Computer algebra independent integration tests

8-Special-functions/8.8-Polylogarithm-function

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [198]. This is test number [208].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (198)	% 0. (0)
Mathematica	% 97.47 (193)	% 2.53 (5)
Maple	% 72.73 (144)	% 27.27 (54)
Maxima	% 44.44 (88)	% 55.56 (110)
Fricas	% 51.52 (102)	% 48.48 (96)
Sympy	% 18.18 (36)	% 81.82 (162)
Giac	% 8.08 (16)	% 91.92 (182)

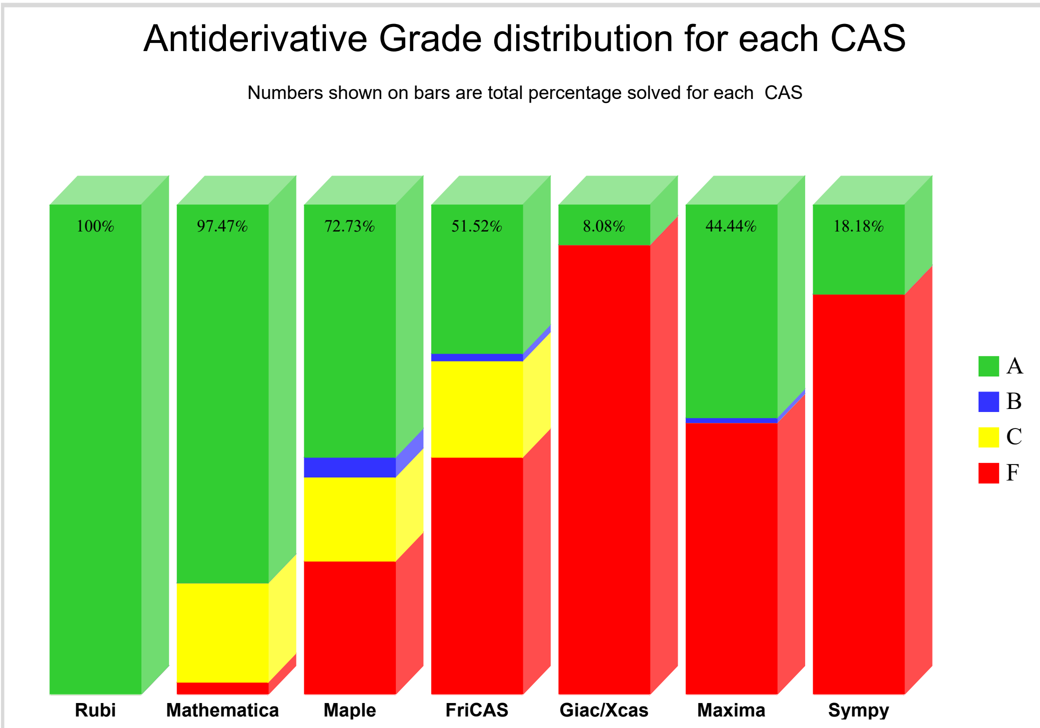
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

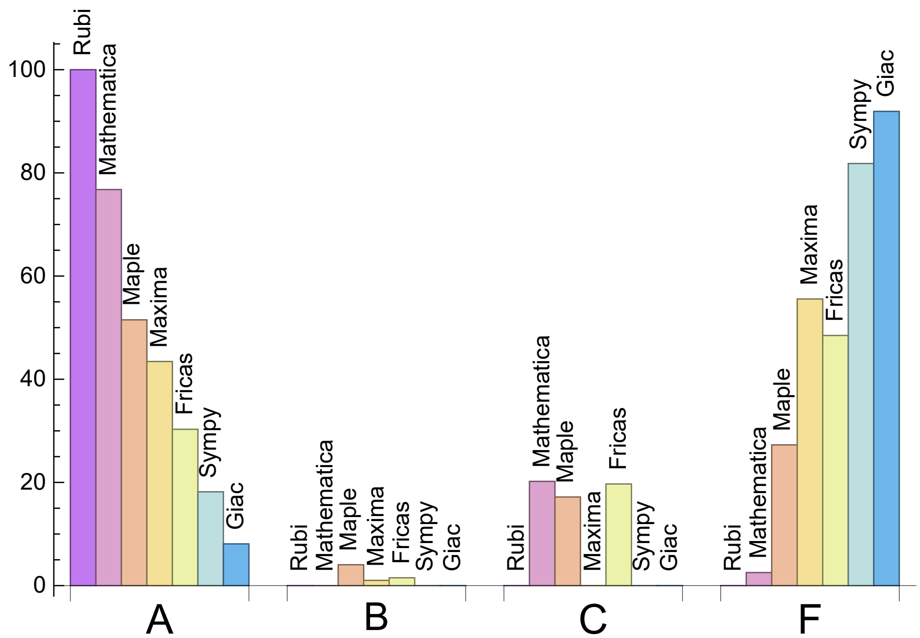
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	76.77	0.	20.2	2.53
Maple	51.52	4.04	17.17	27.27
Maxima	43.43	1.01	0.	55.56
Fricas	30.3	1.52	19.7	48.48
Sympy	18.18	0.	0.	81.82
Giac	8.08	0.	0.	91.92

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	215.93	0.92	91.	1.
Mathematica	0.3	137.58	0.73	63.	0.8
Maple	0.18	116.35	1.09	101.5	1.11
Maxima	0.86	187.28	1.08	80.	1.09
Fricas	2.27	294.7	3.41	228.	2.78
Sympy	11.27	28.83	0.57	13.5	0.69
Giac	0.	0.	0.	0.	0.

1.4 list of integrals that has no closed form antiderivative

$\{96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{17, 18, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

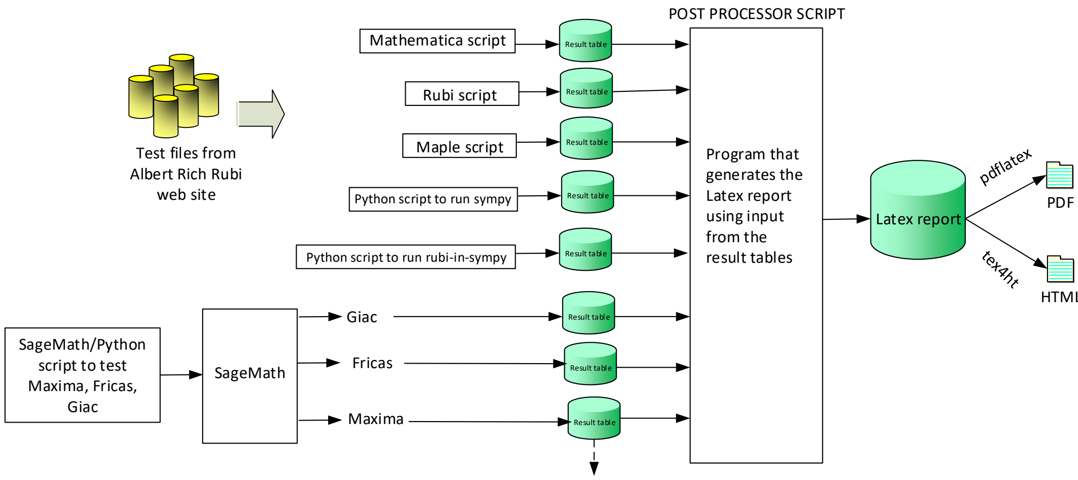
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 86, 87, 96, 97, 98, 99, 100, 102, 105, 108, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198 }

B grade: { }

C grade: { 17, 18, 30, 31, 37, 38, 52, 53, 54, 56, 57, 58, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 103, 104, 106, 107, 109, 110, 112, 113 }

F grade: { 101, 181, 182, 183, 196}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 55, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 134, 138, 139, 140, 142, 143, 153, 154, 155, 159, 160, 165, 173, 180 }

B grade: { 39, 40, 41, 42, 43, 44, 137, 144 }

C grade: { 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { 101, 127, 131, 132, 133, 135, 136, 141, 145, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 32, 33, 34, 36, 37, 38, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 142, 143, 145, 146, 147, 148, 153, 154, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 180, 184, 185, 186, 189, 190, 191, 192, 193, 197, 198

B grade: { 144, 155 }

C grade: { }

F grade: { 22, 26, 27, 28, 29, 30, 31, 35, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 135, 136, 141, 149, 150, 151, 152, 156, 157, 158, 159, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 187, 188, 194, 195, 196 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 59, 60, 61, 62, 63, 64, 72, 73, 74, 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 124, 125, 126, 134, 137, 138, 139, 140, 151, 152, 153, 154, 155, 160, 165, 180 }

B grade: { 75, 76, 77 }

C grade: { 11, 12, 13, 14, 16, 17, 18, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 55, 65, 66, 67, 68, 69, 70, 71, 78, 79, 80, 81, 82, 83, 84, 85, 131, 132, 133, 150 }

F grade: { 6, 15, 22, 35, 45, 46, 47, 49, 50, 51, 52, 53, 54, 56, 57, 58, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 127, 128, 129, 130, 135, 136, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 159, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 19, 20, 21, 23, 24, 25, 27, 28, 29, 96, 97, 98, 99, 100, 114, 115, 116, 117, 118, 119, 120, 122, 123, 134, 160 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 16, 17, 18, 22, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.1.7 Giac

A grade: { 96, 97, 98, 99, 100, 114, 115, 117, 118, 119, 120, 122, 123, 134, 160, 180 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 121, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	73	76	97	177	66	0
normalized size	1	1.	0.85	0.88	1.13	2.06	0.77	0.
time (sec)	N/A	0.053	0.042	0.046	0.964	2.689	15.264	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	65	68	86	153	58	0
normalized size	1	1.	0.86	0.89	1.13	2.01	0.76	0.
time (sec)	N/A	0.045	0.033	0.046	0.983	2.662	8.741	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	57	60	76	132	49	0
normalized size	1	1.	0.86	0.91	1.15	2.	0.74	0.
time (sec)	N/A	0.04	0.029	0.047	0.973	2.599	4.428	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	52	65	111	41	0
normalized size	1	1.	0.86	0.93	1.16	1.98	0.73	0.
time (sec)	N/A	0.028	0.024	0.046	0.965	2.582	2.437	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	36	39	70	22	0
normalized size	1	1.	0.9	1.24	1.34	2.41	0.76	0.
time (sec)	N/A	0.009	0.012	0.045	0.981	2.635	1.209	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	0	3	0
normalized size	1	1.	1.	1.2	1.4	0.	0.6	0.
time (sec)	N/A	0.009	0.001	0.043	0.975	0.	1.712	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	40	38	88	24	0
normalized size	1	1.	1.	1.11	1.06	2.44	0.67	0.
time (sec)	N/A	0.022	0.01	0.119	1.009	2.782	1.563	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	52	54	117	42	0
normalized size	1	1.	0.86	0.9	0.93	2.02	0.72	0.
time (sec)	N/A	0.033	0.022	0.118	0.981	2.629	2.965	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	60	66	143	51	0
normalized size	1	1.	0.76	0.88	0.97	2.1	0.75	0.
time (sec)	N/A	0.036	0.037	0.119	0.99	2.688	5.569	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	60	68	78	163	60	0
normalized size	1	1.	0.77	0.87	1.	2.09	0.77	0.
time (sec)	N/A	0.039	0.035	0.128	0.986	2.786	11.28	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	86	78	104	244	0	0
normalized size	1	1.	0.98	0.89	1.18	2.77	0.	0.
time (sec)	N/A	0.057	0.012	0.161	0.999	2.549	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	69	93	224	0	0
normalized size	1	1.	1.	0.88	1.19	2.87	0.	0.
time (sec)	N/A	0.051	0.011	0.164	0.996	2.676	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	69	62	82	201	0	0
normalized size	1	1.	1.01	0.91	1.21	2.96	0.	0.
time (sec)	N/A	0.035	0.009	0.155	1.014	2.616	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	41	53	151	0	0
normalized size	1	1.	1.15	1.21	1.56	4.44	0.	0.
time (sec)	N/A	0.011	0.012	0.092	0.991	2.598	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	0	3	0
normalized size	1	1.	1.	1.2	1.4	0.	0.6	0.
time (sec)	N/A	0.009	0.001	0.04	0.984	0.	0.552	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	57	45	162	0	0
normalized size	1	1.	0.96	1.24	0.98	3.52	0.	0.
time (sec)	N/A	0.03	0.032	0.08	0.995	2.662	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	25	90	63	194	0	0
normalized size	1	1.	0.36	1.29	0.9	2.77	0.	0.
time (sec)	N/A	0.043	0.009	0.165	0.993	2.718	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	25	106	76	221	0	0
normalized size	1	1.	0.31	1.32	0.95	2.76	0.	0.
time (sec)	N/A	0.048	0.009	0.168	0.988	2.814	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	62	84	142	56	0
normalized size	1	1.	0.88	0.84	1.14	1.92	0.76	0.
time (sec)	N/A	0.061	0.022	0.044	1.008	2.527	42.518	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	54	73	120	48	0
normalized size	1	1.	0.88	0.84	1.14	1.88	0.75	0.
time (sec)	N/A	0.05	0.017	0.045	0.969	2.584	13.184	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	52	54	89	39	0
normalized size	1	1.	0.93	1.13	1.17	1.93	0.85	0.
time (sec)	N/A	0.025	0.009	0.045	0.985	2.57	4.027	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.009	0.001	0.115	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	43	46	112	37	0
normalized size	1	1.	1.	0.88	0.94	2.29	0.76	0.
time (sec)	N/A	0.041	0.012	0.05	0.96	2.68	4.88	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	51	54	62	131	49	0
normalized size	1	1.	0.8	0.84	0.97	2.05	0.77	0.
time (sec)	N/A	0.05	0.028	0.05	0.988	2.649	14.997	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	62	74	154	58	0
normalized size	1	1.	0.81	0.84	1.	2.08	0.78	0.
time (sec)	N/A	0.054	0.031	0.05	0.981	2.657	43.062	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	58	0	398	0	0
normalized size	1	1.	0.89	0.79	0.	5.45	0.	0.
time (sec)	N/A	0.045	0.072	0.046	0.	2.698	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	50	0	351	83	0
normalized size	1	1.	0.9	0.79	0.	5.57	1.32	0.
time (sec)	N/A	0.039	0.051	0.046	0.	2.639	119.376	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	39	37	0	269	60	0
normalized size	1	1.	0.98	0.92	0.	6.72	1.5	0.
time (sec)	N/A	0.017	0.028	0.046	0.	2.631	23.224	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	39	0	232	184	0
normalized size	1	1.	0.98	0.93	0.	5.52	4.38	0.
time (sec)	N/A	0.026	0.018	0.049	0.	2.792	84.144	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	47	45	0	289	0	0
normalized size	1	1.	0.84	0.8	0.	5.16	0.	0.
time (sec)	N/A	0.032	0.014	0.053	0.	2.746	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	47	53	0	333	0	0
normalized size	1	1.	0.71	0.8	0.	5.05	0.	0.
time (sec)	N/A	0.038	0.015	0.052	0.	2.661	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	80	104	240	0	0
normalized size	1	1.	1.	0.91	1.18	2.73	0.	0.
time (sec)	N/A	0.075	0.018	0.058	0.98	2.717	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	79	72	93	217	0	0
normalized size	1	1.	1.01	0.92	1.19	2.78	0.	0.
time (sec)	N/A	0.061	0.014	0.054	0.968	2.706	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	56	72	181	0	0
normalized size	1	1.	0.87	0.93	1.2	3.02	0.	0.
time (sec)	N/A	0.03	0.011	0.055	1.001	2.664	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	0	0	0
normalized size	1	1.	1.	0.91	0.	0.	0.	0.
time (sec)	N/A	0.009	0.001	0.154	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	68	55	194	0	0
normalized size	1	1.	0.95	1.08	0.87	3.08	0.	0.
time (sec)	N/A	0.047	0.027	0.061	0.99	2.719	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	30	98	74	217	0	0
normalized size	1	1.	0.38	1.26	0.95	2.78	0.	0.
time (sec)	N/A	0.064	0.012	0.065	1.003	2.718	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	30	115	86	242	0	0
normalized size	1	1.	0.34	1.31	0.98	2.75	0.	0.
time (sec)	N/A	0.065	0.012	0.066	0.998	2.797	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	144	0	608	0	0
normalized size	1	1.	0.89	1.66	0.	6.99	0.	0.
time (sec)	N/A	0.053	0.161	0.178	0.	2.777	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	69	136	0	555	0	0
normalized size	1	1.	0.9	1.77	0.	7.21	0.	0.
time (sec)	N/A	0.049	0.138	0.175	0.	2.743	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	119	0	452	0	0
normalized size	1	1.	1.	2.38	0.	9.04	0.	0.
time (sec)	N/A	0.023	0.088	0.173	0.	2.694	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	112	0	402	0	0
normalized size	1	1.	0.93	2.07	0.	7.44	0.	0.
time (sec)	N/A	0.036	0.081	0.172	0.	2.871	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	125	0	462	0	0
normalized size	1	1.	0.87	1.79	0.	6.6	0.	0.
time (sec)	N/A	0.041	0.089	0.177	0.	3.227	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	138	0	513	0	0
normalized size	1	1.	0.86	1.72	0.	6.41	0.	0.
time (sec)	N/A	0.047	0.101	0.18	0.	3.241	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0
normalized size	1	1.	0.97	1.52	0.	0.	0.	0.
time (sec)	N/A	0.037	0.044	0.26	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	69	108	0	0	0	0
normalized size	1	1.	0.97	1.52	0.	0.	0.	0.
time (sec)	N/A	0.032	0.036	0.207	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	88	0	0	0	0
normalized size	1	1.	0.94	1.63	0.	0.	0.	0.
time (sec)	N/A	0.023	0.045	0.207	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	28	0	0
normalized size	1	1.	1.	1.09	0.	2.55	0.	0.
time (sec)	N/A	0.01	0.002	0.044	0.	2.839	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	60	106	0	0	0	0
normalized size	1	1.	0.87	1.54	0.	0.	0.	0.
time (sec)	N/A	0.039	0.053	0.208	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	108	0	0	0	0
normalized size	1	1.	0.78	1.38	0.	0.	0.	0.
time (sec)	N/A	0.042	0.052	0.209	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	108	0	0	0	0
normalized size	1	1.	0.8	1.42	0.	0.	0.	0.
time (sec)	N/A	0.04	0.052	0.205	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0
normalized size	1	1.	0.47	1.5	0.	0.	0.	0.
time (sec)	N/A	0.052	0.009	0.345	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	41	132	0	0	0	0
normalized size	1	1.	0.47	1.5	0.	0.	0.	0.
time (sec)	N/A	0.043	0.008	0.338	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	39	105	0	0	0	0
normalized size	1	1.	0.57	1.52	0.	0.	0.	0.
time (sec)	N/A	0.028	0.006	0.334	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	209	0	0
normalized size	1	1.	1.	1.09	0.	19.	0.	0.
time (sec)	N/A	0.01	0.001	0.041	0.	2.724	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	37	129	0	0	0	0
normalized size	1	1.	0.44	1.54	0.	0.	0.	0.
time (sec)	N/A	0.05	0.009	0.333	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	41	132	0	0	0	0
normalized size	1	1.	0.43	1.39	0.	0.	0.	0.
time (sec)	N/A	0.05	0.009	0.343	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	41	132	0	0	0	0
normalized size	1	1.	0.44	1.42	0.	0.	0.	0.
time (sec)	N/A	0.051	0.01	0.34	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	90	96	0	470	0	0
normalized size	1	1.	0.77	0.82	0.	4.02	0.	0.
time (sec)	N/A	0.072	0.102	0.194	0.	2.82	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	75	83	0	362	0	0
normalized size	1	1.	0.74	0.81	0.	3.55	0.	0.
time (sec)	N/A	0.053	0.082	0.05	0.	2.569	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	69	0	331	0	0
normalized size	1	1.	0.79	0.86	0.	4.14	0.	0.
time (sec)	N/A	0.046	0.079	0.05	0.	2.615	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	63	0	316	0	0
normalized size	1	1.	0.75	0.93	0.	4.65	0.	0.
time (sec)	N/A	0.043	0.071	0.053	0.	2.712	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	57	76	0	365	0	0
normalized size	1	1.	0.64	0.85	0.	4.1	0.	0.
time (sec)	N/A	0.05	0.074	0.055	0.	2.665	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	65	89	0	412	0	0
normalized size	1	1.	0.61	0.84	0.	3.89	0.	0.
time (sec)	N/A	0.057	0.095	0.054	0.	2.593	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	98	149	0	822	0	0
normalized size	1	1.	0.64	0.97	0.	5.37	0.	0.
time (sec)	N/A	0.098	0.26	0.182	0.	2.975	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	88	141	0	721	0	0
normalized size	1	1.	0.65	1.04	0.	5.3	0.	0.
time (sec)	N/A	0.083	0.218	0.061	0.	2.874	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	73	133	0	593	0	0
normalized size	1	1.	0.6	1.1	0.	4.9	0.	0.
time (sec)	N/A	0.067	0.188	0.058	0.	2.848	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	57	127	0	547	0	0
normalized size	1	1.	0.59	1.31	0.	5.64	0.	0.
time (sec)	N/A	0.062	0.139	0.057	0.	2.726	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	58	111	0	518	0	0
normalized size	1	1.	0.68	1.31	0.	6.09	0.	0.
time (sec)	N/A	0.059	0.102	0.056	0.	2.589	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	64	122	0	575	0	0
normalized size	1	1.	0.59	1.13	0.	5.32	0.	0.
time (sec)	N/A	0.066	0.096	0.06	0.	2.789	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	72	135	0	629	0	0
normalized size	1	1.	0.58	1.08	0.	5.03	0.	0.
time (sec)	N/A	0.076	0.13	0.062	0.	2.86	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	150	0	471	0	0
normalized size	1	1.	0.72	1.07	0.	3.36	0.	0.
time (sec)	N/A	0.107	0.104	0.214	0.	2.846	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	91	139	0	440	0	0
normalized size	1	1.	0.73	1.11	0.	3.52	0.	0.
time (sec)	N/A	0.086	0.07	0.052	0.	2.703	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	57	137	0	408	0	0
normalized size	1	1.	0.5	1.19	0.	3.55	0.	0.
time (sec)	N/A	0.08	0.074	0.053	0.	2.882	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	62	127	0	425	0	0
normalized size	1	1.	0.6	1.23	0.	4.13	0.	0.
time (sec)	N/A	0.074	0.07	0.052	0.	2.664	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	62	129	0	470	0	0
normalized size	1	1.	0.56	1.16	0.	4.23	0.	0.
time (sec)	N/A	0.072	0.067	0.054	0.	2.735	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	70	140	0	512	0	0
normalized size	1	1.	0.56	1.11	0.	4.06	0.	0.
time (sec)	N/A	0.086	0.077	0.056	0.	2.755	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	0	682	0	0
normalized size	1	1.	0.55	0.96	0.	4.24	0.	0.
time (sec)	N/A	0.127	0.103	0.193	0.	3.106	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	89	155	0	606	0	0
normalized size	1	1.	0.55	0.96	0.	3.76	0.	0.
time (sec)	N/A	0.116	0.1	0.208	0.	2.915	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	68	147	0	579	0	0
normalized size	1	1.	0.47	1.01	0.	3.97	0.	0.
time (sec)	N/A	0.097	0.092	0.224	0.	2.852	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	68	147	0	521	0	0
normalized size	1	1.	0.51	1.1	0.	3.89	0.	0.
time (sec)	N/A	0.101	0.086	0.191	0.	2.862	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	71	131	0	549	0	0
normalized size	1	1.	0.58	1.07	0.	4.5	0.	0.
time (sec)	N/A	0.093	0.09	0.191	0.	3.007	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	71	131	0	591	0	0
normalized size	1	1.	0.54	0.99	0.	4.48	0.	0.
time (sec)	N/A	0.091	0.094	0.194	0.	2.818	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	79	142	0	641	0	0
normalized size	1	1.	0.54	0.97	0.	4.36	0.	0.
time (sec)	N/A	0.107	0.099	0.206	0.	3.003	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	84	142	0	629	0	0
normalized size	1	1.	0.57	0.97	0.	4.28	0.	0.
time (sec)	N/A	0.106	0.096	0.191	0.	3.057	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	82	121	0	0	0	0
normalized size	1	1.	0.81	1.2	0.	0.	0.	0.
time (sec)	N/A	0.059	0.124	0.255	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	82	121	0	0	0	0
normalized size	1	1.	0.82	1.21	0.	0.	0.	0.
time (sec)	N/A	0.055	0.106	0.229	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	48	109	0	0	0	0
normalized size	1	1.	0.52	1.17	0.	0.	0.	0.
time (sec)	N/A	0.054	0.021	0.232	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	48	121	0	0	0	0
normalized size	1	1.	0.49	1.25	0.	0.	0.	0.
time (sec)	N/A	0.06	0.029	0.224	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	48	121	0	0	0	0
normalized size	1	1.	0.46	1.15	0.	0.	0.	0.
time (sec)	N/A	0.061	0.026	0.23	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	125	125	50	145	0	0	0	0
normalized size	1	1.	0.4	1.16	0.	0.	0.	0.
time (sec)	N/A	0.075	0.03	0.379	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	50	145	0	0	0	0
normalized size	1	1.	0.4	1.17	0.	0.	0.	0.
time (sec)	N/A	0.07	0.026	0.381	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	50	133	0	0	0	0
normalized size	1	1.	0.43	1.16	0.	0.	0.	0.
time (sec)	N/A	0.069	0.02	0.372	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	50	145	0	0	0	0
normalized size	1	1.	0.42	1.22	0.	0.	0.	0.
time (sec)	N/A	0.078	0.025	0.372	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	50	145	0	0	0	0
normalized size	1	1.	0.39	1.12	0.	0.	0.	0.
time (sec)	N/A	0.075	0.024	0.381	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.043	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.006	0.046	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.002	0.007	0.048	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.006	0.062	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.047	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.007	0.056	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	53	144	0	0	0	0
normalized size	1	1.	0.68	1.85	0.	0.	0.	0.
time (sec)	N/A	0.049	0.039	0.227	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	88	173	0	0	0	0
normalized size	1	1.	0.86	1.7	0.	0.	0.	0.
time (sec)	N/A	0.064	0.06	0.388	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	119	198	0	0	0	0
normalized size	1	1.	0.98	1.64	0.	0.	0.	0.
time (sec)	N/A	0.086	0.073	0.794	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0
normalized size	1	1.	0.77	1.88	0.	0.	0.	0.
time (sec)	N/A	0.055	0.045	0.236	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0
normalized size	1	1.	1.07	1.85	0.	0.	0.	0.
time (sec)	N/A	0.071	0.079	0.41	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0
normalized size	1	1.	1.17	1.82	0.	0.	0.	0.
time (sec)	N/A	0.093	0.1	0.792	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	72	177	0	0	0	0
normalized size	1	1.	0.77	1.88	0.	0.	0.	0.
time (sec)	N/A	0.055	0.045	0.277	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	126	218	0	0	0	0
normalized size	1	1.	1.07	1.85	0.	0.	0.	0.
time (sec)	N/A	0.071	0.08	0.414	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	166	259	0	0	0	0
normalized size	1	1.	1.17	1.82	0.	0.	0.	0.
time (sec)	N/A	0.094	0.097	0.856	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	80	148	0	0	0	0
normalized size	1	1.	0.79	1.47	0.	0.	0.	0.
time (sec)	N/A	0.06	0.064	0.293	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	50	180	0	0	0	0
normalized size	1	1.	0.38	1.38	0.	0.	0.	0.
time (sec)	N/A	0.076	0.025	0.529	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	52	217	0	0	0	0
normalized size	1	1.	0.34	1.41	0.	0.	0.	0.
time (sec)	N/A	0.102	0.02	3.379	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.022	0.05	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.002	0.001	0.047	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	0	0	5	0
normalized size	1	1.	1.	1.14	0.	0.	0.71	0.
time (sec)	N/A	0.009	0.001	0.044	0.	0.	0.528	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.05	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.049	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.023	0.046	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.003	0.004	0.049	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	0	0	0
normalized size	1	1.	1.	1.08	0.	0.	0.	0.
time (sec)	N/A	0.01	0.001	0.046	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.021	0.046	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.009	0.022	0.046	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	144	269	270	366	0	0
normalized size	1	1.	0.55	1.03	1.04	1.41	0.	0.
time (sec)	N/A	0.321	0.208	0.008	0.995	2.393	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	96	177	196	242	0	0
normalized size	1	1.	0.63	1.16	1.29	1.59	0.	0.
time (sec)	N/A	0.169	0.101	0.004	0.993	2.348	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	96	122	126	0	0
normalized size	1	1.	0.88	1.6	2.03	2.1	0.	0.
time (sec)	N/A	0.049	0.018	0.003	0.983	2.416	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	422	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.146	0.01	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	85	154	0	0	0
normalized size	1	1.	0.87	1.01	1.83	0.	0.	0.
time (sec)	N/A	0.117	0.052	0.13	0.982	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	195	261	0	0	0
normalized size	1	1.	0.76	1.13	1.51	0.	0.	0.
time (sec)	N/A	0.178	0.18	0.227	0.994	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	210	376	408	0	0	0
normalized size	1	1.	0.76	1.36	1.48	0.	0.	0.
time (sec)	N/A	0.273	0.304	0.214	1.027	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	296	0	356	675	0	0
normalized size	1	1.	0.85	0.	1.03	1.95	0.	0.
time (sec)	N/A	0.638	0.068	0.005	1.03	2.345	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	198	0	261	510	0	0
normalized size	1	1.	1.	0.	1.32	2.58	0.	0.
time (sec)	N/A	0.287	0.047	0.006	1.011	2.511	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	0	162	355	0	0
normalized size	1	1.	0.79	0.	1.93	4.23	0.	0.
time (sec)	N/A	0.069	0.023	0.005	1.03	2.355	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.037	0.004	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	477	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.742	0.005	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	629	629	573	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.664	1.931	0.004	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	605	605	485	1177	919	1339	0	0
normalized size	1	1.	0.8	1.95	1.52	2.21	0.	0.
time (sec)	N/A	0.587	0.562	0.064	1.043	2.378	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	274	687	548	779	0	0
normalized size	1	1.	0.71	1.78	1.42	2.02	0.	0.
time (sec)	N/A	0.339	0.183	0.058	1.034	2.321	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	161	292	286	377	0	0
normalized size	1	1.	0.77	1.39	1.36	1.8	0.	0.
time (sec)	N/A	0.197	0.09	0.049	1.019	2.491	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	96	122	126	0	0
normalized size	1	1.	0.88	1.6	2.03	2.1	0.	0.
time (sec)	N/A	0.052	0.016	0.006	1.003	2.436	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	591	591	622	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.518	0.308	0.401	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	108	189	224	0	0	0
normalized size	1	1.	0.78	1.37	1.62	0.	0.	0.
time (sec)	N/A	0.184	0.141	0.38	1.031	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	190	437	512	0	0	0
normalized size	1	1.	0.68	1.57	1.84	0.	0.	0.
time (sec)	N/A	0.27	0.421	0.462	1.008	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	313	1075	1928	0	0	0
normalized size	1	1.	0.7	2.4	4.3	0.	0.	0.
time (sec)	N/A	0.426	0.632	0.394	1.224	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	59	0	0	0
normalized size	1	1.	1.	0.	1.28	0.	0.	0.
time (sec)	N/A	0.066	0.039	0.322	1.022	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	59	0	0	0
normalized size	1	1.	1.	0.	1.28	0.	0.	0.
time (sec)	N/A	0.067	0.007	0.281	0.957	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	66	0	0	0
normalized size	1	1.	1.	0.	1.29	0.	0.	0.
time (sec)	N/A	0.133	0.039	0.302	0.965	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	66	0	0	0
normalized size	1	1.	1.	0.	1.29	0.	0.	0.
time (sec)	N/A	0.134	0.015	0.282	0.978	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.02	0.891	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	782	0	0
normalized size	1	1.	0.97	0.	0.	23.7	0.	0.
time (sec)	N/A	0.061	0.007	0.703	0.	3.456	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	0	0	74	0	0
normalized size	1	1.	0.97	0.	0.	2.24	0.	0.
time (sec)	N/A	0.061	0.007	0.805	0.	2.71	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	40	0	0	68	0	0
normalized size	1	1.	1.21	0.	0.	2.06	0.	0.
time (sec)	N/A	0.058	1.823	0.702	0.	2.445	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	37	78	72	0	0
normalized size	1	1.	1.06	1.03	2.17	2.	0.	0.
time (sec)	N/A	0.317	0.091	0.121	1.014	2.34	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	56	70	84	0	0
normalized size	1	1.	0.97	1.56	1.94	2.33	0.	0.
time (sec)	N/A	0.367	0.103	0.171	1.045	2.478	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	57	285	182	0	0
normalized size	1	1.	1.	1.1	5.48	3.5	0.	0.
time (sec)	N/A	2.025	0.265	0.276	1.179	2.276	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.012	0.185	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.006	0.053	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.005	0.055	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	0	0	0	0
normalized size	1	1.	1.	1.03	0.	0.	0.	0.
time (sec)	N/A	0.018	0.005	0.061	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.051	0.088	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	223	0	508	0	0	0
normalized size	1	1.	0.74	0.	1.69	0.	0.	0.
time (sec)	N/A	0.523	0.573	0.052	1.129	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	192	0	400	0	0	0
normalized size	1	1.	0.74	0.	1.55	0.	0.	0.
time (sec)	N/A	0.405	0.357	0.051	1.119	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	160	0	300	0	0	0
normalized size	1	1.	0.61	0.	1.15	0.	0.	0.
time (sec)	N/A	0.255	0.3	0.049	1.152	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	119	0	190	0	0	0
normalized size	1	1.	0.9	0.	1.44	0.	0.	0.
time (sec)	N/A	0.212	0.023	0.049	1.102	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	26	0	0
normalized size	1	1.	1.	0.91	1.	2.36	0.	0.
time (sec)	N/A	0.026	0.009	0.046	0.966	2.534	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	115	0	153	0	0	0
normalized size	1	1.	1.04	0.	1.38	0.	0.	0.
time (sec)	N/A	0.164	0.136	0.049	1.166	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	185	0	219	0	0	0
normalized size	1	1.	0.97	0.	1.15	0.	0.	0.
time (sec)	N/A	0.282	0.275	0.05	1.664	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	246	0	254	0	0	0
normalized size	1	1.	1.	0.	1.04	0.	0.	0.
time (sec)	N/A	0.355	0.235	0.05	1.653	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	277	0	289	0	0	0
normalized size	1	1.	0.97	0.	1.01	0.	0.	0.
time (sec)	N/A	0.449	0.236	0.05	1.74	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	366	252	0	0	0	0	0
normalized size	1	0.87	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.611	0.461	0.23	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	287	211	0	0	0	0	0
normalized size	1	0.87	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	0.339	0.227	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	149	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.068	0.199	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	0	0	0
normalized size	1	1.	1.	0.95	0.	0.	0.	0.
time (sec)	N/A	0.058	0.011	0.155	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	165	150	0	0	0	0	0
normalized size	1	1.06	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	0.172	0.276	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	278	238	0	0	0	0	0
normalized size	1	1.05	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.496	0.256	0.333	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	351	301	0	0	0	0	0
normalized size	1	1.03	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.651	0.201	0.349	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2995	2995	2610	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	4.6	11.481	0.493	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2252	2252	1996	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	2.904	8.832	0.289	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1653	1653	1546	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	3.186	4.922	0.285	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.553	0.302	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2498	2498	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.667	10.192	0.428	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3119	3119	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	3.238	10.412	0.521	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	3733	3733	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	4.308	10.519	0.537	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	661	425	0	560	0	0	0
normalized size	1	1.	0.64	0.	0.85	0.	0.	0.
time (sec)	N/A	0.982	0.755	0.007	1.04	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	362	0	466	0	0	0
normalized size	1	1.	0.66	0.	0.85	0.	0.	0.
time (sec)	N/A	0.718	0.611	0.007	1.038	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	390	285	0	348	0	0	0
normalized size	1	1.	0.73	0.	0.89	0.	0.	0.
time (sec)	N/A	0.449	0.49	0.005	1.007	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	137	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.235	0.005	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	0.714	0.008	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	285	0	288	0	0	0
normalized size	1	1.	0.86	0.	0.87	0.	0.	0.
time (sec)	N/A	0.541	1.188	0.007	1.164	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	389	0	387	0	0	0
normalized size	1	1.	0.85	0.	0.84	0.	0.	0.
time (sec)	N/A	0.672	1.304	0.006	1.201	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	584	584	505	0	460	0	0	0
normalized size	1	1.	0.86	0.	0.79	0.	0.	0.
time (sec)	N/A	0.845	1.518	0.007	1.189	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	900	900	583	0	699	0	0	0
normalized size	1	1.	0.65	0.	0.78	0.	0.	0.
time (sec)	N/A	1.185	1.28	0.047	1.054	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	645	472	0	556	0	0	0
normalized size	1	1.	0.73	0.	0.86	0.	0.	0.
time (sec)	N/A	0.818	1.033	0.05	1.05	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	298	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.619	0.356	0.049	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	280	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	0.834	0.049	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	343	343	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.74	1.986	0.049	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	488	0	431	0	0	0
normalized size	1	1.	0.95	0.	0.84	0.	0.	0.
time (sec)	N/A	0.823	1.632	0.048	1.218	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	767	767	621	0	544	0	0	0
normalized size	1	1.	0.81	0.	0.71	0.	0.	0.
time (sec)	N/A	1.125	1.953	0.052	1.234	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [163] had the largest ratio of [1.214]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	9	0.333
2	A	4	3	1.	9	0.333
3	A	4	3	1.	9	0.333
4	A	4	3	1.	7	0.429
5	A	3	3	1.	5	0.6
6	A	1	1	1.	9	0.111
7	A	5	5	1.	9	0.556
8	A	4	3	1.	9	0.333
9	A	4	3	1.	9	0.333
10	A	4	3	1.	9	0.333
11	A	5	3	1.	9	0.333
12	A	5	3	1.	9	0.333
13	A	5	3	1.	7	0.429
14	A	4	3	1.	5	0.6
15	A	1	1	1.	9	0.111
16	A	6	5	1.	9	0.556
17	A	5	3	1.	9	0.333
18	A	5	3	1.	9	0.333
19	A	5	4	1.	11	0.364
20	A	5	4	1.	11	0.364
21	A	4	4	1.	9	0.444
22	A	1	1	1.	11	0.091
23	A	6	6	1.	11	0.546

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	5	4	1.	11	0.364
25	A	5	4	1.	11	0.364
26	A	5	4	1.	11	0.364
27	A	5	4	1.	11	0.364
28	A	4	4	1.	7	0.571
29	A	3	3	1.	11	0.273
30	A	4	4	1.	11	0.364
31	A	5	4	1.	11	0.364
32	A	6	4	1.	11	0.364
33	A	6	4	1.	11	0.364
34	A	5	4	1.	9	0.444
35	A	1	1	1.	11	0.091
36	A	7	6	1.	11	0.546
37	A	6	4	1.	11	0.364
38	A	6	4	1.	11	0.364
39	A	6	4	1.	11	0.364
40	A	6	4	1.	11	0.364
41	A	5	4	1.	7	0.571
42	A	4	3	1.	11	0.273
43	A	5	4	1.	11	0.364
44	A	6	4	1.	11	0.364
45	A	3	3	1.	11	0.273
46	A	3	3	1.	9	0.333
47	A	3	3	1.	7	0.429
48	A	1	1	1.	11	0.091
49	A	3	3	1.	11	0.273
50	A	3	3	1.	11	0.273
51	A	3	3	1.	11	0.273
52	A	4	3	1.	11	0.273
53	A	4	3	1.	9	0.333
54	A	4	3	1.	7	0.429
55	A	1	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	4	3	1.	11	0.273
57	A	4	3	1.	11	0.273
58	A	4	3	1.	11	0.273
59	A	7	5	1.	13	0.385
60	A	6	5	1.	13	0.385
61	A	5	5	1.	13	0.385
62	A	4	4	1.	13	0.308
63	A	5	5	1.	13	0.385
64	A	6	5	1.	13	0.385
65	A	9	5	1.	13	0.385
66	A	8	5	1.	13	0.385
67	A	7	5	1.	13	0.385
68	A	6	5	1.	13	0.385
69	A	5	4	1.	13	0.308
70	A	6	5	1.	13	0.385
71	A	7	5	1.	13	0.385
72	A	9	8	1.	15	0.533
73	A	8	8	1.	15	0.533
74	A	8	8	1.	15	0.533
75	A	7	7	1.	15	0.467
76	A	7	7	1.	15	0.467
77	A	8	8	1.	15	0.533
78	A	10	8	1.	15	0.533
79	A	10	8	1.	15	0.533
80	A	9	8	1.	15	0.533
81	A	9	8	1.	15	0.533
82	A	8	7	1.	15	0.467
83	A	8	7	1.	15	0.467
84	A	9	8	1.	15	0.533
85	A	9	8	1.	15	0.533
86	A	4	4	1.	15	0.267
87	A	4	4	1.	15	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	4	4	1.	15	0.267
89	A	4	4	1.	15	0.267
90	A	4	4	1.	15	0.267
91	A	5	4	1.	15	0.267
92	A	5	4	1.	15	0.267
93	A	5	4	1.	15	0.267
94	A	5	4	1.	15	0.267
95	A	5	4	1.	15	0.267
96	A	0	0	0.	0	0.
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	0	0	0.	0	0.
100	A	0	0	0.	0	0.
101	A	2	1	1.	15	0.067
102	A	3	3	1.	11	0.273
103	A	4	3	1.	11	0.273
104	A	5	3	1.	11	0.273
105	A	4	4	1.	13	0.308
106	A	5	4	1.	13	0.308
107	A	6	4	1.	13	0.308
108	A	4	4	1.	13	0.308
109	A	5	4	1.	13	0.308
110	A	6	4	1.	13	0.308
111	A	4	4	1.	13	0.308
112	A	5	4	1.	13	0.308
113	A	6	4	1.	13	0.308
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.
116	A	1	1	1.	9	0.111
117	A	0	0	0.	0	0.
118	A	0	0	0.	0	0.
119	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	0	0	0.	0	0.
121	A	1	1	1.	11	0.091
122	A	0	0	0.	0	0.
123	A	0	0	0.	0	0.
124	A	13	8	1.	13	0.615
125	A	10	8	1.	11	0.727
126	A	7	7	1.	9	0.778
127	A	3	3	1.	13	0.231
128	A	7	9	1.	13	0.692
129	A	11	11	1.	13	0.846
130	A	14	11	1.	13	0.846
131	A	33	13	1.	13	1.
132	A	19	12	1.	11	1.091
133	A	9	8	1.	9	0.889
134	A	0	0	0.	0	0.
135	A	6	5	1.	13	0.385
136	A	12	13	1.	13	1.
137	A	16	8	1.	17	0.471
138	A	13	8	1.	17	0.471
139	A	10	8	1.	15	0.533
140	A	7	7	1.	9	0.778
141	A	3	3	1.	17	0.176
142	A	8	5	1.	17	0.294
143	A	12	8	1.	17	0.471
144	A	15	9	1.	17	0.529
145	A	5	5	1.	9	0.556
146	A	5	5	1.	12	0.417
147	A	8	6	1.	12	0.5
148	A	8	6	1.	15	0.4
149	A	1	1	1.	34	0.029
150	A	1	1	1.	34	0.029
151	A	1	1	1.	34	0.029

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	1	1	1.	37	0.027
153	A	2	2	1.	53	0.038
154	A	2	2	1.	53	0.038
155	A	4	4	1.	76	0.053
156	A	5	3	1.	19	0.158
157	A	4	3	1.	19	0.158
158	A	3	3	1.	17	0.176
159	A	2	2	1.	15	0.133
160	A	0	0	0.	0	0.
161	A	38	16	1.	16	1.
162	A	31	16	1.	16	1.
163	A	22	17	1.	14	1.214
164	A	15	12	1.	13	0.923
165	A	1	2	1.	16	0.125
166	A	10	13	1.	16	0.812
167	A	23	17	1.	16	1.062
168	A	30	17	1.	16	1.062
169	A	37	17	1.	16	1.062
170	A	37	20	0.87	20	1.
171	A	30	20	0.87	18	1.111
172	A	18	14	1.	17	0.824
173	A	3	3	1.	20	0.15
174	A	19	15	1.06	20	0.75
175	A	31	22	1.05	20	1.1
176	A	42	24	1.03	20	1.2
177	A	108	20	1.	27	0.741
178	A	67	20	1.	25	0.8
179	A	42	17	1.	24	0.708
180	A	0	0	0.	0	0.
181	A	22	9	1.	27	0.333
182	A	44	16	1.	27	0.593
183	A	78	18	1.	27	0.667

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	52	17	1.	21	0.81
185	A	40	21	1.	19	1.105
186	A	26	20	1.	18	1.111
187	A	18	15	1.	21	0.714
188	A	13	17	1.	21	0.81
189	A	30	20	1.	21	0.952
190	A	41	19	1.	21	0.905
191	A	51	19	1.	21	0.905
192	A	60	22	1.	24	0.917
193	A	43	21	1.	23	0.913
194	A	29	24	1.	26	0.923
195	A	19	21	1.	26	0.808
196	A	32	22	1.	26	0.846
197	A	43	20	1.	26	0.769
198	A	61	19	1.	26	0.731

Chapter 3

Listing of integrals

3.1 $\int x^4 \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=86

$$\frac{1}{5}x^5\text{PolyLog}(2,ax) - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} - \frac{\log(1-ax)}{25a^5} - \frac{x^4}{100a} + \frac{1}{25}x^5\log(1-ax) - \frac{x^5}{125}$$

```
[Out] -x/(25*a^4) - x^2/(50*a^3) - x^3/(75*a^2) - x^4/(100*a) - x^5/125 - Log[1 -
 a*x]/(25*a^5) + (x^5*Log[1 - a*x])/25 + (x^5*PolyLog[2, a*x])/5
```

Rubi [A] time = 0.0533019, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 43}

$$\frac{1}{5}x^5\text{PolyLog}(2,ax) - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} - \frac{\log(1-ax)}{25a^5} - \frac{x^4}{100a} + \frac{1}{25}x^5\log(1-ax) - \frac{x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[2, a*x],x]
```



```
[Out] -x/(25*a^4) - x^2/(50*a^3) - x^3/(75*a^2) - x^4/(100*a) - x^5/125 - Log[1 -
 a*x]/(25*a^5) + (x^5*Log[1 - a*x])/25 + (x^5*PolyLog[2, a*x])/5
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
```

```
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \mathrm{Li}_2(ax)\,dx &= \frac{1}{5}x^5\mathrm{Li}_2(ax) + \frac{1}{5}\int x^4\log(1-ax)\,dx \\
&= \frac{1}{25}x^5\log(1-ax) + \frac{1}{5}x^5\mathrm{Li}_2(ax) + \frac{1}{25}a\int \frac{x^5}{1-ax}\,dx \\
&= \frac{1}{25}x^5\log(1-ax) + \frac{1}{5}x^5\mathrm{Li}_2(ax) + \frac{1}{25}a\int \left(-\frac{1}{a^5} - \frac{x}{a^4} - \frac{x^2}{a^3} - \frac{x^3}{a^2} - \frac{x^4}{a} - \frac{1}{a^5(-1+ax)}\right)dx \\
&= -\frac{x}{25a^4} - \frac{x^2}{50a^3} - \frac{x^3}{75a^2} - \frac{x^4}{100a} - \frac{x^5}{125} - \frac{\log(1-ax)}{25a^5} + \frac{1}{25}x^5\log(1-ax) + \frac{1}{5}x^5\mathrm{Li}_2(ax)
\end{aligned}
$$

Mathematica [A] time = 0.04193, size = 73, normalized size = 0.85

$$
\frac{300a^5x^5\mathrm{PolyLog}(2,ax) - ax\left(12a^4x^4 + 15a^3x^3 + 20a^2x^2 + 30ax + 60\right) + 60\left(a^5x^5 - 1\right)\log(1-ax)}{1500a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(60 + 30*a*x + 20*a^2*x^2 + 15*a^3*x^3 + 12*a^4*x^4)) + 60*(-1 + a^5
*x^5)*Log[1 - a*x] + 300*a^5*x^5*PolyLog[2, a*x])/(1500*a^5)
```

Maple [A] time = 0.046, size = 76, normalized size = 0.9

$$-\frac{x}{25\,a^{4}}-\frac{\ln\left(-ax+1\right)}{25\,a^{5}}-\frac{x^{2}}{50\,a^{3}}+\frac{x^{5}\mathrm{polylog}\left(2,ax\right)}{5}+\frac{x^{5}\ln\left(-ax+1\right)}{25}+\frac{137}{1500\,a^{5}}-\frac{x^{4}}{100\,a}-\frac{x^{3}}{75\,a^{2}}-\frac{x^{5}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2,a*x),x)
```

```
[Out] -1/25*x/a^4-1/25*ln(-a*x+1)/a^5-1/50*x^2/a^3+1/5*x^5*polylog(2,a*x)+1/25*x^
5*ln(-a*x+1)+137/1500/a^5-1/100*x^4/a-1/75*x^3/a^2-1/125*x^5
```

Maxima [A] time = 0.964112, size = 97, normalized size = 1.13

$$\frac{300\,a^{5}x^{5}\mathrm{Li}_{2}\left(ax\right)-12\,a^{5}x^{5}-15\,a^{4}x^{4}-20\,a^{3}x^{3}-30\,a^{2}x^{2}-60\,ax+60\left(a^{5}x^{5}-1\right)\log\left(-ax+1\right)}{1500\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30*
a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

Fricas [A] time = 2.68893, size = 177, normalized size = 2.06

$$\frac{300\,a^{5}x^{5}\mathrm{Li}_{2}\left(ax\right)-12\,a^{5}x^{5}-15\,a^{4}x^{4}-20\,a^{3}x^{3}-30\,a^{2}x^{2}-60\,ax+60\left(a^{5}x^{5}-1\right)\log\left(-ax+1\right)}{1500\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/1500*(300*a^5*x^5*dilog(a*x) - 12*a^5*x^5 - 15*a^4*x^4 - 20*a^3*x^3 - 30*
a^2*x^2 - 60*a*x + 60*(a^5*x^5 - 1)*log(-a*x + 1))/a^5
```

Sympy [A] time = 15.264, size = 66, normalized size = 0.77

$$\begin{cases} -\frac{x^5 \operatorname{Li}_1(ax)}{25} + \frac{x^5 \operatorname{Li}_2(ax)}{5} - \frac{x^5}{125} - \frac{x^4}{100a} - \frac{x^3}{75a^2} - \frac{x^2}{50a^3} - \frac{x}{25a^4} + \frac{\operatorname{Li}_1(ax)}{25a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**5*polylog(1, a*x)/25 + x**5*polylog(2, a*x)/5 - x**5/125 - x
**4/(100*a) - x**3/(75*a**2) - x**2/(50*a**3) - x/(25*a**4) + polylog(1, a*
x)/(25*a**5), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4*dilog(a*x), x)
```

3.2 $\int x^3 \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=76

$$\frac{1}{4}x^4\text{PolyLog}(2, ax) - \frac{x^2}{32a^2} - \frac{x}{16a^3} - \frac{\log(1-ax)}{16a^4} - \frac{x^3}{48a} + \frac{1}{16}x^4\log(1-ax) - \frac{x^4}{64}$$

```
[Out] -x/(16*a^3) - x^2/(32*a^2) - x^3/(48*a) - x^4/64 - Log[1 - a*x]/(16*a^4) +
(x^4*Log[1 - a*x])/16 + (x^4*PolyLog[2, a*x])/4
```

Rubi [A] time = 0.0447573, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 43}

$$\frac{1}{4}x^4\text{PolyLog}(2, ax) - \frac{x^2}{32a^2} - \frac{x}{16a^3} - \frac{\log(1-ax)}{16a^4} - \frac{x^3}{48a} + \frac{1}{16}x^4\log(1-ax) - \frac{x^4}{64}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[2, a*x],x]
```

```
[Out] -x/(16*a^3) - x^2/(32*a^2) - x^3/(48*a) - x^4/64 - Log[1 - a*x]/(16*a^4) +
(x^4*Log[1 - a*x])/16 + (x^4*PolyLog[2, a*x])/4
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\text{Li}_2(ax)\,dx &= \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{4}\int x^3\log(1-ax)\,dx \\
&= \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{16}a\int \frac{x^4}{1-ax}\,dx \\
&= \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax) + \frac{1}{16}a\int \left(-\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)}\right)dx \\
&= -\frac{x}{16a^3} - \frac{x^2}{32a^2} - \frac{x^3}{48a} - \frac{x^4}{64} - \frac{\log(1-ax)}{16a^4} + \frac{1}{16}x^4\log(1-ax) + \frac{1}{4}x^4\text{Li}_2(ax)
\end{aligned}$$

Mathematica [A] time = 0.0327425, size = 65, normalized size = 0.86

$$\frac{48a^4x^4\text{PolyLog}(2,ax) - ax\left(3a^3x^3 + 4a^2x^2 + 6ax + 12\right) + 12\left(a^4x^4 - 1\right)\log(1-ax)}{192a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(12 + 6*a*x + 4*a^2*x^2 + 3*a^3*x^3)) + 12*(-1 + a^4*x^4)*Log[1 - a*
x] + 48*a^4*x^4*PolyLog[2, a*x])/(192*a^4)
```

Maple [A] time = 0.046, size = 68, normalized size = 0.9

$$\frac{x^4\text{polylog}(2,ax)}{4} + \frac{x^4\ln(-ax+1)}{16} - \frac{\ln(-ax+1)}{16\,a^4} - \frac{x^4}{64} - \frac{x^3}{48\,a} - \frac{x^2}{32\,a^2} - \frac{x}{16\,a^3} + \frac{25}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2,a*x),x)
```

```
[Out] 1/4*x^4*polylog(2,a*x)+1/16*x^4*ln(-a*x+1)-1/16*ln(-a*x+1)/a^4-1/64*x^4-1/4
8*x^3/a-1/32*x^2/a^2-1/16*x/a^3+25/192/a^4
```

Maxima [A] time = 0.98341, size = 86, normalized size = 1.13

$$\frac{48\,a^4x^4\mathrm{Li}_2\,(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\left(a^4x^4 - 1\right)\log\left(-ax + 1\right)}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x +
 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Fricas [A] time = 2.66203, size = 153, normalized size = 2.01

$$\frac{48\,a^4x^4\mathrm{Li}_2\,(ax) - 3\,a^4x^4 - 4\,a^3x^3 - 6\,a^2x^2 - 12\,ax + 12\left(a^4x^4 - 1\right)\log\left(-ax + 1\right)}{192\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/192*(48*a^4*x^4*dilog(a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x +
 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Sympy [A] time = 8.74118, size = 58, normalized size = 0.76

$$\begin{cases} -\frac{x^4\,\mathrm{Li}_1(ax)}{16} + \frac{x^4\,\mathrm{Li}_2(ax)}{4} - \frac{x^4}{64} - \frac{x^3}{48a} - \frac{x^2}{32a^2} - \frac{x}{16a^3} + \frac{\mathrm{Li}_1(ax)}{16a^4} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**4*polylog(1, a*x)/16 + x**4*polylog(2, a*x)/4 - x**4/64 - x*
*3/(48*a) - x**2/(32*a**2) - x/(16*a**3) + polylog(1, a*x)/(16*a**4), Ne(a,
 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \mathrm{Li}_2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(a*x), x)
```

3.3 $\int x^2 \textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=66

$$\frac{1}{3}x^3\text{PolyLog}(2, ax) - \frac{x}{9a^2} - \frac{\log(1 - ax)}{9a^3} - \frac{x^2}{18a} + \frac{1}{9}x^3\log(1 - ax) - \frac{x^3}{27}$$

```
[Out] -x/(9*a^2) - x^2/(18*a) - x^3/27 - Log[1 - a*x]/(9*a^3) + (x^3*Log[1 - a*x]
)/9 + (x^3*PolyLog[2, a*x])/3
```

Rubi [A] time = 0.0396548, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 43}

$$\frac{1}{3}x^3\text{PolyLog}(2, ax) - \frac{x}{9a^2} - \frac{\log(1 - ax)}{9a^3} - \frac{x^2}{18a} + \frac{1}{9}x^3\log(1 - ax) - \frac{x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x],x]
```

```
[Out] -x/(9*a^2) - x^2/(18*a) - x^3/27 - Log[1 - a*x]/(9*a^3) + (x^3*Log[1 - a*x]
)/9 + (x^3*PolyLog[2, a*x])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\text{Li}_2(ax)\,dx &= \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{3}\int x^2\log(1-ax)\,dx \\
&= \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{9}a\int \frac{x^3}{1-ax}\,dx \\
&= \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax) + \frac{1}{9}a\int \left(-\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)}\right)dx \\
&= -\frac{x}{9a^2} - \frac{x^2}{18a} - \frac{x^3}{27} - \frac{\log(1-ax)}{9a^3} + \frac{1}{9}x^3\log(1-ax) + \frac{1}{3}x^3\text{Li}_2(ax)
\end{aligned}$$

Mathematica [A] time = 0.0292432, size = 57, normalized size = 0.86

$$\frac{18a^3x^3\text{PolyLog}(2,ax) - ax\left(2a^2x^2+3ax+6\right) + 6\left(a^3x^3-1\right)\log(1-ax)}{54a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(6 + 3*a*x + 2*a^2*x^2)) + 6*(-1 + a^3*x^3)*Log[1 - a*x] + 18*a^3*x^
3*PolyLog[2, a*x])/(54*a^3)
```

Maple [A] time = 0.047, size = 60, normalized size = 0.9

$$\frac{x^3\text{polylog}(2,ax)}{3} + \frac{x^3\ln(-ax+1)}{9} - \frac{\ln(-ax+1)}{9a^3} - \frac{x^3}{27} - \frac{x^2}{18a} - \frac{x}{9a^2} + \frac{11}{54a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x),x)
```

```
[Out] 1/3*x^3*polylog(2,a*x)+1/9*x^3*ln(-a*x+1)-1/9*ln(-a*x+1)/a^3-1/27*x^3-1/18*
x^2/a-1/9*x/a^2+11/54/a^3
```

Maxima [A] time = 0.973299, size = 76, normalized size = 1.15

$$\frac{18\,a^3x^3\text{Li}_2\,(ax) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\left(a^3x^3 - 1\right)\log\left(-ax + 1\right)}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
1)*log(-a*x + 1))/a^3
```

Fricas [A] time = 2.59888, size = 132, normalized size = 2.

$$\frac{18\,a^3x^3\text{Li}_2\,(ax) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\left(a^3x^3 - 1\right)\log\left(-ax + 1\right)}{54\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/54*(18*a^3*x^3*dilog(a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 -
1)*log(-a*x + 1))/a^3
```

Sympy [A] time = 4.42809, size = 49, normalized size = 0.74

$$\begin{cases} -\frac{x^3\operatorname{Li}_1(ax)}{9} + \frac{x^3\operatorname{Li}_2(ax)}{3} - \frac{x^3}{27} - \frac{x^2}{18a} - \frac{x}{9a^2} + \frac{\operatorname{Li}_1(ax)}{9a^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**3*polylog(1, a*x)/9 + x**3*polylog(2, a*x)/3 - x**3/27 - x**
2/(18*a) - x/(9*a**2) + polylog(1, a*x)/(9*a**3), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_2\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x), x)
```

3.4 $\int x\textbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=56

$$\frac{1}{2}x^2\text{PolyLog}(2, ax) - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2\log(1-ax) - \frac{x}{4a} - \frac{x^2}{8}$$

```
[Out] -x/(4*a) - x^2/8 - Log[1 - a*x]/(4*a^2) + (x^2*Log[1 - a*x])/4 + (x^2*PolyL
og[2, a*x])/2
```

Rubi [A] time = 0.0282725, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6591, 2395, 43}

$$\frac{1}{2}x^2\text{PolyLog}(2, ax) - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2\log(1-ax) - \frac{x}{4a} - \frac{x^2}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x],x]
```

```
[Out] -x/(4*a) - x^2/8 - Log[1 - a*x]/(4*a^2) + (x^2*Log[1 - a*x])/4 + (x^2*PolyL
og[2, a*x])/2
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\text{Li}_2(ax)\,dx &= \frac{1}{2}x^2\text{Li}_2(ax) + \frac{1}{2}\int x\log(1-ax)\,dx \\
&= \frac{1}{4}x^2\log(1-ax) + \frac{1}{2}x^2\text{Li}_2(ax) + \frac{1}{4}a\int \frac{x^2}{1-ax}\,dx \\
&= \frac{1}{4}x^2\log(1-ax) + \frac{1}{2}x^2\text{Li}_2(ax) + \frac{1}{4}a\int \left(-\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)}\right)dx \\
&= -\frac{x}{4a} - \frac{x^2}{8} - \frac{\log(1-ax)}{4a^2} + \frac{1}{4}x^2\log(1-ax) + \frac{1}{2}x^2\text{Li}_2(ax)
\end{aligned}$$

Mathematica [A] time = 0.0238254, size = 48, normalized size = 0.86

$$\frac{4a^2x^2\text{PolyLog}(2,ax) + 2\left(a^2x^2-1\right)\log(1-ax) - ax(ax+2)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x],x]
```

```
[Out] (-(a*x*(2 + a*x)) + 2*(-1 + a^2*x^2)*Log[1 - a*x] + 4*a^2*x^2*PolyLog[2, a*
x])/(8*a^2)
```

Maple [A] time = 0.046, size = 52, normalized size = 0.9

$$\frac{x^2\text{polylog}\,(2,ax)}{2} + \frac{x^2\ln(-ax+1)}{4} - \frac{\ln(-ax+1)}{4\,a^2} - \frac{x^2}{8} - \frac{x}{4\,a} + \frac{3}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x),x)
```

```
[Out] 1/2*x^2*polylog(2,a*x)+1/4*x^2*ln(-a*x+1)-1/4*ln(-a*x+1)/a^2-1/8*x^2-1/4*x/
a+3/8/a^2
```

Maxima [A] time = 0.965084, size = 65, normalized size = 1.16

$$\frac{4\,a^2x^2\mathrm{Li}_2\,(ax) - a^2x^2 - 2\,ax + 2\left(a^2x^2 - 1\right)\log\left(-ax + 1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1)
)/a^2
```

Fricas [A] time = 2.58187, size = 111, normalized size = 1.98

$$\frac{4\,a^2x^2\mathrm{Li}_2\,(ax) - a^2x^2 - 2\,ax + 2\left(a^2x^2 - 1\right)\log\left(-ax + 1\right)}{8\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*x^2*dilog(a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1)
)/a^2
```

Sympy [A] time = 2.43658, size = 41, normalized size = 0.73

$$\begin{cases} -\frac{x^2\,\mathrm{Li}_1(ax)}{4} + \frac{x^2\,\mathrm{Li}_2(ax)}{2} - \frac{x^2}{8} - \frac{x}{4a} + \frac{\mathrm{Li}_1(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x)
```

```
[Out] Piecewise((-x**2*polylog(1, a*x)/4 + x**2*polylog(2, a*x)/2 - x**2/8 - x/(4
*a) + polylog(1, a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x), x)
```

3.5 $\int \text{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=29

$$x\text{PolyLog}(2, ax) - \frac{(1 - ax)\log(1 - ax)}{a} - x$$

```
[Out] -x - ((1 - a*x)*Log[1 - a*x])/a + x*PolyLog[2, a*x]
```

Rubi [A] time = 0.0085078, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6586, 2389, 2295}

$$x\text{PolyLog}(2, ax) - \frac{(1 - ax)\log(1 - ax)}{a} - x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x],x]
```

```
[Out] -x - ((1 - a*x)*Log[1 - a*x])/a + x*PolyLog[2, a*x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_2(ax)\,dx &= x\mathrm{Li}_2(ax) + \int \log(1-ax)\,dx \\ &= x\mathrm{Li}_2(ax) - \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ax)}{a} \\ &= -x - \frac{(1-ax)\log(1-ax)}{a} + x\mathrm{Li}_2(ax)\end{aligned}$$

Mathematica [A] time = 0.011601, size = 26, normalized size = 0.9

$$x\mathrm{PolyLog}(2,ax) + \left(x - \frac{1}{a}\right)\log(1-ax) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x],x]
```

```
[Out] -x + (-a^(-1) + x)*Log[1 - a*x] + x*PolyLog[2, a*x]
```

Maple [A] time = 0.045, size = 36, normalized size = 1.2

$$x\mathrm{polylog}\,(2,ax) + \ln\left(-ax+1\right)x - x - \frac{\ln\left(-ax+1\right)}{a} + a^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x),x)
```

```
[Out] x*polylog(2,a*x)+ln(-a*x+1)*x-x-1/a*ln(-a*x+1)+1/a
```

Maxima [A] time = 0.980908, size = 39, normalized size = 1.34

$$\frac{ax\mathrm{Li}_2\left(ax\right) - ax + (ax-1)\log\left(-ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] (a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Fricas [A] time = 2.63468, size = 70, normalized size = 2.41

$$\frac{ax\mathrm{Li}_2\,(ax) - ax + (ax - 1)\log\left(-ax + 1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] (a*x*dilog(a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Sympy [A] time = 1.20944, size = 22, normalized size = 0.76

$$\begin{cases} -x\,\mathrm{Li}_1\,(ax) + x\,\mathrm{Li}_2\,(ax) - x + \frac{\mathrm{Li}_1(ax)}{a} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x)
```

```
[Out] Piecewise((-x*polylog(1, a*x) + x*polylog(2, a*x) - x + polylog(1, a*x)/a,
Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x), x)
```

3.6 $\int \frac{\mathbf{PolyLog}(2,ax)}{x}\,dx$

Optimal. Leaf size=5

$$\text{PolyLog}(3,ax)$$

[Out] PolyLog[3, a*x]

Rubi [A] time = 0.0090331, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6589}

$$\text{PolyLog}(3,ax)$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a*x]/x,x]

[Out] PolyLog[3, a*x]

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax)}{x}\,dx=\text{Li}_3(ax)$$

Mathematica [A] time = 0.0012581, size = 5, normalized size = 1.

$$\text{PolyLog}(3,ax)$$

Antiderivative was successfully verified.

[In] Integrate[PolyLog[2, a*x]/x,x]

```
[Out] PolyLog[3, a*x]
```

Maple [A] time = 0.043, size = 6, normalized size = 1.2

$$\text{polylog}(3, ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x,x)

[Out] polylog(3,a*x)
```

Maxima [A] time = 0.975433, size = 7, normalized size = 1.4

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="maxima")

[Out] polylog(3, a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="fricas")

[Out] integral(dilog(a*x)/x, x)
```

Sympy [A] time = 1.71222, size = 3, normalized size = 0.6

$$\text{Li}_3(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x)
```

```
[Out] polylog(3, a*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x, x)
```

3.7 $\int \frac{\textbf{PolyLog}(2,ax)}{x^2}\,dx$

Optimal. Leaf size=36

$$-\frac{\text{PolyLog}(2,ax)}{x}+a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}$$

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

Rubi [A] time = 0.0218984, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6591, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2,ax)}{x}+a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^2,x]
```

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^2}\,dx &= -\frac{\text{Li}_2(ax)}{x} - \int \frac{\log(1-ax)}{x^2}\,dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} + a\int \frac{1}{x(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} + a\int \frac{1}{x}\,dx + a^2\int \frac{1}{1-ax}\,dx \\
&= a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0097221, size = 36, normalized size = 1.

$$-\frac{\text{PolyLog}(2,ax)}{x} + a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^2,x]
```

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x
```

Maple [A] time = 0.119, size = 40, normalized size = 1.1

$$-\frac{\text{polylog}(2,ax)}{x} + a\ln(-ax) - a\ln(-ax+1) + \frac{\ln(-ax+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^2,x)
```

```
[Out] -polylog(2,a*x)/x+a*ln(-a*x)-a*ln(-a*x+1)+ln(-a*x+1)/x
```

Maxima [A] time = 1.00914, size = 38, normalized size = 1.06

$$a \log(x) - \frac{(ax-1)\log(-ax+1) + \mathrm{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="maxima")
```

```
[Out] a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x))/x
```

Fricas [A] time = 2.78236, size = 88, normalized size = 2.44

$$-\frac{ax\log(ax-1) - ax\log(x) + \mathrm{Li}_2(ax) - \log(-ax+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + dilog(a*x) - log(-a*x + 1))/x
```

Sympy [A] time = 1.56346, size = 24, normalized size = 0.67

$$a \log(x) + a \mathrm{Li}_1(ax) - \frac{\mathrm{Li}_1(ax)}{x} - \frac{\mathrm{Li}_2(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**2,x)
```

```
[Out] a*log(x) + a*polylog(1, a*x) - polylog(1, a*x)/x - polylog(2, a*x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^2, x)
```

3.8 $\int \frac{\textbf{PolyLog}(2,ax)}{x^3}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{PolyLog}(2,ax)}{2x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{a}{4x}$$

```
[Out] -a/(4*x) + (a^2*Log[x])/4 - (a^2*Log[1 - a*x])/4 + Log[1 - a*x]/(4*x^2) - P
olyLog[2, a*x]/(2*x^2)
```

Rubi [A] time = 0.0332573, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{2x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{a}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^3,x]
```

```
[Out] -a/(4*x) + (a^2*Log[x])/4 - (a^2*Log[1 - a*x])/4 + Log[1 - a*x]/(4*x^2) - P
olyLog[2, a*x]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax)}{x^3}\,dx &= -\frac{\text{Li}_2(ax)}{2x^2}-\frac{1}{2}\int \frac{\log(1-ax)}{x^3}\,dx\\ &= \frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}+\frac{1}{4}a\int \frac{1}{x^2(1-ax)}\,dx\\ &= \frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}+\frac{1}{4}a\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx\\ &= -\frac{a}{4x}+\frac{1}{4}a^2\log(x)-\frac{1}{4}a^2\log(1-ax)+\frac{\log(1-ax)}{4x^2}-\frac{\text{Li}_2(ax)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0221772, size = 50, normalized size = 0.86

$$\frac{-2\text{PolyLog}(2,ax)+a^2x^2\log(x)-a^2x^2\log(1-ax)-ax+\log(1-ax)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^3,x]
```

```
[Out] (-(a*x) + a^2*x^2*Log[x] + Log[1 - a*x] - a^2*x^2*Log[1 - a*x] - 2*PolyLog[
2, a*x])/(4*x^2)
```

Maple [A] time = 0.118, size = 52, normalized size = 0.9

$$-\frac{\text{polylog}(2,ax)}{2x^2}-\frac{a}{4x}+\frac{a^2\ln(-ax)}{4}-\frac{a^2\ln(-ax+1)}{4}+\frac{\ln(-ax+1)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^3,x)
```

```
[Out] -1/2*polylog(2,a*x)/x^2-1/4*a/x+1/4*a^2*ln(-a*x)-1/4*a^2*ln(-a*x+1)+1/4*ln(
-a*x+1)/x^2
```

Maxima [A] time = 0.981041, size = 54, normalized size = 0.93

$$\frac{1}{4}a^2\log(x) - \frac{ax + \left(a^2x^2 - 1\right)\log(-ax + 1) + 2\,\mathrm{Li}_2(ax)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*log(x) - 1/4*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x))/x^2
```

Fricas [A] time = 2.62905, size = 117, normalized size = 2.02

$$-\frac{a^2x^2\log(ax - 1) - a^2x^2\log(x) + ax + 2\,\mathrm{Li}_2(ax) - \log(-ax + 1)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*dilog(a*x) - log(-a*x
 + 1))/x^2
```

Sympy [A] time = 2.96484, size = 42, normalized size = 0.72

$$\frac{a^2\log(x)}{4} + \frac{a^2\,\mathrm{Li}_1(ax)}{4} - \frac{a}{4x} - \frac{\mathrm{Li}_1(ax)}{4x^2} - \frac{\mathrm{Li}_2(ax)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**3,x)
```

```
[Out] a**2*log(x)/4 + a**2*polylog(1, a*x)/4 - a/(4*x) - polylog(1, a*x)/(4*x**2)
 - polylog(2, a*x)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^3, x)
```

3.9 $\int \frac{\textbf{PolyLog}(2,ax)}{x^4}\,dx$

Optimal. Leaf size=68

$$-\frac{\text{PolyLog}(2,ax)}{3x^3}-\frac{a^2}{9x}+\frac{1}{9}a^3\log(x)-\frac{1}{9}a^3\log(1-ax)-\frac{a}{18x^2}+\frac{\log(1-ax)}{9x^3}$$

```
[Out] -a/(18*x^2) - a^2/(9*x) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x])/9 + Log[1 - a
*x]/(9*x^3) - PolyLog[2, a*x]/(3*x^3)
```

Rubi [A] time = 0.0363225, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{3x^3}-\frac{a^2}{9x}+\frac{1}{9}a^3\log(x)-\frac{1}{9}a^3\log(1-ax)-\frac{a}{18x^2}+\frac{\log(1-ax)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^4,x]
```

```
[Out] -a/(18*x^2) - a^2/(9*x) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x])/9 + Log[1 - a
*x]/(9*x^3) - PolyLog[2, a*x]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^4}\,dx &= -\frac{\text{Li}_2(ax)}{3x^3}-\frac{1}{3}\int \frac{\log(1-ax)}{x^4}\,dx\\
&= \frac{\log(1-ax)}{9x^3}-\frac{\text{Li}_2(ax)}{3x^3}+\frac{1}{9}a\int \frac{1}{x^3(1-ax)}\,dx\\
&= \frac{\log(1-ax)}{9x^3}-\frac{\text{Li}_2(ax)}{3x^3}+\frac{1}{9}a\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx\\
&= -\frac{a}{18x^2}-\frac{a^2}{9x}+\frac{1}{9}a^3\log(x)-\frac{1}{9}a^3\log(1-ax)+\frac{\log(1-ax)}{9x^3}-\frac{\text{Li}_2(ax)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0371128, size = 52, normalized size = 0.76

$$-\frac{6\text{PolyLog}(2,ax)-2a^3x^3\log(x)+2\left(a^3x^3-1\right)\log(1-ax)+ax(2ax+1)}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^4,x]
```

```
[Out] -(a*x*(1 + 2*a*x) - 2*a^3*x^3*Log[x] + 2*(-1 + a^3*x^3)*Log[1 - a*x] + 6*Po
lyLog[2, a*x])/(18*x^3)
```

Maple [A] time = 0.119, size = 60, normalized size = 0.9

$$-\frac{\text{polylog}\,(2,ax)}{3\,x^3}-\frac{a^2}{9\,x}+\frac{a^3\ln(-ax)}{9}-\frac{a}{18\,x^2}-\frac{a^3\ln(-ax+1)}{9}+\frac{\ln(-ax+1)}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^4,x)
```

```
[Out] -1/3*polylog(2,a*x)/x^3-1/9*a^2/x+1/9*a^3*ln(-a*x)-1/18*a/x^2-1/9*a^3*ln(-a
*x+1)+1/9*ln(-a*x+1)/x^3
```

Maxima [A] time = 0.989735, size = 66, normalized size = 0.97

$$\frac{1}{9}\,a^3\log\left(x\right) - \frac{2\,a^2x^2 + ax + 2\left(a^3x^3 - 1\right)\log\left(-ax + 1\right) + 6\,\mathrm{Li}_2\left(ax\right)}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*log(x) - 1/18*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) + 6*
dilog(a*x))/x^3
```

Fricas [A] time = 2.68791, size = 143, normalized size = 2.1

$$-\frac{2\,a^3x^3\log\left(ax - 1\right) - 2\,a^3x^3\log\left(x\right) + 2\,a^2x^2 + ax + 6\,\mathrm{Li}_2\left(ax\right) - 2\,\log\left(-ax + 1\right)}{18\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*dilo
g(a*x) - 2*log(-a*x + 1))/x^3
```

Sympy [A] time = 5.56873, size = 51, normalized size = 0.75

$$\frac{a^3\log\left(x\right)}{9} + \frac{a^3\,\mathrm{Li}_1\left(ax\right)}{9} - \frac{a^2}{9x} - \frac{a}{18x^2} - \frac{\mathrm{Li}_1\left(ax\right)}{9x^3} - \frac{\mathrm{Li}_2\left(ax\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**4,x)
```

```
[Out] a**3*log(x)/9 + a**3*polylog(1, a*x)/9 - a**2/(9*x) - a/(18*x**2) - polylog
(1, a*x)/(9*x**3) - polylog(2, a*x)/(3*x**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^4, x)
```

3.10 $\int \frac{\textbf{PolyLog}(2,ax)}{x^5}\,dx$

Optimal. Leaf size=78

$$-\frac{\text{PolyLog}(2,ax)}{4x^4}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)-\frac{a}{48x^3}+\frac{\log(1-ax)}{16x^4}$$

```
[Out] -a/(48*x^3) - a^2/(32*x^2) - a^3/(16*x) + (a^4*Log[x])/16 - (a^4*Log[1 - a*
x])/16 + Log[1 - a*x]/(16*x^4) - PolyLog[2, a*x]/(4*x^4)
```

Rubi [A] time = 0.0392817, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{4x^4}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)-\frac{a}{48x^3}+\frac{\log(1-ax)}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/x^5,x]
```

```
[Out] -a/(48*x^3) - a^2/(32*x^2) - a^3/(16*x) + (a^4*Log[x])/16 - (a^4*Log[1 - a*
x])/16 + Log[1 - a*x]/(16*x^4) - PolyLog[2, a*x]/(4*x^4)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax)}{x^5}\,dx &= -\frac{\text{Li}_2(ax)}{4x^4}-\frac{1}{4}\int \frac{\log(1-ax)}{x^5}\,dx\\
&= \frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}+\frac{1}{16}a\int \frac{1}{x^4(1-ax)}\,dx\\
&= \frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}+\frac{1}{16}a\int \left(\frac{1}{x^4}+\frac{a}{x^3}+\frac{a^2}{x^2}+\frac{a^3}{x}-\frac{a^4}{-1+ax}\right)dx\\
&= -\frac{a}{48x^3}-\frac{a^2}{32x^2}-\frac{a^3}{16x}+\frac{1}{16}a^4\log(x)-\frac{1}{16}a^4\log(1-ax)+\frac{\log(1-ax)}{16x^4}-\frac{\text{Li}_2(ax)}{4x^4}
\end{aligned}
$$

Mathematica [A] time = 0.034528, size = 60, normalized size = 0.77

$$
-\frac{24\text{PolyLog}(2,ax)+ax\left(6a^2x^2+3ax+2\right)-6a^4x^4\log(x)+6\left(a^4x^4-1\right)\log(1-ax)}{96x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/x^5,x]
```

```
[Out] -(a*x*(2 + 3*a*x + 6*a^2*x^2) - 6*a^4*x^4*Log[x] + 6*(-1 + a^4*x^4)*Log[1 -
 a*x] + 24*PolyLog[2, a*x])/(96*x^4)
```

Maple [A] time = 0.128, size = 68, normalized size = 0.9

$$
-\frac{\text{polylog}(2,ax)}{4x^4}-\frac{a^3}{16x}-\frac{a}{48x^3}-\frac{a^2}{32x^2}+\frac{a^4\ln(-ax)}{16}-\frac{a^4\ln(-ax+1)}{16}+\frac{\ln(-ax+1)}{16x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/x^5,x)
```

```
[Out] -1/4*polylog(2,a*x)/x^4-1/16*a^3/x-1/48*a/x^3-1/32*a^2/x^2+1/16*a^4*ln(-a*x
)-1/16*a^4*ln(-a*x+1)+1/16*ln(-a*x+1)/x^4
```

Maxima [A] time = 0.986238, size = 78, normalized size = 1.

$$\frac{1}{16}\,a^4\log\left(x\right)-\frac{6\,a^3x^3+3\,a^2x^2+2\,ax+6\,\left(a^4x^4-1\right)\log\left(-ax+1\right)+24\,\mathrm{Li}_2\left(ax\right)}{96\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*a^4*log(x) - 1/96*(6*a^3*x^3 + 3*a^2*x^2 + 2*a*x + 6*(a^4*x^4 - 1)*log
(-a*x + 1) + 24*dilog(a*x))/x^4
```

Fricas [A] time = 2.78575, size = 163, normalized size = 2.09

$$-\frac{6\,a^4x^4\log\left(ax-1\right)-6\,a^4x^4\log\left(x\right)+6\,a^3x^3+3\,a^2x^2+2\,ax+24\,\mathrm{Li}_2\left(ax\right)-6\,\log\left(-ax+1\right)}{96\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/96*(6*a^4*x^4*log(a*x - 1) - 6*a^4*x^4*log(x) + 6*a^3*x^3 + 3*a^2*x^2 +
2*a*x + 24*dilog(a*x) - 6*log(-a*x + 1))/x^4
```

Sympy [A] time = 11.2801, size = 60, normalized size = 0.77

$$\frac{a^4\log\left(x\right)}{16}+\frac{a^4\,\mathrm{Li}_1\left(ax\right)}{16}-\frac{a^3}{16x}-\frac{a^2}{32x^2}-\frac{a}{48x^3}-\frac{\mathrm{Li}_1\left(ax\right)}{16x^4}-\frac{\mathrm{Li}_2\left(ax\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x**5,x)
```

```
[Out] a**4*log(x)/16 + a**4*polylog(1, a*x)/16 - a**3/(16*x) - a**2/(32*x**2) - a
/(48*x**3) - polylog(1, a*x)/(16*x**4) - polylog(2, a*x)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/x^5, x)
```

3.11 $\int x^3\mathbf{PolyLog}(3,ax)\,dx$

Optimal. Leaf size=88

$$-\frac{1}{16}x^4\text{PolyLog}(2,ax)+\frac{1}{4}x^4\text{PolyLog}(3,ax)+\frac{x^2}{128a^2}+\frac{x}{64a^3}+\frac{\log(1-ax)}{64a^4}+\frac{x^3}{192a}-\frac{1}{64}x^4\log(1-ax)+\frac{x^4}{256}$$

```
[Out] x/(64*a^3) + x^2/(128*a^2) + x^3/(192*a) + x^4/256 + Log[1 - a*x]/(64*a^4)
- (x^4*Log[1 - a*x])/64 - (x^4*PolyLog[2, a*x])/16 + (x^4*PolyLog[3, a*x])/
4
```

Rubi [A] time = 0.0573359, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 43}

$$-\frac{1}{16}x^4\text{PolyLog}(2,ax)+\frac{1}{4}x^4\text{PolyLog}(3,ax)+\frac{x^2}{128a^2}+\frac{x}{64a^3}+\frac{\log(1-ax)}{64a^4}+\frac{x^3}{192a}-\frac{1}{64}x^4\log(1-ax)+\frac{x^4}{256}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[3, a*x],x]
```

```
[Out] x/(64*a^3) + x^2/(128*a^2) + x^3/(192*a) + x^4/256 + Log[1 - a*x]/(64*a^4)
- (x^4*Log[1 - a*x])/64 - (x^4*PolyLog[2, a*x])/16 + (x^4*PolyLog[3, a*x])/
4
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \mathrm{Li}_3(ax)\,dx &= \frac{1}{4}x^4\mathrm{Li}_3(ax) - \frac{1}{4}\int x^3\mathrm{Li}_2(ax)\,dx \\
&= -\frac{1}{16}x^4\mathrm{Li}_2(ax) + \frac{1}{4}x^4\mathrm{Li}_3(ax) - \frac{1}{16}\int x^3\log(1-ax)\,dx \\
&= -\frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\mathrm{Li}_2(ax) + \frac{1}{4}x^4\mathrm{Li}_3(ax) - \frac{1}{64}a\int \frac{x^4}{1-ax}\,dx \\
&= -\frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\mathrm{Li}_2(ax) + \frac{1}{4}x^4\mathrm{Li}_3(ax) - \frac{1}{64}a\int \left(-\frac{1}{a^4} - \frac{x}{a^3} - \frac{x^2}{a^2} - \frac{x^3}{a} - \frac{1}{a^4(-1+ax)}\right)dx \\
&= \frac{x}{64a^3} + \frac{x^2}{128a^2} + \frac{x^3}{192a} + \frac{x^4}{256} + \frac{\log(1-ax)}{64a^4} - \frac{1}{64}x^4\log(1-ax) - \frac{1}{16}x^4\mathrm{Li}_2(ax) + \frac{1}{4}x^4\mathrm{Li}_3(ax)
\end{aligned}$$

Mathematica [A] time = 0.0118425, size = 86, normalized size = 0.98

$$\frac{-48a^4x^4\mathrm{PolyLog}(2,ax) + 192a^4x^4\mathrm{PolyLog}(3,ax) + 3a^4x^4 + 4a^3x^3 + 6a^2x^2 - 12a^4x^4\log(1-ax) + 12ax + 12\log(1-ax}{768a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[3, a*x],x]
```

```
[Out] (12*a*x + 6*a^2*x^2 + 4*a^3*x^3 + 3*a^4*x^4 + 12*Log[1 - a*x] - 12*a^4*x^4*
Log[1 - a*x] - 48*a^4*x^4*PolyLog[2, a*x] + 192*a^4*x^4*PolyLog[3, a*x])/(7
68*a^4)
```

Maple [A] time = 0.161, size = 78, normalized size = 0.9

$$-\frac{1}{a^4}\left(-\frac{xa\left(15x^3a^3 + 20a^2x^2 + 30ax + 60\right)}{3840} - \frac{\left(-5x^4a^4 + 5\right)\ln(-ax+1)}{320} + \frac{x^4a^4\mathrm{polylog}(2,ax)}{16} - \frac{x^4a^4\mathrm{polylog}(3,ax)}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3,a*x),x)
```

```
[Out] -1/a^4*(-1/3840*x*a*(15*a^3*x^3+20*a^2*x^2+30*a*x+60)-1/320*(-5*a^4*x^4+5)*
ln(-a*x+1)+1/16*x^4*a^4*polylog(2,a*x)-1/4*x^4*a^4*polylog(3,a*x))
```

Maxima [A] time = 0.998854, size = 104, normalized size = 1.18

$$-\frac{48\,a^4x^4\mathrm{Li}_2\left(ax\right)-192\,a^4x^4\mathrm{Li}_3(ax)-3\,a^4x^4-4\,a^3x^3-6\,a^2x^2-12\,ax+12\left(a^4x^4-1\right)\log\left(-ax+1\right)}{768\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/768*(48*a^4*x^4*dilog(a*x) - 192*a^4*x^4*polylog(3, a*x) - 3*a^4*x^4 - 4
*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^4*x^4 - 1)*log(-a*x + 1))/a^4
```

Fricas [C] time = 2.54882, size = 244, normalized size = 2.77

$$-\frac{48\,a^4x^4\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-192\,a^4x^4\mathrm{polylog}\left(3,ax\right)-3\,a^4x^4-4\,a^3x^3-6\,a^2x^2-12\,ax+12\left(a^4x^4-1\right.}{768\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/768*(48*a^4*x^4*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 192*a^
4*x^4*polylog(3, a*x) - 3*a^4*x^4 - 4*a^3*x^3 - 6*a^2*x^2 - 12*a*x + 12*(a^
4*x^4 - 1)*log(-a*x + 1))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(3,a*x),x)
```

```
[Out] Integral(x**3*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \mathrm{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*polylog(3, a*x), x)
```

3.12 $\int x^2 \mathbf{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=78

$$-\frac{1}{9}x^3\text{PolyLog}(2,ax)+\frac{1}{3}x^3\text{PolyLog}(3,ax)+\frac{x}{27a^2}+\frac{\log(1-ax)}{27a^3}+\frac{x^2}{54a}-\frac{1}{27}x^3\log(1-ax)+\frac{x^3}{81}$$

```
[Out] x/(27*a^2) + x^2/(54*a) + x^3/81 + Log[1 - a*x]/(27*a^3) - (x^3*Log[1 - a*x
])/27 - (x^3*PolyLog[2, a*x])/9 + (x^3*PolyLog[3, a*x])/3
```

Rubi [A] time = 0.0506273, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 43}

$$-\frac{1}{9}x^3\text{PolyLog}(2,ax)+\frac{1}{3}x^3\text{PolyLog}(3,ax)+\frac{x}{27a^2}+\frac{\log(1-ax)}{27a^3}+\frac{x^2}{54a}-\frac{1}{27}x^3\log(1-ax)+\frac{x^3}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, a*x],x]
```

```
[Out] x/(27*a^2) + x^2/(54*a) + x^3/81 + Log[1 - a*x]/(27*a^3) - (x^3*Log[1 - a*x
])/27 - (x^3*PolyLog[2, a*x])/9 + (x^3*PolyLog[3, a*x])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_3(ax)\,dx &= \frac{1}{3}x^3\text{Li}_3(ax) - \frac{1}{3}\int x^2\text{Li}_2(ax)\,dx\\ &= -\frac{1}{9}x^3\text{Li}_2(ax) + \frac{1}{3}x^3\text{Li}_3(ax) - \frac{1}{9}\int x^2\log(1-ax)\,dx\\ &= -\frac{1}{27}x^3\log(1-ax) - \frac{1}{9}x^3\text{Li}_2(ax) + \frac{1}{3}x^3\text{Li}_3(ax) - \frac{1}{27}a\int\frac{x^3}{1-ax}\,dx\\ &= -\frac{1}{27}x^3\log(1-ax) - \frac{1}{9}x^3\text{Li}_2(ax) + \frac{1}{3}x^3\text{Li}_3(ax) - \frac{1}{27}a\int\left(-\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)}\right)dx\\ &= \frac{x}{27a^2} + \frac{x^2}{54a} + \frac{x^3}{81} + \frac{\log(1-ax)}{27a^3} - \frac{1}{27}x^3\log(1-ax) - \frac{1}{9}x^3\text{Li}_2(ax) + \frac{1}{3}x^3\text{Li}_3(ax)\end{aligned}$$

Mathematica [A] time = 0.0113884, size = 78, normalized size = 1.

$$\frac{-18a^3x^3\text{PolyLog}(2,ax) + 54a^3x^3\text{PolyLog}(3,ax) + 2a^3x^3 + 3a^2x^2 - 6a^3x^3\log(1-ax) + 6ax + 6\log(1-ax)}{162a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, a*x],x]
```

```
[Out] (6*a*x + 3*a^2*x^2 + 2*a^3*x^3 + 6*Log[1 - a*x] - 6*a^3*x^3*Log[1 - a*x] -
18*a^3*x^3*PolyLog[2, a*x] + 54*a^3*x^3*PolyLog[3, a*x])/(162*a^3)
```

Maple [A] time = 0.164, size = 69, normalized size = 0.9

$$\frac{1}{a^3}\left(\frac{xa\left(4a^2x^2 + 6ax + 12\right)}{324} + \frac{\left(-4x^3a^3 + 4\right)\ln(-ax+1)}{108} - \frac{x^3a^3\text{polylog}(2,ax)}{9} + \frac{x^3a^3\text{polylog}(3,ax)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x),x)
```

```
[Out] 1/a^3*(1/324*x*a*(4*a^2*x^2+6*a*x+12)+1/108*(-4*a^3*x^3+4)*ln(-a*x+1)-1/9*x
^3*a^3*polylog(2,a*x)+1/3*x^3*a^3*polylog(3,a*x))
```

Maxima [A] time = 0.995622, size = 93, normalized size = 1.19

$$-\frac{18\,a^3x^3\mathrm{Li}_2\left(ax\right) - 54\,a^3x^3\mathrm{Li}_3(ax) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\left(a^3x^3 - 1\right)\log\left(-ax + 1\right)}{162\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/162*(18*a^3*x^3*dilog(a*x) - 54*a^3*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3*
a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(-a*x + 1))/a^3
```

Fricas [C] time = 2.67641, size = 224, normalized size = 2.87

$$-\frac{18\,a^3x^3\%\mathrm{iint}\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 54\,a^3x^3\mathrm{polylog}\left(3, ax\right) - 2\,a^3x^3 - 3\,a^2x^2 - 6\,ax + 6\left(a^3x^3 - 1\right)\log\left(-ax +\right.}{162\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/162*(18*a^3*x^3*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 54*a^3
*x^3*polylog(3, a*x) - 2*a^3*x^3 - 3*a^2*x^2 - 6*a*x + 6*(a^3*x^3 - 1)*log(
-a*x + 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x),x)
```

