/4) - sqrt(a\*d^4\*sqrt(a/d^6) + a^2\*d\*x\*d\*(a/d^6)^(1/4)))/a) + 16\*d^2\*x\*(a/d^6)^(1/4)\*log(32768\*d^5\*(a/d^6)^(3/4) + 32768\*sqrt(d\*x)\*a) - 16\*d^2\*x\*(a/d^6)^(1/4)\*log(-32768\*d^5\*(a/d^6)^(3/4) + 32768\*sqrt(d\*x)\*a) - 4\*sqrt(d\*x)\*%iint(a, x, -log(-a\*x^2 + 1)/a, -2\*log(-a\*x^2 + 1)/x) + 16\*sqrt(d\*x)\*log(-a\*x^2 + 1) - sqrt(d\*x)\*polylog(3, a\*x^2)/(d^2\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(polylog(3,a*x**2)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

### 3.83 $\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=132

$$-\frac{8\text{PolyLog}(2, ax^2)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax^2)}{3d(dx)^{3/2}} + \frac{64a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32 \log(1 - ax^2)}{27d(dx)^{3/2}}$$

[Out] (64\*a^(3/4)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*d^(5/2)) + (64\*a^(3/4)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*d^(5/2)) + (32\*Log[1 - a\*x^2])/(27\*d\*(d\*x)^(3/2)) - (8\*PolyLog[2, a\*x^2])/(9\*d\*(d\*x)^(3/2)) - (2\*PolyLog[3, a\*x^2])/(3\*d\*(d\*x)^(3/2))

**Rubi [A]** time = 0.0910165, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$ , Rules used = {6591, 2455, 16, 329, 212, 208, 205}

$$-\frac{8\text{PolyLog}(2, ax^2)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax^2)}{3d(dx)^{3/2}} + \frac{64a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32 \log(1 - ax^2)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/(d\*x)^(5/2), x]

[Out] (64\*a^(3/4)\*ArcTan[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*d^(5/2)) + (64\*a^(3/4)\*ArcTanh[(a^(1/4)\*Sqrt[d\*x])/Sqrt[d]])/(27\*d^(5/2)) + (32\*Log[1 - a\*x^2])/(27\*d\*(d\*x)^(3/2)) - (8\*PolyLog[2, a\*x^2])/(9\*d\*(d\*x)^(3/2)) - (2\*PolyLog[3, a\*x^2])/(3\*d\*(d\*x)^(3/2))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a + (b\*x^(k\*n)))/c^n]^(p), x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
  Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
  /b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{4}{3} \int \frac{\text{Li}_2(ax^2)}{(dx)^{5/2}} dx \\
&= -\frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} - \frac{16}{9} \int \frac{\log(1-ax^2)}{(dx)^{5/2}} dx \\
&= \frac{32 \log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a) \int \frac{x}{(dx)^{3/2}(1-ax^2)} dx}{27d} \\
&= \frac{32 \log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a) \int \frac{1}{\sqrt{dx}(1-ax^2)} dx}{27d^2} \\
&= \frac{32 \log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(128a) \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{27d^3} \\
&= \frac{32 \log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}} + \frac{(64a) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{27d^2} + \frac{(64a) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{27d^2} \\
&= \frac{64a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32 \log(1-ax^2)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2(ax^2)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^2)}{3d(dx)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0935473, size = 71, normalized size = 0.54

$$\frac{x\text{Gamma}\left(\frac{1}{4}\right)\left(64ax^2\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) - 12\text{PolyLog}(2, ax^2) - 9\text{PolyLog}(3, ax^2) + 16 \log(1-ax^2)\right)}{54\text{Gamma}\left(\frac{5}{4}\right)(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(5/2), x]
```

```
[Out] (x*Gamma[1/4]*(64*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1 -
  a*x^2] - 12*PolyLog[2, a*x^2] - 9*PolyLog[3, a*x^2]))/(54*(d*x)^(5/2)*Gamma
  [5/4])
```

**Maple [A]** time = 0.194, size = 131, normalized size = 1.

$$-\frac{1}{2}x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{64}{27}\sqrt{x}\sqrt[4]{-a}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)-2\arctan\left(\sqrt[4]{ax^2}\right)\right)\frac{1}{\sqrt[4]{ax^2}}+\frac{64\ln(-ax^2+1)}{27a}\sqrt[4]{-ax}^{-\frac{3}{2}}-\frac{16}{9}\sqrt[4]{-ax}^{-\frac{3}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/(d\*x)^(5/2),x)

[Out] -1/2/(d\*x)^(5/2)\*x^(5/2)\*(-a)^(3/4)\*(-64/27\*x^(1/2)\*(-a)^(1/4)/(a\*x^2)^(1/4))\*(ln(1-(a\*x^2)^(1/4))-ln(1+(a\*x^2)^(1/4))-2\*arctan((a\*x^2)^(1/4)))+64/27/x^(3/2)\*(-a)^(1/4)/a\*ln(-a\*x^2+1)-16/9/x^(3/2)\*(-a)^(1/4)\*polylog(2,a\*x^2)/a-4/3/x^(3/2)\*(-a)^(1/4)/a\*polylog(3,a\*x^2)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.81842, size = 591, normalized size = 4.48

$$2\left(64d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{d}xad^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^6\sqrt{\frac{a^3}{d^{10}}+a^2}dxd^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}}}{a^3}\right)-16d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(32d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+32\sqrt{d}xa\right)+16d^3x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(5/2),x, algorithm="fricas")

[Out] -2/27\*(64\*d^3\*x^2\*(a^3/d^10)^(1/4)\*arctan(-(sqrt(d\*x)\*a\*d^7\*(a^3/d^10)^(3/4))-sqrt(d^6\*sqrt(a^3/d^10)+a^2\*d\*x)\*d^7\*(a^3/d^10)^(3/4))/a^3)-16\*d^3\*x^2\*(a^3/d^10)^(1/4)\*log(32\*d^3\*(a^3/d^10)^(1/4)+32\*sqrt(d\*x)\*a)+16\*d^3\*x^2\*(a^3/d^10)^(1/4)\*log(-32\*d^3\*(a^3/d^10)^(1/4)+32\*sqrt(d\*x)\*a)+12\*sqrt(d\*x)\*%iint(a,x,-log(-a\*x^2+1)/a,-2\*log(-a\*x^2+1)/x)-16\*sqrt(d\*x)\*log(-a\*x^2+1)+9\*sqrt(d\*x)\*polylog(3,a\*x^2)/(d^3\*x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/(d\*x)\*\*(5/2),x)

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

### 3.84 $\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{7/2}} dx$

**Optimal.** Leaf size=147

$$-\frac{8\text{PolyLog}(2, ax^2)}{25d(dx)^{5/2}} - \frac{2\text{PolyLog}(3, ax^2)}{5d(dx)^{5/2}} - \frac{64a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} - \frac{128a}{125d^3\sqrt{dx}} + \frac{32 \log(1 - ax^2)}{125d(dx)^{5/2}}$$

[Out]  $(-128*a)/(125*d^3*\text{Sqrt}[d*x]) - (64*a^{(5/4)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (64*a^{(5/4)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (32*\text{Log}[1 - a*x^2])/(125*d*(d*x)^{(5/2)}) - (8*\text{PolyLog}[2, a*x^2])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[3, a*x^2])/(5*d*(d*x)^{(5/2)})$

**Rubi [A]** time = 0.106859, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 325, 329, 298, 205, 208}

$$-\frac{8\text{PolyLog}(2, ax^2)}{25d(dx)^{5/2}} - \frac{2\text{PolyLog}(3, ax^2)}{5d(dx)^{5/2}} - \frac{64a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} - \frac{128a}{125d^3\sqrt{dx}} + \frac{32 \log(1 - ax^2)}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/(d\*x)^(7/2), x]

[Out]  $(-128*a)/(125*d^3*\text{Sqrt}[d*x]) - (64*a^{(5/4)}*\text{ArcTan}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (64*a^{(5/4)}*\text{ArcTanh}[(a^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(125*d^{(7/2)}) + (32*\text{Log}[1 - a*x^2])/(125*d*(d*x)^{(5/2)}) - (8*\text{PolyLog}[2, a*x^2])/(25*d*(d*x)^{(5/2)}) - (2*\text{PolyLog}[3, a*x^2])/(5*d*(d*x)^{(5/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p]

x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{(dx)^{7/2}} dx &= -\frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{4}{5} \int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}} dx \\
&= -\frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} - \frac{16}{25} \int \frac{\log(1-ax^2)}{(dx)^{7/2}} dx \\
&= \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a) \int \frac{x}{(dx)^{5/2}(1-ax^2)} dx}{125d} \\
&= \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a) \int \frac{1}{(dx)^{3/2}(1-ax^2)} dx}{125d^2} \\
&= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^2) \int \frac{\sqrt{dx}}{1-ax^2} dx}{125d^4} \\
&= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(128a^2) \text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{125d^5} \\
&= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^{3/2}) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{125d^3} \\
&= -\frac{128a}{125d^3\sqrt{dx}} - \frac{64a^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{32 \log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0986821, size = 79, normalized size = 0.54

$$\frac{x\text{Gamma}\left(-\frac{1}{4}\right)\left(64a^2x^4\text{Hypergeometric2F1}\left(\frac{3}{4}, 1, \frac{7}{4}, ax^2\right) - 60\text{PolyLog}\left(2, ax^2\right) - 75\text{PolyLog}\left(3, ax^2\right) - 192ax^2\right)}{750\text{Gamma}\left(\frac{3}{4}\right)(dx)^{7/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/(d\*x)^(7/2),x]

[Out]  $-(x*\Gamma[-1/4]*(-192*a*x^2 + 64*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 48*\text{Log}[1 - a*x^2] - 60*\text{PolyLog}[2, a*x^2] - 75*\text{PolyLog}[3, a*x^2]))/(750*(d*x)^(7/2)*\Gamma[3/4])$

**Maple [A]** time = 0.206, size = 142, normalized size = 1.

$$-\frac{1}{2}x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{256}{125}\frac{1}{\sqrt{x}}\frac{1}{\sqrt[4]{-a}}-\frac{64a}{125}x^{\frac{3}{2}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)\frac{1}{\sqrt[4]{-a}}(ax^2)^{-\frac{3}{4}}+\frac{64\ln(-ax^2)}{125a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/(d\*x)^(7/2),x)

[Out]  $-1/2/(d*x)^(7/2)*x^(7/2)*(-a)^(5/4)*(-256/125/x^(1/2)/(-a)^(1/4)-64/125*x^(3/2)/(-a)^(1/4)*a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))+64/125/x^(5/2)/(-a)^(1/4)/a*ln(-a*x^2+1)-16/25/x^(5/2)/(-a)^(1/4)*polylog(2,a*x^2)/a-4/5/x^(5/2)/(-a)^(1/4)/a*polylog(3,a*x^2)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.00294, size = 641, normalized size = 4.36

$$2\left(64d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dxa^4}d^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}-\sqrt{a^5d^8}\sqrt{\frac{a^5}{d^{14}}+a^8dxd^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}}}{a^5}\right)+16d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(32768d^{11}\left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}}+32768\sqrt{dxa^4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(7/2),x, algorithm="fricas")

[Out]  $2/125*(64*d^4*x^3*(a^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*a^4*d^3*(a^5/d^14)^(1/4) - sqrt(a^5*d^8*sqrt(a^5/d^14) + a^8*d*x)*d^3*(a^5/d^14)^(1/4))/a^5) + 16*d^4*x^3*(a^5/d^14)^(1/4)*log(32768*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x)*a^4) - 16*d^4*x^3*(a^5/d^14)^(1/4)*log(-32768*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x)*a^4) - 16*(4*a*x^2 - log(-a*x^2 + 1))*sqrt(d*x) - 20*sqrt(d*x)*%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 25*sqrt(d*x)*polyl$



$\text{og}(3, a*x^2)/(d^4*x^3)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/(d\*x)\*\*(7/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(7/2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2)/(d\*x)^(7/2), x)

### 3.85 $\int \frac{\text{PolyLog}(3, ax^2)}{(dx)^{9/2}} dx$

**Optimal.** Leaf size=147

$$-\frac{8\text{PolyLog}(2, ax^2)}{49d(dx)^{7/2}} - \frac{2\text{PolyLog}(3, ax^2)}{7d(dx)^{7/2}} + \frac{64a^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} - \frac{128a}{1029d^3(dx)^{3/2}} + \frac{32 \log(1 - ax^2)}{343d(dx)^{3/2}}$$

[Out]  $(-128*a)/(1029*d^3*(d*x)^{(3/2)}) + (64*a^{(7/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(343*d^{(9/2)}) + (64*a^{(7/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(343*d^{(9/2)}) + (32*Log[1 - a*x^2])/(343*d*(d*x)^{(7/2)}) - (8*PolyLog[2, a*x^2])/(49*d*(d*x)^{(7/2)}) - (2*PolyLog[3, a*x^2])/(7*d*(d*x)^{(7/2)})$

**Rubi [A]** time = 0.105684, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6591, 2455, 16, 325, 329, 212, 208, 205}

$$-\frac{8\text{PolyLog}(2, ax^2)}{49d(dx)^{7/2}} - \frac{2\text{PolyLog}(3, ax^2)}{7d(dx)^{7/2}} + \frac{64a^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} - \frac{128a}{1029d^3(dx)^{3/2}} + \frac{32 \log(1 - ax^2)}{343d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^2]/(d\*x)^(9/2), x]

[Out]  $(-128*a)/(1029*d^3*(d*x)^{(3/2)}) + (64*a^{(7/4)}*ArcTan[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(343*d^{(9/2)}) + (64*a^{(7/4)}*ArcTanh[(a^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(343*d^{(9/2)}) + (32*Log[1 - a*x^2])/(343*d*(d*x)^{(7/2)}) - (8*PolyLog[2, a*x^2])/(49*d*(d*x)^{(7/2)}) - (2*PolyLog[3, a*x^2])/(7*d*(d*x)^{(7/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_.)^(m\_.)\*((b\_.)\*(v\_.)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 325

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^2)}{(dx)^{9/2}} dx &= -\frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{4}{7} \int \frac{\text{Li}_2(ax^2)}{(dx)^{9/2}} dx \\
&= -\frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} - \frac{16}{49} \int \frac{\log(1-ax^2)}{(dx)^{9/2}} dx \\
&= \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a) \int \frac{x}{(dx)^{7/2}(1-ax^2)} dx}{343d} \\
&= \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a) \int \frac{1}{(dx)^{5/2}(1-ax^2)} dx}{343d^2} \\
&= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a^2) \int \frac{1}{\sqrt{dx}(1-ax^2)} dx}{343d^4} \\
&= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(128a^2) \text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}} dx, x, \sqrt{dx}\right)}{343d^5} \\
&= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3(ax^2)}{7d(dx)^{7/2}} + \frac{(64a^2) \text{Subst}\left(\int \frac{1}{d-\sqrt{ax^2}} dx, x, \sqrt{dx}\right)}{343d^4} \\
&= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{64a^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{32 \log(1-ax^2)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2(ax^2)}{49d(dx)^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0961169, size = 84, normalized size = 0.57

$$\frac{\Gamma\left(-\frac{3}{4}\right)\sqrt{dx}\left(192a^2x^4\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, ax^2\right) - 84\text{PolyLog}\left(2, ax^2\right) - 147\text{PolyLog}\left(3, ax^2\right) - 64\right)}{686d^5x^4\Gamma\left(\frac{1}{4}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^2]/(d\*x)^(9/2), x]

[Out]  $-(\text{Sqrt}[d*x]*\text{Gamma}[-3/4]*(-64*a*x^2 + 192*a^2*x^4*\text{Hypergeometric2F1}[1/4, 1, 5/4, a*x^2] + 48*\text{Log}[1 - a*x^2] - 84*\text{PolyLog}[2, a*x^2] - 147*\text{PolyLog}[3, a*x^2]))/(686*d^5*x^4*\text{Gamma}[1/4])$

**Maple [A]** time = 0.191, size = 142, normalized size = 1.

$$-\frac{1}{2}x^{\frac{9}{2}}(-a)^{\frac{7}{4}}\left(-\frac{256}{1029}x^{-\frac{3}{2}}(-a)^{-\frac{3}{4}}-\frac{64a}{343}\sqrt{x}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)-2\arctan\left(\sqrt[4]{ax^2}\right)\right)\right)(-a)^{-\frac{3}{4}}\frac{1}{\sqrt[4]{ax^2}}+\frac{64\ln(-a)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^2)/(d\*x)^(9/2), x)

[Out]  $-1/2/(d*x)^{(9/2)}*x^{(9/2)}*(-a)^{(7/4)}*(-256/1029/x^{(3/2)}/(-a)^{(3/4)}-64/343*x^{(1/2)}/(-a)^{(3/4)}*a/(a*x^2)^{(1/4)}*(\ln(1-(a*x^2)^{(1/4)})-\ln(1+(a*x^2)^{(1/4)})-2*\arctan((a*x^2)^{(1/4)}))+64/343/x^{(7/2)}/(-a)^{(3/4)}/a*\ln(-a*x^2+1)-16/49/x^{(7/2)}/(-a)^{(3/4)}*polylog(2,a*x^2)/a-4/7/x^{(7/2)}/(-a)^{(3/4)}/a*polylog(3,a*x^2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(9/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.05735, size = 629, normalized size = 4.28

$$2\left(192d^5x^4\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{dxa^2}d^{13}\left(\frac{a^7}{d^{18}}\right)^{\frac{3}{4}}-\sqrt{d^{10}\sqrt{\frac{a^7}{d^{18}}+a^4dxd^{13}}\left(\frac{a^7}{d^{18}}\right)^{\frac{3}{4}}}}{a^7}\right)-48d^5x^4\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(32d^5\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}+32\sqrt{dxa^2}\right)+48\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(9/2), x, algorithm="fricas")

[Out]  $-2/1029*(192*d^5*x^4*(a^7/d^18)^{(1/4)}*\arctan(-(\text{sqrt}(d*x)*a^2*d^{13}*(a^7/d^18)^{(3/4)} - \text{sqrt}(d^{10}*\text{sqrt}(a^7/d^18) + a^4*d*x)*d^{13}*(a^7/d^18)^{(3/4)})/a^7) - 48*d^5*x^4*(a^7/d^18)^{(1/4)}*\log(32*d^5*(a^7/d^18)^{(1/4)} + 32*\text{sqrt}(d*x)*a^2) + 48*d^5*x^4*(a^7/d^18)^{(1/4)}*\log(-32*d^5*(a^7/d^18)^{(1/4)} + 32*\text{sqrt}(d*x)*a^2) + 16*(4*a*x^2 - 3*\log(-a*x^2 + 1))*\text{sqrt}(d*x) + 84*\text{sqrt}(d*x)*\%iint(a, x, -\log(-a*x^2 + 1)/a, -2*\log(-a*x^2 + 1)/x) + 147*\text{sqrt}(d*x)*polylog(3, a*x^2))/(d^5*x^4)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*2)/(d\*x)\*\*(9/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^2)/(d\*x)^(9/2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^2)/(d\*x)^(9/2), x)

### 3.86 $\int (dx)^{3/2} \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=101

$$\frac{8adq^2\sqrt{d}xx^{q+2}\text{Hypergeometric2F1}\left(1, \frac{q+\frac{5}{2}}{q}, \frac{1}{2}\left(\frac{5}{q}+4\right), ax^q\right)}{25(2q+5)} + \frac{2(dx)^{5/2}\text{PolyLog}(2, ax^q)}{5d} + \frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}$$

[Out]  $(8*a*d*q^2*x^{(2+q)}*Sqrt[d*x]*Hypergeometric2F1[1, (5/2+q)/q, (4+5/q)/2, a*x^q])/(25*(5+2*q)) + (4*q*(d*x)^{(5/2)}*Log[1-a*x^q])/(25*d) + (2*(d*x)^{(5/2)}*PolyLog[2, a*x^q])/(5*d)$

**Rubi [A]** time = 0.0585402, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{2(dx)^{5/2}\text{PolyLog}(2, ax^q)}{5d} + \frac{8adq^2\sqrt{d}xx^{q+2}{}_2F_1\left(1, \frac{q+\frac{5}{2}}{q}; \frac{1}{2}\left(4+\frac{5}{q}\right); ax^q\right)}{25(2q+5)} + \frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*PolyLog[2, a*x^q], x]$

[Out]  $(8*a*d*q^2*x^{(2+q)}*Sqrt[d*x]*Hypergeometric2F1[1, (5/2+q)/q, (4+5/q)/2, a*x^q])/(25*(5+2*q)) + (4*q*(d*x)^{(5/2)}*Log[1-a*x^q])/(25*d) + (2*(d*x)^{(5/2)}*PolyLog[2, a*x^q])/(5*d)$

#### Rule 6591

$\text{Int}[(d_*)*(x_*)^{(m_*)}*PolyLog[n_*, (a_*)*((b_*)*(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*PolyLog[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

$\text{Int}[(a_*) + \text{Log}[(c_*)*(d_*) + (e_*)*(x_*)^{(n_*)})^{(p_*)}]* (b_*)*((f_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])/(f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^{(n-1)}*(f*x)^{(m+1)})/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

$\text{Int}[(u_*)*((a_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[(b*\text{IntPart}[n]*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$  FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])]/(c*(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2} \text{Li}_2(ax^q) dx &= \frac{2(dx)^{5/2} \text{Li}_2(ax^q)}{5d} + \frac{1}{5}(2q) \int (dx)^{3/2} \log(1 - ax^q) dx \\
&= \frac{4q(dx)^{5/2} \log(1 - ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^q)}{5d} + \frac{(4aq^2) \int \frac{x^{-1+q}(dx)^{5/2}}{1-ax^q} dx}{25d} \\
&= \frac{4q(dx)^{5/2} \log(1 - ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^q)}{5d} + \frac{(4adq^2 \sqrt{dx}) \int \frac{x^{2+q}}{1-ax^q} dx}{25\sqrt{x}} \\
&= \frac{8adq^2 x^{2+q} \sqrt{dx} {}_2F_1\left(1, \frac{5+q}{q}; \frac{1}{2}\left(4 + \frac{5}{q}\right); ax^q\right)}{25(5+2q)} + \frac{4q(dx)^{5/2} \log(1 - ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_2(ax^q)}{5d}
\end{aligned}$$

**Mathematica [A]** time = 0.124333, size = 82, normalized size = 0.81

$$\frac{2x(dx)^{3/2} \left( 4aq^2 x^q \text{Hypergeometric2F1}\left(1, \frac{q+\frac{5}{2}}{q}, \frac{5}{2q} + 2, ax^q\right) + (2q+5) (5\text{PolyLog}(2, ax^q) + 2q \log(1 - ax^q)) \right)}{25(2q+5)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*PolyLog[2, a\*x^q], x]

[Out] (2\*x\*(d\*x)^(3/2)\*(4\*a\*q^2\*x^q\*Hypergeometric2F1[1, (5/2 + q)/q, 2 + 5/(2\*q), a\*x^q] + (5 + 2\*q)\*(2\*q\*Log[1 - a\*x^q] + 5\*PolyLog[2, a\*x^q]))/(25\*(5 + 2\*q))

**Maple [C]** time = 0.255, size = 121, normalized size = 1.2

$$-\frac{1}{q} (dx)^{\frac{3}{2}} (-a)^{-\frac{5}{2q}} \left( -\frac{4q^2 \ln(1 - ax^q)}{25} x^{\frac{5}{2}} (-a)^{\frac{5}{2q}} - 2 \frac{qx^{5/2} (1 + 2/5q) \text{polylog}(2, ax^q)}{5 + 2q} (-a)^{5/2q-1} - \frac{4q^2 a}{25} x^{5/2+q} (-a)^{\frac{5}{2q}} \text{LerchPhi}(ax^q, 1, 1/2(5+2q)/q) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*polylog(2, a\*x^q), x)

[Out] -(d\*x)^(3/2)/x^(3/2)\*(-a)^(-5/2/q)/q\*(-4/25\*q^2\*x^(5/2)\*(-a)^(5/2/q)\*ln(1-a\*x^q)-2\*q/(5+2\*q)\*x^(5/2)\*(-a)^(5/2/q)\*(1+2/5\*q)\*polylog(2, a\*x^q)-4/25\*q^2\*x^(5/2+q)\*a\*(-a)^(5/2/q)\*LerchPhi(a\*x^q, 1, 1/2\*(5+2\*q)/q)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8d^{\frac{3}{2}}q^3 \int \frac{x^{\frac{3}{2}}}{25((2a^2q - 5a^2)x^{2q} - 2(2aq - 5a)x^q + 2q - 5)} dx + \frac{2\left(25\left(\left(2ad^{\frac{3}{2}}q - 5ad^{\frac{3}{2}}\right)xx^q - \left(2d^{\frac{3}{2}}q - 5d^{\frac{3}{2}}\right)x\right)x^{\frac{3}{2}}\text{Li}_2\right)}{25((2a^2q - 5a^2)x^{2q} - 2(2aq - 5a)x^q + 2q - 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*polylog(2, a\*x^q), x, algorithm="maxima")

```
[Out] 8*d^(3/2)*q^3*integrate(1/25*x^(3/2)/((2*a^2*q - 5*a^2)*x^(2*q) - 2*(2*a*q - 5*a)*x^q + 2*q - 5), x) + 2/125*(25*((2*a*d^(3/2)*q - 5*a*d^(3/2))*x*x^q - (2*d^(3/2)*q - 5*d^(3/2))*x)*x^(3/2)*dilog(a*x^q) + 10*((2*a*d^(3/2)*q^2 - 5*a*d^(3/2)*q)*x*x^q - (2*d^(3/2)*q^2 - 5*d^(3/2)*q)*x)*x^(3/2)*log(-a*x^q + 1) + 4*(2*d^(3/2)*q^3*x - (2*a*d^(3/2)*q^3 - 5*a*d^(3/2)*q^2)*x*x^q)*x^(3/2)/((2*a*q - 5*a)*x^q - 2*q + 5)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}dx\text{Li}_2(ax^q), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*dilog(a*x^q), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x^q), x)
```



### 3.87 $\int \sqrt{dx} \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=100

$$\frac{8aq^2 \sqrt{dx} x^{q+1} \text{Hypergeometric2F1}\left(1, \frac{q+\frac{3}{2}}{q}, \frac{1}{2}\left(\frac{3}{q}+4\right), ax^q\right)}{9(2q+3)} + \frac{2(dx)^{3/2} \text{PolyLog}(2, ax^q)}{3d} + \frac{4q(dx)^{3/2} \log(1-ax^q)}{9d}$$

[Out]  $(8*a*q^2*x^{(1+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (3/2+q)/q, (4+3/q)/2, a*x^q])/(9*(3+2*q)) + (4*q*(d*x)^{(3/2)}*\text{Log}[1-a*x^q])/(9*d) + (2*(d*x)^{(3/2)}*\text{PolyLog}[2, a*x^q])/(3*d)$

**Rubi [A]** time = 0.0546737, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{2(dx)^{3/2} \text{PolyLog}(2, ax^q)}{3d} + \frac{8aq^2 \sqrt{dx} x^{q+1} {}_2F_1\left(1, \frac{q+\frac{3}{2}}{q}; \frac{1}{2}\left(4+\frac{3}{q}\right); ax^q\right)}{9(2q+3)} + \frac{4q(dx)^{3/2} \log(1-ax^q)}{9d}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[d*x]*PolyLog[2, a*x^q], x]`

[Out]  $(8*a*q^2*x^{(1+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (3/2+q)/q, (4+3/q)/2, a*x^q])/(9*(3+2*q)) + (4*q*(d*x)^{(3/2)}*\text{Log}[1-a*x^q])/(9*d) + (2*(d*x)^{(3/2)}*\text{PolyLog}[2, a*x^q])/(3*d)$

#### Rule 6591

`Int[((d_.)*(x_.))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_.)^(p_.))^(q_.)], x_Symbol] :> Simp[((d*x)^(m+1)*PolyLog[n, a*(b*x^p)^q])/(d*(m+1)), x] - Dist[(p*q)/(m+1), Int[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]`

#### Rule 2455

`Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.)^(n_.))^(p_.)]*(b_.))*((f_.)*(x_.))^(m_.), x_Symbol] :> Simp[((f*x)^(m+1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m+1)), x] - Dist[(b*e*n*p)/(f*(m+1)), Int[(x^(n-1)*(f*x)^(m+1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]`

#### Rule 20

`Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]`

#### Rule 364

`Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])`

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} \operatorname{Li}_2(ax^q) dx &= \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^q)}{3d} + \frac{1}{3}(2q) \int \sqrt{dx} \log(1 - ax^q) dx \\
&= \frac{4q(dx)^{3/2} \log(1 - ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^q)}{3d} + \frac{(4aq^2) \int \frac{x^{-1+q}(dx)^{3/2}}{1-ax^q} dx}{9d} \\
&= \frac{4q(dx)^{3/2} \log(1 - ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^q)}{3d} + \frac{(4aq^2 \sqrt{dx}) \int \frac{x^{\frac{1}{2}+q}}{1-ax^q} dx}{9\sqrt{x}} \\
&= \frac{8aq^2 x^{1+q} \sqrt{dx} {}_2F_1\left(1, \frac{\frac{3}{2}+q}{q}; \frac{1}{2}\left(4 + \frac{3}{q}\right); ax^q\right)}{9(3+2q)} + \frac{4q(dx)^{3/2} \log(1 - ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_2(ax^q)}{3d}
\end{aligned}$$

**Mathematica [A]** time = 0.106325, size = 82, normalized size = 0.82

$$\frac{2x\sqrt{dx} \left( 4aq^2 x^q \operatorname{Hypergeometric2F1}\left(1, \frac{q+\frac{3}{2}}{q}, \frac{3}{2q} + 2, ax^q\right) + (2q+3) (3 \operatorname{PolyLog}(2, ax^q) + 2q \log(1 - ax^q)) \right)}{9(2q+3)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*PolyLog[2, a\*x^q], x]

[Out] (2\*x\*Sqrt[d\*x]\*(4\*a\*q^2\*x^q\*Hypergeometric2F1[1, (3/2 + q)/q, 2 + 3/(2\*q), a\*x^q] + (3 + 2\*q)\*(2\*q\*Log[1 - a\*x^q] + 3\*PolyLog[2, a\*x^q])))/(9\*(3 + 2\*q))

**Maple [C]** time = 0.229, size = 121, normalized size = 1.2

$$-\frac{1}{q} \sqrt{dx} (-a)^{-\frac{3}{2q}} \left( -\frac{4q^2 \ln(1 - ax^q)}{9} x^{\frac{3}{2}} (-a)^{\frac{3}{2q}} - 2 \frac{qx^{3/2} (1 + 2/3q) \operatorname{polylog}(2, ax^q)}{3 + 2q} (-a)^{3/2q-1} - \frac{4q^2 a}{9} x^{\frac{3}{2}+q} (-a)^{\frac{3}{2q}} \operatorname{LerchPhi} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*polylog(2,a\*x^q), x)

[Out] -(d\*x)^(1/2)/x^(1/2)\*(-a)^(-3/2/q)/q\*(-4/9\*q^2\*x^(3/2)\*(-a)^(3/2/q)\*ln(1-a\*x^q)-2\*q/(3+2\*q)\*x^(3/2)\*(-a)^(3/2/q)\*(1+2/3\*q)\*polylog(2,a\*x^q)-4/9\*q^2\*x^(3/2+q)\*a\*(-a)^(3/2/q)\*LerchPhi(a\*x^q, 1, 1/2\*(3+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8\sqrt{dq^3} \int \frac{\sqrt{x}}{9((2a^2q - 3a^2)x^{2q} - 2(2aq - 3a)x^q + 2q - 3)} dx + \frac{2(9((2a\sqrt{dq} - 3a\sqrt{d})xx^q - (2\sqrt{dq} - 3\sqrt{d})x)\sqrt{x}\operatorname{Li}_2(ax^q))}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*polylog(2,a\*x^q), x, algorithm="maxima")

```
[Out] 8*sqrt(d)*q^3*integrate(1/9*sqrt(x)/((2*a^2*q - 3*a^2)*x^(2*q) - 2*(2*a*q - 3*a)*x^q + 2*q - 3), x) + 2/27*(9*((2*a*sqrt(d)*q - 3*a*sqrt(d))*x*x^q - (2*sqrt(d)*q - 3*sqrt(d))*x)*sqrt(x)*dilog(a*x^q) + 6*((2*a*sqrt(d)*q^2 - 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - 3*sqrt(d)*q)*x)*sqrt(x)*log(-a*x^q + 1) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - 3*a*sqrt(d)*q^2)*x*x^q)*sqrt(x))/((2*a*q - 3*a)*x^q - 2*q + 3)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\text{Li}_2(ax^q), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x^q), x)
```

$$3.88 \quad \int \frac{\text{PolyLog}(2, ax^q)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=93

$$\frac{8aq^2\sqrt{dx}x^q\text{Hypergeometric2F1}\left(1, \frac{q+\frac{1}{2}}{q}, \frac{1}{2}\left(\frac{1}{q}+4\right), ax^q\right)}{d(2q+1)} + \frac{2\sqrt{dx}\text{PolyLog}(2, ax^q)}{d} + \frac{4q\sqrt{dx}\log(1-ax^q)}{d}$$

[Out] (8\*a\*q^2\*x^q\*Sqrt[d\*x]\*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2, a\*x^q])/(d\*(1 + 2\*q)) + (4\*q\*Sqrt[d\*x]\*Log[1 - a\*x^q])/d + (2\*Sqrt[d\*x]\*PolyLog[2, a\*x^q])/d

**Rubi [A]** time = 0.0540382, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{2\sqrt{dx}\text{PolyLog}(2, ax^q)}{d} + \frac{8aq^2\sqrt{dx}x^q {}_2F_1\left(1, \frac{q+\frac{1}{2}}{q}; \frac{1}{2}\left(4 + \frac{1}{q}\right); ax^q\right)}{d(2q+1)} + \frac{4q\sqrt{dx}\log(1-ax^q)}{d}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/Sqrt[d\*x], x]

[Out] (8\*a\*q^2\*x^q\*Sqrt[d\*x]\*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2, a\*x^q])/(d\*(1 + 2\*q)) + (4\*q\*Sqrt[d\*x]\*Log[1 - a\*x^q])/d + (2\*Sqrt[d\*x]\*PolyLog[2, a\*x^q])/d

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + (2q) \int \frac{\log(1-ax^q)}{\sqrt{dx}} dx \\
&= \frac{4q\sqrt{dx} \log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{(4aq^2) \int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q} dx}{d} \\
&= \frac{4q\sqrt{dx} \log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{(4aq^2\sqrt{dx}) \int \frac{x^{-\frac{1}{2}+q}}{1-ax^q} dx}{d\sqrt{x}} \\
&= \frac{8aq^2x^q\sqrt{dx} {}_2F_1\left(1, \frac{1}{2}+q; \frac{1}{2}\left(4+\frac{1}{q}\right); ax^q\right)}{d(1+2q)} + \frac{4q\sqrt{dx} \log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.0209061, size = 48, normalized size = 0.52

$$\frac{xG_{4,4}^{1,4}\left(-ax^q \left| \begin{matrix} 1, 1, 1, 1 - \frac{1}{2q} \\ 1, 0, 0, -\frac{1}{2q} \end{matrix} \right. \right)}{q\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[2, a\*x^q]/Sqrt[d\*x], x]

[Out] -(x\*MeijerG[{{1, 1, 1, 1 - 1/(2\*q)}, {}}, {{1}, {0, 0, -1/(2\*q)}}], -(a\*x^q)))/(q\*Sqrt[d\*x]))

**Maple [C]** time = 0.232, size = 109, normalized size = 1.2

$$-\frac{1}{q}\sqrt{x}(-a)^{-\frac{1}{2q}}\left(-4q^2\sqrt{x}(-a)^{1/2q-1}\ln(1-ax^q)-2q\sqrt{x}(-a)^{1/2q-1}\text{polylog}(2,ax^q)-4q^2x^{1/2+q}a(-a)^{1/2q-1}\text{LerchPhi}\left(\frac{ax^q}{1-ax^q},1,\frac{1}{2}+\frac{1}{q}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/(d\*x)^(1/2),x)

[Out] -1/(d\*x)^(1/2)\*x^(1/2)\*(-a)^(-1/2/q)/q\*(-4\*q^2\*x^(1/2)\*(-a)^(1/2/q)\*ln(1-a\*x^q)-2\*q\*x^(1/2)\*(-a)^(1/2/q)\*polylog(2,a\*x^q)-4\*q^2\*x^(1/2+q)\*a\*(-a)^(1/2/q)\*LerchPhi(a\*x^q,1,1/2\*(1+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8q^3 \int \frac{1}{\left((2a^2\sqrt{dq}-a^2\sqrt{d})x^{2q}-2(2a\sqrt{dq}-a\sqrt{d})x^q+2\sqrt{dq}-\sqrt{d}\right)\sqrt{x}} dx - \frac{2\left(\frac{((2a\sqrt{dq}-a\sqrt{d})x^q-(2\sqrt{dq}-\sqrt{d})x)\text{Li}_2(ax^q)}{\sqrt{x}} + \frac{2\sqrt{dq}-\sqrt{d}}{\sqrt{x}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/(d\*x)^(1/2),x, algorithm="maxima")

```
[Out] 8*q^3*integrate(1/(((2*a^2*sqrt(d)*q - a^2*sqrt(d))*x^(2*q) - 2*(2*a*sqrt(d)
)*q - a*sqrt(d))*x^q + 2*sqrt(d)*q - sqrt(d))*sqrt(x)), x) - 2*(((2*a*sqrt(
d)*q - a*sqrt(d))*x*x^q - (2*sqrt(d)*q - sqrt(d))*x)*dilog(a*x^q)/sqrt(x) +
2*((2*a*sqrt(d)*q^2 - a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - sqrt(d)*q)*x)*
log(-a*x^q + 1)/sqrt(x) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - a*sqrt(d)
*q^2)*x*x^q)/sqrt(x))/(2*d*q - (2*a*d*q - a*d)*x^q - d)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{Li}_2(ax^q)}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d*x), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**q)/sqrt(d*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/sqrt(d*x), x)
```

$$3.89 \quad \int \frac{\text{PolyLog}(2, ax^q)}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{8aq^2x^q \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(2 - \frac{1}{q}\right), \frac{1}{2}\left(4 - \frac{1}{q}\right), ax^q\right)}{d(1-2q)\sqrt{dx}} - \frac{2\text{PolyLog}(2, ax^q)}{d\sqrt{dx}} + \frac{4q \log(1 - ax^q)}{d\sqrt{dx}}$$

[Out]  $(-8*a*q^2*x^q*\text{Hypergeometric2F1}[1, (2 - q^{(-1)})/2, (4 - q^{(-1)})/2, a*x^q])/(d*(1 - 2*q)*\text{Sqrt}[d*x]) + (4*q*\text{Log}[1 - a*x^q])/(d*\text{Sqrt}[d*x]) - (2*\text{PolyLog}[2, a*x^q])/(d*\text{Sqrt}[d*x])$

**Rubi [A]** time = 0.0599145, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$-\frac{2\text{PolyLog}(2, ax^q)}{d\sqrt{dx}} - \frac{8aq^2x^q {}_2F_1\left(1, \frac{1}{2}\left(2 - \frac{1}{q}\right); \frac{1}{2}\left(4 - \frac{1}{q}\right); ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{4q \log(1 - ax^q)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/(d\*x)^(3/2), x]

[Out]  $(-8*a*q^2*x^q*\text{Hypergeometric2F1}[1, (2 - q^{(-1)})/2, (4 - q^{(-1)})/2, a*x^q])/(d*(1 - 2*q)*\text{Sqrt}[d*x]) + (4*q*\text{Log}[1 - a*x^q])/(d*\text{Sqrt}[d*x]) - (2*\text{PolyLog}[2, a*x^q])/(d*\text{Sqrt}[d*x])$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} - (2q) \int \frac{\log(1-ax^q)}{(dx)^{3/2}} dx \\
&= \frac{4q \log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{(4aq^2) \int \frac{x^{-1+q}}{\sqrt{dx}(1-ax^q)} dx}{d} \\
&= \frac{4q \log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{(4aq^2\sqrt{x}) \int \frac{x^{-\frac{3}{2}+q}}{1-ax^q} dx}{d\sqrt{dx}} \\
&= -\frac{8aq^2x^q {}_2F_1\left(1, \frac{1}{2}\left(2-\frac{1}{q}\right); \frac{1}{2}\left(4-\frac{1}{q}\right); ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{4q \log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}}
\end{aligned}$$

**Mathematica [C]** time = 0.0290786, size = 48, normalized size = 0.49

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1 + \frac{1}{2q} \\ 1, 0, 0, \frac{1}{2q} \end{matrix}\right)}{q(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[2, a\*x^q]/(d\*x)^(3/2), x]

[Out] -((x\*MeijerG[{{1, 1, 1, 1 + 1/(2\*q)}}, {}], {{1}, {0, 0, 1/(2\*q)}}, -(a\*x^q)])/ (q\*(d\*x)^(3/2)))

**Maple [C]** time = 0.224, size = 121, normalized size = 1.3

$$-\frac{1}{q}x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-4\frac{q^2\ln(1-ax^q)}{\sqrt{x}}(-a)^{-1/2q-1}-2\frac{q(1-2q)\text{polylog}(2,ax^q)}{(2q-1)\sqrt{x}}(-a)^{-1/2q-1}-4q^2x^{q-1/2}a(-a)^{-1/2q-1}\text{LerchPhi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/(d\*x)^(3/2),x)

[Out] -1/(d\*x)^(3/2)\*x^(3/2)\*(-a)^(1/2/q)/q\*(-4\*q^2/x^(1/2)\*(-a)^(-1/2/q)\*ln(1-a\*x^q)-2\*q/(2\*q-1)/x^(1/2)\*(-a)^(-1/2/q)\*(1-2\*q)\*polylog(2,a\*x^q)-4\*q^2\*x^(q-1/2)\*a\*(-a)^(-1/2/q)\*LerchPhi(a\*x^q,1,1/2\*(2\*q-1)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8q^3 \int \frac{1}{\left(2d^{\frac{3}{2}}q + \left(2a^2d^{\frac{3}{2}}q + a^2d^{\frac{3}{2}}\right)x^{2q} - 2\left(2ad^{\frac{3}{2}}q + ad^{\frac{3}{2}}\right)x^q + d^{\frac{3}{2}}\right)x^{\frac{3}{2}}} dx + \frac{2\left(\frac{\left(\left(2a\sqrt{d}q+a\sqrt{d}\right)xx^q - \left(2\sqrt{d}q+\sqrt{d}\right)x\right)\text{Li}_2(ax^q)}{x^{\frac{3}{2}}} - \frac{2\left(2a\sqrt{d}q+a\sqrt{d}\right)}{2d^2q}\right)}{2d^2q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/(d\*x)^(3/2),x, algorithm="maxima")



```
[Out] 8*q^3*integrate(1/((2*d^(3/2)*q + (2*a^2*d^(3/2)*q + a^2*d^(3/2))*x^(2*q) -
2*(2*a*d^(3/2)*q + a*d^(3/2))*x^q + d^(3/2))*x^(3/2)), x) + 2*(((2*a*sqrt(
d)*q + a*sqrt(d))*x*x^q - (2*sqrt(d)*q + sqrt(d))*x)*dilog(a*x^q)/x^(3/2) -
2*((2*a*sqrt(d)*q^2 + a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 + sqrt(d)*q)*x)*
log(-a*x^q + 1)/x^(3/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 + a*sqrt(d)
*q^2)*x*x^q)/x^(3/2))/(2*d^2*q + d^2 - (2*a*d^2*q + a*d^2)*x^q)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{Li}_2(ax^q)}{d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^2*x^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/(d*x)^(3/2), x)
```

### 3.90 $\int \frac{\text{PolyLog}(2, ax^q)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=105

$$\frac{8aq^2x^{q-1}\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(2 - \frac{3}{q}\right), \frac{1}{2}\left(4 - \frac{3}{q}\right), ax^q\right)}{9d^2(3 - 2q)\sqrt{dx}} - \frac{2\text{PolyLog}(2, ax^q)}{3d(dx)^{3/2}} + \frac{4q \log(1 - ax^q)}{9d(dx)^{3/2}}$$

[Out]  $(-8*a*q^2*x^{(-1 + q)}*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]) / (9*d^2*(3 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q]) / (9*d*(d*x)^{(3/2)}) - (2*PolyLog[2, a*x^q]) / (3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.0605675, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{2\text{PolyLog}(2, ax^q)}{3d(dx)^{3/2}} - \frac{8aq^2x^{q-1} {}_2F_1\left(1, \frac{1}{2}\left(2 - \frac{3}{q}\right); \frac{1}{2}\left(4 - \frac{3}{q}\right); ax^q\right)}{9d^2(3 - 2q)\sqrt{dx}} + \frac{4q \log(1 - ax^q)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a\*x^q]/(d\*x)^(5/2), x]

[Out]  $(-8*a*q^2*x^{(-1 + q)}*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]) / (9*d^2*(3 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q]) / (9*d*(d*x)^{(3/2)}) - (2*PolyLog[2, a*x^q]) / (3*d*(d*x)^{(3/2)})$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} - \frac{1}{3}(2q) \int \frac{\log(1-ax^q)}{(dx)^{5/2}} dx \\
&= \frac{4q \log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} + \frac{(4aq^2) \int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)} dx}{9d} \\
&= \frac{4q \log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} + \frac{(4aq^2\sqrt{x}) \int \frac{x^{-\frac{5}{2}+q}}{1-ax^q} dx}{9d^2\sqrt{dx}} \\
&= -\frac{8aq^2x^{-1+q} {}_2F_1\left(1, \frac{1}{2}\left(2-\frac{3}{q}\right); \frac{1}{2}\left(4-\frac{3}{q}\right); ax^q\right)}{9d^2(3-2q)\sqrt{dx}} + \frac{4q \log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0263274, size = 48, normalized size = 0.46

$$\frac{xG_{4,4}^{1,4}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1 + \frac{3}{2q} \\ 1, 0, 0, \frac{3}{2q} \end{matrix}\right)}{q(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[2, a\*x^q]/(d\*x)^(5/2), x]

[Out] -(x\*MeijerG[{{1, 1, 1, 1 + 3/(2\*q)}, {}}, {{1}, {0, 0, 3/(2\*q)}}, -(a\*x^q)]/(q\*(d\*x)^(5/2)))

**Maple [C]** time = 0.23, size = 121, normalized size = 1.2

$$-\frac{1}{q}x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{4q^2\ln(1-ax^q)}{9}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}}-2\frac{q(1-2/3q)\text{polylog}(2,ax^q)}{(-3+2q)x^{3/2}}(-a)^{-3/2q-1}-\frac{4q^2a}{9}x^{q-\frac{3}{2}}(-a)^{-\frac{3}{2q}}\text{LerchPhi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,a\*x^q)/(d\*x)^(5/2), x)

[Out] -1/(d\*x)^(5/2)\*x^(5/2)\*(-a)^(3/2/q)/q\*(-4/9\*q^2/x^(3/2)\*(-a)^(-3/2/q)\*ln(1-a\*x^q)-2\*q/(-3+2\*q)/x^(3/2)\*(-a)^(-3/2/q)\*(1-2/3\*q)\*polylog(2,a\*x^q)-4/9\*q^2\*x^(q-3/2)\*a\*(-a)^(-3/2/q)\*LerchPhi(a\*x^q,1,1/2\*(-3+2\*q)/q)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8q^3 \int \frac{1}{9\left(2d^{\frac{5}{2}}q + 3d^{\frac{5}{2}} + \left(2a^2d^{\frac{5}{2}}q + 3a^2d^{\frac{5}{2}}\right)x^{2q} - 2\left(2ad^{\frac{5}{2}}q + 3ad^{\frac{5}{2}}\right)x^q\right)x^{\frac{5}{2}}} dx + \frac{2\left(\frac{9\left((2a\sqrt{d}q+3a\sqrt{d})xx^q-(2\sqrt{d}q+3\sqrt{d})x\right)\text{Li}_2\left(\frac{ax^q}{1-ax^q}\right)}{x^{\frac{5}{2}}}\right)}{x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,a\*x^q)/(d\*x)^(5/2), x, algorithm="maxima")

```
[Out] 8*q^3*integrate(1/9/((2*d^(5/2)*q + 3*d^(5/2) + (2*a^2*d^(5/2)*q + 3*a^2*d^(5/2))*x^(2*q) - 2*(2*a*d^(5/2)*q + 3*a*d^(5/2))*x^q)*x^(5/2)), x) + 2/27*(9*((2*a*sqrt(d)*q + 3*a*sqrt(d))*x*x^q - (2*sqrt(d)*q + 3*sqrt(d))*x)*dilog(a*x^q)/x^(5/2) - 6*((2*a*sqrt(d)*q^2 + 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 + 3*sqrt(d)*q)*x)*log(-a*x^q + 1)/x^(5/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 + 3*a*sqrt(d)*q^2)*x*x^q)/x^(5/2))/(2*d^3*q + 3*d^3 - (2*a*d^3*q + 3*a*d^3)*x^q)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{Li}_2(ax^q)}{d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^3*x^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/(d*x)^(5/2), x)
```

### 3.91 $\int (dx)^{3/2} \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=125

$$\frac{16adq^3 \sqrt{d} x^{q+2} \text{Hypergeometric2F1}\left(1, \frac{q+\frac{5}{2}}{q}, \frac{1}{2}\left(\frac{5}{q}+4\right), ax^q\right)}{125(2q+5)} - \frac{4q(dx)^{5/2} \text{PolyLog}(2, ax^q)}{25d} + \frac{2(dx)^{5/2} \text{PolyLog}(3, ax^q)}{5d}$$

[Out]  $(-16*a*d*q^3*x^{(2+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (5/2+q)/q, (4+5/q)/2, a*x^q])/(125*(5+2*q)) - (8*q^2*(d*x)^{(5/2)}*\text{Log}[1-a*x^q])/(125*d) - (4*q*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x^q])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[3, a*x^q])/(5*d)$

**Rubi [A]** time = 0.0748302, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{4q(dx)^{5/2} \text{PolyLog}(2, ax^q)}{25d} + \frac{2(dx)^{5/2} \text{PolyLog}(3, ax^q)}{5d} - \frac{16adq^3 \sqrt{d} x^{q+2} {}_2F_1\left(1, \frac{q+\frac{5}{2}}{q}; \frac{1}{2}\left(4+\frac{5}{q}\right); ax^q\right)}{125(2q+5)} - \frac{8q^2(dx)^{5/2} \text{Log}[1-ax^q]}{125d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*\text{PolyLog}[3, a*x^q], x]$

[Out]  $(-16*a*d*q^3*x^{(2+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (5/2+q)/q, (4+5/q)/2, a*x^q])/(125*(5+2*q)) - (8*q^2*(d*x)^{(5/2)}*\text{Log}[1-a*x^q])/(125*d) - (4*q*(d*x)^{(5/2)}*\text{PolyLog}[2, a*x^q])/(25*d) + (2*(d*x)^{(5/2)}*\text{PolyLog}[3, a*x^q])/(5*d)$

#### Rule 6591

$\text{Int}[(d_*)*(x_*)^{(m_*)}*\text{PolyLog}[n_*, (a_*)*((b_*)*(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})^{(p_*)}]]*(b_*)*((f_*)*(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])]/(f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^(n-1)*(f*x)^(m+1))/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

$\text{Int}[(u_*)*((a_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$  FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

$\text{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/((c*(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt

Q[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int (dx)^{3/2} \text{Li}_3(ax^q) dx &= \frac{2(dx)^{5/2} \text{Li}_3(ax^q)}{5d} - \frac{1}{5}(2q) \int (dx)^{3/2} \text{Li}_2(ax^q) dx \\
 &= -\frac{4q(dx)^{5/2} \text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^q)}{5d} - \frac{1}{25}(4q^2) \int (dx)^{3/2} \log(1-ax^q) dx \\
 &= -\frac{8q^2(dx)^{5/2} \log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2} \text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^q)}{5d} - \frac{(8aq^3) \int \frac{x^{-1+q}(dx)^{5/2}}{1-ax^q} dx}{125d} \\
 &= -\frac{8q^2(dx)^{5/2} \log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2} \text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^q)}{5d} - \frac{(8adq^3 \sqrt{dx}) \int \frac{x^{\frac{3}{2}+q}}{1-ax^q} dx}{125\sqrt{x}} \\
 &= -\frac{16adq^3 x^{2+q} \sqrt{dx} {}_2F_1\left(1, \frac{5}{2}+q; \frac{1}{2}\left(4+\frac{5}{q}\right); ax^q\right)}{125(5+2q)} - \frac{8q^2(dx)^{5/2} \log(1-ax^q)}{125d} - \frac{4q(dx)^{5/2} \text{Li}_2(ax^q)}{25d} + \frac{2(dx)^{5/2} \text{Li}_3(ax^q)}{5d}
 \end{aligned}$$

**Mathematica [C]** time = 0.0297026, size = 50, normalized size = 0.4

$$\frac{x(dx)^{3/2} G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 - \frac{5}{2q} \\ 1, 0, 0, 0, -\frac{5}{2q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^(3/2)\*PolyLog[3, a\*x^q], x]

[Out] -((x\*(d\*x)^(3/2)\*MeijerG[{{1, 1, 1, 1, 1 - 5/(2\*q)}}, {}], {{1}, {0, 0, 0, - 5/(2\*q)}}}, -(a\*x^q])/q)

**Maple [C]** time = 0.379, size = 145, normalized size = 1.2

$$-\frac{1}{q} (dx)^{\frac{3}{2}} (-a)^{-\frac{5}{2q}} \left( \frac{8q^3 \ln(1-ax^q)}{125} x^{\frac{5}{2}} (-a)^{\frac{5}{2q}} + \frac{4q^2 \text{polylog}(2, ax^q)}{25} x^{\frac{5}{2}} (-a)^{\frac{5}{2q}} - 2 \frac{qx^{5/2} (1 + 2/5q) \text{polylog}(3, ax^q)}{5+2q} (-a)^{5/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*polylog(3,a\*x^q), x)

[Out] -(d\*x)^(3/2)/x^(3/2)\*(-a)^(-5/2/q)/q\*(8/125\*q^3\*x^(5/2)\*(-a)^(5/2/q)\*ln(1-a\*x^q)+4/25\*q^2\*x^(5/2)\*(-a)^(5/2/q)\*polylog(2,a\*x^q)-2\*q/(5+2\*q)\*x^(5/2)\*(-a)^(5/2/q)\*(1+2/5\*q)\*polylog(3,a\*x^q)+8/125\*q^3\*x^(5/2+q)\*a\*(-a)^(5/2/q)\*LerchPhi(a\*x^q, 1, 1/2\*(5+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-16d^{\frac{3}{2}}q^4 \int \frac{x^{\frac{3}{2}}}{125(a^2(2q-5)x^{2q} - 2a(2q-5)x^q + 2q-5)} dx - \frac{2\left(50\left((2q^2-5q)ad^{\frac{3}{2}}xx^q - (2q^2-5q)d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}\text{Li}_2(ax^q)\right)}{125(5+2q)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*d^(3/2)*q^4*integrate(1/125*x^(3/2)/(a^2*(2*q - 5)*x^(2*q) - 2*a*(2*q - 5)*x^q + 2*q - 5), x) - 2/625*(50*((2*q^2 - 5*q)*a*d^(3/2)*x*x^q - (2*q^2 - 5*q)*d^(3/2)*x)*x^(3/2)*dilog(a*x^q) + 20*((2*q^3 - 5*q^2)*a*d^(3/2)*x*x^q - (2*q^3 - 5*q^2)*d^(3/2)*x)*x^(3/2)*log(-a*x^q + 1) - 125*(a*d^(3/2)*(2*q - 5)*x*x^q - d^(3/2)*(2*q - 5)*x)*x^(3/2)*polylog(3, a*x^q) + 8*(2*d^(3/2)*q^4*x - (2*q^4 - 5*q^3)*a*d^(3/2)*x*x^q)*x^(3/2))/(a*(2*q - 5)*x^q - 2*q + 5)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}dx\text{polylog}(3,ax^q),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*polylog(3, a*x^q), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x**q),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x^q), x)
```

### 3.92 $\int \sqrt{dx} \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=124

$$\frac{16aq^3 \sqrt{dx} x^{q+1} \text{Hypergeometric2F1}\left(1, \frac{q+\frac{3}{2}}{q}, \frac{1}{2}\left(\frac{3}{q}+4\right), ax^q\right)}{27(2q+3)} - \frac{4q(dx)^{3/2} \text{PolyLog}(2, ax^q)}{9d} + \frac{2(dx)^{3/2} \text{PolyLog}(3, ax^q)}{3d}$$

[Out]  $(-16*a*q^3*x^{(1+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (3/2+q)/q, (4+3/q)/2, a*x^q])/(27*(3+2*q)) - (8*q^2*(d*x)^{(3/2)}*\text{Log}[1-a*x^q])/(27*d) - (4*q*(d*x)^{(3/2)}*\text{PolyLog}[2, a*x^q])/(9*d) + (2*(d*x)^{(3/2)}*\text{PolyLog}[3, a*x^q])/(3*d)$

**Rubi [A]** time = 0.0703269, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$-\frac{4q(dx)^{3/2} \text{PolyLog}(2, ax^q)}{9d} + \frac{2(dx)^{3/2} \text{PolyLog}(3, ax^q)}{3d} - \frac{16aq^3 \sqrt{dx} x^{q+1} {}_2F_1\left(1, \frac{q+\frac{3}{2}}{q}; \frac{1}{2}\left(4+\frac{3}{q}\right); ax^q\right)}{27(2q+3)} - \frac{8q^2(dx)^{3/2} \log(1 - ax^q)}{27d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*PolyLog[3, a\*x^q], x]

[Out]  $(-16*a*q^3*x^{(1+q)}*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (3/2+q)/q, (4+3/q)/2, a*x^q])/(27*(3+2*q)) - (8*q^2*(d*x)^{(3/2)}*\text{Log}[1-a*x^q])/(27*d) - (4*q*(d*x)^{(3/2)}*\text{PolyLog}[2, a*x^q])/(9*d) + (2*(d*x)^{(3/2)}*\text{PolyLog}[3, a*x^q])/(3*d)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt



Q[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int \sqrt{dx} \operatorname{Li}_3(ax^q) dx &= \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^q)}{3d} - \frac{1}{3}(2q) \int \sqrt{dx} \operatorname{Li}_2(ax^q) dx \\
 &= -\frac{4q(dx)^{3/2} \operatorname{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^q)}{3d} - \frac{1}{9}(4q^2) \int \sqrt{dx} \log(1-ax^q) dx \\
 &= -\frac{8q^2(dx)^{3/2} \log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2} \operatorname{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^q)}{3d} - \frac{(8aq^3) \int \frac{x^{-1+q}(dx)^{3/2}}{1-ax^q} dx}{27d} \\
 &= -\frac{8q^2(dx)^{3/2} \log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2} \operatorname{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^q)}{3d} - \frac{(8aq^3 \sqrt{dx}) \int \frac{x^{\frac{1}{2}+q}}{1-ax^q} dx}{27\sqrt{x}} \\
 &= -\frac{16aq^3 x^{1+q} \sqrt{dx} {}_2F_1\left(1, \frac{3}{2} + \frac{q}{q}; \frac{1}{2} \left(4 + \frac{3}{q}\right); ax^q\right)}{27(3+2q)} - \frac{8q^2(dx)^{3/2} \log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2} \operatorname{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2} \operatorname{Li}_3(ax^q)}{3d}
 \end{aligned}$$

**Mathematica [C]** time = 0.0262881, size = 50, normalized size = 0.4

$$\frac{x \sqrt{dx} G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 - \frac{3}{2q} \\ 1, 0, 0, 0, -\frac{3}{2q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d\*x]\*PolyLog[3, a\*x^q], x]

[Out] -((x\*Sqrt[d\*x]\*MeijerG[{{1, 1, 1, 1, 1 - 3/(2\*q)}, {}}, {{1}}, {0, 0, 0, -3/(2\*q)}], -(a\*x^q)))/q

**Maple [C]** time = 0.381, size = 145, normalized size = 1.2

$$-\frac{1}{q} \sqrt{dx} (-a)^{-\frac{3}{2q}} \left( \frac{8q^3 \ln(1-ax^q)}{27} x^{\frac{3}{2}} (-a)^{\frac{3}{2q}} + \frac{4q^2 \operatorname{polylog}(2, ax^q)}{9} x^{\frac{3}{2}} (-a)^{\frac{3}{2q}} - 2 \frac{qx^{3/2} (1 + 2/3q) \operatorname{polylog}(3, ax^q)}{3 + 2q} (-a)^{\frac{3}{2q}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*polylog(3,a\*x^q), x)

[Out] -(d\*x)^(1/2)/x^(1/2)\*(-a)^(-3/2/q)/q\*(8/27\*q^3\*x^(3/2)\*(-a)^(3/2/q)\*ln(1-a\*x^q)+4/9\*q^2\*x^(3/2)\*(-a)^(3/2/q)\*polylog(2,a\*x^q)-2\*q/(3+2\*q)\*x^(3/2)\*(-a)^(3/2/q)\*(1+2/3\*q)\*polylog(3,a\*x^q)+8/27\*q^3\*x^(3/2+q)\*a\*(-a)^(3/2/q)\*LerchPhi(a\*x^q,1,1/2\*(3+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-16 \sqrt{dq^4} \int \frac{\sqrt{x}}{27(a^2(2q-3)x^{2q} - 2a(2q-3)x^q + 2q-3)} dx - \frac{2(18((2q^2-3q)a\sqrt{d}xx^q - (2q^2-3q)\sqrt{d}x)\sqrt{x}\operatorname{Li}_2(ax^q) + 2(2q-3)\sqrt{d}x)}{27(a^2(2q-3)x^{2q} - 2a(2q-3)x^q + 2q-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*sqrt(d)*q^4*integrate(1/27*sqrt(x)/(a^2*(2*q - 3)*x^(2*q) - 2*a*(2*q - 3)*x^q + 2*q - 3), x) - 2/81*(18*((2*q^2 - 3*q)*a*sqrt(d)*x*x^q - (2*q^2 - 3*q)*sqrt(d)*x)*sqrt(x)*dilog(a*x^q) + 12*((2*q^3 - 3*q^2)*a*sqrt(d)*x*x^q - (2*q^3 - 3*q^2)*sqrt(d)*x)*sqrt(x)*log(-a*x^q + 1) - 27*(a*sqrt(d)*(2*q - 3)*x*x^q - sqrt(d)*(2*q - 3)*x)*sqrt(x)*polylog(3, a*x^q) + 8*(2*sqrt(d)*q^4*x - (2*q^4 - 3*q^3)*a*sqrt(d)*x*x^q)*sqrt(x))/(a*(2*q - 3)*x^q - 2*q + 3)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\text{polylog}(3, ax^q), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x**q),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x**q), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x^q), x)
```

### 3.93 $\int \frac{\text{PolyLog}(3, ax^q)}{\sqrt{dx}} dx$

**Optimal.** Leaf size=115

$$\frac{16aq^3\sqrt{dx}x^q\text{Hypergeometric2F1}\left(1, \frac{q+\frac{1}{2}}{q}, \frac{1}{2}\left(\frac{1}{q}+4\right), ax^q\right)}{d(2q+1)} - \frac{4q\sqrt{dx}\text{PolyLog}(2, ax^q)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax^q)}{d} - \frac{8\sqrt{dx}\text{PolyLog}(2, ax^q)}{d}$$

[Out]  $(-16*a*q^3*x^q*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (1/2 + q)/q, (4 + q^(-1))/2, a*x^q])/(d*(1 + 2*q)) - (8*q^2*\text{Sqrt}[d*x]*\text{Log}[1 - a*x^q])/d - (4*q*\text{Sqrt}[d*x]*\text{PolyLog}[2, a*x^q])/d + (2*\text{Sqrt}[d*x]*\text{PolyLog}[3, a*x^q])/d$

**Rubi [A]** time = 0.0690147, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$\frac{4q\sqrt{dx}\text{PolyLog}(2, ax^q)}{d} + \frac{2\sqrt{dx}\text{PolyLog}(3, ax^q)}{d} - \frac{16aq^3\sqrt{dx}x^q {}_2F_1\left(1, \frac{q+\frac{1}{2}}{q}; \frac{1}{2}\left(4 + \frac{1}{q}\right); ax^q\right)}{d(2q+1)} - \frac{8q^2\sqrt{dx}\log(1 - ax^q)}{d}$$

Antiderivative was successfully verified.

[In] Int [PolyLog [3, a\*x^q] / Sqrt [d\*x], x]

[Out]  $(-16*a*q^3*x^q*\text{Sqrt}[d*x]*\text{Hypergeometric2F1}[1, (1/2 + q)/q, (4 + q^(-1))/2, a*x^q])/(d*(1 + 2*q)) - (8*q^2*\text{Sqrt}[d*x]*\text{Log}[1 - a*x^q])/d - (4*q*\text{Sqrt}[d*x]*\text{PolyLog}[2, a*x^q])/d + (2*\text{Sqrt}[d*x]*\text{PolyLog}[3, a*x^q])/d$

#### Rule 6591

Int [((d\_.)\*(x\_.))^(m\_.)\*PolyLog [n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] := Simp [((d\*x)^(m+1)\*PolyLog [n, a\*(b\*x^p)^q]) / (d\*(m+1)), x] - Dist [(p\*q) / (m+1), Int [(d\*x)^m \* PolyLog [n-1, a\*(b\*x^p)^q], x], x] /; FreeQ [{a, b, d, m, p, q}, x] && NeQ [m, -1] && GtQ [n, 0]

#### Rule 2455

Int [((a\_.) + Log [(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp [((f\*x)^(m+1)\*(a + b\*Log [c\*(d + e\*x^n)^p])) / (f\*(m+1)), x] - Dist [(b\*e\*n\*p) / (f\*(m+1)), Int [(x^(n-1)\*(f\*x)^(m+1)) / (d + e\*x^n), x], x] /; FreeQ [{a, b, c, d, e, f, m, n, p}, x] && NeQ [m, -1]

#### Rule 20

Int [(u\_.)\*((a\_.)\*(v\_.))^(m\_.)\*((b\_.)\*(v\_.))^(n\_.), x\_Symbol] := Dist [(b^IntPart [n]\*(b\*v)^FracPart [n]) / (a^IntPart [n]\*(a\*v)^FracPart [n]), Int [u\*(a\*v)^(m+n), x], x] /; FreeQ [{a, b, m, n}, x] && !IntegerQ [m] && !IntegerQ [n] && !IntegerQ [m+n]

#### Rule 364

Int [((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Simp [(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1 [-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a]) / (c\*(m+1)), x] /; FreeQ [{a, b, c, m, n, p}, x] && !IGtQ [p, 0] && (ILtQ [p, 0] || GtQ [a, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}\text{Li}_3(ax^q)}{d} - (2q) \int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}} dx \\
&= -\frac{4q\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^q)}{d} - (4q^2) \int \frac{\log(1-ax^q)}{\sqrt{dx}} dx \\
&= -\frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^q)}{d} - \frac{(8aq^3) \int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q} dx}{d} \\
&= -\frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^q)}{d} - \frac{(8aq^3\sqrt{dx}) \int \frac{x^{-\frac{1}{2}+q}}{1-ax^q} dx}{d\sqrt{x}} \\
&= -\frac{16aq^3x^q\sqrt{dx} {}_2F_1\left(1, \frac{1}{2}+q; \frac{1}{2}\left(4+\frac{1}{q}\right); ax^q\right)}{d(1+2q)} - \frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax^q)}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.02007, size = 50, normalized size = 0.43

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 - \frac{1}{2q} \\ 1, 0, 0, 0, -\frac{1}{2q} \end{matrix}\right)}{q\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/Sqrt[d\*x], x]

[Out] -((x\*MeijerG[{{1, 1, 1, 1, 1 - 1/(2\*q)}}, {}], {{1}}, {0, 0, 0, -1/(2\*q)}}, - (a\*x^q)))/(q\*Sqrt[d\*x]))

**Maple [C]** time = 0.372, size = 133, normalized size = 1.2

$$-\frac{1}{q}\sqrt{x}(-a)^{-\frac{1}{2q}}\left(8q^3\sqrt{x}(-a)^{1/2q-1}\ln(1-ax^q)+4q^2\sqrt{x}(-a)^{1/2q-1}\text{polylog}(2,ax^q)-2q\sqrt{x}(-a)^{1/2q-1}\text{polylog}(3,ax^q)+8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^q)/(d\*x)^(1/2), x)

[Out] -1/(d\*x)^(1/2)\*x^(1/2)\*(-a)^(-1/2/q)/q\*(8\*q^3\*x^(1/2)\*(-a)^(1/2/q)\*ln(1-a\*x^q)+4\*q^2\*x^(1/2)\*(-a)^(1/2/q)\*polylog(2,a\*x^q)-2\*q\*x^(1/2)\*(-a)^(1/2/q)\*polylog(3,a\*x^q)+8\*q^3\*x^(1/2+q)\*a\*(-a)^(1/2/q)\*LerchPhi(a\*x^q,1,1/2\*(1+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-16q^4 \int \frac{1}{(a^2\sqrt{d}(2q-1)x^{2q} - 2a\sqrt{d}(2q-1)x^q + \sqrt{d}(2q-1))\sqrt{x}} dx - \frac{2\left(\frac{2((2q^2-q)axx^q - (2q^2-q)x)\text{Li}_2(ax^q)}{\sqrt{x}} + \frac{4((2q^3-q^2)axx^q - (2q^3-q^2)x)\text{Li}_3(ax^q)}{a\sqrt{d}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -16*q^4*integrate(1/((a^2*sqrt(d)*(2*q - 1)*x^(2*q) - 2*a*sqrt(d)*(2*q - 1)
*x^q + sqrt(d)*(2*q - 1))*sqrt(x)), x) - 2*(2*((2*q^2 - q)*a*x*x^q - (2*q^2
- q)*x)*dilog(a*x^q)/sqrt(x) + 4*((2*q^3 - q^2)*a*x*x^q - (2*q^3 - q^2)*x)
*log(-a*x^q + 1)/sqrt(x) - (a*(2*q - 1)*x*x^q - (2*q - 1)*x)*polylog(3, a*x
^q)/sqrt(x) + 8*(2*q^4*x - (2*q^4 - q^3)*a*x*x^q)/sqrt(x))/(a*sqrt(d)*(2*q
- 1)*x^q - sqrt(d)*(2*q - 1))
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{polylog}(3,ax^q)}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d*x), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/sqrt(d*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/sqrt(d*x), x)
```

### 3.94 $\int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=119

$$\frac{16aq^3x^q \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(2 - \frac{1}{q}\right), \frac{1}{2}\left(4 - \frac{1}{q}\right), ax^q\right)}{d(1-2q)\sqrt{dx}} - \frac{4q \text{PolyLog}(2, ax^q)}{d\sqrt{dx}} - \frac{2 \text{PolyLog}(3, ax^q)}{d\sqrt{dx}} + \frac{8q^2 \log(1 - ax^q)}{d\sqrt{dx}}$$

[Out]  $(-16*a*q^3*x^q*Hypergeometric2F1[1, (2 - q^{(-1)})/2, (4 - q^{(-1)})/2, a*x^q]) / (d*(1 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q]) / (d*Sqrt[d*x]) - (4*q*PolyLog[2, a*x^q]) / (d*Sqrt[d*x]) - (2*PolyLog[3, a*x^q]) / (d*Sqrt[d*x])$

**Rubi [A]** time = 0.0780859, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$-\frac{4q \text{PolyLog}(2, ax^q)}{d\sqrt{dx}} - \frac{2 \text{PolyLog}(3, ax^q)}{d\sqrt{dx}} - \frac{16aq^3x^q {}_2F_1\left(1, \frac{1}{2}\left(2 - \frac{1}{q}\right); \frac{1}{2}\left(4 - \frac{1}{q}\right); ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{8q^2 \log(1 - ax^q)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a\*x^q]/(d\*x)^(3/2), x]

[Out]  $(-16*a*q^3*x^q*Hypergeometric2F1[1, (2 - q^{(-1)})/2, (4 - q^{(-1)})/2, a*x^q]) / (d*(1 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q]) / (d*Sqrt[d*x]) - (4*q*PolyLog[2, a*x^q]) / (d*Sqrt[d*x]) - (2*PolyLog[3, a*x^q]) / (d*Sqrt[d*x])$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{(dx)^{3/2}} dx &= -\frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + (2q) \int \frac{\text{Li}_2(ax^q)}{(dx)^{3/2}} dx \\
&= -\frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} - (4q^2) \int \frac{\log(1-ax^q)}{(dx)^{3/2}} dx \\
&= \frac{8q^2 \log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + \frac{(8aq^3) \int \frac{x^{-1+q}}{\sqrt{dx}(1-ax^q)} dx}{d} \\
&= \frac{8q^2 \log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}} + \frac{(8aq^3 \sqrt{x}) \int \frac{x^{-\frac{3}{2}+q}}{1-ax^q} dx}{d\sqrt{dx}} \\
&= -\frac{16aq^3 x^q {}_2F_1\left(1, \frac{1}{2}\left(2-\frac{1}{q}\right); \frac{1}{2}\left(4-\frac{1}{q}\right); ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{8q^2 \log(1-ax^q)}{d\sqrt{dx}} - \frac{4q\text{Li}_2(ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^q)}{d\sqrt{dx}}
\end{aligned}$$

**Mathematica [C]** time = 0.0248691, size = 50, normalized size = 0.42

$$\frac{x G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 + \frac{1}{2q} \\ 1, 0, 0, 0, \frac{1}{2q} \end{matrix}\right)}{q(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/(d\*x)^(3/2), x]

[Out] -((x\*MeijerG[{{1, 1, 1, 1, 1 + 1/(2\*q)}, {}}, {{1}, {0, 0, 0, 1/(2\*q)}}], -(a\*x^q)))/(q\*(d\*x)^(3/2))

**Maple [C]** time = 0.372, size = 145, normalized size = 1.2

$$-\frac{1}{q} x^{\frac{3}{2}} (-a)^{\frac{1}{2q}} \left( -8 \frac{q^3 \ln(1-ax^q)}{\sqrt{x}} (-a)^{-1/2q-1} + 4 \frac{q^2 \text{polylog}(2, ax^q)}{\sqrt{x}} (-a)^{-1/2q-1} - 2 \frac{q(1-2q) \text{polylog}(3, ax^q)}{(2q-1)\sqrt{x}} (-a)^{-1/2q} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,a\*x^q)/(d\*x)^(3/2), x)

[Out] -1/(d\*x)^(3/2)\*x^(3/2)\*(-a)^(1/2/q)/q\*(-8\*q^3/x^(1/2)\*(-a)^(-1/2/q)\*ln(1-a\*x^q)+4\*q^2/x^(1/2)\*(-a)^(-1/2/q)\*polylog(2,a\*x^q)-2\*q/(2\*q-1)/x^(1/2)\*(-a)^(-1/2/q)\*(1-2\*q)\*polylog(3,a\*x^q)-8\*q^3\*x^(q-1/2)\*a\*(-a)^(-1/2/q)\*LerchPhi(a\*x^q,1,1/2\*(2\*q-1)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$16q^4 \int \frac{1}{\left(a^2 d^{\frac{3}{2}}(2q+1)x^{2q} - 2ad^{\frac{3}{2}}(2q+1)x^q + d^{\frac{3}{2}}(2q+1)\right)x^{\frac{3}{2}}} dx - \frac{2\left(\frac{2((2q^2+q)ax^q-(2q^2+q)x)\text{Li}_2(ax^q)}{x^{\frac{3}{2}}} - \frac{4((2q^3+q^2)ax^q-(2q^3+q^2)x)}{ad^{\frac{3}{2}}}\right)}{ad^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 16*q^4*integrate(1/((a^2*d^(3/2)*(2*q + 1)*x^(2*q) - 2*a*d^(3/2)*(2*q + 1)*
x^q + d^(3/2)*(2*q + 1))*x^(3/2)), x) - 2*(2*((2*q^2 + q)*a*x*x^q - (2*q^2
+ q)*x)*dilog(a*x^q)/x^(3/2) - 4*((2*q^3 + q^2)*a*x*x^q - (2*q^3 + q^2)*x)*
log(-a*x^q + 1)/x^(3/2) + (a*(2*q + 1)*x*x^q - (2*q + 1)*x)*polylog(3, a*x^
q)/x^(3/2) + 8*(2*q^4*x - (2*q^4 + q^3)*a*x*x^q)/x^(3/2))/(a*d^(3/2)*(2*q +
1)*x^q - d^(3/2)*(2*q + 1))
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{polylog}(3,ax^q)}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d^2*x^2), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/(d*x)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/(d*x)^(3/2), x)
```



$$3.95 \quad \int \frac{\text{PolyLog}(3, ax^q)}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=129

$$\frac{16aq^3x^{q-1}\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(2 - \frac{3}{q}\right), \frac{1}{2}\left(4 - \frac{3}{q}\right), ax^q\right)}{27d^2(3 - 2q)\sqrt{dx}} - \frac{4q\text{PolyLog}(2, ax^q)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax^q)}{3d(dx)^{3/2}} + \frac{8q^2 \log(1 - ax^q)}{27d(dx)^{3/2}}$$

[Out]  $(-16*a*q^3*x^{(-1 + q)}*\text{Hypergeometric2F1}[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]) / (27*d^2*(3 - 2*q)*\text{Sqrt}[d*x]) + (8*q^2*\text{Log}[1 - a*x^q]) / (27*d*(d*x)^{(3/2)}) - (4*q*\text{PolyLog}[2, a*x^q]) / (9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[3, a*x^q]) / (3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.0749586, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6591, 2455, 20, 364}

$$-\frac{4q\text{PolyLog}(2, ax^q)}{9d(dx)^{3/2}} - \frac{2\text{PolyLog}(3, ax^q)}{3d(dx)^{3/2}} - \frac{16aq^3x^{q-1}{}_2F_1\left(1, \frac{1}{2}\left(2 - \frac{3}{q}\right); \frac{1}{2}\left(4 - \frac{3}{q}\right); ax^q\right)}{27d^2(3 - 2q)\sqrt{dx}} + \frac{8q^2 \log(1 - ax^q)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{PolyLog}[3, a*x^q]/(d*x)^{(5/2)}, x]$

[Out]  $(-16*a*q^3*x^{(-1 + q)}*\text{Hypergeometric2F1}[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]) / (27*d^2*(3 - 2*q)*\text{Sqrt}[d*x]) + (8*q^2*\text{Log}[1 - a*x^q]) / (27*d*(d*x)^{(3/2)}) - (4*q*\text{PolyLog}[2, a*x^q]) / (9*d*(d*x)^{(3/2)}) - (2*\text{PolyLog}[3, a*x^q]) / (3*d*(d*x)^{(3/2)})$

#### Rule 6591

$\text{Int}[(d_*)(x_*)^{(m_*)}*\text{PolyLog}[n_*, (a_*)*((b_*)(x_*)^{(p_*)})^{(q_*)}], x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*\text{PolyLog}[n, a*(b*x^p)^q]/(d*(m+1)), x] - \text{Dist}[(p*q)/(m+1), \text{Int}[(d*x)^m*\text{PolyLog}[n-1, a*(b*x^p)^q], x], x] /;$  FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

$\text{Int}[(a_*) + \text{Log}[(c_*)*((d_*) + (e_*)(x_*)^{(n_*)})^{(p_*)}]]*(b_*)*((f_*)(x_*)^{(m_*)}), x\_Symbol] \rightarrow \text{Simp}[(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])]/(f*(m+1)), x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^(n-1)*(f*x)^{(m+1)})/(d + e*x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

$\text{Int}[(u_*)*((a_*)(v_*)^{(m_*)}*((b_*)(v_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[(b*\text{IntPart}[n]*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$  FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

$\text{Int}[(c_*)(x_*)^{(m_*)}*((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{(m+1)}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])]/(c*(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt

Q[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_3(ax^q)}{(dx)^{5/2}} dx &= -\frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{1}{3}(2q) \int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}} dx \\
 &= -\frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} - \frac{1}{9}(4q^2) \int \frac{\log(1-ax^q)}{(dx)^{5/2}} dx \\
 &= \frac{8q^2 \log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{(8aq^3) \int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)} dx}{27d} \\
 &= \frac{8q^2 \log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{(8aq^3\sqrt{x}) \int \frac{x^{-\frac{5}{2}+q}}{1-ax^q} dx}{27d^2\sqrt{dx}} \\
 &= -\frac{16aq^3x^{-1+q} {}_2F_1\left(1, \frac{1}{2}\left(2-\frac{3}{q}\right); \frac{1}{2}\left(4-\frac{3}{q}\right); ax^q\right)}{27d^2(3-2q)\sqrt{dx}} + \frac{8q^2 \log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.0240634, size = 50, normalized size = 0.39

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q \begin{matrix} 1, 1, 1, 1, 1 + \frac{3}{2q} \\ 1, 0, 0, 0, \frac{3}{2q} \end{matrix}\right)}{q(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[PolyLog[3, a\*x^q]/(d\*x)^(5/2), x]

[Out] -((x\*MeijerG[{{1, 1, 1, 1, 1 + 3/(2\*q)}, {}}, {{1}, {0, 0, 0, 3/(2\*q)}}, -(a\*x^q)])/(q\*(d\*x)^(5/2)))

**Maple [C]** time = 0.381, size = 145, normalized size = 1.1

$$-\frac{1}{q}x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{8q^3 \ln(1-ax^q)}{27}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}} + \frac{4q^2 \text{polylog}(2, ax^q)}{9}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}} - 2\frac{q(1-2/3q) \text{polylog}(3, ax^q)}{(-3+2q)x^{3/2}}(-a)^{-3/2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, a\*x^q)/(d\*x)^(5/2), x)

[Out] -1/(d\*x)^(5/2)\*x^(5/2)\*(-a)^(3/2/q)/q\*(-8/27\*q^3/x^(3/2)\*(-a)^(-3/2/q)\*ln(1-a\*x^q)+4/9\*q^2/x^(3/2)\*(-a)^(-3/2/q)\*polylog(2, a\*x^q)-2\*q/(-3+2\*q)/x^(3/2)\*(-a)^(-3/2/q)\*(1-2/3\*q)\*polylog(3, a\*x^q)-8/27\*q^3\*x^(q-3/2)\*a\*(-a)^(-3/2/q)\*LerchPhi(a\*x^q, 1, 1/2\*(-3+2\*q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$16q^4 \int \frac{1}{27\left(a^2d^{\frac{5}{2}}(2q+3)x^{2q} - 2ad^{\frac{5}{2}}(2q+3)x^q + d^{\frac{5}{2}}(2q+3)\right)x^{\frac{5}{2}}} dx - \frac{2\left(\frac{18((2q^2+3q)axx^q - (2q^2+3q)x)\text{Li}_2(ax^q)}{x^{\frac{5}{2}}} - \frac{12((2q^3+3q^2)}{x^{\frac{5}{2}}}\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/(d\*x)^(5/2),x, algorithm="maxima")

[Out] 16\*q^4\*integrate(1/27/((a^2\*d^(5/2)\*(2\*q + 3)\*x^(2\*q) - 2\*a\*d^(5/2)\*(2\*q + 3)\*x^q + d^(5/2)\*(2\*q + 3))\*x^(5/2)), x) - 2/81\*(18\*((2\*q^2 + 3\*q)\*a\*x\*x^q - (2\*q^2 + 3\*q)\*x)\*dilog(a\*x^q)/x^(5/2) - 12\*((2\*q^3 + 3\*q^2)\*a\*x\*x^q - (2\*q^3 + 3\*q^2)\*x)\*log(-a\*x^q + 1)/x^(5/2) + 27\*(a\*(2\*q + 3)\*x\*x^q - (2\*q + 3)\*x)\*polylog(3, a\*x^q)/x^(5/2) + 8\*(2\*q^4\*x - (2\*q^4 + 3\*q^3)\*a\*x\*x^q)/x^(5/2)/(a\*d^(5/2)\*(2\*q + 3)\*x^q - d^(5/2)\*(2\*q + 3))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\text{polylog}(3,ax^q)}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/(d\*x)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*polylog(3, a\*x^q)/(d^3\*x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x\*\*q)/(d\*x)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,a\*x^q)/(d\*x)^(5/2),x, algorithm="giac")

[Out] integrate(polylog(3, a\*x^q)/(d\*x)^(5/2), x)

### 3.96 $\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right) - x\text{PolyLog}\left(\frac{1}{2}, ax\right) + x\text{PolyLog}\left(\frac{3}{2}, ax\right)$$

[Out]  $-(x*\text{PolyLog}[1/2, a*x]) + x*\text{PolyLog}[3/2, a*x] + \text{Unintegrable}[\text{PolyLog}[-1/2, a*x], x]$

**Rubi [A]** time = 0.0076448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[\text{PolyLog}[3/2, a*x], x]$

[Out]  $-(x*\text{PolyLog}[1/2, a*x]) + x*\text{PolyLog}[3/2, a*x] + \text{Defer}[\text{Int}][\text{PolyLog}[-1/2, a*x], x]$

Rubi steps

$$\begin{aligned} \int \text{Li}_{\frac{3}{2}}(ax) dx &= x\text{Li}_{\frac{3}{2}}(ax) - \int \text{Li}_{\frac{1}{2}}(ax) dx \\ &= -x\text{Li}_{\frac{1}{2}}(ax) + x\text{Li}_{\frac{3}{2}}(ax) + \int \text{Li}_{-\frac{1}{2}}(ax) dx \end{aligned}$$

**Mathematica [A]** time = 0.0066395, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In]  $\text{Integrate}[\text{PolyLog}[3/2, a*x], x]$

[Out]  $\text{Integrate}[\text{PolyLog}[3/2, a*x], x]$

**Maple [A]** time = 0.043, size = 0, normalized size = 0.

$$\int \text{polylog}\left(\frac{3}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\text{polylog}(3/2, a*x), x)$

[Out] `int(polylog(3/2,a*x),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3/2,a*x),x, algorithm="maxima")`

[Out] `integrate(polylog(3/2, a*x), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3/2,a*x),x, algorithm="fricas")`

[Out] `integral(polylog(3/2, a*x), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3/2,a*x),x)`

[Out] `Integral(polylog(3/2, a*x), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3/2,a*x),x, algorithm="giac")`

[Out] `integrate(polylog(3/2, a*x), x)`

### 3.97 $\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$

**Optimal.** Leaf size=21

$$x\text{PolyLog}\left(\frac{1}{2}, ax\right) - \text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

[Out] x\*PolyLog[1/2, a\*x] - Unintegrable[PolyLog[-1/2, a\*x], x]

**Rubi [A]** time = 0.0047659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[1/2, a\*x], x]

[Out] x\*PolyLog[1/2, a\*x] - Defer[Int][PolyLog[-1/2, a\*x], x]

Rubi steps

$$\int \text{Li}_{\frac{1}{2}}(ax) dx = x\text{Li}_{\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax) dx$$

**Mathematica [A]** time = 0.0063639, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[1/2, a\*x], x]

[Out] Integrate[PolyLog[1/2, a\*x], x]

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int \text{polylog}\left(\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(1/2, a\*x), x)

[Out] int(polylog(1/2, a\*x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(1/2,a\*x),x, algorithm="maxima")

[Out] integrate(polylog(1/2, a\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(\frac{1}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(1/2,a\*x),x, algorithm="fricas")

[Out] integral(polylog(1/2, a\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(1/2,a\*x),x)

[Out] Integral(polylog(1/2, a\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(1/2,a\*x),x, algorithm="giac")

[Out] integrate(polylog(1/2, a\*x), x)

### 3.98 $\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$

**Optimal.** Leaf size=9

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

[Out] Unintegrable[PolyLog[-1/2, a\*x], x]

**Rubi [A]** time = 0.0023648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[-1/2, a\*x], x]

[Out] Defer[Int][PolyLog[-1/2, a\*x], x]

Rubi steps

$$\int \text{Li}_{-\frac{1}{2}}(ax) dx = \int \text{Li}_{-\frac{1}{2}}(ax) dx$$

**Mathematica [A]** time = 0.0065787, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[-1/2, a\*x], x]

[Out] Integrate[PolyLog[-1/2, a\*x], x]

**Maple [A]** time = 0.048, size = 0, normalized size = 0.

$$\int \text{polylog}\left(-\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(-1/2, a\*x), x)

[Out] int(polylog(-1/2, a\*x), x)



**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-1/2,a\*x),x, algorithm="maxima")

[Out] integrate(polylog(-1/2, a\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{1}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-1/2,a\*x),x, algorithm="fricas")

[Out] integral(polylog(-1/2, a\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-1/2,a\*x),x)

[Out] Integral(polylog(-1/2, a\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{1}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-1/2,a\*x),x, algorithm="giac")

[Out] integrate(polylog(-1/2, a\*x), x)

### 3.99 $\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$

**Optimal.** Leaf size=21

$$x\text{PolyLog}\left(-\frac{1}{2}, ax\right) - \text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

[Out] x\*PolyLog[-1/2, a\*x] - Unintegrable[PolyLog[-1/2, a\*x], x]

**Rubi [A]** time = 0.0047009, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[-3/2, a\*x], x]

[Out] x\*PolyLog[-1/2, a\*x] - Defer[Int][PolyLog[-1/2, a\*x], x]

Rubi steps

$$\int \text{Li}_{-\frac{3}{2}}(ax) dx = x\text{Li}_{-\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax) dx$$

**Mathematica [A]** time = 0.0064057, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[-3/2, a\*x], x]

[Out] Integrate[PolyLog[-3/2, a\*x], x]

**Maple [A]** time = 0.062, size = 0, normalized size = 0.

$$\int \text{polylog}\left(-\frac{3}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(-3/2, a\*x), x)

[Out] int(polylog(-3/2, a\*x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-3/2,a\*x),x, algorithm="maxima")

[Out] integrate(polylog(-3/2, a\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-3/2,a\*x),x, algorithm="fricas")

[Out] integral(polylog(-3/2, a\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-3/2,a\*x),x)

[Out] Integral(polylog(-3/2, a\*x), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(-3/2,a\*x),x, algorithm="giac")

[Out] integrate(polylog(-3/2, a\*x), x)

### 3.100 $\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right) + x\text{PolyLog}\left(-\frac{3}{2}, ax\right) - x\text{PolyLog}\left(-\frac{1}{2}, ax\right)$$

[Out] x\*PolyLog[-3/2, a\*x] - x\*PolyLog[-1/2, a\*x] + Unintegrable[PolyLog[-1/2, a\*x], x]

**Rubi [A]** time = 0.007823, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[-5/2, a\*x], x]

[Out] x\*PolyLog[-3/2, a\*x] - x\*PolyLog[-1/2, a\*x] + Defer[Int][PolyLog[-1/2, a\*x], x]

Rubi steps

$$\begin{aligned} \int \text{Li}_{-\frac{5}{2}}(ax) dx &= x\text{Li}_{-\frac{3}{2}}(ax) - \int \text{Li}_{-\frac{3}{2}}(ax) dx \\ &= x\text{Li}_{-\frac{3}{2}}(ax) - x\text{Li}_{-\frac{1}{2}}(ax) + \int \text{Li}_{-\frac{1}{2}}(ax) dx \end{aligned}$$

**Mathematica [A]** time = 0.0071344, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[-5/2, a\*x], x]

[Out] Integrate[PolyLog[-5/2, a\*x], x]

**Maple [A]** time = 0.047, size = 0, normalized size = 0.

$$\int \text{polylog}\left(-\frac{5}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(-5/2, a\*x), x)

[Out] `int(polylog(-5/2,a*x),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-5/2,a*x),x, algorithm="maxima")`

[Out] `integrate(polylog(-5/2, a*x), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{5}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-5/2,a*x),x, algorithm="fricas")`

[Out] `integral(polylog(-5/2, a*x), x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-5/2,a*x),x)`

[Out] `Integral(polylog(-5/2, a*x), x)`

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{5}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-5/2,a*x),x, algorithm="giac")`

[Out] `integrate(polylog(-5/2, a*x), x)`

### 3.101 $\int \left( \mathbf{PolyLog} \left( -\frac{3}{2}, ax \right) + \mathbf{PolyLog} \left( -\frac{1}{2}, ax \right) \right) dx$

**Optimal.** Leaf size=9

$$x \mathbf{PolyLog} \left( -\frac{1}{2}, ax \right)$$

[Out] x\*PolyLog[-1/2, a\*x]

**Rubi [A]** time = 0.0079878, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {6587}

$$x \mathbf{PolyLog} \left( -\frac{1}{2}, ax \right)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[-3/2, a\*x] + PolyLog[-1/2, a\*x], x]

[Out] x\*PolyLog[-1/2, a\*x]

#### Rule 6587

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[(x\*PolyLog[n + 1, a\*(b\*x^p)^q]/(p\*q), x] - Dist[1/(p\*q), Int[PolyLog[n + 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && LtQ[n, -1]

#### Rubi steps

$$\begin{aligned} \int \left( \mathbf{Li}_{-\frac{3}{2}}(ax) + \mathbf{Li}_{-\frac{1}{2}}(ax) \right) dx &= \int \mathbf{Li}_{-\frac{3}{2}}(ax) dx + \int \mathbf{Li}_{-\frac{1}{2}}(ax) dx \\ &= x \mathbf{Li}_{-\frac{1}{2}}(ax) \end{aligned}$$

**Mathematica [F]** time = 0.0069411, size = 0, normalized size = 0.

$$\int \left( \mathbf{PolyLog} \left( -\frac{3}{2}, ax \right) + \mathbf{PolyLog} \left( -\frac{1}{2}, ax \right) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[-3/2, a\*x] + PolyLog[-1/2, a\*x], x]

[Out] Integrate[PolyLog[-3/2, a\*x] + PolyLog[-1/2, a\*x], x]

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int \text{polylog} \left( -\frac{3}{2}, ax \right) + \text{polylog} \left( -\frac{1}{2}, ax \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)`

[Out] `int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{1}{2}}(ax) + \operatorname{Li}_{-\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="maxima")`

[Out] `integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{1}{2}, ax\right) + \operatorname{polylog}\left(-\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="fricas")`

[Out] `integral(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\operatorname{Li}_{-\frac{3}{2}}(ax) + \operatorname{Li}_{-\frac{1}{2}}(ax)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)`

[Out] `Integral(polylog(-3/2, a*x) + polylog(-1/2, a*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{1}{2}}(ax) + \operatorname{Li}_{-\frac{3}{2}}(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="giac")`

[Out] `integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)`

### 3.102 $\int (dx)^m \text{PolyLog}(2, ax) dx$

**Optimal.** Leaf size=78

$$\frac{a(dx)^{m+2} \text{Hypergeometric2F1}(1, m+2, m+3, ax)}{d^2(m+1)^2(m+2)} + \frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)} + \frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^2}$$

[Out] (a\*(d\*x)^(2+m)\*Hypergeometric2F1[1, 2+m, 3+m, a\*x])/(d^2\*(1+m)^2\*(2+m)) + ((d\*x)^(1+m)\*Log[1-a\*x])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x])/(d\*(1+m))

**Rubi [A]** time = 0.0487671, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2395, 64}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)} + \frac{a(dx)^{m+2} {}_2F_1(1, m+2; m+3; ax)}{d^2(m+1)^2(m+2)} + \frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[2, a\*x], x]

[Out] (a\*(d\*x)^(2+m)\*Hypergeometric2F1[1, 2+m, 3+m, a\*x])/(d^2\*(1+m)^2\*(2+m)) + ((d\*x)^(1+m)\*Log[1-a\*x])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x])/(d\*(1+m))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q+1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q+1)), x] - Dist[(b\*e\*n)/(g\*(q+1)), Int[(f + g\*x)^(q+1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 64

Int[((b\_.)\*(x\_)^(m\_.))\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c^n\*(b\*x)^(m+1)\*Hypergeometric2F1[-n, m+1, m+2, -(d\*x)/c])/(b\*(m+1)), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b\*c)), 0])))

#### Rubi steps



$$\begin{aligned} \int (dx)^m \text{Li}_2(ax) dx &= \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax) dx}{1+m} \\ &= \frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)} + \frac{a \int \frac{(dx)^{1+m}}{1-ax} dx}{d(1+m)^2} \\ &= \frac{a(dx)^{2+m} {}_2F_1(1, 2+m; 3+m; ax)}{d^2(1+m)^2(2+m)} + \frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.0387445, size = 53, normalized size = 0.68

$$\frac{x(dx)^m(ax\text{Hypergeometric2F1}(1, m+2, m+3, ax) + (m+2)((m+1)\text{PolyLog}(2, ax) + \log(1-ax)))}{(m+1)^2(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*PolyLog[2, a\*x], x]

[Out] (x\*(d\*x)^m\*(a\*x\*Hypergeometric2F1[1, 2 + m, 3 + m, a\*x] + (2 + m)\*(Log[1 - a\*x] + (1 + m)\*PolyLog[2, a\*x]))) / ((1 + m)^2\*(2 + m))

**Maple [C]** time = 0.227, size = 144, normalized size = 1.9

$$\frac{(dx)^m x^{-m} (-a)^{-m}}{a} \left( \frac{x^m (-a)^m (-am^2x - 2amx - m^2 - 3m - 2)}{(1+m)^3(2+m)m} - \frac{x^{1+m} a (-a)^m (-m-2) \ln(-ax+1)}{(1+m)^2(2+m)} + \frac{x^{1+m} a (-a)^m}{1+m} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(2,a\*x), x)

[Out] (d\*x)^m\*x^(-m)\*(-a)^(-m)/a\*(1/(2+m)\*x^m\*(-a)^m\*(-a\*m^2\*x-2\*a\*m\*x-m^2-3\*m-2)/(1+m)^3/m-1/(2+m)\*x^(1+m)\*a\*(-a)^m\*(-m-2)/(1+m)^2\*ln(-a\*x+1)+x^(1+m)\*a\*(-a)^m/(1+m)\*polylog(2,a\*x)+x^m\*(-a)^m/(1+m)^2\*LerchPhi(a\*x, 1, m))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-ad^m \int -\frac{xx^m}{m^2 - (am^2 + 2am + a)x + 2m + 1} dx + \frac{(d^m m + d^m)xx^m \text{Li}_2(ax) + d^m xx^m \log(-ax + 1)}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x), x, algorithm="maxima")

[Out] -a\*d^m\*integrate(-x\*x^m/(m^2 - (a\*m^2 + 2\*a\*m + a)\*x + 2\*m + 1), x) + ((d^m\*m + d^m)\*x\*x^m\*dilog(a\*x) + d^m\*x\*x^m\*log(-a\*x + 1))/(m^2 + 2\*m + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m \text{Li}_2(ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(2, a*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x), x)
```