```
[Out] Integral(x**2*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_3(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, a*x), x)
```

3.13 $\int x\textbf{PolyLog}(3, ax)\,dx$

Optimal. Leaf size=68

$$-\frac{1}{4}x^2\text{PolyLog}(2,ax)+\frac{1}{2}x^2\text{PolyLog}(3,ax)+\frac{\log(1-ax)}{8a^2}-\frac{1}{8}x^2\log(1-ax)+\frac{x}{8a}+\frac{x^2}{16}$$

```
[Out] x/(8*a) + x^2/16 + Log[1 - a*x]/(8*a^2) - (x^2*Log[1 - a*x])/8 - (x^2*PolyL
og[2, a*x])/4 + (x^2*PolyLog[3, a*x])/2
```

Rubi [A] time = 0.0351582, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6591, 2395, 43}

$$-\frac{1}{4}x^2\text{PolyLog}(2,ax)+\frac{1}{2}x^2\text{PolyLog}(3,ax)+\frac{\log(1-ax)}{8a^2}-\frac{1}{8}x^2\log(1-ax)+\frac{x}{8a}+\frac{x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x],x]
```

```
[Out] x/(8*a) + x^2/16 + Log[1 - a*x]/(8*a^2) - (x^2*Log[1 - a*x])/8 - (x^2*PolyL
og[2, a*x])/4 + (x^2*PolyLog[3, a*x])/2
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x\text{Li}_3(ax)\,dx &= \frac{1}{2}x^2\text{Li}_3(ax)-\frac{1}{2}\int x\text{Li}_2(ax)\,dx\\
&= -\frac{1}{4}x^2\text{Li}_2(ax)+\frac{1}{2}x^2\text{Li}_3(ax)-\frac{1}{4}\int x\log(1-ax)\,dx\\
&= -\frac{1}{8}x^2\log(1-ax)-\frac{1}{4}x^2\text{Li}_2(ax)+\frac{1}{2}x^2\text{Li}_3(ax)-\frac{1}{8}a\int\frac{x^2}{1-ax}\,dx\\
&= -\frac{1}{8}x^2\log(1-ax)-\frac{1}{4}x^2\text{Li}_2(ax)+\frac{1}{2}x^2\text{Li}_3(ax)-\frac{1}{8}a\int\left(-\frac{1}{a^2}-\frac{x}{a}-\frac{1}{a^2(-1+ax)}\right)dx\\
&= \frac{x}{8a}+\frac{x^2}{16}+\frac{\log(1-ax)}{8a^2}-\frac{1}{8}x^2\log(1-ax)-\frac{1}{4}x^2\text{Li}_2(ax)+\frac{1}{2}x^2\text{Li}_3(ax)
\end{aligned}$$

Mathematica [A] time = 0.0090438, size = 69, normalized size = 1.01

$$\frac{-4a^2x^2\text{PolyLog}(2,ax)+8a^2x^2\text{PolyLog}(3,ax)+a^2x^2-2a^2x^2\log(1-ax)+2ax+2\log(1-ax)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, a*x],x]
```

```
[Out] (2*a*x + a^2*x^2 + 2*Log[1 - a*x] - 2*a^2*x^2*Log[1 - a*x] - 4*a^2*x^2*Poly
Log[2, a*x] + 8*a^2*x^2*PolyLog[3, a*x])/(16*a^2)
```

Maple [A] time = 0.155, size = 62, normalized size = 0.9

$$-\frac{1}{a^2}\left(-\frac{xa\,(3\,ax+6)}{48}-\frac{\left(-3\,a^2x^2+3\right)\ln(-ax+1)}{24}+\frac{x^2a^2\text{polylog}(2,ax)}{4}-\frac{x^2a^2\text{polylog}(3,ax)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x),x)
```

```
[Out] -1/a^2*(-1/48*x*a*(3*a*x+6)-1/24*(-3*a^2*x^2+3)*ln(-a*x+1)+1/4*x^2*a^2*poly
log(2,a*x)-1/2*x^2*a^2*polylog(3,a*x))
```

Maxima [A] time = 1.01375, size = 82, normalized size = 1.21

$$-\frac{4\,a^2x^2\mathrm{Li}_2\left(ax\right)-8\,a^2x^2\mathrm{Li}_3(ax)-a^2x^2-2\,ax+2\left(a^2x^2-1\right)\log\left(-ax+1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a^2*x^2*dilog(a*x) - 8*a^2*x^2*polylog(3, a*x) - a^2*x^2 - 2*a*x +
 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

Fricas [C] time = 2.61559, size = 201, normalized size = 2.96

$$-\frac{4\,a^2x^2\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-8\,a^2x^2\mathrm{polylog}\left(3,ax\right)-a^2x^2-2\,ax+2\left(a^2x^2-1\right)\log\left(-ax+1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^2*x^2*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 8*a^2*x^
2*polylog(3, a*x) - a^2*x^2 - 2*a*x + 2*(a^2*x^2 - 1)*log(-a*x + 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x)
```

```
[Out] Integral(x*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, a*x), x)
```

3.14 $\int \mathrm{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=34

$$x(-\mathrm{PolyLog}(2, ax)) + x\mathrm{PolyLog}(3, ax) + \frac{(1 - ax)\log(1 - ax)}{a} + x$$

```
[Out] x + ((1 - a*x)*Log[1 - a*x])/a - x*PolyLog[2, a*x] + x*PolyLog[3, a*x]
```

Rubi [A] time = 0.0108174, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6586, 2389, 2295}

$$x(-\mathrm{PolyLog}(2, ax)) + x\mathrm{PolyLog}(3, ax) + \frac{(1 - ax)\log(1 - ax)}{a} + x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x],x]
```

```
[Out] x + ((1 - a*x)*Log[1 - a*x])/a - x*PolyLog[2, a*x] + x*PolyLog[3, a*x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_3(ax)\,dx &= x\mathrm{Li}_3(ax) - \int \mathrm{Li}_2(ax)\,dx\\ &= -x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax) - \int \log(1-ax)\,dx\\ &= -x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax) + \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ax)}{a}\\ &= x + \frac{(1-ax)\log(1-ax)}{a} - x\mathrm{Li}_2(ax) + x\mathrm{Li}_3(ax)\end{aligned}$$

Mathematica [A] time = 0.0116823, size = 39, normalized size = 1.15

$$x\left(-\mathrm{PolyLog}(2,ax) + \mathrm{PolyLog}(3,ax) + \frac{\log(1-ax)}{ax} - \log(1-ax) + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x],x]
```

```
[Out] x*(1 - Log[1 - a*x] + Log[1 - a*x]/(a*x) - PolyLog[2, a*x] + PolyLog[3, a*x
])
```

Maple [A] time = 0.092, size = 41, normalized size = 1.2

$$\frac{1}{a}\left(ax + \frac{(-2\,ax+2)\ln(-ax+1)}{2} - ax\mathrm{polylog}(2,ax) + ax\mathrm{polylog}(3,ax)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x),x)
```

```
[Out] 1/a*(a*x+1/2*(-2*a*x+2)*ln(-a*x+1)-a*x*polylog(2,a*x)+a*x*polylog(3,a*x))
```

Maxima [A] time = 0.990846, size = 53, normalized size = 1.56

$$-\frac{ax\mathrm{Li}_2(ax) - ax\mathrm{Li}_3(ax) - ax + (ax-1)\log(-ax+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] -(a*x*dilog(a*x) - a*x*polylog(3, a*x) - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Fricas [C] time = 2.5983, size = 151, normalized size = 4.44

$$-\frac{ax\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-ax\mathrm{polylog}\left(3,ax\right)-ax+(ax-1)\log\left(-ax+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] -(a*x*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - a*x*polylog(3, a*x)
 - a*x + (a*x - 1)*log(-a*x + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x)
```

```
[Out] Integral(polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x), x)
```

3.15 $\int \frac{\textbf{PolyLog}(3,ax)}{x}\,dx$

Optimal. Leaf size=5

$$\text{PolyLog}(4,ax)$$

```
[Out] PolyLog[4, a*x]
```

Rubi [A] time = 0.0085138, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6589}

$$\text{PolyLog}(4,ax)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x,x]
```

```
[Out] PolyLog[4, a*x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{x}\,dx = \text{Li}_4(ax)$$

Mathematica [A] time = 0.0012345, size = 5, normalized size = 1.

$$\text{PolyLog}(4,ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/x,x]
```

```
[Out] PolyLog[4, a*x]
```

Maple [A] time = 0.04, size = 6, normalized size = 1.2

$$\text{polylog}\,(4, ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x,x)
```

```
[Out] polylog(4,a*x)
```

Maxima [A] time = 0.983743, size = 7, normalized size = 1.4

$$\text{Li}_4(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="maxima")
```

```
[Out] polylog(4, a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\,(3, ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x)/x, x)
```

Sympy [A] time = 0.551546, size = 3, normalized size = 0.6

$$\text{Li}_4\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x)
```

```
[Out] polylog(4, a*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x, x)
```

3.16 $\int \frac{\text{PolyLog}(3,ax)}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}$$

```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x - PolyLog[3,
 a*x]/x
```

Rubi [A] time = 0.0304426, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6591, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2,ax)}{x}-\frac{\text{PolyLog}(3,ax)}{x}+a\log(x)-a\log(1-ax)+\frac{\log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^2,x]
```


```
[Out] a*Log[x] - a*Log[1 - a*x] + Log[1 - a*x]/x - PolyLog[2, a*x]/x - PolyLog[3,
 a*x]/x
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^2}\,dx &= -\frac{\text{Li}_3(ax)}{x} + \int \frac{\text{Li}_2(ax)}{x^2}\,dx \\
&= -\frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} - \int \frac{\log(1-ax)}{x^2}\,dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} + a\int \frac{1}{x(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x} + a\int \frac{1}{x}\,dx + a^2\int \frac{1}{1-ax}\,dx \\
&= a\log(x) - a\log(1-ax) + \frac{\log(1-ax)}{x} - \frac{\text{Li}_2(ax)}{x} - \frac{\text{Li}_3(ax)}{x}
\end{aligned}$$

Mathematica [A] time = 0.0324637, size = 44, normalized size = 0.96

$$-\frac{\text{PolyLog}(2,ax) + \text{PolyLog}(3,ax) - ax\log(-ax) + ax\log(1-ax) - \log(1-ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/x^2,x]
```

```
[Out] -((-(a*x*Log[-(a*x)]) - Log[1 - a*x] + a*x*Log[1 - a*x] + PolyLog[2, a*x] +
 PolyLog[3, a*x])/x)
```

Maple [A] time = 0.08, size = 57, normalized size = 1.2

$$a\left(\ln(x) + \ln(-a) + \frac{(-8ax+8)\ln(-ax+1)}{8ax} - \frac{\text{polylog}(2,ax)}{ax} - \frac{\text{polylog}(3,ax)}{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^2,x)
```

```
[Out] a*(ln(x)+ln(-a)+1/8/a/x*(-8*a*x+8)*ln(-a*x+1)-polylog(2,a*x)/a/x-1/a/x*poly
log(3,a*x))
```

Maxima [A] time = 0.995112, size = 45, normalized size = 0.98

$$a\log(x) - \frac{(ax-1)\log(-ax+1) + \mathrm{Li}_2(ax) + \mathrm{Li}_3(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="maxima")
```

```
[Out] a*log(x) - ((a*x - 1)*log(-a*x + 1) + dilog(a*x) + polylog(3, a*x))/x
```

Fricas [C] time = 2.66231, size = 162, normalized size = 3.52

$$-\frac{ax\log(ax-1) - ax\log(x) + \%\mathrm{iint}\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - \log(-ax+1) + \mathrm{polylog}(3, ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(a*x - 1) - a*x*log(x) + \%iint(a, x, -log(-a*x + 1)/a, -log(-a*x +
 1)/x) - log(-a*x + 1) + polylog(3, a*x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**2,x)
```

```
[Out] Integral(polylog(3, a*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^2, x)
```

3.17 $\int \frac{\text{PolyLog}(3,ax)}{x^3} dx$

Optimal. Leaf size=70

$$-\frac{\text{PolyLog}(2,ax)}{4x^2}-\frac{\text{PolyLog}(3,ax)}{2x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)+\frac{\log(1-ax)}{8x^2}-\frac{a}{8x}$$

```
[Out] -a/(8*x) + (a^2*Log[x])/8 - (a^2*Log[1 - a*x])/8 + Log[1 - a*x]/(8*x^2) - P
olyLog[2, a*x]/(4*x^2) - PolyLog[3, a*x]/(2*x^2)
```

Rubi [A] time = 0.0431429, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{4x^2}-\frac{\text{PolyLog}(3,ax)}{2x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)+\frac{\log(1-ax)}{8x^2}-\frac{a}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^3,x]
```

```
[Out] -a/(8*x) + (a^2*Log[x])/8 - (a^2*Log[1 - a*x])/8 + Log[1 - a*x]/(8*x^2) - P
olyLog[2, a*x]/(4*x^2) - PolyLog[3, a*x]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax)}{x^3}\,dx &= -\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{2}\int \frac{\text{Li}_2(ax)}{x^3}\,dx \\
&= -\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}-\frac{1}{4}\int \frac{\log(1-ax)}{x^3}\,dx \\
&= \frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{8}a\int \frac{1}{x^2(1-ax)}\,dx \\
&= \frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}+\frac{1}{8}a\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx \\
&= -\frac{a}{8x}+\frac{1}{8}a^2\log(x)-\frac{1}{8}a^2\log(1-ax)+\frac{\log(1-ax)}{8x^2}-\frac{\text{Li}_2(ax)}{4x^2}-\frac{\text{Li}_3(ax)}{2x^2}
\end{aligned}
$$

Mathematica [C] time = 0.0090975, size = 25, normalized size = 0.36

$$
\frac{G_{5,5}^{2,4}\left(-ax\left|\begin{array}{c}1,1,1,1,3\\1,2,0,0,0\end{array}\right.\right)}{x^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x]/x^3,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x)]/x^2
```

Maple [A] time = 0.165, size = 90, normalized size = 1.3

$$
-a^2\left(\frac{1}{ax}+\frac{3}{16}-\frac{\ln(x)}{8}-\frac{\ln(-a)}{8}-\frac{81\,ax+378}{432\,ax}-\frac{\left(-27\,a^2x^2+27\right)\ln(-ax+1)}{216\,a^2x^2}+\frac{\text{polylog}(2,ax)}{4\,a^2x^2}+\frac{\text{polylog}(3,ax)}{2\,a^2x^2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^3,x)
```

```
[Out] -a^2*(1/a/x+3/16-1/8*ln(x)-1/8*ln(-a)-1/432/a/x*(81*a*x+378)-1/216/a^2/x^2*
(-27*a^2*x^2+27)*ln(-a*x+1)+1/4/a^2/x^2*polylog(2,a*x)+1/2/a^2/x^2*polylog(
3,a*x))
```

Maxima [A] time = 0.992882, size = 63, normalized size = 0.9

$$\frac{1}{8}\,a^2\log\left(x\right)-\frac{ax+\left(a^2x^2-1\right)\log\left(-ax+1\right)+2\,\mathrm{Li}_2\left(ax\right)+4\,\mathrm{Li}_3(ax)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(x) - 1/8*(a*x + (a^2*x^2 - 1)*log(-a*x + 1) + 2*dilog(a*x) + 4*
polylog(3, a*x))/x^2
```

Fricas [C] time = 2.71799, size = 194, normalized size = 2.77

$$-\frac{a^2x^2\log\left(ax-1\right)-a^2x^2\log\left(x\right)+ax+2\,\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-\log\left(-ax+1\right)+4\,\mathrm{polylog}\left(3,ax\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^2*log(a*x - 1) - a^2*x^2*log(x) + a*x + 2*\%iint(a, x, -log(-a*x
 + 1)/a, -log(-a*x + 1)/x) - log(-a*x + 1) + 4*polylog(3, a*x))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{Li}_3\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**3,x)
```

```
[Out] Integral(polylog(3, a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^3, x)
```

3.18 $\int \frac{\textbf{PolyLog}(3,ax)}{x^4}\,dx$

Optimal. Leaf size=80

$$-\frac{\text{PolyLog}(2,ax)}{9x^3}-\frac{\text{PolyLog}(3,ax)}{3x^3}-\frac{a^2}{27x}+\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)-\frac{a}{54x^2}+\frac{\log(1-ax)}{27x^3}$$

```
[Out] -a/(54*x^2) - a^2/(27*x) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x])/27 + Log[1
- a*x]/(27*x^3) - PolyLog[2, a*x]/(9*x^3) - PolyLog[3, a*x]/(3*x^3)
```

Rubi [A] time = 0.047769, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax)}{9x^3}-\frac{\text{PolyLog}(3,ax)}{3x^3}-\frac{a^2}{27x}+\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)-\frac{a}{54x^2}+\frac{\log(1-ax)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/x^4,x]
```

```
[Out] -a/(54*x^2) - a^2/(27*x) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x])/27 + Log[1
- a*x]/(27*x^3) - PolyLog[2, a*x]/(9*x^3) - PolyLog[3, a*x]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax)}{x^4}\,dx &= -\frac{\text{Li}_3(ax)}{3x^3}+\frac{1}{3}\int \frac{\text{Li}_2(ax)}{x^4}\,dx\\ &= -\frac{\text{Li}_2(ax)}{9x^3}-\frac{\text{Li}_3(ax)}{3x^3}-\frac{1}{9}\int \frac{\log(1-ax)}{x^4}\,dx\\ &= \frac{\log(1-ax)}{27x^3}-\frac{\text{Li}_2(ax)}{9x^3}-\frac{\text{Li}_3(ax)}{3x^3}+\frac{1}{27}a\int \frac{1}{x^3(1-ax)}\,dx\\ &= \frac{\log(1-ax)}{27x^3}-\frac{\text{Li}_2(ax)}{9x^3}-\frac{\text{Li}_3(ax)}{3x^3}+\frac{1}{27}a\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx\\ &= -\frac{a}{54x^2}-\frac{a^2}{27x}+\frac{1}{27}a^3\log(x)-\frac{1}{27}a^3\log(1-ax)+\frac{\log(1-ax)}{27x^3}-\frac{\text{Li}_2(ax)}{9x^3}-\frac{\text{Li}_3(ax)}{3x^3}\end{aligned}$$

Mathematica [C] time = 0.00906, size = 25, normalized size = 0.31

$$\frac{G_{5,5}^{2,4}\left(-ax\,\middle|\,\begin{array}{c}1,1,1,1,4\\1,3,0,0,0\end{array}\right)}{x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x]/x^4,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x)]/x^3
```

Maple [A] time = 0.168, size = 106, normalized size = 1.3

$$a^3\left(-\frac{1}{2\,a^2x^2}-\frac{1}{8\,ax}-\frac{1}{27}+\frac{\ln(x)}{27}+\frac{\ln(-a)}{27}+\frac{64\,a^2x^2+152\,ax+832}{1728\,a^2x^2}+\frac{\left(-64\,x^3a^3+64\right)\ln(-ax+1)}{1728\,x^3a^3}-\frac{\text{polylog}(2,ax)}{9\,x^3a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/x^4,x)
```

```
[Out] a^3*(-1/2/a^2/x^2-1/8/a/x-1/27+1/27*ln(x)+1/27*ln(-a)+1/1728/a^2/x^2*(64*a^
2*x^2+152*a*x+832)+1/1728/a^3/x^3*(-64*a^3*x^3+64)*ln(-a*x+1)-1/9/a^3/x^3*p
olylog(2,a*x)-1/3/a^3/x^3*polylog(3,a*x))
```

Maxima [A] time = 0.988239, size = 76, normalized size = 0.95

$$\frac{1}{27}a^3\log(x) - \frac{2a^2x^2 + ax + 2(a^3x^3 - 1)\log(-ax + 1) + 6\,\mathrm{Li}_2(ax) + 18\,\mathrm{Li}_3(ax)}{54x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/27*a^3*log(x) - 1/54*(2*a^2*x^2 + a*x + 2*(a^3*x^3 - 1)*log(-a*x + 1) + 6
*dilog(a*x) + 18*polylog(3, a*x))/x^3
```

Fricas [C] time = 2.81394, size = 221, normalized size = 2.76

$$-\frac{2a^3x^3\log(ax-1) - 2a^3x^3\log(x) + 2a^2x^2 + ax + 6\,\%\mathrm{iint}\left(a, x, -\frac{\log(-ax+1)}{a}, -\frac{\log(-ax+1)}{x}\right) - 2\log(-ax+1) + 18\,\mathrm{polyl}}{54x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="fricas")
```

```
[Out] -1/54*(2*a^3*x^3*log(a*x - 1) - 2*a^3*x^3*log(x) + 2*a^2*x^2 + a*x + 6*%iin
t(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 2*log(-a*x + 1) + 18*polylog(
3, a*x))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x**4,x)
```

```
[Out] Integral(polylog(3, a*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/x^4, x)
```

3.19 $\int x^5 \text{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=74

$$\frac{1}{6}x^6\text{PolyLog}\left(2,ax^2\right)-\frac{x^2}{18a^2}-\frac{\log\left(1-ax^2\right)}{18a^3}-\frac{x^4}{36a}+\frac{1}{18}x^6\log\left(1-ax^2\right)-\frac{x^6}{54}$$

```
[Out] -x^2/(18*a^2) - x^4/(36*a) - x^6/54 - Log[1 - a*x^2]/(18*a^3) + (x^6*Log[1
- a*x^2])/18 + (x^6*PolyLog[2, a*x^2])/6
```

Rubi [A] time = 0.0614215, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 43}

$$\frac{1}{6}x^6\text{PolyLog}\left(2,ax^2\right)-\frac{x^2}{18a^2}-\frac{\log\left(1-ax^2\right)}{18a^3}-\frac{x^4}{36a}+\frac{1}{18}x^6\log\left(1-ax^2\right)-\frac{x^6}{54}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*PolyLog[2, a*x^2],x]
```



```
[Out] -x^2/(18*a^2) - x^4/(36*a) - x^6/54 - Log[1 - a*x^2]/(18*a^3) + (x^6*Log[1
- a*x^2])/18 + (x^6*PolyLog[2, a*x^2])/6
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^5 \text{Li}_2\left(ax^2\right) dx &= \frac{1}{6}x^6\text{Li}_2\left(ax^2\right) + \frac{1}{3}\int x^5 \log\left(1-ax^2\right) dx \\
&= \frac{1}{6}x^6\text{Li}_2\left(ax^2\right) + \frac{1}{6}\text{Subst}\left(\int x^2\log(1-ax)\,dx, x, x^2\right) \\
&= \frac{1}{18}x^6\log\left(1-ax^2\right) + \frac{1}{6}x^6\text{Li}_2\left(ax^2\right) + \frac{1}{18}a\,\text{Subst}\left(\int \frac{x^3}{1-ax}\,dx, x, x^2\right) \\
&= \frac{1}{18}x^6\log\left(1-ax^2\right) + \frac{1}{6}x^6\text{Li}_2\left(ax^2\right) + \frac{1}{18}a\,\text{Subst}\left(\int \left(-\frac{1}{a^3} - \frac{x}{a^2} - \frac{x^2}{a} - \frac{1}{a^3(-1+ax)}\right) dx, x, x^2\right) \\
&= -\frac{x^2}{18a^2} - \frac{x^4}{36a} - \frac{x^6}{54} - \frac{\log\left(1-ax^2\right)}{18a^3} + \frac{1}{18}x^6\log\left(1-ax^2\right) + \frac{1}{6}x^6\text{Li}_2\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0220162, size = 65, normalized size = 0.88

$$\frac{18a^3x^6\text{PolyLog}\left(2,ax^2\right) - ax^2\left(2a^2x^4+3ax^2+6\right) + 6\left(a^3x^6-1\right)\log\left(1-ax^2\right)}{108a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*PolyLog[2, a*x^2],x]
```

```
[Out] (-(a*x^2*(6 + 3*a*x^2 + 2*a^2*x^4)) + 6*(-1 + a^3*x^6)*Log[1 - a*x^2] + 18*
a^3*x^6*PolyLog[2, a*x^2])/(108*a^3)
```

Maple [A] time = 0.044, size = 62, normalized size = 0.8

$$\frac{x^{6}\mathrm{polylog}\left(2,ax^{2}\right)}{6}+\frac{x^{6}\ln\left(-ax^{2}+1\right)}{18}-\frac{x^{6}}{54}-\frac{x^{4}}{36\,a}-\frac{x^{2}}{18\,a^{2}}-\frac{\ln\left(ax^{2}-1\right)}{18\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(2,a*x^2),x)
```

```
[Out] 1/6*x^6*polylog(2,a*x^2)+1/18*x^6*ln(-a*x^2+1)-1/54*x^6-1/36*x^4/a-1/18*x^2
/a^2-1/18/a^3*ln(a*x^2-1)
```

Maxima [A] time = 1.00801, size = 84, normalized size = 1.14

$$\frac{18\,a^{3}x^{6}\mathrm{Li}_{2}\left(ax^{2}\right)-2\,a^{3}x^{6}-3\,a^{2}x^{4}-6\,ax^{2}+6\left(a^{3}x^{6}-1\right)\log\left(-ax^{2}+1\right)}{108\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x
^6 - 1)*log(-a*x^2 + 1))/a^3
```

Fricas [A] time = 2.5273, size = 142, normalized size = 1.92

$$\frac{18\,a^{3}x^{6}\mathrm{Li}_{2}\left(ax^{2}\right)-2\,a^{3}x^{6}-3\,a^{2}x^{4}-6\,ax^{2}+6\left(a^{3}x^{6}-1\right)\log\left(-ax^{2}+1\right)}{108\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/108*(18*a^3*x^6*dilog(a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x
^6 - 1)*log(-a*x^2 + 1))/a^3
```

Sympy [A] time = 42.5184, size = 56, normalized size = 0.76

$$\begin{cases} -\frac{x^6 \operatorname{Li}_1(ax^2)}{18} + \frac{x^6 \operatorname{Li}_2(ax^2)}{6} - \frac{x^6}{54} - \frac{x^4}{36a} - \frac{x^2}{18a^2} + \frac{\operatorname{Li}_1(ax^2)}{18a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**6*polylog(1, a*x**2)/18 + x**6*polylog(2, a*x**2)/6 - x**6/5
4 - x**4/(36*a) - x**2/(18*a**2) + polylog(1, a*x**2)/(18*a**3), Ne(a, 0)),
 (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*dilog(a*x^2), x)
```

3.20 $\int x^3\textbf{PolyLog}\left(2,ax^2\right)\,dx$

Optimal. Leaf size=64

$$\frac{1}{4}x^4\text{PolyLog}\left(2,ax^2\right)-\frac{\log\left(1-ax^2\right)}{8a^2}-\frac{x^2}{8a}+\frac{1}{8}x^4\log\left(1-ax^2\right)-\frac{x^4}{16}$$

```
[Out] -x^2/(8*a) - x^4/16 - Log[1 - a*x^2]/(8*a^2) + (x^4*Log[1 - a*x^2])/8 + (x^
4*PolyLog[2, a*x^2])/4
```

Rubi [A] time = 0.0495752, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 43}

$$\frac{1}{4}x^4\text{PolyLog}\left(2,ax^2\right)-\frac{\log\left(1-ax^2\right)}{8a^2}-\frac{x^2}{8a}+\frac{1}{8}x^4\log\left(1-ax^2\right)-\frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[2, a*x^2],x]
```

```
[Out] -x^2/(8*a) - x^4/16 - Log[1 - a*x^2]/(8*a^2) + (x^4*Log[1 - a*x^2])/8 + (x^
4*PolyLog[2, a*x^2])/4
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \text{Li}_2\left(ax^2\right) dx &= \frac{1}{4}x^4\text{Li}_2\left(ax^2\right) + \frac{1}{2}\int x^3 \log\left(1-ax^2\right) dx \\
&= \frac{1}{4}x^4\text{Li}_2\left(ax^2\right) + \frac{1}{4}\,\text{Subst}\left(\int x\log(1-ax)\,dx, x, x^2\right) \\
&= \frac{1}{8}x^4\log\left(1-ax^2\right) + \frac{1}{4}x^4\text{Li}_2\left(ax^2\right) + \frac{1}{8}a\,\text{Subst}\left(\int \frac{x^2}{1-ax}\,dx, x, x^2\right) \\
&= \frac{1}{8}x^4\log\left(1-ax^2\right) + \frac{1}{4}x^4\text{Li}_2\left(ax^2\right) + \frac{1}{8}a\,\text{Subst}\left(\int \left(-\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)}\right) dx, x, x^2\right) \\
&= -\frac{x^2}{8a} - \frac{x^4}{16} - \frac{\log\left(1-ax^2\right)}{8a^2} + \frac{1}{8}x^4\log\left(1-ax^2\right) + \frac{1}{4}x^4\text{Li}_2\left(ax^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0172903, size = 56, normalized size = 0.88

$$\frac{4a^2x^4\text{PolyLog}\left(2,ax^2\right) + 2\left(a^2x^4-1\right)\log\left(1-ax^2\right) - ax^2\left(ax^2+2\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[2, a*x^2],x]
```

```
[Out] (-(a*x^2*(2 + a*x^2)) + 2*(-1 + a^2*x^4)*Log[1 - a*x^2] + 4*a^2*x^4*PolyLog
[2, a*x^2])/(16*a^2)
```

Maple [A] time = 0.045, size = 54, normalized size = 0.8

$$\frac{x^4 \mathrm{polylog}\left(2, a x^2\right)}{4}+\frac{x^4 \ln\left(-a x^2+1\right)}{8}-\frac{x^4}{16}-\frac{x^2}{8 a}-\frac{\ln\left(a x^2-1\right)}{8 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(2,a*x^2),x)
```

```
[Out] 1/4*x^4*polylog(2,a*x^2)+1/8*x^4*ln(-a*x^2+1)-1/16*x^4-1/8*x^2/a-1/8/a^2*ln
(a*x^2-1)
```

Maxima [A] time = 0.96931, size = 73, normalized size = 1.14

$$\frac{4 a^2 x^4 \mathrm{Li}_2\left(a x^2\right)-a^2 x^4-2 a x^2+2\left(a^2 x^4-1\right) \log\left(-a x^2+1\right)}{16 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x
^2 + 1))/a^2
```

Fricas [A] time = 2.58406, size = 120, normalized size = 1.88

$$\frac{4 a^2 x^4 \mathrm{Li}_2\left(a x^2\right)-a^2 x^4-2 a x^2+2\left(a^2 x^4-1\right) \log\left(-a x^2+1\right)}{16 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*x^4*dilog(a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x
^2 + 1))/a^2
```

Sympy [A] time = 13.1842, size = 48, normalized size = 0.75

$$\begin{cases} -\frac{x^4 \operatorname{Li}_1(ax^2)}{8} + \frac{x^4 \operatorname{Li}_2(ax^2)}{4} - \frac{x^4}{16} - \frac{x^2}{8a} + \frac{\operatorname{Li}_1(ax^2)}{8a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**4*polylog(1, a*x**2)/8 + x**4*polylog(2, a*x**2)/4 - x**4/16
 - x**2/(8*a) + polylog(1, a*x**2)/(8*a**2), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(a*x^2), x)
```

3.21 $\int x\mathbf{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=46

$$\frac{1}{2}x^2\text{PolyLog}\left(2, ax^2\right) - \frac{\left(1 - ax^2\right)\log\left(1 - ax^2\right)}{2a} - \frac{x^2}{2}$$

```
[Out] -x^2/2 - ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) + (x^2*PolyLog[2, a*x^2])/2
```

Rubi [A] time = 0.0252799, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6591, 2454, 2389, 2295}

$$\frac{1}{2}x^2\text{PolyLog}\left(2, ax^2\right) - \frac{\left(1 - ax^2\right)\log\left(1 - ax^2\right)}{2a} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x^2],x]
```

```
[Out] -x^2/2 - ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) + (x^2*PolyLog[2, a*x^2])/2
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_2\left(ax^2\right)\,dx &= \frac{1}{2}x^2\text{Li}_2\left(ax^2\right)+\int x\log\left(1-ax^2\right)\,dx\\ &= \frac{1}{2}x^2\text{Li}_2\left(ax^2\right)+\frac{1}{2}\,\text{Subst}\left(\int\log(1-ax)\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\text{Li}_2\left(ax^2\right)-\frac{\text{Subst}\left(\int\log(x)\,dx,x,1-ax^2\right)}{2a}\\ &= -\frac{x^2}{2}-\frac{\left(1-ax^2\right)\log\left(1-ax^2\right)}{2a}+\frac{1}{2}x^2\text{Li}_2\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.0091547, size = 43, normalized size = 0.93

$$\frac{ax^2\text{PolyLog}\left(2,ax^2\right)-ax^2+\left(ax^2-1\right)\log\left(1-ax^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x^2],x]
```

```
[Out] (-(a*x^2) + (-1 + a*x^2)*Log[1 - a*x^2] + a*x^2*PolyLog[2, a*x^2])/(2*a)
```

Maple [A] time = 0.045, size = 52, normalized size = 1.1

$$\frac{x^2\text{polylog}\left(2,ax^2\right)}{2}+\frac{\ln\left(-ax^2+1\right)x^2}{2}-\frac{x^2}{2}-\frac{\ln\left(-ax^2+1\right)}{2\,a}+\frac{1}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x^2),x)
```

```
[Out] 1/2*x^2*polylog(2,a*x^2)+1/2*ln(-a*x^2+1)*x^2-1/2*x^2-1/2/a*ln(-a*x^2+1)+1/
2/a
```

Maxima [A] time = 0.985418, size = 54, normalized size = 1.17

$$\frac{ax^2\mathrm{Li}_2\left(ax^2\right)-ax^2+\left(ax^2-1\right)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

Fricas [A] time = 2.57043, size = 89, normalized size = 1.93

$$\frac{ax^2\mathrm{Li}_2\left(ax^2\right)-ax^2+\left(ax^2-1\right)\log\left(-ax^2+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2*dilog(a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

Sympy [A] time = 4.02697, size = 39, normalized size = 0.85

$$\begin{cases} -\frac{x^2\,\mathrm{Li}_1\left(ax^2\right)}{2}+\frac{x^2\,\mathrm{Li}_2\left(ax^2\right)}{2}-\frac{x^2}{2}+\frac{\mathrm{Li}_1\left(ax^2\right)}{2a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-x**2*polylog(1, a*x**2)/2 + x**2*polylog(2, a*x**2)/2 - x**2/2
+ polylog(1, a*x**2)/(2*a), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_2\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x^2), x)
```

3.22 $\int \frac{\text{PolyLog}(2,ax^2)}{x}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\text{PolyLog}\left(3,ax^2\right)$$

```
[Out] PolyLog[3, a*x^2]/2
```

Rubi [A] time = 0.0094764, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6589}

$$\frac{1}{2}\text{PolyLog}\left(3,ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x,x]
```

```
[Out] PolyLog[3, a*x^2]/2
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(ax^2\right)}{x}\,dx=\frac{\text{Li}_3\left(ax^2\right)}{2}$$

Mathematica [A] time = 0.0012499, size = 11, normalized size = 1.

$$\frac{1}{2}\text{PolyLog}\left(3,ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x,x]
```

```
[Out] PolyLog[3, a*x^2]/2
```

Maple [A] time = 0.115, size = 10, normalized size = 0.9

$$\frac{\text{polylog}\left(3, ax^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x,x)
```

```
[Out] 1/2*polylog(3,a*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(dilog(a*x^2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\left(ax^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x,x)
```

```
[Out] Integral(polylog(2, a*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x, x)
```

3.23 $\int \frac{\text{PolyLog}(2,ax^2)}{x^3}\,dx$

Optimal. Leaf size=49

$$-\frac{\text{PolyLog}\left(2,ax^2\right)}{2x^2}-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}+a\log(x)$$

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2)
```

Rubi [A] time = 0.040811, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6591, 2454, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}\left(2,ax^2\right)}{2x^2}-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2\left(ax^2\right)}{x^3}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{2x^2} - \int \frac{\log\left(1-ax^2\right)}{x^3}\,dx \\
&= -\frac{\text{Li}_2\left(ax^2\right)}{2x^2} - \frac{1}{2}\,\text{Subst}\left(\int \frac{\log(1-ax)}{x^2}\,dx, x, x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2} - \frac{\text{Li}_2\left(ax^2\right)}{2x^2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x(1-ax)}\,dx, x, x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2} - \frac{\text{Li}_2\left(ax^2\right)}{2x^2} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) + \frac{1}{2}a^2\,\text{Subst}\left(\int \frac{1}{1-ax}\,dx, x, x^2\right) \\
&= a\log(x) - \frac{1}{2}a\log\left(1-ax^2\right) + \frac{\log\left(1-ax^2\right)}{2x^2} - \frac{\text{Li}_2\left(ax^2\right)}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.0123956, size = 49, normalized size = 1.

$$-\frac{\text{PolyLog}\left(2,ax^2\right)}{2x^2} - \frac{1}{2}a\log\left(1-ax^2\right) + \frac{\log\left(1-ax^2\right)}{2x^2} + a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2)
```

Maple [A] time = 0.05, size = 43, normalized size = 0.9

$$-\frac{\mathrm{polylog}\left(2,ax^2\right)}{2\,x^2}+\frac{\ln\left(-ax^2+1\right)}{2\,x^2}+a\ln\left(x\right)-\frac{a\ln\left(ax^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^3,x)
```

```
[Out] -1/2*polylog(2,a*x^2)/x^2+1/2*ln(-a*x^2+1)/x^2+a*ln(x)-1/2*a*ln(a*x^2-1)
```

Maxima [A] time = 0.960105, size = 46, normalized size = 0.94

$$a\log\left(x\right)-\frac{\left(ax^2-1\right)\log\left(-ax^2+1\right)+\mathrm{Li}_2\left(ax^2\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2))/x^2
```

Fricas [A] time = 2.68007, size = 112, normalized size = 2.29

$$-\frac{ax^2\log\left(ax^2-1\right)-2\,ax^2\log\left(x\right)+\mathrm{Li}_2\left(ax^2\right)-\log\left(-ax^2+1\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + dilog(a*x^2) - log(-a*x^2 + 1
))/x^2
```

Sympy [A] time = 4.87995, size = 37, normalized size = 0.76

$$a\log(x) + \frac{a\operatorname{Li}_1\left(ax^2\right)}{2} - \frac{\operatorname{Li}_1\left(ax^2\right)}{2x^2} - \frac{\operatorname{Li}_2\left(ax^2\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**3,x)
```

```
[Out] a*log(x) + a*polylog(1, a*x**2)/2 - polylog(1, a*x**2)/(2*x**2) - polylog(2
, a*x**2)/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(ax^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^3, x)
```

3.24 $\int \frac{\text{PolyLog}(2,ax^2)}{x^5} dx$

Optimal. Leaf size=64

$$-\frac{\text{PolyLog}(2,ax^2)}{4x^4}-\frac{1}{8}a^2\log(1-ax^2)+\frac{1}{4}a^2\log(x)-\frac{a}{8x^2}+\frac{\log(1-ax^2)}{8x^4}$$

```
[Out] -a/(8*x^2) + (a^2*Log[x])/4 - (a^2*Log[1 - a*x^2])/8 + Log[1 - a*x^2]/(8*x^
4) - PolyLog[2, a*x^2]/(4*x^4)
```

Rubi [A] time = 0.0500673, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax^2)}{4x^4}-\frac{1}{8}a^2\log(1-ax^2)+\frac{1}{4}a^2\log(x)-\frac{a}{8x^2}+\frac{\log(1-ax^2)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^5,x]
```

```
[Out] -a/(8*x^2) + (a^2*Log[x])/4 - (a^2*Log[1 - a*x^2])/8 + Log[1 - a*x^2]/(8*x^
4) - PolyLog[2, a*x^2]/(4*x^4)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2\left(ax^2\right)}{x^5}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{4x^4}-\frac{1}{2}\int \frac{\log\left(1-ax^2\right)}{x^5}\,dx\\
&= -\frac{\text{Li}_2\left(ax^2\right)}{4x^4}-\frac{1}{4}\,\text{Subst}\left(\int \frac{\log(1-ax)}{x^3}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{8x^4}-\frac{\text{Li}_2\left(ax^2\right)}{4x^4}+\frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{x^2(1-ax)}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{8x^4}-\frac{\text{Li}_2\left(ax^2\right)}{4x^4}+\frac{1}{8}a\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx,x,x^2\right)\\
&= -\frac{a}{8x^2}+\frac{1}{4}a^2\log(x)-\frac{1}{8}a^2\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{8x^4}-\frac{\text{Li}_2\left(ax^2\right)}{4x^4}
\end{aligned}$$

Mathematica [A] time = 0.0283499, size = 51, normalized size = 0.8

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)-2a^2x^4\log(x)+\left(a^2x^4-1\right)\log\left(1-ax^2\right)+ax^2}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^5,x]
```

```
[Out] -(a*x^2 - 2*a^2*x^4*Log[x] + (-1 + a^2*x^4)*Log[1 - a*x^2] + 2*PolyLog[2, a
*x^2])/(8*x^4)
```

Maple [A] time = 0.05, size = 54, normalized size = 0.8

$$-\frac{\operatorname{polylog}\left(2, a x^{2}\right)}{4 x^{4}}+\frac{\ln \left(-a x^{2}+1\right)}{8 x^{4}}-\frac{a}{8 x^{2}}+\frac{a^{2} \ln (x)}{4}-\frac{a^{2} \ln \left(a x^{2}-1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^5,x)
```

```
[Out] -1/4*polylog(2,a*x^2)/x^4+1/8*ln(-a*x^2+1)/x^4-1/8*a/x^2+1/4*a^2*ln(x)-1/8*
a^2*ln(a*x^2-1)
```

Maxima [A] time = 0.987978, size = 62, normalized size = 0.97

$$\frac{1}{4} a^{2} \log (x)-\frac{a x^{2}+\left(a^{2} x^{4}-1\right) \log \left(-a x^{2}+1\right)+2 \operatorname{Li}_{2}\left(a x^{2}\right)}{8 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*log(x) - 1/8*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x^2
))/x^4
```

Fricas [A] time = 2.64939, size = 131, normalized size = 2.05

$$-\frac{a^{2} x^{4} \log \left(a x^{2}-1\right)-2 a^{2} x^{4} \log (x)+a x^{2}+2 \operatorname{Li}_{2}\left(a x^{2}\right)-\log \left(-a x^{2}+1\right)}{8 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*dilog(a*x^2) -
log(-a*x^2 + 1))/x^4
```

Sympy [A] time = 14.9968, size = 49, normalized size = 0.77

$$\frac{a^2\log(x)}{4}+\frac{a^2\operatorname{Li}_1(ax^2)}{8}-\frac{a}{8x^2}-\frac{\operatorname{Li}_1(ax^2)}{8x^4}-\frac{\operatorname{Li}_2(ax^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**5,x)
```

```
[Out] a**2*log(x)/4 + a**2*polylog(1, a*x**2)/8 - a/(8*x**2) - polylog(1, a*x**2)
/(8*x**4) - polylog(2, a*x**2)/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{Li}_2(ax^2)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^5, x)
```

3.25 $\int \frac{\text{PolyLog}(2,ax^2)}{x^7}\,dx$

Optimal. Leaf size=74

$$-\frac{\text{PolyLog}\left(2,ax^2\right)}{6x^6}-\frac{a^2}{18x^2}-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{1}{9}a^3\log(x)-\frac{a}{36x^4}+\frac{\log\left(1-ax^2\right)}{18x^6}$$

```
[Out] -a/(36*x^4) - a^2/(18*x^2) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x^2])/18 + Log
[1 - a*x^2]/(18*x^6) - PolyLog[2, a*x^2]/(6*x^6)
```

Rubi [A] time = 0.0541201, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}\left(2,ax^2\right)}{6x^6}-\frac{a^2}{18x^2}-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{1}{9}a^3\log(x)-\frac{a}{36x^4}+\frac{\log\left(1-ax^2\right)}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^7,x]
```

```
[Out] -a/(36*x^4) - a^2/(18*x^2) + (a^3*Log[x])/9 - (a^3*Log[1 - a*x^2])/18 + Log
[1 - a*x^2]/(18*x^6) - PolyLog[2, a*x^2]/(6*x^6)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2\left(ax^2\right)}{x^7}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{6x^6}-\frac{1}{3}\int \frac{\log\left(1-ax^2\right)}{x^7}\,dx\\ &= -\frac{\text{Li}_2\left(ax^2\right)}{6x^6}-\frac{1}{6}\,\text{Subst}\left(\int \frac{\log(1-ax)}{x^4}\,dx,x,x^2\right)\\ &= \frac{\log\left(1-ax^2\right)}{18x^6}-\frac{\text{Li}_2\left(ax^2\right)}{6x^6}+\frac{1}{18}a\,\text{Subst}\left(\int \frac{1}{x^3(1-ax)}\,dx,x,x^2\right)\\ &= \frac{\log\left(1-ax^2\right)}{18x^6}-\frac{\text{Li}_2\left(ax^2\right)}{6x^6}+\frac{1}{18}a\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx,x,x^2\right)\\ &= -\frac{a}{36x^4}-\frac{a^2}{18x^2}+\frac{1}{9}a^3\log(x)-\frac{1}{18}a^3\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{18x^6}-\frac{\text{Li}_2\left(ax^2\right)}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.0314649, size = 60, normalized size = 0.81

$$-\frac{6\text{PolyLog}\left(2,ax^2\right)-4a^3x^6\log(x)+2\left(a^3x^6-1\right)\log\left(1-ax^2\right)+ax^2\left(2ax^2+1\right)}{36x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^7,x]
```

```
[Out] -(a*x^2*(1 + 2*a*x^2) - 4*a^3*x^6*Log[x] + 2*(-1 + a^3*x^6)*Log[1 - a*x^2]
+ 6*PolyLog[2, a*x^2])/(36*x^6)
```

Maple [A] time = 0.05, size = 62, normalized size = 0.8

$$-\frac{\mathrm{polylog}\left(2,a x^{2}\right)}{6 x^{6}}+\frac{\ln\left(-a x^{2}+1\right)}{18 x^{6}}-\frac{a}{36 x^{4}}-\frac{a^{2}}{18 x^{2}}+\frac{a^{3} \ln\left(x\right)}{9}-\frac{a^{3} \ln\left(a x^{2}-1\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^7,x)
```

```
[Out] -1/6*polylog(2,a*x^2)/x^6+1/18*ln(-a*x^2+1)/x^6-1/36*a/x^4-1/18*a^2/x^2+1/9
*a^3*ln(x)-1/18*a^3*ln(a*x^2-1)
```

Maxima [A] time = 0.981148, size = 74, normalized size = 1.

$$\frac{1}{9} a^{3} \log\left(x\right)-\frac{2 a^{2} x^{4}+a x^{2}+2 \left(a^{3} x^{6}-1\right) \log\left(-a x^{2}+1\right)+6 \,\mathrm{Li}_{2}\left(a x^{2}\right)}{36 x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/9*a^3*log(x) - 1/36*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 + 1)
+ 6*dilog(a*x^2))/x^6
```

Fricas [A] time = 2.65693, size = 154, normalized size = 2.08

$$-\frac{2 a^{3} x^{6} \log\left(a x^{2}-1\right)-4 a^{3} x^{6} \log\left(x\right)+2 a^{2} x^{4}+a x^{2}+6 \,\mathrm{Li}_{2}\left(a x^{2}\right)-2 \log\left(-a x^{2}+1\right)}{36 x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/36*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 + 6*
dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^6
```

Sympy [A] time = 43.0619, size = 58, normalized size = 0.78

$$\frac{a^3\log(x)}{9}+\frac{a^3\operatorname{Li}_1(ax^2)}{18}-\frac{a^2}{18x^2}-\frac{a}{36x^4}-\frac{\operatorname{Li}_1(ax^2)}{18x^6}-\frac{\operatorname{Li}_2(ax^2)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**7,x)
```

```
[Out] a**3*log(x)/9 + a**3*polylog(1, a*x**2)/18 - a**2/(18*x**2) - a/(36*x**4) -
 polylog(1, a*x**2)/(18*x**6) - polylog(2, a*x**2)/(6*x**6)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^2)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^7, x)
```

3.26 $\int x^4 \mathbf{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5\text{PolyLog}\left(2,ax^2\right)-\frac{4x}{25a^2}+\frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{25a^{5/2}}-\frac{4x^3}{75a}+\frac{2}{25}x^5\log\left(1-ax^2\right)-\frac{4x^5}{125}$$

```
[Out] (-4*x)/(25*a^2) - (4*x^3)/(75*a) - (4*x^5)/125 + (4*ArcTanh[Sqrt[a]*x])/(25
*a^(5/2)) + (2*x^5*Log[1 - a*x^2])/25 + (x^5*PolyLog[2, a*x^2])/5
```

Rubi [A] time = 0.0450937, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 302, 206}

$$\frac{1}{5}x^5\text{PolyLog}\left(2,ax^2\right)-\frac{4x}{25a^2}+\frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{25a^{5/2}}-\frac{4x^3}{75a}+\frac{2}{25}x^5\log\left(1-ax^2\right)-\frac{4x^5}{125}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[2, a*x^2],x]
```

```
[Out] (-4*x)/(25*a^2) - (4*x^3)/(75*a) - (4*x^5)/125 + (4*ArcTanh[Sqrt[a]*x])/(25
*a^(5/2)) + (2*x^5*Log[1 - a*x^2])/25 + (x^5*PolyLog[2, a*x^2])/5
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4\mathrm{Li}_2\left(ax^2\right)dx &= \frac{1}{5}x^5\mathrm{Li}_2\left(ax^2\right)+\frac{2}{5}\int x^4\log\left(1-ax^2\right)dx\\
&= \frac{2}{25}x^5\log\left(1-ax^2\right)+\frac{1}{5}x^5\mathrm{Li}_2\left(ax^2\right)+\frac{1}{25}(4a)\int\frac{x^6}{1-ax^2}dx\\
&= \frac{2}{25}x^5\log\left(1-ax^2\right)+\frac{1}{5}x^5\mathrm{Li}_2\left(ax^2\right)+\frac{1}{25}(4a)\int\left(-\frac{1}{a^3}-\frac{x^2}{a^2}-\frac{x^4}{a}+\frac{1}{a^3\left(1-ax^2\right)}\right)dx\\
&= -\frac{4x}{25a^2}-\frac{4x^3}{75a}-\frac{4x^5}{125}+\frac{2}{25}x^5\log\left(1-ax^2\right)+\frac{1}{5}x^5\mathrm{Li}_2\left(ax^2\right)+\frac{4\int\frac{1}{1-ax^2}dx}{25a^2}\\
&= -\frac{4x}{25a^2}-\frac{4x^3}{75a}-\frac{4x^5}{125}+\frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{25a^{5/2}}+\frac{2}{25}x^5\log\left(1-ax^2\right)+\frac{1}{5}x^5\mathrm{Li}_2\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0723123, size = 65, normalized size = 0.89

$$\frac{1}{375}\left(75x^5\mathrm{PolyLog}\left(2,ax^2\right)-\frac{60x}{a^2}+\frac{60\tanh^{-1}\left(\sqrt{a}x\right)}{a^{5/2}}-\frac{20x^3}{a}+30x^5\log\left(1-ax^2\right)-12x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[2, a*x^2],x]
```

```
[Out] ((-60*x)/a^2 - (20*x^3)/a - 12*x^5 + (60*ArcTanh[Sqrt[a]*x])/a^(5/2) + 30*x
^5*Log[1 - a*x^2] + 75*x^5*PolyLog[2, a*x^2])/375
```

Maple [A] time = 0.046, size = 58, normalized size = 0.8

$$-\frac{4x}{25a^2}-\frac{4x^3}{75a}-\frac{4x^5}{125}+\frac{4}{25}\mathrm{Artanh}\left(x\sqrt{a}\right)a^{-\frac{5}{2}}+\frac{2x^5\ln\left(-ax^2+1\right)}{25}+\frac{x^5\mathrm{polylog}\left(2,ax^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(2,a*x^2),x)
```

```
[Out] -4/25*x/a^2-4/75*x^3/a-4/125*x^5+4/25*arctanh(x*a^(1/2))/a^(5/2)+2/25*x^5*l
n(-a*x^2+1)+1/5*x^5*polylog(2,a*x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.69839, size = 398, normalized size = 5.45

$$\left[\frac{75\,a^3x^5\mathrm{Li}_2\left(ax^2\right)+30\,a^3x^5\log\left(-ax^2+1\right)-12\,a^3x^5-20\,a^2x^3-60\,ax+30\,\sqrt{a}\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{375\,a^3},\frac{75\,a^3x^5\mathrm{Li}_2\left(ax^2\right)+30\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12*a^3*x^5 -
 20*a^2*x^3 - 60*a*x + 30*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)
))/a^3, 1/375*(75*a^3*x^5*dilog(a*x^2) + 30*a^3*x^5*log(-a*x^2 + 1) - 12*a^
3*x^5 - 20*a^2*x^3 - 60*a*x - 60*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(2,a*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \mathrm{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*dilog(a*x^2), x)
```

3.27 $\int x^2 \textbf{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=63

$$\frac{1}{3}x^3\text{PolyLog}\left(2,ax^2\right)+\frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{9a^{3/2}}+\frac{2}{9}x^3\log\left(1-ax^2\right)-\frac{4x}{9a}-\frac{4x^3}{27}$$

```
[Out] (-4*x)/(9*a) - (4*x^3)/27 + (4*ArcTanh[Sqrt[a]*x])/(9*a^(3/2)) + (2*x^3*Log
[1 - a*x^2])/9 + (x^3*PolyLog[2, a*x^2])/3
```

Rubi [A] time = 0.0387699, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 302, 206}

$$\frac{1}{3}x^3\text{PolyLog}\left(2,ax^2\right)+\frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{9a^{3/2}}+\frac{2}{9}x^3\log\left(1-ax^2\right)-\frac{4x}{9a}-\frac{4x^3}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x^2],x]
```

```
[Out] (-4*x)/(9*a) - (4*x^3)/27 + (4*ArcTanh[Sqrt[a]*x])/(9*a^(3/2)) + (2*x^3*Log
[1 - a*x^2])/9 + (x^3*PolyLog[2, a*x^2])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \text{Li}_2\left(ax^2\right) dx &= \frac{1}{3}x^3\text{Li}_2\left(ax^2\right) + \frac{2}{3}\int x^2 \log\left(1-ax^2\right) dx \\
&= \frac{2}{9}x^3\log\left(1-ax^2\right) + \frac{1}{3}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{9}(4a)\int \frac{x^4}{1-ax^2} dx \\
&= \frac{2}{9}x^3\log\left(1-ax^2\right) + \frac{1}{3}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{9}(4a)\int \left(-\frac{1}{a^2} - \frac{x^2}{a} + \frac{1}{a^2\left(1-ax^2\right)}\right) dx \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{2}{9}x^3\log\left(1-ax^2\right) + \frac{1}{3}x^3\text{Li}_2\left(ax^2\right) + \frac{4\int \frac{1}{1-ax^2} dx}{9a} \\
&= -\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{9a^{3/2}} + \frac{2}{9}x^3\log\left(1-ax^2\right) + \frac{1}{3}x^3\text{Li}_2\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0514052, size = 57, normalized size = 0.9

$$
\frac{1}{27}\left(9x^3\text{PolyLog}\left(2,ax^2\right) + \frac{12\tanh^{-1}\left(\sqrt{a}x\right)}{a^{3/2}} + 6x^3\log\left(1-ax^2\right) - \frac{12x}{a} - 4x^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x^2],x]
```

```
[Out] ((-12*x)/a - 4*x^3 + (12*ArcTanh[Sqrt[a]*x])/a^(3/2) + 6*x^3*Log[1 - a*x^2]
 + 9*x^3*PolyLog[2, a*x^2])/27
```

Maple [A] time = 0.046, size = 50, normalized size = 0.8

$$
-\frac{4x}{9a} - \frac{4x^3}{27} + \frac{4}{9}\text{Artanh}\left(x\sqrt{a}\right)a^{-\frac{3}{2}} + \frac{2x^3\ln\left(-ax^2+1\right)}{9} + \frac{x^3\text{polylog}\left(2,ax^2\right)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x^2),x)
```

```
[Out] -4/9*x/a-4/27*x^3+4/9*arctanh(x*a^(1/2))/a^(3/2)+2/9*x^3*ln(-a*x^2+1)+1/3*x
^3*polylog(2,a*x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63943, size = 351, normalized size = 5.57

$$\left[\frac{9\,a^2x^3\mathrm{Li}_2\left(ax^2\right)+6\,a^2x^3\log\left(-ax^2+1\right)-4\,a^2x^3-12\,ax+6\,\sqrt{a}\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{27\,a^2},\ \frac{9\,a^2x^3\mathrm{Li}_2\left(ax^2\right)+6\,a^2x^3\log\left(-ax^2+1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] [1/27*(9*a^2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12*
a*x + 6*sqrt(a)*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^2, 1/27*(9*a^
2*x^3*dilog(a*x^2) + 6*a^2*x^3*log(-a*x^2 + 1) - 4*a^2*x^3 - 12*a*x - 12*sq
rt(-a)*arctan(sqrt(-a)*x))/a^2]
```

Sympy [A] time = 119.376, size = 83, normalized size = 1.32

$$\begin{cases} -\frac{2x^3\,\mathrm{Li}_1\left(ax^2\right)}{9}+\frac{x^3\,\mathrm{Li}_2\left(ax^2\right)}{3}-\frac{4x^3}{27}-\frac{4x}{9a}-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{9a^2\sqrt{\frac{1}{a}}}-\frac{2\,\mathrm{Li}_1\left(ax^2\right)}{9a^2\sqrt{\frac{1}{a}}} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x**3*polylog(1, a*x**2)/9 + x**3*polylog(2, a*x**2)/3 - 4*x**
3/27 - 4*x/(9*a) - 4*log(x - sqrt(1/a))/(9*a**2*sqrt(1/a)) - 2*polylog(1, a
*x**2)/(9*a**2*sqrt(1/a)), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x^2), x)
```

3.28 $\int \mathrm{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=40

$$x\mathrm{PolyLog}\left(2, ax^2\right) + 2x\log\left(1 - ax^2\right) + \frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}} - 4x$$

```
[Out] -4*x + (4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*Log[1 - a*x^2] + x*PolyLog[2, a
*x^2]
```

Rubi [A] time = 0.0174073, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6586, 2448, 321, 206}

$$x\mathrm{PolyLog}\left(2, ax^2\right) + 2x\log\left(1 - ax^2\right) + \frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}} - 4x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2],x]
```

```
[Out] -4*x + (4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*Log[1 - a*x^2] + x*PolyLog[2, a
*x^2]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_2\left(ax^2\right) dx &= x\mathrm{Li}_2\left(ax^2\right) + 2\int \log\left(1-ax^2\right) dx \\ &= 2x\log\left(1-ax^2\right) + x\mathrm{Li}_2\left(ax^2\right) + (4a)\int \frac{x^2}{1-ax^2}\,dx \\ &= -4x + 2x\log\left(1-ax^2\right) + x\mathrm{Li}_2\left(ax^2\right) + 4\int \frac{1}{1-ax^2}\,dx \\ &= -4x + \frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}} + 2x\log\left(1-ax^2\right) + x\mathrm{Li}_2\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.0278312, size = 39, normalized size = 0.98

$$x\mathrm{PolyLog}\left(2,ax^2\right) + 2x\left(\log\left(1-ax^2\right)-2\right) + \frac{4\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2],x]
```

```
[Out] (4*ArcTanh[Sqrt[a]*x])/Sqrt[a] + 2*x*(-2 + Log[1 - a*x^2]) + x*PolyLog[2, a
*x^2]
```

Maple [A] time = 0.046, size = 37, normalized size = 0.9

$$-4\,x + 2\,x\ln\left(-ax^2+1\right) + x\mathrm{polylog}\left(2,ax^2\right) + 4\,\frac{\mathrm{Artanh}\left(x\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2),x)

[Out] -4*x+2*x*ln(-a*x^2+1)+x*polylog(2,a*x^2)+4*arctanh(x*a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63136, size = 269, normalized size = 6.72

$$\left[\frac{ax\mathrm{Li}_2\left(ax^2\right)+2\,ax\log\left(-ax^2+1\right)-4\,ax+2\,\sqrt{a}\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)}{a},\frac{ax\mathrm{Li}_2\left(ax^2\right)+2\,ax\log\left(-ax^2+1\right)-4\,ax-4\,\sqrt{-a}\,\mathrm{arc}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="fricas")

[Out] [(a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1) - 4*a*x + 2*sqrt(a)*log((a*x^2 +
 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a, (a*x*dilog(a*x^2) + 2*a*x*log(-a*x^2 + 1
) - 4*a*x - 4*sqrt(-a)*arctan(sqrt(-a)*x))/a]
```

Sympy [A] time = 23.2238, size = 60, normalized size = 1.5

$$\begin{cases} -2x\,\mathrm{Li}_1\left(ax^2\right)+x\,\mathrm{Li}_2\left(ax^2\right)-4x-\frac{4\log\left(x-\sqrt{\frac{1}{a}}\right)}{a\sqrt{\frac{1}{a}}}-\frac{2\,\mathrm{Li}_1\left(ax^2\right)}{a\sqrt{\frac{1}{a}}} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2),x)
```

```
[Out] Piecewise((-2*x*polylog(1, a*x**2) + x*polylog(2, a*x**2) - 4*x - 4*log(x -
 sqrt(1/a))/(a*sqrt(1/a)) - 2*polylog(1, a*x**2)/(a*sqrt(1/a)), Ne(a, 0)),
(0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_2\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2), x)
```

3.29 $\int \frac{\text{PolyLog}(2,ax^2)}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\text{PolyLog}(2,ax^2)}{x}+\frac{2\log(1-ax^2)}{x}+4\sqrt{a}\tanh^{-1}(\sqrt{a}x)$$

```
[Out] 4*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (2*Log[1 - a*x^2])/x - PolyLog[2, a*x^2]/x
```

Rubi [A] time = 0.0261791, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 206}

$$-\frac{\text{PolyLog}(2,ax^2)}{x}+\frac{2\log(1-ax^2)}{x}+4\sqrt{a}\tanh^{-1}(\sqrt{a}x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^2,x]
```

```
[Out] 4*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (2*Log[1 - a*x^2])/x - PolyLog[2, a*x^2]/x
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2\left(ax^2\right)}{x^2}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{x}-2\int \frac{\log\left(1-ax^2\right)}{x^2}\,dx\\ &= \frac{2\log\left(1-ax^2\right)}{x}-\frac{\text{Li}_2\left(ax^2\right)}{x}+(4a)\int \frac{1}{1-ax^2}\,dx\\ &= 4\sqrt{a}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{2\log\left(1-ax^2\right)}{x}-\frac{\text{Li}_2\left(ax^2\right)}{x}\end{aligned}$$

Mathematica [A] time = 0.0177403, size = 41, normalized size = 0.98

$$\frac{-\text{PolyLog}\left(2,ax^2\right)+2\log\left(1-ax^2\right)+4\sqrt{a}x\tanh^{-1}\left(\sqrt{a}x\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^2,x]
```

```
[Out] (4*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 2*Log[1 - a*x^2] - PolyLog[2, a*x^2])/x
```

Maple [A] time = 0.049, size = 39, normalized size = 0.9

$$2\,\frac{\ln\left(-ax^2+1\right)}{x}-\frac{\text{polylog}\left(2,ax^2\right)}{x}+4\,\text{Artanh}\left(x\sqrt{a}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^2,x)
```

```
[Out] 2*ln(-a*x^2+1)/x-polylog(2,a*x^2)/x+4*arctanh(x*a^(1/2))*a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.79204, size = 232, normalized size = 5.52

$$\left[\frac{2\sqrt{a}x\log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right)-\mathrm{Li}_2\left(ax^2\right)+2\log\left(-ax^2+1\right)}{x},-\frac{4\sqrt{-a}x\arctan\left(\sqrt{-a}x\right)+\mathrm{Li}_2\left(ax^2\right)-2\log\left(-ax^2+1\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [(2*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - dilog(a*x^2) + 2
*log(-a*x^2 + 1))/x, -(4*sqrt(-a)*x*arctan(sqrt(-a)*x) + dilog(a*x^2) - 2*l
og(-a*x^2 + 1))/x]
```

Sympy [A] time = 84.144, size = 184, normalized size = 4.38

$$\begin{cases} 0 & \text{for} \\ -\frac{\pi^2}{6x} & \text{for} \\ -\frac{4ax^3\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}}-\frac{2ax^3\sqrt{\frac{1}{a}}\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}-\frac{2x^2\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}-\frac{x^2\mathrm{Li}_2\left(ax^2\right)}{x^3-\frac{x}{a}}+\frac{4x\sqrt{\frac{1}{a}}\log\left(x-\sqrt{\frac{1}{a}}\right)}{x^3-\frac{x}{a}}+\frac{2x\sqrt{\frac{1}{a}}\mathrm{Li}_1\left(ax^2\right)}{x^3-\frac{x}{a}}+\frac{2\mathrm{Li}_1\left(ax^2\right)}{ax^3-x}+\frac{\mathrm{Li}_2\left(ax^2\right)}{ax^3-x} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**2,x)
```

```
[Out] Piecewise((0, Eq(a, 0)), (-pi**2/(6*x), Eq(a, x**(-2))), (-4*a*x**3*sqrt(1/
a)*log(x - sqrt(1/a))/(x**3 - x/a) - 2*a*x**3*sqrt(1/a)*polylog(1, a*x**2)/
(x**3 - x/a) - 2*x**2*polylog(1, a*x**2)/(x**3 - x/a) - x**2*polylog(2, a*x
**2)/(x**3 - x/a) + 4*x*sqrt(1/a)*log(x - sqrt(1/a))/(x**3 - x/a) + 2*x*sqr
t(1/a)*polylog(1, a*x**2)/(x**3 - x/a) + 2*polylog(1, a*x**2)/(a*x**3 - x)
+ polylog(2, a*x**2)/(a*x**3 - x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^2, x)
```

3.30 $\int \frac{\textbf{PolyLog}(2,ax^2)}{x^4} dx$

Optimal. Leaf size=56

$$-\frac{\text{PolyLog}(2,ax^2)}{3x^3}+\frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{4a}{9x}$$

```
[Out] (-4*a)/(9*x) + (4*a^(3/2)*ArcTanh[Sqrt[a]*x])/9 + (2*Log[1 - a*x^2])/(9*x^3
) - PolyLog[2, a*x^2]/(3*x^3)
```

Rubi [A] time = 0.0321837, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 325, 206}

$$-\frac{\text{PolyLog}(2,ax^2)}{3x^3}+\frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{4a}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^4,x]
```

```
[Out] (-4*a)/(9*x) + (4*a^(3/2)*ArcTanh[Sqrt[a]*x])/9 + (2*Log[1 - a*x^2])/(9*x^3
) - PolyLog[2, a*x^2]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2\left(ax^2\right)}{x^4}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{3x^3}-\frac{2}{3}\int \frac{\log\left(1-ax^2\right)}{x^4}\,dx\\
&= \frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{\text{Li}_2\left(ax^2\right)}{3x^3}+\frac{1}{9}(4a)\int \frac{1}{x^2\left(1-ax^2\right)}\,dx\\
&= -\frac{4a}{9x}+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{\text{Li}_2\left(ax^2\right)}{3x^3}+\frac{1}{9}\left(4a^2\right)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{4a}{9x}+\frac{4}{9}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{2\log\left(1-ax^2\right)}{9x^3}-\frac{\text{Li}_2\left(ax^2\right)}{3x^3}
\end{aligned}$$

Mathematica [C] time = 0.0136452, size = 47, normalized size = 0.84

$$-\frac{4ax^2\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},ax^2\right)+3\text{PolyLog}\left(2,ax^2\right)-2\log\left(1-ax^2\right)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^4,x]
```

```
[Out] -(4*a*x^2*Hypergeometric2F1[-1/2, 1, 1/2, a*x^2] - 2*Log[1 - a*x^2] + 3*Pol
yLog[2, a*x^2])/(9*x^3)
```

Maple [A] time = 0.053, size = 45, normalized size = 0.8

$$-\frac{4\,a}{9\,x}+\frac{4}{9}a^{\frac{3}{2}}\text{Artanh}\left(x\sqrt{a}\right)+\frac{2\,\ln\left(-ax^2+1\right)}{9\,x^3}-\frac{\text{polylog}\left(2,ax^2\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^4,x)
```

```
[Out] -4/9*a/x+4/9*a^(3/2)*arctanh(x*a^(1/2))+2/9*ln(-a*x^2+1)/x^3-1/3*polylog(2,
a*x^2)/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74604, size = 289, normalized size = 5.16

$$\left[\frac{2\,a^{\frac{3}{2}}x^3\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)-4\,ax^2-3\,\mathrm{Li}_2\left(ax^2\right)+2\,\log\left(-ax^2+1\right)}{9\,x^3},-\frac{4\,\sqrt{-a}ax^3\arctan\left(\sqrt{-a}x\right)+4\,ax^2+3\,\mathrm{Li}_2\left(ax^2\right)-2\,}{9\,x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(2*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 4*a*x^2 -
3*dilog(a*x^2) + 2*log(-a*x^2 + 1))/x^3, -1/9*(4*sqrt(-a)*a*x^3*arctan(sqrt
(-a)*x) + 4*a*x^2 + 3*dilog(a*x^2) - 2*log(-a*x^2 + 1))/x^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^4, x)
```

3.31 $\int \frac{\text{PolyLog}(2,ax^2)}{x^6} dx$

Optimal. Leaf size=66

$$-\frac{\text{PolyLog}(2,ax^2)}{5x^5} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right) - \frac{4a}{75x^3} + \frac{2\log(1-ax^2)}{25x^5}$$

```
[Out] (-4*a)/(75*x^3) - (4*a^2)/(25*x) + (4*a^(5/2)*ArcTanh[Sqrt[a]*x])/25 + (2*L
og[1 - a*x^2])/(25*x^5) - PolyLog[2, a*x^2]/(5*x^5)
```

Rubi [A] time = 0.0376341, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 325, 206}

$$-\frac{\text{PolyLog}(2,ax^2)}{5x^5} - \frac{4a^2}{25x} + \frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right) - \frac{4a}{75x^3} + \frac{2\log(1-ax^2)}{25x^5}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/x^6,x]
```

```
[Out] (-4*a)/(75*x^3) - (4*a^2)/(25*x) + (4*a^(5/2)*ArcTanh[Sqrt[a]*x])/25 + (2*L
og[1 - a*x^2])/(25*x^5) - PolyLog[2, a*x^2]/(5*x^5)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2\left(ax^2\right)}{x^6}\,dx &= -\frac{\text{Li}_2\left(ax^2\right)}{5x^5}-\frac{2}{5}\int \frac{\log\left(1-ax^2\right)}{x^6}\,dx\\
&= \frac{2\log\left(1-ax^2\right)}{25x^5}-\frac{\text{Li}_2\left(ax^2\right)}{5x^5}+\frac{1}{25}(4a)\int \frac{1}{x^4\left(1-ax^2\right)}\,dx\\
&= -\frac{4a}{75x^3}+\frac{2\log\left(1-ax^2\right)}{25x^5}-\frac{\text{Li}_2\left(ax^2\right)}{5x^5}+\frac{1}{25}\left(4a^2\right)\int \frac{1}{x^2\left(1-ax^2\right)}\,dx\\
&= -\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{2\log\left(1-ax^2\right)}{25x^5}-\frac{\text{Li}_2\left(ax^2\right)}{5x^5}+\frac{1}{25}\left(4a^3\right)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{4}{25}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{2\log\left(1-ax^2\right)}{25x^5}-\frac{\text{Li}_2\left(ax^2\right)}{5x^5}
\end{aligned}$$

Mathematica [C] time = 0.0151209, size = 47, normalized size = 0.71

$$-\frac{4ax^2\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},ax^2\right)+15\text{PolyLog}\left(2,ax^2\right)-6\log\left(1-ax^2\right)}{75x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^2]/x^6,x]
```

```
[Out] -(4*a*x^2*Hypergeometric2F1[-3/2, 1, -1/2, a*x^2] - 6*Log[1 - a*x^2] + 15*P
olyLog[2, a*x^2])/(75*x^5)
```

Maple [A] time = 0.052, size = 53, normalized size = 0.8

$$-\frac{4a}{75x^3}-\frac{4a^2}{25x}+\frac{4}{25}a^{\frac{5}{2}}\mathrm{Artanh}\left(x\sqrt{a}\right)+\frac{2\ln\left(-ax^2+1\right)}{25x^5}-\frac{\mathrm{polylog}\left(2,ax^2\right)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/x^6,x)
```

```
[Out] -4/75*a/x^3-4/25*a^2/x+4/25*a^(5/2)*arctanh(x*a^(1/2))+2/25*ln(-a*x^2+1)/x^
5-1/5*polylog(2,a*x^2)/x^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66061, size = 333, normalized size = 5.05

$$\left[\frac{6a^{\frac{5}{2}}x^5\log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right)-12a^2x^4-4ax^2-15\,\mathrm{Li}_2\left(ax^2\right)+6\log\left(-ax^2+1\right)}{75x^5},-\frac{12\sqrt{-a}a^2x^5\arctan\left(\sqrt{-a}x\right)+12a^2x^4+4\right.}{75x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/75*(6*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 12*a^2*x^
4 - 4*a*x^2 - 15*dilog(a*x^2) + 6*log(-a*x^2 + 1))/x^5, -1/75*(12*sqrt(-a)*
a^2*x^5*arctan(sqrt(-a)*x) + 12*a^2*x^4 + 4*a*x^2 + 15*dilog(a*x^2) - 6*log
(-a*x^2 + 1))/x^5]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/x**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/x^6, x)
```

3.32 $\int x^5\textbf{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=88

$$-\frac{1}{18}x^6\text{PolyLog}\left(2,ax^2\right)+\frac{1}{6}x^6\text{PolyLog}\left(3,ax^2\right)+\frac{x^2}{54a^2}+\frac{\log\left(1-ax^2\right)}{54a^3}+\frac{x^4}{108a}-\frac{1}{54}x^6\log\left(1-ax^2\right)+\frac{x^6}{162}$$

```
[Out] x^2/(54*a^2) + x^4/(108*a) + x^6/162 + Log[1 - a*x^2]/(54*a^3) - (x^6*Log[1
 - a*x^2])/54 - (x^6*PolyLog[2, a*x^2])/18 + (x^6*PolyLog[3, a*x^2])/6
```

Rubi [A] time = 0.075215, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 43}

$$-\frac{1}{18}x^6\text{PolyLog}\left(2,ax^2\right)+\frac{1}{6}x^6\text{PolyLog}\left(3,ax^2\right)+\frac{x^2}{54a^2}+\frac{\log\left(1-ax^2\right)}{54a^3}+\frac{x^4}{108a}-\frac{1}{54}x^6\log\left(1-ax^2\right)+\frac{x^6}{162}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/(54*a^2) + x^4/(108*a) + x^6/162 + Log[1 - a*x^2]/(54*a^3) - (x^6*Log[1
 - a*x^2])/54 - (x^6*PolyLog[2, a*x^2])/18 + (x^6*PolyLog[3, a*x^2])/6
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5\mathrm{Li}_3\left(ax^2\right)dx &= \frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)-\frac{1}{3}\int x^5\mathrm{Li}_2\left(ax^2\right)dx\\
&= -\frac{1}{18}x^6\mathrm{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)-\frac{1}{9}\int x^5\log\left(1-ax^2\right)dx\\
&= -\frac{1}{18}x^6\mathrm{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)-\frac{1}{18}\,\mathrm{Subst}\left(\int x^2\log(1-ax)\,dx,x,x^2\right)\\
&= -\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\mathrm{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)-\frac{1}{54}a\,\mathrm{Subst}\left(\int\frac{x^3}{1-ax}\,dx,x,x^2\right)\\
&= -\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\mathrm{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)-\frac{1}{54}a\,\mathrm{Subst}\left(\int\left(-\frac{1}{a^3}-\frac{x}{a^2}-\frac{x^2}{a}-\frac{1}{a^3(-1+\right.\right.\\
&= \frac{x^2}{54a^2}+\frac{x^4}{108a}+\frac{x^6}{162}+\frac{\log\left(1-ax^2\right)}{54a^3}-\frac{1}{54}x^6\log\left(1-ax^2\right)-\frac{1}{18}x^6\mathrm{Li}_2\left(ax^2\right)+\frac{1}{6}x^6\mathrm{Li}_3\left(ax^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0180466, size = 88, normalized size = 1.

$$\frac{-18a^3x^6\mathrm{PolyLog}\left(2,ax^2\right)+54a^3x^6\mathrm{PolyLog}\left(3,ax^2\right)+2a^3x^6+3a^2x^4-6a^3x^6\log\left(1-ax^2\right)+6ax^2+6\log\left(1-ax^2\right)}{324a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*PolyLog[3, a*x^2],x]
```

```
[Out] (6*a*x^2 + 3*a^2*x^4 + 2*a^3*x^6 + 6*Log[1 - a*x^2] - 6*a^3*x^6*Log[1 - a*x
^2] - 18*a^3*x^6*PolyLog[2, a*x^2] + 54*a^3*x^6*PolyLog[3, a*x^2])/(324*a^3
)
```

Maple [A] time = 0.058, size = 80, normalized size = 0.9

$$\frac{1}{2a^{3}}\left(\frac{x^{2}a\left(4a^{2}x^{4}+6ax^{2}+12\right)}{324}+\frac{\left(-4x^{6}a^{3}+4\right)\ln\left(-ax^{2}+1\right)}{108}-\frac{x^{6}a^{3}\mathrm{polylog}\left(2,ax^{2}\right)}{9}+\frac{x^{6}a^{3}\mathrm{polylog}\left(3,ax^{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*polylog(3,a*x^2),x)
```

```
[Out] 1/2/a^3*(1/324*x^2*a*(4*a^2*x^4+6*a*x^2+12)+1/108*(-4*a^3*x^6+4)*ln(-a*x^2+
1)-1/9*x^6*a^3*polylog(2,a*x^2)+1/3*x^6*a^3*polylog(3,a*x^2))
```

Maxima [A] time = 0.97977, size = 104, normalized size = 1.18

$$-\frac{18a^{3}x^{6}\mathrm{Li}_{2}\left(ax^{2}\right)-54a^{3}x^{6}\mathrm{Li}_{3}(ax^{2})-2a^{3}x^{6}-3a^{2}x^{4}-6ax^{2}+6\left(a^{3}x^{6}-1\right)\log\left(-ax^{2}+1\right)}{324a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/324*(18*a^3*x^6*dilog(a*x^2) - 54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6
- 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6 - 1)*log(-a*x^2 + 1))/a^3
```

Fricas [C] time = 2.71681, size = 240, normalized size = 2.73

$$-\frac{18a^{3}x^{6}\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^{2}+1\right)}{a},-\frac{2\log\left(-ax^{2}+1\right)}{x}\right)-54a^{3}x^{6}\mathrm{polylog}\left(3,ax^{2}\right)-2a^{3}x^{6}-3a^{2}x^{4}-6ax^{2}+6\left(a^{3}x^{6}-1\right)\log\left(-\right.}{324a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/324*(18*a^3*x^6*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) -
54*a^3*x^6*polylog(3, a*x^2) - 2*a^3*x^6 - 3*a^2*x^4 - 6*a*x^2 + 6*(a^3*x^6
 - 1)*log(-a*x^2 + 1))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**5*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \operatorname{Li}_3(ax^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5*polylog(3, a*x^2), x)
```

3.33 $\int x^3 \textbf{PolyLog}\left(3, ax^2\right) dx$

Optimal. Leaf size=78

$$-\frac{1}{8}x^4\text{PolyLog}\left(2, ax^2\right) + \frac{1}{4}x^4\text{PolyLog}\left(3, ax^2\right) + \frac{\log\left(1 - ax^2\right)}{16a^2} + \frac{x^2}{16a} - \frac{1}{16}x^4\log\left(1 - ax^2\right) + \frac{x^4}{32}$$

```
[Out] x^2/(16*a) + x^4/32 + Log[1 - a*x^2]/(16*a^2) - (x^4*Log[1 - a*x^2])/16 - (
x^4*PolyLog[2, a*x^2])/8 + (x^4*PolyLog[3, a*x^2])/4
```

Rubi [A] time = 0.0606042, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 43}

$$-\frac{1}{8}x^4\text{PolyLog}\left(2, ax^2\right) + \frac{1}{4}x^4\text{PolyLog}\left(3, ax^2\right) + \frac{\log\left(1 - ax^2\right)}{16a^2} + \frac{x^2}{16a} - \frac{1}{16}x^4\log\left(1 - ax^2\right) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/(16*a) + x^4/32 + Log[1 - a*x^2]/(16*a^2) - (x^4*Log[1 - a*x^2])/16 - (
x^4*PolyLog[2, a*x^2])/8 + (x^4*PolyLog[3, a*x^2])/4
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3 \mathrm{Li}_3\left(ax^2\right) dx &= \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right) - \frac{1}{2}\int x^3\mathrm{Li}_2\left(ax^2\right) dx \\
&= -\frac{1}{8}x^4\mathrm{Li}_2\left(ax^2\right) + \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right) - \frac{1}{4}\int x^3\log\left(1-ax^2\right) dx \\
&= -\frac{1}{8}x^4\mathrm{Li}_2\left(ax^2\right) + \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right) - \frac{1}{8}\,\mathrm{Subst}\left(\int x\log(1-ax)\,dx, x, x^2\right) \\
&= -\frac{1}{16}x^4\log\left(1-ax^2\right) - \frac{1}{8}x^4\mathrm{Li}_2\left(ax^2\right) + \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right) - \frac{1}{16}a\,\mathrm{Subst}\left(\int \frac{x^2}{1-ax}\,dx, x, x^2\right) \\
&= -\frac{1}{16}x^4\log\left(1-ax^2\right) - \frac{1}{8}x^4\mathrm{Li}_2\left(ax^2\right) + \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right) - \frac{1}{16}a\,\mathrm{Subst}\left(\int\left(-\frac{1}{a^2} - \frac{x}{a} - \frac{1}{a^2(-1+ax)}\right) dx \right. \\
&= \frac{x^2}{16a} + \frac{x^4}{32} + \frac{\log\left(1-ax^2\right)}{16a^2} - \frac{1}{16}x^4\log\left(1-ax^2\right) - \frac{1}{8}x^4\mathrm{Li}_2\left(ax^2\right) + \frac{1}{4}x^4\mathrm{Li}_3\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0144214, size = 79, normalized size = 1.01

$$
\frac{-4a^2x^4\mathrm{PolyLog}\left(2,ax^2\right) + 8a^2x^4\mathrm{PolyLog}\left(3,ax^2\right) + a^2x^4 - 2a^2x^4\log\left(1-ax^2\right) + 2ax^2 + 2\log\left(1-ax^2\right)}{32a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[3, a*x^2],x]
```

```
[Out] (2*a*x^2 + a^2*x^4 + 2*Log[1 - a*x^2] - 2*a^2*x^4*Log[1 - a*x^2] - 4*a^2*x^
4*PolyLog[2, a*x^2] + 8*a^2*x^4*PolyLog[3, a*x^2])/(32*a^2)
```

Maple [A] time = 0.054, size = 72, normalized size = 0.9

$$-\frac{1}{2\,a^{2}}\left(-\frac{x^{2}a\left(3\,ax^{2}+6\right)}{48}-\frac{\left(-3\,a^{2}x^{4}+3\right)\ln\left(-ax^{2}+1\right)}{24}+\frac{x^{4}a^{2}\mathrm{polylog}\left(2,ax^{2}\right)}{4}-\frac{x^{4}a^{2}\mathrm{polylog}\left(3,ax^{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(3,a*x^2),x)
```

```
[Out] -1/2/a^2*(-1/48*x^2*a*(3*a*x^2+6)-1/24*(-3*a^2*x^4+3)*ln(-a*x^2+1)+1/4*x^4*
a^2*polylog(2,a*x^2)-1/2*x^4*a^2*polylog(3,a*x^2))
```

Maxima [A] time = 0.968293, size = 93, normalized size = 1.19

$$-\frac{4\,a^{2}x^{4}\mathrm{Li}_{2}\left(ax^{2}\right)-8\,a^{2}x^{4}\mathrm{Li}_{3}(ax^{2})-a^{2}x^{4}-2\,ax^{2}+2\left(a^{2}x^{4}-1\right)\log\left(-ax^{2}+1\right)}{32\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/32*(4*a^2*x^4*dilog(a*x^2) - 8*a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*a
*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2 + 1))/a^2
```

Fricas [C] time = 2.70568, size = 217, normalized size = 2.78

$$-\frac{4\,a^{2}x^{4}\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^{2}+1\right)}{a},-\frac{2\,\log\left(-ax^{2}+1\right)}{x}\right)-8\,a^{2}x^{4}\mathrm{polylog}\left(3,ax^{2}\right)-a^{2}x^{4}-2\,ax^{2}+2\left(a^{2}x^{4}-1\right)\log\left(-ax^{2}+1\right)}{32\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/32*(4*a^2*x^4*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 8*
a^2*x^4*polylog(3, a*x^2) - a^2*x^4 - 2*a*x^2 + 2*(a^2*x^4 - 1)*log(-a*x^2
+ 1))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**3*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^3*polylog(3, a*x^2), x)
```

3.34 $\int x\text{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=60

$$-\frac{1}{2}x^2\text{PolyLog}\left(2,ax^2\right)+\frac{1}{2}x^2\text{PolyLog}\left(3,ax^2\right)+\frac{\left(1-ax^2\right)\log\left(1-ax^2\right)}{2a}+\frac{x^2}{2}$$

```
[Out] x^2/2 + ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) - (x^2*PolyLog[2, a*x^2])/2 + (x
^2*PolyLog[3, a*x^2])/2
```

Rubi [A] time = 0.029869, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6591, 2454, 2389, 2295}

$$-\frac{1}{2}x^2\text{PolyLog}\left(2,ax^2\right)+\frac{1}{2}x^2\text{PolyLog}\left(3,ax^2\right)+\frac{\left(1-ax^2\right)\log\left(1-ax^2\right)}{2a}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x^2],x]
```

```
[Out] x^2/2 + ((1 - a*x^2)*Log[1 - a*x^2])/(2*a) - (x^2*PolyLog[2, a*x^2])/2 + (x
^2*PolyLog[3, a*x^2])/2
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_3\left(ax^2\right)\,dx &= \frac{1}{2}x^2\text{Li}_3\left(ax^2\right) - \int x\text{Li}_2\left(ax^2\right)\,dx\\ &= -\frac{1}{2}x^2\text{Li}_2\left(ax^2\right) + \frac{1}{2}x^2\text{Li}_3\left(ax^2\right) - \int x\log\left(1-ax^2\right)\,dx\\ &= -\frac{1}{2}x^2\text{Li}_2\left(ax^2\right) + \frac{1}{2}x^2\text{Li}_3\left(ax^2\right) - \frac{1}{2}\,\text{Subst}\left(\int \log(1-ax)\,dx, x, x^2\right)\\ &= -\frac{1}{2}x^2\text{Li}_2\left(ax^2\right) + \frac{1}{2}x^2\text{Li}_3\left(ax^2\right) + \frac{\text{Subst}\left(\int \log(x)\,dx, x, 1-ax^2\right)}{2a}\\ &= \frac{x^2}{2} + \frac{\left(1-ax^2\right)\log\left(1-ax^2\right)}{2a} - \frac{1}{2}x^2\text{Li}_2\left(ax^2\right) + \frac{1}{2}x^2\text{Li}_3\left(ax^2\right)\end{aligned}$$

Mathematica [A] time = 0.0112108, size = 52, normalized size = 0.87

$$\frac{1}{2}x^2\left(-\text{PolyLog}\left(2,ax^2\right) + \text{PolyLog}\left(3,ax^2\right) + \frac{\log\left(1-ax^2\right)}{ax^2} - \log\left(1-ax^2\right) + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, a*x^2],x]
```

```
[Out] (x^2*(1 - Log[1 - a*x^2] + Log[1 - a*x^2]/(a*x^2) - PolyLog[2, a*x^2] + Pol
yLog[3, a*x^2]))/2
```

Maple [A] time = 0.055, size = 56, normalized size = 0.9

$$\frac{1}{2\,a}\left(ax^2 + \frac{\left(-2\,ax^2+2\right)\ln\left(-ax^2+1\right)}{2} - ax^2\text{polylog}\left(2,ax^2\right) + ax^2\text{polylog}\left(3,ax^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x^2),x)
```

```
[Out] 1/2/a*(a*x^2+1/2*(-2*a*x^2+2)*ln(-a*x^2+1)-a*x^2*polylog(2,a*x^2)+a*x^2*pol
ylog(3,a*x^2))
```

Maxima [A] time = 1.00071, size = 72, normalized size = 1.2

$$-\frac{ax^2\text{Li}_2\left(ax^2\right) - ax^2\text{Li}_3(ax^2) - ax^2 + \left(ax^2 - 1\right)\log\left(-ax^2 + 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*x^2*dilog(a*x^2) - a*x^2*polylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*lo
g(-a*x^2 + 1))/a
```

Fricas [C] time = 2.66414, size = 181, normalized size = 3.02

$$-\frac{ax^2\%\text{iint}\left(a, x, -\frac{\log\left(-ax^2+1\right)}{a}, -\frac{2\,\log\left(-ax^2+1\right)}{x}\right) - ax^2\text{polylog}\left(3, ax^2\right) - ax^2 + \left(ax^2 - 1\right)\log\left(-ax^2 + 1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - a*x^2*p
olylog(3, a*x^2) - a*x^2 + (a*x^2 - 1)*log(-a*x^2 + 1))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\text{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x**2),x)
```

```
[Out] Integral(x*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ax^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, a*x^2), x)
```

3.35 $\int \frac{\text{PolyLog}(3,ax^2)}{x}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\text{PolyLog}\left(4,ax^2\right)$$

```
[Out] PolyLog[4, a*x^2]/2
```

Rubi [A] time = 0.0090189, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6589}

$$\frac{1}{2}\text{PolyLog}\left(4,ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x,x]

[Out] PolyLog[4, a*x^2]/2
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3\left(ax^2\right)}{x}\,dx=\frac{\text{Li}_4\left(ax^2\right)}{2}$$

Mathematica [A] time = 0.0014221, size = 11, normalized size = 1.

$$\frac{1}{2}\text{PolyLog}\left(4,ax^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x,x]
```

```
[Out] PolyLog[4, a*x^2]/2
```

Maple [A] time = 0.154, size = 10, normalized size = 0.9

$$\frac{\text{polylog}\left(4, ax^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x,x)
```

```
[Out] 1/2*polylog(4,a*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\left(3, ax^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^2)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x, x)
```

3.36 $\int \frac{\text{PolyLog}(3,ax^2)}{x^3}\,dx$

Optimal. Leaf size=63

$$-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}-\frac{1}{2}a\log(1-ax^2)+\frac{\log(1-ax^2)}{2x^2}+a\log(x)$$

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2) - PolyLog[3, a*x^2]/(2*x^2)
```

Rubi [A] time = 0.047025, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6591, 2454, 2395, 36, 29, 31}

$$-\frac{\text{PolyLog}(2,ax^2)}{2x^2}-\frac{\text{PolyLog}(3,ax^2)}{2x^2}-\frac{1}{2}a\log(1-ax^2)+\frac{\log(1-ax^2)}{2x^2}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^3,x]
```

```
[Out] a*Log[x] - (a*Log[1 - a*x^2])/2 + Log[1 - a*x^2]/(2*x^2) - PolyLog[2, a*x^2
]/(2*x^2) - PolyLog[3, a*x^2]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x^3}\,dx &= -\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}+\int \frac{\mathrm{Li}_2\left(ax^2\right)}{x^3}\,dx \\
&= -\frac{\mathrm{Li}_2\left(ax^2\right)}{2x^2}-\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}-\int \frac{\log\left(1-ax^2\right)}{x^3}\,dx \\
&= -\frac{\mathrm{Li}_2\left(ax^2\right)}{2x^2}-\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log(1-ax)}{x^2}\,dx,x,x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\mathrm{Li}_2\left(ax^2\right)}{2x^2}-\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x(1-ax)}\,dx,x,x^2\right) \\
&= \frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\mathrm{Li}_2\left(ax^2\right)}{2x^2}-\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^2\,\mathrm{Subst}\left(\int \frac{1}{1-ax}\,dx,x,x^2\right) \\
&= a\log(x)-\frac{1}{2}a\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{2x^2}-\frac{\mathrm{Li}_2\left(ax^2\right)}{2x^2}-\frac{\mathrm{Li}_3\left(ax^2\right)}{2x^2}
\end{aligned}
$$

Mathematica [A] time = 0.0269275, size = 60, normalized size = 0.95

$$
-\frac{\mathrm{PolyLog}\left(2,ax^2\right)+\mathrm{PolyLog}\left(3,ax^2\right)-ax^2\log\left(-ax^2\right)+ax^2\log\left(1-ax^2\right)-\log\left(1-ax^2\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^3,x]
```

```
[Out] -(-(a*x^2*Log[-(a*x^2)]) - Log[1 - a*x^2] + a*x^2*Log[1 - a*x^2] + PolyLog[
2, a*x^2] + PolyLog[3, a*x^2])/(2*x^2)
```

Maple [A] time = 0.061, size = 68, normalized size = 1.1

$$\frac{a}{2}\left(2\ln(x)+\ln(-a)+\frac{\left(-8ax^2+8\right)\ln\left(-ax^2+1\right)}{8ax^2}-\frac{\operatorname{polylog}\left(2,ax^2\right)}{ax^2}-\frac{\operatorname{polylog}\left(3,ax^2\right)}{ax^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^3,x)
```

```
[Out] 1/2*a*(2*ln(x)+ln(-a)+1/8/a/x^2*(-8*a*x^2+8)*ln(-a*x^2+1)-polylog(2,a*x^2)/
a/x^2-1/a/x^2*polylog(3,a*x^2))
```

Maxima [A] time = 0.989681, size = 55, normalized size = 0.87

$$a\log(x)-\frac{\left(ax^2-1\right)\log\left(-ax^2+1\right)+\operatorname{Li}_2\left(ax^2\right)+\operatorname{Li}_3(ax^2)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] a*log(x) - 1/2*((a*x^2 - 1)*log(-a*x^2 + 1) + dilog(a*x^2) + polylog(3, a*x
^2))/x^2
```

Fricas [C] time = 2.71896, size = 194, normalized size = 3.08

$$-\frac{ax^2\log\left(ax^2-1\right)-2ax^2\log(x)+\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)-\log\left(-ax^2+1\right)+\operatorname{polylog}\left(3,ax^2\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*x^2*log(a*x^2 - 1) - 2*a*x^2*log(x) + \%iint(a, x, -log(-a*x^2 + 1)/
a, -2*log(-a*x^2 + 1)/x) - log(-a*x^2 + 1) + polylog(3, a*x^2))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3\left(ax^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**3,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^3, x)
```

3.37 $\int \frac{\text{PolyLog}(3,ax^2)}{x^5} dx$

Optimal. Leaf size=78

$$-\frac{\text{PolyLog}(2,ax^2)}{8x^4}-\frac{\text{PolyLog}(3,ax^2)}{4x^4}-\frac{1}{16}a^2\log(1-ax^2)+\frac{1}{8}a^2\log(x)-\frac{a}{16x^2}+\frac{\log(1-ax^2)}{16x^4}$$

```
[Out] -a/(16*x^2) + (a^2*Log[x])/8 - (a^2*Log[1 - a*x^2])/16 + Log[1 - a*x^2]/(16
*x^4) - PolyLog[2, a*x^2]/(8*x^4) - PolyLog[3, a*x^2]/(4*x^4)
```

Rubi [A] time = 0.0637554, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax^2)}{8x^4}-\frac{\text{PolyLog}(3,ax^2)}{4x^4}-\frac{1}{16}a^2\log(1-ax^2)+\frac{1}{8}a^2\log(x)-\frac{a}{16x^2}+\frac{\log(1-ax^2)}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^5,x]
```

```
[Out] -a/(16*x^2) + (a^2*Log[x])/8 - (a^2*Log[1 - a*x^2])/16 + Log[1 - a*x^2]/(16
*x^4) - PolyLog[2, a*x^2]/(8*x^4) - PolyLog[3, a*x^2]/(4*x^4)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{x^5}\,dx &= -\frac{\text{Li}_3\left(ax^2\right)}{4x^4}+\frac{1}{2}\int \frac{\text{Li}_2\left(ax^2\right)}{x^5}\,dx\\
&= -\frac{\text{Li}_2\left(ax^2\right)}{8x^4}-\frac{\text{Li}_3\left(ax^2\right)}{4x^4}-\frac{1}{4}\int \frac{\log\left(1-ax^2\right)}{x^5}\,dx\\
&= -\frac{\text{Li}_2\left(ax^2\right)}{8x^4}-\frac{\text{Li}_3\left(ax^2\right)}{4x^4}-\frac{1}{8}\text{Subst}\left(\int \frac{\log(1-ax)}{x^3}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{16x^4}-\frac{\text{Li}_2\left(ax^2\right)}{8x^4}-\frac{\text{Li}_3\left(ax^2\right)}{4x^4}+\frac{1}{16}a\,\text{Subst}\left(\int \frac{1}{x^2(1-ax)}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{16x^4}-\frac{\text{Li}_2\left(ax^2\right)}{8x^4}-\frac{\text{Li}_3\left(ax^2\right)}{4x^4}+\frac{1}{16}a\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a}{x}-\frac{a^2}{-1+ax}\right)dx,x,x^2\right)\\
&= -\frac{a}{16x^2}+\frac{1}{8}a^2\log(x)-\frac{1}{16}a^2\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{16x^4}-\frac{\text{Li}_2\left(ax^2\right)}{8x^4}-\frac{\text{Li}_3\left(ax^2\right)}{4x^4}
\end{aligned}$$

Mathematica [C] time = 0.0118626, size = 30, normalized size = 0.38

$$\frac{G_{5,5}^{2,4}\left(-ax^2\middle|\begin{array}{c}1,1,1,1,3\\1,2,0,0,0\end{array}\right)}{2x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/x^5,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {3}}, {{1, 2}, {0, 0, 0}}, -(a*x^2)]/(2*x^4)
```

Maple [A] time = 0.065, size = 98, normalized size = 1.3

$$-\frac{a^2}{2}\left(\frac{1}{a x^2}+\frac{3}{16}-\frac{\ln(x)}{4}-\frac{\ln(-a)}{8}-\frac{81 a x^2+378}{432 a x^2}-\frac{(-27 a^2 x^4+27)\ln(-a x^2+1)}{216 a^2 x^4}+\frac{\mathrm{polylog}(2, a x^2)}{4 a^2 x^4}+\frac{\mathrm{polylog}(3, a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^5,x)
```

```
[Out] -1/2*a^2*(1/a/x^2+3/16-1/4*ln(x)-1/8*ln(-a)-1/432/a/x^2*(81*a*x^2+378)-1/21
6/a^2/x^4*(-27*a^2*x^4+27)*ln(-a*x^2+1)+1/4/a^2/x^4*polylog(2,a*x^2)+1/2/a^
2/x^4*polylog(3,a*x^2))
```

Maxima [A] time = 1.0028, size = 74, normalized size = 0.95

$$\frac{1}{8}a^2\log(x)-\frac{a x^2+(a^2x^4-1)\log(-a x^2+1)+2\,\mathrm{Li}_2(a x^2)+4\,\mathrm{Li}_3(a x^2)}{16 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(x) - 1/16*(a*x^2 + (a^2*x^4 - 1)*log(-a*x^2 + 1) + 2*dilog(a*x^
2) + 4*polylog(3, a*x^2))/x^4
```

Fricas [C] time = 2.71783, size = 217, normalized size = 2.78

$$-\frac{a^2x^4\log(a x^2-1)-2 a^2x^4\log(x)+a x^2+2\,\%\mathrm{iint}\left(a, x, -\frac{\log(-a x^2+1)}{a}, -\frac{2\log(-a x^2+1)}{x}\right)-\log(-a x^2+1)+4\,\mathrm{polylog}(3,}{16 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(a^2*x^4*log(a*x^2 - 1) - 2*a^2*x^4*log(x) + a*x^2 + 2*\%iint(a, x, -l
og(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - log(-a*x^2 + 1) + 4*polylog(3, a*
```

```
x^2))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3\left(ax^2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**5,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^5, x)
```

3.38 $\int \frac{\text{PolyLog}(3,ax^2)}{x^7}\,dx$

Optimal. Leaf size=88

$$-\frac{\text{PolyLog}(2,ax^2)}{18x^6}-\frac{\text{PolyLog}(3,ax^2)}{6x^6}-\frac{a^2}{54x^2}-\frac{1}{54}a^3\log(1-ax^2)+\frac{1}{27}a^3\log(x)-\frac{a}{108x^4}+\frac{\log(1-ax^2)}{54x^6}$$

```
[Out] -a/(108*x^4) - a^2/(54*x^2) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x^2])/54 + L
og[1 - a*x^2]/(54*x^6) - PolyLog[2, a*x^2]/(18*x^6) - PolyLog[3, a*x^2]/(6*
x^6)
```

Rubi [A] time = 0.0654848, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2454, 2395, 44}

$$-\frac{\text{PolyLog}(2,ax^2)}{18x^6}-\frac{\text{PolyLog}(3,ax^2)}{6x^6}-\frac{a^2}{54x^2}-\frac{1}{54}a^3\log(1-ax^2)+\frac{1}{27}a^3\log(x)-\frac{a}{108x^4}+\frac{\log(1-ax^2)}{54x^6}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^7,x]
```


```
[Out] -a/(108*x^4) - a^2/(54*x^2) + (a^3*Log[x])/27 - (a^3*Log[1 - a*x^2])/54 + L
og[1 - a*x^2]/(54*x^6) - PolyLog[2, a*x^2]/(18*x^6) - PolyLog[3, a*x^2]/(6*
x^6)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{x^7}\,dx &= -\frac{\text{Li}_3\left(ax^2\right)}{6x^6}+\frac{1}{3}\int \frac{\text{Li}_2\left(ax^2\right)}{x^7}\,dx\\
&= -\frac{\text{Li}_2\left(ax^2\right)}{18x^6}-\frac{\text{Li}_3\left(ax^2\right)}{6x^6}-\frac{1}{9}\int \frac{\log\left(1-ax^2\right)}{x^7}\,dx\\
&= -\frac{\text{Li}_2\left(ax^2\right)}{18x^6}-\frac{\text{Li}_3\left(ax^2\right)}{6x^6}-\frac{1}{18}\text{Subst}\left(\int \frac{\log(1-ax)}{x^4}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{54x^6}-\frac{\text{Li}_2\left(ax^2\right)}{18x^6}-\frac{\text{Li}_3\left(ax^2\right)}{6x^6}+\frac{1}{54}a\,\text{Subst}\left(\int \frac{1}{x^3(1-ax)}\,dx,x,x^2\right)\\
&= \frac{\log\left(1-ax^2\right)}{54x^6}-\frac{\text{Li}_2\left(ax^2\right)}{18x^6}-\frac{\text{Li}_3\left(ax^2\right)}{6x^6}+\frac{1}{54}a\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{a}{x^2}+\frac{a^2}{x}-\frac{a^3}{-1+ax}\right)dx,x,x^2\right)\\
&= -\frac{a}{108x^4}-\frac{a^2}{54x^2}+\frac{1}{27}a^3\log(x)-\frac{1}{54}a^3\log\left(1-ax^2\right)+\frac{\log\left(1-ax^2\right)}{54x^6}-\frac{\text{Li}_2\left(ax^2\right)}{18x^6}-\frac{\text{Li}_3\left(ax^2\right)}{6x^6}
\end{aligned}
$$

Mathematica [C] time = 0.0119825, size = 30, normalized size = 0.34

$$
\frac{G_{5,5}^{2,4}\left(-ax^2\left|\begin{array}{c} 1,1,1,1,4 \\ 1,3,0,0,0 \end{array}\right.\right)}{2x^6}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/x^7,x]
```

```
[Out] MeijerG[{{1, 1, 1, 1}, {4}}, {{1, 3}, {0, 0, 0}}, -(a*x^2)]/(2*x^6)
```

Maple [A] time = 0.066, size = 115, normalized size = 1.3

$$\frac{a^3}{2}\left(-\frac{1}{2\,a^2x^4}-\frac{1}{8\,ax^2}-\frac{1}{27}+\frac{2\,\ln(x)}{27}+\frac{\ln(-a)}{27}+\frac{64\,a^2x^4+152\,ax^2+832}{1728\,a^2x^4}+\frac{\left(-64\,x^6a^3+64\right)\ln\left(-ax^2+1\right)}{1728\,x^6a^3}-\frac{\text{polylog}}{9\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^7,x)
```

```
[Out] 1/2*a^3*(-1/2/a^2/x^4-1/8/a/x^2-1/27+2/27*ln(x)+1/27*ln(-a)+1/1728/a^2/x^4*
(64*a^2*x^4+152*a*x^2+832)+1/1728/a^3/x^6*(-64*a^3*x^6+64)*ln(-a*x^2+1)-1/9
/a^3/x^6*polylog(2,a*x^2)-1/3/a^3/x^6*polylog(3,a*x^2))
```

Maxima [A] time = 0.997547, size = 86, normalized size = 0.98

$$\frac{1}{27}\,a^3\log(x)-\frac{2\,a^2x^4+ax^2+2\left(a^3x^6-1\right)\log\left(-ax^2+1\right)+6\,\text{Li}_2\left(ax^2\right)+18\,\text{Li}_3(ax^2)}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/27*a^3*log(x) - 1/108*(2*a^2*x^4 + a*x^2 + 2*(a^3*x^6 - 1)*log(-a*x^2 + 1
) + 6*dilog(a*x^2) + 18*polylog(3, a*x^2))/x^6
```

Fricas [C] time = 2.79682, size = 242, normalized size = 2.75

$$-\frac{2\,a^3x^6\log\left(ax^2-1\right)-4\,a^3x^6\log(x)+2\,a^2x^4+ax^2+6\,\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\,\log\left(-ax^2+1\right)}{x}\right)-2\,\log\left(-ax^2+1\right)+1}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/108*(2*a^3*x^6*log(a*x^2 - 1) - 4*a^3*x^6*log(x) + 2*a^2*x^4 + a*x^2 + 6
*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 2*log(-a*x^2 + 1)
+ 18*polylog(3, a*x^2))/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3\left(ax^2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**7,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^2)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^7, x)
```

3.39 $\int x^4\textbf{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=87

$$-\frac{2}{25}x^5\text{PolyLog}\left(2,ax^2\right)+\frac{1}{5}x^5\text{PolyLog}\left(3,ax^2\right)+\frac{8x}{125a^2}-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{125a^{5/2}}+\frac{8x^3}{375a}-\frac{4}{125}x^5\log\left(1-ax^2\right)+\frac{8x^5}{625}$$

```
[Out] (8*x)/(125*a^2) + (8*x^3)/(375*a) + (8*x^5)/625 - (8*ArcTanh[Sqrt[a]*x])/(1
25*a^(5/2)) - (4*x^5*Log[1 - a*x^2])/125 - (2*x^5*PolyLog[2, a*x^2])/25 + (
x^5*PolyLog[3, a*x^2])/5
```

Rubi [A] time = 0.0530276, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 302, 206}

$$-\frac{2}{25}x^5\text{PolyLog}\left(2,ax^2\right)+\frac{1}{5}x^5\text{PolyLog}\left(3,ax^2\right)+\frac{8x}{125a^2}-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{125a^{5/2}}+\frac{8x^3}{375a}-\frac{4}{125}x^5\log\left(1-ax^2\right)+\frac{8x^5}{625}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*PolyLog[3, a*x^2],x]
```

```
[Out] (8*x)/(125*a^2) + (8*x^3)/(375*a) + (8*x^5)/625 - (8*ArcTanh[Sqrt[a]*x])/(1
25*a^(5/2)) - (4*x^5*Log[1 - a*x^2])/125 - (2*x^5*PolyLog[2, a*x^2])/25 + (
x^5*PolyLog[3, a*x^2])/5
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \mathrm{Li}_3\left(ax^2\right) dx &= \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right) - \frac{2}{5}\int x^4\mathrm{Li}_2\left(ax^2\right) dx \\
&= -\frac{2}{25}x^5\mathrm{Li}_2\left(ax^2\right) + \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right) - \frac{4}{25}\int x^4\log\left(1-ax^2\right) dx \\
&= -\frac{4}{125}x^5\log\left(1-ax^2\right) - \frac{2}{25}x^5\mathrm{Li}_2\left(ax^2\right) + \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right) - \frac{1}{125}(8a)\int \frac{x^6}{1-ax^2} dx \\
&= -\frac{4}{125}x^5\log\left(1-ax^2\right) - \frac{2}{25}x^5\mathrm{Li}_2\left(ax^2\right) + \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right) - \frac{1}{125}(8a)\int \left(-\frac{1}{a^3} - \frac{x^2}{a^2} - \frac{x^4}{a} + \frac{1}{a^3\left(1-ax^2\right)}\right. \\
&= \frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{4}{125}x^5\log\left(1-ax^2\right) - \frac{2}{25}x^5\mathrm{Li}_2\left(ax^2\right) + \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right) - \frac{8\int \frac{1}{1-ax^2} dx}{125a^2} \\
&= \frac{8x}{125a^2} + \frac{8x^3}{375a} + \frac{8x^5}{625} - \frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{125a^{5/2}} - \frac{4}{125}x^5\log\left(1-ax^2\right) - \frac{2}{25}x^5\mathrm{Li}_2\left(ax^2\right) + \frac{1}{5}x^5\mathrm{Li}_3\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.161049, size = 77, normalized size = 0.89

$$
\frac{-150x^5\mathrm{PolyLog}\left(2,ax^2\right) + 375x^5\mathrm{PolyLog}\left(3,ax^2\right) + \frac{120x}{a^2} - \frac{120\tanh^{-1}\left(\sqrt{a}x\right)}{a^{5/2}} + \frac{40x^3}{a} - 60x^5\log\left(1-ax^2\right) + 24x^5}{1875}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*PolyLog[3, a*x^2],x]
```

```
[Out] ((120*x)/a^2 + (40*x^3)/a + 24*x^5 - (120*ArcTanh[Sqrt[a]*x])/a^(5/2) - 60*
x^5*Log[1 - a*x^2] - 150*x^5*PolyLog[2, a*x^2] + 375*x^5*PolyLog[3, a*x^2])
/1875
```

Maple [B] time = 0.178, size = 144, normalized size = 1.7

$$-\frac{1}{2a^2}\left(\frac{2x\left(168a^2x^4+280ax^2+840\right)}{13125a^3}(-a)^{\frac{7}{2}}+\frac{8x}{125a^3}(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{ax^2}\right)-\ln\left(1+\sqrt{ax^2}\right)\right)\frac{1}{\sqrt{ax^2}}-\frac{8x^5\ln\left(-ax^2+1\right.}{125a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*polylog(3,a*x^2),x)
```

```
[Out] -1/2/a^2/(-a)^(1/2)*(2/13125*x*(-a)^(7/2)*(168*a^2*x^4+280*a*x^2+840)/a^3+8
/125*x*(-a)^(7/2)/a^3/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)
))-8/125*x^5*(-a)^(7/2)/a*ln(-a*x^2+1)-4/25*x^5*(-a)^(7/2)*polylog(2,a*x^2)
/a+2/5*x^5*(-a)^(7/2)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.77746, size = 608, normalized size = 6.99

$$\left[-\frac{150a^3x^5\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)+60a^3x^5\log\left(-ax^2+1\right)-375a^3x^5\text{polylog}\left(3,ax^2\right)-24a^3x^5-40\right.}{1875a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/1875*(150*a^3*x^5*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x)
 + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*x^5*polylog(3, a*x^2) - 24*a^3*x^5
- 40*a^2*x^3 - 120*a*x - 60*sqrt(a)*log((a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 -
```

```
1)))/a^3, -1/1875*(150*a^3*x^5*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^
2 + 1)/x) + 60*a^3*x^5*log(-a*x^2 + 1) - 375*a^3*x^5*polylog(3, a*x^2) - 24
*a^3*x^5 - 40*a^2*x^3 - 120*a*x - 120*sqrt(-a)*arctan(sqrt(-a)*x))/a^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**4*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4*polylog(3, a*x^2), x)
```

3.40 $\int x^2 \textbf{PolyLog}\left(3, ax^2\right) dx$

Optimal. Leaf size=77

$$-\frac{2}{9}x^3\text{PolyLog}\left(2,ax^2\right)+\frac{1}{3}x^3\text{PolyLog}\left(3,ax^2\right)-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{27a^{3/2}}-\frac{4}{27}x^3\log\left(1-ax^2\right)+\frac{8x}{27a}+\frac{8x^3}{81}$$

```
[Out] (8*x)/(27*a) + (8*x^3)/81 - (8*ArcTanh[Sqrt[a]*x])/(27*a^(3/2)) - (4*x^3*Lo
g[1 - a*x^2])/27 - (2*x^3*PolyLog[2, a*x^2])/9 + (x^3*PolyLog[3, a*x^2])/3
```

Rubi [A] time = 0.0489431, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 302, 206}

$$-\frac{2}{9}x^3\text{PolyLog}\left(2,ax^2\right)+\frac{1}{3}x^3\text{PolyLog}\left(3,ax^2\right)-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{27a^{3/2}}-\frac{4}{27}x^3\log\left(1-ax^2\right)+\frac{8x}{27a}+\frac{8x^3}{81}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, a*x^2],x]
```

```
[Out] (8*x)/(27*a) + (8*x^3)/81 - (8*ArcTanh[Sqrt[a]*x])/(27*a^(3/2)) - (4*x^3*Lo
g[1 - a*x^2])/27 - (2*x^3*PolyLog[2, a*x^2])/9 + (x^3*PolyLog[3, a*x^2])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\text{Li}_3\left(ax^2\right)\,dx &= \frac{1}{3}x^3\text{Li}_3\left(ax^2\right) - \frac{2}{3}\int x^2\text{Li}_2\left(ax^2\right)\,dx\\
&= -\frac{2}{9}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{3}x^3\text{Li}_3\left(ax^2\right) - \frac{4}{9}\int x^2\log\left(1-ax^2\right)\,dx\\
&= -\frac{4}{27}x^3\log\left(1-ax^2\right) - \frac{2}{9}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{3}x^3\text{Li}_3\left(ax^2\right) - \frac{1}{27}(8a)\int \frac{x^4}{1-ax^2}\,dx\\
&= -\frac{4}{27}x^3\log\left(1-ax^2\right) - \frac{2}{9}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{3}x^3\text{Li}_3\left(ax^2\right) - \frac{1}{27}(8a)\int \left(-\frac{1}{a^2} - \frac{x^2}{a} + \frac{1}{a^2\left(1-ax^2\right)}\right)dx\\
&= \frac{8x}{27a} + \frac{8x^3}{81} - \frac{4}{27}x^3\log\left(1-ax^2\right) - \frac{2}{9}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{3}x^3\text{Li}_3\left(ax^2\right) - \frac{8\int \frac{1}{1-ax^2}\,dx}{27a}\\
&= \frac{8x}{27a} + \frac{8x^3}{81} - \frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{27a^{3/2}} - \frac{4}{27}x^3\log\left(1-ax^2\right) - \frac{2}{9}x^3\text{Li}_2\left(ax^2\right) + \frac{1}{3}x^3\text{Li}_3\left(ax^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.138001, size = 69, normalized size = 0.9

$$
\frac{1}{81}\left(-18x^3\text{PolyLog}\left(2,ax^2\right) + 27x^3\text{PolyLog}\left(3,ax^2\right) - \frac{24\tanh^{-1}\left(\sqrt{a}x\right)}{a^{3/2}} - 12x^3\log\left(1-ax^2\right) + \frac{24x}{a} + 8x^3\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, a*x^2],x]
```

```
[Out] ((24*x)/a + 8*x^3 - (24*ArcTanh[Sqrt[a]*x])/a^(3/2) - 12*x^3*Log[1 - a*x^2]
 - 18*x^3*PolyLog[2, a*x^2] + 27*x^3*PolyLog[3, a*x^2])/81
```

Maple [B] time = 0.175, size = 136, normalized size = 1.8

$$
\frac{1}{2a}\left(\frac{2x\left(40ax^2+120\right)}{405a^2}(-a)^{\frac{5}{2}} + \frac{8x}{27a^2}(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{ax^2}\right) - \ln\left(1+\sqrt{ax^2}\right)\right)\frac{1}{\sqrt{ax^2}} - \frac{8x^3\ln\left(-ax^2+1\right)}{27a}(-a)^{\frac{5}{2}} - \frac{4x^3\text{pol}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x^2),x)
```

```
[Out] 1/2/a/(-a)^(1/2)*(2/405*x*(-a)^(5/2)*(40*a*x^2+120)/a^2+8/27*x*(-a)^(5/2)/a
^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8/27*x^3*(-a)^(5
/2)/a*ln(-a*x^2+1)-4/9*x^3*(-a)^(5/2)*polylog(2,a*x^2)/a+2/3*x^3*(-a)^(5/2)
/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.74317, size = 555, normalized size = 7.21

$$\left[-\frac{18\,a^2x^3\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\,\log\left(-ax^2+1\right)}{x}\right)+12\,a^2x^3\log\left(-ax^2+1\right)-27\,a^2x^3\text{polylog}\left(3,ax^2\right)-8\,a^2x^3-24\,ax-}{81\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/81*(18*a^2*x^3*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) +
12*a^2*x^3*log(-a*x^2 + 1) - 27*a^2*x^3*polylog(3, a*x^2) - 8*a^2*x^3 - 24*
a*x - 12*sqrt(a)*log((a*x^2 - 2*sqrt(a)*x + 1)/(a*x^2 - 1)))/a^2, -1/81*(18
*a^2*x^3*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 12*a^2*x^3
*log(-a*x^2 + 1) - 27*a^2*x^3*polylog(3, a*x^2) - 8*a^2*x^3 - 24*a*x - 24*s
qrt(-a)*arctan(sqrt(-a)*x))/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x**2),x)
```

```
[Out] Integral(x**2*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, a*x^2), x)
```

3.41 $\int \text{PolyLog}\left(3, ax^2\right) dx$

Optimal. Leaf size=50

$$-2x\text{PolyLog}\left(2, ax^2\right) + x\text{PolyLog}\left(3, ax^2\right) - 4x\log\left(1 - ax^2\right) - \frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}} + 8x$$

```
[Out] 8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2,
a*x^2] + x*PolyLog[3, a*x^2]
```

Rubi [A] time = 0.0233193, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6586, 2448, 321, 206}

$$-2x\text{PolyLog}\left(2, ax^2\right) + x\text{PolyLog}\left(3, ax^2\right) - 4x\log\left(1 - ax^2\right) - \frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}} + 8x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2],x]
```

```
[Out] 8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2,
a*x^2] + x*PolyLog[3, a*x^2]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \mathrm{Li}_3\left(ax^2\right)dx &= x\mathrm{Li}_3\left(ax^2\right)-2\int \mathrm{Li}_2\left(ax^2\right)dx \\
&= -2x\mathrm{Li}_2\left(ax^2\right)+x\mathrm{Li}_3\left(ax^2\right)-4\int \log\left(1-ax^2\right)dx \\
&= -4x\log\left(1-ax^2\right)-2x\mathrm{Li}_2\left(ax^2\right)+x\mathrm{Li}_3\left(ax^2\right)-(8a)\int \frac{x^2}{1-ax^2}dx \\
&= 8x-4x\log\left(1-ax^2\right)-2x\mathrm{Li}_2\left(ax^2\right)+x\mathrm{Li}_3\left(ax^2\right)-8\int \frac{1}{1-ax^2}dx \\
&= 8x-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}}-4x\log\left(1-ax^2\right)-2x\mathrm{Li}_2\left(ax^2\right)+x\mathrm{Li}_3\left(ax^2\right)
\end{aligned}$$

Mathematica [A] time = 0.087684, size = 50, normalized size = 1.

$$-2x\mathrm{PolyLog}\left(2,ax^2\right)+x\mathrm{PolyLog}\left(3,ax^2\right)-4x\log\left(1-ax^2\right)-\frac{8\tanh^{-1}\left(\sqrt{a}x\right)}{\sqrt{a}}+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2],x]
```

```
[Out] 8*x - (8*ArcTanh[Sqrt[a]*x])/Sqrt[a] - 4*x*Log[1 - a*x^2] - 2*x*PolyLog[2,
a*x^2] + x*PolyLog[3, a*x^2]
```

Maple [B] time = 0.173, size = 119, normalized size = 2.4

$$-\frac{1}{2}\left(16\frac{x(-a)^{3/2}}{a}+8\frac{x(-a)^{3/2}\left(\ln\left(1-\sqrt{ax^2}\right)-\ln\left(1+\sqrt{ax^2}\right)\right)}{a\sqrt{ax^2}}-8\frac{x(-a)^{3/2}\ln\left(-ax^2+1\right)}{a}-4\frac{x(-a)^{3/2}\mathrm{polylog}\left(2,ax^2\right)}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2),x)
```

```
[Out] -1/2/(-a)^(1/2)*(16*x*(-a)^(3/2)/a+8*x*(-a)^(3/2)/a/(a*x^2)^(1/2)*(ln(1-(a*
x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))-8*x*(-a)^(3/2)/a*ln(-a*x^2+1)-4*x*(-a)^(3/
2)*polylog(2,a*x^2)/a+2*x*(-a)^(3/2)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.69418, size = 452, normalized size = 9.04

$$\left[-\frac{2\,ax\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\,\log\left(-ax^2+1\right)}{x}\right)+4\,ax\log\left(-ax^2+1\right)-ax\text{polylog}\left(3,ax^2\right)-8\,ax-4\,\sqrt{a}\log\left(\frac{ax^2-2\,\sqrt{a}x+1}{ax^2-1}\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] [-(2*a*x*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 4*a*x*log(
-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*a*x - 4*sqrt(a)*log((a*x^2 - 2*sqrt
(a)*x + 1)/(a*x^2 - 1)))/a, -(2*a*x*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(
-a*x^2 + 1)/x) + 4*a*x*log(-a*x^2 + 1) - a*x*polylog(3, a*x^2) - 8*a*x - 8*
sqrt(-a)*arctan(sqrt(-a)*x))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2),x)
```

```
[Out] Integral(polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_3(ax^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2), x)
```

3.42 $\int \frac{\text{PolyLog}(3,ax^2)}{x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{x}-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}+\frac{4\log\left(1-ax^2\right)}{x}+8\sqrt{a}\tanh^{-1}\left(\sqrt{a}x\right)$$

```
[Out] 8*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (4*Log[1 - a*x^2])/x - (2*PolyLog[2, a*x^2])
/x - PolyLog[3, a*x^2]/x
```

Rubi [A] time = 0.0357649, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 206}

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{x}-\frac{\text{PolyLog}\left(3,ax^2\right)}{x}+\frac{4\log\left(1-ax^2\right)}{x}+8\sqrt{a}\tanh^{-1}\left(\sqrt{a}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^2,x]
```

```
[Out] 8*Sqrt[a]*ArcTanh[Sqrt[a]*x] + (4*Log[1 - a*x^2])/x - (2*PolyLog[2, a*x^2])
/x - PolyLog[3, a*x^2]/x
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{x^2}\,dx &= -\frac{\text{Li}_3\left(ax^2\right)}{x}+2\int \frac{\text{Li}_2\left(ax^2\right)}{x^2}\,dx\\
&= -\frac{2\text{Li}_2\left(ax^2\right)}{x}-\frac{\text{Li}_3\left(ax^2\right)}{x}-4\int \frac{\log\left(1-ax^2\right)}{x^2}\,dx\\
&= \frac{4\log\left(1-ax^2\right)}{x}-\frac{2\text{Li}_2\left(ax^2\right)}{x}-\frac{\text{Li}_3\left(ax^2\right)}{x}+(8a)\int \frac{1}{1-ax^2}\,dx\\
&= 8\sqrt{a}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{4\log\left(1-ax^2\right)}{x}-\frac{2\text{Li}_2\left(ax^2\right)}{x}-\frac{\text{Li}_3\left(ax^2\right)}{x}
\end{aligned}
$$

Mathematica [A] time = 0.0806658, size = 50, normalized size = 0.93

$$\frac{-2\text{PolyLog}\left(2,ax^2\right)-\text{PolyLog}\left(3,ax^2\right)+4\log\left(1-ax^2\right)+8\sqrt{a}x\tanh^{-1}\left(\sqrt{a}x\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^2,x]
```

```
[Out] (8*Sqrt[a]*x*ArcTanh[Sqrt[a]*x] + 4*Log[1 - a*x^2] - 2*PolyLog[2, a*x^2] -
PolyLog[3, a*x^2])/x
```

Maple [B] time = 0.172, size = 112, normalized size = 2.1

$$\frac{a}{2}\left(-8\,\frac{x\sqrt{-a}\left(\ln\left(1-\sqrt{ax^2}\right)-\ln\left(1+\sqrt{ax^2}\right)\right)}{\sqrt{ax^2}}+8\,\frac{\sqrt{-a}\ln\left(-ax^2+1\right)}{ax}-4\,\frac{\sqrt{-a}\text{polylog}\left(2,ax^2\right)}{ax}-2\,\frac{\sqrt{-a}\text{polylog}\left(3,ax^2\right)}{ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^2,x)
```

```
[Out] 1/2*a/(-a)^(1/2)*(-8*x*(-a)^(1/2)/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(
a*x^2)^(1/2)))+8/x*(-a)^(1/2)/a*ln(-a*x^2+1)-4/x*(-a)^(1/2)*polylog(2,a*x^2
```

```
)/a-2/x*(-a)^(1/2)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.87069, size = 402, normalized size = 7.44

$$\left[\frac{4\sqrt{a}x\log\left(\frac{ax^2+2\sqrt{a}x+1}{ax^2-1}\right)-2\,\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)+4\log\left(-ax^2+1\right)-\mathrm{polylog}\left(3,ax^2\right)}{x},-\frac{8\sqrt{-a}x\,\mathrm{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [(4*sqrt(a)*x*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 2*\%iint(a, x, -l
og(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 4*log(-a*x^2 + 1) - polylog(3, a*
x^2))/x, -(8*sqrt(-a)*x*arctan(sqrt(-a)*x) + 2*\%iint(a, x, -log(-a*x^2 + 1)
/a, -2*log(-a*x^2 + 1)/x) - 4*log(-a*x^2 + 1) + polylog(3, a*x^2))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\mathrm{Li}_3\left(ax^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**2,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^2, x)
```

3.43 $\int \frac{\text{PolyLog}(3,ax^2)}{x^4} dx$

Optimal. Leaf size=70

$$-\frac{2\text{PolyLog}(2,ax^2)}{9x^3}-\frac{\text{PolyLog}(3,ax^2)}{3x^3}+\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{4\log(1-ax^2)}{27x^3}-\frac{8a}{27x}$$

```
[Out] (-8*a)/(27*x) + (8*a^(3/2)*ArcTanh[Sqrt[a]*x])/27 + (4*Log[1 - a*x^2])/(27*
x^3) - (2*PolyLog[2, a*x^2])/(9*x^3) - PolyLog[3, a*x^2]/(3*x^3)
```

Rubi [A] time = 0.0409967, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 325, 206}

$$-\frac{2\text{PolyLog}(2,ax^2)}{9x^3}-\frac{\text{PolyLog}(3,ax^2)}{3x^3}+\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{4\log(1-ax^2)}{27x^3}-\frac{8a}{27x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^4,x]
```

```
[Out] (-8*a)/(27*x) + (8*a^(3/2)*ArcTanh[Sqrt[a]*x])/27 + (4*Log[1 - a*x^2])/(27*
x^3) - (2*PolyLog[2, a*x^2])/(9*x^3) - PolyLog[3, a*x^2]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{x^4}\,dx &= -\frac{\text{Li}_3\left(ax^2\right)}{3x^3}+\frac{2}{3}\int \frac{\text{Li}_2\left(ax^2\right)}{x^4}\,dx\\
&= -\frac{2\text{Li}_2\left(ax^2\right)}{9x^3}-\frac{\text{Li}_3\left(ax^2\right)}{3x^3}-\frac{4}{9}\int \frac{\log\left(1-ax^2\right)}{x^4}\,dx\\
&= \frac{4\log\left(1-ax^2\right)}{27x^3}-\frac{2\text{Li}_2\left(ax^2\right)}{9x^3}-\frac{\text{Li}_3\left(ax^2\right)}{3x^3}+\frac{1}{27}(8a)\int \frac{1}{x^2\left(1-ax^2\right)}\,dx\\
&= -\frac{8a}{27x}+\frac{4\log\left(1-ax^2\right)}{27x^3}-\frac{2\text{Li}_2\left(ax^2\right)}{9x^3}-\frac{\text{Li}_3\left(ax^2\right)}{3x^3}+\frac{1}{27}\left(8a^2\right)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{8a}{27x}+\frac{8}{27}a^{3/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{4\log\left(1-ax^2\right)}{27x^3}-\frac{2\text{Li}_2\left(ax^2\right)}{9x^3}-\frac{\text{Li}_3\left(ax^2\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.088813, size = 61, normalized size = 0.87

$$-\frac{6\text{PolyLog}\left(2,ax^2\right)+9\text{PolyLog}\left(3,ax^2\right)-8a^{3/2}x^3\tanh^{-1}\left(\sqrt{a}x\right)+8ax^2-4\log\left(1-ax^2\right)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^4,x]
```

```
[Out] -(8*a*x^2 - 8*a^(3/2)*x^3*ArcTanh[Sqrt[a]*x] - 4*Log[1 - a*x^2] + 6*PolyLog
[2, a*x^2] + 9*PolyLog[3, a*x^2])/(27*x^3)
```

Maple [B] time = 0.177, size = 125, normalized size = 1.8

$$-\frac{a^2}{2}\left(-\frac{16}{27\,x}\frac{1}{\sqrt{-a}}-\frac{8\,ax}{27}\left(\ln\left(1-\sqrt{ax^2}\right)-\ln\left(1+\sqrt{ax^2}\right)\right)\frac{1}{\sqrt{-a}}\frac{1}{\sqrt{ax^2}}+\frac{8\,\ln\left(-ax^2+1\right)}{27\,x^3a}\frac{1}{\sqrt{-a}}-\frac{4\,\text{polylog}\left(2,ax^2\right)}{9\,x^3a}\frac{1}{\sqrt{-a}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^4,x)
```

```
[Out] -1/2*a^2/(-a)^(1/2)*(-16/27/x/(-a)^(1/2)-8/27*x/(-a)^(1/2)*a/(a*x^2)^(1/2)*
(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/27/x^3/(-a)^(1/2)/a*ln(-a*x^2+1
)-4/9/x^3/(-a)^(1/2)*polylog(2,a*x^2)/a-2/3/x^3/(-a)^(1/2)/a*polylog(3,a*x^
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.22669, size = 462, normalized size = 6.6

$$\left[\frac{4\,a^{\frac{3}{2}}x^3\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)-8\,ax^2-6\,\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\,\log\left(-ax^2+1\right)}{x}\right)+4\,\log\left(-ax^2+1\right)-9\,\mathrm{polylog}\left(3,ax^2\right)}{27\,x^3},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/27*(4*a^(3/2)*x^3*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 8*a*x^2 -
 6*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 4*log(-a*x^2 + 1
) - 9*polylog(3, a*x^2))/x^3, -1/27*(8*sqrt(-a)*a*x^3*arctan(sqrt(-a)*x) +
8*a*x^2 + 6*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 4*log(-
a*x^2 + 1) + 9*polylog(3, a*x^2))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**4,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^4, x)
```

3.44 $\int \frac{\text{PolyLog}(3,ax^2)}{x^6}\,dx$

Optimal. Leaf size=80

$$-\frac{2\text{PolyLog}(2,ax^2)}{25x^5}-\frac{\text{PolyLog}(3,ax^2)}{5x^5}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right)-\frac{8a}{375x^3}+\frac{4\log(1-ax^2)}{125x^5}$$

```
[Out] (-8*a)/(375*x^3) - (8*a^2)/(125*x) + (8*a^(5/2)*ArcTanh[Sqrt[a]*x])/125 + (
4*Log[1 - a*x^2])/(125*x^5) - (2*PolyLog[2, a*x^2])/(25*x^5) - PolyLog[3, a
*x^2]/(5*x^5)
```

Rubi [A] time = 0.0469064, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6591, 2455, 325, 206}

$$-\frac{2\text{PolyLog}(2,ax^2)}{25x^5}-\frac{\text{PolyLog}(3,ax^2)}{5x^5}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right)-\frac{8a}{375x^3}+\frac{4\log(1-ax^2)}{125x^5}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/x^6,x]
```


```
[Out] (-8*a)/(375*x^3) - (8*a^2)/(125*x) + (8*a^(5/2)*ArcTanh[Sqrt[a]*x])/125 + (
4*Log[1 - a*x^2])/(125*x^5) - (2*PolyLog[2, a*x^2])/(25*x^5) - PolyLog[3, a
*x^2]/(5*x^5)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{x^6}\,dx &= -\frac{\text{Li}_3\left(ax^2\right)}{5x^5}+\frac{2}{5}\int \frac{\text{Li}_2\left(ax^2\right)}{x^6}\,dx\\
&= -\frac{2\text{Li}_2\left(ax^2\right)}{25x^5}-\frac{\text{Li}_3\left(ax^2\right)}{5x^5}-\frac{4}{25}\int \frac{\log\left(1-ax^2\right)}{x^6}\,dx\\
&= \frac{4\log\left(1-ax^2\right)}{125x^5}-\frac{2\text{Li}_2\left(ax^2\right)}{25x^5}-\frac{\text{Li}_3\left(ax^2\right)}{5x^5}+\frac{1}{125}(8a)\int \frac{1}{x^4\left(1-ax^2\right)}\,dx\\
&= -\frac{8a}{375x^3}+\frac{4\log\left(1-ax^2\right)}{125x^5}-\frac{2\text{Li}_2\left(ax^2\right)}{25x^5}-\frac{\text{Li}_3\left(ax^2\right)}{5x^5}+\frac{1}{125}\left(8a^2\right)\int \frac{1}{x^2\left(1-ax^2\right)}\,dx\\
&= -\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{4\log\left(1-ax^2\right)}{125x^5}-\frac{2\text{Li}_2\left(ax^2\right)}{25x^5}-\frac{\text{Li}_3\left(ax^2\right)}{5x^5}+\frac{1}{125}\left(8a^3\right)\int \frac{1}{1-ax^2}\,dx\\
&= -\frac{8a}{375x^3}-\frac{8a^2}{125x}+\frac{8}{125}a^{5/2}\tanh^{-1}\left(\sqrt{a}x\right)+\frac{4\log\left(1-ax^2\right)}{125x^5}-\frac{2\text{Li}_2\left(ax^2\right)}{25x^5}-\frac{\text{Li}_3\left(ax^2\right)}{5x^5}
\end{aligned}$$

Mathematica [A] time = 0.100693, size = 69, normalized size = 0.86

$$-\frac{30\text{PolyLog}\left(2,ax^2\right)+75\text{PolyLog}\left(3,ax^2\right)+24a^2x^4-24a^{5/2}x^5\tanh^{-1}\left(\sqrt{a}x\right)+8ax^2-12\log\left(1-ax^2\right)}{375x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^2]/x^6,x]
```

```
[Out] -(8*a*x^2 + 24*a^2*x^4 - 24*a^(5/2)*x^5*ArcTanh[Sqrt[a]*x] - 12*Log[1 - a*x
^2] + 30*PolyLog[2, a*x^2] + 75*PolyLog[3, a*x^2])/(375*x^5)
```

Maple [B] time = 0.18, size = 138, normalized size = 1.7

$$\frac{a^3}{2}\left(-\frac{16}{375\,x^3}\,(-a)^{-\frac{3}{2}}-\frac{16\,a}{125\,x}\,(-a)^{-\frac{3}{2}}-\frac{8\,a^2x}{125}\left(\ln\left(1-\sqrt{ax^2}\right)-\ln\left(1+\sqrt{ax^2}\right)\right)(-a)^{-\frac{3}{2}}\,\frac{1}{\sqrt{ax^2}}+\frac{8\,\ln\left(-ax^2+1\right)}{125\,ax^5}\,(-a)^{-\frac{3}{2}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/x^6,x)
```

```
[Out] 1/2*a^3/(-a)^(1/2)*(-16/375/x^3/(-a)^(3/2)-16/125/x/(-a)^(3/2)*a-8/125*x/(-
a)^(3/2)*a^2/(a*x^2)^(1/2)*(ln(1-(a*x^2)^(1/2))-ln(1+(a*x^2)^(1/2)))+8/125/
x^5/(-a)^(3/2)/a*ln(-a*x^2+1)-4/25/x^5/(-a)^(3/2)*polylog(2,a*x^2)/a-2/5/x^
5/(-a)^(3/2)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.24127, size = 513, normalized size = 6.41

$$\left[\frac{12\,a^{\frac{5}{2}}x^5\log\left(\frac{ax^2+2\,\sqrt{a}x+1}{ax^2-1}\right)-24\,a^2x^4-8\,ax^2-30\,\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\,\log\left(-ax^2+1\right)}{x}\right)+12\,\log\left(-ax^2+1\right)-75\,\mathrm{poly}}{375\,x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/375*(12*a^(5/2)*x^5*log((a*x^2 + 2*sqrt(a)*x + 1)/(a*x^2 - 1)) - 24*a^2*
x^4 - 8*a*x^2 - 30*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) +
```

```
12*log(-a*x^2 + 1) - 75*polylog(3, a*x^2))/x^5, -1/375*(24*sqrt(-a)*a^2*x^5
*arctan(sqrt(-a)*x) + 24*a^2*x^4 + 8*a*x^2 + 30*\%iint(a, x, -log(-a*x^2 + 1
)/a, -2*log(-a*x^2 + 1)/x) - 12*log(-a*x^2 + 1) + 75*polylog(3, a*x^2))/x^5
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/x**6,x)
```

```
[Out] Integral(polylog(3, a*x**2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/x^6, x)
```

3.45 $\int x^2 \textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=71

$$\frac{aq^2x^{q+3}\text{Hypergeometric2F1}\left(1,\frac{q+3}{q},\frac{3}{q}+2,ax^q\right)}{9(q+3)}+\frac{1}{3}x^3\text{PolyLog}(2,ax^q)+\frac{1}{9}qx^3\log(1-ax^q)$$

```
[Out] (a*q^2*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(9*(3 + q
)) + (q*x^3*Log[1 - a*x^q])/9 + (x^3*PolyLog[2, a*x^q])/3
```

Rubi [A] time = 0.0373625, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$\frac{1}{3}x^3\text{PolyLog}(2,ax^q)+\frac{aq^2x^{q+3}\,{}_2F_1\left(1,\frac{q+3}{q};2+\frac{3}{q};ax^q\right)}{9(q+3)}+\frac{1}{9}qx^3\log(1-ax^q)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(9*(3 + q
)) + (q*x^3*Log[1 - a*x^q])/9 + (x^3*PolyLog[2, a*x^q])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_2(ax^q)\,dx &= \frac{1}{3}x^3\text{Li}_2(ax^q)+\frac{1}{3}q\int x^2\log(1-ax^q)\,dx\\ &= \frac{1}{9}qx^3\log(1-ax^q)+\frac{1}{3}x^3\text{Li}_2(ax^q)+\frac{1}{9}\left(aq^2\right)\int\frac{x^{2+q}}{1-ax^q}\,dx\\ &= \frac{aq^2x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{9(3+q)}+\frac{1}{9}qx^3\log(1-ax^q)+\frac{1}{3}x^3\text{Li}_2(ax^q)\end{aligned}$$

Mathematica [A] time = 0.0439455, size = 69, normalized size = 0.97

$$\frac{qx^3\left(aqx^q\text{Hypergeometric2F1}\left(1,\frac{q+3}{q},\frac{3}{q}+2,ax^q\right)+(q+3)\log(1-ax^q)\right)}{9(q+3)}+\frac{1}{3}x^3\text{PolyLog}(2,ax^q)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, a*x^q],x]
```

```
[Out] (q*x^3*(a*q*x^q*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q] + (3 + q)*L
og[1 - a*x^q]))/(9*(3 + q)) + (x^3*PolyLog[2, a*x^q])/3
```

Maple [C] time = 0.26, size = 108, normalized size = 1.5

$$-\frac{1}{q}(-a)^{-3q^{-1}}\left(-\frac{q^2x^3\ln(1-ax^q)}{9}(-a)^{3q^{-1}}-\frac{qx^3\text{polylog}(2,ax^q)}{3+q}(-a)^{3q^{-1}}\left(1+\frac{q}{3}\right)-\frac{q^2x^{3+q}a}{9}(-a)^{3q^{-1}}\text{LerchPhi}\left(ax^q,1,\frac{3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,a*x^q),x)
```

```
[Out] -(-a)^(-3/q)/q*(-1/9*q^2*x^3*(-a)^(3/q)*ln(1-a*x^q)-q/(3+q)*x^3*(-a)^(3/q)*
(1+1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,(3
```

```
+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{27}\,q^{2}x^{3}+\frac{1}{9}\,qx^{3}\log\left(-ax^{q}+1\right)+\frac{1}{3}\,x^{3}\mathrm{Li}_{2}\left(ax^{q}\right)-q^{2}\int\frac{x^{2}}{9\left(ax^{q}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -1/27*q^2*x^3 + 1/9*q*x^3*log(-a*x^q + 1) + 1/3*x^3*dilog(a*x^q) - q^2*inte
grate(1/9*x^2/(a*x^q - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{2}\mathrm{Li}_{2}\left(ax^{q}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x^2*dilog(a*x^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_2(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(a*x^q), x)
```

3.46 $\int x\textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=71

$$\frac{aq^2x^{q+2}\text{Hypergeometric2F1}\left(1,\frac{q+2}{q},2\left(\frac{1}{q}+1\right),ax^q\right)}{4(q+2)}+\frac{1}{2}x^2\text{PolyLog}\left(2,ax^q\right)+\frac{1}{4}qx^2\log\left(1-ax^q\right)$$

```
[Out] (a*q^2*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])/(4
*(2 + q)) + (q*x^2*Log[1 - a*x^q])/4 + (x^2*PolyLog[2, a*x^q])/2
```

Rubi [A] time = 0.0317548, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2455, 364}

$$\frac{1}{2}x^2\text{PolyLog}\left(2,ax^q\right)+\frac{aq^2x^{q+2}\,{}_2F_1\left(1,\frac{q+2}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{4(q+2)}+\frac{1}{4}qx^2\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, a*x^q],x]
```


```
[Out] (a*q^2*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])/(4
*(2 + q)) + (q*x^2*Log[1 - a*x^q])/4 + (x^2*PolyLog[2, a*x^q])/2
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\mathrm{Li}_2\left(ax^q\right)\,dx &= \frac{1}{2}x^2\mathrm{Li}_2\left(ax^q\right)+\frac{1}{2}q\int x\log\left(1-ax^q\right)\,dx\\ &= \frac{1}{4}qx^2\log\left(1-ax^q\right)+\frac{1}{2}x^2\mathrm{Li}_2\left(ax^q\right)+\frac{1}{4}\left(aq^2\right)\int\frac{x^{1+q}}{1-ax^q}\,dx\\ &= \frac{aq^2x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{4(2+q)}+\frac{1}{4}qx^2\log\left(1-ax^q\right)+\frac{1}{2}x^2\mathrm{Li}_2\left(ax^q\right)\end{aligned}$$

Mathematica [A] time = 0.036441, size = 69, normalized size = 0.97

$$\frac{qx^2\left(aqx^q\text{Hypergeometric2F1}\left(1,\frac{q+2}{q},\frac{2}{q}+2,ax^q\right)+(q+2)\log\left(1-ax^q\right)\right)}{4(q+2)}+\frac{1}{2}x^2\text{PolyLog}\left(2,ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, a*x^q],x]
```

```
[Out] (q*x^2*(a*q*x^q*Hypergeometric2F1[1, (2 + q)/q, 2 + 2/q, a*x^q] + (2 + q)*L
og[1 - a*x^q]))/(4*(2 + q)) + (x^2*PolyLog[2, a*x^q])/2
```

Maple [C] time = 0.207, size = 108, normalized size = 1.5

$$-\frac{1}{q}\left(-a\right)^{-2\,q^{-1}}\left(-\frac{q^2x^2\ln\left(1-ax^q\right)}{4}\left(-a\right)^{2\,q^{-1}}-\frac{qx^2\text{polylog}\left(2,ax^q\right)}{2+q}\left(-a\right)^{2\,q^{-1}}\left(1+\frac{q}{2}\right)-\frac{q^2x^{2+q}a}{4}\left(-a\right)^{2\,q^{-1}}\text{LerchPhi}\left(ax^q,1,\frac{2}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,a*x^q),x)
```

```
[Out] -(-a)^(-2/q)/q*(-1/4*q^2*x^2*(-a)^(2/q)*ln(1-a*x^q)-q/(2+q)*x^2*(-a)^(2/q)*
(1+1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,(2
```

```
+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}q^2x^2 + \frac{1}{4}qx^2\log(-ax^q + 1) + \frac{1}{2}x^2\mathrm{Li}_2(ax^q) - q^2\int \frac{x}{4(ax^q - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -1/8*q^2*x^2 + 1/4*q*x^2*log(-a*x^q + 1) + 1/2*x^2*dilog(a*x^q) - q^2*integ
rate(1/4*x/(a*x^q - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{Li}_2(ax^q), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x*dilog(a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{Li}_2(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x**q),x)
```

```
[Out] Integral(x*polylog(2, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_2(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(a*x^q), x)
```

3.47 $\int \textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=54

$$\frac{aq^2x^{q+1}\text{Hypergeometric2F1}\left(1, \frac{1}{q}+1, \frac{1}{q}+2, ax^q\right)}{q+1} + x\text{PolyLog}(2, ax^q) + qx\log(1-ax^q)$$

```
[Out] (a*q^2*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 +
q) + q*x*Log[1 - a*x^q] + x*PolyLog[2, a*x^q]
```

Rubi [A] time = 0.023219, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6586, 2448, 364}

$$x\text{PolyLog}(2, ax^q) + \frac{aq^2x^{q+1}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{q+1} + qx\log(1-ax^q)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 +
q) + q*x*Log[1 - a*x^q] + x*PolyLog[2, a*x^q]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_2\left(ax^q\right)\,dx &= x\mathrm{Li}_2\left(ax^q\right)+q\int \log\left(1-ax^q\right)\,dx\\ &= qx\log\left(1-ax^q\right)+x\mathrm{Li}_2\left(ax^q\right)+\left(aq^2\right)\int \frac{x^q}{1-ax^q}\,dx\\ &= \frac{aq^2x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q}+qx\log\left(1-ax^q\right)+x\mathrm{Li}_2\left(ax^q\right)\end{aligned}$$

Mathematica [A] time = 0.0450111, size = 51, normalized size = 0.94

$$qx\left(\frac{aqx^q\text{Hypergeometric2F1}\left(1,\frac{1}{q}+1,\frac{1}{q}+2,ax^q\right)}{q+1}+\log\left(1-ax^q\right)\right)+x\text{PolyLog}\left(2,ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q],x]
```

```
[Out] q*x*((a*q*x^q*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1 + q)
+ Log[1 - a*x^q]) + x*PolyLog[2, a*x^q]
```

Maple [C] time = 0.207, size = 88, normalized size = 1.6

$$-\frac{1}{q}\left(-a\right)^{-q^{-1}}\left(-q^2x\sqrt[q]{-a}\ln\left(1-ax^q\right)-qx\sqrt[q]{-a}\text{polylog}\left(2,ax^q\right)-q^2x^{1+q}a\sqrt[q]{-a}\text{LerchPhi}\left(ax^q,1,\frac{1+q}{q}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q),x)
```

```
[Out] -1/q*(-a)^(-1/q)*(-q^2*x*(-a)^(1/q)*ln(1-a*x^q)-q*x*(-a)^(1/q)*polylog(2,a*
x^q)-q^2*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a*x^q,1,(1+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^2x - q^2 \int \frac{1}{ax^q - 1}\, dx + qx \log\left(-ax^q + 1\right) + x\mathrm{Li}_2\left(ax^q\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -q^2*x - q^2*integrate(1/(a*x^q - 1), x) + q*x*log(-a*x^q + 1) + x*dilog(a*
x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{Li}_2\left(ax^q\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_2\left(ax^q\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q),x)
```

```
[Out] Integral(polylog(2, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_2\left(ax^q\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q), x)
```

3.48 $\int \frac{\textbf{PolyLog}(2,ax^q)}{x} \, dx$

Optimal. Leaf size=11

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

[Out] PolyLog[3, a*x^q]/q

Rubi [A] time = 0.0102243, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6589}

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[2, a*x^q]/x,x]

[Out] PolyLog[3, a*x^q]/q

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^q)}{x} \, dx = \frac{\text{Li}_3(ax^q)}{q}$$

Mathematica [A] time = 0.0017468, size = 11, normalized size = 1.

$$\frac{\text{PolyLog}(3, ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x,x]
```

```
[Out] PolyLog[3, a*x^q]/q
```

Maple [A] time = 0.044, size = 12, normalized size = 1.1

$$\frac{\text{polylog}(3, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x,x)
```

```
[Out] polylog(3,a*x^q)/q
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}q^2\log(x)^3 + \frac{1}{2}q\log(-ax^q+1)\log(x)^2 - q^2\int\frac{\log(x)^2}{2(axx^q-x)}\,dx + \text{Li}_2(ax^q)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] -1/6*q^2*log(x)^3 + 1/2*q*log(-a*x^q + 1)*log(x)^2 - q^2*integrate(1/2*log(
x)^2/(a*x*x^q - x), x) + dilog(a*x^q)*log(x)
```

Fricas [A] time = 2.83852, size = 28, normalized size = 2.55

$$\frac{\text{polylog}(3, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] polylog(3, a*x^q)/q
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax^q)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x, x)
```

3.49 $\int \frac{\text{PolyLog}(2,ax^q)}{x^2}\,dx$

Optimal. Leaf size=69

$$-\frac{aq^2x^{q-1}\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q}-\frac{\text{PolyLog}\left(2,ax^q\right)}{x}+\frac{q\log\left(1-ax^q\right)}{x}$$

```
[Out] -((a*q^2*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q*Log[1 - a*x^q])/x - PolyLog[2, a*x^q]/x
```

Rubi [A] time = 0.0393505, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}\left(2,ax^q\right)}{x}-\frac{aq^2x^{q-1}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}+\frac{q\log\left(1-ax^q\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^2,x]
```

```
[Out] -((a*q^2*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q*Log[1 - a*x^q])/x - PolyLog[2, a*x^q]/x
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{x^2}\,dx &= -\frac{\text{Li}_2(ax^q)}{x} - q\int \frac{\log(1-ax^q)}{x^2}\,dx\\ &= \frac{q\log(1-ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x} + \left(aq^2\right)\int \frac{x^{-2+q}}{1-ax^q}\,dx\\ &= -\frac{aq^2x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q} + \frac{q\log(1-ax^q)}{x} - \frac{\text{Li}_2(ax^q)}{x}\end{aligned}$$

Mathematica [A] time = 0.052905, size = 60, normalized size = 0.87

$$\frac{q\left(\frac{aqx^q\text{Hypergeometric2F1}\left(1,\frac{q-1}{q},2-\frac{1}{q},ax^q\right)}{q-1}+\log(1-ax^q)\right)}{x}-\frac{\text{PolyLog}(2,ax^q)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^2,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-1 + q)/q, 2 - q^(-1), a*x^q])/(-1 + q)
+ Log[1 - a*x^q]))/x - PolyLog[2, a*x^q]/x
```

Maple [C] time = 0.208, size = 106, normalized size = 1.5

$$-\frac{\sqrt[q]{-a}}{q}\left(-\frac{q^2\ln(1-ax^q)}{x}(-a)^{-q^{-1}}-\frac{\left(1-q\right)q\text{polylog}(2,ax^q)}{\left(-1+q\right)x}(-a)^{-q^{-1}}-q^2x^{-1+q}a(-a)^{-q^{-1}}\text{LerchPhi}\left(ax^q,1,\frac{-1+q}{q}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^2,x)
```

```
[Out] -(-a)^(1/q)/q*(-q^2/x*(-a)^(-1/q)*ln(1-a*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)*
polylog(2,a*x^q)-q^2*x^(-1+q)*a*(-a)^(-1/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{ax^2x^q - x^2}\,dx + \frac{q^2 + q\log(-ax^q + 1) - \mathrm{Li}_2(ax^q)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/(a*x^2*x^q - x^2), x) + (q^2 + q*log(-a*x^q + 1) - dilog(a
*x^q))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2(ax^q)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**2,x)
```

```
[Out] Integral(polylog(2, a*x**q)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left(ax^q\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^2, x)
```

3.50 $\int \frac{\text{PolyLog}(2,ax^q)}{x^3} dx$

Optimal. Leaf size=78

$$-\frac{aq^2x^{q-2}\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{4(2-q)}-\frac{\text{PolyLog}\left(2,ax^q\right)}{2x^2}+\frac{q\log\left(1-ax^q\right)}{4x^2}$$

```
[Out] -(a*q^2*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a*x^q
])/(4*(2 - q)) + (q*Log[1 - a*x^q])/(4*x^2) - PolyLog[2, a*x^q]/(2*x^2)
```

Rubi [A] time = 0.0418889, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}\left(2,ax^q\right)}{2x^2}-\frac{aq^2x^{q-2}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{4(2-q)}+\frac{q\log\left(1-ax^q\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^3,x]
```

```
[Out] -(a*q^2*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a*x^q
])/(4*(2 - q)) + (q*Log[1 - a*x^q])/(4*x^2) - PolyLog[2, a*x^q]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{x^3}\,dx &= -\frac{\text{Li}_2(ax^q)}{2x^2}-\frac{1}{2}q\int \frac{\log(1-ax^q)}{x^3}\,dx\\ &= \frac{q\log(1-ax^q)}{4x^2}-\frac{\text{Li}_2(ax^q)}{2x^2}+\frac{1}{4}\left(aq^2\right)\int \frac{x^{-3+q}}{1-ax^q}\,dx\\ &= -\frac{aq^2x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{4(2-q)}+\frac{q\log(1-ax^q)}{4x^2}-\frac{\text{Li}_2(ax^q)}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.0521594, size = 61, normalized size = 0.78

$$\frac{q\left(\frac{aqx^q\text{Hypergeometric2F1}\left(1,\frac{q-2}{q},2-\frac{2}{q},ax^q\right)}{q-2}+\log(1-ax^q)\right)-2\text{PolyLog}(2,ax^q)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^3,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-2 + q)/q, 2 - 2/q, a*x^q])/(-2 + q) + L
og[1 - a*x^q]) - 2*PolyLog[2, a*x^q])/(4*x^2)
```

Maple [C] time = 0.209, size = 108, normalized size = 1.4

$$-\frac{1}{q}(-a)^{2q^{-1}}\left(-\frac{q^2\ln(1-ax^q)}{4x^2}(-a)^{-2q^{-1}}-\frac{q\text{polylog}(2,ax^q)}{(-2+q)x^2}(-a)^{-2q^{-1}}\left(1-\frac{q}{2}\right)-\frac{q^2x^{-2+q}a}{4}(-a)^{-2q^{-1}}\text{LerchPhi}\left(ax^q,1,-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^3,x)
```

```
[Out] -(-a)^(2/q)/q*(-1/4*q^2/x^2*(-a)^(-2/q)*ln(1-a*x^q)-q/(-2+q)/x^2*(-a)^(-2/q
)*(1-1/2*q)*polylog(2,a*x^q)-1/4*q^2*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^q,
```

```
1,(-2+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{4\left(ax^3x^q - x^3\right)}\,dx + \frac{q^2 + 2\,q\log\left(-ax^q + 1\right) - 4\,\mathrm{Li}_2\left(ax^q\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/4/(a*x^3*x^q - x^3), x) + 1/8*(q^2 + 2*q*log(-a*x^q + 1) -
 4*dilog(a*x^q))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2\left(ax^q\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^q\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^3, x)
```

3.51 $\int \frac{\text{PolyLog}(2,ax^q)}{x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{aq^2x^{q-3}\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{9(3-q)}-\frac{\text{PolyLog}(2,ax^q)}{3x^3}+\frac{q\log(1-ax^q)}{9x^3}$$

```
[Out] -(a*q^2*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(9*(
3 - q)) + (q*Log[1 - a*x^q])/(9*x^3) - PolyLog[2, a*x^q]/(3*x^3)
```

Rubi [A] time = 0.040045, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{\text{PolyLog}(2,ax^q)}{3x^3}-\frac{aq^2x^{q-3}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{9(3-q)}+\frac{q\log(1-ax^q)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/x^4,x]
```

```
[Out] -(a*q^2*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(9*(
3 - q)) + (q*Log[1 - a*x^q])/(9*x^3) - PolyLog[2, a*x^q]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{x^4}\,dx &= -\frac{\text{Li}_2(ax^q)}{3x^3}-\frac{1}{3}q\int \frac{\log(1-ax^q)}{x^4}\,dx\\ &= \frac{q\log(1-ax^q)}{9x^3}-\frac{\text{Li}_2(ax^q)}{3x^3}+\frac{1}{9}\left(aq^2\right)\int \frac{x^{-4+q}}{1-ax^q}\,dx\\ &= -\frac{aq^2x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{9(3-q)}+\frac{q\log(1-ax^q)}{9x^3}-\frac{\text{Li}_2(ax^q)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0518445, size = 61, normalized size = 0.8

$$\frac{q\left(\frac{aqx^q\text{Hypergeometric2F1}\left(1,\frac{q-3}{q},2-\frac{3}{q},ax^q\right)}{q-3}+\log(1-ax^q)\right)-3\text{PolyLog}(2,ax^q)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x^q]/x^4,x]
```

```
[Out] (q*((a*q*x^q*Hypergeometric2F1[1, (-3 + q)/q, 2 - 3/q, a*x^q])/(-3 + q) + L
og[1 - a*x^q]) - 3*PolyLog[2, a*x^q])/(9*x^3)
```

Maple [C] time = 0.205, size = 108, normalized size = 1.4

$$-\frac{1}{q}(-a)^{3q^{-1}}\left(-\frac{q^2\ln(1-ax^q)}{9x^3}(-a)^{-3q^{-1}}-\frac{q\text{polylog}(2,ax^q)}{(-3+q)x^3}(-a)^{-3q^{-1}}\left(1-\frac{q}{3}\right)-\frac{q^2x^{-3+q}a}{9}(-a)^{-3q^{-1}}\text{LerchPhi}\left(ax^q,1,-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/x^4,x)
```

```
[Out] -(-a)^(3/q)/q*(-1/9*q^2/x^3*(-a)^(-3/q)*ln(1-a*x^q)-q/(-3+q)/x^3*(-a)^(-3/q
)*(1-1/3*q)*polylog(2,a*x^q)-1/9*q^2*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^q,
```

```
1,(-3+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^2 \int \frac{1}{9\left(ax^4x^q - x^4\right)}\,dx + \frac{q^2 + 3\,q\log\left(-ax^q + 1\right) - 9\,\mathrm{Li}_2\left(ax^q\right)}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="maxima")
```

```
[Out] -q^2*integrate(1/9/(a*x^4*x^q - x^4), x) + 1/27*(q^2 + 3*q*log(-a*x^q + 1)
- 9*dilog(a*x^q))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\mathrm{Li}_2\left(ax^q\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(a*x^q)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^q)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/x^4, x)
```

3.52 $\int x^2 \textbf{PolyLog}(3, ax^q) \, dx$

Optimal. Leaf size=88

$$-\frac{aq^3x^{q+3}\text{Hypergeometric2F1}\left(1,\frac{q+3}{q},\frac{3}{q}+2,ax^q\right)}{27(q+3)}-\frac{1}{9}qx^3\text{PolyLog}\left(2,ax^q\right)+\frac{1}{3}x^3\text{PolyLog}\left(3,ax^q\right)-\frac{1}{27}q^2x^3\log\left(1-ax^q\right)$$

```
[Out] -(a*q^3*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(27*(3 +
 q)) - (q^2*x^3*Log[1 - a*x^q])/27 - (q*x^3*PolyLog[2, a*x^q])/9 + (x^3*Pol
yLog[3, a*x^q])/3
```

Rubi [A] time = 0.0515477, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{1}{9}qx^3\text{PolyLog}\left(2,ax^q\right)+\frac{1}{3}x^3\text{PolyLog}\left(3,ax^q\right)-\frac{aq^3x^{q+3}\,{}_2F_1\left(1,\frac{q+3}{q};2+\frac{3}{q};ax^q\right)}{27(q+3)}-\frac{1}{27}q^2x^3\log\left(1-ax^q\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, a*x^q],x]
```

```
[Out] -(a*q^3*x^(3 + q)*Hypergeometric2F1[1, (3 + q)/q, 2 + 3/q, a*x^q])/(27*(3 +
 q)) - (q^2*x^3*Log[1 - a*x^q])/27 - (q*x^3*PolyLog[2, a*x^q])/9 + (x^3*Pol
yLog[3, a*x^q])/3
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\text{Li}_3\left(ax^q\right)\,dx &= \frac{1}{3}x^3\text{Li}_3\left(ax^q\right)-\frac{1}{3}q\int x^2\text{Li}_2\left(ax^q\right)\,dx\\ &= -\frac{1}{9}qx^3\text{Li}_2\left(ax^q\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^q\right)-\frac{1}{9}q^2\int x^2\log\left(1-ax^q\right)\,dx\\ &= -\frac{1}{27}q^2x^3\log\left(1-ax^q\right)-\frac{1}{9}qx^3\text{Li}_2\left(ax^q\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^q\right)-\frac{1}{27}\left(aq^3\right)\int\frac{x^{2+q}}{1-ax^q}\,dx\\ &= -\frac{aq^3x^{3+q}\,{}_2F_1\left(1,\frac{3+q}{q};2+\frac{3}{q};ax^q\right)}{27(3+q)}-\frac{1}{27}q^2x^3\log\left(1-ax^q\right)-\frac{1}{9}qx^3\text{Li}_2\left(ax^q\right)+\frac{1}{3}x^3\text{Li}_3\left(ax^q\right)\end{aligned}$$

Mathematica [C] time = 0.0089413, size = 41, normalized size = 0.47

$$-\frac{x^3G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{q-3}{q}\\ 1,0,0,0,-\frac{3}{q}\end{array}\right)}{q}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*PolyLog[3, a*x^q],x]
```

```
[Out] -((x^3*MeijerG[{{1, 1, 1, 1, (-3 + q)/q}, {}}, {{1}, {0, 0, 0, -3/q}}, -(a*
x^q)])/q)
```

Maple [C] time = 0.345, size = 132, normalized size = 1.5

$$-\frac{1}{q}\left(-a\right)^{-3\,q^{-1}}\left(\frac{q^3x^3\ln\left(1-ax^q\right)}{27}\left(-a\right)^{3\,q^{-1}}+\frac{q^2x^3\text{polylog}\left(2,ax^q\right)}{9}\left(-a\right)^{3\,q^{-1}}-\frac{qx^3\text{polylog}\left(3,ax^q\right)}{3+q}\left(-a\right)^{3\,q^{-1}}\left(1+\frac{q}{3}\right)+\frac{q^3x^{3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,a*x^q),x)
```

```
[Out] -(-a)^(-3/q)/q*(1/27*q^3*x^3*(-a)^(3/q)*ln(1-a*x^q)+1/9*q^2*x^3*(-a)^(3/q)*
polylog(2,a*x^q)-q/(3+q)*x^3*(-a)^(3/q)*(1+1/3*q)*polylog(3,a*x^q)+1/27*q^3
*x^(3+q)*a*(-a)^(3/q)*LerchPhi(a*x^q,1,(3+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{81}\,q^3x^3 - \frac{1}{27}\,q^2x^3\log\left(-ax^q+1\right) - \frac{1}{9}\,qx^3\mathrm{Li}_2\left(ax^q\right) + q^3\int\frac{x^2}{27\left(ax^q-1\right)}\,dx + \frac{1}{3}\,x^3\mathrm{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] 1/81*q^3*x^3 - 1/27*q^2*x^3*log(-a*x^q + 1) - 1/9*q*x^3*dilog(a*x^q) + q^3*
integrate(1/27*x^2/(a*x^q - 1), x) + 1/3*x^3*polylog(3, a*x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2\mathrm{polylog}\left(3,ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x^2*polylog(3, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,\mathrm{Li}_3\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,a*x**q),x)
```

```
[Out] Integral(x**2*polylog(3, a*x**q), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \text{Li}_3(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, a*x^q), x)
```

3.53 $\int x\mathbf{PolyLog}(3, ax^q)\,dx$

Optimal. Leaf size=88

$$-\frac{aq^3x^{q+2}\text{Hypergeometric2F1}\left(1,\frac{q+2}{q},2\left(\frac{1}{q}+1\right),ax^q\right)}{8(q+2)}-\frac{1}{4}qx^2\text{PolyLog}(2,ax^q)+\frac{1}{2}x^2\text{PolyLog}(3,ax^q)-\frac{1}{8}q^2x^2\log(1-$$

```
[Out] -(a*q^3*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])/(
8*(2 + q)) - (q^2*x^2*Log[1 - a*x^q])/8 - (q*x^2*PolyLog[2, a*x^q])/4 + (x^
2*PolyLog[3, a*x^q])/2
```

Rubi [A] time = 0.0425662, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6591, 2455, 364}

$$-\frac{1}{4}qx^2\text{PolyLog}(2,ax^q)+\frac{1}{2}x^2\text{PolyLog}(3,ax^q)-\frac{aq^3x^{q+2}\,{}_2F_1\left(1,\frac{q+2}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{8(q+2)}-\frac{1}{8}q^2x^2\log(1-ax^q)$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, a*x^q],x]
```

```
[Out] -(a*q^3*x^(2 + q)*Hypergeometric2F1[1, (2 + q)/q, 2*(1 + q^(-1)), a*x^q])/(
8*(2 + q)) - (q^2*x^2*Log[1 - a*x^q])/8 - (q*x^2*PolyLog[2, a*x^q])/4 + (x^
2*PolyLog[3, a*x^q])/2
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\text{Li}_3(ax^q)\,dx &= \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{2}q\int x\text{Li}_2(ax^q)\,dx \\
&= -\frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{4}q^2\int x\log(1-ax^q)\,dx \\
&= -\frac{1}{8}q^2x^2\log(1-ax^q) - \frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax^q) - \frac{1}{8}\left(aq^3\right)\int \frac{x^{1+q}}{1-ax^q}\,dx \\
&= -\frac{aq^3x^{2+q}\,{}_2F_1\left(1,\frac{2+q}{q};2\left(1+\frac{1}{q}\right);ax^q\right)}{8(2+q)} - \frac{1}{8}q^2x^2\log(1-ax^q) - \frac{1}{4}qx^2\text{Li}_2(ax^q) + \frac{1}{2}x^2\text{Li}_3(ax^q)
\end{aligned}
$$

Mathematica [C] time = 0.0076607, size = 41, normalized size = 0.47

$$
-\frac{x^2 G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{q-2}{q}\\ 1,0,0,0,-\frac{2}{q}\end{array}\right)}{q}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*PolyLog[3, a*x^q],x]
```

```
[Out] -((x^2*MeijerG[{{1, 1, 1, 1, (-2 + q)/q}, {}}, {{1}, {0, 0, 0, -2/q}}, -(a*
x^q)])/q)
```

Maple [C] time = 0.338, size = 132, normalized size = 1.5

$$
-\frac{1}{q}(-a)^{-2q^{-1}}\left(\frac{q^3x^2\ln(1-ax^q)}{8}(-a)^{2q^{-1}} + \frac{q^2x^2\text{polylog}(2,ax^q)}{4}(-a)^{2q^{-1}} - \frac{qx^2\text{polylog}(3,ax^q)}{2+q}(-a)^{2q^{-1}}\left(1+\frac{q}{2}\right) + \frac{q^3x^2}{8}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,a*x^q),x)
```

```
[Out] -(-a)^(-2/q)/q*(1/8*q^3*x^2*(-a)^(2/q)*ln(1-a*x^q)+1/4*q^2*x^2*(-a)^(2/q)*p
olylog(2,a*x^q)-q/(2+q)*x^2*(-a)^(2/q)*(1+1/2*q)*polylog(3,a*x^q)+1/8*q^3*x
^(2+q)*a*(-a)^(2/q)*LerchPhi(a*x^q,1,(2+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\,q^3x^2 - \frac{1}{8}\,q^2x^2\log\left(-ax^q + 1\right) - \frac{1}{4}\,qx^2\mathrm{Li}_2\left(ax^q\right) + q^3\int \frac{x}{8\left(ax^q - 1\right)}\,dx + \frac{1}{2}\,x^2\mathrm{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] 1/16*q^3*x^2 - 1/8*q^2*x^2*log(-a*x^q + 1) - 1/4*q*x^2*dilog(a*x^q) + q^3*i
ntegrate(1/8*x/(a*x^q - 1), x) + 1/2*x^2*polylog(3, a*x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{polylog}\left(3, ax^q\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(3, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{Li}_3\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x**q),x)
```

```
[Out] Integral(x*polylog(3, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, a*x^q), x)
```

3.54 $\int \text{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=69

$$-\frac{aq^3x^{q+1}\text{Hypergeometric2F1}\left(1, \frac{1}{q}+1, \frac{1}{q}+2, ax^q\right)}{q+1} - qx\text{PolyLog}(2, ax^q) + x\text{PolyLog}(3, ax^q) - q^2x\log(1-ax^q)$$

```
[Out] -((a*q^3*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1
+ q)) - q^2*x*Log[1 - a*x^q] - q*x*PolyLog[2, a*x^q] + x*PolyLog[3, a*x^q]
```

Rubi [A] time = 0.0280803, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6586, 2448, 364}

$$-qx\text{PolyLog}(2, ax^q) + x\text{PolyLog}(3, ax^q) - \frac{aq^3x^{q+1}\,{}_2F_1\left(1, 1+\frac{1}{q}; 2+\frac{1}{q}; ax^q\right)}{q+1} - q^2x\log(1-ax^q)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q],x]
```

```
[Out] -((a*q^3*x^(1 + q)*Hypergeometric2F1[1, 1 + q^(-1), 2 + q^(-1), a*x^q])/(1
+ q)) - q^2*x*Log[1 - a*x^q] - q*x*PolyLog[2, a*x^q] + x*PolyLog[3, a*x^q]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
```

```
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \mathrm{Li}_3(ax^q)\,dx &= x\mathrm{Li}_3(ax^q) - q\int \mathrm{Li}_2(ax^q)\,dx\\ &= -qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q) - q^2\int \log(1-ax^q)\,dx\\ &= -q^2x\log(1-ax^q) - qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q) - \left(aq^3\right)\int \frac{x^q}{1-ax^q}\,dx\\ &= -\frac{aq^3x^{1+q}\,{}_2F_1\left(1,1+\frac{1}{q};2+\frac{1}{q};ax^q\right)}{1+q} - q^2x\log(1-ax^q) - qx\mathrm{Li}_2(ax^q) + x\mathrm{Li}_3(ax^q)\end{aligned}$$

Mathematica [C] time = 0.0060684, size = 39, normalized size = 0.57

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{q-1}{q}\\1,0,0,0,-\frac{1}{q}\end{array}\right)}{q}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, (-1 + q)/q}, {}}, {{1}, {0, 0, 0, -q^(-1)}}, -(a
*x^q)])/q)
```

Maple [C] time = 0.334, size = 105, normalized size = 1.5

$$-\frac{1}{q}(-a)^{-q^{-1}}\left(q^3x\sqrt[q]{-a}\ln(1-ax^q) + q^2x\sqrt[q]{-a}\mathrm{polylog}(2,ax^q) - qx\sqrt[q]{-a}\mathrm{polylog}(3,ax^q) + q^3x^{1+q}a\sqrt[q]{-a}\mathrm{LerchPhi}\left(ax^q,1,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q),x)
```

```
[Out] -1/q*(-a)^(-1/q)*(q^3*x*(-a)^(1/q)*ln(1-a*x^q)+q^2*x*(-a)^(1/q)*polylog(2,a
*x^q)-q*x*(-a)^(1/q)*polylog(3,a*x^q)+q^3*x^(1+q)*a*(-a)^(1/q)*LerchPhi(a*x
```

```
^q,1,(1+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$q^3x + q^3 \int \frac{1}{ax^q - 1}\,dx - q^2x\log\left(-ax^q + 1\right) - qx\mathrm{Li}_2\left(ax^q\right) + x\mathrm{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] q^3*x + q^3*integrate(1/(a*x^q - 1), x) - q^2*x*log(-a*x^q + 1) - q*x*dilog
(a*x^q) + x*polylog(3, a*x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{polylog}\left(3, ax^q\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_3\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q),x)
```

```
[Out] Integral(polylog(3, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_3(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q), x)
```

3.55 $\int \frac{\textbf{PolyLog}(3,ax^q)}{x} \, dx$

Optimal. Leaf size=11

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

[Out] PolyLog[4, a*x^q]/q

Rubi [A] time = 0.0097364, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6589}

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

Antiderivative was successfully verified.

[In] Int[PolyLog[3, a*x^q]/x,x]

[Out] PolyLog[4, a*x^q]/q

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^q)}{x} \, dx = \frac{\text{Li}_4(ax^q)}{q}$$

Mathematica [A] time = 0.0014392, size = 11, normalized size = 1.

$$\frac{\text{PolyLog}(4, ax^q)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x^q]/x,x]
```

```
[Out] PolyLog[4, a*x^q]/q
```

Maple [A] time = 0.041, size = 12, normalized size = 1.1

$$\frac{\text{polylog}(4, ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x,x)
```

```
[Out] polylog(4,a*x^q)/q
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}q^3\log(x)^4 - \frac{1}{6}q^2\log(-ax^q+1)\log(x)^3 + q^3\int\frac{\log(x)^3}{6(axx^q-x)}\,dx - \frac{1}{2}q\text{Li}_2(ax^q)\log(x)^2 + \log(x)\,\text{Li}_3(ax^q)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] 1/24*q^3*log(x)^4 - 1/6*q^2*log(-a*x^q + 1)*log(x)^3 + q^3*integrate(1/6*lo
g(x)^3/(a*x*x^q - x), x) - 1/2*q*dilog(a*x^q)*log(x)^2 + log(x)*polylog(3,
a*x^q)
```

Fricas [C] time = 2.72366, size = 209, normalized size = 19.

$$-\frac{q^2\%\text{iint}\left(a, q, x, -\frac{\log(-ax^q+1)}{a}, -\log(-ax^q+1)\log(x), -\frac{q\log(-ax^q+1)}{x}\right)\log(x)^2 - q^2\text{Li}_2(ax^q)\log(x)^2 - 2\,\text{polylog}(4, ax^q)}{2q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(q^2*\%iint(a, q, x, -log(-a*x^q + 1)/a, -log(-a*x^q + 1)*log(x), -q*lo
g(-a*x^q + 1)/x)*log(x)^2 - q^2*dilog(a*x^q)*log(x)^2 - 2*polylog(4, a*x^q)
)/q
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x,x)
```

```
[Out] Integral(polylog(3, a*x**q)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/x, x)
```

3.56 $\int \frac{\text{PolyLog}(3,ax^q)}{x^2} dx$

Optimal. Leaf size=84

$$-\frac{aq^3x^{q-1}\text{Hypergeometric2F1}\left(1,-\frac{1-q}{q},2-\frac{1}{q},ax^q\right)}{1-q}-\frac{q\text{PolyLog}(2,ax^q)}{x}-\frac{\text{PolyLog}(3,ax^q)}{x}+\frac{q^2\log(1-ax^q)}{x}$$

```
[Out] -((a*q^3*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q^2*Log[1 - a*x^q])/x - (q*PolyLog[2, a*x^q])/x - PolyLog[3, a*
x^q]/x
```

Rubi [A] time = 0.0501811, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}(2,ax^q)}{x}-\frac{\text{PolyLog}(3,ax^q)}{x}-\frac{aq^3x^{q-1}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}+\frac{q^2\log(1-ax^q)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^2,x]
```


```
[Out] -((a*q^3*x^(-1 + q)*Hypergeometric2F1[1, -((1 - q)/q), 2 - q^(-1), a*x^q])/
(1 - q)) + (q^2*Log[1 - a*x^q])/x - (q*PolyLog[2, a*x^q])/x - PolyLog[3, a*
x^q]/x
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax^q)}{x^2}\,dx &= -\frac{\text{Li}_3(ax^q)}{x}+q\int \frac{\text{Li}_2(ax^q)}{x^2}\,dx\\ &= -\frac{q\text{Li}_2(ax^q)}{x}-\frac{\text{Li}_3(ax^q)}{x}-q^2\int \frac{\log(1-ax^q)}{x^2}\,dx\\ &= \frac{q^2\log(1-ax^q)}{x}-\frac{q\text{Li}_2(ax^q)}{x}-\frac{\text{Li}_3(ax^q)}{x}+\left(aq^3\right)\int \frac{x^{-2+q}}{1-ax^q}\,dx\\ &= -\frac{aq^3x^{-1+q}\,{}_2F_1\left(1,-\frac{1-q}{q};2-\frac{1}{q};ax^q\right)}{1-q}+\frac{q^2\log(1-ax^q)}{x}-\frac{q\text{Li}_2(ax^q)}{x}-\frac{\text{Li}_3(ax^q)}{x}\end{aligned}$$

Mathematica [C] time = 0.0088089, size = 37, normalized size = 0.44

$$-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{1}{q}\\ 1,0,0,0,\frac{1}{q}\end{array}\right)}{qx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/x^2,x]
```

```
[Out] -(MeijerG[{{1, 1, 1, 1, 1 + q^(-1)}, {}}, {{1}, {0, 0, 0, q^(-1)}}, -(a*x^q
)]/(q*x))
```

Maple [C] time = 0.333, size = 129, normalized size = 1.5

$$-\frac{\sqrt[q]{-a}}{q}\left(-\frac{q^3\ln(1-ax^q)}{x}(-a)^{-q^{-1}}+\frac{q^2\text{polylog}(2,ax^q)}{x}(-a)^{-q^{-1}}-\frac{\left(1-q\right)q\text{polylog}(3,ax^q)}{\left(-1+q\right)x}(-a)^{-q^{-1}}-q^3x^{-1+q}a(-a)^{-q^{-1}}\text{L}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^2,x)
```

```
[Out] -(-a)^(1/q)/q*(-q^3/x*(-a)^(-1/q)*ln(1-a*x^q)+q^2/x*(-a)^(-1/q)*polylog(2,a
*x^q)-q/(-1+q)/x*(-a)^(-1/q)*(1-q)*polylog(3,a*x^q)-q^3*x^(-1+q)*a*(-a)^(-1
/q)*LerchPhi(a*x^q,1,(-1+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{ax^2x^q - x^2}\,dx + \frac{q^3 + q^2 \log\left(-ax^q + 1\right) - q\mathrm{Li}_2\left(ax^q\right) - \mathrm{Li}_3(ax^q)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="maxima")
```

```
[Out] -q^3*integrate(1/(a*x^2*x^q - x^2), x) + (q^3 + q^2*log(-a*x^q + 1) - q*dil
og(a*x^q) - polylog(3, a*x^q))/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{polylog}\left(3, ax^q\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^q)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**2,x)
```

```
[Out] Integral(polylog(3, a*x**q)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/x^2, x)
```

3.57 $\int \frac{\mathbf{PolyLog}(3,ax^q)}{x^3}\,dx$

Optimal. Leaf size=95

$$-\frac{aq^3x^{q-2}\text{Hypergeometric2F1}\left(1,-\frac{2-q}{q},2\left(1-\frac{1}{q}\right),ax^q\right)}{8(2-q)}-\frac{q\text{PolyLog}\left(2,ax^q\right)}{4x^2}-\frac{\text{PolyLog}\left(3,ax^q\right)}{2x^2}+\frac{q^2\log\left(1-ax^q\right)}{8x^2}$$

```
[Out] -(a*q^3*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a*x^q
])/(8*(2 - q)) + (q^2*Log[1 - a*x^q])/(8*x^2) - (q*PolyLog[2, a*x^q])/(4*x^
2) - PolyLog[3, a*x^q]/(2*x^2)
```

Rubi [A] time = 0.0502811, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}\left(2,ax^q\right)}{4x^2}-\frac{\text{PolyLog}\left(3,ax^q\right)}{2x^2}-\frac{aq^3x^{q-2}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{8(2-q)}+\frac{q^2\log\left(1-ax^q\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^3,x]
```

```
[Out] -(a*q^3*x^(-2 + q)*Hypergeometric2F1[1, -((2 - q)/q), 2*(1 - q^(-1)), a*x^q
])/(8*(2 - q)) + (q^2*Log[1 - a*x^q])/(8*x^2) - (q*PolyLog[2, a*x^q])/(4*x^
2) - PolyLog[3, a*x^q]/(2*x^2)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{x^3}\,dx &= -\frac{\text{Li}_3(ax^q)}{2x^2}+\frac{1}{2}q\int \frac{\text{Li}_2(ax^q)}{x^3}\,dx \\
&= -\frac{q\text{Li}_2(ax^q)}{4x^2}-\frac{\text{Li}_3(ax^q)}{2x^2}-\frac{1}{4}q^2\int \frac{\log(1-ax^q)}{x^3}\,dx \\
&= \frac{q^2\log(1-ax^q)}{8x^2}-\frac{q\text{Li}_2(ax^q)}{4x^2}-\frac{\text{Li}_3(ax^q)}{2x^2}+\frac{1}{8}\left(aq^3\right)\int \frac{x^{-3+q}}{1-ax^q}\,dx \\
&= -\frac{aq^3x^{-2+q}\,{}_2F_1\left(1,-\frac{2-q}{q};2\left(1-\frac{1}{q}\right);ax^q\right)}{8(2-q)}+\frac{q^2\log(1-ax^q)}{8x^2}-\frac{q\text{Li}_2(ax^q)}{4x^2}-\frac{\text{Li}_3(ax^q)}{2x^2}
\end{aligned}
$$

Mathematica [C] time = 0.0085129, size = 41, normalized size = 0.43

$$
-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|
\begin{array}{c}
1,1,1,1,\frac{q+2}{q} \\
1,0,0,0,\frac{2}{q}
\end{array}
\right)}{qx^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/x^3,x]
```

```
[Out] -(MeijerG[{{1, 1, 1, 1, (2 + q)/q}, {}}, {{1}, {0, 0, 0, 2/q}}, -(a*x^q)]/(
q*x^2))
```

Maple [C] time = 0.343, size = 132, normalized size = 1.4

$$
-\frac{1}{q}(-a)^{2q^{-1}}\left(-\frac{q^3\ln(1-ax^q)}{8x^2}(-a)^{-2q^{-1}}+\frac{q^2\text{polylog}(2,ax^q)}{4x^2}(-a)^{-2q^{-1}}-\frac{q\text{polylog}(3,ax^q)}{(-2+q)x^2}(-a)^{-2q^{-1}}\left(1-\frac{q}{2}\right)-\frac{q^3x^{-2+q}a}{8}
\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^3,x)
```

```
[Out] -(-a)^(2/q)/q*(-1/8*q^3/x^2*(-a)^(-2/q)*ln(1-a*x^q)+1/4*q^2/x^2*(-a)^(-2/q)
*polylog(2,a*x^q)-q/(-2+q)/x^2*(-a)^(-2/q)*(1-1/2*q)*polylog(3,a*x^q)-1/8*q
^3*x^(-2+q)*a*(-a)^(-2/q)*LerchPhi(a*x^q,1,(-2+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{8\left(ax^3x^q - x^3\right)}\,dx + \frac{q^3 + 2\,q^2\log\left(-ax^q + 1\right) - 4\,q\mathrm{Li}_2\left(ax^q\right) - 8\,\mathrm{Li}_3(ax^q)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="maxima")
```

```
[Out] -q^3*integrate(1/8/(a*x^3*x^q - x^3), x) + 1/16*(q^3 + 2*q^2*log(-a*x^q + 1
) - 4*q*dilog(a*x^q) - 8*polylog(3, a*x^q))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{polylog}\left(3, ax^q\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^q)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**3,x)
```

```
[Out] Integral(polylog(3, a*x**q)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/x^3, x)
```