### 3.103 $\int (dx)^m \text{PolyLog}(3, ax) dx$

**Optimal.** Leaf size=102

$$\frac{a(dx)^{m+2} \text{Hypergeometric2F1}(1, m+2, m+3, ax)}{d^2(m+1)^3(m+2)} - \frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax)}{d(m+1)} - \frac{\log(1 - ax)}{d(m+1)}$$

```
[Out] -((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^3*(2 + m))) - ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[2, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m))
```

**Rubi [A]** time = 0.0636762, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2395, 64}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax)}{d(m+1)} - \frac{a(dx)^{m+2} {}_2F_1(1, m+2; m+3; ax)}{d^2(m+1)^3(m+2)} - \frac{\log(1 - ax)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x],x]
```

```
[Out] -((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^3*(2 + m))) - ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[2, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m))
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^q_.], x_Symbol] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 64

```
Int[((b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c^n*(b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*x)/c])/(b*(m + 1)), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

#### Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_3(ax) dx &= \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)} - \frac{\int (dx)^m \text{Li}_2(ax) dx}{1+m} \\
&= -\frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)} - \frac{\int (dx)^m \log(1-ax) dx}{(1+m)^2} \\
&= -\frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^3} - \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)} - \frac{a \int \frac{(dx)^{1+m}}{1-ax} dx}{d(1+m)^3} \\
&= -\frac{a(dx)^{2+m} {}_2F_1(1, 2+m; 3+m; ax)}{d^2(1+m)^3(2+m)} - \frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^3} - \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0598987, size = 88, normalized size = 0.86

$$\frac{x \text{Gamma}(m+2) (dx)^m \left( a(m+1) x \text{Gamma}(m+1) {}_2\tilde{F}_1(1, m+2; m+3; ax) + m^2 (-\text{PolyLog}(3, ax)) - 2m \text{PolyLog}(3, ax) \right)}{(m+1)^4 \text{Gamma}(m+1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[3, a\*x], x]

[Out] -((x\*(d\*x)^m\*Gamma[2+m]\*(a\*(1+m)\*x\*Gamma[1+m]\*HypergeometricPFQRegularized[{1, 2+m}, {3+m}, a\*x] + Log[1-a\*x] + (1+m)\*PolyLog[2, a\*x] - PolyLog[3, a\*x] - 2\*m\*PolyLog[3, a\*x] - m^2\*PolyLog[3, a\*x]))/((1+m)^4\*Gamma[1+m]))

**Maple [C]** time = 0.388, size = 173, normalized size = 1.7

$$\frac{(dx)^m x^{-m} (-a)^{-m} \left( \frac{x^m (-a)^m (am^2x + 2amx + m^2 + 3m + 2)}{(1+m)^4(2+m)m} - \frac{x^{1+m} a (-a)^m \ln(-ax+1)}{(1+m)^3} + \frac{x^{1+m} a (-a)^m (-m-2) \text{polylog}(3, a*x)}{(1+m)^2(2+m)} \right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(3,a\*x), x)

[Out] (d\*x)^m\*x^(-m)\*(-a)^(-m)/a\*(1/(2+m)\*x^m\*(-a)^m\*(a\*m^2\*x+2\*a\*m\*x+m^2+3\*m+2)/(1+m)^4/m-x^(1+m)\*a\*(-a)^m/(1+m)^3\*ln(-a\*x+1)+1/(2+m)\*x^(1+m)\*a\*(-a)^m\*(-m-2)/(1+m)^2\*polylog(2,a\*x)+x^(1+m)\*a\*(-a)^m/(1+m)\*polylog(3,a\*x)+1/(2+m)\*x^m\*(-a)^m\*(-m-2)/(1+m)^3\*LerchPhi(a\*x, 1, m))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$ad^m \int -\frac{xx^m}{m^3 - (m^3 + 3m^2 + 3m + 1)ax + 3m^2 + 3m + 1} dx - \frac{d^m(m+1)xx^m \text{Li}_2(ax) - (m^2 + 2m + 1)d^m xx^m \text{Li}_3(ax) + a}{m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x), x, algorithm="maxima")

[Out] a\*d^m\*integrate(-x\*x^m/(m^3 - (m^3 + 3\*m^2 + 3\*m + 1)\*a\*x + 3\*m^2 + 3\*m + 1), x) - (d^m\*(m+1)\*x\*x^m\*dilog(a\*x) - (m^2 + 2\*m + 1)\*d^m\*x\*x^m\*polylog(3

,  $a*x$ ) +  $d^m*x*x^m*\log(-a*x + 1)/(m^3 + 3*m^2 + 3*m + 1)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m \text{polylog}(3, ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(3, a\*x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(3,a\*x),x)

[Out] Integral((d\*x)\*\*m\*polylog(3, a\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(3, a\*x), x)

### 3.104 $\int (dx)^m \text{PolyLog}(4, ax) dx$

**Optimal.** Leaf size=121

$$\frac{a(dx)^{m+2} \text{Hypergeometric2F1}(1, m+2, m+3, ax)}{d^2(m+1)^4(m+2)} + \frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)^3} - \frac{(dx)^{m+1} \text{PolyLog}(3, ax)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax)}{d(m+1)}$$

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^4*(2 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^4) + ((d*x)^(1 + m)*PolyLog[2, a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

**Rubi [A]** time = 0.0857405, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6591, 2395, 64}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax)}{d(m+1)^3} - \frac{(dx)^{m+1} \text{PolyLog}(3, ax)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax)}{d(m+1)} + \frac{a(dx)^{m+2} {}_2F_1(1, m+2; m+3; ax)}{d^2(m+1)^4(m+2)} + \frac{\log(ax)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x], x]
```

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^4*(2 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^4) + ((d*x)^(1 + m)*PolyLog[2, a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m+1)*PolyLog[n, a*(b*x^p)^q])/(d*(m+1)), x] - Dist[(p*q)/(m+1), Int[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q+1)*(a + b*Log[c*(d + e*x)^n])/(g*(q+1)), x] - Dist[(b*e*n)/(g*(q+1)), Int[(f + g*x)^(q+1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 64

```
Int[((b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[(c^n*(b*x)^(m+1)*Hypergeometric2F1[-n, m+1, m+2, -(d*x)/c])/(b*(m+1)), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0] && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0]))
```

#### Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_4(ax) dx &= \frac{(dx)^{1+m} \text{Li}_4(ax)}{d(1+m)} - \frac{\int (dx)^m \text{Li}_3(ax) dx}{1+m} \\
&= -\frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \text{Li}_2(ax) dx}{(1+m)^2} \\
&= \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax) dx}{(1+m)^3} \\
&= \frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax)}{d(1+m)} + \frac{a \int \frac{(dx)^{1+m}}{1-ax} dx}{d(1+m)^4} \\
&= \frac{a(dx)^{2+m} {}_2F_1(1, 2+m; 3+m; ax)}{d^2(1+m)^4(2+m)} + \frac{(dx)^{1+m} \log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m} \text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m} \text{Li}_3(ax)}{d(1+m)^2} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.0726869, size = 119, normalized size = 0.98

$$x\text{Gamma}(m+2)(dx)^m \left( a(m+1)x\text{Gamma}(m+1) {}_2\tilde{F}_1(1, m+2; m+3; ax) + m^3\text{PolyLog}(4, ax) - m^2\text{PolyLog}(3, ax) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[4, a\*x], x]

[Out] (x\*(d\*x)^m\*Gamma[2+m]\*(a\*(1+m)\*x\*Gamma[1+m]\*HypergeometricPFQRegularized[{1, 2+m}, {3+m}, a\*x] + Log[1-a\*x] + (1+m)\*PolyLog[2, a\*x] - PolyLog[3, a\*x] - 2\*m\*PolyLog[3, a\*x] - m^2\*PolyLog[3, a\*x] + PolyLog[4, a\*x] + 3\*m\*PolyLog[4, a\*x] + 3\*m^2\*PolyLog[4, a\*x] + m^3\*PolyLog[4, a\*x]))/(1+m)^5\*Gamma[1+m])

**Maple [C]** time = 0.794, size = 198, normalized size = 1.6

$$\frac{(dx)^m x^{-m} (-a)^{-m}}{a} \left( \frac{x^m (-a)^m (-am^2x - 2amx - m^2 - 3m - 2)}{(1+m)^5 (2+m)m} - \frac{x^{1+m} a (-a)^m (-m-2) \ln(-ax+1)}{(1+m)^4 (2+m)} + \frac{x^{1+m} a (-a)^m \text{polylog}(4, a*x)}{(1+m)^4 (2+m)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(4,a\*x), x)

[Out] (d\*x)^m\*x^(-m)\*(-a)^(-m)/a\*(1/(2+m)\*x^m\*(-a)^m\*(-a\*m^2\*x-2\*a\*m\*x-m^2-3\*m-2)/(1+m)^5/m-1/(2+m)\*x^(1+m)\*a\*(-a)^m\*(-m-2)/(1+m)^4\*ln(-a\*x+1)+x^(1+m)\*a\*(-a)^m/(1+m)^3\*polylog(2,a\*x)+1/(2+m)\*x^(1+m)\*a\*(-a)^m\*(-m-2)/(1+m)^2\*polylog(3,a\*x)+x^(1+m)\*a\*(-a)^m/(1+m)\*polylog(4,a\*x)+x^m\*(-a)^m/(1+m)^4\*LerchPhi(a\*x,1,m))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-ad^m \int \frac{xx^m}{m^4 + 4m^3 + 6m^2 - (am^4 + 4am^3 + 6am^2 + 4am + a)x + 4m + 1} dx + \frac{(d^m m + d^m)xx^m \text{Li}_2(ax) + d^m xx^m \text{Li}_3(ax)}{d^m m + d^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x), x, algorithm="maxima")

```
[Out] -a*d^m*integrate(-x*x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2 +
4*a*m + a)*x + 4*m + 1), x) + ((d^m*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*
log(-a*x + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*polylog(4, a*x)
- (d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x))/(m^4 + 4*m^3 + 6*m^2 +
4*m + 1)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m \text{polylog}(4, ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_4(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_4(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x), x)
```



### 3.105 $\int (dx)^m \text{PolyLog}(2, ax^2) dx$

**Optimal.** Leaf size=94

$$\frac{4a(dx)^{m+3} \text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, ax^2\right)}{d^3(m+1)^2(m+3)} + \frac{(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)} + \frac{2 \log(1-ax^2)(dx)^{m+1}}{d(m+1)^2}$$

[Out] (4\*a\*(d\*x)^(3+m)\*Hypergeometric2F1[1, (3+m)/2, (5+m)/2, a\*x^2])/(d^3\*(1+m)^2\*(3+m)) + (2\*(d\*x)^(1+m)\*Log[1-a\*x^2])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x^2])/(d\*(1+m))

**Rubi [A]** time = 0.0549094, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)} + \frac{4a(dx)^{m+3} {}_2F_1\left(1, \frac{m+3}{2}; \frac{m+5}{2}; ax^2\right)}{d^3(m+1)^2(m+3)} + \frac{2 \log(1-ax^2)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[2, a\*x^2], x]

[Out] (4\*a\*(d\*x)^(3+m)\*Hypergeometric2F1[1, (3+m)/2, (5+m)/2, a\*x^2])/(d^3\*(1+m)^2\*(3+m)) + (2\*(d\*x)^(1+m)\*Log[1-a\*x^2])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x^2])/(d\*(1+m))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])]/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int (dx)^m \operatorname{Li}_2(ax^2) dx &= \frac{(dx)^{1+m} \operatorname{Li}_2(ax^2)}{d(1+m)} + \frac{2 \int (dx)^m \log(1-ax^2) dx}{1+m} \\
&= \frac{2(dx)^{1+m} \log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{Li}_2(ax^2)}{d(1+m)} + \frac{(4a) \int \frac{x(dx)^{1+m}}{1-ax^2} dx}{d(1+m)^2} \\
&= \frac{2(dx)^{1+m} \log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{Li}_2(ax^2)}{d(1+m)} + \frac{(4a) \int \frac{(dx)^{2+m}}{1-ax^2} dx}{d^2(1+m)^2} \\
&= \frac{4a(dx)^{3+m} {}_2F_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; ax^2\right)}{d^3(1+m)^2(3+m)} + \frac{2(dx)^{1+m} \log(1-ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \operatorname{Li}_2(ax^2)}{d(1+m)}
\end{aligned}$$

**Mathematica [A]** time = 0.0454423, size = 72, normalized size = 0.77

$$\frac{x(dx)^m \left( 4ax^2 \operatorname{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, ax^2\right) + (m+3) \left( (m+1) \operatorname{PolyLog}(2, ax^2) + 2 \log(1-ax^2) \right) \right)}{(m+1)^2(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*PolyLog[2, a\*x^2], x]

[Out] (x\*(d\*x)^m\*(4\*a\*x^2\*Hypergeometric2F1[1, (3+m)/2, (5+m)/2, a\*x^2] + (3+m)\*(2\*Log[1-a\*x^2] + (1+m)\*PolyLog[2, a\*x^2]))) / ((1+m)^2\*(3+m))

**Maple [C]** time = 0.236, size = 177, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{2} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left( 2 \frac{x^{1+m} (-a)^{3/2+m/2} (-12-4m)}{(1+m)^3 (3+m) a} - 2 \frac{x^{1+m} (-a)^{3/2+m/2} (-6-2m) \ln(-ax^2+1)}{(1+m)^2 (3+m) a} + 2 \frac{x^{1+m} (-a)^{3/2+m/2}}{(1+m)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(2,a\*x^2), x)

[Out] -1/2\*(d\*x)^m\*x^(-m)\*(-a)^(-1/2-1/2\*m)\*(2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(-1-2-4\*m)/(1+m)^3/a-2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(-6-2\*m)/(1+m)^2/a\*ln(-a\*x^2+1)+2\*x^(1+m)\*(-a)^(3/2+1/2\*m)/(1+m)\*polylog(2,a\*x^2)/a+2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(6+2\*m)/(1+m)^2/a\*LerchPhi(a\*x^2,1,1/2+1/2\*m))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-4ad^m \int \frac{x^2 x^m}{(am^2 + 2am + a)x^2 - m^2 - 2m - 1} dx + \frac{(d^m m + d^m) x x^m \operatorname{Li}_2(ax^2) + 2d^m x x^m \log(-ax^2 + 1)}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^2), x, algorithm="maxima")

[Out] -4\*a\*d^m\*integrate(x^2\*x^m/((a\*m^2 + 2\*a\*m + a)\*x^2 - m^2 - 2\*m - 1), x) + ((d^m\*m + d^m)\*x\*x^m\*dilog(a\*x^2) + 2\*d^m\*x\*x^m\*log(-a\*x^2 + 1))/(m^2 + 2\*m)

+ 1)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \text{Li}_2(ax^2), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^2),x, algorithm="fricas")

[Out] integral((d\*x)^m\*dilog(a\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(2,a\*x\*\*2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_2(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^m\*dilog(a\*x^2), x)

### 3.106 $\int (dx)^m \text{PolyLog}(3, ax^2) dx$

**Optimal.** Leaf size=118

$$\frac{8a(dx)^{m+3} \text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, ax^2\right)}{d^3(m+1)^3(m+3)} - \frac{2(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^2)}{d(m+1)} - \frac{4 \log(1-ax^2)(dx)^{m+1}}{d(m+1)^3}$$

[Out] (-8\*a\*(d\*x)^(3+m)\*Hypergeometric2F1[1, (3+m)/2, (5+m)/2, a\*x^2])/(d^3\*(1+m)^3\*(m+3)) - (4\*(d\*x)^(1+m)\*Log[1-a\*x^2])/(d\*(1+m)^3) - (2\*(d\*x)^(1+m)\*PolyLog[2, a\*x^2])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[3, a\*x^2])/(d\*(1+m))

**Rubi [A]** time = 0.0712051, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$-\frac{2(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^2)}{d(m+1)} - \frac{8a(dx)^{m+3} {}_2F_1\left(1, \frac{m+3}{2}; \frac{m+5}{2}; ax^2\right)}{d^3(m+1)^3(m+3)} - \frac{4 \log(1-ax^2)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[3, a\*x^2], x]

[Out] (-8\*a\*(d\*x)^(3+m)\*Hypergeometric2F1[1, (3+m)/2, (5+m)/2, a\*x^2])/(d^3\*(1+m)^3\*(m+3)) - (4\*(d\*x)^(1+m)\*Log[1-a\*x^2])/(d\*(1+m)^3) - (2\*(d\*x)^(1+m)\*PolyLog[2, a\*x^2])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[3, a\*x^2])/(d\*(1+m))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])]/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_3(ax^2) dx &= \frac{(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)} - \frac{2 \int (dx)^m \text{Li}_2(ax^2) dx}{1+m} \\
&= -\frac{2(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)} - \frac{4 \int (dx)^m \log(1-ax^2) dx}{(1+m)^2} \\
&= -\frac{4(dx)^{1+m} \log(1-ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)} - \frac{(8a) \int \frac{x(dx)^{1+m}}{1-ax^2} dx}{d(1+m)^3} \\
&= -\frac{4(dx)^{1+m} \log(1-ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)} - \frac{(8a) \int \frac{(dx)^{2+m}}{1-ax^2} dx}{d^2(1+m)^3} \\
&= -\frac{8a(dx)^{3+m} {}_2F_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; ax^2\right)}{d^3(1+m)^3(3+m)} - \frac{4(dx)^{1+m} \log(1-ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0792053, size = 126, normalized size = 1.07

$$\frac{2x \text{Gamma}\left(\frac{m+3}{2}\right) (dx)^m \left(2a(m+1)x^2 \text{Gamma}\left(\frac{m+1}{2}\right) {}_2\tilde{F}_1\left(1, \frac{m+3}{2}; \frac{m+5}{2}; ax^2\right) + m^2 (-\text{PolyLog}(3, ax^2)) - 2m \text{PolyLog}(2, ax^2)\right)}{(m+1)^4 \text{Gamma}\left(\frac{m+1}{2}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[3, a\*x^2], x]

[Out]  $(-2*x*(d*x)^m*\text{Gamma}[(3+m)/2]*(2*a*(1+m)*x^2*\text{Gamma}[(1+m)/2]*\text{HypergeometricPFQRegularized}[\{1, (3+m)/2\}, \{(5+m)/2\}, a*x^2] + 4*\text{Log}[1 - a*x^2] + 2*(1+m)*\text{PolyLog}[2, a*x^2] - \text{PolyLog}[3, a*x^2] - 2*m*\text{PolyLog}[3, a*x^2] - m^2*\text{PolyLog}[3, a*x^2]))/((1+m)^4*\text{Gamma}[(1+m)/2])$

**Maple [C]** time = 0.41, size = 218, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{2} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left( 2 \frac{x^{1+m} (-a)^{3/2+m/2} (24+8m)}{(1+m)^4 (3+m)a} - 2 \frac{x^{1+m} (-a)^{3/2+m/2} (12+4m) \ln(-ax^2+1)}{(1+m)^3 (3+m)a} + 2 \frac{x^{1+m} (-a)^{3/2+m/2}}{(1+m)^3 (3+m)a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(3, a\*x^2), x)

[Out]  $-1/2*(d*x)^m*x^{(-m)}*(-a)^{(-1/2-1/2*m)}*(2/(3+m)*x^{(1+m)}*(-a)^{(3/2+1/2*m)}*(24+8*m)/(1+m)^4/a-2/(3+m)*x^{(1+m)}*(-a)^{(3/2+1/2*m)}*(12+4*m)/(1+m)^3/a*\ln(-a*x^2+1)+2/(3+m)*x^{(1+m)}*(-a)^{(3/2+1/2*m)}*(-6-2*m)/(1+m)^2*\text{polylog}(2, a*x^2)/a+2*x^{(1+m)}*(-a)^{(3/2+1/2*m)}/(1+m)/a*\text{polylog}(3, a*x^2)+2/(3+m)*x^{(1+m)}*(-a)^{(3/2+1/2*m)}*(-12-4*m)/(1+m)^3/a*\text{LerchPhi}(a*x^2, 1, 1/2+1/2*m))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8ad^m \int \frac{x^2 x^m}{(m^3 + 3m^2 + 3m + 1)ax^2 - m^3 - 3m^2 - 3m - 1} dx - \frac{2d^m(m+1)xx^m \text{Li}_2(ax^2) - (m^2 + 2m + 1)d^m xx^m \text{Li}_3(ax^2)}{m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^2),x, algorithm="maxima")

[Out]  $8*a*d^m*\int(x^2*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^2 - m^3 - 3*m^2 - 3*m - 1), x) - (2*d^m*(m + 1)*x*x^m*\operatorname{dilog}(a*x^2) - (m^2 + 2*m + 1)*d^m*x*x^m*\operatorname{polylog}(3, a*x^2) + 4*d^m*x*x^m*\log(-a*x^2 + 1))/(m^3 + 3*m^2 + 3*m + 1)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{polylog}(3, ax^2), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^2),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(3, a\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(3,a\*x\*\*2),x)

[Out] Integral((d\*x)\*\*m\*polylog(3, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(3, a\*x^2), x)

### 3.107 $\int (dx)^m \mathbf{PolyLog}(4, ax^2) dx$

**Optimal.** Leaf size=142

$$\frac{16a(dx)^{m+3} \text{Hypergeometric2F1}\left(1, \frac{m+3}{2}, \frac{m+5}{2}, ax^2\right)}{d^3(m+1)^4(m+3)} + \frac{4(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)^3} - \frac{2(dx)^{m+1} \text{PolyLog}(3, ax^2)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax^2)}{d(m+1)}$$

[Out] (16\*a\*(d\*x)^(3 + m)\*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a\*x^2])/(d^3\*(1 + m)^4\*(3 + m)) + (8\*(d\*x)^(1 + m)\*Log[1 - a\*x^2])/(d\*(1 + m)^4) + (4\*(d\*x)^(1 + m)\*PolyLog[2, a\*x^2])/(d\*(1 + m)^3) - (2\*(d\*x)^(1 + m)\*PolyLog[3, a\*x^2])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[4, a\*x^2])/(d\*(1 + m))

**Rubi [A]** time = 0.0934271, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$\frac{4(dx)^{m+1} \text{PolyLog}(2, ax^2)}{d(m+1)^3} - \frac{2(dx)^{m+1} \text{PolyLog}(3, ax^2)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax^2)}{d(m+1)} + \frac{16a(dx)^{m+3} {}_2F_1\left(1, \frac{m+3}{2}; \frac{m+5}{2}; ax^2\right)}{d^3(m+1)^4(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[4, a\*x^2], x]

[Out] (16\*a\*(d\*x)^(3 + m)\*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a\*x^2])/(d^3\*(1 + m)^4\*(3 + m)) + (8\*(d\*x)^(1 + m)\*Log[1 - a\*x^2])/(d\*(1 + m)^4) + (4\*(d\*x)^(1 + m)\*PolyLog[2, a\*x^2])/(d\*(1 + m)^3) - (2\*(d\*x)^(1 + m)\*PolyLog[3, a\*x^2])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[4, a\*x^2])/(d\*(1 + m))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_))^(n\_), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_4(ax^2) dx &= \frac{(dx)^{1+m} \text{Li}_4(ax^2)}{d(1+m)} - \frac{2 \int (dx)^m \text{Li}_3(ax^2) dx}{1+m} \\
&= -\frac{2(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^2)}{d(1+m)} + \frac{4 \int (dx)^m \text{Li}_2(ax^2) dx}{(1+m)^2} \\
&= \frac{4(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^2)}{d(1+m)} + \frac{8 \int (dx)^m \log(1-ax^2) dx}{(1+m)^3} \\
&= \frac{8(dx)^{1+m} \log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^2)}{d(1+m)} + \frac{(16a) \int}{d(1+m)} \\
&= \frac{8(dx)^{1+m} \log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_3(ax^2)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^2)}{d(1+m)} + \frac{(16a) \int}{d^2(1+m)} \\
&= \frac{16a(dx)^{3+m} {}_2F_1\left(1, \frac{3+m}{2}; \frac{5+m}{2}; ax^2\right)}{d^3(1+m)^4(3+m)} + \frac{8(dx)^{1+m} \log(1-ax^2)}{d(1+m)^4} + \frac{4(dx)^{1+m} \text{Li}_2(ax^2)}{d(1+m)^3} - \frac{2(dx)^{1+m} \text{Li}_3}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0995597, size = 166, normalized size = 1.17

$$2x\text{Gamma}\left(\frac{m+3}{2}\right)(dx)^m\left(4a(m+1)x^2\text{Gamma}\left(\frac{m+1}{2}\right) {}_2\tilde{F}_1\left(1, \frac{m+3}{2}; \frac{m+5}{2}; ax^2\right) + m^3\text{PolyLog}\left(4, ax^2\right) - 2m^2\text{PolyLog}\left(3, a\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[4, a\*x^2], x]

[Out] (2\*x\*(d\*x)^m\*Gamma[(3+m)/2]\*(4\*a\*(1+m)\*x^2\*Gamma[(1+m)/2]\*HypergeometricPFQRegularized[{1, (3+m)/2}, {(5+m)/2}, a\*x^2] + 8\*Log[1 - a\*x^2] + 4\*(1+m)\*PolyLog[2, a\*x^2] - 2\*PolyLog[3, a\*x^2] - 4\*m\*PolyLog[3, a\*x^2] - 2\*m^2\*PolyLog[3, a\*x^2] + PolyLog[4, a\*x^2] + 3\*m\*PolyLog[4, a\*x^2] + 3\*m^2\*PolyLog[4, a\*x^2] + m^3\*PolyLog[4, a\*x^2]))/((1+m)^5\*Gamma[(1+m)/2])

**Maple [C]** time = 0.792, size = 259, normalized size = 1.8

$$-\frac{(dx)^m x^{-m}}{2} (-a)^{-\frac{1}{2}-\frac{m}{2}} \left( 2 \frac{x^{1+m} (-a)^{3/2+m/2} (-48-16m)}{(1+m)^5 (3+m) a} - 2 \frac{x^{1+m} (-a)^{3/2+m/2} (-24-8m) \ln(-ax^2+1)}{(1+m)^4 (3+m) a} + 2 \frac{x^{1+m} (-a)^{3/2+m/2}}{(1+m)^4 (3+m) a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(4,a\*x^2), x)

[Out] -1/2\*(d\*x)^m\*x^(-m)\*(-a)^(-1/2-1/2\*m)\*(2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(-48-16\*m)/(1+m)^5/a-2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(-24-8\*m)/(1+m)^4/a\*ln(-a\*x^2+1)+2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(12+4\*m)/(1+m)^3\*polylog(2,a\*x^2)/a+2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(-6-2\*m)/(1+m)^2/a\*polylog(3,a\*x^2)+2\*x^(1+m)\*(-a)^(3/2+1/2\*m)/(1+m)/a\*polylog(4,a\*x^2)+2/(3+m)\*x^(1+m)\*(-a)^(3/2+1/2\*m)\*(24+8\*m)/(1+m)^4/a\*LerchPhi(a\*x^2, 1, 1/2+1/2\*m))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-16ad^m \int \frac{x^2 x^m}{m^4 + 4m^3 - (am^4 + 4am^3 + 6am^2 + 4am + a)x^2 + 6m^2 + 4m + 1} dx + \frac{4(d^m m + d^m) x x^m \text{Li}_2(ax^2) + 8d^m}{m^4 + 4m^3 - (am^4 + 4am^3 + 6am^2 + 4am + a)x^2 + 6m^2 + 4m + 1}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^2),x, algorithm="maxima")

[Out]  $-16*a*d^m*\int(-x^2*x^m/(m^4 + 4*m^3 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*a*m + a)*x^2 + 6*m^2 + 4*m + 1), x) + (4*(d^m*m + d^m)*x*x^m*\operatorname{dilog}(a*x^2) + 8*d^m*x*x^m*\log(-a*x^2 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*\operatorname{polylog}(4, a*x^2) - 2*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*\operatorname{polylog}(3, a*x^2))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx)^m \operatorname{polylog}\left(4, ax^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^2),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(4, a\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(4,a\*x\*\*2),x)

[Out] Integral((d\*x)\*\*m\*polylog(4, a\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^2),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(4, a\*x^2), x)

### 3.108 $\int (dx)^m \text{PolyLog}(2, ax^3) dx$

**Optimal.** Leaf size=94

$$\frac{9a(dx)^{m+4} \text{Hypergeometric2F1}\left(1, \frac{m+4}{3}, \frac{m+7}{3}, ax^3\right)}{d^4(m+1)^2(m+4)} + \frac{(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)} + \frac{3 \log(1 - ax^3)(dx)^{m+1}}{d(m+1)^2}$$

[Out] (9\*a\*(d\*x)^(4 + m)\*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a\*x^3])/(d^4\*(1 + m)^2\*(4 + m)) + (3\*(d\*x)^(1 + m)\*Log[1 - a\*x^3])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[2, a\*x^3])/(d\*(1 + m))

**Rubi [A]** time = 0.0547537, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)} + \frac{9a(dx)^{m+4} {}_2F_1\left(1, \frac{m+4}{3}; \frac{m+7}{3}; ax^3\right)}{d^4(m+1)^2(m+4)} + \frac{3 \log(1 - ax^3)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[2, a\*x^3], x]

[Out] (9\*a\*(d\*x)^(4 + m)\*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a\*x^3])/(d^4\*(1 + m)^2\*(4 + m)) + (3\*(d\*x)^(1 + m)\*Log[1 - a\*x^3])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[2, a\*x^3])/(d\*(1 + m))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_2(ax^3) dx &= \frac{(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)} + \frac{3 \int (dx)^m \log(1-ax^3) dx}{1+m} \\
&= \frac{3(dx)^{1+m} \log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)} + \frac{(9a) \int \frac{x^2(dx)^{1+m}}{1-ax^3} dx}{d(1+m)^2} \\
&= \frac{3(dx)^{1+m} \log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)} + \frac{(9a) \int \frac{(dx)^{3+m}}{1-ax^3} dx}{d^3(1+m)^2} \\
&= \frac{9a(dx)^{4+m} {}_2F_1\left(1, \frac{4+m}{3}; \frac{7+m}{3}; ax^3\right)}{d^4(1+m)^2(4+m)} + \frac{3(dx)^{1+m} \log(1-ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)}
\end{aligned}$$

**Mathematica [A]** time = 0.0448169, size = 72, normalized size = 0.77

$$\frac{x(dx)^m \left( 9ax^3 \text{Hypergeometric2F1}\left(1, \frac{m+4}{3}, \frac{m+7}{3}, ax^3\right) + (m+4) \left( (m+1) \text{PolyLog}(2, ax^3) + 3 \log(1-ax^3) \right) \right)}{(m+1)^2(m+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*PolyLog[2, a\*x^3], x]

[Out] (x\*(d\*x)^m\*(9\*a\*x^3\*Hypergeometric2F1[1, (4+m)/3, (7+m)/3, a\*x^3] + (4+m)\*(3\*Log[1-a\*x^3] + (1+m)\*PolyLog[2, a\*x^3]))/((1+m)^2\*(4+m))

**Maple [C]** time = 0.277, size = 177, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{3} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left( 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-36-9m)}{(1+m)^3 (4+m)a} - 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-12-3m) \ln(-x^3a+1)}{(1+m)^2 (4+m)a} + 3 \frac{x^{1+m} (-a)^{4/3}}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(2,x^3\*a), x)

[Out] -1/3\*(d\*x)^m\*x^(-m)\*(-a)^(-1/3-1/3\*m)\*(3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-36-9\*m)/(1+m)^3/a-3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-12-3\*m)/(1+m)^2\*ln(-a\*x^3+1)/a+3\*x^(1+m)\*(-a)^(4/3+1/3\*m)/(1+m)/a\*polylog(2,x^3\*a)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(12+3\*m)/(1+m)^2/a\*LerchPhi(x^3\*a,1,1/3\*m+1/3))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-9ad^m \int \frac{x^3 x^m}{(am^2 + 2am + a)x^3 - m^2 - 2m - 1} dx + \frac{(d^m m + d^m) x x^m \text{Li}_2(ax^3) + 3d^m x x^m \log(-ax^3 + 1)}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^3), x, algorithm="maxima")

[Out] -9\*a\*d^m\*integrate(x^3\*x^m/((a\*m^2 + 2\*a\*m + a)\*x^3 - m^2 - 2\*m - 1), x) + ((d^m\*m + d^m)\*x\*x^m\*dilog(a\*x^3) + 3\*d^m\*x\*x^m\*log(-a\*x^3 + 1))/(m^2 + 2\*m)

+ 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \text{Li}_2(ax^3), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^3),x, algorithm="fricas")

[Out] integral((d\*x)^m\*dilog(a\*x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(2,a\*x\*\*3),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_2(ax^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^3),x, algorithm="giac")

[Out] integrate((d\*x)^m\*dilog(a\*x^3), x)

### 3.109 $\int (dx)^m \text{PolyLog}(3, ax^3) dx$

**Optimal.** Leaf size=118

$$\frac{27a(dx)^{m+4} \text{Hypergeometric2F1}\left(1, \frac{m+4}{3}, \frac{m+7}{3}, ax^3\right)}{d^4(m+1)^3(m+4)} - \frac{3(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^3)}{d(m+1)} - 9 \log(1-ax^3) \frac{(dx)^m}{d(m+1)^3}$$

[Out]  $(-27*a*(d*x)^(4+m)*\text{Hypergeometric2F1}[1, (4+m)/3, (7+m)/3, a*x^3])/(d^4*(1+m)^3*(4+m)) - (9*(d*x)^(1+m)*\text{Log}[1-a*x^3])/(d*(1+m)^3) - (3*(d*x)^(1+m)*\text{PolyLog}[2, a*x^3])/(d*(1+m)^2) + ((d*x)^(1+m)*\text{PolyLog}[3, a*x^3])/(d*(1+m))$

**Rubi [A]** time = 0.0712691, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$\frac{3(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^3)}{d(m+1)} - \frac{27a(dx)^{m+4} {}_2F_1\left(1, \frac{m+4}{3}; \frac{m+7}{3}; ax^3\right)}{d^4(m+1)^3(m+4)} - \frac{9 \log(1-ax^3) (dx)^m}{d(m+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[3, a\*x^3], x]

[Out]  $(-27*a*(d*x)^(4+m)*\text{Hypergeometric2F1}[1, (4+m)/3, (7+m)/3, a*x^3])/(d^4*(1+m)^3*(4+m)) - (9*(d*x)^(1+m)*\text{Log}[1-a*x^3])/(d*(1+m)^3) - (3*(d*x)^(1+m)*\text{PolyLog}[2, a*x^3])/(d*(1+m)^2) + ((d*x)^(1+m)*\text{PolyLog}[3, a*x^3])/(d*(1+m))$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_3(ax^3) dx &= \frac{(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)} - \frac{3 \int (dx)^m \text{Li}_2(ax^3) dx}{1+m} \\
&= -\frac{3(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)} - \frac{9 \int (dx)^m \log(1-ax^3) dx}{(1+m)^2} \\
&= -\frac{9(dx)^{1+m} \log(1-ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)} - \frac{(27a) \int \frac{x^2(dx)^{1+m}}{1-ax^3} dx}{d(1+m)^3} \\
&= -\frac{9(dx)^{1+m} \log(1-ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)} - \frac{(27a) \int \frac{(dx)^{3+m}}{1-ax^3} dx}{d^3(1+m)^3} \\
&= -\frac{27a(dx)^{4+m} {}_2F_1\left(1, \frac{4+m}{3}; \frac{7+m}{3}; ax^3\right)}{d^4(1+m)^3(4+m)} - \frac{9(dx)^{1+m} \log(1-ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0802421, size = 126, normalized size = 1.07

$$\frac{3x \text{Gamma}\left(\frac{m+4}{3}\right) (dx)^m \left(3a(m+1)x^3 \text{Gamma}\left(\frac{m+1}{3}\right) {}_2\tilde{F}_1\left(1, \frac{m+4}{3}; \frac{m+7}{3}; ax^3\right) - m^2 \text{PolyLog}\left(3, ax^3\right) + 3(m+1) \text{PolyLog}\left(2, ax^3\right)\right)}{(m+1)^4 \text{Gamma}\left(\frac{m+1}{3}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[3, a\*x^3], x]

[Out] (-3\*x\*(d\*x)^m\*Gamma[(4+m)/3]\*(3\*a\*(1+m)\*x^3\*Gamma[(1+m)/3]\*HypergeometricPFQRegularized[{1, (4+m)/3}, {(7+m)/3}, a\*x^3] + 9\*Log[1 - a\*x^3] + 3\*(1+m)\*PolyLog[2, a\*x^3] - PolyLog[3, a\*x^3] - 2\*m\*PolyLog[3, a\*x^3] - m^2\*PolyLog[3, a\*x^3])/((1+m)^4\*Gamma[(1+m)/3])

**Maple [C]** time = 0.414, size = 218, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{3} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left( 3 \frac{x^{1+m} (-a)^{4/3+m/3} (108+27m)}{(1+m)^4 (4+m)a} - 3 \frac{x^{1+m} (-a)^{4/3+m/3} (36+9m) \ln(-x^3a+1)}{(1+m)^3 (4+m)a} + 3 \frac{x^{1+m} (-a)^{4/3+m/3}}{(1+m)^3 (4+m)a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(3,x^3\*a), x)

[Out] -1/3\*(d\*x)^m\*x^(-m)\*(-a)^(-1/3-1/3\*m)\*(3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(108+27\*m)/(1+m)^4/a-3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(36+9\*m)/(1+m)^3/a\*ln(-a\*x^3+1)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-12-3\*m)/(1+m)^2\*polylog(2,x^3\*a)/a+3\*x^(1+m)\*(-a)^(4/3+1/3\*m)/(1+m)/a\*polylog(3,x^3\*a)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-36-9\*m)/(1+m)^3/a\*LerchPhi(x^3\*a,1,1/3\*m+1/3))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$27 ad^m \int \frac{x^3 x^m}{(m^3 + 3m^2 + 3m + 1)ax^3 - m^3 - 3m^2 - 3m - 1} dx - \frac{3d^m(m+1)xx^m \text{Li}_2(ax^3) - (m^2 + 2m + 1)d^m xx^m \text{Li}_3(ax^3)}{m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^3),x, algorithm="maxima")

[Out]  $27*a*d^m*\int (x^3*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^3 - m^3 - 3*m^2 - 3*m - 1), x) - (3*d^m*(m + 1)*x*x^m*\operatorname{dilog}(a*x^3) - (m^2 + 2*m + 1)*d^m*x*x^m*\operatorname{polylog}(3, a*x^3) + 9*d^m*x*x^m*\log(-a*x^3 + 1))/(m^3 + 3*m^2 + 3*m + 1)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((dx)^m \operatorname{polylog}(3, ax^3), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^3),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(3, a\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(3,a\*x\*\*3),x)

[Out] Integral((d\*x)\*\*m\*polylog(3, a\*x\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3,a\*x^3),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(3, a\*x^3), x)

### 3.110 $\int (dx)^m \text{PolyLog}(4, ax^3) dx$

**Optimal.** Leaf size=142

$$\frac{81a(dx)^{m+4} \text{Hypergeometric2F1}\left(1, \frac{m+4}{3}, \frac{m+7}{3}, ax^3\right)}{d^4(m+1)^4(m+4)} + \frac{9(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)^3} - \frac{3(dx)^{m+1} \text{PolyLog}(3, ax^3)}{d(m+1)^2} + \frac{(dx)^m}{d}$$

[Out] (81\*a\*(d\*x)^(4 + m)\*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a\*x^3])/(d^4\*(1 + m)^4\*(4 + m)) + (27\*(d\*x)^(1 + m)\*Log[1 - a\*x^3])/(d\*(1 + m)^4) + (9\*(d\*x)^(1 + m)\*PolyLog[2, a\*x^3])/(d\*(1 + m)^3) - (3\*(d\*x)^(1 + m)\*PolyLog[3, a\*x^3])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[4, a\*x^3])/(d\*(1 + m))

**Rubi [A]** time = 0.0943941, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 16, 364}

$$\frac{9(dx)^{m+1} \text{PolyLog}(2, ax^3)}{d(m+1)^3} - \frac{3(dx)^{m+1} \text{PolyLog}(3, ax^3)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax^3)}{d(m+1)} + \frac{81a(dx)^{m+4} {}_2F_1\left(1, \frac{m+4}{3}; \frac{m+7}{3}; ax^3\right)}{d^4(m+1)^4(m+4)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[4, a\*x^3], x]

[Out] (81\*a\*(d\*x)^(4 + m)\*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a\*x^3])/(d^4\*(1 + m)^4\*(4 + m)) + (27\*(d\*x)^(1 + m)\*Log[1 - a\*x^3])/(d\*(1 + m)^4) + (9\*(d\*x)^(1 + m)\*PolyLog[2, a\*x^3])/(d\*(1 + m)^3) - (3\*(d\*x)^(1 + m)\*PolyLog[3, a\*x^3])/(d\*(1 + m)^2) + ((d\*x)^(1 + m)\*PolyLog[4, a\*x^3])/(d\*(1 + m))

#### Rule 6591

Int[((d\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_.)^(m\_.)\*((b\_.)\*(v\_.))^(n\_.), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps



$$\begin{aligned}
\int (dx)^m \text{Li}_4(ax^3) dx &= \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)} - \frac{3 \int (dx)^m \text{Li}_3(ax^3) dx}{1+m} \\
&= -\frac{3(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)} + \frac{9 \int (dx)^m \text{Li}_2(ax^3) dx}{(1+m)^2} \\
&= \frac{9(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)} + \frac{27 \int (dx)^m \log(1-ax^3) dx}{(1+m)^3} \\
&= \frac{27(dx)^{1+m} \log(1-ax^3)}{d(1+m)^4} + \frac{9(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)} + \frac{81 \int (dx)^m \log(1-ax^3) dx}{(1+m)^4} \\
&= \frac{27(dx)^{1+m} \log(1-ax^3)}{d(1+m)^4} + \frac{9(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)} + \frac{81 \int (dx)^m \log(1-ax^3) dx}{(1+m)^4} \\
&= \frac{81a(dx)^{4+m} {}_2F_1\left(1, \frac{4+m}{3}; \frac{7+m}{3}; ax^3\right)}{d^4(1+m)^4(4+m)} + \frac{27(dx)^{1+m} \log(1-ax^3)}{d(1+m)^4} + \frac{9(dx)^{1+m} \text{Li}_2(ax^3)}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{Li}_3(ax^3)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^3)}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0967328, size = 166, normalized size = 1.17

$$3x \text{Gamma}\left(\frac{m+4}{3}\right) (dx)^m \left(9a(m+1)x^3 \text{Gamma}\left(\frac{m+1}{3}\right) {}_2\tilde{F}_1\left(1, \frac{m+4}{3}; \frac{m+7}{3}; ax^3\right) + m^3 \text{PolyLog}(4, ax^3) - 3m^2 \text{PolyLog}(3, ax^3) - 6m \text{PolyLog}(2, ax^3) + \text{PolyLog}(1, ax^3)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[4, a\*x^3], x]

[Out] (3\*x\*(d\*x)^m\*Gamma[(4+m)/3]\*(9\*a\*(1+m)\*x^3\*Gamma[(1+m)/3]\*HypergeometricPFQRegularized[{1, (4+m)/3}, {(7+m)/3}, a\*x^3] + 27\*Log[1-a\*x^3] + 9\*(1+m)\*PolyLog[2, a\*x^3] - 3\*PolyLog[3, a\*x^3] - 6\*m\*PolyLog[3, a\*x^3] - 3\*m^2\*PolyLog[3, a\*x^3] + PolyLog[4, a\*x^3] + 3\*m\*PolyLog[4, a\*x^3] + 3\*m^2\*PolyLog[4, a\*x^3] + m^3\*PolyLog[4, a\*x^3])/((1+m)^5\*Gamma[(1+m)/3])

**Maple [C]** time = 0.856, size = 259, normalized size = 1.8

$$-\frac{(dx)^m x^{-m}}{3} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left( 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-324-81m)}{(1+m)^5 (4+m)a} - 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-108-27m) \ln(-x^3a+1)}{(1+m)^4 (4+m)a} + 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-108-27m)}{(1+m)^4 (4+m)a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(4, x^3\*a), x)

[Out] -1/3\*(d\*x)^m\*x^(-m)\*(-a)^(-1/3-1/3\*m)\*(3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-324-81\*m)/(1+m)^5/a-3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-108-27\*m)/(1+m)^4/a\*ln(-a\*x^3+1)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(36+9\*m)/(1+m)^3/a\*polylog(2, x^3\*a)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(-12-3\*m)/(1+m)^2\*polylog(3, x^3\*a)/a+3\*x^(1+m)\*(-a)^(4/3+1/3\*m)/(1+m)/a\*polylog(4, x^3\*a)+3/(4+m)\*x^(1+m)\*(-a)^(4/3+1/3\*m)\*(108+27\*m)/(1+m)^4/a\*LerchPhi(x^3\*a, 1, 1/3\*m+1/3))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-81 ad^m \int \frac{x^3 x^m}{m^4 - (am^4 + 4am^3 + 6am^2 + 4am + a)x^3 + 4m^3 + 6m^2 + 4m + 1} dx + \frac{9(d^m m + d^m) x x^m \text{Li}_2(ax^3) + 27 d^m \log(1-ax^3)}{d(1+m)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^3),x, algorithm="maxima")

[Out]  $-81*a*d^m*\int(-x^3*x^m/(m^4 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*a*m + a)*x^3 + 4*m^3 + 6*m^2 + 4*m + 1), x) + (9*(d^m*m + d^m)*x*x^m*\operatorname{dilog}(a*x^3) + 27*d^m*x*x^m*\log(-a*x^3 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*\operatorname{polylog}(4, a*x^3) - 3*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*\operatorname{polylog}(3, a*x^3))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{polylog}(4, ax^3), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^3),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(4, a\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(4,a\*x\*\*3),x)

[Out] Integral((d\*x)\*\*m\*polylog(4, a\*x\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^3),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(4, a\*x^3), x)

### 3.111 $\int (dx)^m \text{PolyLog}(2, ax^q) dx$

**Optimal.** Leaf size=101

$$\frac{aq^2x^{q+1}(dx)^m \text{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right)}{(m+1)^2(m+q+1)} + \frac{(dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)} + \frac{q(dx)^{m+1} \log(1-ax^q)}{d(m+1)^2}$$

[Out] (a\*q^2\*x^(1+q)\*(d\*x)^m\*Hypergeometric2F1[1, (1+m+q)/q, (1+m+2\*q)/q, a\*x^q])/((1+m)^2\*(1+m+q)) + (q\*(d\*x)^(1+m)\*Log[1-a\*x^q])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x^q])/(d\*(1+m))

**Rubi [A]** time = 0.0597507, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 20, 364}

$$\frac{(dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)} + \frac{aq^2x^{q+1}(dx)^m {}_2F_1\left(1, \frac{m+q+1}{q}; \frac{m+2q+1}{q}; ax^q\right)}{(m+1)^2(m+q+1)} + \frac{q(dx)^{m+1} \log(1-ax^q)}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[2, a\*x^q], x]

[Out] (a\*q^2\*x^(1+q)\*(d\*x)^m\*Hypergeometric2F1[1, (1+m+q)/q, (1+m+2\*q)/q, a\*x^q])/((1+m)^2\*(1+m+q)) + (q\*(d\*x)^(1+m)\*Log[1-a\*x^q])/(d\*(1+m)^2) + ((d\*x)^(1+m)\*PolyLog[2, a\*x^q])/(d\*(1+m))

#### Rule 6591

Int[((d\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m+1)), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_.))^(m\_.)\*((b\_.)\*(v\_.))^(n\_.), x\_Symbol] :> Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_2(ax^q) dx &= \frac{(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)} + \frac{q \int (dx)^m \log(1-ax^q) dx}{1+m} \\
&= \frac{q(dx)^{1+m} \log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)} + \frac{(aq^2) \int \frac{x^{-1+q}(dx)^{1+m}}{1-ax^q} dx}{d(1+m)^2} \\
&= \frac{q(dx)^{1+m} \log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)} + \frac{(aq^2 x^{-m} (dx)^m) \int \frac{x^{m+q}}{1-ax^q} dx}{(1+m)^2} \\
&= \frac{aq^2 x^{1+q} (dx)^m {}_2F_1\left(1, \frac{1+m+q}{q}; \frac{1+m+2q}{q}; ax^q\right)}{(1+m)^2(1+m+q)} + \frac{q(dx)^{1+m} \log(1-ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)}
\end{aligned}$$

**Mathematica [A]** time = 0.0642053, size = 80, normalized size = 0.79

$$\frac{x(dx)^m \left( aq^2 x^q \text{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right) + (m+q+1) \left( (m+1) \text{PolyLog}(2, ax^q) + q \log(1-ax^q) \right) \right)}{(m+1)^2(m+q+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*PolyLog[2, a\*x^q], x]

[Out] (x\*(d\*x)^m\*(a\*q^2\*x^q\*Hypergeometric2F1[1, (1+m+q)/q, (1+m+2\*q)/q, a\*x^q] + (1+m+q)\*(q\*Log[1-a\*x^q] + (1+m)\*PolyLog[2, a\*x^q]))/((1+m)^2\*(1+m+q))

**Maple [C]** time = 0.293, size = 148, normalized size = 1.5

$$-\frac{(dx)^m x^{-m}}{q} (-a)^{-\frac{m}{q}-q^{-1}} \left( -\frac{q^2 x^{1+m} \ln(1-ax^q)}{(1+m)^2} (-a)^{\frac{m}{q}+q^{-1}} - \frac{qx^{1+m} \text{polylog}(2, ax^q)}{1+m} (-a)^{\frac{m}{q}+q^{-1}} - \frac{q^2 x^{1+m+q} a}{(1+m)^2} (-a)^{\frac{m}{q}+q^{-1}} \text{LerchPhi}(a*x^q, 1, (1+m+q)/q) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(2,a\*x^q), x)

[Out] -(d\*x)^m\*x^(-m)\*(-a)^(-m/q-1/q)/q\*(-q^2\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^2\*ln(1-a\*x^q)-q\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)\*polylog(2,a\*x^q)-q^2\*x^(1+m+q)\*a\*(-a)^(m/q+1/q)/(1+m)^2\*LerchPhi(a\*x^q, 1, (1+m+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-d^m q^2 \int -\frac{x^m}{m^2 - (am^2 + 2am + a)x^q + 2m + 1} dx - \frac{d^m q^2 x x^m - (d^m m + d^m) q x x^m \log(-ax^q + 1) - (d^m m^2 + 2d^m m + d^m)}{m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^q), x, algorithm="maxima")

[Out] -d^m\*q^2\*integrate(-x^m/(m^2 - (a\*m^2 + 2\*a\*m + a)\*x^q + 2\*m + 1), x) - (d^m\*q^2\*x\*x^m - (d^m\*m + d^m)\*q\*x\*x^m\*log(-a\*x^q + 1) - (d^m\*m^2 + 2\*d^m\*m + d^m)\*x\*x^m\*dilog(a\*x^q))/(m^3 + 3\*m^2 + 3\*m + 1)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m \text{Li}_2(ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^q),x, algorithm="fricas")

[Out] integral((d\*x)^m\*dilog(a\*x^q), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(2,a\*x\*\*q),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_2(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(2,a\*x^q),x, algorithm="giac")

[Out] integrate((d\*x)^m\*dilog(a\*x^q), x)

### 3.112 $\int (dx)^m \text{PolyLog}(3, ax^q) dx$

**Optimal.** Leaf size=130

$$\frac{aq^3x^{q+1}(dx)^m \text{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right)}{(m+1)^3(m+q+1)} - \frac{q(dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^q)}{d(m+1)}$$

[Out]  $-\left(\frac{a^3q^3x^{q+1}(dx)^m \text{Hypergeometric2F1}\left[1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, ax^q\right]}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m} \text{Log}[1-ax^q]}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{PolyLog}[2, ax^q]}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}[3, ax^q]}{d(1+m)}\right)$

**Rubi [A]** time = 0.0758585, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 20, 364}

$$-\frac{q(dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(3, ax^q)}{d(m+1)} - \frac{aq^3x^{q+1}(dx)^m {}_2F_1\left(1, \frac{m+q+1}{q}; \frac{m+2q+1}{q}; ax^q\right)}{(m+1)^3(m+q+1)} - \frac{q^2(dx)^{m+1} \log(1-ax^q)}{d(m+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*PolyLog[3, a\*x^q], x]

[Out]  $-\left(\frac{a^3q^3x^{q+1}(dx)^m \text{Hypergeometric2F1}\left[1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, ax^q\right]}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m} \text{Log}[1-ax^q]}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{PolyLog}[2, ax^q]}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}[3, ax^q]}{d(1+m)}\right)$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m+1)\*PolyLog[n, a\*(b\*x^p)^q])/d(m+1), x] - Dist[(p\*q)/(m+1), Int[(d\*x)^m\*PolyLog[n-1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m+1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m+1)), x] - Dist[(b\*e\*n\*p)/(f\*(m+1)), Int[(x^(n-1)\*(f\*x)^(m+1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

#### Rule 20

Int[(u\_.)\*((a\_.)\*(v\_))^(m\_.)\*((b\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b^IntPart[n]\*(b\*v)^FracPart[n])/(a^IntPart[n]\*(a\*v)^FracPart[n]), Int[u\*(a\*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/c\*(m+1), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_3(ax^q) dx &= \frac{(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)} - \frac{q \int (dx)^m \text{Li}_2(ax^q) dx}{1+m} \\
&= -\frac{q(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)} - \frac{q^2 \int (dx)^m \log(1-ax^q) dx}{(1+m)^2} \\
&= -\frac{q^2(dx)^{1+m} \log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)} - \frac{(aq^3) \int \frac{x^{-1+q}(dx)^{1+m}}{1-ax^q} dx}{d(1+m)^3} \\
&= -\frac{q^2(dx)^{1+m} \log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)} - \frac{(aq^3 x^{-m}(dx)^m) \int \frac{x^{m+q}}{1-ax^q} dx}{(1+m)^3} \\
&= -\frac{aq^3 x^{1+q}(dx)^m {}_2F_1\left(1, \frac{1+m+q}{q}; \frac{1+m+2q}{q}; ax^q\right)}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m} \log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)}
\end{aligned}$$

**Mathematica [C]** time = 0.0248751, size = 50, normalized size = 0.38

$$\frac{x(dx)^m G_{5,5}^{1,5}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1 - \frac{m+1}{q} \\ 1, 0, 0, 0, -\frac{m+1}{q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[3, a\*x^q], x]

[Out] -((x\*(d\*x)^m\*MeijerG[{{1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}}, {0, 0, 0, -((1 + m)/q)}}, -(a\*x^q)]/q)

**Maple [C]** time = 0.529, size = 180, normalized size = 1.4

$$-\frac{(dx)^m x^{-m}}{q} (-a)^{-\frac{m}{q}-q^{-1}} \left( \frac{q^3 x^{1+m} \ln(1-ax^q)}{(1+m)^3} (-a)^{\frac{m}{q}+q^{-1}} + \frac{q^2 x^{1+m} \text{polylog}(2, ax^q)}{(1+m)^2} (-a)^{\frac{m}{q}+q^{-1}} - \frac{q x^{1+m} \text{polylog}(3, ax^q)}{1+m} (-a)^{\frac{m}{q}+q^{-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(3, a\*x^q), x)

[Out] -(d\*x)^m\*x^(-m)\*(-a)^(-m/q-1/q)/q\*(q^3\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^3\*ln(1-a\*x^q)+q^2\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^2\*polylog(2, a\*x^q)-q\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)\*polylog(3, a\*x^q)+q^3\*x^(1+m+q)\*a\*(-a)^(m/q+1/q)/(1+m)^3\*Lerc hPhi(a\*x^q, 1, (1+m+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^m q^3 \int -\frac{x^m}{m^3 - (m^3 + 3m^2 + 3m + 1)ax^q + 3m^2 + 3m + 1} dx + \frac{d^m q^3 x x^m - (m^2 q + 2mq + q)d^m x x^m \text{Li}_2(ax^q) - (mq^2 + 2mq + q)d^m x x^m \text{Li}_3(ax^q)}{m^4 + 3m^3 + 3m^2 + 3m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(3, a\*x^q), x, algorithm="maxima")

```
[Out] d^m*q^3*integrate(-x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x^q + 3*m^2 + 3*m + 1), x) + (d^m*q^3*x*x^m - (m^2*q + 2*m*q + q)*d^m*x*x^m*dilog(a*x^q) - (m*q^2 + q^2)*d^m*x*x^m*log(-a*x^q + 1) + (m^3 + 3*m^2 + 3*m + 1)*d^m*x*x^m*polylog(3, a*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m \text{polylog}(3, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^q), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**q), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \text{Li}_3(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x^q), x)
```



### 3.113 $\int (dx)^m \text{PolyLog}(4, ax^q) dx$

**Optimal.** Leaf size=154

$$\frac{aq^4 x^{q+1} (dx)^m \text{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right)}{(m+1)^4(m+q+1)} + \frac{q^2 (dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)^3} - \frac{q (dx)^{m+1} \text{PolyLog}(3, ax^q)}{d(m+1)^2}$$

```
[Out] (a*q^4*x^(1+q)*(d*x)^m*Hypergeometric2F1[1, (1+m+q)/q, (1+m+2*q)/q, a*x^q])/((1+m)^4*(1+m+q)) + (q^3*(d*x)^(1+m)*Log[1-a*x^q])/(d*(1+m)^4) + (q^2*(d*x)^(1+m)*PolyLog[2, a*x^q])/(d*(1+m)^3) - (q*(d*x)^(1+m)*PolyLog[3, a*x^q])/(d*(1+m)^2) + ((d*x)^(1+m)*PolyLog[4, a*x^q])/d*(1+m)
```

**Rubi [A]** time = 0.101742, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6591, 2455, 20, 364}

$$\frac{q^2 (dx)^{m+1} \text{PolyLog}(2, ax^q)}{d(m+1)^3} - \frac{q (dx)^{m+1} \text{PolyLog}(3, ax^q)}{d(m+1)^2} + \frac{(dx)^{m+1} \text{PolyLog}(4, ax^q)}{d(m+1)} + \frac{aq^4 x^{q+1} (dx)^m {}_2F_1\left(1, \frac{m+q+1}{q}; \frac{m+2q+1}{q}, ax^q\right)}{(m+1)^4(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^q], x]
```

```
[Out] (a*q^4*x^(1+q)*(d*x)^m*Hypergeometric2F1[1, (1+m+q)/q, (1+m+2*q)/q, a*x^q])/((1+m)^4*(1+m+q)) + (q^3*(d*x)^(1+m)*Log[1-a*x^q])/(d*(1+m)^4) + (q^2*(d*x)^(1+m)*PolyLog[2, a*x^q])/(d*(1+m)^3) - (q*(d*x)^(1+m)*PolyLog[3, a*x^q])/(d*(1+m)^2) + ((d*x)^(1+m)*PolyLog[4, a*x^q])/d*(1+m)
```

#### Rule 6591

```
Int[((d_.)*(x_.))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_.)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m+1)*PolyLog[n, a*(b*x^p)^q])/d*(m+1), x] - Dist[(p*q)/(m+1), Int[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.)^(n_.))^(p_.)]*(b_.))*((f_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((f*x)^(m+1)*(a + b*Log[c*(d + e*x^n)^p]))/f*(m+1), x] - Dist[(b*e*n*p)/f*(m+1), Int[(x^(n-1)*(f*x)^(m+1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

#### Rule 20

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol]
:> Dist[(b^IntPart[n]*(a*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]
```

#### Rule 364

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol]
:> Simp[(a^IntPart[p]*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a
```

)]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (dx)^m \text{Li}_4(ax^q) dx &= \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} - \frac{q \int (dx)^m \text{Li}_3(ax^q) dx}{1+m} \\ &= -\frac{q(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} + \frac{q^2 \int (dx)^m \text{Li}_2(ax^q) dx}{(1+m)^2} \\ &= \frac{q^2(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} + \frac{q^3 \int (dx)^m \log(1-ax^q) dx}{(1+m)^3} \\ &= \frac{q^3(dx)^{1+m} \log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} + \frac{(aq^4) \int (dx)^m \log(1-ax^q) dx}{d(1+m)^3} \\ &= \frac{q^3(dx)^{1+m} \log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} + \frac{(aq^4 x^{-m}) \int (dx)^m \log(1-ax^q) dx}{d(1+m)^3} \\ &= \frac{aq^4 x^{1+q} (dx)^m {}_2F_1\left(1, \frac{1+m+q}{q}; \frac{1+m+2q}{q}; ax^q\right)}{(1+m)^4(1+m+q)} + \frac{q^3(dx)^{1+m} \log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m} \text{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m} \text{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m} \text{Li}_4(ax^q)}{d(1+m)} \end{aligned}$$

**Mathematica [C]** time = 0.0204511, size = 52, normalized size = 0.34

$$\frac{x(dx)^m G_{6,6}^{1,6}\left(-ax^q \mid \begin{matrix} 1, 1, 1, 1, 1, 1 - \frac{m+1}{q} \\ 1, 0, 0, 0, 0, -\frac{m+1}{q} \end{matrix}\right)}{q}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d\*x)^m\*PolyLog[4, a\*x^q], x]

[Out] -((x\*(d\*x)^m\*MeijerG[{{1, 1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0, 0, -((1 + m)/q)}}], -(a\*x^q)]/q)

**Maple [C]** time = 3.379, size = 217, normalized size = 1.4

$$-\frac{(dx)^m x^{-m}}{q} (-a)^{-\frac{m}{q}-q^{-1}} \left( -\frac{q^4 x^{1+m} \ln(1-ax^q)}{(1+m)^4} (-a)^{\frac{m}{q}+q^{-1}} - \frac{q^3 x^{1+m} \text{polylog}(2, ax^q)}{(1+m)^3} (-a)^{\frac{m}{q}+q^{-1}} + \frac{q^2 x^{1+m} \text{polylog}(3, ax^q)}{(1+m)^2} (-a)^{\frac{m}{q}+q^{-1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*polylog(4,a\*x^q), x)

[Out] -(d\*x)^m\*x^(-m)\*(-a)^(-m/q-1/q)/q\*(-q^4\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^4\*ln(1-a\*x^q)-q^3\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^3\*polylog(2,a\*x^q)+q^2\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)^2\*polylog(3,a\*x^q)-q\*x^(1+m)\*(-a)^(m/q+1/q)/(1+m)\*polylog(4,a\*x^q)-q^4\*x^(1+m+q)\*a\*(-a)^(m/q+1/q)/(1+m)^4\*LerchPhi(a\*x^q,1,(1+m+q)/q))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-d^m q^4 \int \frac{x^m}{m^4 + 4m^3 + 6m^2 - (am^4 + 4am^3 + 6am^2 + 4am + a)x^q + 4m + 1} dx - \frac{d^m q^4 x x^m - (d^m m + d^m) q^3 x x^m \log(-a x^q + 1) - (d^m m^2 + 2d^m m + d^m) q^2 x x^m \operatorname{dilog}(a x^q) + (d^m m^3 + 3d^m m^2 + 3d^m m + d^m) q x x^m \operatorname{polylog}(3, a x^q) - (d^m m^4 + 4d^m m^3 + 6d^m m^2 + 4d^m m + d^m) x x^m \operatorname{polylog}(4, a x^q)}{(m^5 + 5m^4 + 10m^3 + 10m^2 + 5m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^q),x, algorithm="maxima")

[Out] -d^m\*q^4\*integrate(-x^m/(m^4 + 4\*m^3 + 6\*m^2 - (a\*m^4 + 4\*a\*m^3 + 6\*a\*m^2 + 4\*a\*m + a)\*x^q + 4\*m + 1), x) - (d^m\*q^4\*x\*x^m - (d^m\*m + d^m)\*q^3\*x\*x^m\*log(-a\*x^q + 1) - (d^m\*m^2 + 2\*d^m\*m + d^m)\*q^2\*x\*x^m\*dilog(a\*x^q) + (d^m\*m^3 + 3\*d^m\*m^2 + 3\*d^m\*m + d^m)\*q\*x\*x^m\*polylog(3, a\*x^q) - (d^m\*m^4 + 4\*d^m\*m^3 + 6\*d^m\*m^2 + 4\*d^m\*m + d^m)\*x\*x^m\*polylog(4, a\*x^q))/(m^5 + 5\*m^4 + 10\*m^3 + 10\*m^2 + 5\*m + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx)^m \operatorname{polylog}(4, ax^q), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^q),x, algorithm="fricas")

[Out] integral((d\*x)^m\*polylog(4, a\*x^q), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*polylog(4,a\*x\*\*q),x)

[Out] Integral((d\*x)\*\*m\*polylog(4, a\*x\*\*q), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*polylog(4,a\*x^q),x, algorithm="giac")

[Out] integrate((d\*x)^m\*polylog(4, a\*x^q), x)

### 3.114 $\int x \text{PolyLog}(n, ax) dx$

**Optimal.** Leaf size=9

Unintegrable( $x \text{PolyLog}(n, ax), x$ )

[Out] Unintegrable[x\*PolyLog[n, a\*x], x]

**Rubi [A]** time = 0.0054963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \text{PolyLog}(n, ax) dx$$

Verification is Not applicable to the result.

[In] Int[x\*PolyLog[n, a\*x], x]

[Out] Defer[Int][x\*PolyLog[n, a\*x], x]

Rubi steps

$$\int x \text{Li}_n(ax) dx = \int x \text{Li}_n(ax) dx$$

**Mathematica [A]** time = 0.0216945, size = 0, normalized size = 0.

$$\int x \text{PolyLog}(n, ax) dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*PolyLog[n, a\*x], x]

[Out] Integrate[x\*PolyLog[n, a\*x], x]

**Maple [A]** time = 0.05, size = 0, normalized size = 0.

$$\int x \text{polylog}(n, ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(n, a\*x), x)

[Out] int(x\*polylog(n, a\*x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="maxima")
```

```
[Out] integrate(x*polylog(n, a*x), x)
```

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x\text{polylog}(n, ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(n, a*x), x)
```

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x)
```

```
[Out] Integral(x*polylog(n, a*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(n, a*x), x)
```

### 3.115 $\int \text{PolyLog}(n, ax) dx$

**Optimal.** Leaf size=7

Unintegrable(PolyLog( $n, ax$ ),  $x$ )

[Out] Unintegrable[PolyLog[n, a\*x], x]

**Rubi [A]** time = 0.0022881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}(n, ax) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x], x]

[Out] Defer[Int][PolyLog[n, a\*x], x]

Rubi steps

$$\int \text{Li}_n(ax) dx = \int \text{Li}_n(ax) dx$$

**Mathematica [A]** time = 0.0007752, size = 0, normalized size = 0.

$$\int \text{PolyLog}(n, ax) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x], x]

[Out] Integrate[PolyLog[n, a\*x], x]

**Maple [A]** time = 0.047, size = 0, normalized size = 0.

$$\int \text{polylog}(n, ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x), x)

[Out] int(polylog(n, a\*x), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x), x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{polylog}(n, ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x)
```

```
[Out] Integral(polylog(n, a*x), x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x), x)
```

$$3.116 \quad \int \frac{\text{PolyLog}(n, ax)}{x} dx$$

**Optimal.** Leaf size=7

$$\text{PolyLog}(n + 1, ax)$$

[Out] PolyLog[1 + n, a\*x]

**Rubi [A]** time = 0.0089991, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {6589}

$$\text{PolyLog}(n + 1, ax)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[n, a\*x]/x, x]

[Out] PolyLog[1 + n, a\*x]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x} dx = \text{Li}_{1+n}(ax)$$

**Mathematica [A]** time = 0.0011286, size = 7, normalized size = 1.

$$\text{PolyLog}(n + 1, ax)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[n, a\*x]/x, x]

[Out] PolyLog[1 + n, a\*x]

**Maple [A]** time = 0.044, size = 8, normalized size = 1.1

$$\text{polylog}(1 + n, ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x)/x, x)

[Out] polylog(1+n, a\*x)



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x,x, algorithm="maxima")

[Out] integrate(polylog(n, a\*x)/x, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x,x, algorithm="fricas")

[Out] integral(polylog(n, a\*x)/x, x)

---

**Sympy [A]** time = 0.528206, size = 5, normalized size = 0.71

$$\text{Li}_{n+1}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x,x)

[Out] polylog(n + 1, a\*x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x,x, algorithm="giac")

[Out] integrate(polylog(n, a\*x)/x, x)

$$3.117 \quad \int \frac{\text{PolyLog}(n, ax)}{x^2} dx$$

**Optimal.** Leaf size=11

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n, ax)}{x^2}, x\right)$$

[Out] Unintegrable[PolyLog[n, a\*x]/x^2, x]

**Rubi [A]** time = 0.0093445, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x]/x^2, x]

[Out] Defer[Int][PolyLog[n, a\*x]/x^2, x]

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^2} dx = \int \frac{\text{Li}_n(ax)}{x^2} dx$$

**Mathematica [A]** time = 0.0207728, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n, ax)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x]/x^2, x]

[Out] Integrate[PolyLog[n, a\*x]/x^2, x]

**Maple [A]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(n, ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x)/x^2, x)

[Out] int(polylog(n, a\*x)/x^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^2,x, algorithm="maxima")

[Out] integrate(polylog(n, a\*x)/x^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^2,x, algorithm="fricas")

[Out] integral(polylog(n, a\*x)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x\*\*2,x)

[Out] Integral(polylog(n, a\*x)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^2,x, algorithm="giac")

[Out] integrate(polylog(n, a\*x)/x^2, x)

$$3.118 \quad \int \frac{\text{PolyLog}(n, ax)}{x^3} dx$$

**Optimal.** Leaf size=11

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n, ax)}{x^3}, x\right)$$

[Out] Unintegrable[PolyLog[n, a\*x]/x^3, x]

**Rubi [A]** time = 0.0089528, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}(n, ax)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x]/x^3, x]

[Out] Defer[Int][PolyLog[n, a\*x]/x^3, x]

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^3} dx = \int \frac{\text{Li}_n(ax)}{x^3} dx$$

**Mathematica [A]** time = 0.0208255, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n, ax)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x]/x^3, x]

[Out] Integrate[PolyLog[n, a\*x]/x^3, x]

**Maple [A]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(n, ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x)/x^3, x)

[Out] int(polylog(n, a\*x)/x^3, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^3,x, algorithm="maxima")

[Out] integrate(polylog(n, a\*x)/x^3, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^3,x, algorithm="fricas")

[Out] integral(polylog(n, a\*x)/x^3, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x\*\*3,x)

[Out] Integral(polylog(n, a\*x)/x\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x)/x^3,x, algorithm="giac")

[Out] integrate(polylog(n, a\*x)/x^3, x)

### 3.119 $\int x \text{PolyLog}(n, ax^q) dx$

**Optimal.** Leaf size=11

Unintegrable( $x \text{PolyLog}(n, ax^q), x$ )

[Out] Unintegrable[x\*PolyLog[n, a\*x^q], x]

**Rubi [A]** time = 0.0056386, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \text{PolyLog}(n, ax^q) dx$$

Verification is Not applicable to the result.

[In] Int[x\*PolyLog[n, a\*x^q], x]

[Out] Defer[Int][x\*PolyLog[n, a\*x^q], x]

Rubi steps

$$\int x \text{Li}_n(ax^q) dx = \int x \text{Li}_n(ax^q) dx$$

**Mathematica [A]** time = 0.0234105, size = 0, normalized size = 0.

$$\int x \text{PolyLog}(n, ax^q) dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*PolyLog[n, a\*x^q], x]

[Out] Integrate[x\*PolyLog[n, a\*x^q], x]

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int x \text{polylog}(n, ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(n, a\*x^q), x)

[Out] int(x\*polylog(n, a\*x^q), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(n,a\*x^q),x, algorithm="maxima")

[Out] integrate(x\*polylog(n, a\*x^q), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x \text{polylog}(n, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(n,a\*x^q),x, algorithm="fricas")

[Out] integral(x\*polylog(n, a\*x^q), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(n,a\*x\*\*q),x)

[Out] Integral(x\*polylog(n, a\*x\*\*q), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int x \text{Li}_n(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(n,a\*x^q),x, algorithm="giac")

[Out] integrate(x\*polylog(n, a\*x^q), x)

### 3.120 $\int \text{PolyLog}(n, ax^q) dx$

**Optimal.** Leaf size=9

Unintegrable(PolyLog(n, ax<sup>q</sup>), x)

[Out] Unintegrable[PolyLog[n, a\*x<sup>q</sup>], x]

**Rubi [A]** time = 0.0025715, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \text{PolyLog}(n, ax^q) dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x<sup>q</sup>], x]

[Out] Defer[Int][PolyLog[n, a\*x<sup>q</sup>], x]

Rubi steps

$$\int \text{Li}_n(ax^q) dx = \int \text{Li}_n(ax^q) dx$$

**Mathematica [A]** time = 0.0037649, size = 0, normalized size = 0.

$$\int \text{PolyLog}(n, ax^q) dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x<sup>q</sup>], x]

[Out] Integrate[PolyLog[n, a\*x<sup>q</sup>], x]

**Maple [A]** time = 0.049, size = 0, normalized size = 0.

$$\int \text{polylog}(n, ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x<sup>q</sup>), x)

[Out] int(polylog(n, a\*x<sup>q</sup>), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax^q) dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q), x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{polylog}(n, ax^q), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q),x)
```

```
[Out] Integral(polylog(n, a*x**q), x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n(ax^q) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q), x)
```

$$3.121 \quad \int \frac{\text{PolyLog}(n, ax^q)}{x} dx$$

**Optimal.** Leaf size=13

$$\frac{\text{PolyLog}(n+1, ax^q)}{q}$$

[Out] PolyLog[1 + n, a\*x^q]/q

**Rubi [A]** time = 0.0098521, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6589}

$$\frac{\text{PolyLog}(n+1, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[n, a\*x^q]/x,x]

[Out] PolyLog[1 + n, a\*x^q]/q

**Rule 6589**

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

**Rubi steps**

$$\int \frac{\text{Li}_n(ax^q)}{x} dx = \frac{\text{Li}_{1+n}(ax^q)}{q}$$

**Mathematica [A]** time = 0.0012072, size = 13, normalized size = 1.

$$\frac{\text{PolyLog}(n+1, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[n, a\*x^q]/x,x]

[Out] PolyLog[1 + n, a\*x^q]/q

**Maple [A]** time = 0.046, size = 14, normalized size = 1.1

$$\frac{\text{polylog}(1+n, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(n,a*x^q)/x,x)`

[Out] `polylog(1+n,a*x^q)/q`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x^q)/x,x, algorithm="maxima")`

[Out] `integrate(polylog(n, a*x^q)/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax^q)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x^q)/x,x, algorithm="fricas")`

[Out] `integral(polylog(n, a*x^q)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x**q)/x,x)`

[Out] `Integral(polylog(n, a*x**q)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,a*x^q)/x,x, algorithm="giac")`

[Out] `integrate(polylog(n, a*x^q)/x, x)`

$$3.122 \quad \int \frac{\text{PolyLog}(n, ax^q)}{x^2} dx$$

**Optimal.** Leaf size=13

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n, ax^q)}{x^2}, x\right)$$

[Out] Unintegrable[PolyLog[n, a\*x^q]/x^2, x]

**Rubi [A]** time = 0.0093166, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x^q]/x^2, x]

[Out] Defer[Int][PolyLog[n, a\*x^q]/x^2, x]

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^2} dx = \int \frac{\text{Li}_n(ax^q)}{x^2} dx$$

**Mathematica [A]** time = 0.0214191, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x^q]/x^2, x]

[Out] Integrate[PolyLog[n, a\*x^q]/x^2, x]

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(n, ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x^q)/x^2, x)

[Out] int(polylog(n, a\*x^q)/x^2, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^2,x, algorithm="maxima")

[Out] integrate(polylog(n, a\*x^q)/x^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax^q)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^2,x, algorithm="fricas")

[Out] integral(polylog(n, a\*x^q)/x^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x\*\*q)/x\*\*2,x)

[Out] Integral(polylog(n, a\*x\*\*q)/x\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^2,x, algorithm="giac")

[Out] integrate(polylog(n, a\*x^q)/x^2, x)

$$3.123 \quad \int \frac{\text{PolyLog}(n, ax^q)}{x^3} dx$$

**Optimal.** Leaf size=13

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n, ax^q)}{x^3}, x\right)$$

[Out] Unintegrable[PolyLog[n, a\*x^q]/x^3, x]

**Rubi [A]** time = 0.0094303, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, a\*x^q]/x^3, x]

[Out] Defer[Int][PolyLog[n, a\*x^q]/x^3, x]

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^3} dx = \int \frac{\text{Li}_n(ax^q)}{x^3} dx$$

**Mathematica [A]** time = 0.0219503, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n, ax^q)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, a\*x^q]/x^3, x]

[Out] Integrate[PolyLog[n, a\*x^q]/x^3, x]

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(n, ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, a\*x^q)/x^3, x)

[Out] int(polylog(n, a\*x^q)/x^3, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^3,x, algorithm="maxima")

[Out] integrate(polylog(n, a\*x^q)/x^3, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(n, ax^q)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^3,x, algorithm="fricas")

[Out] integral(polylog(n, a\*x^q)/x^3, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x\*\*q)/x\*\*3,x)

[Out] Integral(polylog(n, a\*x\*\*q)/x\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n,a\*x^q)/x^3,x, algorithm="giac")

[Out] integrate(polylog(n, a\*x^q)/x^3, x)

### 3.124 $\int x^2 \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=260

$$\frac{a^3 \text{PolyLog}(2, c(a + bx))}{3b^3} + \frac{1}{3} x^3 \text{PolyLog}(2, c(a + bx)) - \frac{a^2(-ac - bcx + 1) \log(-ac - bcx + 1)}{3b^3c} - \frac{a^2x}{3b^2} - \frac{x(1 - ac)^2}{9b^2c^2} + \frac{a(1 - ac)}{9b^2c^2}$$

[Out]  $-(a^2x)/(3b^2) + (a(1 - ac)x)/(6b^2c) - ((1 - ac)^2x)/(9b^2c^2) + (ax^2)/(12b) - ((1 - ac)x^2)/(18bc) - x^3/27 + (a(1 - ac)^2 \text{Log}[1 - ac - bcx])/(6b^3c^2) - ((1 - ac)^3 \text{Log}[1 - ac - bcx])/(9b^3c^3) - (ax^2 \text{Log}[1 - ac - bcx])/(6b) + (x^3 \text{Log}[1 - ac - bcx])/9 - (a^2(1 - ac - bcx) \text{Log}[1 - ac - bcx])/(3b^3c) + (a^3 \text{PolyLog}[2, c(a + bx)])/(3b^3) + (x^3 \text{PolyLog}[2, c(a + bx)])/3$

**Rubi [A]** time = 0.320951, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$ , Rules used = {6598, 43, 2416, 2389, 2295, 2395, 2393, 2391}

$$\frac{a^3 \text{PolyLog}(2, c(a + bx))}{3b^3} + \frac{1}{3} x^3 \text{PolyLog}(2, c(a + bx)) - \frac{a^2(-ac - bcx + 1) \log(-ac - bcx + 1)}{3b^3c} - \frac{a^2x}{3b^2} - \frac{x(1 - ac)^2}{9b^2c^2} + \frac{a(1 - ac)}{9b^2c^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*PolyLog[2, c\*(a + b\*x)],x]

[Out]  $-(a^2x)/(3b^2) + (a(1 - ac)x)/(6b^2c) - ((1 - ac)^2x)/(9b^2c^2) + (ax^2)/(12b) - ((1 - ac)x^2)/(18bc) - x^3/27 + (a(1 - ac)^2 \text{Log}[1 - ac - bcx])/(6b^3c^2) - ((1 - ac)^3 \text{Log}[1 - ac - bcx])/(9b^3c^3) - (ax^2 \text{Log}[1 - ac - bcx])/(6b) + (x^3 \text{Log}[1 - ac - bcx])/9 - (a^2(1 - ac - bcx) \text{Log}[1 - ac - bcx])/(3b^3c) + (a^3 \text{PolyLog}[2, c(a + bx)])/(3b^3) + (x^3 \text{PolyLog}[2, c(a + bx)])/3$

#### Rule 6598

Int[((d.) + (e.)\*(x.))^(m.)\*PolyLog[2, (c.)\*((a.) + (b.)\*(x.))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 43

Int[((a.) + (b.)\*(x.))^(m.)\*((c.) + (d.)\*(x.))^(n.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2416

Int[((a.) + Log[(c.)\*((d.) + (e.)\*(x.))^(n.)])\*(b.))^(p.)\*((h.)\*(x.))^(m.)\*((f.) + (g.)\*(x.))^(q.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2389

Int[((a.) + Log[(c.)\*((d.) + (e.)\*(x.))^(n.)])\*(b.))^(p.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a



, b, c, d, e, n, p}, x]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned} \int x^2 \text{Li}_2(c(a+bx)) dx &= \frac{1}{3} x^3 \text{Li}_2(c(a+bx)) + \frac{1}{3} b \int \frac{x^3 \log(1-ac-bcx)}{a+bx} dx \\ &= \frac{1}{3} x^3 \text{Li}_2(c(a+bx)) + \frac{1}{3} b \int \left( \frac{a^2 \log(1-ac-bcx)}{b^3} - \frac{ax \log(1-ac-bcx)}{b^2} + \frac{x^2 \log(1-ac-bcx)}{b} \right) dx \\ &= \frac{1}{3} x^3 \text{Li}_2(c(a+bx)) + \frac{1}{3} \int x^2 \log(1-ac-bcx) dx + \frac{a^2 \int \log(1-ac-bcx) dx}{3b^2} - \frac{a^3 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{3b^2} \\ &= -\frac{ax^2 \log(1-ac-bcx)}{6b} + \frac{1}{9} x^3 \log(1-ac-bcx) + \frac{1}{3} x^3 \text{Li}_2(c(a+bx)) - \frac{a^3 \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, \frac{a+bx}{b}\right)}{3b^3} \\ &= -\frac{a^2 x}{3b^2} - \frac{ax^2 \log(1-ac-bcx)}{6b} + \frac{1}{9} x^3 \log(1-ac-bcx) - \frac{a^2(1-ac-bcx) \log(1-ac-bcx)}{3b^3 c} + \frac{a^3 \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, \frac{a+bx}{b}\right)}{3b^3} \\ &= -\frac{a^2 x}{3b^2} + \frac{a(1-ac)x}{6b^2 c} - \frac{(1-ac)^2 x}{9b^2 c^2} + \frac{ax^2}{12b} - \frac{(1-ac)x^2}{18bc} - \frac{x^3}{27} + \frac{a(1-ac)^2 \log(1-ac-bcx)}{6b^3 c^2} - \frac{a^3 \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, \frac{a+bx}{b}\right)}{3b^3} \end{aligned}$$

**Mathematica [A]** time = 0.208213, size = 144, normalized size = 0.55

$$\frac{36c^3 (a^3 + b^3 x^3) \text{PolyLog}(2, c(a+bx)) - bcx (66a^2 c^2 - 3ac(5bcx + 14) + 4b^2 c^2 x^2 + 6bcx + 12) + 6 (6a^2 c^2 (bcx - 3) + 36c^3 (a^3 + b^3 x^3) \text{PolyLog}(2, c(a+bx)))}{108b^3 c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[2, c\*(a + b\*x)], x]

[Out] (- (b\*c\*x\*(12 + 66\*a^2\*c^2 + 6\*b\*c\*x + 4\*b^2\*c^2\*x^2 - 3\*a\*c\*(14 + 5\*b\*c\*x)) + 6\*(-2 + 11\*a^3\*c^3 + 2\*b^3\*c^3\*x^3 + 6\*a^2\*c^2\*(-3 + b\*c\*x) + a\*(9\*c - 3\*b^2\*c^3\*x^2))\*Log[1 - a\*c - b\*c\*x] + 36\*c^3\*(a^3 + b^3\*x^3)\*PolyLog[2, c\*(a + b\*x)]) / (108\*b^3\*c^3)

$(a + b*x)])/(108*b^3*c^3)$

**Maple [A]** time = 0.008, size = 269, normalized size = 1.

$$-\frac{31a}{36b^3c^2} - \frac{\ln(-xbc - ac + 1)}{9b^3c^3} - \frac{85a^3}{108b^3} + \frac{a^3 \operatorname{dilog}(-xbc - ac + 1)}{3b^3} - \frac{x^3}{27} + \frac{11}{54b^3c^3} + \frac{11 \ln(-xbc - ac + 1)a^3}{18b^3} - \frac{\ln(-xbc - ac + 1)}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(2,c\*(b\*x+a)),x)

[Out]  $-31/36/b^3/c^2*a-1/9/b^3/c^3*\ln(-b*c*x-a*c+1)-85/108/b^3*a^3+1/3/b^3*a^3*\operatorname{dilog}(-b*c*x-a*c+1)-1/27*x^3+11/54/b^3/c^3+11/18/b^3*\ln(-b*c*x-a*c+1)*a^3-1/b^3/c*\ln(-b*c*x-a*c+1)*a^2+1/2/b^3/c^2*\ln(-b*c*x-a*c+1)*a-1/18/b/c*x^2-1/6*a*x^2*\ln(-b*c*x-a*c+1)/b+1/3/b^2*\ln(-b*c*x-a*c+1)*x*a^2+7/18/b^2/c*x*a-11/18*a^2*x/b^2+5/36*a*x^2/b-1/9/b^2/c^2*x+1/3*polylog(2,b*c*x+a*c)*x^3+1/9*x^3*\ln(-b*c*x-a*c+1)+13/9/b^3/c*a^2$

**Maxima [A]** time = 0.994565, size = 270, normalized size = 1.04

$$-\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \operatorname{Li}_2(-bc x - ac + 1))a^3}{3b^3} + \frac{36b^3c^3x^3\operatorname{Li}_2(bc x + ac) - 4b^3c^3x^3 + 3(5ab^2c^3 - 2b^2c^2)x^2}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/3*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \operatorname{dilog}(-b*c*x - a*c + 1))*a^3/b^3 + 1/108*(36*b^3*c^3*x^3*\operatorname{dilog}(b*c*x + a*c) - 4*b^3*c^3*x^3 + 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 - 6*(11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*x + 6*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*\log(-b*c*x - a*c + 1))/(b^3*c^3)$

**Fricas [A]** time = 2.39335, size = 366, normalized size = 1.41

$$-\frac{4b^3c^3x^3 - 3(5ab^2c^3 - 2b^2c^2)x^2 + 6(11a^2bc^3 - 7abc^2 + 2bc)x - 36(b^3c^3x^3 + a^3c^3)\operatorname{Li}_2(bc x + ac) - 6(2b^3c^3x^3 - 3ab^2c^3x^2 + 6a^2b^2c^3x + 11a^3c^3 - 18a^2c^2 + 9a^2c - 2)*\log(-b*c*x - a*c + 1)}{108b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-1/108*(4*b^3*c^3*x^3 - 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 + 6*(11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*x - 36*(b^3*c^3*x^3 + a^3*c^3)*\operatorname{dilog}(b*c*x + a*c) - 6*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9*a^2*c - 2)*\log(-b*c*x - a*c + 1))/(b^3*c^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate(x^2\*dilog((b\*x + a)\*c), x)

### 3.125 $\int x \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=152

$$-\frac{a^2 \text{PolyLog}(2, c(a + bx))}{2b^2} + \frac{1}{2} x^2 \text{PolyLog}(2, c(a + bx)) - \frac{(1 - ac)^2 \log(-ac - bcx + 1)}{4b^2 c^2} + \frac{a(-ac - bcx + 1) \log(-ac - bcx)}{2b^2 c}$$

[Out] (a\*x)/(2\*b) - ((1 - a\*c)\*x)/(4\*b\*c) - x^2/8 - ((1 - a\*c)^2\*Log[1 - a\*c - b\*c\*x])/(4\*b^2\*c^2) + (x^2\*Log[1 - a\*c - b\*c\*x])/4 + (a\*(1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(2\*b^2\*c) - (a^2\*PolyLog[2, c\*(a + b\*x)])/(2\*b^2) + (x^2\*PolyLog[2, c\*(a + b\*x)])/2

**Rubi [A]** time = 0.169219, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$ , Rules used = {6598, 43, 2416, 2389, 2295, 2395, 2393, 2391}

$$-\frac{a^2 \text{PolyLog}(2, c(a + bx))}{2b^2} + \frac{1}{2} x^2 \text{PolyLog}(2, c(a + bx)) - \frac{(1 - ac)^2 \log(-ac - bcx + 1)}{4b^2 c^2} + \frac{a(-ac - bcx + 1) \log(-ac - bcx)}{2b^2 c}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[2, c\*(a + b\*x)], x]

[Out] (a\*x)/(2\*b) - ((1 - a\*c)\*x)/(4\*b\*c) - x^2/8 - ((1 - a\*c)^2\*Log[1 - a\*c - b\*c\*x])/(4\*b^2\*c^2) + (x^2\*Log[1 - a\*c - b\*c\*x])/4 + (a\*(1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(2\*b^2\*c) - (a^2\*PolyLog[2, c\*(a + b\*x)])/(2\*b^2) + (x^2\*PolyLog[2, c\*(a + b\*x)])/2

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)])/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.)]^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
]; FreeQ[{c, n}, x]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int x \operatorname{Li}_2(c(a+bx)) dx &= \frac{1}{2} x^2 \operatorname{Li}_2(c(a+bx)) + \frac{1}{2} b \int \frac{x^2 \log(1-ac-bcx)}{a+bx} dx \\
&= \frac{1}{2} x^2 \operatorname{Li}_2(c(a+bx)) + \frac{1}{2} b \int \left( -\frac{a \log(1-ac-bcx)}{b^2} + \frac{x \log(1-ac-bcx)}{b} + \frac{a^2 \log(1-ac-bcx)}{b^2(a+bx)} \right) dx \\
&= \frac{1}{2} x^2 \operatorname{Li}_2(c(a+bx)) + \frac{1}{2} \int x \log(1-ac-bcx) dx - \frac{a \int \log(1-ac-bcx) dx}{2b} + \frac{a^2 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{2b} \\
&= \frac{1}{4} x^2 \log(1-ac-bcx) + \frac{1}{2} x^2 \operatorname{Li}_2(c(a+bx)) + \frac{a^2 \operatorname{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{2b^2} + \frac{a \operatorname{Subst}\left(\int \frac{\log(1-cx)}{a+bx} dx, x, a+bx\right)}{2b} \\
&= \frac{ax}{2b} + \frac{1}{4} x^2 \log(1-ac-bcx) + \frac{a(1-ac-bcx) \log(1-ac-bcx)}{2b^2 c} - \frac{a^2 \operatorname{Li}_2(c(a+bx))}{2b^2} + \frac{1}{2} x^2 \operatorname{Li}_2(c(a+bx)) \\
&= \frac{ax}{2b} - \frac{(1-ac)x}{4bc} - \frac{x^2}{8} - \frac{(1-ac)^2 \log(1-ac-bcx)}{4b^2 c^2} + \frac{1}{4} x^2 \log(1-ac-bcx) + \frac{a(1-ac-bcx) \log(1-ac-bcx)}{2b^2 c}
\end{aligned}$$

**Mathematica [A]** time = 0.100904, size = 96, normalized size = 0.63

$$\frac{-4c^2(a^2 - b^2x^2) \operatorname{PolyLog}(2, c(a+bx)) + (-6a^2c^2 - 4ac(bcx - 2) + 2b^2c^2x^2 - 2) \log(-ac - bcx + 1) - bcx(-6ac + bcx)}{8b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] (-b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]/(8*b^2*c^2)
```

**Maple [A]** time = 0.004, size = 177, normalized size = 1.2

$$\frac{\text{polylog}(2, xbc + ac) a^2}{2b^2} + \frac{\text{polylog}(2, xbc + ac) x^2}{2} - \frac{\ln(-xbc - ac + 1) xa}{2b} - \frac{3 \ln(-xbc - ac + 1) a^2}{4b^2} + \frac{3ax}{4b} + \frac{7a^2}{8b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(2,c\*(b\*x+a)),x)

[Out]  $-1/2/b^2*\text{polylog}(2,b*c*x+a*c)*a^2+1/2*\text{polylog}(2,b*c*x+a*c)*x^2-1/2/b*\ln(-b*c*x-a*c+1)*x*a-3/4/b^2*\ln(-b*c*x-a*c+1)*a^2+3/4*a*x/b+7/8/b^2*a^2+1/b^2/c*\ln(-b*c*x-a*c+1)*a-5/4/b^2/c*a+1/4*x^2*\ln(-b*c*x-a*c+1)-1/4/b^2/c^2*\ln(-b*c*x-a*c+1)-1/8*x^2-1/4/b/c*x+3/8/b^2/c^2$

**Maxima [A]** time = 0.992845, size = 196, normalized size = 1.29

$$\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1)) a^2}{2b^2} + \frac{4b^2c^2x^2\text{Li}_2(bc x + ac) - b^2c^2x^2 + 2(3abc^2 - bc)x + 2(b^2c^2x^2 - 2a^2c^2)}{8b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $1/2*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \text{dilog}(-b*c*x - a*c + 1))*a^2/b^2 + 1/8*(4*b^2*c^2*x^2*\text{dilog}(b*c*x + a*c) - b^2*c^2*x^2 + 2*(3*a*b*c^2 - b*c)*x + 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*\log(-b*c*x - a*c + 1))/(b^2*c^2)$

**Fricas [A]** time = 2.348, size = 242, normalized size = 1.59

$$\frac{b^2c^2x^2 - 2(3abc^2 - bc)x - 4(b^2c^2x^2 - a^2c^2)\text{Li}_2(bc x + ac) - 2(b^2c^2x^2 - 2abc^2x - 3a^2c^2 + 4ac - 1)\log(-bc x - ac + 1)}{8b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-1/8*(b^2*c^2*x^2 - 2*(3*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - a^2*c^2)*\text{dilog}(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*\log(-b*c*x - a*c + 1))/(b^2*c^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*dilog((b*x + a)*c), x)
```

### 3.126 $\int \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=60

$$x \text{PolyLog}(2, c(a + bx)) + \frac{a \text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1) \log(-ac - bcx + 1)}{bc} - x$$

[Out] -x - ((1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(b\*c) + (a\*PolyLog[2, c\*(a + b\*x)]/b + x\*PolyLog[2, c\*(a + b\*x)])

**Rubi [A]** time = 0.0486846, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391}

$$x \text{PolyLog}(2, c(a + bx)) + \frac{a \text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1) \log(-ac - bcx + 1)}{bc} - x$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, c\*(a + b\*x)], x]

[Out] -x - ((1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(b\*c) + (a\*PolyLog[2, c\*(a + b\*x)]/b + x\*PolyLog[2, c\*(a + b\*x)])

#### Rule 6595

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)], x\_Symbol] := Simp[x\*PolyLog[n, c\*(a + b\*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c\*(a + b\*x)^p], x], x] + Dist[a\*p, Int[PolyLog[n - 1, c\*(a + b\*x)^p]/(a + b\*x), x], x]) /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]

#### Rule 2444

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_.), x\_Symbol] := Int[u\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c\*v, (e\_.)\*((f\_) + (g\_.)\*x)] /; FreeQ[{e, f, g}, x])

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2421

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_)^(q\_.), x\_Symbol] := Int[ExpandToSum[u, x]^q\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatchQ[u, x] && LinearMatchQ[v, x])

#### Rule 2393



```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \text{Li}_2(c(a+bx)) dx &= x\text{Li}_2(c(a+bx)) - a \int \frac{\log(1-c(a+bx))}{a+bx} dx + \int \log(1-c(a+bx)) dx \\ &= x\text{Li}_2(c(a+bx)) - a \int \frac{\log(1-ac-bcx)}{a+bx} dx + \int \log(1-ac-bcx) dx \\ &= x\text{Li}_2(c(a+bx)) - \frac{a \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{b} - \frac{\text{Subst}\left(\int \log(x) dx, x, 1-ac-bcx\right)}{bc} \\ &= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\text{Li}_2(c(a+bx))}{b} + x\text{Li}_2(c(a+bx)) \end{aligned}$$

**Mathematica [A]** time = 0.0177979, size = 53, normalized size = 0.88

$$\frac{c(a+bx)\text{PolyLog}[2, c(a+bx)] - c(a+bx) + (c(a+bx) - 1)\log(1 - c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)], x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

**Maple [A]** time = 0.003, size = 96, normalized size = 1.6

$$\ln(-xbc - ac + 1)x + \text{polylog}(2, xbc + ac)x + \frac{\ln(-xbc - ac + 1)a}{b} + \frac{\text{polylog}(2, xbc + ac)a}{b} - x - \frac{a}{b} - \frac{\ln(-xbc - ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(b*x+a)), x)
```

```
[Out] ln(-b*c*x-a*c+1)*x+polylog(2,b*c*x+a*c)*x+1/b*ln(-b*c*x-a*c+1)*a+1/b*polylo
g(2,b*c*x+a*c)*a-x-a/b-1/b/c*ln(-b*c*x-a*c+1)+1/b/c
```

**Maxima [A]** time = 0.982883, size = 122, normalized size = 2.03

$$\frac{(\log(bcx + ac)\log(-bcx - ac + 1) + \text{Li}_2(-bcx - ac + 1))a}{b} + \frac{bcx\text{Li}_2(bcx + ac) - bcx + (bcx + ac - 1)\log(-bcx - ac)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $-(\log(bc^2x + ac) \log(-bc^2x - ac + 1) + \operatorname{dilog}(-bc^2x - ac + 1)) \frac{a}{b} + (bc^2x \operatorname{dilog}(bc^2x + ac) - bc^2x + (bc^2x + ac - 1) \log(-bc^2x - ac + 1)) / (bc)$

**Fricas [A]** time = 2.41585, size = 126, normalized size = 2.1

$$\frac{bcx - (bcx + ac) \operatorname{Li}_2(bcx + ac) - (bcx + ac - 1) \log(-bcx - ac + 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-(bc^2x - (bc^2x + ac) \operatorname{dilog}(bc^2x + ac) - (bc^2x + ac - 1) \log(-bc^2x - ac + 1)) / (bc)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c), x)

$$3.127 \quad \int \frac{\text{PolyLog}(2, c(a+bx))}{x} dx$$

**Optimal.** Leaf size=401

$$\text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right) - \text{PolyLog}\left(3, -\frac{bcx}{1-c(a+bx)}\right) - \text{PolyLog}(3, 1-c(a+bx)) + \log\left(-\frac{a(1-c(a+bx))}{bx}\right)$$

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)] + ((Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2/2 + ((Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2/2 + (Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)] + Log[x]*PolyLog[2, c*(a + b*x)] + Log[-((a*(1 - c*(a + b*x)))/(b*x))*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))] + (Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, -((b*x)/a)] + PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x)))] - PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

**Rubi [A]** time = 0.355029, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {6597, 2440, 2435}

$$\text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right) - \text{PolyLog}\left(3, -\frac{bcx}{1-c(a+bx)}\right) - \text{PolyLog}(3, 1-c(a+bx)) + \log\left(-\frac{a(1-c(a+bx))}{bx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x, x]
```