3.58 $\int \frac{\textbf{PolyLog}(3,ax^q)}{x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{aq^3x^{q-3}\text{Hypergeometric2F1}\left(1,-\frac{3-q}{q},2-\frac{3}{q},ax^q\right)}{27(3-q)}-\frac{q\text{PolyLog}\left(2,ax^q\right)}{9x^3}-\frac{\text{PolyLog}\left(3,ax^q\right)}{3x^3}+\frac{q^2\log\left(1-ax^q\right)}{27x^3}$$

```
[Out] -(a*q^3*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(27*
(3 - q)) + (q^2*Log[1 - a*x^q])/(27*x^3) - (q*PolyLog[2, a*x^q])/(9*x^3) -
PolyLog[3, a*x^q]/(3*x^3)
```

Rubi [A] time = 0.0508152, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2455, 364}

$$-\frac{q\text{PolyLog}\left(2,ax^q\right)}{9x^3}-\frac{\text{PolyLog}\left(3,ax^q\right)}{3x^3}-\frac{aq^3x^{q-3}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{27(3-q)}+\frac{q^2\log\left(1-ax^q\right)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/x^4,x]
```

```
[Out] -(a*q^3*x^(-3 + q)*Hypergeometric2F1[1, -((3 - q)/q), 2 - 3/q, a*x^q])/(27*
(3 - q)) + (q^2*Log[1 - a*x^q])/(27*x^3) - (q*PolyLog[2, a*x^q])/(9*x^3) -
PolyLog[3, a*x^q]/(3*x^3)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_3(ax^q)}{x^4}\,dx &= -\frac{\text{Li}_3(ax^q)}{3x^3}+\frac{1}{3}q\int \frac{\text{Li}_2(ax^q)}{x^4}\,dx\\ &= -\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}-\frac{1}{9}q^2\int \frac{\log(1-ax^q)}{x^4}\,dx\\ &= \frac{q^2\log(1-ax^q)}{27x^3}-\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}+\frac{1}{27}\left(aq^3\right)\int \frac{x^{-4+q}}{1-ax^q}\,dx\\ &= -\frac{aq^3x^{-3+q}\,{}_2F_1\left(1,-\frac{3-q}{q};2-\frac{3}{q};ax^q\right)}{27(3-q)}+\frac{q^2\log(1-ax^q)}{27x^3}-\frac{q\text{Li}_2(ax^q)}{9x^3}-\frac{\text{Li}_3(ax^q)}{3x^3}\end{aligned}$$

Mathematica [C] time = 0.0104082, size = 41, normalized size = 0.44

$$-\frac{G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,\frac{q+3}{q}\\ 1,0,0,0,\frac{3}{q}\end{array}\right)}{qx^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/x^4,x]
```

```
[Out] -(MeijerG[{{1, 1, 1, 1, (3 + q)/q}, {}}, {{1}, {0, 0, 0, 3/q}}, -(a*x^q)]/(
q*x^3))
```

Maple [C] time = 0.34, size = 132, normalized size = 1.4

$$-\frac{1}{q}(-a)^{3q^{-1}}\left(-\frac{q^3\ln(1-ax^q)}{27\,x^3}(-a)^{-3q^{-1}}+\frac{q^2\text{polylog}(2,ax^q)}{9\,x^3}(-a)^{-3q^{-1}}-\frac{q\text{polylog}(3,ax^q)}{\left(-3+q\right)x^3}(-a)^{-3q^{-1}}\left(1-\frac{q}{3}\right)-\frac{q^3x^{-3+q}a}{27}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/x^4,x)
```

```
[Out] -(-a)^(3/q)/q*(-1/27*q^3/x^3*(-a)^(-3/q)*ln(1-a*x^q)+1/9*q^2/x^3*(-a)^(-3/q
)*polylog(2,a*x^q)-q/(-3+q)/x^3*(-a)^(-3/q)*(1-1/3*q)*polylog(3,a*x^q)-1/27
*q^3*x^(-3+q)*a*(-a)^(-3/q)*LerchPhi(a*x^q,1,(-3+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-q^3 \int \frac{1}{27\left(ax^4x^q - x^4\right)}\,dx + \frac{q^3 + 3\,q^2\log\left(-ax^q + 1\right) - 9\,q\mathrm{Li}_2\left(ax^q\right) - 27\,\mathrm{Li}_3(ax^q)}{81\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="maxima")
```

```
[Out] -q^3*integrate(1/27/(a*x^4*x^q - x^4), x) + 1/81*(q^3 + 3*q^2*log(-a*x^q +
1) - 9*q*dilog(a*x^q) - 27*polylog(3, a*x^q))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{polylog}\left(3, ax^q\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, a*x^q)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^q\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/x**4,x)
```

```
[Out] Integral(polylog(3, a*x**q)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/x^4,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/x^4, x)
```

3.59 $\int (dx)^{3/2}\mathbf{PolyLog}(2,ax)\,dx$

Optimal. Leaf size=117

$$\frac{2(dx)^{5/2}\text{PolyLog}(2,ax)}{5d}+\frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}}-\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}-\frac{8(dx)^{5/2}}{125d}$$

```
[Out] (-8*d*Sqrt[d*x])/(25*a^2) - (8*(d*x)^(3/2))/(75*a) - (8*(d*x)^(5/2))/(125*d
) + (8*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/2)) + (4*(d*x
)^(5/2)*Log[1 - a*x])/(25*d) + (2*(d*x)^(5/2)*PolyLog[2, a*x])/(5*d)
```

Rubi [A] time = 0.0721405, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2(dx)^{5/2}\text{PolyLog}(2,ax)}{5d}+\frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}}-\frac{8d\sqrt{dx}}{25a^2}-\frac{8(dx)^{3/2}}{75a}+\frac{4(dx)^{5/2}\log(1-ax)}{25d}-\frac{8(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

```
[Out] (-8*d*Sqrt[d*x])/(25*a^2) - (8*(d*x)^(3/2))/(75*a) - (8*(d*x)^(5/2))/(125*d
) + (8*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/2)) + (4*(d*x
)^(5/2)*Log[1 - a*x])/(25*d) + (2*(d*x)^(5/2)*PolyLog[2, a*x])/(5*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (dx)^{3/2}\mathrm{Li}_2(ax)\,dx = \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{2}{5}\int (dx)^{3/2}\log(1-ax)\,dx$$

$$= \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{(4a)\int \frac{(dx)^{5/2}}{1-ax}\,dx}{25d}$$

$$= -\frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{4}{25}\int \frac{(dx)^{3/2}}{1-ax}\,dx$$

$$= -\frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{(4d)\int \frac{\sqrt{dx}}{1-ax}\,dx}{25a}$$

$$= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{\left(4d^2\right)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{25a^2}$$

$$= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d} + \frac{(8d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx\right.}{25a^2}$$

$$= -\frac{8d\sqrt{dx}}{25a^2} - \frac{8(dx)^{3/2}}{75a} - \frac{8(dx)^{5/2}}{125d} + \frac{8d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/2}} + \frac{4(dx)^{5/2}\log(1-ax)}{25d} + \frac{2(dx)^{5/2}\mathrm{Li}_2(ax)}{5d}$$

Mathematica [A] time = 0.102173, size = 90, normalized size = 0.77

$$\frac{2(dx)^{3/2}\left(x^{5/2}\text{PolyLog}(2,ax)+\frac{2}{75}\sqrt{x}\left(15x^2\log(1-ax)-\frac{2(3a^2x^2+5ax+15)}{a^2}\right)+\frac{4\tanh^{-1}(\sqrt{a}\sqrt{x})}{5a^{5/2}}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x],x]
```

```
[Out] (2*(d*x)^(3/2)*((4*ArcTanh[Sqrt[a]*Sqrt[x]])/(5*a^(5/2)) + (2*Sqrt[x]*((-2*
(15 + 5*a*x + 3*a^2*x^2))/a^2 + 15*x^2*Log[1 - a*x]))/75 + x^(5/2)*PolyLog[
2, a*x]))/(5*x^(3/2))
```

Maple [A] time = 0.194, size = 96, normalized size = 0.8

$$\frac{2\,\text{polylog}(2,ax)}{5\,d}(dx)^{\frac{5}{2}}+\frac{4}{25\,d}(dx)^{\frac{5}{2}}\ln\left(\frac{-adx+d}{d}\right)-\frac{8}{125\,d}(dx)^{\frac{5}{2}}-\frac{8}{75\,a}(dx)^{\frac{3}{2}}-\frac{8\,d}{25\,a^2}\sqrt{dx}+\frac{8\,d^2}{25\,a^2}\text{Artanh}\left(a\sqrt{dx}\frac{1}{\sqrt{ad}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x),x)
```

```
[Out] 2/5*(d*x)^(5/2)*polylog(2,a*x)/d+4/25/d*(d*x)^(5/2)*ln((-a*d*x+d)/d)-8/125*
(d*x)^(5/2)/d-8/75*(d*x)^(3/2)/a-8/25*d*(d*x)^(1/2)/a^2+8/25*d^2/a^2/(a*d)^
(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.82002, size = 470, normalized size = 4.02

$$\left[\frac{2\left(30\,d\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)+\left(75\,a^2dx^2\mathrm{Li}_2\left(ax\right)+30\,a^2dx^2\log\left(-ax+1\right)-12\,a^2dx^2-20\,adx-60\,d\right)\sqrt{dx}\right)}{375\,a^2},-\frac{2\left(60\,c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] [2/375*(30*d*sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1))
 + (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) - 12*a^2*d*x^2 - 2
0*a*d*x - 60*d)*sqrt(d*x))/a^2, -2/375*(60*d*sqrt(-d/a)*arctan(sqrt(d*x)*a*
sqrt(-d/a)/d) - (75*a^2*d*x^2*dilog(a*x) + 30*a^2*d*x^2*log(-a*x + 1) - 12*
a^2*d*x^2 - 20*a*d*x - 60*d)*sqrt(d*x))/a^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\,\mathrm{Li}_2\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x), x)
```

3.60 $\int \sqrt{dx}\,\textbf{PolyLog}(2,ax)\,dx$

Optimal. Leaf size=102

$$\frac{2(dx)^{3/2}\text{PolyLog}(2,ax)}{3d}+\frac{8\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}}-\frac{8\sqrt{dx}}{9a}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}-\frac{8(dx)^{3/2}}{27d}$$

```
[Out] (-8*Sqrt[d*x])/(9*a) - (8*(d*x)^(3/2))/(27*d) + (8*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/2)) + (4*(d*x)^(3/2)*Log[1 - a*x])/(9*d) + (2
*(d*x)^(3/2)*PolyLog[2, a*x])/(3*d)
```

Rubi [A] time = 0.0530667, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2(dx)^{3/2}\text{PolyLog}(2,ax)}{3d}+\frac{8\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}}-\frac{8\sqrt{dx}}{9a}+\frac{4(dx)^{3/2}\log(1-ax)}{9d}-\frac{8(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x],x]
```

```
[Out] (-8*Sqrt[d*x])/(9*a) - (8*(d*x)^(3/2))/(27*d) + (8*Sqrt[d]*ArcTanh[(Sqrt[a]
*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/2)) + (4*(d*x)^(3/2)*Log[1 - a*x])/(9*d) + (2
*(d*x)^(3/2)*PolyLog[2, a*x])/(3*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\,\text{Li}_2(ax)\,dx &= \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d} + \frac{2}{3}\int \sqrt{dx}\log(1-ax)\,dx \\
&= \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d} + \frac{(4a)\int \frac{(dx)^{3/2}}{1-ax}\,dx}{9d} \\
&= -\frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d} + \frac{4}{9}\int \frac{\sqrt{dx}}{1-ax}\,dx \\
&= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d} + \frac{(4d)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{9a} \\
&= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d} + \frac{8\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{9a} \\
&= -\frac{8\sqrt{dx}}{9a} - \frac{8(dx)^{3/2}}{27d} + \frac{8\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/2}} + \frac{4(dx)^{3/2}\log(1-ax)}{9d} + \frac{2(dx)^{3/2}\text{Li}_2(ax)}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.0816363, size = 75, normalized size = 0.74

$$\frac{2\sqrt{dx}\left(9x^{3/2}\text{PolyLog}(2,ax)+\frac{12\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}}+\frac{2\sqrt{x}(-2ax+3ax\log(1-ax)-6)}{a}\right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x],x]
```

```
[Out] (2*Sqrt[d*x]*((12*ArcTanh[Sqrt[a]*Sqrt[x]])/a^(3/2) + (2*Sqrt[x]*(-6 - 2*a*
x + 3*a*x*Log[1 - a*x]))/a + 9*x^(3/2)*PolyLog[2, a*x]))/(27*Sqrt[x])
```

Maple [A] time = 0.05, size = 83, normalized size = 0.8

$$\frac{2\,\text{polylog}\,(2,ax)}{3\,d}(dx)^{\frac{3}{2}}+\frac{4}{9\,d}(dx)^{\frac{3}{2}}\ln\left(\frac{-adx+d}{d}\right)-\frac{8}{27\,d}(dx)^{\frac{3}{2}}-\frac{8}{9\,a}\sqrt{dx}+\frac{8\,d}{9\,a}\text{Artanh}\left(a\sqrt{dx}\frac{1}{\sqrt{ad}}\right)\frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x),x)
```

```
[Out] 2/3*(d*x)^(3/2)*polylog(2,a*x)/d+4/9/d*(d*x)^(3/2)*ln((-a*d*x+d)/d)-8/27*(d
*x)^(3/2)/d-8/9*(d*x)^(1/2)/a+8/9*d/a/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*
d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56879, size = 362, normalized size = 3.55

$$\left[\frac{2\left((9\,ax\mathrm{Li}_2\,(ax) + 6\,ax\log\,(-ax+1) - 4\,ax - 12)\sqrt{dx} + 6\,\sqrt{\frac{d}{a}}\log\left(\frac{adx+2\,\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)\right)}{27\,a}, \frac{2\left((9\,ax\mathrm{Li}_2\,(ax) + 6\,ax\log\,(-ax+1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] [2/27*((9*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) + 6*
sqrt(d/a)*log((a*d*x + 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)))/a, 2/27*((9
*a*x*dilog(a*x) + 6*a*x*log(-a*x + 1) - 4*a*x - 12)*sqrt(d*x) - 12*sqrt(-d/
a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\mathrm{Li}_2\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x), x)
```

3.61 $\int \frac{\textbf{PolyLog}(2,ax)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=80

$$\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}+\frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}-\frac{8\sqrt{dx}}{d}$$

```
[Out] (-8*Sqrt[d*x])/d + (8*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[d
]) + (4*Sqrt[d*x]*Log[1 - a*x])/d + (2*Sqrt[d*x]*PolyLog[2, a*x])/d
```

Rubi [A] time = 0.0458829, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$\frac{2\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}+\frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}-\frac{8\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/Sqrt[d*x],x]
```

```
[Out] (-8*Sqrt[d*x])/d + (8*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[d
]) + (4*Sqrt[d*x]*Log[1 - a*x])/d + (2*Sqrt[d*x]*PolyLog[2, a*x])/d
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+2\int \frac{\log(1-ax)}{\sqrt{dx}}\,dx\\
&= \frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{(4a)\int \frac{\sqrt{dx}}{1-ax}\,dx}{d}\\
&= -\frac{8\sqrt{dx}}{d}+\frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+4\int \frac{1}{\sqrt{dx}(1-ax)}\,dx\\
&= -\frac{8\sqrt{dx}}{d}+\frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}+\frac{8\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}\\
&= -\frac{8\sqrt{dx}}{d}+\frac{8\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}+\frac{4\sqrt{dx}\log(1-ax)}{d}+\frac{2\sqrt{dx}\,\text{Li}_2(ax)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.078857, size = 63, normalized size = 0.79

$$\frac{2\sqrt{ax}\text{PolyLog}(2,ax)+4\sqrt{ax}(\log(1-ax)-2)+8\sqrt{x}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/Sqrt[d*x],x]
```

```
[Out] (8*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Sqrt[a]*x*(-2 + Log[1 - a*x]) + 2*S
qrt[a]*x*PolyLog[2, a*x])/(Sqrt[a]*Sqrt[d*x])
```

Maple [A] time = 0.05, size = 69, normalized size = 0.9

$$2\,\frac{\sqrt{dx}\text{polylog}\left(2,ax\right)}{d}+4\,\frac{\sqrt{dx}}{d}\ln\left(\frac{-adx+d}{d}\right)-8\,\frac{\sqrt{dx}}{d}+8\,\frac{1}{\sqrt{ad}}\text{Artanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(1/2),x)
```

```
[Out] 2*polylog(2,a*x)*(d*x)^(1/2)/d+4/d*(d*x)^(1/2)*ln((-a*d*x+d)/d)-8*(d*x)^(1/
2)/d+8/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.61497, size = 331, normalized size = 4.14

$$\left[\frac{2\left(\sqrt{dx}(a\text{Li}_2(ax)+2\,a\log(-ax+1)-4\,a)+2\,\sqrt{ad}\log\left(\frac{adx+2\,\sqrt{ad}\sqrt{dx}+d}{ax-1}\right)\right)}{ad},\frac{2\left(\sqrt{dx}(a\text{Li}_2(ax)+2\,a\log(-ax+1)-4\,a)-\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(d*x)*(a*dilog(a*x) + 2*a*log(-a*x + 1) - 4*a) + 2*sqrt(a*d)*log((a
*d*x + 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)))/(a*d), 2*(sqrt(d*x)*(a*dilog(
a*x) + 2*a*log(-a*x + 1) - 4*a) - 4*sqrt(-a*d)*arctan(sqrt(-a*d)*sqrt(d*x)/
(a*d*x)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/sqrt(d*x), x)
```

3.62 $\int \frac{\textbf{PolyLog}(2,ax)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\text{PolyLog}(2,ax)}{d\sqrt{dx}}+\frac{8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{4\log(1-ax)}{d\sqrt{dx}}$$

```
[Out] (8*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (4*Log[1 - a*x])
/(d*Sqrt[d*x]) - (2*PolyLog[2, a*x])/(d*Sqrt[d*x])
```

Rubi [A] time = 0.0431147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2395, 63, 206}

$$-\frac{2\text{PolyLog}(2,ax)}{d\sqrt{dx}}+\frac{8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{4\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

```
[Out] (8*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (4*Log[1 - a*x])
/(d*Sqrt[d*x]) - (2*PolyLog[2, a*x])/(d*Sqrt[d*x])
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_2(ax)}{(dx)^{3/2}}\,dx &= -\frac{2\mathrm{Li}_2(ax)}{d\sqrt{dx}} - 2\int \frac{\log(1-ax)}{(dx)^{3/2}}\,dx\\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\mathrm{Li}_2(ax)}{d\sqrt{dx}} + \frac{(4a)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{d}\\
&= \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\mathrm{Li}_2(ax)}{d\sqrt{dx}} + \frac{(8a)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2}\\
&= \frac{8\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{4\log(1-ax)}{d\sqrt{dx}} - \frac{2\mathrm{Li}_2(ax)}{d\sqrt{dx}}
\end{aligned}
$$

Mathematica [A] time = 0.0714846, size = 51, normalized size = 0.75

$$
\frac{2x\left(-\mathrm{PolyLog}(2,ax) + 2\log(1-ax) + 4\sqrt{a}\sqrt{x}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{(dx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(4*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 2*Log[1 - a*x] - PolyLog
[2, a*x]))/(d*x)^(3/2)
```

Maple [A] time = 0.053, size = 63, normalized size = 0.9

$$-2\frac{\mathrm{polylog}(2,ax)}{d\sqrt{dx}}+4\frac{1}{d\sqrt{dx}}\ln\left(\frac{-adx+d}{d}\right)+8\frac{a}{d\sqrt{ad}}\mathrm{Artanh}\left(\frac{a\sqrt{dx}}{\sqrt{ad}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(3/2),x)
```

```
[Out] -2*polylog(2,a*x)/d/(d*x)^(1/2)+4/d/(d*x)^(1/2)*ln((-a*d*x+d)/d)+8/d*a/(a*d
)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.71198, size = 316, normalized size = 4.65

$$\left[\frac{2\left(2\,dx\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\,\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\sqrt{dx}(\mathrm{Li}_2\,(ax)-2\,\log\,(-ax+1))\right)}{d^2x},-\frac{2\left(4\,dx\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+\sqrt{dx}(\mathrm{Li}_2\,(ax)-2\right.}{d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*(2*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - sqrt
(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/(d^2*x), -2*(4*d*x*sqrt(-a/d)*arctan(
sqrt(d*x)*sqrt(-a/d)/(a*x)) + sqrt(d*x)*(dilog(a*x) - 2*log(-a*x + 1)))/(d^
```

```
2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(2, a*x)/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(3/2), x)
```

3.63 $\int \frac{\textbf{PolyLog}(2,ax)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}-\frac{8a}{9d^2\sqrt{dx}}+\frac{4\log(1-ax)}{9d(dx)^{3/2}}$$

```
[Out] (-8*a)/(9*d^2*Sqrt[d*x]) + (8*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])
/(9*d^(5/2)) + (4*Log[1 - a*x])/(9*d*(d*x)^(3/2)) - (2*PolyLog[2, a*x])/(3*
d*(d*x)^(3/2))
```

Rubi [A] time = 0.0504307, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{2\text{PolyLog}(2,ax)}{3d(dx)^{3/2}}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}-\frac{8a}{9d^2\sqrt{dx}}+\frac{4\log(1-ax)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-8*a)/(9*d^2*Sqrt[d*x]) + (8*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])
/(9*d^(5/2)) + (4*Log[1 - a*x])/(9*d*(d*x)^(3/2)) - (2*PolyLog[2, a*x])/(3*
d*(d*x)^(3/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_2(ax)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} - \frac{2}{3}\int \frac{\log(1-ax)}{(dx)^{5/2}}\,dx \\
&= \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{(4a)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{9d} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{\left(4a^2\right)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{9d^2} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}} + \frac{\left(8a^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{9d^3} \\
&= -\frac{8a}{9d^2\sqrt{dx}} + \frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{4\log(1-ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax)}{3d(dx)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.0741523, size = 57, normalized size = 0.64

$$-\frac{2x\left(3\text{PolyLog}(2,ax)-4a^{3/2}x^{3/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)+4ax-2\log(1-ax)\right)}{9(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(4*a*x - 4*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 2*Log[1 - a*x]
+ 3*PolyLog[2, a*x]))/(9*(d*x)^(5/2))
```

Maple [A] time = 0.055, size = 76, normalized size = 0.9

$$-\frac{2\,\mathrm{polylog}\left(2,ax\right)}{3\,d}(dx)^{-\frac{3}{2}}+\frac{4}{9\,d}\ln\left(\frac{-adx+d}{d}\right)(dx)^{-\frac{3}{2}}-\frac{8\,a}{9\,d^{2}}\frac{1}{\sqrt{dx}}+\frac{8\,a^{2}}{9\,d^{2}}\mathrm{Artanh}\left(a\sqrt{dx}\frac{1}{\sqrt{ad}}\right)\frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(5/2),x)
```

```
[Out] -2/3*polylog(2,a*x)/d/(d*x)^(3/2)+4/9/d/(d*x)^(3/2)*ln((-a*d*x+d)/d)-8/9*a/
d^2/(d*x)^(1/2)+8/9/d^2*a^2/(a*d)^(1/2)*arctanh(a*(d*x)^(1/2)/(a*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.66517, size = 365, normalized size = 4.1

$$\left[\frac{2\left(2\,adx^2\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-(4\,ax+3\,\mathrm{Li}_2\,(ax)-2\,\log\,(-ax+1))\sqrt{dx}\right)}{9\,d^3x^2},-\frac{2\left(4\,adx^2\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{a}{d}}}{ax}\right)+(4\,ax\right.}{9\,d^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1))
- (4*a*x + 3*dilog(a*x) - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2), -2/9*(4*a*
d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + (4*a*x + 3*dilog(a*x)
 - 2*log(-a*x + 1))*sqrt(d*x))/(d^3*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\,(ax)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(5/2), x)
```

3.64 $\int \frac{\textbf{PolyLog}(2,ax)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}-\frac{8a^2}{25d^3\sqrt{dx}}+\frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{8a}{75d^2(dx)^{3/2}}+\frac{4\log(1-ax)}{25d(dx)^{5/2}}$$

```
[Out] (-8*a)/(75*d^2*(d*x)^(3/2)) - (8*a^2)/(25*d^3*Sqrt[d*x]) + (8*a^(5/2)*ArcTa
nh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*d^(7/2)) + (4*Log[1 - a*x])/(25*d*(d*x
)^(5/2)) - (2*PolyLog[2, a*x])/(5*d*(d*x)^(5/2))
```

Rubi [A] time = 0.057124, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{2\text{PolyLog}(2,ax)}{5d(dx)^{5/2}}-\frac{8a^2}{25d^3\sqrt{dx}}+\frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{8a}{75d^2(dx)^{3/2}}+\frac{4\log(1-ax)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x]/(d*x)^(7/2),x]
```


```
[Out] (-8*a)/(75*d^2*(d*x)^(3/2)) - (8*a^2)/(25*d^3*Sqrt[d*x]) + (8*a^(5/2)*ArcTa
nh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(25*d^(7/2)) + (4*Log[1 - a*x])/(25*d*(d*x
)^(5/2)) - (2*PolyLog[2, a*x])/(5*d*(d*x)^(5/2))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{Li}_2(ax)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} - \frac{2}{5}\int \frac{\log(1-ax)}{(dx)^{7/2}}\,dx$$

$$= \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{(4a)\int \frac{1}{(dx)^{5/2}(1-ax)}\,dx}{25d}$$

$$= -\frac{8a}{75d^2(dx)^{3/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{\left(4a^2\right)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{25d^2}$$

$$= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{\left(4a^3\right)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{25d^3}$$

$$= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}} + \frac{\left(8a^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{25d^4}$$

$$= -\frac{8a}{75d^2(dx)^{3/2}} - \frac{8a^2}{25d^3\sqrt{dx}} + \frac{8a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{4\log(1-ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax)}{5d(dx)^{5/2}}$$

Mathematica [A] time = 0.0946373, size = 65, normalized size = 0.61

$$-\frac{2x\left(15\text{PolyLog}(2,ax) + 12a^2x^2 - 12a^{5/2}x^{5/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right) + 4ax - 6\log(1-ax)\right)}{75(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*(4*a*x + 12*a^2*x^2 - 12*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 6
*Log[1 - a*x] + 15*PolyLog[2, a*x]))/(75*(d*x)^(7/2))
```

Maple [A] time = 0.054, size = 89, normalized size = 0.8

$$-\frac{2\,\text{polylog}\,(2,ax)}{5\,d}(dx)^{-\frac{5}{2}} + \frac{4}{25\,d}\ln\left(\frac{-adx+d}{d}\right)(dx)^{-\frac{5}{2}} - \frac{8\,a}{75\,d^2}(dx)^{-\frac{3}{2}} - \frac{8\,a^2}{25\,d^3}\frac{1}{\sqrt{dx}} + \frac{8\,a^3}{25\,d^3}\text{Artanh}\left(a\sqrt{dx}\frac{1}{\sqrt{ad}}\right)\frac{1}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x)/(d*x)^(7/2),x)
```

```
[Out] -2/5*polylog(2,a*x)/d/(d*x)^(5/2)+4/25/d/(d*x)^(5/2)*ln((-a*d*x+d)/d)-8/75*
a/d^2/(d*x)^(3/2)-8/25*a^2/d^3/(d*x)^(1/2)+8/25/d^3*a^3/(a*d)^(1/2)*arctanh
(a*(d*x)^(1/2)/(a*d)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.59325, size = 412, normalized size = 3.89

$$\left[\frac{2\left(6\,a^2dx^3\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\,\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-\left(12\,a^2x^2+4\,ax+15\,\mathrm{Li}_2\left(ax\right)-6\,\log\left(-ax+1\right)\right)\sqrt{dx}\right)}{75\,d^4x^3},-\frac{2\left(12\,a^2dx^3\sqrt{-\frac{a}{d}}\arctan\left(\frac{\sqrt{d\cdots}}{\cdots}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [2/75*(6*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1
)) - (12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d^4
*x^3), -2/75*(12*a^2*d*x^3*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) +
(12*a^2*x^2 + 4*a*x + 15*dilog(a*x) - 6*log(-a*x + 1))*sqrt(d*x))/(d^4*x^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(ax)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x)/(d*x)^(7/2), x)
```

3.65 $\int (dx)^{5/2}\textbf{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=153

$$-\frac{4(dx)^{7/2}\text{PolyLog}(2,ax)}{49d}+\frac{2(dx)^{7/2}\text{PolyLog}(3,ax)}{7d}+\frac{16d^2\sqrt{dx}}{343a^3}-\frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}}+\frac{16d(dx)^{3/2}}{1029a^2}+\frac{16(dx)^{5/2}}{1715a}-8($$

```
[Out] (16*d^2*Sqrt[d*x])/(343*a^3) + (16*d*(d*x)^(3/2))/(1029*a^2) + (16*(d*x)^(5
/2))/(1715*a) + (16*(d*x)^(7/2))/(2401*d) - (16*d^(5/2)*ArcTanh[(Sqrt[a]*Sq
rt[d*x])/Sqrt[d]])/(343*a^(7/2)) - (8*(d*x)^(7/2)*Log[1 - a*x])/(343*d) - (
4*(d*x)^(7/2)*PolyLog[2, a*x])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3, a*x])/(7*
d)
```

Rubi [A] time = 0.0982584, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$-\frac{4(dx)^{7/2}\text{PolyLog}(2,ax)}{49d}+\frac{2(dx)^{7/2}\text{PolyLog}(3,ax)}{7d}+\frac{16d^2\sqrt{dx}}{343a^3}-\frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}}+\frac{16d(dx)^{3/2}}{1029a^2}+\frac{16(dx)^{5/2}}{1715a}-8($$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*PolyLog[3, a*x],x]
```


```
[Out] (16*d^2*Sqrt[d*x])/(343*a^3) + (16*d*(d*x)^(3/2))/(1029*a^2) + (16*(d*x)^(5
/2))/(1715*a) + (16*(d*x)^(7/2))/(2401*d) - (16*d^(5/2)*ArcTanh[(Sqrt[a]*Sq
rt[d*x])/Sqrt[d]])/(343*a^(7/2)) - (8*(d*x)^(7/2)*Log[1 - a*x])/(343*d) - (
4*(d*x)^(7/2)*PolyLog[2, a*x])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3, a*x])/(7*
d)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (dx)^{5/2}\mathrm{Li}_3(ax)\,dx = \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{2}{7}\int (dx)^{5/2}\mathrm{Li}_2(ax)\,dx$$
$$= -\frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{4}{49}\int (dx)^{5/2}\log(1-ax)\,dx$$
$$= -\frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{(8a)\int \frac{(dx)^{7/2}}{1-ax}\,dx}{343d}$$
$$= \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{8}{343}\int \frac{(dx)^{5/2}}{1-ax}\,dx$$
$$= \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} - \frac{(8d)\int \frac{(dx)^{3/2}}{1-ax}\,d}{343a}$$
$$= \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/2}\mathrm{Li}_3(ax)}{7d} -$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/}}{}$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} - \frac{4(dx)^{7/2}\mathrm{Li}_2(ax)}{49d} + \frac{2(dx)^{7/}}{}$$
$$= \frac{16d^2\sqrt{dx}}{343a^3} + \frac{16d(dx)^{3/2}}{1029a^2} + \frac{16(dx)^{5/2}}{1715a} + \frac{16(dx)^{7/2}}{2401d} - \frac{16d^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/2}} - \frac{8(dx)^{7/2}\log(1-ax)}{343d} -$$

<u>Mathematica</u> [A] time = 0.260023, size = 98, normalized size = 0.64

$$\frac{2(dx)^{5/2}\left(-1470x^3\mathrm{PolyLog}(2,ax) + 5145x^3\mathrm{PolyLog}(3,ax) + \frac{8(15a^3x^3+21a^2x^2+35ax+105)}{a^3} - \frac{840\tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{7/2}\sqrt{x}} - 420x^3\log(1-\right.}{36015x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(5/2)*PolyLog[3, a*x],x]
```

```
[Out] (2*(d*x)^(5/2)*((8*(105 + 35*a*x + 21*a^2*x^2 + 15*a^3*x^3))/a^3 - (840*Arc
Tanh[Sqrt[a]*Sqrt[x]])/(a^(7/2)*Sqrt[x]) - 420*x^3*Log[1 - a*x] - 1470*x^3*
PolyLog[2, a*x] + 5145*x^3*PolyLog[3, a*x]))/(36015*x^2)
```

Maple [A] time = 0.182, size = 149, normalized size = 1.

$$\frac{1}{a}(dx)^{\frac{5}{2}}\left(\frac{720\,x^3a^3+1008\,a^2x^2+1680\,ax+5040}{108045\,a^4}\sqrt{x}\,(-a)^{\frac{9}{2}}+\frac{8}{343\,a^4}\sqrt{x}\,(-a)^{\frac{9}{2}}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)\frac{1}{\sqrt{ax}}-\frac{8\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3,a*x),x)
```

```
[Out] (d*x)^(5/2)/x^(5/2)/(-a)^(5/2)/a*(2/108045*x^(1/2)*(-a)^(9/2)*(360*a^3*x^3+
504*a^2*x^2+840*a*x+2520)/a^4+8/343*x^(1/2)*(-a)^(9/2)/a^4/(a*x)^(1/2)*(ln(
1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8/343*x^(7/2)*(-a)^(9/2)/a*ln(-a*x+1)-4/4
9*x^(7/2)*(-a)^(9/2)*polylog(2,a*x)/a+2/7*x^(7/2)*(-a)^(9/2)/a*polylog(3,a*
x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.97483, size = 822, normalized size = 5.37

$$\left[-\frac{2\left(1470\,\sqrt{dx}a^3d^2x^3\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-5145\,\sqrt{dx}a^3d^2x^3\mathrm{polylog}\,(3,ax)-420\,d^2\sqrt{\frac{d}{a}}\log\left(\frac{adx-2\,\sqrt{dx}a}{ax-1}\right.\right.}{36015\,a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [-2/36015*(1470*sqrt(d*x)*a^3*d^2*x^3*\%iint(a, x, -log(-a*x + 1)/a, -log(-a
*x + 1)/x) - 5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) - 420*d^2*sqrt(d/a)
```

```
*log((a*d*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) + 4*(105*a^3*d^2*x^3*
log(-a*x + 1) - 30*a^3*d^2*x^3 - 42*a^2*d^2*x^2 - 70*a*d^2*x - 210*d^2)*sqr
t(d*x))/a^3, -2/36015*(1470*sqrt(d*x)*a^3*d^2*x^3*\%iint(a, x, -log(-a*x + 1
)/a, -log(-a*x + 1)/x) - 5145*sqrt(d*x)*a^3*d^2*x^3*polylog(3, a*x) - 840*d
^2*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d) + 4*(105*a^3*d^2*x^3*log(-a*
x + 1) - 30*a^3*d^2*x^3 - 42*a^2*d^2*x^2 - 70*a*d^2*x - 210*d^2)*sqrt(d*x))
/a^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*polylog(3,a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \operatorname{Li}_3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*polylog(3, a*x), x)
```

3.66 $\int (dx)^{3/2}\text{PolyLog}(3,ax)\,dx$

Optimal. Leaf size=136

$$-\frac{4(dx)^{5/2}\text{PolyLog}(2,ax)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(3,ax)}{5d}-\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}}+\frac{16d\sqrt{dx}}{125a^2}+\frac{16(dx)^{3/2}}{375a}-\frac{8(dx)^{5/2}\log(1-$$

```
[Out] (16*d*Sqrt[d*x])/(125*a^2) + (16*(d*x)^(3/2))/(375*a) + (16*(d*x)^(5/2))/(6
25*d) - (16*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/2)) - (
8*(d*x)^(5/2)*Log[1 - a*x])/(125*d) - (4*(d*x)^(5/2)*PolyLog[2, a*x])/(25*d
) + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

Rubi [A] time = 0.0834981, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$-\frac{4(dx)^{5/2}\text{PolyLog}(2,ax)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(3,ax)}{5d}-\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}}+\frac{16d\sqrt{dx}}{125a^2}+\frac{16(dx)^{3/2}}{375a}-\frac{8(dx)^{5/2}\log(1-$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

```
[Out] (16*d*Sqrt[d*x])/(125*a^2) + (16*(d*x)^(3/2))/(375*a) + (16*(d*x)^(5/2))/(6
25*d) - (16*d^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/2)) - (
8*(d*x)^(5/2)*Log[1 - a*x])/(125*d) - (4*(d*x)^(5/2)*PolyLog[2, a*x])/(25*d
) + (2*(d*x)^(5/2)*PolyLog[3, a*x])/(5*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_3(ax)\,dx = \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{2}{5}\int (dx)^{3/2}\text{Li}_2(ax)\,dx$$
$$= -\frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{4}{25}\int (dx)^{3/2}\log(1-ax)\,dx$$
$$= -\frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{(8a)\int \frac{(dx)^{5/2}}{1-ax}\,dx}{125d}$$
$$= \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{8}{125}\int \frac{(dx)^{3/2}}{1-ax}\,dx$$
$$= \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{(8d)\int \frac{\sqrt{dx}}{1-ax}\,d}{125a}$$
$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{(}{}$$
$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(ax)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3(ax)}{5d} - \frac{(}{}$$
$$= \frac{16d\sqrt{dx}}{125a^2} + \frac{16(dx)^{3/2}}{375a} + \frac{16(dx)^{5/2}}{625d} - \frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/2}} - \frac{8(dx)^{5/2}\log(1-ax)}{125d} - \frac{4(dx)^{5/2}\text{Li}_2(}{25d}$$

Mathematica [A] time = 0.217824, size = 88, normalized size = 0.65

$$\frac{2d\sqrt{dx}\left(-150x^2\text{PolyLog}(2,ax) + 375x^2\text{PolyLog}(3,ax) + 4\left(-\frac{30\tanh^{-1}(\sqrt{a}\sqrt{x})}{a^{5/2}\sqrt{x}} + \frac{30}{a^2} - 15x^2\log(1-ax) + \frac{10x}{a} + 6x^2\right)\right)}{1875}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x],x]
```

```
[Out] (2*d*Sqrt[d*x]*(4*(30/a^2 + (10*x)/a + 6*x^2 - (30*ArcTanh[Sqrt[a]*Sqrt[x]]
)/(a^(5/2)*Sqrt[x]) - 15*x^2*Log[1 - a*x]) - 150*x^2*PolyLog[2, a*x] + 375*
x^2*PolyLog[3, a*x]))/1875
```

Maple [A] time = 0.061, size = 141, normalized size = 1.

$$\frac{1}{a}(dx)^{\frac{3}{2}}\left(\frac{336a^2x^2 + 560ax + 1680}{13125a^3}\sqrt{x}(-a)^{\frac{7}{2}} + \frac{8}{125a^3}\sqrt{x}(-a)^{\frac{7}{2}}\left(\ln\left(1-\sqrt{ax}\right) - \ln\left(1+\sqrt{ax}\right)\right)\frac{1}{\sqrt{ax}} - \frac{8\ln(-ax+1)}{125a}x^{\frac{5}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x),x)
```

```
[Out] (d*x)^(3/2)/x^(3/2)/(-a)^(3/2)/a*(2/13125*x^(1/2)*(-a)^(7/2)*(168*a^2*x^2+2
80*a*x+840)/a^3+8/125*x^(1/2)*(-a)^(7/2)/a^3/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))
-ln(1+(a*x)^(1/2)))-8/125*x^(5/2)*(-a)^(7/2)/a*ln(-a*x+1)-4/25*x^(5/2)*(-a)
^(7/2)*polylog(2,a*x)/a+2/5*x^(5/2)*(-a)^(7/2)/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.87358, size = 721, normalized size = 5.3

$$\left[-\frac{2\left(150\sqrt{dx}a^2dx^2\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-375\sqrt{dx}a^2dx^2\mathrm{polylog}\left(3,ax\right)-60\,d\sqrt{\frac{d}{a}}\log\left(\frac{adx-2\sqrt{dx}a\sqrt{\frac{d}{a}}+d}{ax-1}\right)+\right.}{1875\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [-2/1875*(150*sqrt(d*x)*a^2*d*x^2*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x +
 1)/x) - 375*sqrt(d*x)*a^2*d*x^2*polylog(3, a*x) - 60*d*sqrt(d/a)*log((a*d*
x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)) + 4*(15*a^2*d*x^2*log(-a*x + 1)
 - 6*a^2*d*x^2 - 10*a*d*x - 30*d)*sqrt(d*x))/a^2, -2/1875*(150*sqrt(d*x)*a^
2*d*x^2*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 375*sqrt(d*x)*a^2
*d*x^2*polylog(3, a*x) - 120*d*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d)
+ 4*(15*a^2*d*x^2*log(-a*x + 1) - 6*a^2*d*x^2 - 10*a*d*x - 30*d)*sqrt(d*x))
/a^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x),x)
```

```
[Out] Integral((d*x)**(3/2)*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x), x)
```

3.67 $\int \sqrt{dx}\,\mathbf{PolyLog}(3, ax)\,dx$

Optimal. Leaf size=121

$$-\frac{4(dx)^{3/2}\text{PolyLog}(2,ax)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}(3,ax)}{3d}-\frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}}+\frac{16\sqrt{dx}}{27a}-\frac{8(dx)^{3/2}\log(1-ax)}{27d}+\frac{16(dx)^{3/}}{81d}$$

```
[Out] (16*Sqrt[d*x])/(27*a) + (16*(d*x)^(3/2))/(81*d) - (16*Sqrt[d]*ArcTanh[(Sqrt
[a]*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/2)) - (8*(d*x)^(3/2)*Log[1 - a*x])/(27*d)
 - (4*(d*x)^(3/2)*PolyLog[2, a*x])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x])/
(3*d)
```

Rubi [A] time = 0.066747, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$-\frac{4(dx)^{3/2}\text{PolyLog}(2,ax)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}(3,ax)}{3d}-\frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}}+\frac{16\sqrt{dx}}{27a}-\frac{8(dx)^{3/2}\log(1-ax)}{27d}+\frac{16(dx)^{3/}}{81d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x],x]
```

```
[Out] (16*Sqrt[d*x])/(27*a) + (16*(d*x)^(3/2))/(81*d) - (16*Sqrt[d]*ArcTanh[(Sqrt
[a]*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/2)) - (8*(d*x)^(3/2)*Log[1 - a*x])/(27*d)
 - (4*(d*x)^(3/2)*PolyLog[2, a*x])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x])/
(3*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\mathrm{Li}_3(ax)\,dx &= \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{2}{3}\int \sqrt{dx}\mathrm{Li}_2(ax)\,dx \\
&= -\frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{4}{9}\int \sqrt{dx}\log(1-ax)\,dx \\
&= -\frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{(8a)\int \frac{(dx)^{3/2}}{1-ax}\,dx}{27d} \\
&= \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{8}{27}\int \frac{\sqrt{dx}}{1-ax}\,dx \\
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{(8d)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{27a} \\
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax)}{3d} - \frac{16\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\right.}{27a} \\
&= \frac{16\sqrt{dx}}{27a} + \frac{16(dx)^{3/2}}{81d} - \frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/2}} - \frac{8(dx)^{3/2}\log(1-ax)}{27d} - \frac{4(dx)^{3/2}\mathrm{Li}_2(ax)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(}{3d}
\end{aligned}
$$

Mathematica [A] time = 0.188443, size = 73, normalized size = 0.6

$$
\frac{2}{81}\sqrt{dx}\left(-18x\mathrm{PolyLog}(2,ax) + 27x\mathrm{PolyLog}(3,ax) + 4\left(-\frac{6\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{a^{3/2}\sqrt{x}} - 3x\log(1-ax) + \frac{6}{a} + 2x\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x],x]
```

```
[Out] (2*Sqrt[d*x]*(4*(6/a + 2*x - (6*ArcTanh[Sqrt[a]*Sqrt[x]])/(a^(3/2)*Sqrt[x])
 - 3*x*Log[1 - a*x]) - 18*x*PolyLog[2, a*x] + 27*x*PolyLog[3, a*x]))/81
```

Maple [A] time = 0.058, size = 133, normalized size = 1.1

$$
\frac{1}{a}\sqrt{dx}\left(\frac{80\,ax+240}{405\,a^2}\sqrt{x}\,(-a)^{\frac{5}{2}} + \frac{8}{27\,a^2}\sqrt{x}\,(-a)^{\frac{5}{2}}\left(\ln\left(1-\sqrt{ax}\right) - \ln\left(1+\sqrt{ax}\right)\right)\frac{1}{\sqrt{ax}} - \frac{8\,\ln(-ax+1)}{27\,a}x^{\frac{3}{2}}(-a)^{\frac{5}{2}} - \frac{4\,\mathrm{polylog}}{9\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x),x)
```

```
[Out] (d*x)^(1/2)/x^(1/2)/(-a)^(1/2)/a*(2/405*x^(1/2)*(-a)^(5/2)*(40*a*x+120)/a^2
+8/27*x^(1/2)*(-a)^(5/2)/a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2
)))-8/27*x^(3/2)*(-a)^(5/2)/a*ln(-a*x+1)-4/9*x^(3/2)*(-a)^(5/2)*polylog(2,a
*x)/a+2/3*x^(3/2)*(-a)^(5/2)/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.8478, size = 593, normalized size = 4.9

$$\left[-\frac{2\left(18\sqrt{dx}ax\%\text{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-27\sqrt{dx}ax\text{polylog}\left(3,ax\right)+4\left(3\,ax\log\left(-ax+1\right)-2\,ax-6\right)\sqrt{dx}-\right.}{81\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] [-2/81*(18*sqrt(d*x)*a*x*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) -
27*sqrt(d*x)*a*x*polylog(3, a*x) + 4*(3*a*x*log(-a*x + 1) - 2*a*x - 6)*sqrt
(d*x) - 12*sqrt(d/a)*log((a*d*x - 2*sqrt(d*x)*a*sqrt(d/a) + d)/(a*x - 1)))/
a, -2/81*(18*sqrt(d*x)*a*x*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x)
- 27*sqrt(d*x)*a*x*polylog(3, a*x) + 4*(3*a*x*log(-a*x + 1) - 2*a*x - 6)*sq
rt(d*x) - 24*sqrt(-d/a)*arctan(sqrt(d*x)*a*sqrt(-d/a)/d))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx}\,\text{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,\mathrm{Li}_3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x), x)
```

3.68 $\int \frac{\text{PolyLog}(3,ax)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=97

$$-\frac{4\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-\frac{8\sqrt{dx}\log(1-ax)}{d}-\frac{16\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}+\frac{16\sqrt{dx}}{d}$$

```
[Out] (16*Sqrt[d*x])/d - (16*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[
d]) - (8*Sqrt[d*x]*Log[1 - a*x])/d - (4*Sqrt[d*x]*PolyLog[2, a*x])/d + (2*S
qrt[d*x]*PolyLog[3, a*x])/d
```

Rubi [A] time = 0.0621491, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 50, 63, 206}

$$-\frac{4\sqrt{dx}\,\text{PolyLog}(2,ax)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}(3,ax)}{d}-\frac{8\sqrt{dx}\log(1-ax)}{d}-\frac{16\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}}+\frac{16\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/Sqrt[d*x],x]
```

```
[Out] (16*Sqrt[d*x])/d - (16*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[a]*Sqrt[
d]) - (8*Sqrt[d*x]*Log[1 - a*x])/d - (4*Sqrt[d*x]*PolyLog[2, a*x])/d + (2*S
qrt[d*x]*PolyLog[3, a*x])/d
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 2\int \frac{\text{Li}_2(ax)}{\sqrt{dx}}\,dx$$
$$= -\frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 4\int \frac{\log(1-ax)}{\sqrt{dx}}\,dx$$
$$= -\frac{8\sqrt{dx}\log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - \frac{(8a)\int \frac{\sqrt{dx}}{1-ax}\,dx}{d}$$
$$= \frac{16\sqrt{dx}}{d} - \frac{8\sqrt{dx}\log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - 8\int \frac{1}{\sqrt{dx}(1-ax)}\,dx$$
$$= \frac{16\sqrt{dx}}{d} - \frac{8\sqrt{dx}\log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d} - \frac{16\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{d}$$
$$= \frac{16\sqrt{dx}}{d} - \frac{16\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{a}\sqrt{d}} - \frac{8\sqrt{dx}\log(1-ax)}{d} - \frac{4\sqrt{dx}\text{Li}_2(ax)}{d} + \frac{2\sqrt{dx}\text{Li}_3(ax)}{d}$$

Mathematica [A] time = 0.138969, size = 57, normalized size = 0.59

$$\frac{2x\left(-2\text{PolyLog}(2,ax) + \text{PolyLog}(3,ax) - 4\log(1-ax) - \frac{8\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{x}} + 8\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/Sqrt[d*x],x]
```

```
[Out] (2*x*(8 - (8*ArcTanh[Sqrt[a]*Sqrt[x]])/(Sqrt[a]*Sqrt[x]) - 4*Log[1 - a*x] -
 2*PolyLog[2, a*x] + PolyLog[3, a*x]))/Sqrt[d*x]
```

Maple [A] time = 0.057, size = 127, normalized size = 1.3

$$\frac{1}{a}\sqrt{x}\sqrt{-a}\left(16\,\frac{\sqrt{x}\,(-a)^{3/2}}{a} + 8\,\frac{\sqrt{x}\,(-a)^{3/2}\left(\ln\left(1-\sqrt{ax}\right) - \ln\left(1+\sqrt{ax}\right)\right)}{a\sqrt{ax}} - 8\,\frac{\sqrt{x}\,(-a)^{3/2}\ln(-ax+1)}{a} - 4\,\frac{\sqrt{x}\,(-a)^{3/2}\,\text{polyl}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(1/2),x)
```

```
[Out] 1/(d*x)^(1/2)*x^(1/2)*(-a)^(1/2)/a*(16*x^(1/2)*(-a)^(3/2)/a+8*x^(1/2)*(-a)^
(3/2)/a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))-8*x^(1/2)*(-a)^(3
/2)/a*ln(-a*x+1)-4*x^(1/2)*(-a)^(3/2)*polylog(2,a*x)/a+2*x^(1/2)*(-a)^(3/2)
/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.72649, size = 547, normalized size = 5.64

$$\left[-\frac{2\left(2\sqrt{dx}a\%\text{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-\sqrt{dx}a\text{polylog}(3,ax)+4\sqrt{dx}(a\log(-ax+1)-2a)-4\sqrt{ad}\log\left(\frac{adx-2}{\ldots}\right.\right.}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-2*(2*sqrt(d*x)*a*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - sqrt(d
*x)*a*polylog(3, a*x) + 4*sqrt(d*x)*(a*log(-a*x + 1) - 2*a) - 4*sqrt(a*d)*l
og((a*d*x - 2*sqrt(a*d)*sqrt(d*x) + d)/(a*x - 1)))/(a*d), -2*(2*sqrt(d*x)*a
*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - sqrt(d*x)*a*polylog(3, a
*x) + 4*sqrt(d*x)*(a*log(-a*x + 1) - 2*a) - 8*sqrt(-a*d)*arctan(sqrt(-a*d)*
sqrt(d*x)/(a*d*x)))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/sqrt(d*x), x)
```

3.69 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{4\text{PolyLog}(2,ax)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax)}{d\sqrt{dx}}+\frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log(1-ax)}{d\sqrt{dx}}$$

```
[Out] (16*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x]
)/(d*Sqrt[d*x]) - (4*PolyLog[2, a*x])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x])/(
d*Sqrt[d*x])
```

Rubi [A] time = 0.0588706, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2395, 63, 206}

$$-\frac{4\text{PolyLog}(2,ax)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax)}{d\sqrt{dx}}+\frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log(1-ax)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

```
[Out] (16*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x]
)/(d*Sqrt[d*x]) - (4*PolyLog[2, a*x])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x])/(
d*Sqrt[d*x])
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3(ax)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + 2\int \frac{\text{Li}_2(ax)}{(dx)^{3/2}}\,dx \\
&= -\frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} - 4\int \frac{\log(1-ax)}{(dx)^{3/2}}\,dx \\
&= \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(8a)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{d} \\
&= \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}} + \frac{(16a)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} \\
&= \frac{16\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log(1-ax)}{d\sqrt{dx}} - \frac{4\text{Li}_2(ax)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax)}{d\sqrt{dx}}
\end{aligned}
$$

Mathematica [A] time = 0.101918, size = 58, normalized size = 0.68

$$
\frac{2x\left(-2\text{PolyLog}(2,ax) - \text{PolyLog}(3,ax) + 4\log(1-ax) + 8\sqrt{a}\sqrt{x}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right)\right)}{(dx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(8*Sqrt[a]*Sqrt[x]*ArcTanh[Sqrt[a]*Sqrt[x]] + 4*Log[1 - a*x] - 2*PolyL
og[2, a*x] - PolyLog[3, a*x]))/(d*x)^(3/2)
```

Maple [A] time = 0.056, size = 111, normalized size = 1.3

$$\frac{1}{a}x^{\frac{3}{2}}(-a)^{\frac{3}{2}}\left(-8\,\frac{\sqrt{x}\sqrt{-a}\left(\ln\left(1-\sqrt{ax}\right)-\ln\left(1+\sqrt{ax}\right)\right)}{\sqrt{ax}}+8\,\frac{\sqrt{-a}\ln\left(-ax+1\right)}{\sqrt{x}a}-4\,\frac{\sqrt{-a}\mathrm{polylog}\left(2,ax\right)}{\sqrt{x}a}-2\,\frac{\sqrt{-a}\mathrm{polylog}\left(3,\right.}{\sqrt{x}a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(3/2),x)
```

```
[Out] 1/(d*x)^(3/2)*x^(3/2)*(-a)^(3/2)/a*(-8*x^(1/2)*(-a)^(1/2)/(a*x)^(1/2)*(ln(1
-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/x^(1/2)*(-a)^(1/2)/a*ln(-a*x+1)-4/x^(1/2
)*(-a)^(1/2)*polylog(2,a*x)/a-2/x^(1/2)*(-a)^(1/2)/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.5892, size = 518, normalized size = 6.09

$$\left[\frac{2\left(4\,dx\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\,\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-2\,\sqrt{dx}\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)+4\,\sqrt{dx}\log\left(-ax+1\right)-\sqrt{dx}\mathrm{polylog}\left(3,ax\right)\right)}{d^{2}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [2*(4*d*x*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1)) - 2*sq
rt(d*x)*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) + 4*sqrt(d*x)*log(-
a*x + 1) - sqrt(d*x)*polylog(3, a*x))/(d^2*x), -2*(8*d*x*sqrt(-a/d)*arctan(
sqrt(d*x)*sqrt(-a/d)/(a*x)) + 2*sqrt(d*x)*\%iint(a, x, -log(-a*x + 1)/a, -lo
g(-a*x + 1)/x) - 4*sqrt(d*x)*log(-a*x + 1) + sqrt(d*x)*polylog(3, a*x))/(d^
2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x)/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(3/2), x)
```

3.70 $\int \frac{\text{PolyLog}(3,ax)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=108

$$-\frac{4\text{PolyLog}(2,ax)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(3,ax)}{3d(dx)^{3/2}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}-\frac{16a}{27d^2\sqrt{dx}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}$$

```
[Out] (-16*a)/(27*d^2*Sqrt[d*x]) + (16*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d
]])/(27*d^(5/2)) + (8*Log[1 - a*x])/(27*d*(d*x)^(3/2)) - (4*PolyLog[2, a*x]
)/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x])/(3*d*(d*x)^(3/2))
```

Rubi [A] time = 0.0655397, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{4\text{PolyLog}(2,ax)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}(3,ax)}{3d(dx)^{3/2}}+\frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}-\frac{16a}{27d^2\sqrt{dx}}+\frac{8\log(1-ax)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(5/2),x]
```


```
[Out] (-16*a)/(27*d^2*Sqrt[d*x]) + (16*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d
]])/(27*d^(5/2)) + (8*Log[1 - a*x])/(27*d*(d*x)^(3/2)) - (4*PolyLog[2, a*x]
)/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x])/(3*d*(d*x)^(3/2))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

eQ[q, -1]

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{(dx)^{5/2}} \, dx = -\frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{2}{3} \int \frac{\text{Li}_2(ax)}{(dx)^{5/2}} \, dx$$
$$= -\frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} - \frac{4}{9} \int \frac{\log(1-ax)}{(dx)^{5/2}} \, dx$$
$$= \frac{8\log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{(8a)\int \frac{1}{(dx)^{3/2}(1-ax)} \, dx}{27d}$$
$$= -\frac{16a}{27d^2\sqrt{dx}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{\left(8a^2\right)\int \frac{1}{\sqrt{dx}(1-ax)} \, dx}{27d^2}$$
$$= -\frac{16a}{27d^2\sqrt{dx}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}} + \frac{\left(16a^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}} \, dx, x, \sqrt{dx}\right)}{27d^3}$$
$$= -\frac{16a}{27d^2\sqrt{dx}} + \frac{16a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{8\log(1-ax)}{27d(dx)^{3/2}} - \frac{4\text{Li}_2(ax)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax)}{3d(dx)^{3/2}}$$

Mathematica [A] time = 0.0961699, size = 64, normalized size = 0.59

$$-\frac{2x\left(6\text{PolyLog}(2,ax) + 9\text{PolyLog}(3,ax) - 8a^{3/2}x^{3/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right) + 8ax - 4\log(1-ax)\right)}{27(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(8*a*x - 8*a^(3/2)*x^(3/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 4*Log[1 - a*x]
+ 6*PolyLog[2, a*x] + 9*PolyLog[3, a*x]))/(27*(d*x)^(5/2))
```

Maple [A] time = 0.06, size = 122, normalized size = 1.1

$$\frac{1}{a}x^{\frac{5}{2}}(-a)^{\frac{5}{2}}\left(-\frac{16}{27}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{-a}} - \frac{8a}{27}\sqrt{x}\left(\ln\left(1-\sqrt{ax}\right) - \ln\left(1+\sqrt{ax}\right)\right)\frac{1}{\sqrt{-a}}\frac{1}{\sqrt{ax}} + \frac{8\ln(-ax+1)}{27a}x^{-\frac{3}{2}}\frac{1}{\sqrt{-a}} - \frac{4\,\text{polylog}(2,ax)}{9a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(5/2),x)
```

```
[Out] 1/(d*x)^(5/2)*x^(5/2)*(-a)^(5/2)/a*(-16/27/x^(1/2)/(-a)^(1/2)-8/27*x^(1/2)/
(-a)^(1/2)*a/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))-ln(1+(a*x)^(1/2)))+8/27/x^(3/2)
/(-a)^(1/2)/a*ln(-a*x+1)-4/9/x^(3/2)/(-a)^(1/2)*polylog(2,a*x)/a-2/3/x^(3/2
)/(-a)^(1/2)/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.78928, size = 575, normalized size = 5.32

$$\left[\frac{2\left(4\,adx^2\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\,\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-4\,(2\,ax-\log\left(-ax+1\right))\sqrt{dx}-6\,\sqrt{dx}\%\text{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right)-9\,\sqrt{dx}\text{po}\right.}{27\,d^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [2/27*(4*a*d*x^2*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x - 1))
 - 4*(2*a*x - log(-a*x + 1))*sqrt(d*x) - 6*sqrt(d*x)*\%iint(a, x, -log(-a*x
+ 1)/a, -log(-a*x + 1)/x) - 9*sqrt(d*x)*polylog(3, a*x))/(d^3*x^2), -2/27*(
8*a*d*x^2*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a*x)) + 4*(2*a*x - log(-a
*x + 1))*sqrt(d*x) + 6*sqrt(d*x)*\%iint(a, x, -log(-a*x + 1)/a, -log(-a*x +
1)/x) + 9*sqrt(d*x)*polylog(3, a*x))/(d^3*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{Li}_3\left(ax\right)}{\left(dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(3, a*x)/(d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(5/2), x)
```

3.71 $\int \frac{\textbf{PolyLog}(3,ax)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{4\text{PolyLog}(2,ax)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}-\frac{16a^2}{125d^3\sqrt{dx}}+\frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{16a}{375d^2(dx)^{3/2}}+\frac{8\log(1-ax)}{125d(dx)^{5/2}}$$

```
[Out] (-16*a)/(375*d^2*(d*x)^(3/2)) - (16*a^2)/(125*d^3*Sqrt[d*x]) + (16*a^(5/2)*
ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*d^(7/2)) + (8*Log[1 - a*x])/(125
*d*(d*x)^(5/2)) - (4*PolyLog[2, a*x])/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*
x])/(5*d*(d*x)^(5/2))
```

Rubi [A] time = 0.0763217, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6591, 2395, 51, 63, 206}

$$-\frac{4\text{PolyLog}(2,ax)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}(3,ax)}{5d(dx)^{5/2}}-\frac{16a^2}{125d^3\sqrt{dx}}+\frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{16a}{375d^2(dx)^{3/2}}+\frac{8\log(1-ax)}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-16*a)/(375*d^2*(d*x)^(3/2)) - (16*a^2)/(125*d^3*Sqrt[d*x]) + (16*a^(5/2)*
ArcTanh[(Sqrt[a]*Sqrt[d*x])/Sqrt[d]])/(125*d^(7/2)) + (8*Log[1 - a*x])/(125
*d*(d*x)^(5/2)) - (4*PolyLog[2, a*x])/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*
x])/(5*d*(d*x)^(5/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{Li}_3(ax)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{2}{5}\int \frac{\text{Li}_2(ax)}{(dx)^{7/2}}\,dx$$
$$= -\frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} - \frac{4}{25}\int \frac{\log(1-ax)}{(dx)^{7/2}}\,dx$$
$$= \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a)\int \frac{1}{(dx)^{5/2}(1-ax)}\,dx}{125d}$$
$$= -\frac{16a}{375d^2(dx)^{3/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^2)\int \frac{1}{(dx)^{3/2}(1-ax)}\,dx}{125d^2}$$
$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(8a^3)\int \frac{1}{\sqrt{dx}(1-ax)}\,dx}{125d^3}$$
$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}} + \frac{(16a^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^2}{d}}\,dx, x, \sqrt{\right.}}{125d^4}$$
$$= -\frac{16a}{375d^2(dx)^{3/2}} - \frac{16a^2}{125d^3\sqrt{dx}} + \frac{16a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{8\log(1-ax)}{125d(dx)^{5/2}} - \frac{4\text{Li}_2(ax)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax)}{5d(dx)^{5/2}}$$

Mathematica [A] time = 0.13028, size = 72, normalized size = 0.58

$$-\frac{2x\left(30\text{PolyLog}(2,ax) + 75\text{PolyLog}(3,ax) + 24a^2x^2 - 24a^{5/2}x^{5/2}\tanh^{-1}\left(\sqrt{a}\sqrt{x}\right) + 8ax - 12\log(1-ax)\right)}{375(dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, a*x]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*(8*a*x + 24*a^2*x^2 - 24*a^(5/2)*x^(5/2)*ArcTanh[Sqrt[a]*Sqrt[x]] - 1
2*Log[1 - a*x] + 30*PolyLog[2, a*x] + 75*PolyLog[3, a*x]))/(375*(d*x)^(7/2)
)
```

Maple [A] time = 0.062, size = 135, normalized size = 1.1

$$\frac{1}{a}x^{\frac{7}{2}}(-a)^{\frac{7}{2}}\left(-\frac{16}{375}x^{-\frac{3}{2}}(-a)^{-\frac{3}{2}} - \frac{16a}{125}\frac{1}{\sqrt{x}}(-a)^{-\frac{3}{2}} - \frac{8a^2}{125}\sqrt{x}\left(\ln\left(1-\sqrt{ax}\right) - \ln\left(1+\sqrt{ax}\right)\right)(-a)^{-\frac{3}{2}}\frac{1}{\sqrt{ax}} + \frac{8\ln(-ax+1)}{125a}x^{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x)/(d*x)^(7/2),x)
```

```
[Out] 1/(d*x)^(7/2)*x^(7/2)*(-a)^(7/2)/a*(-16/375/x^(3/2)/(-a)^(3/2)-16/125/x^(1/
2)/(-a)^(3/2)*a-8/125*x^(1/2)/(-a)^(3/2)*a^2/(a*x)^(1/2)*(ln(1-(a*x)^(1/2))
-ln(1+(a*x)^(1/2)))+8/125/x^(5/2)/(-a)^(3/2)/a*ln(-a*x+1)-4/25/x^(5/2)/(-a)
^(3/2)*polylog(2,a*x)/a-2/5/x^(5/2)/(-a)^(3/2)/a*polylog(3,a*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.85994, size = 629, normalized size = 5.03

$$\left[\frac{2\left(12\,a^2dx^3\sqrt{\frac{a}{d}}\log\left(\frac{ax+2\,\sqrt{dx}\sqrt{\frac{a}{d}}+1}{ax-1}\right)-4\left(6\,a^2x^2+2\,ax-3\,\log\left(-ax+1\right)\right)\sqrt{dx}-30\,\sqrt{dx}\%\mathrm{iint}\left(a,x,-\frac{\log(-ax+1)}{a},-\frac{\log(-ax+1)}{x}\right.\right.}{375\,d^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [2/375*(12*a^2*d*x^3*sqrt(a/d)*log((a*x + 2*sqrt(d*x)*sqrt(a/d) + 1)/(a*x -
 1)) - 4*(6*a^2*x^2 + 2*a*x - 3*log(-a*x + 1))*sqrt(d*x) - 30*sqrt(d*x)*%ii
nt(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) - 75*sqrt(d*x)*polylog(3, a*x)
)/(d^4*x^3), -2/375*(24*a^2*d*x^3*sqrt(-a/d)*arctan(sqrt(d*x)*sqrt(-a/d)/(a
*x)) + 4*(6*a^2*x^2 + 2*a*x - 3*log(-a*x + 1))*sqrt(d*x) + 30*sqrt(d*x)*%ii
nt(a, x, -log(-a*x + 1)/a, -log(-a*x + 1)/x) + 75*sqrt(d*x)*polylog(3, a*x)
)/(d^4*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x)/(d*x)^(7/2), x)
```

3.72 $\int (dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)\,dx$

Optimal. Leaf size=140

$$\frac{2(dx)^{5/2}\text{PolyLog}\left(2,ax^2\right)}{5d}+\frac{16d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d}-\frac{32d\sqrt{dx}}{25a}-\frac{32(dx)}{125}$$

```
[Out] (-32*d*Sqrt[d*x])/(25*a) - (32*(d*x)^(5/2))/(125*d) + (16*d^(3/2)*ArcTan[(a
^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/4)) + (16*d^(3/2)*ArcTanh[(a^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(25*a^(5/4)) + (8*(d*x)^(5/2)*Log[1 - a*x^2])/(25*d) + (2
*(d*x)^(5/2)*PolyLog[2, a*x^2])/(5*d)
```

Rubi [A] time = 0.10671, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{5/2}\text{PolyLog}\left(2,ax^2\right)}{5d}+\frac{16d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}}+\frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d}-\frac{32d\sqrt{dx}}{25a}-\frac{32(dx)}{125}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
```


```
[Out] (-32*d*Sqrt[d*x])/(25*a) - (32*(d*x)^(5/2))/(125*d) + (16*d^(3/2)*ArcTan[(a
^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*a^(5/4)) + (16*d^(3/2)*ArcTanh[(a^(1/4)*Sqr
t[d*x])/Sqrt[d]])/(25*a^(5/4)) + (8*(d*x)^(5/2)*Log[1 - a*x^2])/(25*d) + (2
*(d*x)^(5/2)*PolyLog[2, a*x^2])/(5*d)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_2\left(ax^2\right)dx = \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{4}{5}\int (dx)^{3/2}\log\left(1-ax^2\right)dx$$
$$= \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{(16a)\int \frac{x(dx)^{5/2}}{1-ax^2}dx}{25d}$$
$$= \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{(16a)\int \frac{(dx)^{7/2}}{1-ax^2}dx}{25d^2}$$
$$= -\frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{16}{25}\int \frac{(dx)^{3/2}}{1-ax^2}dx$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{\left(16d^2\right)\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}dx}{25a}$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{(32d)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}dx, x\right.}{25a}$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{5d} + \frac{\left(16d^2\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}dx\right.}{25a}$$
$$= -\frac{32d\sqrt{dx}}{25a} - \frac{32(dx)^{5/2}}{125d} + \frac{16d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{16d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25a^{5/4}} + \frac{8(dx)^{5/2}\log\left(1-ax^2\right.}{25d}$$

Mathematica [A] time = 0.104193, size = 101, normalized size = 0.72

$$\frac{2(dx)^{3/2}\left(25x^{5/2}\text{PolyLog}\left(2,ax^2\right) + \frac{4\sqrt[4]{a}\sqrt{x}\left(-4ax^2+5ax^2\log\left(1-ax^2\right)-20\right)+40\tan^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)+40\tanh^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)}{a^{5/4}}\right)}{125x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x^2],x]
```

```
[Out] (2*(d*x)^(3/2)*((40*ArcTan[a^(1/4)*Sqrt[x]] + 40*ArcTanh[a^(1/4)*Sqrt[x]] +
 4*a^(1/4)*Sqrt[x]*(-20 - 4*a*x^2 + 5*a*x^2*Log[1 - a*x^2]))/a^(5/4) + 25*x
^(5/2)*PolyLog[2, a*x^2]))/(125*x^(3/2))
```

Maple [A] time = 0.214, size = 150, normalized size = 1.1

$$\frac{2\,\mathrm{polylog}\left(2,ax^{2}\right)}{5\,d}(dx)^{\frac{5}{2}}+\frac{8}{25\,d}(dx)^{\frac{5}{2}}\ln\left(\frac{-ad^{2}x^{2}+d^{2}}{d^{2}}\right)-\frac{32}{125\,d}(dx)^{\frac{5}{2}}-\frac{32\,d}{25\,a}\sqrt{dx}+\frac{8\,d}{25\,a}\sqrt[4]{\frac{d^{2}}{a}}\ln\left(\left(\sqrt{dx}+\sqrt[4]{\frac{d^{2}}{a}}\right)\left(\sqrt{dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x^2),x)
```

```
[Out] 2/5*(d*x)^(5/2)*polylog(2,a*x^2)/d+8/25/d*(d*x)^(5/2)*ln((-a*d^2*x^2+d^2)/d
^2)-32/125*(d*x)^(5/2)/d-32/25*d*(d*x)^(1/2)/a+8/25*d/a*(d^2/a)^(1/4)*ln(((
d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))+16/25*d/a*(d^2/a)^(1
/4)*arctan((d*x)^(1/2)/(d^2/a)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.84643, size = 471, normalized size = 3.36

$$-\frac{2\left(80\,a\left(\frac{d^{6}}{a^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^{4}d\left(\frac{d^{6}}{a^{5}}\right)^{\frac{3}{4}}-\sqrt{d^{3}x+a^{2}\sqrt{\frac{d^{6}}{a^{5}}}}a^{4}\left(\frac{d^{6}}{a^{5}}\right)^{\frac{3}{4}}}{d^{6}}\right)-20\,a\left(\frac{d^{6}}{a^{5}}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}d+8\,a\left(\frac{d^{6}}{a^{5}}\right)^{\frac{1}{4}}\right)+20\,a\left(\frac{d^{6}}{a^{5}}\right)^{\frac{1}{4}}\log\left(8\sqrt{dx}\right.\right.}{125\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] -2/125*(80*a*(d^6/a^5)^(1/4)*arctan(-(sqrt(d*x)*a^4*d*(d^6/a^5)^(3/4) - sqr
t(d^3*x + a^2*sqrt(d^6/a^5))*a^4*(d^6/a^5)^(3/4))/d^6) - 20*a*(d^6/a^5)^(1/
```

```
4)*log(8*sqrt(d*x)*d + 8*a*(d^6/a^5)^(1/4)) + 20*a*(d^6/a^5)^(1/4)*log(8*sq
rt(d*x)*d - 8*a*(d^6/a^5)^(1/4)) - (25*a*d*x^2*dilog(a*x^2) + 20*a*d*x^2*lo
g(-a*x^2 + 1) - 16*a*d*x^2 - 80*d)*sqrt(d*x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x^2), x)
```

3.73 $\int \sqrt{dx}\,\mathbf{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=125

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{3d}-\frac{16\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}-\frac{32(dx)^{3/2}}{27d}$$

```
[Out] (-32*(d*x)^(3/2))/(27*d) - (16*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(9*a^(3/4)) + (16*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)
) + (8*(d*x)^(3/2)*Log[1 - a*x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[2, a*x^2]
)/(3*d)
```

Rubi [A] time = 0.086485, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{3d}-\frac{16\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}-\frac{32(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

```
[Out] (-32*(d*x)^(3/2))/(27*d) - (16*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(9*a^(3/4)) + (16*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*a^(3/4)
) + (8*(d*x)^(3/2)*Log[1 - a*x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[2, a*x^2]
)/(3*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{dx}\text{Li}_2\left(ax^2\right)\,dx &= \frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{4}{3}\int \sqrt{dx}\log\left(1-ax^2\right)\,dx\\ &= \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{(16a)\int \frac{x(dx)^{3/2}}{1-ax^2}\,dx}{9d}\\ &= \frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{(16a)\int \frac{(dx)^{5/2}}{1-ax^2}\,dx}{9d^2}\\ &= -\frac{32(dx)^{3/2}}{27d}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{16}{9}\int \frac{\sqrt{dx}}{1-ax^2}\,dx\\ &= -\frac{32(dx)^{3/2}}{27d}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{32\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx,x,\sqrt{dx}\right)}{9d}\\ &= -\frac{32(dx)^{3/2}}{27d}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{3d}+\frac{(16d)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx,x,\sqrt{dx}\right)}{9\sqrt{a}}-\\ &= -\frac{32(dx)^{3/2}}{27d}-\frac{16\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{16\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9a^{3/4}}+\frac{8(dx)^{3/2}\log\left(1-ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{I}}{3}\end{aligned}$$

Mathematica [A] time = 0.0696545, size = 91, normalized size = 0.73

$$\frac{2\sqrt{dx}\left(9x^{3/2}\text{PolyLog}\left(2,ax^2\right)+\frac{4\left(a^{3/4}x^{3/2}\left(3\log\left(1-ax^2\right)-4\right)-6\tan^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)+6\tanh^{-1}\left(\sqrt[4]{a}\sqrt{x}\right)\right)}{a^{3/4}}\right)}{27\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x^2],x]
```

```
[Out] (2*Sqrt[d*x]*((4*(-6*ArcTan[a^(1/4)*Sqrt[x]] + 6*ArcTanh[a^(1/4)*Sqrt[x]] +
 a^(3/4)*x^(3/2)*(-4 + 3*Log[1 - a*x^2])))/a^(3/4) + 9*x^(3/2)*PolyLog[2, a
*x^2]))/(27*Sqrt[x])
```

Maple [A] time = 0.052, size = 139, normalized size = 1.1

$$\frac{2\,\text{polylog}\left(2,ax^2\right)}{3\,d}(dx)^{\frac{3}{2}}+\frac{8}{9\,d}(dx)^{\frac{3}{2}}\ln\left(\frac{-ad^2x^2+d^2}{d^2}\right)-\frac{32}{27\,d}(dx)^{\frac{3}{2}}-\frac{16\,d}{9\,a}\arctan\left(\sqrt{dx}\frac{1}{\sqrt[4]{\frac{d^2}{a}}}\right)\frac{1}{\sqrt[4]{\frac{d^2}{a}}}+\frac{8\,d}{9\,a}\ln\left(\left(\sqrt{dx}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x^2),x)
```

```
[Out] 2/3*(d*x)^(3/2)*polylog(2,a*x^2)/d+8/9/d*(d*x)^(3/2)*ln((-a*d^2*x^2+d^2)/d^
2)-32/27*(d*x)^(3/2)/d-16/9*d/a/(d^2/a)^(1/4)*arctan((d*x)^(1/2)/(d^2/a)^(1
/4))+8/9*d/a/(d^2/a)^(1/4)*ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2
/a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.70334, size = 440, normalized size = 3.52

$$\frac{2}{27}\sqrt{dx}\left(9\,x\mathrm{Li}_2\left(ax^2\right)+12\,x\log\left(-ax^2+1\right)-16\,x\right)+\frac{32}{9}\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}ad\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}-\sqrt{d^3x+ad^2\sqrt{\frac{d^2}{a^3}}a\left(\frac{d^2}{a^3}\right)^{\frac{1}{4}}}}{d^2}\right)+\frac{8}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] 2/27*sqrt(d*x)*(9*x*dilog(a*x^2) + 12*x*log(-a*x^2 + 1) - 16*x) + 32/9*(d^2
/a^3)^(1/4)*arctan(-(sqrt(d*x)*a*d*(d^2/a^3)^(1/4) - sqrt(d^3*x + a*d^2*sqr
t(d^2/a^3))*a*(d^2/a^3)^(1/4))/d^2) + 8/9*(d^2/a^3)^(1/4)*log(512*a^2*(d^2/
a^3)^(3/4) + 512*sqrt(d*x)*d) - 8/9*(d^2/a^3)^(1/4)*log(-512*a^2*(d^2/a^3)^
(3/4) + 512*sqrt(d*x)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\mathrm{Li}_2\left(ax^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x^2), x)
```

3.74 $\int \frac{\mathbf{PolyLog}(2,ax^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}+\frac{8\sqrt{dx}\log\left(1-ax^2\right)}{d}+\frac{16\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{32\sqrt{dx}}{d}$$

```
[Out] (-32*Sqrt[d*x])/d + (16*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) + (16*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) + (8*Sqrt
[d*x]*Log[1 - a*x^2])/d + (2*Sqrt[d*x]*PolyLog[2, a*x^2])/d
```

Rubi [A] time = 0.0803542, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$\frac{2\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}+\frac{8\sqrt{dx}\log\left(1-ax^2\right)}{d}+\frac{16\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+\frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{32\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (-32*Sqrt[d*x])/d + (16*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) + (16*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) + (8*Sqrt
[d*x]*Log[1 - a*x^2])/d + (2*Sqrt[d*x]*PolyLog[2, a*x^2])/d
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 4\int \frac{\log(1-ax^2)}{\sqrt{dx}}\,dx$$

$$= \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{(16a)\int \frac{x\sqrt{dx}}{1-ax^2}\,dx}{d}$$

$$= \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{(16a)\int \frac{(dx)^{3/2}}{1-ax^2}\,dx}{d^2}$$

$$= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 16\int \frac{1}{\sqrt{dx}(1-ax^2)}\,dx$$

$$= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + \frac{32\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d}$$

$$= -\frac{32\sqrt{dx}}{d} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d} + 16\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right) + 16\,\text{Subst}\left(\int\right.$$

$$= -\frac{32\sqrt{dx}}{d} + \frac{16\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{16\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} + \frac{8\sqrt{dx}\log(1-ax^2)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^2)}{d}$$

Mathematica [C] time = 0.0738958, size = 57, normalized size = 0.5

$$\frac{5x\text{Gamma}\left(\frac{5}{4}\right)\left(16\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right) + \text{PolyLog}(2,ax^2) + 4\log(1-ax^2) - 16\right)}{2\text{Gamma}\left(\frac{9}{4}\right)\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (5*x*Gamma[5/4]*(-16 + 16*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 -
 a*x^2] + PolyLog[2, a*x^2]))/(2*Sqrt[d*x]*Gamma[9/4])
```

Maple [A] time = 0.053, size = 137, normalized size = 1.2

$$2\frac{\sqrt{dx}\text{polylog}(2,ax^2)}{d} + 8\frac{\sqrt{dx}}{d}\ln\left(\frac{-ad^2x^2+d^2}{d^2}\right) + 16\frac{1}{d}\sqrt[4]{\frac{d^2}{a}}\arctan\left(\sqrt{dx}\frac{1}{\sqrt[4]{\frac{d^2}{a}}}\right) + 8\frac{1}{d}\sqrt[4]{\frac{d^2}{a}}\ln\left(\left(\sqrt{dx}+\sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(1/2),x)
```

```
[Out] 2*polylog(2,a*x^2)*(d*x)^(1/2)/d+8/d*(d*x)^(1/2)*ln((-a*d^2*x^2+d^2)/d^2)+1
6/d*(d^2/a)^(1/4)*arctan((d*x)^(1/2)/(d^2/a)^(1/4))+8/d*(d^2/a)^(1/4)*ln(((
d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))-32*(d*x)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.88189, size = 408, normalized size = 3.55

$$-\frac{2\left(16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{d^2\sqrt{\frac{1}{ad^2}}+dxad}\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}-\sqrt{dx}ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right)-4\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+4\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(\cdots\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(16*d*(1/(a*d^2))^(1/4)*arctan(sqrt(d^2*sqrt(1/(a*d^2)) + d*x)*a*d*(1/(a
*d^2))^(3/4) - sqrt(d*x)*a*d*(1/(a*d^2))^(3/4)) - 4*d*(1/(a*d^2))^(1/4)*log
(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 4*d*(1/(a*d^2))^(1/4)*log(-d*(1/(a*d^2)
)^(1/4) + sqrt(d*x)) - sqrt(d*x)*(dilog(a*x^2) + 4*log(-a*x^2 + 1) - 16))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{Li}_2\left(ax^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**2)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/sqrt(d*x), x)
```

3.75 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{3/2}} dx$

Optimal. Leaf size=103

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}-\frac{16\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{16\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}}$$

```
[Out] (-16*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (16*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x^2])/(d*Sqrt[d*x
]) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])
```

Rubi [A] time = 0.0738395, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6591, 2455, 16, 329, 298, 205, 208}

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}-\frac{16\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{16\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (-16*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (16*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (8*Log[1 - a*x^2])/(d*Sqrt[d*x
]) - (2*PolyLog[2, a*x^2])/(d*Sqrt[d*x])
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{3/2}}\,dx = -\frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}} - 4\int \frac{\log\left(1-ax^2\right)}{(dx)^{3/2}}\,dx$$
$$= \frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}} + \frac{(16a)\int \frac{x}{\sqrt{dx}(1-ax^2)}\,dx}{d}$$
$$= \frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}} + \frac{(16a)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{d^2}$$
$$= \frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}} + \frac{(32a)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^3}$$
$$= \frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}} + \frac{\left(16\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{d} - \frac{\left(16\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{d+\sqrt{a}x^2}\,dx, x\right.}{d}$$
$$= -\frac{16\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{16\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{8\log\left(1-ax^2\right)}{d\sqrt{dx}} - \frac{2\text{Li}_2\left(ax^2\right)}{d\sqrt{dx}}$$

Mathematica [C] time = 0.0700504, size = 62, normalized size = 0.6

$$\frac{x\text{Gamma}\left(\frac{3}{4}\right)\left(16ax^2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right) - 3\text{PolyLog}\left(2,ax^2\right) + 12\log\left(1-ax^2\right)\right)}{2\text{Gamma}\left(\frac{7}{4}\right)(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (x*Gamma[3/4]*(16*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 12*Log[1 -
a*x^2] - 3*PolyLog[2, a*x^2]))/(2*(d*x)^(3/2)*Gamma[7/4])
```

Maple [A] time = 0.052, size = 127, normalized size = 1.2

$$-2\,\frac{\text{polylog}\left(2,ax^2\right)}{d\sqrt{dx}} + 8\,\frac{1}{d\sqrt{dx}}\ln\left(\frac{-ad^2x^2+d^2}{d^2}\right) - 16\,\frac{1}{d}\arctan\left(\sqrt{dx}\frac{1}{\sqrt[4]{\frac{d^2}{a}}}\right)\frac{1}{\sqrt[4]{\frac{d^2}{a}}} + 8\,\frac{1}{d}\ln\left(\left(\sqrt{dx}+\sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx}-\sqrt[4]{\frac{d^2}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(3/2),x)
```

```
[Out] -2*polylog(2,a*x^2)/d/(d*x)^(1/2)+8/d/(d*x)^(1/2)*ln((-a*d^2*x^2+d^2)/d^2)-
16/d/(d^2/a)^(1/4)*arctan((d*x)^(1/2)/(d^2/a)^(1/4))+8/d/(d^2/a)^(1/4)*ln((
(d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^2/a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66427, size = 425, normalized size = 4.13

$$\frac{2\left(16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}ad\left(\frac{a}{d^6}\right)^{\frac{1}{4}}-\sqrt{ad^4\sqrt{\frac{a}{d^6}}+a^2dx}\,d\left(\frac{a}{d^6}\right)^{\frac{1}{4}}}{a}\right)+4\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(512\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+512\,\sqrt{dx}a\right)-4\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(\right.\right.}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(16*d^2*x*(a/d^6)^(1/4)*arctan(-(sqrt(d*x)*a*d*(a/d^6)^(1/4) - sqrt(a*d^4
*sqrt(a/d^6) + a^2*d*x)*d*(a/d^6)^(1/4))/a) + 4*d^2*x*(a/d^6)^(1/4)*log(512
*d^5*(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - 4*d^2*x*(a/d^6)^(1/4)*log(-512*d^5*
(a/d^6)^(3/4) + 512*sqrt(d*x)*a) - sqrt(d*x)*(dilog(a*x^2) - 4*log(-a*x^2 +
 1)))/(d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(3/2), x)
```

3.76 $\int \frac{\textbf{PolyLog}(2,ax^2)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}+\frac{16a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}}$$

```
[Out] (16*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (16*a^(3/4)*
ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (8*Log[1 - a*x^2])/(9*d
*(d*x)^(3/2)) - (2*PolyLog[2, a*x^2])/(3*d*(d*x)^(3/2))
```

Rubi [A] time = 0.0720639, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6591, 2455, 16, 329, 212, 208, 205}

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{3d(dx)^{3/2}}+\frac{16a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}}+\frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (16*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (16*a^(3/4)*
ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(9*d^(5/2)) + (8*Log[1 - a*x^2])/(9*d
*(d*x)^(3/2)) - (2*PolyLog[2, a*x^2])/(3*d*(d*x)^(3/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{5/2}}\,dx = -\frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}} - \frac{4}{3}\int \frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}\,dx$$

$$= \frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(16a)\int \frac{x}{(dx)^{3/2}\left(1-ax^2\right)}\,dx}{9d}$$

$$= \frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(16a)\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}\,dx}{9d^2}$$

$$= \frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(32a)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{9d^3}$$

$$= \frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(16a)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{9d^2} + \frac{(16a)\,\text{Subst}\left(\int \frac{1}{d+\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right.}{9d^2}$$

$$= \frac{16a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{16a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{9d^{5/2}} + \frac{8\log\left(1-ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2\left(ax^2\right)}{3d(dx)^{3/2}}$$

Mathematica [C] time = 0.0673089, size = 62, normalized size = 0.56

$$\frac{x\text{Gamma}\left(\frac{1}{4}\right)\left(16ax^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right) - 3\text{PolyLog}\left(2,ax^2\right) + 4\log\left(1-ax^2\right)\right)}{18\text{Gamma}\left(\frac{5}{4}\right)(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (x*Gamma[1/4]*(16*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 4*Log[1 - a
*x^2] - 3*PolyLog[2, a*x^2]))/(18*(d*x)^(5/2)*Gamma[5/4])
```

Maple [A] time = 0.054, size = 129, normalized size = 1.2

$$-\frac{2\,\text{polylog}\left(2,ax^2\right)}{3\,d}(dx)^{-\frac{3}{2}} + \frac{8}{9\,d}\ln\left(\frac{-ad^2x^2+d^2}{d^2}\right)(dx)^{-\frac{3}{2}} + \frac{8\,a}{9\,d^3}\sqrt[4]{\frac{d^2}{a}}\ln\left(\left(\sqrt{dx}+\sqrt[4]{\frac{d^2}{a}}\right)\left(\sqrt{dx}-\sqrt[4]{\frac{d^2}{a}}\right)^{-1}\right) + \frac{16\,a}{9\,d^3}\sqrt[4]{\frac{d^2}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(5/2),x)
```

```
[Out] -2/3*polylog(2,a*x^2)/d/(d*x)^(3/2)+8/9/d/(d*x)^(3/2)*ln((-a*d^2*x^2+d^2)/d
^2)+8/9/d^3*a*(d^2/a)^(1/4)*ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x)^(1/2)-(d^
2/a)^(1/4)))+16/9/d^3*a*(d^2/a)^(1/4)*arctan((d*x)^(1/2)/(d^2/a)^(1/4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.73533, size = 470, normalized size = 4.23

$$-\frac{2\left(16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}ad^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^6\sqrt{\frac{a^3}{d^{10}}}+a^2dx}\,d^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}}{a^3}\right)-4\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(8\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+8\sqrt{dx}a\right)+4\,d^3x^2\left(\frac{a^3}{d^{1}}\right.\right.}{9\,d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(16*d^3*x^2*(a^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*a*d^7*(a^3/d^10)^(3/4)
 - sqrt(d^6*sqrt(a^3/d^10) + a^2*d*x)*d^7*(a^3/d^10)^(3/4))/a^3) - 4*d^3*x^
2*(a^3/d^10)^(1/4)*log(8*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x)*a) + 4*d^3*x^2*
(a^3/d^10)^(1/4)*log(-8*d^3*(a^3/d^10)^(1/4) + 8*sqrt(d*x)*a) + sqrt(d*x)*(
3*dilog(a*x^2) - 4*log(-a*x^2 + 1)))/(d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left(ax^2\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(5/2), x)
```

3.77 $\int \frac{\text{PolyLog}(2,ax^2)}{(dx)^{7/2}} dx$

Optimal. Leaf size=126

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}-\frac{16a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{32a}{25d^3\sqrt{dx}}+\frac{8\log\left(1-ax^2\right)}{25d(dx)^{5/2}}$$

```
[Out] (-32*a)/(25*d^3*Sqrt[d*x]) - (16*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]
])/(25*d^(7/2)) + (16*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*d^(
7/2)) + (8*Log[1 - a*x^2])/(25*d*(d*x)^(5/2)) - (2*PolyLog[2, a*x^2])/(5*d*
(d*x)^(5/2))
```

Rubi [A] time = 0.0858323, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 325, 329, 298, 205, 208}

$$-\frac{2\text{PolyLog}\left(2,ax^2\right)}{5d(dx)^{5/2}}-\frac{16a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}+\frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}}-\frac{32a}{25d^3\sqrt{dx}}+\frac{8\log\left(1-ax^2\right)}{25d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```


```
[Out] (-32*a)/(25*d^3*Sqrt[d*x]) - (16*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]
])/(25*d^(7/2)) + (16*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(25*d^(
7/2)) + (8*Log[1 - a*x^2])/(25*d*(d*x)^(5/2)) - (2*PolyLog[2, a*x^2])/(5*d*
(d*x)^(5/2))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} - \frac{4}{5}\int \frac{\log(1-ax^2)}{(dx)^{7/2}}\,dx$$
$$= \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a)\int \frac{x}{(dx)^{5/2}(1-ax^2)}\,dx}{25d}$$
$$= \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a)\int \frac{1}{(dx)^{3/2}(1-ax^2)}\,dx}{25d^2}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^2)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{25d^4}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(32a^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{25d^5}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}} + \frac{(16a^{3/2})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{25d^3} - \frac{(16a^{3/2})\,\text{Subst}}{}$$
$$= -\frac{32a}{25d^3\sqrt{dx}} - \frac{16a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{16a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{25d^{7/2}} + \frac{8\log(1-ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_2(ax^2)}{5d(dx)^{5/2}}$$

Mathematica [C] time = 0.0765584, size = 70, normalized size = 0.56

$$-\frac{x\text{Gamma}\left(-\frac{1}{4}\right)\left(16a^2x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right) - 15\text{PolyLog}(2,ax^2) - 48ax^2 + 12\log(1-ax^2)\right)}{150\text{Gamma}\left(\frac{3}{4}\right)(dx)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] -(x*Gamma[-1/4]*(-48*a*x^2 + 16*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/4, a*x^
2] + 12*Log[1 - a*x^2] - 15*PolyLog[2, a*x^2]))/(150*(d*x)^(7/2)*Gamma[3/4]
)
```

Maple [A] time = 0.056, size = 140, normalized size = 1.1

$$-\frac{2\,\text{polylog}(2,ax^2)}{5\,d}(dx)^{-\frac{5}{2}} + \frac{8}{25\,d}\ln\left(\frac{-ad^2x^2+d^2}{d^2}\right)(dx)^{-\frac{5}{2}} - \frac{32\,a}{25\,d^3}\frac{1}{\sqrt{dx}} - \frac{16\,a}{25\,d^3}\arctan\left(\sqrt{dx}\frac{1}{\sqrt[4]{\frac{d^2}{a}}}\right)\frac{1}{\sqrt[4]{\frac{d^2}{a}}} + \frac{8\,a}{25\,d^3}\ln\Bigg(\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^2)/(d*x)^(7/2),x)
```

```
[Out] -2/5*polylog(2,a*x^2)/d/(d*x)^(5/2)+8/25/d/(d*x)^(5/2)*ln((-a*d^2*x^2+d^2)/
d^2)-32/25*a/d^3/(d*x)^(1/2)-16/25/d^3*a/(d^2/a)^(1/4)*arctan((d*x)^(1/2)/(
d^2/a)^(1/4))+8/25/d^3*a/(d^2/a)^(1/4)*ln(((d*x)^(1/2)+(d^2/a)^(1/4))/((d*x
)^(1/2)-(d^2/a)^(1/4)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.75536, size = 512, normalized size = 4.06

$$\frac{2\left(16\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^4d^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}-\sqrt{a^5d^8\sqrt{\frac{a^5}{d^{14}}}+a^8dx}\,d^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}}{a^5}\right)+4\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(512\,d^{11}\left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}}+512\,\sqrt{dx}a^4\right)-4\,d^4 \cdots}{25\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/25*(16*d^4*x^3*(a^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*a^4*d^3*(a^5/d^14)^(1/
4) - sqrt(a^5*d^8*sqrt(a^5/d^14) + a^8*d*x)*d^3*(a^5/d^14)^(1/4))/a^5) + 4*
d^4*x^3*(a^5/d^14)^(1/4)*log(512*d^11*(a^5/d^14)^(3/4) + 512*sqrt(d*x)*a^4)
 - 4*d^4*x^3*(a^5/d^14)^(1/4)*log(-512*d^11*(a^5/d^14)^(3/4) + 512*sqrt(d*x
)*a^4) - (16*a*x^2 + 5*dilog(a*x^2) - 4*log(-a*x^2 + 1))*sqrt(d*x))/(d^4*x^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^2)/(d*x)^(7/2), x)
```

3.78 $\int (dx)^{5/2}\text{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=161

$$-\frac{8(dx)^{7/2}\text{PolyLog}\left(2,ax^2\right)}{49d}+\frac{2(dx)^{7/2}\text{PolyLog}\left(3,ax^2\right)}{7d}+\frac{64d^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{32(dx)^{7/2}\text{lo}}{343}$$

```
[Out] (128*d*(d*x)^(3/2))/(1029*a) + (128*(d*x)^(7/2))/(2401*d) + (64*d^(5/2)*Arc
Tan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (64*d^(5/2)*ArcTanh[(a^(1
/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (32*(d*x)^(7/2)*Log[1 - a*x^2])/(3
43*d) - (8*(d*x)^(7/2)*PolyLog[2, a*x^2])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3
, a*x^2])/(7*d)
```

Rubi [A] time = 0.126749, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$-\frac{8(dx)^{7/2}\text{PolyLog}\left(2,ax^2\right)}{49d}+\frac{2(dx)^{7/2}\text{PolyLog}\left(3,ax^2\right)}{7d}+\frac{64d^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}}-\frac{32(dx)^{7/2}\text{lo}}{343}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```


```
[Out] (128*d*(d*x)^(3/2))/(1029*a) + (128*(d*x)^(7/2))/(2401*d) + (64*d^(5/2)*Arc
Tan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (64*d^(5/2)*ArcTanh[(a^(1
/4)*Sqrt[d*x])/Sqrt[d]])/(343*a^(7/4)) - (32*(d*x)^(7/2)*Log[1 - a*x^2])/(3
43*d) - (8*(d*x)^(7/2)*PolyLog[2, a*x^2])/(49*d) + (2*(d*x)^(7/2)*PolyLog[3
, a*x^2])/(7*d)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int (dx)^{5/2}\text{Li}_3\left(ax^2\right)dx = \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} - \frac{4}{7}\int (dx)^{5/2}\text{Li}_2\left(ax^2\right)dx$$
$$= -\frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} - \frac{16}{49}\int (dx)^{5/2}\log\left(1-ax^2\right)dx$$
$$= -\frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} - \frac{(64a)\int \frac{x(dx)^{7/2}}{1-ax^2}dx}{343d}$$
$$= -\frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} - \frac{(64a)\int \frac{(dx)^{9/2}}{1-ax^2}dx}{343d^2}$$
$$= \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} - \frac{64}{343}\int \frac{(dx)^{5/2}}{1-ax^2}$$
$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} -$$
$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} -$$
$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} - \frac{32(dx)^{7/2}\log\left(1-ax^2\right)}{343d} - \frac{8(dx)^{7/2}\text{Li}_2\left(ax^2\right)}{49d} + \frac{2(dx)^{7/2}\text{Li}_3\left(ax^2\right)}{7d} -$$
$$= \frac{128d(dx)^{3/2}}{1029a} + \frac{128(dx)^{7/2}}{2401d} + \frac{64d^{5/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{64d^{5/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343a^{7/4}} - \frac{32(dx)^{7/2}\log\left(1-\right.}{343d}$$

Mathematica [C] time = 0.103023, size = 89, normalized size = 0.55

$$-\frac{11d\text{Gamma}\left(\frac{11}{4}\right)(dx)^{3/2}\left(448\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+588ax^2\text{PolyLog}\left(2,ax^2\right)-1029ax^2\text{PolyLog}\left(3,ax\right.\right.}{14406a\text{Gamma}\left(\frac{15}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(5/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (-11*d*(d*x)^(3/2)*Gamma[11/4]*(-448 - 192*a*x^2 + 448*Hypergeometric2F1[3/
4, 1, 7/4, a*x^2] + 336*a*x^2*Log[1 - a*x^2] + 588*a*x^2*PolyLog[2, a*x^2]
- 1029*a*x^2*PolyLog[3, a*x^2]))/(14406*a*Gamma[15/4])
```

Maple [A] time = 0.193, size = 155, normalized size = 1.

$$-\frac{1}{2}(dx)^{\frac{5}{2}}\left(\frac{8448\,ax^2+19712}{79233\,a^2}x^{\frac{3}{2}}(-a)^{\frac{11}{4}}+\frac{64}{343\,a^2}x^{\frac{3}{2}}(-a)^{\frac{11}{4}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)\left(ax^2\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(5/2)*polylog(3,a*x^2),x)
```

```
[Out] -1/2*(d*x)^(5/2)/x^(5/2)/(-a)^(7/4)*(4/79233*x^(3/2)*(-a)^(11/4)*(2112*a*x^
2+4928)/a^2+64/343*x^(3/2)*(-a)^(11/4)/a^2/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4
))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))-64/343*x^(7/2)*(-a)^(11/4)/
a*ln(-a*x^2+1)-16/49*x^(7/2)*(-a)^(11/4)*polylog(2,a*x^2)/a+4/7*x^(7/2)*(-a
)^(11/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.10612, size = 682, normalized size = 4.24

$$-\frac{2\left(588\sqrt{dx}ad^2x^3\%\mathrm{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)-1029\sqrt{dx}ad^2x^3\mathrm{polylog}\left(3,ax^2\right)+1344\left(\frac{d^{10}}{a^7}\right)^{\frac{1}{4}}a\arctan\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -2/7203*(588*sqrt(d*x)*a*d^2*x^3*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*
x^2 + 1)/x) - 1029*sqrt(d*x)*a*d^2*x^3*polylog(3, a*x^2) + 1344*(d^10/a^7)^
```

```
(1/4)*a*arctan(-((d^10/a^7)^(1/4)*sqrt(d*x)*a^2*d^7 - sqrt(d^15*x + sqrt(d^
10/a^7)*a^3*d^10)*(d^10/a^7)^(1/4)*a^2)/d^10) + 336*(d^10/a^7)^(1/4)*a*log(
32768*sqrt(d*x)*d^7 + 32768*(d^10/a^7)^(3/4)*a^5) - 336*(d^10/a^7)^(1/4)*a*
log(32768*sqrt(d*x)*d^7 - 32768*(d^10/a^7)^(3/4)*a^5) + 16*(21*a*d^2*x^3*lo
g(-a*x^2 + 1) - 12*a*d^2*x^3 - 28*d^2*x)*sqrt(d*x))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*polylog(3,a*x**2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*polylog(3,a*x^2),x, algorithm="giac")

[Out] integrate((d*x)^(5/2)*polylog(3, a*x^2), x)
```

3.79 $\int (dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=161

$$-\frac{8(dx)^{5/2}\text{PolyLog}\left(2,ax^2\right)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}\left(3,ax^2\right)}{5d}-\frac{64d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{32(dx)^{5/2}\,\mathrm{l}}{12}$$

```
[Out] (128*d*Sqrt[d*x])/(125*a) + (128*(d*x)^(5/2))/(625*d) - (64*d^(3/2)*ArcTan[
(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (64*d^(3/2)*ArcTanh[(a^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (32*(d*x)^(5/2)*Log[1 - a*x^2])/(125*d
) - (8*(d*x)^(5/2)*PolyLog[2, a*x^2])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*
x^2])/(5*d)
```

Rubi [A] time = 0.115868, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$-\frac{8(dx)^{5/2}\text{PolyLog}\left(2,ax^2\right)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}\left(3,ax^2\right)}{5d}-\frac{64d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}}-\frac{32(dx)^{5/2}\,\mathrm{l}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```


```
[Out] (128*d*Sqrt[d*x])/(125*a) + (128*(d*x)^(5/2))/(625*d) - (64*d^(3/2)*ArcTan[
(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (64*d^(3/2)*ArcTanh[(a^(1/4)*
Sqrt[d*x])/Sqrt[d]])/(125*a^(5/4)) - (32*(d*x)^(5/2)*Log[1 - a*x^2])/(125*d
) - (8*(d*x)^(5/2)*PolyLog[2, a*x^2])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*
x^2])/(5*d)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int (dx)^{3/2}\text{Li}_3\left(ax^2\right)dx = \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{4}{5}\int (dx)^{3/2}\text{Li}_2\left(ax^2\right)dx$$
$$= -\frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{16}{25}\int (dx)^{3/2}\log\left(1-ax^2\right)dx$$
$$= -\frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{(64a)\int \frac{x(dx)^{5/2}}{1-ax^2}dx}{125d}$$
$$= -\frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{(64a)\int \frac{(dx)^{7/2}}{1-ax^2}dx}{125d^2}$$
$$= \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{64}{125}\int \frac{(dx)^{3/2}}{1-ax^2}$$
$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{(6}{}$$
$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{(1}{}$$
$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{32(dx)^{5/2}\log\left(1-ax^2\right)}{125d} - \frac{8(dx)^{5/2}\text{Li}_2\left(ax^2\right)}{25d} + \frac{2(dx)^{5/2}\text{Li}_3\left(ax^2\right)}{5d} - \frac{(6}{}$$
$$= \frac{128d\sqrt{dx}}{125a} + \frac{128(dx)^{5/2}}{625d} - \frac{64d^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{64d^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125a^{5/4}} - \frac{32(dx)^{5/2}\log(1-}{125d}$$

Mathematica [C] time = 0.0996985, size = 89, normalized size = 0.55

$$-\frac{9d\text{Gamma}\left(\frac{9}{4}\right)\sqrt{dx}\left(320\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+100ax^2\text{PolyLog}\left(2,ax^2\right)-125ax^2\text{PolyLog}\left(3,ax^2\right)-\right.}{1250a\text{Gamma}\left(\frac{13}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x^2],x]
```

```
[Out] (-9*d*Sqrt[d*x]*Gamma[9/4]*(-320 - 64*a*x^2 + 320*Hypergeometric2F1[1/4, 1,
 5/4, a*x^2] + 80*a*x^2*Log[1 - a*x^2] + 100*a*x^2*PolyLog[2, a*x^2] - 125*
a*x^2*PolyLog[3, a*x^2]))/(1250*a*Gamma[13/4])
```

Maple [A] time = 0.208, size = 155, normalized size = 1.

$$-\frac{1}{2}(dx)^{\frac{3}{2}}\left(\frac{2304\,ax^2+11520}{5625\,a^2}\sqrt{x}\,(-a)^{\frac{9}{4}}+\frac{64}{125\,a^2}\sqrt{x}\,(-a)^{\frac{9}{4}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)-2\arctan\left(\sqrt[4]{ax^2}\right)\right)\frac{1}{\sqrt[4]{ax^2}}-\frac{64}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x^2),x)
```

```
[Out] -1/2*(d*x)^(3/2)/x^(3/2)/(-a)^(5/4)*(4/5625*x^(1/2)*(-a)^(9/4)*(576*a*x^2+2
880)/a^2+64/125*x^(1/2)*(-a)^(9/4)/a^2/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-l
n(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))-64/125*x^(5/2)*(-a)^(9/4)/a*ln(
-a*x^2+1)-16/25*x^(5/2)*(-a)^(9/4)*polylog(2,a*x^2)/a+4/5*x^(5/2)*(-a)^(9/4
)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.91474, size = 606, normalized size = 3.76

$$-\frac{2\left(100\sqrt{dx}adx^2\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)-125\sqrt{dx}adx^2\text{polylog}\left(3,ax^2\right)-320\,a\left(\frac{d^6}{a^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^4d(}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -2/625*(100*sqrt(d*x)*a*d*x^2*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2
 + 1)/x) - 125*sqrt(d*x)*a*d*x^2*polylog(3, a*x^2) - 320*a*(d^6/a^5)^(1/4)*
```

```
arctan(-(sqrt(d*x)*a^4*d*(d^6/a^5)^(3/4) - sqrt(d^3*x + a^2*sqrt(d^6/a^5))*
a^4*(d^6/a^5)^(3/4))/d^6) + 80*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*x)*d + 32*a*
(d^6/a^5)^(1/4)) - 80*a*(d^6/a^5)^(1/4)*log(32*sqrt(d*x)*d - 32*a*(d^6/a^5)
^(1/4)) + 16*(5*a*d*x^2*log(-a*x^2 + 1) - 4*a*d*x^2 - 20*d)*sqrt(d*x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x**2),x)
```

```
[Out] Integral((d*x)**(3/2)*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x^2), x)
```

3.80 $\int \sqrt{dx}\,\textbf{PolyLog}\left(3,ax^2\right)\,dx$

Optimal. Leaf size=146

$$-\frac{8(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}+\frac{64\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{64\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{32(dx)^{3/2}\log}{27a}$$

```
[Out] (128*(d*x)^(3/2))/(81*d) + (64*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(27*a^(3/4)) - (64*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/
4)) - (32*(d*x)^(3/2)*Log[1 - a*x^2])/(27*d) - (8*(d*x)^(3/2)*PolyLog[2, a*
x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^2])/(3*d)
```

Rubi [A] time = 0.0974576, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 298, 205, 208}

$$-\frac{8(dx)^{3/2}\text{PolyLog}\left(2,ax^2\right)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^2\right)}{3d}+\frac{64\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{64\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}}-\frac{32(dx)^{3/2}\log}{27a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

```
[Out] (128*(d*x)^(3/2))/(81*d) + (64*Sqrt[d]*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])
/(27*a^(3/4)) - (64*Sqrt[d]*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*a^(3/
4)) - (32*(d*x)^(3/2)*Log[1 - a*x^2])/(27*d) - (8*(d*x)^(3/2)*PolyLog[2, a*
x^2])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^2])/(3*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{dx}\text{Li}_3\left(ax^2\right)\,dx = \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{4}{3}\int \sqrt{dx}\text{Li}_2\left(ax^2\right)\,dx$$
$$= -\frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{16}{9}\int \sqrt{dx}\log\left(1-ax^2\right)\,dx$$
$$= -\frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{(64a)\int \frac{x(dx)^{3/2}}{1-ax^2}\,dx}{27d}$$
$$= -\frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{(64a)\int \frac{(dx)^{5/2}}{1-ax^2}\,dx}{27d^2}$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{64}{27}\int \frac{\sqrt{dx}}{1-ax^2}\,dx$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{128\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax}{d^{\cdots}}}\cdots\right)}{27d}$$
$$= \frac{128(dx)^{3/2}}{81d} - \frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{Li}_2\left(ax^2\right)}{9d} + \frac{2(dx)^{3/2}\text{Li}_3\left(ax^2\right)}{3d} - \frac{(64d)\,\text{Subst}\left(\int \frac{\cdots}{d-\cdots}\cdots\right)}{27\sqrt{\cdots}}$$
$$= \frac{128(dx)^{3/2}}{81d} + \frac{64\sqrt{d}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{64\sqrt{d}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27a^{3/4}} - \frac{32(dx)^{3/2}\log\left(1-ax^2\right)}{27d} - \frac{8(dx)^{3/2}\text{L}\cdots}{9d}$$

Mathematica [C] time = 0.0915019, size = 68, normalized size = 0.47

$$-\frac{7x\text{Gamma}\left(\frac{7}{4}\right)\sqrt{dx}\left(64\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right)+36\text{PolyLog}\left(2,ax^2\right)-27\text{PolyLog}\left(3,ax^2\right)+48\log\left(1-\cdots\right.\right.}{162\text{Gamma}\left(\frac{11}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x^2],x]
```

```
[Out] (-7*x*Sqrt[d*x]*Gamma[7/4]*(-64 + 64*Hypergeometric2F1[3/4, 1, 7/4, a*x^2]
+ 48*Log[1 - a*x^2] + 36*PolyLog[2, a*x^2] - 27*PolyLog[3, a*x^2]))/(162*Ga
mma[11/4])
```

Maple [A] time = 0.224, size = 147, normalized size = 1.

$$-\frac{1}{2}\sqrt{dx}\left(\frac{256}{81\,a}x^{\frac{3}{2}}(-a)^{\frac{7}{4}} + \frac{64}{27\,a}x^{\frac{3}{2}}(-a)^{\frac{7}{4}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)\left(ax^2\right)^{-\frac{3}{4}} - \frac{64\ln\left(-ax^2+1\right)}{27\,a}x\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x^2),x)
```

```
[Out] -1/2*(d*x)^(1/2)/x^(1/2)/(-a)^(3/4)*(256/81*x^(3/2)*(-a)^(7/4)/a+64/27*x^(3
/2)*(-a)^(7/4)/a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*a
rctan((a*x^2)^(1/4)))-64/27*x^(3/2)*(-a)^(7/4)/a*ln(-a*x^2+1)-16/9*x^(3/2)*
(-a)^(7/4)*polylog(2,a*x^2)/a+4/3*x^(3/2)*(-a)^(7/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.8515, size = 579, normalized size = 3.97

$$-\frac{8}{9}\sqrt{dx}x\%\text{iint}\left(a,x,-\frac{\log\left(-ax^2+1\right)}{a},-\frac{2\log\left(-ax^2+1\right)}{x}\right)+\frac{2}{3}\sqrt{dx}x\text{polylog}\left(3,ax^2\right)-\frac{32}{81}\sqrt{dx}\left(3x\log\left(-ax^2+1\right)-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] -8/9*sqrt(d*x)*x*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 2/
3*sqrt(d*x)*x*polylog(3, a*x^2) - 32/81*sqrt(d*x)*(3*x*log(-a*x^2 + 1) - 4*
x) - 128/27*(d^2/a^3)^(1/4)*arctan(-(sqrt(d*x)*a*d*(d^2/a^3)^(1/4) - sqrt(d
^3*x + a*d^2*sqrt(d^2/a^3))*a*(d^2/a^3)^(1/4))/d^2) - 32/27*(d^2/a^3)^(1/4)
*log(32768*a^2*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d) + 32/27*(d^2/a^3)^(1/4)
*log(-32768*a^2*(d^2/a^3)^(3/4) + 32768*sqrt(d*x)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x**2),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\operatorname{Li}_3(ax^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x^2), x)
```

3.81 $\int \frac{\textbf{PolyLog}(3,ax^2)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=134

$$-\frac{8\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)}{d}-\frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d}-\frac{64\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+$$

```
[Out] (128*Sqrt[d*x])/d - (64*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) - (64*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) - (32*Sqr
t[d*x]*Log[1 - a*x^2])/d - (8*Sqrt[d*x]*PolyLog[2, a*x^2])/d + (2*Sqrt[d*x]
*PolyLog[3, a*x^2])/d
```

Rubi [A] time = 0.101164, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 321, 329, 212, 208, 205}

$$-\frac{8\sqrt{dx}\,\text{PolyLog}\left(2,ax^2\right)}{d}+\frac{2\sqrt{dx}\,\text{PolyLog}\left(3,ax^2\right)}{d}-\frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d}-\frac{64\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}-\frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}}+$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (128*Sqrt[d*x])/d - (64*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[
d]) - (64*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(a^(1/4)*Sqrt[d]) - (32*Sqr
t[d*x]*Log[1 - a*x^2])/d - (8*Sqrt[d*x]*PolyLog[2, a*x^2])/d + (2*Sqrt[d*x]
*PolyLog[3, a*x^2])/d
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3\left(ax^2\right)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - 4\int \frac{\text{Li}_2\left(ax^2\right)}{\sqrt{dx}}\,dx \\
&= -\frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - 16\int \frac{\log\left(1-ax^2\right)}{\sqrt{dx}}\,dx \\
&= -\frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - \frac{(64a)\int \frac{x\sqrt{dx}}{1-ax^2}\,dx}{d} \\
&= -\frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - \frac{(64a)\int \frac{(dx)^{3/2}}{1-ax^2}\,dx}{d^2} \\
&= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - 64\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}\,dx \\
&= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - \frac{128\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right.}{d} \\
&= \frac{128\sqrt{dx}}{d} - \frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{dx}\,\text{Li}_3\left(ax^2\right)}{d} - 64\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \right. \\
&= \frac{128\sqrt{dx}}{d} - \frac{64\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{64\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{a}\sqrt{d}} - \frac{32\sqrt{dx}\log\left(1-ax^2\right)}{d} - \frac{8\sqrt{dx}\,\text{Li}_2\left(ax^2\right)}{d} + \frac{2\sqrt{d}}{}
\end{aligned}
$$

Mathematica [C] time = 0.0861366, size = 68, normalized size = 0.51

$$
-\frac{5x\text{Gamma}\left(\frac{5}{4}\right)\left(64\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right)+4\text{PolyLog}\left(2,ax^2\right)-\text{PolyLog}\left(3,ax^2\right)+16\log\left(1-ax^2\right)-\right.}{2\text{Gamma}\left(\frac{9}{4}\right)\sqrt{dx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/Sqrt[d*x],x]
```

```
[Out] (-5*x*Gamma[5/4]*(-64 + 64*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1
 - a*x^2] + 4*PolyLog[2, a*x^2] - PolyLog[3, a*x^2]))/(2*Sqrt[d*x]*Gamma[9/
4])
```

Maple [A] time = 0.191, size = 147, normalized size = 1.1

$$-\frac{1}{2}\sqrt{x}\left(256\,\frac{\sqrt{x}\,(-a)^{5/4}}{a}+64\,\frac{\sqrt{x}\,(-a)^{5/4}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)-2\,\arctan\left(\sqrt[4]{ax^2}\right)\right)}{a\sqrt[4]{ax^2}}-64\,\frac{\sqrt{x}\,(-a)^{5/4}\ln\left(-ax^2+1\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(1/2),x)
```

```
[Out] -1/2/(d*x)^(1/2)*x^(1/2)/(-a)^(1/4)*(256*x^(1/2)*(-a)^(5/4)/a+64*x^(1/2)*(-
a)^(5/4)/a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan(
(a*x^2)^(1/4)))-64*x^(1/2)*(-a)^(5/4)/a*ln(-a*x^2+1)-16*x^(1/2)*(-a)^(5/4)*
polylog(2,a*x^2)/a+4*x^(1/2)*(-a)^(5/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.8615, size = 521, normalized size = 3.89

$$\frac{2\left(64\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\arctan\left(\sqrt{d^2\sqrt{\frac{1}{ad^2}}+dx}ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}-\sqrt{dx}ad\left(\frac{1}{ad^2}\right)^{\frac{3}{4}}\right)-16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+16\,d\left(\frac{1}{ad^2}\right)^{\frac{1}{4}}\log\left(\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(64*d*(1/(a*d^2))^(1/4)*arctan(sqrt(d^2*sqrt(1/(a*d^2)) + d*x)*a*d*(1/(a*
d^2))^(3/4) - sqrt(d*x)*a*d*(1/(a*d^2))^(3/4)) - 16*d*(1/(a*d^2))^(1/4)*log
(d*(1/(a*d^2))^(1/4) + sqrt(d*x)) + 16*d*(1/(a*d^2))^(1/4)*log(-d*(1/(a*d^2
```

```
))^(1/4) + sqrt(d*x)) - 16*sqrt(d*x)*(log(-a*x^2 + 1) - 4) - 4*sqrt(d*x)*%i
int(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + sqrt(d*x)*polylog(3,
a*x^2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/sqrt(d*x), x)
```

3.82 $\int \frac{\textbf{PolyLog}(3,ax^2)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}-\frac{64\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{64\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{32\log\left(1-ax^2\right)}{d\sqrt{dx}}$$

```
[Out] (-64*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (64*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (32*Log[1 - a*x^2])/(d*Sqrt[d*
x]) - (8*PolyLog[2, a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^2])/(d*Sqrt[d
*x])
```

Rubi [A] time = 0.0932373, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6591, 2455, 16, 329, 298, 205, 208}

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{d\sqrt{dx}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{d\sqrt{dx}}-\frac{64\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{64\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{32\log\left(1-ax^2\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```


```
[Out] (-64*a^(1/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (64*a^(1/4)*Arc
Tanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/d^(3/2) + (32*Log[1 - a*x^2])/(d*Sqrt[d*
x]) - (8*PolyLog[2, a*x^2])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^2])/(d*Sqrt[d
*x])
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{3/2}}\,dx = -\frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + 4\int \frac{\text{Li}_2(ax^2)}{(dx)^{3/2}}\,dx$$
$$= -\frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} - 16\int \frac{\log(1-ax^2)}{(dx)^{3/2}}\,dx$$
$$= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a)\int \frac{x}{\sqrt{dx}(1-ax^2)}\,dx}{d}$$
$$= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(64a)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{d^2}$$
$$= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{(128a)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{d^3}$$
$$= \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}} + \frac{\left(64\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{d} - \frac{\left(64\sqrt{a}\right)\text{Subst}\left(\right.}{}$$
$$= -\frac{64\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{64\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{32\log(1-ax^2)}{d\sqrt{dx}} - \frac{8\text{Li}_2(ax^2)}{d\sqrt{dx}} - \frac{2\text{Li}_3(ax^2)}{d\sqrt{dx}}$$

Mathematica [C] time = 0.0897261, size = 71, normalized size = 0.58

$$\frac{x\text{Gamma}\left(\frac{3}{4}\right)\left(64ax^2\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right) - 12\text{PolyLog}\left(2,ax^2\right) - 3\text{PolyLog}\left(3,ax^2\right) + 48\log\left(1-ax^2\right)\right)}{2\text{Gamma}\left(\frac{7}{4}\right)(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(3/2),x]
```

```
[Out] (x*Gamma[3/4]*(64*a*x^2*Hypergeometric2F1[3/4, 1, 7/4, a*x^2] + 48*Log[1 -
a*x^2] - 12*PolyLog[2, a*x^2] - 3*PolyLog[3, a*x^2]))/(2*(d*x)^(3/2)*Gamma[
7/4])
```

Maple [A] time = 0.191, size = 131, normalized size = 1.1

$$-\frac{1}{2}x^{\frac{3}{2}}\sqrt[4]{-a}\left(-64\frac{x^{3/2}(-a)^{3/4}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)}{\left(ax^2\right)^{3/4}} + 64\frac{(-a)^{3/4}\ln\left(-ax^2+1\right)}{\sqrt{x}a} - 16\frac{(-a)^{3/4}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(3/2),x)
```

```
[Out] -1/2/(d*x)^(3/2)*x^(3/2)*(-a)^(1/4)*(-64*x^(3/2)*(-a)^(3/4)/(a*x^2)^(3/4)*(
ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*arctan((a*x^2)^(1/4)))+64/x^(1/2)
*(-a)^(3/4)/a*ln(-a*x^2+1)-16/x^(1/2)*(-a)^(3/4)*polylog(2,a*x^2)/a-4/x^(1/
2)*(-a)^(3/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.0072, size = 549, normalized size = 4.5

$$2\left(64\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}ad\left(\frac{a}{d^6}\right)^{\frac{1}{4}}-\sqrt{ad^4\sqrt{\frac{a}{d^6}}+a^2dx}\,d\left(\frac{a}{d^6}\right)^{\frac{1}{4}}}{a}\right)+16\,d^2x\left(\frac{a}{d^6}\right)^{\frac{1}{4}}\log\left(32768\,d^5\left(\frac{a}{d^6}\right)^{\frac{3}{4}}+32768\,\sqrt{dx}a\right)-16\,d^2x\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(64*d^2*x*(a/d^6)^(1/4)*arctan(-(sqrt(d*x)*a*d*(a/d^6)^(1/4) - sqrt(a*d^4
*sqrt(a/d^6) + a^2*d*x)*d*(a/d^6)^(1/4))/a) + 16*d^2*x*(a/d^6)^(1/4)*log(32
768*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x)*a) - 16*d^2*x*(a/d^6)^(1/4)*log(-32
768*d^5*(a/d^6)^(3/4) + 32768*sqrt(d*x)*a) - 4*sqrt(d*x)*\%iint(a, x, -log(-
a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 16*sqrt(d*x)*log(-a*x^2 + 1) - sqrt(d
*x)*polylog(3, a*x^2))/(d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(3/2), x)
```

3.83 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}+\frac{64a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}}$$

```
[Out] (64*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (64*a^(3/4)
*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (32*Log[1 - a*x^2])/(
27*d*(d*x)^(3/2)) - (8*PolyLog[2, a*x^2])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3,
 a*x^2])/(3*d*(d*x)^(3/2))
```

Rubi [A] time = 0.0910165, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6591, 2455, 16, 329, 212, 208, 205}

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{3d(dx)^{3/2}}+\frac{64a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}}+\frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (64*a^(3/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (64*a^(3/4)
*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(27*d^(5/2)) + (32*Log[1 - a*x^2])/(
27*d*(d*x)^(3/2)) - (8*PolyLog[2, a*x^2])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3,
 a*x^2])/(3*d*(d*x)^(3/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_3\left(ax^2\right)}{(dx)^{5/2}}\,dx = -\frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{4}{3}\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{5/2}}\,dx$$
$$= -\frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} - \frac{16}{9}\int \frac{\log\left(1-ax^2\right)}{(dx)^{5/2}}\,dx$$
$$= \frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(64a)\int \frac{x}{(dx)^{3/2}\left(1-ax^2\right)}\,dx}{27d}$$
$$= \frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(64a)\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}\,dx}{27d^2}$$
$$= \frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(128a)\,\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{27d^3}$$
$$= \frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}} + \frac{(64a)\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{27d^2} + \frac{(64a)\,\text{Subst}\left(\int \frac{}{}\right.}{27}$$
$$= \frac{64a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{64a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{27d^{5/2}} + \frac{32\log\left(1-ax^2\right)}{27d(dx)^{3/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{3d(dx)^{3/2}}$$

Mathematica [C] time = 0.0935473, size = 71, normalized size = 0.54

$$\frac{x\text{Gamma}\left(\frac{1}{4}\right)\left(64ax^2\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right) - 12\text{PolyLog}\left(2,ax^2\right) - 9\text{PolyLog}\left(3,ax^2\right) + 16\log\left(1-ax^2\right)\right.}{54\text{Gamma}\left(\frac{5}{4}\right)(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(5/2),x]
```

```
[Out] (x*Gamma[1/4]*(64*a*x^2*Hypergeometric2F1[1/4, 1, 5/4, a*x^2] + 16*Log[1 -
a*x^2] - 12*PolyLog[2, a*x^2] - 9*PolyLog[3, a*x^2]))/(54*(d*x)^(5/2)*Gamma
[5/4])
```

Maple [A] time = 0.194, size = 131, normalized size = 1.

$$-\frac{1}{2}x^{\frac{5}{2}}(-a)^{\frac{3}{4}}\left(-\frac{64}{27}\sqrt{x}\sqrt[4]{-a}\left(\ln\left(1-\sqrt[4]{ax^2}\right) - \ln\left(1+\sqrt[4]{ax^2}\right) - 2\arctan\left(\sqrt[4]{ax^2}\right)\right)\frac{1}{\sqrt[4]{ax^2}} + \frac{64\ln\left(-ax^2+1\right)}{27a}\sqrt[4]{-a}x^{-\frac{3}{2}} - \frac{16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(5/2),x)
```

```
[Out] -1/2/(d*x)^(5/2)*x^(5/2)*(-a)^(3/4)*(-64/27*x^(1/2)*(-a)^(1/4)/(a*x^2)^(1/4
)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2*arctan((a*x^2)^(1/4)))+64/27/x
^(3/2)*(-a)^(1/4)/a*ln(-a*x^2+1)-16/9/x^(3/2)*(-a)^(1/4)*polylog(2,a*x^2)/a
-4/3/x^(3/2)*(-a)^(1/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 2.81842, size = 591, normalized size = 4.48

$$-\frac{2\left(64\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}ad^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}-\sqrt{d^6\sqrt{\frac{a^3}{d^{10}}}+a^2dx}\,d^7\left(\frac{a^3}{d^{10}}\right)^{\frac{3}{4}}}{a^3}\right)-16\,d^3x^2\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(32\,d^3\left(\frac{a^3}{d^{10}}\right)^{\frac{1}{4}}+32\sqrt{dx}a\right)+16\,d^3x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(64*d^3*x^2*(a^3/d^10)^(1/4)*arctan(-(sqrt(d*x)*a*d^7*(a^3/d^10)^(3/4
) - sqrt(d^6*sqrt(a^3/d^10) + a^2*d*x)*d^7*(a^3/d^10)^(3/4))/a^3) - 16*d^3*
x^2*(a^3/d^10)^(1/4)*log(32*d^3*(a^3/d^10)^(1/4) + 32*sqrt(d*x)*a) + 16*d^3
*x^2*(a^3/d^10)^(1/4)*log(-32*d^3*(a^3/d^10)^(1/4) + 32*sqrt(d*x)*a) + 12*s
qrt(d*x)*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 16*sqrt(d*
x)*log(-a*x^2 + 1) + 9*sqrt(d*x)*polylog(3, a*x^2))/(d^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(ax^2\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(5/2),x)
```

```
[Out] Integral(polylog(3, a*x**2)/(d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(5/2), x)
```

3.84 $\int \frac{\mathbf{PolyLog}(3,ax^2)}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}-\frac{64a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{128a}{125d^3\sqrt{dx}}+\frac{32\log\left(1-\right.}{125d(dx)^{5/2}}$$

```
[Out] (-128*a)/(125*d^3*Sqrt[d*x]) - (64*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[
d]])/(125*d^(7/2)) + (64*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125
*d^(7/2)) + (32*Log[1 - a*x^2])/(125*d*(d*x)^(5/2)) - (8*PolyLog[2, a*x^2])
/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*x^2])/(5*d*(d*x)^(5/2))
```

Rubi [A] time = 0.106859, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 325, 329, 298, 205, 208}

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{25d(dx)^{5/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{5d(dx)^{5/2}}-\frac{64a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}+\frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}}-\frac{128a}{125d^3\sqrt{dx}}+\frac{32\log\left(1-\right.}{125d(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] (-128*a)/(125*d^3*Sqrt[d*x]) - (64*a^(5/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sqrt[
d]])/(125*d^(7/2)) + (64*a^(5/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(125
*d^(7/2)) + (32*Log[1 - a*x^2])/(125*d*(d*x)^(5/2)) - (8*PolyLog[2, a*x^2])
/(25*d*(d*x)^(5/2)) - (2*PolyLog[3, a*x^2])/(5*d*(d*x)^(5/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_3(ax^2)}{(dx)^{7/2}}\,dx = -\frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{4}{5}\int \frac{\text{Li}_2(ax^2)}{(dx)^{7/2}}\,dx$$
$$= -\frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} - \frac{16}{25}\int \frac{\log(1-ax^2)}{(dx)^{7/2}}\,dx$$
$$= \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a)\int \frac{x}{(dx)^{5/2}(1-ax^2)}\,dx}{125d}$$
$$= \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a)\int \frac{1}{(dx)^{3/2}(1-ax^2)}\,dx}{125d^2}$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^2)\int \frac{\sqrt{dx}}{1-ax^2}\,dx}{125d^4}$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(128a^2)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{125d^5}$$
$$= -\frac{128a}{125d^3\sqrt{dx}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}_3(ax^2)}{5d(dx)^{5/2}} + \frac{(64a^{3/2})\,\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{125d^3} - \cdots$$
$$= -\frac{128a}{125d^3\sqrt{dx}} - \frac{64a^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{64a^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{125d^{7/2}} + \frac{32\log(1-ax^2)}{125d(dx)^{5/2}} - \frac{8\text{Li}_2(ax^2)}{25d(dx)^{5/2}} - \frac{2\text{Li}\cdots}{5d(\cdots}$$

Mathematica [C] time = 0.0986821, size = 79, normalized size = 0.54

$$-\frac{x\text{Gamma}\left(-\frac{1}{4}\right)\left(64a^2x^4\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},ax^2\right) - 60\text{PolyLog}\left(2,ax^2\right) - 75\text{PolyLog}\left(3,ax^2\right) - 192ax^2 + 4\cdots\right.}{750\text{Gamma}\left(\frac{3}{4}\right)(dx)^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(7/2),x]
```

```
[Out] -(x*Gamma[-1/4]*(-192*a*x^2 + 64*a^2*x^4*Hypergeometric2F1[3/4, 1, 7/4, a*x
^2] + 48*Log[1 - a*x^2] - 60*PolyLog[2, a*x^2] - 75*PolyLog[3, a*x^2]))/(75
0*(d*x)^(7/2)*Gamma[3/4])
```

Maple [A] time = 0.206, size = 142, normalized size = 1.

$$-\frac{1}{2}x^{\frac{7}{2}}(-a)^{\frac{5}{4}}\left(-\frac{256}{125}\frac{1}{\sqrt{x}}\frac{1}{\sqrt[4]{-a}}-\frac{64\,a}{125}x^{\frac{3}{2}}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)+2\arctan\left(\sqrt[4]{ax^2}\right)\right)\frac{1}{\sqrt[4]{-a}}\left(ax^2\right)^{-\frac{3}{4}}+\frac{64\ln\left(-ax^2\right.}{125\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(7/2),x)
```

```
[Out] -1/2/(d*x)^(7/2)*x^(7/2)*(-a)^(5/4)*(-256/125/x^(1/2)/(-a)^(1/4)-64/125*x^(
3/2)/(-a)^(1/4)*a/(a*x^2)^(3/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))+2*
arctan((a*x^2)^(1/4)))+64/125/x^(5/2)/(-a)^(1/4)/a*ln(-a*x^2+1)-16/25/x^(5/
2)/(-a)^(1/4)*polylog(2,a*x^2)/a-4/5/x^(5/2)/(-a)^(1/4)/a*polylog(3,a*x^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.00294, size = 641, normalized size = 4.36

$$\frac{2\left(64\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^4d^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}-\sqrt{a^5d^8\sqrt{\frac{a^5}{d^{14}}}+a^8dx}d^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}}{a^5}\right)+16\,d^4x^3\left(\frac{a^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(32768\,d^{11}\left(\frac{a^5}{d^{14}}\right)^{\frac{3}{4}}+32768\sqrt{dx}a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/125*(64*d^4*x^3*(a^5/d^14)^(1/4)*arctan(-(sqrt(d*x)*a^4*d^3*(a^5/d^14)^(1
/4) - sqrt(a^5*d^8*sqrt(a^5/d^14) + a^8*d*x)*d^3*(a^5/d^14)^(1/4))/a^5) + 1
6*d^4*x^3*(a^5/d^14)^(1/4)*log(32768*d^11*(a^5/d^14)^(3/4) + 32768*sqrt(d*x
```

```
)*a^4) - 16*d^4*x^3*(a^5/d^14)^(1/4)*log(-32768*d^11*(a^5/d^14)^(3/4) + 327
68*sqrt(d*x)*a^4) - 16*(4*a*x^2 - log(-a*x^2 + 1))*sqrt(d*x) - 20*sqrt(d*x)
*\%iint(a, x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) - 25*sqrt(d*x)*polyl
og(3, a*x^2))/(d^4*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{(dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(7/2), x)
```

3.85 $\int \frac{\text{PolyLog}(3,ax^2)}{(dx)^{9/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{49d(dx)^{7/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}+\frac{64a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}+\frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}-\frac{128a}{1029d^3(dx)^{3/2}}+\frac{32\log}{343d}$$

```
[Out] (-128*a)/(1029*d^3*(d*x)^(3/2)) + (64*a^(7/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(343*d^(9/2)) + (64*a^(7/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(
343*d^(9/2)) + (32*Log[1 - a*x^2])/(343*d*(d*x)^(7/2)) - (8*PolyLog[2, a*x^
2])/(49*d*(d*x)^(7/2)) - (2*PolyLog[3, a*x^2])/(7*d*(d*x)^(7/2))
```

Rubi [A] time = 0.105684, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6591, 2455, 16, 325, 329, 212, 208, 205}

$$-\frac{8\text{PolyLog}\left(2,ax^2\right)}{49d(dx)^{7/2}}-\frac{2\text{PolyLog}\left(3,ax^2\right)}{7d(dx)^{7/2}}+\frac{64a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}+\frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}}-\frac{128a}{1029d^3(dx)^{3/2}}+\frac{32\log}{343d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```


```
[Out] (-128*a)/(1029*d^3*(d*x)^(3/2)) + (64*a^(7/4)*ArcTan[(a^(1/4)*Sqrt[d*x])/Sq
rt[d]])/(343*d^(9/2)) + (64*a^(7/4)*ArcTanh[(a^(1/4)*Sqrt[d*x])/Sqrt[d]])/(
343*d^(9/2)) + (32*Log[1 - a*x^2])/(343*d*(d*x)^(7/2)) - (8*PolyLog[2, a*x^
2])/(49*d*(d*x)^(7/2)) - (2*PolyLog[3, a*x^2])/(7*d*(d*x)^(7/2))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{Li}_3\left(ax^2\right)}{(dx)^{9/2}}\,dx = -\frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{4}{7}\int \frac{\text{Li}_2\left(ax^2\right)}{(dx)^{9/2}}\,dx$$

$$= -\frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} - \frac{16}{49}\int \frac{\log\left(1-ax^2\right)}{(dx)^{9/2}}\,dx$$

$$= \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{(64a)\int \frac{x}{(dx)^{7/2}\left(1-ax^2\right)}\,dx}{343d}$$

$$= \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{(64a)\int \frac{1}{(dx)^{5/2}\left(1-ax^2\right)}\,dx}{343d^2}$$

$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{\left(64a^2\right)\int \frac{1}{\sqrt{dx}\left(1-ax^2\right)}\,dx}{343d^4}$$

$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{\left(128a^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{ax^4}{d^2}}\,dx, x, \sqrt{dx}\right)}{343d^5}$$

$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} - \frac{2\text{Li}_3\left(ax^2\right)}{7d(dx)^{7/2}} + \frac{\left(64a^2\right)\text{Subst}\left(\int \frac{1}{d-\sqrt{a}x^2}\,dx, x, \sqrt{dx}\right)}{343d^4}$$

$$= -\frac{128a}{1029d^3(dx)^{3/2}} + \frac{64a^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{64a^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{a}\sqrt{dx}}{\sqrt{d}}\right)}{343d^{9/2}} + \frac{32\log\left(1-ax^2\right)}{343d(dx)^{7/2}} - \frac{8\text{Li}_2\left(ax^2\right)}{49d(dx)^{7/2}} -$$

Mathematica [C] time = 0.0961169, size = 84, normalized size = 0.57

$$-\frac{\text{Gamma}\left(-\frac{3}{4}\right)\sqrt{dx}\left(192a^2x^4\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},ax^2\right) - 84\text{PolyLog}\left(2,ax^2\right) - 147\text{PolyLog}\left(3,ax^2\right) - 64a\right.}{686d^5x^4\text{Gamma}\left(\frac{1}{4}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^2]/(d*x)^(9/2),x]
```

```
[Out] -(Sqrt[d*x]*Gamma[-3/4]*(-64*a*x^2 + 192*a^2*x^4*Hypergeometric2F1[1/4, 1,
5/4, a*x^2] + 48*Log[1 - a*x^2] - 84*PolyLog[2, a*x^2] - 147*PolyLog[3, a*x
^2]))/(686*d^5*x^4*Gamma[1/4])
```

Maple [A] time = 0.191, size = 142, normalized size = 1.

$$-\frac{1}{2}x^{\frac{9}{2}}(-a)^{\frac{7}{4}}\left(-\frac{256}{1029}x^{-\frac{3}{2}}(-a)^{-\frac{3}{4}}-\frac{64\,a}{343}\sqrt{x}\left(\ln\left(1-\sqrt[4]{ax^2}\right)-\ln\left(1+\sqrt[4]{ax^2}\right)-2\arctan\left(\sqrt[4]{ax^2}\right)\right)(-a)^{-\frac{3}{4}}\frac{1}{\sqrt[4]{ax^2}}+\frac{64\ln\left(-a\right.}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^2)/(d*x)^(9/2),x)
```

```
[Out] -1/2/(d*x)^(9/2)*x^(9/2)*(-a)^(7/4)*(-256/1029/x^(3/2)/(-a)^(3/4)-64/343*x^
(1/2)/(-a)^(3/4)*a/(a*x^2)^(1/4)*(ln(1-(a*x^2)^(1/4))-ln(1+(a*x^2)^(1/4))-2
*arctan((a*x^2)^(1/4)))+64/343/x^(7/2)/(-a)^(3/4)/a*ln(-a*x^2+1)-16/49/x^(7
/2)/(-a)^(3/4)*polylog(2,a*x^2)/a-4/7/x^(7/2)/(-a)^(3/4)/a*polylog(3,a*x^2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [C] time = 3.05735, size = 629, normalized size = 4.28

$$-\frac{2\left(192\,d^5x^4\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{dx}a^2d^{13}\left(\frac{a^7}{d^{18}}\right)^{\frac{3}{4}}-\sqrt{d^{10}\sqrt{\frac{a^7}{d^{18}}}+a^4dx}\,d^{13}\left(\frac{a^7}{d^{18}}\right)^{\frac{3}{4}}}{a^7}\right)-48\,d^5x^4\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(32\,d^5\left(\frac{a^7}{d^{18}}\right)^{\frac{1}{4}}+32\sqrt{dx}a^2\right)+48\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/1029*(192*d^5*x^4*(a^7/d^18)^(1/4)*arctan(-(sqrt(d*x)*a^2*d^13*(a^7/d^18
)^(3/4) - sqrt(d^10*sqrt(a^7/d^18) + a^4*d*x)*d^13*(a^7/d^18)^(3/4))/a^7) -
```

```
 48*d^5*x^4*(a^7/d^18)^(1/4)*log(32*d^5*(a^7/d^18)^(1/4) + 32*sqrt(d*x)*a^2
) + 48*d^5*x^4*(a^7/d^18)^(1/4)*log(-32*d^5*(a^7/d^18)^(1/4) + 32*sqrt(d*x)
*a^2) + 16*(4*a*x^2 - 3*log(-a*x^2 + 1))*sqrt(d*x) + 84*sqrt(d*x)*\%iint(a,
x, -log(-a*x^2 + 1)/a, -2*log(-a*x^2 + 1)/x) + 147*sqrt(d*x)*polylog(3, a*x
^2))/(d^5*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**2)/(d*x)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3(ax^2)}{(dx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^2)/(d*x)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^2)/(d*x)^(9/2), x)
```

3.86 $\int (dx)^{3/2}\textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=101

$$\frac{8adq^2\sqrt{dx}x^{q+2}\text{Hypergeometric2F1}\left(1,\frac{q+\frac{5}{2}}{q},\frac{1}{2}\left(\frac{5}{q}+4\right),ax^q\right)}{25(2q+5)}+\frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}+\frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}$$

```
[Out] (8*a*d*q^2*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q)/
2, a*x^q])/(25*(5 + 2*q)) + (4*q*(d*x)^(5/2)*Log[1 - a*x^q])/(25*d) + (2*(d
*x)^(5/2)*PolyLog[2, a*x^q])/(5*d)
```

Rubi [A] time = 0.0585402, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$\frac{2(dx)^{5/2}\text{PolyLog}(2,ax^q)}{5d}+\frac{8adq^2\sqrt{dx}x^{q+2}\,{}_2F_1\left(1,\frac{q+\frac{5}{2}}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{25(2q+5)}+\frac{4q(dx)^{5/2}\log(1-ax^q)}{25d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

```
[Out] (8*a*d*q^2*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q)/
2, a*x^q])/(25*(5 + 2*q)) + (4*q*(d*x)^(5/2)*Log[1 - a*x^q])/(25*d) + (2*(d
*x)^(5/2)*PolyLog[2, a*x^q])/(5*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^{3/2}\text{Li}_2\left(ax^q\right)\,dx &= \frac{2(dx)^{5/2}\text{Li}_2\left(ax^q\right)}{5d}+\frac{1}{5}(2q)\int (dx)^{3/2}\log\left(1-ax^q\right)\,dx\\ &= \frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2\left(ax^q\right)}{5d}+\frac{\left(4aq^2\right)\int\frac{x^{-1+q}(dx)^{5/2}}{1-ax^q}\,dx}{25d}\\ &= \frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2\left(ax^q\right)}{5d}+\frac{\left(4adq^2\sqrt{dx}\right)\int\frac{x^{\frac{3}{2}+q}}{1-ax^q}\,dx}{25\sqrt{x}}\\ &= \frac{8adq^2x^{2+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{25(5+2q)}+\frac{4q(dx)^{5/2}\log\left(1-ax^q\right)}{25d}+\frac{2(dx)^{5/2}\text{Li}_2\left(ax^q\right)}{5d}\end{aligned}$$

Mathematica [A] time = 0.124333, size = 82, normalized size = 0.81

$$\frac{2x(dx)^{3/2}\left(4aq^2x^q\text{Hypergeometric2F1}\left(1,\frac{q+\frac{5}{2}}{q},\frac{5}{2q}+2,ax^q\right)+(2q+5)\left(5\text{PolyLog}\left(2,ax^q\right)+2q\log\left(1-ax^q\right)\right)\right)}{25(2q+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[2, a*x^q],x]
```

```
[Out] (2*x*(d*x)^(3/2)*(4*a*q^2*x^q*Hypergeometric2F1[1, (5/2 + q)/q, 2 + 5/(2*q)
, a*x^q] + (5 + 2*q)*(2*q*Log[1 - a*x^q] + 5*PolyLog[2, a*x^q])))/(25*(5 +
2*q))
```

Maple [C] time = 0.255, size = 121, normalized size = 1.2

$$-\frac{1}{q}(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(-\frac{4q^2\ln(1-ax^q)}{25}x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}-2\frac{qx^{5/2}\left(1+2/5\,q\right)\mathrm{polylog}\left(2,ax^q\right)}{5+2q}(-a)^{5/2\,q^{-1}}-\frac{4q^2a}{25}x^{\frac{5}{2}+q}(-a)^{\frac{5}{2q}}\,\mathrm{LerchI}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(2,a*x^q),x)
```

```
[Out] -(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(-4/25*q^2*x^(5/2)*(-a)^(5/2/q)*ln(1-a
*x^q)-2*q/(5+2*q)*x^(5/2)*(-a)^(5/2/q)*(1+2/5*q)*polylog(2,a*x^q)-4/25*q^2*
x^(5/2+q)*a*(-a)^(5/2/q)*LerchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8d^{\frac{3}{2}}q^3\int\frac{x^{\frac{3}{2}}}{25\left(\left(2a^2q-5a^2\right)x^{2q}-2\left(2aq-5a\right)x^q+2q-5\right)}\,dx+\frac{2\left(25\left(\left(2ad^{\frac{3}{2}}q-5ad^{\frac{3}{2}}\right)xx^q-\left(2d^{\frac{3}{2}}q-5d^{\frac{3}{2}}\right)x\right)x^{\frac{3}{2}}\mathrm{Li}_2\left(ax\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] 8*d^(3/2)*q^3*integrate(1/25*x^(3/2)/((2*a^2*q - 5*a^2)*x^(2*q) - 2*(2*a*q
- 5*a)*x^q + 2*q - 5), x) + 2/125*(25*((2*a*d^(3/2)*q - 5*a*d^(3/2))*x*x^q
- (2*d^(3/2)*q - 5*d^(3/2))*x)*x^(3/2)*dilog(a*x^q) + 10*((2*a*d^(3/2)*q^2
- 5*a*d^(3/2)*q)*x*x^q - (2*d^(3/2)*q^2 - 5*d^(3/2)*q)*x)*x^(3/2)*log(-a*x^
q + 1) + 4*(2*d^(3/2)*q^3*x - (2*a*d^(3/2)*q^3 - 5*a*d^(3/2)*q^2)*x*x^q)*x^
(3/2))/((2*a*q - 5*a)*x^q - 2*q + 5)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx}dx\mathrm{Li}_2\left(ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*dilog(a*x^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \operatorname{Li}_2(ax^q) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*dilog(a*x^q), x)
```

3.87 $\int \sqrt{dx}\,\text{PolyLog}\left(2,ax^q\right)\,dx$

Optimal. Leaf size=100

$$\frac{8aq^2\sqrt{dx}x^{q+1}\text{Hypergeometric2F1}\left(1,\frac{q+\frac{3}{2}}{q},\frac{1}{2}\left(\frac{3}{q}+4\right),ax^q\right)}{9(2q+3)}+\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^q\right)}{3d}+\frac{4q(dx)^{3/2}\log\left(1-ax^q\right)}{9d}$$

```
[Out] (8*a*q^2*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/2,
 a*x^q])/(9*(3 + 2*q)) + (4*q*(d*x)^(3/2)*Log[1 - a*x^q])/(9*d) + (2*(d*x)^
(3/2)*PolyLog[2, a*x^q])/(3*d)
```

Rubi [A] time = 0.0546737, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$\frac{2(dx)^{3/2}\text{PolyLog}\left(2,ax^q\right)}{3d}+\frac{8aq^2\sqrt{dx}x^{q+1}\,{}_2F_1\left(1,\frac{q+\frac{3}{2}}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{9(2q+3)}+\frac{4q(dx)^{3/2}\log\left(1-ax^q\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[2, a*x^q],x]
```


```
[Out] (8*a*q^2*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/2,
 a*x^q])/(9*(3 + 2*q)) + (4*q*(d*x)^(3/2)*Log[1 - a*x^q])/(9*d) + (2*(d*x)^
(3/2)*PolyLog[2, a*x^q])/(3*d)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\,\mathrm{Li}_2(ax^q)\,dx &= \frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d} + \frac{1}{3}(2q)\int \sqrt{dx}\log(1-ax^q)\,dx \\
&= \frac{4q(dx)^{3/2}\log(1-ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d} + \frac{\left(4aq^2\right)\int \frac{x^{-1+q}(dx)^{3/2}}{1-ax^q}\,dx}{9d} \\
&= \frac{4q(dx)^{3/2}\log(1-ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d} + \frac{\left(4aq^2\sqrt{dx}\right)\int \frac{x^{\frac{1}{2}+q}}{1-ax^q}\,dx}{9\sqrt{x}} \\
&= \frac{8aq^2x^{1+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{9(3+2q)} + \frac{4q(dx)^{3/2}\log(1-ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_2(ax^q)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.106325, size = 82, normalized size = 0.82

$$\frac{2x\sqrt{dx}\left(4aq^2x^q\,\text{Hypergeometric2F1}\left(1,\frac{q+\frac{3}{2}}{q},\frac{3}{2q}+2,ax^q\right)+(2q+3)\left(3\,\text{PolyLog}(2,ax^q)+2q\log(1-ax^q)\right)\right)}{9(2q+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*PolyLog[2, a*x^q],x]
```

```
[Out] (2*x*Sqrt[d*x]*(4*a*q^2*x^q*Hypergeometric2F1[1, (3/2 + q)/q, 2 + 3/(2*q),
a*x^q] + (3 + 2*q)*(2*q*Log[1 - a*x^q] + 3*PolyLog[2, a*x^q])))/(9*(3 + 2*q
))
```

Maple [C] time = 0.229, size = 121, normalized size = 1.2

$$-\frac{1}{q}\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(-\frac{4q^2\ln(1-ax^q)}{9}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}-2\,\frac{qx^{3/2}\left(1+2/3\,q\right)\operatorname{polylog}(2,ax^q)}{3+2q}(-a)^{3/2\,q^{-1}}-\frac{4q^2a}{9}x^{\frac{3}{2}+q}(-a)^{\frac{3}{2q}}\,\text{LerchPh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(2,a*x^q),x)
```

```
[Out] -(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(-4/9*q^2*x^(3/2)*(-a)^(3/2/q)*ln(1-a*
x^q)-2*q/(3+2*q)*x^(3/2)*(-a)^(3/2/q)*(1+2/3*q)*polylog(2,a*x^q)-4/9*q^2*x^
(3/2+q)*a*(-a)^(3/2/q)*LerchPhi(a*x^q,1,1/2*(3+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\sqrt{d}q^3\int\frac{\sqrt{x}}{9\left(\left(2a^2q-3a^2\right)x^{2q}-2\left(2aq-3a\right)x^q+2q-3\right)}\,dx+\frac{2\left(9\left(\left(2a\sqrt{d}q-3a\sqrt{d}\right)xx^q-\left(2\sqrt{d}q-3\sqrt{d}\right)x\right)\sqrt{x}\operatorname{Li}_2\left(ax\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] 8*sqrt(d)*q^3*integrate(1/9*sqrt(x)/((2*a^2*q - 3*a^2)*x^(2*q) - 2*(2*a*q -
 3*a)*x^q + 2*q - 3), x) + 2/27*(9*((2*a*sqrt(d)*q - 3*a*sqrt(d))*x*x^q - (
2*sqrt(d)*q - 3*sqrt(d))*x)*sqrt(x)*dilog(a*x^q) + 6*((2*a*sqrt(d)*q^2 - 3*
a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - 3*sqrt(d)*q)*x)*sqrt(x)*log(-a*x^q +
1) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - 3*a*sqrt(d)*q^2)*x*x^q)*sqrt(x
))/((2*a*q - 3*a)*x^q - 2*q + 3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{dx}\operatorname{Li}_2\left(ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\text{Li}_2\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*dilog(a*x^q), x)
```

3.88 $\int \frac{\text{PolyLog}(2,ax^q)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=93

$$\frac{8aq^2\sqrt{dx}x^q\text{Hypergeometric2F1}\left(1,\frac{q+\frac{1}{2}}{q},\frac{1}{2}\left(\frac{1}{q}+4\right),ax^q\right)}{d(2q+1)}+\frac{2\sqrt{dx}\text{PolyLog}\left(2,ax^q\right)}{d}+\frac{4q\sqrt{dx}\log\left(1-ax^q\right)}{d}$$

```
[Out] (8*a*q^2*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2, a*
x^q])/(d*(1 + 2*q)) + (4*q*Sqrt[d*x]*Log[1 - a*x^q])/d + (2*Sqrt[d*x]*PolyL
og[2, a*x^q])/d
```

Rubi [A] time = 0.0540382, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$\frac{2\sqrt{dx}\text{PolyLog}\left(2,ax^q\right)}{d}+\frac{8aq^2\sqrt{dx}x^q\,{}_2F_1\left(1,\frac{q+\frac{1}{2}}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(2q+1)}+\frac{4q\sqrt{dx}\log\left(1-ax^q\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

```
[Out] (8*a*q^2*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2, a*
x^q])/(d*(1 + 2*q)) + (4*q*Sqrt[d*x]*Log[1 - a*x^q])/d + (2*Sqrt[d*x]*PolyL
og[2, a*x^q])/d
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(ax^q)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + (2q)\int \frac{\log(1-ax^q)}{\sqrt{dx}}\,dx \\
&= \frac{4q\sqrt{dx}\log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{\left(4aq^2\right)\int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q}\,dx}{d} \\
&= \frac{4q\sqrt{dx}\log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d} + \frac{\left(4aq^2\sqrt{dx}\right)\int \frac{x^{-\frac{1}{2}+q}}{1-ax^q}\,dx}{d\sqrt{x}} \\
&= \frac{8aq^2x^q\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)} + \frac{4q\sqrt{dx}\log(1-ax^q)}{d} + \frac{2\sqrt{dx}\text{Li}_2(ax^q)}{d}
\end{aligned}$$

Mathematica [C] time = 0.0209061, size = 48, normalized size = 0.52

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1-\frac{1}{2q}\\1,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^q]/Sqrt[d*x],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, -1/(2*q)}}, -(a*x^q
)])/(q*Sqrt[d*x]))
```

Maple [C] time = 0.232, size = 109, normalized size = 1.2

$$-\frac{1}{q}\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(-4\,q^{2}\sqrt{x}\,(-a)^{1/2\,q^{-1}}\ln\left(1-ax^{q}\right)-2\,q\sqrt{x}\,(-a)^{1/2\,q^{-1}}\,\mathrm{polylog}\left(2,ax^{q}\right)-4\,q^{2}x^{1/2+q}a\,(-a)^{1/2\,q^{-1}}\,\mathrm{LerchPhi}\left(ax^{q}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(1/2),x)
```

```
[Out] -1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(-4*q^2*x^(1/2)*(-a)^(1/2/q)*ln(1-a*
x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-4*q^2*x^(1/2+q)*a*(-a)^(1/2/
q)*LerchPhi(a*x^q,1,1/2*(1+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\,q^{3}\int\frac{1}{\left(\left(2\,a^{2}\sqrt{d}q-a^{2}\sqrt{d}\right)x^{2\,q}-2\left(2\,a\sqrt{d}q-a\sqrt{d}\right)x^{q}+2\,\sqrt{d}q-\sqrt{d}\right)\sqrt{x}}\,dx-\frac{2\left(\frac{\left(\left(2\,a\sqrt{d}q-a\sqrt{d}\right)xx^{q}-\left(2\,\sqrt{d}q-\sqrt{d}\right)x\right)\mathrm{Li}_{2}(ax^{q})}{\sqrt{x}}+\frac{2\left((2\,a\right.}{}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/(((2*a^2*sqrt(d)*q - a^2*sqrt(d))*x^(2*q) - 2*(2*a*sqrt(d
)*q - a*sqrt(d))*x^q + 2*sqrt(d)*q - sqrt(d))*sqrt(x)), x) - 2*(((2*a*sqrt(
d)*q - a*sqrt(d))*x*x^q - (2*sqrt(d)*q - sqrt(d))*x)*dilog(a*x^q)/sqrt(x) +
 2*((2*a*sqrt(d)*q^2 - a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 - sqrt(d)*q)*x)*
log(-a*x^q + 1)/sqrt(x) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 - a*sqrt(d)
*q^2)*x*x^q)/sqrt(x))/(2*d*q - (2*a*d*q - a*d)*x^q - d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}\mathrm{Li}_{2}\left(ax^{q}\right)}{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^q)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(2, a*x**q)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(ax^q)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/sqrt(d*x), x)
```

3.89 $\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{8aq^2x^q\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right),\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{2\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}+\frac{4q\log(1-ax^q)}{d\sqrt{dx}}$$

```
[Out] (-8*a*q^2*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])/
(d*(1 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[2
, a*x^q])/(d*Sqrt[d*x])
```

Rubi [A] time = 0.0599145, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{2\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}-\frac{8aq^2x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}+\frac{4q\log(1-ax^q)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] (-8*a*q^2*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])/
(d*(1 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[2
, a*x^q])/(d*Sqrt[d*x])
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} - (2q)\int \frac{\log(1-ax^q)}{(dx)^{3/2}}\,dx \\ &= \frac{4q\log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{(4aq^2)\int \frac{x^{-1+q}}{\sqrt{dx}(1-ax^q)}\,dx}{d} \\ &= \frac{4q\log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}} + \frac{(4aq^2\sqrt{x})\int \frac{x^{-\frac{3}{2}+q}}{1-ax^q}\,dx}{d\sqrt{dx}} \\ &= -\frac{8aq^2x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}} + \frac{4q\log(1-ax^q)}{d\sqrt{dx}} - \frac{2\text{Li}_2(ax^q)}{d\sqrt{dx}}\end{aligned}$$

Mathematica [C] time = 0.0290786, size = 48, normalized size = 0.49

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1+\frac{1}{2q}\\1,0,0,\frac{1}{2q}\end{array}\right)}{q(dx)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 1/(2*q)}}, -(a*x^q)
])/(q*(d*x)^(3/2)))
```

Maple [C] time = 0.224, size = 121, normalized size = 1.3

$$-\frac{1}{q}x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-4\frac{q^2\ln(1-ax^q)}{\sqrt{x}}(-a)^{-1/2q^{-1}}-2\frac{q\left(1-2q\right)\mathrm{polylog}(2,ax^q)}{\left(2q-1\right)\sqrt{x}}(-a)^{-1/2q^{-1}}-4q^2x^{q-1/2}a(-a)^{-1/2q^{-1}}\mathrm{LerchPhi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(3/2),x)
```

```
[Out] -1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-4*q^2/x^(1/2)*(-a)^(-1/2/q)*ln(1-a*
x^q)-2*q/(2*q-1)/x^(1/2)*(-a)^(-1/2/q)*(1-2*q)*polylog(2,a*x^q)-4*q^2*x^(q-
1/2)*a*(-a)^(-1/2/q)*LerchPhi(a*x^q,1,1/2*(2*q-1)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8q^3\int\frac{1}{\left(2d^{\frac{3}{2}}q+\left(2a^2d^{\frac{3}{2}}q+a^2d^{\frac{3}{2}}\right)x^{2q}-2\left(2ad^{\frac{3}{2}}q+ad^{\frac{3}{2}}\right)x^q+d^{\frac{3}{2}}\right)x^{\frac{3}{2}}}\,dx+\frac{2\left(\frac{\left(\left(2a\sqrt{d}q+a\sqrt{d}\right)xx^q-\left(2\sqrt{d}q+\sqrt{d}\right)x\right)\mathrm{Li}_2(ax^q)}{x^{\frac{3}{2}}}-\frac{2\left(\left(2a\sqrt{d}\right.\right.}{}\right.}{2d^2q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/((2*d^(3/2)*q + (2*a^2*d^(3/2)*q + a^2*d^(3/2))*x^(2*q) -
 2*(2*a*d^(3/2)*q + a*d^(3/2))*x^q + d^(3/2))*x^(3/2)), x) + 2*(((2*a*sqrt(
d)*q + a*sqrt(d))*x*x^q - (2*sqrt(d)*q + sqrt(d))*x)*dilog(a*x^q)/x^(3/2) -
 2*((2*a*sqrt(d)*q^2 + a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q^2 + sqrt(d)*q)*x)*
log(-a*x^q + 1)/x^(3/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sqrt(d)*q^3 + a*sqrt(d)
*q^2)*x*x^q)/x^(3/2))/(2*d^2*q + d^2 - (2*a*d^2*q + a*d^2)*x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}\,\mathrm{Li}_2\left(ax^q\right)}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/(d*x)^(3/2), x)
```

3.90 $\int \frac{\text{PolyLog}(2,ax^q)}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=105

$$-\frac{8aq^2x^{q-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right),\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{9d^2(3-2q)\sqrt{dx}}-\frac{2\text{PolyLog}\left(2,ax^q\right)}{3d(dx)^{3/2}}+\frac{4q\log\left(1-ax^q\right)}{9d(dx)^{3/2}}$$

```
[Out] (-8*a*q^2*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q])
/(9*d^2*(3 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(9*d*(d*x)^(3/2)) - (2*
PolyLog[2, a*x^q])/(3*d*(d*x)^(3/2))
```

Rubi [A] time = 0.0605675, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{2\text{PolyLog}\left(2,ax^q\right)}{3d(dx)^{3/2}}-\frac{8aq^2x^{q-1}\,_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{9d^2(3-2q)\sqrt{dx}}+\frac{4q\log\left(1-ax^q\right)}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] (-8*a*q^2*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q])
/(9*d^2*(3 - 2*q)*Sqrt[d*x]) + (4*q*Log[1 - a*x^q])/(9*d*(d*x)^(3/2)) - (2*
PolyLog[2, a*x^q])/(3*d*(d*x)^(3/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} - \frac{1}{3}(2q)\int \frac{\log(1-ax^q)}{(dx)^{5/2}}\,dx \\ &= \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} + \frac{\left(4aq^2\right)\int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)}\,dx}{9d} \\ &= \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}} + \frac{\left(4aq^2\sqrt{x}\right)\int \frac{x^{-\frac{5}{2}+q}}{1-ax^q}\,dx}{9d^2\sqrt{dx}} \\ &= -\frac{8aq^2x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{9d^2(3-2q)\sqrt{dx}} + \frac{4q\log(1-ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_2(ax^q)}{3d(dx)^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.0263274, size = 48, normalized size = 0.46

$$-\frac{xG_{4,4}^{1,4}\left(-ax^q\middle|\begin{array}{c}1,1,1,1+\frac{3}{2q}\\1,0,0,\frac{3}{2q}\end{array}\right)}{q(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[2, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 3/(2*q)}}, -(a*x^q)
])/(q*(d*x)^(5/2)))
```

Maple [C] time = 0.23, size = 121, normalized size = 1.2

$$-\frac{1}{q}x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{4q^2\ln(1-ax^q)}{9}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}}-2\frac{q\left(1-2/3q\right)\mathrm{polylog}(2,ax^q)}{\left(-3+2q\right)x^{3/2}}(-a)^{-3/2q^{-1}}-\frac{4q^2a}{9}x^{q-\frac{3}{2}}(-a)^{-\frac{3}{2q}}\mathrm{LerchPhi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,a*x^q)/(d*x)^(5/2),x)
```

```
[Out] -1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*ln(1-
a*x^q)-2*q/(-3+2*q)/x^(3/2)*(-a)^(-3/2/q)*(1-2/3*q)*polylog(2,a*x^q)-4/9*q^
2*x^(q-3/2)*a*(-a)^(-3/2/q)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8q^3\int\frac{1}{9\left(2d^{\frac{5}{2}}q+3d^{\frac{5}{2}}+\left(2a^2d^{\frac{5}{2}}q+3a^2d^{\frac{5}{2}}\right)x^{2q}-2\left(2ad^{\frac{5}{2}}q+3ad^{\frac{5}{2}}\right)x^q\right)x^{\frac{5}{2}}}\,dx+\frac{2\left(\frac{9\left(\left(2a\sqrt{d}q+3a\sqrt{d}\right)xx^q-\left(2\sqrt{d}q+3\sqrt{d}\right)x\right)\mathrm{Li}_2(ax^q)}{x^{\frac{5}{2}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8*q^3*integrate(1/9/((2*d^(5/2)*q + 3*d^(5/2) + (2*a^2*d^(5/2)*q + 3*a^2*d^
(5/2))*x^(2*q) - 2*(2*a*d^(5/2)*q + 3*a*d^(5/2))*x^q)*x^(5/2)), x) + 2/27*(
9*((2*a*sqrt(d)*q + 3*a*sqrt(d))*x*x^q - (2*sqrt(d)*q + 3*sqrt(d))*x)*dilog
(a*x^q)/x^(5/2) - 6*((2*a*sqrt(d)*q^2 + 3*a*sqrt(d)*q)*x*x^q - (2*sqrt(d)*q
^2 + 3*sqrt(d)*q)*x)*log(-a*x^q + 1)/x^(5/2) + 4*(2*sqrt(d)*q^3*x - (2*a*sq
rt(d)*q^3 + 3*a*sqrt(d)*q^2)*x*x^q)/x^(5/2))/(2*d^3*q + 3*d^3 - (2*a*d^3*q
+ 3*a*d^3)*x^q)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}\mathrm{Li}_2(ax^q)}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*dilog(a*x^q)/(d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x**q)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(ax^q\right)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(dilog(a*x^q)/(d*x)^(5/2), x)
```

3.91 $\int (dx)^{3/2}\text{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=125

$$-\frac{16adq^3\sqrt{dx}x^{q+2}\text{Hypergeometric2F1}\left(1,\frac{q+\frac{5}{2}}{q},\frac{1}{2}\left(\frac{5}{q}+4\right),ax^q\right)}{125(2q+5)}-\frac{4q(dx)^{5/2}\text{PolyLog}(2,ax^q)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d}$$

```
[Out] (-16*a*d*q^3*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q
)/2, a*x^q])/(125*(5 + 2*q)) - (8*q^2*(d*x)^(5/2)*Log[1 - a*x^q])/(125*d) -
 (4*q*(d*x)^(5/2)*PolyLog[2, a*x^q])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*x
^q])/(5*d)
```

Rubi [A] time = 0.0748302, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{4q(dx)^{5/2}\text{PolyLog}(2,ax^q)}{25d}+\frac{2(dx)^{5/2}\text{PolyLog}(3,ax^q)}{5d}-\frac{16adq^3\sqrt{dx}x^{q+2}\,{}_2F_1\left(1,\frac{q+\frac{5}{2}}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{125(2q+5)}-\frac{8q^2(dx)^{5/2}\log(}{125d}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

```
[Out] (-16*a*d*q^3*x^(2 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (5/2 + q)/q, (4 + 5/q
)/2, a*x^q])/(125*(5 + 2*q)) - (8*q^2*(d*x)^(5/2)*Log[1 - a*x^q])/(125*d) -
 (4*q*(d*x)^(5/2)*PolyLog[2, a*x^q])/(25*d) + (2*(d*x)^(5/2)*PolyLog[3, a*x
^q])/(5*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2}\mathrm{Li}_3\left(ax^q\right)\,dx &= \frac{2(dx)^{5/2}\mathrm{Li}_3\left(ax^q\right)}{5d}-\frac{1}{5}(2q)\int (dx)^{3/2}\mathrm{Li}_2\left(ax^q\right)\,dx\\
&= -\frac{4q(dx)^{5/2}\mathrm{Li}_2\left(ax^q\right)}{25d}+\frac{2(dx)^{5/2}\mathrm{Li}_3\left(ax^q\right)}{5d}-\frac{1}{25}\left(4q^2\right)\int (dx)^{3/2}\log\left(1-ax^q\right)\,dx\\
&= -\frac{8q^2(dx)^{5/2}\log\left(1-ax^q\right)}{125d}-\frac{4q(dx)^{5/2}\mathrm{Li}_2\left(ax^q\right)}{25d}+\frac{2(dx)^{5/2}\mathrm{Li}_3\left(ax^q\right)}{5d}-\frac{\left(8aq^3\right)\int \frac{x^{-1+q}(dx)^{5/2}}{1-ax^q}\,dx}{125d}\\
&= -\frac{8q^2(dx)^{5/2}\log\left(1-ax^q\right)}{125d}-\frac{4q(dx)^{5/2}\mathrm{Li}_2\left(ax^q\right)}{25d}+\frac{2(dx)^{5/2}\mathrm{Li}_3\left(ax^q\right)}{5d}-\frac{\left(8adq^3\sqrt{dx}\right)\int \frac{x^{\frac{3}{2}+q}}{1-ax^q}\,dx}{125\sqrt{x}}\\
&= -\frac{16adq^3x^{2+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{5}{2}+q}{q};\frac{1}{2}\left(4+\frac{5}{q}\right);ax^q\right)}{125(5+2q)}-\frac{8q^2(dx)^{5/2}\log\left(1-ax^q\right)}{125d}-\frac{4q(dx)^{5/2}\mathrm{Li}_2\left(ax^q\right)}{25d}+
\end{aligned}$$

Mathematica [C] time = 0.0297026, size = 50, normalized size = 0.4

$$-\frac{x(dx)^{3/2}G_{5,5}^{1,5}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1-\frac{5}{2q}\\ 1,0,0,0,-\frac{5}{2q}\end{array}\right.\right)}{q}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^(3/2)*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*(d*x)^(3/2)*MeijerG[{{1, 1, 1, 1, 1 - 5/(2*q)}, {}}, {{1}, {0, 0, 0, -
5/(2*q)}}, -(a*x^q)])/q)
```

Maple [C] time = 0.379, size = 145, normalized size = 1.2

$$-\frac{1}{q}(dx)^{\frac{3}{2}}(-a)^{-\frac{5}{2q}}\left(\frac{8q^3\ln(1-ax^q)}{125}x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}+\frac{4q^2\mathrm{polylog}(2,ax^q)}{25}x^{\frac{5}{2}}(-a)^{\frac{5}{2q}}-2\frac{qx^{5/2}\left(1+2/5q\right)\mathrm{polylog}(3,ax^q)}{5+2q}(-a)^{5/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*polylog(3,a*x^q),x)
```

```
[Out] -(d*x)^(3/2)/x^(3/2)*(-a)^(-5/2/q)/q*(8/125*q^3*x^(5/2)*(-a)^(5/2/q)*ln(1-a
*x^q)+4/25*q^2*x^(5/2)*(-a)^(5/2/q)*polylog(2,a*x^q)-2*q/(5+2*q)*x^(5/2)*(-
a)^(5/2/q)*(1+2/5*q)*polylog(3,a*x^q)+8/125*q^3*x^(5/2+q)*a*(-a)^(5/2/q)*Le
rchPhi(a*x^q,1,1/2*(5+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,d^{\frac{3}{2}}q^4\int\frac{x^{\frac{3}{2}}}{125\left(a^2\left(2q-5\right)x^{2q}-2a\left(2q-5\right)x^q+2q-5\right)}\,dx-\frac{2\left(50\left(\left(2q^2-5q\right)ad^{\frac{3}{2}}xx^q-\left(2q^2-5q\right)d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}\mathrm{Li}_2\left(ax^q\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*d^(3/2)*q^4*integrate(1/125*x^(3/2)/(a^2*(2*q - 5)*x^(2*q) - 2*a*(2*q -
 5)*x^q + 2*q - 5), x) - 2/625*(50*((2*q^2 - 5*q)*a*d^(3/2)*x*x^q - (2*q^2
- 5*q)*d^(3/2)*x)*x^(3/2)*dilog(a*x^q) + 20*((2*q^3 - 5*q^2)*a*d^(3/2)*x*x^
q - (2*q^3 - 5*q^2)*d^(3/2)*x)*x^(3/2)*log(-a*x^q + 1) - 125*(a*d^(3/2)*(2*
q - 5)*x*x^q - d^(3/2)*(2*q - 5)*x)*x^(3/2)*polylog(3, a*x^q) + 8*(2*d^(3/2
)*q^4*x - (2*q^4 - 5*q^3)*a*d^(3/2)*x*x^q)*x^(3/2))/(a*(2*q - 5)*x^q - 2*q
+ 5)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{dx}dx\mathrm{polylog}\left(3,ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x*polylog(3, a*x^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*polylog(3,a*x**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\operatorname{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*polylog(3, a*x^q), x)
```

3.92 $\int \sqrt{dx}\textbf{PolyLog}\left(3, ax^q\right)\,dx$

Optimal. Leaf size=124

$$-\frac{16aq^3\sqrt{dx}x^{q+1}\text{Hypergeometric2F1}\left(1,\frac{q+\frac{3}{2}}{q},\frac{1}{2}\left(\frac{3}{q}+4\right),ax^q\right)}{27(2q+3)}-\frac{4q(dx)^{3/2}\text{PolyLog}\left(2,ax^q\right)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^q\right)}{3d}$$

```
[Out] (-16*a*q^3*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/
2, a*x^q])/(27*(3 + 2*q)) - (8*q^2*(d*x)^(3/2)*Log[1 - a*x^q])/(27*d) - (4*
q*(d*x)^(3/2)*PolyLog[2, a*x^q])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^q])/
(3*d)
```

Rubi [A] time = 0.0703269, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{4q(dx)^{3/2}\text{PolyLog}\left(2,ax^q\right)}{9d}+\frac{2(dx)^{3/2}\text{PolyLog}\left(3,ax^q\right)}{3d}-\frac{16aq^3\sqrt{dx}x^{q+1}\,{}_2F_1\left(1,\frac{q+\frac{3}{2}}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{27(2q+3)}-\frac{8q^2(dx)^{3/2}\log(1}{27d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

```
[Out] (-16*a*q^3*x^(1 + q)*Sqrt[d*x]*Hypergeometric2F1[1, (3/2 + q)/q, (4 + 3/q)/
2, a*x^q])/(27*(3 + 2*q)) - (8*q^2*(d*x)^(3/2)*Log[1 - a*x^q])/(27*d) - (4*
q*(d*x)^(3/2)*PolyLog[2, a*x^q])/(9*d) + (2*(d*x)^(3/2)*PolyLog[3, a*x^q])/
(3*d)
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{dx}\,\mathrm{Li}_3(ax^q)\,dx &= \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{1}{3}(2q)\int \sqrt{dx}\,\mathrm{Li}_2(ax^q)\,dx \\
&= -\frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{1}{9}\left(4q^2\right)\int \sqrt{dx}\log(1-ax^q)\,dx \\
&= -\frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{\left(8aq^3\right)\int \frac{x^{-1+q}(dx)^{3/2}}{1-ax^q}\,dx}{27d} \\
&= -\frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(dx)^{3/2}\mathrm{Li}_3(ax^q)}{3d} - \frac{\left(8aq^3\sqrt{dx}\right)\int \frac{x^{\frac{1}{2}+q}}{1-ax^q}\,dx}{27\sqrt{x}} \\
&= -\frac{16aq^3x^{1+q}\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{3}{2}+q}{q};\frac{1}{2}\left(4+\frac{3}{q}\right);ax^q\right)}{27(3+2q)} - \frac{8q^2(dx)^{3/2}\log(1-ax^q)}{27d} - \frac{4q(dx)^{3/2}\mathrm{Li}_2(ax^q)}{9d} + \frac{2(
\end{aligned}
$$

Mathematica [C] time = 0.0262881, size = 50, normalized size = 0.4

$$
-\frac{x\sqrt{dx}\,G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{3}{2q}\\1,0,0,0,-\frac{3}{2q}\end{array}\right)}{q}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*x]*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*Sqrt[d*x]*MeijerG[{{1, 1, 1, 1, 1 - 3/(2*q)}, {}}, {{1}, {0, 0, 0, -3/
(2*q)}}, -(a*x^q)])/q)
```

Maple [C] time = 0.381, size = 145, normalized size = 1.2

$$-\frac{1}{q}\sqrt{dx}\,(-a)^{-\frac{3}{2q}}\left(\frac{8\,q^3\ln(1-ax^q)}{27}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}+\frac{4\,q^2\mathrm{polylog}(2,ax^q)}{9}x^{\frac{3}{2}}(-a)^{\frac{3}{2q}}-2\,\frac{qx^{3/2}\left(1+2/3\,q\right)\mathrm{polylog}(3,ax^q)}{3+2\,q}(-a)^{3/2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(1/2)*polylog(3,a*x^q),x)
```

```
[Out] -(d*x)^(1/2)/x^(1/2)*(-a)^(-3/2/q)/q*(8/27*q^3*x^(3/2)*(-a)^(3/2/q)*ln(1-a*
x^q)+4/9*q^2*x^(3/2)*(-a)^(3/2/q)*polylog(2,a*x^q)-2*q/(3+2*q)*x^(3/2)*(-a)
^(3/2/q)*(1+2/3*q)*polylog(3,a*x^q)+8/27*q^3*x^(3/2+q)*a*(-a)^(3/2/q)*Lerch
Phi(a*x^q,1,1/2*(3+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,\sqrt{d}q^4\int\frac{\sqrt{x}}{27\left(a^2\left(2\,q-3\right)x^{2\,q}-2\,a\left(2\,q-3\right)x^q+2\,q-3\right)}\,dx-\frac{2\left(18\left(\left(2\,q^2-3\,q\right)a\sqrt{d}xx^q-\left(2\,q^2-3\,q\right)\sqrt{d}x\right)\sqrt{x}\mathrm{Li}_2\left(ax^q\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] -16*sqrt(d)*q^4*integrate(1/27*sqrt(x)/(a^2*(2*q - 3)*x^(2*q) - 2*a*(2*q -
3)*x^q + 2*q - 3), x) - 2/81*(18*((2*q^2 - 3*q)*a*sqrt(d)*x*x^q - (2*q^2 -
3*q)*sqrt(d)*x)*sqrt(x)*dilog(a*x^q) + 12*((2*q^3 - 3*q^2)*a*sqrt(d)*x*x^q
- (2*q^3 - 3*q^2)*sqrt(d)*x)*sqrt(x)*log(-a*x^q + 1) - 27*(a*sqrt(d)*(2*q -
 3)*x*x^q - sqrt(d)*(2*q - 3)*x)*sqrt(x)*polylog(3, a*x^q) + 8*(2*sqrt(d)*q
^4*x - (2*q^4 - 3*q^3)*a*sqrt(d)*x*x^q)*sqrt(x))/(a*(2*q - 3)*x^q - 2*q + 3
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\text{polylog}\left(3,ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,\text{Li}_3\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)*polylog(3,a*x**q),x)
```

```
[Out] Integral(sqrt(d*x)*polylog(3, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\text{Li}_3(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*polylog(3, a*x^q), x)
```

3.93 $\int \frac{\textbf{PolyLog}(3,ax^q)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=115

$$-\frac{16aq^3\sqrt{dx}x^q\text{Hypergeometric2F1}\left(1,\frac{q+\frac{1}{2}}{q},\frac{1}{2}\left(\frac{1}{q}+4\right),ax^q\right)}{d(2q+1)}-\frac{4q\sqrt{dx}\text{PolyLog}\left(2,ax^q\right)}{d}+\frac{2\sqrt{dx}\text{PolyLog}\left(3,ax^q\right)}{d}-\frac{8q^2\cdot}{}$$

```
[Out] (-16*a*q^3*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2,
a*x^q])/(d*(1 + 2*q)) - (8*q^2*Sqrt[d*x]*Log[1 - a*x^q])/d - (4*q*Sqrt[d*x]
*PolyLog[2, a*x^q])/d + (2*Sqrt[d*x]*PolyLog[3, a*x^q])/d
```

Rubi [A] time = 0.0690147, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{4q\sqrt{dx}\text{PolyLog}\left(2,ax^q\right)}{d}+\frac{2\sqrt{dx}\text{PolyLog}\left(3,ax^q\right)}{d}-\frac{16aq^3\sqrt{dx}x^q\,{}_2F_1\left(1,\frac{q+\frac{1}{2}}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(2q+1)}-\frac{8q^2\sqrt{dx}\log\left(1-ax^q\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```


```
[Out] (-16*a*q^3*x^q*Sqrt[d*x]*Hypergeometric2F1[1, (1/2 + q)/q, (4 + q^(-1))/2,
a*x^q])/(d*(1 + 2*q)) - (8*q^2*Sqrt[d*x]*Log[1 - a*x^q])/d - (4*q*Sqrt[d*x]
*PolyLog[2, a*x^q])/d + (2*Sqrt[d*x]*PolyLog[3, a*x^q])/d
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\mathrm{Li}_3(ax^q)}{\sqrt{dx}}\,dx &= \frac{2\sqrt{dx}\,\mathrm{Li}_3(ax^q)}{d} - (2q)\int \frac{\mathrm{Li}_2(ax^q)}{\sqrt{dx}}\,dx \\
&= -\frac{4q\sqrt{dx}\,\mathrm{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\mathrm{Li}_3(ax^q)}{d} - \left(4q^2\right)\int \frac{\log(1-ax^q)}{\sqrt{dx}}\,dx \\
&= -\frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\mathrm{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\mathrm{Li}_3(ax^q)}{d} - \frac{\left(8aq^3\right)\int \frac{x^{-1+q}\sqrt{dx}}{1-ax^q}\,dx}{d} \\
&= -\frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\mathrm{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\mathrm{Li}_3(ax^q)}{d} - \frac{\left(8aq^3\sqrt{dx}\right)\int \frac{x^{-\frac{1}{2}+q}}{1-ax^q}\,dx}{d\sqrt{x}} \\
&= -\frac{16aq^3x^q\sqrt{dx}\,{}_2F_1\left(1,\frac{\frac{1}{2}+q}{q};\frac{1}{2}\left(4+\frac{1}{q}\right);ax^q\right)}{d(1+2q)} - \frac{8q^2\sqrt{dx}\log(1-ax^q)}{d} - \frac{4q\sqrt{dx}\,\mathrm{Li}_2(ax^q)}{d} + \frac{2\sqrt{dx}\,\mathrm{Li}_3(ax^q)}{d}
\end{aligned}$$

Mathematica [C] time = 0.02007, size = 50, normalized size = 0.43

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{1}{2q}\\1,0,0,0,-\frac{1}{2q}\end{array}\right)}{q\sqrt{dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/Sqrt[d*x],x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 - 1/(2*q)}, {}}, {{1}, {0, 0, 0, -1/(2*q)}}, -
(a*x^q)])/(q*Sqrt[d*x]))
```

Maple [C] time = 0.372, size = 133, normalized size = 1.2

$$-\frac{1}{q}\sqrt{x}\,(-a)^{-\frac{1}{2q}}\left(8\,q^3\sqrt{x}\,(-a)^{1/2\,q^{-1}}\ln(1-ax^q)+4\,q^2\sqrt{x}\,(-a)^{1/2\,q^{-1}}\operatorname{polylog}(2,ax^q)-2\,q\sqrt{x}\,(-a)^{1/2\,q^{-1}}\operatorname{polylog}(3,ax^q)+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(1/2),x)
```

```
[Out] -1/(d*x)^(1/2)*x^(1/2)*(-a)^(-1/2/q)/q*(8*q^3*x^(1/2)*(-a)^(1/2/q)*ln(1-a*x
^q)+4*q^2*x^(1/2)*(-a)^(1/2/q)*polylog(2,a*x^q)-2*q*x^(1/2)*(-a)^(1/2/q)*po
lylog(3,a*x^q)+8*q^3*x^(1/2+q)*a*(-a)^(1/2/q)*LerchPhi(a*x^q,1,1/2*(1+2*q)/
q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,q^4\int\frac{1}{\left(a^2\sqrt{d}\left(2\,q-1\right)x^{2\,q}-2\,a\sqrt{d}\left(2\,q-1\right)x^q+\sqrt{d}\left(2\,q-1\right)\right)\sqrt{x}}\,dx-\frac{2\left(\frac{2\left(\left(2\,q^2-q\right)axx^q-\left(2\,q^2-q\right)x\right)\mathrm{Li}_2(ax^q)}{\sqrt{x}}+\frac{4\left(\left(2\,q^3-q^2\right)axx^q-\right.}{}\right.}{a\sqrt{d}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -16*q^4*integrate(1/((a^2*sqrt(d)*(2*q - 1)*x^(2*q) - 2*a*sqrt(d)*(2*q - 1)
*x^q + sqrt(d)*(2*q - 1))*sqrt(x)), x) - 2*(2*((2*q^2 - q)*a*x*x^q - (2*q^2
 - q)*x)*dilog(a*x^q)/sqrt(x) + 4*((2*q^3 - q^2)*a*x*x^q - (2*q^3 - q^2)*x)
*log(-a*x^q + 1)/sqrt(x) - (a*(2*q - 1)*x*x^q - (2*q - 1)*x)*polylog(3, a*x
^q)/sqrt(x) + 8*(2*q^4*x - (2*q^4 - q^3)*a*x*x^q)/sqrt(x))/(a*sqrt(d)*(2*q
- 1)*x^q - sqrt(d)*(2*q - 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\,\text{polylog}\left(3, ax^q\right)}{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3\left(ax^q\right)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(1/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/sqrt(d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/sqrt(d*x), x)
```

3.94 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{3/2}} \, dx$

Optimal. Leaf size=119

$$-\frac{16aq^3x^q\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right),\frac{1}{2}\left(4-\frac{1}{q}\right),ax^q\right)}{d(1-2q)\sqrt{dx}}-\frac{4q\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}+\frac{8q^2\log(1-}{d\sqrt{dx}}$$