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)] + ((Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2/2 + ((Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2/2 + (Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)] + Log[x]*PolyLog[2, c*(a + b*x)] + Log[-((a*(1 - c*(a + b*x)))/(b*x))*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))] + (Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, -((b*x)/a)] + PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x)))] - PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

#### Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c*(b*d - a*e) + e, 0]
```

#### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.) * ((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Symbol]
:> Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f + g*Log[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; FreeQ[{a,
```

b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] := Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
+ (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))],
x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(c(a+bx))}{x} dx &= \log(x) \text{Li}_2(c(a+bx)) + b \int \frac{\log(x) \log(1-ac-bcx)}{a+bx} dx \\ &= \log(x) \text{Li}_2(c(a+bx)) + \text{Subst} \left( \int \frac{\log\left(-\frac{a}{b} + \frac{x}{b}\right) \log\left(\frac{-abc-b(1-ac)-cx}{b}\right)}{x} dx, x, a+bx \right) \\ &= \log(x) \log\left(1 + \frac{bx}{a}\right) \log(1-c(a+bx)) + \frac{1}{2} \left( \log\left(1 + \frac{bx}{a}\right) + \log\left(\frac{1-ac}{1-c(a+bx)}\right) - \log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right) \right) \end{aligned}$$

**Mathematica [A]** time = 0.145861, size = 422, normalized size = 1.05

$$-\text{PolyLog}(3, -ac - bcx + 1) + \text{PolyLog}\left(3, \frac{a(ac + bcx - 1)}{bx}\right) - \text{PolyLog}\left(3, \frac{ac + bcx - 1}{bcx}\right) + \log\left(\frac{a(ac + bcx - 1)}{bx}\right) \left(\text{PolyLog}(2, \frac{a(ac + bcx - 1)}{bx}) - \text{PolyLog}(2, \frac{ac + bcx - 1}{bcx})\right)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, c\*(a + b\*x)]/x,x]

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1
+ (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (L
og[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c +
b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[-((( -1 + a*c)*(a + b*x))/(
b*x))]) + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1
- a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)]
+ (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x]
+ Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*
x)] + PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + Log[x]*PolyLog[2, a*c + b*c
*x] - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*
(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]
```

**Maple [F]** time = 0.01, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, c(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(2,c*(b*x+a))/x,x)`

[Out] `int(polylog(2,c*(b*x+a))/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx+a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x,x, algorithm="maxima")`

[Out] `integrate(dilog((b*x + a)*c)/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bc x + ac)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x,x, algorithm="fricas")`

[Out] `integral(dilog(b*c*x + a*c)/x, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx+a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x,x, algorithm="giac")`

[Out] `integrate(dilog((b*x + a)*c)/x, x)`

### 3.128 $\int \frac{\text{PolyLog}(2, c(a+bx))}{x^2} dx$

**Optimal.** Leaf size=84

$$\frac{b \text{PolyLog}(2, c(a+bx))}{a} - \frac{\text{PolyLog}(2, c(a+bx))}{x} - \frac{b \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{a} - \frac{b \log\left(\frac{bcx}{1-ac}\right) \log(-ac - bcx + 1)}{a}$$

[Out] -((b\*Log[(b\*c\*x)/(1 - a\*c)]\*Log[1 - a\*c - b\*c\*x])/a) - (b\*PolyLog[2, c\*(a + b\*x)]/a - PolyLog[2, c\*(a + b\*x)]/x - (b\*PolyLog[2, 1 - (b\*c\*x)/(1 - a\*c)])/a

**Rubi [A]** time = 0.116564, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$ , Rules used = {6598, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$\frac{b \text{PolyLog}(2, c(a+bx))}{a} - \frac{\text{PolyLog}(2, c(a+bx))}{x} - \frac{b \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{a} - \frac{b \log\left(\frac{bcx}{1-ac}\right) \log(-ac - bcx + 1)}{a}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, c\*(a + b\*x)]/x^2, x]

[Out] -((b\*Log[(b\*c\*x)/(1 - a\*c)]\*Log[1 - a\*c - b\*c\*x])/a) - (b\*PolyLog[2, c\*(a + b\*x)]/a - PolyLog[2, c\*(a + b\*x)]/x - (b\*PolyLog[2, 1 - (b\*c\*x)/(1 - a\*c)])/a

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.)\*((q\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(c(a+bx))}{x^2} dx &= -\frac{\text{Li}_2(c(a+bx))}{x} - b \int \frac{\log(1-ac-bcx)}{x(a+bx)} dx \\ &= -\frac{\text{Li}_2(c(a+bx))}{x} - b \int \left( \frac{\log(1-ac-bcx)}{ax} - \frac{b \log(1-ac-bcx)}{a(a+bx)} \right) dx \\ &= -\frac{\text{Li}_2(c(a+bx))}{x} - \frac{b \int \frac{\log(1-ac-bcx)}{x} dx}{a} + \frac{b^2 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{a} \\ &= -\frac{b \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{a} - \frac{\text{Li}_2(c(a+bx))}{x} + \frac{b \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{a} - \frac{(b^2c)}{a} \\ &= -\frac{b \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{a} - \frac{b \text{Li}_2(c(a+bx))}{a} - \frac{\text{Li}_2(c(a+bx))}{x} - \frac{b \text{Li}_2\left(1 - \frac{bcx}{1-ac}\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.0519712, size = 73, normalized size = 0.87

$$\frac{(a+bx)\text{PolyLog}[2, c(a+bx)] + bx \left( \text{PolyLog}\left[2, \frac{ac+bcx-1}{ac-1}\right] + \log\left(\frac{bcx}{1-ac}\right) \log(-ac-bcx+1) \right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^2, x]
```

```
[Out] -(((a + b*x)*PolyLog[2, c*(a + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x] + PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)])))/(a*x)
```

**Maple [A]** time = 0.13, size = 85, normalized size = 1.

$$-\frac{\text{polylog}(2, xbc + ac)}{x} - \frac{b \ln(-xbc - ac + 1)}{a} \ln\left(-\frac{xbc}{ac-1}\right) - \frac{b}{a} \text{dilog}\left(-\frac{xbc}{ac-1}\right) - \frac{b \text{dilog}(-xbc - ac + 1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(2,c*(b*x+a))/x^2,x)`

[Out] `-polylog(2,b*c*x+a*c)/x-b/a*ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1))-b/a*dilog(-b*c*x/(a*c-1))-b/a*dilog(-b*c*x-a*c+1)`

**Maxima [A]** time = 0.981607, size = 154, normalized size = 1.83

$$\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1))b}{a} - \frac{\left(\log(-bc x - ac + 1) \log\left(-\frac{bc x + ac - 1}{ac - 1} + 1\right) + \text{Li}_2\left(\frac{bc x + ac - 1}{ac - 1}\right)\right)b}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="maxima")`

[Out] `(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/a - (log(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x + a*c - 1)/(a*c - 1)))*b/a - dilog(b*c*x + a*c)/x`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bc x + ac)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="fricas")`

[Out] `integral(dilog(b*c*x + a*c)/x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx + a)c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="giac")`

[Out] `integrate(dilog((b*x + a)*c)/x^2, x)`



### 3.129 $\int \frac{\text{PolyLog}(2, c(a+bx))}{x^3} dx$

**Optimal.** Leaf size=173

$$\frac{b^2 \text{PolyLog}(2, c(a+bx))}{2a^2} + \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{2a^2} - \frac{\text{PolyLog}(2, c(a+bx))}{2x^2} + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(-ac - bcx + 1)}{2a^2} + \frac{b^2}{2}$$

[Out] (b^2\*c\*Log[x])/(2\*a\*(1 - a\*c)) - (b^2\*c\*Log[1 - a\*c - b\*c\*x])/(2\*a\*(1 - a\*c)) + (b\*Log[1 - a\*c - b\*c\*x])/(2\*a\*x) + (b^2\*Log[(b\*c\*x)/(1 - a\*c)]\*Log[1 - a\*c - b\*c\*x])/(2\*a^2) + (b^2\*PolyLog[2, c\*(a + b\*x)])/(2\*a^2) - PolyLog[2, c\*(a + b\*x)]/(2\*x^2) + (b^2\*PolyLog[2, 1 - (b\*c\*x)/(1 - a\*c)])/(2\*a^2)

**Rubi [A]** time = 0.177859, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$ , Rules used = {6598, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{b^2 \text{PolyLog}(2, c(a+bx))}{2a^2} + \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{2a^2} - \frac{\text{PolyLog}(2, c(a+bx))}{2x^2} + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(-ac - bcx + 1)}{2a^2} + \frac{b^2}{2}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, c\*(a + b\*x)]/x^3, x]

[Out] (b^2\*c\*Log[x])/(2\*a\*(1 - a\*c)) - (b^2\*c\*Log[1 - a\*c - b\*c\*x])/(2\*a\*(1 - a\*c)) + (b\*Log[1 - a\*c - b\*c\*x])/(2\*a\*x) + (b^2\*Log[(b\*c\*x)/(1 - a\*c)]\*Log[1 - a\*c - b\*c\*x])/(2\*a^2) + (b^2\*PolyLog[2, c\*(a + b\*x)])/(2\*a^2) - PolyLog[2, c\*(a + b\*x)]/(2\*x^2) + (b^2\*PolyLog[2, 1 - (b\*c\*x)/(1 - a\*c)])/(2\*a^2)

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)])/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_.))^(m\_.)\*((f\_.) + (g\_.)\*(x\_.)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && N

eQ[q, -1]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(c(a+bx))}{x^3} dx &= -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b \int \frac{\log(1-ac-bcx)}{x^2(a+bx)} dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b \int \left( \frac{\log(1-ac-bcx)}{ax^2} - \frac{b \log(1-ac-bcx)}{a^2x} + \frac{b^2 \log(1-ac-bcx)}{a^2(a+bx)} \right) dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{b \int \frac{\log(1-ac-bcx)}{x^2} dx}{2a} + \frac{b^2 \int \frac{\log(1-ac-bcx)}{x} dx}{2a^2} - \frac{b^3 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{2a^2} \\
&= \frac{b \log(1-ac-bcx)}{2ax} + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{b^2 \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx\right)}{2a^2} \\
&= \frac{b \log(1-ac-bcx)}{2ax} + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{2a^2} + \frac{b^2 \text{Li}_2(c(a+bx))}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2} + \frac{b^2 \log(x)}{2a(1-ac)} \\
&= \frac{b^2 c \log(x)}{2a(1-ac)} - \frac{b^2 c \log(1-ac-bcx)}{2a(1-ac)} + \frac{b \log(1-ac-bcx)}{2ax} + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{2a^2} + \frac{b^2 \text{Li}_2(c(a+bx))}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.180152, size = 131, normalized size = 0.76

$$\frac{bx \left( bx(ac-1) \text{PolyLog}\left(2, \frac{ac+bcx-1}{ac-1}\right) - abcx \log(x) + \left( a(ac+bcx-1) + bx(ac-1) \log\left(\frac{bcx}{1-ac}\right) \right) \log(-ac-bcx+1) \right) - (a^2 c \log(x) - b^2 c \log(1-ac-bcx) + b \log(1-ac-bcx) + \frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{2a^2} + \frac{b^2 \text{Li}_2(c(a+bx))}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2})}{2a^2 x^2 (ac-1)}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, c\*(a + b\*x)]/x^3, x]

[Out]  $(-((-1 + a*c)*(a^2 - b^2*x^2)*\text{PolyLog}[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*\text{Log}[x]) + (a*(-1 + a*c + b*c*x) + b*(-1 + a*c)*x*\text{Log}[(b*c*x)/(1 - a*c)])*\text{Log}[1 - a*c - b*c*x] + b*(-1 + a*c)*x*\text{PolyLog}[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/(2*a^2*(-1 + a*c)*x^2)$

**Maple [A]** time = 0.227, size = 195, normalized size = 1.1

$$-\frac{\text{polylog}(2, xbc + ac)}{2x^2} + \frac{b^2 \ln(-xbc - ac + 1)}{2a^2} \ln\left(-\frac{xbc}{ac-1}\right) + \frac{b^2}{2a^2} \text{dilog}\left(-\frac{xbc}{ac-1}\right) + \frac{b^2 \text{dilog}(-xbc - ac + 1)}{2a^2} - \frac{b^2 c \log(x)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, c\*(b\*x+a))/x^3, x)

[Out]  $-1/2*\text{polylog}(2, b*c*x+a*c)/x^2 + 1/2*b^2/a^2*\ln(-b*c*x-a*c+1)*\ln(-b*c*x/(a*c-1)) + 1/2*b^2/a^2*\text{dilog}(-b*c*x/(a*c-1)) + 1/2*b^2/a^2*\text{dilog}(-b*c*x-a*c+1) - 1/2*b^2*c/a/(a*c-1)*\ln(-x*b*c) + 1/2*b^2*c/a*\ln(-b*c*x-a*c+1)/(a*c-1) + 1/2*b*c*\ln(-b*c*x-a*c+1)/(a*c-1)/x - 1/2*b/a*\ln(-b*c*x-a*c+1)/(a*c-1)/x$

**Maxima [A]** time = 0.994426, size = 261, normalized size = 1.51

$$\frac{b^2 c \log(x)}{2(a^2 c - a)} - \frac{(\log(bcx + ac) \log(-bcx - ac + 1) + \text{Li}_2(-bcx - ac + 1))b^2}{2a^2} + \frac{(\log(-bcx - ac + 1) \log\left(-\frac{bcx+ac-1}{ac-1}\right) + 1)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^3,x, algorithm="maxima")

[Out] 
$$-1/2*b^2*c*\log(x)/(a^2*c - a) - 1/2*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \operatorname{dilog}(-b*c*x - a*c + 1))*b^2/a^2 + 1/2*(\log(-b*c*x - a*c + 1)*\log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + \operatorname{dilog}((b*c*x + a*c - 1)/(a*c - 1)))*b^2/a^2 - 1/2*((a^2*c - a)*\operatorname{dilog}(b*c*x + a*c) - (b^2*c*x^2 + (a*b*c - b)*x)*\log(-b*c*x - a*c + 1))/((a^2*c - a)*x^2)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(bc x + ac)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^3,x, algorithm="fricas")

[Out] integral(dilog(b\*c\*x + a\*c)/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2((bx + a)c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^3,x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c)/x^3, x)

### 3.130 $\int \frac{\text{PolyLog}(2, c(a+bx))}{x^4} dx$

**Optimal.** Leaf size=276

$$\frac{b^3 \text{PolyLog}(2, c(a+bx))}{3a^3} - \frac{b^3 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{3a^3} - \frac{\text{PolyLog}(2, c(a+bx))}{3x^3} - \frac{b^3 c \log(x)}{3a^2(1-ac)} - \frac{b^3 \log\left(\frac{bcx}{1-ac}\right) \log(-ac)}{3a^3}$$

```
[Out] -(b^2*c)/(6*a*(1 - a*c)*x) + (b^3*c^2*Log[x])/(6*a*(1 - a*c)^2) - (b^3*c*Log[x])/(3*a^2*(1 - a*c)) - (b^3*c^2*Log[1 - a*c - b*c*x])/(6*a*(1 - a*c)^2) + (b^3*c*Log[1 - a*c - b*c*x])/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x])/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x])/(3*a^2*x) - (b^3*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) - (b^3*PolyLog[2, c*(a + b*x)])/(3*a^3) - PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^3*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(3*a^3)
```

**Rubi [A]** time = 0.272902, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$ , Rules used = {6598, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{b^3 \text{PolyLog}(2, c(a+bx))}{3a^3} - \frac{b^3 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{3a^3} - \frac{\text{PolyLog}(2, c(a+bx))}{3x^3} - \frac{b^3 c \log(x)}{3a^2(1-ac)} - \frac{b^3 \log\left(\frac{bcx}{1-ac}\right) \log(-ac)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^4, x]
```

```
[Out] -(b^2*c)/(6*a*(1 - a*c)*x) + (b^3*c^2*Log[x])/(6*a*(1 - a*c)^2) - (b^3*c*Log[x])/(3*a^2*(1 - a*c)) - (b^3*c^2*Log[1 - a*c - b*c*x])/(6*a*(1 - a*c)^2) + (b^3*c*Log[1 - a*c - b*c*x])/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x])/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x])/(3*a^2*x) - (b^3*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) - (b^3*PolyLog[2, c*(a + b*x)])/(3*a^3) - PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^3*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(3*a^3)
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)]/(e*(m + 1)), x] + Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(r_.)]^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{Li}_2(c(a+bx))}{x^4} dx &= -\frac{\operatorname{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b \int \frac{\log(1-ac-bcx)}{x^3(a+bx)} dx \\
&= -\frac{\operatorname{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b \int \left( \frac{\log(1-ac-bcx)}{ax^3} - \frac{b \log(1-ac-bcx)}{a^2x^2} + \frac{b^2 \log(1-ac-bcx)}{a^3x} - \frac{b^3 \log(1-ac-bcx)}{a^3} \right) dx \\
&= -\frac{\operatorname{Li}_2(c(a+bx))}{3x^3} - \frac{b \int \frac{\log(1-ac-bcx)}{x^3} dx}{3a} + \frac{b^2 \int \frac{\log(1-ac-bcx)}{x^2} dx}{3a^2} - \frac{b^3 \int \frac{\log(1-ac-bcx)}{x} dx}{3a^3} + \frac{b^4 \int \frac{\log(1-ac-bcx)}{1} dx}{3a^3} \\
&= \frac{b \log(1-ac-bcx)}{6ax^2} - \frac{b^2 \log(1-ac-bcx)}{3a^2x} - \frac{b^3 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{3a^3} - \frac{\operatorname{Li}_2(c(a+bx))}{3x^3} + \frac{b^4 \log(1-ac-bcx)}{3a^3} \\
&= \frac{b \log(1-ac-bcx)}{6ax^2} - \frac{b^2 \log(1-ac-bcx)}{3a^2x} - \frac{b^3 \log\left(\frac{bcx}{1-ac}\right) \log(1-ac-bcx)}{3a^3} - \frac{b^3 \operatorname{Li}_2(c(a+bx))}{3a^3} + \frac{b^4 \log(1-ac-bcx)}{3a^3} \\
&= -\frac{b^2c}{6a(1-ac)x} + \frac{b^3c^2 \log(x)}{6a(1-ac)^2} - \frac{b^3c \log(x)}{3a^2(1-ac)} - \frac{b^3c^2 \log(1-ac-bcx)}{6a(1-ac)^2} + \frac{b^3c \log(1-ac-bcx)}{3a^2(1-ac)} + \frac{b^4 \log(1-ac-bcx)}{3a^3}
\end{aligned}$$

**Mathematica [A]** time = 0.303672, size = 210, normalized size = 0.76

$$\frac{b \left( 2b^2 \operatorname{PolyLog}(2, c(a+bx)) + 2b^2 \operatorname{PolyLog}\left(2, \frac{ac+bcx-1}{ac-1}\right) - \frac{a^2 \log(-ac-bcx+1)}{x^2} - \frac{a^2 bc(-bcx \log(-ac-bcx+1) + ac+bcx \log(x)-1)}{x(ac-1)^2} - \frac{2a^3 \log(1-ac-bcx)}{3a^3} \right)}{6a^3}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, c\*(a + b\*x)]/x^4, x]

[Out]  $-(b*((-2*a*b^2*c*(\operatorname{Log}[x] - \operatorname{Log}[1 - a*c - b*c*x])))/(-1 + a*c) - (a^2*\operatorname{Log}[1 - a*c - b*c*x])/x^2 + (2*a*b*\operatorname{Log}[1 - a*c - b*c*x])/x + 2*b^2*\operatorname{Log}[(b*c*x)/(1 - a*c)]*\operatorname{Log}[1 - a*c - b*c*x] - (a^2*b*c*(-1 + a*c + b*c*x*\operatorname{Log}[x] - b*c*x*\operatorname{Log}[1 - a*c - b*c*x]))/((-1 + a*c)^2*x) + 2*b^2*\operatorname{PolyLog}[2, c*(a + b*x)] + 2*b^2*\operatorname{PolyLog}[2, (-1 + a*c + b*c*x)/(-1 + a*c)])/(6*a^3) - \operatorname{PolyLog}[2, a*c + b*c*x]/(3*x^3)$

**Maple [A]** time = 0.214, size = 376, normalized size = 1.4

$$-\frac{\operatorname{polylog}(2, xbc + ac)}{3x^3} - \frac{b^3 \ln(-xbc - ac + 1)}{3a^3} \ln\left(-\frac{xbc}{ac-1}\right) - \frac{b^3}{3a^3} \operatorname{dilog}\left(-\frac{xbc}{ac-1}\right) + \frac{b^3c^2 \ln(-xbc)}{6a(ac-1)^2} + \frac{b^2c^2}{6(ac-1)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, c\*(b\*x+a))/x^4, x)

[Out]  $-1/3*\operatorname{polylog}(2, b*c*x+a*c)/x^3 - 1/3*b^3/a^3*\ln(-b*c*x-a*c+1)*\ln(-b*c*x/(a*c-1)) - 1/3*b^3/a^3*\operatorname{dilog}(-b*c*x/(a*c-1)) + 1/6*b^3*c^2/a/(a*c-1)^2*\ln(-x*b*c) + 1/6*b^2*c^2/(a*c-1)^2/x - 1/6*b^2*c/a/(a*c-1)^2/x - 1/6*b^3*c^2/a*\ln(-b*c*x-a*c+1)/(a*c-1)^2 + 1/6*b*c^2*\ln(-b*c*x-a*c+1)/x^2/(a*c-1)^2*a - 1/3*b*c*\ln(-b*c*x-a*c+1)/x^2/(a*c-1)^2 + 1/6*b/a*\ln(-b*c*x-a*c+1)/x^2/(a*c-1)^2 - 1/3*b^3/a^3*\operatorname{dilog}(-b*c*x-a*c+1) + 1/3*b^3*c/a^2/(a*c-1)*\ln(-x*b*c) - 1/3*b^3*c/a^2*\ln(-b*c*x-a*c+1)/(a*c-1) - 1/3*b^2*c/a*\ln(-b*c*x-a*c+1)/(a*c-1)/x + 1/3*b^2/a^2*\ln(-b*c*x-a*c+1)/(a*c-1)/x$

**Maxima [A]** time = 1.02696, size = 408, normalized size = 1.48

$$\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1)) b^3}{3 a^3} - \frac{\left(\log(-bc x - ac + 1) \log\left(-\frac{bc x + ac - 1}{ac - 1} + 1\right) + \text{Li}_2\left(\frac{bc x + ac - 1}{ac - 1}\right)\right) b}{3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^4,x, algorithm="maxima")

[Out] 1/3\*(log(b\*c\*x + a\*c)\*log(-b\*c\*x - a\*c + 1) + dilog(-b\*c\*x - a\*c + 1))\*b^3/a^3 - 1/3\*(log(-b\*c\*x - a\*c + 1)\*log(-(b\*c\*x + a\*c - 1)/(a\*c - 1) + 1) + dilog((b\*c\*x + a\*c - 1)/(a\*c - 1)))\*b^3/a^3 + 1/6\*(3\*a\*b^3\*c^2 - 2\*b^3\*c)\*log(x)/(a^4\*c^2 - 2\*a^3\*c + a^2) + 1/6\*((a^2\*b^2\*c^2 - a\*b^2\*c)\*x^2 - 2\*(a^4\*c^2 - 2\*a^3\*c + a^2)\*dilog(b\*c\*x + a\*c) - ((3\*a\*b^3\*c^2 - 2\*b^3\*c)\*x^3 + 2\*(a^2\*b^2\*c^2 - 2\*a\*b^2\*c + b^2)\*x^2 - (a^3\*b\*c^2 - 2\*a^2\*b\*c + a\*b)\*x)\*log(-b\*c\*x - a\*c + 1)/((a^4\*c^2 - 2\*a^3\*c + a^2)\*x^3)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bc x + ac)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^4,x, algorithm="fricas")

[Out] integral(dilog(b\*c\*x + a\*c)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx + a)c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/x^4,x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c)/x^4, x)



### 3.131 $\int x^2 \text{PolyLog}(3, c(a + bx)) dx$

**Optimal.** Leaf size=347

$$\frac{(a^3 - b^3 x^3) \text{PolyLog}(3, c(a + bx))}{3b^3} - \frac{11a^3 \text{PolyLog}(2, c(a + bx))}{18b^3} + \frac{2a^3 \text{PolyLog}(3, c(a + bx))}{3b^3} - \frac{a^2 x \text{PolyLog}(2, c(a + bx))}{3b^2}$$

```
[Out] (11*a^2*x)/(18*b^2) - (5*a*(1 - a*c)*x)/(36*b^2*c) + ((1 - a*c)^2*x)/(27*b^2*c^2) - (5*a*x^2)/(72*b) + ((1 - a*c)*x^2)/(54*b*c) + x^3/81 - (5*a*(1 - a*c)^2*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(27*b^3*c^3) + (5*a*x^2*Log[1 - a*c - b*c*x])/(36*b) - (x^3*Log[1 - a*c - b*c*x])/27 + (11*a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(18*b^3*c) - (11*a^3*PolyLog[2, c*(a + b*x)])/(18*b^3) - (a^2*x*PolyLog[2, c*(a + b*x)])/(3*b^2) + (a*x^2*PolyLog[2, c*(a + b*x)])/(6*b) - (x^3*PolyLog[2, c*(a + b*x)])/9 + (2*a^3*PolyLog[3, c*(a + b*x)])/(3*b^3) - ((a^3 - b^3*x^3)*PolyLog[3, c*(a + b*x)])/(3*b^3)
```

**Rubi [A]** time = 0.637739, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6599, 6595, 2444, 2389, 2295, 2421, 2393, 2391, 6598, 43, 2416, 2395, 6589}

$$\frac{(a^3 - b^3 x^3) \text{PolyLog}(3, c(a + bx))}{3b^3} - \frac{11a^3 \text{PolyLog}(2, c(a + bx))}{18b^3} + \frac{2a^3 \text{PolyLog}(3, c(a + bx))}{3b^3} - \frac{a^2 x \text{PolyLog}(2, c(a + bx))}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, c*(a + b*x)], x]
```

```
[Out] (11*a^2*x)/(18*b^2) - (5*a*(1 - a*c)*x)/(36*b^2*c) + ((1 - a*c)^2*x)/(27*b^2*c^2) - (5*a*x^2)/(72*b) + ((1 - a*c)*x^2)/(54*b*c) + x^3/81 - (5*a*(1 - a*c)^2*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(27*b^3*c^3) + (5*a*x^2*Log[1 - a*c - b*c*x])/(36*b) - (x^3*Log[1 - a*c - b*c*x])/27 + (11*a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(18*b^3*c) - (11*a^3*PolyLog[2, c*(a + b*x)])/(18*b^3) - (a^2*x*PolyLog[2, c*(a + b*x)])/(3*b^2) + (a*x^2*PolyLog[2, c*(a + b*x)])/(6*b) - (x^3*PolyLog[2, c*(a + b*x)])/9 + (2*a^3*PolyLog[3, c*(a + b*x)])/(3*b^3) - ((a^3 - b^3*x^3)*PolyLog[3, c*(a + b*x)])/(3*b^3)
```

#### Rule 6599

```
Int[(x_)^(m_)*PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)], x_Symbol] :=
-Simp[((a^(m + 1) - b^(m + 1))*x^(m + 1))*PolyLog[n, c*(a + b*x)^p]/((m + 1)*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1, c*(a + b*x)^p], (a^(m + 1) - b^(m + 1))*x^(m + 1)/(a + b*x), x], x] /;
FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

#### Rule 6595

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)], x_Symbol] := Simp[x*PolyLog[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x], x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x) /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

#### Rule 2444

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_.), x\_Symbol] := Int[u\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c\*v, (e\_.)\*(f\_.) + (g\_.)\*x]) /; FreeQ[{e, f, g}, x]]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2421

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_)^(q\_.), x\_Symbol] := Int[ExpandToSum[u, x]^q\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatchQ[u, x] && LinearMatchQ[v, x])

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

**Rule 6589**

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

**Rubi steps**

$$\begin{aligned} \int x^2 \text{Li}_3(c(a+bx)) dx &= -\frac{(a^3 - b^3 x^3) \text{Li}_3(c(a+bx))}{3b^3} + \frac{\int (-a^2 \text{Li}_2(c(a+bx)) + abx \text{Li}_2(c(a+bx)) - b^2 x^2 \text{Li}_2(c(a+bx))) dx}{3b^2} \\ &= -\frac{(a^3 - b^3 x^3) \text{Li}_3(c(a+bx))}{3b^3} - \frac{1}{3} \int x^2 \text{Li}_2(c(a+bx)) dx - \frac{a^2 \int \text{Li}_2(c(a+bx)) dx}{3b^2} + \frac{(2a^3) \int \frac{L}{3}}{3} \\ &= -\frac{a^2 x \text{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2 \text{Li}_2(c(a+bx))}{6b} - \frac{1}{9} x^3 \text{Li}_2(c(a+bx)) + \frac{2a^3 \text{Li}_3(c(a+bx))}{3b^3} - \frac{(a^3 - b^3)}{3} \\ &= -\frac{a^2 x \text{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2 \text{Li}_2(c(a+bx))}{6b} - \frac{1}{9} x^3 \text{Li}_2(c(a+bx)) + \frac{2a^3 \text{Li}_3(c(a+bx))}{3b^3} - \frac{(a^3 - b^3)}{3} \\ &= -\frac{a^2 x \text{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2 \text{Li}_2(c(a+bx))}{6b} - \frac{1}{9} x^3 \text{Li}_2(c(a+bx)) + \frac{2a^3 \text{Li}_3(c(a+bx))}{3b^3} - \frac{(a^3 - b^3)}{3} \\ &= \frac{a^2 x}{3b^2} + \frac{5ax^2 \log(1 - ac - bcx)}{36b} - \frac{1}{27} x^3 \log(1 - ac - bcx) + \frac{a^2(1 - ac - bcx) \log(1 - ac - bcx)}{3b^3 c} \\ &= \frac{11a^2 x}{18b^2} + \frac{5ax^2 \log(1 - ac - bcx)}{36b} - \frac{1}{27} x^3 \log(1 - ac - bcx) + \frac{11a^2(1 - ac - bcx) \log(1 - ac - bcx)}{18b^3 c} \\ &= \frac{11a^2 x}{18b^2} - \frac{5a(1 - ac)x}{36b^2 c} + \frac{(1 - ac)^2 x}{27b^2 c^2} - \frac{5ax^2}{72b} + \frac{(1 - ac)x^2}{54bc} + \frac{x^3}{81} - \frac{5a(1 - ac)^2 \log(1 - ac - bcx)}{36b^3 c^2} \end{aligned}$$

**Mathematica [A]** time = 0.0681564, size = 296, normalized size = 0.85

$$-36c^3 (6a^2bx + 11a^3 - 3ab^2x^2 + 2b^3x^3) \text{PolyLog}(2, c(a+bx)) + 216c^3 (a^3 + b^3x^3) \text{PolyLog}(3, c(a+bx)) + 510a^2bc^3$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, c*(a + b*x)], x]
```

```
[Out] (24*a*c - 150*a^2*c^2 + 575*a^3*c^3 + 24*b*c*x - 138*a*b*c^2*x + 510*a^2*b*c^3*x + 12*b^2*c^2*x^2 - 57*a*b^2*c^3*x^2 + 8*b^3*c^3*x^3 + 24*Log[1 - a*c - b*c*x] - 162*a*c*Log[1 - a*c - b*c*x] + 648*a^2*c^2*Log[1 - a*c - b*c*x] - 510*a^3*c^3*Log[1 - a*c - b*c*x] - 396*a^2*b*c^3*x*Log[1 - a*c - b*c*x] + 90*a*b^2*c^3*x^2*Log[1 - a*c - b*c*x] - 24*b^3*c^3*x^3*Log[1 - a*c - b*c*x] - 36*c^3*(11*a^3 + 6*a^2*b*x - 3*a*b^2*x^2 + 2*b^3*x^3)*PolyLog[2, c*(a + b*x)] + 216*c^3*(a^3 + b^3*x^3)*PolyLog[3, c*(a + b*x)]/(648*b^3*c^3)
```

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int x^2 \text{polylog}(3, c(bx+a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*polylog(3,c*(b*x+a)),x)`

[Out] `int(x^2*polylog(3,c*(b*x+a)),x)`

**Maxima [A]** time = 1.03005, size = 356, normalized size = 1.03

$$\frac{11(\log(bc x + ac)\log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1))a^3}{18b^3} + \frac{a^3\text{Li}_3(bc x + ac)}{3b^3} + \frac{216b^3c^3x^3\text{Li}_3(bc x + ac) + 8b^3c^3x^3 - \dots}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="maxima")`

[Out] `11/18*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^3/b^3 + 1/3*a^3*polylog(3, b*c*x + a*c)/b^3 + 1/648*(216*b^3*c^3*x^3*polylog(3, b*c*x + a*c) + 8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(8*5*a^2*b*c^3 - 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 + 66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c + 1))/(b^3*c^3)`

**Fricas [C]** time = 2.34524, size = 675, normalized size = 1.95

$$8b^3c^3x^3 - 3(19ab^2c^3 - 4b^2c^2)x^2 + 6(85a^2bc^3 - 23abc^2 + 4bc)x - 36(2b^3c^3x^3 - 3ab^2c^3x^2 + 6a^2bc^3x + 11a^3c^3)\%iint$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="fricas")`

[Out] `1/648*(8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(85*a^2*b*c^3 - 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3)*%iint(a, b, c, x, -log(-b*c*x - a*c + 1)/(b*x + a), -x*log(-b*c*x - a*c + 1)/(b*x + a), -log(-b*c*x - a*c + 1)/c, -b*log(-b*c*x - a*c + 1)/(b*x + a)) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 + 66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c + 1) + 216*(b^3*c^3*x^3 + a^3*c^3)*polylog(3, b*c*x + a*c))/(b^3*c^3)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_3(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*polylog(3,c*(b*x+a)),x)`

[Out] `Integral(x**2*polylog(3, a*c + b*c*x), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_3((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, (b*x + a)*c), x)
```

### 3.132 $\int x \text{PolyLog}(3, c(a + bx)) dx$

**Optimal.** Leaf size=198

$$-\frac{(a^2 - b^2x^2) \text{PolyLog}(3, c(a + bx))}{2b^2} + \frac{3a^2 \text{PolyLog}(2, c(a + bx))}{4b^2} - \frac{1}{4}x^2 \text{PolyLog}(2, c(a + bx)) + \frac{ax \text{PolyLog}(2, c(a + bx))}{2b}$$

[Out]  $(-3ax)/(4b) + ((1 - ac)x)/(8bc) + x^2/16 + ((1 - ac)^2 \text{Log}[1 - ac - bcx])/(8b^2c^2) - (x^2 \text{Log}[1 - ac - bcx])/8 - (3a(1 - ac - bcx) \text{Log}[1 - ac - bcx])/(4b^2c) + (3a^2 \text{PolyLog}[2, c(a + bx)])/(4b^2) + (ax \text{PolyLog}[2, c(a + bx)])/(2b) - (x^2 \text{PolyLog}[2, c(a + bx)])/4 - ((a^2 - b^2x^2) \text{PolyLog}[3, c(a + bx)])/(2b^2)$

**Rubi [A]** time = 0.2869, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.091$ , Rules used = {6599, 6595, 2444, 2389, 2295, 2421, 2393, 2391, 6598, 43, 2416, 2395}

$$-\frac{(a^2 - b^2x^2) \text{PolyLog}(3, c(a + bx))}{2b^2} + \frac{3a^2 \text{PolyLog}(2, c(a + bx))}{4b^2} - \frac{1}{4}x^2 \text{PolyLog}(2, c(a + bx)) + \frac{ax \text{PolyLog}(2, c(a + bx))}{2b}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[3, c\*(a + b\*x)],x]

[Out]  $(-3ax)/(4b) + ((1 - ac)x)/(8bc) + x^2/16 + ((1 - ac)^2 \text{Log}[1 - ac - bcx])/(8b^2c^2) - (x^2 \text{Log}[1 - ac - bcx])/8 - (3a(1 - ac - bcx) \text{Log}[1 - ac - bcx])/(4b^2c) + (3a^2 \text{PolyLog}[2, c(a + bx)])/(4b^2) + (ax \text{PolyLog}[2, c(a + bx)])/(2b) - (x^2 \text{PolyLog}[2, c(a + bx)])/4 - ((a^2 - b^2x^2) \text{PolyLog}[3, c(a + bx)])/(2b^2)$

#### Rule 6599

Int[(x\_)^(m\_)\*PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)], x\_Symbol] :> -Simp[((a^(m + 1) - b^(m + 1))\*x^(m + 1))\*PolyLog[n, c\*(a + b\*x)^p]/((m + 1)\*b^(m + 1)), x] + Dist[p/((m + 1)\*b^m), Int[ExpandIntegrand[PolyLog[n - 1, c\*(a + b\*x)^p], (a^(m + 1) - b^(m + 1))\*x^(m + 1)/(a + b\*x), x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]

#### Rule 6595

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)], x\_Symbol] :> Simp[x\*PolyLog[n, c\*(a + b\*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c\*(a + b\*x)^p], x], x] + Dist[a\*p, Int[PolyLog[n - 1, c\*(a + b\*x)^p]/(a + b\*x), x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]

#### Rule 2444

Int[((a\_) + Log[(c\_)\*(v\_)^(n\_)])\*(b\_)^(p\_)\*(u\_), x\_Symbol] :> Int[u\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c\*v, (e\_)\*((f\_) + (g\_)\*x)] /; FreeQ[{e, f, g}, x]

#### Rule 2389

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^(p\_), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a

, b, c, d, e, n, p}, x]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2421

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_)^(q\_.), x\_Symbol] := Int[ExpandToSum[u, x]^q\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatchQ[u, x] && LinearMatchQ[v, x])

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_)^(m\_.))\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.))\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rubi steps

$$\begin{aligned}
\int x \operatorname{Li}_3(c(a+bx)) dx &= -\frac{(a^2 - b^2x^2) \operatorname{Li}_3(c(a+bx))}{2b^2} + \frac{\int (a \operatorname{Li}_2(c(a+bx)) - bx \operatorname{Li}_2(c(a+bx))) dx}{2b} \\
&= -\frac{(a^2 - b^2x^2) \operatorname{Li}_3(c(a+bx))}{2b^2} - \frac{1}{2} \int x \operatorname{Li}_2(c(a+bx)) dx + \frac{a \int \operatorname{Li}_2(c(a+bx)) dx}{2b} \\
&= \frac{ax \operatorname{Li}_2(c(a+bx))}{2b} - \frac{1}{4} x^2 \operatorname{Li}_2(c(a+bx)) - \frac{(a^2 - b^2x^2) \operatorname{Li}_3(c(a+bx))}{2b^2} + \frac{a \int \log(1 - c(a+bx)) dx}{2b} \\
&= \frac{ax \operatorname{Li}_2(c(a+bx))}{2b} - \frac{1}{4} x^2 \operatorname{Li}_2(c(a+bx)) - \frac{(a^2 - b^2x^2) \operatorname{Li}_3(c(a+bx))}{2b^2} + \frac{a \int \log(1 - ac - bcx) dx}{2b} \\
&= \frac{ax \operatorname{Li}_2(c(a+bx))}{2b} - \frac{1}{4} x^2 \operatorname{Li}_2(c(a+bx)) - \frac{(a^2 - b^2x^2) \operatorname{Li}_3(c(a+bx))}{2b^2} - \frac{1}{4} \int x \log(1 - ac - bcx) dx \\
&= -\frac{ax}{2b} - \frac{1}{8} x^2 \log(1 - ac - bcx) - \frac{a(1 - ac - bcx) \log(1 - ac - bcx)}{2b^2c} + \frac{a^2 \operatorname{Li}_2(c(a+bx))}{2b^2} + \frac{ax \operatorname{Li}_2(c(a+bx))}{2b} \\
&= -\frac{3ax}{4b} - \frac{1}{8} x^2 \log(1 - ac - bcx) - \frac{3a(1 - ac - bcx) \log(1 - ac - bcx)}{4b^2c} + \frac{3a^2 \operatorname{Li}_2(c(a+bx))}{4b^2} + \frac{ax \operatorname{Li}_2(c(a+bx))}{2b} \\
&= -\frac{3ax}{4b} + \frac{(1 - ac)x}{8bc} + \frac{x^2}{16} + \frac{(1 - ac)^2 \log(1 - ac - bcx)}{8b^2c^2} - \frac{1}{8} x^2 \log(1 - ac - bcx) - \frac{3a(1 - ac - bcx) \log(1 - ac - bcx)}{4b^2c}
\end{aligned}$$

**Mathematica [A]** time = 0.0466226, size = 198, normalized size = 1.

$$4c^2 (3a^2 + 2abx - b^2x^2) \operatorname{PolyLog}(2, c(a+bx)) - 8c^2 (a^2 - b^2x^2) \operatorname{PolyLog}(3, c(a+bx)) + 14a^2c^2 \log(-ac - bcx + 1) - 15a^2c^2 \log^2(-ac - bcx + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[3, c\*(a + b\*x)], x]

[Out] (2\*a\*c - 15\*a^2\*c^2 + 2\*b\*c\*x - 14\*a\*b\*c^2\*x + b^2\*c^2\*x^2 + 2\*Log[1 - a\*c - b\*c\*x] - 16\*a\*c\*Log[1 - a\*c - b\*c\*x] + 14\*a^2\*c^2\*Log[1 - a\*c - b\*c\*x] + 12\*a\*b\*c^2\*x\*Log[1 - a\*c - b\*c\*x] - 2\*b^2\*c^2\*x^2\*Log[1 - a\*c - b\*c\*x] + 4\*c^2\*(3\*a^2 + 2\*a\*b\*x - b^2\*x^2)\*PolyLog[2, c\*(a + b\*x)] - 8\*c^2\*(a^2 - b^2\*x^2)\*PolyLog[3, c\*(a + b\*x)]/(16\*b^2\*c^2)

**Maple [F]** time = 0.006, size = 0, normalized size = 0.

$$\int x \operatorname{polylog}(3, c(bx+a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(3, c\*(b\*x+a)), x)

[Out] int(x\*polylog(3, c\*(b\*x+a)), x)

**Maxima [A]** time = 1.01089, size = 261, normalized size = 1.32

$$\frac{3(\log(bcx+ac)\log(-bcx-ac+1) + \operatorname{Li}_2(-bcx-ac+1))a^2}{4b^2} - \frac{a^2 \operatorname{Li}_3(bcx+ac)}{2b^2} + \frac{8b^2c^2x^2 \operatorname{Li}_3(bcx+ac) + b^2c^2x^2 - 2}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*polylog(3,c\*(b\*x+a)),x, algorithm="maxima")

[Out] 
$$-3/4*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \operatorname{dilog}(-b*c*x - a*c + 1))*a^2/b^2 - 1/2*a^2*\operatorname{polylog}(3, b*c*x + a*c)/b^2 + 1/16*(8*b^2*c^2*x^2*\operatorname{polylog}(3, b*c*x + a*c) + b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*b*c^2*x)*\operatorname{dilog}(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 + 8*a*c - 1)*\log(-b*c*x - a*c + 1))/(b^2*c^2)$$

**Fricas [C]** time = 2.51134, size = 510, normalized size = 2.58

$$b^2c^2x^2 - 2(7abc^2 - bc)x - 4(b^2c^2x^2 - 2abc^2x - 3a^2c^2)\%iint\left(a, b, c, x, -\frac{\log(-bcx-ac+1)}{bx+a}, -\frac{x\log(-bcx-ac+1)}{bx+a}, -\frac{\log(-bcx-ac+1)}{c}\right)$$

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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,c\*(b\*x+a)),x, algorithm="fricas")

[Out] 
$$1/16*(b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2))*\%iint(a, b, c, x, -\log(-b*c*x - a*c + 1)/(b*x + a), -x*\log(-b*c*x - a*c + 1)/(b*x + a), -\log(-b*c*x - a*c + 1)/c, -b*\log(-b*c*x - a*c + 1)/(b*x + a)) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 + 8*a*c - 1)*\log(-b*c*x - a*c + 1) + 8*(b^2*c^2*x^2 - a^2*c^2)*\operatorname{polylog}(3, b*c*x + a*c))/(b^2*c^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ac + bcx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,c\*(b\*x+a)),x)

[Out] Integral(x\*polylog(3, a\*c + b\*c\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(3,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate(x\*polylog(3, (b\*x + a)\*c), x)

### 3.133 $\int \text{PolyLog}(3, c(a + bx)) dx$

**Optimal.** Leaf size=84

$$x(-\text{PolyLog}(2, c(a + bx))) + x\text{PolyLog}(3, c(a + bx)) - \frac{a\text{PolyLog}(2, c(a + bx))}{b} + \frac{a\text{PolyLog}(3, c(a + bx))}{b} + \frac{(-ac - bcx)}{b}$$

[Out] x + ((1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(b\*c) - (a\*PolyLog[2, c\*(a + b\*x)])/b - x\*PolyLog[2, c\*(a + b\*x)] + (a\*PolyLog[3, c\*(a + b\*x)])/b + x\*PolyLog[3, c\*(a + b\*x)]

**Rubi [A]** time = 0.0692237, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$ , Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391, 6589}

$$x(-\text{PolyLog}(2, c(a + bx))) + x\text{PolyLog}(3, c(a + bx)) - \frac{a\text{PolyLog}(2, c(a + bx))}{b} + \frac{a\text{PolyLog}(3, c(a + bx))}{b} + \frac{(-ac - bcx)}{b}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, c\*(a + b\*x)], x]

[Out] x + ((1 - a\*c - b\*c\*x)\*Log[1 - a\*c - b\*c\*x])/(b\*c) - (a\*PolyLog[2, c\*(a + b\*x)])/b - x\*PolyLog[2, c\*(a + b\*x)] + (a\*PolyLog[3, c\*(a + b\*x)])/b + x\*PolyLog[3, c\*(a + b\*x)]

#### Rule 6595

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)], x\_Symbol] := Simp[x\*PolyLog[n, c\*(a + b\*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c\*(a + b\*x)^p], x], x] + Dist[a\*p, Int[PolyLog[n - 1, c\*(a + b\*x)^p]/(a + b\*x), x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]

#### Rule 2444

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_.), x\_Symbol] := Int[u\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c\*v, (e\_.)\*((f\_) + (g\_.)\*x)] /; FreeQ[{e, f, g}, x]]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2421

Int[((a\_.) + Log[(c\_.)\*(v\_)^(n\_.)]\*(b\_.))^(p\_.)\*(u\_)^(q\_.), x\_Symbol] := Int[ExpandToSum[u, x]^q\*(a + b\*Log[c\*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatchQ[u, x] && LinearMatchQ[v, x])

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \text{Li}_3(c(a+bx)) dx &= x\text{Li}_3(c(a+bx)) + a \int \frac{\text{Li}_2(c(a+bx))}{a+bx} dx - \int \text{Li}_2(c(a+bx)) dx \\
&= -x\text{Li}_2(c(a+bx)) + \frac{a\text{Li}_3(c(a+bx))}{b} + x\text{Li}_3(c(a+bx)) + a \int \frac{\log(1-c(a+bx))}{a+bx} dx - \int \log(1-c(a+bx)) dx \\
&= -x\text{Li}_2(c(a+bx)) + \frac{a\text{Li}_3(c(a+bx))}{b} + x\text{Li}_3(c(a+bx)) + a \int \frac{\log(1-ac-bcx)}{a+bx} dx - \int \log(1-c(a+bx)) dx \\
&= -x\text{Li}_2(c(a+bx)) + \frac{a\text{Li}_3(c(a+bx))}{b} + x\text{Li}_3(c(a+bx)) + \frac{a \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{b} - \int \log(1-c(a+bx)) dx \\
&= x + \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} - \frac{a\text{Li}_2(c(a+bx))}{b} - x\text{Li}_2(c(a+bx)) + \frac{a\text{Li}_3(c(a+bx))}{b} + \int \log(1-c(a+bx)) dx
\end{aligned}$$

**Mathematica [A]** time = 0.0232919, size = 66, normalized size = 0.79

$$\frac{(a+bx)\left(-\text{PolyLog}(2, c(a+bx)) + \text{PolyLog}(3, c(a+bx)) + \frac{\log(1-c(a+bx))}{c(a+bx)} - \log(1-c(a+bx)) + 1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)], x]
```

```
[Out] ((a + b*x)*(1 - Log[1 - c*(a + b*x)] + Log[1 - c*(a + b*x)]/(c*(a + b*x)) -
PolyLog[2, c*(a + b*x)] + PolyLog[3, c*(a + b*x)]))/b
```

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int \text{polylog}(3, c(bx+a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(b*x+a)), x)
```

```
[Out] int(polylog(3, c*(b*x+a)), x)
```

---

**Maxima [A]** time = 1.03029, size = 162, normalized size = 1.93

$$\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1))a}{b} + \frac{a \text{Li}_3(bc x + ac)}{b} - \frac{bc x \text{Li}_2(bc x + ac) - bc x \text{Li}_3(bc x + ac) - bc}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a)),x, algorithm="maxima")

[Out] (log(b\*c\*x + a\*c)\*log(-b\*c\*x - a\*c + 1) + dilog(-b\*c\*x - a\*c + 1))\*a/b + a\*polylog(3, b\*c\*x + a\*c)/b - (b\*c\*x\*dilog(b\*c\*x + a\*c) - b\*c\*x\*polylog(3, b\*c\*x + a\*c) - b\*c\*x + (b\*c\*x + a\*c - 1)\*log(-b\*c\*x - a\*c + 1))/(b\*c)

---

**Fricas [C]** time = 2.3553, size = 355, normalized size = 4.23

$$\frac{bcx - (bcx + ac)\%iint\left(a, b, c, x, -\frac{\log(-bcx-ac+1)}{bx+a}, -\frac{x \log(-bcx-ac+1)}{bx+a}, -\frac{\log(-bcx-ac+1)}{c}, -\frac{b \log(-bcx-ac+1)}{bx+a}\right) - (bcx + ac - 1) \log(-bcx - ac + 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a)),x, algorithm="fricas")

[Out] (b\*c\*x - (b\*c\*x + a\*c)\*%iint(a, b, c, x, -log(-b\*c\*x - a\*c + 1)/(b\*x + a), -x\*log(-b\*c\*x - a\*c + 1)/(b\*x + a), -log(-b\*c\*x - a\*c + 1)/c, -b\*log(-b\*c\*x - a\*c + 1)/(b\*x + a)) - (b\*c\*x + a\*c - 1)\*log(-b\*c\*x - a\*c + 1) + (b\*c\*x + a\*c)\*polylog(3, b\*c\*x + a\*c))/(b\*c)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3(c(a + bx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a)),x)

[Out] Integral(polylog(3, c\*(a + b\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate(polylog(3, (b\*x + a)\*c), x)

$$3.134 \quad \int \frac{\text{PolyLog}(3, c(a+bx))}{x} dx$$

**Optimal.** Leaf size=16

$$\text{Int}\left(\frac{\text{PolyLog}(3, ac + bcx)}{x}, x\right)$$

[Out] Int[PolyLog[3, a\*c + b\*c\*x]/x, x]

**Rubi [A]** time = 0.0377706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}(3, c(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[3, c\*(a + b\*x)]/x,x]

[Out] Defer[Int][PolyLog[3, a\*c + b\*c\*x]/x, x]

Rubi steps

$$\int \frac{\text{Li}_3(c(a + bx))}{x} dx = \int \frac{\text{Li}_3(ac + bcx)}{x} dx$$

**Mathematica [A]** time = 0.0369997, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(3, c(a + bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[3, c\*(a + b\*x)]/x,x]

[Out] Integrate[PolyLog[3, c\*(a + b\*x)]/x, x]

**Maple [A]** time = 0.004, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(3, c(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3, c\*(b\*x+a))/x, x)

[Out] int(polylog(3, c\*(b\*x+a))/x, x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3((bx+a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate(polylog(3, (b\*x + a)\*c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, bcx + ac)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x,x, algorithm="fricas")

[Out] integral(polylog(3, b\*c\*x + a\*c)/x, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ac + bcx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x,x)

[Out] Integral(polylog(3, a\*c + b\*c\*x)/x, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3((bx+a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x,x, algorithm="giac")

[Out] integrate(polylog(3, (b\*x + a)\*c)/x, x)

### 3.135 $\int \frac{\text{PolyLog}(3, c(a+bx))}{x^2} dx$

**Optimal.** Leaf size=486

$$\frac{2b\text{PolyLog}(3, c(a+bx))}{a} + \frac{\left(b - \frac{a}{x}\right)\text{PolyLog}(3, c(a+bx))}{a} + \frac{b\text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right)}{a} - \frac{b\text{PolyLog}\left(3, -\frac{bcx}{1-c(a+bx)}\right)}{a}$$

```
[Out] (b*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)]/a + (b*(Log[1 + (b*x)/a] +
Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))]*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*a) + (b*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(
2*a) + (b*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*Poly
Log[2, -((b*x)/a)]/a + (b*Log[x]*PolyLog[2, c*(a + b*x)]/a + (b*Log[-((a*
(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))]/a -
(b*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x
)))]/a + (b*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 -
c*(a + b*x)]/a - (b*PolyLog[3, -((b*x)/a)]/a - (2*b*PolyLog[3, c*(a + b*x
)]/a + ((b - a/x)*PolyLog[3, c*(a + b*x)]/a + (b*PolyLog[3, -((b*x)/(a*(1
- c*(a + b*x)))]/a - (b*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/a - (b
*PolyLog[3, 1 - c*(a + b*x)]/a
```

**Rubi [A]** time = 0.559953, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {6599, 6597, 2440, 2435, 6589}

$$\frac{2b\text{PolyLog}(3, c(a+bx))}{a} + \frac{\left(b - \frac{a}{x}\right)\text{PolyLog}(3, c(a+bx))}{a} + \frac{b\text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right)}{a} - \frac{b\text{PolyLog}\left(3, -\frac{bcx}{1-c(a+bx)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x^2, x]
```

```
[Out] (b*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)]/a + (b*(Log[1 + (b*x)/a] +
Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))]*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*a) + (b*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(
2*a) + (b*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*Poly
Log[2, -((b*x)/a)]/a + (b*Log[x]*PolyLog[2, c*(a + b*x)]/a + (b*Log[-((a*
(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))]/a -
(b*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x
)))]/a + (b*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 -
c*(a + b*x)]/a - (b*PolyLog[3, -((b*x)/a)]/a - (2*b*PolyLog[3, c*(a + b*x
)]/a + ((b - a/x)*PolyLog[3, c*(a + b*x)]/a + (b*PolyLog[3, -((b*x)/(a*(1
- c*(a + b*x)))]/a - (b*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/a - (b
*PolyLog[3, 1 - c*(a + b*x)]/a
```

#### Rule 6599

```
Int[(x_)^(m_)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :=
-Simp[((a^(m + 1) - b^(m + 1))*x^(m + 1))*PolyLog[n, c*(a + b*x)^p]/((m + 1)
*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1,
c*(a + b*x)^p], (a^(m + 1) - b^(m + 1))*x^(m + 1)/(a + b*x), x], x] /;
FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

#### Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
+ e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Symbol] :>
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f +
g*Log[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-((b*c - a*d)*x)/(a*(c + d*x))]) + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
+ (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))],
x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(c(a+bx))}{x^2} dx &= \frac{\left(b - \frac{a}{x}\right) \text{Li}_3(c(a+bx))}{a} - b^2 \int \left( -\frac{\text{Li}_2(c(a+bx))}{abx} + \frac{2\text{Li}_2(c(a+bx))}{a(a+bx)} \right) dx \\
&= \frac{\left(b - \frac{a}{x}\right) \text{Li}_3(c(a+bx))}{a} + \frac{b \int \frac{\text{Li}_2(c(a+bx))}{x} dx}{a} - \frac{(2b^2) \int \frac{\text{Li}_2(c(a+bx))}{a+bx} dx}{a} \\
&= \frac{b \log(x) \text{Li}_2(c(a+bx))}{a} - \frac{2b \text{Li}_3(c(a+bx))}{a} + \frac{\left(b - \frac{a}{x}\right) \text{Li}_3(c(a+bx))}{a} + \frac{b^2 \int \frac{\log(x) \log(1-ac-bcx)}{a+bx} dx}{a} \\
&= \frac{b \log(x) \text{Li}_2(c(a+bx))}{a} - \frac{2b \text{Li}_3(c(a+bx))}{a} + \frac{\left(b - \frac{a}{x}\right) \text{Li}_3(c(a+bx))}{a} + \frac{b \text{Subst} \left( \int \frac{\log\left(-\frac{a}{b} + \frac{x}{b}\right) \log\left(-\frac{a}{b} + \frac{x}{b}\right)}{x} dx \right)}{a} \\
&= \frac{b \log(x) \log\left(1 + \frac{bx}{a}\right) \log(1 - c(a+bx))}{a} + \frac{b \left( \log\left(1 + \frac{bx}{a}\right) + \log\left(\frac{1-ac}{1-c(a+bx)}\right) - \log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right) \right) \log\left(-\frac{a}{b} + \frac{x}{b}\right)}{2a}
\end{aligned}$$

**Mathematica [A]** time = 0.741879, size = 477, normalized size = 0.98

$$\frac{b \left( -\text{PolyLog}(3, c(a+bx)) - \text{PolyLog}(3, -ac - bcx + 1) + \text{PolyLog}\left(3, \frac{a(ac+bcx-1)}{bx}\right) - \text{PolyLog}\left(3, \frac{ac+bcx-1}{bcx}\right) + \log\left(\frac{a(ac+bcx-1)}{bx}\right) \right)}{2a}$$



Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x^2, x]
```

```
[Out] -(PolyLog[3, c*(a + b*x)]/x) + (b*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[-(((-1 + a*c)*(a + b*x))/(b*x))]) + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -(b*x)/a] + (Log[x] - Log[a + b*x])*PolyLog[2, c*(a + b*x)] + Log[a + b*x]*PolyLog[2, c*(a + b*x)] + (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) - PolyLog[3, -(b*x)/a] - PolyLog[3, c*(a + b*x)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)])/a
```

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(3, c(bx + a))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3, c*(b*x+a))/x^2, x)
```

```
[Out] int(polylog(3, c*(b*x+a))/x^2, x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{\text{Li}_2(bcx + ac)}{bx^2 + ax} dx - \frac{\text{Li}_3(bcx + ac)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3, c*(b*x+a))/x^2, x, algorithm="maxima")
```

```
[Out] b*integrate(dilog(b*c*x + a*c)/(b*x^2 + a*x), x) - polylog(3, b*c*x + a*c)/x
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, bcx + ac)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3, c*(b*x+a))/x^2, x, algorithm="fricas")
```

```
[Out] integral(polylog(3, b*c*x + a*c)/x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ac + bcx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x**2,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3((bx + a)c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x^2, x)
```

### 3.136 $\int \frac{\text{PolyLog}(3, c(a+bx))}{x^3} dx$

**Optimal.** Leaf size=629

$$\frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{PolyLog}(3, c(a+bx))}{2a^2} - \frac{b^2 \text{PolyLog}(2, c(a+bx))}{2a^2} - \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{2a^2} - \frac{b^2 \text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}$$

[Out]  $-(b^2 \cdot \text{Log}[(b \cdot c \cdot x)/(1 - a \cdot c)] \cdot \text{Log}[1 - a \cdot c - b \cdot c \cdot x])/(2 \cdot a^2) - (b^2 \cdot \text{Log}[x] \cdot \text{Log}[1 + (b \cdot x)/a] \cdot \text{Log}[1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot (\text{Log}[1 + (b \cdot x)/a] + \text{Log}[(1 - a \cdot c)/(1 - c \cdot (a + b \cdot x))]) - \text{Log}[(1 - a \cdot c) \cdot (a + b \cdot x)]/(a \cdot (1 - c \cdot (a + b \cdot x)))) \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]^2)/(4 \cdot a^2) - (b^2 \cdot (\text{Log}[c \cdot (a + b \cdot x)] - \text{Log}[1 + (b \cdot x)/a]) \cdot (\text{Log}[x] + \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))])^2)/(4 \cdot a^2) - (b^2 \cdot (\text{Log}[1 - c \cdot (a + b \cdot x)] - \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]) \cdot \text{PolyLog}[2, -((b \cdot x)/a)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a \cdot x) - (b^2 \cdot \text{Log}[x] \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[2, 1 - (b \cdot c \cdot x)/(1 - a \cdot c)])/(2 \cdot a^2) - (b^2 \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))] \cdot \text{PolyLog}[2, -((b \cdot x)/(a \cdot (1 - c \cdot (a + b \cdot x))))])/(2 \cdot a^2) + (b^2 \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))] \cdot \text{PolyLog}[2, -((b \cdot c \cdot x)/(1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) - (b^2 \cdot (\text{Log}[x] + \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]) \cdot \text{PolyLog}[2, 1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, -((b \cdot x)/a)])/(2 \cdot a^2) + ((b^2 - a^2/x^2) \cdot \text{PolyLog}[3, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[3, -((b \cdot x)/(a \cdot (1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, -((b \cdot c \cdot x)/(1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, 1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2)$

**Rubi [A]** time = 0.663908, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6599, 6598, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 6597, 2440, 2435}

$$\frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{PolyLog}(3, c(a+bx))}{2a^2} - \frac{b^2 \text{PolyLog}(2, c(a+bx))}{2a^2} - \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{bcx}{1-ac}\right)}{2a^2} - \frac{b^2 \text{PolyLog}\left(3, -\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, c\*(a + b\*x)]/x^3, x]

[Out]  $-(b^2 \cdot \text{Log}[(b \cdot c \cdot x)/(1 - a \cdot c)] \cdot \text{Log}[1 - a \cdot c - b \cdot c \cdot x])/(2 \cdot a^2) - (b^2 \cdot \text{Log}[x] \cdot \text{Log}[1 + (b \cdot x)/a] \cdot \text{Log}[1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot (\text{Log}[1 + (b \cdot x)/a] + \text{Log}[(1 - a \cdot c)/(1 - c \cdot (a + b \cdot x))]) - \text{Log}[(1 - a \cdot c) \cdot (a + b \cdot x)]/(a \cdot (1 - c \cdot (a + b \cdot x)))) \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]^2)/(4 \cdot a^2) - (b^2 \cdot (\text{Log}[c \cdot (a + b \cdot x)] - \text{Log}[1 + (b \cdot x)/a]) \cdot (\text{Log}[x] + \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))])^2)/(4 \cdot a^2) - (b^2 \cdot (\text{Log}[1 - c \cdot (a + b \cdot x)] - \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]) \cdot \text{PolyLog}[2, -((b \cdot x)/a)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a \cdot x) - (b^2 \cdot \text{Log}[x] \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[2, 1 - (b \cdot c \cdot x)/(1 - a \cdot c)])/(2 \cdot a^2) - (b^2 \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))] \cdot \text{PolyLog}[2, -((b \cdot x)/(a \cdot (1 - c \cdot (a + b \cdot x))))])/(2 \cdot a^2) + (b^2 \cdot \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))] \cdot \text{PolyLog}[2, -((b \cdot c \cdot x)/(1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) - (b^2 \cdot (\text{Log}[x] + \text{Log}[-((a \cdot (1 - c \cdot (a + b \cdot x)))/(b \cdot x))]) \cdot \text{PolyLog}[2, 1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, -((b \cdot x)/a)])/(2 \cdot a^2) + ((b^2 - a^2/x^2) \cdot \text{PolyLog}[3, c \cdot (a + b \cdot x)])/(2 \cdot a^2) - (b^2 \cdot \text{PolyLog}[3, -((b \cdot x)/(a \cdot (1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, -((b \cdot c \cdot x)/(1 - c \cdot (a + b \cdot x)))]/(2 \cdot a^2) + (b^2 \cdot \text{PolyLog}[3, 1 - c \cdot (a + b \cdot x)])/(2 \cdot a^2)$

**Rule 6599**

Int[(x\_)^(m\_)\*PolyLog[n\_, (c\_)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)], x\_Symbol] := -Simp[((a^(m + 1) - b^(m + 1))\*x^(m + 1))\*PolyLog[n, c\*(a + b\*x)^p]/(m + 1

) $\cdot b^{(m + 1)}$ ), x] + Dist[p/((m + 1) $\cdot b^m$ ), Int[ExpandIntegrand[PolyLog[n - 1, c\*(a + b\*x)<sup>p</sup>], (a<sup>(m + 1)</sup> - b<sup>(m + 1)</sup> $\cdot x^{(m + 1)$ )/(a + b\*x), x], x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]

#### Rule 6598

Int[((d\_) + (e\_)\*(x\_))<sup>(m\_)</sup>\*PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)<sup>(m + 1)</sup>\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)<sup>(m + 1)</sup>\*Log[1 - a\*c - b\*c\*x]/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)<sup>(-1)</sup>, x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2416

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)</sup>]\*(b\_))<sup>(p\_)</sup>\*((h\_)\*(x\_))<sup>(m\_)</sup>\*((f\_) + (g\_)\*(x\_))<sup>(r\_)</sup>]<sup>(q\_)</sup>, x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>p</sup>, (h\*x)<sup>m</sup>(f + g\*x)<sup>r</sup>]<sup>q</sup>, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)</sup>]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2315

Int[Log[(c\_)\*(x\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)</sup>]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x<sup>n</sup>)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
+ e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Symbol] :>
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f +
g*Log[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-((b*c - a*d)*x)/(a*(c + d*x))]) + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1
+ (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))],
x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(c(a+bx))}{x^3} dx &= \frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{Li}_3(c(a+bx))}{2a^2} - \frac{1}{2} b^3 \int \left( -\frac{\text{Li}_2(c(a+bx))}{ab^2x^2} + \frac{\text{Li}_2(c(a+bx))}{a^2bx} \right) dx \\
&= \frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{Li}_3(c(a+bx))}{2a^2} + \frac{b \int \frac{\text{Li}_2(c(a+bx))}{x^2} dx}{2a} - \frac{b^2 \int \frac{\text{Li}_2(c(a+bx))}{x} dx}{2a^2} \\
&= -\frac{b \text{Li}_2(c(a+bx))}{2ax} - \frac{b^2 \log(x) \text{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{Li}_3(c(a+bx))}{2a^2} - \frac{b^2 \int \frac{\log(1-ac-bcx)}{x(a+bx)} dx}{2a} \\
&= -\frac{b \text{Li}_2(c(a+bx))}{2ax} - \frac{b^2 \log(x) \text{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2 - \frac{a^2}{x^2}\right) \text{Li}_3(c(a+bx))}{2a^2} - \frac{b^2 \text{Subst}\left(\int \frac{\log\left(-\frac{a}{b} + \frac{x}{b}\right)}{x(a+bx)} dx\right)}{4a^2} \\
&= -\frac{b^2 \log(x) \log\left(1 + \frac{bx}{a}\right) \log(1 - c(a+bx))}{2a^2} - \frac{b^2 \left(\log\left(1 + \frac{bx}{a}\right) + \log\left(\frac{1-ac}{1-c(a+bx)}\right) - \log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)}{4a^2} \\
&= -\frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} - \frac{b^2 \log(x) \log\left(1 + \frac{bx}{a}\right) \log(1 - c(a+bx))}{2a^2} - \frac{b^2 \left(\log\left(1 + \frac{bx}{a}\right)\right)}{4a^2} \\
&= -\frac{b^2 \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} - \frac{b^2 \log(x) \log\left(1 + \frac{bx}{a}\right) \log(1 - c(a+bx))}{2a^2} - \frac{b^2 \left(\log\left(1 + \frac{bx}{a}\right)\right)}{4a^2}
\end{aligned}$$

**Mathematica [A]** time = 1.93103, size = 573, normalized size = 0.91

$$bx \left( bx \left( \text{PolyLog}\left(2, \frac{bcx}{1-ac}\right) + \text{PolyLog}(2, -ac-bcx+1) + \text{PolyLog}(3, c(a+bx)) + \text{PolyLog}(3, -ac-bcx+1) - \text{PolyLog}\left(3, \frac{a(ac+bcx-1)}{bx}\right) + \text{PolyLog}\left(3, \frac{ac+bcx-1}{bcx}\right) + \log\left(\frac{a(ac+bcx-1)}{bx}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, c\*(a + b\*x)]/x^3,x]

[Out] (-PolyLog[3, c\*(a + b\*x)] + (b\*x\*(-((a + b\*x\*Log[x] - b\*x\*Log[a + b\*x])\*PolyLog[2, c\*(a + b\*x)]) + b\*x\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x] - Log[x]\*Log[1 + (b\*x)/a]\*Log[1 - a\*c - b\*c\*x] + ((Log[c\*(a + b\*x)] - Log[1 + (b\*x)/a])\*Log[1 - a\*c - b\*c\*x]\*(-2\*Log[x] + Log[1 - a\*c - b\*c\*x]))/2 - (Log[c\*(a + b\*x)] - Log[1 + (b\*x)/a])\*Log[1 - a\*c - b\*c\*x]\*Log[(a\*(-1 + a\*c + b\*c\*x))/(b\*x)] - ((Log[(1 - a\*c)/(b\*c\*x)] - Log[-(((-1 + a\*c)\*(a + b\*x))/(b\*x))]) + Log[1 + (b\*x)/a])\*Log[(a\*(-1 + a\*c + b\*c\*x))/(b\*x)]^2)/2 - Log[x]\*(Log[1 - a\*c - b\*c\*x] - Log[1 + (b\*c\*x)/(-1 + a\*c)]) - (Log[1 - a\*c - b\*c\*x] - Log[(a\*(-1 + a\*c + b\*c\*x))/(b\*x)])\*PolyLog[2, -(b\*x)/a] + PolyLog[2, (b\*c\*x)/(1 - a\*c)] - Log[a + b\*x]\*PolyLog[2, c\*(a + b\*x)] + PolyLog[2, 1 - a\*c - b\*c\*x] - (Log[x] + Log[(a\*(-1 + a\*c + b\*c\*x))/(b\*x)])\*PolyLog[2, 1 - a\*c - b\*c\*x] + Log[(a\*(-1 + a\*c + b\*c\*x))/(b\*x)]\*(PolyLog[2, (a\*(-1 + a\*c + b\*c\*x))/(b\*x)] - PolyLog[2, (-1 + a\*c + b\*c\*x)/(b\*c\*x)]) + PolyLog[3, -(b\*x)/a] + PolyLog[3, c\*(a + b\*x)] + PolyLog[3, 1 - a\*c - b\*c\*x] - PolyLog[3, (a\*(-1 + a\*c + b\*c\*x))/(b\*x)] + PolyLog[3, (-1 + a\*c + b\*c\*x)/(b\*c\*x)])))/a^2)/(2\*x^2)

**Maple [F]** time = 0.004, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(3, c(bx + a))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,c\*(b\*x+a))/x^3,x)

[Out] int(polylog(3,c\*(b\*x+a))/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{\text{Li}_2(bcx + ac)}{2(bx^3 + ax^2)} dx - \frac{\text{Li}_3(bcx + ac)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x^3,x, algorithm="maxima")

[Out] b\*integrate(1/2\*dilog(b\*c\*x + a\*c)/(b\*x^3 + a\*x^2), x) - 1/2\*polylog(3, b\*c\*x + a\*c)/x^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, bcx + ac)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,c\*(b\*x+a))/x^3,x, algorithm="fricas")

[Out] `integral(polylog(3, b*c*x + a*c)/x^3, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ac + bcx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,c*(b*x+a))/x**3,x)`

[Out] `Integral(polylog(3, a*c + b*c*x)/x**3, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3((bx + a)c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="giac")`

[Out] `integrate(polylog(3, (b*x + a)*c)/x^3, x)`

### 3.137 $\int (d + ex)^3 \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=605

$$-\frac{(bd - ae)^4 \text{PolyLog}(2, c(a + bx))}{4b^4e} + \frac{(d + ex)^4 \text{PolyLog}(2, c(a + bx))}{4e} - \frac{x(bd - ae)(-ace + bcd + e)^2}{12b^3c^2} - \frac{(d + ex)^2(-ace + bcd + e)^2}{32b^2c^2e}$$

```
[Out] -((b*d - a*e)^3*x)/(4*b^3) - ((b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(8*b^3*c)
) - ((b*d - a*e)*(b*c*d + e - a*c*e)^2*x)/(12*b^3*c^2) - ((b*c*d + e - a*c*
e)^3*x)/(16*b^3*c^3) - ((b*d - a*e)^2*(d + e*x)^2)/(16*b^2*e) - ((b*d - a*e
)*(b*c*d + e - a*c*e)*(d + e*x)^2)/(24*b^2*c*e) - ((b*c*d + e - a*c*e)^2*(d
+ e*x)^2)/(32*b^2*c^2*e) - ((b*d - a*e)*(d + e*x)^3)/(36*b*e) - ((b*c*d +
e - a*c*e)*(d + e*x)^3)/(48*b*c*e) - (d + e*x)^4/(64*e) - ((b*d - a*e)^2*(b
*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(8*b^4*c^2*e) - ((b*d - a*e)*(b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(12*b^4*c^3*e) - ((b*c*d + e - a*c*
e)^4*Log[1 - a*c - b*c*x])/(16*b^4*c^4*e) - ((b*d - a*e)^3*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(4*b^4*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c
- b*c*x])/(8*b^2*e) + ((b*d - a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(12*b*
e) + ((d + e*x)^4*Log[1 - a*c - b*c*x])/(16*e) - ((b*d - a*e)^4*PolyLog[2,
c*(a + b*x)])/(4*b^4*e) + ((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e)
```

**Rubi [A]** time = 0.587251, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$-\frac{(bd - ae)^4 \text{PolyLog}(2, c(a + bx))}{4b^4e} + \frac{(d + ex)^4 \text{PolyLog}(2, c(a + bx))}{4e} - \frac{x(bd - ae)(-ace + bcd + e)^2}{12b^3c^2} - \frac{(d + ex)^2(-ace + bcd + e)^2}{32b^2c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] -((b*d - a*e)^3*x)/(4*b^3) - ((b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(8*b^3*c)
) - ((b*d - a*e)*(b*c*d + e - a*c*e)^2*x)/(12*b^3*c^2) - ((b*c*d + e - a*c*
e)^3*x)/(16*b^3*c^3) - ((b*d - a*e)^2*(d + e*x)^2)/(16*b^2*e) - ((b*d - a*e
)*(b*c*d + e - a*c*e)*(d + e*x)^2)/(24*b^2*c*e) - ((b*c*d + e - a*c*e)^2*(d
+ e*x)^2)/(32*b^2*c^2*e) - ((b*d - a*e)*(d + e*x)^3)/(36*b*e) - ((b*c*d +
e - a*c*e)*(d + e*x)^3)/(48*b*c*e) - (d + e*x)^4/(64*e) - ((b*d - a*e)^2*(b
*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(8*b^4*c^2*e) - ((b*d - a*e)*(b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(12*b^4*c^3*e) - ((b*c*d + e - a*c*
e)^4*Log[1 - a*c - b*c*x])/(16*b^4*c^4*e) - ((b*d - a*e)^3*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(4*b^4*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c
- b*c*x])/(8*b^2*e) + ((b*d - a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(12*b*
e) + ((d + e*x)^4*Log[1 - a*c - b*c*x])/(16*e) - ((b*d - a*e)^4*PolyLog[2,
c*(a + b*x)])/(4*b^4*e) + ((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e)
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
```



```
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (d+ex)^3 \text{Li}_2(c(a+bx)) dx &= \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{b \int \frac{(d+ex)^4 \log(1-ac-bcx)}{a+bx} dx}{4e} \\
&= \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{b \int \left( \frac{e(bd-ae)^3 \log(1-ac-bcx)}{b^4} + \frac{(bd-ae)^4 \log(1-ac-bcx)}{b^4(a+bx)} + \frac{e(bd-ae)^2 (d+ex) \log(1-ac-bcx)}{b^3} \right) dx}{4e} \\
&= \frac{(d+ex)^4 \text{Li}_2(c(a+bx))}{4e} + \frac{1}{4} \int (d+ex)^3 \log(1-ac-bcx) dx + \frac{(bd-ae) \int (d+ex)^2 \log(1-ac-bcx) dx}{4b} \\
&= \frac{(bd-ae)^2 (d+ex)^2 \log(1-ac-bcx)}{8b^2e} + \frac{(bd-ae)(d+ex)^3 \log(1-ac-bcx)}{12be} + \frac{(d+ex)^4 \log(1-ac-bcx)}{4e} \\
&= -\frac{(bd-ae)^3 x}{4b^3} - \frac{(bd-ae)^3 (1-ac-bcx) \log(1-ac-bcx)}{4b^4 c} + \frac{(bd-ae)^2 (d+ex)^2 \log(1-ac-bcx)}{8b^2 e} \\
&= -\frac{(bd-ae)^3 x}{4b^3} - \frac{(bd-ae)^2 (bcd+e-ace)x}{8b^3 c} - \frac{(bd-ae)(bcd+e-ace)^2 x}{12b^3 c^2} - \frac{(bcd+e-ace)^3}{16b^3 c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.562469, size = 485, normalized size = 0.8

$$\frac{-144c^4 (6a^2b^2d^2e - 4a^3bde^2 + a^4e^3 - 4ab^3d^3 - b^4x(6d^2ex + 4d^3 + 4de^2x^2 + e^3x^3)) \text{PolyLog}(2, c(a+bx)) + bc(-6a^2c^2e^2)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*PolyLog[2, c\*(a + b\*x)], x]

[Out] (12\*e\*(-1 + a\*c + b\*c\*x)\*((3 - 13\*a\*c + 23\*a^2\*c^2 - 25\*a^3\*c^3)\*e^2 + b\*c\*e\*(8\*(2 - 7\*a\*c + 11\*a^2\*c^2)\*d + (3 - 10\*a\*c + 13\*a^2\*c^2)\*e\*x) + b^3\*c^3\*x\*(36\*d^2 + 16\*d\*e\*x + 3\*e^2\*x^2) + b^2\*c^2\*(-36\*(-1 + 3\*a\*c)\*d^2 - 8\*(-2 + 5\*a\*c)\*d\*e\*x + (3 - 7\*a\*c)\*e^2\*x^2))\*Log[1 - a\*c - b\*c\*x] + b\*c\*(300\*a^3\*c^3\*e^3\*x - 6\*a^2\*c^2\*e^2\*x\*(46\*e + b\*c\*(176\*d + 13\*e\*x)) + 4\*a\*c\*(39\*e^3\*x + 3\*b\*c\*e^2\*x\*(56\*d + 5\*e\*x) + b^2\*c^2\*(-144\*d^3 + 324\*d^2\*e\*x + 60\*d\*e^2\*x^2 + 7\*e^3\*x^3)) - x\*(36\*e^3 + 6\*b\*c\*e^2\*(32\*d + 3\*e\*x) + 12\*b^2\*c^2\*e\*(36\*d^2 + 8\*d\*e\*x + e^2\*x^2) + b^3\*c^3\*(576\*d^3 + 216\*d^2\*e\*x + 64\*d\*e^2\*x^2 + 9\*e^3\*x^3)) + 576\*b^2\*c^2\*d^3\*(-1 + a\*c + b\*c\*x)\*Log[1 - c\*(a + b\*x)]) - 144\*c^4\*(-4\*a\*b^3\*d^3 + 6\*a^2\*b^2\*d^2\*e - 4\*a^3\*b\*d\*e^2 + a^4\*e^3 - b^4\*x\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3))\*PolyLog[2, c\*(a + b\*x)]/(576\*b^4\*c^4)

**Maple [B]** time = 0.064, size = 1177, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*polylog(2,c\*(b\*x+a)), x)

[Out] 1/4\*e^3\*polylog(2,b\*c\*x+a\*c)\*x^4-1/4/e\*dilog(-b\*c\*x-a\*c+1)\*d^4+1/4/e\*polylog(2,b\*c\*x+a\*c)\*d^4-1/9\*e^2\*x^3\*d-3/8\*e\*x^2\*d^2+1/b/c\*d^3+25/192/b^4/c^4\*e^3+1/16\*e^3\*ln(-b\*c\*x-a\*c+1)\*x^4+ln(-b\*c\*x-a\*c+1)\*x\*d^3+polylog(2,b\*c\*x+a\*c)\*x\*d^3-23/48/b^3/c\*e^3\*x\*a^2-1/32/b^2/c^2\*e^3\*x^2-1/16/b^3/c^3\*e^3\*x-1/b\*c\*ln(-b\*c\*x-a\*c+1)\*d^3-13/96/b^2\*e^3\*x^2\*a^2+25/48/b^3\*e^3\*x\*a^3+7/144/b\*e^3\*x^3\*a-1/4/b^4\*e^3\*dilog(-b\*c\*x-a\*c+1)\*a^4+3/4\*e\*ln(-b\*c\*x-a\*c+1)\*x^2\*d^2+3/2\*e\*polylog(2,b\*c\*x+a\*c)\*d^2\*x^2+e^2\*polylog(2,b\*c\*x+a\*c)\*d\*x^3+1/3\*e^2\*ln(-

$$\begin{aligned}
& b^3 c^3 x^{-a^3 c+1} x^{3d-1/48} / b^4 c^4 e^3 \ln(-b^3 c^3 x - a^3 c + 1) - 25/48 \\
& / b^4 e^3 \ln(-b^3 c^3 x - a^3 c + 1) a^4 + 1/b \ln(-b^3 c^3 x - a^3 c + 1) a^3 d^3 + 1/b \operatorname{dilog}(-b^3 c^3 x - a^3 c + 1) \\
& a^3 d^3 + 415/576 / b^4 e^3 a^4 + 137/96 / b^4 c^2 e^3 a^2 + 9/8 / b^2 c^2 e^3 d^2 - 97/144 / b^4 c^3 e^3 a - 1/b a^3 d^3 - 77/48 / b^4 c e^3 a^3 + 11/18 / b^3 c^3 e^2 d - d^3 x - \\
& 1/64 e^3 x^4 - 3/2 / b^2 e^3 \operatorname{dilog}(-b^3 c^3 x - a^3 c + 1) a^2 d^2 + 11/6 / b^3 e^2 \ln(-b^3 c^3 x - a^3 c + 1) a^3 d - 1/12 / b e^3 \ln(-b^3 c^3 x - a^3 c + 1) x^3 a + 1/8 / b^2 e^3 \ln(-b^3 c^3 x - a^3 c + 1) x^2 a^2 + 13/48 / b^3 c^2 e^3 x a - 31/12 / b^3 c^2 e^2 a d - 85/36 / b^3 e^2 a^3 d + 21/8 / b^2 e^2 a^2 d^2 - 15/4 / b^2 c e^2 a^3 d^2 - 3/4 / b^4 c^2 e^3 \ln(-b^3 c^3 x - a^3 c + 1) a^2 + 1/3 / b^4 c^3 e^3 \ln(-b^3 c^3 x - a^3 c + 1) a - 3/4 / b^2 c^2 e^3 \ln(-b^3 c^3 x - a^3 c + 1) d^2 - 1/3 / b^3 c^3 e^2 \ln(-b^3 c^3 x - a^3 c + 1) d + 1/b^3 e^2 \operatorname{dilog}(-b^3 c^3 x - a^3 c + 1) a^3 d - 1/6 / b c e^2 x^2 d - 1/3 / b^2 c^2 e^2 x^2 d + 5/48 / b^2 c e^3 x^2 a - 3/4 / b c e^2 x^2 d + 9/4 / b e^2 x^2 a^2 d^2 + 5/12 / b e^2 x^2 a^2 d - 11/6 / b^2 e^2 x^2 a^2 d - 1/4 / b^3 e^3 \ln(-b^3 c^3 x - a^3 c + 1) x^2 a^3 - 9/4 / b^2 e^3 \ln(-b^3 c^3 x - a^3 c + 1) a^2 d^2 + 1/b^4 c e^3 \ln(-b^3 c^3 x - a^3 c + 1) a^3 + 13/3 / b^3 c e^2 a^2 d - 3/2 / b e^3 \ln(-b^3 c^3 x - a^3 c + 1) x^2 a^2 d - 1/2 / b e^2 \ln(-b^3 c^3 x - a^3 c + 1) x^2 a^2 d + 1/b^2 e^2 \ln(-b^3 c^3 x - a^3 c + 1) x^2 a^2 d + 3/b^2 c e^3 \ln(-b^3 c^3 x - a^3 c + 1) a^2 d^2 - 3/b^3 c e^2 \ln(-b^3 c^3 x - a^3 c + 1) a^2 d + 3/2 / b^3 c^2 e^2 \ln(-b^3 c^3 x - a^3 c + 1) a^2 d + 7/6 / b^2 c e^2 x^2 a^2 d
\end{aligned}$$

**Maxima [A]** time = 1.04257, size = 919, normalized size = 1.52

$$\frac{(4ab^3d^3 - 6a^2b^2d^2e + 4a^3bde^2 - a^4e^3)(\log(bcx + ac)\log(-bcx - ac + 1) + \operatorname{Li}_2(-bcx - ac + 1))}{4b^4} - \frac{9b^4c^4e^3x^4 + 4(1)}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/4*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e^2 - a^4*e^3)*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \operatorname{dilog}(-b*c*x - a*c + 1))/b^4 - 1/576*(9*b^4*c^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3)*e^3)*x^3 + 6*(36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13*a^2*b^2*c^4 - 10*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 36*(3*a*b^3*c^4 - b^3*c^3)*d^2*e + 8*(11*a^2*b^2*c^4 - 7*a*b^2*c^3 + 2*b^2*c^2)*d*e^2 - (25*a^3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*e^3)*x - 144*(b^4*c^4*e^3*x^4 + 4*b^4*c^4*d*e^2*x^3 + 6*b^4*c^4*d^2*e*x^2 + 4*b^4*c^4*d^3*x)*\operatorname{dilog}(b*c*x + a*c) - 12*(3*b^4*c^4*e^3*x^4 + 48*(a*b^3*c^4 - b^3*c^3)*d^3 - 36*(3*a^2*b^2*c^4 - 4*a*b^2*c^3 + b^2*c^2)*d^2*e + 8*(11*a^3*b*c^4 - 18*a^2*b*c^3 + 9*a*b*c^2 - 2*b*c)*d*e^2 - (25*a^4*c^4 - 48*a^3*c^3 + 36*a^2*c^2 - 16*a*c + 3)*e^3 + 4*(4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 + 6*(6*b^4*c^4*d^2*e - 4*a*b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3)*x^2 + 12*(4*b^4*c^4*d^3 - 6*a*b^3*c^4*d^2*e + 4*a^2*b^2*c^4*d*e^2 - a^3*b*c^4*e^3)*x*\log(-b*c*x - a*c + 1))/(b^4*c^4)$

**Fricas [A]** time = 2.37849, size = 1339, normalized size = 2.21

$$\frac{9b^4c^4e^3x^4 + 4(16b^4c^4de^2 - (7ab^3c^4 - 3b^3c^3)e^3)x^3 + 6(36b^4c^4d^2e - 8(5ab^3c^4 - 2b^3c^3)de^2 + (13a^2b^2c^4 - 10ab^2c^3))}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-1/576*(9*b^4*c^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3)*e^3)*x^3 + 6*(36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13*a^2*b^2*c^4 - 10*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 36*($

$$3ab^3c^4 - b^3c^3)d^2e + 8(11a^2b^2c^4 - 7ab^2c^3 + 2b^2c^2) * d^2e^2 - (25a^3b^2c^4 - 23a^2b^2c^3 + 13ab^2c^2 - 3b^2c) * e^3 * x - 144(b^4c^4e^3x^4 + 4b^4c^4d^2e^2x^3 + 6b^4c^4d^2e^2x^2 + 4b^4c^4d^3x + 4ab^3c^4d^3 - 6a^2b^2c^4d^2e + 4a^3b^2c^4d^2e^2 - a^4c^4e^3) * \text{dilog}(b^2cx + ac) - 12(3b^4c^4e^3x^4 + 48(ab^3c^4 - b^3c^3)d^3 - 36(3a^2b^2c^4 - 4ab^2c^3 + b^2c^2)d^2e + 8(11a^3b^2c^4 - 18a^2b^2c^3 + 9ab^2c^2 - 2b^2c) * d^2e^2 - (25a^4c^4 - 48a^3c^3 + 36a^2c^2 - 16ac + 3) * e^3 + 4(4b^4c^4d^2e^2 - ab^3c^4e^3) * x^3 + 6(6b^4c^4d^2e - 4ab^3c^4d^2e^2 + a^2b^2c^4e^3) * x^2 + 12(4b^4c^4d^3 - 6ab^3c^4d^2e + 4a^2b^2c^4d^2e^2 - a^3b^2c^4e^3) * x) * \log(-b^2cx - ac + 1) / (b^4c^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 \text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*dilog((b\*x + a)\*c), x)

### 3.138 $\int (d + ex)^2 \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=385

$$\frac{(bd - ae)^3 \text{PolyLog}(2, c(a + bx))}{3b^3e} + \frac{(d + ex)^3 \text{PolyLog}(2, c(a + bx))}{3e} - \frac{x(-ace + bcd + e)^2}{9b^2c^2} - \frac{(bd - ae)(-ace + bcd + e)}{6b^3c^2}$$

```
[Out] -((b*d - a*e)^2*x)/(3*b^2) - ((b*d - a*e)*(b*c*d + e - a*c*e)*x)/(6*b^2*c)
- ((b*c*d + e - a*c*e)^2*x)/(9*b^2*c^2) - ((b*d - a*e)*(d + e*x)^2)/(12*b*e)
- ((b*c*d + e - a*c*e)*(d + e*x)^2)/(18*b*c*e) - (d + e*x)^3/(27*e) - ((b
*d - a*e)*(b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^3*e) - ((b*d - a*e)^2*(1 -
a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + ((b*d - a*e)*(d + e*x)^2*Lo
g[1 - a*c - b*c*x])/(6*b*e) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(9*e) - ((
b*d - a*e)^3*PolyLog[2, c*(a + b*x)])/(3*b^3*e) + ((d + e*x)^3*PolyLog[2, c
*(a + b*x)])/(3*e)
```

**Rubi [A]** time = 0.338798, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$\frac{(bd - ae)^3 \text{PolyLog}(2, c(a + bx))}{3b^3e} + \frac{(d + ex)^3 \text{PolyLog}(2, c(a + bx))}{3e} - \frac{x(-ace + bcd + e)^2}{9b^2c^2} - \frac{(bd - ae)(-ace + bcd + e)}{6b^3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] -((b*d - a*e)^2*x)/(3*b^2) - ((b*d - a*e)*(b*c*d + e - a*c*e)*x)/(6*b^2*c)
- ((b*c*d + e - a*c*e)^2*x)/(9*b^2*c^2) - ((b*d - a*e)*(d + e*x)^2)/(12*b*e)
- ((b*c*d + e - a*c*e)*(d + e*x)^2)/(18*b*c*e) - (d + e*x)^3/(27*e) - ((b
*d - a*e)*(b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^3*e) - ((b*d - a*e)^2*(1 -
a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + ((b*d - a*e)*(d + e*x)^2*Lo
g[1 - a*c - b*c*x])/(6*b*e) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(9*e) - ((
b*d - a*e)^3*PolyLog[2, c*(a + b*x)])/(3*b^3*e) + ((d + e*x)^3*PolyLog[2, c
*(a + b*x)])/(3*e)
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
]; FreeQ[{c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))])*(b_.)*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int (d + ex)^2 \text{Li}_2(c(a + bx)) dx &= \frac{(d + ex)^3 \text{Li}_2(c(a + bx))}{3e} + \frac{b \int \frac{(d+ex)^3 \log(1-ac-bcx)}{a+bx} dx}{3e} \\ &= \frac{(d + ex)^3 \text{Li}_2(c(a + bx))}{3e} + \frac{b \int \left( \frac{e(bd-ae)^2 \log(1-ac-bcx)}{b^3} + \frac{(bd-ae)^3 \log(1-ac-bcx)}{b^3(a+bx)} + \frac{e(bd-ae)(d+ex) \log(1-ac-bcx)}{b^2} \right) dx}{3e} \\ &= \frac{(d + ex)^3 \text{Li}_2(c(a + bx))}{3e} + \frac{1}{3} \int (d + ex)^2 \log(1 - ac - bcx) dx + \frac{(bd - ae) \int (d + ex) \log(1 - ac - bcx) dx}{3b} \\ &= \frac{(bd - ae)(d + ex)^2 \log(1 - ac - bcx)}{6be} + \frac{(d + ex)^3 \log(1 - ac - bcx)}{9e} + \frac{(d + ex)^3 \text{Li}_2(c(a + bx))}{3e} \\ &= -\frac{(bd - ae)^2 x}{3b^2} - \frac{(bd - ae)^2 (1 - ac - bcx) \log(1 - ac - bcx)}{3b^3 c} + \frac{(bd - ae)(d + ex)^2 \log(1 - ac - bcx)}{6be} \\ &= -\frac{(bd - ae)^2 x}{3b^2} - \frac{(bd - ae)(bcd + e - ace)x}{6b^2 c} - \frac{(bcd + e - ace)^2 x}{9b^2 c^2} - \frac{(bd - ae)(d + ex)^2}{12be} - \frac{(bcd + e - ace)^2}{12b^2 c} \end{aligned}$$

**Mathematica [A]** time = 0.182727, size = 274, normalized size = 0.71

$$36c^3 (-3a^2 bde + a^3 e^2 + 3ab^2 d^2 + b^3 x (3d^2 + 3dex + e^2 x^2)) \text{PolyLog}(2, c(a + bx)) + bc (-66a^2 c^2 e^2 x + 3ac (bc (-36d^2 + 5$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*PolyLog[2, c\*(a + b\*x)],x]

[Out] (6\*e\*(-1 + a\*c + b\*c\*x)\*((2 - 7\*a\*c + 11\*a^2\*c^2)\*e + b^2\*c^2\*x\*(9\*d + 2\*e\*x) + b\*c\*((9 - 27\*a\*c)\*d + (2 - 5\*a\*c)\*e\*x))\*Log[1 - a\*c - b\*c\*x] + b\*c\*(-6\*a^2\*c^2\*e^2\*x - x\*(12\*e^2 + 6\*b\*c\*e\*(9\*d + e\*x) + b^2\*c^2\*(108\*d^2 + 27\*d\*e\*x + 4\*e^2\*x^2)) + 3\*a\*c\*(14\*e^2\*x + b\*c\*(-36\*d^2 + 54\*d\*e\*x + 5\*e^2\*x^2)) + 108\*b\*c\*d^2\*(-1 + a\*c + b\*c\*x)\*Log[1 - c\*(a + b\*x)]) + 36\*c^3\*(3\*a\*b^2\*d^2 - 3\*a^2\*b\*d\*e + a^3\*e^2 + b^3\*x\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2))\*PolyLog[2, c\*(a + b\*x)]/(108\*b^3\*c^3)

**Maple [A]** time = 0.058, size = 687, normalized size = 1.8

$$-d^2x - \frac{x^3e^2}{27} - \frac{e^2 \ln(-xbc - ac + 1)a^2}{cb^3} + \frac{3axde}{2b} + \frac{7e^2xa}{18b^2c} - \frac{\operatorname{edilog}(-xbc - ac + 1)a^2d}{b^2} - \frac{dx}{2bc} - \frac{3e \ln(-xbc - ac + 1)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*polylog(2,c\*(b\*x+a)),x)

[Out] -d^2\*x-1/27\*x^3\*e^2-1/b^3/c\*e^2\*ln(-b\*c\*x-a\*c+1)\*a^2+3/2/b\*e\*x\*a\*d+7/18/b^2/c\*e^2\*x\*a-1/b^2\*e\*dilog(-b\*c\*x-a\*c+1)\*a^2\*d-1/2/b/c\*x\*d\*e-3/2/b^2\*e\*ln(-b\*c\*x-a\*c+1)\*a^2\*d-1/6/b\*e^2\*ln(-b\*c\*x-a\*c+1)\*x^2\*a+1/3/b^2\*e^2\*ln(-b\*c\*x-a\*c+1)\*x\*a^2-1/2/b^2/c^2\*e\*ln(-b\*c\*x-a\*c+1)\*d-5/2/b^2/c\*e\*a\*d+7/4/b^2\*e\*a^2\*d+1/2/b^3/c^2\*e^2\*ln(-b\*c\*x-a\*c+1)\*a+1/b/c\*d^2+11/54/b^3/c^3\*e^2+1/9\*e^2\*ln(-b\*c\*x-a\*c+1)\*x^3-1/3/e\*dilog(-b\*c\*x-a\*c+1)\*d^3+1/3/e\*polylog(2,b\*c\*x+a\*c)\*d^3+ln(-b\*c\*x-a\*c+1)\*x\*d^2+1/3\*e^2\*polylog(2,b\*c\*x+a\*c)\*x^3+polylog(2,b\*c\*x+a\*c)\*x\*d^2-1/4\*x^2\*d\*e+2/b^2/c\*e\*ln(-b\*c\*x-a\*c+1)\*a\*d-1/b\*e\*ln(-b\*c\*x-a\*c+1)\*x\*a\*d+13/9/b^3/c\*e^2\*a^2+3/4/b^2/c^2\*e\*d-31/36/b^3/c^2\*e^2\*a-85/108/b^3\*e^2\*a^3-1/b\*a\*d^2+5/36/b\*e^2\*x^2\*a-11/18/b^2\*e^2\*x\*a^2-1/18/b/c\*x^2\*e^2-1/9/b^2/c^2\*e^2\*x+1/2\*e\*ln(-b\*c\*x-a\*c+1)\*x^2\*d+e\*polylog(2,b\*c\*x+a\*c)\*d\*x^2+1/3/b^3\*e^2\*dilog(-b\*c\*x-a\*c+1)\*a^3+11/18/b^3\*e^2\*ln(-b\*c\*x-a\*c+1)\*a^3+1/b\*dilog(-b\*c\*x-a\*c+1)\*a\*d^2+1/b\*ln(-b\*c\*x-a\*c+1)\*a\*d^2-1/b/c\*ln(-b\*c\*x-a\*c+1)\*d^2-1/9/b^3/c^3\*e^2\*ln(-b\*c\*x-a\*c+1)

**Maxima [A]** time = 1.03377, size = 548, normalized size = 1.42

$$\frac{(3ab^2d^2 - 3a^2bde + a^3e^2)(\log(bcx + ac)\log(-bcx - ac + 1) + \operatorname{Li}_2(-bcx - ac + 1))}{3b^3} - \frac{4b^3c^3e^2x^3 + 3(9b^3c^3de - (5a^2b^2c^3 - 2b^2c^2)e^2)x^2 + 6(18b^3c^3d^2 - 9(3ab^2c^3 - b^2c^2)d*e + (11a^2b^2c^3 - 7a^2b^2c^2 + 2b^2c^2)e^2)x - 36(b^3c^3e^2x^3 + 3b^3c^3d^2e*x^2 + 3b^3c^3d^2*x)*\operatorname{dilog}(b*c*x + a*c) - 6(2b^3c^3e^2x^3 + 18(a*b^2c^3 - b^2c^2)d^2 - 9(3a^2b^2c^3 - 4a^2b^2c^2 + b^2c^2)d*e + (11a^3c^3 - 18a^2c^2 + 9a^2c - 2)e^2 + 3(3b^3c^3d^2e - a*b^2c^3e^2)x^2 + 6(3b^3c^3d^2 - 3a*b^2c^3d*e + a^2b^2c^3e^2)x)*\log(-b*c*x - a*c + 1)}{(b^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out] -1/3\*(3\*a\*b^2\*d^2 - 3\*a^2\*b\*d\*e + a^3\*e^2)\*(log(b\*c\*x + a\*c)\*log(-b\*c\*x - a\*c + 1) + dilog(-b\*c\*x - a\*c + 1))/b^3 - 1/108\*(4\*b^3\*c^3\*e^2\*x^3 + 3\*(9\*b^3\*c^3\*d\*e - (5\*a\*b^2\*c^3 - 2\*b^2\*c^2)\*e^2)\*x^2 + 6\*(18\*b^3\*c^3\*d^2 - 9\*(3\*a\*b^2\*c^3 - b^2\*c^2)\*d\*e + (11\*a^2\*b^2\*c^3 - 7\*a^2\*b^2\*c^2 + 2\*b^2\*c^2)\*e^2)\*x - 36\*(b^3\*c^3\*e^2\*x^3 + 3\*b^3\*c^3\*d^2\*e\*x^2 + 3\*b^3\*c^3\*d^2\*x)\*dilog(b\*c\*x + a\*c) - 6\*(2\*b^3\*c^3\*e^2\*x^3 + 18\*(a\*b^2\*c^3 - b^2\*c^2)\*d^2 - 9\*(3\*a^2\*b^2\*c^3 - 4\*a^2\*b^2\*c^2 + b^2\*c^2)\*d\*e + (11\*a^3\*c^3 - 18\*a^2\*c^2 + 9\*a^2\*c - 2)\*e^2 + 3\*(3\*b^3\*c^3\*d^2\*e - a\*b^2\*c^3\*e^2)\*x^2 + 6\*(3\*b^3\*c^3\*d^2 - 3\*a\*b^2\*c^3\*d\*e + a^2\*b^2\*c^3\*e^2)\*x)\*log(-b\*c\*x - a\*c + 1)/(b^3\*c^3)

**Fricas [A]** time = 2.32108, size = 779, normalized size = 2.02

$$4b^3c^3e^2x^3 + 3(9b^3c^3de - (5ab^2c^3 - 2b^2c^2)e^2)x^2 + 6(18b^3c^3d^2 - 9(3ab^2c^3 - b^2c^2)de + (11a^2bc^3 - 7abc^2 + 2bc)e^2)x$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out] 
$$-1/108*(4*b^3*c^3*e^2*x^3 + 3*(9*b^3*c^3*d*e - (5*a*b^2*c^3 - 2*b^2*c^2)*e^2)*x^2 + 6*(18*b^3*c^3*d^2 - 9*(3*a*b^2*c^3 - b^2*c^2)*d*e + (11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*e^2)*x - 36*(b^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 + 3*b^3*c^3*d^2*x + 3*a*b^2*c^3*d^2 - 3*a^2*b*c^3*d*e + a^3*c^3*e^2)*\text{dilog}(b*c*x + a*c) - 6*(2*b^3*c^3*e^2*x^3 + 18*(a*b^2*c^3 - b^2*c^2)*d^2 - 9*(3*a^2*b*c^3 - 4*a*b*c^2 + b*c)*d*e + (11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^2 + 3*(3*b^3*c^3*d*e - a*b^2*c^3*e^2)*x^2 + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3*d*e + a^2*b*c^3*e^2)*x*\log(-b*c*x - a*c + 1))/(b^3*c^3)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 \text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*dilog((b\*x + a)\*c), x)



### 3.139 $\int (d + ex) \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=210

$$\frac{(bd - ae)^2 \text{PolyLog}(2, c(a + bx))}{2b^2e} + \frac{(d + ex)^2 \text{PolyLog}(2, c(a + bx))}{2e} - \frac{(-ace + bcd + e)^2 \log(-ac - bcx + 1)}{4b^2c^2e} - \frac{(-ac - bcx + 1)^2 \log(-ac - bcx + 1)}{4b^2c^2e}$$

```
[Out] -((b*d - a*e)*x)/(2*b) - ((b*c*d + e - a*c*e)*x)/(4*b*c) - (d + e*x)^2/(8*e)
) - ((b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) - ((b*d - a*
e)*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) + ((d + e*x)^2*Log[1 -
a*c - b*c*x])/(4*e) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) +
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e)
```

**Rubi [A]** time = 0.196508, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$ , Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$\frac{(bd - ae)^2 \text{PolyLog}(2, c(a + bx))}{2b^2e} + \frac{(d + ex)^2 \text{PolyLog}(2, c(a + bx))}{2e} - \frac{(-ace + bcd + e)^2 \log(-ac - bcx + 1)}{4b^2c^2e} - \frac{(-ac - bcx + 1)^2 \log(-ac - bcx + 1)}{4b^2c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] -((b*d - a*e)*x)/(2*b) - ((b*c*d + e - a*c*e)*x)/(4*b*c) - (d + e*x)^2/(8*e)
) - ((b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) - ((b*d - a*
e)*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) + ((d + e*x)^2*Log[1 -
a*c - b*c*x])/(4*e) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) +
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e)
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_.))], x_Sy
mbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

#### Rule 2295

```
Int[Log[(c_.)*(x_.)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^ (q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (d + ex)\text{Li}_2(c(a + bx)) dx &= \frac{(d + ex)^2 \text{Li}_2(c(a + bx))}{2e} + \frac{b \int \frac{(d+ex)^2 \log(1-ac-bcx)}{a+bx} dx}{2e} \\
&= \frac{(d + ex)^2 \text{Li}_2(c(a + bx))}{2e} + \frac{b \int \left( \frac{e(bd-ae) \log(1-ac-bcx)}{b^2} + \frac{(bd-ae)^2 \log(1-ac-bcx)}{b^2(a+bx)} + \frac{e(d+ex) \log(1-ac-bcx)}{b} \right) dx}{2e} \\
&= \frac{(d + ex)^2 \text{Li}_2(c(a + bx))}{2e} + \frac{1}{2} \int (d + ex) \log(1 - ac - bcx) dx + \frac{(bd - ae) \int \log(1 - ac - bcx) dx}{2b} \\
&= \frac{(d + ex)^2 \log(1 - ac - bcx)}{4e} + \frac{(d + ex)^2 \text{Li}_2(c(a + bx))}{2e} + \frac{(bc) \int \frac{(d+ex)^2}{1-ac-bcx} dx}{4e} - \frac{(bd - ae) \text{Subst}}{4e} \\
&= -\frac{(bd - ae)x}{2b} - \frac{(bd - ae)(1 - ac - bcx) \log(1 - ac - bcx)}{2b^2c} + \frac{(d + ex)^2 \log(1 - ac - bcx)}{4e} - \frac{(bd - ae) \text{Subst}}{4e} \\
&= -\frac{(bd - ae)x}{2b} - \frac{(bcd + e - ace)x}{4bc} - \frac{(d + ex)^2}{8e} - \frac{(bcd + e - ace)^2 \log(1 - ac - bcx)}{4b^2c^2e} - \frac{(bd - ae) \text{Subst}}{4e}
\end{aligned}$$

**Mathematica [A]** time = 0.0900849, size = 161, normalized size = 0.77

$$\frac{e(-4a^2c^2 \text{PolyLog}(2, c(a + bx)) + (-6a^2c^2 - 4ac(bcx - 2) + 2b^2c^2x^2 - 2) \log(-ac - bcx + 1) - bcx(-6ac + bcx + 2))}{8b^2c^2} + \frac{d}{8b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] (e*(-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*
(-2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, c*(a + b*x)]))/(8
*b^2*c^2) + (d*(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] +
c*(a + b*x)*PolyLog[2, c*(a + b*x)])/(b*c) + (e*x^2*PolyLog[2, a*c + b*c*x
```

]/2

**Maple [A]** time = 0.049, size = 292, normalized size = 1.4

$$-\frac{\operatorname{polylog}(2, xbc + ac) a^2 e}{2b^2} + \operatorname{polylog}(2, xbc + ac) xd + \frac{\operatorname{polylog}(2, xbc + ac) ad}{b} + \frac{\operatorname{polylog}(2, xbc + ac) ex^2}{2} + \frac{3e}{8b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*polylog(2,c\*(b\*x+a)),x)

[Out]  $-1/2/b^2*\operatorname{polylog}(2,b*c*x+a*c)*a^2*e+\operatorname{polylog}(2,b*c*x+a*c)*x*d+1/b*\operatorname{polylog}(2,b*c*x+a*c)*a*d+1/2*\operatorname{polylog}(2,b*c*x+a*c)*e*x^2+3/8/b^2/c^2*e-5/4/b^2/c*a*e+1/b/c*d+1/4*e*\ln(-b*c*x-a*c+1)*x^2+1/b*\ln(-b*c*x-a*c+1)*a*d+\ln(-b*c*x-a*c+1)*x*d+3/4/b*a*e*x+7/8/b^2*a^2*e-1/2/b*\ln(-b*c*x-a*c+1)*x*a*e-1/4/b^2/c^2*e*\ln(-b*c*x-a*c+1)-1/8*e*x^2-1/4/b/c*x*e-d*x-a*d/b-1/b/c*\ln(-b*c*x-a*c+1)*d-3/4/b^2*\ln(-b*c*x-a*c+1)*a^2*e+1/b^2/c*\ln(-b*c*x-a*c+1)*a*e$

**Maxima [A]** time = 1.01942, size = 286, normalized size = 1.36

$$\frac{(2abd - a^2e)(\log(bc x + ac) \log(-bc x - ac + 1) + \operatorname{Li}_2(-bc x - ac + 1))}{2b^2} - \frac{b^2c^2ex^2 + 2(4b^2c^2d - (3abc^2 - bc)e)x - 4}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $-1/2*(2*a*b*d - a^2*e)*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \operatorname{dilog}(-b*c*x - a*c + 1))/b^2 - 1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b*c)*e)*x - 4*(b^2*c^2*e*x^2 + 2*b^2*c^2*d*x)*\operatorname{dilog}(b*c*x + a*c) - 2*(b^2*c^2*e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2*c^2*d - a*b*c^2*e)*x)*\log(-b*c*x - a*c + 1)/(b^2*c^2)$

**Fricas [A]** time = 2.49073, size = 377, normalized size = 1.8

$$\frac{b^2c^2ex^2 + 2(4b^2c^2d - (3abc^2 - bc)e)x - 4(b^2c^2ex^2 + 2b^2c^2dx + 2abc^2d - a^2c^2e)\operatorname{Li}_2(bc x + ac) - 2(b^2c^2ex^2 + 4(ab}}{8b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b*c)*e)*x - 4*(b^2*c^2*e*x^2 + 2*b^2*c^2*d*x + 2*a*b*c^2*d - a^2*c^2*e)*\operatorname{dilog}(b*c*x + a*c) - 2*(b^2*c^2*e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2*c^2*d - a*b*c^2*e)*x)*\log(-b*c*x - a*c + 1)/(b^2*c^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)\text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate((e\*x + d)\*dilog((b\*x + a)\*c), x)

### 3.140 $\int \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=60

$$x \text{PolyLog}(2, c(a + bx)) + \frac{a \text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1) \log(-ac - bcx + 1)}{bc} - x$$

[Out]  $-x - ((1 - a*c - b*c*x)*\text{Log}[1 - a*c - b*c*x])/(b*c) + (a*\text{PolyLog}[2, c*(a + b*x)])/b + x*\text{PolyLog}[2, c*(a + b*x)]$

**Rubi [A]** time = 0.0519429, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391}

$$x \text{PolyLog}(2, c(a + bx)) + \frac{a \text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1) \log(-ac - bcx + 1)}{bc} - x$$

Antiderivative was successfully verified.