```
[Out] (-16*a*q^3*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])
/(d*(1 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (4*q*Poly
Log[2, a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^q])/(d*Sqrt[d*x])
```

Rubi [A] time = 0.0780859, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{4q\text{PolyLog}(2,ax^q)}{d\sqrt{dx}}-\frac{2\text{PolyLog}(3,ax^q)}{d\sqrt{dx}}-\frac{16aq^3x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}+\frac{8q^2\log(1-ax^q)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```


```
[Out] (-16*a*q^3*x^q*Hypergeometric2F1[1, (2 - q^(-1))/2, (4 - q^(-1))/2, a*x^q])
/(d*(1 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(d*Sqrt[d*x]) - (4*q*Poly
Log[2, a*x^q])/(d*Sqrt[d*x]) - (2*PolyLog[3, a*x^q])/(d*Sqrt[d*x])
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\text{Li}_3\left(ax^q\right)}{(dx)^{3/2}}\,dx &= -\frac{2\text{Li}_3\left(ax^q\right)}{d\sqrt{dx}}+(2q)\int \frac{\text{Li}_2\left(ax^q\right)}{(dx)^{3/2}}\,dx\\
&= -\frac{4q\text{Li}_2\left(ax^q\right)}{d\sqrt{dx}}-\frac{2\text{Li}_3\left(ax^q\right)}{d\sqrt{dx}}-\left(4q^2\right)\int \frac{\log\left(1-ax^q\right)}{(dx)^{3/2}}\,dx\\
&= \frac{8q^2\log\left(1-ax^q\right)}{d\sqrt{dx}}-\frac{4q\text{Li}_2\left(ax^q\right)}{d\sqrt{dx}}-\frac{2\text{Li}_3\left(ax^q\right)}{d\sqrt{dx}}+\frac{\left(8aq^3\right)\int \frac{x^{-1+q}}{\sqrt{dx}\left(1-ax^q\right)}\,dx}{d}\\
&= \frac{8q^2\log\left(1-ax^q\right)}{d\sqrt{dx}}-\frac{4q\text{Li}_2\left(ax^q\right)}{d\sqrt{dx}}-\frac{2\text{Li}_3\left(ax^q\right)}{d\sqrt{dx}}+\frac{\left(8aq^3\sqrt{x}\right)\int \frac{x^{-\frac{3}{2}+q}}{1-ax^q}\,dx}{d\sqrt{dx}}\\
&= -\frac{16aq^3x^q\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{1}{q}\right);\frac{1}{2}\left(4-\frac{1}{q}\right);ax^q\right)}{d(1-2q)\sqrt{dx}}+\frac{8q^2\log\left(1-ax^q\right)}{d\sqrt{dx}}-\frac{4q\text{Li}_2\left(ax^q\right)}{d\sqrt{dx}}-\frac{2\text{Li}_3\left(ax^q\right)}{d\sqrt{dx}}
\end{aligned}
$$

Mathematica [C] time = 0.0248691, size = 50, normalized size = 0.42

$$
-\frac{xG_{5,5}^{1,5}\left(-ax^q\left|
\begin{array}{c}
1,1,1,1,1+\frac{1}{2q}\\
1,0,0,0,\frac{1}{2q}
\end{array}\right.\right)}{q(dx)^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/(d*x)^(3/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 + 1/(2*q)}, {}}, {{1}, {0, 0, 0, 1/(2*q)}}, -(
a*x^q)])/(q*(d*x)^(3/2)))
```

Maple [C] time = 0.372, size = 145, normalized size = 1.2

$$-\frac{1}{q}x^{\frac{3}{2}}(-a)^{\frac{1}{2q}}\left(-8\frac{q^3\ln(1-ax^q)}{\sqrt{x}}(-a)^{-1/2q^{-1}}+4\frac{q^2\mathrm{polylog}(2,ax^q)}{\sqrt{x}}(-a)^{-1/2q^{-1}}-2\frac{q(1-2q)\mathrm{polylog}(3,ax^q)}{(2q-1)\sqrt{x}}(-a)^{-1/2q^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(3/2),x)
```

```
[Out] -1/(d*x)^(3/2)*x^(3/2)*(-a)^(1/2/q)/q*(-8*q^3/x^(1/2)*(-a)^(-1/2/q)*ln(1-a*
x^q)+4*q^2/x^(1/2)*(-a)^(-1/2/q)*polylog(2,a*x^q)-2*q/(2*q-1)/x^(1/2)*(-a)^
(-1/2/q)*(1-2*q)*polylog(3,a*x^q)-8*q^3*x^(q-1/2)*a*(-a)^(-1/2/q)*LerchPhi(
a*x^q,1,1/2*(2*q-1)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16q^4\int\frac{1}{\left(a^2d^{\frac{3}{2}}(2q+1)x^{2q}-2ad^{\frac{3}{2}}(2q+1)x^q+d^{\frac{3}{2}}(2q+1)\right)x^{\frac{3}{2}}}\,dx-\frac{2\left(\frac{2\left((2q^2+q)axx^q-(2q^2+q)x\right)\mathrm{Li}_2(ax^q)}{x^{\frac{3}{2}}}-\frac{4\left((2q^3+q^2)axx^q-(2q^3+\right.}{x^{\frac{3}{2}}}\right.}{ad^{\frac{3}{2}}(2q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 16*q^4*integrate(1/((a^2*d^(3/2)*(2*q + 1)*x^(2*q) - 2*a*d^(3/2)*(2*q + 1)*
x^q + d^(3/2)*(2*q + 1))*x^(3/2)), x) - 2*(2*((2*q^2 + q)*a*x*x^q - (2*q^2
+ q)*x)*dilog(a*x^q)/x^(3/2) - 4*((2*q^3 + q^2)*a*x*x^q - (2*q^3 + q^2)*x)*
log(-a*x^q + 1)/x^(3/2) + (a*(2*q + 1)*x*x^q - (2*q + 1)*x)*polylog(3, a*x^
q)/x^(3/2) + 8*(2*q^4*x - (2*q^4 + q^3)*a*x*x^q)/x^(3/2))/(a*d^(3/2)*(2*q +
 1)*x^q - d^(3/2)*(2*q + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx}\,\mathrm{polylog}(3,ax^q)}{d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(3/2),x)
```

```
[Out] Integral(polylog(3, a*x**q)/(d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ax^q)}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/(d*x)^(3/2), x)
```

3.95 $\int \frac{\text{PolyLog}(3,ax^q)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{16aq^3x^{q-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right),\frac{1}{2}\left(4-\frac{3}{q}\right),ax^q\right)}{27d^2(3-2q)\sqrt{dx}}-\frac{4q\text{PolyLog}\left(2,ax^q\right)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}\left(3,ax^q\right)}{3d(dx)^{3/2}}+\frac{8q^2\log\left(1\right.}{27d(dx}$$

```
[Out] (-16*a*q^3*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]
)/(27*d^2*(3 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(27*d*(d*x)^(3/2))
- (4*q*PolyLog[2, a*x^q])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x^q])/(3*d*(d
*x)^(3/2))
```

Rubi [A] time = 0.0749586, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6591, 2455, 20, 364}

$$-\frac{4q\text{PolyLog}\left(2,ax^q\right)}{9d(dx)^{3/2}}-\frac{2\text{PolyLog}\left(3,ax^q\right)}{3d(dx)^{3/2}}-\frac{16aq^3x^{q-1}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{27d^2(3-2q)\sqrt{dx}}+\frac{8q^2\log\left(1-ax^q\right)}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] (-16*a*q^3*x^(-1 + q)*Hypergeometric2F1[1, (2 - 3/q)/2, (4 - 3/q)/2, a*x^q]
)/(27*d^2*(3 - 2*q)*Sqrt[d*x]) + (8*q^2*Log[1 - a*x^q])/(27*d*(d*x)^(3/2))
- (4*q*PolyLog[2, a*x^q])/(9*d*(d*x)^(3/2)) - (2*PolyLog[3, a*x^q])/(3*d*(d
*x)^(3/2))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_3(ax^q)}{(dx)^{5/2}}\,dx &= -\frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{1}{3}(2q)\int \frac{\text{Li}_2(ax^q)}{(dx)^{5/2}}\,dx \\
&= -\frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} - \frac{1}{9}\left(4q^2\right)\int \frac{\log(1-ax^q)}{(dx)^{5/2}}\,dx \\
&= \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{\left(8aq^3\right)\int \frac{x^{-1+q}}{(dx)^{3/2}(1-ax^q)}\,dx}{27d} \\
&= \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}} + \frac{\left(8aq^3\sqrt{x}\right)\int \frac{x^{-\frac{5}{2}+q}}{1-ax^q}\,dx}{27d^2\sqrt{dx}} \\
&= -\frac{16aq^3x^{-1+q}\,{}_2F_1\left(1,\frac{1}{2}\left(2-\frac{3}{q}\right);\frac{1}{2}\left(4-\frac{3}{q}\right);ax^q\right)}{27d^2(3-2q)\sqrt{dx}} + \frac{8q^2\log(1-ax^q)}{27d(dx)^{3/2}} - \frac{4q\text{Li}_2(ax^q)}{9d(dx)^{3/2}} - \frac{2\text{Li}_3(ax^q)}{3d(dx)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.0240634, size = 50, normalized size = 0.39

$$-\frac{xG_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1+\frac{3}{2q}\\1,0,0,0,\frac{3}{2q}\end{array}\right)}{q(dx)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[PolyLog[3, a*x^q]/(d*x)^(5/2),x]
```

```
[Out] -((x*MeijerG[{{1, 1, 1, 1, 1 + 3/(2*q)}, {}}, {{1}, {0, 0, 0, 3/(2*q)}}, -(
a*x^q)])/(q*(d*x)^(5/2)))
```

Maple [C] time = 0.381, size = 145, normalized size = 1.1

$$-\frac{1}{q}x^{\frac{5}{2}}(-a)^{\frac{3}{2q}}\left(-\frac{8q^3\ln(1-ax^q)}{27}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}}+\frac{4q^2\mathrm{polylog}(2,ax^q)}{9}(-a)^{-\frac{3}{2q}}x^{-\frac{3}{2}}-2\frac{q\left(1-2/3q\right)\mathrm{polylog}(3,ax^q)}{\left(-3+2q\right)x^{3/2}}(-a)^{-3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,a*x^q)/(d*x)^(5/2),x)
```

```
[Out] -1/(d*x)^(5/2)*x^(5/2)*(-a)^(3/2/q)/q*(-8/27*q^3/x^(3/2)*(-a)^(-3/2/q)*ln(1
-a*x^q)+4/9*q^2/x^(3/2)*(-a)^(-3/2/q)*polylog(2,a*x^q)-2*q/(-3+2*q)/x^(3/2)
*(-a)^(-3/2/q)*(1-2/3*q)*polylog(3,a*x^q)-8/27*q^3*x^(q-3/2)*a*(-a)^(-3/2/q
)*LerchPhi(a*x^q,1,1/2*(-3+2*q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$16q^4\int\frac{1}{27\left(a^2d^{\frac{5}{2}}\left(2q+3\right)x^{2q}-2ad^{\frac{5}{2}}\left(2q+3\right)x^q+d^{\frac{5}{2}}\left(2q+3\right)\right)x^{\frac{5}{2}}}\,dx-\frac{2\left(\frac{18\left(\left(2q^2+3q\right)axx^q-\left(2q^2+3q\right)x\right)\mathrm{Li}_2(ax^q)}{x^{\frac{5}{2}}}-\frac{12\left(\left(2q^3+3q^2\right)\right.}{}\right.}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16*q^4*integrate(1/27/((a^2*d^(5/2)*(2*q + 3)*x^(2*q) - 2*a*d^(5/2)*(2*q +
3)*x^q + d^(5/2)*(2*q + 3))*x^(5/2)), x) - 2/81*(18*((2*q^2 + 3*q)*a*x*x^q
- (2*q^2 + 3*q)*x)*dilog(a*x^q)/x^(5/2) - 12*((2*q^3 + 3*q^2)*a*x*x^q - (2*
q^3 + 3*q^2)*x)*log(-a*x^q + 1)/x^(5/2) + 27*(a*(2*q + 3)*x*x^q - (2*q + 3)
*x)*polylog(3, a*x^q)/x^(5/2) + 8*(2*q^4*x - (2*q^4 + 3*q^3)*a*x*x^q)/x^(5/
2))/(a*d^(5/2)*(2*q + 3)*x^q - d^(5/2)*(2*q + 3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}\,\text{polylog}\,(3,ax^q)}{d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*polylog(3, a*x^q)/(d^3*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x**q)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ax^q)}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,a*x^q)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, a*x^q)/(d*x)^(5/2), x)
```

3.96 $\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right) - x\text{PolyLog}\left(\frac{1}{2}, ax\right) + x\text{PolyLog}\left(\frac{3}{2}, ax\right)$$

```
[Out] -(x*PolyLog[1/2, a*x]) + x*PolyLog[3/2, a*x] + Unintegrable[PolyLog[-1/2, a
*x], x]
```

Rubi [A] time = 0.0076448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[3/2, a*x],x]
```

```
[Out] -(x*PolyLog[1/2, a*x]) + x*PolyLog[3/2, a*x] + Defer[Int][PolyLog[-1/2, a*x
], x]
```

Rubi steps

$$\begin{aligned} \int \text{Li}_{\frac{3}{2}}(ax)\,dx &= x\text{Li}_{\frac{3}{2}}(ax) - \int \text{Li}_{\frac{1}{2}}(ax)\,dx \\ &= -x\text{Li}_{\frac{1}{2}}(ax) + x\text{Li}_{\frac{3}{2}}(ax) + \int \text{Li}_{-\frac{1}{2}}(ax)\,dx \end{aligned}$$

Mathematica [A] time = 0.0066395, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[3/2, a*x],x]
```

```
[Out] Integrate[PolyLog[3/2, a*x], x]
```

Maple [A] time = 0.043, size = 0, normalized size = 0.

$$\int \mathrm{polylog}\left(\frac{3}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3/2,a*x),x)
```

```
[Out] int(polylog(3/2,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(3/2, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{polylog}\left(\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(3/2, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x)
```

```
[Out] Integral(polylog(3/2, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(3/2, a*x), x)
```

3.97 $\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$

Optimal. Leaf size=21

$$x\text{PolyLog}\left(\frac{1}{2}, ax\right) - \text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] x*PolyLog[1/2, a*x] - Unintegrable[PolyLog[-1/2, a*x], x]
```

Rubi [A] time = 0.0047659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[1/2, a*x],x]
```

```
[Out] x*PolyLog[1/2, a*x] - Defer[Int][PolyLog[-1/2, a*x], x]
```

Rubi steps

$$\int \text{Li}_{\frac{1}{2}}(ax)\,dx = x\text{Li}_{\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax)\,dx$$

Mathematica [A] time = 0.0063639, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[1/2, a*x],x]
```

```
[Out] Integrate[PolyLog[1/2, a*x], x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \mathrm{polylog}\left(\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(1/2,a*x),x)
```

```
[Out] int(polylog(1/2,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{\frac{1}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(1/2, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{polylog}\left(\frac{1}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(1/2, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{\frac{1}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x)
```

```
[Out] Integral(polylog(1/2, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_{\frac{1}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(1/2, a*x), x)
```

3.98 $\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$

Optimal. Leaf size=9

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] Unintegrable[PolyLog[-1/2, a*x], x]
```

Rubi [A] time = 0.0023648, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[-1/2, a*x],x]
```

```
[Out] Defer[Int][PolyLog[-1/2, a*x], x]
```

Rubi steps

$$\int \text{Li}_{-\frac{1}{2}}(ax)\, dx = \int \text{Li}_{-\frac{1}{2}}(ax)\, dx$$

Mathematica [A] time = 0.0065787, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{1}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[-1/2, a*x],x]
```

```
[Out] Integrate[PolyLog[-1/2, a*x], x]
```

Maple [A] time = 0.048, size = 0, normalized size = 0.

$$\int \text{polylog}\left(-\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-1/2,a*x),x)
```

```
[Out] int(polylog(-1/2,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_{-\frac{1}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-1/2, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{polylog}\left(-\frac{1}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-1/2, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_{-\frac{1}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x)
```

```
[Out] Integral(polylog(-1/2, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{1}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-1/2, a*x), x)
```

3.99 $\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$

Optimal. Leaf size=21

$$x\text{PolyLog}\left(-\frac{1}{2}, ax\right) - \text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right)$$

```
[Out] x*PolyLog[-1/2, a*x] - Unintegrable[PolyLog[-1/2, a*x], x]
```

Rubi [A] time = 0.0047009, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[-3/2, a*x],x]
```

```
[Out] x*PolyLog[-1/2, a*x] - Defer[Int][PolyLog[-1/2, a*x], x]
```

Rubi steps

$$\int \text{Li}_{-\frac{3}{2}}(ax)\, dx = x\text{Li}_{-\frac{1}{2}}(ax) - \int \text{Li}_{-\frac{1}{2}}(ax)\, dx$$

Mathematica [A] time = 0.0064057, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{3}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[-3/2, a*x],x]
```

```
[Out] Integrate[PolyLog[-3/2, a*x], x]
```

Maple [A] time = 0.062, size = 0, normalized size = 0.

$$\int \operatorname{polylog}\left(-\frac{3}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-3/2,a*x),x)
```

```
[Out] int(polylog(-3/2,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-3/2, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-3/2, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{-\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x)
```

```
[Out] Integral(polylog(-3/2, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-3/2, a*x), x)
```

3.100 $\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\text{PolyLog}\left(-\frac{1}{2}, ax\right), x\right) + x\text{PolyLog}\left(-\frac{3}{2}, ax\right) - x\text{PolyLog}\left(-\frac{1}{2}, ax\right)$$

```
[Out] x*PolyLog[-3/2, a*x] - x*PolyLog[-1/2, a*x] + Unintegrable[PolyLog[-1/2, a*
x], x]
```

Rubi [A] time = 0.007823, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[-5/2, a*x],x]
```

```
[Out] x*PolyLog[-3/2, a*x] - x*PolyLog[-1/2, a*x] + Defer[Int][PolyLog[-1/2, a*x]
, x]
```

Rubi steps

$$\begin{aligned}\int \text{Li}_{-\frac{5}{2}}(ax)\,dx &= x\text{Li}_{-\frac{3}{2}}(ax) - \int \text{Li}_{-\frac{3}{2}}(ax)\,dx\\ &= x\text{Li}_{-\frac{3}{2}}(ax) - x\text{Li}_{-\frac{1}{2}}(ax) + \int \text{Li}_{-\frac{1}{2}}(ax)\,dx\end{aligned}$$

Mathematica [A] time = 0.0071344, size = 0, normalized size = 0.

$$\int \text{PolyLog}\left(-\frac{5}{2}, ax\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[-5/2, a*x],x]
```

```
[Out] Integrate[PolyLog[-5/2, a*x], x]
```

Maple [A] time = 0.047, size = 0, normalized size = 0.

$$\int \mathrm{polylog}\left(-\frac{5}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-5/2,a*x),x)
```

```
[Out] int(polylog(-5/2,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{5}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-5/2, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{polylog}\left(-\frac{5}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-5/2, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x)
```

```
[Out] Integral(polylog(-5/2, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{5}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-5/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-5/2, a*x), x)
```

3.101 $\int \left(\text{PolyLog}\left(-\frac{3}{2}, ax\right) + \text{PolyLog}\left(-\frac{1}{2}, ax\right)\right) dx$

Optimal. Leaf size=9

$$x\text{PolyLog}\left(-\frac{1}{2}, ax\right)$$

```
[Out] x*PolyLog[-1/2, a*x]
```

Rubi [A] time = 0.0079878, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6587}

$$x\text{PolyLog}\left(-\frac{1}{2}, ax\right)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]
```

```
[Out] x*PolyLog[-1/2, a*x]
```

Rule 6587

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[(x*PolyL
og[n + 1, a*(b*x^p)^q])/(p*q), x] - Dist[1/(p*q), Int[PolyLog[n + 1, a*(b*x
^p)^q], x], x] /; FreeQ[{a, b, p, q}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \left(\text{Li}_{-\frac{3}{2}}(ax) + \text{Li}_{-\frac{1}{2}}(ax)\right) dx &= \int \text{Li}_{-\frac{3}{2}}(ax)\, dx + \int \text{Li}_{-\frac{1}{2}}(ax)\, dx \\ &= x\text{Li}_{-\frac{1}{2}}(ax)\end{aligned}$$

Mathematica [F] time = 0.0069411, size = 0, normalized size = 0.

$$\int \left(\text{PolyLog}\left(-\frac{3}{2}, ax\right) + \text{PolyLog}\left(-\frac{1}{2}, ax\right)\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x],x]
```

```
[Out] Integrate[PolyLog[-3/2, a*x] + PolyLog[-1/2, a*x], x]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \operatorname{polylog}\left(-\frac{3}{2}, ax\right) + \operatorname{polylog}\left(-\frac{1}{2}, ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

```
[Out] int(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_{-\frac{1}{2}}(ax) + \mathrm{Li}_{-\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(-\frac{1}{2}, ax\right) + \operatorname{polylog}\left(-\frac{3}{2}, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\text{Li}_{-\frac{3}{2}}(ax) + \text{Li}_{-\frac{1}{2}}(ax)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x)
```

```
[Out] Integral(polylog(-3/2, a*x) + polylog(-1/2, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_{-\frac{1}{2}}(ax) + \text{Li}_{-\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(-3/2,a*x)+polylog(-1/2,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(-1/2, a*x) + polylog(-3/2, a*x), x)
```

3.102 $\int (dx)^m \mathbf{PolyLog}(2, ax)\, dx$

Optimal. Leaf size=78

$$\frac{a(dx)^{m+2}\text{Hypergeometric2F1}(1, m+2, m+3, ax)}{d^2(m+1)^2(m+2)} + \frac{(dx)^{m+1}\text{PolyLog}(2, ax)}{d(m+1)} + \frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^2}$$

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^2*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m))
```

Rubi [A] time = 0.0487671, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2395, 64}

$$\frac{(dx)^{m+1}\text{PolyLog}(2, ax)}{d(m+1)} + \frac{a(dx)^{m+2}\,{}_2F_1(1, m+2; m+3; ax)}{d^2(m+1)^2(m+2)} + \frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x],x]
```

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^2*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (dx)^m \mathrm{Li}_2(ax)\,dx &= \frac{(dx)^{1+m}\mathrm{Li}_2(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax)\,dx}{1+m}\\ &= \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2(ax)}{d(1+m)} + \frac{a\int \frac{(dx)^{1+m}}{1-ax}\,dx}{d(1+m)^2}\\ &= \frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^2(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2(ax)}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0387445, size = 53, normalized size = 0.68

$$\frac{x(dx)^m(ax\text{Hypergeometric2F1}(1,m+2,m+3,ax)+(m+2)((m+1)\text{PolyLog}(2,ax)+\log(1-ax)))}{(m+1)^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x],x]
```

```
[Out] (x*(d*x)^m*(a*x*Hypergeometric2F1[1, 2 + m, 3 + m, a*x] + (2 + m)*(Log[1 -
a*x] + (1 + m)*PolyLog[2, a*x])))/((1 + m)^2*(2 + m))
```

Maple [C] time = 0.227, size = 144, normalized size = 1.9

$$\frac{(dx)^m x^{-m}(-a)^{-m}}{a}\left(\frac{x^m(-a)^m\left(-am^2x-2amx-m^2-3m-2\right)}{(1+m)^3(2+m)m} - \frac{x^{1+m}a(-a)^m(-m-2)\ln(-ax+1)}{(1+m)^2(2+m)} + \frac{x^{1+m}a(-a)^m \mathrm{pol}}{1+m}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x),x)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2)
/(1+m)^3/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*ln(-a*x+1)+x^(1+m)*a*(-a
```

```
)^m/(1+m)*polylog(2,a*x)+x^m*(-a)^m/(1+m)^2*LerchPhi(a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-ad^m \int -\frac{xx^m}{m^2 - (am^2 + 2\,am + a)x + 2\,m + 1}\,dx + \frac{(d^m m + d^m)xx^m \mathrm{Li}_2\,(ax) + d^m xx^m \log\left(-ax + 1\right)}{m^2 + 2\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="maxima")
```

```
[Out] -a*d^m*integrate(-x*x^m/(m^2 - (a*m^2 + 2*a*m + a)*x + 2*m + 1), x) + ((d^m
*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*log(-a*x + 1))/(m^2 + 2*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{Li}_2\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\,\mathrm{Li}_2\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(2, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x), x)
```

3.103 $\int (dx)^m \textbf{PolyLog}(3, ax)\, dx$

Optimal. Leaf size=102

$$-\frac{a(dx)^{m+2}\text{Hypergeometric2F1}(1,m+2,m+3,ax)}{d^2(m+1)^3(m+2)}-\frac{(dx)^{m+1}\text{PolyLog}(2,ax)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ax)}{d(m+1)}-\frac{\log(1-ax)}{d(m+}$$

```
[Out] -((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^3*
(2 + m))) - ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*Pol
yLog[2, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m))
```

Rubi [A] time = 0.0636762, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2395, 64}

$$-\frac{(dx)^{m+1}\text{PolyLog}(2,ax)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ax)}{d(m+1)}-\frac{a(dx)^{m+2}\,{}_2F_1(1,m+2;m+3;ax)}{d^2(m+1)^3(m+2)}-\frac{\log(1-ax)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x],x]
```

```
[Out] -((a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^3*
(2 + m))) - ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*Pol
yLog[2, a*x])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (dx)^m\text{Li}_3(ax)\,dx &= \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)}-\frac{\int (dx)^m\text{Li}_2(ax)\,dx}{1+m}\\ &= -\frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)}-\frac{\int (dx)^m\log(1-ax)\,dx}{(1+m)^2}\\ &= -\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3}-\frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)}-\frac{a\int \frac{(dx)^{1+m}}{1-ax}\,dx}{d(1+m)^3}\\ &= -\frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^3(2+m)}-\frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^3}-\frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^2}+\frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)}\end{aligned}$$

Mathematica [C] time = 0.0598987, size = 88, normalized size = 0.86

$$-\frac{x\text{Gamma}(m+2)(dx)^m\left(a(m+1)x\text{Gamma}(m+1)\,{}_2\tilde{F}_1(1,m+2;m+3;ax)+m^2(-\text{PolyLog}(3,ax))-2m\text{PolyLog}(3\right.}{(m+1)^4\text{Gamma}(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x],x]
```

```
[Out] -((x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegula
rized[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x] -
PolyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x]))/((1 + m)^4*Ga
mma[1 + m]))
```

Maple [C] time = 0.388, size = 173, normalized size = 1.7

$$\frac{(dx)^m x^{-m}(-a)^{-m}}{a}\left(\frac{x^m(-a)^m\left(am^2x+2\,amx+m^2+3\,m+2\right)}{(1+m)^4(2+m)\,m}-\frac{x^{1+m}a(-a)^m\ln(-ax+1)}{(1+m)^3}+\frac{x^{1+m}a(-a)^m(-m-2)\,\text{poly}}{(1+m)^2(2+m)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x),x)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(a*m^2*x+2*a*m*x+m^2+3*m+2)/
(1+m)^4/m-x^(1+m)*a*(-a)^m/(1+m)^3*ln(-a*x+1)+1/(2+m)*x^(1+m)*a*(-a)^m*(-m-
2)/(1+m)^2*polylog(2,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(3,a*x)+1/(2+m)*x^m
*(-a)^m*(-m-2)/(1+m)^3*LerchPhi(a*x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ad^m \int -\frac{xx^m}{m^3 - \left(m^3 + 3\,m^2 + 3\,m + 1\right)ax + 3\,m^2 + 3\,m + 1}\,dx - \frac{d^m(m+1)xx^m\mathrm{Li}_2\left(ax\right) - \left(m^2 + 2\,m + 1\right)d^m xx^m \mathrm{Li}_3(ax) + \cdots}{m^3 + 3\,m^2 + 3\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="maxima")
```

```
[Out] a*d^m*integrate(-x*x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x + 3*m^2 + 3*m + 1
), x) - (d^m*(m + 1)*x*x^m*dilog(a*x) - (m^2 + 2*m + 1)*d^m*x*x^m*polylog(3
, a*x) + d^m*x*x^m*log(-a*x + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{polylog}\left(3, ax\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m\,\mathrm{Li}_3\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x), x)
```

3.104 $\int (dx)^m \mathbf{PolyLog}(4, ax)\, dx$

Optimal. Leaf size=121

$$\frac{a(dx)^{m+2}\text{Hypergeometric2F1}(1, m+2, m+3, ax)}{d^2(m+1)^4(m+2)} + \frac{(dx)^{m+1}\text{PolyLog}(2, ax)}{d(m+1)^3} - \frac{(dx)^{m+1}\text{PolyLog}(3, ax)}{d(m+1)^2} + \frac{(dx)^{m+1}\text{Poly}}{d(m}$$

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^4*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^4) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m)^2) +
((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

Rubi [A] time = 0.0857405, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6591, 2395, 64}

$$\frac{(dx)^{m+1}\text{PolyLog}(2, ax)}{d(m+1)^3} - \frac{(dx)^{m+1}\text{PolyLog}(3, ax)}{d(m+1)^2} + \frac{(dx)^{m+1}\text{PolyLog}(4, ax)}{d(m+1)} + \frac{a(dx)^{m+2}\,{}_2F_1(1, m+2; m+3; ax)}{d^2(m+1)^4(m+2)} + \log($$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x],x]
```

```
[Out] (a*(d*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, a*x])/(d^2*(1 + m)^4*(2
 + m)) + ((d*x)^(1 + m)*Log[1 - a*x])/(d*(1 + m)^4) + ((d*x)^(1 + m)*PolyLo
g[2, a*x])/(d*(1 + m)^3) - ((d*x)^(1 + m)*PolyLog[3, a*x])/(d*(1 + m)^2) +
((d*x)^(1 + m)*PolyLog[4, a*x])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_4(ax)\,dx &= \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} - \frac{\int (dx)^m \text{Li}_3(ax)\,dx}{1+m}\\ &= -\frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \text{Li}_2(ax)\,dx}{(1+m)^2}\\ &= \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{\int (dx)^m \log(1-ax)\,dx}{(1+m)^3}\\ &= \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4(ax)}{d(1+m)} + \frac{a\int \frac{(dx)^{1+m}}{1-ax}\,dx}{d(1+m)^4}\\ &= \frac{a(dx)^{2+m}\,{}_2F_1(1,2+m;3+m;ax)}{d^2(1+m)^4(2+m)} + \frac{(dx)^{1+m}\log(1-ax)}{d(1+m)^4} + \frac{(dx)^{1+m}\text{Li}_2(ax)}{d(1+m)^3} - \frac{(dx)^{1+m}\text{Li}_3(ax)}{d(1+m)^2} + \frac{(}{}\end{aligned}$$

Mathematica [C] time = 0.0726869, size = 119, normalized size = 0.98

$$\frac{x\text{Gamma}(m+2)(dx)^m \left(a(m+1)x\text{Gamma}(m+1)\,{}_2\tilde{F}_1(1,m+2;m+3;ax) + m^3\text{PolyLog}(4,ax) - m^2\text{PolyLog}(3,ax)\right.}{(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x],x]
```

```
[Out] (x*(d*x)^m*Gamma[2 + m]*(a*(1 + m)*x*Gamma[1 + m]*HypergeometricPFQRegulari
zed[{1, 2 + m}, {3 + m}, a*x] + Log[1 - a*x] + (1 + m)*PolyLog[2, a*x] - Po
lyLog[3, a*x] - 2*m*PolyLog[3, a*x] - m^2*PolyLog[3, a*x] + PolyLog[4, a*x]
 + 3*m*PolyLog[4, a*x] + 3*m^2*PolyLog[4, a*x] + m^3*PolyLog[4, a*x]))/((1
+ m)^5*Gamma[1 + m])
```

Maple [C] time = 0.794, size = 198, normalized size = 1.6

$$\frac{(dx)^m x^{-m} (-a)^{-m}}{a}\left(\frac{x^m (-a)^m \left(-am^2x-2\,amx-m^2-3\,m-2\right)}{(1+m)^5 (2+m)\,m}-\frac{x^{1+m}a\,(-a)^m (-m-2)\ln(-ax+1)}{(1+m)^4 (2+m)}+\frac{x^{1+m}a\,(-a)^m \operatorname{poly}}{(1+m)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x),x)
```

```
[Out] (d*x)^m*x^(-m)*(-a)^(-m)/a*(1/(2+m)*x^m*(-a)^m*(-a*m^2*x-2*a*m*x-m^2-3*m-2)
/(1+m)^5/m-1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^4*ln(-a*x+1)+x^(1+m)*a*(-a
)^m/(1+m)^3*polylog(2,a*x)+1/(2+m)*x^(1+m)*a*(-a)^m*(-m-2)/(1+m)^2*polylog(
3,a*x)+x^(1+m)*a*(-a)^m/(1+m)*polylog(4,a*x)+x^m*(-a)^m/(1+m)^4*LerchPhi(a*
x,1,m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-ad^m\int-\frac{xx^m}{m^4+4\,m^3+6\,m^2-\left(am^4+4\,am^3+6\,am^2+4\,am+a\right)x+4\,m+1}\,dx+\frac{(d^m m+d^m)xx^m \operatorname{Li}_2(ax)+d^m xx^m \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="maxima")
```

```
[Out] -a*d^m*integrate(-x*x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2 +
 4*a*m + a)*x + 4*m + 1), x) + ((d^m*m + d^m)*x*x^m*dilog(a*x) + d^m*x*x^m*
log(-a*x + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*polylog(4, a*x)
 - (d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x))/(m^4 + 4*m^3 + 6*m^2 +
4*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx)^m \operatorname{polylog}(4,ax),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x), x)
```

3.105 $\int (dx)^m \mathbf{PolyLog}\left(2, ax^2\right) dx$

Optimal. Leaf size=94

$$\frac{4a(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)^2(m+3)}+\frac{(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)}+\frac{2\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^2}$$

```
[Out] (4*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3*
(1 + m)^2*(3 + m)) + (2*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))
```

Rubi [A] time = 0.0549094, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$\frac{(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)}+\frac{4a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)}{d^3(m+1)^2(m+3)}+\frac{2\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^2],x]
```

```
[Out] (4*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3*
(1 + m)^2*(3 + m)) + (2*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int (dx)^m \mathrm{Li}_2\left(ax^2\right) dx &= \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^2\right)}{d(1+m)} + \frac{2\int (dx)^m \log\left(1-ax^2\right) dx}{1+m} \\
&= \frac{2(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^2\right)}{d(1+m)} + \frac{(4a)\int \frac{x(dx)^{1+m}}{1-ax^2}\,dx}{d(1+m)^2} \\
&= \frac{2(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^2\right)}{d(1+m)} + \frac{(4a)\int \frac{(dx)^{2+m}}{1-ax^2}\,dx}{d^2(1+m)^2} \\
&= \frac{4a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^2(3+m)} + \frac{2(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^2\right)}{d(1+m)}
\end{aligned}
$$

Mathematica [A] time = 0.0454423, size = 72, normalized size = 0.77

$$
\frac{x(dx)^m\left(4ax^2\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)+(m+3)\left((m+1)\text{PolyLog}\left(2,ax^2\right)+2\log\left(1-ax^2\right)\right)\right)}{(m+1)^2(m+3)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^2],x]
```

```
[Out] (x*(d*x)^m*(4*a*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2] + (3
+ m)*(2*Log[1 - a*x^2] + (1 + m)*PolyLog[2, a*x^2])))/((1 + m)^2*(3 + m))
```

Maple [C] time = 0.236, size = 177, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{2}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(2\frac{x^{1+m}(-a)^{3/2+m/2}(-12-4m)}{(1+m)^3(3+m)a}-2\frac{x^{1+m}(-a)^{3/2+m/2}(-6-2m)\ln\left(-ax^2+1\right)}{(1+m)^2(3+m)a}+2\frac{x^{1+m}(-a)^{3/2+m/2}}{(1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x^2),x)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-1
2-4*m)/(1+m)^3/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*ln(-a*
x^2+1)+2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)*polylog(2,a*x^2)/a+2/(3+m)*x^(1+m)*
(-a)^(3/2+1/2*m)*(6+2*m)/(1+m)^2/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,ad^m\int\frac{x^2x^m}{\left(am^2+2\,am+a\right)x^2-m^2-2\,m-1}\,dx+\frac{(d^mm+d^m)xx^m\mathrm{Li}_2\left(ax^2\right)+2\,d^mxx^m\log\left(-ax^2+1\right)}{m^2+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="maxima")
```

```
[Out] -4*a*d^m*integrate(x^2*x^m/((a*m^2 + 2*a*m + a)*x^2 - m^2 - 2*m - 1), x) +
((d^m*m + d^m)*x*x^m*dilog(a*x^2) + 2*d^m*x*x^m*log(-a*x^2 + 1))/(m^2 + 2*m
 + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{Li}_2\left(ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x^2), x)
```

3.106 $\int (dx)^m \textbf{PolyLog}\left(3, ax^2\right) dx$

Optimal. Leaf size=118

$$-\frac{8a(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)^3(m+3)}-\frac{2(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ax^2\right)}{d(m+1)}-\frac{4\log\left(\right.}{}$$

```
[Out] (-8*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^3*(3 + m)) - (4*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^3) - (2*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a
*x^2])/(d*(1 + m))
```

Rubi [A] time = 0.0712051, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$-\frac{2(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ax^2\right)}{d(m+1)}-\frac{8a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)}{d^3(m+1)^3(m+3)}-\frac{4\log\left(1-ax^2\right)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^2],x]
```

```
[Out] (-8*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^3*(3 + m)) - (4*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^3) - (2*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3, a
*x^2])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \text{Li}_3\left(ax^2\right) dx = \frac{(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)} - \frac{2\int (dx)^m \text{Li}_2\left(ax^2\right) dx}{1+m}$$

$$= -\frac{2(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)} - \frac{4\int (dx)^m \log\left(1-ax^2\right) dx}{(1+m)^2}$$

$$= -\frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)} - \frac{(8a)\int \frac{x(dx)^{1+m}}{1-ax^2} dx}{d(1+m)^3}$$

$$= -\frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)} - \frac{(8a)\int \frac{(dx)^{2+m}}{1-ax^2} dx}{d^2(1+m)^3}$$

$$= -\frac{8a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^3(3+m)} - \frac{4(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3}{d(1+m}$$

Mathematica [C] time = 0.0792053, size = 126, normalized size = 1.07

$$-\frac{2x\text{Gamma}\left(\frac{m+3}{2}\right)(dx)^m\left(2a(m+1)x^2\text{Gamma}\left(\frac{m+1}{2}\right)\,{}_2\tilde{F}_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)+m^2\left(-\text{PolyLog}\left(3,ax^2\right)\right)-2m\text{PolyLog}}{(m+1)^4\text{Gamma}\left(\frac{m+1}{2}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^2],x]
```

```
[Out] (-2*x*(d*x)^m*Gamma[(3 + m)/2]*(2*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeome
tricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 4*Log[1 - a*x^2] +
```

```
 2*(1 + m)*PolyLog[2, a*x^2] - PolyLog[3, a*x^2] - 2*m*PolyLog[3, a*x^2] -
m^2*PolyLog[3, a*x^2]))/((1 + m)^4*Gamma[(1 + m)/2])
```

Maple [C] time = 0.41, size = 218, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{2}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(2\frac{x^{1+m}(-a)^{3/2+m/2}(24+8m)}{(1+m)^4(3+m)a}-2\frac{x^{1+m}(-a)^{3/2+m/2}(12+4m)\ln\left(-ax^2+1\right)}{(1+m)^3(3+m)a}+2\frac{x^{1+m}(-a)^{3/2+m/2}(\cdot}{(1+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x^2),x)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(24
+8*m)/(1+m)^4/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3/a*ln(-a*x
^2+1)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2*polylog(2,a*x^2)/a+
2*x^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(3,a*x^2)+2/(3+m)*x^(1+m)*(-a)^(3
/2+1/2*m)*(-12-4*m)/(1+m)^3/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\,ad^m\int\frac{x^2x^m}{\left(m^3+3\,m^2+3\,m+1\right)ax^2-m^3-3\,m^2-3\,m-1}\,dx-\frac{2\,d^m(m+1)xx^m\mathrm{Li}_2\left(ax^2\right)-\left(m^2+2\,m+1\right)d^mxx^m\mathrm{Li}_3(ax^2}{m^3+3\,m^2+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="maxima")
```

```
[Out] 8*a*d^m*integrate(x^2*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^2 - m^3 - 3*m^2 - 3*
m - 1), x) - (2*d^m*(m + 1)*x*x^m*dilog(a*x^2) - (m^2 + 2*m + 1)*d^m*x*x^m*
polylog(3, a*x^2) + 4*d^m*x*x^m*log(-a*x^2 + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{polylog}\left(3,ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**2),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x^2), x)
```

3.107 $\int (dx)^m \mathbf{PolyLog}\left(4, ax^2\right) dx$

Optimal. Leaf size=142

$$\frac{16a(dx)^{m+3}\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},ax^2\right)}{d^3(m+1)^4(m+3)}+\frac{4(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)^3}-\frac{2(dx)^{m+1}\text{PolyLog}\left(3,ax^2\right)}{d(m+1)^2}+\frac{(dx)^m}{}$$

```
[Out] (16*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^4*(3 + m)) + (8*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^4) + (4*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^3) - (2*(d*x)^(1 + m)*PolyLog[3,
 a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^2])/(d*(1 + m))
```

Rubi [A] time = 0.0934271, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$\frac{4(dx)^{m+1}\text{PolyLog}\left(2,ax^2\right)}{d(m+1)^3}-\frac{2(dx)^{m+1}\text{PolyLog}\left(3,ax^2\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(4,ax^2\right)}{d(m+1)}+\frac{16a(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right.}{d^3(m+1)^4(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^2],x]
```


```
[Out] (16*a*(d*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, a*x^2])/(d^3
*(1 + m)^4*(3 + m)) + (8*(d*x)^(1 + m)*Log[1 - a*x^2])/(d*(1 + m)^4) + (4*(
d*x)^(1 + m)*PolyLog[2, a*x^2])/(d*(1 + m)^3) - (2*(d*x)^(1 + m)*PolyLog[3,
 a*x^2])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^2])/(d*(1 + m))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_4\left(ax^2\right) dx &= \frac{(dx)^{1+m}\text{Li}_4\left(ax^2\right)}{d(1+m)} - \frac{2\int (dx)^m \text{Li}_3\left(ax^2\right) dx}{1+m}\\
&= -\frac{2(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^2\right)}{d(1+m)} + \frac{4\int (dx)^m \text{Li}_2\left(ax^2\right) dx}{(1+m)^2}\\
&= \frac{4(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^2\right)}{d(1+m)} + \frac{8\int (dx)^m \log\left(1-ax^2\right) dx}{(1+m)^3}\\
&= \frac{8(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^4} + \frac{4(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^2\right)}{d(1+m)} + \frac{(16a)}{d(}\\
&= \frac{8(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^4} + \frac{4(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}_3\left(ax^2\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^2\right)}{d(1+m)} + \frac{(16a)}{d^2(}\\
&= \frac{16a(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};ax^2\right)}{d^3(1+m)^4(3+m)} + \frac{8(dx)^{1+m}\log\left(1-ax^2\right)}{d(1+m)^4} + \frac{4(dx)^{1+m}\text{Li}_2\left(ax^2\right)}{d(1+m)^3} - \frac{2(dx)^{1+m}\text{Li}}{d(1+m}\end{aligned}$$

Mathematica [C] time = 0.0995597, size = 166, normalized size = 1.17

$$\frac{2x\text{Gamma}\left(\frac{m+3}{2}\right)(dx)^m\left(4a(m+1)x^2\text{Gamma}\left(\frac{m+1}{2}\right){}_2\tilde{F}_1\left(1,\frac{m+3}{2};\frac{m+5}{2};ax^2\right)+m^3\text{PolyLog}\left(4,ax^2\right)-2m^2\text{PolyLog}\left(3,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^2],x]
```

```
[Out] (2*x*(d*x)^m*Gamma[(3 + m)/2]*(4*a*(1 + m)*x^2*Gamma[(1 + m)/2]*Hypergeomet
ricPFQRegularized[{1, (3 + m)/2}, {(5 + m)/2}, a*x^2] + 8*Log[1 - a*x^2] +
4*(1 + m)*PolyLog[2, a*x^2] - 2*PolyLog[3, a*x^2] - 4*m*PolyLog[3, a*x^2] -
 2*m^2*PolyLog[3, a*x^2] + PolyLog[4, a*x^2] + 3*m*PolyLog[4, a*x^2] + 3*m^
2*PolyLog[4, a*x^2] + m^3*PolyLog[4, a*x^2]))/((1 + m)^5*Gamma[(1 + m)/2])
```

Maple [C] time = 0.792, size = 259, normalized size = 1.8

$$-\frac{(dx)^m x^{-m}}{2}(-a)^{-\frac{1}{2}-\frac{m}{2}}\left(2\frac{x^{1+m}(-a)^{3/2+m/2}(-48-16m)}{(1+m)^5(3+m)a}-2\frac{x^{1+m}(-a)^{3/2+m/2}(-24-8m)\ln\left(-ax^2+1\right)}{(1+m)^4(3+m)a}+2\frac{x^{1+m}(-a)^{3/2+m/2}}{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x^2),x)
```

```
[Out] -1/2*(d*x)^m*x^(-m)*(-a)^(-1/2-1/2*m)*(2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-4
8-16*m)/(1+m)^5/a-2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-24-8*m)/(1+m)^4/a*ln(-
a*x^2+1)+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(12+4*m)/(1+m)^3*polylog(2,a*x^2)
/a+2/(3+m)*x^(1+m)*(-a)^(3/2+1/2*m)*(-6-2*m)/(1+m)^2/a*polylog(3,a*x^2)+2*x
^(1+m)*(-a)^(3/2+1/2*m)/(1+m)/a*polylog(4,a*x^2)+2/(3+m)*x^(1+m)*(-a)^(3/2+
1/2*m)*(24+8*m)/(1+m)^4/a*LerchPhi(a*x^2,1,1/2+1/2*m))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,ad^m\int -\frac{x^2x^m}{m^4+4\,m^3-\left(am^4+4\,am^3+6\,am^2+4\,am+a\right)x^2+6\,m^2+4\,m+1}\,dx+\frac{4\,(d^m m+d^m)xx^m\mathrm{Li}_2\left(ax^2\right)+8\,d^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="maxima")
```

```
[Out] -16*a*d^m*integrate(-x^2*x^m/(m^4 + 4*m^3 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*
a*m + a)*x^2 + 6*m^2 + 4*m + 1), x) + (4*(d^m*m + d^m)*x*x^m*dilog(a*x^2) +
 8*d^m*x*x^m*log(-a*x^2 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m*
polylog(4, a*x^2) - 2*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^2))/(m
^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \text{polylog}\left(4, ax^2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4\left(ax^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**2),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x^2), x)
```

3.108 $\int (dx)^m \textbf{PolyLog}\left(2, ax^3\right) dx$

Optimal. Leaf size=94

$$\frac{9a(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)^2(m+4)}+\frac{(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)}+\frac{3\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^2}$$

```
[Out] (9*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4*
(1 + m)^2*(4 + m)) + (3*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))
```

Rubi [A] time = 0.0547537, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$\frac{(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)}+\frac{9a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)}{d^4(m+1)^2(m+4)}+\frac{3\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^3],x]
```


```
[Out] (9*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4*
(1 + m)^2*(4 + m)) + (3*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^2) + ((d*x
)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \text{Li}_2\left(ax^3\right)\,dx &= \frac{(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)} + \frac{3\int (dx)^m \log\left(1-ax^3\right)\,dx}{1+m} \\
&= \frac{3(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)} + \frac{(9a)\int \frac{x^2(dx)^{1+m}}{1-ax^3}\,dx}{d(1+m)^2} \\
&= \frac{3(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)} + \frac{(9a)\int \frac{(dx)^{3+m}}{1-ax^3}\,dx}{d^3(1+m)^2} \\
&= \frac{9a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^2(4+m)} + \frac{3(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.0448169, size = 72, normalized size = 0.77

$$\frac{x(dx)^m\left(9ax^3\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)+(m+4)\left((m+1)\text{PolyLog}\left(2,ax^3\right)+3\log\left(1-ax^3\right)\right)\right)}{(m+1)^2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^3],x]
```

```
[Out] (x*(d*x)^m*(9*a*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3] + (4
+ m)*(3*Log[1 - a*x^3] + (1 + m)*PolyLog[2, a*x^3])))/((1 + m)^2*(4 + m))
```

Maple [C] time = 0.277, size = 177, normalized size = 1.9

$$-\frac{(dx)^{m} x^{-m}}{3} (-a)^{-\frac{1}{3}-\frac{m}{3}} \left(3 \frac{x^{1+m} (-a)^{4/3+m/3} (-36-9 m)}{(1+m)^{3} (4+m) a} - 3 \frac{x^{1+m} (-a)^{4/3+m/3} (-12-3 m) \ln\left(-x^{3} a+1\right)}{(1+m)^{2} (4+m) a} + 3 \frac{x^{1+m} (-a)^{4/3+m/}}{(1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,x^3*a),x)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-3
6-9*m)/(1+m)^3/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*ln(-a*x
^3+1)/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(2,x^3*a)+3/(4+m)*x^(1+m)
*(-a)^(4/3+1/3*m)*(12+3*m)/(1+m)^2/a*LerchPhi(x^3*a,1,1/3*m+1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-9\, ad^{m} \int \frac{x^{3} x^{m}}{\left(am^{2} + 2\, am + a\right)x^{3} - m^{2} - 2\, m - 1}\, dx + \frac{(d^{m} m + d^{m})x x^{m} \mathrm{Li}_2\left(ax^{3}\right) + 3\, d^{m} x x^{m} \log\left(-ax^{3} + 1\right)}{m^{2} + 2\, m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="maxima")
```

```
[Out] -9*a*d^m*integrate(x^3*x^m/((a*m^2 + 2*a*m + a)*x^3 - m^2 - 2*m - 1), x) +
((d^m*m + d^m)*x*x^m*dilog(a*x^3) + 3*d^m*x*x^m*log(-a*x^3 + 1))/(m^2 + 2*m
 + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^{m} \mathrm{Li}_2\left(ax^{3}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2(ax^3)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^3),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x^3), x)
```

3.109 $\int (dx)^m \textbf{PolyLog}\left(3, ax^3\right) dx$

Optimal. Leaf size=118

$$-\frac{27a(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)^3(m+4)}-\frac{3(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ax^3\right)}{d(m+1)}-\frac{9\log}{}$$

```
[Out] (-27*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^
4*(1 + m)^3*(4 + m)) - (9*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^3) - (3*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3,
a*x^3])/(d*(1 + m))
```

Rubi [A] time = 0.0712691, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$-\frac{3(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(3,ax^3\right)}{d(m+1)}-\frac{27a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)}{d^4(m+1)^3(m+4)}-\frac{9\log\left(1-ax^3\right)(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^3],x]
```


```
[Out] (-27*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^
4*(1 + m)^3*(4 + m)) - (9*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^3) - (3*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[3,
a*x^3])/(d*(1 + m))
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_3\left(ax^3\right) dx &= \frac{(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)} - \frac{3\int (dx)^m \text{Li}_2\left(ax^3\right) dx}{1+m}\\ &= -\frac{3(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)} - \frac{9\int (dx)^m \log\left(1-ax^3\right) dx}{(1+m)^2}\\ &= -\frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)} - \frac{(27a)\int \frac{x^2(dx)^{1+m}}{1-ax^3} dx}{d(1+m)^3}\\ &= -\frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)} - \frac{(27a)\int \frac{(dx)^{3+m}}{1-ax^3} dx}{d^3(1+m)^3}\\ &= -\frac{27a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^3(4+m)} - \frac{9(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{L}}{d(1+}\end{aligned}$$

Mathematica [C] time = 0.0802421, size = 126, normalized size = 1.07

$$-\frac{3x\text{Gamma}\left(\frac{m+4}{3}\right)(dx)^m\left(3a(m+1)x^3\text{Gamma}\left(\frac{m+1}{3}\right){}_2\tilde{F}_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right) - m^2\text{PolyLog}\left(3,ax^3\right) + 3(m+1)\text{PolyL}\right.}{(m+1)^4\text{Gamma}\left(\frac{m+1}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^3],x]
```

```
[Out] (-3*x*(d*x)^m*Gamma[(4 + m)/3]*(3*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeome
tricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 9*Log[1 - a*x^3] +
```

```
 3*(1 + m)*PolyLog[2, a*x^3] - PolyLog[3, a*x^3] - 2*m*PolyLog[3, a*x^3] -
m^2*PolyLog[3, a*x^3]))/((1 + m)^4*Gamma[(1 + m)/3])
```

Maple [C] time = 0.414, size = 218, normalized size = 1.9

$$-\frac{(dx)^m x^{-m}}{3}(-a)^{-\frac{1}{3}-\frac{m}{3}}\left(3\,\frac{x^{1+m}(-a)^{4/3+m/3}(108+27\,m)}{(1+m)^4(4+m)\,a}-3\,\frac{x^{1+m}(-a)^{4/3+m/3}(36+9\,m)\ln\left(-x^3a+1\right)}{(1+m)^3(4+m)\,a}+3\,\frac{x^{1+m}(-a)^{4/3+m/}}{(1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,x^3*a),x)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(10
8+27*m)/(1+m)^4/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*ln(-a
*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(2,x^3*a)
/a+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(3,x^3*a)+3/(4+m)*x^(1+m)*(-a)
^(4/3+1/3*m)*(-36-9*m)/(1+m)^3/a*LerchPhi(x^3*a,1,1/3*m+1/3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$27\,ad^m\int\frac{x^3x^m}{\left(m^3+3\,m^2+3\,m+1\right)ax^3-m^3-3\,m^2-3\,m-1}\,dx-\frac{3\,d^m(m+1)xx^m\mathrm{Li}_2\left(ax^3\right)-\left(m^2+2\,m+1\right)d^mxx^m\mathrm{Li}_3(a}{m^3+3\,m^2+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="maxima")
```

```
[Out] 27*a*d^m*integrate(x^3*x^m/((m^3 + 3*m^2 + 3*m + 1)*a*x^3 - m^3 - 3*m^2 - 3
*m - 1), x) - (3*d^m*(m + 1)*x*x^m*dilog(a*x^3) - (m^2 + 2*m + 1)*d^m*x*x^m
*polylog(3, a*x^3) + 9*d^m*x*x^m*log(-a*x^3 + 1))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{polylog}\left(3,ax^3\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3\left(ax^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**3),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^3)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^3),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x^3), x)
```

3.110 $\int (dx)^m \mathbf{PolyLog}\left(4, ax^3\right) dx$

Optimal. Leaf size=142

$$\frac{81a(dx)^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},ax^3\right)}{d^4(m+1)^4(m+4)}+\frac{9(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)^3}-\frac{3(dx)^{m+1}\text{PolyLog}\left(3,ax^3\right)}{d(m+1)^2}+\frac{(dx)^m}{}$$

```
[Out] (81*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4
*(1 + m)^4*(4 + m)) + (27*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^4) + (9*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^3) - (3*(d*x)^(1 + m)*PolyLog[3
, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^3])/(d*(1 + m))
```

Rubi [A] time = 0.0943941, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 16, 364}

$$\frac{9(dx)^{m+1}\text{PolyLog}\left(2,ax^3\right)}{d(m+1)^3}-\frac{3(dx)^{m+1}\text{PolyLog}\left(3,ax^3\right)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}\left(4,ax^3\right)}{d(m+1)}+\frac{81a(dx)^{m+4}\,{}_2F_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right.}{d^4(m+1)^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^3],x]
```

```
[Out] (81*a*(d*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, a*x^3])/(d^4
*(1 + m)^4*(4 + m)) + (27*(d*x)^(1 + m)*Log[1 - a*x^3])/(d*(1 + m)^4) + (9*
(d*x)^(1 + m)*PolyLog[2, a*x^3])/(d*(1 + m)^3) - (3*(d*x)^(1 + m)*PolyLog[3
, a*x^3])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^3])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \text{Li}_4\left(ax^3\right) dx &= \frac{(dx)^{1+m}\text{Li}_4\left(ax^3\right)}{d(1+m)} - \frac{3\int (dx)^m \text{Li}_3\left(ax^3\right) dx}{1+m} \\ &= -\frac{3(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^3\right)}{d(1+m)} + \frac{9\int (dx)^m \text{Li}_2\left(ax^3\right) dx}{(1+m)^2} \\ &= \frac{9(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^3\right)}{d(1+m)} + \frac{27\int (dx)^m \log\left(1-ax^3\right) dx}{(1+m)^3} \\ &= \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^3\right)}{d(1+m)} + \frac{(81a)}{} \\ &= \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{Li}_3\left(ax^3\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\text{Li}_4\left(ax^3\right)}{d(1+m)} + \frac{(81a)}{d^3} \\ &= \frac{81a(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};ax^3\right)}{d^4(1+m)^4(4+m)} + \frac{27(dx)^{1+m}\log\left(1-ax^3\right)}{d(1+m)^4} + \frac{9(dx)^{1+m}\text{Li}_2\left(ax^3\right)}{d(1+m)^3} - \frac{3(dx)^{1+m}\text{I}}{d(1+}\end{aligned}$$

Mathematica [C] time = 0.0967328, size = 166, normalized size = 1.17

$$\frac{3x\text{Gamma}\left(\frac{m+4}{3}\right)(dx)^m\left(9a(m+1)x^3\text{Gamma}\left(\frac{m+1}{3}\right){}_2\tilde{F}_1\left(1,\frac{m+4}{3};\frac{m+7}{3};ax^3\right)+m^3\text{PolyLog}\left(4,ax^3\right)-3m^2\text{PolyLog}\left(3,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^3],x]
```

```
[Out] (3*x*(d*x)^m*Gamma[(4 + m)/3]*(9*a*(1 + m)*x^3*Gamma[(1 + m)/3]*Hypergeomet
ricPFQRegularized[{1, (4 + m)/3}, {(7 + m)/3}, a*x^3] + 27*Log[1 - a*x^3] +
 9*(1 + m)*PolyLog[2, a*x^3] - 3*PolyLog[3, a*x^3] - 6*m*PolyLog[3, a*x^3]
- 3*m^2*PolyLog[3, a*x^3] + PolyLog[4, a*x^3] + 3*m*PolyLog[4, a*x^3] + 3*m
^2*PolyLog[4, a*x^3] + m^3*PolyLog[4, a*x^3]))/((1 + m)^5*Gamma[(1 + m)/3])
```

Maple [C] time = 0.856, size = 259, normalized size = 1.8

$$-\frac{(dx)^m x^{-m}}{3}(-a)^{-\frac{1}{3}-\frac{m}{3}}\left(3\frac{x^{1+m}(-a)^{4/3+m/3}(-324-81m)}{(1+m)^5(4+m)a}-3\frac{x^{1+m}(-a)^{4/3+m/3}(-108-27m)\ln\left(-x^3a+1\right)}{(1+m)^4(4+m)a}+3\frac{x^{1+m}(-a)^{4}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,x^3*a),x)
```

```
[Out] -1/3*(d*x)^m*x^(-m)*(-a)^(-1/3-1/3*m)*(3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-3
24-81*m)/(1+m)^5/a-3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-108-27*m)/(1+m)^4/a*l
n(-a*x^3+1)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(36+9*m)/(1+m)^3/a*polylog(2,x
^3*a)+3/(4+m)*x^(1+m)*(-a)^(4/3+1/3*m)*(-12-3*m)/(1+m)^2*polylog(3,x^3*a)/a
+3*x^(1+m)*(-a)^(4/3+1/3*m)/(1+m)/a*polylog(4,x^3*a)+3/(4+m)*x^(1+m)*(-a)^(
4/3+1/3*m)*(108+27*m)/(1+m)^4/a*LerchPhi(x^3*a,1,1/3*m+1/3))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$-81\,ad^m\int-\frac{x^3x^m}{m^4-\left(am^4+4\,am^3+6\,am^2+4\,am+a\right)x^3+4\,m^3+6\,m^2+4\,m+1}\,dx+\frac{9\,(d^m m+d^m)xx^m\mathrm{Li}_2\left(ax^3\right)+27\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="maxima")
```

```
[Out] -81*a*d^m*integrate(-x^3*x^m/(m^4 - (a*m^4 + 4*a*m^3 + 6*a*m^2 + 4*a*m + a)
*x^3 + 4*m^3 + 6*m^2 + 4*m + 1), x) + (9*(d^m*m + d^m)*x*x^m*dilog(a*x^3) +
 27*d^m*x*x^m*log(-a*x^3 + 1) + (d^m*m^3 + 3*d^m*m^2 + 3*d^m*m + d^m)*x*x^m
*polylog(4, a*x^3) - 3*(d^m*m^2 + 2*d^m*m + d^m)*x*x^m*polylog(3, a*x^3))/(
m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \,\text{polylog}\left(4, ax^3\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4\left(ax^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**3),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^3)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^3),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x^3), x)
```

3.111 $\int (dx)^m \textbf{PolyLog}(2, ax^q)\, dx$

Optimal. Leaf size=101

$$\frac{aq^2x^{q+1}(dx)^m\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ax^q\right)}{(m+1)^2(m+q+1)}+\frac{(dx)^{m+1}\text{PolyLog}(2,ax^q)}{d(m+1)}+\frac{q(dx)^{m+1}\log(1-ax^q)}{d(m+1)^2}$$

```
[Out] (a*q^2*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^2*(1 + m + q)) + (q*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*(1
 + m)^2) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m))
```

Rubi [A] time = 0.0597507, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 20, 364}

$$\frac{(dx)^{m+1}\text{PolyLog}(2,ax^q)}{d(m+1)}+\frac{aq^2x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\frac{m+2q+1}{q};ax^q\right)}{(m+1)^2(m+q+1)}+\frac{q(dx)^{m+1}\log(1-ax^q)}{d(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[2, a*x^q],x]
```

```
[Out] (a*q^2*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^2*(1 + m + q)) + (q*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*(1
 + m)^2) + ((d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \mathrm{Li}_2\left(ax^q\right)\,dx &= \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^q\right)}{d(1+m)} + \frac{q\int (dx)^m \log\left(1-ax^q\right)\,dx}{1+m}\\ &= \frac{q(dx)^{1+m}\log\left(1-ax^q\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^q\right)}{d(1+m)} + \frac{\left(aq^2\right)\int \frac{x^{-1+q}(dx)^{1+m}}{1-ax^q}\,dx}{d(1+m)^2}\\ &= \frac{q(dx)^{1+m}\log\left(1-ax^q\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^q\right)}{d(1+m)} + \frac{\left(aq^2x^{-m}(dx)^m\right)\int \frac{x^{m+q}}{1-ax^q}\,dx}{(1+m)^2}\\ &= \frac{aq^2x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^2(1+m+q)} + \frac{q(dx)^{1+m}\log\left(1-ax^q\right)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_2\left(ax^q\right)}{d(1+m)}\end{aligned}$$

Mathematica [A] time = 0.0642053, size = 80, normalized size = 0.79

$$\frac{x(dx)^m\left(aq^2x^q\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ax^q\right)+(m+q+1)\left((m+1)\text{PolyLog}\left(2,ax^q\right)+q\log\left(1-ax^q\right)\right)\right)}{(m+1)^2(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*PolyLog[2, a*x^q],x]
```

```
[Out] (x*(d*x)^m*(a*q^2*x^q*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/q,
a*x^q] + (1 + m + q)*(q*Log[1 - a*x^q] + (1 + m)*PolyLog[2, a*x^q])))/((1 +
 m)^2*(1 + m + q))
```

Maple [C] time = 0.293, size = 148, normalized size = 1.5

$$-\frac{(dx)^m x^{-m}}{q}(-a)^{-\frac{m}{q}-q^{-1}}\left(-\frac{q^2x^{1+m}\ln(1-ax^q)}{(1+m)^2}(-a)^{\frac{m}{q}+q^{-1}}-\frac{qx^{1+m}\mathrm{polylog}(2,ax^q)}{1+m}(-a)^{\frac{m}{q}+q^{-1}}-\frac{q^2x^{1+m+q}a}{(1+m)^2}(-a)^{\frac{m}{q}+q^{-1}}\mathrm{Lerc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(2,a*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*ln(1
-a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*polylog(2,a*x^q)-q^2*x^(1+m+q)*a*(-a
)^(m/q+1/q)/(1+m)^2*LerchPhi(a*x^q,1,(1+m+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-d^m q^2 \int -\frac{x^m}{m^2-\left(am^2+2\,am+a\right)x^q+2\,m+1}\,dx-\frac{d^m q^2 x x^m-(d^m m+d^m)qxx^m\log\left(-ax^q+1\right)-\left(d^m m^2+2\,d^m m+d^m\right.}{m^3+3\,m^2+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="maxima")
```

```
[Out] -d^m*q^2*integrate(-x^m/(m^2 - (a*m^2 + 2*a*m + a)*x^q + 2*m + 1), x) - (d^
m*q^2*x*x^m - (d^m*m + d^m)*q*x*x^m*log(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m +
d^m)*x*x^m*dilog(a*x^q))/(m^3 + 3*m^2 + 3*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{Li}_2\left(ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*dilog(a*x^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(2,a*x**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_2(ax^q) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(2,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*dilog(a*x^q), x)
```

3.112 $\int (dx)^m \mathbf{PolyLog}(3, ax^q)\, dx$

Optimal. Leaf size=130

$$-\frac{aq^3x^{q+1}(dx)^m\text{Hypergeometric2F1}\left(1,\frac{m+q+1}{q},\frac{m+2q+1}{q},ax^q\right)}{(m+1)^3(m+q+1)}-\frac{q(dx)^{m+1}\text{PolyLog}(2,ax^q)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ax^q)}{d(m+1)}-$$

```
[Out] -((a*q^3*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q
)/q, a*x^q])/((1 + m)^3*(1 + m + q))) - (q^2*(d*x)^(1 + m)*Log[1 - a*x^q])/
(d*(1 + m)^3) - (q*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^2) + ((d*x)^
(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m))
```

Rubi [A] time = 0.0758585, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 20, 364}

$$-\frac{q(dx)^{m+1}\text{PolyLog}(2,ax^q)}{d(m+1)^2}+\frac{(dx)^{m+1}\text{PolyLog}(3,ax^q)}{d(m+1)}-\frac{aq^3x^{q+1}(dx)^m\,{}_2F_1\left(1,\frac{m+q+1}{q};\frac{m+2q+1}{q};ax^q\right)}{(m+1)^3(m+q+1)}-\frac{q^2(dx)^{m+1}\log(1-}{d(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[3, a*x^q],x]
```

```
[Out] -((a*q^3*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q
)/q, a*x^q])/((1 + m)^3*(1 + m + q))) - (q^2*(d*x)^(1 + m)*Log[1 - a*x^q])/
(d*(1 + m)^3) - (q*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^2) + ((d*x)^
(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
```

```
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \mathrm{Li}_3(ax^q)\,dx &= \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{q\int (dx)^m \mathrm{Li}_2(ax^q)\,dx}{1+m}\\
&= -\frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{q^2\int (dx)^m \log(1-ax^q)\,dx}{(1+m)^2}\\
&= -\frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{\left(aq^3\right)\int \frac{x^{-1+q}(dx)^{1+m}}{1-ax^q}\,dx}{d(1+m)^3}\\
&= -\frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)} - \frac{\left(aq^3x^{-m}(dx)^m\right)\int \frac{x^{m+q}}{1-ax^q}\,dx}{(1+m)^3}\\
&= -\frac{aq^3x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^3(1+m+q)} - \frac{q^2(dx)^{1+m}\log(1-ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^2} + \frac{(dx
\end{aligned}$$

Mathematica [C] time = 0.0248751, size = 50, normalized size = 0.38

$$-\frac{x(dx)^m G_{5,5}^{1,5}\left(-ax^q\middle|\begin{array}{c}1,1,1,1,1-\frac{m+1}{q}\\1,0,0,0,-\frac{m+1}{q}\end{array}\right)}{q}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[3, a*x^q],x]
```

```
[Out] -((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0, -((
1 + m)/q)}}, -(a*x^q)])/q)
```

Maple [C] time = 0.529, size = 180, normalized size = 1.4

$$-\frac{(dx)^m x^{-m}}{q}(-a)^{-\frac{m}{q}-q^{-1}}\left(\frac{q^3x^{1+m}\ln(1-ax^q)}{(1+m)^3}(-a)^{\frac{m}{q}+q^{-1}}+\frac{q^2x^{1+m}\mathrm{polylog}(2,ax^q)}{(1+m)^2}(-a)^{\frac{m}{q}+q^{-1}}-\frac{qx^{1+m}\mathrm{polylog}(3,ax^q)}{1+m}(-a)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(3,a*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*ln(1-
a*x^q)+q^2*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^2*polylog(2,a*x^q)-q*x^(1+m)*(-a)^(
m/q+1/q)/(1+m)*polylog(3,a*x^q)+q^3*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^3*Lerc
hPhi(a*x^q,1,(1+m+q)/q))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^m q^3 \int -\frac{x^m}{m^3-\left(m^3+3\,m^2+3\,m+1\right)ax^q+3\,m^2+3\,m+1}\,dx+\frac{d^m q^3 x x^m-\left(m^2q+2\,mq+q\right)d^m x x^m \mathrm{Li}_2\left(ax^q\right)-\left(mq^2+\right.}{m^4+4\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="maxima")
```

```
[Out] d^m*q^3*integrate(-x^m/(m^3 - (m^3 + 3*m^2 + 3*m + 1)*a*x^q + 3*m^2 + 3*m +
 1), x) + (d^m*q^3*x*x^m - (m^2*q + 2*m*q + q)*d^m*x*x^m*dilog(a*x^q) - (m*
q^2 + q^2)*d^m*x*x^m*log(-a*x^q + 1) + (m^3 + 3*m^2 + 3*m + 1)*d^m*x*x^m*po
lylog(3, a*x^q))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx)^m\,\mathrm{polylog}\left(3,ax^q\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(3, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(3,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(3, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_3(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(3,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(3, a*x^q), x)
```

3.113 $\int (dx)^m \mathbf{PolyLog}(4, ax^q)\, dx$

Optimal. Leaf size=154

$$\frac{aq^4x^{q+1}(dx)^m \text{Hypergeometric2F1}\left(1, \frac{m+q+1}{q}, \frac{m+2q+1}{q}, ax^q\right)}{(m+1)^4(m+q+1)} + \frac{q^2(dx)^{m+1}\text{PolyLog}(2, ax^q)}{d(m+1)^3} - \frac{q(dx)^{m+1}\text{PolyLog}(3, ax^q)}{d(m+1)^2} +$$

```
[Out] (a*q^4*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^4*(1 + m + q)) + (q^3*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*
(1 + m)^4) + (q^2*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^3) - (q*(d*x)
^(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^q
])/(d*(1 + m))
```

Rubi [A] time = 0.101742, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6591, 2455, 20, 364}

$$\frac{q^2(dx)^{m+1}\text{PolyLog}(2, ax^q)}{d(m+1)^3} - \frac{q(dx)^{m+1}\text{PolyLog}(3, ax^q)}{d(m+1)^2} + \frac{(dx)^{m+1}\text{PolyLog}(4, ax^q)}{d(m+1)} + \frac{aq^4x^{q+1}(dx)^m\ {}_2F_1\left(1, \frac{m+q+1}{q}; \frac{m+2q}{q}\right.}{(m+1)^4(m+q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*PolyLog[4, a*x^q],x]
```

```
[Out] (a*q^4*x^(1 + q)*(d*x)^m*Hypergeometric2F1[1, (1 + m + q)/q, (1 + m + 2*q)/
q, a*x^q])/((1 + m)^4*(1 + m + q)) + (q^3*(d*x)^(1 + m)*Log[1 - a*x^q])/(d*
(1 + m)^4) + (q^2*(d*x)^(1 + m)*PolyLog[2, a*x^q])/(d*(1 + m)^3) - (q*(d*x)
^(1 + m)*PolyLog[3, a*x^q])/(d*(1 + m)^2) + ((d*x)^(1 + m)*PolyLog[4, a*x^q
])/(d*(1 + m))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
```

```
+ 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (dx)^m \mathrm{Li}_4(ax^q)\,dx = \frac{(dx)^{1+m}\mathrm{Li}_4(ax^q)}{d(1+m)} - \frac{q\int (dx)^m \mathrm{Li}_3(ax^q)\,dx}{1+m}$$
$$= -\frac{q(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^q)}{d(1+m)} + \frac{q^2\int (dx)^m \mathrm{Li}_2(ax^q)\,dx}{(1+m)^2}$$
$$= \frac{q^2(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^q)}{d(1+m)} + \frac{q^3\int (dx)^m \log(1-ax^q)\,dx}{(1+m)^3}$$
$$= \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^q)}{d(1+m)} + \frac{\left(aq^4\right)\ldots}{\ldots}$$
$$= \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(dx)^{1+m}\mathrm{Li}_3(ax^q)}{d(1+m)^2} + \frac{(dx)^{1+m}\mathrm{Li}_4(ax^q)}{d(1+m)} + \frac{\left(aq^4x^{\ldots}\right.}{\ldots}$$
$$= \frac{aq^4x^{1+q}(dx)^m\,{}_2F_1\left(1,\frac{1+m+q}{q};\frac{1+m+2q}{q};ax^q\right)}{(1+m)^4(1+m+q)} + \frac{q^3(dx)^{1+m}\log(1-ax^q)}{d(1+m)^4} + \frac{q^2(dx)^{1+m}\mathrm{Li}_2(ax^q)}{d(1+m)^3} - \frac{q(d\ldots}{\ldots}$$

Mathematica [C] time = 0.0204511, size = 52, normalized size = 0.34

$$-\frac{x(dx)^m G_{6,6}^{1,6}\left(-ax^q\left|\begin{array}{c}1,1,1,1,1,1-\frac{m+1}{q}\\ 1,0,0,0,0,-\frac{m+1}{q}\end{array}\right.\right)}{q}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*PolyLog[4, a*x^q],x]
```

```
[Out] -((x*(d*x)^m*MeijerG[{{1, 1, 1, 1, 1, 1 - (1 + m)/q}, {}}, {{1}, {0, 0, 0,
0, -((1 + m)/q)}}, -(a*x^q)])/q)
```

Maple [C] time = 3.379, size = 217, normalized size = 1.4

$$-\frac{(dx)^m x^{-m}}{q}(-a)^{-\frac{m}{q}-q^{-1}}\left(-\frac{q^4x^{1+m}\ln(1-ax^q)}{(1+m)^4}(-a)^{\frac{m}{q}+q^{-1}}-\frac{q^3x^{1+m}\mathrm{polylog}(2,ax^q)}{(1+m)^3}(-a)^{\frac{m}{q}+q^{-1}}+\frac{q^2x^{1+m}\mathrm{polylog}(3,ax^q)}{(1+m)^2}(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*polylog(4,a*x^q),x)
```

```
[Out] -(d*x)^m*x^(-m)*(-a)^(-m/q-1/q)/q*(-q^4*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^4*ln(1
-a*x^q)-q^3*x^(1+m)*(-a)^(m/q+1/q)/(1+m)^3*polylog(2,a*x^q)+q^2*x^(1+m)*(-a
)^(m/q+1/q)/(1+m)^2*polylog(3,a*x^q)-q*x^(1+m)*(-a)^(m/q+1/q)/(1+m)*polylog
(4,a*x^q)-q^4*x^(1+m+q)*a*(-a)^(m/q+1/q)/(1+m)^4*LerchPhi(a*x^q,1,(1+m+q)/q
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-d^mq^4\int -\frac{x^m}{m^4+4\,m^3+6\,m^2-\left(am^4+4\,am^3+6\,am^2+4\,am+a\right)x^q+4\,m+1}\,dx-\frac{d^mq^4xx^m-(d^mm+d^m)q^3xx^m\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="maxima")
```

```
[Out] -d^m*q^4*integrate(-x^m/(m^4 + 4*m^3 + 6*m^2 - (a*m^4 + 4*a*m^3 + 6*a*m^2 +
 4*a*m + a)*x^q + 4*m + 1), x) - (d^m*q^4*x*x^m - (d^m*m + d^m)*q^3*x*x^m*l
og(-a*x^q + 1) - (d^m*m^2 + 2*d^m*m + d^m)*q^2*x*x^m*dilog(a*x^q) + (d^m*m^
3 + 3*d^m*m^2 + 3*d^m*m + d^m)*q*x*x^m*polylog(3, a*x^q) - (d^m*m^4 + 4*d^m
*m^3 + 6*d^m*m^2 + 4*d^m*m + d^m)*x*x^m*polylog(4, a*x^q))/(m^5 + 5*m^4 + 1
0*m^3 + 10*m^2 + 5*m + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m \text{polylog}\left(4, ax^q\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*polylog(4, a*x^q), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4\left(ax^q\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*polylog(4,a*x**q),x)
```

```
[Out] Integral((d*x)**m*polylog(4, a*x**q), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \operatorname{Li}_4(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*polylog(4,a*x^q),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*polylog(4, a*x^q), x)
```

3.114 $\int x\mathbf{PolyLog}(n,ax)\,dx$

Optimal. Leaf size=9

$$\text{Unintegrable}(x\text{PolyLog}(n,ax),x)$$

```
[Out] Unintegrable[x*PolyLog[n, a*x], x]
```

Rubi [A] time = 0.0054963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x\text{PolyLog}(n,ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*PolyLog[n, a*x],x]
```

```
[Out] Defer[Int][x*PolyLog[n, a*x], x]
```

Rubi steps

$$\int x\text{Li}_n(ax)\,dx=\int x\text{Li}_n(ax)\,dx$$

Mathematica [A] time = 0.0216945, size = 0, normalized size = 0.

$$\int x\text{PolyLog}(n,ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*PolyLog[n, a*x],x]
```

```
[Out] Integrate[x*PolyLog[n, a*x], x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int x\mathrm{polylog}\left(n,ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,a*x),x)
```

```
[Out] int(x*polylog(n,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="maxima")
```

```
[Out] integrate(x*polylog(n, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{polylog}\left(n,ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(n, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x\,\mathrm{Li}_{n}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x)
```

```
[Out] Integral(x*polylog(n, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_n(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(n, a*x), x)
```

3.115 $\int \text{PolyLog}(n, ax)\,dx$

Optimal. Leaf size=7

$$\text{Unintegrable}(\text{PolyLog}(n, ax), x)$$

```
[Out] Unintegrable[PolyLog[n, a*x], x]
```

Rubi [A] time = 0.0022881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \text{PolyLog}(n, ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x],x]
```

```
[Out] Defer[Int][PolyLog[n, a*x], x]
```

Rubi steps

$$\int \text{Li}_n(ax)\,dx = \int \text{Li}_n(ax)\,dx$$

Mathematica [A] time = 0.0007752, size = 0, normalized size = 0.

$$\int \text{PolyLog}(n, ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x],x]
```

```
[Out] Integrate[PolyLog[n, a*x], x]
```

Maple [A] time = 0.047, size = 0, normalized size = 0.

$$\int \operatorname{polylog}(n, ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x),x)
```

```
[Out] int(polylog(n,a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(\operatorname{polylog}(n, ax), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_n(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x)
```

```
[Out] Integral(polylog(n, a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_n(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x), x)
```

3.116 $\int \frac{\textbf{PolyLog}(n,ax)}{x}\,dx$

Optimal. Leaf size=7

$$\text{PolyLog}(n+1,ax)$$

```
[Out] PolyLog[1 + n, a*x]
```

Rubi [A] time = 0.0089991, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6589}

$$\text{PolyLog}(n+1,ax)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, a*x]/x,x]

[Out] PolyLog[1 + n, a*x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x}\,dx = \text{Li}_{1+n}(ax)$$

Mathematica [A] time = 0.0011286, size = 7, normalized size = 1.

$$\text{PolyLog}(n+1,ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, a*x]/x,x]
```

```
[Out] PolyLog[1 + n, a*x]
```

Maple [A] time = 0.044, size = 8, normalized size = 1.1

$$\mathrm{polylog}\,(1+n,ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x)/x,x)
```

```
[Out] polylog(1+n,a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{polylog}\,(n,ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x)/x, x)
```

Sympy [A] time = 0.528206, size = 5, normalized size = 0.71

$$\mathrm{Li}_{n+1}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x)
```

```
[Out] polylog(n + 1, a*x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_n(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x)/x, x)
```

3.117 $\int \frac{\text{PolyLog}(n,ax)}{x^2} \, dx$

Optimal. Leaf size=11

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n,ax)}{x^2},x\right)$$

```
[Out] Unintegrable[PolyLog[n, a*x]/x^2, x]
```

Rubi [A] time = 0.0093445, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{PolyLog}(n,ax)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x]/x^2,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x]/x^2, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^2} \, dx = \int \frac{\text{Li}_n(ax)}{x^2} \, dx$$

Mathematica [A] time = 0.0207728, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n,ax)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x]/x^2,x]
```

```
[Out] Integrate[PolyLog[n, a*x]/x^2, x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}\left(n, ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x)/x^2,x)
```

```
[Out] int(polylog(n,a*x)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\left(n, ax\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x**2,x)
```

```
[Out] Integral(polylog(n, a*x)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_n(ax)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x)/x^2, x)
```

3.118 $\int \frac{\text{PolyLog}(n,ax)}{x^3} dx$

Optimal. Leaf size=11

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n,ax)}{x^3},x\right)$$

```
[Out] Unintegrable[PolyLog[n, a*x]/x^3, x]
```

Rubi [A] time = 0.0089528, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{PolyLog}(n,ax)}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x]/x^3,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x]/x^3, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax)}{x^3} dx = \int \frac{\text{Li}_n(ax)}{x^3} dx$$

Mathematica [A] time = 0.0208255, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n,ax)}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x]/x^3,x]
```

```
[Out] Integrate[PolyLog[n, a*x]/x^3, x]
```

Maple [A] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{\mathrm{polylog}\left(n, ax\right)}{x^{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x)/x^3,x)
```

```
[Out] int(polylog(n,a*x)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_{n}(ax)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{polylog}\left(n, ax\right)}{x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_{n}\left(ax\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x**3,x)
```

```
[Out] Integral(polylog(n, a*x)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_n(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x)/x^3, x)
```

3.119 $\int x\text{PolyLog}(n, ax^q)\,dx$

Optimal. Leaf size=11

$$\text{Unintegrable}\left(x\text{PolyLog}(n, ax^q), x\right)$$

```
[Out] Unintegrable[x*PolyLog[n, a*x^q], x]
```

Rubi [A] time = 0.0056386, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x\text{PolyLog}(n, ax^q)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x*PolyLog[n, a*x^q],x]
```

```
[Out] Defer[Int][x*PolyLog[n, a*x^q], x]
```

Rubi steps

$$\int x\text{Li}_n(ax^q)\,dx = \int x\text{Li}_n(ax^q)\,dx$$

Mathematica [A] time = 0.0234105, size = 0, normalized size = 0.

$$\int x\text{PolyLog}(n, ax^q)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*PolyLog[n, a*x^q],x]
```

```
[Out] Integrate[x*PolyLog[n, a*x^q], x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int x\mathrm{polylog}\left(n, a x^{q}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,a*x^q),x)
```

```
[Out] int(x*polylog(n,a*x^q),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_{n}(ax^{q})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="maxima")
```

```
[Out] integrate(x*polylog(n, a*x^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{polylog}\left(n, a x^{q}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(n, a*x^q), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_{n}\left(a x^{q}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x**q),x)
```

```
[Out] Integral(x*polylog(n, a*x**q), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_n(ax^q)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(n, a*x^q), x)
```

3.120 $\int \text{PolyLog}(n, ax^q)\, dx$

Optimal. Leaf size=9

$$\text{Unintegrable}\left(\text{PolyLog}(n, ax^q), x\right)$$

```
[Out] Unintegrable[PolyLog[n, a*x^q], x]
```

Rubi [A] time = 0.0025715, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \text{PolyLog}(n, ax^q)\, dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x^q],x]
```

```
[Out] Defer[Int][PolyLog[n, a*x^q], x]
```

Rubi steps

$$\int \text{Li}_n(ax^q)\, dx = \int \text{Li}_n(ax^q)\, dx$$

Mathematica [A] time = 0.0037649, size = 0, normalized size = 0.

$$\int \text{PolyLog}(n, ax^q)\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x^q],x]
```

```
[Out] Integrate[PolyLog[n, a*x^q], x]
```

Maple [A] time = 0.049, size = 0, normalized size = 0.

$$\int \operatorname{polylog}\left(n,a x^{q}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q),x)
```

```
[Out] int(polylog(n,a*x^q),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{n}(a x^{q})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\operatorname{polylog}\left(n,a x^{q}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{n}\left(a x^{q}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q),x)
```

```
[Out] Integral(polylog(n, a*x**q), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_n(ax^q)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q), x)
```

3.121 $\int \frac{\textbf{PolyLog}(n,ax^q)}{x}\,dx$

Optimal. Leaf size=13

$$\frac{\text{PolyLog}\left(n+1,ax^q\right)}{q}$$

```
[Out] PolyLog[1 + n, a*x^q]/q
```

Rubi [A] time = 0.0098521, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6589}

$$\frac{\text{PolyLog}\left(n+1,ax^q\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, a*x^q]/x,x]
```

```
[Out] PolyLog[1 + n, a*x^q]/q
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\text{Li}_n\left(ax^q\right)}{x}\,dx=\frac{\text{Li}_{1+n}\left(ax^q\right)}{q}$$

Mathematica [A] time = 0.0012072, size = 13, normalized size = 1.

$$\frac{\text{PolyLog}\left(n+1,ax^q\right)}{q}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, a*x^q]/x,x]
```

```
[Out] PolyLog[1 + n, a*x^q]/q
```

Maple [A] time = 0.046, size = 14, normalized size = 1.1

$$\frac{\text{polylog}\,(1+n,ax^q)}{q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x,x)
```

```
[Out] polylog(1+n,a*x^q)/q
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\,(n,ax^q)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(ax^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x,x)
```

```
[Out] Integral(polylog(n, a*x**q)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q)/x, x)
```

3.122 $\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx$

Optimal. Leaf size=13

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n,ax^q)}{x^2},x\right)$$

```
[Out] Unintegrable[PolyLog[n, a*x^q]/x^2, x]
```

Rubi [A] time = 0.0093166, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x^q]/x^2,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x^q]/x^2, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^2}\,dx = \int \frac{\text{Li}_n(ax^q)}{x^2}\,dx$$

Mathematica [A] time = 0.0214191, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x^q]/x^2,x]
```

```
[Out] Integrate[PolyLog[n, a*x^q]/x^2, x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\operatorname{polylog}(n, a x^{q})}{x^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x^2,x)
```

```
[Out] int(polylog(n,a*x^q)/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_{n}(a x^{q})}{x^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q)/x^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{polylog}(n, a x^{q})}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q)/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_{n}\left(a x^{q}\right)}{x^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x**2,x)
```

```
[Out] Integral(polylog(n, a*x**q)/x**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n(ax^q)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q)/x^2, x)
```

3.123 $\int \frac{\mathbf{PolyLog}(n,ax^q)}{x^3} dx$

Optimal. Leaf size=13

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(n,ax^q)}{x^3},x\right)$$

```
[Out] Unintegrable[PolyLog[n, a*x^q]/x^3, x]
```

Rubi [A] time = 0.0094303, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, a*x^q]/x^3,x]
```

```
[Out] Defer[Int][PolyLog[n, a*x^q]/x^3, x]
```

Rubi steps

$$\int \frac{\text{Li}_n(ax^q)}{x^3} dx = \int \frac{\text{Li}_n(ax^q)}{x^3} dx$$

Mathematica [A] time = 0.0219503, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(n,ax^q)}{x^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, a*x^q]/x^3,x]
```

```
[Out] Integrate[PolyLog[n, a*x^q]/x^3, x]
```

Maple [A] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\operatorname{polylog}(n, a x^q)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,a*x^q)/x^3,x)
```

```
[Out] int(polylog(n,a*x^q)/x^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n(a x^q)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, a*x^q)/x^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{polylog}(n, a x^q)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, a*x^q)/x^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_{n}\left(a x^{q}\right)}{x^{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x**q)/x**3,x)
```

```
[Out] Integral(polylog(n, a*x**q)/x**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(ax^q)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,a*x^q)/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, a*x^q)/x^3, x)
```

3.124 $\int x^2 \mathbf{PolyLog}(2, c(a+bx))\,dx$

Optimal. Leaf size=260

$$\frac{a^3 \text{PolyLog}(2, c(a+bx))}{3b^3} + \frac{1}{3}x^3 \text{PolyLog}(2, c(a+bx)) - \frac{a^2(-ac-bcx+1)\log(-ac-bcx+1)}{3b^3c} - \frac{a^2x}{3b^2} - \frac{x(1-ac)^2}{9b^2c^2} + \frac{a(1-}{}$$

```
[Out] -(a^2*x)/(3*b^2) + (a*(1 - a*c)*x)/(6*b^2*c) - ((1 - a*c)^2*x)/(9*b^2*c^2)
+ (a*x^2)/(12*b) - ((1 - a*c)*x^2)/(18*b*c) - x^3/27 + (a*(1 - a*c)^2*Log[1
 - a*c - b*c*x])/(6*b^3*c^2) - ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^
3) - (a*x^2*Log[1 - a*c - b*c*x])/(6*b) + (x^3*Log[1 - a*c - b*c*x])/9 - (a
^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + (a^3*PolyLog[2, c*(a
 + b*x)])/(3*b^3) + (x^3*PolyLog[2, c*(a + b*x)])/3
```

Rubi [A] time = 0.320951, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {6598, 43, 2416, 2389, 2295, 2395, 2393, 2391}

$$\frac{a^3 \text{PolyLog}(2, c(a+bx))}{3b^3} + \frac{1}{3}x^3 \text{PolyLog}(2, c(a+bx)) - \frac{a^2(-ac-bcx+1)\log(-ac-bcx+1)}{3b^3c} - \frac{a^2x}{3b^2} - \frac{x(1-ac)^2}{9b^2c^2} + \frac{a(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[2, c*(a + b*x)],x]
```


```
[Out] -(a^2*x)/(3*b^2) + (a*(1 - a*c)*x)/(6*b^2*c) - ((1 - a*c)^2*x)/(9*b^2*c^2)
+ (a*x^2)/(12*b) - ((1 - a*c)*x^2)/(18*b*c) - x^3/27 + (a*(1 - a*c)^2*Log[1
 - a*c - b*c*x])/(6*b^3*c^2) - ((1 - a*c)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^
3) - (a*x^2*Log[1 - a*c - b*c*x])/(6*b) + (x^3*Log[1 - a*c - b*c*x])/9 - (a
^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + (a^3*PolyLog[2, c*(a
 + b*x)])/(3*b^3) + (x^3*PolyLog[2, c*(a + b*x)])/3
```


Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\text{Li}_2(c(a+bx))\,dx = \frac{1}{3}x^3\text{Li}_2(c(a+bx)) + \frac{1}{3}b\int \frac{x^3\log(1-ac-bcx)}{a+bx}\,dx$$
$$= \frac{1}{3}x^3\text{Li}_2(c(a+bx)) + \frac{1}{3}b\int\left(\frac{a^2\log(1-ac-bcx)}{b^3} - \frac{ax\log(1-ac-bcx)}{b^2} + \frac{x^2\log(1-ac-bcx)}{b}\right.$$
$$= \frac{1}{3}x^3\text{Li}_2(c(a+bx)) + \frac{1}{3}\int x^2\log(1-ac-bcx)\,dx + \frac{a^2\int\log(1-ac-bcx)\,dx}{3b^2} - \frac{a^3\int\frac{\log(1-ac-bcx)}{a+bx}}{3b^2}$$
$$= -\frac{ax^2\log(1-ac-bcx)}{6b} + \frac{1}{9}x^3\log(1-ac-bcx) + \frac{1}{3}x^3\text{Li}_2(c(a+bx)) - \frac{a^3\,\text{Subst}\left(\int\frac{\log(1-cx)}{x}\,dx,\right.}{3b^3}$$
$$= -\frac{a^2x}{3b^2} - \frac{ax^2\log(1-ac-bcx)}{6b} + \frac{1}{9}x^3\log(1-ac-bcx) - \frac{a^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c} + \frac{a^3}{}$$
$$= -\frac{a^2x}{3b^2} + \frac{a(1-ac)x}{6b^2c} - \frac{(1-ac)^2x}{9b^2c^2} + \frac{ax^2}{12b} - \frac{(1-ac)x^2}{18bc} - \frac{x^3}{27} + \frac{a(1-ac)^2\log(1-ac-bcx)}{6b^3c^2} - \frac{(1-}{}$$