[In] `Int[PolyLog[2, c*(a + b*x)], x]`

[Out]  $-x - ((1 - a*c - b*c*x)*\text{Log}[1 - a*c - b*c*x])/(b*c) + (a*\text{PolyLog}[2, c*(a + b*x)])/b + x*\text{PolyLog}[2, c*(a + b*x)]$

#### Rule 6595

`Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] := Simp[x*PolyLog[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x], x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]`

#### Rule 2444

`Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] := Int[u*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f_) + (g_.)*x)] /; FreeQ[{e, f, g}, x]]`

#### Rule 2389

`Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]`

#### Rule 2295

`Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]`

#### Rule 2421

`Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] := Int[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatchQ[u, x] && LinearMatchQ[v, x])`

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \text{Li}_2(c(a+bx)) dx &= x\text{Li}_2(c(a+bx)) - a \int \frac{\log(1-c(a+bx))}{a+bx} dx + \int \log(1-c(a+bx)) dx \\ &= x\text{Li}_2(c(a+bx)) - a \int \frac{\log(1-ac-bcx)}{a+bx} dx + \int \log(1-ac-bcx) dx \\ &= x\text{Li}_2(c(a+bx)) - \frac{a \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{b} - \frac{\text{Subst}\left(\int \log(x) dx, x, 1-ac-bcx\right)}{bc} \\ &= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\text{Li}_2(c(a+bx))}{b} + x\text{Li}_2(c(a+bx)) \end{aligned}$$

**Mathematica [A]** time = 0.0160818, size = 53, normalized size = 0.88

$$\frac{c(a+bx)\text{PolyLog}[2, c(a+bx)] - c(a+bx) + (c(a+bx) - 1)\log(1 - c(a+bx))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)], x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

**Maple [A]** time = 0.006, size = 96, normalized size = 1.6

$$\ln(-xbc - ac + 1)x + \text{polylog}(2, xbc + ac)x + \frac{\ln(-xbc - ac + 1)a}{b} + \frac{\text{polylog}(2, xbc + ac)a}{b} - x - \frac{a}{b} - \frac{\ln(-xbc - ac + 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(b*x+a)), x)
```

```
[Out] ln(-b*c*x-a*c+1)*x+polylog(2, b*c*x+a*c)*x+1/b*ln(-b*c*x-a*c+1)*a+1/b*polylo
g(2, b*c*x+a*c)*a-x-a/b-1/b/c*ln(-b*c*x-a*c+1)+1/b/c
```

**Maxima [A]** time = 1.00266, size = 122, normalized size = 2.03

$$\frac{(\log(bcx + ac)\log(-bcx - ac + 1) + \text{Li}_2(-bcx - ac + 1))a}{b} + \frac{bcx\text{Li}_2(bcx + ac) - bcx + (bcx + ac - 1)\log(-bcx - ac + 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out]  $-(\log(bc^2x + ac) \cdot \log(-bc^2x - ac + 1) + \operatorname{dilog}(-bc^2x - ac + 1)) \cdot a/b + (bc^2x \cdot \operatorname{dilog}(bc^2x + ac) - bc^2x + (bc^2x + ac - 1) \cdot \log(-bc^2x - ac + 1)) / (bc)$

**Fricas [A]** time = 2.43648, size = 126, normalized size = 2.1

$$\frac{bcx - (bcx + ac)\operatorname{Li}_2(bcx + ac) - (bcx + ac - 1)\log(-bcx - ac + 1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out]  $-(bc^2x - (bc^2x + ac) \cdot \operatorname{dilog}(bc^2x + ac) - (bc^2x + ac - 1) \cdot \log(-bc^2x - ac + 1)) / (bc)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c), x)

### 3.141 $\int \frac{\text{PolyLog}(2, c(a+bx))}{d+ex} dx$

**Optimal.** Leaf size=591

$$\frac{\text{PolyLog}\left(3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e} + \frac{\text{PolyLog}\left(3, \frac{(1-c(a+bx))(bd-ae)}{b(d+ex)}\right)}{e} - \frac{\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right) \text{PolyLog}\left(2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e} + \frac{\log\left(\frac{1-c(a+bx)}{1-c(a+bx)}\right)}{e}$$

```
[Out] ((Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*e) + (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/e - ((Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(2*e) + (Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + ((Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/e + ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e - (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/e + (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e - PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/e - PolyLog[3, 1 - c*(a + b*x)]/e - PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/e + PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/e
```

**Rubi [A]** time = 0.517702, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {6597, 2440, 2435}

$$\frac{\text{PolyLog}\left(3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e} + \frac{\text{PolyLog}\left(3, \frac{(1-c(a+bx))(bd-ae)}{b(d+ex)}\right)}{e} - \frac{\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right) \text{PolyLog}\left(2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e} + \frac{\log\left(\frac{1-c(a+bx)}{1-c(a+bx)}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x), x]
```

```
[Out] ((Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*e) + (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/e - ((Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(2*e) + (Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + ((Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/e + ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e - (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/e + (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e - PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/e - PolyLog[3, 1 - c*(a + b*x)]/e - PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/e + PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/e
```

**Rule 6597**

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d + e*x]*Log[1 - a*c - b*c*x]]/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x]
```



] && NeQ[c\*(b\*d - a\*e) + e, 0]

### Rule 2440

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))\*((f\_.) + Log[(h\_.) \*((i\_.) + (j\_.)\*(x\_.))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_.))^(r\_.), x\_Symbol] := Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*1)/1) + (e\*x)/1)^n])\*(f + g\*Log[h\*(-((j\*k - i\*1)/1) + (j\*x)/1)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

### Rule 2435

Int[(Log[(a\_) + (b\_.)\*(x\_)]\*Log[(c\_) + (d\_.)\*(x\_)])/(x\_), x\_Symbol] := Simp [Log[-((b\*x)/a)]\*Log[a + b\*x]\*Log[c + d\*x], x] + (Simp[(1\*(Log[-((b\*x)/a)] - Log[-((b\*c - a\*d)\*x]/(a\*(c + d\*x)))] + Log[(b\*c - a\*d)/(b\*(c + d\*x))])\*Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2/2, x] - Simp[(1\*(Log[-((b\*x)/a)] - Log[-((d\*x)/c]))\*(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x)]))^2/2, x] + Simp[(Log[c + d\*x] - Log[(a\*(c + d\*x))/(c\*(a + b\*x)])]\*PolyLog[2, 1 + (b\*x)/a], x] + Simp[(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x)])]\*PolyLog[2, 1 + (d\*x)/c], x] + Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))], x] - Simp[PolyLog[3, 1 + (b\*x)/a], x] - Simp[PolyLog[3, 1 + (d\*x)/c], x] + Simp[PolyLog[3, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(c(a+bx))}{d+ex} dx &= \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{b \int \frac{\log(1-ac-bcx)\log(d+ex)}{a+bx} dx}{e} \\ &= \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{\text{Subst}\left(\int \frac{\log\left(-\frac{-abc-b(1-ac)-cx}{b}\right)\log\left(-\frac{-bd+ae}{b}+\frac{ex}{b}\right)}{x} dx, x, a+bx\right)}{e} \\ &= \frac{\left(\log(c(a+bx)) + \log\left(\frac{bcd+e-ace}{bc(d+ex)}\right) - \log\left(\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right) + \log(c(a+bx))}{2e} \end{aligned}$$

**Mathematica [A]** time = 0.307802, size = 622, normalized size = 1.05

$$\frac{-\text{PolyLog}\left(3, \frac{bc(d+ex)}{e(ac+bcx-1)}\right) + \text{PolyLog}\left(3, -\frac{b(d+ex)}{(ac+bcx-1)(bd-ae)}\right) + \log\left(-\frac{b(d+ex)}{(ac+bcx-1)(bd-ae)}\right)\left(\text{PolyLog}\left(2, \frac{bc(d+ex)}{e(ac+bcx-1)}\right) - \text{PolyLog}\left(2, -\frac{b(d+ex)}{(ac+bcx-1)(bd-ae)}\right)\right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, c\*(a + b\*x)]/(d + e\*x), x]

[Out] (Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-b\*d + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*(-2\*Log[1 - a\*c - b\*c\*x] + Log[(b\*(d + e\*x))/(b\*d - a\*e)]))/2 + (-Log[c\*(a + b\*x)] + Log[(e\*(a + b\*x))/(-b\*d + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + (Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]^2\*(Log[c\*(a + b\*x)] - Log[(b\*c\*d + e - a\*c\*e)\*(a + b\*x)/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x))]) + Log[(b\*c\*d + e - a\*c\*e)/(e - a\*c\*e - b\*c\*e\*x)]))/2 + Log[d + e\*x]\*PolyLog[2, c\*(a + b\*x)] + (Log[d + e\*x] - Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))])\*PolyLog[2, 1

$$- a*c - b*c*x] + (\text{Log}[1 - a*c - b*c*x] + \text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*\text{PolyLog}[2, (b*(d + e*x))/(b*d - a*e)] + \text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(\text{PolyLog}[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - \text{PolyLog}[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] - \text{PolyLog}[3, 1 - a*c - b*c*x] - \text{PolyLog}[3, (b*(d + e*x))/(b*d - a*e)] - \text{PolyLog}[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + \text{PolyLog}[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])/e$$

**Maple [F]** time = 0.401, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, c(bx + a))}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,c\*(b\*x+a))/(e\*x+d),x)

[Out] int(polylog(2,c\*(b\*x+a))/(e\*x+d),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx + a)c)}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d),x, algorithm="maxima")

[Out] integrate(dilog((b\*x + a)\*c)/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bcx + ac)}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d),x, algorithm="fricas")

[Out] integral(dilog(b\*c\*x + a\*c)/(e\*x + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx+a)c)}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d),x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c)/(e\*x + d), x)

### 3.142 $\int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^2} dx$

**Optimal.** Leaf size=138

$$\frac{b \text{PolyLog}(2, c(a+bx))}{e(bd-ae)} - \frac{\text{PolyLog}(2, c(a+bx))}{e(d+ex)} + \frac{b \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{e(bd-ae)} + \frac{b \log(-ac-bcx+1) \log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{e(bd-ae)}$$

[Out] (b\*Log[1 - a\*c - b\*c\*x]\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e])/(e\*(b\*d - a\*e)) + (b\*PolyLog[2, c\*(a + b\*x)]/(e\*(b\*d - a\*e)) - PolyLog[2, c\*(a + b\*x)]/(e\*(d + e\*x)) + (b\*PolyLog[2, (e\*(1 - a\*c - b\*c\*x))/(b\*c\*d + e - a\*c\*e)]/(e\*(b\*d - a\*e)))

**Rubi [A]** time = 0.18397, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6598, 2418, 2393, 2391, 2394}

$$\frac{b \text{PolyLog}(2, c(a+bx))}{e(bd-ae)} - \frac{\text{PolyLog}(2, c(a+bx))}{e(d+ex)} + \frac{b \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{e(bd-ae)} + \frac{b \log(-ac-bcx+1) \log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{e(bd-ae)}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, c\*(a + b\*x)]/(d + e\*x)^2, x]

[Out] (b\*Log[1 - a\*c - b\*c\*x]\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e])/(e\*(b\*d - a\*e)) + (b\*PolyLog[2, c\*(a + b\*x)]/(e\*(b\*d - a\*e)) - PolyLog[2, c\*(a + b\*x)]/(e\*(d + e\*x)) + (b\*PolyLog[2, (e\*(1 - a\*c - b\*c\*x))/(b\*c\*d + e - a\*c\*e)]/(e\*(b\*d - a\*e)))

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)]^n)]^p, RFx, x}], Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))* (b_.)]/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\text{Li}_2(c(a+bx))}{(d+ex)^2} dx &= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b \int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)} dx}{e} \\ &= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b \int \left( \frac{b \log(1-ac-bcx)}{(bd-ae)(a+bx)} - \frac{e \log(1-ac-bcx)}{(bd-ae)(d+ex)} \right) dx}{e} \\ &= -\frac{\text{Li}_2(c(a+bx))}{e(d+ex)} + \frac{b \int \frac{\log(1-ac-bcx)}{d+ex} dx}{bd-ae} - \frac{b^2 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{e(bd-ae)} \\ &= \frac{b \log(1-ac-bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} - \frac{\text{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx, x, a+bx\right)}{e(bd-ae)} + \frac{(b^2 c)}{e(bd-ae)} \\ &= \frac{b \log(1-ac-bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} + \frac{b \text{Li}_2(c(a+bx))}{e(bd-ae)} - \frac{\text{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b \text{Subst}\left(\int \frac{\log\left(1+\frac{e}{-bcd+e-ace}\right)}{x} dx, x, a+bx\right)}{e(bd-ae)} \\ &= \frac{b \log(1-ac-bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} + \frac{b \text{Li}_2(c(a+bx))}{e(bd-ae)} - \frac{\text{Li}_2(c(a+bx))}{e(d+ex)} + \frac{b \text{Li}_2\left(\frac{e(1-ac-bcx)}{bcd+e-ace}\right)}{e(bd-ae)} \end{aligned}$$

**Mathematica [A]** time = 0.140869, size = 108, normalized size = 0.78

$$\frac{b \left( \text{PolyLog}\left(2, \frac{e(ac+bcx-1)}{ace-bcd-e}\right) + \text{PolyLog}(2, c(a+bx)) + \log(-ac-bcx+1) \log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right) \right)}{bd-ae} - \frac{\text{PolyLog}(2, c(a+bx))}{d+ex} \cdot e$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^2, x]
```

```
[Out] (-PolyLog[2, c*(a + b*x)]/(d + e*x)) + (b*(Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)] + PolyLog[2, c*(a + b*x)] + PolyLog[2, (e*(-1 + a*c + b*c*x))/(-b*c*d - e + a*c*e)]))/(b*d - a*e)/e
```

**Maple [A]** time = 0.38, size = 189, normalized size = 1.4

$$-\frac{bc \text{polylog}(2, xbc + ac)}{(bcex + bcd)e} - \frac{b}{e(ae - bd)} \text{dilog}\left(\frac{ace - bcd + (-xbc - ac + 1)e - e}{ace - bcd - e}\right) - \frac{b \ln(-xbc - ac + 1)}{e(ae - bd)} \ln\left(\frac{ace - bcd + ac}{ae - bcd - e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2, c*(b*x+a))/(e*x+d)^2, x)
```

```
[Out] -b*c/(b*c*e*x+b*c*d)/e*polylog(2, b*c*x+a*c) - b/e/(a*e-b*d)*dilog((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e)) - b/e/(a*e-b*d)*ln(-b*c*x-a*c+1)*ln((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e)) - b/e/(a*e-b*d)*dilog(-b*c*x-a*c+1)
```

---

**Maxima [A]** time = 1.03118, size = 224, normalized size = 1.62

$$\frac{(\log(bc x + ac) \log(-bc x - ac + 1) + \text{Li}_2(-bc x - ac + 1))b}{bde - ae^2} + \frac{\left(\log(-bc x - ac + 1) \log\left(\frac{bcex + (ac-1)e}{bcd - (ac-1)e} + 1\right) + \text{Li}_2\left(-\frac{bcex + (ac-1)e}{bcd - (ac-1)e}\right)\right)b}{bde - ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \text{dilog}(-b*c*x - a*c + 1))*b/(b*d*e - a*e^2) + (\log(-b*c*x - a*c + 1)*\log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + \text{dilog}(-(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b/(b*d*e - a*e^2) - \text{dilog}(b*c*x + a*c)/(e^2*x + d*e)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bc x + ac)}{e^2 x^2 + 2 dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="fricas")

[Out] integral(dilog(b\*c\*x + a\*c)/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(\frac{(bx + a)c}{ex + d}\right)}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d)^2,x, algorithm="giac")

[Out] integrate(dilog((b\*x + a)\*c)/(e\*x + d)^2, x)

$$3.143 \quad \int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^3} dx$$

**Optimal.** Leaf size=278

$$\frac{b^2 \text{PolyLog}(2, c(a+bx))}{2e(bd-ae)^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{2e(bd-ae)^2} - \frac{\text{PolyLog}(2, c(a+bx))}{2e(d+ex)^2} + \frac{b^2 c \log(-ac-bcx+1)}{2e(bd-ae)(-ace+bcd+e)} - \frac{1}{2e}$$

```
[Out] (b^2*c*Log[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(b*c*d + e - a*c*e)) - (b*Log
[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(d + e*x)) - (b^2*c*Log[d + e*x])/(2*e*
(b*d - a*e)*(b*c*d + e - a*c*e)) + (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d +
e*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2) + (b^2*PolyLog[2, c*(a + b*
x)])/(2*e*(b*d - a*e)^2) - PolyLog[2, c*(a + b*x)]/(2*e*(d + e*x)^2) + (b^2
*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2)
```

**Rubi [A]** time = 0.270028, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {6598, 2418, 2393, 2391, 2395, 36, 31, 2394}

$$\frac{b^2 \text{PolyLog}(2, c(a+bx))}{2e(bd-ae)^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{2e(bd-ae)^2} - \frac{\text{PolyLog}(2, c(a+bx))}{2e(d+ex)^2} + \frac{b^2 c \log(-ac-bcx+1)}{2e(bd-ae)(-ace+bcd+e)} - \frac{1}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^3, x]
```

```
[Out] (b^2*c*Log[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(b*c*d + e - a*c*e)) - (b*Log
[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(d + e*x)) - (b^2*c*Log[d + e*x])/(2*e*
(b*d - a*e)*(b*c*d + e - a*c*e)) + (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d +
e*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2) + (b^2*PolyLog[2, c*(a + b*
x)])/(2*e*(b*d - a*e)^2) - PolyLog[2, c*(a + b*x)]/(2*e*(d + e*x)^2) + (b^2
*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2)
```

#### Rule 6598

```
Int[((d_) + (e_)*(x_)^(m_))*PolyLog[2, (c_)*((a_) + (b_)*(x_))], x_Sy
mbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 2418

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)])*(b_)^(p_)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))])*(b_)/((f_) + (g_)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_))), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(c(a + bx))}{(d + ex)^3} dx &= -\frac{\text{Li}_2(c(a + bx))}{2e(d + ex)^2} - \frac{b \int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^2} dx}{2e} \\
 &= -\frac{\text{Li}_2(c(a + bx))}{2e(d + ex)^2} - \frac{b \int \left( \frac{b^2 \log(1-ac-bcx)}{(bd-ae)^2(a+bx)} - \frac{e \log(1-ac-bcx)}{(bd-ae)(d+ex)^2} - \frac{be \log(1-ac-bcx)}{(bd-ae)^2(d+ex)} \right) dx}{2e} \\
 &= -\frac{\text{Li}_2(c(a + bx))}{2e(d + ex)^2} + \frac{b^2 \int \frac{\log(1-ac-bcx)}{d+ex} dx}{2(bd - ae)^2} - \frac{b^3 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{2e(bd - ae)^2} + \frac{b \int \frac{\log(1-ac-bcx)}{(d+ex)^2} dx}{2(bd - ae)} \\
 &= -\frac{b \log(1 - ac - bcx)}{2e(bd - ae)(d + ex)} + \frac{b^2 \log(1 - ac - bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd - ae)^2} - \frac{\text{Li}_2(c(a + bx))}{2e(d + ex)^2} - \frac{b^2 \text{Subst}\left(\int \frac{\log(1-cx)}{x} dx\right)}{2e(bd - ae)} \\
 &= -\frac{b \log(1 - ac - bcx)}{2e(bd - ae)(d + ex)} + \frac{b^2 \log(1 - ac - bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd - ae)^2} + \frac{b^2 \text{Li}_2(c(a + bx))}{2e(bd - ae)^2} - \frac{\text{Li}_2(c(a + bx))}{2e(d + ex)^2} \\
 &= \frac{b^2 c \log(1 - ac - bcx)}{2e(bd - ae)(bcd + e - ace)} - \frac{b \log(1 - ac - bcx)}{2e(bd - ae)(d + ex)} - \frac{b^2 c \log(d + ex)}{2e(bd - ae)(bcd + e - ace)} + \frac{b^2 \log(1 - ac - bcx)}{2e(bd - ae)}
 \end{aligned}$$

**Mathematica [A]** time = 0.420508, size = 190, normalized size = 0.68

$$\frac{b \left( b \text{PolyLog}\left(2, \frac{e(ac+bcx-1)}{e(ac-1)-bcd}\right) + b \text{PolyLog}(2, c(a+bx)) + b \log(-ac-bcx+1) \log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right) - \frac{(bd-ae) \log(-ac-bcx+1)}{d+ex} + \frac{bc(bd-ae)(\log(-ac-bcx+1)-\log(d+ex))}{-ace+bcd+e} \right)}{(bd-ae)^2} - \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^2}$$

Antiderivative was successfully verified.



[In] Integrate[PolyLog[2, c\*(a + b\*x)]/(d + e\*x)^3, x]

[Out] 
$$\begin{aligned} & -(\text{PolyLog}[2, c*(a + b*x)]/(d + e*x)^2) + (b*(-((b*d - a*e)*\text{Log}[1 - a*c - \\ & b*c*x])/(d + e*x)) + (b*c*(b*d - a*e)*(\text{Log}[1 - a*c - b*c*x] - \text{Log}[d + e*x])) \\ & )/(b*c*d + e - a*c*e) + b*\text{Log}[1 - a*c - b*c*x]*\text{Log}[(b*c*(d + e*x))/(b*c*d + \\ & e - a*c*e)] + b*\text{PolyLog}[2, c*(a + b*x)] + b*\text{PolyLog}[2, (e*(-1 + a*c + b*c* \\ & x))/(-b*c*d + (-1 + a*c)*e))]/(b*d - a*e)^2/(2*e) \end{aligned}$$

**Maple [A]** time = 0.462, size = 437, normalized size = 1.6

$$-\frac{b^2 c^2 \text{polylog}(2, xbc + ac)}{2 (bcex + bcd)^2 e} + \frac{b^2}{2 e (ae - bd)^2} \text{dilog}\left(\frac{ace - bcd + (-xbc - ac + 1)e - e}{ace - bcd - e}\right) + \frac{b^2 \ln(-xbc - ac + 1)}{2 e (ae - bd)^2} \ln\left(\frac{ace - bcd + (-xbc - ac + 1)e - e}{ace - bcd - e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, c\*(b\*x+a))/(e\*x+d)^3, x)

[Out] 
$$\begin{aligned} & -1/2*b^2*c^2/(b*c*e*x+b*c*d)^2/e*\text{polylog}(2, b*c*x+a*c)+1/2*b^2/e/(a*e-b*d)^2 \\ & * \text{dilog}((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))+1/2*b^2/e/(a*e-b*d \\ & )^2*\ln(-b*c*x-a*c+1)*\ln((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))-1 \\ & /2*b^2*c/e/(a*e-b*d)/(a*c*e-b*c*d-e)*\ln(a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)-1/2 \\ & *b^3*c^2/(a*e-b*d)*\ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*x-1/2* \\ & b^2*c^2/(a*e-b*d)*\ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*a+1/2*b \\ & ^2*c/(a*e-b*d)*\ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)+1/2*b^2/e/ \\ & (a*e-b*d)^2*\text{dilog}(-b*c*x-a*c+1) \end{aligned}$$

**Maxima [A]** time = 1.00782, size = 512, normalized size = 1.84

$$\frac{b^2 c \log(bc x + ac - 1)}{2 (b^2 c d^2 e - (2 abc - b) d e^2 + (a^2 c - a) e^3)} - \frac{b^2 c \log(ex + d)}{2 (b^2 c d^2 e - (2 abc - b) d e^2 + (a^2 c - a) e^3)} - \frac{(\log(bc x + ac) \log(-bc x - ac + 1))}{2 (b^2 d^2 e - 2 abcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2, c\*(b\*x+a))/(e\*x+d)^3, x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 1/2*b^2*c*\log(b*c*x + a*c - 1)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (a^2*c \\ & - a)*e^3) - 1/2*b^2*c*\log(e*x + d)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (a^2*c \\ & - a)*e^3) - 1/2*(\log(b*c*x + a*c)*\log(-b*c*x - a*c + 1) + \text{dilog}(-b*c*x \\ & - a*c + 1))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/2*(\log(-b*c*x - a*c \\ & + 1)*\log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + \text{dilog}(-b*c* \\ & e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a \\ & ^2*e^3) - 1/2*((b*d - a*e)*\text{dilog}(b*c*x + a*c) + (b*e*x + b*d)*\log(-b*c*x - \\ & a*c + 1))/(b*d^3*e - a*d^2*e^2 + (b*d*e^3 - a*e^4)*x^2 + 2*(b*d^2*e^2 - a*d \\ & *e^3)*x) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(bc x + ac)}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(\frac{(bx+a)c}{ex+d}\right)}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^3, x)
```

$$3.144 \quad \int \frac{\text{PolyLog}(2, c(a+bx))}{(d+ex)^4} dx$$

**Optimal.** Leaf size=448

$$\frac{b^3 \text{PolyLog}(2, c(a+bx))}{3e(bd-ae)^3} + \frac{b^3 \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{3e(bd-ae)^3} - \frac{\text{PolyLog}(2, c(a+bx))}{3e(d+ex)^3} + \frac{b^3 c^2 \log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2} - \frac{b^3 c^2 \log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2}$$

```
[Out] (b^2*c)/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)*(d + e*x)) + (b^3*c^2*Log[1 - a*c - b*c*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^3*c*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) - (b*Log[1 - a*c - b*c*x])/(6*e*(b*d - a*e)*(d + e*x)^2) - (b^2*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*e)^2*(d + e*x)) - (b^3*c^2*Log[d + e*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) - (b^3*c*Log[d + e*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) + (b^3*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d - a*e)^3) + (b^3*PolyLog[2, c*(a + b*x)])/(3*e*(b*d - a*e)^3) - PolyLog[2, c*(a + b*x)]/(3*e*(d + e*x)^3) + (b^3*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d - a*e)^3)
```

**Rubi [A]** time = 0.426017, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$ , Rules used = {6598, 2418, 2393, 2391, 2395, 44, 36, 31, 2394}

$$\frac{b^3 \text{PolyLog}(2, c(a+bx))}{3e(bd-ae)^3} + \frac{b^3 \text{PolyLog}\left(2, \frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{3e(bd-ae)^3} - \frac{\text{PolyLog}(2, c(a+bx))}{3e(d+ex)^3} + \frac{b^3 c^2 \log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2} - \frac{b^3 c^2 \log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^4, x]
```

```
[Out] (b^2*c)/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)*(d + e*x)) + (b^3*c^2*Log[1 - a*c - b*c*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^3*c*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) - (b*Log[1 - a*c - b*c*x])/(6*e*(b*d - a*e)*(d + e*x)^2) - (b^2*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*e)^2*(d + e*x)) - (b^3*c^2*Log[d + e*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) - (b^3*c*Log[d + e*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) + (b^3*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d - a*e)^3) + (b^3*PolyLog[2, c*(a + b*x)])/(3*e*(b*d - a*e)^3) - PolyLog[2, c*(a + b*x)]/(3*e*(d + e*x)^3) + (b^3*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d - a*e)^3)
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol]
:= Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)]/(e*(m + 1)), x] + Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Symbol]
:= With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(c(a+bx))}{(d+ex)^4} dx &= -\frac{\text{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b \int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^3} dx}{3e} \\
&= -\frac{\text{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b \int \left( \frac{b^3 \log(1-ac-bcx)}{(bd-ae)^3(a+bx)} - \frac{e \log(1-ac-bcx)}{(bd-ae)(d+ex)^3} - \frac{be \log(1-ac-bcx)}{(bd-ae)^2(d+ex)^2} - \frac{b^2 e \log(1-ac-bcx)}{(bd-ae)^3(d+ex)} \right) dx}{3e} \\
&= -\frac{\text{Li}_2(c(a+bx))}{3e(d+ex)^3} + \frac{b^3 \int \frac{\log(1-ac-bcx)}{d+ex} dx}{3(bd-ae)^3} - \frac{b^4 \int \frac{\log(1-ac-bcx)}{a+bx} dx}{3e(bd-ae)^3} + \frac{b^2 \int \frac{\log(1-ac-bcx)}{(d+ex)^2} dx}{3(bd-ae)^2} + \frac{b \int \frac{\log(1-ac-bcx)}{d+ex} dx}{3e} \\
&= -\frac{b \log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2 \log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3 \log(1-ac-bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{3e(bd-ae)^3} - \frac{\text{Li}_2(c(a+bx))}{3e(d+ex)^3} \\
&= -\frac{b \log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2 \log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3 \log(1-ac-bcx) \log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{3e(bd-ae)^3} + \frac{b^3 \text{Li}_2(c(a+bx))}{3e(bd-ae)^3} \\
&= \frac{b^2 c}{6e(bd-ae)(bcd+e-ace)(d+ex)} + \frac{b^3 c^2 \log(1-ac-bcx)}{6e(bd-ae)(bcd+e-ace)^2} + \frac{b^3 c \log(1-ac-bcx)}{3e(bd-ae)^2(bcd+e-ace)}
\end{aligned}$$

**Mathematica [A]** time = 0.631829, size = 313, normalized size = 0.7

$$\frac{b \left( 2b^2 \text{PolyLog}\left(2, \frac{e(ac+bcx-1)}{e(ac-1)-bcd}\right) + 2b^2 \text{PolyLog}(2, c(a+bx)) + \frac{2b^2 c(bd-ae)(\log(-ac-bcx+1)-\log(d+ex))}{-ace+bcd+e} + 2b^2 \log(-ac-bcx+1) \log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right) - \frac{2b(bd-ae) \log(-ac-bcx+1)}{d+ex} \right)}{(bd-ae)^3}$$

6e

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, c\*(a + b\*x)]/(d + e\*x)^4, x]

[Out]  $\left( (-2 \text{PolyLog}[2, c(a + b*x)]) / (d + e*x)^3 + (b * (-((b*d - a*e)^2 * \text{Log}[1 - a*c - b*c*x]) / (d + e*x)^2 - (2*b*(b*d - a*e) * \text{Log}[1 - a*c - b*c*x]) / (d + e*x) + (2*b^2*c*(b*d - a*e) * (\text{Log}[1 - a*c - b*c*x] - \text{Log}[d + e*x])) / (b*c*d + e - a*c*e) + (b*c*(b*d - a*e)^2 * (b*c*d + e - a*c*e + b*c*(d + e*x) * \text{Log}[1 - a*c - b*c*x] - b*c*(d + e*x) * \text{Log}[d + e*x])) / ((b*c*d + e - a*c*e)^2 * (d + e*x)) + 2*b^2 * \text{Log}[1 - a*c - b*c*x] * \text{Log}[(b*c*(d + e*x)) / (b*c*d + e - a*c*e)] + 2*b^2 * \text{PolyLog}[2, c(a + b*x)] + 2*b^2 * \text{PolyLog}[2, (e*(-1 + a*c + b*c*x)) / (-b*c*d + (-1 + a*c)*e)]) / (b*d - a*e)^3 \right) / (6*e)$

**Maple [B]** time = 0.394, size = 1075, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2, c\*(b\*x+a))/(e\*x+d)^4, x)

[Out]  $-1/3*b^3*c^3/(b*c*e*x+b*c*d)^3/e*\text{polylog}(2, b*c*x+a*c) - 1/3*b^3/e/(a*e-b*d)^3 * \text{dilog}((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e)) - 1/3*b^3/e/(a*e-b*d)^3 * \ln(-b*c*x-a*c+1) * \ln((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e)) + 1/3*b^3*c/e/(a*e-b*d)^2/(a*c*e-b*c*d-e) * \ln(a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e) + 1/3*b^4*c^2/(a*e-b*d)^2 * \ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*x + 1/3*b^3*c^2/(a*e-b*d)^2 * \ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*a - 1/3*b^3*c/(a*e-b*d)^2 * \ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d) - 1/$

$$3b^3/e/(a^2e-b^2d)^3 \operatorname{dilog}(-b^2cx-a^2c+1) + 1/6b^3c^2/e/(a^2e-b^2d)/(a^2ce-b^2cd-e)^2 \ln(a^2ce-b^2cd+(-b^2cx-a^2c+1)e-e) - 1/6b^3c^3/(a^2e-b^2d)/(a^2ce-b^2cd-e)^2 / (-b^2ce^2x-b^2cd) * a + 1/6b^4c^3/e/(a^2e-b^2d)/(a^2ce-b^2cd-e)^2 / (-b^2ce^2x-b^2cd) * d + 1/6b^3c^2/(a^2e-b^2d)/(a^2ce-b^2cd-e)^2 / (-b^2ce^2x-b^2cd) - 1/6b^5c^4e/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * x^2 - 1/3b^5c^4/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * x * d + 1/6b^3c^4e/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * a^2 - 1/3b^4c^4/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * d * a - 1/3b^3c^3e/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * a + 1/3b^4c^3/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2 * d + 1/6b^3c^2e/(a^2e-b^2d) * \ln(-b^2cx-a^2c+1) / (-b^2ce^2x-b^2cd)^2 / (a^2ce-b^2cd-e)^2$$

**Maxima [B]** time = 1.22441, size = 1928, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,c\*(b\*x+a))/(e\*x+d)^4,x, algorithm="maxima")

[Out] 
$$-1/3 * (\log(b^2cx + a^2c) * \log(-b^2cx - a^2c + 1) + \operatorname{dilog}(-b^2cx - a^2c + 1)) * b^3 / (b^3d^3e - 3a^2b^2d^2e^2 + 3a^2b^2d^2e^3 - a^3e^4) + 1/3 * (\log(-b^2cx - a^2c + 1) * \log((b^2ce^2x + (a^2c - 1)e) / (b^2cd - (a^2c - 1)e)) + 1) + \operatorname{dilog}((b^2ce^2x + (a^2c - 1)e) / (b^2cd - (a^2c - 1)e))) * b^3 / (b^3d^3e - 3a^2b^2d^2e^2 + 3a^2b^2d^2e^3 - a^3e^4) - 1/6 * (3b^4c^2d - (3a^2b^3c^2 - 2b^3c) * e) * \log(e^2x + d) / (b^4c^2d^4e - 2 * (2a^2b^3c^2 - b^3c) * d^3e^2 + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^2e^3 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d * e^4 + (a^4c^2 - 2a^3c + a^2) * e^5) + 1/6 * (b^4c^2d^4 - (2a^2b^3c^2 - b^3c) * d^3e + (a^2b^2c^2 - a^2b^2c) * d^2e^2 + (b^4c^2d^2e^2 - (2a^2b^3c^2 - b^3c) * d * e^3 + (a^2b^2c^2 - a^2b^2c) * e^4)) * x^2 + 2 * (b^4c^2d^3e - (2a^2b^3c^2 - b^3c) * d^2e^2 + (a^2b^2c^2 - a^2b^2c) * d * e^3) * x - 2 * (b^4c^2d^4 - 2 * (2a^2b^3c^2 - b^3c) * d^3e + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^2e^2 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d * e^3 + (a^4c^2 - 2a^3c + a^2) * e^4) * \operatorname{dilog}(b^2cx + a^2c) + (4 * (a^2b^3c^2 - b^3c) * d^3e - (5a^2b^2c^2 - 8a^2b^2c + 3b^2) * d^2e^2 + (a^3b^2c^2 - 2a^2b^2c + a^2b) * d * e^3 + (3b^4c^2d^2e^3 - (3a^2b^3c^2 - 2b^3c) * e^4)) * x^3 + (7b^4c^2d^2e^2 - (5a^2b^3c^2 - 2b^3c) * d * e^3 - 2 * (a^2b^2c^2 - 2a^2b^2c + b^2) * e^4) * x^2 + (4b^4c^2d^3e + 2 * (a^2b^3c^2 - 2b^3c) * d^2e^2 - (7a^2b^2c^2 - 12a^2b^2c + 5b^2) * d * e^3 + (a^3b^2c^2 - 2a^2b^2c + a^2b) * e^4) * x) * \log(-b^2cx - a^2c + 1) / (b^4c^2d^7e - 2 * (2a^2b^3c^2 - b^3c) * d^6e^2 + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^5e^3 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d^4e^4 + (a^4c^2 - 2a^3c + a^2) * d^3e^5 + (b^4c^2d^4e^4 - 2 * (2a^2b^3c^2 - b^3c) * d^3e^5 + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^2e^6 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d * e^7 + (a^4c^2 - 2a^3c + a^2) * e^8)) * x^3 + 3 * (b^4c^2d^5e^3 - 2 * (2a^2b^3c^2 - b^3c) * d^4e^4 + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^3e^5 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d^2e^6 + (a^4c^2 - 2a^3c + a^2) * d * e^7) * x^2 + 3 * (b^4c^2d^6e^2 - 2 * (2a^2b^3c^2 - b^3c) * d^5e^3 + (6a^2b^2c^2 - 6a^2b^2c + b^2) * d^4e^4 - 2 * (2a^3b^2c^2 - 3a^2b^2c + a^2b) * d^3e^5 + (a^4c^2 - 2a^3c + a^2) * d^2e^6) * x)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(bcx + ac)}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(\frac{(bx+a)c}{ex+d}\right)}{(ex+d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^4, x)
```

### 3.145 $\int \frac{\text{PolyLog}(2,x)}{-1+x} dx$

**Optimal.** Leaf size=46

$-2\text{PolyLog}(3,1-x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

**Rubi [A]** time = 0.0657938, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$ , Rules used = {6596, 2396, 2433, 2374, 6589}

$-2\text{PolyLog}(3,1-x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, x]/(-1 + x), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d



, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(x)}{-1+x} dx &= \log(1-x)\text{Li}_2(x) + \int \frac{\log^2(1-x)}{x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) + 2 \int \frac{\log(1-x)\log(x)}{1-x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - 2 \text{Subst} \left( \int \frac{\log(1-x)\log(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2 \log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2 \text{Subst} \left( \int \frac{\text{Li}_2(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2 \log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2\text{Li}_3(1-x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0385298, size = 46, normalized size = 1.

$$-2\text{PolyLog}(3, 1-x) + 2\log(1-x)\text{PolyLog}(2, 1-x) + \log(1-x)\text{PolyLog}(2, x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, x]/(-1 + x), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, x)}{-1+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,x)/(-1+x), x)

[Out] int(polylog(2,x)/(-1+x), x)

**Maxima [A]** time = 1.0224, size = 59, normalized size = 1.28

$$\log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\text{Li}_2(-x+1)\log(-x+1) - 2\text{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x), x, algorithm="maxima")

[Out] log(x)\*log(-x + 1)^2 + dilog(x)\*log(-x + 1) + 2\*dilog(-x + 1)\*log(-x + 1) - 2\*polylog(3, -x + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x),x, algorithm="fricas")

[Out] integral(dilog(x)/(x - 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x),x)

[Out] Integral(polylog(2, x)/(x - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x),x, algorithm="giac")

[Out] integrate(dilog(x)/(x - 1), x)

$$3.146 \quad \int -\frac{\text{PolyLog}(2,x)}{1-x} dx$$

**Optimal.** Leaf size=46

$$-2\text{PolyLog}(3,1-x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

**Rubi [A]** time = 0.0674004, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Int[-(PolyLog[2, x]/(1 - x)), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d

, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int -\frac{\text{Li}_2(x)}{1-x} dx &= \log(1-x)\text{Li}_2(x) + \int \frac{\log^2(1-x)}{x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) + 2 \int \frac{\log(1-x)\log(x)}{1-x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - 2 \text{Subst} \left( \int \frac{\log(1-x)\log(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2\log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2 \text{Subst} \left( \int \frac{\text{Li}_2(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2\log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2\text{Li}_3(1-x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0065388, size = 46, normalized size = 1.

$$-2\text{PolyLog}(3, 1-x) + 2\log(1-x)\text{PolyLog}(2, 1-x) + \log(1-x)\text{PolyLog}(2, x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[-(PolyLog[2, x]/(1 - x)), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x]

**Maple [F]** time = 0.281, size = 0, normalized size = 0.

$$\int -\frac{\text{polylog}(2, x)}{1-x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-polylog(2,x)/(1-x), x)

[Out] int(-polylog(2,x)/(1-x), x)

**Maxima [A]** time = 0.957218, size = 59, normalized size = 1.28

$$\log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\text{Li}_2(-x+1)\log(-x+1) - 2\text{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x), x, algorithm="maxima")

[Out] log(x)\*log(-x + 1)^2 + dilog(x)\*log(-x + 1) + 2\*dilog(-x + 1)\*log(-x + 1) - 2\*polylog(3, -x + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x),x, algorithm="fricas")

[Out] integral(dilog(x)/(x - 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x),x)

[Out] Integral(polylog(2, x)/(x - 1), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x),x, algorithm="giac")

[Out] integrate(dilog(x)/(x - 1), x)

$$3.147 \quad \int \frac{\text{PolyLog}(2,x)}{(-1+x)x} dx$$

**Optimal.** Leaf size=51

$$-2\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x] - PolyLog[3, x]

**Rubi [A]** time = 0.13268, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6742, 6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, x]/((-1 + x)\*x), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x] - PolyLog[3, x]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*1, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_))^(m\_.)]\*(a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

&& EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{\text{Li}_2(x)}{(-1+x)x} dx &= \int \left( \frac{\text{Li}_2(x)}{-1+x} - \frac{\text{Li}_2(x)}{x} \right) dx \\
 &= \int \frac{\text{Li}_2(x)}{-1+x} dx - \int \frac{\text{Li}_2(x)}{x} dx \\
 &= \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) + \int \frac{\log^2(1-x)}{x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) + 2 \int \frac{\log(1-x)\log(x)}{1-x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) - 2 \text{Subst} \left( \int \frac{\log(1-x)\log(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2 \log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) - 2 \text{Subst} \left( \int \frac{\text{Li}_2(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2 \log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2\text{Li}_3(1-x) - \text{Li}_3(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0387094, size = 51, normalized size = 1.

$$-2\text{PolyLog}(3, 1-x) - \text{PolyLog}(3, x) + 2 \log(1-x)\text{PolyLog}(2, 1-x) + \log(1-x)\text{PolyLog}(2, x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, x]/((-1 + x)\*x), x]

[Out] Log[1 - x]^2\*Log[x] + 2\*Log[1 - x]\*PolyLog[2, 1 - x] + Log[1 - x]\*PolyLog[2, x] - 2\*PolyLog[3, 1 - x] - PolyLog[3, x]

**Maple [F]** time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, x)}{(-1+x)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,x)/(-1+x)/x,x)

[Out] int(polylog(2,x)/(-1+x)/x,x)

**Maxima [A]** time = 0.965157, size = 66, normalized size = 1.29

$$\log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\text{Li}_2(-x+1)\log(-x+1) - \text{Li}_3(x) - 2\text{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="maxima")

[Out] log(x)\*log(-x + 1)^2 + dilog(x)\*log(-x + 1) + 2\*dilog(-x + 1)\*log(-x + 1) - polylog(3, x) - 2\*polylog(3, -x + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(x)}{x^2 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="fricas")

[Out] integral(dilog(x)/(x^2 - x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x(x-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x)/x,x)

[Out] Integral(polylog(2, x)/(x\*(x - 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{(x-1)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="giac")

[Out] integrate(dilog(x)/((x - 1)\*x), x)



$$3.148 \quad \int -\frac{\text{PolyLog}(2,x)}{(1-x)x} dx$$

**Optimal.** Leaf size=51

$$-2\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

**Rubi [A]** time = 0.134478, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6742, 6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x) - \text{PolyLog}(3,x) + 2\log(1-x)\text{PolyLog}(2,1-x) + \log(1-x)\text{PolyLog}(2,x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[-(PolyLog[2, x]/((1 - x)*x)), x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

#### Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] :=> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*(b*d - a*e) + e, 0]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.)*(x_)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Symbol] :=> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] :=> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

&& EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int -\frac{\text{Li}_2(x)}{(1-x)x} dx &= -\int \left( -\frac{\text{Li}_2(x)}{-1+x} + \frac{\text{Li}_2(x)}{x} \right) dx \\
 &= \int \frac{\text{Li}_2(x)}{-1+x} dx - \int \frac{\text{Li}_2(x)}{x} dx \\
 &= \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) + \int \frac{\log^2(1-x)}{x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) + 2 \int \frac{\log(1-x)\log(x)}{1-x} dx \\
 &= \log^2(1-x)\log(x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) - 2 \text{Subst} \left( \int \frac{\log(1-x)\log(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2\log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - \text{Li}_3(x) - 2 \text{Subst} \left( \int \frac{\text{Li}_2(x)}{x} dx, x, 1-x \right) \\
 &= \log^2(1-x)\log(x) + 2\log(1-x)\text{Li}_2(1-x) + \log(1-x)\text{Li}_2(x) - 2\text{Li}_3(1-x) - \text{Li}_3(x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0149876, size = 51, normalized size = 1.

$$-2\text{PolyLog}(3, 1-x) - \text{PolyLog}(3, x) + 2\log(1-x)\text{PolyLog}(2, 1-x) + \log(1-x)\text{PolyLog}(2, x) + \log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

[In] Integrate[-(PolyLog[2, x]/((1-x)\*x)), x]

[Out] Log[1-x]^2\*Log[x] + 2\*Log[1-x]\*PolyLog[2, 1-x] + Log[1-x]\*PolyLog[2, x] - 2\*PolyLog[3, 1-x] - PolyLog[3, x]

**Maple [F]** time = 0.282, size = 0, normalized size = 0.

$$\int -\frac{\text{polylog}(2, x)}{(1-x)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-polylog(2,x)/(1-x)/x,x)

[Out] int(-polylog(2,x)/(1-x)/x,x)

**Maxima [A]** time = 0.977682, size = 66, normalized size = 1.29

$$\log(x)\log(-x+1)^2 + \text{Li}_2(x)\log(-x+1) + 2\text{Li}_2(-x+1)\log(-x+1) - \text{Li}_3(x) - 2\text{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="maxima")

[Out] log(x)\*log(-x + 1)^2 + dilog(x)\*log(-x + 1) + 2\*dilog(-x + 1)\*log(-x + 1) - polylog(3, x) - 2\*polylog(3, -x + 1)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(x)}{x^2 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="fricas")

[Out] integral(dilog(x)/(x^2 - x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x(x-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x)/x,x)

[Out] Integral(polylog(2, x)/(x\*(x - 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{(x-1)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="giac")

[Out] integrate(dilog(x)/((x - 1)\*x), x)

$$3.149 \quad \int \frac{\text{PolyLog}\left(n, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=35

$$\frac{\text{PolyLog}\left(n+1, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

[Out] PolyLog[1 + n, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

**Rubi [A]** time = 0.071321, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {6610}

$$\frac{\text{PolyLog}\left(n+1, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[n, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[1 + n, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

#### Rubi steps

$$\int \frac{\text{Li}_n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx = \frac{\text{Li}_{1+n}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

**Mathematica [A]** time = 0.0198299, size = 34, normalized size = 0.97

$$\frac{\text{PolyLog}\left(n+1, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[n, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[1 + n, e\*((a + b\*x)/(c + d\*x))^n]/(b\*c\*n - a\*d\*n)

**Maple [F]** time = 0.891, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \text{polylog}\left(n, e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)`

[Out] `int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxima")`

[Out] `integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{polylog}\left(n, e\left(\frac{bx+a}{dx+c}\right)^n\right)}{bdx^2 + ac + (bc + ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fricas")`

[Out] `integral(polylog(n, e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,e*((b*x+a)/(d*x+c)**n)/(b*x+a)/(d*x+c),x)`

[Out] `Integral(polylog(n, e*(a/(c + d*x) + b*x/(c + d*x)**n)/((a + b*x)*(c + d*x))), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi  
ac")
```

```
[Out] integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

$$3.150 \quad \int \frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=33

$$\frac{\text{PolyLog}\left(4, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

[Out] PolyLog[4, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

**Rubi [A]** time = 0.0611184, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {6610}

$$\frac{\text{PolyLog}\left(4, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[4, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

**Rule 6610**

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :=> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

**Rubi steps**

$$\int \frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx = \frac{\text{Li}_4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

**Mathematica [A]** time = 0.007177, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(4, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[3, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[4, e\*((a + b\*x)/(c + d\*x))^n]/(b\*c\*n - a\*d\*n)

**Maple [F]** time = 0.703, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \text{polylog}\left(3, e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(3,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x)

[Out] int(polylog(3,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3 \left( n \log (bx + a)^2 - 2 n \log (bx + a) \log (dx + c) + n \log (dx + c)^2 \right) \text{Li}_2 \left( e e^{(n \log (bx + a) - n \log (dx + c))} \right) + \left( n^2 \log (bx + a)^3 - 3 n^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out]  $-1/6*(3*(n*\log(b*x + a)^2 - 2*n*\log(b*x + a)*\log(d*x + c) + n*\log(d*x + c)^2)*\text{dilog}(e*e^{(n*\log(b*x + a) - n*\log(d*x + c))}) + (n^2*\log(b*x + a)^3 - 3*n^2*\log(b*x + a)^2*\log(d*x + c) + 3*n^2*\log(b*x + a)*\log(d*x + c)^2 - n^2*\log(d*x + c)^3)*\log(-(b*x + a)^n*e + (d*x + c)^n) - (n^2*\log(b*x + a)^3 - 3*n^2*\log(b*x + a)^2*\log(d*x + c) + 3*n^2*\log(b*x + a)*\log(d*x + c)^2 - n^2*\log(d*x + c)^3)*\log((d*x + c)^n) - 6*(\log(b*x + a) - \log(d*x + c))*\text{polylog}(3, e*e^{(n*\log(b*x + a) - n*\log(d*x + c))})/(b*c - a*d) + \text{integrate}(1/6*(e*n^3*\log(b*x + a)^3 - 3*e*n^3*\log(b*x + a)^2*\log(d*x + c) + 3*e*n^3*\log(b*x + a)*\log(d*x + c)^2 - e*n^3*\log(d*x + c)^3)*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c*e + a*d*e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)$

**Fricas [C]** time = 3.45558, size = 782, normalized size = 23.7

$$n^2\%iint \left( a, b, c, d, e, n, x, -\frac{n \log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right)}{bx+a}, -\frac{nx \log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right)}{bx+a}, \frac{n \log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right)}{dx+c}, \frac{nx \log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right)}{dx+c}, -\frac{\log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right)}{e}, -\log \left( -e \left( \frac{bx+a}{dx+c} \right)^n + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out]  $-1/2*(n^2*\%iint(a, b, c, d, e, n, x, -n*\log(-e*((b*x + a)/(d*x + c))^n + 1)/(b*x + a), -n*x*\log(-e*((b*x + a)/(d*x + c))^n + 1)/(b*x + a), n*\log(-e*((b*x + a)/(d*x + c))^n + 1)/(d*x + c), n*x*\log(-e*((b*x + a)/(d*x + c))^n + 1)/(d*x + c), -\log(-e*((b*x + a)/(d*x + c))^n + 1)/e, -\log(-e*((b*x + a)/(d*x + c))^n + 1)*\log((b*x + a)/(d*x + c)), -(b*c - a*d)*n*\log(-e*((b*x + a)/(d*x + c))^n + 1)/(b*d*x^2 + a*c + (b*c + a*d)*x))*\log((b*x + a)/(d*x + c))^2 - n^2*\text{dilog}(e*((b*x + a)/(d*x + c))^n)*\log((b*x + a)/(d*x + c))^2 - 2*\text{polylog}(4, e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,e\*((b\*x+a)/(d\*x+c))\*\*n)/(b\*x+a)/(d\*x+c),x)

[Out] Integral(polylog(3, e\*(a/(c + d\*x) + b\*x/(c + d\*x))\*\*n)/((a + b\*x)\*(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(3,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] integrate(polylog(3, e\*((b\*x + a)/(d\*x + c))^n)/((b\*x + a)\*(d\*x + c)), x)

$$3.151 \quad \int \frac{\text{PolyLog}\left(2, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=33

$$\frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

[Out] PolyLog[3, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

**Rubi [A]** time = 0.0612712, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {6610}

$$\frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[3, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

#### Rubi steps

$$\int \frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx = \frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

**Mathematica [A]** time = 0.006995, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(3, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

[Out] PolyLog[3, e\*((a + b\*x)/(c + d\*x))^n]/(b\*c\*n - a\*d\*n)

**Maple [F]** time = 0.805, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)} \text{polylog}\left(2, e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x)

[Out] int(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2(\log(bx+a) - \log(dx+c))\text{Li}_2\left(e^{(n\log(bx+a) - n\log(dx+c))}\right) + (n\log(bx+a)^2 - 2n\log(bx+a)\log(dx+c) + n\log(dx+c)^2)}{2(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out] 1/2\*(2\*(log(b\*x + a) - log(d\*x + c))\*dilog(e\*e^(n\*log(b\*x + a) - n\*log(d\*x + c))) + (n\*log(b\*x + a)^2 - 2\*n\*log(b\*x + a)\*log(d\*x + c) + n\*log(d\*x + c)^2)\*log(-(b\*x + a)^n\*e + (d\*x + c)^n) - (n\*log(b\*x + a)^2 - 2\*n\*log(b\*x + a)\*log(d\*x + c) + n\*log(d\*x + c)^2)\*log((d\*x + c)^n))/(b\*c - a\*d) + integrate(-1/2\*(e\*n^2\*log(b\*x + a)^2 - 2\*e\*n^2\*log(b\*x + a)\*log(d\*x + c) + e\*n^2\*log(d\*x + c)^2)\*(b\*x + a)^n/((b\*d\*e\*x^2 + a\*c\*e + (b\*c + a\*d)\*e\*x)\*(b\*x + a)^n - (b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)\*(d\*x + c)^n), x)

**Fricas [A]** time = 2.7098, size = 74, normalized size = 2.24

$$\frac{\text{polylog}\left(3, e^{\left(\frac{bx+a}{dx+c}\right)^n}\right)}{(bc - ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] polylog(3, e\*((b\*x + a)/(d\*x + c))^n)/((b\*c - a\*d)\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(2,e\*((b\*x+a)/(d\*x+c))^n)/(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] integrate(dilog(e\*((b\*x + a)/(d\*x + c))^n)/((b\*x + a)\*(d\*x + c)), x)

$$3.152 \quad \int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

**Optimal.** Leaf size=33

$$\frac{\text{PolyLog}\left(2, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

[Out] PolyLog[2, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

**Rubi [A]** time = 0.0581217, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$ , Rules used = {2518}

$$\frac{\text{PolyLog}\left(2, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[-(Log[1 - e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x))), x]

[Out] PolyLog[2, e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n)

#### Rule 2518

Int[Log[v\_]\*(u\_), x\_Symbol] :> With[{w = DerivativeDivides[v, u\*(1 - v), x]}, Simp[w\*PolyLog[2, 1 - v], x] /; !FalseQ[w]]

#### Rubi steps

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx = \frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

**Mathematica [F]** time = 1.82302, size = 40, normalized size = 1.21

$$-\int \frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

Antiderivative was successfully verified.

[In] Integrate[-(Log[1 - e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x))), x]

[Out] -Integrate[Log[1 - e\*((a + b\*x)/(c + d\*x))^n]/((a + b\*x)\*(c + d\*x)), x]

**Maple [F]** time = 0.702, size = 0, normalized size = 0.

$$\int -\frac{1}{(bx+a)(dx+c)} \ln\left(1-e\left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)`

[Out] `int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(\log(bx + a) - \log(dx + c)) \log\left(-\frac{(bx + a)^n e + (dx + c)^n}{bc - ad}\right) - (\log(bx + a) - \log(dx + c)) \log\left(\frac{(dx + c)^n}{bc - ad}\right)}{bc - ad} + \int \frac{1}{(bdx^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxima")`

[Out] `-((log(b*x + a) - log(d*x + c))*log(-(b*x + a)^n*e + (d*x + c)^n) - (log(b*x + a) - log(d*x + c))*log((d*x + c)^n))/(b*c - a*d) + integrate((e*n*log(b*x + a) - e*n*log(d*x + c))*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c*e + a*d*e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)`

**Fricas [A]** time = 2.44507, size = 68, normalized size = 2.06

$$\frac{\text{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bc - ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fricas")`

[Out] `dilog(e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n + 1\right)}{(bx + a)(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(-log(-e*((b*x + a)/(d*x + c))^n + 1)/((b*x + a)*(d*x + c)), x)
```

$$3.153 \quad \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=36

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

[Out] -(Log[1 - e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n))

**Rubi [A]** time = 0.316718, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$ , Rules used = {12, 6684}

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(e\*((a + b\*x)/(c + d\*x))^n)/((a + b\*x)\*(c + d\*x)\*(1 - e\*((a + b\*x)/(c + d\*x))^n)), x]

[Out] -(Log[1 - e\*((a + b\*x)/(c + d\*x))^n]/((b\*c - a\*d)\*n))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 6684

Int[(u\_)/(y\_), x\_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q\*L og[RemoveContent[y, x]], x] /; !FalseQ[q]]

#### Rubi steps

$$\begin{aligned} \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= e \int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \\ &= -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n} \end{aligned}$$

**Mathematica [A]** time = 0.0914007, size = 38, normalized size = 1.06

$$-\frac{e \log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcen - aden}$$



Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)),x]
```

```
[Out] -(e*Log[1 - e*((a + b*x)/(c + d*x))^n])/(b*c*e*n - a*d*e*n)
```

**Maple [A]** time = 0.121, size = 37, normalized size = 1.

$$\frac{1}{n(ad - bc)} \ln \left( e^{n \ln \left( \frac{bx+a}{dx+c} \right)} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/n/(a*d-b*c)*ln(e*exp(n*ln((b*x+a)/(d*x+c)))-1)
```

**Maxima [A]** time = 1.01397, size = 78, normalized size = 2.17

$$-e \left( \frac{\log \left( -(bx + a)^n e + (dx + c)^n \right)}{bcen - aden} - \frac{\log(dx + c)}{bce - ade} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -e*(log(-(b*x + a)^n*e + (d*x + c)^n)/(b*c*e*n - a*d*e*n) - log(d*x + c)/(b*c*e - a*d*e))
```

**Fricas [A]** time = 2.34046, size = 72, normalized size = 2.

$$\frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n - 1 \right)}{(bc - ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] -log(e*((b*x + a)/(d*x + c))^n - 1)/((b*c - a*d)*n)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(-e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*x + c))^n - 1)), x)
```

$$3.154 \quad \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2} dx$$

**Optimal.** Leaf size=36

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

[Out] 1/((b\*c - a\*d)\*n\*(1 - e\*((a + b\*x)/(c + d\*x))^n))

**Rubi [A]** time = 0.366682, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$ , Rules used = {12, 6686}

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

Antiderivative was successfully verified.

[In] Int[(e\*((a + b\*x)/(c + d\*x))^n)/((a + b\*x)\*(c + d\*x)\*(1 - e\*((a + b\*x)/(c + d\*x))^n)^2), x]

[Out] 1/((b\*c - a\*d)\*n\*(1 - e\*((a + b\*x)/(c + d\*x))^n))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 6686

Int[(u\_)\*(y\_)^(m\_.), x\_Symbol] := With[{q = DerivativeDivides[y, u, x]}, Simp[(q\*y^(m + 1))/(m + 1), x] /; !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2} dx &= e \int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2} dx \\ &= \frac{1}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)} \end{aligned}$$

**Mathematica [A]** time = 0.103027, size = 35, normalized size = 0.97

$$\frac{1}{n(ad-bc)\left(e\left(\frac{a+bx}{c+dx}\right)^n - 1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^2), x]
```

```
[Out] 1/((-b*c) + a*d)*n*(-1 + e*((a + b*x)/(c + d*x))^n)
```

**Maple [A]** time = 0.171, size = 56, normalized size = 1.6

$$\frac{e}{n(ad - bc)} e^{n \ln\left(\frac{bx+a}{dx+c}\right)} \left( e^{n \ln\left(\frac{bx+a}{dx+c}\right)} - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c), x)
```

```
[Out] e/n/(a*d-b*c)*exp(n*ln((b*x+a)/(d*x+c)))/(e*exp(n*ln((b*x+a)/(d*x+c)))-1)
```

**Maxima [A]** time = 1.04494, size = 70, normalized size = 1.94

$$-\frac{(bx + a)^n e}{(bcen - aden)(bx + a)^n - (bcn - adn)(dx + c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c), x, algorithm="maxima")
```

```
[Out] -(b*x + a)^n*e/((b*c*e*n - a*d*e*n)*(b*x + a)^n - (b*c*n - a*d*n)*(d*x + c)^n)
```

**Fricas [A]** time = 2.47833, size = 84, normalized size = 2.33

$$-\frac{1}{(bc - ad)en \left(\frac{bx+a}{dx+c}\right)^n - (bc - ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c), x, algorithm="fricas")
```

```
[Out] -1/((b*c - a*d)*e*n*((b*x + a)/(d*x + c))^n - (b*c - a*d)*n)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)**2/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n - 1\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*x + c))^n - 1)^2), x)
```

$$3.155 \quad \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n + e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^3} dx$$

**Optimal.** Leaf size=52

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

[Out] (e\*((a + b\*x)/(c + d\*x))^n)/((b\*c - a\*d)\*n\*(1 - e\*((a + b\*x)/(c + d\*x))^n)^2)

**Rubi [A]** time = 2.02456, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 76,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {6741, 12, 6692, 34}

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(e\*((a + b\*x)/(c + d\*x))^n + e^2\*((a + b\*x)/(c + d\*x))^(2\*n))/((a + b\*x)\*(c + d\*x)\*(1 - e\*((a + b\*x)/(c + d\*x))^n)^3), x]

[Out] (e\*((a + b\*x)/(c + d\*x))^n)/((b\*c - a\*d)\*n\*(1 - e\*((a + b\*x)/(c + d\*x))^n)^2)

#### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 6692

Int[(u\_)\*((c\_.) + (d\_.)\*(v\_))^(n\_.)\*((a\_.) + (b\_.)\*(y\_))^(m\_.), x\_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Dist[q, Subst[Int[(a + b\*x)^m\*(c + d\*x)^n, x], x, y], x] /; !FalseQ[q] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[v, y]

#### Rule 34

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x)^(m + 1))/(b\*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a\*d - b\*c\*(m + 2), 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n + e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^3} dx &= \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n \left(1 + e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^3} dx \\
&= e \int \frac{\left(\frac{a+bx}{c+dx}\right)^n \left(1 + e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^3} dx \\
&= \frac{\text{Subst}\left(\int \frac{1+x}{(1-x)^3} dx, x, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n} \\
&= \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(1 - e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.265491, size = 52, normalized size = 1.

$$-\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(ad-bc)\left(e\left(\frac{a+bx}{c+dx}\right)^n - 1\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e\*((a + b\*x)/(c + d\*x))^n + e^2\*((a + b\*x)/(c + d\*x))^(2\*n))/((a + b\*x)\*(c + d\*x)\*(1 - e\*((a + b\*x)/(c + d\*x))^n)^3), x]

[Out] -(e\*((a + b\*x)/(c + d\*x))^n)/((-b\*c) + a\*d)\*n\*(-1 + e\*((a + b\*x)/(c + d\*x))^n)^2)

**Maple [A]** time = 0.276, size = 57, normalized size = 1.1

$$-\frac{e}{n(ad-bc)}e^{n\ln\left(\frac{bx+a}{dx+c}\right)}\left(e^{n\ln\left(\frac{bx+a}{dx+c}\right)} - 1\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(1+e\*((b\*x+a)/(d\*x+c))^n)\*e\*((b\*x+a)/(d\*x+c))^n/(-1+e\*((b\*x+a)/(d\*x+c))^n)^3/(b\*x+a)/(d\*x+c), x)

[Out] -e/n/(a\*d-b\*c)\*exp(n\*ln((b\*x+a)/(d\*x+c)))/(e\*exp(n\*ln((b\*x+a)/(d\*x+c)))-1)^2

**Maxima [B]** time = 1.17927, size = 285, normalized size = 5.48

$$\frac{1}{2} \left( \frac{(bx+a)^{2n}e}{(bce^2n - ade^2n)(bx+a)^{2n} + (bcn - adn)(dx+c)^{2n} - 2(bcen - aden)e^{(n\log(bx+a) + n\log(dx+c))}} - \frac{1}{(bce^2n - ade^2n)(bx+a)^{2n}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(1+e\*((b\*x+a)/(d\*x+c))^n)\*e\*((b\*x+a)/(d\*x+c))^n/(-1+e\*((b\*x+a)/(d\*x+c))^n)^3/(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out]  $\frac{1}{2} * ((b*x + a)^{(2*n)} * e / ((b*c*e^{2*n} - a*d*e^{2*n}) * (b*x + a)^{(2*n)} + (b*c*n - a*d*n) * (d*x + c)^{(2*n)} - 2 * (b*c*e^n - a*d*e^n) * e^{(n*\log(b*x + a) + n*\log(d*x + c))}) - ((b*x + a)^{(2*n)} * e - 2 * e^{(n*\log(b*x + a) + n*\log(d*x + c))}) / ((b*c*e^{2*n} - a*d*e^{2*n}) * (b*x + a)^{(2*n)} + (b*c*n - a*d*n) * (d*x + c)^{(2*n)} - 2 * (b*c*e^n - a*d*e^n) * e^{(n*\log(b*x + a) + n*\log(d*x + c))}) * e$

**Fricas [A]** time = 2.27585, size = 182, normalized size = 3.5

$$\frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bc-ad)e^{2n}\left(\frac{bx+a}{dx+c}\right)^{2n} - 2(bc-ad)en\left(\frac{bx+a}{dx+c}\right)^n + (bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(1+e\*((b\*x+a)/(d\*x+c))^n)\*e\*((b\*x+a)/(d\*x+c))^n/(-1+e\*((b\*x+a)/(d\*x+c))^n)^3/(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out]  $e*((b*x + a)/(d*x + c))^n / ((b*c - a*d)*e^{2*n}*((b*x + a)/(d*x + c))^{(2*n)} - 2*(b*c - a*d)*e^n*((b*x + a)/(d*x + c))^n + (b*c - a*d)*n$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(1+e\*((b\*x+a)/(d\*x+c))^n)\*e\*((b\*x+a)/(d\*x+c))^n/(-1+e\*((b\*x+a)/(d\*x+c))^n)^3/(b\*x+a)/(d\*x+c),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(e\left(\frac{bx+a}{dx+c}\right)^n + 1\right)e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n - 1\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(1+e\*((b\*x+a)/(d\*x+c))^n)\*e\*((b\*x+a)/(d\*x+c))^n/(-1+e\*((b\*x+a)/(d\*x+c))^n)^3/(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] integrate(-(e\*((b\*x + a)/(d\*x + c))^n + 1)\*e\*((b\*x + a)/(d\*x + c))^n/((b\*x + a)\*(d\*x + c)\*(e\*((b\*x + a)/(d\*x + c))^n - 1)^3), x)



### 3.156 $\int x^3 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

**Optimal.** Leaf size=135

$$\frac{3x^2 \text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \text{PolyLog}\left(n+4, d\left(F^{c(a+bx)}\right)^p\right)}{b^4 c^4 p^4 \log^4(F)} + \frac{x^3 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right)}{b^4 c^4 p^4 \log^4(F)}$$

[Out]  $(x^3 \text{PolyLog}[1+n, d(F^{c(a+bx)})^p]) / (b*c*p*\text{Log}[F]) - (3*x^2*\text{PolyLog}[2+n, d(F^{c(a+bx)})^p]) / (b^2*c^2*p^2*\text{Log}[F]^2) + (6*x*\text{PolyLog}[3+n, d(F^{c(a+bx)})^p]) / (b^3*c^3*p^3*\text{Log}[F]^3) - (6*\text{PolyLog}[4+n, d(F^{c(a+bx)})^p]) / (b^4*c^4*p^4*\text{Log}[F]^4)$

**Rubi [A]** time = 0.0883024, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {6609, 2282, 6589}

$$\frac{3x^2 \text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \text{PolyLog}\left(n+4, d\left(F^{c(a+bx)}\right)^p\right)}{b^4 c^4 p^4 \log^4(F)} + \frac{x^3 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right)}{b^4 c^4 p^4 \log^4(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3 \text{PolyLog}[n, d(F^{c(a+bx)})^p], x]$

[Out]  $(x^3 \text{PolyLog}[1+n, d(F^{c(a+bx)})^p]) / (b*c*p*\text{Log}[F]) - (3*x^2*\text{PolyLog}[2+n, d(F^{c(a+bx)})^p]) / (b^2*c^2*p^2*\text{Log}[F]^2) + (6*x*\text{PolyLog}[3+n, d(F^{c(a+bx)})^p]) / (b^3*c^3*p^3*\text{Log}[F]^3) - (6*\text{PolyLog}[4+n, d(F^{c(a+bx)})^p]) / (b^4*c^4*p^4*\text{Log}[F]^4)$

#### Rule 6609

$\text{Int}[(e_. + (f_.)*(x_.))^{(m_.)*\text{PolyLog}[n_, (d_.)*((F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))})^{(p_.)}], x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^m*\text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^p] / (b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m) / (b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{m-1}*\text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^p], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rule 2282

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_.)\*((a\_.)\*(v\_.)^{n\_.})^{m\_.} /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^{(c\_.)\*((a\_.) + (b\_.)\*x)}\*(F\_.)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_.))^{(p_.)}] / ((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int x^3 \text{Li}_n \left( d \left( F^{c(a+bx)} \right)^p \right) dx &= \frac{x^3 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{3 \int x^2 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{bcp \log(F)} \\
&= \frac{x^3 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{3x^2 \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6 \int x \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{b^2 c^2 p^2 \log^2(F)} \\
&= \frac{x^3 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{3x^2 \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \int \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{b^3 c^3 p^3 \log^3(F)} \\
&= \frac{x^3 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{3x^2 \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \text{Subst} \left( \int \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx \right)}{b^4 c^4 p^4 \log^4(F)} \\
&= \frac{x^3 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{3x^2 \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \text{Li}_{4+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^4 c^4 p^4 \log^4(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0115321, size = 135, normalized size = 1.

$$-\frac{3x^2 \text{PolyLog} \left( n+2, d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{6x \text{PolyLog} \left( n+3, d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)} - \frac{6 \text{PolyLog} \left( n+4, d \left( F^{c(a+bx)} \right)^p \right)}{b^4 c^4 p^4 \log^4(F)} + \frac{x^3 \text{PolyLog} \left( n+5, d \left( F^{c(a+bx)} \right)^p \right)}{b^5 c^5 p^5 \log^5(F)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] (x^3\*PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]) - (3\*x^2\*PolyLog[2 + n, d\*(F^(c\*(a + b\*x)))^p])/(b^2\*c^2\*p^2\*Log[F]^2) + (6\*x\*PolyLog[3 + n, d\*(F^(c\*(a + b\*x)))^p])/(b^3\*c^3\*p^3\*Log[F]^3) - (6\*PolyLog[4 + n, d\*(F^(c\*(a + b\*x)))^p])/(b^4\*c^4\*p^4\*Log[F]^4)

**Maple [F]** time = 0.185, size = 0, normalized size = 0.

$$\int x^3 \text{polylog} \left( n, d \left( F^{c(bx+a)} \right)^p \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x)

[Out] int(x^3\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{Li}_n \left( \left( F^{(bx+a)c} \right)^p d \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="maxima")

[Out] integrate(x^3\*polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{polylog}\left(n, (F^{bcx+ac})^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="fricas")

[Out] integral(x^3\*polylog(n, (F^(b\*c\*x + a\*c))^p\*d), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{Li}_n\left(d (F^{ac} F^{bcx})^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*polylog(n,d\*(F\*\*(c\*(b\*x+a)))\*\*p),x)

[Out] Integral(x\*\*3\*polylog(n, d\*(F\*\*(a\*c)\*F\*\*(b\*c\*x))\*\*p), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{Li}_n\left((F^{(bx+a)c})^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="giac")

[Out] integrate(x^3\*polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

### 3.157 $\int x^2 \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

**Optimal.** Leaf size=100

$$-\frac{2x \text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3 c^3 p^3 \log^3(F)} + \frac{x^2 \text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{b c p \log(F)}$$

[Out]  $(x^2 \text{PolyLog}[1+n, d*(F^{c*(a+b*x)})^p]) / (b*c*p*\text{Log}[F]) - (2*x*\text{PolyLog}[2+n, d*(F^{c*(a+b*x)})^p]) / (b^2*c^2*p^2*\text{Log}[F]^2) + (2*\text{PolyLog}[3+n, d*(F^{c*(a+b*x)})^p]) / (b^3*c^3*p^3*\text{Log}[F]^3)$

**Rubi [A]** time = 0.0579408, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {6609, 2282, 6589}

$$-\frac{2x \text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3 c^3 p^3 \log^3(F)} + \frac{x^2 \text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{b c p \log(F)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 \text{PolyLog}[n, d*(F^{c*(a+b*x)})^p], x]$

[Out]  $(x^2 \text{PolyLog}[1+n, d*(F^{c*(a+b*x)})^p]) / (b*c*p*\text{Log}[F]) - (2*x*\text{PolyLog}[2+n, d*(F^{c*(a+b*x)})^p]) / (b^2*c^2*p^2*\text{Log}[F]^2) + (2*\text{PolyLog}[3+n, d*(F^{c*(a+b*x)})^p]) / (b^3*c^3*p^3*\text{Log}[F]^3)$

#### Rule 6609

$\text{Int}[(e_.) + (f_.)*(x_.)^{(m_.)} \text{PolyLog}[n_., (d_.)*((F_.)^{((c_.)*((a_.) + (b_.)*(x_.)))})^p]), x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^m \text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^p] / (b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m) / (b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{m-1} \text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^p], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

#### Rule 2282

$\text{Int}[u_., x\_Symbol] \rightarrow \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$  FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 6589

$\text{Int}[\text{PolyLog}[n_., (c_.)*((a_.) + (b_.)*(x_.)^p)] / ((d_.) + (e_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /;$  FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int x^2 \text{Li}_n \left( d \left( F^{c(a+bx)} \right)^p \right) dx &= \frac{x^2 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{2 \int x \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{bcp \log(F)} \\
&= \frac{x^2 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{2x \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \int \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{b^2 c^2 p^2 \log^2(F)} \\
&= \frac{x^2 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{2x \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \text{Subst} \left( \int \frac{\text{Li}_{2+n}(dx^p)}{x} dx, x, F^{c(a+bx)} \right)}{b^3 c^3 p^2 \log^3(F)} \\
&= \frac{x^2 \text{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)} - \frac{2x \text{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \text{Li}_{3+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)}
\end{aligned}$$

**Mathematica [A]** time = 0.0058507, size = 100, normalized size = 1.

$$\frac{2x \text{PolyLog} \left( n+2, d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} + \frac{2 \text{PolyLog} \left( n+3, d \left( F^{c(a+bx)} \right)^p \right)}{b^3 c^3 p^3 \log^3(F)} + \frac{x^2 \text{PolyLog} \left( n+1, d \left( F^{c(a+bx)} \right)^p \right)}{bcp \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] (x^2\*PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]) - (2\*x\*PolyLog[2 + n, d\*(F^(c\*(a + b\*x)))^p])/(b^2\*c^2\*p^2\*Log[F]^2) + (2\*PolyLog[3 + n, d\*(F^(c\*(a + b\*x)))^p])/(b^3\*c^3\*p^3\*Log[F]^3)

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int x^2 \text{polylog} \left( n, d \left( F^{c(bx+a)} \right)^p \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*polylog(n,d\*(F^(c\*(b\*x+a)))^p), x)

[Out] int(x^2\*polylog(n,d\*(F^(c\*(b\*x+a)))^p), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_n \left( \left( F^{(bx+a)c} \right)^p d \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(n,d\*(F^(c\*(b\*x+a)))^p), x, algorithm="maxima")

[Out] integrate(x^2\*polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^2 \text{polylog}\left(n, (F^{bcx+ac})^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="fricas")

[Out] integral(x^2\*polylog(n, (F^(b\*c\*x + a\*c))^p\*d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_n\left(d (F^{ac} F^{bcx})^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*polylog(n,d\*(F\*\*(c\*(b\*x+a))))\*\*p),x)

[Out] Integral(x\*\*2\*polylog(n, d\*(F\*\*(a\*c)\*F\*\*(b\*c\*x))))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_n\left((F^{(bx+a)c})^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="giac")

[Out] integrate(x^2\*polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

### 3.158 $\int x \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

**Optimal.** Leaf size=65

$$\frac{x \text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bc p \log(F)} - \frac{\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)}$$

[Out] (x\*PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]) - PolyLog[2 + n, d\*(F^(c\*(a + b\*x)))^p]/(b^2\*c^2\*p^2\*Log[F]^2)

**Rubi [A]** time = 0.0331011, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {6609, 2282, 6589}

$$\frac{x \text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bc p \log(F)} - \frac{\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2 c^2 p^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Int[x\*PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] (x\*PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]) - PolyLog[2 + n, d\*(F^(c\*(a + b\*x)))^p]/(b^2\*c^2\*p^2\*Log[F]^2)

#### Rule 6609

```
Int[((e_) + (f_)*(x_)^(m_))*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)
)*(x_)))^(p_)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_)^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned} \int x \operatorname{Li}_n \left( d \left( F^{c(a+bx)} \right)^p \right) dx &= \frac{x \operatorname{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bc p \log(F)} - \frac{\int \operatorname{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right) dx}{bc p \log(F)} \\ &= \frac{x \operatorname{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bc p \log(F)} - \frac{\operatorname{Subst} \left( \int \frac{\operatorname{Li}_{1+n}(dx^p)}{x} dx, x, F^{c(a+bx)} \right)}{b^2 c^2 p \log^2(F)} \\ &= \frac{x \operatorname{Li}_{1+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{bc p \log(F)} - \frac{\operatorname{Li}_{2+n} \left( d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)} \end{aligned}$$

**Mathematica [A]** time = 0.0052208, size = 65, normalized size = 1.

$$\frac{x \operatorname{PolyLog} \left( n+1, d \left( F^{c(a+bx)} \right)^p \right)}{bc p \log(F)} - \frac{\operatorname{PolyLog} \left( n+2, d \left( F^{c(a+bx)} \right)^p \right)}{b^2 c^2 p^2 \log^2(F)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] (x\*PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]) - PolyLog[2 + n, d\*(F^(c\*(a + b\*x)))^p]/(b^2\*c^2\*p^2\*Log[F]^2)

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int x \operatorname{polylog} \left( n, d \left( F^{c(bx+a)} \right)^p \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x)

[Out] int(x\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_n \left( \left( F^{(bx+a)c} \right)^p d \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*polylog(n,d\*(F^(c\*(b\*x+a)))^p),x, algorithm="maxima")

[Out] integrate(x\*polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( x \operatorname{polylog} \left( n, \left( F^{bcx+ac} \right)^p d \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(x*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

### 3.159 $\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

**Optimal.** Leaf size=31

$$\frac{\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)}$$

[Out] PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F])

**Rubi [A]** time = 0.0177012, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2282, 6589}

$$\frac{\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F])

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned} \int \text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right) dx &= \frac{\text{Subst}\left(\int \frac{\text{Li}_n(dx^p)}{x} dx, x, F^{c(a+bx)}\right)}{bc \log(F)} \\ &= \frac{\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)} \end{aligned}$$

**Mathematica [A]** time = 0.004743, size = 31, normalized size = 1.

$$\frac{\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp \log(F)}$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[n, d\*(F^(c\*(a + b\*x)))^p], x]

[Out] PolyLog[1 + n, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F])

**Maple [A]** time = 0.061, size = 32, normalized size = 1.

$$\frac{\text{polylog}\left(1 + n, d\left(F^{c(bx+a)}\right)^p\right)}{pbc \ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, d\*(F^(c\*(b\*x+a)))^p), x)

[Out] polylog(1+n, d\*(F^(c\*(b\*x+a)))^p)/b/c/p/ln(F)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n, d\*(F^(c\*(b\*x+a)))^p), x, algorithm="maxima")

[Out] integrate(polylog(n, (F^((b\*x + a)\*c))^p\*d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{polylog}\left(n, \left(F^{bcx+ac}\right)^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n, d\*(F^(c\*(b\*x+a)))^p), x, algorithm="fricas")

[Out] integral(polylog(n, (F^(b\*c\*x + a\*c))^p\*d), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(polylog(n, d\*(F\*\*(c\*(b\*x+a)))\*\*p), x)

[Out] Integral(polylog(n, d\*(F\*\*(c\*(a + b\*x)))\*\*p), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_n((F^{(bx+a)c})^p d) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d), x)
```

$$3.160 \quad \int \frac{\text{PolyLog}\left(n, d(F^{c(a+bx)})^p\right)}{x} dx$$

**Optimal.** Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\text{PolyLog}\left(n, d(F^{ac+bcx})^p\right)}{x}, x\right)$$

[Out] CannotIntegrate[PolyLog[n, d\*(F^(a\*c + b\*c\*x))^p]/x, x]

**Rubi [A]** time = 0.0684746, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\text{PolyLog}\left(n, d(F^{c(a+bx)})^p\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Int[PolyLog[n, d\*(F^(c\*(a + b\*x)))^p]/x, x]

[Out] Defer[Int][PolyLog[n, d\*(F^(a\*c + b\*c\*x))^p]/x, x]

Rubi steps

$$\int \frac{\text{Li}_n\left(d(F^{c(a+bx)})^p\right)}{x} dx = \int \frac{\text{Li}_n\left(d(F^{ac+bcx})^p\right)}{x} dx$$

**Mathematica [A]** time = 0.0510035, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}\left(n, d(F^{c(a+bx)})^p\right)}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[PolyLog[n, d\*(F^(c\*(a + b\*x)))^p]/x, x]

[Out] Integrate[PolyLog[n, d\*(F^(c\*(a + b\*x)))^p]/x, x]

**Maple [A]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}\left(n, d(F^{c(bx+a)})^p\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(polylog(n, d\*(F^(c\*(b\*x+a)))^p)/x, x)

[Out] `int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="maxima")`

[Out] `integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\left(n,\left(F^{bcx+ac}\right)^p d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="fricas")`

[Out] `integral(polylog(n, (F^(b*c*x + a*c))^p*d)/x, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,d*(F**(c*(b*x+a)))**p)/x,x)`

[Out] `Integral(polylog(n, d*(F**(a*c)*F**(b*c*x))**p)/x, x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="giac")`

[Out] `integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)`

### 3.161 $\int x^3 \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=300

$$\frac{x^2 \text{PolyLog}(2, cx)}{8c^2} - \frac{x \text{PolyLog}(2, cx)}{4c^3} + \frac{\text{PolyLog}(3, 1 - cx)}{2c^4} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{4c^4} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{2c^4}$$

```
[Out] (355*x)/(576*c^3) + (139*x^2)/(1152*c^2) + (67*x^3)/(1728*c) + (3*x^4)/256
+ (139*Log[1 - c*x])/(576*c^4) - (x^2*Log[1 - c*x])/(8*c^2) - (5*x^3*Log[1
- c*x])/(72*c) - (3*x^4*Log[1 - c*x])/64 + (3*(1 - c*x)*Log[1 - c*x])/(8*c^
4) - Log[1 - c*x]^2/(16*c^4) + (x^4*Log[1 - c*x]^2)/16 - (Log[c*x]*Log[1 -
c*x]^2)/(4*c^4) - (x*PolyLog[2, c*x])/(4*c^3) - (x^2*PolyLog[2, c*x])/(8*c^
2) - (x^3*PolyLog[2, c*x])/(12*c) - (x^4*PolyLog[2, c*x])/16 - (Log[1 - c*x
]*PolyLog[2, c*x])/(4*c^4) + (x^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1
- c*x]*PolyLog[2, 1 - c*x])/(2*c^4) + PolyLog[3, 1 - c*x]/(2*c^4)
```

**Rubi [A]** time = 0.523459, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 16, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6591, 2395, 43, 6603, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{x^2 \text{PolyLog}(2, cx)}{8c^2} - \frac{x \text{PolyLog}(2, cx)}{4c^3} + \frac{\text{PolyLog}(3, 1 - cx)}{2c^4} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{4c^4} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] (355*x)/(576*c^3) + (139*x^2)/(1152*c^2) + (67*x^3)/(1728*c) + (3*x^4)/256
+ (139*Log[1 - c*x])/(576*c^4) - (x^2*Log[1 - c*x])/(8*c^2) - (5*x^3*Log[1
- c*x])/(72*c) - (3*x^4*Log[1 - c*x])/64 + (3*(1 - c*x)*Log[1 - c*x])/(8*c^
4) - Log[1 - c*x]^2/(16*c^4) + (x^4*Log[1 - c*x]^2)/16 - (Log[c*x]*Log[1 -
c*x]^2)/(4*c^4) - (x*PolyLog[2, c*x])/(4*c^3) - (x^2*PolyLog[2, c*x])/(8*c^
2) - (x^3*PolyLog[2, c*x])/(12*c) - (x^4*PolyLog[2, c*x])/16 - (Log[1 - c*x
]*PolyLog[2, c*x])/(4*c^4) + (x^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1
- c*x]*PolyLog[2, 1 - c*x])/(2*c^4) + PolyLog[3, 1 - c*x]/(2*c^4)
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(
g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(f\_. + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

### Rule 2410

Int[(Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(f\_. + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

### Rule 6596



```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x^3 \log(1 - cx) \operatorname{Li}_2(cx) dx &= \frac{1}{4} x^4 \log(1 - cx) \operatorname{Li}_2(cx) + \frac{1}{4} \int x^3 \log^2(1 - cx) dx + \frac{1}{4} c \int \left( -\frac{\operatorname{Li}_2(cx)}{c^4} - \frac{x \operatorname{Li}_2(cx)}{c^3} - \frac{x^2 \operatorname{Li}_2(cx)}{c^2} \right. \\
&= \frac{1}{16} x^4 \log^2(1 - cx) + \frac{1}{4} x^4 \log(1 - cx) \operatorname{Li}_2(cx) - \frac{1}{4} \int x^3 \operatorname{Li}_2(cx) dx - \frac{\int \operatorname{Li}_2(cx) dx}{4c^3} - \frac{\int \frac{\operatorname{Li}_2(cx)}{-1+c}}{4c} \\
&= \frac{1}{16} x^4 \log^2(1 - cx) - \frac{x \operatorname{Li}_2(cx)}{4c^3} - \frac{x^2 \operatorname{Li}_2(cx)}{8c^2} - \frac{x^3 \operatorname{Li}_2(cx)}{12c} - \frac{1}{16} x^4 \operatorname{Li}_2(cx) - \frac{\log(1 - cx) \operatorname{Li}_2(cx)}{4c^4} \\
&= -\frac{x^2 \log(1 - cx)}{16c^2} - \frac{x^3 \log(1 - cx)}{36c} - \frac{1}{64} x^4 \log(1 - cx) + \frac{1}{16} x^4 \log^2(1 - cx) - \frac{\log(cx) \log^2(1 - cx)}{4c^4} \\
&= \frac{x}{4c^3} - \frac{x^2 \log(1 - cx)}{8c^2} - \frac{5x^3 \log(1 - cx)}{72c} - \frac{3}{64} x^4 \log(1 - cx) + \frac{(1 - cx) \log(1 - cx)}{4c^4} + \frac{1}{16} x^4 \log^2(1 - cx) \\
&= \frac{277x}{576c^3} + \frac{61x^2}{1152c^2} + \frac{25x^3}{1728c} + \frac{x^4}{256} + \frac{61 \log(1 - cx)}{576c^4} - \frac{x^2 \log(1 - cx)}{8c^2} - \frac{5x^3 \log(1 - cx)}{72c} - \frac{\log(1 - cx) \operatorname{Li}_2(cx)}{4c^4} \\
&= \frac{355x}{576c^3} + \frac{139x^2}{1152c^2} + \frac{67x^3}{1728c} + \frac{3x^4}{256} + \frac{139 \log(1 - cx)}{576c^4} - \frac{x^2 \log(1 - cx)}{8c^2} - \frac{5x^3 \log(1 - cx)}{72c} - \frac{\log(1 - cx) \operatorname{Li}_2(cx)}{4c^4}
\end{aligned}$$

**Mathematica [A]** time = 0.573099, size = 223, normalized size = 0.74

$$144 \left( 12 \left( c^4 x^4 - 1 \right) \log(1 - cx) - cx \left( 3c^3 x^3 + 4c^2 x^2 + 6cx + 12 \right) \right) \text{PolyLog}(2, cx) + 3456 \text{PolyLog}(3, 1 - cx) - 3456 \log(1 -$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[1 - c\*x]\*PolyLog[2, c\*x],x]

[Out] (4260\*c\*x + 834\*c^2\*x^2 + 268\*c^3\*x^3 + 81\*c^4\*x^4 + 4260\*Log[1 - c\*x] - 25  
92\*c\*x\*Log[1 - c\*x] - 864\*c^2\*x^2\*Log[1 - c\*x] - 480\*c^3\*x^3\*Log[1 - c\*x] -  
324\*c^4\*x^4\*Log[1 - c\*x] - 432\*Log[1 - c\*x]^2 + 432\*c^4\*x^4\*Log[1 - c\*x]^2  
- 1728\*Log[c\*x]\*Log[1 - c\*x]^2 + 144\*(-(c\*x\*(12 + 6\*c\*x + 4\*c^2\*x^2 + 3\*c^3\*x^3))  
+ 12\*(-1 + c^4\*x^4)\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 3456\*Log[1 - c\*x]  
x)\*PolyLog[2, 1 - c\*x] + 3456\*PolyLog[3, 1 - c\*x])/(6912\*c^4)

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int x^3 \ln(-cx + 1) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] int(x^3\*ln(-c\*x+1)\*polylog(2,c\*x),x)

**Maxima [A]** time = 1.12854, size = 508, normalized size = 1.69

$$9c^4 \left( \frac{3c^3x^4 + 4c^2x^3 + 6cx^2 + 12x}{c^4} + \frac{12 \log(cx-1)}{c^5} \right) + 24c^3 \left( \frac{2c^2x^3 + 3cx^2 + 6x}{c^3} + \frac{6 \log(cx-1)}{c^4} \right) + 108c^2 \left( \frac{cx^2 + 2x}{c^2} + \frac{2 \log(cx-1)}{c^3} \right) + 432c \left( \frac{x}{c} + \frac{\log(cx-1)}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="maxima")

[Out] 1/6912\*(9\*c^4\*((3\*c^3\*x^4 + 4\*c^2\*x^3 + 6\*c\*x^2 + 12\*x)/c^4 + 12\*log(c\*x -  
1)/c^5) + 24\*c^3\*((2\*c^2\*x^3 + 3\*c\*x^2 + 6\*x)/c^3 + 6\*log(c\*x - 1)/c^4) + 1  
08\*c^2\*((c\*x^2 + 2\*x)/c^2 + 2\*log(c\*x - 1)/c^3) + 432\*c\*(x/c + log(c\*x - 1)  
/c^2) + 2\*(27\*c^4\*x^4 + 92\*c^3\*x^3 + 300\*c^2\*x^2 + 1680\*c\*x - 72\*(3\*c^4\*x^4  
+ 4\*c^3\*x^3 + 6\*c^2\*x^2 + 12\*c\*x + 12\*log(-c\*x + 1))\*dilog(c\*x) - 12\*(9\*c^4  
x^4 + 14\*c^3\*x^3 + 27\*c^2\*x^2 + 90\*c\*x - 140)\*log(-c\*x + 1))/c - 1728\*(lo  
g(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*  
x + 1))/c)/c^3 + 1/192\*(48\*c^4\*x^4\*dilog(c\*x) - 3\*c^4\*x^4 - 4\*c^3\*x^3 - 6\*c  
^2\*x^2 - 12\*c\*x + 12\*(c^4\*x^4 - 1)\*log(-c\*x + 1))\*log(-c\*x + 1)/c^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^3 \text{Li}_2(cx) \log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*dilog(c*x)*log(-c*x + 1), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(c*x)*log(-c*x + 1), x)
```

### 3.162 $\int x^2 \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=258

$$-\frac{x \text{PolyLog}(2, cx)}{3c^2} + \frac{2 \text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{3c^3} - \frac{2 \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{3c^3} - \frac{1}{9} x^3 \text{PolyLog}(2, cx)$$

[Out] (31\*x)/(36\*c^2) + (11\*x^2)/(72\*c) + x^3/27 + (11\*Log[1 - c\*x])/(36\*c^3) - (7\*x^2\*Log[1 - c\*x])/(36\*c) - (x^3\*Log[1 - c\*x])/9 + (5\*(1 - c\*x)\*Log[1 - c\*x])/(9\*c^3) - Log[1 - c\*x]^2/(9\*c^3) + (x^3\*Log[1 - c\*x]^2)/9 - (Log[c\*x]\*Log[1 - c\*x]^2)/(3\*c^3) - (x\*PolyLog[2, c\*x])/(3\*c^2) - (x^2\*PolyLog[2, c\*x])/(6\*c) - (x^3\*PolyLog[2, c\*x])/9 - (Log[1 - c\*x]\*PolyLog[2, c\*x])/(3\*c^3) + (x^3\*Log[1 - c\*x]\*PolyLog[2, c\*x])/3 - (2\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/(3\*c^3) + (2\*PolyLog[3, 1 - c\*x])/(3\*c^3)

**Rubi [A]** time = 0.404814, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6591, 2395, 43, 6603, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x \text{PolyLog}(2, cx)}{3c^2} + \frac{2 \text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{3c^3} - \frac{2 \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{3c^3} - \frac{1}{9} x^3 \text{PolyLog}(2, cx)$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[1 - c\*x]\*PolyLog[2, c\*x], x]

[Out] (31\*x)/(36\*c^2) + (11\*x^2)/(72\*c) + x^3/27 + (11\*Log[1 - c\*x])/(36\*c^3) - (7\*x^2\*Log[1 - c\*x])/(36\*c) - (x^3\*Log[1 - c\*x])/9 + (5\*(1 - c\*x)\*Log[1 - c\*x])/(9\*c^3) - Log[1 - c\*x]^2/(9\*c^3) + (x^3\*Log[1 - c\*x]^2)/9 - (Log[c\*x]\*Log[1 - c\*x]^2)/(3\*c^3) - (x\*PolyLog[2, c\*x])/(3\*c^2) - (x^2\*PolyLog[2, c\*x])/(6\*c) - (x^3\*PolyLog[2, c\*x])/9 - (Log[1 - c\*x]\*PolyLog[2, c\*x])/(3\*c^3) + (x^3\*Log[1 - c\*x]\*PolyLog[2, c\*x])/3 - (2\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/(3\*c^3) + (2\*PolyLog[3, 1 - c\*x])/(3\*c^3)

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(h\_.))\*(x\_.)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))], x\_Symbol] := Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

Rule 2410

Int[(Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(x\_.)^(m\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2295

Int[Log[(c\_.)\*(x\_.)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))/(x\_.), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)]/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&

EqQ[c\*(b\*d - a\*e) + e, 0]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x^2 \log(1 - cx) \text{Li}_2(cx) dx &= \frac{1}{3} x^3 \log(1 - cx) \text{Li}_2(cx) + \frac{1}{3} \int x^2 \log^2(1 - cx) dx + \frac{1}{3} c \int \left( -\frac{\text{Li}_2(cx)}{c^3} - \frac{x \text{Li}_2(cx)}{c^2} - \frac{x^2 \text{Li}_2(cx)}{c} \right. \\
 &= \frac{1}{9} x^3 \log^2(1 - cx) + \frac{1}{3} x^3 \log(1 - cx) \text{Li}_2(cx) - \frac{1}{3} \int x^2 \text{Li}_2(cx) dx - \frac{\int \text{Li}_2(cx) dx}{3c^2} - \frac{\int \frac{\text{Li}_2(cx)}{-1+cx} dx}{3c^2} \\
 &= \frac{1}{9} x^3 \log^2(1 - cx) - \frac{x \text{Li}_2(cx)}{3c^2} - \frac{x^2 \text{Li}_2(cx)}{6c} - \frac{1}{9} x^3 \text{Li}_2(cx) - \frac{\log(1 - cx) \text{Li}_2(cx)}{3c^3} + \frac{1}{3} x^3 \log(1 - cx) \\
 &= -\frac{x^2 \log(1 - cx)}{12c} - \frac{1}{27} x^3 \log(1 - cx) + \frac{1}{9} x^3 \log^2(1 - cx) - \frac{\log(cx) \log^2(1 - cx)}{3c^3} - \frac{x \text{Li}_2(cx)}{3c^2} \\
 &= \frac{x}{3c^2} - \frac{7x^2 \log(1 - cx)}{36c} - \frac{1}{9} x^3 \log(1 - cx) + \frac{(1 - cx) \log(1 - cx)}{3c^3} + \frac{1}{9} x^3 \log^2(1 - cx) - \frac{\log(cx)}{3c^3} \\
 &= \frac{73x}{108c^2} + \frac{13x^2}{216c} + \frac{x^3}{81} + \frac{13 \log(1 - cx)}{108c^3} - \frac{7x^2 \log(1 - cx)}{36c} - \frac{1}{9} x^3 \log(1 - cx) + \frac{5(1 - cx) \log(1 - cx)}{9c^3} \\
 &= \frac{31x}{36c^2} + \frac{11x^2}{72c} + \frac{x^3}{27} + \frac{11 \log(1 - cx)}{36c^3} - \frac{7x^2 \log(1 - cx)}{36c} - \frac{1}{9} x^3 \log(1 - cx) + \frac{5(1 - cx) \log(1 - cx)}{9c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.357232, size = 192, normalized size = 0.74

$12 \left( 6 \left( c^3 x^3 - 1 \right) \log(1 - cx) - cx \left( 2c^2 x^2 + 3cx + 6 \right) \right) \text{PolyLog}(2, cx) + 144 \text{PolyLog}(3, 1 - cx) - 144 \log(1 - cx) \text{PolyLog}($

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[1 - c\*x]\*PolyLog[2, c\*x],x]

[Out] (186\*c\*x + 33\*c^2\*x^2 + 8\*c^3\*x^3 + 186\*Log[1 - c\*x] - 120\*c\*x\*Log[1 - c\*x] - 42\*c^2\*x^2\*Log[1 - c\*x] - 24\*c^3\*x^3\*Log[1 - c\*x] - 24\*Log[1 - c\*x]^2 + 24\*c^3\*x^3\*Log[1 - c\*x]^2 - 72\*Log[c\*x]\*Log[1 - c\*x]^2 + 12\*(-(c\*x\*(6 + 3\*c\*x + 2\*c^2\*x^2)) + 6\*(-1 + c^3\*x^3)\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 144\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 144\*PolyLog[3, 1 - c\*x])/(216\*c^3)

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int x^2 \ln(-cx + 1) \operatorname{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] int(x^2\*ln(-c\*x+1)\*polylog(2,c\*x),x)

**Maxima [A]** time = 1.11877, size = 400, normalized size = 1.55

$$\frac{4c^3 \left( \frac{2c^2x^3 + 3cx^2 + 6x}{c^3} + \frac{6 \log(cx-1)}{c^4} \right) + 18c^2 \left( \frac{cx^2 + 2x}{c^2} + \frac{2 \log(cx-1)}{c^3} \right) + 72c \left( \frac{x}{c} + \frac{\log(cx-1)}{c^2} \right) + \frac{16c^3x^3 + 69c^2x^2 + 426cx - 36(2c^3x^3 + 3c^2x^2 + 6cx - 6 \log(-cx + 1)) \operatorname{dilog}(cx) - 6(8c^3x^3 + 15c^2x^2 + 48cx - 71) \log(-cx + 1)}{c} - 216(\log(cx) \log(-cx + 1))^2 + 2 \operatorname{dilog}(-cx + 1) \log(-cx + 1) - 2 \operatorname{polylog}(3, -cx + 1)}{c} / c^2 + 1/54(18c^3x^3 \operatorname{dilog}(cx) - 2c^3x^3 - 3c^2x^2 - 6cx + 6(c^3x^3 - 1) \log(-cx + 1)) \log(-cx + 1) / c^3}{648c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="maxima")

[Out] 1/648\*(4\*c^3\*((2\*c^2\*x^3 + 3\*c\*x^2 + 6\*x)/c^3 + 6\*log(c\*x - 1)/c^4) + 18\*c^2\*((c\*x^2 + 2\*x)/c^2 + 2\*log(c\*x - 1)/c^3) + 72\*c\*(x/c + log(c\*x - 1)/c^2) + (16\*c^3\*x^3 + 69\*c^2\*x^2 + 426\*c\*x - 36\*(2\*c^3\*x^3 + 3\*c^2\*x^2 + 6\*c\*x + 6\*log(-c\*x + 1))\*dilog(c\*x) - 6\*(8\*c^3\*x^3 + 15\*c^2\*x^2 + 48\*c\*x - 71)\*log(-c\*x + 1))/c - 216\*(log(c\*x)\*log(-c\*x + 1))^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))/c)/c^2 + 1/54\*(18\*c^3\*x^3\*dilog(c\*x) - 2\*c^3\*x^3 - 3\*c^2\*x^2 - 6\*c\*x + 6\*(c^3\*x^3 - 1)\*log(-c\*x + 1))\*log(-c\*x + 1)/c^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^2 \operatorname{Li}_2(cx) \log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="fricas")

[Out] integral(x^2\*dilog(c\*x)\*log(-c\*x + 1), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="giac")

[Out] integrate(x^2\*dilog(c\*x)\*log(-c\*x + 1), x)



### 3.163 $\int x \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=262

$$\frac{\text{PolyLog}(3, 1 - cx)}{c^2} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{2c^2} - \frac{\log(1 - cx) \text{PolyLog}(2, 1 - cx)}{c^2} - \frac{1}{4} x^2 \text{PolyLog}(2, cx) + \frac{1}{2} x^2 \log(1 - cx)$$

```
[Out] (13*x)/(8*c) + x^2/16 + (1 - c*x)^2/(8*c^2) + Log[1 - c*x]/(8*c^2) - (x^2*Log[1 - c*x])/8 + (3*(1 - c*x)*Log[1 - c*x])/(2*c^2) - ((1 - c*x)^2*Log[1 - c*x])/(4*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + ((1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - (Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - (x*PolyLog[2, c*x])/(2*c) - (x^2*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x^2*Log[1 - c*x]*PolyLog[2, c*x])/2 - (Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + PolyLog[3, 1 - c*x]/c^2
```

**Rubi [A]** time = 0.254646, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$ , Rules used = {6591, 2395, 43, 6603, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{\text{PolyLog}(3, 1 - cx)}{c^2} - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{2c^2} - \frac{\log(1 - cx) \text{PolyLog}(2, 1 - cx)}{c^2} - \frac{1}{4} x^2 \text{PolyLog}(2, cx) + \frac{1}{2} x^2 \log(1 - cx)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] (13*x)/(8*c) + x^2/16 + (1 - c*x)^2/(8*c^2) + Log[1 - c*x]/(8*c^2) - (x^2*Log[1 - c*x])/8 + (3*(1 - c*x)*Log[1 - c*x])/(2*c^2) - ((1 - c*x)^2*Log[1 - c*x])/(4*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + ((1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - (Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - (x*PolyLog[2, c*x])/(2*c) - (x^2*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x^2*Log[1 - c*x]*PolyLog[2, c*x])/2 - (Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + PolyLog[3, 1 - c*x]/c^2
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(f\_. + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(f\_. + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EQ[e\*f - d\*g, 0]

Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
 \int x \log(1 - cx) \text{Li}_2(cx) dx &= \frac{1}{2} x^2 \log(1 - cx) \text{Li}_2(cx) + \frac{1}{2} \int x \log^2(1 - cx) dx + \frac{1}{2} c \int \left( -\frac{\text{Li}_2(cx)}{c^2} - \frac{x \text{Li}_2(cx)}{c} - \frac{\text{Li}_2(cx)}{c^2(-1 + cx)} \right) dx \\
 &= \frac{1}{2} x^2 \log(1 - cx) \text{Li}_2(cx) + \frac{1}{2} \int \left( \frac{\log^2(1 - cx)}{c} - \frac{(1 - cx) \log^2(1 - cx)}{c} \right) dx - \frac{1}{2} \int x \text{Li}_2(cx) dx \\
 &= -\frac{x \text{Li}_2(cx)}{2c} - \frac{1}{4} x^2 \text{Li}_2(cx) - \frac{\log(1 - cx) \text{Li}_2(cx)}{2c^2} + \frac{1}{2} x^2 \log(1 - cx) \text{Li}_2(cx) - \frac{1}{4} \int x \log(1 - cx) dx \\
 &= -\frac{1}{8} x^2 \log(1 - cx) - \frac{\log(cx) \log^2(1 - cx)}{2c^2} - \frac{x \text{Li}_2(cx)}{2c} - \frac{1}{4} x^2 \text{Li}_2(cx) - \frac{\log(1 - cx) \text{Li}_2(cx)}{2c^2} + \frac{1}{2} x^2 \log(1 - cx) \text{Li}_2(cx) \\
 &= \frac{x}{2c} - \frac{1}{8} x^2 \log(1 - cx) + \frac{(1 - cx) \log(1 - cx)}{2c^2} - \frac{(1 - cx) \log^2(1 - cx)}{2c^2} + \frac{(1 - cx)^2 \log^2(1 - cx)}{4c^2} \\
 &= \frac{13x}{8c} + \frac{x^2}{16} + \frac{(1 - cx)^2}{8c^2} + \frac{\log(1 - cx)}{8c^2} - \frac{1}{8} x^2 \log(1 - cx) + \frac{3(1 - cx) \log(1 - cx)}{2c^2} - \frac{(1 - cx)^2 \log^2(1 - cx)}{4c^2} \\
 &= \frac{13x}{8c} + \frac{x^2}{16} + \frac{(1 - cx)^2}{8c^2} + \frac{\log(1 - cx)}{8c^2} - \frac{1}{8} x^2 \log(1 - cx) + \frac{3(1 - cx) \log(1 - cx)}{2c^2} - \frac{(1 - cx)^2 \log^2(1 - cx)}{4c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.30023, size = 160, normalized size = 0.61

$$\frac{(8(c^2x^2 - 1)\log(1 - cx) - 4cx(cx + 2))\text{PolyLog}(2, cx) + 16\text{PolyLog}(3, 1 - cx) - 16\log(1 - cx)\text{PolyLog}(2, 1 - cx) + 3c^2}{16c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[1 - c\*x]\*PolyLog[2, c\*x], x]

[Out]  $(-14 + 22cx + 3c^2x^2 + 22\text{Log}[1 - cx] - 16cx\text{Log}[1 - cx] - 6c^2x^2\text{Log}[1 - cx] - 4\text{Log}[1 - cx]^2 + 4c^2x^2\text{Log}[1 - cx]^2 - 8\text{Log}[cx]\text{Log}[1 - cx]^2 + (-4cx(2 + cx) + 8(-1 + c^2x^2)\text{Log}[1 - cx])\text{PolyLog}[2, cx] - 16\text{Log}[1 - cx]\text{PolyLog}[2, 1 - cx] + 16\text{PolyLog}[3, 1 - cx])/(16c^2)$

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int x \ln(-cx + 1) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(-c\*x+1)\*polylog(2,c\*x), x)

[Out] int(x\*ln(-c\*x+1)\*polylog(2,c\*x), x)

**Maxima [A]** time = 1.15223, size = 300, normalized size = 1.15

$$\frac{c^2\left(\frac{cx^2+2x}{c^2} + \frac{2\log(cx-1)}{c^3}\right) + 4c\left(\frac{x}{c} + \frac{\log(cx-1)}{c^2}\right) + \frac{2(c^2x^2+8cx-2(c^2x^2+2cx+2\log(-cx+1))\text{Li}_2(cx)-2(c^2x^2+3cx-4)\log(-cx+1))}{c} - \frac{8(\log(cx)\log(-cx+1))}{16c}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(-c\*x+1)\*polylog(2,c\*x), x, algorithm="maxima")

[Out]  $\frac{1}{16c^2}((c^2x^2 + 2x)/c^2 + 2\log(cx - 1)/c^3) + 4c(x/c + \log(cx - 1)/c^2) + 2(c^2x^2 + 8cx - 2(c^2x^2 + 2cx + 2\log(-cx + 1))\text{dilog}(cx) - 2(c^2x^2 + 3cx - 4)\log(-cx + 1))/c - 8(\log(cx)\log(-cx + 1))^2 + 2\text{dilog}(-cx + 1)\log(-cx + 1) - 2\text{polylog}(3, -cx + 1))/c + 1/8(4c^2x^2\text{dilog}(cx) - c^2x^2 - 2cx + 2(c^2x^2 - 1)\log(-cx + 1))\log(-cx + 1)/c^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x\text{Li}_2(cx)\log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(-c\*x+1)\*polylog(2,c\*x), x, algorithm="fricas")

[Out] integral(x\*dilog(c\*x)\*log(-c\*x + 1), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \log(-cx + 1) \operatorname{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] Integral(x\*log(-c\*x + 1)\*polylog(2, c\*x), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="giac")

[Out] integrate(x\*dilog(c\*x)\*log(-c\*x + 1), x)

### 3.164 $\int \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=132

$$-x \text{PolyLog}(2, cx) + \frac{2 \text{PolyLog}(3, 1 - cx)}{c} + x \log(1 - cx) \text{PolyLog}(2, cx) - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{c} - \frac{2 \log(1 - cx) \text{PolyLog}(2, cx)}{c}$$

```
[Out] 3*x + (3*(1 - c*x)*Log[1 - c*x])/c - ((1 - c*x)*Log[1 - c*x]^2)/c - (Log[c*x]*Log[1 - c*x]^2)/c - x*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/c
+ x*Log[1 - c*x]*PolyLog[2, c*x] - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c
+ (2*PolyLog[3, 1 - c*x])/c
```

**Rubi [A]** time = 0.211916, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$ , Rules used = {6586, 2389, 2295, 6600, 2296, 6688, 6742, 6596, 2396, 2433, 2374, 6589}

$$-x \text{PolyLog}(2, cx) + \frac{2 \text{PolyLog}(3, 1 - cx)}{c} + x \log(1 - cx) \text{PolyLog}(2, cx) - \frac{\log(1 - cx) \text{PolyLog}(2, cx)}{c} - \frac{2 \log(1 - cx) \text{PolyLog}(2, cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] 3*x + (3*(1 - c*x)*Log[1 - c*x])/c - ((1 - c*x)*Log[1 - c*x]^2)/c - (Log[c*x]*Log[1 - c*x]^2)/c - x*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/c
+ x*Log[1 - c*x]*PolyLog[2, c*x] - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c
+ (2*PolyLog[3, 1 - c*x])/c
```

#### Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

#### Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))]*(h_.))*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLog[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
```

#### Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

#### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/x, x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int \log(1-cx)\text{Li}_2(cx) dx &= x \log(1-cx)\text{Li}_2(cx) + c \int \left( -\frac{1}{c} - \frac{1}{c(-1+cx)} \right) \text{Li}_2(cx) dx + \int \log^2(1-cx) dx \\
&= x \log(1-cx)\text{Li}_2(cx) - \frac{\text{Subst}\left(\int \log^2(x) dx, x, 1-cx\right)}{c} + c \int \frac{x\text{Li}_2(cx)}{1-cx} dx \\
&= -\frac{(1-cx)\log^2(1-cx)}{c} + x \log(1-cx)\text{Li}_2(cx) + \frac{2\text{Subst}\left(\int \log(x) dx, x, 1-cx\right)}{c} + c \int \left( -\frac{\text{Li}_2(cx)}{c} \right) dx \\
&= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} + x \log(1-cx)\text{Li}_2(cx) - \int \text{Li}_2(cx) dx - \int \text{Li}_2(cx) dx \\
&= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - x\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{c} + x \log(1-cx) \\
&= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\text{Li}_2(cx) - \frac{\log(1-cx)}{c} \\
&= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\text{Li}_2(cx) - \frac{\log(1-cx)}{c} \\
&= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\text{Li}_2(cx) - \frac{\log(1-cx)}{c} \\
&= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\text{Li}_2(cx) - \frac{\log(1-cx)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.0230911, size = 119, normalized size = 0.9

$$\frac{2\text{PolyLog}(3, 1-cx) - 2\log(1-cx)\text{PolyLog}(2, 1-cx) + ((cx-1)\log(1-cx) - cx)\text{PolyLog}(2, cx) + 3cx + cx\log^2(1-cx)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 - c\*x]\*PolyLog[2, c\*x], x]

[Out] (-2 + 3\*c\*x + 3\*Log[1 - c\*x] - 3\*c\*x\*Log[1 - c\*x] - Log[1 - c\*x]^2 + c\*x\*Log[1 - c\*x]^2 - Log[c\*x]\*Log[1 - c\*x]^2 + (-c\*x) + (-1 + c\*x)\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 2\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 2\*PolyLog[3, 1 - c\*x])/c

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \ln(-cx+1) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(-c\*x+1)\*polylog(2, c\*x), x)

[Out] int(ln(-c\*x+1)\*polylog(2, c\*x), x)

**Maxima [A]** time = 1.10237, size = 190, normalized size = 1.44

$$c\left(\frac{x}{c} + \frac{\log(cx-1)}{c^2}\right) + \frac{(cx\text{Li}_2(cx) - cx + (cx-1)\log(-cx+1))\log(-cx+1)}{c} - \frac{\log(cx)\log(-cx+1)^2 + 2\text{Li}_2(-cx+1)\log(-cx+1)}{c}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] c*(x/c + log(c*x - 1)/c^2) + (c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1))
*log(-c*x + 1)/c - (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1))/c + (2*c*x - (c*x + log(-c*x + 1))*dilog(c*x) - 2*(c*x - 1)*log(-c*x + 1))/c
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{Li}_2(cx) \log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \log(-cx + 1) \text{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1), x)
```

$$3.165 \quad \int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x} dx$$

**Optimal.** Leaf size=11

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

[Out] -PolyLog[2, c\*x]^2/2

**Rubi [A]** time = 0.0257549, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {6589, 6601}

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

[In] Int[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x,x]

[Out] -PolyLog[2, c\*x]^2/2

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6601

Int[(Log[1 + (e\_.)\*(x\_)]\*PolyLog[2, (c\_.)\*(x\_)])/(x\_), x\_Symbol] :> -Simp[PolyLog[2, c\*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]

#### Rubi steps

$$\int \frac{\log(1-cx)\text{Li}_2(cx)}{x} dx = -\frac{1}{2}\text{Li}_2(cx)^2$$

**Mathematica [A]** time = 0.0091507, size = 11, normalized size = 1.

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

[In] Integrate[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x,x]

[Out] -PolyLog[2, c\*x]^2/2

**Maple [A]** time = 0.046, size = 10, normalized size = 0.9

$$-\frac{(\text{polylog}(2,cx))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(-c*x+1)*polylog(2,c*x)/x,x)`

[Out] `-1/2*polylog(2,c*x)^2`

**Maxima [A]** time = 0.966354, size = 11, normalized size = 1.

$$-\frac{1}{2} \operatorname{Li}_2(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")`

[Out] `-1/2*dilog(c*x)^2`

**Fricas [A]** time = 2.53436, size = 26, normalized size = 2.36

$$-\frac{1}{2} \operatorname{Li}_2(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")`

[Out] `-1/2*dilog(c*x)^2`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(-cx + 1) \operatorname{Li}_2(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(-c*x+1)*polylog(2,c*x)/x,x)`

[Out] `Integral(log(-c*x + 1)*polylog(2, c*x)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(cx) \log(-cx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")`

[Out] `integrate(dilog(c*x)*log(-c*x + 1)/x, x)`

$$3.166 \quad \int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^2} dx$$

**Optimal.** Leaf size=111

$$-2c\text{PolyLog}(2,cx) - c\text{PolyLog}(3,cx) - 2c\text{PolyLog}(3,1-cx) + c\log(1-cx)\text{PolyLog}(2,cx) - \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x}$$

```
[Out] ((1 - c*x)*Log[1 - c*x]^2)/x + c*Log[c*x]*Log[1 - c*x]^2 - 2*c*PolyLog[2, c*x] + c*Log[1 - c*x]*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c*x]
```

**Rubi [A]** time = 0.163861, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 13, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$ , Rules used = {6591, 2395, 36, 29, 31, 6603, 2397, 2391, 6589, 6596, 2396, 2433, 2374}

$$-2c\text{PolyLog}(2,cx) - c\text{PolyLog}(3,cx) - 2c\text{PolyLog}(3,1-cx) + c\log(1-cx)\text{PolyLog}(2,cx) - \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] ((1 - c*x)*Log[1 - c*x]^2)/x + c*Log[c*x]*Log[1 - c*x]^2 - 2*c*PolyLog[2, c*x] + c*Log[1 - c*x]*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c*x]
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 6603

Int[((g\_) + Log[(f\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)]\*(h\_))\*(x\_)<sup>(m\_)</sup>\*PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))], x\_Symbol] := Simp[(x<sup>(m+1)</sup>\*(g + h\*Log[f\*(d + e\*x)<sup>n</sup>])\*PolyLog[2, c\*(a + b\*x)]/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)<sup>n</sup>])\*Log[1 - a\*c - b\*c\*x], x<sup>(m+1)</sup>/(a + b\*x), x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x<sup>(m+1)</sup>/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]</sup>

#### Rule 2397

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)]\*(b\_))<sup>(p\_)</sup>/((f\_) + (g\_)\*(x\_))<sup>2</sup>, x\_Symbol] := Simp[((d + e\*x)\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>p</sup>/((e\*f - d\*g)\*(f + g\*x)), x] - Dist[(b\*e\*n\*p)/(e\*f - d\*g), Int[(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>(p-1)</sup>/(f + g\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0]</sup>

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x<sup>n</sup>)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]</sup>

#### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))<sup>(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)<sup>p</sup>]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]</sup>

#### Rule 6596

Int[PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)]/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]<sup>2</sup>/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)]\*(b\_))<sup>(p\_)</sup>/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>p</sup>/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>(p-1)</sup>/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]</sup>

#### Rule 2433

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))<sup>(n\_)]\*(b\_))<sup>(p\_)</sup>\*((f\_) + Log[(h\_)\*((i\_) + (j\_)\*(x\_))<sup>(m\_)]\*(g\_))\*((k\_) + (l\_)\*(x\_))<sup>(r\_)</sup>, x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)<sup>r</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]<sup>m</sup>), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]</sup></sup>

#### Rule 2374

Int[(Log[(d\_)\*((e\_) + (f\_)\*(x\_))<sup>(m\_)])\*((a\_) + Log[(c\_)\*(x\_))<sup>(n\_)]\*(b\_))<sup>(p\_)</sup>/x\_, x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x<sup>m</sup>)]\*(a + b\*Log[c\*x<sup>n</sup>]))<sup>p</sup>/x\_, x\_Symbol]</sup></sup>

$\wedge n])^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[d*e, 1]$

### Rubi steps

$$\begin{aligned}
 \int \frac{\log(1-cx)\text{Li}_2(cx)}{x^2} dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{x} - c \int \left( \frac{\text{Li}_2(cx)}{x} - \frac{c\text{Li}_2(cx)}{-1+cx} \right) dx - \int \frac{\log^2(1-cx)}{x^2} dx \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} - \frac{\log(1-cx)\text{Li}_2(cx)}{x} - c \int \frac{\text{Li}_2(cx)}{x} dx + (2c) \int \frac{\log(1-cx)}{x} dx + c^2 \int \frac{\log^2(1-cx)}{x} dx \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{x} - c\text{Li}_3(cx) + c \int \frac{\log^2(1-cx)}{x} dx \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} + c \log(cx) \log^2(1-cx) - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)}{x} \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} + c \log(cx) \log^2(1-cx) - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)}{x} \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} + c \log(cx) \log^2(1-cx) - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)}{x} \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} + c \log(cx) \log^2(1-cx) - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)}{x} \\
 &= \frac{(1-cx)\log^2(1-cx)}{x} + c \log(cx) \log^2(1-cx) - 2c\text{Li}_2(cx) + c \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)}{x}
 \end{aligned}$$

**Mathematica [A]** time = 0.135536, size = 115, normalized size = 1.04

$$-c\text{PolyLog}(3, cx) - 2c\text{PolyLog}(3, 1-cx) + \frac{(cx-1)\log(1-cx)\text{PolyLog}(2, cx)}{x} + 2c(\log(1-cx) + 1)\text{PolyLog}(2, 1-cx)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^2, x]

[Out] 2\*c\*Log[c\*x]\*Log[1 - c\*x] - c\*Log[1 - c\*x]^2 + Log[1 - c\*x]^2/x + c\*Log[c\*x]\*Log[1 - c\*x]^2 + ((-1 + c\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x + 2\*c\*(1 + Log[1 - c\*x])\*PolyLog[2, 1 - c\*x] - c\*PolyLog[3, c\*x] - 2\*c\*PolyLog[3, 1 - c\*x]

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx+1)\text{polylog}(2, cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(-c\*x+1)\*polylog(2, c\*x)/x^2, x)

[Out] int(ln(-c\*x+1)\*polylog(2, c\*x)/x^2, x)

**Maxima [A]** time = 1.16597, size = 153, normalized size = 1.38

$$(\log(cx) \log(-cx+1))^2 + 2\text{Li}_2(-cx+1) \log(-cx+1) - 2\text{Li}_3(-cx+1)c + 2(\log(cx) \log(-cx+1) + \text{Li}_2(-cx+1))c - c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="maxima")

[Out] (log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))\*c + 2\*(log(c\*x)\*log(-c\*x + 1) + dilog(-c\*x + 1))\*c - c\*polylog(3, c\*x) + ((c\*x - 1)\*dilog(c\*x)\*log(-c\*x + 1) - (c\*x - 1)\*log(-c\*x + 1)^2)/x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(cx)\log(-cx+1)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="fricas")

[Out] integral(dilog(c\*x)\*log(-c\*x + 1)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(-cx+1)\text{Li}_2(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(-c\*x+1)\*polylog(2,c\*x)/x\*\*2,x)

[Out] Integral(log(-c\*x + 1)\*polylog(2, c\*x)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="giac")

[Out] integrate(dilog(c\*x)\*log(-c\*x + 1)/x^2, x)

$$3.167 \quad \int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^3} dx$$

**Optimal.** Leaf size=191

$$-\frac{1}{2}c^2\mathbf{PolyLog}(2,cx) - \frac{1}{2}c^2\mathbf{PolyLog}(3,cx) - c^2\mathbf{PolyLog}(3,1-cx) + \frac{1}{2}c^2\log(1-cx)\mathbf{PolyLog}(2,cx) + c^2\log(1-cx)\mathbf{PolyLog}(2,cx)$$

[Out]  $-(c^2*\text{Log}[x]) + c^2*\text{Log}[1 - c*x] - (c*\text{Log}[1 - c*x])/x - (c^2*\text{Log}[1 - c*x]^2)/4 + \text{Log}[1 - c*x]^2/(4*x^2) + (c^2*\text{Log}[c*x]*\text{Log}[1 - c*x]^2)/2 - (c^2*\text{PolyLog}[2, c*x])/2 + (c*\text{PolyLog}[2, c*x])/(2*x) + (c^2*\text{Log}[1 - c*x]*\text{PolyLog}[2, c*x])/2 - (\text{Log}[1 - c*x]*\text{PolyLog}[2, c*x])/(2*x^2) + c^2*\text{Log}[1 - c*x]*\text{PolyLog}[2, 1 - c*x] - (c^2*\text{PolyLog}[3, c*x])/2 - c^2*\text{PolyLog}[3, 1 - c*x]$

**Rubi [A]** time = 0.281613, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$ , Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}c^2\mathbf{PolyLog}(2,cx) - \frac{1}{2}c^2\mathbf{PolyLog}(3,cx) - c^2\mathbf{PolyLog}(3,1-cx) + \frac{1}{2}c^2\log(1-cx)\mathbf{PolyLog}(2,cx) + c^2\log(1-cx)\mathbf{PolyLog}(2,cx)$$

Antiderivative was successfully verified.

[In] Int[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^3,x]

[Out]  $-(c^2*\text{Log}[x]) + c^2*\text{Log}[1 - c*x] - (c*\text{Log}[1 - c*x])/x - (c^2*\text{Log}[1 - c*x]^2)/4 + \text{Log}[1 - c*x]^2/(4*x^2) + (c^2*\text{Log}[c*x]*\text{Log}[1 - c*x]^2)/2 - (c^2*\text{PolyLog}[2, c*x])/2 + (c*\text{PolyLog}[2, c*x])/(2*x) + (c^2*\text{Log}[1 - c*x]*\text{PolyLog}[2, c*x])/2 - (\text{Log}[1 - c*x]*\text{PolyLog}[2, c*x])/(2*x^2) + c^2*\text{Log}[1 - c*x]*\text{PolyLog}[2, 1 - c*x] - (c^2*\text{PolyLog}[3, c*x])/2 - c^2*\text{PolyLog}[3, 1 - c*x]$

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] :> Simp[(x^(m + 1)\*(g + h\*Log[f



```
*(d + e*x)^n)*PolyLog[2, c*(a + b*x)]/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

#### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

#### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^p_)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_))^(m_.)]*(a_.) + Log[(c_.)*(x_))^(n_.)]*(b
_.))^p_)/x, x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^3} dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2} \int \frac{\log^2(1-cx)}{x^3} dx - \frac{1}{2}c \int \left( \frac{\text{Li}_2(cx)}{x^2} + \frac{c\text{Li}_2(cx)}{x} - \frac{c^2\text{Li}_2(cx)}{-1+cx} \right) dx \\
&= \frac{\log^2(1-cx)}{4x^2} - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} + \frac{1}{2}c \int \frac{\log(1-cx)}{x^2(1-cx)} dx - \frac{1}{2}c \int \frac{\text{Li}_2(cx)}{x^2} dx - \frac{1}{2}c^2 \int \frac{\text{Li}_2(cx)}{x} dx \\
&= \frac{\log^2(1-cx)}{4x^2} + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2 \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2}c^2\text{Li}_3(cx) + \frac{1}{2}c \int \frac{\log(1-cx)}{x} dx \\
&= -\frac{c \log(1-cx)}{2x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2 \log(cx) \log^2(1-cx) + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2 \log(1-cx)\text{Li}_2(cx) \\
&= -\frac{c \log(1-cx)}{x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2 \log(cx) \log^2(1-cx) - \frac{1}{2}c^2\text{Li}_2(cx) + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2 \log(1-cx)\text{Li}_2(cx) \\
&= -\frac{1}{2}c^2 \log(x) + \frac{1}{2}c^2 \log(1-cx) - \frac{c \log(1-cx)}{x} - \frac{1}{4}c^2 \log^2(1-cx) + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2 \log(1-cx)\text{Li}_2(cx) \\
&= -c^2 \log(x) + c^2 \log(1-cx) - \frac{c \log(1-cx)}{x} - \frac{1}{4}c^2 \log^2(1-cx) + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2 \log(1-cx)\text{Li}_2(cx)
\end{aligned}$$

**Mathematica [A]** time = 0.274997, size = 185, normalized size = 0.97

$$\frac{1}{4} \left( \frac{(c^2x^2 - 1) \log(1-cx) + cx}{x^2} \text{PolyLog}(2, cx) - 2c^2 \text{PolyLog}(3, cx) - 4c^2 \text{PolyLog}(3, 1-cx) + 2c^2(2 \log(1-cx) + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^3,x]

[Out]  $(-2*c^2*\text{Log}[x] - 2*c^2*\text{Log}[c*x] + 4*c^2*\text{Log}[1 - c*x] - (4*c*\text{Log}[1 - c*x])/x + 2*c^2*\text{Log}[c*x]*\text{Log}[1 - c*x] - c^2*\text{Log}[1 - c*x]^2 + \text{Log}[1 - c*x]^2/x^2 + 2*c^2*\text{Log}[c*x]*\text{Log}[1 - c*x]^2 + (2*(c*x + (-1 + c^2*x^2))*\text{Log}[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*\text{Log}[1 - c*x])*PolyLog[2, 1 - c*x] - 2*c^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x])/4$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx + 1) \text{polylog}(2, cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^3,x)

[Out] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^3,x)

**Maxima [A]** time = 1.66414, size = 219, normalized size = 1.15

$$\frac{1}{2} (\log(cx) \log(-cx + 1))^2 + 2 \text{Li}_2(-cx + 1) \log(-cx + 1) - 2 \text{Li}_3(-cx + 1) c^2 + \frac{1}{2} (\log(cx) \log(-cx + 1) + \text{Li}_2(-cx + 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^3,x, algorithm="maxima")

[Out]  $1/2*(\text{log}(c*x)*\text{log}(-c*x + 1)^2 + 2*\text{dilog}(-c*x + 1)*\text{log}(-c*x + 1) - 2*\text{polylog}(3, -c*x + 1))*c^2 + 1/2*(\text{log}(c*x)*\text{log}(-c*x + 1) + \text{dilog}(-c*x + 1))*c^2 - c^2*\text{log}(x) - 1/2*c^2*\text{polylog}(3, c*x) - 1/4*((c^2*x^2 - 1)*\text{log}(-c*x + 1)^2 - 2*(c*x + (c^2*x^2 - 1)*\text{log}(-c*x + 1))*\text{dilog}(c*x) - 4*(c^2*x^2 - c*x)*\text{log}(-c*x + 1))/x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(cx) \log(-cx + 1)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^3,x, algorithm="fricas")

[Out] integral(dilog(c\*x)\*log(-c\*x + 1)/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(-cx + 1) \text{Li}_2(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(-c\*x+1)\*polylog(2,c\*x)/x\*\*3,x)

[Out] Integral(log(-c\*x + 1)\*polylog(2, c\*x)/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(cx) \log(-cx + 1)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^3,x, algorithm="giac")

[Out] integrate(dilog(c\*x)\*log(-c\*x + 1)/x^3, x)

$$3.168 \quad \int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^4} dx$$

**Optimal.** Leaf size=245

$$-\frac{2}{9}c^3\text{PolyLog}(2,cx) - \frac{1}{3}c^3\text{PolyLog}(3,cx) - \frac{2}{3}c^3\text{PolyLog}(3,1-cx) + \frac{c^2\text{PolyLog}(2,cx)}{3x} + \frac{1}{3}c^3\log(1-cx)\text{PolyLog}(2,cx)$$

```
[Out] (7*c^2)/(36*x) - (3*c^3*Log[x])/4 + (3*c^3*Log[1 - c*x])/4 - (7*c*Log[1 - c*x])/(36*x^2) - (5*c^2*Log[1 - c*x])/(9*x) - (c^3*Log[1 - c*x]^2)/9 + Log[1 - c*x]^2/(9*x^3) + (c^3*Log[c*x]*Log[1 - c*x]^2)/3 - (2*c^3*PolyLog[2, c*x])/9 + (c*PolyLog[2, c*x])/(6*x^2) + (c^2*PolyLog[2, c*x])/(3*x) + (c^3*Log[1 - c*x]*PolyLog[2, c*x])/3 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*x^3) + (2*c^3*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*PolyLog[3, c*x])/3 - (2*c^3*PolyLog[3, 1 - c*x])/3
```

**Rubi [A]** time = 0.354818, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 17, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$ , Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{2}{9}c^3\text{PolyLog}(2,cx) - \frac{1}{3}c^3\text{PolyLog}(3,cx) - \frac{2}{3}c^3\text{PolyLog}(3,1-cx) + \frac{c^2\text{PolyLog}(2,cx)}{3x} + \frac{1}{3}c^3\log(1-cx)\text{PolyLog}(2,cx)$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^4, x]
```

```
[Out] (7*c^2)/(36*x) - (3*c^3*Log[x])/4 + (3*c^3*Log[1 - c*x])/4 - (7*c*Log[1 - c*x])/(36*x^2) - (5*c^2*Log[1 - c*x])/(9*x) - (c^3*Log[1 - c*x]^2)/9 + Log[1 - c*x]^2/(9*x^3) + (c^3*Log[c*x]*Log[1 - c*x]^2)/3 - (2*c^3*PolyLog[2, c*x])/9 + (c*PolyLog[2, c*x])/(6*x^2) + (c^2*PolyLog[2, c*x])/(3*x) + (c^3*Log[1 - c*x]*PolyLog[2, c*x])/3 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*x^3) + (2*c^3*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*PolyLog[3, c*x])/3 - (2*c^3*PolyLog[3, 1 - c*x])/3
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/(a + b*x), x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_.) + (e_.)*(x_))]*(x_)^(m_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_.))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_.))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_.))^(n\_.))\*((b\_.)^(p\_.))/(x\_.), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{\log(1-cx)\text{Li}_2(cx)}{x^4} dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{3x^3} - \frac{1}{3} \int \frac{\log^2(1-cx)}{x^4} dx - \frac{1}{3}c \int \left( \frac{\text{Li}_2(cx)}{x^3} + \frac{c\text{Li}_2(cx)}{x^2} + \frac{c^2\text{Li}_2(cx)}{x} - \frac{c^3}{3} \right) dx \\
 &= \frac{\log^2(1-cx)}{9x^3} - \frac{\log(1-cx)\text{Li}_2(cx)}{3x^3} + \frac{1}{9}(2c) \int \frac{\log(1-cx)}{x^3(1-cx)} dx - \frac{1}{3}c \int \frac{\text{Li}_2(cx)}{x^3} dx - \frac{1}{3}c^2 \int \frac{\log(1-cx)}{x^2} dx \\
 &= \frac{\log^2(1-cx)}{9x^3} + \frac{c\text{Li}_2(cx)}{6x^2} + \frac{c^2\text{Li}_2(cx)}{3x} + \frac{1}{3}c^3 \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{3x^3} - \frac{1}{3}c^3 \int \frac{\log(1-cx)}{x} dx \\
 &= -\frac{c \log(1-cx)}{12x^2} - \frac{c^2 \log(1-cx)}{3x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3 \log(cx) \log^2(1-cx) + \frac{c\text{Li}_2(cx)}{6x^2} + \frac{c^2}{12x} \\
 &= -\frac{7c \log(1-cx)}{36x^2} - \frac{5c^2 \log(1-cx)}{9x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3 \log(cx) \log^2(1-cx) - \frac{2}{9}c^3 \text{Li}_2(cx) \\
 &= \frac{c^2}{12x} - \frac{5}{12}c^3 \log(x) + \frac{5}{12}c^3 \log(1-cx) - \frac{7c \log(1-cx)}{36x^2} - \frac{5c^2 \log(1-cx)}{9x} - \frac{1}{9}c^3 \log^2(1-cx) \\
 &= \frac{7c^2}{36x} - \frac{3}{4}c^3 \log(x) + \frac{3}{4}c^3 \log(1-cx) - \frac{7c \log(1-cx)}{36x^2} - \frac{5c^2 \log(1-cx)}{9x} - \frac{1}{9}c^3 \log^2(1-cx)
 \end{aligned}$$

**Mathematica [A]** time = 0.23515, size = 246, normalized size = 1.

$$-12c^3x^3\text{PolyLog}(3, cx) - 24c^3x^3\text{PolyLog}(3, 1 - cx) + 8c^3x^3(3\log(1 - cx) + 1)\text{PolyLog}(2, 1 - cx) + 6\left(2(c^3x^3 - 1)\log(1 - cx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^4, x]

[Out] (7\*c^2\*x^2 - 4\*c^3\*x^3 - 15\*c^3\*x^3\*Log[x] - 12\*c^3\*x^3\*Log[c\*x] - 7\*c\*x\*Log[1 - c\*x] - 20\*c^2\*x^2\*Log[1 - c\*x] + 27\*c^3\*x^3\*Log[1 - c\*x] + 8\*c^3\*x^3\*Log[c\*x]\*Log[1 - c\*x] + 4\*Log[1 - c\*x]^2 - 4\*c^3\*x^3\*Log[1 - c\*x]^2 + 12\*c^3\*x^3\*Log[c\*x]\*Log[1 - c\*x]^2 + 6\*(c\*x\*(1 + 2\*c\*x) + 2\*(-1 + c^3\*x^3))\*Log[1 - c\*x])\*PolyLog[2, c\*x] + 8\*c^3\*x^3\*(1 + 3\*Log[1 - c\*x])\*PolyLog[2, 1 - c\*x] - 12\*c^3\*x^3\*PolyLog[3, c\*x] - 24\*c^3\*x^3\*PolyLog[3, 1 - c\*x])/(36\*x^3)

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx + 1) \text{polylog}(2, cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^4,x)

[Out] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^4,x)

**Maxima [A]** time = 1.65255, size = 254, normalized size = 1.04

$$\frac{1}{3} \left( \log(cx) \log(-cx + 1)^2 + 2 \text{Li}_2(-cx + 1) \log(-cx + 1) - 2 \text{Li}_3(-cx + 1) \right) c^3 + \frac{2}{9} \left( \log(cx) \log(-cx + 1) + \text{Li}_2(-cx + 1) \right) c^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^4,x, algorithm="maxima")

[Out] 1/3\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))\*c^3 + 2/9\*(log(c\*x)\*log(-c\*x + 1) + dilog(-c\*x + 1))\*c^3 - 3/4\*c^3\*log(x) - 1/3\*c^3\*polylog(3, c\*x) + 1/36\*(7\*c^2\*x^2 - 4\*(c^3\*x^3 - 1)\*log(-c\*x + 1)^2 + 6\*(2\*c^2\*x^2 + c\*x + 2\*(c^3\*x^3 - 1)\*log(-c\*x + 1))\*dilog(c\*x) + (27\*c^3\*x^3 - 20\*c^2\*x^2 - 7\*c\*x)\*log(-c\*x + 1))/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(cx) \log(-cx + 1)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^4,x, algorithm="fricas")



[Out] `integral(dilog(c*x)*log(-c*x + 1)/x^4, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(-c*x+1)*polylog(2,c*x)/x**4,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(cx) \log(-cx + 1)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")`

[Out] `integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)`

$$3.169 \quad \int \frac{\log(1-cx)\text{PolyLog}(2,cx)}{x^5} dx$$

**Optimal.** Leaf size=287

$$\frac{c^2\text{PolyLog}(2,cx)}{8x^2} - \frac{1}{8}c^4\text{PolyLog}(2,cx) - \frac{1}{4}c^4\text{PolyLog}(3,cx) - \frac{1}{2}c^4\text{PolyLog}(3,1-cx) + \frac{c^3\text{PolyLog}(2,cx)}{4x} + \frac{1}{4}c^4\log(1-cx)$$

[Out] (5\*c^2)/(144\*x^2) + (7\*c^3)/(36\*x) - (41\*c^4\*Log[x])/72 + (41\*c^4\*Log[1 - c\*x])/72 - (5\*c\*Log[1 - c\*x])/(72\*x^3) - (c^2\*Log[1 - c\*x])/(8\*x^2) - (3\*c^3\*Log[1 - c\*x])/(8\*x) - (c^4\*Log[1 - c\*x]^2)/16 + Log[1 - c\*x]^2/(16\*x^4) + (c^4\*Log[c\*x]\*Log[1 - c\*x]^2)/4 - (c^4\*PolyLog[2, c\*x])/8 + (c\*PolyLog[2, c\*x])/(12\*x^3) + (c^2\*PolyLog[2, c\*x])/(8\*x^2) + (c^3\*PolyLog[2, c\*x])/(4\*x) + (c^4\*Log[1 - c\*x]\*PolyLog[2, c\*x])/4 - (Log[1 - c\*x]\*PolyLog[2, c\*x])/(4\*x^4) + (c^4\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/2 - (c^4\*PolyLog[3, c\*x])/4 - (c^4\*PolyLog[3, 1 - c\*x])/2

**Rubi [A]** time = 0.449191, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$ , Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$\frac{c^2\text{PolyLog}(2,cx)}{8x^2} - \frac{1}{8}c^4\text{PolyLog}(2,cx) - \frac{1}{4}c^4\text{PolyLog}(3,cx) - \frac{1}{2}c^4\text{PolyLog}(3,1-cx) + \frac{c^3\text{PolyLog}(2,cx)}{4x} + \frac{1}{4}c^4\log(1-cx)$$

Antiderivative was successfully verified.

[In] Int[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^5,x]

[Out] (5\*c^2)/(144\*x^2) + (7\*c^3)/(36\*x) - (41\*c^4\*Log[x])/72 + (41\*c^4\*Log[1 - c\*x])/72 - (5\*c\*Log[1 - c\*x])/(72\*x^3) - (c^2\*Log[1 - c\*x])/(8\*x^2) - (3\*c^3\*Log[1 - c\*x])/(8\*x) - (c^4\*Log[1 - c\*x]^2)/16 + Log[1 - c\*x]^2/(16\*x^4) + (c^4\*Log[c\*x]\*Log[1 - c\*x]^2)/4 - (c^4\*PolyLog[2, c\*x])/8 + (c\*PolyLog[2, c\*x])/(12\*x^3) + (c^2\*PolyLog[2, c\*x])/(8\*x^2) + (c^3\*PolyLog[2, c\*x])/(4\*x) + (c^4\*Log[1 - c\*x]\*PolyLog[2, c\*x])/4 - (Log[1 - c\*x]\*PolyLog[2, c\*x])/(4\*x^4) + (c^4\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/2 - (c^4\*PolyLog[3, c\*x])/4 - (c^4\*PolyLog[3, 1 - c\*x])/2

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e^n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

### Rule 2410

Int[(Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))])\*(x\_)^(m\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.)/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{\log(1-cx)\text{Li}_2(cx)}{x^5} dx &= -\frac{\log(1-cx)\text{Li}_2(cx)}{4x^4} - \frac{1}{4} \int \frac{\log^2(1-cx)}{x^5} dx - \frac{1}{4}c \int \left( \frac{\text{Li}_2(cx)}{x^4} + \frac{c\text{Li}_2(cx)}{x^3} + \frac{c^2\text{Li}_2(cx)}{x^2} + \frac{c^3\text{Li}_2(cx)}{x} \right) dx \\
 &= \frac{\log^2(1-cx)}{16x^4} - \frac{\log(1-cx)\text{Li}_2(cx)}{4x^4} + \frac{1}{8}c \int \frac{\log(1-cx)}{x^4(1-cx)} dx - \frac{1}{4}c \int \frac{\text{Li}_2(cx)}{x^4} dx - \frac{1}{4}c^2 \int \frac{\text{Li}_2(cx)}{x^3} dx \\
 &= \frac{\log^2(1-cx)}{16x^4} + \frac{c\text{Li}_2(cx)}{12x^3} + \frac{c^2\text{Li}_2(cx)}{8x^2} + \frac{c^3\text{Li}_2(cx)}{4x} + \frac{1}{4}c^4 \log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{4x^4} \\
 &= -\frac{c \log(1-cx)}{36x^3} - \frac{c^2 \log(1-cx)}{16x^2} - \frac{c^3 \log(1-cx)}{4x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4 \log(cx) \log^2(1-cx) + \frac{c^4 \log^3(1-cx)}{4} \\
 &= -\frac{5c \log(1-cx)}{72x^3} - \frac{c^2 \log(1-cx)}{8x^2} - \frac{3c^3 \log(1-cx)}{8x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4 \log(cx) \log^2(1-cx) + \frac{c^4 \log^3(1-cx)}{4} \\
 &= \frac{c^2}{72x^2} + \frac{13c^3}{144x} - \frac{49}{144}c^4 \log(x) + \frac{49}{144}c^4 \log(1-cx) - \frac{5c \log(1-cx)}{72x^3} - \frac{c^2 \log(1-cx)}{8x^2} - \frac{3c^3 \log(1-cx)}{8x} \\
 &= \frac{5c^2}{144x^2} + \frac{7c^3}{36x} - \frac{41}{72}c^4 \log(x) + \frac{41}{72}c^4 \log(1-cx) - \frac{5c \log(1-cx)}{72x^3} - \frac{c^2 \log(1-cx)}{8x^2} - \frac{3c^3 \log(1-cx)}{8x}
 \end{aligned}$$

**Mathematica [A]** time = 0.235598, size = 277, normalized size = 0.97

$$\frac{-36c^4x^4\text{PolyLog}(3, cx) - 72c^4x^4\text{PolyLog}(3, 1 - cx) + 18c^4x^4(4\log(1 - cx) + 1)\text{PolyLog}(2, 1 - cx) + 6\left(cx(6c^2x^2 + 3\right)}{144x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Log[1 - c\*x]\*PolyLog[2, c\*x])/x^5,x]

[Out] (5\*c^2\*x^2 + 28\*c^3\*x^3 - 18\*c^4\*x^4 - 49\*c^4\*x^4\*Log[x] - 33\*c^4\*x^4\*Log[c\*x] - 10\*c\*x\*Log[1 - c\*x] - 18\*c^2\*x^2\*Log[1 - c\*x] - 54\*c^3\*x^3\*Log[1 - c\*x] + 82\*c^4\*x^4\*Log[1 - c\*x] + 18\*c^4\*x^4\*Log[c\*x]\*Log[1 - c\*x] + 9\*Log[1 - c\*x]^2 - 9\*c^4\*x^4\*Log[1 - c\*x]^2 + 36\*c^4\*x^4\*Log[c\*x]\*Log[1 - c\*x]^2 + 6\*(c\*x\*(2 + 3\*c\*x + 6\*c^2\*x^2) + 6\*(-1 + c^4\*x^4)\*Log[1 - c\*x])\*PolyLog[2, c\*x] + 18\*c^4\*x^4\*(1 + 4\*Log[1 - c\*x])\*PolyLog[2, 1 - c\*x] - 36\*c^4\*x^4\*PolyLog[3, c\*x] - 72\*c^4\*x^4\*PolyLog[3, 1 - c\*x])/(144\*x^4)

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx + 1) \text{polylog}(2, cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^5,x)

[Out] int(ln(-c\*x+1)\*polylog(2,c\*x)/x^5,x)

**Maxima [A]** time = 1.73984, size = 289, normalized size = 1.01

$$\frac{1}{4} \left( \log(cx) \log(-cx + 1)^2 + 2 \text{Li}_2(-cx + 1) \log(-cx + 1) - 2 \text{Li}_3(-cx + 1) \right) c^4 + \frac{1}{8} \left( \log(cx) \log(-cx + 1) + \text{Li}_2(-cx + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^5,x, algorithm="maxima")

[Out] 1/4\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))\*c^4 + 1/8\*(log(c\*x)\*log(-c\*x + 1) + dilog(-c\*x + 1))\*c^4 - 4/172\*c^4\*log(x) - 1/4\*c^4\*polylog(3, c\*x) + 1/144\*(28\*c^3\*x^3 + 5\*c^2\*x^2 - 9\*(c^4\*x^4 - 1)\*log(-c\*x + 1)^2 + 6\*(6\*c^3\*x^3 + 3\*c^2\*x^2 + 2\*c\*x + 6\*(c^4\*x^4 - 1)\*log(-c\*x + 1))\*dilog(c\*x) + 2\*(41\*c^4\*x^4 - 27\*c^3\*x^3 - 9\*c^2\*x^2 - 5\*c\*x)\*log(-c\*x + 1))/x^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(cx) \log(-cx + 1)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(-c\*x+1)\*polylog(2,c\*x)/x^5,x, algorithm="fricas")

[Out] `integral(dilog(c*x)*log(-c*x + 1)/x^5, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(-c*x+1)*polylog(2,c*x)/x**5,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(cx) \log(-cx + 1)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")`

[Out] `integrate(dilog(c*x)*log(-c*x + 1)/x^5, x)`

### 3.170 $\int x^2(g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=423

$$-\frac{hx \text{PolyLog}(2, cx)}{3c^2} + \frac{2h \text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{3c^3} - \frac{2h \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{3c^3} + \frac{1}{3}$$

```
[Out] (121*h*x)/(108*c^2) + (13*h*x^2)/(216*c) + (h*x^3)/81 + (h*(1 - c*x)^2)/(6*c^3) - (2*h*(1 - c*x)^3)/(81*c^3) + (13*h*Log[1 - c*x])/(108*c^3) - (h*x^2*Log[1 - c*x])/(12*c) - (h*x^3*Log[1 - c*x])/27 + (h*(1 - c*x)*Log[1 - c*x])/(3*c^3) + (h*Log[1 - c*x]^2)/(9*c^3) - (h*Log[c*x]*Log[1 - c*x]^2)/(3*c^3) + (x^3*Log[1 - c*x]*(g + h*Log[1 - c*x]))/9 + ((1 - c*x)*(g + 2*h*Log[1 - c*x]))/(3*c^3) - ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/(6*c^3) + ((1 - c*x)^3*(g + 2*h*Log[1 - c*x]))/(27*c^3) - (Log[1 - c*x]*(g + 2*h*Log[1 - c*x]))/(9*c^3) - (h*x*PolyLog[2, c*x])/(3*c^2) - (h*x^2*PolyLog[2, c*x])/(6*c) - (h*x^3*PolyLog[2, c*x])/9 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3) + (x^3*(g + h*Log[1 - c*x])*PolyLog[2, c*x])/3 - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + (2*h*PolyLog[3, 1 - c*x])/(3*c^3)
```

**Rubi [A]** time = 0.611175, antiderivative size = 366, normalized size of antiderivative = 0.87, number of steps used = 37, number of rules used = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 1$ , Rules used = {6603, 2439, 2410, 2389, 2295, 2395, 43, 2390, 2301, 2411, 2334, 12, 14, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$-\frac{hx \text{PolyLog}(2, cx)}{3c^2} + \frac{2h \text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{3c^3} - \frac{2h \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{3c^3} + \frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x], x]
```

```
[Out] (107*h*x)/(108*c^2) + (23*h*x^2)/(216*c) + (2*h*x^3)/81 + (h*(1 - c*x)^2)/(12*c^3) - (h*(1 - c*x)^3)/(81*c^3) + (23*h*Log[1 - c*x])/(108*c^3) - (5*h*x^2*Log[1 - c*x])/(36*c) - (2*h*x^3*Log[1 - c*x])/27 + (4*h*(1 - c*x)*Log[1 - c*x])/(9*c^3) - (h*Log[c*x]*Log[1 - c*x]^2)/(3*c^3) + (x^3*Log[1 - c*x]*(g + h*Log[1 - c*x]))/9 + (((18*(1 - c*x))/c^3 - (9*(1 - c*x)^2)/c^3 + (2*(1 - c*x)^3)/c^3 - (6*Log[1 - c*x])/c^3)*(g + h*Log[1 - c*x]))/54 - (h*x*PolyLog[2, c*x])/(3*c^2) - (h*x^2*PolyLog[2, c*x])/(6*c) - (h*x^3*PolyLog[2, c*x])/9 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3) + (x^3*(g + h*Log[1 - c*x])*PolyLog[2, c*x])/3 - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + (2*h*PolyLog[3, 1 - c*x])/(3*c^3)
```

#### Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/(a + b*x), x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

#### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
```

] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m])]/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2334



```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*(b*d - a*e) + e, 0]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

&& EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int x^2(g + h \log(1 - cx))\text{Li}_2(cx) dx &= \frac{1}{3}x^3(g + h \log(1 - cx))\text{Li}_2(cx) + \frac{1}{3} \int x^2 \log(1 - cx)(g + h \log(1 - cx)) dx + \frac{1}{3}(ch) \int \\
 &= \frac{1}{9}x^3 \log(1 - cx)(g + h \log(1 - cx)) + \frac{1}{3}x^3(g + h \log(1 - cx))\text{Li}_2(cx) + \frac{1}{9}c \int \frac{x^3(g + h \log(1 - cx))}{1 - cx} dx \\
 &= \frac{1}{9}x^3 \log(1 - cx)(g + h \log(1 - cx)) - \frac{hx\text{Li}_2(cx)}{3c^2} - \frac{hx^2\text{Li}_2(cx)}{6c} - \frac{1}{9}hx^3\text{Li}_2(cx) - \frac{h \log(1 - cx)}{9} \\
 &= -\frac{hx^2 \log(1 - cx)}{12c} - \frac{1}{27}hx^3 \log(1 - cx) - \frac{h \log(cx) \log^2(1 - cx)}{3c^3} + \frac{1}{9}x^3 \log(1 - cx)(g + h \log(1 - cx)) \\
 &= \frac{hx}{3c^2} - \frac{5hx^2 \log(1 - cx)}{36c} - \frac{2}{27}hx^3 \log(1 - cx) + \frac{h(1 - cx) \log(1 - cx)}{3c^3} - \frac{h \log(cx) \log^2(1 - cx)}{3c^3} \\
 &= \frac{61hx}{108c^2} + \frac{13hx^2}{216c} + \frac{hx^3}{81} + \frac{13h \log(1 - cx)}{108c^3} - \frac{5hx^2 \log(1 - cx)}{36c} - \frac{2}{27}hx^3 \log(1 - cx) + \frac{h(1 - cx) \log(1 - cx)}{3c^3} - \frac{h \log(cx) \log^2(1 - cx)}{3c^3} \\
 &= \frac{107hx}{108c^2} + \frac{23hx^2}{216c} + \frac{2hx^3}{81} + \frac{h(1 - cx)^2}{12c^3} - \frac{h(1 - cx)^3}{81c^3} + \frac{23h \log(1 - cx)}{108c^3} - \frac{5hx^2 \log(1 - cx)}{36c} \\
 &= \frac{107hx}{108c^2} + \frac{23hx^2}{216c} + \frac{2hx^3}{81} + \frac{h(1 - cx)^2}{12c^3} - \frac{h(1 - cx)^3}{81c^3} + \frac{23h \log(1 - cx)}{108c^3} - \frac{5hx^2 \log(1 - cx)}{36c}
 \end{aligned}$$

**Mathematica [A]** time = 0.46092, size = 252, normalized size = 0.6

$$\frac{g(18c^3x^3\text{PolyLog}(2, cx) - cx(2c^2x^2 + 3cx + 6) + 6(c^3x^3 - 1)\log(1 - cx))}{54c^3} + \frac{h(12(6(c^3x^3 - 1)\log(1 - cx) - cx(2c^2x^2 + 3cx + 6) + 6(c^3x^3 - 1)\log(1 - cx)))}{54c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x], x]

[Out] (g\*(-(c\*x\*(6 + 3\*c\*x + 2\*c^2\*x^2)) + 6\*(-1 + c^3\*x^3)\*Log[1 - c\*x] + 18\*c^3\*x^3\*PolyLog[2, c\*x]))/(54\*c^3) + (h\*(186\*c\*x + 33\*c^2\*x^2 + 8\*c^3\*x^3 + 18\*6\*Log[1 - c\*x] - 120\*c\*x\*Log[1 - c\*x] - 42\*c^2\*x^2\*Log[1 - c\*x] - 24\*c^3\*x^3\*Log[1 - c\*x] - 24\*Log[1 - c\*x]^2 + 24\*c^3\*x^3\*Log[1 - c\*x]^2 - 72\*Log[c\*x]\*Log[1 - c\*x]^2 + 12\*(-(c\*x\*(6 + 3\*c\*x + 2\*c^2\*x^2)) + 6\*(-1 + c^3\*x^3)\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 144\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 144\*PolyLog[3, 1 - c\*x]))/(216\*c^3)

**Maple [F]** time = 0.23, size = 0, normalized size = 0.

$$\int x^2 (g + h \ln(-cx + 1)) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)`

[Out] `int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{18}h \left( \frac{(2c^3x^3 + 3c^2x^2 + 6cx - 6(c^3x^3 - 1)\log(-cx + 1))\text{Li}_2(cx)}{c^3} - \frac{\frac{4}{9}c^3x^3 - \frac{1}{9}(6x^3\log(-cx + 1) - c\left(\frac{2c^2x^3 + 3cx^2 + 6x}{c^3}\right))}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")`

[Out] `-1/18*h*((2*c^3*x^3 + 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 - 1)*log(-c*x + 1))*dilog(c*x)/c^3 - integrate((6*(c^3*x^3 - 1)*log(-c*x + 1)^2 - (2*c^3*x^3 + 3*c^2*x^2 + 6*c*x)*log(-c*x + 1))/x, x)/c^3) + 1/54*(18*c^3*x^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*g/c^3`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(hx^2\text{Li}_2(cx)\log(-cx + 1) + gx^2\text{Li}_2(cx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")`

[Out] `integral(h*x^2*dilog(c*x)*log(-c*x + 1) + g*x^2*dilog(c*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (h\log(-cx + 1) + g)x^2\text{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")`

[Out] `integrate((h*log(-c*x + 1) + g)*x^2*dilog(c*x), x)`

### 3.171 $\int x(g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=330

$$\frac{h \text{PolyLog}(3, 1 - cx)}{c^2} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{2c^2} - \frac{h \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{c^2} + \frac{1}{2} x^2 \text{PolyLog}(2, cx) (h \log(1 - cx) + g)$$

[Out] (13\*h\*x)/(8\*c) + (h\*x^2)/16 + (h\*(1 - c\*x)^2)/(8\*c^2) + (h\*Log[1 - c\*x])/(8\*c^2) - (h\*x^2\*Log[1 - c\*x])/8 + (h\*(1 - c\*x)\*Log[1 - c\*x])/(2\*c^2) + (h\*Log[1 - c\*x]^2)/(4\*c^2) - (h\*Log[c\*x]\*Log[1 - c\*x]^2)/(2\*c^2) + (x^2\*Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/4 + ((1 - c\*x)\*(g + 2\*h\*Log[1 - c\*x]))/(2\*c^2) - ((1 - c\*x)^2\*(g + 2\*h\*Log[1 - c\*x]))/(8\*c^2) - (Log[1 - c\*x]\*(g + 2\*h\*Log[1 - c\*x]))/(4\*c^2) - (h\*x\*PolyLog[2, c\*x])/(2\*c) - (h\*x^2\*PolyLog[2, c\*x])/4 - (h\*Log[1 - c\*x]\*PolyLog[2, c\*x])/(2\*c^2) + (x^2\*(g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/2 - (h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/c^2 + (h\*PolyLog[3, 1 - c\*x])/c^2

**Rubi [A]** time = 0.44859, antiderivative size = 287, normalized size of antiderivative = 0.87, number of steps used = 30, number of rules used = 20, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$ , Rules used = {6603, 2439, 2410, 2389, 2295, 2395, 43, 2390, 2301, 2411, 2334, 12, 14, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$\frac{h \text{PolyLog}(3, 1 - cx)}{c^2} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{2c^2} - \frac{h \log(1 - cx) \text{PolyLog}(2, 1 - cx)}{c^2} + \frac{1}{2} x^2 \text{PolyLog}(2, cx) (h \log(1 - cx) + g)$$

Antiderivative was successfully verified.

[In] Int[x\*(g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x], x]

[Out] (3\*h\*x)/(2\*c) + (h\*x^2)/8 + (h\*(1 - c\*x)^2)/(16\*c^2) + (h\*Log[1 - c\*x])/(4\*c^2) - (h\*x^2\*Log[1 - c\*x])/4 + (3\*h\*(1 - c\*x)\*Log[1 - c\*x])/(4\*c^2) - (h\*Log[c\*x]\*Log[1 - c\*x]^2)/(2\*c^2) + (x^2\*Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/4 + (((4\*(1 - c\*x))/c^2 - (1 - c\*x)^2/c^2 - (2\*Log[1 - c\*x])/c^2)\*(g + h\*Log[1 - c\*x]))/8 - (h\*x\*PolyLog[2, c\*x])/(2\*c) - (h\*x^2\*PolyLog[2, c\*x])/4 - (h\*Log[1 - c\*x]\*PolyLog[2, c\*x])/(2\*c^2) + (x^2\*(g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/2 - (h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/c^2 + (h\*PolyLog[3, 1 - c\*x])/c^2

#### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] :> Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

#### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*(x\_)^(r\_.), x\_Symbol] :> Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ

[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] :> Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.)), x\_Symbol] :> Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2334

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] :> With[{u = IntHide[x^m\*(d + e\*x^r)^q, x]}, Simp[u\*(a + b\*Log[c\*x^n]), x] - Dist[b\*n, Int[SimplifyIntegrand[u/x, x], x], x] /;

FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)]/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_))^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_))^(n\_.)]\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int x(g + h \log(1 - cx))\text{Li}_2(cx) dx &= \frac{1}{2}x^2(g + h \log(1 - cx))\text{Li}_2(cx) + \frac{1}{2} \int x \log(1 - cx)(g + h \log(1 - cx)) dx + \frac{1}{2}(ch) \int \frac{x^2(g + h \log(1 - cx))}{1 - cx} dx \\
&= \frac{1}{4}x^2 \log(1 - cx)(g + h \log(1 - cx)) + \frac{1}{2}x^2(g + h \log(1 - cx))\text{Li}_2(cx) + \frac{1}{4}c \int \frac{x^2(g + h \log(1 - cx))}{1 - cx} dx \\
&= \frac{1}{4}x^2 \log(1 - cx)(g + h \log(1 - cx)) - \frac{hx\text{Li}_2(cx)}{2c} - \frac{1}{4}hx^2\text{Li}_2(cx) - \frac{h \log(1 - cx)\text{Li}_2(cx)}{2c^2} \\
&= -\frac{1}{8}hx^2 \log(1 - cx) - \frac{h \log(cx) \log^2(1 - cx)}{2c^2} + \frac{1}{4}x^2 \log(1 - cx)(g + h \log(1 - cx)) + \frac{hx}{2c} \\
&= \frac{hx}{2c} - \frac{1}{4}hx^2 \log(1 - cx) + \frac{h(1 - cx) \log(1 - cx)}{2c^2} - \frac{h \log(cx) \log^2(1 - cx)}{2c^2} + \frac{1}{4}x^2 \log(1 - cx)(g + h \log(1 - cx)) \\
&= \frac{7hx}{8c} + \frac{hx^2}{16} + \frac{h \log(1 - cx)}{8c^2} - \frac{1}{4}hx^2 \log(1 - cx) + \frac{3h(1 - cx) \log(1 - cx)}{4c^2} - \frac{h \log^2(1 - cx)}{8c^2} \\
&= \frac{3hx}{2c} + \frac{hx^2}{8} + \frac{h(1 - cx)^2}{16c^2} + \frac{h \log(1 - cx)}{4c^2} - \frac{1}{4}hx^2 \log(1 - cx) + \frac{3h(1 - cx) \log(1 - cx)}{4c^2} \\
&= \frac{3hx}{2c} + \frac{hx^2}{8} + \frac{h(1 - cx)^2}{16c^2} + \frac{h \log(1 - cx)}{4c^2} - \frac{1}{4}hx^2 \log(1 - cx) + \frac{3h(1 - cx) \log(1 - cx)}{4c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.339235, size = 211, normalized size = 0.64

$$\frac{g(4c^2x^2\text{PolyLog}(2, cx) + 2(c^2x^2 - 1)\log(1 - cx) - cx(cx + 2))}{8c^2} + \frac{h((8(c^2x^2 - 1)\log(1 - cx) - 4cx(cx + 2))\text{PolyLog}(2, cx) + 16c^2x^2\text{PolyLog}(3, 1 - cx))}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x], x]
```

```
[Out] (g*(-(c*x*(2 + c*x)) + 2*(-1 + c^2*x^2)*Log[1 - c*x] + 4*c^2*x^2*PolyLog[2, c*x]))/(8*c^2) + (h*(-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*Log[1 - c*x] - 6*c^2*x^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 - c*x]^2 - 8*Log[c*x]*Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2))*Log[1 - c*x])*PolyLog[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*PolyLog[3, 1 - c*x]))/(16*c^2)
```

**Maple [F]** time = 0.227, size = 0, normalized size = 0.

$$\int x(g + h \ln(-cx + 1)) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g+h*ln(-c*x+1))*polylog(2,c*x), x)
```

[Out] `int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}h\left(\frac{(c^2x^2 + 2cx - 2(c^2x^2 - 1)\log(-cx + 1))\text{Li}_2(cx)}{c^2} - \frac{\frac{1}{2}c^2x^2 - \frac{1}{4}\left(2x^2\log(-cx + 1) - c\left(\frac{cx^2 + 2x}{c^2} + \frac{2\log(cx-1)}{c^3}\right)\right)}{c^2} + (c^2x^2 + 2cx - 2(c^2x^2 - 1)\log(-cx + 1))\text{dilog}(cx)/c^2 - \int \frac{(2(c^2x^2 - 1)\log(-cx + 1)^2 - (c^2x^2 + 2cx)\log(-cx + 1))}{x} dx/c^2 + 1/8(4c^2x^2\text{dilog}(cx) - c^2x^2 - 2cx + 2(c^2x^2 - 1)\log(-cx + 1))g/c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")`

[Out] `-1/4*h*((c^2*x^2 + 2*c*x - 2*(c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x)/c^2 - integrate((2*(c^2*x^2 - 1)*log(-c*x + 1)^2 - (c^2*x^2 + 2*c*x)*log(-c*x + 1))/x, x)/c^2) + 1/8*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*g/c^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(hx\text{Li}_2(cx)\log(-cx + 1) + gx\text{Li}_2(cx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")`

[Out] `integral(h*x*dilog(c*x)*log(-c*x + 1) + g*x*dilog(c*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (h\log(-cx + 1) + g)x\text{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")`

[Out] `integrate((h*log(-c*x + 1) + g)*x*dilog(c*x), x)`



### 3.172 $\int (g + h \log(1 - cx)) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=167

$$x \text{PolyLog}(2, cx)(h \log(1 - cx) + g) - hx \text{PolyLog}(2, cx) + \frac{2h \text{PolyLog}(3, 1 - cx)}{c} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{c} - \frac{2h}{c}$$

```
[Out] -(g*x) + 3*h*x - (g*(1 - c*x)*Log[1 - c*x])/c + (3*h*(1 - c*x)*Log[1 - c*x])/c - (h*(1 - c*x)*Log[1 - c*x]^2)/c - (h*Log[c*x]*Log[1 - c*x]^2)/c - h*x*PolyLog[2, c*x] - (h*Log[1 - c*x]*PolyLog[2, c*x])/c + x*(g + h*Log[1 - c*x])*PolyLog[2, c*x] - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*h*PolyLog[3, 1 - c*x])/c
```

**Rubi [A]** time = 0.210957, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$ , Rules used = {6600, 2364, 2360, 2295, 2296, 6688, 6742, 6586, 2389, 6596, 2396, 2433, 2374, 6589}

$$x \text{PolyLog}(2, cx)(h \log(1 - cx) + g) - hx \text{PolyLog}(2, cx) + \frac{2h \text{PolyLog}(3, 1 - cx)}{c} - \frac{h \log(1 - cx) \text{PolyLog}(2, cx)}{c} - \frac{2h}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[1 - c*x])*PolyLog[2, c*x], x]
```

```
[Out] -(g*x) + 3*h*x - (g*(1 - c*x)*Log[1 - c*x])/c + (3*h*(1 - c*x)*Log[1 - c*x])/c - (h*(1 - c*x)*Log[1 - c*x]^2)/c - (h*Log[c*x]*Log[1 - c*x]^2)/c - h*x*PolyLog[2, c*x] - (h*Log[1 - c*x]*PolyLog[2, c*x])/c + x*(g + h*Log[1 - c*x])*PolyLog[2, c*x] - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*h*PolyLog[3, 1 - c*x])/c
```

#### Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLog[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
```

#### Rule 2364

```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.), x_Symbol] := Dist[1/Coeff[v, x, 1], Subst[Int[(a + b*Log[x])^p*(c + d*Log[x])^q, x], x, v], x] /; FreeQ[{a, b, c, d, p, q}, x] && LinearQ[v, x] && NeQ[Coeff[v, x, 0], 0]
```

#### Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)]^(n_.))*((b_.)^(p_.)*(Log[(c_.)*(x_)]^(n_.))*(e_. + (d_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*x]^n)]^p*(d + e*Log[c*x]^n)^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] && IntegerQ[q]
```

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x]
]; FreeQ[{c, n}, x]
```

#### Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

#### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

#### Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*(b*d - a*e) + e, 0]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_)/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.))*((k_.) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x]]
```

$n])^{(p-1)}/x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$   
 $\&\& \ \text{EqQ}[d*e, 1]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n, (c_.)*((a_.) + (b_.)*(x_.))^{(p_.)}]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \ \&\& \ \text{EqQ}[b*d, a*e]$

### Rubi steps

$$\begin{aligned} \int (g + h \log(1 - cx)) \text{Li}_2(cx) dx &= x(g + h \log(1 - cx)) \text{Li}_2(cx) + (ch) \int \left( -\frac{1}{c} - \frac{1}{c(-1 + cx)} \right) \text{Li}_2(cx) dx + \int \log(1 - cx) dx \\ &= x(g + h \log(1 - cx)) \text{Li}_2(cx) - \frac{\text{Subst}(\int \log(x)(g + h \log(x)) dx, x, 1 - cx)}{c} + (ch) \int \frac{\log(1 - cx)}{c} dx \\ &= x(g + h \log(1 - cx)) \text{Li}_2(cx) - \frac{\text{Subst}(\int (g \log(x) + h \log^2(x)) dx, x, 1 - cx)}{c} + (ch) \int \frac{\log(1 - cx)}{c} dx \\ &= x(g + h \log(1 - cx)) \text{Li}_2(cx) - \frac{g \text{Subst}(\int \log(x) dx, x, 1 - cx)}{c} - h \int \text{Li}_2(cx) dx - h \int \frac{\log(1 - cx)}{c} dx \\ &= -gx - \frac{g(1 - cx) \log(1 - cx)}{c} - \frac{h(1 - cx) \log^2(1 - cx)}{c} - hx \text{Li}_2(cx) - \frac{h \log(1 - cx) \text{Li}_2(cx)}{c} \\ &= -gx + 2hx - \frac{g(1 - cx) \log(1 - cx)}{c} + \frac{2h(1 - cx) \log(1 - cx)}{c} - \frac{h(1 - cx) \log^2(1 - cx)}{c} \\ &= -gx + 3hx - \frac{g(1 - cx) \log(1 - cx)}{c} + \frac{3h(1 - cx) \log(1 - cx)}{c} - \frac{h(1 - cx) \log^2(1 - cx)}{c} \\ &= -gx + 3hx - \frac{g(1 - cx) \log(1 - cx)}{c} + \frac{3h(1 - cx) \log(1 - cx)}{c} - \frac{h(1 - cx) \log^2(1 - cx)}{c} \\ &= -gx + 3hx - \frac{g(1 - cx) \log(1 - cx)}{c} + \frac{3h(1 - cx) \log(1 - cx)}{c} - \frac{h(1 - cx) \log^2(1 - cx)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.0682036, size = 149, normalized size = 0.89

$$g \left( x \text{PolyLog}(2, cx) + \left( x - \frac{1}{c} \right) \log(1 - cx) - x \right) + \frac{h \left( 2 \text{PolyLog}(3, 1 - cx) - 2 \log(1 - cx) \text{PolyLog}(2, 1 - cx) + ((cx - 1) \log(1 - cx))^2 \right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x], x]

[Out] g\*(-x + (-c^(-1) + x)\*Log[1 - c\*x] + x\*PolyLog[2, c\*x]) + (h\*(-2 + 3\*c\*x + 3\*Log[1 - c\*x] - 3\*c\*x\*Log[1 - c\*x] - Log[1 - c\*x]^2 + c\*x\*Log[1 - c\*x]^2 - Log[c\*x]\*Log[1 - c\*x]^2 + (-c\*x) + (-1 + c\*x)\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 2\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 2\*PolyLog[3, 1 - c\*x])/c

**Maple [F]** time = 0.199, size = 0, normalized size = 0.

$$\int (g + h \ln(-cx + 1)) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(-c\*x+1))\*polylog(2,c\*x),x)

[Out] int((g+h\*ln(-c\*x+1))\*polylog(2,c\*x),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-h \left( \frac{(cx - (cx - 1) \log(-cx + 1)) \operatorname{Li}_2(cx)}{c} - \frac{(cx - 1) (\log(-cx + 1)^2 - 2 \log(-cx + 1) + 2) + cx - (cx - 1) \log(-cx + 1) - \int}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x),x, algorithm="maxima")

[Out] -h\*((c\*x - (c\*x - 1)\*log(-c\*x + 1))\*dilog(c\*x)/c - integrate(-(c\*x\*log(-c\*x + 1) - (c\*x - 1)\*log(-c\*x + 1)^2)/x, x)/c) + (c\*x\*dilog(c\*x) - c\*x + (c\*x - 1)\*log(-c\*x + 1))\*g/c

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(h \operatorname{Li}_2(cx) \log(-cx + 1) + g \operatorname{Li}_2(cx), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x),x, algorithm="fricas")

[Out] integral(h\*dilog(c\*x)\*log(-c\*x + 1) + g\*dilog(c\*x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (g + h \log(-cx + 1)) \operatorname{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(-c\*x+1))\*polylog(2,c\*x),x)

[Out] Integral((g + h\*log(-c\*x + 1))\*polylog(2, c\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (h \log(-cx + 1) + g) \operatorname{Li}_2(cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x),x, algorithm="giac")

[Out] integrate((h\*log(-c\*x + 1) + g)\*dilog(c\*x), x)

$$3.173 \quad \int \frac{(g+h \log(1-cx))\text{PolyLog}(2,cx)}{x} dx$$

**Optimal.** Leaf size=20

$$g\text{PolyLog}(3,cx) - \frac{1}{2}h\text{PolyLog}(2,cx)^2$$

[Out]  $-(h*\text{PolyLog}[2, c*x]^2)/2 + g*\text{PolyLog}[3, c*x]$

**Rubi [A]** time = 0.0575455, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {6602, 6589, 6601}

$$g\text{PolyLog}(3,cx) - \frac{1}{2}h\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + h*\text{Log}[1 - c*x])*\text{PolyLog}[2, c*x])/x, x]$

[Out]  $-(h*\text{PolyLog}[2, c*x]^2)/2 + g*\text{PolyLog}[3, c*x]$

#### Rule 6602

$\text{Int}[(\text{Log}[1 + (e_.)*(x_.)]*(h_.) + (g_.))*\text{PolyLog}[2, (c_.)*(x_.)]/(x_.), x\_Symbol] \rightarrow \text{Dist}[g, \text{Int}[\text{PolyLog}[2, c*x]/x, x], x] + \text{Dist}[h, \text{Int}[(\text{Log}[1 + e*x]*\text{PolyLog}[2, c*x])/x, x], x] /; \text{FreeQ}\{c, e, g, h\}, x] \&\& \text{EqQ}[c + e, 0]$

#### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_.))^{(p_.)}]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

#### Rule 6601

$\text{Int}[(\text{Log}[1 + (e_.)*(x_.)]*\text{PolyLog}[2, (c_.)*(x_.)]/(x_.), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, c*x]^2/2, x] /; \text{FreeQ}\{c, e\}, x] \&\& \text{EqQ}[c + e, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{(g+h \log(1-cx))\text{Li}_2(cx)}{x} dx &= g \int \frac{\text{Li}_2(cx)}{x} dx + h \int \frac{\log(1-cx)\text{Li}_2(cx)}{x} dx \\ &= -\frac{1}{2}h\text{Li}_2(cx)^2 + g\text{Li}_3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0110358, size = 20, normalized size = 1.

$$g\text{PolyLog}(3,cx) - \frac{1}{2}h\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(g + h*\text{Log}[1 - c*x])*\text{PolyLog}[2, c*x])/x, x]$

[Out]  $-(h*\text{PolyLog}[2, c*x]^2)/2 + g*\text{PolyLog}[3, c*x]$

**Maple [A]** time = 0.155, size = 19, normalized size = 1.

$$-\frac{h(\text{polylog}(2, cx))^2}{2} + g\text{polylog}(3, cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)`

[Out]  $-1/2*h*\text{polylog}(2,c*x)^2+g*\text{polylog}(3,c*x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log(-cx + 1) + g) \text{Li}_2(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="maxima")`

[Out] `integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h\text{Li}_2(cx) \log(-cx + 1) + g\text{Li}_2(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="fricas")`

[Out] `integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(g + h \log(-cx + 1)) \text{Li}_2(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)`

[Out] `Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log(-cx + 1) + g) \text{Li}_2(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

$$3.174 \quad \int \frac{(g+h \log(1-cx))\text{PolyLog}(2,cx)}{x^2} dx$$

**Optimal.** Leaf size=156

$$-\frac{\text{PolyLog}(2,cx)(h \log(1-cx)+g)}{x} - 2ch\text{PolyLog}\left(2,\frac{1}{1-cx}\right) - ch\text{PolyLog}(3,cx) - 2ch\text{PolyLog}(3,1-cx) + ch \log(1-cx)$$

[Out] c\*h\*Log[c\*x]\*Log[1 - c\*x]^2 + (Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/x + c\*(g + 2\*h\*Log[1 - c\*x])\*Log[1 - (1 - c\*x)^(-1)] + c\*h\*Log[1 - c\*x]\*PolyLog[2, c\*x] - ((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/x - 2\*c\*h\*PolyLog[2, (1 - c\*x)^(-1)] + 2\*c\*h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] - c\*h\*PolyLog[3, c\*x] - 2\*c\*h\*PolyLog[3, 1 - c\*x]

**Rubi [A]** time = 0.355162, antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 15, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6603, 2439, 2410, 2391, 2390, 2301, 2411, 2344, 2316, 2315, 6589, 6596, 2396, 2433, 2374}

$$-\frac{\text{PolyLog}(2,cx)(h \log(1-cx)+g)}{x} - 2ch\text{PolyLog}(2,cx) - ch\text{PolyLog}(3,cx) - 2ch\text{PolyLog}(3,1-cx) + ch \log(1-cx)$$

Antiderivative was successfully verified.