Mathematica [A] time = 0.208213, size = 144, normalized size = 0.55

$$\frac{36c^3\left(a^3+b^3x^3\right)\text{PolyLog}(2,c(a+bx)) - bcx\left(66a^2c^2-3ac(5bcx+14)+4b^2c^2x^2+6bcx+12\right)+6\left(6a^2c^2(bcx-3)+11a\right.}{108b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(b*c*x*(12 + 66*a^2*c^2 + 6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x))
) + 6*(-2 + 11*a^3*c^3 + 2*b^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c -
3*b^2*c^3*x^2))*Log[1 - a*c - b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2, c*
(a + b*x)])/(108*b^3*c^3)
```

Maple [A] time = 0.008, size = 269, normalized size = 1.

$$-\frac{31\,a}{36\,b^3c^2} - \frac{\ln(-xbc-ac+1)}{9\,b^3c^3} - \frac{85\,a^3}{108\,b^3} + \frac{a^3\text{dilog}(-xbc-ac+1)}{3\,b^3} - \frac{x^3}{27} + \frac{11}{54\,b^3c^3} + \frac{11\,\ln(-xbc-ac+1)\,a^3}{18\,b^3} - \frac{\ln(-xbc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(2,c*(b*x+a)),x)
```

```
[Out] -31/36/b^3/c^2*a-1/9/b^3/c^3*ln(-b*c*x-a*c+1)-85/108/b^3*a^3+1/3/b^3*a^3*di
log(-b*c*x-a*c+1)-1/27*x^3+11/54/b^3/c^3+11/18/b^3*ln(-b*c*x-a*c+1)*a^3-1/b
```

```
^3/c*ln(-b*c*x-a*c+1)*a^2+1/2/b^3/c^2*ln(-b*c*x-a*c+1)*a-1/18/b/c*x^2-1/6*a
*x^2*ln(-b*c*x-a*c+1)/b+1/3/b^2*ln(-b*c*x-a*c+1)*x*a^2+7/18/b^2/c*x*a-11/18
*a^2*x/b^2+5/36*a*x^2/b-1/9/b^2/c^2*x+1/3*polylog(2,b*c*x+a*c)*x^3+1/9*x^3*
ln(-b*c*x-a*c+1)+13/9/b^3/c*a^2
```

Maxima [A] time = 0.994565, size = 270, normalized size = 1.04

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a^3}{3\,b^3}+\frac{36\,b^3c^3x^3\mathrm{Li}_2(bcx+ac)-4\,b^3c^3x^3+3\left(5\,ab^2c^3-2\,b^2c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^3
/b^3 + 1/108*(36*b^3*c^3*x^3*dilog(b*c*x + a*c) - 4*b^3*c^3*x^3 + 3*(5*a*b^
2*c^3 - 2*b^2*c^2)*x^2 - 6*(11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*x + 6*(2*b^3*
c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 + 9*a*c
 - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Fricas [A] time = 2.39335, size = 366, normalized size = 1.41

$$-\frac{4\,b^3c^3x^3-3\left(5\,ab^2c^3-2\,b^2c^2\right)x^2+6\left(11\,a^2bc^3-7\,abc^2+2\,bc\right)x-36\left(b^3c^3x^3+a^3c^3\right)\mathrm{Li}_2(bcx+ac)-6\left(2\,b^3c^3x^3-3\,ab\right.}{108\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/108*(4*b^3*c^3*x^3 - 3*(5*a*b^2*c^3 - 2*b^2*c^2)*x^2 + 6*(11*a^2*b*c^3 -
 7*a*b*c^2 + 2*b*c)*x - 36*(b^3*c^3*x^3 + a^3*c^3)*dilog(b*c*x + a*c) - 6*(
2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*x + 11*a^3*c^3 - 18*a^2*c^2 +
 9*a*c - 2)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog((b*x + a)*c), x)
```

3.125 $\int x\textbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=152

$$-\frac{a^2\text{PolyLog}(2,c(a+bx))}{2b^2}+\frac{1}{2}x^2\text{PolyLog}(2,c(a+bx))-\frac{(1-ac)^2\log(-ac-bcx+1)}{4b^2c^2}+\frac{a(-ac-bcx+1)\log(-ac-bc}{2b^2c}$$

```
[Out] (a*x)/(2*b) - ((1 - a*c)*x)/(4*b*c) - x^2/8 - ((1 - a*c)^2*Log[1 - a*c - b*
c*x])/(4*b^2*c^2) + (x^2*Log[1 - a*c - b*c*x])/4 + (a*(1 - a*c - b*c*x)*Log
[1 - a*c - b*c*x])/(2*b^2*c) - (a^2*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2
*PolyLog[2, c*(a + b*x)])/2
```

Rubi [A] time = 0.169219, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {6598, 43, 2416, 2389, 2295, 2395, 2393, 2391}

$$-\frac{a^2\text{PolyLog}(2,c(a+bx))}{2b^2}+\frac{1}{2}x^2\text{PolyLog}(2,c(a+bx))-\frac{(1-ac)^2\log(-ac-bcx+1)}{4b^2c^2}+\frac{a(-ac-bcx+1)\log(-ac-bc}{2b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (a*x)/(2*b) - ((1 - a*c)*x)/(4*b*c) - x^2/8 - ((1 - a*c)^2*Log[1 - a*c - b*
c*x])/(4*b^2*c^2) + (x^2*Log[1 - a*c - b*c*x])/4 + (a*(1 - a*c - b*c*x)*Log
[1 - a*c - b*c*x])/(2*b^2*c) - (a^2*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2
*PolyLog[2, c*(a + b*x)])/2
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x\text{Li}_2(c(a+bx))\,dx = \frac{1}{2}x^2\text{Li}_2(c(a+bx)) + \frac{1}{2}b\int \frac{x^2\log(1-ac-bcx)}{a+bx}\,dx$$
$$= \frac{1}{2}x^2\text{Li}_2(c(a+bx)) + \frac{1}{2}b\int\left(-\frac{a\log(1-ac-bcx)}{b^2} + \frac{x\log(1-ac-bcx)}{b} + \frac{a^2\log(1-ac-bcx)}{b^2(a+bx)}\right)$$
$$= \frac{1}{2}x^2\text{Li}_2(c(a+bx)) + \frac{1}{2}\int x\log(1-ac-bcx)\,dx - \frac{a\int\log(1-ac-bcx)\,dx}{2b} + \frac{a^2\int\frac{\log(1-ac-bcx)}{a+bx}d}{2b}$$
$$= \frac{1}{4}x^2\log(1-ac-bcx) + \frac{1}{2}x^2\text{Li}_2(c(a+bx)) + \frac{a^2\,\text{Subst}\left(\int\frac{\log(1-cx)}{x}dx, x, a+bx\right)}{2b^2} + \frac{a\,\text{Subst}(\int \text{l}}{}$$
$$= \frac{ax}{2b} + \frac{1}{4}x^2\log(1-ac-bcx) + \frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} - \frac{a^2\text{Li}_2(c(a+bx))}{2b^2} + \frac{1}{2}x^2\text{Li}_2(c($$
$$= \frac{ax}{2b} - \frac{(1-ac)x}{4bc} - \frac{x^2}{8} - \frac{(1-ac)^2\log(1-ac-bcx)}{4b^2c^2} + \frac{1}{4}x^2\log(1-ac-bcx) + \frac{a(1-ac-bcx)\log}{2b^2c}$$

Mathematica [A] time = 0.100904, size = 96, normalized size = 0.63

$$\frac{-4c^2\left(a^2-b^2x^2\right)\text{PolyLog}(2,c(a+bx)) + \left(-6a^2c^2-4ac(bcx-2)+2b^2c^2x^2-2\right)\log(-ac-bcx+1) - bcx(-6ac+bcx+}{8b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2
 + b*c*x))*Log[1 - a*c - b*c*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b
*x)])/(8*b^2*c^2)
```

Maple [A] time = 0.004, size = 177, normalized size = 1.2

$$-\frac{\text{polylog}(2, xbc+ac)\,a^2}{2\,b^2} + \frac{\text{polylog}(2, xbc+ac)\,x^2}{2} - \frac{\ln(-xbc-ac+1)\,xa}{2\,b} - \frac{3\,\ln(-xbc-ac+1)\,a^2}{4\,b^2} + \frac{3\,ax}{4\,b} + \frac{7\,a^2}{8\,b^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(2,c*(b*x+a)),x)
```

```
[Out] -1/2/b^2*polylog(2,b*c*x+a*c)*a^2+1/2*polylog(2,b*c*x+a*c)*x^2-1/2/b*ln(-b*
c*x-a*c+1)*x*a-3/4/b^2*ln(-b*c*x-a*c+1)*a^2+3/4*a*x/b+7/8/b^2*a^2+1/b^2/c*l
```

```
n(-b*c*x-a*c+1)*a-5/4/b^2/c*a+1/4*x^2*ln(-b*c*x-a*c+1)-1/4/b^2/c^2*ln(-b*c*
x-a*c+1)-1/8*x^2-1/4/b/c*x+3/8/b^2/c^2
```

Maxima [A] time = 0.992845, size = 196, normalized size = 1.29

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a^2}{2\,b^2}+\frac{4\,b^2c^2x^2\mathrm{Li}_2(bcx+ac)-b^2c^2x^2+2\left(3\,abc^2-bc\right)x+2\left(b^2c\right.}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^2/
b^2 + 1/8*(4*b^2*c^2*x^2*dilog(b*c*x + a*c) - b^2*c^2*x^2 + 2*(3*a*b*c^2 -
b*c)*x + 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log(-b*c*x -
 a*c + 1))/(b^2*c^2)
```

Fricas [A] time = 2.348, size = 242, normalized size = 1.59

$$-\frac{b^2c^2x^2-2\left(3\,abc^2-bc\right)x-4\left(b^2c^2x^2-a^2c^2\right)\mathrm{Li}_2(bcx+ac)-2\left(b^2c^2x^2-2\,abc^2x-3\,a^2c^2+4\,ac-1\right)\log(-bcx-ac+1)}{8\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*c^2*x^2 - 2*(3*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - a^2*c^2)*dilog
(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 2*a*b*c^2*x - 3*a^2*c^2 + 4*a*c - 1)*log(-
b*c*x - a*c + 1))/(b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\text{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*dilog((b*x + a)*c), x)
```

3.126 $\int \text{PolyLog}(2, c(a + bx))\, dx$

Optimal. Leaf size=60

$$x\text{PolyLog}(2, c(a + bx)) + \frac{a\text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1)\log(-ac - bcx + 1)}{bc} - x$$

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rubi [A] time = 0.0486846, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391}

$$x\text{PolyLog}(2, c(a + bx)) + \frac{a\text{PolyLog}(2, c(a + bx))}{b} - \frac{(-ac - bcx + 1)\log(-ac - bcx + 1)}{bc} - x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \text{Li}_2(c(a+bx))\,dx &= x\text{Li}_2(c(a+bx)) - a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx + \int \log(1-c(a+bx))\,dx \\
&= x\text{Li}_2(c(a+bx)) - a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx + \int \log(1-ac-bcx)\,dx \\
&= x\text{Li}_2(c(a+bx)) - \frac{a\,\text{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{b} - \frac{\text{Subst}\left(\int \log(x)\,dx, x, 1-ac-bcx\right)}{bc} \\
&= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\text{Li}_2(c(a+bx))}{b} + x\text{Li}_2(c(a+bx))
\end{aligned}
$$

Mathematica [A] time = 0.0177979, size = 53, normalized size = 0.88

$$
\frac{c(a+bx)\text{PolyLog}(2,c(a+bx)) - c(a+bx) + (c(a+bx)-1)\log(1-c(a+bx))}{bc}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

Maple [A] time = 0.003, size = 96, normalized size = 1.6

$$\ln\left(-xbc-ac+1\right)x+\operatorname{polylog}\left(2,xbc+ac\right)x+\frac{\ln\left(-xbc-ac+1\right)a}{b}+\frac{\operatorname{polylog}\left(2,xbc+ac\right)a}{b}-x-\frac{a}{b}-\frac{\ln\left(-xbc-ac+\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a)),x)
```

```
[Out] ln(-b*c*x-a*c+1)*x+polylog(2,b*c*x+a*c)*x+1/b*ln(-b*c*x-a*c+1)*a+1/b*polylo
g(2,b*c*x+a*c)*a-x-a/b-1/b/c*ln(-b*c*x-a*c+1)+1/b/c
```

Maxima [A] time = 0.982883, size = 122, normalized size = 2.03

$$-\frac{\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\operatorname{Li}_2\left(-bcx-ac+1\right)\right)a}{b}+\frac{bcx\operatorname{Li}_2\left(bcx+ac\right)-bcx+\left(bcx+ac-1\right)\log\left(-bcx-ac+\right.}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + (
b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))
/(b*c)
```

Fricas [A] time = 2.41585, size = 126, normalized size = 2.1

$$-\frac{bcx-\left(bcx+ac\right)\operatorname{Li}_2\left(bcx+ac\right)-\left(bcx+ac-1\right)\log\left(-bcx-ac+1\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
 a*c + 1))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c), x)
```

3.127 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x} dx$

Optimal. Leaf size=401

$$\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)-\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)-\text{PolyLog}(3,1-c(a+bx))+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{Po}$$

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)] + ((Log[1 + (b*x)/a] + Log[(1
- a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))
])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/2 + ((Log[c*(a + b*x)] - Log[1 +
(b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/2 + (Log[1 - c*
(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)] +
Log[x]*PolyLog[2, c*(a + b*x)] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLo
g[2, -((b*x)/(a*(1 - c*(a + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))]*
PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))] + (Log[x] + Log[-((a*(1 - c*(a + b
*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, -((b*x)/a)] + PolyL
og[3, -((b*x)/(a*(1 - c*(a + b*x))))] - PolyLog[3, -((b*c*x)/(1 - c*(a + b*
x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

Rubi [A] time = 0.355029, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6597, 2440, 2435}

$$\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)-\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)-\text{PolyLog}(3,1-c(a+bx))+\log\left(-\frac{a(1-c(a+bx))}{bx}\right)\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x,x]
```


```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)] + ((Log[1 + (b*x)/a] + Log[(1
- a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))
])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/2 + ((Log[c*(a + b*x)] - Log[1 +
(b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/2 + (Log[1 - c*
(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)] +
Log[x]*PolyLog[2, c*(a + b*x)] + Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLo
g[2, -((b*x)/(a*(1 - c*(a + b*x))))] - Log[-((a*(1 - c*(a + b*x)))/(b*x))]*
PolyLog[2, -((b*c*x)/(1 - c*(a + b*x)))] + (Log[x] + Log[-((a*(1 - c*(a + b
*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)] - PolyLog[3, -((b*x)/a)] + PolyL
og[3, -((b*x)/(a*(1 - c*(a + b*x))))] - PolyLog[3, -((b*c*x)/(1 - c*(a + b*
x)))] - PolyLog[3, 1 - c*(a + b*x)]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\mathrm{Li}_2(c(a+bx))}{x}\,dx = \log(x)\mathrm{Li}_2(c(a+bx)) + b\int \frac{\log(x)\log(1-ac-bcx)}{a+bx}\,dx$$

$$= \log(x)\mathrm{Li}_2(c(a+bx)) + \mathrm{Subst}\left(\int \frac{\log\left(-\frac{a}{b}+\frac{x}{b}\right)\log\left(-\frac{-abc-b(1-ac)}{b}-cx\right)}{x}\,dx, x, a+bx\right)$$

$$= \log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx)) + \frac{1}{2}\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a}{a(1-c(a}\right.\right.$$

Mathematica [A] time = 0.145861, size = 422, normalized size = 1.05

$$-\text{PolyLog}(3,-ac-bcx+1)+\text{PolyLog}\left(3,\frac{a(ac+bcx-1)}{bx}\right)-\text{PolyLog}\left(3,\frac{ac+bcx-1}{bcx}\right)+\log\left(\frac{a(ac+bcx-1)}{bx}\right)\left(\text{Poly}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x,x]
```

```
[Out] Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((-Log[c*(a + b*x)] + Log[1
+ (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 + (L
og[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c +
 b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x)] - Log[-(((-1 + a*c)*(a + b*x))/(
b*x))] + Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 + (Log[1
- a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)]
+ (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x]
+ Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*
x)] + PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + Log[x]*PolyLog[2, a*c + b*c
*x] - PolyLog[3, -((b*x)/a)] - PolyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*
(-1 + a*c + b*c*x))/(b*x)] - PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)]
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, c(bx+a))}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x,x)
```

```
[Out] int(polylog(2,c*(b*x+a))/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2((bx+a)c)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(dilog((b*x + a)*c)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\left(bcx+ac\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx+a)c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x, x)
```

3.128 $\int \frac{\text{PolyLog}(2,c(a+bx))}{x^2} dx$

Optimal. Leaf size=84

$$-\frac{b\text{PolyLog}(2,c(a+bx))}{a}-\frac{\text{PolyLog}(2,c(a+bx))}{x}-\frac{b\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a}-\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a}$$

```
[Out] -((b*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*PolyLog[2, c*(a +
 b*x)])/a - PolyLog[2, c*(a + b*x)]/x - (b*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/a
```

Rubi [A] time = 0.116564, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {6598, 36, 29, 31, 2416, 2394, 2315, 2393, 2391}

$$-\frac{b\text{PolyLog}(2,c(a+bx))}{a}-\frac{\text{PolyLog}(2,c(a+bx))}{x}-\frac{b\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{a}-\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^2,x]
```

```
[Out] -((b*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*PolyLog[2, c*(a +
 b*x)])/a - PolyLog[2, c*(a + b*x)]/x - (b*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/a
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{Li}_2(c(a+bx))}{x^2}\,dx &= -\frac{\text{Li}_2(c(a+bx))}{x} - b\int \frac{\log(1-ac-bcx)}{x(a+bx)}\,dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{x} - b\int \left(\frac{\log(1-ac-bcx)}{ax} - \frac{b\log(1-ac-bcx)}{a(a+bx)}\right) dx \\
&= -\frac{\text{Li}_2(c(a+bx))}{x} - \frac{b\int \frac{\log(1-ac-bcx)}{x}\,dx}{a} + \frac{b^2\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{a} \\
&= -\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{\text{Li}_2(c(a+bx))}{x} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{a} - \frac{(b^2c)\int \frac{\text{lo}}{}}{} \\
&= -\frac{b\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{b\text{Li}_2(c(a+bx))}{a} - \frac{\text{Li}_2(c(a+bx))}{x} - \frac{b\text{Li}_2\left(1-\frac{bcx}{1-ac}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0519712, size = 73, normalized size = 0.87

$$-\frac{(a+bx)\text{PolyLog}(2,c(a+bx)) + bx\left(\text{PolyLog}\left(2,\frac{ac+bcx-1}{ac-1}\right) + \log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^2,x]
```

```
[Out] -(((a + b*x)*PolyLog[2, c*(a + b*x)] + b*x*(Log[(b*c*x)/(1 - a*c)]*Log[1 -
a*c - b*c*x] + PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/(a*x))
```

Maple [A] time = 0.13, size = 85, normalized size = 1.

$$-\frac{\text{polylog}(2,xbc+ac)}{x} - \frac{b\ln(-xbc-ac+1)}{a}\ln\left(-\frac{xbc}{ac-1}\right) - \frac{b}{a}\text{dilog}\left(-\frac{xbc}{ac-1}\right) - \frac{b\text{dilog}(-xbc-ac+1)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^2,x)
```

```
[Out] -polylog(2,b*c*x+a*c)/x-b/a*ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1))-b/a*dilog(-
b*c*x/(a*c-1))-b/a*dilog(-b*c*x-a*c+1)
```

Maxima [A] time = 0.981607, size = 154, normalized size = 1.83

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))b}{a}-\frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right)+\mathrm{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)b}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] (log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/a - (l
og(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x +
 a*c - 1)/(a*c - 1)))*b/a - dilog(b*c*x + a*c)/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\mathrm{Li}_2(bcx+ac)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx+a)c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x^2, x)
```

3.129 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{x^3}\,dx$

Optimal. Leaf size=173

$$\frac{b^2\text{PolyLog}(2,c(a+bx))}{2a^2}+\frac{b^2\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{\text{PolyLog}(2,c(a+bx))}{2x^2}+\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{2a^2}+\frac{b^2c}{2a(}$$

```
[Out] (b^2*c*Log[x])/(2*a*(1 - a*c)) - (b^2*c*Log[1 - a*c - b*c*x])/(2*a*(1 - a*c
)) + (b*Log[1 - a*c - b*c*x])/(2*a*x) + (b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 -
 a*c - b*c*x])/(2*a^2) + (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - PolyLog[2,
 c*(a + b*x)]/(2*x^2) + (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2)
```

Rubi [A] time = 0.177859, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {6598, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$\frac{b^2\text{PolyLog}(2,c(a+bx))}{2a^2}+\frac{b^2\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{\text{PolyLog}(2,c(a+bx))}{2x^2}+\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(-ac-bcx+1)}{2a^2}+\frac{b^2c}{2a(}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^3,x]
```

```
[Out] (b^2*c*Log[x])/(2*a*(1 - a*c)) - (b^2*c*Log[1 - a*c - b*c*x])/(2*a*(1 - a*c
)) + (b*Log[1 - a*c - b*c*x])/(2*a*x) + (b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 -
 a*c - b*c*x])/(2*a^2) + (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - PolyLog[2,
 c*(a + b*x)]/(2*x^2) + (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{x^3}\,dx = -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int \frac{\log(1-ac-bcx)}{x^2(a+bx)}\,dx$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int \left(\frac{\log(1-ac-bcx)}{ax^2} - \frac{b\log(1-ac-bcx)}{a^2x} + \frac{b^2\log(1-ac-bcx)}{a^2(a+bx)}\right)dx$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{b\int \frac{\log(1-ac-bcx)}{x^2}\,dx}{2a} + \frac{b^2\int \frac{\log(1-ac-bcx)}{x}\,dx}{2a^2} - \frac{b^3\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{2a^2}$$

$$= \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2} - \frac{b^2\,\text{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx\right.}{2a^2}$$

$$= \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2\text{Li}_2(c(a+bx))}{2a^2} - \frac{\text{Li}_2(c(a+bx))}{2x^2} + \frac{b^2\text{L}}{}$$

$$= \frac{b^2c\log(x)}{2a(1-ac)} - \frac{b^2c\log(1-ac-bcx)}{2a(1-ac)} + \frac{b\log(1-ac-bcx)}{2ax} + \frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2\text{L}}{}$$

Mathematica [A] time = 0.180152, size = 131, normalized size = 0.76

$$\frac{bx\left(bx(ac-1)\text{PolyLog}\left(2,\frac{ac+bcx-1}{ac-1}\right) - abcx\log(x) + \left(a(ac+bcx-1) + bx(ac-1)\log\left(\frac{bcx}{1-ac}\right)\right)\log(-ac-bcx+1)\right) - (ac}{2a^2x^2(ac-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^3,x]
```

```
[Out] (-((-1 + a*c)*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]) + b*x*(-(a*b*c*x*Log
[x]) + (a*(-1 + a*c + b*c*x) + b*(-1 + a*c)*x*Log[(b*c*x)/(1 - a*c)])*Log[1
```

```
 - a*c - b*c*x] + b*(-1 + a*c)*x*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)])
)/(2*a^2*(-1 + a*c)*x^2)
```

Maple [A] time = 0.227, size = 195, normalized size = 1.1

$$-\frac{\operatorname{polylog}(2, xbc + ac)}{2\,x^2} + \frac{b^2 \ln(-xbc - ac + 1)}{2\,a^2}\ln\left(-\frac{xbc}{ac-1}\right) + \frac{b^2}{2\,a^2}\operatorname{dilog}\left(-\frac{xbc}{ac-1}\right) + \frac{b^2 \operatorname{dilog}(-xbc - ac + 1)}{2\,a^2} - \frac{b^2 c \ln(-}{2\,a\,(ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^3,x)
```

```
[Out] -1/2*polylog(2,b*c*x+a*c)/x^2+1/2*b^2/a^2*ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1
))+1/2*b^2/a^2*dilog(-b*c*x/(a*c-1))+1/2*b^2/a^2*dilog(-b*c*x-a*c+1)-1/2*b^
2*c/a/(a*c-1)*ln(-x*b*c)+1/2*b^2*c/a*ln(-b*c*x-a*c+1)/(a*c-1)+1/2*b*c*ln(-b
*c*x-a*c+1)/(a*c-1)/x-1/2*b/a*ln(-b*c*x-a*c+1)/(a*c-1)/x
```

Maxima [A] time = 0.994426, size = 261, normalized size = 1.51

$$-\frac{b^2 c \log(x)}{2\left(a^2 c - a\right)} - \frac{\left(\log(bcx + ac)\log(-bcx - ac + 1) + \operatorname{Li}_2(-bcx - ac + 1)\right)b^2}{2\,a^2} + \frac{\left(\log(-bcx - ac + 1)\log\left(-\frac{bcx+ac-1}{ac-1} + 1\right) + \right.}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^2*c*log(x)/(a^2*c - a) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1)
 + dilog(-b*c*x - a*c + 1))*b^2/a^2 + 1/2*(log(-b*c*x - a*c + 1)*log(-(b*c*
x + a*c - 1)/(a*c - 1) + 1) + dilog((b*c*x + a*c - 1)/(a*c - 1)))*b^2/a^2 -
 1/2*((a^2*c - a)*dilog(b*c*x + a*c) - (b^2*c*x^2 + (a*b*c - b)*x)*log(-b*c
*x - a*c + 1))/((a^2*c - a)*x^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(bcx + ac)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx+a)c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x^3, x)
```

3.130 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{x^4}\,dx$

Optimal. Leaf size=276

$$-\frac{b^3\text{PolyLog}(2,c(a+bx))}{3a^3}-\frac{b^3\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{3a^3}-\frac{\text{PolyLog}(2,c(a+bx))}{3x^3}-\frac{b^3c\log(x)}{3a^2(1-ac)}-\frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(-ac-b}{3a^3}$$

```
[Out] -(b^2*c)/(6*a*(1 - a*c)*x) + (b^3*c^2*Log[x])/(6*a*(1 - a*c)^2) - (b^3*c*Lo
g[x])/(3*a^2*(1 - a*c)) - (b^3*c^2*Log[1 - a*c - b*c*x])/(6*a*(1 - a*c)^2)
+ (b^3*c*Log[1 - a*c - b*c*x])/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x])
/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x])/(3*a^2*x) - (b^3*Log[(b*c*x)/(1 - a
*c)]*Log[1 - a*c - b*c*x])/(3*a^3) - (b^3*PolyLog[2, c*(a + b*x)])/(3*a^3)
- PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^3*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])
/(3*a^3)
```

Rubi [A] time = 0.272902, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {6598, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391}

$$-\frac{b^3\text{PolyLog}(2,c(a+bx))}{3a^3}-\frac{b^3\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{3a^3}-\frac{\text{PolyLog}(2,c(a+bx))}{3x^3}-\frac{b^3c\log(x)}{3a^2(1-ac)}-\frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(-ac-b}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/x^4,x]
```

```
[Out] -(b^2*c)/(6*a*(1 - a*c)*x) + (b^3*c^2*Log[x])/(6*a*(1 - a*c)^2) - (b^3*c*Lo
g[x])/(3*a^2*(1 - a*c)) - (b^3*c^2*Log[1 - a*c - b*c*x])/(6*a*(1 - a*c)^2)
+ (b^3*c*Log[1 - a*c - b*c*x])/(3*a^2*(1 - a*c)) + (b*Log[1 - a*c - b*c*x])
/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*x])/(3*a^2*x) - (b^3*Log[(b*c*x)/(1 - a
*c)]*Log[1 - a*c - b*c*x])/(3*a^3) - (b^3*PolyLog[2, c*(a + b*x)])/(3*a^3)
- PolyLog[2, c*(a + b*x)]/(3*x^3) - (b^3*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])
/(3*a^3)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\mathrm{Li}_2(c(a+bx))}{x^4}\,dx = -\frac{\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int \frac{\log(1-ac-bcx)}{x^3(a+bx)}\,dx$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int\left(\frac{\log(1-ac-bcx)}{ax^3} - \frac{b\log(1-ac-bcx)}{a^2x^2} + \frac{b^2\log(1-ac-bcx)}{a^3x} - \frac{b^3\log(\ldots}{a^3\ldots}\right.$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{b\int\frac{\log(1-ac-bcx)}{x^3}\,dx}{3a} + \frac{b^2\int\frac{\log(1-ac-bcx)}{x^2}\,dx}{3a^2} - \frac{b^3\int\frac{\log(1-ac-bcx)}{x}\,dx}{3a^3} + \frac{b^4\int\frac{\log(1-ac\ldots}{a+bx}\ldots}{3a^3}$$

$$= \frac{b\log(1-ac-bcx)}{6ax^2} - \frac{b^2\log(1-ac-bcx)}{3a^2x} - \frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} - \frac{\mathrm{Li}_2(c(a+bx))}{3x^3} + \frac{b^3\ldots}{\ldots}$$

$$= \frac{b\log(1-ac-bcx)}{6ax^2} - \frac{b^2\log(1-ac-bcx)}{3a^2x} - \frac{b^3\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} - \frac{b^3\mathrm{Li}_2(c(a+bx))}{3a^3} - \ldots$$

$$= -\frac{b^2c}{6a(1-ac)x} + \frac{b^3c^2\log(x)}{6a(1-ac)^2} - \frac{b^3c\log(x)}{3a^2(1-ac)} - \frac{b^3c^2\log(1-ac-bcx)}{6a(1-ac)^2} + \frac{b^3c\log(1-ac-bcx)}{3a^2(1-ac)} + \frac{b\log\ldots}{\ldots}$$

Mathematica [A] time = 0.303672, size = 210, normalized size = 0.76

$$-\frac{b\left(2b^2\mathrm{PolyLog}(2,c(a+bx)) + 2b^2\mathrm{PolyLog}\left(2,\frac{ac+bcx-1}{ac-1}\right) - \frac{a^2\log(-ac-bcx+1)}{x^2} - \frac{a^2bc(-bcx\log(-ac-bcx+1)+ac+bcx\log(x)-1)}{x(ac-1)^2} - \frac{2ab^2c(\ldots}{\ldots}\right.}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/x^4,x]
```

```
[Out] -(b*((-2*a*b^2*c*(Log[x] - Log[1 - a*c - b*c*x]))/(-1 + a*c) - (a^2*Log[1 -
 a*c - b*c*x])/x^2 + (2*a*b*Log[1 - a*c - b*c*x])/x + 2*b^2*Log[(b*c*x)/(1
- a*c)]*Log[1 - a*c - b*c*x] - (a^2*b*c*(-1 + a*c + b*c*x*Log[x] - b*c*x*Lo
g[1 - a*c - b*c*x]))/((-1 + a*c)^2*x) + 2*b^2*PolyLog[2, c*(a + b*x)] + 2*b
^2*PolyLog[2, (-1 + a*c + b*c*x)/(-1 + a*c)]))/(6*a^3) - PolyLog[2, a*c + b
*c*x]/(3*x^3)
```

Maple [A] time = 0.214, size = 376, normalized size = 1.4

$$-\frac{\operatorname{polylog}(2, xbc+ac)}{3x^3}-\frac{b^3\ln(-xbc-ac+1)}{3a^3}\ln\left(-\frac{xbc}{ac-1}\right)-\frac{b^3}{3a^3}\operatorname{dilog}\left(-\frac{xbc}{ac-1}\right)+\frac{b^3c^2\ln(-xbc)}{6a(ac-1)^2}+\frac{b^2c^2}{6(ac-1)^2x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/x^4,x)
```

```
[Out] -1/3*polylog(2,b*c*x+a*c)/x^3-1/3*b^3/a^3*ln(-b*c*x-a*c+1)*ln(-b*c*x/(a*c-1
))-1/3*b^3/a^3*dilog(-b*c*x/(a*c-1))+1/6*b^3*c^2/a/(a*c-1)^2*ln(-x*b*c)+1/6
*b^2*c^2/(a*c-1)^2/x-1/6*b^2*c/a/(a*c-1)^2/x-1/6*b^3*c^2/a*ln(-b*c*x-a*c+1)
/(a*c-1)^2+1/6*b*c^2*ln(-b*c*x-a*c+1)/x^2/(a*c-1)^2*a-1/3*b*c*ln(-b*c*x-a*c
+1)/x^2/(a*c-1)^2+1/6*b/a*ln(-b*c*x-a*c+1)/x^2/(a*c-1)^2-1/3*b^3/a^3*dilog(
-b*c*x-a*c+1)+1/3*b^3*c/a^2/(a*c-1)*ln(-x*b*c)-1/3*b^3*c/a^2*ln(-b*c*x-a*c+
1)/(a*c-1)-1/3*b^2*c/a*ln(-b*c*x-a*c+1)/(a*c-1)/x+1/3*b^2/a^2*ln(-b*c*x-a*c
+1)/(a*c-1)/x
```

Maxima [A] time = 1.02696, size = 408, normalized size = 1.48

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\operatorname{Li}_2(-bcx-ac+1))b^3}{3a^3}-\frac{\left(\log(-bcx-ac+1)\log\left(-\frac{bcx+ac-1}{ac-1}+1\right)+\operatorname{Li}_2\left(\frac{bcx+ac-1}{ac-1}\right)\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^3/
a^3 - 1/3*(log(-b*c*x - a*c + 1)*log(-(b*c*x + a*c - 1)/(a*c - 1) + 1) + di
log((b*c*x + a*c - 1)/(a*c - 1)))*b^3/a^3 + 1/6*(3*a*b^3*c^2 - 2*b^3*c)*log
(x)/(a^4*c^2 - 2*a^3*c + a^2) + 1/6*((a^2*b^2*c^2 - a*b^2*c)*x^2 - 2*(a^4*c
^2 - 2*a^3*c + a^2)*dilog(b*c*x + a*c) - ((3*a*b^3*c^2 - 2*b^3*c)*x^3 + 2*(
```

```
a^2*b^2*c^2 - 2*a*b^2*c + b^2)*x^2 - (a^3*b*c^2 - 2*a^2*b*c + a*b)*x)*log(-
b*c*x - a*c + 1))/((a^4*c^2 - 2*a^3*c + a^2)*x^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\left(bcx + ac\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx + a)c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/x^4, x)
```

3.131 $\int x^2 \mathbf{PolyLog}(3, c(a + bx))\, dx$

Optimal. Leaf size=347

$$-\frac{\left(a^3-b^3x^3\right)\text{PolyLog}(3,c(a+bx))}{3b^3}-\frac{11a^3\text{PolyLog}(2,c(a+bx))}{18b^3}+\frac{2a^3\text{PolyLog}(3,c(a+bx))}{3b^3}-\frac{a^2x\text{PolyLog}(2,c(a+}{3b^2}$$

```
[Out] (11*a^2*x)/(18*b^2) - (5*a*(1 - a*c)*x)/(36*b^2*c) + ((1 - a*c)^2*x)/(27*b^
2*c^2) - (5*a*x^2)/(72*b) + ((1 - a*c)*x^2)/(54*b*c) + x^3/81 - (5*a*(1 - a
*c)^2*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x
])/(27*b^3*c^3) + (5*a*x^2*Log[1 - a*c - b*c*x])/(36*b) - (x^3*Log[1 - a*c
- b*c*x])/27 + (11*a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(18*b^3*c) -
 (11*a^3*PolyLog[2, c*(a + b*x)])/(18*b^3) - (a^2*x*PolyLog[2, c*(a + b*x)]
)/(3*b^2) + (a*x^2*PolyLog[2, c*(a + b*x)])/(6*b) - (x^3*PolyLog[2, c*(a +
b*x)])/9 + (2*a^3*PolyLog[3, c*(a + b*x)])/(3*b^3) - ((a^3 - b^3*x^3)*PolyL
og[3, c*(a + b*x)])/(3*b^3)
```

Rubi [A] time = 0.637739, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6599, 6595, 2444, 2389, 2295, 2421, 2393, 2391, 6598, 43, 2416, 2395, 6589}

$$-\frac{\left(a^3-b^3x^3\right)\text{PolyLog}(3,c(a+bx))}{3b^3}-\frac{11a^3\text{PolyLog}(2,c(a+bx))}{18b^3}+\frac{2a^3\text{PolyLog}(3,c(a+bx))}{3b^3}-\frac{a^2x\text{PolyLog}(2,c(a+}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[3, c*(a + b*x)],x]
```


```
[Out] (11*a^2*x)/(18*b^2) - (5*a*(1 - a*c)*x)/(36*b^2*c) + ((1 - a*c)^2*x)/(27*b^
2*c^2) - (5*a*x^2)/(72*b) + ((1 - a*c)*x^2)/(54*b*c) + x^3/81 - (5*a*(1 - a
*c)^2*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + ((1 - a*c)^3*Log[1 - a*c - b*c*x
])/(27*b^3*c^3) + (5*a*x^2*Log[1 - a*c - b*c*x])/(36*b) - (x^3*Log[1 - a*c
- b*c*x])/27 + (11*a^2*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(18*b^3*c) -
 (11*a^3*PolyLog[2, c*(a + b*x)])/(18*b^3) - (a^2*x*PolyLog[2, c*(a + b*x)]
)/(3*b^2) + (a*x^2*PolyLog[2, c*(a + b*x)])/(6*b) - (x^3*PolyLog[2, c*(a +
b*x)])/9 + (2*a^3*PolyLog[3, c*(a + b*x)])/(3*b^3) - ((a^3 - b^3*x^3)*PolyL
og[3, c*(a + b*x)])/(3*b^3)
```


Rule 6599

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
-Simp[((a^(m + 1) - b^(m + 1)*x^(m + 1))*PolyLog[n, c*(a + b*x)^p])/((m + 1
```

```
)*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1,
 c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x] /;
 FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\mathrm{Li}_3(c(a+bx))\,dx = -\frac{\left(a^3-b^3x^3\right)\mathrm{Li}_3(c(a+bx))}{3b^3} + \frac{\int\left(-a^2\mathrm{Li}_2(c(a+bx)) + abx\mathrm{Li}_2(c(a+bx)) - b^2x^2\mathrm{Li}_2(c(a+bx)) +\right.}{3b^2}$$

$$= -\frac{\left(a^3-b^3x^3\right)\mathrm{Li}_3(c(a+bx))}{3b^3} - \frac{1}{3}\int x^2\mathrm{Li}_2(c(a+bx))\,dx - \frac{a^2\int \mathrm{Li}_2(c(a+bx))\,dx}{3b^2} + \frac{\left(2a^3\right)\int \frac{\mathrm{Li}_2(c(}{a+}}{3b^2}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx))}{3b^3} - \frac{\left(a^3-b^3x^3\right.}{}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx))}{3b^3} - \frac{\left(a^3-b^3x^3\right.}{}$$

$$= -\frac{a^2x\mathrm{Li}_2(c(a+bx))}{3b^2} + \frac{ax^2\mathrm{Li}_2(c(a+bx))}{6b} - \frac{1}{9}x^3\mathrm{Li}_2(c(a+bx)) + \frac{2a^3\mathrm{Li}_3(c(a+bx))}{3b^3} - \frac{\left(a^3-b^3x^3\right.}{}$$

$$= \frac{a^2x}{3b^2} + \frac{5ax^2\log(1-ac-bcx)}{36b} - \frac{1}{27}x^3\log(1-ac-bcx) + \frac{a^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c} - \frac{a^3}{}$$

$$= \frac{11a^2x}{18b^2} + \frac{5ax^2\log(1-ac-bcx)}{36b} - \frac{1}{27}x^3\log(1-ac-bcx) + \frac{11a^2(1-ac-bcx)\log(1-ac-bcx)}{18b^3c}$$

$$= \frac{11a^2x}{18b^2} - \frac{5a(1-ac)x}{36b^2c} + \frac{(1-ac)^2x}{27b^2c^2} - \frac{5ax^2}{72b} + \frac{(1-ac)x^2}{54bc} + \frac{x^3}{81} - \frac{5a(1-ac)^2\log(1-ac-bcx)}{36b^3c^2} + \frac{(}{}$$

Mathematica [A] time = 0.0681564, size = 296, normalized size = 0.85

$$\frac{-36c^3\left(6a^2bx+11a^3-3ab^2x^2+2b^3x^3\right)\mathrm{PolyLog}(2,c(a+bx)) + 216c^3\left(a^3+b^3x^3\right)\mathrm{PolyLog}(3,c(a+bx)) + 510a^2bc^3x -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (24*a*c - 150*a^2*c^2 + 575*a^3*c^3 + 24*b*c*x - 138*a*b*c^2*x + 510*a^2*b*
c^3*x + 12*b^2*c^2*x^2 - 57*a*b^2*c^3*x^2 + 8*b^3*c^3*x^3 + 24*Log[1 - a*c
- b*c*x] - 162*a*c*Log[1 - a*c - b*c*x] + 648*a^2*c^2*Log[1 - a*c - b*c*x]
- 510*a^3*c^3*Log[1 - a*c - b*c*x] - 396*a^2*b*c^3*x*Log[1 - a*c - b*c*x] +
 90*a*b^2*c^3*x^2*Log[1 - a*c - b*c*x] - 24*b^3*c^3*x^3*Log[1 - a*c - b*c*x
] - 36*c^3*(11*a^3 + 6*a^2*b*x - 3*a*b^2*x^2 + 2*b^3*x^3)*PolyLog[2, c*(a +
 b*x)] + 216*c^3*(a^3 + b^3*x^3)*PolyLog[3, c*(a + b*x)])/(648*b^3*c^3)
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int x^2 \mathrm{polylog}\left(3, c\left(bx+a\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(3,c*(b*x+a)),x)
```

```
[Out] int(x^2*polylog(3,c*(b*x+a)),x)
```

Maxima [A] time = 1.03005, size = 356, normalized size = 1.03

$$\frac{11\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2\left(-bcx-ac+1\right)\right)a^3}{18\,b^3}+\frac{a^3\mathrm{Li}_3(bcx+ac)}{3\,b^3}+\frac{216\,b^3c^3x^3\mathrm{Li}_3(bcx+ac)+8\,b^3c^3x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] 11/18*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^
3/b^3 + 1/3*a^3*polylog(3, b*c*x + a*c)/b^3 + 1/648*(216*b^3*c^3*x^3*polylo
g(3, b*c*x + a*c) + 8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(8
5*a^2*b*c^3 - 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 +
 6*a^2*b*c^3*x)*dilog(b*c*x + a*c) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 +
66*a^2*b*c^3*x + 85*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c +
1))/(b^3*c^3)
```

Fricas [C] time = 2.34524, size = 675, normalized size = 1.95

$$\frac{8\,b^3c^3x^3-3\left(19\,ab^2c^3-4\,b^2c^2\right)x^2+6\left(85\,a^2bc^3-23\,abc^2+4\,bc\right)x-36\left(2\,b^3c^3x^3-3\,ab^2c^3x^2+6\,a^2bc^3x+11\,a^3c^3\right)\%\mathrm{iin}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/648*(8*b^3*c^3*x^3 - 3*(19*a*b^2*c^3 - 4*b^2*c^2)*x^2 + 6*(85*a^2*b*c^3 -
 23*a*b*c^2 + 4*b*c)*x - 36*(2*b^3*c^3*x^3 - 3*a*b^2*c^3*x^2 + 6*a^2*b*c^3*
```

```
x + 11*a^3*c^3)*\%iint(a, b, c, x, -log(-b*c*x - a*c + 1)/(b*x + a), -x*log(
-b*c*x - a*c + 1)/(b*x + a), -log(-b*c*x - a*c + 1)/c, -b*log(-b*c*x - a*c
+ 1)/(b*x + a)) - 6*(4*b^3*c^3*x^3 - 15*a*b^2*c^3*x^2 + 66*a^2*b*c^3*x + 85
*a^3*c^3 - 108*a^2*c^2 + 27*a*c - 4)*log(-b*c*x - a*c + 1) + 216*(b^3*c^3*x
^3 + a^3*c^3)*polylog(3, b*c*x + a*c))/(b^3*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3(ac + bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(3,c*(b*x+a)),x)
```

```
[Out] Integral(x**2*polylog(3, a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_3((bx + a)c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(3,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(3, (b*x + a)*c), x)
```

3.132 $\int x\textbf{PolyLog}(3, c(a + bx))\,dx$

Optimal. Leaf size=198

$$-\frac{(a^2 - b^2x^2)\,\text{PolyLog}(3, c(a + bx))}{2b^2} + \frac{3a^2\text{PolyLog}(2, c(a + bx))}{4b^2} - \frac{1}{4}x^2\text{PolyLog}(2, c(a + bx)) + \frac{ax\text{PolyLog}(2, c(a + b}{2b}$$

```
[Out] (-3*a*x)/(4*b) + ((1 - a*c)*x)/(8*b*c) + x^2/16 + ((1 - a*c)^2*Log[1 - a*c
- b*c*x])/(8*b^2*c^2) - (x^2*Log[1 - a*c - b*c*x])/8 - (3*a*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b^2*c) + (3*a^2*PolyLog[2, c*(a + b*x)])/(4*b^2
) + (a*x*PolyLog[2, c*(a + b*x)])/(2*b) - (x^2*PolyLog[2, c*(a + b*x)])/4 -
 ((a^2 - b^2*x^2)*PolyLog[3, c*(a + b*x)])/(2*b^2)
```

Rubi [A] time = 0.2869, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {6599, 6595, 2444, 2389, 2295, 2421, 2393, 2391, 6598, 43, 2416, 2395}

$$-\frac{(a^2 - b^2x^2)\,\text{PolyLog}(3, c(a + bx))}{2b^2} + \frac{3a^2\text{PolyLog}(2, c(a + bx))}{4b^2} - \frac{1}{4}x^2\text{PolyLog}(2, c(a + bx)) + \frac{ax\text{PolyLog}(2, c(a + b}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (-3*a*x)/(4*b) + ((1 - a*c)*x)/(8*b*c) + x^2/16 + ((1 - a*c)^2*Log[1 - a*c
- b*c*x])/(8*b^2*c^2) - (x^2*Log[1 - a*c - b*c*x])/8 - (3*a*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b^2*c) + (3*a^2*PolyLog[2, c*(a + b*x)])/(4*b^2
) + (a*x*PolyLog[2, c*(a + b*x)])/(2*b) - (x^2*PolyLog[2, c*(a + b*x)])/4 -
 ((a^2 - b^2*x^2)*PolyLog[3, c*(a + b*x)])/(2*b^2)
```

Rule 6599

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
-Simp[((a^(m + 1) - b^(m + 1)*x^(m + 1))*PolyLog[n, c*(a + b*x)^p])/((m + 1
)*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1,
 c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x] /;
 FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
```

```
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\int x\text{Li}_3(c(a+bx))\,dx = -\frac{\left(a^2-b^2x^2\right)\text{Li}_3(c(a+bx))}{2b^2} + \frac{\int (a\text{Li}_2(c(a+bx)) - bx\text{Li}_2(c(a+bx)))\,dx}{2b}$$
$$= -\frac{\left(a^2-b^2x^2\right)\text{Li}_3(c(a+bx))}{2b^2} - \frac{1}{2}\int x\text{Li}_2(c(a+bx))\,dx + \frac{a\int \text{Li}_2(c(a+bx))\,dx}{2b}$$
$$= \frac{ax\text{Li}_2(c(a+bx))}{2b} - \frac{1}{4}x^2\text{Li}_2(c(a+bx)) - \frac{\left(a^2-b^2x^2\right)\text{Li}_3(c(a+bx))}{2b^2} + \frac{a\int \log(1-c(a+bx))\,dx}{2b} -$$
$$= \frac{ax\text{Li}_2(c(a+bx))}{2b} - \frac{1}{4}x^2\text{Li}_2(c(a+bx)) - \frac{\left(a^2-b^2x^2\right)\text{Li}_3(c(a+bx))}{2b^2} + \frac{a\int \log(1-ac-bcx)\,dx}{2b} -$$
$$= \frac{ax\text{Li}_2(c(a+bx))}{2b} - \frac{1}{4}x^2\text{Li}_2(c(a+bx)) - \frac{\left(a^2-b^2x^2\right)\text{Li}_3(c(a+bx))}{2b^2} - \frac{1}{4}\int x\log(1-ac-bcx)\,dx$$
$$= -\frac{ax}{2b} - \frac{1}{8}x^2\log(1-ac-bcx) - \frac{a(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} + \frac{a^2\text{Li}_2(c(a+bx))}{2b^2} + \frac{ax\text{Li}_2(c(a}{2b}$$
$$= -\frac{3ax}{4b} - \frac{1}{8}x^2\log(1-ac-bcx) - \frac{3a(1-ac-bcx)\log(1-ac-bcx)}{4b^2c} + \frac{3a^2\text{Li}_2(c(a+bx))}{4b^2} + \frac{ax\text{Li}_2(}{}$$
$$= -\frac{3ax}{4b} + \frac{(1-ac)x}{8bc} + \frac{x^2}{16} + \frac{(1-ac)^2\log(1-ac-bcx)}{8b^2c^2} - \frac{1}{8}x^2\log(1-ac-bcx) - \frac{3a(1-ac-bcx)\text{l}}{4b}$$

Mathematica [A] time = 0.0466226, size = 198, normalized size = 1.

$$\frac{4c^2\left(3a^2+2abx-b^2x^2\right)\text{PolyLog}(2,c(a+bx)) - 8c^2\left(a^2-b^2x^2\right)\text{PolyLog}(3,c(a+bx)) + 14a^2c^2\log(-ac-bcx+1) - 15a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[3, c*(a + b*x)],x]
```

```
[Out] (2*a*c - 15*a^2*c^2 + 2*b*c*x - 14*a*b*c^2*x + b^2*c^2*x^2 + 2*Log[1 - a*c
- b*c*x] - 16*a*c*Log[1 - a*c - b*c*x] + 14*a^2*c^2*Log[1 - a*c - b*c*x] +
12*a*b*c^2*x*Log[1 - a*c - b*c*x] - 2*b^2*c^2*x^2*Log[1 - a*c - b*c*x] + 4*
c^2*(3*a^2 + 2*a*b*x - b^2*x^2)*PolyLog[2, c*(a + b*x)] - 8*c^2*(a^2 - b^2*
x^2)*PolyLog[3, c*(a + b*x)])/(16*b^2*c^2)
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int x\text{polylog}\left(3, c\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(3,c*(b*x+a)),x)
```

```
[Out] int(x*polylog(3,c*(b*x+a)),x)
```

Maxima [A] time = 1.01089, size = 261, normalized size = 1.32

$$-\frac{3\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2\left(-bcx-ac+1\right)\right)a^2}{4\,b^2}-\frac{a^2\mathrm{Li}_3(bcx+ac)}{2\,b^2}+\frac{8\,b^2c^2x^2\mathrm{Li}_3(bcx+ac)+b^2c^2x^2-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -3/4*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a^2
/b^2 - 1/2*a^2*polylog(3, b*c*x + a*c)/b^2 + 1/16*(8*b^2*c^2*x^2*polylog(3,
 b*c*x + a*c) + b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*
b*c^2*x)*dilog(b*c*x + a*c) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 + 8*
a*c - 1)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Fricas [C] time = 2.51134, size = 510, normalized size = 2.58

$$\frac{b^2c^2x^2-2\left(7\,abc^2-bc\right)x-4\left(b^2c^2x^2-2\,abc^2x-3\,a^2c^2\right)\%\mathrm{iint}\left(a,b,c,x,-\frac{\log(-bcx-ac+1)}{bx+a},-\frac{x\log(-bcx-ac+1)}{bx+a},-\frac{\log(-bcx-ac+1)}{c},\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*c^2*x^2 - 2*(7*a*b*c^2 - b*c)*x - 4*(b^2*c^2*x^2 - 2*a*b*c^2*x -
3*a^2*c^2)*\%iint(a, b, c, x, -log(-b*c*x - a*c + 1)/(b*x + a), -x*log(-b*c*
x - a*c + 1)/(b*x + a), -log(-b*c*x - a*c + 1)/c, -b*log(-b*c*x - a*c + 1)/
(b*x + a)) - 2*(b^2*c^2*x^2 - 6*a*b*c^2*x - 7*a^2*c^2 + 8*a*c - 1)*log(-b*c
*x - a*c + 1) + 8*(b^2*c^2*x^2 - a^2*c^2)*polylog(3, b*c*x + a*c))/(b^2*c^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3(ac + bcx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x)
```

```
[Out] Integral(x*polylog(3, a*c + b*c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_3((bx + a)c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(3,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(3, (b*x + a)*c), x)
```

3.133 $\int \mathbf{PolyLog}(3, c(a + bx))\,dx$

Optimal. Leaf size=84

$$x(-\text{PolyLog}(2, c(a + bx))) + x\text{PolyLog}(3, c(a + bx)) - \frac{a\text{PolyLog}(2, c(a + bx))}{b} + \frac{a\text{PolyLog}(3, c(a + bx))}{b} + \frac{(-ac - bc}{}$$

```
[Out] x + ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) - (a*PolyLog[2, c*(a + b
*x)])/b - x*PolyLog[2, c*(a + b*x)] + (a*PolyLog[3, c*(a + b*x)])/b + x*Pol
yLog[3, c*(a + b*x)]
```

Rubi [A] time = 0.0692237, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391, 6589}

$$x(-\text{PolyLog}(2, c(a + bx))) + x\text{PolyLog}(3, c(a + bx)) - \frac{a\text{PolyLog}(2, c(a + bx))}{b} + \frac{a\text{PolyLog}(3, c(a + bx))}{b} + \frac{(-ac - bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)],x]
```

```
[Out] x + ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) - (a*PolyLog[2, c*(a + b
*x)])/b - x*PolyLog[2, c*(a + b*x)] + (a*PolyLog[3, c*(a + b*x)])/b + x*Pol
yLog[3, c*(a + b*x)]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \mathrm{Li}_3(c(a+bx))\,dx = x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\mathrm{Li}_2(c(a+bx))}{a+bx}\,dx - \int \mathrm{Li}_2(c(a+bx))\,dx$$
$$= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx - \int \log(1-c$$
$$= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx - \int \log(1-ac$$
$$= -x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{Li}_3(c(a+bx)) + \frac{a\,\mathrm{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{b} + \frac{\mathrm{Su}}{}$$
$$= x + \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} - \frac{a\mathrm{Li}_2(c(a+bx))}{b} - x\mathrm{Li}_2(c(a+bx)) + \frac{a\mathrm{Li}_3(c(a+bx))}{b} + x\mathrm{L}$$

Mathematica [A] time = 0.0232919, size = 66, normalized size = 0.79

$$\frac{(a+bx)\left(-\mathrm{PolyLog}(2,c(a+bx)) + \mathrm{PolyLog}(3,c(a+bx)) + \frac{\log(1-c(a+bx))}{c(a+bx)} - \log(1-c(a+bx)) + 1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)],x]
```

```
[Out] ((a + b*x)*(1 - Log[1 - c*(a + b*x)] + Log[1 - c*(a + b*x)]/(c*(a + b*x)) -
 PolyLog[2, c*(a + b*x)] + PolyLog[3, c*(a + b*x)]))/b
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int \mathrm{polylog}(3, c(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a)),x)
```

```
[Out] int(polylog(3,c*(b*x+a)),x)
```

Maxima [A] time = 1.03029, size = 162, normalized size = 1.93

$$\frac{(\log(bcx+ac)\log(-bcx-ac+1) + \mathrm{Li}_2(-bcx-ac+1))a}{b} + \frac{a\mathrm{Li}_3(bcx+ac)}{b} - \frac{bcx\mathrm{Li}_2(bcx+ac) - bcx\mathrm{Li}_3(bcx+ac) - b}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] (log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + a*
polylog(3, b*c*x + a*c)/b - (b*c*x*dilog(b*c*x + a*c) - b*c*x*polylog(3, b*
c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))/(b*c)
```

Fricas [C] time = 2.3553, size = 355, normalized size = 4.23

$$\frac{bcx - (bcx + ac)\%\text{iint}\left(a, b, c, x, -\frac{\log(-bcx-ac+1)}{bx+a}, -\frac{x\log(-bcx-ac+1)}{bx+a}, -\frac{\log(-bcx-ac+1)}{c}, -\frac{b\log(-bcx-ac+1)}{bx+a}\right) - (bcx + ac - 1)\log(-bc}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] (b*c*x - (b*c*x + a*c)*\%iint(a, b, c, x, -log(-b*c*x - a*c + 1)/(b*x + a),
-x*log(-b*c*x - a*c + 1)/(b*x + a), -log(-b*c*x - a*c + 1)/c, -b*log(-b*c*x
 - a*c + 1)/(b*x + a)) - (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1) + (b*c*x +
 a*c)*polylog(3, b*c*x + a*c))/(b*c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3\left(c\left(a + bx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x)
```

```
[Out] Integral(polylog(3, c*(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{Li}_3((bx + a)c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c), x)
```

3.134 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x}\,dx$

Optimal. Leaf size=16

$$\text{Int}\left(\frac{\text{PolyLog}(3,ac+bcx)}{x},x\right)$$

```
[Out] Int[PolyLog[3, a*c + b*c*x]/x, x]
```

Rubi [A] time = 0.0377706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x,x]
```

```
[Out] Defer[Int][PolyLog[3, a*c + b*c*x]/x, x]
```

Rubi steps

$$\int \frac{\text{Li}_3(c(a+bx))}{x}\,dx = \int \frac{\text{Li}_3(ac+bcx)}{x}\,dx$$

Mathematica [A] time = 0.0369997, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}(3,c(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x,x]
```

```
[Out] Integrate[PolyLog[3, c*(a + b*x)]/x, x]
```

Maple [A] time = 0.004, size = 0, normalized size = 0.

$$\int \frac{\operatorname{polylog}(3, c(bx + a))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x,x)
```

```
[Out] int(polylog(3,c*(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3((bx + a)c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{polylog}(3, bcx + ac)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, b*c*x + a*c)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_3(ac + bcx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3((bx+a)c)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x, x)
```

3.135 $\int \frac{\textbf{PolyLog}(3,c(a+bx))}{x^2}\,dx$

Optimal. Leaf size=486

$$-\frac{2b\text{PolyLog}(3,c(a+bx))}{a}+\frac{\left(b-\frac{a}{x}\right)\text{PolyLog}(3,c(a+bx))}{a}+\frac{b\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{a}-\frac{b\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{a}$$

```
[Out] (b*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/a + (b*(Log[1 + (b*x)/a] +
 Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*a) + (b*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
2*a) + (b*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*Poly
Log[2, -((b*x)/a)])/a + (b*Log[x]*PolyLog[2, c*(a + b*x)])/a + (b*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/a -
(b*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x
)))])/a + (b*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 -
c*(a + b*x)])/a - (b*PolyLog[3, -((b*x)/a)])/a - (2*b*PolyLog[3, c*(a + b*x
)])/a + ((b - a/x)*PolyLog[3, c*(a + b*x)])/a + (b*PolyLog[3, -((b*x)/(a*(1
 - c*(a + b*x))))])/a - (b*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/a - (b
*PolyLog[3, 1 - c*(a + b*x)])/a
```

Rubi [A] time = 0.559953, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {6599, 6597, 2440, 2435, 6589}

$$-\frac{2b\text{PolyLog}(3,c(a+bx))}{a}+\frac{\left(b-\frac{a}{x}\right)\text{PolyLog}(3,c(a+bx))}{a}+\frac{b\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{a}-\frac{b\text{PolyLog}\left(3,-\frac{bcx}{1-c(a+bx)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x^2,x]
```

```
[Out] (b*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/a + (b*(Log[1 + (b*x)/a] +
 Log[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a
+ b*x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*a) + (b*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
2*a) + (b*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*Poly
Log[2, -((b*x)/a)])/a + (b*Log[x]*PolyLog[2, c*(a + b*x)])/a + (b*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/a -
(b*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a + b*x
```

```
)))])/a + (b*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 -
c*(a + b*x)])/a - (b*PolyLog[3, -((b*x)/a)])/a - (2*b*PolyLog[3, c*(a + b*x
)])/a + ((b - a/x)*PolyLog[3, c*(a + b*x)])/a + (b*PolyLog[3, -((b*x)/(a*(1
 - c*(a + b*x))))])/a - (b*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/a - (b
*PolyLog[3, 1 - c*(a + b*x)])/a
```

Rule 6599

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
-Simp[((a^(m + 1) - b^(m + 1)*x^(m + 1))*PolyLog[n, c*(a + b*x)^p])/((m + 1
)*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1,
 c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x] /;
 FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\mathrm{Li}_3(c(a+bx))}{x^2}\,dx = \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} - b^2\int\left(-\frac{\mathrm{Li}_2(c(a+bx))}{abx} + \frac{2\mathrm{Li}_2(c(a+bx))}{a(a+bx)}\right)dx$$

$$= \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b\int\frac{\mathrm{Li}_2(c(a+bx))}{x}\,dx}{a} - \frac{\left(2b^2\right)\int\frac{\mathrm{Li}_2(c(a+bx))}{a+bx}\,dx}{a}$$

$$= \frac{b\log(x)\mathrm{Li}_2(c(a+bx))}{a} - \frac{2b\mathrm{Li}_3(c(a+bx))}{a} + \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b^2\int\frac{\log(x)\log(1-ac-bcx)}{a+bx}\,dx}{a}$$

$$= \frac{b\log(x)\mathrm{Li}_2(c(a+bx))}{a} - \frac{2b\mathrm{Li}_3(c(a+bx))}{a} + \frac{\left(b-\frac{a}{x}\right)\mathrm{Li}_3(c(a+bx))}{a} + \frac{b\,\mathrm{Subst}\left(\int\frac{\log\left(-\frac{a}{b}+\frac{x}{b}\right)\log\left(-\right.}{x}\right.}{}$$

$$= \frac{b\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{a} + \frac{b\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\right)\mathrm{lo}}{2a}$$

Mathematica [A] time = 0.741879, size = 477, normalized size = 0.98

$$\frac{b\left(-\mathrm{PolyLog}(3,c(a+bx))-\mathrm{PolyLog}(3,-ac-bcx+1)+\mathrm{PolyLog}\left(3,\frac{a(ac+bcx-1)}{bx}\right)-\mathrm{PolyLog}\left(3,\frac{ac+bcx-1}{bcx}\right)+\log\left(\frac{a(ac-}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x^2,x]
```

```
[Out] -(PolyLog[3, c*(a + b*x)]/x) + (b*(Log[x]*Log[1 + (b*x)/a]*Log[1 - a*c - b*
c*x] + ((-Log[c*(a + b*x)] + Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*(-2*Log
[x] + Log[1 - a*c - b*c*x]))/2 + (Log[c*(a + b*x)] - Log[1 + (b*x)/a])*Log[
1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x))/(b*x)] + ((Log[(1 - a*c)/(b*c*x
)] - Log[-(((-1 + a*c)*(a + b*x))/(b*x))] + Log[1 + (b*x)/a])*Log[(a*(-1 +
a*c + b*c*x))/(b*x)]^2)/2 + (Log[1 - a*c - b*c*x] - Log[(a*(-1 + a*c + b*c*
x))/(b*x)])*PolyLog[2, -((b*x)/a)] + (Log[x] - Log[a + b*x])*PolyLog[2, c*(
a + b*x)] + Log[a + b*x]*PolyLog[2, c*(a + b*x)] + (Log[x] + Log[(a*(-1 + a
*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b*c*x] + Log[(a*(-1 + a*c + b*c*x
))/(b*x)]*(-PolyLog[2, (a*(-1 + a*c + b*c*x))/(b*x)] + PolyLog[2, (-1 + a*c
```

```
 + b*c*x)/(b*c*x)]) - PolyLog[3, -((b*x)/a)] - PolyLog[3, c*(a + b*x)] - Po
lyLog[3, 1 - a*c - b*c*x] + PolyLog[3, (a*(-1 + a*c + b*c*x))/(b*x)] - Poly
Log[3, (-1 + a*c + b*c*x)/(b*c*x)]))/a
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}\,(3, c\,(bx + a))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x^2,x)
```

```
[Out] int(polylog(3,c*(b*x+a))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\text{Li}_2\,(bcx + ac)}{bx^2 + ax}\,dx - \frac{\text{Li}_3(bcx + ac)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="maxima")
```

```
[Out] b*integrate(dilog(b*c*x + a*c)/(b*x^2 + a*x), x) - polylog(3, b*c*x + a*c)/
x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\,(3, bcx + ac)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, b*c*x + a*c)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3\left(ac+bcx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x**2,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3((bx+a)c)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^2,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x^2, x)
```

3.136 $\int \frac{\text{PolyLog}(3,c(a+bx))}{x^3} dx$

Optimal. Leaf size=629

$$\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{PolyLog}(3,c(a+bx))}{2a^2}-\frac{b^2\text{PolyLog}(2,c(a+bx))}{2a^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{b^2\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}+$$

```
[Out] -(b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b^2*Log[x]*Lo
g[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/(2*a^2) - (b^2*(Log[1 + (b*x)/a] + Log
[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*
x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*a^2) - (b^2*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
4*a^2) - (b^2*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*
PolyLog[2, -((b*x)/a)])/(2*a^2) - (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - (
b*PolyLog[2, c*(a + b*x)])/(2*a*x) - (b^2*Log[x]*PolyLog[2, c*(a + b*x)])/(
2*a^2) - (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*Log[-((a*(1
 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(2*a^2
) + (b^2*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a
 + b*x)))])/(2*a^2) - (b^2*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, 1 - c*(a + b*x)])/(2*a^2) + (b^2*PolyLog[3, -((b*x)/a)])/(2*a^2)
+ ((b^2 - a^2/x^2)*PolyLog[3, c*(a + b*x)])/(2*a^2) - (b^2*PolyLog[3, -((b*
x)/(a*(1 - c*(a + b*x))))])/(2*a^2) + (b^2*PolyLog[3, -((b*c*x)/(1 - c*(a +
 b*x)))])/(2*a^2) + (b^2*PolyLog[3, 1 - c*(a + b*x)])/(2*a^2)
```

Rubi [A] time = 0.663908, antiderivative size = 629, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6599, 6598, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 6597, 2440, 2435}

$$\frac{\left(b^2-\frac{a^2}{x^2}\right)\text{PolyLog}(3,c(a+bx))}{2a^2}-\frac{b^2\text{PolyLog}(2,c(a+bx))}{2a^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{bcx}{1-ac}\right)}{2a^2}-\frac{b^2\text{PolyLog}\left(3,-\frac{bx}{a(1-c(a+bx))}\right)}{2a^2}+$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, c*(a + b*x)]/x^3,x]
```

```
[Out] -(b^2*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b^2*Log[x]*Lo
g[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/(2*a^2) - (b^2*(Log[1 + (b*x)/a] + Log
[(1 - a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*
x)))])*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*a^2) - (b^2*(Log[c*(a + b*
x)] - Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(
```

```
4*a^2) - (b^2*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*
PolyLog[2, -((b*x)/a)])/(2*a^2) - (b^2*PolyLog[2, c*(a + b*x)])/(2*a^2) - (
b*PolyLog[2, c*(a + b*x)])/(2*a*x) - (b^2*Log[x]*PolyLog[2, c*(a + b*x)])/(
2*a^2) - (b^2*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*Log[-((a*(1
 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(2*a^2
) + (b^2*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a
 + b*x)))])/(2*a^2) - (b^2*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, 1 - c*(a + b*x)])/(2*a^2) + (b^2*PolyLog[3, -((b*x)/a)])/(2*a^2)
+ ((b^2 - a^2/x^2)*PolyLog[3, c*(a + b*x)])/(2*a^2) - (b^2*PolyLog[3, -((b*
x)/(a*(1 - c*(a + b*x))))])/(2*a^2) + (b^2*PolyLog[3, -((b*c*x)/(1 - c*(a +
 b*x)))])/(2*a^2) + (b^2*PolyLog[3, 1 - c*(a + b*x)])/(2*a^2)
```

Rule 6599

```
Int[(x_)^(m_.)*PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :>
-Simp[((a^(m + 1) - b^(m + 1)*x^(m + 1))*PolyLog[n, c*(a + b*x)^p])/((m + 1
)*b^(m + 1)), x] + Dist[p/((m + 1)*b^m), Int[ExpandIntegrand[PolyLog[n - 1,
 c*(a + b*x)^p], (a^(m + 1) - b^(m + 1)*x^(m + 1))/(a + b*x), x], x], x] /;
 FreeQ[{a, b, c, p}, x] && GtQ[n, 0] && IntegerQ[m] && NeQ[m, -1]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\mathrm{Li}_3(c(a+bx))}{x^3}\,dx = \frac{\left(b^2-\frac{a^2}{x^2}\right)\mathrm{Li}_3(c(a+bx))}{2a^2} - \frac{1}{2}b^3\int \left(-\frac{\mathrm{Li}_2(c(a+bx))}{ab^2x^2} + \frac{\mathrm{Li}_2(c(a+bx))}{a^2bx}\right)dx$$

$$= \frac{\left(b^2-\frac{a^2}{x^2}\right)\mathrm{Li}_3(c(a+bx))}{2a^2} + \frac{b\int \frac{\mathrm{Li}_2(c(a+bx))}{x^2}\,dx}{2a} - \frac{b^2\int \frac{\mathrm{Li}_2(c(a+bx))}{x}\,dx}{2a^2}$$

$$= -\frac{b\mathrm{Li}_2(c(a+bx))}{2ax} - \frac{b^2\log(x)\mathrm{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2-\frac{a^2}{x^2}\right)\mathrm{Li}_3(c(a+bx))}{2a^2} - \frac{b^2\int \frac{\log(1-ac-bcx)}{x(a+bx)}\,dx}{2a} - \cdots$$

$$= -\frac{b\mathrm{Li}_2(c(a+bx))}{2ax} - \frac{b^2\log(x)\mathrm{Li}_2(c(a+bx))}{2a^2} + \frac{\left(b^2-\frac{a^2}{x^2}\right)\mathrm{Li}_3(c(a+bx))}{2a^2} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{a}{b}+\frac{x}{b}\right)\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\left(\log\left(1+\frac{bx}{a}\right)+\log\left(\frac{1-ac}{1-c(a+bx)}\right)-\log\left(\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right)\cdots\right.}{4a^2}$$

$$= -\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\left(\log\left(1+\frac{bx}{a}\right)+\cdots\right.}{\cdots}$$

$$= -\frac{b^2\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} - \frac{b^2\log(x)\log\left(1+\frac{bx}{a}\right)\log(1-c(a+bx))}{2a^2} - \frac{b^2\left(\log\left(1+\frac{bx}{a}\right)+\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.93103, size = 573, normalized size = 0.91

$$bx\left(bx\left(\text{PolyLog}\left(2,\frac{bcx}{1-ac}\right)+\text{PolyLog}(2,-ac-bcx+1)+\text{PolyLog}(3,c(a+bx))+\text{PolyLog}(3,-ac-bcx+1)-\text{PolyLog}\left(3,\frac{a(ac+bcx-1)}{bx}\right)+\text{PolyLog}\left(3,\frac{ac+bcx-1}{bcx}\right)+\log\left(\frac{a(ac+bcx-}{bx}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, c*(a + b*x)]/x^3,x]
```

```
[Out] (-PolyLog[3, c*(a + b*x)] + (b*x*(-((a + b*x*Log[x] - b*x*Log[a + b*x])*Pol
yLog[2, c*(a + b*x)]) + b*x*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x] - Log[x]
*Log[1 + (b*x)/a]*Log[1 - a*c - b*c*x] + ((Log[c*(a + b*x)] - Log[1 + (b*x)
/a])*Log[1 - a*c - b*c*x]*(-2*Log[x] + Log[1 - a*c - b*c*x]))/2 - (Log[c*(a
 + b*x)] - Log[1 + (b*x)/a])*Log[1 - a*c - b*c*x]*Log[(a*(-1 + a*c + b*c*x)
)/(b*x)] - ((Log[(1 - a*c)/(b*c*x)] - Log[-(((-1 + a*c)*(a + b*x))/(b*x))]
+ Log[1 + (b*x)/a])*Log[(a*(-1 + a*c + b*c*x))/(b*x)]^2)/2 - Log[x]*(Log[1
- a*c - b*c*x] - Log[1 + (b*c*x)/(-1 + a*c)]) - (Log[1 - a*c - b*c*x] - Log
[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, -((b*x)/a)] + PolyLog[2, (b*c*x)
/(1 - a*c)] - Log[a + b*x]*PolyLog[2, c*(a + b*x)] + PolyLog[2, 1 - a*c - b
*c*x] - (Log[x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)])*PolyLog[2, 1 - a*c - b
*c*x] + Log[(a*(-1 + a*c + b*c*x))/(b*x)]*(PolyLog[2, (a*(-1 + a*c + b*c*x)
)/(b*x)] - PolyLog[2, (-1 + a*c + b*c*x)/(b*c*x)]) + PolyLog[3, -((b*x)/a)]
 + PolyLog[3, c*(a + b*x)] + PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (a*(-
1 + a*c + b*c*x))/(b*x)] + PolyLog[3, (-1 + a*c + b*c*x)/(b*c*x)])))/a^2)/(
2*x^2)
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}\left(3, c\left(bx+a\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,c*(b*x+a))/x^3,x)
```

```
[Out] int(polylog(3,c*(b*x+a))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{\text{Li}_2\left(bcx+ac\right)}{2\left(bx^3+ax^2\right)}\, dx - \frac{\text{Li}_3(bcx+ac)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="maxima")
```

```
[Out] b*integrate(1/2*dilog(b*c*x + a*c)/(b*x^3 + a*x^2), x) - 1/2*polylog(3, b*c
*x + a*c)/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}(3, bcx + ac)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="fricas")
```

```
[Out] integral(polylog(3, b*c*x + a*c)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(ac + bcx)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x**3,x)
```

```
[Out] Integral(polylog(3, a*c + b*c*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3((bx + a)c)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,c*(b*x+a))/x^3,x, algorithm="giac")
```

```
[Out] integrate(polylog(3, (b*x + a)*c)/x^3, x)
```

3.137 $\int (d+ex)^3 \textbf{PolyLog}(2, c(a+bx))\,dx$

Optimal. Leaf size=605

$$-\frac{(bd-ae)^4\text{PolyLog}(2,c(a+bx))}{4b^4e}+\frac{(d+ex)^4\text{PolyLog}(2,c(a+bx))}{4e}-\frac{x(bd-ae)(-ace+bcd+e)^2}{12b^3c^2}-\frac{(d+ex)^2(-ace+bc}{32b^2c^2e}$$

```
[Out] -((b*d - a*e)^3*x)/(4*b^3) - ((b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(8*b^3*c
) - ((b*d - a*e)*(b*c*d + e - a*c*e)^2*x)/(12*b^3*c^2) - ((b*c*d + e - a*c*
e)^3*x)/(16*b^3*c^3) - ((b*d - a*e)^2*(d + e*x)^2)/(16*b^2*e) - ((b*d - a*e
)*(b*c*d + e - a*c*e)*(d + e*x)^2)/(24*b^2*c*e) - ((b*c*d + e - a*c*e)^2*(d
 + e*x)^2)/(32*b^2*c^2*e) - ((b*d - a*e)*(d + e*x)^3)/(36*b*e) - ((b*c*d +
e - a*c*e)*(d + e*x)^3)/(48*b*c*e) - (d + e*x)^4/(64*e) - ((b*d - a*e)^2*(b
*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(8*b^4*c^2*e) - ((b*d - a*e)*(b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(12*b^4*c^3*e) - ((b*c*d + e - a*c*
e)^4*Log[1 - a*c - b*c*x])/(16*b^4*c^4*e) - ((b*d - a*e)^3*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(4*b^4*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c
- b*c*x])/(8*b^2*e) + ((b*d - a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(12*b*
e) + ((d + e*x)^4*Log[1 - a*c - b*c*x])/(16*e) - ((b*d - a*e)^4*PolyLog[2,
c*(a + b*x)])/(4*b^4*e) + ((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e)
```

Rubi [A] time = 0.587251, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$-\frac{(bd-ae)^4\text{PolyLog}(2,c(a+bx))}{4b^4e}+\frac{(d+ex)^4\text{PolyLog}(2,c(a+bx))}{4e}-\frac{x(bd-ae)(-ace+bcd+e)^2}{12b^3c^2}-\frac{(d+ex)^2(-ace+bc}{32b^2c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```


```
[Out] -((b*d - a*e)^3*x)/(4*b^3) - ((b*d - a*e)^2*(b*c*d + e - a*c*e)*x)/(8*b^3*c
) - ((b*d - a*e)*(b*c*d + e - a*c*e)^2*x)/(12*b^3*c^2) - ((b*c*d + e - a*c*
e)^3*x)/(16*b^3*c^3) - ((b*d - a*e)^2*(d + e*x)^2)/(16*b^2*e) - ((b*d - a*e
)*(b*c*d + e - a*c*e)*(d + e*x)^2)/(24*b^2*c*e) - ((b*c*d + e - a*c*e)^2*(d
 + e*x)^2)/(32*b^2*c^2*e) - ((b*d - a*e)*(d + e*x)^3)/(36*b*e) - ((b*c*d +
e - a*c*e)*(d + e*x)^3)/(48*b*c*e) - (d + e*x)^4/(64*e) - ((b*d - a*e)^2*(b
*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(8*b^4*c^2*e) - ((b*d - a*e)*(b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(12*b^4*c^3*e) - ((b*c*d + e - a*c*
e)^4*Log[1 - a*c - b*c*x])/(16*b^4*c^4*e) - ((b*d - a*e)^3*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(4*b^4*c) + ((b*d - a*e)^2*(d + e*x)^2*Log[1 - a*c
```

```
- b*c*x])/(8*b^2*e) + ((b*d - a*e)*(d + e*x)^3*Log[1 - a*c - b*c*x])/(12*b*
e) + ((d + e*x)^4*Log[1 - a*c - b*c*x])/(16*e) - ((b*d - a*e)^4*PolyLog[2,
c*(a + b*x)])/(4*b^4*e) + ((d + e*x)^4*PolyLog[2, c*(a + b*x)])/(4*e)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^3 \text{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^4\text{Li}_2(c(a+bx))}{4e} + \frac{b\int \frac{(d+ex)^4\log(1-ac-bcx)}{a+bx}\,dx}{4e}$$

$$= \frac{(d+ex)^4\text{Li}_2(c(a+bx))}{4e} + \frac{b\int \left(\frac{e(bd-ae)^3\log(1-ac-bcx)}{b^4} + \frac{(bd-ae)^4\log(1-ac-bcx)}{b^4(a+bx)} + \frac{e(bd-ae)^2(d+ex)\log}{b^3}\right.}{4e}$$

$$= \frac{(d+ex)^4\text{Li}_2(c(a+bx))}{4e} + \frac{1}{4}\int (d+ex)^3\log(1-ac-bcx)\,dx + \frac{(bd-ae)\int (d+ex)^2\log(1}{4b}$$

$$= \frac{(bd-ae)^2(d+ex)^2\log(1-ac-bcx)}{8b^2e} + \frac{(bd-ae)(d+ex)^3\log(1-ac-bcx)}{12be} + \frac{(d+ex)^4\text{lo}}{}$$

$$= -\frac{(bd-ae)^3x}{4b^3} - \frac{(bd-ae)^3(1-ac-bcx)\log(1-ac-bcx)}{4b^4c} + \frac{(bd-ae)^2(d+ex)^2\log(1-ac}{8b^2e}$$

$$= -\frac{(bd-ae)^3x}{4b^3} - \frac{(bd-ae)^2(bcd+e-ace)x}{8b^3c} - \frac{(bd-ae)(bcd+e-ace)^2x}{12b^3c^2} - \frac{(bcd+e-ace)^3}{16b^3c^3}$$

Mathematica [A] time = 0.562469, size = 485, normalized size = 0.8

$$\frac{-144c^4\left(6a^2b^2d^2e-4a^3bde^2+a^4e^3-4ab^3d^3-b^4x\left(6d^2ex+4d^3+4de^2x^2+e^3x^3\right)\right)\text{PolyLog}(2,c(a+bx))+bc\left(-6a^2c^2e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (12*e*(-1 + a*c + b*c*x)*((3 - 13*a*c + 23*a^2*c^2 - 25*a^3*c^3)*e^2 + b*c*
e*(8*(2 - 7*a*c + 11*a^2*c^2)*d + (3 - 10*a*c + 13*a^2*c^2)*e*x) + b^3*c^3*
x*(36*d^2 + 16*d*e*x + 3*e^2*x^2) + b^2*c^2*(-36*(-1 + 3*a*c)*d^2 - 8*(-2 +
```

```
 5*a*c)*d*e*x + (3 - 7*a*c)*e^2*x^2))*Log[1 - a*c - b*c*x] + b*c*(300*a^3*c
^3*e^3*x - 6*a^2*c^2*e^2*x*(46*e + b*c*(176*d + 13*e*x)) + 4*a*c*(39*e^3*x
+ 3*b*c*e^2*x*(56*d + 5*e*x) + b^2*c^2*(-144*d^3 + 324*d^2*e*x + 60*d*e^2*x
^2 + 7*e^3*x^3)) - x*(36*e^3 + 6*b*c*e^2*(32*d + 3*e*x) + 12*b^2*c^2*e*(36*
d^2 + 8*d*e*x + e^2*x^2) + b^3*c^3*(576*d^3 + 216*d^2*e*x + 64*d*e^2*x^2 +
9*e^3*x^3)) + 576*b^2*c^2*d^3*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*x)]) - 14
4*c^4*(-4*a*b^3*d^3 + 6*a^2*b^2*d^2*e - 4*a^3*b*d*e^2 + a^4*e^3 - b^4*x*(4*
d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*PolyLog[2, c*(a + b*x)])/(576*b^4
*c^4)
```

Maple [B] time = 0.064, size = 1177, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*polylog(2,c*(b*x+a)),x)
```

```
[Out] 1/4*e^3*polylog(2,b*c*x+a*c)*x^4-1/4/e*dilog(-b*c*x-a*c+1)*d^4+1/4/e*polylo
g(2,b*c*x+a*c)*d^4-1/9*e^2*x^3*d-3/8*e*x^2*d^2+1/b/c*d^3+25/192/b^4/c^4*e^3
+1/16*e^3*ln(-b*c*x-a*c+1)*x^4+ln(-b*c*x-a*c+1)*x*d^3+polylog(2,b*c*x+a*c)*
x*d^3-23/48/b^3/c*e^3*x*a^2-1/32/b^2/c^2*e^3*x^2-1/16/b^3/c^3*e^3*x-1/b/c*l
n(-b*c*x-a*c+1)*d^3-13/96/b^2*e^3*x^2*a^2+25/48/b^3*e^3*x*a^3+7/144/b*e^3*x
^3*a-1/4/b^4*e^3*dilog(-b*c*x-a*c+1)*a^4+3/4*e*ln(-b*c*x-a*c+1)*x^2*d^2+3/2
*e*polylog(2,b*c*x+a*c)*d^2*x^2+e^2*polylog(2,b*c*x+a*c)*d*x^3+1/3*e^2*ln(-
b*c*x-a*c+1)*x^3*d-1/48/b/c*x^3*e^3-1/16/b^4/c^4*e^3*ln(-b*c*x-a*c+1)-25/48
/b^4*e^3*ln(-b*c*x-a*c+1)*a^4+1/b*ln(-b*c*x-a*c+1)*a*d^3+1/b*dilog(-b*c*x-a
*c+1)*a*d^3+415/576/b^4*e^3*a^4+137/96/b^4/c^2*e^3*a^2+9/8/b^2/c^2*e*d^2-97
/144/b^4/c^3*e^3*a-1/b*a*d^3-77/48/b^4/c*e^3*a^3+11/18/b^3/c^3*e^2*d-d^3*x-
1/64*e^3*x^4-3/2/b^2*e*dilog(-b*c*x-a*c+1)*a^2*d^2+11/6/b^3*e^2*ln(-b*c*x-a
*c+1)*a^3*d-1/12/b*e^3*ln(-b*c*x-a*c+1)*x^3*a+1/8/b^2*e^3*ln(-b*c*x-a*c+1)*
x^2*a^2+13/48/b^3/c^2*e^3*x*a-31/12/b^3/c^2*e^2*a*d-85/36/b^3*e^2*a^3*d+21/
8/b^2*e*a^2*d^2-15/4/b^2/c*e*a*d^2-3/4/b^4/c^2*e^3*ln(-b*c*x-a*c+1)*a^2+1/3
/b^4/c^3*e^3*ln(-b*c*x-a*c+1)*a-3/4/b^2/c^2*e*ln(-b*c*x-a*c+1)*d^2-1/3/b^3/
c^3*e^2*ln(-b*c*x-a*c+1)*d+1/b^3*e^2*dilog(-b*c*x-a*c+1)*a^3*d-1/6/b/c*e^2*
x^2*d-1/3/b^2/c^2*e^2*x*d+5/48/b^2/c*e^3*x^2*a-3/4/b/c*e*x*d^2+9/4/b*e*x*a*
d^2+5/12/b*e^2*x^2*a*d-11/6/b^2*e^2*x*a^2*d-1/4/b^3*e^3*ln(-b*c*x-a*c+1)*x*
a^3-9/4/b^2*e*ln(-b*c*x-a*c+1)*a^2*d^2+1/b^4/c*e^3*ln(-b*c*x-a*c+1)*a^3+13/
3/b^3/c*e^2*a^2*d-3/2/b*e*ln(-b*c*x-a*c+1)*x*a*d^2-1/2/b*e^2*ln(-b*c*x-a*c+
1)*x^2*a*d+1/b^2*e^2*ln(-b*c*x-a*c+1)*x*a^2*d+3/b^2/c*e*ln(-b*c*x-a*c+1)*a*
d^2-3/b^3/c*e^2*ln(-b*c*x-a*c+1)*a^2*d+3/2/b^3/c^2*e^2*ln(-b*c*x-a*c+1)*a*d
+7/6/b^2/c*e^2*x*a*d
```

Maxima [A] time = 1.04257, size = 919, normalized size = 1.52

$$-\frac{\left(4\,ab^3d^3-6\,a^2b^2d^2e+4\,a^3bde^2-a^4e^3\right)\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_2\left(-bcx-ac+1\right)\right)}{4\,b^4}-\frac{9\,b^4c^4e^3x^4+4\,\left(16\,b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e^2 - a^4*e^3)*(log(b*c*x +
 a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))/b^4 - 1/576*(9*b^4*c
^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3)*e^3)*x^3 + 6*(
36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13*a^2*b^2*c^4 - 10
*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 36*(3*a*b^3*c^4 - b
^3*c^3)*d^2*e + 8*(11*a^2*b^2*c^4 - 7*a*b^2*c^3 + 2*b^2*c^2)*d*e^2 - (25*a^
3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*e^3)*x - 144*(b^4*c^4*e^3*x^4
+ 4*b^4*c^4*d*e^2*x^3 + 6*b^4*c^4*d^2*e*x^2 + 4*b^4*c^4*d^3*x)*dilog(b*c*x
+ a*c) - 12*(3*b^4*c^4*e^3*x^4 + 48*(a*b^3*c^4 - b^3*c^3)*d^3 - 36*(3*a^2*b
^2*c^4 - 4*a*b^2*c^3 + b^2*c^2)*d^2*e + 8*(11*a^3*b*c^4 - 18*a^2*b*c^3 + 9*
a*b*c^2 - 2*b*c)*d*e^2 - (25*a^4*c^4 - 48*a^3*c^3 + 36*a^2*c^2 - 16*a*c + 3
)*e^3 + 4*(4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 + 6*(6*b^4*c^4*d^2*e - 4*a*
b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3)*x^2 + 12*(4*b^4*c^4*d^3 - 6*a*b^3*c^4*d^2*
e + 4*a^2*b^2*c^4*d*e^2 - a^3*b*c^4*e^3)*x)*log(-b*c*x - a*c + 1))/(b^4*c^4
)
```

Fricas [A] time = 2.37849, size = 1339, normalized size = 2.21

$$-\frac{9\,b^4c^4e^3x^4+4\,\left(16\,b^4c^4de^2-\left(7\,ab^3c^4-3\,b^3c^3\right)e^3\right)x^3+6\,\left(36\,b^4c^4d^2e-8\,\left(5\,ab^3c^4-2\,b^3c^3\right)de^2+\left(13\,a^2b^2c^4-10\,ab^2c^3+3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/576*(9*b^4*c^4*e^3*x^4 + 4*(16*b^4*c^4*d*e^2 - (7*a*b^3*c^4 - 3*b^3*c^3)
*e^3)*x^3 + 6*(36*b^4*c^4*d^2*e - 8*(5*a*b^3*c^4 - 2*b^3*c^3)*d*e^2 + (13*a
^2*b^2*c^4 - 10*a*b^2*c^3 + 3*b^2*c^2)*e^3)*x^2 + 12*(48*b^4*c^4*d^3 - 36*(
3*a*b^3*c^4 - b^3*c^3)*d^2*e + 8*(11*a^2*b^2*c^4 - 7*a*b^2*c^3 + 2*b^2*c^2)
*d*e^2 - (25*a^3*b*c^4 - 23*a^2*b*c^3 + 13*a*b*c^2 - 3*b*c)*e^3)*x - 144*(b
^4*c^4*e^3*x^4 + 4*b^4*c^4*d*e^2*x^3 + 6*b^4*c^4*d^2*e*x^2 + 4*b^4*c^4*d^3*
```

```
x + 4*a*b^3*c^4*d^3 - 6*a^2*b^2*c^4*d^2*e + 4*a^3*b*c^4*d*e^2 - a^4*c^4*e^3
)*dilog(b*c*x + a*c) - 12*(3*b^4*c^4*e^3*x^4 + 48*(a*b^3*c^4 - b^3*c^3)*d^3
 - 36*(3*a^2*b^2*c^4 - 4*a*b^2*c^3 + b^2*c^2)*d^2*e + 8*(11*a^3*b*c^4 - 18*
a^2*b*c^3 + 9*a*b*c^2 - 2*b*c)*d*e^2 - (25*a^4*c^4 - 48*a^3*c^3 + 36*a^2*c^
2 - 16*a*c + 3)*e^3 + 4*(4*b^4*c^4*d*e^2 - a*b^3*c^4*e^3)*x^3 + 6*(6*b^4*c^
4*d^2*e - 4*a*b^3*c^4*d*e^2 + a^2*b^2*c^4*e^3)*x^2 + 12*(4*b^4*c^4*d^3 - 6*
a*b^3*c^4*d^2*e + 4*a^2*b^2*c^4*d*e^2 - a^3*b*c^4*e^3)*x)*log(-b*c*x - a*c
+ 1))/(b^4*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 \mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*dilog((b*x + a)*c), x)
```

3.138 $\int (d+ex)^2 \textbf{PolyLog}(2, c(a+bx))\, dx$

Optimal. Leaf size=385

$$-\frac{(bd-ae)^3\text{PolyLog}(2,c(a+bx))}{3b^3e}+\frac{(d+ex)^3\text{PolyLog}(2,c(a+bx))}{3e}-\frac{x(-ace+bcd+e)^2}{9b^2c^2}-\frac{(bd-ae)(-ace+bcd+e)^2\text{l}}{6b^3c^2e}$$