[In] Int[((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/x^2,x]

[Out] c\*g\*Log[x] - (c\*h\*Log[1 - c\*x]^2)/2 + c\*h\*Log[c\*x]\*Log[1 - c\*x]^2 + (Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/x - (c\*(g + h\*Log[1 - c\*x])^2)/(2\*h) - 2\*c\*h\*PolyLog[2, c\*x] + c\*h\*Log[1 - c\*x]\*PolyLog[2, c\*x] - ((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/x + 2\*c\*h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] - c\*h\*PolyLog[3, c\*x] - 2\*c\*h\*PolyLog[3, 1 - c\*x]

### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] :> Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/ (m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*(x\_)^(r\_.), x\_Symbol] :> Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

### Rule 2410

Int[(Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ



[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

#### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(a + b\*Log[-((c\*d)/e)]\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))]*((a_.) + Log[(c_.)*(x_))^(n_.)]*(b
_.))^(p_.)/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x^2} dx &= -\frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x} - (ch) \int \left( \frac{\text{Li}_2(cx)}{x} - \frac{c \text{Li}_2(cx)}{-1 + cx} \right) dx - \int \frac{\log(1 - cx)(g + h \log(1 - cx))}{x^2} dx \\
&= \frac{\log(1 - cx)(g + h \log(1 - cx))}{x} - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x} + c \int \frac{g + h \log(1 - cx)}{x(1 - cx)} dx \\
&= \frac{\log(1 - cx)(g + h \log(1 - cx))}{x} + ch \log(1 - cx) \text{Li}_2(cx) - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x} - c \int \frac{g + h \log(1 - cx)}{x(1 - cx)} dx \\
&= ch \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{x} + ch \log(1 - cx) \text{Li}_2(cx) - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x} - c \int \frac{g + h \log(1 - cx)}{x(1 - cx)} dx \\
&= cg \log(x) + ch \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{x} - \frac{c(g + h \log(1 - cx)) \text{Li}_2(cx)}{2h} \\
&= cg \log(x) - \frac{1}{2} ch \log^2(1 - cx) + ch \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{x} \\
&= cg \log(x) - \frac{1}{2} ch \log^2(1 - cx) + ch \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.172119, size = 150, normalized size = 0.96

$$\frac{g(-\text{PolyLog}(2, cx) + cx \log(x) + (1 - cx) \log(1 - cx))}{x} + h \left( -c \text{PolyLog}(3, cx) - 2c \text{PolyLog}(3, 1 - cx) + \frac{(cx - 1) \log(1 - cx)}{x} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2, x]
```

```
[Out] (g*(c*x*Log[x] + (1 - c*x)*Log[1 - c*x] - PolyLog[2, c*x]))/x + h*(2*c*Log[
c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x]*Log[1
- c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + Log[1 - c
```

\*x))\*PolyLog[2, 1 - c\*x] - c\*PolyLog[3, c\*x] - 2\*c\*PolyLog[3, 1 - c\*x])

**Maple [F]** time = 0.276, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(-cx + 1)) \operatorname{polylog}(2, cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(-c\*x+1))\*polylog(2,c\*x)/x^2,x)

[Out] int((g+h\*ln(-c\*x+1))\*polylog(2,c\*x)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( c \log(x) - \frac{(cx - 1) \log(-cx + 1) + \operatorname{Li}_2(cx)}{x} \right) g + h \int \frac{\operatorname{Li}_2(cx) \log(-cx + 1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x)/x^2,x, algorithm="maxima")

[Out] (c\*log(x) - ((c\*x - 1)\*log(-c\*x + 1) + dilog(c\*x))/x)\*g + h\*integrate(dilog(c\*x)\*log(-c\*x + 1)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{h \operatorname{Li}_2(cx) \log(-cx + 1) + g \operatorname{Li}_2(cx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x)/x^2,x, algorithm="fricas")

[Out] integral((h\*dilog(c\*x)\*log(-c\*x + 1) + g\*dilog(c\*x))/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(g + h \log(-cx + 1)) \operatorname{Li}_2(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(-c\*x+1))\*polylog(2,c\*x)/x\*\*2,x)

[Out] Integral((g + h\*log(-c\*x + 1))\*polylog(2, c\*x)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log(-cx + 1) + g) \text{Li}_2(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^2, x)
```

$$3.175 \quad \int \frac{(g+h \log(1-cx))\text{PolyLog}(2,cx)}{x^3} dx$$

**Optimal.** Leaf size=266

$$-\frac{1}{2}c^2h\text{PolyLog}\left(2, \frac{1}{1-cx}\right) - \frac{1}{2}c^2h\text{PolyLog}(3, cx) - c^2h\text{PolyLog}(3, 1-cx) + \frac{1}{2}c^2h \log(1-cx)\text{PolyLog}(2, cx) + c^2h \log(1-cx)$$

```
[Out] -(c^2*h*Log[x]) + (c^2*h*Log[1 - c*x])/2 - (c*h*Log[1 - c*x])/(2*x) + (c^2*
h*Log[c*x]*Log[1 - c*x]^2)/2 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/(4*x^2)
- (c*(1 - c*x)*(g + 2*h*Log[1 - c*x]))/(4*x) + (c^2*(g + 2*h*Log[1 - c*x])*
Log[1 - (1 - c*x)^(-1)])/4 + (c*h*PolyLog[2, c*x])/(2*x) + (c^2*h*Log[1 - c
*x]*PolyLog[2, c*x])/2 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(2*x^2) - (
c^2*h*PolyLog[2, (1 - c*x)^(-1)])/2 + c^2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x
] - (c^2*h*PolyLog[3, c*x])/2 - c^2*h*PolyLog[3, 1 - c*x]
```

**Rubi [A]** time = 0.496284, antiderivative size = 278, normalized size of antiderivative = 1.05, number of steps used = 31, number of rules used = 22, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.1$ , Rules used = {6603, 2439, 2410, 2395, 36, 29, 31, 2391, 2390, 2301, 2411, 2347, 2344, 2316, 2315, 2314, 6591, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}c^2h\text{PolyLog}(2, cx) - \frac{1}{2}c^2h\text{PolyLog}(3, cx) - c^2h\text{PolyLog}(3, 1-cx) + \frac{1}{2}c^2h \log(1-cx)\text{PolyLog}(2, cx) + c^2h \log(1-cx)$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3, x]
```

```
[Out] (c^2*g*Log[x])/4 - c^2*h*Log[x] + (3*c^2*h*Log[1 - c*x])/4 - (3*c*h*Log[1 -
c*x])/(4*x) - (c^2*h*Log[1 - c*x]^2)/8 + (c^2*h*Log[c*x]*Log[1 - c*x]^2)/2
- (c*(1 - c*x)*(g + h*Log[1 - c*x]))/(4*x) + (Log[1 - c*x]*(g + h*Log[1 -
c*x]))/(4*x^2) - (c^2*(g + h*Log[1 - c*x])^2)/(8*h) - (c^2*h*PolyLog[2, c*x
])/2 + (c*h*PolyLog[2, c*x])/(2*x) + (c^2*h*Log[1 - c*x]*PolyLog[2, c*x])/2
- ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(2*x^2) + c^2*h*Log[1 - c*x]*Poly
Log[2, 1 - c*x] - (c^2*h*PolyLog[3, c*x])/2 - c^2*h*PolyLog[3, 1 - c*x]
```

### Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

Rule 2347

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.)/(x\_), x\_Symbol] := Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[

{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

#### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\_/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

#### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^q\_, x\_Symbol] :> Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r\*(q + 1) + 1, 0]

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^q\_], x\_Symbol] :> Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^p\_/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^p\_\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Sym

```
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x^3} dx &= -\frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{2x^2} - \frac{1}{2} \int \frac{\log(1 - cx)(g + h \log(1 - cx))}{x^3} dx - \frac{1}{2}(ch) \int \left( \frac{\text{Li}_2(cx)}{x^2} \right. \\ &= \frac{\log(1 - cx)(g + h \log(1 - cx))}{4x^2} - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{2x^2} + \frac{1}{4}c \int \frac{g + h \log(1 - cx)}{x^2(1 - cx)} dx \\ &= \frac{\log(1 - cx)(g + h \log(1 - cx))}{4x^2} + \frac{ch \text{Li}_2(cx)}{2x} + \frac{1}{2}c^2h \log(1 - cx) \text{Li}_2(cx) - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{2x} \\ &= -\frac{ch \log(1 - cx)}{2x} + \frac{1}{2}c^2h \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{4x^2} + \frac{ch \text{Li}_2(cx)}{2x} \\ &= -\frac{3ch \log(1 - cx)}{4x} + \frac{1}{2}c^2h \log(cx) \log^2(1 - cx) - \frac{c(1 - cx)(g + h \log(1 - cx))}{4x} + \frac{\log(1 - cx)(g + h \log(1 - cx))}{4x^2} \\ &= \frac{1}{4}c^2g \log(x) - \frac{3}{4}c^2h \log(x) + \frac{1}{2}c^2h \log(1 - cx) - \frac{3ch \log(1 - cx)}{4x} - \frac{1}{8}c^2h \log^2(1 - cx) + \frac{ch \text{Li}_2(cx)}{2x} \\ &= \frac{1}{4}c^2g \log(x) - c^2h \log(x) + \frac{3}{4}c^2h \log(1 - cx) - \frac{3ch \log(1 - cx)}{4x} - \frac{1}{8}c^2h \log^2(1 - cx) + \frac{ch \text{Li}_2(cx)}{2x} \end{aligned}$$

**Mathematica [A]** time = 0.256103, size = 238, normalized size = 0.89

$$\frac{g(-2\text{PolyLog}[2, cx] + c^2x^2 \log(x) - c^2x^2 \log(1 - cx) - cx + \log(1 - cx))}{4x^2} + \frac{1}{4}h \left( \frac{2((c^2x^2 - 1) \log(1 - cx) + cx) \text{PolyLog}[2, cx]}{x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3, x]
```

```
[Out] (g*(-(c*x) + c^2*x^2*Log[x] + Log[1 - c*x] - c^2*x^2*Log[1 - c*x] - 2*PolyLog[2, c*x]))/(4*x^2) + (h*(-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c*x] - (4*c*Log[1 - c*x])/x + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]^2 + Log[1 - c*x]^2/x^2 + 2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c^2*x^2)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 2*c^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x]))/4
```

**Maple [F]** time = 0.333, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(-cx + 1)) \text{polylog}(2, cx)}{x^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)`

[Out] `int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( c^2 \log(x) - \frac{cx + (c^2x^2 - 1) \log(-cx + 1) + 2 \operatorname{Li}_2(cx)}{x^2} \right) g + h \int \frac{\operatorname{Li}_2(cx) \log(-cx + 1)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="maxima")`

[Out] `1/4*(c^2*log(x) - (c*x + (c^2*x^2 - 1)*log(-c*x + 1) + 2*dilog(c*x))/x^2)*g + h*integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{h \operatorname{Li}_2(cx) \log(-cx + 1) + g \operatorname{Li}_2(cx)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="fricas")`

[Out] `integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log(-cx + 1) + g) \operatorname{Li}_2(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="giac")`

[Out] `integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^3, x)`

$$3.176 \quad \int \frac{(g+h \log(1-cx)) \text{PolyLog}(2, cx)}{x^4} dx$$

**Optimal.** Leaf size=340

$$-\frac{2}{9}c^3h \text{PolyLog}\left(2, \frac{1}{1-cx}\right) - \frac{1}{3}c^3h \text{PolyLog}(3, cx) - \frac{2}{3}c^3h \text{PolyLog}(3, 1-cx) + \frac{c^2h \text{PolyLog}(2, cx)}{3x} + \frac{1}{3}c^3h \log(1-cx) \text{PolyLog}(2, cx)$$

[Out] (7\*c^2\*h)/(36\*x) - (3\*c^3\*h\*Log[x])/4 + (19\*c^3\*h\*Log[1 - c\*x])/36 - (c\*h\*Log[1 - c\*x])/(12\*x^2) - (c^2\*h\*Log[1 - c\*x])/(3\*x) + (c^3\*h\*Log[c\*x]\*Log[1 - c\*x]^2)/3 + (Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/(9\*x^3) - (c\*(g + 2\*h\*Log[1 - c\*x]))/(18\*x^2) - (c^2\*(1 - c\*x)\*(g + 2\*h\*Log[1 - c\*x]))/(9\*x) + (c^3\*(g + 2\*h\*Log[1 - c\*x])\*Log[1 - (1 - c\*x)^(-1)])/9 + (c\*h\*PolyLog[2, c\*x])/(6\*x^2) + (c^2\*h\*PolyLog[2, c\*x])/(3\*x) + (c^3\*h\*Log[1 - c\*x]\*PolyLog[2, c\*x])/3 - ((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/(3\*x^3) - (2\*c^3\*h\*PolyLog[2, (1 - c\*x)^(-1)])/9 + (2\*c^3\*h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/3 - (c^3\*h\*PolyLog[3, c\*x])/3 - (2\*c^3\*h\*PolyLog[3, 1 - c\*x])/3

**Rubi [A]** time = 0.650804, antiderivative size = 351, normalized size of antiderivative = 1.03, number of steps used = 42, number of rules used = 24, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.2$ , Rules used = {6603, 2439, 2410, 2395, 44, 36, 29, 31, 2391, 2390, 2301, 2411, 2347, 2344, 2316, 2315, 2314, 2319, 6591, 6589, 6596, 2396, 2433, 2374}

$$-\frac{2}{9}c^3h \text{PolyLog}(2, cx) - \frac{1}{3}c^3h \text{PolyLog}(3, cx) - \frac{2}{3}c^3h \text{PolyLog}(3, 1-cx) + \frac{c^2h \text{PolyLog}(2, cx)}{3x} + \frac{1}{3}c^3h \log(1-cx) \text{PolyLog}(2, cx)$$

Antiderivative was successfully verified.

[In] Int[((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/x^4, x]

[Out] (7\*c^2\*h)/(36\*x) + (c^3\*g\*Log[x])/9 - (3\*c^3\*h\*Log[x])/4 + (23\*c^3\*h\*Log[1 - c\*x])/36 - (5\*c\*h\*Log[1 - c\*x])/(36\*x^2) - (4\*c^2\*h\*Log[1 - c\*x])/(9\*x) - (c^3\*h\*Log[1 - c\*x]^2)/18 + (c^3\*h\*Log[c\*x]\*Log[1 - c\*x]^2)/3 - (c\*(g + h\*Log[1 - c\*x]))/(18\*x^2) - (c^2\*(1 - c\*x)\*(g + h\*Log[1 - c\*x]))/(9\*x) + (Log[1 - c\*x]\*(g + h\*Log[1 - c\*x]))/(9\*x^3) - (c^3\*(g + h\*Log[1 - c\*x])^2)/(18\*h) - (2\*c^3\*h\*PolyLog[2, c\*x])/9 + (c\*h\*PolyLog[2, c\*x])/(6\*x^2) + (c^2\*h\*PolyLog[2, c\*x])/(3\*x) + (c^3\*h\*Log[1 - c\*x]\*PolyLog[2, c\*x])/3 - ((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/(3\*x^3) + (2\*c^3\*h\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/3 - (c^3\*h\*PolyLog[3, c\*x])/3 - (2\*c^3\*h\*PolyLog[3, 1 - c\*x])/3

### Rule 6603

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(h\_.))\*(x\_.)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))], x\_Symbol] :> Simp[(x^(m + 1)\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], x^(m + 1)/(a + b\*x), x], x] - Dist[(e\*h\*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], x^(m + 1)/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_.))^(m\_.)]\*(g\_.))\*(x\_.)^(r\_.), x\_Symbol] :> Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a

, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]^(n\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]^(n\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)]^(n\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2347

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_))/(x\_), x\_Symbol] := Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

#### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

#### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_))^(r\_.))^(q\_), x\_Symbol] := Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r\*(q + 1) + 1, 0]

#### Rule 2319

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/(e\*(q + 1)), x] - Dist[(b\*n\*p)/(e\*(q + 1)), Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_))^(p\_.)]^(q\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q])/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d

, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/x, x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{x^4} dx &= -\frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{3x^3} - \frac{1}{3} \int \frac{\log(1 - cx)(g + h \log(1 - cx))}{x^4} dx - \frac{1}{3}(ch) \int \left( \frac{\text{Li}_2(cx)}{x^3} \right) dx \\
&= \frac{\log(1 - cx)(g + h \log(1 - cx))}{9x^3} - \frac{(g + h \log(1 - cx)) \text{Li}_2(cx)}{3x^3} + \frac{1}{9}c \int \frac{g + h \log(1 - cx)}{x^3(1 - cx)} dx \\
&= \frac{\log(1 - cx)(g + h \log(1 - cx))}{9x^3} + \frac{ch \text{Li}_2(cx)}{6x^2} + \frac{c^2 h \text{Li}_2(cx)}{3x} + \frac{1}{3}c^3 h \log(1 - cx) \text{Li}_2(cx) - \frac{1}{3}c^3 h \log^2(1 - cx) \\
&= -\frac{ch \log(1 - cx)}{12x^2} - \frac{c^2 h \log(1 - cx)}{3x} + \frac{1}{3}c^3 h \log(cx) \log^2(1 - cx) + \frac{\log(1 - cx)(g + h \log(1 - cx))}{9x^3} \\
&= -\frac{5ch \log(1 - cx)}{36x^2} - \frac{4c^2 h \log(1 - cx)}{9x} + \frac{1}{3}c^3 h \log(cx) \log^2(1 - cx) - \frac{c(g + h \log(1 - cx))}{18x^2} \\
&= \frac{c^2 h}{12x} - \frac{5}{12}c^3 h \log(x) + \frac{5}{12}c^3 h \log(1 - cx) - \frac{5ch \log(1 - cx)}{36x^2} - \frac{4c^2 h \log(1 - cx)}{9x} - \frac{1}{18}c^3 h \log^2(1 - cx) \\
&= \frac{7c^2 h}{36x} + \frac{1}{9}c^3 g \log(x) - \frac{3}{4}c^3 h \log(x) + \frac{23}{36}c^3 h \log(1 - cx) - \frac{5ch \log(1 - cx)}{36x^2} - \frac{4c^2 h \log(1 - cx)}{9x} \\
&= \frac{7c^2 h}{36x} + \frac{1}{9}c^3 g \log(x) - \frac{3}{4}c^3 h \log(x) + \frac{23}{36}c^3 h \log(1 - cx) - \frac{5ch \log(1 - cx)}{36x^2} - \frac{4c^2 h \log(1 - cx)}{9x}
\end{aligned}$$

**Mathematica [A]** time = 0.200583, size = 301, normalized size = 0.89

$$h(-12c^3x^3\text{PolyLog}(3, cx) - 24c^3x^3\text{PolyLog}(3, 1 - cx) + 8c^3x^3(3\log(1 - cx) + 1)\text{PolyLog}(2, 1 - cx) + 6(2(c^3x^3 - 1)\log(1 - cx) - c^3x^3\log^2(1 - cx))) / (36x^3)$$

Antiderivative was successfully verified.

[In] Integrate[((g + h\*Log[1 - c\*x])\*PolyLog[2, c\*x])/x^4, x]

[Out] -(g\*(c\*x\*(1 + 2\*c\*x) - 2\*c^3\*x^3\*Log[x] + 2\*(-1 + c^3\*x^3)\*Log[1 - c\*x] + 6\*PolyLog[2, c\*x]))/(18\*x^3) + (h\*(7\*c^2\*x^2 - 4\*c^3\*x^3 - 15\*c^3\*x^3\*Log[x] - 12\*c^3\*x^3\*Log[c\*x] - 7\*c\*x\*Log[1 - c\*x] - 20\*c^2\*x^2\*Log[1 - c\*x] + 27\*c^3\*x^3\*Log[1 - c\*x] + 8\*c^3\*x^3\*Log[c\*x]\*Log[1 - c\*x] + 4\*Log[1 - c\*x]^2 - 4\*c^3\*x^3\*Log[1 - c\*x]^2 + 12\*c^3\*x^3\*Log[c\*x]\*Log[1 - c\*x]^2 + 6\*(c\*x\*(1 + 2\*c\*x) + 2\*(-1 + c^3\*x^3)\*Log[1 - c\*x])\*PolyLog[2, c\*x] + 8\*c^3\*x^3\*(1 + 3\*Log[1 - c\*x])\*PolyLog[2, 1 - c\*x] - 12\*c^3\*x^3\*PolyLog[3, c\*x] - 24\*c^3\*x^3\*PolyLog[3, 1 - c\*x]))/(36\*x^3)

**Maple [F]** time = 0.349, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(-cx + 1)) \text{polylog}(2, cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(-c\*x+1))\*polylog(2, c\*x)/x^4, x)

[Out] int((g+h\*ln(-c\*x+1))\*polylog(2, c\*x)/x^4, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{18} \left( 2c^3 \log(x) - \frac{2c^2x^2 + cx + 2(c^3x^3 - 1) \log(-cx + 1) + 6 \operatorname{Li}_2(cx)}{x^3} \right) g + h \int \frac{\operatorname{Li}_2(cx) \log(-cx + 1)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x)/x^4,x, algorithm="maxima")

[Out] 1/18\*(2\*c^3\*log(x) - (2\*c^2\*x^2 + c\*x + 2\*(c^3\*x^3 - 1)\*log(-c\*x + 1) + 6\*dilog(c\*x))/x^3)\*g + h\*integrate(dilog(c\*x)\*log(-c\*x + 1)/x^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{h \operatorname{Li}_2(cx) \log(-cx + 1) + g \operatorname{Li}_2(cx)}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x)/x^4,x, algorithm="fricas")

[Out] integral((h\*dilog(c\*x)\*log(-c\*x + 1) + g\*dilog(c\*x))/x^4, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(-c\*x+1))\*polylog(2,c\*x)/x\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log(-cx + 1) + g) \operatorname{Li}_2(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(-c\*x+1))\*polylog(2,c\*x)/x^4,x, algorithm="giac")

[Out] integrate((h\*log(-c\*x + 1) + g)\*dilog(c\*x)/x^4, x)

$$3.177 \quad \int x^2 \left( g + h \log \left( f(d + ex)^n \right) \right) \text{PolyLog}(2, c(a + bx)) dx$$

Optimal. Leaf size=2995

result too large to display

```
[Out] -(a^2*g*x)/(3*b^2) + (a*(1 - a*c)*g*x)/(6*b^2*c) - ((1 - a*c)^2*g*x)/(9*b^2*c^2) + (7*a^2*h*n*x)/(9*b^2) - (11*a*(1 - a*c)*h*n*x)/(36*b^2*c) + (5*(1 - a*c)^2*h*n*x)/(27*b^2*c^2) + (13*d^2*h*n*x)/(27*e^2) + (5*a*d*h*n*x)/(12*b*e) - (7*(1 - a*c)*d*h*n*x)/(36*b*c*e) - (a*h*n*x^2)/(9*b) + (7*(1 - a*c)*h*n*x^2)/(108*b*c) - (19*d*h*n*x^2)/(216*e) + (h*n*x^3)/27 - (5*a*(1 - a*c)^2*h*n*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + (2*(1 - a*c)^3*h*n*Log[1 - a*c - b*c*x])/(27*b^3*c^3) - (5*(1 - a*c)^2*d*h*n*Log[1 - a*c - b*c*x])/(36*b^2*c^2*e) + (5*a*h*n*x^2*Log[1 - a*c - b*c*x])/(36*b) + (5*d*h*n*x^2*Log[1 - a*c - b*c*x])/(36*e) - (2*h*n*x^3*Log[1 - a*c - b*c*x])/27 + (4*a^2*h*n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b^3*c) + (4*d^2*h*n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b*c*e^2) + (a*d*h*n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^2*c*e) - (d^3*h*n*Log[d + e*x])/(27*e^3) - (a*d^2*h*n*Log[d + e*x])/(12*b*e^2) + ((1 - a*c)*d^2*h*n*Log[d + e*x])/(18*b*c*e^2) + (d^3*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(9*e^3) + (a*d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(6*b*e^2) + (a^2*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*b^2*e) - (a^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(3*b^2*e) + (a*(1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(6*b^2*c*e) - ((1 - a*c)^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(9*b^2*c^2*e) + (a*x^2*(g + h*Log[f*(d + e*x)^n]))/(12*b) - ((1 - a*c)*x^2*(g + h*Log[f*(d + e*x)^n]))/(18*b*c) - (x^3*(g + h*Log[f*(d + e*x)^n]))/27 + (a^2*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(3*b^2) - (a*x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(6*b) + (x^3*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/9 - (a^2*(1 - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(3*b^3*c) + (a*(1 - a*c)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*b^3*c^2) - ((1 - a*c)^3*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(9*b^3*c^3) - (a^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*b^3) + (d^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*e^3) - (a^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*b^3) + (d^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*e^3) + (a^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(6*b^3) - (d^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(6*e^3) + (a^3*g*PolyLog[2, c*(a + b*x)])/(3*b^3) - (a^3*h*n*PolyLog[2, c*(a + b*x)])/(9*b^3) - (a*d^2*h*n*PolyLog[2, c*(a + b*x)])/(3*b*e^2) - (a^2*d*h*n*PolyLog[2, c*(a + b*x)])/(6*b^2*e) - (d^2*h*n*x*PolyLog[2, c*(a + b*x)])/(3*e^2) + (d*h*n*x^2*PolyLog[2, c*(a + b*x)])/(6*e) - (h*n*x^3*PolyLog[2, c*(a + b*x)])/9 + (d^3*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*e^3) - (a^3*h*n*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*b^3) + (x^3*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/3 + (d^3*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(9*e^3) + (a*d^2*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(6*b*e^2) + (a^2*d*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(3*b^2*e) - (a^3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/(3*b^3) + (d^3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
```



$$\begin{aligned}
& e*x)/(b*d - a*e)]/(3*e^3) - (a^2*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x)) \\
& )/(b*c*d + e - a*c*e)]/(3*b^3*c) + (a*(1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + \\
& e*x))/(b*c*d + e - a*c*e)]/(6*b^3*c^2) - ((1 - a*c)^3*h*n*PolyLog[2, (b*c \\
& *(d + e*x))/(b*c*d + e - a*c*e)]/(9*b^3*c^3) - (a^3*h*n*(Log[d + e*x] - Lo \\
& g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))]))*PolyLog[2, 1 - c*(a + b*x \\
& )]/(3*b^3) + (d^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - \\
& c*(a + b*x))]))*PolyLog[2, 1 - c*(a + b*x)]/(3*e^3) + (a^3*h*n*Log[(b*(d + \\
& e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/ \\
& (b*c*(d + e*x)))]/(3*b^3) - (d^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c \\
& *(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(3*e^3) \\
& - (a^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ( \\
& (b*d - a*e)*(1 - c*(a + b*x))/(b*(d + e*x))]/(3*b^3) + (d^3*h*n*Log[(b*(d \\
& + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a \\
& + b*x))/(b*(d + e*x)))]/(3*e^3) + (a^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d \\
& - a*e)]/(3*b^3) - (d^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/(3*e^3) \\
& + (a^3*h*n*PolyLog[3, 1 - c*(a + b*x)]/(3*b^3) - (d^3*h*n*PolyLog[3, 1 - c \\
& *(a + b*x)]/(3*e^3) + (a^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d \\
& + e*x)))]/(3*b^3) - (d^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + \\
& e*x)))]/(3*e^3) - (a^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*( \\
& d + e*x)))]/(3*b^3) + (d^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/( \\
& b*(d + e*x)))]/(3*e^3)
\end{aligned}$$

**Rubi [A]** time = 4.59961, antiderivative size = 2995, normalized size of antiderivative = 1., number of steps used = 108, number of rules used = 20, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.741$ , Rules used = {6603, 2430, 43, 2416, 2389, 2295, 2394, 2393, 2391, 2439, 2395, 2440, 2438, 2437, 2435, 6595, 2444, 2421, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^2\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)],x]

[Out]  $\begin{aligned}
& -(a^2*g*x)/(3*b^2) + (a*(1 - a*c)*g*x)/(6*b^2*c) - ((1 - a*c)^2*g*x)/(9*b^2 \\
& *c^2) + (7*a^2*h*n*x)/(9*b^2) - (11*a*(1 - a*c)*h*n*x)/(36*b^2*c) + (5*(1 - \\
& a*c)^2*h*n*x)/(27*b^2*c^2) + (13*d^2*h*n*x)/(27*e^2) + (5*a*d*h*n*x)/(12*b \\
& *e) - (7*(1 - a*c)*d*h*n*x)/(36*b*c*e) - (a*h*n*x^2)/(9*b) + (7*(1 - a*c)*h \\
& *n*x^2)/(108*b*c) - (19*d*h*n*x^2)/(216*e) + (h*n*x^3)/27 - (5*a*(1 - a*c)^ \\
& 2*h*n*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + (2*(1 - a*c)^3*h*n*Log[1 - a*c - \\
& b*c*x])/(27*b^3*c^3) - (5*(1 - a*c)^2*d*h*n*Log[1 - a*c - b*c*x])/(36*b^2* \\
& c^2*e) + (5*a*h*n*x^2*Log[1 - a*c - b*c*x])/(36*b) + (5*d*h*n*x^2*Log[1 - a \\
& *c - b*c*x])/(36*e) - (2*h*n*x^3*Log[1 - a*c - b*c*x])/27 + (4*a^2*h*n*(1 - \\
& a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b^3*c) + (4*d^2*h*n*(1 - a*c - b*c*x \\
& )*Log[1 - a*c - b*c*x])/(9*b*c*e^2) + (a*d*h*n*(1 - a*c - b*c*x)*Log[1 - a* \\
& c - b*c*x])/(3*b^2*c*e) - (d^3*h*n*Log[d + e*x])/(27*e^3) - (a*d^2*h*n*Log[ \\
& d + e*x])/(12*b*e^2) + ((1 - a*c)*d^2*h*n*Log[d + e*x])/(18*b*c*e^2) + (d^3 \\
& *h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]/(9*e^3) \\
& + (a*d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)] \\
& )/(6*b*e^2) + (a^2*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + \\
& e - a*c*e)]/(3*b^2*e) - (a^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(3*b^2*e) + ( \\
& a*(1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(6*b^2*c*e) - ((1 - a*c)^2*h*(d \\
& + e*x)*Log[f*(d + e*x)^n])/(9*b^2*c^2*e) + (a*x^2*(g + h*Log[f*(d + e*x)^n \\
& ]))/(12*b) - ((1 - a*c)*x^2*(g + h*Log[f*(d + e*x)^n]))/(18*b*c) - (x^3*(g \\
& + h*Log[f*(d + e*x)^n])/27 + (a^2*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + \\
& e*x)^n]))/(3*b^2) - (a*x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]) \\
& )/(6*b) + (x^3*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/9 - (a^2*(1 \\
& - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*
\end{aligned}$

$$\begin{aligned}
& x^n) / (3b^3c) + (a(1 - ac)^2 \text{Log}[(e(1 - ac - bcx)) / (b^3cd + e - ac^2e)] * (g + h \text{Log}[f(d + ex)^n]) / (6b^3c^2) - ((1 - ac)^3 \text{Log}[(e(1 - ac - bcx)) / (b^3cd + e - ac^2e)] * (g + h \text{Log}[f(d + ex)^n]) / (9b^3c^3) - \\
& (a^3h \text{Log}[c(a + bx)] + \text{Log}[(b^3cd + e - ac^2e) / (b^3c(d + ex))] - \text{Log}[(b^3cd + e - ac^2e) * (a + bx) / (b^3c(d + ex))] * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))]^2) / (6b^3) + (d^3h \text{Log}[c(a + bx)] + \text{Log}[(b^3cd + e - ac^2e) / (b^3c(d + ex))] - \text{Log}[(b^3cd + e - ac^2e) * (a + bx) / (b^3c(d + ex))] * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))]^2) / (6e^3) - \\
& (a^3h \text{Log}[c(a + bx)] * \text{Log}[d + ex] * \text{Log}[1 - c(a + bx)]) / (3b^3) + (d^3h \text{Log}[c(a + bx)] * \text{Log}[d + ex] * \text{Log}[1 - c(a + bx)]) / (3e^3) + (a^3h \text{Log}[c(a + bx)] - \text{Log}[-(e(a + bx)) / (b^3d - a^3e)]) * (\text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] + \text{Log}[1 - c(a + bx)]^2) / (6b^3) - (d^3h \text{Log}[c(a + bx)] - \text{Log}[-(e(a + bx)) / (b^3d - a^3e)]) * (\text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] + \text{Log}[1 - c(a + bx)]^2) / (6e^3) + \\
& (a^3g \text{PolyLog}[2, c(a + bx)]) / (3b^3) - (a^3h \text{PolyLog}[2, c(a + bx)]) / (9b^3) - (a^3d^2h \text{PolyLog}[2, c(a + bx)]) / (3b^3e^2) - (a^2d^2h \text{PolyLog}[2, c(a + bx)]) / (6b^2e) - (d^2h \text{PolyLog}[2, c(a + bx)]) / (3e^2) + (d^2h \text{PolyLog}[2, c(a + bx)]) / (6e) - (h \text{PolyLog}[2, c(a + bx)]) / 9 + (d^3h \text{Log}[d + ex] * \text{PolyLog}[2, c(a + bx)]) / (3e^3) - (a^3h * (\text{Log}[d + ex] - \text{Log}[f(d + ex)^n]) * \text{PolyLog}[2, c(a + bx)]) / (3b^3) + (x^3 * (g + h \text{Log}[f(d + ex)^n]) * \text{PolyLog}[2, c(a + bx)]) / 3 + (d^3h \text{PolyLog}[2, (e(1 - ac - bcx)) / (b^3cd + e - ac^2e)]) / (9e^3) + (a^3d^2h \text{PolyLog}[2, (e(1 - ac - bcx)) / (b^3cd + e - ac^2e)]) / (6b^3e^2) + (a^2d^2h \text{PolyLog}[2, (e(1 - ac - bcx)) / (b^3cd + e - ac^2e)]) / (3b^2e) - (a^3h * (\text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] + \text{Log}[1 - c(a + bx)]) * \text{PolyLog}[2, (b(d + ex)) / (b^3d - a^3e)]) / (3b^3) + (d^3h * (\text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] + \text{Log}[1 - c(a + bx)]) * \text{PolyLog}[2, (b(d + ex)) / (b^3d - a^3e)]) / (3e^3) - (a^2 * (1 - ac) * h * \text{PolyLog}[2, (b^3c(d + ex)) / (b^3cd + e - ac^2e)]) / (3b^3c) + (a(1 - ac)^2 * h * \text{PolyLog}[2, (b^3c(d + ex)) / (b^3cd + e - ac^2e)]) / (6b^3c^2) - ((1 - ac)^3 * h * \text{PolyLog}[2, (b^3c(d + ex)) / (b^3cd + e - ac^2e)]) / (9b^3c^3) - (a^3h * (\text{Log}[d + ex] - \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))])) * \text{PolyLog}[2, 1 - c(a + bx)] / (3b^3) + (d^3h * (\text{Log}[d + ex] - \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))])) * \text{PolyLog}[2, 1 - c(a + bx)] / (3e^3) + (a^3h * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] * \text{PolyLog}[2, -(e(1 - c(a + bx))) / (b^3c(d + ex))]) / (3b^3) - (d^3h * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] * \text{PolyLog}[2, -(e(1 - c(a + bx))) / (b^3c(d + ex))]) / (3e^3) - (a^3h * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] * \text{PolyLog}[2, ((b^3d - a^3e) * (1 - c(a + bx))) / (b^3c(d + ex))]) / (3b^3) + (d^3h * \text{Log}[(b(d + ex)) / ((b^3d - a^3e) * (1 - c(a + bx)))] * \text{PolyLog}[2, ((b^3d - a^3e) * (1 - c(a + bx))) / (b^3c(d + ex))]) / (3e^3) + (a^3h * \text{PolyLog}[3, (b(d + ex)) / (b^3d - a^3e)]) / (3b^3) - (d^3h * \text{PolyLog}[3, (b(d + ex)) / (b^3d - a^3e)]) / (3e^3) + (a^3h * \text{PolyLog}[3, 1 - c(a + bx)]) / (3b^3) - (d^3h * \text{PolyLog}[3, 1 - c(a + bx)]) / (3e^3) + (a^3h * \text{PolyLog}[3, -(e(1 - c(a + bx))) / (b^3c(d + ex))]) / (3b^3) - (d^3h * \text{PolyLog}[3, -(e(1 - c(a + bx))) / (b^3c(d + ex))]) / (3e^3) - (a^3h * \text{PolyLog}[3, ((b^3d - a^3e) * (1 - c(a + bx))) / (b^3c(d + ex))]) / (3b^3) + (d^3h * \text{PolyLog}[3, ((b^3d - a^3e) * (1 - c(a + bx))) / (b^3c(d + ex))]) / (3e^3)
\end{aligned}$$

### Rule 6603

```

Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_)), x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f*(d + ex)^n])*PolyLog[2, c*(a + bx)]/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h*Log[f*(d + ex)^n])*Log[1 - ac - bcx], x^(m + 1)/(a + bx), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c*(a + bx)], x^(m + 1)/(d + ex), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

```

Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.)), x\_Symbol] := Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m))]/(d + e\*x), x], x]) / ; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] / ; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] / ; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] / ; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2295

Int[Log[(c\_.)\*(x\_))^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] / ; FreeQ[{c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] / ; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] / ; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] / ; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*(x\_))^(r\_.), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d +

$e^x)^n)^{(p-1)*(f+g*\text{Log}[h*(i+j*x)^m])/(d+e*x), x, x) /;$  FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 2440

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*l)/l) + (e\*x)/l)^n])\*(f + g\*Log[h\*(-((j\*k - i\*l)/l) + (j\*x)/l)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

#### Rule 2438

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.) + (f\_.)))/(x\_), x\_Symbol] := Dist[f, Int[(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] + Dist[g, Int[(Log[h\*(i + j\*x)^m]\*(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0]

#### Rule 2437

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)))/(x\_), x\_Symbol] := Dist[m, Int[(Log[i + j\*x]\*Log[c\*(d + e\*x)^n])/x, x], x] - Dist[m\*Log[i + j\*x] - Log[h\*(i + j\*x)^m], Int[Log[c\*(d + e\*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0] && NeQ[i + j\*x, h\*(i + j\*x)^m]

#### Rule 2435

Int[(Log[(a\_) + (b\_.)\*(x\_)]\*Log[(c\_) + (d\_.)\*(x\_)])/(x\_), x\_Symbol] := Simp[Log[-((b\*x)/a)]\*Log[a + b\*x]\*Log[c + d\*x], x] + (Simp[(1\*(Log[-((b\*x)/a)] - Log[-((b\*c - a\*d)\*x]/(a\*(c + d\*x)))) + Log[(b\*c - a\*d)/(b\*(c + d\*x))])\*Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2/2, x] - Simp[(1\*(Log[-((b\*x)/a)] - Log[-((d\*x)/c]))\*(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2)/2, x] + Simp[(Log[c + d\*x] - Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (b\*x)/a], x] + Simp[(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (d\*x)/c], x] + Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))], x] - Simp[PolyLog[3, 1 + (b\*x)/a], x] - Simp[PolyLog[3, 1 + (d\*x)/c], x] + Simp[PolyLog[3, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 6595

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)], x\_Symbol] := Simp[x\*PolyLog[n, c\*(a + b\*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c\*(a + b\*x)^p], x], x] + Dist[a\*p, Int[PolyLog[n - 1, c\*(a + b\*x)^p]/(a + b\*x), x], x]) /; FreeQ[{a, b, c, p}, x] && GtQ[n, 0]

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^p_.*(u_.), x_Symbol] := Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*(f
_) + (g_.)*x] /; FreeQ[{e, f, g}, x])
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^p_.*(u_)^q_.), x_Symbol] := In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_)^(m_.))*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)]/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)]/e, x] + Dist[b/e, Int[(Log[d
+ e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 (g + h \log(f(d + ex^n))) \operatorname{Li}_2(c(a + bx)) dx &= \frac{1}{3} x^3 (g + h \log(f(d + ex^n))) \operatorname{Li}_2(c(a + bx)) + \frac{1}{3} b \int \left( \frac{a^2 \log(1 - ac - bcx)}{1 - ac - bcx} \right) dx \\
&= \frac{1}{3} x^3 (g + h \log(f(d + ex^n))) \operatorname{Li}_2(c(a + bx)) + \frac{1}{3} \int x^2 \log(1 - ac - bcx) dx \\
&= \frac{a^2 x \log(1 - ac - bcx) (g + h \log(f(d + ex^n)))}{3b^2} - \frac{ax^2 \log(1 - ac - bcx)}{3b^2} \\
&= \frac{a^2 x \log(1 - ac - bcx) (g + h \log(f(d + ex^n)))}{3b^2} - \frac{ax^2 \log(1 - ac - bcx)}{3b^2} \\
&= \frac{a^2 x \log(1 - ac - bcx) (g + h \log(f(d + ex^n)))}{3b^2} - \frac{ax^2 \log(1 - ac - bcx)}{3b^2} \\
&= -\frac{a^2 gx}{3b^2} + \frac{a(1 - ac)gx}{6b^2 c} - \frac{(1 - ac)^2 gx}{9b^2 c^2} + \frac{d^2 hnx}{3e^2} + \frac{5ahnx^2 \log(1 - ac - bcx)}{36b} \\
&= -\frac{a^2 gx}{3b^2} + \frac{a(1 - ac)gx}{6b^2 c} - \frac{(1 - ac)^2 gx}{9b^2 c^2} + \frac{4a^2 hnx}{9b^2} + \frac{4d^2 hnx}{9e^2} + \frac{5ahnx^2}{36b} \\
&= -\frac{a^2 gx}{3b^2} + \frac{a(1 - ac)gx}{6b^2 c} - \frac{(1 - ac)^2 gx}{9b^2 c^2} + \frac{7a^2 hnx}{9b^2} - \frac{11a(1 - ac)hnx}{36b^2 c} + \frac{5ahnx^2}{36b}
\end{aligned}$$

**Mathematica [A]** time = 11.4813, size = 2610, normalized size = 0.87

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[x^2\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)],x]

[Out] ((g - h\*n\*Log[d + e\*x] + h\*Log[f\*(d + e\*x)^n])\*(-(b\*c\*x\*(12 + 66\*a^2\*c^2 + 6\*b\*c\*x + 4\*b^2\*c^2\*x^2 - 3\*a\*c\*(14 + 5\*b\*c\*x))) + 6\*(-2 + 11\*a^3\*c^3 + 2\*b^3\*c^3\*x^3 + 6\*a^2\*c^2\*(-3 + b\*c\*x) + a\*(9\*c - 3\*b^2\*c^3\*x^2))\*Log[1 - a\*c - b\*c\*x] + 36\*c^3\*(a^3 + b^3\*x^3)\*PolyLog[2, c\*(a + b\*x)])/(108\*b^3\*c^3) + (h\*n\*(36\*b^3\*c^3\*(e\*x\*(-6\*d^2 + 3\*d\*e\*x - 2\*e^2\*x^2) + 6\*(d^3 + e^3\*x^3)\*Log[d + e\*x])\*PolyLog[2, c\*(a + b\*x)] - 216\*b^2\*c^2\*d^2\*e\*(1 - a\*c - b\*c\*x + (-1 + a\*c + b\*c\*x - a\*c\*Log[c\*(a + b\*x)]))\*Log[1 - a\*c - b\*c\*x] - a\*c\*PolyLog[2, 1 - a\*c - b\*c\*x]) - 27\*b\*c\*d\*e^2\*(c\*(-4\*a^2\*c + a\*(4 - 6\*b\*c\*x) + b\*x\*(2 + b\*c\*x)) + (2 + 6\*a^2\*c^2 - 2\*b^2\*c^2\*x^2 + 4\*a\*c\*(-2 + b\*c\*x) - 4\*a^2\*c^2\*Log[c\*(a + b\*x)]))\*Log[1 - a\*c - b\*c\*x] - 4\*a^2\*c^2\*PolyLog[2, 1 - a\*c - b\*c\*x]) - 2\*e^3\*(-(c\*(36\*a^3\*c^2 - 3\*a\*b\*c\*x\*(14 + 5\*b\*c\*x) + 6\*a^2\*c\*(-6 + 11\*b\*c\*x) + 2\*b\*x\*(6 + 3\*b\*c\*x + 2\*b^2\*c^2\*x^2))) - 6\*(2 - 11\*a^3\*c^3 - 2\*b^3\*c^3\*x^3 - 6\*a^2\*c^2\*(-3 + b\*c\*x) + 3\*a\*c\*(-3 + b^2\*c^2\*x^2) + 6\*a^3\*c^3\*Log[c\*(a + b\*x)]))\*Log[1 - a\*c - b\*c\*x] - 36\*a^3\*c^3\*PolyLog[2, 1 - a\*c - b\*c\*x]) + 216\*b^3\*c^3\*d^3\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-b\*d) + a\*e]))\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]/(b\*d - a\*e))\*(-2\*Log[1 - a\*c - b\*c\*x] + Log[(b\*(d + e\*x))/(b\*d - a\*e)]))/2 + (-Log[c\*(a + b\*x)] + Log[(e\*(a + b\*x))/(-b\*d) + a\*e])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + (Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]^2\*(Log[c\*(a + b\*x)] - Log[((b\*c\*d + e - a\*c\*e)\*(a + b\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x))]) + Log[(b\*c\*d + e - a\*c\*e)/(e - a\*c\*e - b\*c\*e\*x)]))/2 + (Log[d + e\*x] - Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]))\*PolyLog[2, 1 - a\*c - b\*c\*x] + (Log[1 - a\*c - b\*c\*x] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]))\*PolyLog[2, (b\*(d + e\*x))/(b\*d - a\*e)] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]\*(PolyLog[2, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] - PolyLog[2, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))])) - PolyLog[3, 1 - a\*c - b\*c\*x] - PolyLog[3, (b\*(d + e\*x))/(b\*d - a\*e)] - PolyLog[3, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] + PolyLog[3, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + 2\*(b\*c\*(e\*(48\*(-1 + a\*c)^2\*e^2\*x + 3\*b\*c\*(-1 + a\*c)\*(12\*d^2 + 12\*d\*e\*x - 5\*e^2\*x^2) + b^2\*c^2\*x\*(48\*d^2 - 15\*d\*e\*x + 8\*e^2\*x^2)) - 6\*(d + e\*x)\*(6\*(-1 + a\*c)^2\*e^2 + 3\*b\*c\*(-1 + a\*c)\*e\*(d - e\*x) + 2\*b^2\*c^2\*(d^2 - d\*e\*x + e^2\*x^2))\*Log[d + e\*x]) + 6\*Log[1 - a\*c - b\*c\*x]\*(-(e\*(-1 + a\*c + b\*c\*x)\*(2\*(-1 + a\*c)^2\*e^2 + b\*c\*(-1 + a\*c)\*e\*(3\*d - 2\*e\*x) + b^2\*c^2\*(6\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2))) + 6\*e^3\*(-1 + 3\*a\*c - 3\*a^2\*c^2 + a^3\*c^3 + b^3\*c^3\*x^3)\*Log[d + e\*x] + 6\*(b^3\*c^3\*d^3 - (-1 + a\*c)^3\*e^3)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)]) + 36\*(b^3\*c^3\*d^3 - (-1 + a\*c)^3\*e^3)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-b\*c\*d) + (-1 + a\*c)\*e] - 108\*a^2\*c^2\*e^2\*(e - a\*c\*e - 2\*b\*c\*e\*x + b\*c\*d\*Log[d + e\*x] + b\*c\*e\*x\*Log[d + e\*x] - Log[1 - a\*c - b\*c\*x]\*(-(e\*(-1 + a\*c + b\*c\*x)) + e\*(-1 + a\*c + b\*c\*x))\*Log[d + e\*x] + (b\*c\*d + e - a\*c\*e)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)]) - (b\*c\*d + e - a\*c\*e)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-b\*c\*d) + (-1 + a\*c)\*e]) - 27\*a\*c\*e\*(b\*c\*(e\*(d\*(2 - 2\*a\*c - 3\*b\*c\*x) + e\*x\*(3 - 3\*a\*c + b\*c\*x)) + (d + e\*x)\*(2\*(-1 + a\*c)\*e + b\*c\*(d - e\*x))\*Log[d + e\*x]) + Log[1 - a\*c - b\*c\*x]\*(e\*(-1 + a\*c + b\*c\*x)\*((-1 + a\*c)\*e + b\*c\*(2\*d - e\*x)) - 2\*e^2\*(1 - 2\*a\*c + a^2\*c^2 - b^2\*c^2\*x^2)\*Log[d + e\*x] + 2\*(-(b^2\*c^2\*d^2) + (-1 + a\*c)^2\*e^2)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)]) + 2\*(-(b^2\*c^2\*d^2) + (-1 + a\*c)^2\*e^2)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-b\*c\*d) + (-1 + a\*c)\*e]) - 108\*a^3\*c^3\*e^3\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-b\*d) + a\*e]))\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*(-2\*Log[1 - a\*c - b\*c\*x] + Log[(b\*(d + e\*x))/(b\*d

$$\begin{aligned}
& - a*e)))/2 + (-\text{Log}[c*(a + b*x)] + \text{Log}[(e*(a + b*x))/(-(b*d) + a*e)])*\text{Log}[(b*(d + e*x))/(b*d - a*e)]*\text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] \\
& + (\text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(\text{Log}[c*(a + b*x)] - \text{Log}[(b*c*d + e - a*c*e)*(a + b*x)/((b*d - a*e)*(-1 + a*c + b*c*x))]) \\
& + \text{Log}[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (\text{Log}[d + e*x] - \text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*\text{PolyLog}[2, 1 - a*c - b*c*x] \\
& + (\text{Log}[1 - a*c - b*c*x] + \text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*\text{PolyLog}[2, (b*(d + e*x))/(b*d - a*e)] + \text{Log}[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(\text{PolyLog}[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - \text{PolyLog}[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])) - \text{PolyLog}[3, 1 - a*c - b*c*x] - \text{PolyLog}[3, (b*(d + e*x))/(b*d - a*e)] - \text{PolyLog}[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + \text{PolyLog}[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])))/(648*b^3*c^3*e^3)
\end{aligned}$$

**Maple [F]** time = 0.493, size = 0, normalized size = 0.

$$\int x^2 (g + h \ln(f(e^x + d)^n)) \text{polylog}(2, c(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

[Out] int(x^2\*(g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(6e^3hx^3 \log((ex + d)^n) + 3de^2hnx^2 - 6d^2ehnx + 6d^3hn \log(ex + d) - 2(e^3hn - 3e^3h \log(f) - 3e^3g)x^3)\text{Li}_2(bcx + a)}{18e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out] 1/18\*(6\*e^3\*h\*x^3\*log((e\*x + d)^n) + 3\*d\*e^2\*h\*n\*x^2 - 6\*d^2\*e\*h\*n\*x + 6\*d^3\*h\*n\*log(e\*x + d) - 2\*(e^3\*h\*n - 3\*e^3\*h\*log(f) - 3\*e^3\*g)\*x^3)\*dilog(b\*c\*x + a\*c)/e^3 + integrate(1/18\*(6\*b\*e^3\*h\*x^3\*log(-b\*c\*x - a\*c + 1)\*log((e\*x + d)^n) + (3\*b\*d\*e^2\*h\*n\*x^2 - 6\*b\*d^2\*e\*h\*n\*x + 6\*b\*d^3\*h\*n\*log(e\*x + d) - 2\*(b\*e^3\*h\*n - 3\*b\*e^3\*h\*log(f) - 3\*b\*e^3\*g)\*x^3)\*log(-b\*c\*x - a\*c + 1))/(b\*e^3\*x + a\*e^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(hx^2\text{Li}_2(bcx + ac) \log((ex + d)^n f) + gx^2\text{Li}_2(bcx + ac), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out] `integral(h*x^2*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*x^2*dilog(b*c*x + a*c), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (h \log((ex + d)^n f) + g) x^2 \operatorname{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac")`

[Out] `integrate((h*log((e*x + d)^n*f) + g)*x^2*dilog((b*x + a)*c), x)`



### 3.178 $\int x \left( g + h \log \left( f(d + ex)^n \right) \right) \text{PolyLog}(2, c(a + bx)) dx$

**Optimal.** Leaf size=2252

result too large to display

```
[Out] (a*g*x)/(2*b) - ((1 - a*c)*g*x)/(4*b*c) - (5*a*h*n*x)/(4*b) + ((1 - a*c)*h*
n*x)/(2*b*c) - (7*d*h*n*x)/(8*e) + (3*h*n*x^2)/16 + ((1 - a*c)^2*h*n*Log[1
- a*c - b*c*x])/(4*b^2*c^2) - (h*n*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*h*n*(
1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b^2*c) - (3*d*h*n*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b*c*e) + (d^2*h*n*Log[d + e*x])/(8*e^2) - (d^2*
h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*e^2)
- (a*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(
2*b*e) + (a*h*(d + e*x)*Log[f*(d + e*x)^n])/(2*b*e) - ((1 - a*c)*h*(d + e*x
)*Log[f*(d + e*x)^n])/(4*b*c*e) - (x^2*(g + h*Log[f*(d + e*x)^n]))/8 - (a*x
*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*b) + (x^2*Log[1 - a*c
- b*c*x]*(g + h*Log[f*(d + e*x)^n]))/4 + (a*(1 - a*c)*Log[(e*(1 - a*c - b*c
*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*b^2*c) - ((1 - a*c
)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(4*b^2*c^2) + (a^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c
*(d + e*x)]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*b^2) - (d^2*h*n*(Log[c*(a
+ b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)]) - Log[((b*c*d + e - a*c*
e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))]^2)/(4*e^2) + (a^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*
x)])/(2*b^2) - (d^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(2*e^2) - (a^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*
(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])
^2)/(4*b^2) + (d^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))
])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x
)]^2)/(4*e^2) - (a^2*g*PolyLog[2, c*(a + b*x)])/(2*b^2) + (a^2*h*n*PolyLog
[2, c*(a + b*x)])/(4*b^2) + (a*d*h*n*PolyLog[2, c*(a + b*x)])/(2*b*e) + (d*
h*n*x*PolyLog[2, c*(a + b*x)])/(2*e) - (h*n*x^2*PolyLog[2, c*(a + b*x)])/4
- (d^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*e^2) + (a^2*h*(n*Log[d
+ e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*(g + h
*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (d^2*h*n*PolyLog[2, (e*(1
- a*c - b*c*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*PolyLog[2, (e*(1
- a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a^2*h*n*(Log[(b*(d + e*x))
/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d
+ e*x))/(b*d - a*e)])/(2*b^2) - (d^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))] + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)])/(2*e^2) + (a*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*
c*e)])/(2*b^2*c) - ((1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e -
a*c*e)])/(4*b^2*c^2) + (a^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d -
a*e)*(1 - c*(a + b*x)))]*PolyLog[2, 1 - c*(a + b*x)])/(2*b^2) - (d^2*h*n*(
Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[
2, 1 - c*(a + b*x)])/(2*e^2) - (a^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^
2) + (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
-((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*h*n*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a +
b*x)))/(b*(d + e*x)))])/(2*b^2) - (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))
])/(2*e^2) - (a^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*b^2) + (d^2*
h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*e^2) - (a^2*h*n*PolyLog[3, 1
- c*(a + b*x)])/(2*b^2) + (d^2*h*n*PolyLog[3, 1 - c*(a + b*x)])/(2*e^2) - (
a^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^2) + (d^
2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*
h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*b^2) - (d
```

$$^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/(2*e^2)$$

**Rubi [A]** time = 2.90446, antiderivative size = 2252, normalized size of antiderivative = 1., number of steps used = 67, number of rules used = 20, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$ , Rules used = {6603, 2430, 43, 2416, 2389, 2295, 2394, 2393, 2391, 2439, 2395, 2440, 2438, 2437, 2435, 6595, 2444, 2421, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

[In] Int[x\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)],x]

[Out]  $(a*g*x)/(2*b) - ((1 - a*c)*g*x)/(4*b*c) - (5*a*h*n*x)/(4*b) + ((1 - a*c)*h*n*x)/(2*b*c) - (7*d*h*n*x)/(8*e) + (3*h*n*x^2)/16 + ((1 - a*c)^2*h*n*Log[1 - a*c - b*c*x])/(4*b^2*c^2) - (h*n*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*h*n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b^2*c) - (3*d*h*n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b*c*e) + (d^2*h*n*Log[d + e*x])/(8*e^2) - (d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a*h*(d + e*x)*Log[f*(d + e*x)^n])/(2*b*e) - ((1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(4*b*c*e) - (x^2*(g + h*Log[f*(d + e*x)^n]))/8 - (a*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*b) + (x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/4 + (a*(1 - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*b^2*c) - ((1 - a*c)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(4*b^2*c^2) + (a^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)])) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2/(4*b^2) - (d^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)])) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2/(4*e^2) + (a^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(2*b^2) - (d^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(2*e^2) - (a^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2/(4*b^2) + (d^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2/(4*e^2) - (a^2*g*PolyLog[2, c*(a + b*x)])/(2*b^2) + (a^2*h*n*PolyLog[2, c*(a + b*x)])/(4*b^2) + (a*d*h*n*PolyLog[2, c*(a + b*x)])/(2*b*e) + (d*h*n*x*PolyLog[2, c*(a + b*x)])/(2*e) - (h*n*x^2*PolyLog[2, c*(a + b*x)])/4 - (d^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*e^2) + (a^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (d^2*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*b^2) - (d^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*e^2) + (a*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*b^2*c) - ((1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*b^2*c^2) + (a^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]))*PolyLog[2, 1 - c*(a + b*x)]/(2*b^2) - (d^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]))*PolyLog[2, 1 - c*(a + b*x)]/(2*e^2) - (a^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*b^2) + (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*e^2) + (a^2*h*n*Log[(b*(d +$

$$e*x)/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/(2*b^2) - (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/(2*e^2) - (a^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/(2*b^2) + (d^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/(2*e^2) - (a^2*h*n*PolyLog[3, 1 - c*(a + b*x)]/(2*b^2) + (d^2*h*n*PolyLog[3, 1 - c*(a + b*x)]/(2*e^2) - (a^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*b^2) + (d^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*e^2) + (a^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*b^2) - (d^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*e^2)$$
Rule 6603

$$\text{Int}[(g_.) + \text{Log}[(f_.)*((d_.) + (e_.)*(x_.))^n]* (h_.)*(x_.)^m * \text{PolyLog}[2, (c_.)*((a_.) + (b_.)*(x_.))], x\_Symbol] \rightarrow \text{Simp}[(x^{m+1})*(g + h*\text{Log}[f*(d + e*x)^n]) * \text{PolyLog}[2, c*(a + b*x)]/(m + 1), x] + (\text{Dist}[b/(m + 1), \text{Int}[\text{ExpandIntegrand}[(g + h*\text{Log}[f*(d + e*x)^n]) * \text{Log}[1 - a*c - b*c*x], x^{m+1}/(a + b*x), x], x] - \text{Dist}[(e*h*n)/(m + 1), \text{Int}[\text{ExpandIntegrand}[\text{PolyLog}[2, c*(a + b*x)], x^{m+1}/(d + e*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{NeQ}[m, -1]$$
Rule 2430

$$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^n]* (b_.)^p * ((f_.) + \text{Log}[(h_.)*((i_.) + (j_.)*(x_.))^m]* (g_.)), x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*(d + e*x)^n])^p * (f + g*\text{Log}[h*(i + j*x)^m]), x] + (-\text{Dist}[g*j*m, \text{Int}[(x*(a + b*\text{Log}[c*(d + e*x)^n])^p)/(i + j*x), x], x] - \text{Dist}[b*e*n*p, \text{Int}[(x*(a + b*\text{Log}[c*(d + e*x)^n])^p - 1)*(f + g*\text{Log}[h*(i + j*x)^m])]/(d + e*x), x], x)) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, m, n\}, x] \&\& \text{IGtQ}[p, 0]$$
Rule 43

$$\text{Int}[(a_.) + (b_.)*(x_.)^m * ((c_.) + (d_.)*(x_.))^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$$
Rule 2416

$$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^n]* (b_.)^p * ((h_.)*(x_.))^m * ((f_.) + (g_.)*(x_.)^r)^q, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, (h*x)^m * (f + g*x^r)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, n, p, q, r\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[q]$$
Rule 2389

$$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^n]* (b_.)^p, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$$
Rule 2295

$$\text{Int}[\text{Log}[(c_.)*(x_.)^n], x\_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n\}, x]$$
Rule 2394

$$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^n]* (b_.)/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]) * (a + b*\text{Log}[c*(d + e*x)$$

)^n))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*(x\_)^(r\_.), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 2440

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*l)/l) + (e\*x)/l)^n])\*(f + g\*Log[h\*(-((j\*k - i\*l)/l) + (j\*x)/l)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

#### Rule 2438

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.) + (f\_.)))/(x\_), x\_Symbol] := Dist[f, Int[(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] + Dist[g, Int[(Log[h\*(i + j\*x)^m]\*(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0]

#### Rule 2437

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]/(x\_), x\_Symbol] := Dist[m, Int[(Log[i + j\*x]\*Log[c\*(d + e\*x)^n])/x, x], x] - Dist[m\*Log[i + j\*x] - Log[h\*(i + j\*x)^m], Int[Log[c\*(d + e\*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0] && NeQ[i + j\*x, h\*(i + j\*x)^m]

#### Rule 2435

```

Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] := Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-((b*c - a*d)*x)/(a*(c + d*x))]) + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
+ (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))],
x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]

```

#### Rule 6595

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] := Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]

```

#### Rule 2444

```

Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] := Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] && !LinearMatchQ[v, x] && !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])

```

#### Rule 2421

```

Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] := In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] && !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])

```

#### Rule 6598

```

Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)]/(e*(m + 1)), x] +
Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

```

#### Rule 6597

```

Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)]/e, x] + Dist[b/e, Int[(Log[d
+ e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]

```

#### Rubi steps

$$\begin{aligned}
\int x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) dx &= \frac{1}{2}x^2(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + \frac{1}{2}b \int \left( -\frac{a \log(1 - ac - bcx)}{1 - ac - bcx} \right) dx \\
&= \frac{1}{2}x^2(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + \frac{1}{2} \int x \log(1 - ac - bcx) (g + h \log(f(d + ex)^n)) dx \\
&= -\frac{ax \log(1 - ac - bcx) (g + h \log(f(d + ex)^n))}{2b} + \frac{1}{4}x^2 \log(1 - ac - bcx) (g + h \log(f(d + ex)^n)) \\
&= -\frac{ax \log(1 - ac - bcx) (g + h \log(f(d + ex)^n))}{2b} + \frac{1}{4}x^2 \log(1 - ac - bcx) (g + h \log(f(d + ex)^n)) \\
&= -\frac{ax \log(1 - ac - bcx) (g + h \log(f(d + ex)^n))}{2b} + \frac{1}{4}x^2 \log(1 - ac - bcx) (g + h \log(f(d + ex)^n)) \\
&= \frac{agx}{2b} - \frac{(1 - ac)gx}{4bc} - \frac{dhnx}{2e} - \frac{dhn(1 - ac - bcx) \log(1 - ac - bcx)}{2bce} - \frac{d^2hn}{4b^2c} \\
&= \frac{agx}{2b} - \frac{(1 - ac)gx}{4bc} - \frac{3ahnx}{4b} - \frac{3dhnx}{4e} - \frac{3ahn(1 - ac - bcx) \log(1 - ac - bcx)}{4b^2c} \\
&= \frac{agx}{2b} - \frac{(1 - ac)gx}{4bc} - \frac{5ahnx}{4b} + \frac{(1 - ac)hnx}{4bc} - \frac{7dhnx}{8e} + \frac{1}{16}hnx^2 - \frac{3ahn(1 - ac - bcx) \log(1 - ac - bcx)}{4b^2c}
\end{aligned}$$

**Mathematica [A]** time = 8.83211, size = 1996, normalized size = 0.89

result too large to display

Antiderivative was successfully verified.

[In] Integrate[x\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x]

[Out] ((g - h\*n\*Log[d + e\*x] + h\*Log[f\*(d + e\*x)^n])\*(-(b\*c\*x\*(2 - 6\*a\*c + b\*c\*x)) + (-2 - 6\*a^2\*c^2 + 2\*b^2\*c^2\*x^2 - 4\*a\*c\*(-2 + b\*c\*x))\*Log[1 - a\*c - b\*c\*x] - 4\*c^2\*(a^2 - b^2\*x^2)\*PolyLog[2, c\*(a + b\*x)])/(8\*b^2\*c^2) + (h\*n\*(4\*b^2\*c^2\*(e\*x\*(2\*d - e\*x) - 2\*(d^2 - e^2\*x^2))\*Log[d + e\*x])\*PolyLog[2, c\*(a + b\*x)] + 8\*b\*c\*d\*e\*(1 - a\*c - b\*c\*x + (-1 + a\*c + b\*c\*x - a\*c\*Log[c\*(a + b\*x)]))\*Log[1 - a\*c - b\*c\*x] - a\*c\*PolyLog[2, 1 - a\*c - b\*c\*x] + e^2\*(c\*(-4\*a^2\*c + a\*(4 - 6\*b\*c\*x) + b\*x\*(2 + b\*c\*x)) + (2 + 6\*a^2\*c^2 - 2\*b^2\*c^2\*x^2 + 4\*a\*c\*(-2 + b\*c\*x) - 4\*a^2\*c^2\*Log[c\*(a + b\*x)]))\*Log[1 - a\*c - b\*c\*x] - 4\*a^2\*c^2\*PolyLog[2, 1 - a\*c - b\*c\*x] - 8\*b^2\*c^2\*d^2\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-b\*d + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*(-2\*Log[1 - a\*c - b\*c\*x] + Log[(b\*(d + e\*x))/(b\*d - a\*e)]))/2 + (-Log[c\*(a + b\*x)] + Log[(e\*(a + b\*x))/(-b\*d + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + (Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]^2\*(Log[c\*(a + b\*x)] - Log[((b\*c\*d + e - a\*c\*e)\*(a + b\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x))]) + Log[(b\*c\*d + e - a\*c\*e)/(e - a\*c\*e - b\*c\*e\*x]))/2 + (Log[d + e\*x] - Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]))\*PolyLog[2, 1 - a\*c - b\*c\*x] + (Log[1 - a\*c - b\*c\*x] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]))\*PolyLog[2, (b\*(d + e\*x))/(b\*d - a\*e)

)] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]\*(PolyLog[2, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] - PolyLog[2, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] - PolyLog[3, 1 - a\*c - b\*c\*x] - PolyLog[3, (b\*(d + e\*x))/(b\*d - a\*e)] - PolyLog[3, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] + PolyLog[3, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + 2\*(b\*c\*(e\*(d\*(2 - 2\*a\*c - 3\*b\*c\*x) + e\*x\*(3 - 3\*a\*c + b\*c\*x)) + (d + e\*x)\*(2\*(-1 + a\*c)\*e + b\*c\*(d - e\*x))\*Log[d + e\*x]) + Log[1 - a\*c - b\*c\*x]\*(e\*(-1 + a\*c + b\*c\*x)\*((-1 + a\*c)\*e + b\*c\*(2\*d - e\*x)) - 2\*e^2\*(1 - 2\*a\*c + a^2\*c^2 - b^2\*c^2\*x^2)\*Log[d + e\*x] + 2\*(-(b^2\*c^2\*d^2) + (-1 + a\*c)^2\*e^2)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)] + 2\*(-(b^2\*c^2\*d^2) + (-1 + a\*c)^2\*e^2)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-(b\*c\*d) + (-1 + a\*c)\*e)] + 4\*a\*c\*e\*(e - a\*c\*e - 2\*b\*c\*e\*x + b\*c\*d\*Log[d + e\*x] + b\*c\*e\*x\*Log[d + e\*x] - Log[1 - a\*c - b\*c\*x]\*(-(e\*(-1 + a\*c + b\*c\*x)) + e\*(-1 + a\*c + b\*c\*x)\*Log[d + e\*x] + (b\*c\*d + e - a\*c\*e)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)] - (b\*c\*d + e - a\*c\*e)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-(b\*c\*d) + (-1 + a\*c)\*e)] + 4\*a^2\*c^2\*e^2\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-(b\*d) + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*(-2\*Log[1 - a\*c - b\*c\*x] + Log[(b\*(d + e\*x))/(b\*d - a\*e)]))/2 + (-Log[c\*(a + b\*x)] + Log[(e\*(a + b\*x))/(-(b\*d) + a\*e)])\*Log[(b\*(d + e\*x))/(b\*d - a\*e)]\*Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] + (Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]^2\*(Log[c\*(a + b\*x)] - Log[(b\*c\*d + e - a\*c\*e)\*(a + b\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x))] + Log[(b\*c\*d + e - a\*c\*e)/(e - a\*c\*e - b\*c\*e\*x)]))/2 + (Log[d + e\*x] - Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))])\*PolyLog[2, 1 - a\*c - b\*c\*x] + (Log[1 - a\*c - b\*c\*x] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))])\*PolyLog[2, (b\*(d + e\*x))/(b\*d - a\*e)] + Log[-((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))]\*(PolyLog[2, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] - PolyLog[2, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))] - PolyLog[3, 1 - a\*c - b\*c\*x] - PolyLog[3, (b\*(d + e\*x))/(b\*d - a\*e)] - PolyLog[3, (b\*c\*(d + e\*x))/(e\*(-1 + a\*c + b\*c\*x))] + PolyLog[3, -((b\*(d + e\*x))/((b\*d - a\*e)\*(-1 + a\*c + b\*c\*x)))])))/(16\*b^2\*c^2\*e^2)

**Maple [F]** time = 0.289, size = 0, normalized size = 0.

$$\int x(g + h \ln(f(ex + d)^n)) \operatorname{polylog}(2, c(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

[Out] int(x\*(g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(2e^2hx^2 \log((ex + d)^n) + 2dehnx - 2d^2hn \log(ex + d) - (e^2hm - 2e^2h \log(f) - 2e^2g)x^2) \operatorname{Li}_2(bcx + ac)}{4e^2} + \int \frac{2be^2hx}{e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out] 1/4\*(2\*e^2\*h\*x^2\*log((e\*x + d)^n) + 2\*d\*e\*h\*n\*x - 2\*d^2\*h\*n\*log(e\*x + d) - (e^2\*h\*n - 2\*e^2\*h\*log(f) - 2\*e^2\*g)\*x^2)\*dilog(b\*c\*x + a\*c)/e^2 + integrat

$$e(1/4*(2*b*e^2*h*x^2*\log(-b*c*x - a*c + 1)*\log((e*x + d)^n) + (2*b*d*e*h*n*x - 2*b*d^2*h*n*\log(e*x + d) - (b*e^2*h*n - 2*b*e^2*h*\log(f) - 2*b*e^2*g)*x^2)*\log(-b*c*x - a*c + 1))/(b*e^2*x + a*e^2), x)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(hx\text{Li}_2(bc x + ac) \log((ex + d)^n f) + gx\text{Li}_2(bc x + ac), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out] integral(h\*x\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*x\*dilog(b\*c\*x + a\*c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (h \log((ex + d)^n f) + g)x\text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*x\*dilog((b\*x + a)\*c), x)



### 3.179 $\int (g + h \log(f(d + ex)^n)) \text{PolyLog}(2, c(a+bx)) dx$

**Optimal.** Leaf size=1653

result too large to display

```
[Out] -(g*x) + 3*h*n*x - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (2*h*
n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (d*h*n*Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[-d - e*x])/e + (d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/e + (d*h*n*(Log[c*(a + b*x)] + Log[(b*c*d
+ e - a*c*e)/(b*c*(d + e*x)]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d
+ e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (d*
h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - a*c - b
*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (h*(
d + e*x)*Log[f*(d + e*x)^n])/e + h*x*Log[1 - a*c - b*c*x]*Log[f*(d + e*x)^n
] - ((1 - a*c)*h*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*Log[f*(d +
e*x)^n])/(b*c) - (a*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
+ e*x)]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2)/(2*b) - (a*h*n*Log[c*(a + b*x)]*
Log[d + e*x]*Log[1 - c*(a + b*x)]/b + (a*h*n*(Log[c*(a + b*x)] - Log[-((e*
(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)] + Log[1 - c*(a + b*x)]^2)/(2*b) + (a*g*PolyLog[2, c*(a + b*x)]/b - (a*
h*n*PolyLog[2, c*(a + b*x)]/b - (a*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])
*PolyLog[2, c*(a + b*x)]/b + x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a
+ b*x)] + (d*h*n*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c -
b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e + (d*h*n*PolyLog[2, (e*(1 - a*c -
b*c*x))/(b*c*d + e - a*c*e)])/e - h*n*x*PolyLog[2, a*c + b*c*x] + (d*h*n*L
og[-d - e*x]*PolyLog[2, a*c + b*c*x])/e - (d*h*n*Log[(b*(d + e*x))/((b*d -
a*e)*(1 - a*c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)
))]/e + (d*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2
, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))]/e + (d*h*n*(Log[1 - a*c -
b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, (b
*(d + e*x))/(b*d - a*e)])/e - (a*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))] + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d + e*x))/(b*d - a*e)
])/b - ((1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/b*c)
- (a*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)])*PolyLog[2, 1 - c*(a + b*x)]/b + (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(
1 - c*(a + b*x))])*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))])/b
- (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])*PolyLog[2, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b - (d*h*n*PolyLog[3, 1 - a*c -
b*c*x])/e - (d*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))])/e
+ (d*h*n*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (a
*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/b - (d*h*n*PolyLog[3, (b*(d + e
*x))/(b*d - a*e)])/e + (a*h*n*PolyLog[3, 1 - c*(a + b*x)]/b + (a*h*n*PolyL
og[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))])/b - (a*h*n*PolyLog[3, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b
```

---

**Rubi [A]** time = 3.18592, antiderivative size = 1653, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 17, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$ , Rules used = {6600, 2418, 2389, 2295, 2394, 2393, 2391, 6688, 43, 2416, 6742, 2430, 2440, 2437, 2435, 6595, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x]
```

```
[Out] -(g*x) + 3*h*n*x - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (2*h*
n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (d*h*n*Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[-d - e*x])/e + (d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e])/e + (d*h*n*(Log[c*(a + b*x)] + Log[(b*c*d
+ e - a*c*e)/(b*c*(d + e*x))]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d
+ e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (d*
h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[1 - a*c - b
*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (h*(
d + e*x)*Log[f*(d + e*x)^n])/e + h*x*Log[1 - a*c - b*c*x]*Log[f*(d + e*x)^n
] - ((1 - a*c)*h*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*Log[f*(d +
e*x)^n])/b - (a*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
+ e*x))]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2)/(2*b) - (a*h*n*Log[c*(a + b*x)]*
Log[d + e*x]*Log[1 - c*(a + b*x)]/b + (a*h*n*(Log[c*(a + b*x)] - Log[-((e
(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)
))] + Log[1 - c*(a + b*x)]^2)/(2*b) + (a*g*PolyLog[2, c*(a + b*x)]/b - (a*
h*n*PolyLog[2, c*(a + b*x)]/b - (a*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])
*PolyLog[2, c*(a + b*x)]/b + x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a
+ b*x)] + (d*h*n*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c -
b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e + (d*h*n*PolyLog[2, (e*(1 - a*c -
b*c*x))/(b*c*d + e - a*c*e)]/e - h*n*x*PolyLog[2, a*c + b*c*x] + (d*h*n*L
og[-d - e*x]*PolyLog[2, a*c + b*c*x])/e - (d*h*n*Log[(b*(d + e*x))/((b*d -
a*e)*(1 - a*c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)
))]/e + (d*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2
, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))]/e + (d*h*n*(Log[1 - a*c -
b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, (b
*(d + e*x))/(b*d - a*e)]/e - (a*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x))]) + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d + e*x))/(b*d - a*e]
])/b - ((1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)]/b - (a
*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])
)*PolyLog[2, 1 - c*(a + b*x)]/b + (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(
1 - c*(a + b*x))])*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))])/b
- (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])*PolyLog[2, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/b - (d*h*n*PolyLog[3, 1 - a*c -
b*c*x])/e - (d*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))])/e
+ (d*h*n*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (a
*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/b - (d*h*n*PolyLog[3, (b*(d + e
*x))/(b*d - a*e)]/e + (a*h*n*PolyLog[3, 1 - c*(a + b*x)]/b + (a*h*n*PolyL
og[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))])/b - (a*h*n*PolyLog[3, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b
```

### Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] := Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_))^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6688

Int[u\_, x\_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.)), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m))]/(d + e\*x), x], x]) /

; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

#### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n)]*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

#### Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.))]/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
+ j*x, h*(i + j*x)^m]
```

#### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-((b*c - a*d)*x)/(a*(c + d*x))]) + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
+ (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))],
x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

#### Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
+ e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

#### Rubi steps

$$\begin{aligned}
\int (g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) dx &= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + b \int \left( \frac{1}{b} - \frac{a}{b(a + bx)} \right) \log(1 - ac - bcx) dx \\
&= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + b \int \frac{x \log(1 - ac - bcx) (g + h \log(f(d + ex)^n))}{a + bx} dx \\
&= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + b \int \left( \frac{gx \log(1 - ac - bcx)}{a + bx} + \frac{hx \log(f(d + ex)^n) \log(1 - ac - bcx)}{a + bx} \right) dx \\
&= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) + (bg) \int \frac{x \log(1 - ac - bcx)}{a + bx} dx + (bh) \int \frac{x \log(f(d + ex)^n) \log(1 - ac - bcx)}{a + bx} dx \\
&= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) - hnx \operatorname{Li}_2(ac + bcx) + \frac{dhn \log(c(a + bx)) \log(1 - ac - bcx)}{e} \\
&= x(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx)) - hnx \operatorname{Li}_2(ac + bcx) + \frac{dhn \log(c(a + bx)) \log(1 - ac - bcx)}{e} \\
&= hnx + \frac{hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{dhn \log(c(a + bx)) \log(1 - ac - bcx)}{e} \\
&= -gx + hnx - \frac{g(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc} \\
&= -gx + hnx - \frac{g(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc} \\
&= -gx + hnx - \frac{g(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc} \\
&= -gx + 3hnx - \frac{g(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{2hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc} \\
&= -gx + 3hnx - \frac{g(1 - ac - bcx) \log(1 - ac - bcx)}{bc} + \frac{2hn(1 - ac - bcx) \log(1 - ac - bcx)}{bc}
\end{aligned}$$

**Mathematica [A]** time = 4.92197, size = 1546, normalized size = 0.94

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x]

[Out] ((g - h\*n\*Log[d + e\*x] + h\*Log[f\*(d + e\*x)^n])\*(-(b\*c\*x) + (-1 + a\*c + b\*c\*x)\*Log[1 - a\*c - b\*c\*x] + c\*(a + b\*x)\*PolyLog[2, c\*(a + b\*x)])/(b\*c) + (h\*n\*((-e\*x) + (d + e\*x)\*Log[d + e\*x])\*PolyLog[2, c\*(a + b\*x)] + (-e + a\*c\*e + 2\*b\*c\*e\*x - b\*c\*d\*Log[d + e\*x] - b\*c\*e\*x\*Log[d + e\*x] + Log[1 - a\*c - b\*c\*x])\*(-(e\*(-1 + a\*c + b\*c\*x)) + e\*(-1 + a\*c + b\*c\*x)\*Log[d + e\*x] + (b\*c\*d + e - a\*c\*e)\*Log[(b\*c\*(d + e\*x))/(b\*c\*d + e - a\*c\*e)]) + e\*(-1 + a\*c + b\*c\*x + (1 - a\*c - b\*c\*x + a\*c\*Log[c\*(a + b\*x)])\*Log[1 - a\*c - b\*c\*x] + a\*c\*PolyLog[2, 1 - a\*c - b\*c\*x]) + (b\*c\*d + e - a\*c\*e)\*PolyLog[2, (e\*(-1 + a\*c + b\*c\*x))/(-b\*c\*d + (-1 + a\*c)\*e)] + b\*c\*d\*(Log[c\*(a + b\*x)]\*Log[1 - a\*c - b\*c\*x]\*Log[d + e\*x] + ((Log[c\*(a + b\*x)] - Log[(e\*(a + b\*x))/(-b\*d + a\*e)]))

$$\begin{aligned}
& * \text{Log}[(b*(d + e*x))/(b*d - a*e)] * (-2 * \text{Log}[1 - a*c - b*c*x] + \text{Log}[(b*(d + e*x)) / (b*d - a*e)]) / 2 + (-\text{Log}[c*(a + b*x)] + \text{Log}[(e*(a + b*x)) / (-b*d + a*e)]) \\
& * \text{Log}[(b*(d + e*x))/(b*d - a*e)] * \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] + (\text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))]^2 * (\text{Log}[c*(a + b*x)] - \text{Log}[(b*c*d + e - a*c*e) * (a + b*x) / ((b*d - a*e) * (-1 + a*c + b*c*x))]) + \text{Log}[(b*c*d + e - a*c*e) / (e - a*c*e - b*c*e*x)]) / 2 + (\text{Log}[d + e*x] - \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * \text{PolyLog}[2, 1 - a*c - b*c*x] + (\text{Log}[1 - a*c - b*c*x] + \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * \text{PolyLog}[2, (b*(d + e*x)) / (b*d - a*e)] + \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * (\text{PolyLog}[2, (b*c*(d + e*x)) / (e * (-1 + a*c + b*c*x))] - \text{PolyLog}[2, -((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))])) - \text{PolyLog}[3, 1 - a*c - b*c*x] - \text{PolyLog}[3, (b*(d + e*x)) / (b*d - a*e)] - \text{PolyLog}[3, (b*c*(d + e*x)) / (e * (-1 + a*c + b*c*x))] + \text{PolyLog}[3, -((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] - a*c*e * (\text{Log}[c*(a + b*x)] * \text{Log}[1 - a*c - b*c*x] * \text{Log}[d + e*x] + ((\text{Log}[c*(a + b*x)] - \text{Log}[(e*(a + b*x)) / (-b*d + a*e)]) * \text{Log}[(b*(d + e*x)) / (b*d - a*e)] * (-2 * \text{Log}[1 - a*c - b*c*x] + \text{Log}[(b*(d + e*x)) / (b*d - a*e)]) / 2 + (-\text{Log}[c*(a + b*x)] + \text{Log}[(e*(a + b*x)) / (-b*d + a*e)]) * \text{Log}[(b*(d + e*x)) / (b*d - a*e)] * \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] + (\text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))]^2 * (\text{Log}[c*(a + b*x)] - \text{Log}[(b*c*d + e - a*c*e) * (a + b*x) / ((b*d - a*e) * (-1 + a*c + b*c*x))]) + \text{Log}[(b*c*d + e - a*c*e) / (e - a*c*e - b*c*e*x)]) / 2 + (\text{Log}[d + e*x] - \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * \text{PolyLog}[2, 1 - a*c - b*c*x] + (\text{Log}[1 - a*c - b*c*x] + \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * \text{PolyLog}[2, (b*(d + e*x)) / (b*d - a*e)] + \text{Log}[-((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))] * (\text{PolyLog}[2, (b*c*(d + e*x)) / (e * (-1 + a*c + b*c*x))] - \text{PolyLog}[2, -((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))])) - \text{PolyLog}[3, 1 - a*c - b*c*x] - \text{PolyLog}[3, (b*(d + e*x)) / (b*d - a*e)] - \text{PolyLog}[3, (b*c*(d + e*x)) / (e * (-1 + a*c + b*c*x))] + \text{PolyLog}[3, -((b*(d + e*x)) / ((b*d - a*e) * (-1 + a*c + b*c*x)))])) / (b*c)) / e
\end{aligned}$$

**Maple [F]** time = 0.285, size = 0, normalized size = 0.

$$\int (g + h \ln(f(ex + d)^n)) \text{polylog}(2, c(bx + a)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

[Out] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(dhn \log(ex + d) + ehx \log((ex + d)^n) - (ehn - eh \log(f) - eg)x) \text{Li}_2(bcx + ac)}{e} + \int \frac{behx \log(-bcx - ac + 1) \log((ex + d)^n)}{e} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="maxima")

[Out] (d\*h\*n\*log(e\*x + d) + e\*h\*x\*log((e\*x + d)^n) - (e\*h\*n - e\*h\*log(f) - e\*g)\*x) \* dilog(b\*c\*x + a\*c) / e + integrate((b\*e\*h\*x\*log(-b\*c\*x - a\*c + 1) \* log((e\*x + d)^n) + (b\*d\*h\*n\*log(e\*x + d) - (b\*e\*h\*n - b\*e\*h\*log(f) - b\*e\*g)\*x) \* log(-

$b*c*x - a*c + 1)/(b*e*x + a*e), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(h\text{Li}_2(bc x + ac) \log\left((ex + d)^n f\right) + g\text{Li}_2(bc x + ac), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="fricas")

[Out] integral(h\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*dilog(b\*c\*x + a\*c), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( h \log\left((ex + d)^n f\right) + g \right) \text{Li}_2((bx + a)c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a)),x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c), x)

$$3.180 \quad \int \frac{(g+h \log(f(d+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x} dx$$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(2,c(a+bx))(h \log(f(d+ex)^n)+g)}{x}, x\right)$$

[Out] Unintegrable[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x, x]

**Rubi [A]** time = 0.0250694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(g+h \log(f(d+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x,x]

[Out] Defer[Int][((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x, x]

Rubi steps

$$\int \frac{(g+h \log(f(d+ex)^n)) \text{Li}_2(c(a+bx))}{x} dx = \int \frac{(g+h \log(f(d+ex)^n)) \text{Li}_2(c(a+bx))}{x} dx$$

**Mathematica [A]** time = 0.552935, size = 0, normalized size = 0.

$$\int \frac{(g+h \log(f(d+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x,x]

[Out] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x, x]

**Maple [A]** time = 0.302, size = 0, normalized size = 0.

$$\int \frac{(g+h \ln(f(ex+d)^n)) \text{polylog}(2,c(bx+a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x,x)

[Out] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x,x)



---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x,x, algorithm="maxima")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h \text{Li}_2(bc x + ac) \log((ex + d)^n f) + g \text{Li}_2(bc x + ac)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x,x, algorithm="fricas")

[Out] integral((h\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*dilog(b\*c\*x + a\*c))/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a))/x,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x,x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x, x)

$$3.181 \quad \int \frac{(g+h \log(f(d+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x^2} dx$$

Optimal. Leaf size=2498

result too large to display

```
[Out] -((b*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/a - (b*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(2*a) + (b*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(2*a) + (b*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n])/a + (b*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2)/(2*a) - (e*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2)/(2*d) + (e*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)]/d + (b*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)]/a - (e*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)]/d - (b*h*n*(Log[c*(a + b*x)] - Log[-(e*(a + b*x))/(b*d - a*e)]))*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(2*a) + (e*h*n*(Log[c*(a + b*x)] - Log[-(e*(a + b*x))/(b*d - a*e)]))*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(2*d) + (e*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -(b*x)/a])/d - (b*g*PolyLog[2, c*(a + b*x)]/a + (e*h*n*Log[x]*PolyLog[2, c*(a + b*x)]/d - (e*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)]/d + (b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/a - ((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/x - (b*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a - (b*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a + (b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a - (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/a + (b*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]))*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/a - (e*h*n*(Log[(b*(d + e*x))/(b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]))*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/d - (b*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/a + (e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x)))]/d - (e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))]/d + (b*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]))*PolyLog[2, 1 - c*(a + b*x)]/a - (e*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]))*PolyLog[2, 1 - c*(a + b*x)]/d + (e*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)]/d - (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/a + (e*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/d + (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d - (e*h*n*PolyLog[3, -(b*x)/a])/d + (b*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)]/a - (b*h*n*PolyLog[3, (d*(1 - a
```

$$\begin{aligned} & c - b*c*x))/((1 - a*c)*(d + e*x)))/a + (b*h*n*PolyLog[3, -((e*(1 - a*c - b \\ & *c*x))/(b*c*(d + e*x)))]/a - (b*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)] \\ & /a + (e*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/d + (b*h*n*PolyLog[3, 1 \\ & + (e*x)/d])/a + (e*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))]/d - (e*h \\ & *n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/d - (b*h*n*PolyLog[3, 1 - c*(a \\ & + b*x)]/a - (b*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/ \\ & a + (e*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/d + (b*h*n \\ & *PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/a - (e*h*n*Poly \\ & Log[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/d \end{aligned}$$

**Rubi [A]** time = 2.66747, antiderivative size = 2498, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6603, 2438, 2394, 2315, 2437, 2435, 2440, 2391, 6597}

result too large to display

Antiderivative was successfully verified.

[In] Int[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^2,x]

[Out] 
$$\begin{aligned} & -((b*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a - (b*h*n*Log[(b*c*x) \\ & /((1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/a - (b*h*n*(Log[(b*c*x)/(1 - \\ & a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e \\ & )x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x) \\ & )]^2)/(2*a) + (b*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-(e*x)/d])*Log[1 - a* \\ & c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(2*a) + ( \\ & b*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d \\ & + e*x)^n])/a + (b*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d \\ & + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + \\ & e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2)/(2*a) - (e*h*n*(Log[c*(a + b*x)] \\ & + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + \\ & b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))]^2) \\ & /((2*d) + (e*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)]/d + (b*h*n*Lo \\ & g[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)]/a - (e*h*n*Log[c*(a + b*x) \\ & ]*Log[d + e*x]*Log[1 - c*(a + b*x)]/d - (b*h*n*(Log[c*(a + b*x)] - Log[-( \\ & e*(a + b*x))/(b*d - a*e)]))*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b* \\ & x))] + Log[1 - c*(a + b*x)]^2)/(2*a) + (e*h*n*(Log[c*(a + b*x)] - Log[-( \\ & e*(a + b*x))/(b*d - a*e)]))*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b* \\ & x))] + Log[1 - c*(a + b*x)]^2)/(2*d) + (e*h*n*(Log[1 + (b*x)/a] + Log[(1 - \\ & a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))] \\ & )*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[c*(a + b*x)] - \\ & Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) \\ & + (e*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyL \\ & og[2, -((b*x)/a)]/d - (b*g*PolyLog[2, c*(a + b*x)]/a + (e*h*n*Log[x]*Poly \\ & Log[2, c*(a + b*x)]/d - (e*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)]/d + ( \\ & b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/a - ((g \\ & + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/x - (b*g*PolyLog[2, 1 - (b \\ & *c*x)/(1 - a*c)]/a - (b*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*( \\ & 1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a + (b*h*(n*Log[d + \\ & e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/a - (b*h*n*Lo \\ & g[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, (d*(1 - a*c - b*c \\ & *x))/((1 - a*c)*(d + e*x))]/a + (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a \\ & *c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/a + (b* \\ & h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b* \\ & x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/a - (e*h*n*(Log[(b*(d + e*x))/ \\ & (b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + \\ & e*x))/(b*d - a*e)]/d - (b*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + \end{aligned}$$

```

e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, 1 + (e*x)/d])/a + (e*h*n*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))]/d -
(e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a +
b*x)))]/d + (b*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*
(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)]/a - (e*h*n*(Log[d + e*x] - Log[(
b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)]
/d + (e*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, 1 - c
*(a + b*x)]/d - (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*
PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/a + (e*h*n*Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))]/d + (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b
*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/a - (e*h*
n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e
)*(1 - c*(a + b*x)))/(b*(d + e*x))]/d - (e*h*n*PolyLog[3, -((b*x)/a)]/d +
(b*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)]/a - (b*h*n*PolyLog[3, (d*(1 - a*
c - b*c*x))/((1 - a*c)*(d + e*x))]/a + (b*h*n*PolyLog[3, -((e*(1 - a*c - b
*c*x))/(b*c*(d + e*x)))]/a - (b*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]
/a + (e*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/d + (b*h*n*PolyLog[3, 1
+ (e*x)/d])/a + (e*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x)))]/d - (e*h
*n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/d - (b*h*n*PolyLog[3, 1 - c*(a
+ b*x)]/a - (b*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/
a + (e*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/d + (b*h*n
*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/a - (e*h*n*Poly
Log[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]/d

```

#### Rule 6603

```

Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

```

#### Rule 2438

```

Int[(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
+ (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] := Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n])/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]

```

#### Rule 2394

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

```

#### Rule 2315

```

Int[Log[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

```

#### Rule 2437

```

Int[(Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)))/(x_), x_Symbol] := Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x

```

], x] - Dist[m\*Log[i + j\*x] - Log[h\*(i + j\*x)^m], Int[Log[c\*(d + e\*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0] && NeQ[i + j\*x, h\*(i + j\*x)^m]

#### Rule 2435

Int[(Log[(a\_) + (b\_.)\*(x\_)])\*Log[(c\_) + (d\_.)\*(x\_)])/(x\_), x\_Symbol] := Simp[Log[-((b\*x)/a)]\*Log[a + b\*x]\*Log[c + d\*x], x] + (Simp[(1\*(Log[-((b\*x)/a)] - Log[-(((b\*c - a\*d)\*x)/(a\*(c + d\*x))]) + Log[(b\*c - a\*d)/(b\*(c + d\*x))])\*Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2)/2, x] - Simp[(1\*(Log[-((b\*x)/a)] - Log[-((d\*x)/c)])\*(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2)/2, x] + Simp[(Log[c + d\*x] - Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (b\*x)/a], x] + Simp[(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (d\*x)/c], x] + Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))], x] - Simp[PolyLog[3, 1 + (b\*x)/a], x] - Simp[PolyLog[3, 1 + (d\*x)/c], x] + Simp[PolyLog[3, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2440

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*1)/1) + (e\*x)/1)^n])\*(f + g\*Log[h\*(-((j\*k - i\*1)/1) + (j\*x)/1)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.)))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6597

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*PolyLog[2, c\*(a + b\*x)]/e, x] + Dist[b/e, Int[(Log[d + e\*x]\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c\*(b\*d - a\*e) + e, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{x^2} dx &= -\frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{x} - b \int \left( \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{ax} \right) dx \\
&= -\frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{x} - \frac{b \int \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{x} dx}{a} \\
&= \frac{ehn \log(x) \operatorname{Li}_2(c(a + bx))}{d} - \frac{ehn \log(d + ex) \operatorname{Li}_2(c(a + bx))}{d} - \frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{d} \\
&= -\frac{bg \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{a} + \frac{ehn \log(x) \operatorname{Li}_2(c(a + bx))}{d} - \frac{ehn \log(d + ex) \operatorname{Li}_2(c(a + bx))}{d} \\
&= -\frac{bg \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{a} - \frac{bhn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(d + ex)}{a} \\
&= -\frac{bg \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{a} - \frac{bhn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(d + ex)}{a}
\end{aligned}$$

**Mathematica [F]** time = 10.1921, size = 0, normalized size = 0.

$$\int \frac{(g + h \log(f(d + ex)^n)) \operatorname{PolyLog}(2, c(a + bx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^2, x]

[Out] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^2, x]

**Maple [F]** time = 0.428, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(f(ex + d)^n)) \operatorname{polylog}(2, c(bx + a))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^2,x)

[Out] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \operatorname{Li}_2((bx + a)c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^2,x, algorithm="maxima")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h\text{Li}_2(bc x + ac) \log((ex + d)^n f) + g\text{Li}_2(bc x + ac)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^2,x, algorithm="fricas")

[Out] integral((h\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*dilog(b\*c\*x + a\*c))/x^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a))/x\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^2,x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^2, x)

$$3.182 \quad \int \frac{(g+h \log(f(d+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x^3} dx$$

Optimal. Leaf size=3119

result too large to display

```
[Out] (b^2*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b*e*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(a*d) + (b^2*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/(2*a^2) + (b*e*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x)))^2)/(4*a^2) - (b^2*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(4*a^2) - (b^2*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n]))/(2*a^2) + (b^2*c*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(2*a*(1 - a*c)) + (b*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*a*x) - (b^2*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*a*(1 - a*c)) - (b^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*a^2) + (e^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)])) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*d^2) - (e^2*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/(2*d^2) - (b^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(2*a^2) + (e^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(4*a^2) - (e^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(4*d^2) - (e^2*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x)] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]))*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2) - (e^2*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(4*d^2) - (e^2*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)])/(2*d^2) + (b^2*g*PolyLog[2, c*(a + b*x)])/(2*a^2) - (b*e*h*n*PolyLog[2, c*(a + b*x)])/(2*a*d) - (e*h*n*PolyLog[2, c*(a + b*x)])/(2*d*x) - (e^2*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(2*d^2) + (e^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*d^2) - (b^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*a^2) - ((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*x^2) + (b*e*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(a*d) + (b^2*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x)))]/(2*a^2) - (b^2*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) - (b^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) + (e^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*d^2) - (b^2*c*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*a*(1 - a*c)) + (b^2*c*h*n*PolyLog[2, 1 + (e*x)/d])/(2*a*(1 - a*c)) + (b^2*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(2*a^2) - (e^2*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x)))])/(2*d^2) + (e^2*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)
```



$$\begin{aligned}
& c*x)/(1 - c*(a + b*x)))]/(2*d^2) - (b^2*h*n*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, 1 - c*(a + b*x)]/(2*a^2) \\
& + (e^2*h*n*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, 1 - c*(a + b*x)]/(2*d^2) - (e^2*h*n*(\text{Log}[x] + \text{Log}[-((a*(1 - c*(a + b*x)))/(b*x))]) *PolyLog[2, 1 - c*(a + b*x)]/(2*d^2) + (b^2*h*n*\text{Log} \\
& [(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*a^2) - (e^2*h*n*\text{Log}[(b*(d + e*x))/((b*d - a*e) \\
& )*(1 - c*(a + b*x))]) *PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(2*d^2) - (b^2*h*n*\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*Poly \\
& \text{Log}[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*a^2) + (e^2*h*n*\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*( \\
& 1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*d^2) + (e^2*h*n*PolyLog[3, -((b*x)/a)]/(2*d^2) - (b^2*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)]/(2*a^2) + (b^2*h*n* \\
& \text{PolyLog}[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x)))]/(2*a^2) - (b^2*h*n* \\
& \text{PolyLog}[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/(2*a^2) + (b^2*h*n*P \\
& \text{olyLog}[3, (b*(d + e*x))/(b*d - a*e)]/(2*a^2) - (e^2*h*n*PolyLog[3, (b*(d + \\
& e*x))/(b*d - a*e)]/(2*d^2) - (b^2*h*n*PolyLog[3, 1 + (e*x)/d)]/(2*a^2) - \\
& (e^2*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x)))]/(2*d^2) + (e^2*h*n*Pol \\
& yLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/(2*d^2) + (b^2*h*n*PolyLog[3, 1 - c* \\
& (a + b*x)]/(2*a^2) + (b^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + \\
& e*x)))]/(2*a^2) - (e^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e \\
& *x)))]/(2*d^2) - (b^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d \\
& + e*x)))]/(2*a^2) + (e^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b \\
& *(d + e*x)))]/(2*d^2)
\end{aligned}$$

**Rubi [A]** time = 3.23828, antiderivative size = 3119, normalized size of antiderivative = 1, number of steps used = 44, number of rules used = 16, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$ , Rules used = {6603, 2439, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 2438, 2437, 2435, 2440, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

[In] Int[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^3,x]

[Out]  $(b^2*g*\text{Log}[(b*c*x)/(1 - a*c)]*\text{Log}[1 - a*c - b*c*x]/(2*a^2) - (b*e*h*n*\text{Log}[(b*c*x)/(1 - a*c)]*\text{Log}[1 - a*c - b*c*x]/(a*d) + (b^2*h*n*\text{Log}[(b*c*x)/(1 - a*c)]*\text{Log}[1 - a*c - b*c*x]*\text{Log}[d + e*x]/(2*a^2) + (b*e*h*n*\text{Log}[1 - a*c - b*c*x]*\text{Log}[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]/(2*a*d) + (b^2*h*n*(\text{Log}[(b*c*x)/(1 - a*c)] + \text{Log}[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - \text{Log}[(b*c*d + e - a*c*e)*x]/((1 - a*c)*(d + e*x)))*\text{Log}[(1 - a*c)*(d + e*x)/(d*(1 - a*c - b*c*x))]^2)/(4*a^2) - (b^2*h*n*(\text{Log}[(b*c*x)/(1 - a*c)] - \text{Log}[-((e*x)/d)])*(\text{Log}[1 - a*c - b*c*x] + \text{Log}[(1 - a*c)*(d + e*x)/(d*(1 - a*c - b*c*x))])^2)/(4*a^2) - (b^2*h*\text{Log}[(b*c*x)/(1 - a*c)]*\text{Log}[1 - a*c - b*c*x]*(n*\text{Log}[d + e*x] - \text{Log}[f*(d + e*x)^n]))/(2*a^2) + (b^2*c*\text{Log}[-((e*x)/d)]*(g + h*\text{Log}[f*(d + e*x)^n]))/(2*a*(1 - a*c)) + (b*\text{Log}[1 - a*c - b*c*x]*(g + h*\text{Log}[f*(d + e*x)^n]))/(2*a*x) - (b^2*c*\text{Log}[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*\text{Log}[f*(d + e*x)^n]))/(2*a*(1 - a*c)) - (b^2*h*n*(\text{Log}[c*(a + b*x)] + \text{Log}[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - \text{Log}[(b*c*d + e - a*c*e)*(a + b*x)]/(b*(d + e*x)))*\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*a^2) + (e^2*h*n*(\text{Log}[c*(a + b*x)] + \text{Log}[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - \text{Log}[(b*c*d + e - a*c*e)*(a + b*x)]/(b*(d + e*x)))*\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*d^2) - (e^2*h*n*\text{Log}[x]*\text{Log}[1 + (b*x)/a]*\text{Log}[1 - c*(a + b*x)]/(2*d^2) - (b^2*h*n*\text{Log}[c*(a + b*x)]*\text{Log}[d + e*x]*\text{Log}[1 - c*(a + b*x)]/(2*a^2) + (e^2*h*n*\text{Log}[c*(a + b*x)]*\text{Log}[d + e*x]*\text{Log}[1 - c*(a + b*x)]/(2*d^2) + (b^2*h*n*(\text{Log}[c*(a + b*x)] - \text{Log}[-((e*(a + b*x))$

$$\begin{aligned}
&)/(b*d - a*e)))]*(\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + \text{Log}[ \\
&1 - c*(a + b*x)]^2)/(4*a^2) - (e^{2*h*n}*(\text{Log}[c*(a + b*x)] - \text{Log}[-((e*(a + b \\
&*x))/(b*d - a*e)))]*(\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + \text{L} \\
&\text{og}[1 - c*(a + b*x)]^2)/(4*d^2) - (e^{2*h*n}*(\text{Log}[1 + (b*x)/a] + \text{Log}[(1 - a*c \\
&)/(1 - c*(a + b*x))] - \text{Log}[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*\text{Lo} \\
&\text{g}[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2) - (e^{2*h*n}*(\text{Log}[c*(a + b*x)] - \\
&\text{Log}[1 + (b*x)/a])*(\text{Log}[x] + \text{Log}[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2 \\
&) - (e^{2*h*n}*(\text{Log}[1 - c*(a + b*x)] - \text{Log}[-((a*(1 - c*(a + b*x)))/(b*x))])*P \\
&\text{olyLog}[2, -((b*x)/a)]/(2*d^2) + (b^2*g*\text{PolyLog}[2, c*(a + b*x)]/(2*a^2) - \\
&(b*e*h*n*\text{PolyLog}[2, c*(a + b*x)]/(2*a*d) - (e*h*n*\text{PolyLog}[2, c*(a + b*x)] \\
&)/(2*d*x) - (e^{2*h*n}*\text{Log}[x]*\text{PolyLog}[2, c*(a + b*x)]/(2*d^2) + (e^{2*h*n}*\text{Log}[ \\
&d + e*x]*\text{PolyLog}[2, c*(a + b*x)]/(2*d^2) - (b^2*h*(n*\text{Log}[d + e*x] - \text{Log}[f* \\
&(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/(2*a^2) - ((g + h*\text{Log}[f*(d + e*x)^n] \\
&)*PolyLog[2, c*(a + b*x)]/(2*x^2) + (b*e*h*n*\text{PolyLog}[2, (e*(1 - a*c - b*c* \\
&x))/(b*c*d + e - a*c*e)]/(2*a*d) + (b^2*g*\text{PolyLog}[2, 1 - (b*c*x)/(1 - a*c) \\
&)]/(2*a^2) - (b*e*h*n*\text{PolyLog}[2, 1 - (b*c*x)/(1 - a*c)]/(a*d) + (b^2*h*n*( \\
&\text{Log}[d + e*x] - \text{Log}[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, \\
&1 - (b*c*x)/(1 - a*c)]/(2*a^2) - (b^2*h*(n*\text{Log}[d + e*x] - \text{Log}[f*(d + e*x) \\
&^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(2*a^2) + (b^2*h*n*\text{Log}[((1 - a*c)*( \\
&d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c \\
&)*(d + e*x)))]/(2*a^2) - (b^2*h*n*\text{Log}[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b \\
&*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/(2*a^2) - (b^ \\
&2*h*n*(\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + \text{Log}[1 - c*(a + \\
&b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/(2*a^2) + (e^{2*h*n}*(\text{Log}[(b*(d \\
&+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + \text{Log}[1 - c*(a + b*x)])*PolyLog[2 \\
&, (b*(d + e*x))/(b*d - a*e)]/(2*d^2) - (b^2*c*h*n*\text{PolyLog}[2, (b*c*(d + e*x) \\
&)/(b*c*d + e - a*c*e)]/(2*a*(1 - a*c)) + (b^2*c*h*n*\text{PolyLog}[2, 1 + (e*x)/ \\
&d)]/(2*a*(1 - a*c)) + (b^2*h*n*(\text{Log}[1 - a*c - b*c*x] + \text{Log}[((1 - a*c)*(d + \\
&e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d)]/(2*a^2) - (e^{2*h*n}*L \\
&\text{og}[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))) \\
&)]/(2*d^2) + (e^{2*h*n}*\text{Log}[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b* \\
&c*x)/(1 - c*(a + b*x)))]/(2*d^2) - (b^2*h*n*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e* \\
&x))/((b*d - a*e)*(1 - c*(a + b*x))])*PolyLog[2, 1 - c*(a + b*x)]/(2*a^2 \\
&+ (e^{2*h*n}*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x) \\
&)]))*PolyLog[2, 1 - c*(a + b*x)]/(2*d^2) - (e^{2*h*n}*(\text{Log}[x] + \text{Log}[-((a*(1 - \\
&c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)]/(2*d^2) + (b^2*h*n*\text{Log} \\
&[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])*PolyLog[2, -((e*(1 - c*(a + \\
&b*x)))/(b*c*(d + e*x)))]/(2*a^2) - (e^{2*h*n}*\text{Log}[(b*(d + e*x))/((b*d - a*e \\
&)*(1 - c*(a + b*x))])*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))] \\
&)/(2*d^2) - (b^2*h*n*\text{Log}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])*Poly \\
&\text{Log}[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*a^2) + (e^{2*h*n}*L \\
&\text{og}[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))])*PolyLog[2, ((b*d - a*e)*( \\
&1 - c*(a + b*x)))/(b*(d + e*x)))]/(2*d^2) + (e^{2*h*n}*\text{PolyLog}[3, -((b*x)/a) \\
&)]/(2*d^2) - (b^2*h*n*\text{PolyLog}[3, 1 - (b*c*x)/(1 - a*c)]/(2*a^2) + (b^2*h*n* \\
&\text{PolyLog}[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x)))]/(2*a^2) - (b^2*h*n* \\
&\text{PolyLog}[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/(2*a^2) + (b^2*h*n*P \\
&\text{olyLog}[3, (b*(d + e*x))/(b*d - a*e)]/(2*a^2) - (e^{2*h*n}*\text{PolyLog}[3, (b*(d + \\
&e*x))/(b*d - a*e)]/(2*d^2) - (b^2*h*n*\text{PolyLog}[3, 1 + (e*x)/d)]/(2*a^2) - \\
&(e^{2*h*n}*\text{PolyLog}[3, -((b*x)/(a*(1 - c*(a + b*x)))]/(2*d^2) + (e^{2*h*n}*Pol \\
&yLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/(2*d^2) + (b^2*h*n*\text{PolyLog}[3, 1 - c* \\
&(a + b*x)]/(2*a^2) + (b^2*h*n*\text{PolyLog}[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + \\
&e*x)))]/(2*a^2) - (e^{2*h*n}*\text{PolyLog}[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e \\
&*x)))]/(2*d^2) - (b^2*h*n*\text{PolyLog}[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d \\
&+ e*x)))]/(2*a^2) + (e^{2*h*n}*\text{PolyLog}[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b \\
&*(d + e*x)))]/(2*d^2)
\end{aligned}$$

### Rule 6603

$$\text{Int}(((g_.) + \text{Log}[(f_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]*(h_.))*(x_.)^{(m_.)}*\text{PolyLo}$$

```
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

#### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((h_.)*(x_)
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2438

Int[(((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*(Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.) + (f\_.)))/(x\_), x\_Symbol] := Dist[f, Int[(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] + Dist[g, Int[(Log[h\*(i + j\*x)^m]\*(a + b\*Log[c\*(d + e\*x)^n))]/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0]

Rule 2437

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]/(x\_), x\_Symbol] := Dist[m, Int[(Log[i + j\*x]\*Log[c\*(d + e\*x)^n])/x, x], x] - Dist[m\*Log[i + j\*x] - Log[h\*(i + j\*x)^m], Int[Log[c\*(d + e\*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0] && NeQ[i + j\*x, h\*(i + j\*x)^m]

Rule 2435

Int[(Log[(a\_) + (b\_.)\*(x\_)]\*Log[(c\_) + (d\_.)\*(x\_)])/(x\_), x\_Symbol] := Simp[Log[-((b\*x)/a)]\*Log[a + b\*x]\*Log[c + d\*x], x] + (Simp[(1\*(Log[-((b\*x)/a)] - Log[-((b\*c - a\*d)\*x]/(a\*(c + d\*x)))] + Log[(b\*c - a\*d)/(b\*(c + d\*x))])\*Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2/2, x] - Simp[(1\*(Log[-((b\*x)/a)] - Log[-((d\*x)/c)])\*(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))]^2)/2, x] + Simp[(Log[c + d\*x] - Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (b\*x)/a], x] + Simp[(Log[a + b\*x] + Log[(a\*(c + d\*x))/(c\*(a + b\*x))])\*PolyLog[2, 1 + (d\*x)/c], x] + Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[Log[(a\*(c + d\*x))/(c\*(a + b\*x))]\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))], x] - Simp[PolyLog[3, 1 + (b\*x)/a], x] - Simp[PolyLog[3, 1 + (d\*x)/c], x] + Simp[PolyLog[3, (c\*(a + b\*x))/(a\*(c + d\*x))], x] - Simp[PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 2440

Int[(((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_) + (l\_.)\*(x\_)^(r\_.)))/(x\_), x\_Symbol] := Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*l)/l) + (e\*x)/l)^n])\*(f + g\*Log[h\*(-((j\*k - i\*l)/l) + (j\*x)/l)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

Rule 6598

Int[(((d\_.) + (e\_.)\*(x\_)^(m\_.))\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)])/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

Rule 6597

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*PolyLog[2, c\*(a + b\*x)]/e, x] + Dist[b/e, Int[(Log[d + e\*x]\*Log[1 - a\*c - b\*c\*x])/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c\*(b\*d - a\*e) + e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{x^3} dx &= -\frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{2x^2} - \frac{1}{2}b \int \left( \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{x^2} \right) dx \\
&= -\frac{(g + h \log(f(d + ex)^n)) \operatorname{Li}_2(c(a + bx))}{2x^2} - \frac{b \int \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{x^2} dx}{2a} \\
&= \frac{b \log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{2ax} - \frac{ehn \operatorname{Li}_2(c(a + bx))}{2dx} - \frac{e^2 h}{2a} \\
&= \frac{b^2 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} + \frac{b \log(1 - ac - bcx)(g + h \log(f(d + ex)^n))}{2ax} \\
&= \frac{b^2 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} + \frac{b^2 hn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(f(d + ex)^n)}{2a^2} \\
&= \frac{b^2 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} + \frac{b^2 hn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(f(d + ex)^n)}{2a^2} \\
&= \frac{b^2 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} + \frac{b^2 hn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(f(d + ex)^n)}{2a^2} \\
&= \frac{b^2 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2} + \frac{b^2 hn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx) \log(f(d + ex)^n)}{2a^2}
\end{aligned}$$

**Mathematica [F]** time = 10.4117, size = 0, normalized size = 0.

$$\int \frac{(g + h \log(f(d + ex)^n)) \operatorname{PolyLog}(2, c(a + bx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^3, x]

[Out] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^3, x]

**Maple [F]** time = 0.521, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(f(ex + d)^n)) \operatorname{polylog}(2, c(bx + a))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^3,x)

[Out] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^3,x, algorithm="maxima")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h \text{Li}_2(bc x + ac) \log((ex + d)^n f) + g \text{Li}_2(bc x + ac)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^3,x, algorithm="fricas")

[Out] integral((h\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*dilog(b\*c\*x + a\*c))/x^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a))/x\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^3,x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^3, x)

$$3.183 \quad \int \frac{(g+h \log(f(dx+ex)^n)) \text{PolyLog}(2,c(a+bx))}{x^4} dx$$

Optimal. Leaf size=3733

result too large to display

```
[Out] (b^2*c*e*h*n*Log[x])/(2*a*(1 - a*c)*d) - (b^2*c*e*h*n*Log[1 - a*c - b*c*x])
/(3*a*(1 - a*c)*d) + (b*e*h*n*Log[1 - a*c - b*c*x])/(3*a*d*x) - (b^3*g*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) + (b^2*e*h*n*Log[(b*c*x)/(
1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2*d) + (b*e^2*h*n*Log[(b*c*x)/(1 - a*c
)]*Log[1 - a*c - b*c*x])/(2*a*d^2) - (b^2*c*e*h*n*Log[d + e*x])/(6*a*(1 - a
*c)*d) - (b^3*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])
/(3*a^3) - (b^2*e*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e -
a*c*e)])/(3*a^2*d) - (b*e^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(
b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x)]) - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*
(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) +
(b^3*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*c - b*c*x] +
Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) + (b^3*h*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n
]))/(3*a^3) - (b^2*c*(g + h*Log[f*(d + e*x)^n])/(6*a*(1 - a*c)*x) + (b^3*c
^2*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n])/(6*a*(1 - a*c)^2) - (b^3*c*L
og[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n])/(3*a^2*(1 - a*c)) + (b*Log[1 - a
*c - b*c*x]*(g + h*Log[f*(d + e*x)^n])/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*
x]*(g + h*Log[f*(d + e*x)^n])/(3*a^2*x) - (b^3*c^2*Log[(e*(1 - a*c - b*c*x
))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n])/(6*a*(1 - a*c)^2) + (b^
3*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(3*a^2*(1 - a*c)) + (b^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e
)/(b*c*(d + e*x)]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*a^3) - (e^3*h*n*(Log
[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)]) - Log[((b*c*d + e
- a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(
a + b*x)))]^2)/(6*d^3) + (e^3*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*
x)])/(3*d^3) + (b^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(3*a^3) - (e^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*
d^3) - (b^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/
(6*a^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*
(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^
2)/(6*d^3) + (e^3*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))
] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]*Log[-((a*(1 - c*(a + b
*x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(L
og[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[1 -
c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)]
)/(3*d^3) - (b^3*g*PolyLog[2, c*(a + b*x)]/(3*a^3) + (b^2*e*h*n*PolyLog[2,
c*(a + b*x)]/(6*a^2*d) + (b*e^2*h*n*PolyLog[2, c*(a + b*x)]/(3*a*d^2) -
(e*h*n*PolyLog[2, c*(a + b*x)]/(6*d*x^2) + (e^2*h*n*PolyLog[2, c*(a + b*x)
])/(3*d^2*x) + (e^3*h*n*Log[x]*PolyLog[2, c*(a + b*x)]/(3*d^3) - (e^3*h*n*
Log[d + e*x]*PolyLog[2, c*(a + b*x)]/(3*d^3) + (b^3*h*(n*Log[d + e*x] - Lo
g[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/(3*a^3) - ((g + h*Log[f*(d + e*x)
^n])*PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^2*e*h*n*PolyLog[2, (e*(1 - a*c
- b*c*x))/(b*c*d + e - a*c*e)]/(3*a^2*d) - (b*e^2*h*n*PolyLog[2, (e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)]/(6*a*d^2) - (b^3*g*PolyLog[2, 1 - (b*c*x
)/(1 - a*c)]/(3*a^3) + (b^2*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(2*a^
2*d) + (b*e^2*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(2*a*d^2) - (b^3*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
1 - (b*c*x)/(1 - a*c)]/(3*a^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(3*a^3) - (b^3*h*n*Log[((1 - a*c)*(
```

$$\begin{aligned}
& d + e*x)) / (d*(1 - a*c - b*c*x))] * \text{PolyLog}[2, (d*(1 - a*c - b*c*x)) / ((1 - a*c) * (d + e*x))] / (3*a^3) + (b^3*h*n*\text{Log}[(1 - a*c)*(d + e*x)) / (d*(1 - a*c - b*c*x))] * \text{PolyLog}[2, -((e*(1 - a*c - b*c*x)) / (b*c*(d + e*x)))] / (3*a^3) + (b^3*h*n*(\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] + \text{Log}[1 - c*(a + b*x)]) * \text{PolyLog}[2, (b*(d + e*x)) / (b*d - a*e)] / (3*a^3) - (e^3*h*n*(\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] + \text{Log}[1 - c*(a + b*x)]) * \text{PolyLog}[2, (b*(d + e*x)) / (b*d - a*e)] / (3*d^3) - (b^3*c^2*h*n*\text{PolyLog}[2, (b*c*(d + e*x)) / (b*c*d + e - a*c*e)] / (6*a*(1 - a*c)^2) + (b^3*c^2*h*n*\text{PolyLog}[2, (b*c*(d + e*x)) / (b*c*d + e - a*c*e)] / (3*a^2*(1 - a*c)) + (b^3*c^2*h*n*\text{PolyLog}[2, 1 + (e*x)/d]) / (6*a*(1 - a*c)^2) - (b^3*c^2*h*n*\text{PolyLog}[2, 1 + (e*x)/d]) / (3*a^2*(1 - a*c)) - (b^3*h*n*(\text{Log}[1 - a*c - b*c*x] + \text{Log}[(1 - a*c)*(d + e*x)) / (d*(1 - a*c - b*c*x)])) * \text{PolyLog}[2, 1 + (e*x)/d] / (3*a^3) + (e^3*h*n*\text{Log}[-((a*(1 - c*(a + b*x))) / (b*x))] * \text{PolyLog}[2, -((b*x) / (a*(1 - c*(a + b*x))))] / (3*d^3) - (e^3*h*n*\text{Log}[-((a*(1 - c*(a + b*x))) / (b*x))] * \text{PolyLog}[2, -((b*c*x) / (1 - c*(a + b*x)))] / (3*d^3) + (b^3*h*n*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))])) * \text{PolyLog}[2, 1 - c*(a + b*x)] / (3*a^3) - (e^3*h*n*(\text{Log}[d + e*x] - \text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))])) * \text{PolyLog}[2, 1 - c*(a + b*x)] / (3*d^3) + (e^3*h*n*(\text{Log}[x] + \text{Log}[-((a*(1 - c*(a + b*x))) / (b*x)])) * \text{PolyLog}[2, 1 - c*(a + b*x)] / (3*d^3) - (b^3*h*n*\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] * \text{PolyLog}[2, -((e*(1 - c*(a + b*x))) / (b*c*(d + e*x)))] / (3*a^3) + (e^3*h*n*\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] * \text{PolyLog}[2, -((e*(1 - c*(a + b*x))) / (b*c*(d + e*x)))] / (3*d^3) + (b^3*h*n*\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] * \text{PolyLog}[2, ((b*d - a*e)*(1 - c*(a + b*x))) / (b*(d + e*x))] / (3*a^3) - (e^3*h*n*\text{Log}[(b*(d + e*x)) / ((b*d - a*e)*(1 - c*(a + b*x)))] * \text{PolyLog}[2, ((b*d - a*e)*(1 - c*(a + b*x))) / (b*(d + e*x))] / (3*d^3) - (e^3*h*n*\text{PolyLog}[3, -((b*x)/a)] / (3*d^3) + (b^3*h*n*\text{PolyLog}[3, 1 - (b*c*x)/(1 - a*c)] / (3*a^3) - (b^3*h*n*\text{PolyLog}[3, (d*(1 - a*c - b*c*x)) / ((1 - a*c)*(d + e*x))] / (3*a^3) + (b^3*h*n*\text{PolyLog}[3, -((e*(1 - a*c - b*c*x)) / (b*c*(d + e*x)))] / (3*a^3) - (b^3*h*n*\text{PolyLog}[3, (b*(d + e*x)) / (b*d - a*e)] / (3*a^3) + (e^3*h*n*\text{PolyLog}[3, (b*(d + e*x)) / (b*d - a*e)] / (3*d^3) + (b^3*h*n*\text{PolyLog}[3, 1 + (e*x)/d]) / (3*a^3) + (e^3*h*n*\text{PolyLog}[3, -((b*x) / (a*(1 - c*(a + b*x))))] / (3*d^3) - (e^3*h*n*\text{PolyLog}[3, -((b*c*x) / (1 - c*(a + b*x)))] / (3*d^3) - (b^3*h*n*\text{PolyLog}[3, 1 - c*(a + b*x)] / (3*a^3) - (b^3*h*n*\text{PolyLog}[3, -((e*(1 - c*(a + b*x))) / (b*c*(d + e*x)))] / (3*a^3) + (e^3*h*n*\text{PolyLog}[3, -((e*(1 - c*(a + b*x))) / (b*c*(d + e*x)))] / (3*d^3) + (b^3*h*n*\text{PolyLog}[3, ((b*d - a*e)*(1 - c*(a + b*x))) / (b*(d + e*x)))] / (3*a^3) - (e^3*h*n*\text{PolyLog}[3, ((b*d - a*e)*(1 - c*(a + b*x))) / (b*(d + e*x)))] / (3*d^3)
\end{aligned}$$

**Rubi [A]** time = 4.30785, antiderivative size = 3733, normalized size of antiderivative = 1., number of steps used = 78, number of rules used = 18, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6603, 2439, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391, 2438, 2437, 2435, 2440, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

[In] Int[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^4,x]

[Out]  $(b^2*c*e*h*n*\text{Log}[x]) / (2*a*(1 - a*c)*d) - (b^2*c*e*h*n*\text{Log}[1 - a*c - b*c*x]) / (3*a*(1 - a*c)*d) + (b*e*h*n*\text{Log}[1 - a*c - b*c*x]) / (3*a*d*x) - (b^3*g*\text{Log}[(b*c*x) / (1 - a*c)] * \text{Log}[1 - a*c - b*c*x]) / (3*a^3) + (b^2*e*h*n*\text{Log}[(b*c*x) / (1 - a*c)] * \text{Log}[1 - a*c - b*c*x]) / (2*a^2*d) + (b*e^2*h*n*\text{Log}[(b*c*x) / (1 - a*c)] * \text{Log}[1 - a*c - b*c*x]) / (2*a*d^2) - (b^2*c*e*h*n*\text{Log}[d + e*x]) / (6*a*(1 - a*c)*d) - (b^3*h*n*\text{Log}[(b*c*x) / (1 - a*c)] * \text{Log}[1 - a*c - b*c*x] * \text{Log}[d + e*x]) / (3*a^3) - (b^2*e*h*n*\text{Log}[1 - a*c - b*c*x] * \text{Log}[(b*c*(d + e*x)) / (b*c*d + e -$



$$\begin{aligned}
& a*c*e)]/(3*a^2*d) - (b*e^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]/(6*a*d^2) - (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) + \\
& (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) + (b^3*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n]))/(3*a^3) - (b^2*c*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)*x) + (b^3*c^2*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) - (b^3*c*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*x) - (b^3*c^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) + (b^3*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*(1 - a*c)) + (b^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)]) - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*a^3) - (e^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*d^3) + (e^3*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)]/(3*d^3) + (b^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)]/(3*a^3) - (e^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)]/(3*d^3) - (b^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(6*a^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]^2)/(6*d^3) + (e^3*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))]) - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -(b*x)/a])/(3*d^3) - (b^3*g*PolyLog[2, c*(a + b*x)]/(3*a^3) + (b^2*e*h*n*PolyLog[2, c*(a + b*x)]/(6*a^2*d) + (b*e^2*h*n*PolyLog[2, c*(a + b*x)]/(3*a*d^2) - (e*h*n*PolyLog[2, c*(a + b*x)]/(6*d*x^2) + (e^2*h*n*PolyLog[2, c*(a + b*x)]/(3*d^2*x) + (e^3*h*n*Log[x]*PolyLog[2, c*(a + b*x)]/(3*d^3) - (e^3*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)]/(3*d^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^2*e*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(3*a^2*d) - (b*e^2*h*n*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]/(6*a*d^2) - (b^3*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(3*a^3) + (b^2*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(2*a^2*d) + (b*e^2*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(2*a*d^2) - (b^3*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(3*a^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)]/(3*a^3) - (b^3*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x)))]/(3*a^3) + (b^3*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/(3*a^3) + (b^3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/(3*a^3) - (e^3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)]*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]/(3*d^3) - (b^3*c^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)]/(6*a*(1 - a*c)^2) + (b^3*c*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)]/(3*a^2*(1 - a*c)) + (b^3*c^2*h*n*PolyLog[2, 1 + (e*x)/d]/(6*a*(1 - a*c)^2) - (b^3*c*h*n*PolyLog[2, 1 + (e*x)/d]/(3*a^2*(1 - a*c)) - (b^3*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d]/(3*a^3) + (e^3*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))/(3*d^3) - (e^3*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*c*x)/(
\end{aligned}$$

$$\begin{aligned}
& 1 - c*(a + b*x)))]/(3*d^3) + (b^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, 1 - c*(a + b*x)]/(3*a^3) - (e^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, 1 - c*(a + b*x)]/(3*d^3) + (e^3*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)]/(3*d^3) - (b^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(3*a^3) + (e^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(3*d^3) + (b^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(3*a^3) - (e^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(3*d^3) - (e^3*h*n*PolyLog[3, -((b*x)/a)]/(3*d^3) + (b^3*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)]/(3*a^3) - (b^3*h*n*PolyLog[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x)))]/(3*a^3) + (b^3*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))]/(3*a^3) - (b^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/(3*a^3) + (e^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)]/(3*d^3) + (b^3*h*n*PolyLog[3, 1 + (e*x)/d]/(3*a^3) + (e^3*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x)))]/(3*d^3) - (e^3*h*n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))]/(3*d^3) - (b^3*h*n*PolyLog[3, 1 - c*(a + b*x)]/(3*a^3) - (b^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(3*a^3) + (e^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]/(3*d^3) + (b^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(3*a^3) - (e^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x)))]/(3*d^3)))/(3*d^3)
\end{aligned}$$

#### Rule 6603

```

Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[(x^(m + 1)*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)]/(m + 1), x] + (Dist[b/(m + 1), Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/(a + b*x), x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

```

#### Rule 2439

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x] + (-Dist[(g*j^m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p/(i + j*x), x], x] - Dist[(b*e^n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p - 1*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

```

#### Rule 44

```

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

```

#### Rule 2416

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

```

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^( -1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_))), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2438

Int((((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.) + (f\_.)))/(x\_)), x\_Symbol] :> Dist[f, Int[(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] + Dist[g, Int[(Log[h\*(i + j\*x)^m]\*(a + b\*Log[c\*(d + e\*x)^n])/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && NeQ[e\*i - d\*j, 0]

Rule 2437

```
Int[(Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]/(x_), x_Symbol] := Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i + j*x, h*(i + j*x)^m]
```

#### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/x, x_Symbol] := Simp[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)] - Log[-((b*c - a*d)*x]/(a*(c + d*x)))) + Log[(b*c - a*d)/(b*(c + d*x))]*Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] + Simp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 2440

```
Int[((a_.) + Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]*(b_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f + g*Log[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_)^(m_.))*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] + Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c*(b*d - a*e) + e, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(g + h \log(f(d + ex^n)) \operatorname{Li}_2(c(a + bx)))}{x^4} dx &= -\frac{(g + h \log(f(d + ex^n)) \operatorname{Li}_2(c(a + bx)))}{3x^3} - \frac{1}{3}b \int \left( \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex^n)))}{x^3} \right. \\
&= -\frac{(g + h \log(f(d + ex^n)) \operatorname{Li}_2(c(a + bx)))}{3x^3} - \frac{b \int \frac{\log(1 - ac - bcx)(g + h \log(f(d + ex^n)))}{x^3}}{3a} \\
&= \frac{b \log(1 - ac - bcx)(g + h \log(f(d + ex^n)))}{6ax^2} - \frac{b^2 \log(1 - ac - bcx)(g + h \log(f(d + ex^n)))}{3a^2} \\
&= -\frac{b^3 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} + \frac{b \log(1 - ac - bcx)(g + h \log(f(d + ex^n)))}{6ax^2} \\
&= -\frac{b^3 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} - \frac{b^3 hn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} \\
&= -\frac{b^3 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} + \frac{b^2 ehn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2 d} \\
&= -\frac{b^3 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} + \frac{b^2 ehn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2 d} \\
&= \frac{b^2 cehn \log(x)}{6a(1 - ac)d} - \frac{b^3 g \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{3a^3} + \frac{b^2 ehn \log\left(\frac{bcx}{1-ac}\right) \log(1 - ac - bcx)}{2a^2 d}
\end{aligned}$$

**Mathematica [F]** time = 10.5193, size = 0, normalized size = 0.

$$\int \frac{(g + h \log(f(d + ex^n)) \operatorname{PolyLog}(2, c(a + bx)))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^4, x]

[Out] Integrate[((g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)])/x^4, x]

**Maple [F]** time = 0.537, size = 0, normalized size = 0.

$$\int \frac{(g + h \ln(f(ex + d)^n)) \operatorname{polylog}(2, c(bx + a))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^4, x)

[Out] int((g+h\*ln(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^4,x, algorithm="maxima")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h\text{Li}_2(bc x + ac) \log((ex + d)^n f) + g\text{Li}_2(bc x + ac)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^4,x, algorithm="fricas")

[Out] integral((h\*dilog(b\*c\*x + a\*c)\*log((e\*x + d)^n\*f) + g\*dilog(b\*c\*x + a\*c))/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*ln(f\*(e\*x+d)\*\*n))\*polylog(2,c\*(b\*x+a))/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(h \log((ex + d)^n f) + g) \text{Li}_2((bx + a)c)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g+h\*log(f\*(e\*x+d)^n))\*polylog(2,c\*(b\*x+a))/x^4,x, algorithm="giac")

[Out] integrate((h\*log((e\*x + d)^n\*f) + g)\*dilog((b\*x + a)\*c)/x^4, x)

### 3.184 $\int x^2(a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=661

$$\frac{x^2(4ac + 3b)\text{PolyLog}(2, cx)}{24c^2} - \frac{x(4ac + 3b)\text{PolyLog}(2, cx)}{12c^3} + \frac{(4ac + 3b)\text{PolyLog}(3, 1 - cx)}{6c^4} - \frac{(4ac + 3b)\log(1 - cx)}{12c^4}$$

```
[Out] (53*b*x)/(192*c^3) + (11*a*x)/(27*c^2) + (49*(3*b + 4*a*c)*x)/(432*c^3) + (
29*b*x^2)/(384*c^2) + (5*a*x^2)/(54*c) + (13*(3*b + 4*a*c)*x^2)/(864*c^2) +
(2*a*x^3)/81 + (17*b*x^3)/(576*c) + ((3*b + 4*a*c)*x^3)/(324*c) + (3*b*x^4
)/256 + (29*b*Log[1 - c*x])/(192*c^4) + (5*a*Log[1 - c*x])/(27*c^3) + (13*(
3*b + 4*a*c)*Log[1 - c*x])/(432*c^4) - (b*x^2*Log[1 - c*x])/(16*c^2) - (a*x
^2*Log[1 - c*x])/(9*c) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^2) - (2*a*x
^3*Log[1 - c*x])/27 - (b*x^3*Log[1 - c*x])/(24*c) - ((3*b + 4*a*c)*x^3*Log[
1 - c*x])/(108*c) - (3*b*x^4*Log[1 - c*x])/64 + (b*(1 - c*x)*Log[1 - c*x])/
(8*c^4) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) + ((3*b + 4*a*c)*(1 - c*x)*L
og[1 - c*x])/(12*c^4) - (b*Log[1 - c*x]^2)/(16*c^4) - (a*Log[1 - c*x]^2)/(9
*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]^2)/16 - ((3*b + 4*a*
c)*Log[c*x]*Log[1 - c*x]^2)/(12*c^4) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(1
2*c^3) - ((3*b + 4*a*c)*x^2*PolyLog[2, c*x])/(24*c^2) - ((3*b + 4*a*c)*x^3*
PolyLog[2, c*x])/(36*c) - (b*x^4*PolyLog[2, c*x])/16 - ((3*b + 4*a*c)*Log[1
 - c*x]*PolyLog[2, c*x])/(12*c^4) + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyL
og[2, c*x])/12 - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6*c^4) +
((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^4)
```

**Rubi [A]** time = 0.981897, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.81$ , Rules used = {6742, 6591, 2395, 43, 6604, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{x^2(4ac + 3b)\text{PolyLog}(2, cx)}{24c^2} - \frac{x(4ac + 3b)\text{PolyLog}(2, cx)}{12c^3} + \frac{(4ac + 3b)\text{PolyLog}(3, 1 - cx)}{6c^4} - \frac{(4ac + 3b)\log(1 - cx)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] (53*b*x)/(192*c^3) + (11*a*x)/(27*c^2) + (49*(3*b + 4*a*c)*x)/(432*c^3) + (
29*b*x^2)/(384*c^2) + (5*a*x^2)/(54*c) + (13*(3*b + 4*a*c)*x^2)/(864*c^2) +
(2*a*x^3)/81 + (17*b*x^3)/(576*c) + ((3*b + 4*a*c)*x^3)/(324*c) + (3*b*x^4
)/256 + (29*b*Log[1 - c*x])/(192*c^4) + (5*a*Log[1 - c*x])/(27*c^3) + (13*(
3*b + 4*a*c)*Log[1 - c*x])/(432*c^4) - (b*x^2*Log[1 - c*x])/(16*c^2) - (a*x
^2*Log[1 - c*x])/(9*c) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^2) - (2*a*x
^3*Log[1 - c*x])/27 - (b*x^3*Log[1 - c*x])/(24*c) - ((3*b + 4*a*c)*x^3*Log[
1 - c*x])/(108*c) - (3*b*x^4*Log[1 - c*x])/64 + (b*(1 - c*x)*Log[1 - c*x])/
(8*c^4) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) + ((3*b + 4*a*c)*(1 - c*x)*L
og[1 - c*x])/(12*c^4) - (b*Log[1 - c*x]^2)/(16*c^4) - (a*Log[1 - c*x]^2)/(9
*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]^2)/16 - ((3*b + 4*a*
c)*Log[c*x]*Log[1 - c*x]^2)/(12*c^4) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(1
2*c^3) - ((3*b + 4*a*c)*x^2*PolyLog[2, c*x])/(24*c^2) - ((3*b + 4*a*c)*x^3*
PolyLog[2, c*x])/(36*c) - (b*x^4*PolyLog[2, c*x])/16 - ((3*b + 4*a*c)*Log[1
 - c*x]*PolyLog[2, c*x])/(12*c^4) + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyL
og[2, c*x])/12 - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6*c^4) +
((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^4)
```

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q]/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))]*(h_.))*((a_.) + (b_.)*(x_))^(p_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{u = IntHide[Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x]
```

### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
```



] /; FreeQ[{c, n}, x]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int x^2(a+bx)\log(1-cx)\operatorname{Li}_2(cx)dx &= \frac{1}{12}(4ax^3+3bx^4)\log(1-cx)\operatorname{Li}_2(cx)+c\int\left(\frac{(-3b-4ac)\operatorname{Li}_2(cx)}{12c^4}-\frac{(3b+4ac)x\operatorname{Li}_2(cx)}{12c^3}\right)dx \\
&= \frac{1}{12}(4ax^3+3bx^4)\log(1-cx)\operatorname{Li}_2(cx)+\frac{1}{3}a\int x^2\log^2(1-cx)dx+\frac{1}{4}b\int x^3\log^2(1-cx)dx \\
&= \frac{1}{9}ax^3\log^2(1-cx)+\frac{1}{16}bx^4\log^2(1-cx)-\frac{(3b+4ac)x\operatorname{Li}_2(cx)}{12c^3}-\frac{(3b+4ac)x^2\operatorname{Li}_2(cx)}{24c^2} \\
&= -\frac{(3b+4ac)x^2\log(1-cx)}{48c^2}-\frac{(3b+4ac)x^3\log(1-cx)}{108c}-\frac{1}{64}bx^4\log(1-cx)+\frac{1}{9}ax^3\log(1-cx) \\
&= \frac{(3b+4ac)x}{12c^3}-\frac{(3b+4ac)x^2\log(1-cx)}{48c^2}-\frac{(3b+4ac)x^3\log(1-cx)}{108c}-\frac{1}{64}bx^4\log(1-cx) \\
&= \frac{bx}{64c^3}+\frac{49(3b+4ac)x}{432c^3}+\frac{bx^2}{128c^2}+\frac{13(3b+4ac)x^2}{864c^2}+\frac{bx^3}{192c}+\frac{(3b+4ac)x^3}{324c}+\frac{bx^4}{256} \\
&= \frac{9bx}{64c^3}+\frac{2ax}{9c^2}+\frac{49(3b+4ac)x}{432c^3}+\frac{bx^2}{128c^2}+\frac{13(3b+4ac)x^2}{864c^2}+\frac{bx^3}{192c}+\frac{(3b+4ac)x^3}{324c} \\
&= \frac{53bx}{192c^3}+\frac{11ax}{27c^2}+\frac{49(3b+4ac)x}{432c^3}+\frac{29bx^2}{384c^2}+\frac{5ax^2}{54c}+\frac{13(3b+4ac)x^2}{864c^2}+\frac{2ax^3}{81}+\frac{17bx^4}{576c}
\end{aligned}$$

**Mathematica [A]** time = 0.755005, size = 425, normalized size = 0.64

$$48\operatorname{PolyLog}(2, cx)\left(12\log(1-cx)\left(4ac(c^3x^3-1)+3b(c^4x^4-1)\right)-cx\left(8ac(2c^2x^2+3cx+6)+3b(3c^3x^3+4c^2x^2+6cx+3)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x], x]

[Out] (4260\*b\*c\*x + 5952\*a\*c^2\*x + 834\*b\*c^2\*x^2 + 1056\*a\*c^3\*x^2 + 268\*b\*c^3\*x^3 + 256\*a\*c^4\*x^3 + 81\*b\*c^4\*x^4 + 4260\*b\*Log[1 - c\*x] + 5952\*a\*c\*Log[1 - c\*x] - 2592\*b\*c\*x\*Log[1 - c\*x] - 3840\*a\*c^2\*x\*Log[1 - c\*x] - 864\*b\*c^2\*x^2\*Log[1 - c\*x] - 1344\*a\*c^3\*x^2\*Log[1 - c\*x] - 480\*b\*c^3\*x^3\*Log[1 - c\*x] - 768\*a\*c^4\*x^3\*Log[1 - c\*x] - 324\*b\*c^4\*x^4\*Log[1 - c\*x] - 432\*b\*Log[1 - c\*x]^2 - 768\*a\*c\*Log[1 - c\*x]^2 + 768\*a\*c^4\*x^3\*Log[1 - c\*x]^2 + 432\*b\*c^4\*x^4\*Log[1 - c\*x]^2 - 1728\*b\*Log[c\*x]\*Log[1 - c\*x]^2 - 2304\*a\*c\*Log[c\*x]\*Log[1 - c\*x]^2 + 48\*(-(c\*x\*(8\*a\*c\*(6 + 3\*c\*x + 2\*c^2\*x^2) + 3\*b\*(12 + 6\*c\*x + 4\*c^2\*x^2 + 3\*c^3\*x^3))) + 12\*(4\*a\*c\*(-1 + c^3\*x^3) + 3\*b\*(-1 + c^4\*x^4))\*Log[1 - c\*x])\*PolyLog[2, c\*x] - 1152\*(3\*b + 4\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 3456\*b\*PolyLog[3, 1 - c\*x] + 4608\*a\*c\*PolyLog[3, 1 - c\*x])/(6912\*c^4)

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int x^2(bx+a)\ln(-cx+1)\operatorname{polylog}(2, cx)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x+a)\*ln(-c\*x+1)\*polylog(2, c\*x), x)

[Out] int(x^2\*(b\*x+a)\*ln(-c\*x+1)\*polylog(2, c\*x), x)

---

**Maxima [A]** time = 1.04004, size = 560, normalized size = 0.85

$$-\frac{1}{6912} c \left( \frac{576 (\log(cx) \log(-cx+1)^2 + 2 \operatorname{Li}_2(-cx+1) \log(-cx+1) - 2 \operatorname{Li}_3(-cx+1)) (4ac + 3b)}{c^5} - \frac{81 bc^4 x^4 + 4 (64 a^2 c^4 + 3 b^2 c^4)}{c^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="maxima")

[Out] -1/6912\*c\*(576\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))\*(4\*a\*c + 3\*b)/c^5 - (81\*b\*c^4\*x^4 + 4\*(64\*a\*c^4 + 67\*b\*c^3)\*x^3 + 6\*(176\*a\*c^3 + 139\*b\*c^2)\*x^2 + 12\*(496\*a\*c^2 + 355\*b\*c)\*x - 48\*(9\*b\*c^4\*x^4 + 4\*(4\*a\*c^4 + 3\*b\*c^3)\*x^3 + 6\*(4\*a\*c^3 + 3\*b\*c^2)\*x^2 + 12\*(4\*a\*c^2 + 3\*b\*c)\*x + 12\*(4\*a\*c + 3\*b)\*log(-c\*x + 1))\*dilog(c\*x) - 4\*(54\*b\*c^4\*x^4 + 4\*(32\*a\*c^4 + 21\*b\*c^3)\*x^3 + 6\*(40\*a\*c^3 + 27\*b\*c^2)\*x^2 - 1488\*a\*c + 12\*(64\*a\*c^2 + 45\*b\*c)\*x - 1065\*b)\*log(-c\*x + 1))/c^5 + 1/1728\*(32\*(18\*c^3\*x^3\*dilog(c\*x) - 2\*c^3\*x^3 - 3\*c^2\*x^2 - 6\*c\*x + 6\*(c^3\*x^3 - 1)\*log(-c\*x + 1))\*a/c^3 + 9\*(48\*c^4\*x^4\*dilog(c\*x) - 3\*c^4\*x^4 - 4\*c^3\*x^3 - 6\*c^2\*x^2 - 12\*c\*x + 12\*(c^4\*x^4 - 1)\*log(-c\*x + 1))\*b/c^4)\*log(-c\*x + 1)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((bx^3 + ax^2)\operatorname{Li}_2(cx) \log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="fricas")

[Out] integral((b\*x^3 + a\*x^2)\*dilog(c\*x)\*log(-c\*x + 1), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)x^2 \operatorname{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="giac")

[Out] integrate((b\*x + a)\*x^2\*dilog(c\*x)\*log(-c\*x + 1), x)

### 3.185 $\int x(a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=546

$$-\frac{x(3ac + 2b)\text{PolyLog}(2, cx)}{6c^2} + \frac{(3ac + 2b)\text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{(3ac + 2b) \log(1 - cx)\text{PolyLog}(2, cx)}{6c^3} - \frac{(3ac + 2b) \log(1 - cx)}{3c^3}$$

```
[Out] (4*b*x)/(9*c^2) + (a*x)/c + (5*(2*b + 3*a*c)*x)/(24*c^2) + (b*x^2)/(9*c) +
((2*b + 3*a*c)*x^2)/(48*c) + (b*x^3)/27 + (a*(1 - c*x)^2)/(8*c^2) + (2*b*Log[1 - c*x])/(9*c^3) +
((2*b + 3*a*c)*Log[1 - c*x])/(24*c^3) - (b*x^2*Log[1 - c*x])/(9*c) - ((2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c) -
(b*x^3*Log[1 - c*x])/9 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1 - c*x)*Log[1 - c*x])/c^2 +
((2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^3) - (a*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) -
(b*Log[1 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (a*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) +
(a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - ((2*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/(6*c^3) -
((2*b + 3*a*c)*x*PolyLog[2, c*x])/(6*c^2) - ((2*b + 3*a*c)*x^2*PolyLog[2, c*x])/(12*c) -
(b*x^3*PolyLog[2, c*x])/9 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^3) +
((3*a*x^2 + 2*b*x^3)*Log[1 - c*x]*PolyLog[2, c*x])/6 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) +
((2*b + 3*a*c)*PolyLog[3, 1 - c*x])/(3*c^3)
```

**Rubi [A]** time = 0.71802, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 21, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.105$ , Rules used = {6742, 6591, 2395, 43, 6604, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2410, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x(3ac + 2b)\text{PolyLog}(2, cx)}{6c^2} + \frac{(3ac + 2b)\text{PolyLog}(3, 1 - cx)}{3c^3} - \frac{(3ac + 2b) \log(1 - cx)\text{PolyLog}(2, cx)}{6c^3} - \frac{(3ac + 2b) \log(1 - cx)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] (4*b*x)/(9*c^2) + (a*x)/c + (5*(2*b + 3*a*c)*x)/(24*c^2) + (b*x^2)/(9*c) +
((2*b + 3*a*c)*x^2)/(48*c) + (b*x^3)/27 + (a*(1 - c*x)^2)/(8*c^2) + (2*b*Log[1 - c*x])/(9*c^3) +
((2*b + 3*a*c)*Log[1 - c*x])/(24*c^3) - (b*x^2*Log[1 - c*x])/(9*c) - ((2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c) -
(b*x^3*Log[1 - c*x])/9 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1 - c*x)*Log[1 - c*x])/c^2 +
((2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^3) - (a*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) -
(b*Log[1 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (a*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) +
(a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - ((2*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/(6*c^3) -
((2*b + 3*a*c)*x*PolyLog[2, c*x])/(6*c^2) - ((2*b + 3*a*c)*x^2*PolyLog[2, c*x])/(12*c) -
(b*x^3*PolyLog[2, c*x])/9 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^3) +
((3*a*x^2 + 2*b*x^3)*Log[1 - c*x]*PolyLog[2, c*x])/6 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) +
((2*b + 3*a*c)*PolyLog[3, 1 - c*x])/(3*c^3)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q]/(d*(m + 1)), x] - Dist[(
```

$p \cdot q / (m + 1)$ ,  $\text{Int}[(d \cdot x)^m \cdot \text{PolyLog}[n - 1, a \cdot (b \cdot x^p)^q], x], x] /;$   $\text{FreeQ}\{a, b, d, m, p, q\}, x\}$  &&  $\text{NeQ}[m, -1]$  &&  $\text{GtQ}[n, 0]$

Rule 2395

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x)^n] \cdot b) \cdot (f + g \cdot x)^q, x\_Symbol] := \text{Simp}[(f + g \cdot x)^{q+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n]) / (g \cdot (q + 1)), x] - \text{Dist}[(b \cdot e \cdot n) / (g \cdot (q + 1)), \text{Int}[(f + g \cdot x)^{q+1} / (d + e \cdot x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\}$  &&  $\text{NeQ}[e \cdot f - d \cdot g, 0]$  &&  $\text{NeQ}[q, -1]$

Rule 43

$\text{Int}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d, n\}, x\}$  &&  $\text{NeQ}[b \cdot c - a \cdot d, 0]$  &&  $\text{IGtQ}[m, 0]$  &&  $(! \text{IntegerQ}[n] \mid \mid (\text{EqQ}[c, 0] \&\& \text{LeQ}[7 \cdot m + 4 \cdot n + 4, 0]) \mid \mid \text{LtQ}[9 \cdot m + 5 \cdot (n + 1), 0] \mid \mid \text{GtQ}[m + n + 2, 0])$

Rule 6604

$\text{Int}[(g + \text{Log}[f \cdot (d + e \cdot x)^n] \cdot h) \cdot (P_x) \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)], x\_Symbol] := \text{With}\{u = \text{IntHide}[P_x, x]\}, \text{Simp}[u \cdot (g + h \cdot \text{Log}[f \cdot (d + e \cdot x)^n]) \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)], x] + (\text{Dist}[b, \text{Int}[\text{ExpandIntegrand}[(g + h \cdot \text{Log}[f \cdot (d + e \cdot x)^n]) \cdot \text{Log}[1 - a \cdot c - b \cdot c \cdot x], u / (a + b \cdot x), x], x] - \text{Dist}[e \cdot h \cdot n, \text{Int}[\text{ExpandIntegrand}[\text{PolyLog}[2, c \cdot (a + b \cdot x)], u / (d + e \cdot x), x], x], x]) /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, h, n\}, x\}$  &&  $\text{PolyQ}[P_x, x]$

Rule 2401

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x)^n] \cdot b)^p \cdot (f + g \cdot x)^q, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(f + g \cdot x)^q \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x\}$  &&  $\text{NeQ}[e \cdot f - d \cdot g, 0]$  &&  $\text{IGtQ}[q, 0]$

Rule 2389

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x)^n] \cdot b)^p, x\_Symbol] := \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b \cdot \text{Log}[c \cdot x^n])^p, x], x, d + e \cdot x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, n, p\}, x\}$

Rule 2296

$\text{Int}[(a + \text{Log}[c \cdot x^n] \cdot b)^p, x\_Symbol] := \text{Simp}[x \cdot (a + b \cdot \text{Log}[c \cdot x^n])^p, x] - \text{Dist}[b \cdot n \cdot p, \text{Int}[(a + b \cdot \text{Log}[c \cdot x^n])^{p-1}, x], x] /;$   $\text{FreeQ}\{a, b, c, n\}, x\}$  &&  $\text{GtQ}[p, 0]$  &&  $\text{IntegerQ}[2 \cdot p]$

Rule 2295

$\text{Int}[\text{Log}[c \cdot x^n], x\_Symbol] := \text{Simp}[x \cdot \text{Log}[c \cdot x^n], x] - \text{Simp}[n \cdot x, x] /;$   $\text{FreeQ}\{c, n\}, x\}$

Rule 2390

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x)^n] \cdot b)^p \cdot (f + g \cdot x)^q, x\_Symbol] := \text{Dist}[1/e, \text{Subst}[\text{Int}[(f \cdot x)/d]^q \cdot (a + b \cdot \text{Log}[c \cdot x^n])^p, x], x, d + e \cdot x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\}$  &&  $\text{EqQ}[e \cdot f - d \cdot g, 0]$

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_)^(q_.), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol]
:> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.))/((f_.) + (g_.
)*(x_)), x_Symbol]
:> Simp[(Log[(e*(f + g*x))]/(e*f - d*g))*(a + b*Log[c*(d
+ e*x)^n])^p/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))]/(e*f - d*g)
)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.), x_Sym
bol]
:> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```



$2*b*(-1 + c^3*x^3))*\text{Log}[1 - c*x]*\text{PolyLog}[2, c*x] - 144*(2*b + 3*a*c)*\text{Log}[1 - c*x]*\text{PolyLog}[2, 1 - c*x] + 288*b*\text{PolyLog}[3, 1 - c*x] + 432*a*c*\text{PolyLog}[3, 1 - c*x])/(432*c^3)$

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int x(bx + a) \ln(-cx + 1) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)`

[Out] `int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)`

**Maxima [A]** time = 1.03758, size = 466, normalized size = 0.85

$$-\frac{1}{432}c \left( \frac{72(\log(cx)\log(-cx+1)^2 + 2\text{Li}_2(-cx+1)\log(-cx+1) - 2\text{Li}_3(-cx+1))(3ac+2b)}{c^4} - \frac{16bc^3x^3 + 3(27ac^3 + 27a^2c^2 + 27ab^2c + 27b^3)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")`

[Out] `-1/432*c*(72*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1))*(3*a*c + 2*b)/c^4 - (16*b*c^3*x^3 + 3*(27*a*c^3 + 27*a^2*c^2 + 27*b*c^2)*x^2 + 6*(99*a*c^2 + 62*b*c)*x - 12*(4*b*c^3*x^3 + 3*(3*a*c^3 + 2*b*c^2)*x^2 + 6*(3*a*c^2 + 2*b*c)*x + 6*(3*a*c + 2*b)*log(-c*x + 1))*dilog(c*x) - 2*(16*b*c^3*x^3 + 6*(9*a*c^3 + 5*b*c^2)*x^2 - 297*a*c + 6*(27*a*c^2 + 16*b*c)*x - 186*b)*log(-c*x + 1))/c^4 + 1/216*(27*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*a/c^2 + 4*(18*c^3*x^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*b/c^3)*log(-c*x + 1)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((bx^2 + ax)\text{Li}_2(cx)\log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a*x)*dilog(c*x)*log(-c*x + 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)x\text{Li}_2(cx) \log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x*dilog(c*x)*log(-c*x + 1), x)
```

### 3.186 $\int (a + bx) \log(1 - cx) \text{PolyLog}(2, cx) dx$

**Optimal.** Leaf size=390

$$\frac{(2ac + b)\text{PolyLog}(3, 1 - cx)}{c^2} - \frac{(2ac + b) \log(1 - cx)\text{PolyLog}(2, cx)}{2c^2} - \frac{(2ac + b) \log(1 - cx)\text{PolyLog}(2, 1 - cx)}{c^2} + \frac{1}{2} (2ax$$

```
[Out] 2*a*x + (9*b*x)/(8*c) + ((b + 2*a*c)*x)/(2*c) + (b*x^2)/16 + (b*(1 - c*x)^2)/(8*c^2) + (b*Log[1 - c*x])/(8*c^2) - (b*x^2*Log[1 - c*x])/8 + (b*(1 - c*x)*Log[1 - c*x])/c^2 + (2*a*(1 - c*x)*Log[1 - c*x])/c + ((b + 2*a*c)*(1 - c*x)*Log[1 - c*x])/(2*c^2) - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (b*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - ((b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - ((b + 2*a*c)*x*PolyLog[2, c*x])/(2*c) - (b*x^2*PolyLog[2, c*x])/4 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + ((2*a*x + b*x^2)*Log[1 - c*x]*PolyLog[2, c*x])/2 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + ((b + 2*a*c)*PolyLog[3, 1 - c*x])/c^2
```

**Rubi [A]** time = 0.44939, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 20, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$ , Rules used = {6598, 43, 2416, 2389, 2295, 2391, 2395, 6604, 2296, 2401, 2390, 2305, 2304, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$\frac{(2ac + b)\text{PolyLog}(3, 1 - cx)}{c^2} - \frac{(2ac + b) \log(1 - cx)\text{PolyLog}(2, cx)}{2c^2} - \frac{(2ac + b) \log(1 - cx)\text{PolyLog}(2, 1 - cx)}{c^2} + \frac{1}{2} (2ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] 2*a*x + (9*b*x)/(8*c) + ((b + 2*a*c)*x)/(2*c) + (b*x^2)/16 + (b*(1 - c*x)^2)/(8*c^2) + (b*Log[1 - c*x])/(8*c^2) - (b*x^2*Log[1 - c*x])/8 + (b*(1 - c*x)*Log[1 - c*x])/c^2 + (2*a*(1 - c*x)*Log[1 - c*x])/c + ((b + 2*a*c)*(1 - c*x)*Log[1 - c*x])/(2*c^2) - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (b*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2) - ((b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - ((b + 2*a*c)*x*PolyLog[2, c*x])/(2*c) - (b*x^2*PolyLog[2, c*x])/4 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + ((2*a*x + b*x^2)*Log[1 - c*x]*PolyLog[2, c*x])/2 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + ((b + 2*a*c)*PolyLog[3, 1 - c*x])/c^2
```

#### Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)]/(e*(m + 1)), x] + Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.))\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 6604

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(h\_.))\*(Px\_)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := With[{u = IntHide[Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6591

```
Int[((d_.)*(x_)^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/d*(m + 1), x] - Dist[(p
*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.), x_Sym
bol]
:> Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol]
:> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx) \log(1 - cx) \text{Li}_2(cx) dx &= \frac{1}{2} (2ax + bx^2) \log(1 - cx) \text{Li}_2(cx) + c \int \left( \frac{(-b - 2ac) \text{Li}_2(cx)}{2c^2} - \frac{bx \text{Li}_2(cx)}{2c} + \frac{(-b - 2ac)}{2c^2} \right) dx \\
&= \frac{1}{2} (2ax + bx^2) \log(1 - cx) \text{Li}_2(cx) + a \int \log^2(1 - cx) dx + \frac{1}{2} b \int x \log^2(1 - cx) dx - \\
&= -\frac{(b + 2ac)x \text{Li}_2(cx)}{2c} - \frac{1}{4} bx^2 \text{Li}_2(cx) - \frac{(b + 2ac) \log(1 - cx) \text{Li}_2(cx)}{2c^2} + \frac{1}{2} (2ax + bx^2) \log(1 - cx) \\
&= -\frac{1}{8} bx^2 \log(1 - cx) - \frac{a(1 - cx) \log^2(1 - cx)}{c} - \frac{(b + 2ac) \log(cx) \log^2(1 - cx)}{2c^2} - \frac{(b + 2ac)}{2c^2} \log(1 - cx) \\
&= 2ax + \frac{(b + 2ac)x}{2c} - \frac{1}{8} bx^2 \log(1 - cx) + \frac{2a(1 - cx) \log(1 - cx)}{c} + \frac{(b + 2ac)(1 - cx) \log(1 - cx)}{2c^2} \\
&= 2ax + \frac{bx}{8c} + \frac{(b + 2ac)x}{2c} + \frac{bx^2}{16} + \frac{b \log(1 - cx)}{8c^2} - \frac{1}{8} bx^2 \log(1 - cx) + \frac{2a(1 - cx) \log(1 - cx)}{c} \\
&= 2ax + \frac{9bx}{8c} + \frac{(b + 2ac)x}{2c} + \frac{bx^2}{16} + \frac{b(1 - cx)^2}{8c^2} + \frac{b \log(1 - cx)}{8c^2} - \frac{1}{8} bx^2 \log(1 - cx) + \frac{2a(1 - cx) \log(1 - cx)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.490164, size = 285, normalized size = 0.73

$4\text{PolyLog}(2, cx)(2(cx - 1) \log(1 - cx)(2ac + bcx + b) - cx(4ac + bcx + 2b)) - 16(2ac + b) \log(1 - cx)\text{PolyLog}(2, 1 - cx)$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x], x]
```

```
[Out] (-14*b - 32*a*c + 22*b*c*x + 48*a*c^2*x + 3*b*c^2*x^2 + 22*b*Log[1 - c*x] + 48*a*c*Log[1 - c*x] - 16*b*c*x*Log[1 - c*x] - 48*a*c^2*x*Log[1 - c*x] - 6*b*c^2*x^2*Log[1 - c*x] - 4*b*Log[1 - c*x]^2 - 16*a*c*Log[1 - c*x]^2 + 16*a*c^2*x*Log[1 - c*x]^2 + 4*b*c^2*x^2*Log[1 - c*x]^2 - 8*b*Log[c*x]*Log[1 - c*x]^2 - 16*a*c*Log[c*x]*Log[1 - c*x]^2 + 4*(-(c*x*(2*b + 4*a*c + b*c*x)) + 2*(-1 + c*x)*(b + 2*a*c + b*c*x))*Log[1 - c*x]*PolyLog[2, c*x] - 16*(b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*b*PolyLog[3, 1 - c*x] + 32*a*c*PolyLog[3, 1 - c*x])/(16*c^2)
```

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int (bx + a) \ln(-cx + 1) \text{polylog}(2, cx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x), x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x), x)
```

---

**Maxima [A]** time = 1.00725, size = 348, normalized size = 0.89

$$-\frac{1}{16}c \left( \frac{8(\log(cx)\log(-cx+1)^2 + 2\operatorname{Li}_2(-cx+1)\log(-cx+1) - 2\operatorname{Li}_3(-cx+1))(2ac+b)}{c^3} - \frac{3bc^2x^2 + 2(24ac^2 + 11bc)}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="maxima")

[Out] -1/16\*c\*(8\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1))\*(2\*a\*c + b)/c^3 - (3\*b\*c^2\*x^2 + 2\*(24\*a\*c^2 + 11\*b\*c)\*x - 4\*(b\*c^2\*x^2 + 2\*(2\*a\*c^2 + b\*c)\*x + 2\*(2\*a\*c + b)\*log(-c\*x + 1))\*dilog(c\*x) - 2\*(2\*b\*c^2\*x^2 - 24\*a\*c + 2\*(8\*a\*c^2 + 3\*b\*c)\*x - 11\*b)\*log(-c\*x + 1))/c^3 + 1/8\*(8\*(c\*x\*dilog(c\*x) - c\*x + (c\*x - 1)\*log(-c\*x + 1))\*a/c + (4\*c^2\*x^2\*dilog(c\*x) - c^2\*x^2 - 2\*c\*x + 2\*(c^2\*x^2 - 1)\*log(-c\*x + 1))\*b/c^2)\*log(-c\*x + 1)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((bx + a)\operatorname{Li}_2(cx)\log(-cx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="fricas")

[Out] integral((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)\operatorname{Li}_2(cx)\log(-cx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x),x, algorithm="giac")

[Out] integrate((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1), x)

$$3.187 \quad \int \frac{(a+bx) \log(1-cx) \mathbf{PolyLog}(2, cx)}{x} dx$$

**Optimal.** Leaf size=153

$$-\frac{1}{2}a \mathbf{PolyLog}(2, cx)^2 - bx \mathbf{PolyLog}(2, cx) + \frac{2b \mathbf{PolyLog}(3, 1-cx)}{c} + bx \log(1-cx) \mathbf{PolyLog}(2, cx) - \frac{b \log(1-cx) \mathbf{PolyLog}(2, cx)}{c}$$

[Out] 3\*b\*x + (3\*b\*(1 - c\*x)\*Log[1 - c\*x])/c - (b\*(1 - c\*x)\*Log[1 - c\*x]^2)/c - (b\*Log[c\*x]\*Log[1 - c\*x]^2)/c - b\*x\*PolyLog[2, c\*x] - (b\*Log[1 - c\*x]\*PolyLog[2, c\*x])/c + b\*x\*Log[1 - c\*x]\*PolyLog[2, c\*x] - (a\*PolyLog[2, c\*x]^2)/2 - (2\*b\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/c + (2\*b\*PolyLog[3, 1 - c\*x])/c

**Rubi [A]** time = 0.338379, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {6742, 6586, 2389, 2295, 6589, 6605, 6601, 12, 6600, 2296, 6688, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}a \mathbf{PolyLog}(2, cx)^2 - bx \mathbf{PolyLog}(2, cx) + \frac{2b \mathbf{PolyLog}(3, 1-cx)}{c} + bx \log(1-cx) \mathbf{PolyLog}(2, cx) - \frac{b \log(1-cx) \mathbf{PolyLog}(2, cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x, x]

[Out] 3\*b\*x + (3\*b\*(1 - c\*x)\*Log[1 - c\*x])/c - (b\*(1 - c\*x)\*Log[1 - c\*x]^2)/c - (b\*Log[c\*x]\*Log[1 - c\*x]^2)/c - b\*x\*PolyLog[2, c\*x] - (b\*Log[1 - c\*x]\*PolyLog[2, c\*x])/c + b\*x\*Log[1 - c\*x]\*PolyLog[2, c\*x] - (a\*PolyLog[2, c\*x]^2)/2 - (2\*b\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/c + (2\*b\*PolyLog[3, 1 - c\*x])/c

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)])*(h_.)*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x_)], x_Symbol] := Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*PolyLog[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)])*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] := -Simp[PolyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))]*(h_.))*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLog[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)])*(b_.)^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 - a*c - b*c*x])*PolyLog[2, c*(a + b*x)]/e, x] + Dist[b/e, Int[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.)^(p_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.)^(p_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.))*((k_.) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```



f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*1, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/x], x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx) \log(1 - cx) \text{Li}_2(cx)}{x} dx &= a \int \frac{\log(1 - cx) \text{Li}_2(cx)}{x} dx + \int b \log(1 - cx) \text{Li}_2(cx) dx \\
 &= -\frac{1}{2} a \text{Li}_2(cx)^2 + b \int \log(1 - cx) \text{Li}_2(cx) dx \\
 &= bx \log(1 - cx) \text{Li}_2(cx) - \frac{1}{2} a \text{Li}_2(cx)^2 + b \int \log^2(1 - cx) dx + (bc) \int \left( -\frac{1}{c} - \frac{1}{c(-1 + cx)} \right) dx \\
 &= bx \log(1 - cx) \text{Li}_2(cx) - \frac{1}{2} a \text{Li}_2(cx)^2 - \frac{b \text{Subst}\left(\int \log^2(x) dx, x, 1 - cx\right)}{c} + (bc) \int \frac{x}{-1 + cx} dx \\
 &= -\frac{b(1 - cx) \log^2(1 - cx)}{c} + bx \log(1 - cx) \text{Li}_2(cx) - \frac{1}{2} a \text{Li}_2(cx)^2 + \frac{(2b) \text{Subst}\left(\int \log(x) dx, x, 1 - cx\right)}{c} \\
 &= 2bx + \frac{2b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} + bx \log(1 - cx) \text{Li}_2(cx) - \frac{1}{2} a \text{Li}_2(cx)^2 \\
 &= 2bx + \frac{2b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} - bx \text{Li}_2(cx) - \frac{b \log(1 - cx) \text{Li}_2(cx)}{c} \\
 &= 2bx + \frac{2b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} - \frac{b \log(cx) \log^2(1 - cx)}{c} - bx \text{Li}_2(cx) \\
 &= 3bx + \frac{3b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} - \frac{b \log(cx) \log^2(1 - cx)}{c} - bx \text{Li}_2(cx) \\
 &= 3bx + \frac{3b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} - \frac{b \log(cx) \log^2(1 - cx)}{c} - bx \text{Li}_2(cx) \\
 &= 3bx + \frac{3b(1 - cx) \log(1 - cx)}{c} - \frac{b(1 - cx) \log^2(1 - cx)}{c} - \frac{b \log(cx) \log^2(1 - cx)}{c} - bx \text{Li}_2(cx)
 \end{aligned}$$

**Mathematica [A]** time = 0.234598, size = 137, normalized size = 0.9

$$-\frac{1}{2} a \text{PolyLog}(2, cx)^2 + \frac{b \left( 2 \text{PolyLog}(3, 1 - cx) - 2 \log(1 - cx) \text{PolyLog}(2, 1 - cx) + 3cx + cx \log^2(1 - cx) - \log(cx) \log(1 - cx) \right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x,x]

[Out] (b\*(-(c\*x) + (-1 + c\*x)\*Log[1 - c\*x])\*PolyLog[2, c\*x])/c - (a\*PolyLog[2, c\*x]^2)/2 + (b\*(-2 + 3\*c\*x + 3\*Log[1 - c\*x] - 3\*c\*x\*Log[1 - c\*x] - Log[1 - c\*x]^2 + c\*x\*Log[1 - c\*x]^2 - Log[c\*x]\*Log[1 - c\*x]^2 - 2\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x] + 2\*PolyLog[3, 1 - c\*x]))/c

**Maple [F]** time = 0.005, size = 0, normalized size = 0.

$$\int \frac{(bx + a) \ln(-cx + 1) \operatorname{polylog}(2, cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x,x)

[Out] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a) \operatorname{Li}_2(cx) \log(-cx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x,x, algorithm="maxima")

[Out] integrate((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx + a) \operatorname{Li}_2(cx) \log(-cx + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x,x, algorithm="fricas")

[Out] integral((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx) \log(-cx + 1) \operatorname{Li}_2(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x,x)

[Out] Integral((a + b\*x)\*log(-c\*x + 1)\*polylog(2, c\*x)/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a) \operatorname{Li}_2(cx) \log(-cx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

$$3.188 \quad \int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^2} dx$$

**Optimal.** Leaf size=131

$$-2ac \text{PolyLog}(2, cx) - ac \text{PolyLog}(3, cx) - 2ac \text{PolyLog}(3, 1 - cx) + ac \log(1 - cx) \text{PolyLog}(2, cx) - \frac{a \log(1 - cx) \text{PolyLog}(2, cx)}{x}$$

```
[Out] (a*(1 - c*x)*Log[1 - c*x]^2)/x + a*c*Log[c*x]*Log[1 - c*x]^2 - 2*a*c*PolyLog[2, c*x] + a*c*Log[1 - c*x]*PolyLog[2, c*x] - (a*Log[1 - c*x]*PolyLog[2, c*x])/x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

**Rubi [A]** time = 0.313536, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.81$ , Rules used = {6742, 6591, 2395, 36, 29, 31, 6589, 6605, 6601, 12, 6603, 2397, 2391, 6596, 2396, 2433, 2374}

$$-2ac \text{PolyLog}(2, cx) - ac \text{PolyLog}(3, cx) - 2ac \text{PolyLog}(3, 1 - cx) + ac \log(1 - cx) \text{PolyLog}(2, cx) - \frac{a \log(1 - cx) \text{PolyLog}(2, cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2, x]
```

```
[Out] (a*(1 - c*x)*Log[1 - c*x]^2)/x + a*c*Log[c*x]*Log[1 - c*x]^2 - 2*a*c*PolyLog[2, c*x] + a*c*Log[1 - c*x]*PolyLog[2, c*x] - (a*Log[1 - c*x]*PolyLog[2, c*x])/x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

$\text{Int}[(x\_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

### Rule 31

$\text{Int}[(a\_ + (b\_)(x\_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n, (c\_)((a\_ + (b\_)(x\_))^{p\_})]/((d\_ + (e\_)(x\_)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

### Rule 6605

$\text{Int}[(g\_ + \text{Log}[1 + (e\_)(x\_)]*(h\_))*(P_x)(x\_)^{m\_} \text{PolyLog}[2, (c\_)(x\_)], x\_Symbol] \rightarrow \text{Dist}[\text{Coeff}[P_x, x, -m - 1], \text{Int}[(g + h*\text{Log}[1 + e*x])*\text{PolyLog}[2, c*x]/x, x], x] + \text{Int}[x^m*(P_x - \text{Coeff}[P_x, x, -m - 1]*x^{-(m + 1)})*(g + h*\text{Log}[1 + e*x])*\text{PolyLog}[2, c*x], x] /; \text{FreeQ}[\{c, e, g, h\}, x] \&\& \text{PolyQ}[P_x, x] \&\& \text{ILtQ}[m, 0] \&\& \text{EqQ}[c + e, 0] \&\& \text{NeQ}[\text{Coeff}[P_x, x, -m - 1], 0]$

### Rule 6601

$\text{Int}[(\text{Log}[1 + (e\_)(x\_)]*\text{PolyLog}[2, (c\_)(x_)])/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, c*x]^2/2, x] /; \text{FreeQ}[\{c, e\}, x] \&\& \text{EqQ}[c + e, 0]$

### Rule 12

$\text{Int}[(a_)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)(v_)] /; \text{FreeQ}[b, x]$

### Rule 6603

$\text{Int}[(g\_ + \text{Log}[(f\_)((d\_ + (e\_)(x\_))^{n\_})]*(h\_))(x\_)^{m\_} \text{PolyLog}[2, (c\_)((a\_ + (b\_)(x_))], x\_Symbol] \rightarrow \text{Simp}[(x^{m+1}*(g + h*\text{Log}[f*(d + e*x)^n])*\text{PolyLog}[2, c*(a + b*x)])/(m + 1), x] + (\text{Dist}[b/(m + 1), \text{Int}[\text{ExpandIntegrand}[(g + h*\text{Log}[f*(d + e*x)^n])*\text{Log}[1 - a*c - b*c*x], x^{m+1}/(a + b*x), x], x] - \text{Dist}[(e*h*n)/(m + 1), \text{Int}[\text{ExpandIntegrand}[\text{PolyLog}[2, c*(a + b*x)], x^{m+1}/(d + e*x), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{NeQ}[m, -1]$

### Rule 2397

$\text{Int}[(a\_ + \text{Log}[(c\_)((d\_ + (e\_)(x\_))^{n\_})]*(b\_))^{p\_}/((f\_ + (g\_)(x_))^{2}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)*(a + b*\text{Log}[c*(d + e*x)^n])^p]/((e*f - d*g)*(f + g*x)), x] - \text{Dist}[(b*e*n*p)/(e*f - d*g), \text{Int}[(a + b*\text{Log}[c*(d + e*x)^n])^{p-1}/(f + g*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{GtQ}[p, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_)((d\_ + (e\_)(x_))^{n_})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 6596

$\text{Int}[\text{PolyLog}[2, (c\_)((a\_ + (b\_)(x_))]/((d\_ + (e\_)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 - a*c - b*c*x]*\text{PolyLog}[2, c*(a + b*x)])/e, x] + \text{Dist}[b/e, \text{Int}[\text{Log}[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\&$

EqQ[c\*(b\*d - a\*e) + e, 0]

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx) \log(1 - cx) \text{Li}_2(cx)}{x^2} dx &= b \int \frac{\log(1 - cx) \text{Li}_2(cx)}{x} dx + \int \frac{a \log(1 - cx) \text{Li}_2(cx)}{x^2} dx \\
 &= -\frac{1}{2} b \text{Li}_2(cx)^2 + a \int \frac{\log(1 - cx) \text{Li}_2(cx)}{x^2} dx \\
 &= -\frac{a \log(1 - cx) \text{Li}_2(cx)}{x} - \frac{1}{2} b \text{Li}_2(cx)^2 - a \int \frac{\log^2(1 - cx)}{x^2} dx - (ac) \int \left( \frac{\text{Li}_2(cx)}{x} - \frac{c \text{Li}_2}{-1} \right) dx \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} - \frac{a \log(1 - cx) \text{Li}_2(cx)}{x} - \frac{1}{2} b \text{Li}_2(cx)^2 - (ac) \int \frac{\text{Li}_2(cx)}{x} dx + (2ac) \int \frac{\log(1 - cx)}{x} dx \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} - 2ac \text{Li}_2(cx) + ac \log(1 - cx) \text{Li}_2(cx) - \frac{a \log(1 - cx) \text{Li}_2(cx)}{x} \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} + ac \log(cx) \log^2(1 - cx) - 2ac \text{Li}_2(cx) + ac \log(1 - cx) \text{Li}_2(cx) \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} + ac \log(cx) \log^2(1 - cx) - 2ac \text{Li}_2(cx) + ac \log(1 - cx) \text{Li}_2(cx) \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} + ac \log(cx) \log^2(1 - cx) - 2ac \text{Li}_2(cx) + ac \log(1 - cx) \text{Li}_2(cx) \\
 &= \frac{a(1 - cx) \log^2(1 - cx)}{x} + ac \log(cx) \log^2(1 - cx) - 2ac \text{Li}_2(cx) + ac \log(1 - cx) \text{Li}_2(cx)
 \end{aligned}$$

**Mathematica [A]** time = 0.713885, size = 135, normalized size = 1.03

$$-ac \text{PolyLog}(3, cx) - 2ac \text{PolyLog}(3, 1 - cx) + \frac{a(cx - 1) \log(1 - cx) \text{PolyLog}(2, cx)}{x} + 2ac(\log(1 - cx) + 1) \text{PolyLog}(2, 1 - cx)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^2,x]

[Out] 2\*a\*c\*Log[c\*x]\*Log[1 - c\*x] - a\*c\*Log[1 - c\*x]^2 + (a\*Log[1 - c\*x]^2)/x + a\*c\*Log[c\*x]\*Log[1 - c\*x]^2 + (a\*(-1 + c\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x - (b\*PolyLog[2, c\*x]^2)/2 + 2\*a\*c\*(1 + Log[1 - c\*x])\*PolyLog[2, 1 - c\*x] - a\*c\*PolyLog[3, c\*x] - 2\*a\*c\*PolyLog[3, 1 - c\*x]

**Maple [F]** time = 0.008, size = 0, normalized size = 0.

$$\int \frac{(bx + a) \ln(-cx + 1) \operatorname{polylog}(2, cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x^2,x)

[Out] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a) \operatorname{Li}_2(cx) \log(-cx + 1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="maxima")

[Out] integrate((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx + a) \operatorname{Li}_2(cx) \log(-cx + 1)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="fricas")

[Out] integral((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx) \log(-cx + 1) \operatorname{Li}_2(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x\*\*2,x)

[Out] Integral((a + b\*x)\*log(-c\*x + 1)\*polylog(2, c\*x)/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)\text{Li}_2(cx)\log(-cx + 1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^2,x, algorithm="giac")

[Out] integrate((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^2, x)



$$3.189 \quad \int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^3} dx$$

**Optimal.** Leaf size=331

$$\frac{b^2 \text{PolyLog}(3, 1-cx)}{a} - \frac{b^2 \log(1-cx) \text{PolyLog}(2, 1-cx)}{a} - \frac{(a+bx)^2 \log(1-cx) \text{PolyLog}(2, cx)}{2ax^2} - \frac{1}{2}c(ac+2b) \text{PolyLog}(2, cx)$$

```
[Out] -(a*c^2*Log[x]) + a*c^2*Log[1 - c*x] - (a*c*Log[1 - c*x])/x - (a*c^2*Log[1 - c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1 - c*x]^2)/x - (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) - 2*b*c*PolyLog[2, c*x] - (a*c^2*PolyLog[2, c*x])/2 + (a*c*PolyLog[2, c*x])/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a) - ((a + b*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a - (c*(2*b + a*c)*PolyLog[3, c*x])/2 + (b^2*PolyLog[3, 1 - c*x])/a - ((b + a*c)^2*PolyLog[3, 1 - c*x])/a
```

**Rubi [A]** time = 0.540904, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 20, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$ , Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 37, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 2396, 2433, 2374, 6589, 6596}

$$\frac{b^2 \text{PolyLog}(3, 1-cx)}{a} - \frac{b^2 \log(1-cx) \text{PolyLog}(2, 1-cx)}{a} - \frac{(a+bx)^2 \log(1-cx) \text{PolyLog}(2, cx)}{2ax^2} - \frac{1}{2}c(ac+2b) \text{PolyLog}(2, cx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3, x]
```

```
[Out] -(a*c^2*Log[x]) + a*c^2*Log[1 - c*x] - (a*c*Log[1 - c*x])/x - (a*c^2*Log[1 - c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1 - c*x]^2)/x - (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) - 2*b*c*PolyLog[2, c*x] - (a*c^2*PolyLog[2, c*x])/2 + (a*c*PolyLog[2, c*x])/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a) - ((a + b*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a - (c*(2*b + a*c)*PolyLog[3, c*x])/2 + (b^2*PolyLog[3, 1 - c*x])/a - ((b + a*c)^2*PolyLog[3, 1 - c*x])/a
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6591

```
Int[((d_.)*(x_.))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_.)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m+1)*PolyLog[n, a*(b*x^p)^q])/(d*(m+1)), x] - Dist[(p*q)/(m+1), Int[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Simp[((f + g*x)^(q+1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q+1)), x] - Dist[(b*e*n)/(g*(q+1)), Int[(f + g*x)^(q+1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

eQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 37

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rule 6606

Int[((g\_) + Log[(f\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(h\_)\*(Px\_)\*(x\_)^m\_)\*PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))], x\_Symbol] := With[{u = IntHide[x^m\*Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x] && IntegerQ[m]

#### Rule 2398

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 2410

Int[(Log[(c\_)\*((d\_) + (e\_)\*(x\_))])\*(x\_)^m\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2397

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)^2, x\_Symbol] := Simp[((d + e\*x)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/((e\*f - d\*g)\*(f + g\*x)), x] - Dist[(b\*e\*n\*p)/(e\*f - d\*g), Int[(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)/(f + g\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&

EqQ[c\*(b\*d - a\*e) + e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^3} dx &= -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} + c \int \left( -\frac{a\text{Li}_2(cx)}{2x^2} + \frac{(-2b-ac)\text{Li}_2(cx)}{2x} + \frac{(b+ac)^2\text{Li}_2(cx)}{2a(-1+cx)} \right) dx \\
&= -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} - \frac{1}{2}a \int \frac{\log^2(1-cx)}{x^3} dx - b \int \frac{\log^2(1-cx)}{x^2} dx - \frac{b^2}{2} \int \frac{\log^2(1-cx)}{x} dx \\
&= \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a} + \frac{ac\text{Li}_2(cx)}{2x} + \frac{(b+ac)^2\log(1-cx)\text{Li}_2(cx)}{2a(-1+cx)} \\
&= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a} + \frac{(b+ac)^2\log(1-cx)\text{Li}_2(cx)}{2a(-1+cx)} \\
&= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a} + \frac{(b+ac)^2\log(1-cx)\text{Li}_2(cx)}{2a(-1+cx)} \\
&= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} \\
&= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx) + \frac{a\log^2(1-cx)}{4x^2} \\
&= -ac^2\log(x) + ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx) + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x}
\end{aligned}$$

**Mathematica [A]** time = 1.1877, size = 285, normalized size = 0.86

$$\frac{1}{4} \left( \frac{2\text{PolyLog}(2, cx)((cx-1)\log(1-cx)(acx+a+2bx)+acx)}{x^2} + 2c\text{PolyLog}(2, 1-cx)(2(ac+2b)\log(1-cx)+ac+4b) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^3, x]

[Out]  $(-4*a*c^2*\text{Log}[c*x] + 4*a*c^2*\text{Log}[1 - c*x] - (4*a*c*\text{Log}[1 - c*x]))/x + 8*b*c*\text{Log}[c*x]*\text{Log}[1 - c*x] + 2*a*c^2*\text{Log}[c*x]*\text{Log}[1 - c*x] - 4*b*c*\text{Log}[1 - c*x]^2 - a*c^2*\text{Log}[1 - c*x]^2 + (a*\text{Log}[1 - c*x]^2)/x^2 + (4*b*\text{Log}[1 - c*x]^2)/x + 4*b*c*\text{Log}[c*x]*\text{Log}[1 - c*x]^2 + 2*a*c^2*\text{Log}[c*x]*\text{Log}[1 - c*x]^2 + (2*(a*c*x + (-1 + c*x)*(a + 2*b*x + a*c*x))*\text{Log}[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c*(4*b + a*c + 2*(2*b + a*c))*\text{Log}[1 - c*x])*PolyLog[2, 1 - c*x] - 4*b*c*PolyLog[3, c*x] - 2*a*c^2*PolyLog[3, c*x] - 8*b*c*PolyLog[3, 1 - c*x] - 4*a*c^2*PolyLog[3, 1 - c*x])/4$

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\text{polylog}(2, cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2, c\*x)/x^3, x)

[Out]  $\int (b*x+a)*\ln(-c*x+1)*\text{polylog}(2,c*x)/x^3,x$

**Maxima [A]** time = 1.16395, size = 288, normalized size = 0.87

$$-ac^2 \log(x) + \frac{1}{2}(ac^2 + 2bc)\left(\log(cx) \log(-cx + 1)^2 + 2 \text{Li}_2(-cx + 1) \log(-cx + 1) - 2 \text{Li}_3(-cx + 1)\right) + \frac{1}{2}(ac^2 + 4bc)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*\log(-c*x+1)*\text{polylog}(2,c*x)/x^3,x, \text{algorithm}="maxima")$

[Out]  $-a*c^2*\log(x) + 1/2*(a*c^2 + 2*b*c)*( \log(c*x)*\log(-c*x + 1)^2 + 2*\text{dilog}(-c*x + 1)*\log(-c*x + 1) - 2*\text{polylog}(3, -c*x + 1)) + 1/2*(a*c^2 + 4*b*c)*( \log(c*x)*\log(-c*x + 1) + \text{dilog}(-c*x + 1)) - 1/2*(a*c^2 + 2*b*c)*\text{polylog}(3, c*x) - 1/4*((a*c^2 + 4*b*c)*x^2 - 4*b*x - a)*\log(-c*x + 1)^2 - 2*(a*c*x + (a*c^2 + 2*b*c)*x^2 - 2*b*x - a)*\log(-c*x + 1)*\text{dilog}(c*x) - 4*(a*c^2*x^2 - a*c*x)*\log(-c*x + 1))/x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*\log(-c*x+1)*\text{polylog}(2,c*x)/x^3,x, \text{algorithm}="fricas")$

[Out]  $\text{integral}((b*x + a)*\text{dilog}(c*x)*\log(-c*x + 1)/x^3, x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*\ln(-c*x+1)*\text{polylog}(2,c*x)/x^{**3},x)$

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*\log(-c*x+1)*\text{polylog}(2,c*x)/x^3,x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((b*x + a)*\text{dilog}(c*x)*\log(-c*x + 1)/x^3, x)$

$$3.190 \quad \int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^4} dx$$

**Optimal.** Leaf size=460

$$-\frac{1}{6}c^2(2ac + 3b)\text{PolyLog}(3, cx) - \frac{1}{3}c^2(2ac + 3b)\text{PolyLog}(3, 1 - cx) + \frac{1}{6}c^2(2ac + 3b) \log(1 - cx)\text{PolyLog}(2, cx) + \frac{1}{3}c^2(2a$$

[Out] (7\*a\*c^2)/(36\*x) - (b\*c^2\*Log[x])/2 - (5\*a\*c^3\*Log[x])/12 - (c^2\*(3\*b + 2\*a\*c)\*Log[x])/6 + (b\*c^2\*Log[1 - c\*x])/2 + (5\*a\*c^3\*Log[1 - c\*x])/12 + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x])/6 - (7\*a\*c\*Log[1 - c\*x])/(36\*x^2) - (b\*c\*Log[1 - c\*x])/(2\*x) - (2\*a\*c^2\*Log[1 - c\*x])/(9\*x) - (c\*(3\*b + 2\*a\*c)\*Log[1 - c\*x])/(6\*x) - (b\*c^2\*Log[1 - c\*x]^2)/4 - (a\*c^3\*Log[1 - c\*x]^2)/9 + (a\*Log[1 - c\*x]^2)/(9\*x^3) + (b\*Log[1 - c\*x]^2)/(4\*x^2) + (c^2\*(3\*b + 2\*a\*c)\*Log[c\*x]\*Log[1 - c\*x]^2)/6 - (b\*c^2\*PolyLog[2, c\*x])/2 - (2\*a\*c^3\*PolyLog[2, c\*x])/9 + (a\*c\*PolyLog[2, c\*x])/(6\*x^2) + (c\*(3\*b + 2\*a\*c)\*PolyLog[2, c\*x])/(6\*x) + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/6 - (((2\*a)/x^3 + (3\*b)/x^2)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/6 + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/3 - (c^2\*(3\*b + 2\*a\*c)\*PolyLog[3, c\*x])/6 - (c^2\*(3\*b + 2\*a\*c)\*PolyLog[3, 1 - c\*x])/3

**Rubi [A]** time = 0.671801, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 19, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$ , Rules used = {6742, 6591, 2395, 44, 43, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{6}c^2(2ac + 3b)\text{PolyLog}(3, cx) - \frac{1}{3}c^2(2ac + 3b)\text{PolyLog}(3, 1 - cx) + \frac{1}{6}c^2(2ac + 3b) \log(1 - cx)\text{PolyLog}(2, cx) + \frac{1}{3}c^2(2a$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^4, x]

[Out] (7\*a\*c^2)/(36\*x) - (b\*c^2\*Log[x])/2 - (5\*a\*c^3\*Log[x])/12 - (c^2\*(3\*b + 2\*a\*c)\*Log[x])/6 + (b\*c^2\*Log[1 - c\*x])/2 + (5\*a\*c^3\*Log[1 - c\*x])/12 + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x])/6 - (7\*a\*c\*Log[1 - c\*x])/(36\*x^2) - (b\*c\*Log[1 - c\*x])/(2\*x) - (2\*a\*c^2\*Log[1 - c\*x])/(9\*x) - (c\*(3\*b + 2\*a\*c)\*Log[1 - c\*x])/(6\*x) - (b\*c^2\*Log[1 - c\*x]^2)/4 - (a\*c^3\*Log[1 - c\*x]^2)/9 + (a\*Log[1 - c\*x]^2)/(9\*x^3) + (b\*Log[1 - c\*x]^2)/(4\*x^2) + (c^2\*(3\*b + 2\*a\*c)\*Log[c\*x]\*Log[1 - c\*x]^2)/6 - (b\*c^2\*PolyLog[2, c\*x])/2 - (2\*a\*c^3\*PolyLog[2, c\*x])/9 + (a\*c\*PolyLog[2, c\*x])/(6\*x^2) + (c\*(3\*b + 2\*a\*c)\*PolyLog[2, c\*x])/(6\*x) + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/6 - (((2\*a)/x^3 + (3\*b)/x^2)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/6 + (c^2\*(3\*b + 2\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/3 - (c^2\*(3\*b + 2\*a\*c)\*PolyLog[3, c\*x])/6 - (c^2\*(3\*b + 2\*a\*c)\*PolyLog[3, 1 - c\*x])/3

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 6591

Int[((d\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6606

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(h\_.))\*(Px\_)\*(x\_))^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := With[{u = IntHide[x^m\*Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x]]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x] && IntegerQ[m]

Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(x\_))^(m\_.)/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)<sup>(n\_)</sup>)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x<sup>n</sup>)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)<sup>(n\_)</sup>)]\*(b\_)<sup>(p\_)</sup>\*((f\_) + (g\_)\*(x\_)<sup>(q\_)</sup>), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)<sup>q</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)<sup>(n\_)</sup>]\*(b\_))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x<sup>n</sup>])<sup>2</sup>/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_)<sup>(p\_)</sup>)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)<sup>p</sup>]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]<sup>2</sup>/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)<sup>(n\_)</sup>)]\*(b\_)<sup>(p\_)</sup>)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>p</sup>/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)<sup>n</sup>])<sup>(p - 1)</sup>/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)<sup>(n\_)</sup>)]\*(b\_)<sup>(p\_)</sup>\*((f\_) + Log[(h\_)\*((i\_) + (j\_)\*(x\_)<sup>(m\_)</sup>)]\*(g\_))\*((k\_) + (l\_)\*(x\_)<sup>(r\_)</sup>), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)<sup>r</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]<sup>m</sup>), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_)\*((e\_) + (f\_)\*(x\_)<sup>(m\_)</sup>)]\*((a\_) + Log[(c\_)\*(x\_)<sup>(n\_)]\*(b\_)<sup>(p\_)</sup>)]/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x<sup>m</sup>)]\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>]/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x<sup>m</sup>)]\*(a + b\*Log[c\*x<sup>n</sup>])<sup>(p - 1)</sup>]/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]</sup>

#### Rubi steps



$$\begin{aligned}
\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^4} dx &= -\frac{1}{6}\left(\frac{2a}{x^3} + \frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) + c \int \left(-\frac{a\text{Li}_2(cx)}{3x^3} + \frac{(-3b-2ac)\text{Li}_2(cx)}{6x^2} - \frac{c(3b+2ac)\text{Li}_2(cx)}{6x}\right) dx \\
&= -\frac{1}{6}\left(\frac{2a}{x^3} + \frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) - \frac{1}{3}a \int \frac{\log^2(1-cx)}{x^4} dx - \frac{1}{2}b \int \frac{\log^2(1-cx)}{x^3} dx \\
&= \frac{a \log^2(1-cx)}{9x^3} + \frac{b \log^2(1-cx)}{4x^2} + \frac{ac\text{Li}_2(cx)}{6x^2} + \frac{c(3b+2ac)\text{Li}_2(cx)}{6x} + \frac{1}{6}c^2(3b+2ac)\log(1-cx) \\
&= -\frac{ac \log(1-cx)}{12x^2} - \frac{c(3b+2ac) \log(1-cx)}{6x} + \frac{a \log^2(1-cx)}{9x^3} + \frac{b \log^2(1-cx)}{4x^2} + \frac{1}{6}c^2 \\
&= -\frac{ac \log(1-cx)}{12x^2} - \frac{c(3b+2ac) \log(1-cx)}{6x} + \frac{a \log^2(1-cx)}{9x^3} + \frac{b \log^2(1-cx)}{4x^2} + \frac{1}{6}c^2 \\
&= \frac{ac^2}{12x} - \frac{1}{12}ac^3 \log(x) - \frac{1}{6}c^2(3b+2ac) \log(x) + \frac{1}{12}ac^3 \log(1-cx) + \frac{1}{6}c^2(3b+2ac) \log(1-cx) \\
&= \frac{ac^2}{12x} - \frac{1}{12}ac^3 \log(x) - \frac{1}{6}c^2(3b+2ac) \log(x) + \frac{1}{12}ac^3 \log(1-cx) + \frac{1}{6}c^2(3b+2ac) \log(1-cx) \\
&= \frac{7ac^2}{36x} - \frac{1}{2}bc^2 \log(x) - \frac{5}{12}ac^3 \log(x) - \frac{1}{6}c^2(3b+2ac) \log(x) + \frac{1}{2}bc^2 \log(1-cx) + \frac{5}{12}ac^3 \log(1-cx) + \frac{1}{6}c^2(3b+2ac) \log(1-cx)
\end{aligned}$$

**Mathematica [A]** time = 1.30438, size = 389, normalized size = 0.85

$$\frac{1}{36} \left( \frac{6\text{PolyLog}(2, cx) (\log(1-cx) (2ac^3x^3 - 2a + 3bc^2x^3 - 3bx) + cx(2acx + a + 3bx))}{x^3} + 2c^2\text{PolyLog}(2, 1-cx)(6(2a+bx)\log(1-cx) + 2c^2\text{PolyLog}(2, 1-cx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^4, x]

[Out]  $(-7ac^3 + (7ac^2)/x - 36b^2c^2\text{Log}[c*x] - 27ac^3\text{Log}[c*x] + 36b^2c^2\text{Log}[1-c*x] + 27ac^3\text{Log}[1-c*x] - (7ac\text{Log}[1-c*x])/x^2 - (36b^2c^2\text{Log}[1-c*x])/x - (20ac^2\text{Log}[1-c*x])/x + 18b^2c^2\text{Log}[c*x]\text{Log}[1-c*x] + 8ac^3\text{Log}[c*x]\text{Log}[1-c*x] - 9b^2c^2\text{Log}[1-c*x]^2 - 4ac^3\text{Log}[1-c*x]^2 + (4a\text{Log}[1-c*x]^2)/x^3 + (9b\text{Log}[1-c*x]^2)/x^2 + 18b^2c^2\text{Log}[c*x]\text{Log}[1-c*x]^2 + 12ac^3\text{Log}[c*x]\text{Log}[1-c*x]^2 + (6(c*x(a + 3bx + 2ac*x) + (-2a - 3bx + 3b^2c^2x^3 + 2ac^3x^3))\text{Log}[1-c*x])\text{PolyLog}[2, c*x])/x^3 + 2c^2(9b + 4ac + 6(3b + 2ac)\text{Log}[1-c*x])\text{PolyLog}[2, 1-c*x] - 18b^2c^2\text{PolyLog}[3, c*x] - 12ac^3\text{PolyLog}[3, c*x] - 36b^2c^2\text{PolyLog}[3, 1-c*x] - 24ac^3\text{PolyLog}[3, 1-c*x])/36$

**Maple [F]** time = 0.006, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\text{polylog}(2, cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2, c\*x)/x^4, x)

[Out] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2, c\*x)/x^4, x)

**Maxima [A]** time = 1.20069, size = 387, normalized size = 0.84

$$\frac{1}{6} (2ac^3 + 3bc^2) (\log(cx) \log(-cx + 1)^2 + 2 \operatorname{Li}_2(-cx + 1) \log(-cx + 1) - 2 \operatorname{Li}_3(-cx + 1)) + \frac{1}{18} (4ac^3 + 9bc^2) (\log(cx) \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^4,x, algorithm="maxima")

[Out] 1/6\*(2\*a\*c^3 + 3\*b\*c^2)\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1)) + 1/18\*(4\*a\*c^3 + 9\*b\*c^2)\*(log(c\*x)\*log(-c\*x + 1) + dilog(-c\*x + 1)) - 1/4\*(3\*a\*c^3 + 4\*b\*c^2)\*log(x) - 1/6\*(2\*a\*c^3 + 3\*b\*c^2)\*polylog(3, c\*x) + 1/36\*(7\*a\*c^2\*x^2 - ((4\*a\*c^3 + 9\*b\*c^2)\*x^3 - 9\*b\*x - 4\*a)\*log(-c\*x + 1)^2 + 6\*(a\*c\*x + (2\*a\*c^2 + 3\*b\*c)\*x^2 + ((2\*a\*c^3 + 3\*b\*c^2)\*x^3 - 3\*b\*x - 2\*a)\*log(-c\*x + 1))\*dilog(c\*x) + (9\*(3\*a\*c^3 + 4\*b\*c^2)\*x^3 - 7\*a\*c\*x - 4\*(5\*a\*c^2 + 9\*b\*c)\*x^2)\*log(-c\*x + 1))/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx + a)\operatorname{Li}_2(cx) \log(-cx + 1)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^4,x, algorithm="fricas")

[Out] integral((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx + a)\operatorname{Li}_2(cx) \log(-cx + 1)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^4,x, algorithm="giac")

[Out] integrate((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^4, x)

$$3.191 \quad \int \frac{(a+bx) \log(1-cx) \text{PolyLog}(2, cx)}{x^5} dx$$

**Optimal.** Leaf size=584

$$-\frac{1}{12}c^3(3ac + 4b)\text{PolyLog}(3, cx) - \frac{1}{6}c^3(3ac + 4b)\text{PolyLog}(3, 1 - cx) + \frac{c^2(3ac + 4b)\text{PolyLog}(2, cx)}{12x} + \frac{1}{12}c^3(3ac + 4b)$$

[Out] (5\*a\*c^2)/(144\*x^2) + (b\*c^2)/(9\*x) + (19\*a\*c^3)/(144\*x) + (c^2\*(4\*b + 3\*a\*c))/(48\*x) - (b\*c^3\*Log[x])/3 - (37\*a\*c^4\*Log[x])/144 - (5\*c^3\*(4\*b + 3\*a\*c)\*Log[x])/48 + (b\*c^3\*Log[1 - c\*x])/3 + (37\*a\*c^4\*Log[1 - c\*x])/144 + (5\*c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/48 - (5\*a\*c\*Log[1 - c\*x])/(72\*x^3) - (b\*c\*Log[1 - c\*x])/(9\*x^2) - (a\*c^2\*Log[1 - c\*x])/(16\*x^2) - (c\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/(48\*x^2) - (2\*b\*c^2\*Log[1 - c\*x])/(9\*x) - (a\*c^3\*Log[1 - c\*x])/(8\*x) - (c^2\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/(12\*x) - (b\*c^3\*Log[1 - c\*x]^2)/9 - (a\*c^4\*Log[1 - c\*x]^2)/16 + (a\*Log[1 - c\*x]^2)/(16\*x^4) + (b\*Log[1 - c\*x]^2)/(9\*x^3) + (c^3\*(4\*b + 3\*a\*c)\*Log[c\*x]\*Log[1 - c\*x]^2)/12 - (2\*b\*c^3\*PolyLog[2, c\*x])/9 - (a\*c^4\*PolyLog[2, c\*x])/8 + (a\*c\*PolyLog[2, c\*x])/(12\*x^3) + (c\*(4\*b + 3\*a\*c)\*PolyLog[2, c\*x])/(24\*x^2) + (c^2\*(4\*b + 3\*a\*c)\*PolyLog[2, c\*x])/(12\*x) + (c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/12 - ((3\*a)/x^4 + (4\*b)/x^3)\*Log[1 - c\*x]\*PolyLog[2, c\*x]/12 + (c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/6 - (c^3\*(4\*b + 3\*a\*c)\*PolyLog[3, c\*x])/12 - (c^3\*(4\*b + 3\*a\*c)\*PolyLog[3, 1 - c\*x])/6

**Rubi [A]** time = 0.844662, antiderivative size = 584, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 19, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$ , Rules used = {6742, 6591, 2395, 44, 43, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{12}c^3(3ac + 4b)\text{PolyLog}(3, cx) - \frac{1}{6}c^3(3ac + 4b)\text{PolyLog}(3, 1 - cx) + \frac{c^2(3ac + 4b)\text{PolyLog}(2, cx)}{12x} + \frac{1}{12}c^3(3ac + 4b)$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^5, x]

[Out] (5\*a\*c^2)/(144\*x^2) + (b\*c^2)/(9\*x) + (19\*a\*c^3)/(144\*x) + (c^2\*(4\*b + 3\*a\*c))/(48\*x) - (b\*c^3\*Log[x])/3 - (37\*a\*c^4\*Log[x])/144 - (5\*c^3\*(4\*b + 3\*a\*c)\*Log[x])/48 + (b\*c^3\*Log[1 - c\*x])/3 + (37\*a\*c^4\*Log[1 - c\*x])/144 + (5\*c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/48 - (5\*a\*c\*Log[1 - c\*x])/(72\*x^3) - (b\*c\*Log[1 - c\*x])/(9\*x^2) - (a\*c^2\*Log[1 - c\*x])/(16\*x^2) - (c\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/(48\*x^2) - (2\*b\*c^2\*Log[1 - c\*x])/(9\*x) - (a\*c^3\*Log[1 - c\*x])/(8\*x) - (c^2\*(4\*b + 3\*a\*c)\*Log[1 - c\*x])/(12\*x) - (b\*c^3\*Log[1 - c\*x]^2)/9 - (a\*c^4\*Log[1 - c\*x]^2)/16 + (a\*Log[1 - c\*x]^2)/(16\*x^4) + (b\*Log[1 - c\*x]^2)/(9\*x^3) + (c^3\*(4\*b + 3\*a\*c)\*Log[c\*x]\*Log[1 - c\*x]^2)/12 - (2\*b\*c^3\*PolyLog[2, c\*x])/9 - (a\*c^4\*PolyLog[2, c\*x])/8 + (a\*c\*PolyLog[2, c\*x])/(12\*x^3) + (c\*(4\*b + 3\*a\*c)\*PolyLog[2, c\*x])/(24\*x^2) + (c^2\*(4\*b + 3\*a\*c)\*PolyLog[2, c\*x])/(12\*x) + (c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/12 - ((3\*a)/x^4 + (4\*b)/x^3)\*Log[1 - c\*x]\*PolyLog[2, c\*x]/12 + (c^3\*(4\*b + 3\*a\*c)\*Log[1 - c\*x]\*PolyLog[2, 1 - c\*x])/6 - (c^3\*(4\*b + 3\*a\*c)\*PolyLog[3, c\*x])/12 - (c^3\*(4\*b + 3\*a\*c)\*PolyLog[3, 1 - c\*x])/6

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q]/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))]*(h_.))*((Px_)*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]), x_Symbol]
:> With[{u = IntHide[x^m*Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol]
:> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol]
:> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

$\text{Int}[(x\_)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 31

$\text{Int}[(a\_ + (b\_)(x\_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 2391

$\text{Int}[\text{Log}[(c\_)((d\_ + (e\_)(x\_)^{n\_})]/(x\_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

Rule 2390

$\text{Int}[(a\_ + \text{Log}[(c\_)((d\_ + (e\_)(x\_)^{n\_})]*b\_)]^{p\_}((f\_ + (g\_)(x\_))^{q\_}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \ \&\& \ \text{EqQ}[e*f - d*g, 0]$

Rule 2301

$\text{Int}[(a\_ + \text{Log}[(c\_)(x\_)^{n\_}]*b\_)]/(x\_), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}[\{a, b, c, n\}, x]$

Rule 6589

$\text{Int}[\text{PolyLog}[n, (c\_)((a\_ + (b\_)(x\_))^{p\_})]/((d\_ + (e\_)(x\_))), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \ \&\& \ \text{EqQ}[b*d, a*e]$

Rule 6596

$\text{Int}[\text{PolyLog}[2, (c\_)((a\_ + (b\_)(x\_))]/((d\_ + (e\_)(x\_))), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 - a*c - b*c*x]*\text{PolyLog}[2, c*(a + b*x)])/e, x] + \text{Dist}[b/e, \text{Int}[\text{Log}[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*(b*d - a*e) + e, 0]$

Rule 2396

$\text{Int}[(a\_ + \text{Log}[(c\_)((d\_ + (e\_)(x\_))^{n\_})]*b\_)]^{p\_}/((f\_ + (g\_)(x\_))), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^p)/g, x] - \text{Dist}[(b*e*n*p)/g, \text{Int}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^{p-1})/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{IGtQ}[p, 1]$

Rule 2433

$\text{Int}[(a\_ + \text{Log}[(c\_)((d\_ + (e\_)(x\_))^{n\_})]*b\_)]^{p\_}((f\_ + \text{Log}[(h\_)(i\_ + (j\_)(x\_))^{m\_}]*g\_)]^{r\_}((k\_ + (l\_)(x\_))^{r\_}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(k*x)/d]^r*(a + b*\text{Log}[c*x^n])^p*(f + g*\text{Log}[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r\}, x] \ \&\& \ \text{EqQ}[e*k - d*l, 0]$

Rule 2374

$\text{Int}[(\text{Log}[(d\_)(e\_ + (f\_)(x\_)^{m\_})]*a\_ + \text{Log}[(c\_)(x\_)^{n\_}]*b\_)]^{p\_}/(x\_), x\_Symbol] \rightarrow -\text{Simp}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^p)/m, x]]$

$n])^{(p-1)}/x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x\} \&\& \text{IGtQ}[p, 0]$   
 $\&\& \text{EqQ}[d*e, 1]$

### Rubi steps

$$\begin{aligned} \int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^5} dx &= -\frac{1}{12} \left( \frac{3a}{x^4} + \frac{4b}{x^3} \right) \log(1-cx)\text{Li}_2(cx) + c \int \left( -\frac{a\text{Li}_2(cx)}{4x^4} + \frac{(-4b-3ac)\text{Li}_2(cx)}{12x^3} - \frac{c(4b+3ac)\text{Li}_2(cx)}{12x^2} \right) dx \\ &= -\frac{1}{12} \left( \frac{3a}{x^4} + \frac{4b}{x^3} \right) \log(1-cx)\text{Li}_2(cx) - \frac{1}{4}a \int \frac{\log^2(1-cx)}{x^5} dx - \frac{1}{3}b \int \frac{\log^2(1-cx)}{x^4} dx - \frac{c}{12} \int \frac{\log^2(1-cx)}{x^3} dx \\ &= \frac{a \log^2(1-cx)}{16x^4} + \frac{b \log^2(1-cx)}{9x^3} + \frac{ac\text{Li}_2(cx)}{12x^3} + \frac{c(4b+3ac)\text{Li}_2(cx)}{24x^2} + \frac{c^2(4b+3ac)\text{Li}_2(cx)}{12x} \\ &= -\frac{ac \log(1-cx)}{36x^3} - \frac{c(4b+3ac) \log(1-cx)}{48x^2} - \frac{c^2(4b+3ac) \log(1-cx)}{12x} + \frac{a \log^2(1-cx)}{16x^4} \\ &= -\frac{ac \log(1-cx)}{36x^3} - \frac{c(4b+3ac) \log(1-cx)}{48x^2} - \frac{c^2(4b+3ac) \log(1-cx)}{12x} + \frac{a \log^2(1-cx)}{16x^4} \\ &= \frac{ac^2}{72x^2} + \frac{ac^3}{36x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{36}ac^4 \log(x) - \frac{5}{48}c^3(4b+3ac) \log(x) + \frac{1}{36}ac^4 \log(1-cx) \\ &= \frac{ac^2}{72x^2} + \frac{ac^3}{36x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{36}ac^4 \log(x) - \frac{5}{48}c^3(4b+3ac) \log(x) + \frac{1}{36}ac^4 \log(1-cx) \\ &= \frac{5ac^2}{144x^2} + \frac{bc^2}{9x} + \frac{19ac^3}{144x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{3}bc^3 \log(x) - \frac{37}{144}ac^4 \log(x) - \frac{5}{48}c^3(4b+3ac) \log(1-cx) \end{aligned}$$

**Mathematica [A]** time = 1.51805, size = 505, normalized size = 0.86

$$-2c^3x^4\text{PolyLog}(2, 1-cx)(12(3ac+4b)\log(1-cx)+9ac+16b)-6\text{PolyLog}(2, cx)\left(cx\left(a(6c^2x^2+3cx+2)+4bx(2cx-1)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Log[1 - c\*x]\*PolyLog[2, c\*x])/x^5, x]

[Out] 
$$\begin{aligned} & -(-5*a*c^2*x^2 - 28*b*c^2*x^3 - 28*a*c^3*x^3 + 28*b*c^3*x^4 + 33*a*c^4*x^4 \\ & + 108*b*c^3*x^4*\text{Log}[c*x] + 82*a*c^4*x^4*\text{Log}[c*x] + 10*a*c*x*\text{Log}[1 - c*x] + \\ & 28*b*c*x^2*\text{Log}[1 - c*x] + 18*a*c^2*x^2*\text{Log}[1 - c*x] + 80*b*c^2*x^3*\text{Log}[1 - \\ & c*x] + 54*a*c^3*x^3*\text{Log}[1 - c*x] - 108*b*c^3*x^4*\text{Log}[1 - c*x] - 82*a*c^4*x^4 \\ & * \text{Log}[1 - c*x] - 32*b*c^3*x^4*\text{Log}[c*x]*\text{Log}[1 - c*x] - 18*a*c^4*x^4*\text{Log}[c*x] \\ & * \text{Log}[1 - c*x] - 9*a*\text{Log}[1 - c*x]^2 - 16*b*x*\text{Log}[1 - c*x]^2 + 16*b*c^3*x^4*L \\ & \text{og}[1 - c*x]^2 + 9*a*c^4*x^4*\text{Log}[1 - c*x]^2 - 48*b*c^3*x^4*\text{Log}[c*x]*\text{Log}[1 - \\ & c*x]^2 - 36*a*c^4*x^4*\text{Log}[c*x]*\text{Log}[1 - c*x]^2 - 6*(c*x*(4*b*x*(1 + 2*c*x) + \\ & a*(2 + 3*c*x + 6*c^2*x^2)) + (8*b*x*(-1 + c^3*x^3) + 6*a*(-1 + c^4*x^4))*L \\ & \text{og}[1 - c*x])* \text{PolyLog}[2, c*x] - 2*c^3*x^4*(16*b + 9*a*c + 12*(4*b + 3*a*c)*L \\ & \text{og}[1 - c*x])* \text{PolyLog}[2, 1 - c*x] + 48*b*c^3*x^4*\text{PolyLog}[3, c*x] + 36*a*c^4* \\ & x^4*\text{PolyLog}[3, c*x] + 96*b*c^3*x^4*\text{PolyLog}[3, 1 - c*x] + 72*a*c^4*x^4*\text{PolyL} \\ & \text{og}[3, 1 - c*x])/(144*x^4) \end{aligned}$$

**Maple [F]** time = 0.007, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\text{polylog}(2, cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x^5,x)

[Out] int((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x^5,x)

**Maxima [A]** time = 1.18925, size = 460, normalized size = 0.79

$$\frac{1}{12} (3ac^4 + 4bc^3) (\log(cx) \log(-cx+1)^2 + 2\text{Li}_2(-cx+1) \log(-cx+1) - 2\text{Li}_3(-cx+1)) + \frac{1}{72} (9ac^4 + 16bc^3) (\log(cx) \log(-cx+1) + \text{dilog}(-cx+1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^5,x, algorithm="maxima")

[Out] 1/12\*(3\*a\*c^4 + 4\*b\*c^3)\*(log(c\*x)\*log(-c\*x + 1)^2 + 2\*dilog(-c\*x + 1)\*log(-c\*x + 1) - 2\*polylog(3, -c\*x + 1)) + 1/72\*(9\*a\*c^4 + 16\*b\*c^3)\*(log(c\*x)\*log(-c\*x + 1) + dilog(-c\*x + 1)) - 1/72\*(41\*a\*c^4 + 54\*b\*c^3)\*log(x) - 1/12\*(3\*a\*c^4 + 4\*b\*c^3)\*polylog(3, c\*x) + 1/144\*(5\*a\*c^2\*x^2 + 28\*(a\*c^3 + b\*c^2)\*x^3 - ((9\*a\*c^4 + 16\*b\*c^3)\*x^4 - 16\*b\*x - 9\*a)\*log(-c\*x + 1)^2 + 6\*(2\*(3\*a\*c^3 + 4\*b\*c^2)\*x^3 + 2\*a\*c\*x + (3\*a\*c^2 + 4\*b\*c)\*x^2 + 2\*((3\*a\*c^4 + 4\*b\*c^3)\*x^4 - 4\*b\*x - 3\*a)\*log(-c\*x + 1))\*dilog(c\*x) + 2\*((41\*a\*c^4 + 54\*b\*c^3)\*x^4 - (27\*a\*c^3 + 40\*b\*c^2)\*x^3 - 5\*a\*c\*x - (9\*a\*c^2 + 14\*b\*c)\*x^2)\*log(-c\*x + 1))/x^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*log(-c\*x+1)\*polylog(2,c\*x)/x^5,x, algorithm="fricas")

[Out] integral((b\*x + a)\*dilog(c\*x)\*log(-c\*x + 1)/x^5, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*ln(-c\*x+1)\*polylog(2,c\*x)/x\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```



### 3.192 $\int x(a + bx + cx^2) \log(1-dx) \text{PolyLog}(2, dx) dx$

**Optimal.** Leaf size=900

$$\frac{1}{16}c \log^2(1-dx)x^4 + \frac{3cx^4}{256} - \frac{3}{64}c \log(1-dx)x^4 - \frac{1}{16}c \text{PolyLog}(2, dx)x^4 + \frac{1}{9}b \log^2(1-dx)x^3 + \frac{2bx^3}{81} + \frac{(3c + 4bd)x^3}{324d}$$

[Out]  $(53*c*x)/(192*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + ((3*c + 4*b*d)*x)/(108*d^3) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^3) + (29*c*x^2)/(384*d^2) + (5*b*x^2)/(54*d) + ((3*c + 4*b*d)*x^2)/(216*d^2) + ((3*c + 4*b*d + 6*a*d^2)*x^2)/(96*d^2) + (2*b*x^3)/81 + (17*c*x^3)/(576*d) + ((3*c + 4*b*d)*x^3)/(324*d) + (3*c*x^4)/256 + (a*(1 - d*x)^2)/(8*d^2) + (29*c*\text{Log}[1 - d*x])/(192*d^4) + (5*b*\text{Log}[1 - d*x])/(27*d^3) + ((3*c + 4*b*d)*\text{Log}[1 - d*x])/(108*d^4) + ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x])/(48*d^4) - (c*x^2*\text{Log}[1 - d*x])/(16*d^2) - (b*x^2*\text{Log}[1 - d*x])/(9*d) - ((3*c + 4*b*d + 6*a*d^2)*x^2*\text{Log}[1 - d*x])/(48*d^2) - (2*b*x^3*\text{Log}[1 - d*x])/27 - (c*x^3*\text{Log}[1 - d*x])/(24*d) - ((3*c + 4*b*d)*x^3*\text{Log}[1 - d*x])/(108*d) - (3*c*x^4*\text{Log}[1 - d*x])/64 + (c*(1 - d*x)*\text{Log}[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*\text{Log}[1 - d*x])/(9*d^3) + (a*(1 - d*x)*\text{Log}[1 - d*x])/d^2 + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*\text{Log}[1 - d*x])/(12*d^4) - (a*(1 - d*x)^2*\text{Log}[1 - d*x])/(4*d^2) - (c*\text{Log}[1 - d*x]^2)/(16*d^4) - (b*\text{Log}[1 - d*x]^2)/(9*d^3) + (b*x^3*\text{Log}[1 - d*x]^2)/9 + (c*x^4*\text{Log}[1 - d*x]^2)/16 - (a*(1 - d*x)*\text{Log}[1 - d*x]^2)/(2*d^2) + (a*(1 - d*x)^2*\text{Log}[1 - d*x]^2)/(4*d^2) - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[d*x]*\text{Log}[1 - d*x]^2)/(12*d^4) - ((3*c + 4*b*d + 6*a*d^2)*x*\text{PolyLog}[2, d*x])/(12*d^3) - ((3*c + 4*b*d + 6*a*d^2)*x^2*\text{PolyLog}[2, d*x])/(24*d^2) - ((3*c + 4*b*d)*x^3*\text{PolyLog}[2, d*x])/(36*d) - (c*x^4*\text{PolyLog}[2, d*x])/16 - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x]*\text{PolyLog}[2, d*x])/(12*d^4) + ((6*a*x^2 + 4*b*x^3 + 3*c*x^4)*\text{Log}[1 - d*x]*\text{PolyLog}[2, d*x])/12 - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x]*\text{PolyLog}[2, 1 - d*x])/(6*d^4) + ((3*c + 4*b*d + 6*a*d^2)*\text{PolyLog}[3, 1 - d*x])/(6*d^4)$

**Rubi [A]** time = 1.18491, antiderivative size = 900, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 22, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$ , Rules used = {6742, 6591, 2395, 43, 14, 6604, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2410, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{1}{16}c \log^2(1-dx)x^4 + \frac{3cx^4}{256} - \frac{3}{64}c \log(1-dx)x^4 - \frac{1}{16}c \text{PolyLog}(2, dx)x^4 + \frac{1}{9}b \log^2(1-dx)x^3 + \frac{2bx^3}{81} + \frac{(3c + 4bd)x^3}{324d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x], x]

[Out]  $(53*c*x)/(192*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + ((3*c + 4*b*d)*x)/(108*d^3) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^3) + (29*c*x^2)/(384*d^2) + (5*b*x^2)/(54*d) + ((3*c + 4*b*d)*x^2)/(216*d^2) + ((3*c + 4*b*d + 6*a*d^2)*x^2)/(96*d^2) + (2*b*x^3)/81 + (17*c*x^3)/(576*d) + ((3*c + 4*b*d)*x^3)/(324*d) + (3*c*x^4)/256 + (a*(1 - d*x)^2)/(8*d^2) + (29*c*\text{Log}[1 - d*x])/(192*d^4) + (5*b*\text{Log}[1 - d*x])/(27*d^3) + ((3*c + 4*b*d)*\text{Log}[1 - d*x])/(108*d^4) + ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x])/(48*d^4) - (c*x^2*\text{Log}[1 - d*x])/(16*d^2) - (b*x^2*\text{Log}[1 - d*x])/(9*d) - ((3*c + 4*b*d + 6*a*d^2)*x^2*\text{Log}[1 - d*x])/(48*d^2) - (2*b*x^3*\text{Log}[1 - d*x])/27 - (c*x^3*\text{Log}[1 - d*x])/(24*d) - ((3*c + 4*b*d)*x^3*\text{Log}[1 - d*x])/(108*d) - (3*c*x^4*\text{Log}[1 - d*x])/64 + (c*(1 - d*x)*\text{Log}[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*\text{Log}[1 - d*x])/(9*d^3) + (a*(1 - d*x)*\text{Log}[1 - d*x])/d^2 + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*\text{Log}[1 - d*x])/(12*d^4) - (a*(1 - d*x)^2*\text{Log}[1 - d*x])/(4*d^2) - (c*\text{Log}[1 - d*x]^2)/(16*d^4) - (b*\text{Log}[1 - d*x]^2)/(9*d^3) + (b*x^3*\text{Log}[1 - d*x]^2)/9 + (c*x^4*\text{Log}[1 - d*x]^2)/16 - (a*(1 - d*x)*\text{Log}[1 - d*x]^2)/(2*d^2) + (a*(1 - d*x)^2*\text{Log}[1 -$

$$\begin{aligned} & d*x]^2)/(4*d^2) - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[d*x]*\text{Log}[1 - d*x]^2)/(12*d^4) \\ & - ((3*c + 4*b*d + 6*a*d^2)*x*\text{PolyLog}[2, d*x])/(12*d^3) - ((3*c + 4*b*d + 6*a*d^2)*x^2*\text{PolyLog}[2, d*x])/(24*d^2) \\ & - ((3*c + 4*b*d)*x^3*\text{PolyLog}[2, d*x])/(36*d) - (c*x^4*\text{PolyLog}[2, d*x])/16 - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x]*\text{PolyLog}[2, d*x])/(12*d^4) \\ & + ((6*a*x^2 + 4*b*x^3 + 3*c*x^4)*\text{Log}[1 - d*x]*\text{PolyLog}[2, d*x])/12 - ((3*c + 4*b*d + 6*a*d^2)*\text{Log}[1 - d*x]*\text{PolyLog}[2, 1 - d*x])/(6*d^4) \\ & + ((3*c + 4*b*d + 6*a*d^2)*\text{PolyLog}[3, 1 - d*x])/(6*d^4) \end{aligned}$$
Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))]*(h_.))*P(x)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] := With[{u = IntHide[P(x), x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[P(x), x]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[q, 0]
```

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(x\_)^(m\_.)/((f\_) + (g\_.)\*(x\_)), x\_Symbol] :> Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] :> Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /

; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^p)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^p)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p)/(x\_), x\_Symbol] :> -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps

$$\begin{aligned}
\int x(a + bx + cx^2) \log(1 - dx) \text{Li}_2(dx) dx &= \frac{1}{12} (6ax^2 + 4bx^3 + 3cx^4) \log(1 - dx) \text{Li}_2(dx) + d \int \left( \frac{-3c - 4bd - 6ad^2}{12d^4} \right) dx \\
&= \frac{1}{12} (6ax^2 + 4bx^3 + 3cx^4) \log(1 - dx) \text{Li}_2(dx) + \frac{1}{2} a \int x \log^2(1 - dx) dx + \frac{1}{2} b \int x^2 \log^2(1 - dx) dx + \frac{1}{2} c \int x^3 \log^2(1 - dx) dx \\
&= \frac{1}{9} bx^3 \log^2(1 - dx) + \frac{1}{16} cx^4 \log^2(1 - dx) - \frac{(3c + 4bd + 6ad^2) x \text{Li}_2(dx)}{12d^3} - \frac{(3c + 4bd + 6ad^2) x^2 \log(1 - dx)}{48d^2} \\
&\quad - \frac{(3c + 4bd) x^3 \log(1 - dx)}{108d} - \frac{1}{64} cx^4 \log(1 - dx) \\
&= \frac{(3c + 4bd + 6ad^2) x}{12d^3} - \frac{(3c + 4bd + 6ad^2) x^2 \log(1 - dx)}{48d^2} - \frac{(3c + 4bd) x^3 \log(1 - dx)}{108d} \\
&= \frac{cx}{64d^3} + \frac{(3c + 4bd)x}{108d^3} + \frac{5(3c + 4bd + 6ad^2)x}{48d^3} + \frac{cx^2}{128d^2} + \frac{(3c + 4bd)x^2}{216d^2} + \frac{cx^3}{192d} \\
&= \frac{9cx}{64d^3} + \frac{2bx}{9d^2} + \frac{ax}{d} + \frac{(3c + 4bd)x}{108d^3} + \frac{5(3c + 4bd + 6ad^2)x}{48d^3} + \frac{cx^2}{128d^2} + \frac{(3c + 4bd)x^2}{216d^2} + \frac{cx^3}{192d} \\
&= \frac{53cx}{192d^3} + \frac{11bx}{27d^2} + \frac{ax}{d} + \frac{(3c + 4bd)x}{108d^3} + \frac{5(3c + 4bd + 6ad^2)x}{48d^3} + \frac{29cx^2}{384d^2} + \frac{cx^3}{192d}
\end{aligned}$$

**Mathematica [A]** time = 1.27973, size = 583, normalized size = 0.65

$$\text{PolyLog}(2, dx) \left( 12 \log(1 - dx) (6ad^4x^2 - 6ad^2 + 4bd^4x^3 - 4bd + 3c(d^4x^4 - 1)) - dx (4d(9ad(dx + 2) + 2b(2d^2x^2 + 3c))) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x], x]

[Out] ((355\*c\*d\*x)/4 + 124\*b\*d^2\*x + 198\*a\*d^3\*x + (139\*c\*d^2\*x^2)/8 + 22\*b\*d^3\*x^2 + 27\*a\*d^4\*x^2 + (67\*c\*d^3\*x^3)/12 + (16\*b\*d^4\*x^3)/3 + (27\*c\*d^4\*x^4)/16 + (355\*c\*Log[1 - d\*x])/4 + 124\*b\*d\*Log[1 - d\*x] + 198\*a\*d^2\*Log[1 - d\*x] - 54\*c\*d\*x\*Log[1 - d\*x] - 80\*b\*d^2\*x\*Log[1 - d\*x] - 144\*a\*d^3\*x\*Log[1 - d\*x] - 18\*c\*d^2\*x^2\*Log[1 - d\*x] - 28\*b\*d^3\*x^2\*Log[1 - d\*x] - 54\*a\*d^4\*x^2\*Log[1 - d\*x] - 10\*c\*d^3\*x^3\*Log[1 - d\*x] - 16\*b\*d^4\*x^3\*Log[1 - d\*x] - (27\*c\*d^4\*x^4\*Log[1 - d\*x])/4 - 9\*c\*Log[1 - d\*x]^2 - 16\*b\*d\*Log[1 - d\*x]^2 - 36\*a\*d^2\*Log[1 - d\*x]^2 + 36\*a\*d^4\*x^2\*Log[1 - d\*x]^2 + 16\*b\*d^4\*x^3\*Log[1 - d\*x]^2 + 9\*c\*d^4\*x^4\*Log[1 - d\*x]^2 - 36\*c\*Log[d\*x]\*Log[1 - d\*x]^2 - 48\*b\*d\*Log[d\*x]\*Log[1 - d\*x]^2 - 72\*a\*d^2\*Log[d\*x]\*Log[1 - d\*x]^2 + ((-d\*x\*(3\*c\*(12 + 6\*d\*x + 4\*d^2\*x^2 + 3\*d^3\*x^3) + 4\*d\*(9\*a\*d\*(2 + d\*x) + 2\*b\*(6 + 3\*d\*x + 2\*d^2\*x^2)))) + 12\*(-4\*b\*d - 6\*a\*d^2 + 6\*a\*d^4\*x^2 + 4\*b\*d^4\*x^3 + 3\*c\*(-1 + d^4\*x^4))\*Log[1 - d\*x])\*PolyLog[2, d\*x] - 24\*(3\*c + 4\*b\*d + 6\*a\*d^2)\*Log[1 - d\*x]\*PolyLog[2, 1 - d\*x] + 24\*(3\*c + 4\*b\*d + 6\*a\*d^2)\*PolyLog[3, 1 - d\*x])/(144\*d^4)

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int x(cx^2 + bx + a) \ln(-dx + 1) \text{polylog}(2, dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)`

[Out] `int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)`

**Maxima [A]** time = 1.05354, size = 699, normalized size = 0.78

$$\frac{1}{6912} d \left( \frac{576 (6 a d^2 + 4 b d + 3 c) (\log(dx) \log(-dx + 1)^2 + 2 \operatorname{Li}_2(-dx + 1) \log(-dx + 1) - 2 \operatorname{Li}_3(-dx + 1))}{d^5} - \frac{81 c d^4 x^4 + \dots}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")`

[Out] `-1/6912*d*(576*(6*a*d^2 + 4*b*d + 3*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^5 - (81*c*d^4*x^4 + 4*(64*b*d^4 + 67*c*d^3)*x^3 + 6*(216*a*d^4 + 176*b*d^3 + 139*c*d^2)*x^2 + 12*(792*a*d^3 + 496*b*d^2 + 355*c*d)*x - 48*(9*c*d^4*x^4 + 4*(4*b*d^4 + 3*c*d^3)*x^3 + 6*(6*a*d^4 + 4*b*d^3 + 3*c*d^2)*x^2 + 12*(6*a*d^3 + 4*b*d^2 + 3*c*d)*x + 12*(6*a*d^2 + 4*b*d + 3*c)*log(-d*x + 1))*dilog(d*x) - 4*(54*c*d^4*x^4 + 4*(32*b*d^4 + 21*c*d^3)*x^3 - 2376*a*d^2 + 6*(72*a*d^4 + 40*b*d^3 + 27*c*d^2)*x^2 - 1488*b*d + 12*(108*a*d^3 + 64*b*d^2 + 45*c*d)*x - 1065*c)*log(-d*x + 1))/d^5 + 1/1728*(216*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(d^2*x^2 - 1)*log(-d*x + 1))*a/d^2 + 32*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 - 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*b/d^3 + 9*(48*d^4*x^4*dilog(d*x) - 3*d^4*x^4 - 4*d^3*x^3 - 6*d^2*x^2 - 12*d*x + 12*(d^4*x^4 - 1)*log(-d*x + 1))*c/d^4)*log(-d*x + 1)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((cx^3 + bx^2 + ax)\operatorname{Li}_2(dx) \log(-dx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")`

[Out] `integral((c*x^3 + b*x^2 + a*x)*dilog(d*x)*log(-d*x + 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)x\operatorname{Li}_2(dx) \log(-dx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*x*dilog(d*x)*log(-d*x + 1), x)
```

### 3.193 $\int (a + bx + cx^2) \log(1 - dx) \mathbf{PolyLog}(2, dx) dx$

**Optimal.** Leaf size=645

$$\frac{x \mathbf{PolyLog}(2, dx)(3d(2ad + b) + 2c)}{6d^2} + \frac{\mathbf{PolyLog}(3, 1 - dx)(3d(2ad + b) + 2c)}{3d^3} - \frac{\log(1 - dx) \mathbf{PolyLog}(2, dx)(3d(2ad + b) + 2c)}{6d^3}$$

```
[Out] 2*a*x + (4*c*x)/(9*d^2) + (b*x)/d + ((2*c + 3*b*d)*x)/(24*d^2) + ((2*c + 3*d*(b + 2*a*d))*x)/(6*d^2) + (c*x^2)/(9*d) + ((2*c + 3*b*d)*x^2)/(48*d) + (c*x^3)/27 + (b*(1 - d*x)^2)/(8*d^2) + (2*c*Log[1 - d*x])/(9*d^3) + ((2*c + 3*b*d)*Log[1 - d*x])/(24*d^3) - (c*x^2*Log[1 - d*x])/(9*d) - ((2*c + 3*b*d)*x^2*Log[1 - d*x])/(24*d) - (c*x^3*Log[1 - d*x])/9 + (2*c*(1 - d*x)*Log[1 - d*x])/(9*d^3) + (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x])/d + ((2*c + 3*d*(b + 2*a*d))*(1 - d*x)*Log[1 - d*x])/(6*d^3) - (b*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1 - d*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 - d*x]^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((2*c + 3*d*(b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/(6*d^3) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x])/(6*d^2) - ((2*c + 3*b*d)*x^2*PolyLog[2, d*x])/(12*d) - (c*x^3*PolyLog[2, d*x])/9 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^3) + ((6*a*x + 3*b*x^2 + 2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^3) + ((2*c + 3*d*(b + 2*a*d))*PolyLog[3, 1 - d*x])/(3*d^3)
```

**Rubi [A]** time = 0.818251, antiderivative size = 645, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 21, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$ , Rules used = {6742, 6586, 2389, 2295, 6591, 2395, 43, 6604, 2296, 2401, 2390, 2305, 2304, 2398, 2410, 2301, 6596, 2396, 2433, 2374, 6589}

$$\frac{x \mathbf{PolyLog}(2, dx)(3d(2ad + b) + 2c)}{6d^2} + \frac{\mathbf{PolyLog}(3, 1 - dx)(3d(2ad + b) + 2c)}{3d^3} - \frac{\log(1 - dx) \mathbf{PolyLog}(2, dx)(3d(2ad + b) + 2c)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x], x]
```

```
[Out] 2*a*x + (4*c*x)/(9*d^2) + (b*x)/d + ((2*c + 3*b*d)*x)/(24*d^2) + ((2*c + 3*d*(b + 2*a*d))*x)/(6*d^2) + (c*x^2)/(9*d) + ((2*c + 3*b*d)*x^2)/(48*d) + (c*x^3)/27 + (b*(1 - d*x)^2)/(8*d^2) + (2*c*Log[1 - d*x])/(9*d^3) + ((2*c + 3*b*d)*Log[1 - d*x])/(24*d^3) - (c*x^2*Log[1 - d*x])/(9*d) - ((2*c + 3*b*d)*x^2*Log[1 - d*x])/(24*d) - (c*x^3*Log[1 - d*x])/9 + (2*c*(1 - d*x)*Log[1 - d*x])/(9*d^3) + (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x])/d + ((2*c + 3*d*(b + 2*a*d))*(1 - d*x)*Log[1 - d*x])/(6*d^3) - (b*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1 - d*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 - d*x]^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((2*c + 3*d*(b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/(6*d^3) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x])/(6*d^2) - ((2*c + 3*b*d)*x^2*PolyLog[2, d*x])/(12*d) - (c*x^3*PolyLog[2, d*x])/9 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^3) + ((6*a*x + 3*b*x^2 + 2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^3) + ((2*c + 3*d*(b + 2*a*d))*PolyLog[3, 1 - d*x])/(3*d^3)
```

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```



Rule 6586

Int[PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[x\*PolyLog[n, a\*(b\*x^p)^q], x] - Dist[p\*q, Int[PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 6591

Int[((d\_.)\*(x\_)^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6604

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(h\_.))\*Px)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := With[{u = IntHide[Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x]

Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f -

$d * g, 0 \ \&\& \text{IGtQ}[q, 0]$

### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.) * (x_.))^{(n_.)}] * (b_.)]^{(p_.)} * ((f_.) + (g_.) * (x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f * x)/d]^q * (a + b * \text{Log}[c * x^n])^p, x], x, d + e * x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \ \&\& \ \text{EqQ}[e * f - d * g, 0]$

### Rule 2305

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)]^{(p_.)} * ((d_.) * (x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d * x)^{(m + 1)} * (a + b * \text{Log}[c * x^n])^p / (d * (m + 1)), x] - \text{Dist}[(b * n * p) / (m + 1), \text{Int}[(d * x)^m * (a + b * \text{Log}[c * x^n])^{(p - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{GtQ}[p, 0]$

### Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)] * ((d_.) * (x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d * x)^{(m + 1)} * (a + b * \text{Log}[c * x^n]) / (d * (m + 1)), x] - \text{Simp}[(b * n * (d * x)^{(m + 1)}) / (d * (m + 1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[m, -1]$

### Rule 2398

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.) * (x_.))^{(n_.)}] * (b_.)]^{(p_.)} * ((f_.) + (g_.) * (x_.))^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(f + g * x)^{(q + 1)} * (a + b * \text{Log}[c * (d + e * x)^n])^p / (g * (q + 1)), x] - \text{Dist}[(b * e * n * p) / (g * (q + 1)), \text{Int}[(f + g * x)^{(q + 1)} * (a + b * \text{Log}[c * (d + e * x)^n])^{(p - 1)} / (d + e * x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\} \ \&\& \ \text{NeQ}[e * f - d * g, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[q, -1] \ \&\& \ \text{IntegersQ}[2 * p, 2 * q] \ \&\& \ (\ !\text{IGtQ}[q, 0] \ || \ (\text{EqQ}[p, 2] \ \&\& \ \text{NeQ}[q, 1]))$

### Rule 2410

$\text{Int}[(\text{Log}[(c_.) * ((d_.) + (e_.) * (x_.))] * (x_.))^{(m_.)} / ((f_.) + (g_.) * (x_.)), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Log}[c * (d + e * x)], x^m / (f + g * x), x], x] /; \text{FreeQ}\{c, d, e, f, g\}, x\} \ \&\& \ \text{EqQ}[e * f - d * g, 0] \ \&\& \ \text{EqQ}[c * d, 1] \ \&\& \ \text{IntegerQ}[m]$

### Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)] / (x_.), x\_Symbol] \rightarrow \text{Simp}[(a + b * \text{Log}[c * x^n])^2 / (2 * b * n), x] /; \text{FreeQ}\{a, b, c, n\}, x\}$

### Rule 6596

$\text{Int}[\text{PolyLog}[2, (c_.) * ((a_.) + (b_.) * (x_.))] / ((d_.) + (e_.) * (x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 - a * c - b * c * x] * \text{PolyLog}[2, c * (a + b * x)]) / e, x] + \text{Dist}[b/e, \text{Int}[\text{Log}[1 - a * c - b * c * x]^2 / (a + b * x), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c * (b * d - a * e) + e, 0]$

### Rule 2396

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.) * (x_.))^{(n_.)}] * (b_.)]^{(p_.)} / ((f_.) + (g_.) * (x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e * (f + g * x)) / (e * f - d * g)]) * (a + b * \text{Log}[c * (d + e * x)^n])^p / g, x] - \text{Dist}[(b * e * n * p) / g, \text{Int}[(\text{Log}[(e * (f + g * x)) / (e * f - d * g)]) * (a + b * \text{Log}[c * (d + e * x)^n])^{(p - 1)} / (d + e * x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x\} \ \&\& \ \text{NeQ}[e * f - d * g, 0] \ \&\& \ \text{IGtQ}[p, 1]$

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_.))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*1, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_.))^(m_.))]*((a_.) + Log[(c_.)*(x_.))^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rubi steps

$$\begin{aligned}
\int (a + bx + cx^2) \log(1 - dx) \text{Li}_2(dx) dx &= \frac{1}{6} (6ax + 3bx^2 + 2cx^3) \log(1 - dx) \text{Li}_2(dx) + d \int \left( \frac{(-2c - 3d(b + 2ad)) \text{Li}_2(dx)}{6d^3} \right. \\
&= \frac{1}{6} (6ax + 3bx^2 + 2cx^3) \log(1 - dx) \text{Li}_2(dx) + a \int \log^2(1 - dx) dx + \frac{1}{2} b \int x \log(1 - dx) dx \\
&= \frac{1}{9} cx^3 \log^2(1 - dx) - \frac{(2c + 3d(b + 2ad))x \text{Li}_2(dx)}{6d^2} - \frac{(2c + 3bd)x^2 \text{Li}_2(dx)}{12d} - \frac{1}{9} cx^3 \log(1 - dx) \\
&= -\frac{(2c + 3bd)x^2 \log(1 - dx)}{24d} - \frac{1}{27} cx^3 \log(1 - dx) + \frac{1}{9} cx^3 \log^2(1 - dx) - \frac{a(1 - dx)}{9d} \\
&= 2ax + \frac{(2c + 3d(b + 2ad))x}{6d^2} - \frac{(2c + 3bd)x^2 \log(1 - dx)}{24d} - \frac{1}{27} cx^3 \log(1 - dx) \\
&= 2ax + \frac{cx}{27d^2} + \frac{(2c + 3bd)x}{24d^2} + \frac{(2c + 3d(b + 2ad))x}{6d^2} + \frac{cx^2}{54d} + \frac{(2c + 3bd)x^2}{48d} + \\
&= 2ax + \frac{7cx}{27d^2} + \frac{bx}{d} + \frac{(2c + 3bd)x}{24d^2} + \frac{(2c + 3d(b + 2ad))x}{6d^2} + \frac{cx^2}{54d} + \frac{(2c + 3bd)x^2}{48d} \\
&= 2ax + \frac{4cx}{9d^2} + \frac{bx}{d} + \frac{(2c + 3bd)x}{24d^2} + \frac{(2c + 3d(b + 2ad))x}{6d^2} + \frac{cx^2}{9d} + \frac{(2c + 3bd)x^2}{48d}
\end{aligned}$$

**Mathematica [A]** time = 1.03267, size = 472, normalized size = 0.73

---

PolyLog(2, dx) (6(dx - 1) log(1 - dx) (3d(2ad + bdx + b) + 2c(d^2x^2 + dx + 1)) - dx(9d(4ad + bdx + 2b) + 2c(2d^2x^2

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x], x]
```

```
[Out] (31*c*d*x + (99*b*d^2*x)/2 + 108*a*d^3*x + (11*c*d^2*x^2)/2 + (27*b*d^3*x^2
)/4 + (4*c*d^3*x^3)/3 + 31*c*Log[1 - d*x] + (99*b*d*Log[1 - d*x])/2 + 108*a
*d^2*Log[1 - d*x] - 20*c*d*x*Log[1 - d*x] - 36*b*d^2*x*Log[1 - d*x] - 108*a
```

$$\begin{aligned} & d^3 x \operatorname{Log}[1 - dx] - 7c d^2 x^2 \operatorname{Log}[1 - dx] - (27b d^3 x^2 \operatorname{Log}[1 - dx] \\ & )/2 - 4c d^3 x^3 \operatorname{Log}[1 - dx] - 4c \operatorname{Log}[1 - dx]^2 - 9b d \operatorname{Log}[1 - dx]^2 \\ & - 36a d^2 \operatorname{Log}[1 - dx]^2 + 36a d^3 x \operatorname{Log}[1 - dx]^2 + 9b d^3 x^2 \operatorname{Log}[1 - \\ & dx]^2 + 4c d^3 x^3 \operatorname{Log}[1 - dx]^2 - 12c \operatorname{Log}[dx] \operatorname{Log}[1 - dx]^2 - 18b d \\ & \operatorname{Log}[dx] \operatorname{Log}[1 - dx]^2 - 36a d^2 \operatorname{Log}[dx] \operatorname{Log}[1 - dx]^2 + (-(dx(9d \\ & (2b + 4ad + bdx) + 2c(6 + 3dx + 2d^2 x^2))) + 6(-1 + dx)(3d( \\ & b + 2ad + bdx) + 2c(1 + dx + d^2 x^2)) \operatorname{Log}[1 - dx]) \operatorname{PolyLog}[2, dx] \\ & - 12(2c + 3d(b + 2ad)) \operatorname{Log}[1 - dx] \operatorname{PolyLog}[2, 1 - dx] + 12(2c + \\ & 3d(b + 2ad)) \operatorname{PolyLog}[3, 1 - dx]) / (36d^3) \end{aligned}$$

**Maple [F]** time = 0.05, size = 0, normalized size = 0.

$$\int (cx^2 + bx + a) \ln(-dx + 1) \operatorname{polylog}(2, dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)`

[Out] `int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)`

**Maxima [A]** time = 1.05019, size = 556, normalized size = 0.86

$$-\frac{1}{432} d \left( \frac{72(6ad^2 + 3bd + 2c)(\log(dx) \log(-dx + 1)^2 + 2 \operatorname{Li}_2(-dx + 1) \log(-dx + 1) - 2 \operatorname{Li}_3(-dx + 1))}{d^4} - \frac{16cd^3 x^3 + 3(27b d^3 + 22c d^2) x^2 + 6(216a d^3 + 99b d^2 + 62c d) x - 12(4c d^3 x^3 + 3(3b d^3 + 2c d^2) x^2 + 6(6a d^3 + 3b d^2 + 2c d) x + 6(6a d^2 + 3b d + 2c) \log(-dx + 1)) \operatorname{dilog}(dx) - 2(16c d^3 x^3 - 648a d^2 + 6(9b d^3 + 5c d^2) x^2 - 297b d + 6(72a d^3 + 27b d^2 + 16c d) x - 186c) \log(-dx + 1)}{d^4} + \frac{1}{216} (216(d x \operatorname{dilog}(dx) - dx + (dx - 1) \log(-dx + 1)) a/d + 27(4d^2 x^2 \operatorname{dilog}(dx) - d^2 x^2 - 2dx + 2(d^2 x^2 - 1) \log(-dx + 1)) b/d^2 + 4(18d^3 x^3 \operatorname{dilog}(dx) - 2d^3 x^3 - 3d^2 x^2 - 6dx + 6(d^3 x^3 - 1) \log(-dx + 1)) c/d^3) \log(-dx + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")`

[Out] `-1/432*d*(72*(6*a*d^2 + 3*b*d + 2*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^4 - (16*c*d^3*x^3 + 3*(27*b*d^3 + 22*c*d^2)*x^2 + 6*(216*a*d^3 + 99*b*d^2 + 62*c*d)*x - 12*(4*c*d^3*x^3 + 3*(3*b*d^3 + 2*c*d^2)*x^2 + 6*(6*a*d^3 + 3*b*d^2 + 2*c*d)*x + 6*(6*a*d^2 + 3*b*d + 2*c)*log(-d*x + 1))*dilog(d*x) - 2*(16*c*d^3*x^3 - 648*a*d^2 + 6*(9*b*d^3 + 5*c*d^2)*x^2 - 297*b*d + 6*(72*a*d^3 + 27*b*d^2 + 16*c*d)*x - 186*c)*log(-d*x + 1))/d^4 + 1/216*(216*(d*x*dilog(d*x) - d*x + (d*x - 1)*log(-d*x + 1))*a/d + 27*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(d^2*x^2 - 1)*log(-d*x + 1))*b/d^2 + 4*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 - 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*c/d^3)*log(-d*x + 1)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x),x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1), x)

$$3.194 \quad \int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2,dx)}{x} dx$$

**Optimal.** Leaf size=402

$$-\frac{1}{2}a \text{PolyLog}(2,dx)^2 + \frac{(2bd+c) \text{PolyLog}(3,1-dx)}{d^2} - \frac{(2bd+c) \log(1-dx) \text{PolyLog}(2,dx)}{2d^2} - \frac{(2bd+c) \log(1-dx) \text{PolyLog}(2,dx)}{d^2}$$

```
[Out] 2*b*x + (9*c*x)/(8*d) + ((c + 2*b*d)*x)/(2*d) + (c*x^2)/16 + (c*(1 - d*x)^2)/(8*d^2) + (c*Log[1 - d*x])/(8*d^2) - (c*x^2*Log[1 - d*x])/8 + (c*(1 - d*x)*Log[1 - d*x])/d^2 + (2*b*(1 - d*x)*Log[1 - d*x])/d + ((c + 2*b*d)*(1 - d*x)*Log[1 - d*x])/(2*d^2) - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (c*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((c + 2*b*d)*Log[d*x]*Log[1 - d*x]^2)/(2*d^2) - ((c + 2*b*d)*x*PolyLog[2, d*x])/(2*d) - (c*x^2*PolyLog[2, d*x])/4 - ((c + 2*b*d)*Log[1 - d*x]*PolyLog[2, d*x])/(2*d^2) + ((2*b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/2 - (a*PolyLog[2, d*x]^2)/2 - ((c + 2*b*d)*Log[1 - d*x]*PolyLog[2, 1 - d*x])/d^2 + ((c + 2*b*d)*PolyLog[3, 1 - d*x])/d^2
```

**Rubi [A]** time = 0.618594, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 24, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$ , Rules used = {6742, 6586, 2389, 2295, 6589, 6591, 2395, 43, 6605, 6601, 1584, 6598, 2416, 2391, 6604, 2296, 2401, 2390, 2305, 2304, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}a \text{PolyLog}(2,dx)^2 + \frac{(2bd+c) \text{PolyLog}(3,1-dx)}{d^2} - \frac{(2bd+c) \log(1-dx) \text{PolyLog}(2,dx)}{2d^2} - \frac{(2bd+c) \log(1-dx) \text{PolyLog}(2,dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

```
[Out] 2*b*x + (9*c*x)/(8*d) + ((c + 2*b*d)*x)/(2*d) + (c*x^2)/16 + (c*(1 - d*x)^2)/(8*d^2) + (c*Log[1 - d*x])/(8*d^2) - (c*x^2*Log[1 - d*x])/8 + (c*(1 - d*x)*Log[1 - d*x])/d^2 + (2*b*(1 - d*x)*Log[1 - d*x])/d + ((c + 2*b*d)*(1 - d*x)*Log[1 - d*x])/(2*d^2) - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (c*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((c + 2*b*d)*Log[d*x]*Log[1 - d*x]^2)/(2*d^2) - ((c + 2*b*d)*x*PolyLog[2, d*x])/(2*d) - (c*x^2*PolyLog[2, d*x])/4 - ((c + 2*b*d)*Log[1 - d*x]*PolyLog[2, d*x])/(2*d^2) + ((2*b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/2 - (a*PolyLog[2, d*x]^2)/2 - ((c + 2*b*d)*Log[1 - d*x]*PolyLog[2, 1 - d*x])/d^2 + ((c + 2*b*d)*PolyLog[3, 1 - d*x])/d^2
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

#### Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^q_.], x_Symbol] := Simp[x*PolyLog[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

, b, c, d, e, n, p}, x]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6591

Int[((d\_.)\*(x\_)^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(p\*q)/(m + 1), Int[(d\*x)^m\*PolyLog[n - 1, a\*(b\*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 6605

Int[((g\_.) + Log[1 + (e\_.)\*(x\_)])\*(h\_.)\*(Px\_)\*(x\_)^(m\_)\*PolyLog[2, (c\_.)\*(x\_)], x\_Symbol] := Dist[Coeff[Px, x, -m - 1], Int[((g + h\*Log[1 + e\*x])\*PolyLog[2, c\*x])/x, x], x] + Int[x^m\*(Px - Coeff[Px, x, -m - 1]\*x^(-m - 1))\*(g + h\*Log[1 + e\*x])\*PolyLog[2, c\*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]

#### Rule 6601

Int[(Log[1 + (e\_.)\*(x\_)])\*PolyLog[2, (c\_.)\*(x\_)])/(x\_), x\_Symbol] := -Simp[PolyLog[2, c\*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]

#### Rule 1584

Int[(u\_.)\*(x\_)^(m\_.)\*((a\_.)\*(x\_)^(p\_.) + (b\_.)\*(x\_)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 6598

Int[((d\_.) + (e\_.)\*(x\_)^(m\_.)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*PolyLog[2, c\*(a + b\*x)]/(e\*(m + 1)), x] + Dist[b/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*Log[1 - a\*c - b\*c\*x])/(a + b\*x)

, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.))\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6604

Int[((g\_.) + Log[(f\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]\*(h\_.))\*(Px\_)\*PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))], x\_Symbol] := With[{u = IntHide[Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 6596



```
Int[PolyLog[2, (c_.)*(a_.) + (b_.)*(x_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_.)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_.))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2) \log(1 - dx) \operatorname{Li}_2(dx)}{x} dx &= a \int \frac{\log(1 - dx) \operatorname{Li}_2(dx)}{x} dx + \int \frac{(bx + cx^2) \log(1 - dx) \operatorname{Li}_2(dx)}{x} dx \\
&= -\frac{1}{2} a \operatorname{Li}_2(dx)^2 + \int (b + cx) \log(1 - dx) \operatorname{Li}_2(dx) dx \\
&= \frac{1}{2} (2bx + cx^2) \log(1 - dx) \operatorname{Li}_2(dx) - \frac{1}{2} a \operatorname{Li}_2(dx)^2 + d \int \left( \frac{(-c - 2bd) \operatorname{Li}_2(dx)}{2d^2} \right. \\
&= \frac{1}{2} (2bx + cx^2) \log(1 - dx) \operatorname{Li}_2(dx) - \frac{1}{2} a \operatorname{Li}_2(dx)^2 + b \int \log^2(1 - dx) dx + \frac{1}{2} \\
&= -\frac{(c + 2bd)x \operatorname{Li}_2(dx)}{2d} - \frac{1}{4} cx^2 \operatorname{Li}_2(dx) - \frac{(c + 2bd) \log(1 - dx) \operatorname{Li}_2(dx)}{2d^2} + \frac{1}{2} (2b \\
&= -\frac{1}{8} cx^2 \log(1 - dx) - \frac{b(1 - dx) \log^2(1 - dx)}{d} - \frac{(c + 2bd) \log(dx) \log^2(1 - dx)}{2d^2} \\
&= 2bx + \frac{(c + 2bd)x}{2d} - \frac{1}{8} cx^2 \log(1 - dx) + \frac{2b(1 - dx) \log(1 - dx)}{d} + \frac{(c + 2bd)}{2d} \\
&= 2bx + \frac{cx}{8d} + \frac{(c + 2bd)x}{2d} + \frac{cx^2}{16} + \frac{c \log(1 - dx)}{8d^2} - \frac{1}{8} cx^2 \log(1 - dx) + \frac{2b(1 - dx)}{d} \\
&= 2bx + \frac{9cx}{8d} + \frac{(c + 2bd)x}{2d} + \frac{cx^2}{16} + \frac{c(1 - dx)^2}{8d^2} + \frac{c \log(1 - dx)}{8d^2} - \frac{1}{8} cx^2 \log(1 - dx)
\end{aligned}$$

**Mathematica [A]** time = 0.355676, size = 298, normalized size = 0.74

$$\frac{-8ad^2\text{PolyLog}(2, dx)^2 + 4\text{PolyLog}(2, dx)(2(dx-1)\log(1-dx)(2bd+cdx+c) - dx(4bd+cdx+2c)) - 16(2bd+c)\log(1-dx)}{16d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x, x]

[Out] (-14\*c - 32\*b\*d + 22\*c\*d\*x + 48\*b\*d^2\*x + 3\*c\*d^2\*x^2 + 22\*c\*Log[1 - d\*x] + 48\*b\*d\*Log[1 - d\*x] - 16\*c\*d\*x\*Log[1 - d\*x] - 48\*b\*d^2\*x\*Log[1 - d\*x] - 6\*c\*d^2\*x^2\*Log[1 - d\*x] - 4\*c\*Log[1 - d\*x]^2 - 16\*b\*d\*Log[1 - d\*x]^2 + 16\*b\*d^2\*x\*Log[1 - d\*x]^2 + 4\*c\*d^2\*x^2\*Log[1 - d\*x]^2 - 8\*c\*Log[d\*x]\*Log[1 - d\*x]^2 - 16\*b\*d\*Log[d\*x]\*Log[1 - d\*x]^2 + 4\*(-(d\*x\*(2\*c + 4\*b\*d + c\*d\*x)) + 2\*(-1 + d\*x)\*(c + 2\*b\*d + c\*d\*x))\*Log[1 - d\*x])\*PolyLog[2, d\*x] - 8\*a\*d^2\*PolyLog[2, d\*x]^2 - 16\*(c + 2\*b\*d)\*Log[1 - d\*x]\*PolyLog[2, 1 - d\*x] + 16\*c\*PolyLog[3, 1 - d\*x] + 32\*b\*d\*PolyLog[3, 1 - d\*x])/(16\*d^2)

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \ln(-dx + 1) \text{polylog}(2, dx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x,x)

[Out] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \text{Li}_2(dx) \log(-dx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x,x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a) \text{Li}_2(dx) \log(-dx + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x,x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x,x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x, x)

$$3.195 \quad \int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2,dx)}{x^2} dx$$

**Optimal.** Leaf size=218

$$-2d \left( a - \frac{c}{d^2} \right) \text{PolyLog}(3,1-dx) + d \left( a - \frac{c}{d^2} \right) \log(1-dx) \text{PolyLog}(2,dx) + 2d \left( a - \frac{c}{d^2} \right) \log(1-dx) \text{PolyLog}(2,1-dx) -$$

```
[Out] 3*c*x + (3*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d + (
a*(1 - d*x)*Log[1 - d*x]^2)/x + (a - c/d^2)*d*Log[d*x]*Log[1 - d*x]^2 - 2*a
*d*PolyLog[2, d*x] - c*x*PolyLog[2, d*x] + (a - c/d^2)*d*Log[1 - d*x]*PolyL
og[2, d*x] - (a/x - c*x)*Log[1 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^
2)/2 + 2*(a - c/d^2)*d*Log[1 - d*x]*PolyLog[2, 1 - d*x] - a*d*PolyLog[3, d*
x] - 2*(a - c/d^2)*d*PolyLog[3, 1 - d*x]
```

**Rubi [A]** time = 0.507956, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 21, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.808$ , Rules used = {6742, 6586, 2389, 2295, 6591, 2395, 36, 29, 31, 6589, 6605, 6601, 14, 6606, 2296, 2397, 2391, 6596, 2396, 2433, 2374}

$$-2d \left( a - \frac{c}{d^2} \right) \text{PolyLog}(3,1-dx) + d \left( a - \frac{c}{d^2} \right) \log(1-dx) \text{PolyLog}(2,dx) + 2d \left( a - \frac{c}{d^2} \right) \log(1-dx) \text{PolyLog}(2,1-dx) -$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

```
[Out] 3*c*x + (3*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d + (
a*(1 - d*x)*Log[1 - d*x]^2)/x + (a - c/d^2)*d*Log[d*x]*Log[1 - d*x]^2 - 2*a
*d*PolyLog[2, d*x] - c*x*PolyLog[2, d*x] + (a - c/d^2)*d*Log[1 - d*x]*PolyL
og[2, d*x] - (a/x - c*x)*Log[1 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^
2)/2 + 2*(a - c/d^2)*d*Log[1 - d*x]*PolyLog[2, 1 - d*x] - a*d*PolyLog[3, d*
x] - 2*(a - c/d^2)*d*PolyLog[3, 1 - d*x]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] := Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

#### Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

#### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

#### Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol]
:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q]/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))* (b_.)]*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol]
:> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol]
:> Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol]
:> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)])*(h_.)*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x_)], x_Symbol]
:> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*PolyLog[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g + h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

#### Rule 6601

```
Int[(Log[1 + (e_.)*(x_)])*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol]
:> -Simp[PolyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol]
:> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

#### Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_)^(n_.))* (h_.)]*(Px_)*(x_)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol]
:> With[{u = IntHide[x^m*Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
```

$u/(a + b*x), x], x], x] - \text{Dist}[e*h*n, \text{Int}[\text{ExpandIntegrand}[\text{PolyLog}[2, c*(a + b*x)], u/(d + e*x), x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, n\}, x] \&\& \text{PolyQ}[Px, x] \&\& \text{IntegerQ}[m]$

#### Rule 2296

$\text{Int}[(a + \text{Log}[c*(x)^n]*b)^p, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{p-1}, x], x] /; \text{FreeQ}[\{a, b, c, n\}, x] \&\& \text{GtQ}[p, 0] \&\& \text{IntegerQ}[2*p]$

#### Rule 2397

$\text{Int}[(a + \text{Log}[c*((d) + (e)*(x))^n]*b)^p/((f) + (g)*(x))^2, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)*(a + b*\text{Log}[c*(d + e*x)^n])^p/((e*f - d*g)*(f + g*x)), x] - \text{Dist}[(b*e*n*p)/(e*f - d*g), \text{Int}[(a + b*\text{Log}[c*(d + e*x)^n])^{p-1}/(f + g*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{GtQ}[p, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[c*((d) + (e)*(x))^n]/(x), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

#### Rule 6596

$\text{Int}[\text{PolyLog}[2, c*(a + b*x)]/((d) + (e)*(x)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 - a*c - b*c*x]*\text{PolyLog}[2, c*(a + b*x)])/e, x] + \text{Dist}[b/e, \text{Int}[\text{Log}[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[c*(b*d - a*e) + e, 0]$

#### Rule 2396

$\text{Int}[(a + \text{Log}[c*((d) + (e)*(x))^n]*b)^p/((f) + (g)*(x)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^p)/g, x] - \text{Dist}[(b*e*n*p)/g, \text{Int}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^{p-1})/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n, p\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{IGtQ}[p, 1]$

#### Rule 2433

$\text{Int}[(a + \text{Log}[c*((d) + (e)*(x))^n]*b)^p*((f) + \text{Log}[(h)*(i) + (j)*(x))^m]*g*((k) + (l)*(x))^r, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(k*x)/d]^r*(a + b*\text{Log}[c*x^n])^p*(f + g*\text{Log}[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r\}, x] \&\& \text{EqQ}[e*k - d*l, 0]$

#### Rule 2374

$\text{Int}[(\text{Log}[d*(e) + (f)*(x))^m]*(a + \text{Log}[c*(x)^n]*b)^p/(x), x\_Symbol] \rightarrow -\text{Simp}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^{p-1})/x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[d*e, 1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2) \log(1 - dx) \text{Li}_2(dx)}{x^2} dx &= b \int \frac{\log(1 - dx) \text{Li}_2(dx)}{x} dx + \int \frac{(a + cx^2) \log(1 - dx) \text{Li}_2(dx)}{x^2} dx \\
&= -\left(\frac{a}{x} - cx\right) \log(1 - dx) \text{Li}_2(dx) - \frac{1}{2} b \text{Li}_2(dx)^2 + d \int \left( -\frac{c \text{Li}_2(dx)}{d} - \frac{a \text{Li}_2(dx)}{x} \right) dx \\
&= -\left(\frac{a}{x} - cx\right) \log(1 - dx) \text{Li}_2(dx) - \frac{1}{2} b \text{Li}_2(dx)^2 - a \int \frac{\log^2(1 - dx)}{x^2} dx + c \int \frac{\log(1 - dx)}{x} dx \\
&= \frac{a(1 - dx) \log^2(1 - dx)}{x} - cx \text{Li}_2(dx) - \frac{(c - ad^2) \log(1 - dx) \text{Li}_2(dx)}{d} - \left(\frac{a}{x} - cx\right) \log(1 - dx) \\
&= -\frac{c(1 - dx) \log^2(1 - dx)}{d} + \frac{a(1 - dx) \log^2(1 - dx)}{x} - \frac{(c - ad^2) \log(dx) \log^2(1 - dx)}{d} \\
&= 3cx + \frac{3c(1 - dx) \log(1 - dx)}{d} - \frac{c(1 - dx) \log^2(1 - dx)}{d} + \frac{a(1 - dx) \log^2(1 - dx)}{x} \\
&= 3cx + \frac{3c(1 - dx) \log(1 - dx)}{d} - \frac{c(1 - dx) \log^2(1 - dx)}{d} + \frac{a(1 - dx) \log^2(1 - dx)}{x} \\
&= 3cx + \frac{3c(1 - dx) \log(1 - dx)}{d} - \frac{c(1 - dx) \log^2(1 - dx)}{d} + \frac{a(1 - dx) \log^2(1 - dx)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.83377, size = 280, normalized size = 1.28

$$2(2x \text{PolyLog}(2, 1 - dx) ((ad^2 - c) \log(1 - dx) + ad^2) - ad^2 x \text{PolyLog}(3, dx) - 2ad^2 x \text{PolyLog}(3, 1 - dx) + 2cx \text{PolyLog}(3, 1 - dx)) / (2d^2 x)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x^2, x]

[Out] (2\*(-(c\*d\*x^2) + (a\*d + c\*x)\*(-1 + d\*x)\*Log[1 - d\*x])\*PolyLog[2, d\*x] - b\*d\*x\*PolyLog[2, d\*x]^2 + 2\*(-2\*c\*x + 3\*c\*d\*x^2 + 3\*c\*x\*Log[1 - d\*x] - 3\*c\*d\*x^2\*Log[1 - d\*x] + 2\*a\*d^2\*x\*Log[d\*x]\*Log[1 - d\*x] + a\*d\*Log[1 - d\*x]^2 - c\*x\*Log[1 - d\*x]^2 - a\*d^2\*x\*Log[1 - d\*x]^2 + c\*d\*x^2\*Log[1 - d\*x]^2 - c\*x\*Log[d\*x]\*Log[1 - d\*x]^2 + a\*d^2\*x\*Log[d\*x]\*Log[1 - d\*x]^2 + 2\*x\*(a\*d^2 + (-c + a\*d^2)\*Log[1 - d\*x])\*PolyLog[2, 1 - d\*x] - a\*d^2\*x\*PolyLog[3, d\*x] + 2\*c\*x\*PolyLog[3, 1 - d\*x] - 2\*a\*d^2\*x\*PolyLog[3, 1 - d\*x]))/(2\*d\*x)

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \ln(-dx + 1) \text{polylog}(2, dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^2,x)

[Out] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \text{Li}_2(dx) \log(-dx + 1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a)\text{Li}_2(dx)\log(-dx + 1)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)\text{Li}_2(dx)\log(-dx + 1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```



$$3.196 \quad \int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2,dx)}{x^3} dx$$

**Optimal.** Leaf size=343

$$\frac{b^2 \text{PolyLog}(3,1-dx)}{a} - \frac{b^2 \log(1-dx) \text{PolyLog}(2,1-dx)}{a} - \frac{(a+bx)^2 \log(1-dx) \text{PolyLog}(2,dx)}{2ax^2} - \frac{1}{2} d(ad+2b) \text{Poly}$$

```
[Out] -(a*d^2*Log[x]) + a*d^2*Log[1 - d*x] - (a*d*Log[1 - d*x])/x - (a*d^2*Log[1 - d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1 - d*x]^2)/x - (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) - 2*b*d*PolyLog[2, d*x] - (a*d^2*PolyLog[2, d*x])/2 + (a*d*PolyLog[2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a) - ((a + b*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a - (d*(2*b + a*d)*PolyLog[3, d*x])/2 + (b^2*PolyLog[3, 1 - d*x])/a - ((b + a*d)^2*PolyLog[3, 1 - d*x])/a
```

**Rubi [A]** time = 0.740443, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 22, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$ , Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 6589, 6605, 6601, 37, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 2396, 2433, 2374, 6596}

$$\frac{b^2 \text{PolyLog}(3,1-dx)}{a} - \frac{b^2 \log(1-dx) \text{PolyLog}(2,1-dx)}{a} - \frac{(a+bx)^2 \log(1-dx) \text{PolyLog}(2,dx)}{2ax^2} - \frac{1}{2} d(ad+2b) \text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```

```
[Out] -(a*d^2*Log[x]) + a*d^2*Log[1 - d*x] - (a*d*Log[1 - d*x])/x - (a*d^2*Log[1 - d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1 - d*x]^2)/x - (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) - 2*b*d*PolyLog[2, d*x] - (a*d^2*PolyLog[2, d*x])/2 + (a*d*PolyLog[2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a) - ((a + b*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a - (d*(2*b + a*d)*PolyLog[3, d*x])/2 + (b^2*PolyLog[3, 1 - d*x])/a - ((b + a*d)^2*PolyLog[3, 1 - d*x])/a
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 6591

```
Int[((d_.)*(x_.))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_.)^(p_.))^(q_.)], x_Symbol] := Simp[((d*x)^(m+1)*PolyLog[n, a*(b*x^p)^q])/(d*(m+1)), x] - Dist[(p*q)/(m+1), Int[(d*x)^m*PolyLog[n-1, a*(b*x^p)^q], x], x] /; FreeQ[{a, b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

#### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Simp[((f + g*x)^(q+1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q+1)), x] - Dist[(b*e*n)/(g*(q+1)), Int[(f + g*x)^(q+1)/(d + e*x)
```

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6605

Int[((g\_) + Log[1 + (e\_)\*(x\_)]\*(h\_))\*(Px\_)\*(x\_)^m)\*PolyLog[2, (c\_)\*(x\_)], x\_Symbol] := Dist[Coeff[Px, x, -m - 1], Int[((g + h\*Log[1 + e\*x])\*PolyLog[2, c\*x])/x, x], x] + Int[x^m\*(Px - Coeff[Px, x, -m - 1]\*x^(-m - 1))\*(g + h\*Log[1 + e\*x])\*PolyLog[2, c\*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]

#### Rule 6601

Int[(Log[1 + (e\_)\*(x\_)]\*PolyLog[2, (c\_)\*(x\_)])/(x\_), x\_Symbol] := -Simp[PolyLog[2, c\*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]

#### Rule 37

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rule 6606

Int[((g\_) + Log[(f\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(h\_))\*(Px\_)\*(x\_)^m)\*PolyLog[2, (c\_)\*((a\_) + (b\_)\*(x\_))], x\_Symbol] := With[{u = IntHide[x^m\*Px, x]}, Simp[u\*(g + h\*Log[f\*(d + e\*x)^n])\*PolyLog[2, c\*(a + b\*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h\*Log[f\*(d + e\*x)^n])\*Log[1 - a\*c - b\*c\*x], u/(a + b\*x), x], x], x] - Dist[e\*h\*n, Int[ExpandIntegrand[PolyLog[2, c\*(a + b\*x)], u/(d + e\*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]

&& PolyQ[Px, x] && IntegerQ[m]

#### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^ (p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(x\_)^(m\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^ (p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2397

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^ (p\_.)/((f\_.) + (g\_.)\*(x\_))^2, x\_Symbol] := Simp[((d + e\*x)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/((e\*f - d\*g)\*(f + g\*x)), x] - Dist[(b\*e\*n\*p)/(e\*f - d\*g), Int[(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)/(f + g\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^ (p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^ (p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p-1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

### Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*(b*d - a*e) + e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2) \log(1 - dx) \text{Li}_2(dx)}{x^3} dx &= c \int \frac{\log(1 - dx) \text{Li}_2(dx)}{x} dx + \int \frac{(a + bx) \log(1 - dx) \text{Li}_2(dx)}{x^3} dx \\
&= -\frac{(a + bx)^2 \log(1 - dx) \text{Li}_2(dx)}{2ax^2} - \frac{1}{2} c \text{Li}_2(dx)^2 + d \int \left( -\frac{a \text{Li}_2(dx)}{2x^2} + \frac{(-2b - ad) \log(1 - dx)}{2x} \right) dx \\
&= -\frac{(a + bx)^2 \log(1 - dx) \text{Li}_2(dx)}{2ax^2} - \frac{1}{2} c \text{Li}_2(dx)^2 - \frac{1}{2} a \int \frac{\log^2(1 - dx)}{x^3} dx - b \int \frac{\log(1 - dx)}{x^2} dx \\
&= \frac{a \log^2(1 - dx)}{4x^2} + \frac{b(1 - dx) \log^2(1 - dx)}{x} - \frac{b^2 \log(dx) \log^2(1 - dx)}{2a} + \frac{ad \text{Li}_2(dx)}{2x} \\
&= -\frac{ad \log(1 - dx)}{2x} + \frac{a \log^2(1 - dx)}{4x^2} + \frac{b(1 - dx) \log^2(1 - dx)}{x} - \frac{b^2 \log(dx) \log^2(1 - dx)}{2a} \\
&= -\frac{ad \log(1 - dx)}{2x} + \frac{a \log^2(1 - dx)}{4x^2} + \frac{b(1 - dx) \log^2(1 - dx)}{x} - \frac{b^2 \log(dx) \log^2(1 - dx)}{2a} \\
&= -\frac{1}{2} ad^2 \log(x) + \frac{1}{2} ad^2 \log(1 - dx) - \frac{ad \log(1 - dx)}{x} + \frac{a \log^2(1 - dx)}{4x^2} + \frac{b(1 - dx) \log^2(1 - dx)}{x} \\
&= -\frac{1}{2} ad^2 \log(x) + \frac{1}{2} ad^2 \log(1 - dx) - \frac{ad \log(1 - dx)}{x} - \frac{1}{4} ad^2 \log^2(1 - dx) + \frac{a \log^2(1 - dx)}{4x^2} \\
&= -ad^2 \log(x) + ad^2 \log(1 - dx) - \frac{ad \log(1 - dx)}{x} - \frac{1}{4} ad^2 \log^2(1 - dx) + \frac{a \log^2(1 - dx)}{4x^2}
\end{aligned}$$

**Mathematica [F]** time = 1.98638, size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2) \log(1 - dx) \text{PolyLog}(2, dx)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x^3, x]

[Out] Integrate[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x^3, x]

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \ln(-dx + 1) \text{polylog}(2, dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)`

[Out] `int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**3,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)`

$$3.197 \quad \int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2,dx)}{x^4} dx$$

**Optimal.** Leaf size=515

$$-\frac{1}{6} \log(1-dx) \text{PolyLog}(2,dx) \left( \frac{2a}{x^3} + \frac{3b}{x^2} + \frac{6c}{x} \right) - \frac{1}{6} d \text{PolyLog}(3,dx)(d(2ad+3b)+6c) - \frac{1}{3} d \text{PolyLog}(3,1-dx)(d(2ad+$$

[Out] (7\*a\*d^2)/(36\*x) - (b\*d^2\*Log[x])/2 - (5\*a\*d^3\*Log[x])/12 - (d^2\*(3\*b + 2\*a\*d)\*Log[x])/6 + (b\*d^2\*Log[1 - d\*x])/2 + (5\*a\*d^3\*Log[1 - d\*x])/12 + (d^2\*(3\*b + 2\*a\*d)\*Log[1 - d\*x])/6 - (7\*a\*d\*Log[1 - d\*x])/(36\*x^2) - (b\*d\*Log[1 - d\*x])/(2\*x) - (2\*a\*d^2\*Log[1 - d\*x])/(9\*x) - (d\*(3\*b + 2\*a\*d)\*Log[1 - d\*x])/(6\*x) - (b\*d^2\*Log[1 - d\*x]^2)/4 - (a\*d^3\*Log[1 - d\*x]^2)/9 + (a\*Log[1 - d\*x]^2)/(9\*x^3) + (b\*Log[1 - d\*x]^2)/(4\*x^2) + (c\*(1 - d\*x)\*Log[1 - d\*x]^2)/x + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[d\*x]\*Log[1 - d\*x]^2)/6 - 2\*c\*d\*PolyLog[2, d\*x] - (b\*d^2\*PolyLog[2, d\*x])/2 - (2\*a\*d^3\*PolyLog[2, d\*x])/9 + (a\*d\*PolyLog[2, d\*x])/(6\*x^2) + (d\*(3\*b + 2\*a\*d)\*PolyLog[2, d\*x])/(6\*x) + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[1 - d\*x]\*PolyLog[2, d\*x])/6 - ((2\*a)/x^3 + (3\*b)/x^2 + (6\*c)/x)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/6 + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[1 - d\*x]\*PolyLog[2, 1 - d\*x])/3 - (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*PolyLog[3, d\*x])/6 - (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*PolyLog[3, 1 - d\*x])/3

**Rubi [A]** time = 0.823474, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 20, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$ , Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 14, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{6} \log(1-dx) \text{PolyLog}(2,dx) \left( \frac{2a}{x^3} + \frac{3b}{x^2} + \frac{6c}{x} \right) - \frac{1}{6} d \text{PolyLog}(3,dx)(d(2ad+3b)+6c) - \frac{1}{3} d \text{PolyLog}(3,1-dx)(d(2ad+$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x^4,x]

[Out] (7\*a\*d^2)/(36\*x) - (b\*d^2\*Log[x])/2 - (5\*a\*d^3\*Log[x])/12 - (d^2\*(3\*b + 2\*a\*d)\*Log[x])/6 + (b\*d^2\*Log[1 - d\*x])/2 + (5\*a\*d^3\*Log[1 - d\*x])/12 + (d^2\*(3\*b + 2\*a\*d)\*Log[1 - d\*x])/6 - (7\*a\*d\*Log[1 - d\*x])/(36\*x^2) - (b\*d\*Log[1 - d\*x])/(2\*x) - (2\*a\*d^2\*Log[1 - d\*x])/(9\*x) - (d\*(3\*b + 2\*a\*d)\*Log[1 - d\*x])/(6\*x) - (b\*d^2\*Log[1 - d\*x]^2)/4 - (a\*d^3\*Log[1 - d\*x]^2)/9 + (a\*Log[1 - d\*x]^2)/(9\*x^3) + (b\*Log[1 - d\*x]^2)/(4\*x^2) + (c\*(1 - d\*x)\*Log[1 - d\*x]^2)/x + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[d\*x]\*Log[1 - d\*x]^2)/6 - 2\*c\*d\*PolyLog[2, d\*x] - (b\*d^2\*PolyLog[2, d\*x])/2 - (2\*a\*d^3\*PolyLog[2, d\*x])/9 + (a\*d\*PolyLog[2, d\*x])/(6\*x^2) + (d\*(3\*b + 2\*a\*d)\*PolyLog[2, d\*x])/(6\*x) + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[1 - d\*x]\*PolyLog[2, d\*x])/6 - ((2\*a)/x^3 + (3\*b)/x^2 + (6\*c)/x)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/6 + (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*Log[1 - d\*x]\*PolyLog[2, 1 - d\*x])/3 - (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*PolyLog[3, d\*x])/6 - (d\*(6\*c + d\*(3\*b + 2\*a\*d))\*PolyLog[3, 1 - d\*x])/3

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

**Rule 6591**

Int[((d\_.)\*(x\_.))^(m\_.)\*PolyLog[n\_, (a\_.)\*((b\_.)\*(x\_.)^(p\_.))^(q\_.)], x\_Symbol] := Simp[((d\*x)^(m + 1)\*PolyLog[n, a\*(b\*x^p)^q]/(d\*(m + 1)), x] - Dist[(

$p \cdot q / (m + 1)$ ,  $\text{Int}[(d \cdot x)^m \cdot \text{PolyLog}[n - 1, a \cdot (b \cdot x^p)^q], x], x] /;$   $\text{FreeQ}\{a, b, d, m, p, q\}, x\}$  &&  $\text{NeQ}[m, -1]$  &&  $\text{GtQ}[n, 0]$

Rule 2395

$\text{Int}[(a + \text{Log}[c \cdot (d + (e \cdot x)^n]) \cdot b] \cdot (f + (g \cdot x)^q))^q, x\_Symbol] := \text{Simp}[(f + g \cdot x)^{q+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n]) / (g \cdot (q + 1)), x] - \text{Dist}[(b \cdot e \cdot n) / (g \cdot (q + 1)), \text{Int}[(f + g \cdot x)^{q+1} / (d + e \cdot x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\}$  &&  $\text{NeQ}[e \cdot f - d \cdot g, 0]$  &&  $\text{NeQ}[q, -1]$

Rule 44

$\text{Int}[(a + (b \cdot x)^m) \cdot (c + (d \cdot x)^n), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b \cdot x)^m \cdot (c + d \cdot x)^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x\}$  &&  $\text{NeQ}[b \cdot c - a \cdot d, 0]$  &&  $\text{ILtQ}[m, 0]$  &&  $\text{IntegerQ}[n]$  &&  $!(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

Rule 36

$\text{Int}[1 / ((a + (b \cdot x)^m) \cdot (c + (d \cdot x)^n)), x\_Symbol] := \text{Dist}[b / (b \cdot c - a \cdot d), \text{Int}[1 / (a + b \cdot x), x], x] - \text{Dist}[d / (b \cdot c - a \cdot d), \text{Int}[1 / (c + d \cdot x), x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x\}$  &&  $\text{NeQ}[b \cdot c - a \cdot d, 0]$

Rule 29

$\text{Int}[(x)^{-1}, x\_Symbol] := \text{Simp}[\text{Log}[x], x]$

Rule 31

$\text{Int}[(a + (b \cdot x)^m)^{-1}, x\_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x, x]] / b, x] /;$   $\text{FreeQ}\{a, b\}, x\}$

Rule 14

$\text{Int}[(u + (c + (d \cdot x)^m))^u, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(c \cdot x)^m \cdot u, x], x] /;$   $\text{FreeQ}\{c, m\}, x\}$  &&  $\text{SumQ}[u]$  &&  $!\text{LinearQ}[u, x]$  &&  $!\text{MatchQ}[u, (a + (b \cdot x)^m) \cdot (v)] /;$   $\text{FreeQ}\{a, b\}, x\}$  &&  $\text{InverseFunctionQ}[v]$

Rule 6606

$\text{Int}[(g + \text{Log}[f \cdot (d + (e \cdot x)^n)] \cdot h) \cdot (P \cdot x)^m \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])^p, x\_Symbol] := \text{With}\{u = \text{IntHide}[x^m \cdot P \cdot x, x]\}, \text{Simp}[u \cdot (g + h \cdot \text{Log}[f \cdot (d + e \cdot x)^n]) \cdot \text{PolyLog}[2, c \cdot (a + b \cdot x)], x] + (\text{Dist}[b, \text{Int}[\text{ExpandIntegrand}[(g + h \cdot \text{Log}[f \cdot (d + e \cdot x)^n]) \cdot \text{Log}[1 - a \cdot c - b \cdot c \cdot x], u / (a + b \cdot x), x], x], x] - \text{Dist}[e \cdot h \cdot n, \text{Int}[\text{ExpandIntegrand}[\text{PolyLog}[2, c \cdot (a + b \cdot x)], u / (d + e \cdot x), x], x], x]) /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, h, n\}, x\}$  &&  $\text{PolyQ}[P \cdot x, x]$  &&  $\text{IntegerQ}[m]$

Rule 2398

$\text{Int}[(a + \text{Log}[c \cdot (d + (e \cdot x)^n)] \cdot b)^p \cdot (f + (g \cdot x)^q)^q, x\_Symbol] := \text{Simp}[(f + g \cdot x)^{q+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])^p / (g \cdot (q + 1)), x] - \text{Dist}[(b \cdot e \cdot n \cdot p) / (g \cdot (q + 1)), \text{Int}[(f + g \cdot x)^{q+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x)^n])^{p-1} / (d + e \cdot x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\}$  &&  $\text{NeQ}[e \cdot f - d \cdot g, 0]$  &&  $\text{GtQ}[p, 0]$  &&  $\text{NeQ}[q, -1]$  &&  $\text{IntegersQ}[2 \cdot p, 2 \cdot q]$  &&  $(\text{!IGtQ}[q, 0] \mid \mid (\text{EqQ}[p, 2] \&\& \text{NeQ}[q, 1]))$

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2397

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)^2, x\_Symbol] := Simp[((d + e\*x)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/((e\*f - d\*g)\*(f + g\*x)), x] - Dist[(b\*e\*n\*p)/(e\*f - d\*g), Int[(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)/(f + g\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 6596

Int[PolyLog[2, (c\_.)\*((a\_.) + (b\_.)\*(x\_))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 - a\*c - b\*c\*x]\*PolyLog[2, c\*(a + b\*x)])/e, x] + Dist[b/e, Int[Log[1 - a\*c - b\*c\*x]^2/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c\*(b\*d - a\*e) + e, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*1, 0]

#### Rule 2374



```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/x, x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^m])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^m])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx + cx^2) \log(1 - dx) \operatorname{Li}_2(dx)}{x^4} dx &= -\frac{1}{6} \left( \frac{2a}{x^3} + \frac{3b}{x^2} + \frac{6c}{x} \right) \log(1 - dx) \operatorname{Li}_2(dx) + d \int \left( -\frac{a \operatorname{Li}_2(dx)}{3x^3} + \frac{(-3b - 2ad) \operatorname{Li}_2(dx)}{6x^2} \right) dx \\
&= -\frac{1}{6} \left( \frac{2a}{x^3} + \frac{3b}{x^2} + \frac{6c}{x} \right) \log(1 - dx) \operatorname{Li}_2(dx) - \frac{1}{3} a \int \frac{\log^2(1 - dx)}{x^4} dx - \frac{1}{2} b \int \frac{\log^2(1 - dx)}{x^3} dx \\
&= \frac{a \log^2(1 - dx)}{9x^3} + \frac{b \log^2(1 - dx)}{4x^2} + \frac{c(1 - dx) \log^2(1 - dx)}{x} + \frac{ad \operatorname{Li}_2(dx)}{6x^2} + \frac{d \log^2(1 - dx)}{2x} \\
&= -\frac{ad \log(1 - dx)}{12x^2} - \frac{d(3b + 2ad) \log(1 - dx)}{6x} + \frac{a \log^2(1 - dx)}{9x^3} + \frac{b \log^2(1 - dx)}{4x^2} \\
&= -\frac{ad \log(1 - dx)}{12x^2} - \frac{d(3b + 2ad) \log(1 - dx)}{6x} + \frac{a \log^2(1 - dx)}{9x^3} + \frac{b \log^2(1 - dx)}{4x^2} \\
&= \frac{ad^2}{12x} - \frac{1}{12} ad^3 \log(x) - \frac{1}{6} d^2(3b + 2ad) \log(x) + \frac{1}{12} ad^3 \log(1 - dx) + \frac{1}{6} d^2(3b + 2ad) \log(1 - dx) \\
&= \frac{ad^2}{12x} - \frac{1}{12} ad^3 \log(x) - \frac{1}{6} d^2(3b + 2ad) \log(x) + \frac{1}{12} ad^3 \log(1 - dx) + \frac{1}{6} d^2(3b + 2ad) \log(1 - dx) \\
&= \frac{7ad^2}{36x} - \frac{1}{2} bd^2 \log(x) - \frac{5}{12} ad^3 \log(x) - \frac{1}{6} d^2(3b + 2ad) \log(x) + \frac{1}{2} bd^2 \log(1 - dx) + \frac{1}{6} d^2(3b + 2ad) \log(1 - dx)
\end{aligned}$$

**Mathematica [A]** time = 1.63239, size = 488, normalized size = 0.95

$$\frac{1}{36} \left( \frac{6 \operatorname{PolyLog}(2, dx) ((dx - 1) \log(1 - dx) (2a(d^2x^2 + dx + 1) + 3x(bdx + b + 2cx)) + dx(2adx + a + 3bx))}{x^3} + 2d \operatorname{PolyLog}(2, 1 - dx) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

```
[Out] (-7*a*d^3 + (7*a*d^2)/x - 36*b*d^2*Log[d*x] - 27*a*d^3*Log[d*x] + 36*b*d^2*
Log[1 - d*x] + 27*a*d^3*Log[1 - d*x] - (7*a*d*Log[1 - d*x])/x^2 - (36*b*d*L
og[1 - d*x])/x - (20*a*d^2*Log[1 - d*x])/x + 72*c*d*Log[d*x]*Log[1 - d*x] +
18*b*d^2*Log[d*x]*Log[1 - d*x] + 8*a*d^3*Log[d*x]*Log[1 - d*x] - 36*c*d*Lo
g[1 - d*x]^2 - 9*b*d^2*Log[1 - d*x]^2 - 4*a*d^3*Log[1 - d*x]^2 + (4*a*Log[1
- d*x]^2)/x^3 + (9*b*Log[1 - d*x]^2)/x^2 + (36*c*Log[1 - d*x]^2)/x + 36*c*
d*Log[d*x]*Log[1 - d*x]^2 + 18*b*d^2*Log[d*x]*Log[1 - d*x]^2 + 12*a*d^3*Log
[d*x]*Log[1 - d*x]^2 + (6*(d*x*(a + 3*b*x + 2*a*d*x) + (-1 + d*x)*(3*x*(b +
2*c*x + b*d*x) + 2*a*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2, d*x])/x
^3 + 2*d*(36*c + 9*b*d + 4*a*d^2 + 6*(6*c + 3*b*d + 2*a*d^2)*Log[1 - d*x])*
PolyLog[2, 1 - d*x] - 36*c*d*PolyLog[3, d*x] - 18*b*d^2*PolyLog[3, d*x] - 1
2*a*d^3*PolyLog[3, d*x] - 72*c*d*PolyLog[3, 1 - d*x] - 36*b*d^2*PolyLog[3,
1 - d*x] - 24*a*d^3*PolyLog[3, 1 - d*x])/36
```

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \ln(-dx + 1) \operatorname{polylog}(2, dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^4,x)

[Out] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^4,x)

**Maxima [A]** time = 1.21818, size = 431, normalized size = 0.84

$$\frac{1}{6} (2ad^3 + 3bd^2 + 6cd) (\log(dx) \log(-dx + 1)^2 + 2 \operatorname{Li}_2(-dx + 1) \log(-dx + 1) - 2 \operatorname{Li}_3(-dx + 1)) + \frac{1}{18} (4ad^3 + 9bd^2 + 3cd)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^4,x, algorithm="maxima")

[Out] 1/6\*(2\*a\*d^3 + 3\*b\*d^2 + 6\*c\*d)\*(log(d\*x)\*log(-d\*x + 1)^2 + 2\*dilog(-d\*x + 1)\*log(-d\*x + 1) - 2\*polylog(3, -d\*x + 1)) + 1/18\*(4\*a\*d^3 + 9\*b\*d^2 + 36\*c\*d)\*(log(d\*x)\*log(-d\*x + 1) + dilog(-d\*x + 1)) - 1/4\*(3\*a\*d^3 + 4\*b\*d^2)\*log(x) - 1/6\*(2\*a\*d^3 + 3\*b\*d^2 + 6\*c\*d)\*polylog(3, d\*x) + 1/36\*(7\*a\*d^2\*x^2 - ((4\*a\*d^3 + 9\*b\*d^2 + 36\*c\*d)\*x^3 - 36\*c\*x^2 - 9\*b\*x - 4\*a)\*log(-d\*x + 1)^2 + 6\*(a\*d\*x + (2\*a\*d^2 + 3\*b\*d)\*x^2 + ((2\*a\*d^3 + 3\*b\*d^2 + 6\*c\*d)\*x^3 - 6\*c\*x^2 - 3\*b\*x - 2\*a)\*log(-d\*x + 1))\*dilog(d\*x) + (9\*(3\*a\*d^3 + 4\*b\*d^2)\*x^3 - 7\*a\*d\*x - 4\*(5\*a\*d^2 + 9\*b\*d)\*x^2)\*log(-d\*x + 1))/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^4,x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^4,x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x^4, x)

$$3.198 \quad \int \frac{(a+bx+cx^2) \log(1-dx) \text{PolyLog}(2,dx)}{x^5} dx$$

**Optimal.** Leaf size=767

$$-\frac{1}{12}d^2 \text{PolyLog}(3,dx)(d(3ad+4b)+6c) - \frac{1}{6}d^2 \text{PolyLog}(3,1-dx)(d(3ad+4b)+6c) + \frac{1}{12}d^2 \log(1-dx) \text{PolyLog}(2,dx)$$

[Out]  $(5*a*d^2)/(144*x^2) + (b*d^2)/(9*x) + (19*a*d^3)/(144*x) + (d^2*(4*b + 3*a*d))/(48*x) - (c*d^2*\text{Log}[x])/2 - (b*d^3*\text{Log}[x])/3 - (37*a*d^4*\text{Log}[x])/144 - (d^3*(4*b + 3*a*d)*\text{Log}[x])/48 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[x])/12 + (c*d^2*\text{Log}[1-d*x])/2 + (b*d^3*\text{Log}[1-d*x])/3 + (37*a*d^4*\text{Log}[1-d*x])/144 + (d^3*(4*b + 3*a*d)*\text{Log}[1-d*x])/48 + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x])/12 - (5*a*d*\text{Log}[1-d*x])/(72*x^3) - (b*d*\text{Log}[1-d*x])/(9*x^2) - (a*d^2*\text{Log}[1-d*x])/(16*x^2) - (d*(4*b + 3*a*d)*\text{Log}[1-d*x])/(48*x^2) - (c*d*\text{Log}[1-d*x])/(2*x) - (2*b*d^2*\text{Log}[1-d*x])/(9*x) - (a*d^3*\text{Log}[1-d*x])/(8*x) - (d*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x])/(12*x) - (c*d^2*\text{Log}[1-d*x]^2)/4 - (b*d^3*\text{Log}[1-d*x]^2)/9 - (a*d^4*\text{Log}[1-d*x]^2)/16 + (a*\text{Log}[1-d*x]^2)/(16*x^4) + (b*\text{Log}[1-d*x]^2)/(9*x^3) + (c*\text{Log}[1-d*x]^2)/(4*x^2) + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[d*x]*\text{Log}[1-d*x]^2)/12 - (c*d^2*\text{PolyLog}[2,d*x])/2 - (2*b*d^3*\text{PolyLog}[2,d*x])/9 - (a*d^4*\text{PolyLog}[2,d*x])/8 + (a*d*\text{PolyLog}[2,d*x])/(12*x^3) + (d*(4*b + 3*a*d)*\text{PolyLog}[2,d*x])/(24*x^2) + (d*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[2,d*x])/(12*x) + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x]*\text{PolyLog}[2,d*x])/12 - (((3*a)/x^4 + (4*b)/x^3 + (6*c)/x^2)*\text{Log}[1-d*x]*\text{PolyLog}[2,d*x])/12 + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x]*\text{PolyLog}[2,1-d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[3,d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[3,1-d*x])/6$

**Rubi [A]** time = 1.12465, antiderivative size = 767, normalized size of antiderivative = 1., number of steps used = 61, number of rules used = 19, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$ , Rules used = {6742, 6591, 2395, 44, 14, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{12}d^2 \text{PolyLog}(3,dx)(d(3ad+4b)+6c) - \frac{1}{6}d^2 \text{PolyLog}(3,1-dx)(d(3ad+4b)+6c) + \frac{1}{12}d^2 \log(1-dx) \text{PolyLog}(2,dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x + c\*x^2)\*Log[1 - d\*x]\*PolyLog[2, d\*x])/x^5, x]

[Out]  $(5*a*d^2)/(144*x^2) + (b*d^2)/(9*x) + (19*a*d^3)/(144*x) + (d^2*(4*b + 3*a*d))/(48*x) - (c*d^2*\text{Log}[x])/2 - (b*d^3*\text{Log}[x])/3 - (37*a*d^4*\text{Log}[x])/144 - (d^3*(4*b + 3*a*d)*\text{Log}[x])/48 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[x])/12 + (c*d^2*\text{Log}[1-d*x])/2 + (b*d^3*\text{Log}[1-d*x])/3 + (37*a*d^4*\text{Log}[1-d*x])/144 + (d^3*(4*b + 3*a*d)*\text{Log}[1-d*x])/48 + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x])/12 - (5*a*d*\text{Log}[1-d*x])/(72*x^3) - (b*d*\text{Log}[1-d*x])/(9*x^2) - (a*d^2*\text{Log}[1-d*x])/(16*x^2) - (d*(4*b + 3*a*d)*\text{Log}[1-d*x])/(48*x^2) - (c*d*\text{Log}[1-d*x])/(2*x) - (2*b*d^2*\text{Log}[1-d*x])/(9*x) - (a*d^3*\text{Log}[1-d*x])/(8*x) - (d*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x])/(12*x) - (c*d^2*\text{Log}[1-d*x]^2)/4 - (b*d^3*\text{Log}[1-d*x]^2)/9 - (a*d^4*\text{Log}[1-d*x]^2)/16 + (a*\text{Log}[1-d*x]^2)/(16*x^4) + (b*\text{Log}[1-d*x]^2)/(9*x^3) + (c*\text{Log}[1-d*x]^2)/(4*x^2) + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[d*x]*\text{Log}[1-d*x]^2)/12 - (c*d^2*\text{PolyLog}[2,d*x])/2 - (2*b*d^3*\text{PolyLog}[2,d*x])/9 - (a*d^4*\text{PolyLog}[2,d*x])/8 + (a*d*\text{PolyLog}[2,d*x])/(12*x^3) + (d*(4*b + 3*a*d)*\text{PolyLog}[2,d*x])/(24*x^2) + (d*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[2,d*x])/(12*x) + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x]*\text{PolyLog}[2,d*x])/12 - (((3*a)/x^4 + (4*b)/x^3 + (6*c)/x^2)*\text{Log}[1-d*x]*\text{PolyLog}[2,d*x])/12 + (d^2*(6*c + d*(4*b + 3*a*d))*\text{Log}[1-d*x]*\text{PolyLog}[2,1-d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[3,d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*\text{PolyLog}[3,1-d*x])/6$

$$g[1 - d*x]*PolyLog[2, 1 - d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3, d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3, 1 - d*x])/6$$

#### Rule 6742

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]`  
`]`

#### Rule 6591

`Int[((d_.)*(x_.))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_.)^(p_.))^(q_.)], x_Symbol]`  
`]:> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q]/(d*(m + 1)), x] - Dist[(p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,`  
`b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]`

#### Rule 2395

`Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol]`  
`]:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]`

#### Rule 44

`Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

#### Rule 14

`Int[(u_)*((c_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

#### Rule 6606

`Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(h_.))*((Px_)*(x_.)^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_.))], x_Symbol] := With[{u = IntHide[x^m*Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x] && IntegerQ[m]`

#### Rule 2398

`Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))`

#### Rule 2410

`Int[(Log[(c_.)*((d_) + (e_.)*(x_.))]*(x_.)^(m_.))/((f_) + (g_.)*(x_.)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ`

$\{c, d, e, f, g\}, x \} \&\& \text{EqQ}[e*f - d*g, 0] \&\& \text{EqQ}[c*d, 1] \&\& \text{IntegerQ}[m]$

### Rule 36

$\text{Int}[1/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))], x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x \} \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 29

$\text{Int}[(x_-)^{-1}], x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_-)^{-1}], x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$   $\text{FreeQ}\{a, b\}, x \}$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_-)^{n_.})]/(x_-), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\text{FreeQ}\{c, d, e, n\}, x \} \&\& \text{EqQ}[c*d, 1]$

### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_-)^{n_.})]*(b_.))^{p_.}*((f_.) + (g_.)*(x_-)^{q_.}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x \} \&\& \text{EqQ}[e*f - d*g, 0]$

### Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_-)^{n_.}]]*(b_.)/(x_-), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /;$   $\text{FreeQ}\{a, b, c, n\}, x \}$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_-)^{p_.})]/((d_.) + (e_.)*(x_-)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /;$   $\text{FreeQ}\{a, b, c, d, e, n, p\}, x \} \&\& \text{EqQ}[b*d, a*e]$

### Rule 6596

$\text{Int}[\text{PolyLog}[2, (c_.)*((a_.) + (b_.)*(x_-))]/((d_.) + (e_.)*(x_-)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 - a*c - b*c*x]*\text{PolyLog}[2, c*(a + b*x)])/e, x] + \text{Dist}[b/e, \text{Int}[\text{Log}[1 - a*c - b*c*x]^2/(a + b*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e\}, x \} \&\& \text{EqQ}[c*(b*d - a*e) + e, 0]$

### Rule 2396

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_-)^{n_.})]*(b_.))^{p_.}*((f_.) + (g_.)*(x_-)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^p)/g, x] - \text{Dist}[(b*e*n*p)/g, \text{Int}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n])^{p-1})/(d + e*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x \} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{IGtQ}[p, 1]$

### Rule 2433

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_-)^{n_.})]*(b_.))^{p_.}*((f_.) + \text{Log}[(h_.)*((i_.) + (j_.)*(x_-)^{m_.})]*(g_.))*((k_.) + (l_.)*(x_-)^{r_.}), x\_Sym$

```
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx + cx^2) \log(1 - dx) \text{Li}_2(dx)}{x^5} dx &= -\frac{1}{12} \left( \frac{3a}{x^4} + \frac{4b}{x^3} + \frac{6c}{x^2} \right) \log(1 - dx) \text{Li}_2(dx) + d \int \left( -\frac{a \text{Li}_2(dx)}{4x^4} + \frac{(-4b - 3ad)}{12x^3} \right) dx \\ &= -\frac{1}{12} \left( \frac{3a}{x^4} + \frac{4b}{x^3} + \frac{6c}{x^2} \right) \log(1 - dx) \text{Li}_2(dx) - \frac{1}{4} a \int \frac{\log^2(1 - dx)}{x^5} dx - \frac{1}{3} b \int \frac{\log(1 - dx)}{x^4} dx \\ &= \frac{a \log^2(1 - dx)}{16x^4} + \frac{b \log^2(1 - dx)}{9x^3} + \frac{c \log^2(1 - dx)}{4x^2} + \frac{ad \text{Li}_2(dx)}{12x^3} + \frac{d(4b + 3ad)}{24x^2} \\ &= -\frac{ad \log(1 - dx)}{36x^3} - \frac{d(4b + 3ad) \log(1 - dx)}{48x^2} - \frac{d(6c + d(4b + 3ad)) \log(1 - dx)}{12x} \\ &= -\frac{ad \log(1 - dx)}{36x^3} - \frac{d(4b + 3ad) \log(1 - dx)}{48x^2} - \frac{d(6c + d(4b + 3ad)) \log(1 - dx)}{12x} \\ &= \frac{ad^2}{72x^2} + \frac{ad^3}{36x} + \frac{d^2(4b + 3ad)}{48x} - \frac{1}{36} ad^4 \log(x) - \frac{1}{48} d^3(4b + 3ad) \log(x) - \frac{1}{12} d^2(6c + d(4b + 3ad)) \log(x) \\ &= \frac{ad^2}{72x^2} + \frac{ad^3}{36x} + \frac{d^2(4b + 3ad)}{48x} - \frac{1}{36} ad^4 \log(x) - \frac{1}{48} d^3(4b + 3ad) \log(x) - \frac{1}{12} d^2(6c + d(4b + 3ad)) \log(x) \\ &= \frac{5ad^2}{144x^2} + \frac{bd^2}{9x} + \frac{19ad^3}{144x} + \frac{d^2(4b + 3ad)}{48x} - \frac{1}{2} cd^2 \log(x) - \frac{1}{3} bd^3 \log(x) - \frac{37}{144} ad^2 \log(x) \end{aligned}$$

**Mathematica [A]** time = 1.95317, size = 621, normalized size = 0.81

$$\frac{1}{144} \left( \frac{6 \text{PolyLog}(2, dx) (dx (a (6d^2x^2 + 3dx + 2) + 4x(2bdx + b + 3cx)) + 2 \log(1 - dx) (3a(d^4x^4 - 1) + 4bd^3x^4 - 4bx^4))}{x^4} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5, x]
```

```
[Out] (-28*b*d^3 - 33*a*d^4 + (5*a*d^2)/x^2 + (28*b*d^2)/x + (28*a*d^3)/x - 144*c
*d^2*Log[d*x] - 108*b*d^3*Log[d*x] - 82*a*d^4*Log[d*x] + 144*c*d^2*Log[1 -
d*x] + 108*b*d^3*Log[1 - d*x] + 82*a*d^4*Log[1 - d*x] - (10*a*d*Log[1 - d*x
])/x^3 - (28*b*d*Log[1 - d*x])/x^2 - (18*a*d^2*Log[1 - d*x])/x^2 - (144*c*d
*Log[1 - d*x])/x - (80*b*d^2*Log[1 - d*x])/x - (54*a*d^3*Log[1 - d*x])/x +
72*c*d^2*Log[d*x]*Log[1 - d*x] + 32*b*d^3*Log[d*x]*Log[1 - d*x] + 18*a*d^4*
Log[d*x]*Log[1 - d*x] - 36*c*d^2*Log[1 - d*x]^2 - 16*b*d^3*Log[1 - d*x]^2 -
9*a*d^4*Log[1 - d*x]^2 + (9*a*Log[1 - d*x]^2)/x^4 + (16*b*Log[1 - d*x]^2)/
x^3 + (36*c*Log[1 - d*x]^2)/x^2 + 72*c*d^2*Log[d*x]*Log[1 - d*x]^2 + 48*b*d
^3*Log[d*x]*Log[1 - d*x]^2 + 36*a*d^4*Log[d*x]*Log[1 - d*x]^2 + (6*(d*x*(4*
x*(b + 3*c*x + 2*b*d*x) + a*(2 + 3*d*x + 6*d^2*x^2)) + 2*(-4*b*x - 6*c*x^2
```

+ 6\*c\*d^2\*x^4 + 4\*b\*d^3\*x^4 + 3\*a\*(-1 + d^4\*x^4))\*Log[1 - d\*x])\*PolyLog[2, d\*x])/x^4 + 2\*d^2\*(36\*c + 16\*b\*d + 9\*a\*d^2 + 12\*(6\*c + 4\*b\*d + 3\*a\*d^2))\*Log[1 - d\*x])\*PolyLog[2, 1 - d\*x] - 72\*c\*d^2\*PolyLog[3, d\*x] - 48\*b\*d^3\*PolyLog[3, d\*x] - 36\*a\*d^4\*PolyLog[3, d\*x] - 144\*c\*d^2\*PolyLog[3, 1 - d\*x] - 96\*b\*d^3\*PolyLog[3, 1 - d\*x] - 72\*a\*d^4\*PolyLog[3, 1 - d\*x])/144

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \ln(-dx + 1) \operatorname{polylog}(2, dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^5,x)

[Out] int((c\*x^2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x^5,x)

**Maxima [A]** time = 1.23365, size = 544, normalized size = 0.71

$$\frac{1}{12} (3ad^4 + 4bd^3 + 6cd^2) (\log(dx) \log(-dx + 1)^2 + 2\operatorname{Li}_2(-dx + 1) \log(-dx + 1) - 2\operatorname{Li}_3(-dx + 1)) + \frac{1}{72} (9ad^4 + 16bd^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^5,x, algorithm="maxima")

[Out] 1/12\*(3\*a\*d^4 + 4\*b\*d^3 + 6\*c\*d^2)\*(log(d\*x)\*log(-d\*x + 1)^2 + 2\*dilog(-d\*x + 1)\*log(-d\*x + 1) - 2\*polylog(3, -d\*x + 1)) + 1/72\*(9\*a\*d^4 + 16\*b\*d^3 + 36\*c\*d^2)\*(log(d\*x)\*log(-d\*x + 1) + dilog(-d\*x + 1)) - 1/72\*(41\*a\*d^4 + 54\*b\*d^3 + 72\*c\*d^2)\*log(x) - 1/12\*(3\*a\*d^4 + 4\*b\*d^3 + 6\*c\*d^2)\*polylog(3, d\*x) + 1/144\*(5\*a\*d^2\*x^2 + 28\*(a\*d^3 + b\*d^2)\*x^3 - ((9\*a\*d^4 + 16\*b\*d^3 + 36\*c\*d^2)\*x^4 - 36\*c\*x^2 - 16\*b\*x - 9\*a)\*log(-d\*x + 1)^2 + 6\*(2\*(3\*a\*d^3 + 4\*b\*d^2 + 6\*c\*d)\*x^3 + 2\*a\*d\*x + (3\*a\*d^2 + 4\*b\*d)\*x^2 + 2\*((3\*a\*d^4 + 4\*b\*d^3 + 6\*c\*d^2)\*x^4 - 6\*c\*x^2 - 4\*b\*x - 3\*a)\*log(-d\*x + 1))\*dilog(d\*x) + 2\*((41\*a\*d^4 + 54\*b\*d^3 + 72\*c\*d^2)\*x^4 - (27\*a\*d^3 + 40\*b\*d^2 + 72\*c\*d)\*x^3 - 5\*a\*d\*x - (9\*a\*d^2 + 14\*b\*d)\*x^2)\*log(-d\*x + 1))/x^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^5,x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x^5, x)



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*ln(-d\*x+1)\*polylog(2,d\*x)/x\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a) \operatorname{Li}_2(dx) \log(-dx + 1)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+b\*x+a)\*log(-d\*x+1)\*polylog(2,d\*x)/x^5,x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*dilog(d\*x)\*log(-d\*x + 1)/x^5, x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+`') or type(expn,'*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```



```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```