```
[Out] -((b*d - a*e)^2*x)/(3*b^2) - ((b*d - a*e)*(b*c*d + e - a*c*e)*x)/(6*b^2*c)
- ((b*c*d + e - a*c*e)^2*x)/(9*b^2*c^2) - ((b*d - a*e)*(d + e*x)^2)/(12*b*e
) - ((b*c*d + e - a*c*e)*(d + e*x)^2)/(18*b*c*e) - (d + e*x)^3/(27*e) - ((b
*d - a*e)*(b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^3*e) - ((b*d - a*e)^2*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + ((b*d - a*e)*(d + e*x)^2*Lo
g[1 - a*c - b*c*x])/(6*b*e) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(9*e) - ((
b*d - a*e)^3*PolyLog[2, c*(a + b*x)])/(3*b^3*e) + ((d + e*x)^3*PolyLog[2, c
*(a + b*x)])/(3*e)
```

Rubi [A] time = 0.338798, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$-\frac{(bd-ae)^3\text{PolyLog}(2,c(a+bx))}{3b^3e}+\frac{(d+ex)^3\text{PolyLog}(2,c(a+bx))}{3e}-\frac{x(-ace+bcd+e)^2}{9b^2c^2}-\frac{(bd-ae)(-ace+bcd+e)^2\text{l}}{6b^3c^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -((b*d - a*e)^2*x)/(3*b^2) - ((b*d - a*e)*(b*c*d + e - a*c*e)*x)/(6*b^2*c)
- ((b*c*d + e - a*c*e)^2*x)/(9*b^2*c^2) - ((b*d - a*e)*(d + e*x)^2)/(12*b*e
) - ((b*c*d + e - a*c*e)*(d + e*x)^2)/(18*b*c*e) - (d + e*x)^3/(27*e) - ((b
*d - a*e)*(b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(6*b^3*c^2*e) - ((b*c
*d + e - a*c*e)^3*Log[1 - a*c - b*c*x])/(9*b^3*c^3*e) - ((b*d - a*e)^2*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(3*b^3*c) + ((b*d - a*e)*(d + e*x)^2*Lo
g[1 - a*c - b*c*x])/(6*b*e) + ((d + e*x)^3*Log[1 - a*c - b*c*x])/(9*e) - ((
b*d - a*e)^3*PolyLog[2, c*(a + b*x)])/(3*b^3*e) + ((d + e*x)^3*PolyLog[2, c
*(a + b*x)])/(3*e)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
```

```
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)^2 \mathrm{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{b\int \frac{(d+ex)^3\log(1-ac-bcx)}{a+bx}\,dx}{3e}$$

$$= \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{b\int \left(\frac{e(bd-ae)^2\log(1-ac-bcx)}{b^3} + \frac{(bd-ae)^3\log(1-ac-bcx)}{b^3(a+bx)} + \frac{e(bd-ae)(d+ex)\log(}{b^2}\right.}{3e}$$

$$= \frac{(d+ex)^3 \mathrm{Li}_2(c(a+bx))}{3e} + \frac{1}{3}\int (d+ex)^2\log(1-ac-bcx)\,dx + \frac{(bd-ae)\int (d+ex)\log(1-}{3b}$$

$$= \frac{(bd-ae)(d+ex)^2\log(1-ac-bcx)}{6be} + \frac{(d+ex)^3\log(1-ac-bcx)}{9e} + \frac{(d+ex)^3\mathrm{Li}_2(c(a+bx}{3e}$$

$$= -\frac{(bd-ae)^2x}{3b^2} - \frac{(bd-ae)^2(1-ac-bcx)\log(1-ac-bcx)}{3b^3c} + \frac{(bd-ae)(d+ex)^2\log(1-ac}{6be}$$

$$= -\frac{(bd-ae)^2x}{3b^2} - \frac{(bd-ae)(bcd+e-ace)x}{6b^2c} - \frac{(bcd+e-ace)^2x}{9b^2c^2} - \frac{(bd-ae)(d+ex)^2}{12be} - \frac{(bcd}{}$$

Mathematica [A] time = 0.182727, size = 274, normalized size = 0.71

$$\frac{36c^3\left(-3a^2bde+a^3e^2+3ab^2d^2+b^3x\left(3d^2+3dex+e^2x^2\right)\right)\mathrm{PolyLog}(2,c(a+bx))+bc\left(-66a^2c^2e^2x+3ac\left(bc\left(-36d^2+5\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (6*e*(-1 + a*c + b*c*x)*((2 - 7*a*c + 11*a^2*c^2)*e + b^2*c^2*x*(9*d + 2*e*
x) + b*c*((9 - 27*a*c)*d + (2 - 5*a*c)*e*x))*Log[1 - a*c - b*c*x] + b*c*(-6
6*a^2*c^2*e^2*x - x*(12*e^2 + 6*b*c*e*(9*d + e*x) + b^2*c^2*(108*d^2 + 27*d
*e*x + 4*e^2*x^2)) + 3*a*c*(14*e^2*x + b*c*(-36*d^2 + 54*d*e*x + 5*e^2*x^2)
) + 108*b*c*d^2*(-1 + a*c + b*c*x)*Log[1 - c*(a + b*x)]) + 36*c^3*(3*a*b^2*
d^2 - 3*a^2*b*d*e + a^3*e^2 + b^3*x*(3*d^2 + 3*d*e*x + e^2*x^2))*PolyLog[2,
 c*(a + b*x)])/(108*b^3*c^3)
```

Maple [A] time = 0.058, size = 687, normalized size = 1.8

$$-d^{2}x-\frac{x^{3}e^{2}}{27}-\frac{e^{2}\ln\left(-xbc-ac+1\right)a^{2}}{cb^{3}}+\frac{3\,axde}{2\,b}+\frac{7\,e^{2}xa}{18\,b^{2}c}-\frac{e\mathrm{dilog}\left(-xbc-ac+1\right)a^{2}d}{b^{2}}-\frac{dxe}{2\,bc}-\frac{3\,e\ln\left(-xbc-ac+1\right)a}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*polylog(2,c*(b*x+a)),x)
```

```
[Out] -d^2*x-1/27*x^3*e^2-1/b^3/c*e^2*ln(-b*c*x-a*c+1)*a^2+3/2/b*e*x*a*d+7/18/b^2
/c*e^2*x*a-1/b^2*e*dilog(-b*c*x-a*c+1)*a^2*d-1/2/b/c*x*d*e-3/2/b^2*e*ln(-b*
c*x-a*c+1)*a^2*d-1/6/b*e^2*ln(-b*c*x-a*c+1)*x^2*a+1/3/b^2*e^2*ln(-b*c*x-a*c
+1)*x*a^2-1/2/b^2/c^2*e*ln(-b*c*x-a*c+1)*d-5/2/b^2/c*e*a*d+7/4/b^2*e*a^2*d+
1/2/b^3/c^2*e^2*ln(-b*c*x-a*c+1)*a+1/b/c*d^2+11/54/b^3/c^3*e^2+1/9*e^2*ln(-
b*c*x-a*c+1)*x^3-1/3/e*dilog(-b*c*x-a*c+1)*d^3+1/3/e*polylog(2,b*c*x+a*c)*d
^3+ln(-b*c*x-a*c+1)*x*d^2+1/3*e^2*polylog(2,b*c*x+a*c)*x^3+polylog(2,b*c*x+
a*c)*x*d^2-1/4*x^2*d*e+2/b^2/c*e*ln(-b*c*x-a*c+1)*a*d-1/b*e*ln(-b*c*x-a*c+1
)*x*a*d+13/9/b^3/c*e^2*a^2+3/4/b^2/c^2*e*d-31/36/b^3/c^2*e^2*a-85/108/b^3*e
^2*a^3-1/b*a*d^2+5/36/b*e^2*x^2*a-11/18/b^2*e^2*x*a^2-1/18/b/c*x^2*e^2-1/9/
b^2/c^2*e^2*x+1/2*e*ln(-b*c*x-a*c+1)*x^2*d+e*polylog(2,b*c*x+a*c)*d*x^2+1/3
/b^3*e^2*dilog(-b*c*x-a*c+1)*a^3+11/18/b^3*e^2*ln(-b*c*x-a*c+1)*a^3+1/b*dil
og(-b*c*x-a*c+1)*a*d^2+1/b*ln(-b*c*x-a*c+1)*a*d^2-1/b/c*ln(-b*c*x-a*c+1)*d^
2-1/9/b^3/c^3*e^2*ln(-b*c*x-a*c+1)
```

Maxima [A] time = 1.03377, size = 548, normalized size = 1.42

$$-\frac{\left(3\,ab^{2}d^{2}-3\,a^{2}bde+a^{3}e^{2}\right)\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_{2}\left(-bcx-ac+1\right)\right)}{3\,b^{3}}-\frac{4\,b^{3}c^{3}e^{2}x^{3}+3\left(9\,b^{3}c^{3}de-\left(5\,ab^{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*(log(b*c*x + a*c)*log(-b*c*x - a
*c + 1) + dilog(-b*c*x - a*c + 1))/b^3 - 1/108*(4*b^3*c^3*e^2*x^3 + 3*(9*b^
3*c^3*d*e - (5*a*b^2*c^3 - 2*b^2*c^2)*e^2)*x^2 + 6*(18*b^3*c^3*d^2 - 9*(3*a
*b^2*c^3 - b^2*c^2)*d*e + (11*a^2*b*c^3 - 7*a*b*c^2 + 2*b*c)*e^2)*x - 36*(b
^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 + 3*b^3*c^3*d^2*x)*dilog(b*c*x + a*c) -
6*(2*b^3*c^3*e^2*x^3 + 18*(a*b^2*c^3 - b^2*c^2)*d^2 - 9*(3*a^2*b*c^3 - 4*a*
b*c^2 + b*c)*d*e + (11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^2 + 3*(3*b^3*c^3
*d*e - a*b^2*c^3*e^2)*x^2 + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3*d*e + a^2*b*c^3*
```

```
e^2)*x)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Fricas [A] time = 2.32108, size = 779, normalized size = 2.02

$$-\frac{4\,b^3c^3e^2x^3+3\left(9\,b^3c^3de-\left(5\,ab^2c^3-2\,b^2c^2\right)e^2\right)x^2+6\left(18\,b^3c^3d^2-9\left(3\,ab^2c^3-b^2c^2\right)de+\left(11\,a^2bc^3-7\,abc^2+2\,bc\right)e^2\right)x+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/108*(4*b^3*c^3*e^2*x^3 + 3*(9*b^3*c^3*d*e - (5*a*b^2*c^3 - 2*b^2*c^2)*e^
2)*x^2 + 6*(18*b^3*c^3*d^2 - 9*(3*a*b^2*c^3 - b^2*c^2)*d*e + (11*a^2*b*c^3
- 7*a*b*c^2 + 2*b*c)*e^2)*x - 36*(b^3*c^3*e^2*x^3 + 3*b^3*c^3*d*e*x^2 + 3*b
^3*c^3*d^2*x + 3*a*b^2*c^3*d^2 - 3*a^2*b*c^3*d*e + a^3*c^3*e^2)*dilog(b*c*x
 + a*c) - 6*(2*b^3*c^3*e^2*x^3 + 18*(a*b^2*c^3 - b^2*c^2)*d^2 - 9*(3*a^2*b*
c^3 - 4*a*b*c^2 + b*c)*d*e + (11*a^3*c^3 - 18*a^2*c^2 + 9*a*c - 2)*e^2 + 3*
(3*b^3*c^3*d*e - a*b^2*c^3*e^2)*x^2 + 6*(3*b^3*c^3*d^2 - 3*a*b^2*c^3*d*e +
a^2*b*c^3*e^2)*x)*log(-b*c*x - a*c + 1))/(b^3*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^2 \mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*dilog((b*x + a)*c), x)
```

3.139 $\int (d+ex)\mathbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=210

$$-\frac{(bd-ae)^2\text{PolyLog}(2,c(a+bx))}{2b^2e}+\frac{(d+ex)^2\text{PolyLog}(2,c(a+bx))}{2e}-\frac{(-ace+bcd+e)^2\log(-ac-bcx+1)}{4b^2c^2e}-\frac{(-ac-bc</mathrm>}{}$$

```
[Out] -((b*d - a*e)*x)/(2*b) - ((b*c*d + e - a*c*e)*x)/(4*b*c) - (d + e*x)^2/(8*e
) - ((b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) - ((b*d - a*
e)*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) + ((d + e*x)^2*Log[1 -
 a*c - b*c*x])/(4*e) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) +
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e)
```

Rubi [A] time = 0.196508, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {6598, 2418, 2389, 2295, 2393, 2391, 2395, 43}

$$-\frac{(bd-ae)^2\text{PolyLog}(2,c(a+bx))}{2b^2e}+\frac{(d+ex)^2\text{PolyLog}(2,c(a+bx))}{2e}-\frac{(-ace+bcd+e)^2\log(-ac-bcx+1)}{4b^2c^2e}-\frac{(-ac-bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -((b*d - a*e)*x)/(2*b) - ((b*c*d + e - a*c*e)*x)/(4*b*c) - (d + e*x)^2/(8*e
) - ((b*c*d + e - a*c*e)^2*Log[1 - a*c - b*c*x])/(4*b^2*c^2*e) - ((b*d - a*
e)*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(2*b^2*c) + ((d + e*x)^2*Log[1 -
 a*c - b*c*x])/(4*e) - ((b*d - a*e)^2*PolyLog[2, c*(a + b*x)])/(2*b^2*e) +
((d + e*x)^2*PolyLog[2, c*(a + b*x)])/(2*e)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
```

```
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+ex)\text{Li}_2(c(a+bx))\,dx = \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{b\int \frac{(d+ex)^2\log(1-ac-bcx)}{a+bx}dx}{2e}$$
$$= \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{b\int \left(\frac{e(bd-ae)\log(1-ac-bcx)}{b^2} + \frac{(bd-ae)^2\log(1-ac-bcx)}{b^2(a+bx)} + \frac{e(d+ex)\log(1-ac-bcx)}{b}\right.}{2e}$$
$$= \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{1}{2}\int (d+ex)\log(1-ac-bcx)\,dx + \frac{(bd-ae)\int \log(1-ac-bcx)}{2b}$$
$$= \frac{(d+ex)^2\log(1-ac-bcx)}{4e} + \frac{(d+ex)^2\text{Li}_2(c(a+bx))}{2e} + \frac{(bc)\int \frac{(d+ex)^2}{1-ac-bcx}dx}{4e} - \frac{(bd-ae)\,\text{Subs}}{}$$
$$= -\frac{(bd-ae)x}{2b} - \frac{(bd-ae)(1-ac-bcx)\log(1-ac-bcx)}{2b^2c} + \frac{(d+ex)^2\log(1-ac-bcx)}{4e} - \frac{(bo}{}$$
$$= -\frac{(bd-ae)x}{2b} - \frac{(bcd+e-ace)x}{4bc} - \frac{(d+ex)^2}{8e} - \frac{(bcd+e-ace)^2\log(1-ac-bcx)}{4b^2c^2e} - \frac{(bd-ae}{}$$

Mathematica [A] time = 0.0900849, size = 161, normalized size = 0.77

$$\frac{e\left(-4a^2c^2\text{PolyLog}(2,c(a+bx)) + \left(-6a^2c^2-4ac(bcx-2)+2b^2c^2x^2-2\right)\log(-ac-bcx+1) - bcx(-6ac+bcx+2)\right)}{8b^2c^2} + \frac{d(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (e*(-(b*c*x*(2 - 6*a*c + b*c*x)) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*
(-2 + b*c*x))*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, c*(a + b*x)]))/(8
*b^2*c^2) + (d*(-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] +
c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (e*x^2*PolyLog[2, a*c + b*c*x
])/2
```

Maple [A] time = 0.049, size = 292, normalized size = 1.4

$$-\frac{\text{polylog}(2,xbc+ac)\,a^2e}{2\,b^2} + \text{polylog}(2,xbc+ac)\,xd + \frac{\text{polylog}(2,xbc+ac)\,ad}{b} + \frac{\text{polylog}(2,xbc+ac)\,ex^2}{2} + \frac{3\,e}{8\,b^2c^2} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*polylog(2,c*(b*x+a)),x)
```

```
[Out] -1/2/b^2*polylog(2,b*c*x+a*c)*a^2*e+polylog(2,b*c*x+a*c)*x*d+1/b*polylog(2,
b*c*x+a*c)*a*d+1/2*polylog(2,b*c*x+a*c)*e*x^2+3/8/b^2/c^2*e-5/4/b^2/c*a*e+1
/b/c*d+1/4*e*ln(-b*c*x-a*c+1)*x^2+1/b*ln(-b*c*x-a*c+1)*a*d+ln(-b*c*x-a*c+1)
*x*d+3/4/b*a*e*x+7/8/b^2*a^2*e-1/2/b*ln(-b*c*x-a*c+1)*x*a*e-1/4/b^2/c^2*e*l
n(-b*c*x-a*c+1)-1/8*e*x^2-1/4/b/c*x*e-d*x-a*d/b-1/b/c*ln(-b*c*x-a*c+1)*d-3/
4/b^2*ln(-b*c*x-a*c+1)*a^2*e+1/b^2/c*ln(-b*c*x-a*c+1)*a*e
```

Maxima [A] time = 1.01942, size = 286, normalized size = 1.36

$$-\frac{\left(2\,abd-a^{2}e\right)\left(\log\left(bcx+ac\right)\log\left(-bcx-ac+1\right)+\mathrm{Li}_{2}\left(-bcx-ac+1\right)\right)}{2\,b^{2}}-\frac{b^{2}c^{2}ex^{2}+2\left(4\,b^{2}c^{2}d-\left(3\,abc^{2}-bc\right)e\right)x-4\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b*d - a^2*e)*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c
*x - a*c + 1))/b^2 - 1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b*c
)*e)*x - 4*(b^2*c^2*e*x^2 + 2*b^2*c^2*d*x)*dilog(b*c*x + a*c) - 2*(b^2*c^2*
e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2*c^2*d -
a*b*c^2*e)*x)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Fricas [A] time = 2.49073, size = 377, normalized size = 1.8

$$-\frac{b^{2}c^{2}ex^{2}+2\left(4\,b^{2}c^{2}d-\left(3\,abc^{2}-bc\right)e\right)x-4\left(b^{2}c^{2}ex^{2}+2\,b^{2}c^{2}dx+2\,abc^{2}d-a^{2}c^{2}e\right)\mathrm{Li}_{2}\left(bcx+ac\right)-2\left(b^{2}c^{2}ex^{2}+4\left(abc^{2}\right.\right.}{8\,b^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*c^2*e*x^2 + 2*(4*b^2*c^2*d - (3*a*b*c^2 - b*c)*e)*x - 4*(b^2*c^2*
e*x^2 + 2*b^2*c^2*d*x + 2*a*b*c^2*d - a^2*c^2*e)*dilog(b*c*x + a*c) - 2*(b^
2*c^2*e*x^2 + 4*(a*b*c^2 - b*c)*d - (3*a^2*c^2 - 4*a*c + 1)*e + 2*(2*b^2*c^
2*d - a*b*c^2*e)*x)*log(-b*c*x - a*c + 1))/(b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)\mathrm{Li}_2\left((bx + a)c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*dilog((b*x + a)*c), x)
```

3.140 $\int \text{PolyLog}(2, c(a + bx))\, dx$

Optimal. Leaf size=60

$$x\text{PolyLog}(2, c(a+bx)) + \frac{a\text{PolyLog}(2, c(a+bx))}{b} - \frac{(-ac-bcx+1)\log(-ac-bcx+1)}{bc} - x$$

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rubi [A] time = 0.0519429, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6595, 2444, 2389, 2295, 2421, 2393, 2391}

$$x\text{PolyLog}(2, c(a+bx)) + \frac{a\text{PolyLog}(2, c(a+bx))}{b} - \frac{(-ac-bcx+1)\log(-ac-bcx+1)}{bc} - x$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -x - ((1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (a*PolyLog[2, c*(a +
b*x)])/b + x*PolyLog[2, c*(a + b*x)]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \mathrm{Li}_2(c(a+bx))\,dx &= x\mathrm{Li}_2(c(a+bx)) - a\int \frac{\log(1-c(a+bx))}{a+bx}\,dx + \int \log(1-c(a+bx))\,dx \\
&= x\mathrm{Li}_2(c(a+bx)) - a\int \frac{\log(1-ac-bcx)}{a+bx}\,dx + \int \log(1-ac-bcx)\,dx \\
&= x\mathrm{Li}_2(c(a+bx)) - \frac{a\,\mathrm{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{b} - \frac{\mathrm{Subst}(\int \log(x)\,dx, x, 1-ac-bcx)}{bc} \\
&= -x - \frac{(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{a\mathrm{Li}_2(c(a+bx))}{b} + x\mathrm{Li}_2(c(a+bx))
\end{aligned}
$$

Mathematica [A] time = 0.0160818, size = 53, normalized size = 0.88

$$
\frac{c(a+bx)\mathrm{PolyLog}(2,c(a+bx)) - c(a+bx) + (c(a+bx)-1)\log(1-c(a+bx))}{bc}
$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (-(c*(a + b*x)) + (-1 + c*(a + b*x))*Log[1 - c*(a + b*x)] + c*(a + b*x)*Pol
yLog[2, c*(a + b*x)])/(b*c)
```

Maple [A] time = 0.006, size = 96, normalized size = 1.6

$$\ln(-xbc-ac+1)\,x+\mathrm{polylog}(2,xbc+ac)\,x+\frac{\ln(-xbc-ac+1)\,a}{b}+\frac{\mathrm{polylog}(2,xbc+ac)\,a}{b}-x-\frac{a}{b}-\frac{\ln(-xbc-ac}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a)),x)
```

```
[Out] ln(-b*c*x-a*c+1)*x+polylog(2,b*c*x+a*c)*x+1/b*ln(-b*c*x-a*c+1)*a+1/b*polylo
g(2,b*c*x+a*c)*a-x-a/b-1/b/c*ln(-b*c*x-a*c+1)+1/b/c
```

Maxima [A] time = 1.00266, size = 122, normalized size = 2.03

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\mathrm{Li}_2(-bcx-ac+1))a}{b}+\frac{bcx\mathrm{Li}_2(bcx+ac)-bcx+(bcx+ac-1)\log(-bcx-ac}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*a/b + (
b*c*x*dilog(b*c*x + a*c) - b*c*x + (b*c*x + a*c - 1)*log(-b*c*x - a*c + 1))
/(b*c)
```

Fricas [A] time = 2.43648, size = 126, normalized size = 2.1

$$-\frac{bcx-(bcx+ac)\mathrm{Li}_2(bcx+ac)-(bcx+ac-1)\log(-bcx-ac+1)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -(b*c*x - (b*c*x + a*c)*dilog(b*c*x + a*c) - (b*c*x + a*c - 1)*log(-b*c*x -
 a*c + 1))/(b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c), x)
```

3.141 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{d+ex}\,dx$

Optimal. Leaf size=591

$$-\frac{\text{PolyLog}\left(3,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\text{PolyLog}\left(3,\frac{(1-c(a+bx))(bd-ae)}{b(d+ex)}\right)}{e}-\frac{\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)\text{PolyLog}\left(2,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\log(}{}$$

```
[Out] ((Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d
 + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))]^2)/(2*e) + (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b
*x)])/e - ((Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*e)
 + (Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + ((Log[(b*(d + e*x))/((b*d - a
*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b
*d - a*e)])/e + ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a +
 b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e - (Log[(b*(d + e*x))/((b*d - a*e)*
(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/e
 + (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a
*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e - PolyLog[3, (b*(d + e*x))/(b*d -
a*e)]/e - PolyLog[3, 1 - c*(a + b*x)]/e - PolyLog[3, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))]/e + PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d +
e*x))]/e
```

Rubi [A] time = 0.517702, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6597, 2440, 2435}

$$-\frac{\text{PolyLog}\left(3,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\text{PolyLog}\left(3,\frac{(1-c(a+bx))(bd-ae)}{b(d+ex)}\right)}{e}-\frac{\log\left(\frac{b(d+ex)}{(1-c(a+bx))(bd-ae)}\right)\text{PolyLog}\left(2,-\frac{e(1-c(a+bx))}{bc(d+ex)}\right)}{e}+\frac{\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```


```
[Out] ((Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d
 + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1
- c*(a + b*x)))]^2)/(2*e) + (Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b
*x)])/e - ((Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(2*e)
 + (Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e + ((Log[(b*(d + e*x))/((b*d - a
```

```
*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b
*d - a*e)])/e + ((Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a +
 b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/e - (Log[(b*(d + e*x))/((b*d - a*e)*
(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/e
 + (Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a
*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/e - PolyLog[3, (b*(d + e*x))/(b*d -
a*e)]/e - PolyLog[3, 1 - c*(a + b*x)]/e - PolyLog[3, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))]/e + PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d +
e*x))]/e
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{d+ex}\,dx = \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{b\int \frac{\log(1-ac-bcx)\log(d+ex)}{a+bx}\,dx}{e}$$

$$= \frac{\log(d+ex)\text{Li}_2(c(a+bx))}{e} + \frac{\text{Subst}\left(\int \frac{\log\left(-\frac{-abc-b(1-ac)}{b}-cx\right)\log\left(-\frac{-bd+ae}{b}+\frac{ex}{b}\right)}{x}\,dx, x, a+bx\right)}{e}$$

$$= \frac{\left(\log(c(a+bx)) + \log\left(\frac{bcd+e-ace}{bc(d+ex)}\right) - \log\left(\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right)\right)\log^2\left(\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right)}{2e} + \frac{\log(c(a+bx)}{}$$

Mathematica [A] time = 0.307802, size = 622, normalized size = 1.05

$$\frac{-\text{PolyLog}\left(3, \frac{bc(d+ex)}{e(ac+bcx-1)}\right) + \text{PolyLog}\left(3, -\frac{b(d+ex)}{(ac+bcx-1)(bd-ae)}\right) + \log\left(-\frac{b(d+ex)}{(ac+bcx-1)(bd-ae)}\right)\left(\text{PolyLog}\left(2, \frac{bc(d+ex)}{e(ac+bcx-1)}\right) - \text{PolyLo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] -
Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1
 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] +
 Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b
*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d
 - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e
)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e
 - a*c*e - b*c*e*x)]))/2 + Log[d + e*x]*PolyLog[2, c*(a + b*x)] + (Log[d +
e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1
 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*
(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d
+ e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(
-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b
*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a
*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b
*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])/e
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2, c(bx+a))}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

```
[Out] int(polylog(2,c*(b*x+a))/(e*x+d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left((bx+a)c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2\left(bcx+ac\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx+a)c\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d), x)
```

3.142 $\int \frac{\text{PolyLog}(2,c(a+bx))}{(d+ex)^2} dx$

Optimal. Leaf size=138

$$\frac{b\text{PolyLog}(2,c(a+bx))}{e(bd-ae)}-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}+\frac{b\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{e(bd-ae)}+\frac{b\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{e(bd-ae)}$$

```
[Out] (b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(e*(b*d -
 a*e)) + (b*PolyLog[2, c*(a + b*x)])/(e*(b*d - a*e)) - PolyLog[2, c*(a + b*
x)]/(e*(d + e*x)) + (b*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)
])/(e*(b*d - a*e))
```

Rubi [A] time = 0.18397, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6598, 2418, 2393, 2391, 2394}

$$\frac{b\text{PolyLog}(2,c(a+bx))}{e(bd-ae)}-\frac{\text{PolyLog}(2,c(a+bx))}{e(d+ex)}+\frac{b\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{e(bd-ae)}+\frac{b\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)}{e(bd-ae)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

```
[Out] (b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(e*(b*d -
 a*e)) + (b*PolyLog[2, c*(a + b*x)])/(e*(b*d - a*e)) - PolyLog[2, c*(a + b*
x)]/(e*(d + e*x)) + (b*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)
])/(e*(b*d - a*e))
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
```

```
RFx, x] && IntegerQ[p]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_2(c(a+bx))}{(d+ex)^2}\,dx &= -\frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b\int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)}\,dx}{e}\\
&= -\frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b\int \left(\frac{b\log(1-ac-bcx)}{(bd-ae)(a+bx)} - \frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)}\right)dx}{e}\\
&= -\frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} + \frac{b\int \frac{\log(1-ac-bcx)}{d+ex}\,dx}{bd-ae} - \frac{b^2\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{e(bd-ae)}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} - \frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\log(1-cx)}{x}\,dx, x, a+bx\right)}{e(bd-ae)} + \frac{\left(b^2c\right)\cdots}{\cdots}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} + \frac{b\mathrm{Li}_2(c(a+bx))}{e(bd-ae)} - \frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} - \frac{b\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ex}{-bcd-(1-\cdots}\right)}{x}\cdots\right)}{e(bd\cdots}\\
&= \frac{b\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{e(bd-ae)} + \frac{b\mathrm{Li}_2(c(a+bx))}{e(bd-ae)} - \frac{\mathrm{Li}_2(c(a+bx))}{e(d+ex)} + \frac{b\mathrm{Li}_2\left(\frac{e(1-ac-bcx)}{bcd+e-ace}\right)}{e(bd-ae)}
\end{aligned}
$$

Mathematica [A] time = 0.140869, size = 108, normalized size = 0.78

$$\frac{\frac{b\left(\text{PolyLog}\left(2,\frac{e(ac+bcx-1)}{ace-bcd-e}\right)+\text{PolyLog}(2,c(a+bx))+\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)\right)}{bd-ae}-\frac{\text{PolyLog}(2,c(a+bx))}{d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^2,x]
```

```
[Out] (-(PolyLog[2, c*(a + b*x)]/(d + e*x)) + (b*(Log[1 - a*c - b*c*x]*Log[(b*c*(
d + e*x))/(b*c*d + e - a*c*e)] + PolyLog[2, c*(a + b*x)] + PolyLog[2, (e*(-
1 + a*c + b*c*x))/(-(b*c*d) - e + a*c*e)]))/(b*d - a*e))/e
```

Maple [A] time = 0.38, size = 189, normalized size = 1.4

$$-\frac{bc\text{polylog}(2,xbc+ac)}{(bcex+bcd)e}-\frac{b}{e(ae-bd)}\text{dilog}\left(\frac{ace-bcd+(-xbc-ac+1)e-e}{ace-bcd-e}\right)-\frac{b\ln(-xbc-ac+1)}{e(ae-bd)}\ln\left(\frac{ace-bcd+(-}{ace-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^2,x)
```

```
[Out] -b*c/(b*c*e*x+b*c*d)/e*polylog(2,b*c*x+a*c)-b/e/(a*e-b*d)*dilog((a*c*e-b*c*
d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))-b/e/(a*e-b*d)*ln(-b*c*x-a*c+1)*ln((a
*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))-b/e/(a*e-b*d)*dilog(-b*c*x-
a*c+1)
```

Maxima [A] time = 1.03118, size = 224, normalized size = 1.62

$$-\frac{(\log(bcx+ac)\log(-bcx-ac+1)+\text{Li}_2(-bcx-ac+1))b}{bde-ae^2}+\frac{\left(\log(-bcx-ac+1)\log\left(\frac{bcex+(ac-1)e}{bcd-(ac-1)e}+1\right)+\text{Li}_2\left(-\frac{bcex+(ac-}{bcd-(ac-}\right.\right.}{bde-ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b/(b*d*
e - a*e^2) + (log(-b*c*x - a*c + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a
```

```
*c - 1)*e) + 1) + dilog(-(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b/
(b*d*e - a*e^2) - dilog(b*c*x + a*c)/(e^2*x + d*e)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\,(bcx+ac)}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\,((bx+a)c)}{(ex+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^2, x)
```

3.143 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{(d+ex)^3} \, dx$

Optimal. Leaf size=278

$$\frac{b^2\text{PolyLog}(2,c(a+bx))}{2e(bd-ae)^2}+\frac{b^2\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{2e(bd-ae)^2}-\frac{\text{PolyLog}(2,c(a+bx))}{2e(d+ex)^2}+\frac{b^2c\log(-ac-bcx+1)}{2e(bd-ae)(-ace+bcd+e)}-\frac{}{2e(bd}$$

```
[Out] (b^2*c*Log[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(b*c*d + e - a*c*e)) - (b*Log
[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(d + e*x)) - (b^2*c*Log[d + e*x])/(2*e*
(b*d - a*e)*(b*c*d + e - a*c*e)) + (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d +
e*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2) + (b^2*PolyLog[2, c*(a + b*
x)])/(2*e*(b*d - a*e)^2) - PolyLog[2, c*(a + b*x)]/(2*e*(d + e*x)^2) + (b^2
*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2)
```

Rubi [A] time = 0.270028, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6598, 2418, 2393, 2391, 2395, 36, 31, 2394}

$$\frac{b^2\text{PolyLog}(2,c(a+bx))}{2e(bd-ae)^2}+\frac{b^2\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{2e(bd-ae)^2}-\frac{\text{PolyLog}(2,c(a+bx))}{2e(d+ex)^2}+\frac{b^2c\log(-ac-bcx+1)}{2e(bd-ae)(-ace+bcd+e)}-\frac{}{2e(bd}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

```
[Out] (b^2*c*Log[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(b*c*d + e - a*c*e)) - (b*Log
[1 - a*c - b*c*x])/(2*e*(b*d - a*e)*(d + e*x)) - (b^2*c*Log[d + e*x])/(2*e*
(b*d - a*e)*(b*c*d + e - a*c*e)) + (b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d +
e*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2) + (b^2*PolyLog[2, c*(a + b*
x)])/(2*e*(b*d - a*e)^2) - PolyLog[2, c*(a + b*x)]/(2*e*(d + e*x)^2) + (b^2
*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*e*(b*d - a*e)^2)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\text{Li}_2(c(a+bx))}{(d+ex)^3}\,dx = -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b\int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^2}\,dx}{2e}$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b\int \left(\frac{b^2\log(1-ac-bcx)}{(bd-ae)^2(a+bx)} - \frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)^2} - \frac{be\log(1-ac-bcx)}{(bd-ae)^2(d+ex)}\right)dx}{2e}$$

$$= -\frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} + \frac{b^2\int \frac{\log(1-ac-bcx)}{d+ex}\,dx}{2(bd-ae)^2} - \frac{b^3\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{2e(bd-ae)^2} + \frac{b\int \frac{\log(1-ac-bcx)}{(d+ex)^2}\,dx}{2(bd-ae)}$$

$$= -\frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} + \frac{b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd-ae)^2} - \frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} - \frac{b^2\,\text{Subst}\left(\int \frac{\log(1-c}{x}\right.}{2e(bd-}$$

$$= -\frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} + \frac{b^2\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{2e(bd-ae)^2} + \frac{b^2\text{Li}_2(c(a+bx))}{2e(bd-ae)^2} - \frac{\text{Li}_2(c(a+bx))}{2e(d+ex)^2} -$$

$$= \frac{b^2c\log(1-ac-bcx)}{2e(bd-ae)(bcd+e-ace)} - \frac{b\log(1-ac-bcx)}{2e(bd-ae)(d+ex)} - \frac{b^2c\log(d+ex)}{2e(bd-ae)(bcd+e-ace)} + \frac{b^2\log(1-ac-bc}{2e(bd}$$

Mathematica [A] time = 0.420508, size = 190, normalized size = 0.68

$$\frac{\frac{b\left(b\text{PolyLog}\left(2,\frac{e(ac+bcx-1)}{e(ac-1)-bcd}\right)+b\text{PolyLog}(2,c(a+bx))+b\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)-\frac{(bd-ae)\log(-ac-bcx+1)}{d+ex}+\frac{bc(bd-ae)(\log(-ac-bcx+1)-\log(d+ex))}{-ace+bcd+e}\right)}{(bd-ae)^2} - \frac{\text{PolyLog}(2,}{(d+ex}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^3,x]
```

```
[Out] (-(PolyLog[2, c*(a + b*x)]/(d + e*x)^2) + (b*(-(((b*d - a*e)*Log[1 - a*c -
b*c*x])/(d + e*x)) + (b*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x])
)/(b*c*d + e - a*c*e) + b*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d +
 e - a*c*e)] + b*PolyLog[2, c*(a + b*x)] + b*PolyLog[2, (e*(-1 + a*c + b*c*
x))/(-(b*c*d) + (-1 + a*c)*e)]))/(b*d - a*e)^2)/(2*e)
```

Maple [A] time = 0.462, size = 437, normalized size = 1.6

$$-\frac{b^2c^2\text{polylog}(2,xbc+ac)}{2(bcex+bcd)^2e} + \frac{b^2}{2e(ae-bd)^2}\text{dilog}\left(\frac{ace-bcd+(-xbc-ac+1)e-e}{ace-bcd-e}\right) + \frac{b^2\ln(-xbc-ac+1)}{2e(ae-bd)^2}\ln\left(\frac{ace-bc}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^3,x)
```

```
[Out] -1/2*b^2*c^2/(b*c*e*x+b*c*d)^2/e*polylog(2,b*c*x+a*c)+1/2*b^2/e/(a*e-b*d)^2
*dilog((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))+1/2*b^2/e/(a*e-b*d
)^2*ln(-b*c*x-a*c+1)*ln((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))-1
/2*b^2*c/e/(a*e-b*d)/(a*c*e-b*c*d-e)*ln(a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)-1/2
*b^3*c^2/(a*e-b*d)*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*x-1/2*
b^2*c^2/(a*e-b*d)*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*a+1/2*b
^2*c/(a*e-b*d)*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)+1/2*b^2/e/
(a*e-b*d)^2*dilog(-b*c*x-a*c+1)
```

Maxima [A] time = 1.00782, size = 512, normalized size = 1.84

$$\frac{b^2c\log(bcx+ac-1)}{2\left(b^2cd^2e-(2\,abc-b)de^2+\left(a^2c-a\right)e^3\right)}-\frac{b^2c\log(ex+d)}{2\left(b^2cd^2e-(2\,abc-b)de^2+\left(a^2c-a\right)e^3\right)}-\frac{(\log(bcx+ac)\log(-bcx-ac+\cdots}{2\left(b^2d^2e-2\,abd\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*c*log(b*c*x + a*c - 1)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (a^2*c
- a)*e^3) - 1/2*b^2*c*log(e*x + d)/(b^2*c*d^2*e - (2*a*b*c - b)*d*e^2 + (a^
2*c - a)*e^3) - 1/2*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x
- a*c + 1))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) + 1/2*(log(-b*c*x - a*c
 + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + dilog(-(b*c*
e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b^2/(b^2*d^2*e - 2*a*b*d*e^2 + a
^2*e^3) - 1/2*((b*d - a*e)*dilog(b*c*x + a*c) + (b*e*x + b*d)*log(-b*c*x -
a*c + 1))/(b*d^3*e - a*d^2*e^2 + (b*d*e^3 - a*e^4)*x^2 + 2*(b*d^2*e^2 - a*d
*e^3)*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2\left(bcx+ac\right)}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left((bx+a)c\right)}{(ex+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^3, x)
```

3.144 $\int \frac{\textbf{PolyLog}(2,c(a+bx))}{(d+ex)^4}\,dx$

Optimal. Leaf size=448

$$\frac{b^3\text{PolyLog}(2,c(a+bx))}{3e(bd-ae)^3}+\frac{b^3\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{3e(bd-ae)^3}-\frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3}+\frac{b^3c^2\log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2}-\frac{}{6e(b}$$

```
[Out] (b^2*c)/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)*(d + e*x)) + (b^3*c^2*Log[1 -
a*c - b*c*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^3*c*Log[1 - a*c
- b*c*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) - (b*Log[1 - a*c - b*c*x]
)/(6*e*(b*d - a*e)*(d + e*x)^2) - (b^2*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*
e)^2*(d + e*x)) - (b^3*c^2*Log[d + e*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*
e)^2) - (b^3*c*Log[d + e*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) + (b^3
*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d -
 a*e)^3) + (b^3*PolyLog[2, c*(a + b*x)])/(3*e*(b*d - a*e)^3) - PolyLog[2, c
*(a + b*x)]/(3*e*(d + e*x)^3) + (b^3*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*
d + e - a*c*e)])/(3*e*(b*d - a*e)^3)
```

Rubi [A] time = 0.426017, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6598, 2418, 2393, 2391, 2395, 44, 36, 31, 2394}

$$\frac{b^3\text{PolyLog}(2,c(a+bx))}{3e(bd-ae)^3}+\frac{b^3\text{PolyLog}\left(2,\frac{e(-ac-bcx+1)}{-ace+bcd+e}\right)}{3e(bd-ae)^3}-\frac{\text{PolyLog}(2,c(a+bx))}{3e(d+ex)^3}+\frac{b^3c^2\log(-ac-bcx+1)}{6e(bd-ae)(-ace+bcd+e)^2}-\frac{}{6e(b}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```


```
[Out] (b^2*c)/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)*(d + e*x)) + (b^3*c^2*Log[1 -
a*c - b*c*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*e)^2) + (b^3*c*Log[1 - a*c
- b*c*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) - (b*Log[1 - a*c - b*c*x]
)/(6*e*(b*d - a*e)*(d + e*x)^2) - (b^2*Log[1 - a*c - b*c*x])/(3*e*(b*d - a*
e)^2*(d + e*x)) - (b^3*c^2*Log[d + e*x])/(6*e*(b*d - a*e)*(b*c*d + e - a*c*
e)^2) - (b^3*c*Log[d + e*x])/(3*e*(b*d - a*e)^2*(b*c*d + e - a*c*e)) + (b^3
*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(3*e*(b*d -
 a*e)^3) + (b^3*PolyLog[2, c*(a + b*x)])/(3*e*(b*d - a*e)^3) - PolyLog[2, c
*(a + b*x)]/(3*e*(d + e*x)^3) + (b^3*PolyLog[2, (e*(1 - a*c - b*c*x))/(b*c*
d + e - a*c*e)])/(3*e*(b*d - a*e)^3)
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\int \frac{\mathrm{Li}_2(c(a+bx))}{(d+ex)^4}\,dx = -\frac{\mathrm{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b\int \frac{\log(1-ac-bcx)}{(a+bx)(d+ex)^3}\,dx}{3e}$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{3e(d+ex)^3} - \frac{b\int \left(\frac{b^3\log(1-ac-bcx)}{(bd-ae)^3(a+bx)} - \frac{e\log(1-ac-bcx)}{(bd-ae)(d+ex)^3} - \frac{be\log(1-ac-bcx)}{(bd-ae)^2(d+ex)^2} - \frac{b^2e\log(1-ac-bcx)}{(bd-ae)^3(d+ex)}\right)dx}{3e}$$

$$= -\frac{\mathrm{Li}_2(c(a+bx))}{3e(d+ex)^3} + \frac{b^3\int \frac{\log(1-ac-bcx)}{d+ex}\,dx}{3(bd-ae)^3} - \frac{b^4\int \frac{\log(1-ac-bcx)}{a+bx}\,dx}{3e(bd-ae)^3} + \frac{b^2\int \frac{\log(1-ac-bcx)}{(d+ex)^2}\,dx}{3(bd-ae)^2} + \frac{b\int \frac{\log(1-}{(d+}}{3(bd}$$

$$= -\frac{b\log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2\log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{3e(bd-ae)^3} - \frac{\mathrm{Li}_2(c(a+}{3e(d+ex}$$

$$= -\frac{b\log(1-ac-bcx)}{6e(bd-ae)(d+ex)^2} - \frac{b^2\log(1-ac-bcx)}{3e(bd-ae)^2(d+ex)} + \frac{b^3\log(1-ac-bcx)\log\left(\frac{bc(d+ex)}{bcd+e-ace}\right)}{3e(bd-ae)^3} + \frac{b^3\mathrm{Li}_2(c(a}{3e(bd-}$$

$$= \frac{b^2c}{6e(bd-ae)(bcd+e-ace)(d+ex)} + \frac{b^3c^2\log(1-ac-bcx)}{6e(bd-ae)(bcd+e-ace)^2} + \frac{b^3c\log(1-ac-bcx)}{3e(bd-ae)^2(bcd+e-ace)} -$$

Mathematica [A] time = 0.631829, size = 313, normalized size = 0.7

$$\frac{\frac{b\left(2b^2\mathrm{PolyLog}\left(2,\frac{e(ac+bcx-1)}{e(ac-1)-bcd}\right)+2b^2\mathrm{PolyLog}(2,c(a+bx))+\frac{2b^2c(bd-ae)(\log(-ac-bcx+1)-\log(d+ex))}{-ace+bcd+e}+2b^2\log(-ac-bcx+1)\log\left(\frac{bc(d+ex)}{-ace+bcd+e}\right)-\frac{2b(bd-ae)\log(-ac-bcx+1)}{d+ex}+\frac{bc}{}\right)}{(bd-ae)^3}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, c*(a + b*x)]/(d + e*x)^4,x]
```

```
[Out] ((-2*PolyLog[2, c*(a + b*x)])/(d + e*x)^3 + (b*(-(((b*d - a*e)^2*Log[1 - a*
c - b*c*x])/(d + e*x)^2) - (2*b*(b*d - a*e)*Log[1 - a*c - b*c*x])/(d + e*x)
 + (2*b^2*c*(b*d - a*e)*(Log[1 - a*c - b*c*x] - Log[d + e*x]))/(b*c*d + e -
 a*c*e) + (b*c*(b*d - a*e)^2*(b*c*d + e - a*c*e + b*c*(d + e*x)*Log[1 - a*c
 - b*c*x] - b*c*(d + e*x)*Log[d + e*x]))/((b*c*d + e - a*c*e)^2*(d + e*x))
+ 2*b^2*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)] + 2*b
^2*PolyLog[2, c*(a + b*x)] + 2*b^2*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c
*d) + (-1 + a*c)*e)]))/(b*d - a*e)^3)/(6*e)
```

Maple [B] time = 0.394, size = 1075, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,c*(b*x+a))/(e*x+d)^4,x)
```

```
[Out] -1/3*b^3*c^3/(b*c*e*x+b*c*d)^3/e*polylog(2,b*c*x+a*c)-1/3*b^3/e/(a*e-b*d)^3
*dilog((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))-1/3*b^3/e/(a*e-b*d
)^3*ln(-b*c*x-a*c+1)*ln((a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)/(a*c*e-b*c*d-e))+1
/3*b^3*c/e/(a*e-b*d)^2/(a*c*e-b*c*d-e)*ln(a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)+1
/3*b^4*c^2/(a*e-b*d)^2*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*x+
1/3*b^3*c^2/(a*e-b*d)^2*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)*a
-1/3*b^3*c/(a*e-b*d)^2*ln(-b*c*x-a*c+1)/(a*c*e-b*c*d-e)/(-b*c*e*x-b*c*d)-1/
3*b^3/e/(a*e-b*d)^3*dilog(-b*c*x-a*c+1)+1/6*b^3*c^2/e/(a*e-b*d)/(a*c*e-b*c*
d-e)^2*ln(a*c*e-b*c*d+(-b*c*x-a*c+1)*e-e)-1/6*b^3*c^3/(a*e-b*d)/(a*c*e-b*c*
d-e)^2/(-b*c*e*x-b*c*d)*a+1/6*b^4*c^3/e/(a*e-b*d)/(a*c*e-b*c*d-e)^2/(-b*c*e
*x-b*c*d)*d+1/6*b^3*c^2/(a*e-b*d)/(a*c*e-b*c*d-e)^2/(-b*c*e*x-b*c*d)-1/6*b^
5*c^4*e/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c*d)^2/(a*c*e-b*c*d-e)^2*x^2
-1/3*b^5*c^4/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c*d)^2/(a*c*e-b*c*d-e)^
2*x*d+1/6*b^3*c^4*e/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c*d)^2/(a*c*e-b*
c*d-e)^2*a^2-1/3*b^4*c^4/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c*d)^2/(a*c
*e-b*c*d-e)^2*d*a-1/3*b^3*c^3*e/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c*d)
^2/(a*c*e-b*c*d-e)^2*a+1/3*b^4*c^3/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*x-b*c
*d)^2/(a*c*e-b*c*d-e)^2*d+1/6*b^3*c^2*e/(a*e-b*d)*ln(-b*c*x-a*c+1)/(-b*c*e*
x-b*c*d)^2/(a*c*e-b*c*d-e)^2
```

Maxima [B] time = 1.22441, size = 1928, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(b*c*x + a*c)*log(-b*c*x - a*c + 1) + dilog(-b*c*x - a*c + 1))*b^3
/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) + 1/3*(log(-b*c*x
- a*c + 1)*log((b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e) + 1) + dilog(-
(b*c*e*x + (a*c - 1)*e)/(b*c*d - (a*c - 1)*e)))*b^3/(b^3*d^3*e - 3*a*b^2*d^
2*e^2 + 3*a^2*b*d*e^3 - a^3*e^4) - 1/6*(3*b^4*c^2*d - (3*a*b^3*c^2 - 2*b^3*
c)*e)*log(e*x + d)/(b^4*c^2*d^4*e - 2*(2*a*b^3*c^2 - b^3*c)*d^3*e^2 + (6*a^
2*b^2*c^2 - 6*a*b^2*c + b^2)*d^2*e^3 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d*
e^4 + (a^4*c^2 - 2*a^3*c + a^2)*e^5) + 1/6*(b^4*c^2*d^4 - (2*a*b^3*c^2 - b^
3*c)*d^3*e + (a^2*b^2*c^2 - a*b^2*c)*d^2*e^2 + (b^4*c^2*d^2*e^2 - (2*a*b^3*
c^2 - b^3*c)*d*e^3 + (a^2*b^2*c^2 - a*b^2*c)*e^4)*x^2 + 2*(b^4*c^2*d^3*e -
(2*a*b^3*c^2 - b^3*c)*d^2*e^2 + (a^2*b^2*c^2 - a*b^2*c)*d*e^3)*x - 2*(b^4*c
^2*d^4 - 2*(2*a*b^3*c^2 - b^3*c)*d^3*e + (6*a^2*b^2*c^2 - 6*a*b^2*c + b^2)*
d^2*e^2 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d*e^3 + (a^4*c^2 - 2*a^3*c + a^
2)*e^4)*dilog(b*c*x + a*c) + (4*(a*b^3*c^2 - b^3*c)*d^3*e - (5*a^2*b^2*c^2
- 8*a*b^2*c + 3*b^2)*d^2*e^2 + (a^3*b*c^2 - 2*a^2*b*c + a*b)*d*e^3 + (3*b^4
*c^2*d*e^3 - (3*a*b^3*c^2 - 2*b^3*c)*e^4)*x^3 + (7*b^4*c^2*d^2*e^2 - (5*a*b
^3*c^2 - 2*b^3*c)*d*e^3 - 2*(a^2*b^2*c^2 - 2*a*b^2*c + b^2)*e^4)*x^2 + (4*b
^4*c^2*d^3*e + 2*(a*b^3*c^2 - 2*b^3*c)*d^2*e^2 - (7*a^2*b^2*c^2 - 12*a*b^2*
c + 5*b^2)*d*e^3 + (a^3*b*c^2 - 2*a^2*b*c + a*b)*e^4)*x)*log(-b*c*x - a*c +
 1))/(b^4*c^2*d^7*e - 2*(2*a*b^3*c^2 - b^3*c)*d^6*e^2 + (6*a^2*b^2*c^2 - 6*
a*b^2*c + b^2)*d^5*e^3 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d^4*e^4 + (a^4*c
^2 - 2*a^3*c + a^2)*d^3*e^5 + (b^4*c^2*d^4*e^4 - 2*(2*a*b^3*c^2 - b^3*c)*d^
3*e^5 + (6*a^2*b^2*c^2 - 6*a*b^2*c + b^2)*d^2*e^6 - 2*(2*a^3*b*c^2 - 3*a^2*
b*c + a*b)*d*e^7 + (a^4*c^2 - 2*a^3*c + a^2)*e^8)*x^3 + 3*(b^4*c^2*d^5*e^3
- 2*(2*a*b^3*c^2 - b^3*c)*d^4*e^4 + (6*a^2*b^2*c^2 - 6*a*b^2*c + b^2)*d^3*e
^5 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d^2*e^6 + (a^4*c^2 - 2*a^3*c + a^2)*
d*e^7)*x^2 + 3*(b^4*c^2*d^6*e^2 - 2*(2*a*b^3*c^2 - b^3*c)*d^5*e^3 + (6*a^2*
b^2*c^2 - 6*a*b^2*c + b^2)*d^4*e^4 - 2*(2*a^3*b*c^2 - 3*a^2*b*c + a*b)*d^3*
e^5 + (a^4*c^2 - 2*a^3*c + a^2)*d^2*e^6)*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\left(bcx+ac\right)}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(b*c*x + a*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left((bx+a)c\right)}{(ex+d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,c*(b*x+a))/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] integrate(dilog((b*x + a)*c)/(e*x + d)^4, x)
```

3.145 $\int \frac{\text{PolyLog}(2,x)}{-1+x}\,dx$

Optimal. Leaf size=46

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rubi [A] time = 0.0657938, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, x]/(-1 + x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\text{Li}_2(x)}{-1+x}\,dx &= \log(1-x)\text{Li}_2(x)+\int \frac{\log^2(1-x)}{x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\ &= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)-2\,\text{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\ &= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx,x,1-x\right)\\ &= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\text{Li}_3(1-x)\end{aligned}$$

Mathematica [A] time = 0.0385298, size = 46, normalized size = 1.

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, x]/(-1 + x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int \frac{\mathrm{polylog}\,(2,x)}{-1+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,x)/(-1+x),x)
```

```
[Out] int(polylog(2,x)/(-1+x),x)
```

Maxima [A] time = 1.0224, size = 59, normalized size = 1.28

$$\log(x)\log(-x+1)^2 + \mathrm{Li}_2(x)\log(-x+1) + 2\,\mathrm{Li}_2(-x+1)\log(-x+1) - 2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 2*polylog(3, -x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x)
```

```
[Out] Integral(polylog(2, x)/(x - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(x)}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/(x - 1), x)
```

3.146 $\int -\frac{\text{PolyLog}(2,x)}{1-x}\,dx$

Optimal. Leaf size=46

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rubi [A] time = 0.0674004, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[-(PolyLog[2, x]/(1 - x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\text{Li}_2(x)}{1-x}\,dx &= \log(1-x)\text{Li}_2(x)+\int \frac{\log^2(1-x)}{x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\text{Li}_2(x)-2\,\text{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\
&= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx,x,1-x\right)\\
&= \log^2(1-x)\log(x)+2\log(1-x)\text{Li}_2(1-x)+\log(1-x)\text{Li}_2(x)-2\text{Li}_3(1-x)
\end{aligned}
$$

Mathematica [A] time = 0.0065388, size = 46, normalized size = 1.

$$-2\text{PolyLog}(3,1-x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(PolyLog[2, x]/(1 - x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x]
```

Maple [F] time = 0.281, size = 0, normalized size = 0.

$$\int -\frac{\operatorname{polylog}(2,x)}{1-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-polylog(2,x)/(1-x),x)
```

```
[Out] int(-polylog(2,x)/(1-x),x)
```

Maxima [A] time = 0.957218, size = 59, normalized size = 1.28

$$\log(x)\log(-x+1)^2 + \operatorname{Li}_2(x)\log(-x+1) + 2\operatorname{Li}_2(-x+1)\log(-x+1) - 2\operatorname{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 2*polylog(3, -x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{Li}_2(x)}{x-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x)
```

```
[Out] Integral(polylog(2, x)/(x - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x),x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/(x - 1), x)
```

3.147 $\int \frac{\text{PolyLog}(2,x)}{(-1+x)x}\,dx$

Optimal. Leaf size=51

$$-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rubi [A] time = 0.13268, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6742, 6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, x]/((-1 + x)*x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\mathrm{Li}_2(x)}{(-1+x)x}\,dx &= \int \left(\frac{\mathrm{Li}_2(x)}{-1+x}-\frac{\mathrm{Li}_2(x)}{x}\right)dx\\
&= \int \frac{\mathrm{Li}_2(x)}{-1+x}\,dx-\int \frac{\mathrm{Li}_2(x)}{x}\,dx\\
&= \log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+\int \frac{\log^2(1-x)}{x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\,\mathrm{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(x)}{x}\,dx,x,1-\right.\\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Li}_3(1-x)-\mathrm{Li}_3(x)
\end{aligned}
$$

Mathematica [A] time = 0.0387094, size = 51, normalized size = 1.

$$-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, x]/((-1 + x)*x),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\text{polylog}(2,x)}{(-1+x)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,x)/(-1+x)/x,x)
```

```
[Out] int(polylog(2,x)/(-1+x)/x,x)
```

Maxima [A] time = 0.965157, size = 66, normalized size = 1.29

$$\log(x)\log(-x+1)^2+\text{Li}_2(x)\log(-x+1)+2\,\text{Li}_2(-x+1)\log(-x+1)-\text{Li}_3(x)-2\,\text{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 polylog(3, x) - 2*polylog(3, -x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2(x)}{x^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x^2 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(x)}{x(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x)
```

```
[Out] Integral(polylog(2, x)/(x*(x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2(x)}{(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,x)/(-1+x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/((x - 1)*x), x)
```

3.148 $\int -\frac{\text{PolyLog}(2,x)}{(1-x)x}\,dx$

Optimal. Leaf size=51

$$-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rubi [A] time = 0.134478, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6742, 6596, 2396, 2433, 2374, 6589}

$$-2\text{PolyLog}(3,1-x)-\text{PolyLog}(3,x)+2\log(1-x)\text{PolyLog}(2,1-x)+\log(1-x)\text{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[-(PolyLog[2, x]/((1 - x)*x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int -\frac{\mathrm{Li}_2(x)}{(1-x)x}\,dx &= -\int \left(-\frac{\mathrm{Li}_2(x)}{-1+x}+\frac{\mathrm{Li}_2(x)}{x}\right)dx\\
&= \int \frac{\mathrm{Li}_2(x)}{-1+x}\,dx-\int \frac{\mathrm{Li}_2(x)}{x}\,dx\\
&= \log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+\int \frac{\log^2(1-x)}{x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)+2\int \frac{\log(1-x)\log(x)}{1-x}\,dx\\
&= \log^2(1-x)\log(x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\,\mathrm{Subst}\left(\int \frac{\log(1-x)\log(x)}{x}\,dx,x,1-x\right)\\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-\mathrm{Li}_3(x)-2\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(x)}{x}\,dx,x,1-\right.\\
&= \log^2(1-x)\log(x)+2\log(1-x)\mathrm{Li}_2(1-x)+\log(1-x)\mathrm{Li}_2(x)-2\mathrm{Li}_3(1-x)-\mathrm{Li}_3(x)
\end{aligned}$$

Mathematica [A] time = 0.0149876, size = 51, normalized size = 1.

$$-2\mathrm{PolyLog}(3,1-x)-\mathrm{PolyLog}(3,x)+2\log(1-x)\mathrm{PolyLog}(2,1-x)+\log(1-x)\mathrm{PolyLog}(2,x)+\log(x)\log^2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(PolyLog[2, x]/((1 - x)*x)),x]
```

```
[Out] Log[1 - x]^2*Log[x] + 2*Log[1 - x]*PolyLog[2, 1 - x] + Log[1 - x]*PolyLog[2
, x] - 2*PolyLog[3, 1 - x] - PolyLog[3, x]
```

Maple [F] time = 0.282, size = 0, normalized size = 0.

$$\int -\frac{\mathrm{polylog}(2,x)}{(1-x)\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-polylog(2,x)/(1-x)/x,x)
```

```
[Out] int(-polylog(2,x)/(1-x)/x,x)
```

Maxima [A] time = 0.977682, size = 66, normalized size = 1.29

$$\log(x)\log(-x+1)^2+\mathrm{Li}_2(x)\log(-x+1)+2\,\mathrm{Li}_2(-x+1)\log(-x+1)-\mathrm{Li}_3(x)-2\,\mathrm{Li}_3(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1)^2 + dilog(x)*log(-x + 1) + 2*dilog(-x + 1)*log(-x + 1) -
 polylog(3, x) - 2*polylog(3, -x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2(x)}{x^2-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="fricas")
```

```
[Out] integral(dilog(x)/(x^2 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(x)}{x(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x)
```

```
[Out] Integral(polylog(2, x)/(x*(x - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(x)}{(x-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-polylog(2,x)/(1-x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(x)/((x - 1)*x), x)
```

3.149 $\int \frac{\text{PolyLog}\left(n,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{\text{PolyLog}\left(n+1,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

```
[Out] PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rubi [A] time = 0.071321, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {6610}

$$\frac{\text{PolyLog}\left(n+1,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_n\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{Li}_{1+n}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A] time = 0.0198299, size = 34, normalized size = 0.97

$$\frac{\text{PolyLog}\left(n+1,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[1 + n, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F] time = 0.891, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\text{polylog}\left(n,e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{polylog}\left(n, e\left(\frac{bx+a}{dx+c}\right)^n\right)}{bdx^2 + ac + (bc + ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral(polylog(n, e*((b*x + a)/(d*x + c))^n)/(b*d*x^2 + a*c + (b*c + a*d)
*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n\left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^n\right)}{(a + bx)(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(n, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*x
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_n(e\left(\frac{bx+a}{dx+c}\right)^n)}{(bx + a)(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(polylog(n, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.150 $\int \frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

```
[Out] PolyLog[4, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rubi [A] time = 0.0611184, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {6610}

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[4, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{Li}_4\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A] time = 0.007177, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(4,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[4, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F] time = 0.703, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\text{polylog}\left(3,e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\left(n\log(bx+a)^2-2\,n\log(bx+a)\log(dx+c)+n\log(dx+c)^2\right)\text{Li}_2\left(ee^{(n\log(bx+a)-n\log(dx+c))}\right)+\left(n^2\log(bx+a)^3-3\,n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] -1/6*(3*(n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x + c)^
2)*dilog(e*e^(n*log(b*x + a) - n*log(d*x + c))) + (n^2*log(b*x + a)^3 - 3*n
^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^2 - n^2*lo
g(d*x + c)^3)*log(-(b*x + a)^n*e + (d*x + c)^n) - (n^2*log(b*x + a)^3 - 3*n
^2*log(b*x + a)^2*log(d*x + c) + 3*n^2*log(b*x + a)*log(d*x + c)^2 - n^2*lo
g(d*x + c)^3)*log((d*x + c)^n) - 6*(log(b*x + a) - log(d*x + c))*polylog(3,
```

```
e*e^(n*log(b*x + a) - n*log(d*x + c))))/(b*c - a*d) + integrate(1/6*(e*n^3
*log(b*x + a)^3 - 3*e*n^3*log(b*x + a)^2*log(d*x + c) + 3*e*n^3*log(b*x + a
)*log(d*x + c)^2 - e*n^3*log(d*x + c)^3)*(b*x + a)^n/((b*d*e*x^2 + a*c*e +
(b*c*e + a*d*e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^
n), x)
```

Fricas [C] time = 3.45558, size = 782, normalized size = 23.7

$$-\frac{n^2\%\mathrm{iint}\left(a,b,c,d,e,n,x,-\frac{n\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{bx+a},-\frac{nx\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{bx+a},\frac{n\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{dx+c},\frac{nx\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{dx+c},-\frac{\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{e},-\mathrm{l}\ldots\right.}{2(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] -1/2*(n^2*\%iint(a, b, c, d, e, n, x, -n*log(-e*((b*x + a)/(d*x + c))^n + 1)
/(b*x + a), -n*x*log(-e*((b*x + a)/(d*x + c))^n + 1)/(b*x + a), n*log(-e*((
b*x + a)/(d*x + c))^n + 1)/(d*x + c), n*x*log(-e*((b*x + a)/(d*x + c))^n +
1)/(d*x + c), -log(-e*((b*x + a)/(d*x + c))^n + 1)/e, -log(-e*((b*x + a)/(d
*x + c))^n + 1)*log((b*x + a)/(d*x + c)), -(b*c - a*d)*n*log(-e*((b*x + a)/
(d*x + c))^n + 1)/(b*d*x^2 + a*c + (b*c + a*d)*x))*log((b*x + a)/(d*x + c))
^2 - n^2*dilog(e*((b*x + a)/(d*x + c))^n)*log((b*x + a)/(d*x + c))^2 - 2*po
lylog(4, e*((b*x + a)/(d*x + c))^n))/((b*c - a*d)*n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_3\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(polylog(3, e*(a/(c + d*x) + b*x/(c + d*x))**n)/((a + b*x)*(c + d*x
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_3(e\left(\frac{bx+a}{dx+c}\right)^n)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(3,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(polylog(3, e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.151 $\int \frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

```
[Out] PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rubi [A] time = 0.0612712, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {6610}

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[3, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{Li}_3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A] time = 0.006995, size = 32, normalized size = 0.97

$$\frac{\text{PolyLog}\left(3,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcn-adn}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)),x]
```

```
[Out] PolyLog[3, e*((a + b*x)/(c + d*x))^n]/(b*c*n - a*d*n)
```

Maple [F] time = 0.805, size = 0, normalized size = 0.

$$\int \frac{1}{(bx+a)(dx+c)}\text{polylog}\left(2,e\left(\frac{bx+a}{dx+c}\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\log(bx+a)-\log(dx+c)\right)\text{Li}_2\left(ee^{(n\log(bx+a)-n\log(dx+c))}\right)+\left(n\log(bx+a)^2-2\,n\log(bx+a)\log(dx+c)+n\log(dx+\right.}{2\,(bc-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(log(b*x + a) - log(d*x + c))*dilog(e*e^(n*log(b*x + a) - n*log(d*x
+ c))) + (n*log(b*x + a)^2 - 2*n*log(b*x + a)*log(d*x + c) + n*log(d*x + c)
^2)*log(-(b*x + a)^n*e + (d*x + c)^n) - (n*log(b*x + a)^2 - 2*n*log(b*x + a
)*log(d*x + c) + n*log(d*x + c)^2)*log((d*x + c)^n))/(b*c - a*d) + integrat
e(-1/2*(e*n^2*log(b*x + a)^2 - 2*e*n^2*log(b*x + a)*log(d*x + c) + e*n^2*lo
g(d*x + c)^2)*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c + a*d)*e*x)*(b*x + a)^
```

```
n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

Fricas [A] time = 2.7098, size = 74, normalized size = 2.24

$$\frac{\operatorname{polylog}\left(3, e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

```
[Out] polylog(3, e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(2,e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate(dilog(e*((b*x + a)/(d*x + c))^n)/((b*x + a)*(d*x + c)), x)
```

3.152 $\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

```
[Out] PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rubi [A] time = 0.0581217, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2518}

$$\frac{\text{PolyLog}\left(2,e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

```
[Out] PolyLog[2, e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n)
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\frac{\text{Li}_2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [F] time = 1.82302, size = 40, normalized size = 1.21

$$-\int \frac{\log \left(1-e \left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} \, dx$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x))),x]
```

```
[Out] -Integrate[Log[1 - e*((a + b*x)/(c + d*x))^n]/((a + b*x)*(c + d*x)), x]
```

Maple [F] time = 0.702, size = 0, normalized size = 0.

$$\int -\frac{1}{(bx+a)(dx+c)} \ln \left(1-e \left(\frac{bx+a}{dx+c}\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] int(-ln(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(\log(bx+a)-\log(dx+c))\log\left(-(bx+a)^n e+(dx+c)^n\right)-(\log(bx+a)-\log(dx+c))\log\left((dx+c)^n\right)}{bc-ad}+\int \frac{}{\left(bdex^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="maxim
a")
```

```
[Out] -((log(b*x + a) - log(d*x + c))*log(-(b*x + a)^n*e + (d*x + c)^n) - (log(b*
x + a) - log(d*x + c))*log((d*x + c)^n))/(b*c - a*d) + integrate((e*n*log(b
*x + a) - e*n*log(d*x + c))*(b*x + a)^n/((b*d*e*x^2 + a*c*e + (b*c*e + a*d*
e)*x)*(b*x + a)^n - (b*d*x^2 + a*c + (b*c + a*d)*x)*(d*x + c)^n), x)
```

Fricas [A] time = 2.44507, size = 68, normalized size = 2.06

$$\frac{\text{Li}_2\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="frica
s")
```

```
[Out] dilog(e*((b*x + a)/(d*x + c))^n)/((b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-ln(1-e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(-e\left(\frac{bx+a}{dx+c}\right)^n+1\right)}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-log(1-e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x, algorithm="giac"
)
```

```
[Out] integrate(-log(-e*((b*x + a)/(d*x + c))^n + 1)/((b*x + a)*(d*x + c)), x)
```

3.153 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$

Optimal. Leaf size=36

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

```
[Out] -(Log[1 - e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n))
```

Rubi [A] time = 0.316718, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {12, 6684}

$$-\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c +
 d*x))^n)),x]
```

```
[Out] -(Log[1 - e*((a + b*x)/(c + d*x))^n]/((b*c - a*d)*n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= -\frac{\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n}$$

Mathematica [A] time = 0.0914007, size = 38, normalized size = 1.06

$$-\frac{e\log\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bcen-aden}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x
)/(c + d*x))^n)),x]
```

```
[Out] -((e*Log[1 - e*((a + b*x)/(c + d*x))^n])/(b*c*e*n - a*d*e*n))
```

Maple [A] time = 0.121, size = 37, normalized size = 1.

$$\frac{1}{n\,(ad-bc)}\ln\left(e{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c),x)
```

```
[Out] 1/n/(a*d-b*c)*ln(e*exp(n*ln((b*x+a)/(d*x+c)))-1)
```

Maxima [A] time = 1.01397, size = 78, normalized size = 2.17

$$-e\left(\frac{\log\left(-(bx+a)^n e+(dx+c)^n\right)}{bcen-aden}-\frac{\log(dx+c)}{bce-ade}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="maxima")
```

```
[Out] -e*(log(-(b*x + a)^n*e + (d*x + c)^n)/(b*c*e*n - a*d*e*n) - log(d*x + c)/(b
*c*e - a*d*e))
```

Fricas [A] time = 2.34046, size = 72, normalized size = 2.

$$-\frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}-1\right)}{(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="fricas")
```

```
[Out] -log(e*((b*x + a)/(d*x + c))^n - 1)/((b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)/(b*x+a)/(d*x+
c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{e\left(\frac{bx+a}{dx+c}\right)^{n}}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^{n}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)/(b*x+a)/(d*x+c)
,x, algorithm="giac")
```

```
[Out] integrate(-e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*
x + c))^n - 1)), x)
```

3.154 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

```
[Out] 1/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n))
```

Rubi [A] time = 0.366682, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {12, 6686}

$$\frac{1}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c +
 d*x))^n)^2),x]
```

```
[Out] 1/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx = e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}\,dx$$
$$= \frac{1}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)}$$

Mathematica [A] time = 0.103027, size = 35, normalized size = 0.97

$$\frac{1}{n(ad-bc)\left(e\left(\frac{a+bx}{c+dx}\right)^n-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n)/((a + b*x)*(c + d*x)*(1 - e*((a + b*x
)/(c + d*x))^n)^2),x]
```

```
[Out] 1/((-(b*c) + a*d)*n*(-1 + e*((a + b*x)/(c + d*x))^n))
```

Maple [A] time = 0.171, size = 56, normalized size = 1.6

$$\frac{e}{n\,(ad-bc)}e^{n\ln\left(\frac{bx+a}{dx+c}\right)}\left(ee^{n\ln\left(\frac{bx+a}{dx+c}\right)}-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c),x)
```

```
[Out] e/n/(a*d-b*c)*exp(n*ln((b*x+a)/(d*x+c)))/(e*exp(n*ln((b*x+a)/(d*x+c)))-1)
```

Maxima [A] time = 1.04494, size = 70, normalized size = 1.94

$$-\frac{(bx+a)^n e}{(bcen-aden)(bx+a)^n-(bcn-adn)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="maxima")
```

```
[Out] -(b*x + a)^n*e/((b*c*e*n - a*d*e*n)*(b*x + a)^n - (b*c*n - a*d*n)*(d*x + c)
^n)
```

Fricas [A] time = 2.47833, size = 84, normalized size = 2.33

$$-\frac{1}{(bc-ad)en\left(\frac{bx+a}{dx+c}\right)^n-(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="fricas")
```

```
[Out] -1/((b*c - a*d)*e*n*((b*x + a)/(d*x + c))^n - (b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)/(d*x+c))**n)**2/(b*x+a)/(d*
x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e\left(\frac{bx+a}{dx+c}\right)^n}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^n-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c))^n)^2/(b*x+a)/(d*x+c
),x, algorithm="giac")
```

```
[Out] integrate(e*((b*x + a)/(d*x + c))^n/((b*x + a)*(d*x + c)*(e*((b*x + a)/(d*x
 + c))^n - 1)^2), x)
```

3.155 $\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx$

Optimal. Leaf size=52

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

```
[Out] (e*((a + b*x)/(c + d*x))^n)/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n)^
2)
```

Rubi [A] time = 2.02456, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {6741, 12, 6692, 34}

$$\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(bc-ad)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((a + b*x
)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

```
[Out] (e*((a + b*x)/(c + d*x))^n)/((b*c - a*d)*n*(1 - e*((a + b*x)/(c + d*x))^n)^
2)
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6692

```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((a_.) + (b_.)*(y_))^(m_.), x_Symbol] :
> With[{q = DerivativeDivides[y, u, x]}, Dist[q, Subst[Int[(a + b*x)^m*(c +
 d*x)^n, x], x, y], x] /;  !FalseQ[q]] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[v, y]
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e\left(\frac{a+bx}{c+dx}\right)^n+e^2\left(\frac{a+bx}{c+dx}\right)^{2n}}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx &= \int \frac{e\left(\frac{a+bx}{c+dx}\right)^n\left(1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx \\
&= e\int \frac{\left(\frac{a+bx}{c+dx}\right)^n\left(1+e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{1+x}{(1-x)^3}\,dx, x, e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n} \\
&= \frac{e\left(\frac{a+bx}{c+dx}\right)^n}{(bc-ad)n\left(1-e\left(\frac{a+bx}{c+dx}\right)^n\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.265491, size = 52, normalized size = 1.

$$-\frac{e\left(\frac{a+bx}{c+dx}\right)^n}{n(ad-bc)\left(e\left(\frac{a+bx}{c+dx}\right)^n-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*((a + b*x)/(c + d*x))^n + e^2*((a + b*x)/(c + d*x))^(2*n))/((a
 + b*x)*(c + d*x)*(1 - e*((a + b*x)/(c + d*x))^n)^3),x]
```

```
[Out] -((e*((a + b*x)/(c + d*x))^n)/((-(b*c) + a*d)*n*(-1 + e*((a + b*x)/(c + d*x
))^n)^2))
```

Maple [A] time = 0.276, size = 57, normalized size = 1.1

$$-\frac{e}{n\,(ad-bc)}\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}\left(e\mathrm{e}^{n\ln\left(\frac{bx+a}{dx+c}\right)}-1\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(d*x+c)
)^n)^3/(b*x+a)/(d*x+c),x)
```

```
[Out] -e/n/(a*d-b*c)*exp(n*ln((b*x+a)/(d*x+c)))/(e*exp(n*ln((b*x+a)/(d*x+c)))-1)^
2
```

Maxima [B] time = 1.17927, size = 285, normalized size = 5.48

$$\frac{1}{2}\left(\frac{(bx+a)^{2n}e}{\left(bce^2n-ade^2n\right)(bx+a)^{2n}+(bcn-adn)(dx+c)^{2n}-2\,(bcen-aden)e^{(n\log(bx+a)+n\log(dx+c))}}-\frac{}{\left(bce^2n-ade^2n\right)(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*((b*x + a)^(2*n)*e/((b*c*e^2*n - a*d*e^2*n)*(b*x + a)^(2*n) + (b*c*n -
a*d*n)*(d*x + c)^(2*n) - 2*(b*c*e*n - a*d*e*n)*e^(n*log(b*x + a) + n*log(d*
x + c))) - ((b*x + a)^(2*n)*e - 2*e^(n*log(b*x + a) + n*log(d*x + c)))/((b*
c*e^2*n - a*d*e^2*n)*(b*x + a)^(2*n) + (b*c*n - a*d*n)*(d*x + c)^(2*n) - 2*
(b*c*e*n - a*d*e*n)*e^(n*log(b*x + a) + n*log(d*x + c))))*e
```

Fricas [A] time = 2.27585, size = 182, normalized size = 3.5

$$\frac{e\left(\frac{bx+a}{dx+c}\right)^{n}}{(bc-ad)e^{2}n\left(\frac{bx+a}{dx+c}\right)^{2n}-2\,(bc-ad)en\left(\frac{bx+a}{dx+c}\right)^{n}+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] e*((b*x + a)/(d*x + c))^n/((b*c - a*d)*e^2*n*((b*x + a)/(d*x + c))^(2*n) -
2*(b*c - a*d)*e*n*((b*x + a)/(d*x + c))^n + (b*c - a*d)*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))**n)*e*((b*x+a)/(d*x+c))**n/(-1+e*((b*x+a)
/(d*x+c))**n)**3/(b*x+a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(e\left(\frac{bx+a}{dx+c}\right)^{n}+1\right)e\left(\frac{bx+a}{dx+c}\right)^{n}}{(bx+a)(dx+c)\left(e\left(\frac{bx+a}{dx+c}\right)^{n}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(1+e*((b*x+a)/(d*x+c))^n)*e*((b*x+a)/(d*x+c))^n/(-1+e*((b*x+a)/(
d*x+c))^n)^3/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(-(e*((b*x + a)/(d*x + c))^n + 1)*e*((b*x + a)/(d*x + c))^n/((b*x
+ a)*(d*x + c)*(e*((b*x + a)/(d*x + c))^n - 1)^3), x)
```

3.156 $\int x^3 \mathbf{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)dx$

Optimal. Leaf size=135

$$-\frac{3x^2\text{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\frac{6\text{PolyLog}\left(n+4,d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)}+\frac{x^3\text{PolyLog}}{}$$

```
[Out] (x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLog
[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 + n
, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(F^(
c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```

Rubi [A] time = 0.0883024, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6609, 2282, 6589}

$$-\frac{3x^2\text{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{6x\text{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}-\frac{6\text{PolyLog}\left(n+4,d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)}+\frac{x^3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```


```
[Out] (x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLog
[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 + n
, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(F^(
c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```


Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right) dx = \frac{x^3 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3\int x^2 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right) dx}{bcp\log(F)}$$

$$= \frac{x^3 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6\int x \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right) dx}{b^2c^2p^2\log^2(F)}$$

$$= \frac{x^3 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{6\int \text{Li}_{3+n}\left(\right.}{b^3c^3}$$

$$= \frac{x^3 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{6\,\text{Subst}\left(\int\right.}{b}$$

$$= \frac{x^3 \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{3x^2 \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{6 \text{Li}_{4+n}\left(d\left(\right.\right.}{b^4c^4p^4\,l}$$

Mathematica [A] time = 0.0115321, size = 135, normalized size = 1.

$$-\frac{3x^2 \text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{6x \text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} - \frac{6 \text{PolyLog}\left(n+4, d\left(F^{c(a+bx)}\right)^p\right)}{b^4c^4p^4\log^4(F)} + \frac{x^3 \text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x^3*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (3*x^2*PolyLog
[2 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (6*x*PolyLog[3 + n
, d*(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3) - (6*PolyLog[4 + n, d*(F^(
```

```
c*(a + b*x)))^p])/(b^4*c^4*p^4*Log[F]^4)
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int x^3 \text{polylog}\left(n, d\left(F^{c(bx+a)}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)

[Out] int(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \text{Li}_n(\left(F^{(bx+a)c}\right)^p d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")

[Out] integrate(x^3*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3 \text{polylog}\left(n, \left(F^{bcx+ac}\right)^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")

[Out] integral(x^3*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(x**3*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{Li}_n(\left(F^{(bx+a)c}\right)^p d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(x^3*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.157 $\int x^2\textbf{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)dx$

Optimal. Leaf size=100

$$-\frac{2x\text{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}+\frac{x^2\text{PolyLog}\left(n+1,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

```
[Out] (x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[2
 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n, d*
(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```

Rubi [A] time = 0.0579408, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6609, 2282, 6589}

$$-\frac{2x\text{PolyLog}\left(n+2,d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}+\frac{2\text{PolyLog}\left(n+3,d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}+\frac{x^2\text{PolyLog}\left(n+1,d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```



```
[Out] (x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[2
 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n, d*
(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```


Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)dx &= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2\int x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)} \\
&= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{2\int \text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)dx}{b^2c^2p^2\log^2(F)} \\
&= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{2\,\text{Subst}\left(\int \frac{\text{Li}_{2+n}(dx^p)}{x}dx, x, F^{c(a+bx)}\right)}{b^3c^3p^2\log^3(F)} \\
&= \frac{x^2\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{2x\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{2\text{Li}_{3+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)}
\end{aligned}$$

Mathematica [A] time = 0.0058507, size = 100, normalized size = 1.

$$-\frac{2x\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)} + \frac{2\text{PolyLog}\left(n+3, d\left(F^{c(a+bx)}\right)^p\right)}{b^3c^3p^3\log^3(F)} + \frac{x^2\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x^2*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - (2*x*PolyLog[2
 + n, d*(F^(c*(a + b*x)))^p])/(b^2*c^2*p^2*Log[F]^2) + (2*PolyLog[3 + n, d*
(F^(c*(a + b*x)))^p])/(b^3*c^3*p^3*Log[F]^3)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int x^2 \mathrm{polylog}\left(n, d\left(F^{c(bx+a)}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

```
[Out] int(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_n(\left(F^{(bx+a)c}\right)^p d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

```
[Out] integrate(x^2*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 \mathrm{polylog}\left(n, \left(F^{bcx+ac}\right)^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

```
[Out] integral(x^2*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(x**2*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(x^2*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.158 $\int x\mathbf{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

Optimal. Leaf size=65

$$\frac{x\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

```
[Out] (x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n, d
*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

Rubi [A] time = 0.0331011, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6609, 2282, 6589}

$$\frac{x\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Int[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n, d
*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)dx &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\int \text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)dx}{bcp\log(F)}\\ &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\text{Subst}\left(\int \frac{\text{Li}_{1+n}(dx^p)}{x}dx, x, F^{c(a+bx)}\right)}{b^2c^2p\log^2(F)}\\ &= \frac{x\text{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\text{Li}_{2+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}\end{aligned}$$

Mathematica [A] time = 0.0052208, size = 65, normalized size = 1.

$$\frac{x\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)} - \frac{\text{PolyLog}\left(n+2, d\left(F^{c(a+bx)}\right)^p\right)}{b^2c^2p^2\log^2(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] (x*PolyLog[1 + n, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]) - PolyLog[2 + n, d
*(F^(c*(a + b*x)))^p]/(b^2*c^2*p^2*Log[F]^2)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int x\text{polylog}\left(n, d\left(F^{c(bx+a)}\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

```
[Out] int(x*polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

```
[Out] integrate(x*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{polylog}\left(n, \left(F^{bcx+ac}\right)^p d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

```
[Out] integral(x*polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{Li}_{n}\left(d \left(F^{ac} F^{bcx}\right)^{p}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(x*polylog(n, d*(F**(a*c)*F**(b*c*x))**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_n\left(\left(F^{(bx+a)c}\right)^p d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(x*polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.159 $\int \text{PolyLog}\left(n, d\left(F^{c(a+bx)}\right)^p\right) dx$

Optimal. Leaf size=31

$$\frac{\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

```
[Out] PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

Rubi [A] time = 0.0177012, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2282, 6589}

$$\frac{\text{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \mathrm{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)dx = \frac{\mathrm{Subst}\left(\int \frac{\mathrm{Li}_n(dx^p)}{x}dx, x, F^{c(a+bx)}\right)}{bc\log(F)}$$
$$= \frac{\mathrm{Li}_{1+n}\left(d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Mathematica [A] time = 0.004743, size = 31, normalized size = 1.

$$\frac{\mathrm{PolyLog}\left(n+1, d\left(F^{c(a+bx)}\right)^p\right)}{bcp\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p],x]
```

```
[Out] PolyLog[1 + n, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F])
```

Maple [A] time = 0.061, size = 32, normalized size = 1.

$$\frac{\mathrm{polylog}\left(1+n, d\left(F^{c(bx+a)}\right)^p\right)}{pbc\ln(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,d*(F^(c*(b*x+a)))^p),x)
```

```
[Out] polylog(1+n,d*(F^(c*(b*x+a)))^p)/b/c/p/ln(F)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_n(\left(F^{(bx+a)c}\right)^p d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\text{polylog}\left(n, \left(F^{bcx+ac}\right)^{p} d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="fricas")
```

```
[Out] integral(polylog(n, (F^(b*c*x + a*c))^p*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{n}\left(d \left(F^{c(a+bx)}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F**(c*(b*x+a)))**p),x)
```

```
[Out] Integral(polylog(n, d*(F**(c*(a + b*x)))**p), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{Li}_{n}(\left(F^{(bx+a)c}\right)^{p} d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p),x, algorithm="giac")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d), x)
```

3.160 $\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\text{PolyLog}\left(n,d\left(F^{ac+bcx}\right)^p\right)}{x},x\right)$$

```
[Out] CannotIntegrate[PolyLog[n, d*(F^(a*c + b*c*x))^p]/x, x]
```

Rubi [A] time = 0.0684746, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
```

```
[Out] Defer[Int][PolyLog[n, d*(F^(a*c + b*c*x))^p]/x, x]
```

Rubi steps

$$\int \frac{\text{Li}_n\left(d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx = \int \frac{\text{Li}_n\left(d\left(F^{ac+bcx}\right)^p\right)}{x}\,dx$$

Mathematica [A] time = 0.0510035, size = 0, normalized size = 0.

$$\int \frac{\text{PolyLog}\left(n,d\left(F^{c(a+bx)}\right)^p\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x,x]
```

```
[Out] Integrate[PolyLog[n, d*(F^(c*(a + b*x)))^p]/x, x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\operatorname{polylog}\left(n, d\left(F^{c(bx+a)}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

```
[Out] int(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_{n}(\left(F^{(bx+a)c}\right)^{p} d)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="maxima")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{polylog}\left(n, \left(F^{bcx+ac}\right)^{p} d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="fricas")
```

```
[Out] integral(polylog(n, (F^(b*c*x + a*c))^p*d)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n\left(d\left(F^{ac}F^{bcx}\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F**(c*(b*x+a)))**p)/x,x)
```

```
[Out] Integral(polylog(n, d*(F**(a*c)*F**(b*c*x))**p)/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_n(\left(F^{(bx+a)c}\right)^p d)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(polylog(n,d*(F^(c*(b*x+a)))^p)/x,x, algorithm="giac")
```

```
[Out] integrate(polylog(n, (F^((b*x + a)*c))^p*d)/x, x)
```

3.161 $\int x^3 \log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=300

$$-\frac{x^2\text{PolyLog}(2,cx)}{8c^2}-\frac{x\text{PolyLog}(2,cx)}{4c^3}+\frac{\text{PolyLog}(3,1-cx)}{2c^4}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{4c^4}-\frac{\log(1-cx)\text{PolyLog}(2,1-\ldots}{2c^4}$$

```
[Out] (355*x)/(576*c^3) + (139*x^2)/(1152*c^2) + (67*x^3)/(1728*c) + (3*x^4)/256
+ (139*Log[1 - c*x])/(576*c^4) - (x^2*Log[1 - c*x])/(8*c^2) - (5*x^3*Log[1
- c*x])/(72*c) - (3*x^4*Log[1 - c*x])/64 + (3*(1 - c*x)*Log[1 - c*x])/(8*c^
4) - Log[1 - c*x]^2/(16*c^4) + (x^4*Log[1 - c*x]^2)/16 - (Log[c*x]*Log[1 -
c*x]^2)/(4*c^4) - (x*PolyLog[2, c*x])/(4*c^3) - (x^2*PolyLog[2, c*x])/(8*c^
2) - (x^3*PolyLog[2, c*x])/(12*c) - (x^4*PolyLog[2, c*x])/16 - (Log[1 - c*x
]*PolyLog[2, c*x])/(4*c^4) + (x^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1
- c*x]*PolyLog[2, 1 - c*x])/(2*c^4) + PolyLog[3, 1 - c*x]/(2*c^4)
```

Rubi [A] time = 0.523459, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6591, 2395, 43, 6603, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x^2\text{PolyLog}(2,cx)}{8c^2}-\frac{x\text{PolyLog}(2,cx)}{4c^3}+\frac{\text{PolyLog}(3,1-cx)}{2c^4}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{4c^4}-\frac{\log(1-cx)\text{PolyLog}(2,1-\ldots}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] (355*x)/(576*c^3) + (139*x^2)/(1152*c^2) + (67*x^3)/(1728*c) + (3*x^4)/256
+ (139*Log[1 - c*x])/(576*c^4) - (x^2*Log[1 - c*x])/(8*c^2) - (5*x^3*Log[1
- c*x])/(72*c) - (3*x^4*Log[1 - c*x])/64 + (3*(1 - c*x)*Log[1 - c*x])/(8*c^
4) - Log[1 - c*x]^2/(16*c^4) + (x^4*Log[1 - c*x]^2)/16 - (Log[c*x]*Log[1 -
c*x]^2)/(4*c^4) - (x*PolyLog[2, c*x])/(4*c^3) - (x^2*PolyLog[2, c*x])/(8*c^
2) - (x^3*PolyLog[2, c*x])/(12*c) - (x^4*PolyLog[2, c*x])/16 - (Log[1 - c*x
]*PolyLog[2, c*x])/(4*c^4) + (x^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1
- c*x]*PolyLog[2, 1 - c*x])/(2*c^4) + PolyLog[3, 1 - c*x]/(2*c^4)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \log(1-cx)\mathrm{Li}_2(cx)\,dx = \frac{1}{4}x^4\log(1-cx)\mathrm{Li}_2(cx) + \frac{1}{4}\int x^3\log^2(1-cx)\,dx + \frac{1}{4}c\int\left(-\frac{\mathrm{Li}_2(cx)}{c^4} - \frac{x\mathrm{Li}_2(cx)}{c^3} - \frac{x^2\mathrm{Li}_2(c\right.$$

$$= \frac{1}{16}x^4\log^2(1-cx) + \frac{1}{4}x^4\log(1-cx)\mathrm{Li}_2(cx) - \frac{1}{4}\int x^3\mathrm{Li}_2(cx)\,dx - \frac{\int \mathrm{Li}_2(cx)\,dx}{4c^3} - \frac{\int\frac{\mathrm{Li}_2(cx)}{-1+cx}}{4c^3}$$

$$= \frac{1}{16}x^4\log^2(1-cx) - \frac{x\mathrm{Li}_2(cx)}{4c^3} - \frac{x^2\mathrm{Li}_2(cx)}{8c^2} - \frac{x^3\mathrm{Li}_2(cx)}{12c} - \frac{1}{16}x^4\mathrm{Li}_2(cx) - \frac{\log(1-cx)\mathrm{Li}_2(c}{4c^4}$$

$$= -\frac{x^2\log(1-cx)}{16c^2} - \frac{x^3\log(1-cx)}{36c} - \frac{1}{64}x^4\log(1-cx) + \frac{1}{16}x^4\log^2(1-cx) - \frac{\log(cx)\log^2(1}{4c^4}$$

$$= \frac{x}{4c^3} - \frac{x^2\log(1-cx)}{8c^2} - \frac{5x^3\log(1-cx)}{72c} - \frac{3}{64}x^4\log(1-cx) + \frac{(1-cx)\log(1-cx)}{4c^4} + \frac{1}{16}x^4\mathrm{l}$$

$$= \frac{277x}{576c^3} + \frac{61x^2}{1152c^2} + \frac{25x^3}{1728c} + \frac{x^4}{256} + \frac{61\log(1-cx)}{576c^4} - \frac{x^2\log(1-cx)}{8c^2} - \frac{5x^3\log(1-cx)}{72c} - \frac{3}{64}$$

$$= \frac{355x}{576c^3} + \frac{139x^2}{1152c^2} + \frac{67x^3}{1728c} + \frac{3x^4}{256} + \frac{139\log(1-cx)}{576c^4} - \frac{x^2\log(1-cx)}{8c^2} - \frac{5x^3\log(1-cx)}{72c} - $$

Mathematica [A] time = 0.573099, size = 223, normalized size = 0.74

$$\frac{144\left(12\left(c^4x^4-1\right)\log(1-cx)-cx\left(3c^3x^3+4c^2x^2+6cx+12\right)\right)\text{PolyLog}(2,cx)+3456\text{PolyLog}(3,1-cx)-3456\log(1-\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (4260*c*x + 834*c^2*x^2 + 268*c^3*x^3 + 81*c^4*x^4 + 4260*Log[1 - c*x] - 25
92*c*x*Log[1 - c*x] - 864*c^2*x^2*Log[1 - c*x] - 480*c^3*x^3*Log[1 - c*x] -
 324*c^4*x^4*Log[1 - c*x] - 432*Log[1 - c*x]^2 + 432*c^4*x^4*Log[1 - c*x]^2
 - 1728*Log[c*x]*Log[1 - c*x]^2 + 144*(-(c*x*(12 + 6*c*x + 4*c^2*x^2 + 3*c^
3*x^3)) + 12*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog[2, c*x] - 3456*Log[1 - c*
x]*PolyLog[2, 1 - c*x] + 3456*PolyLog[3, 1 - c*x])/(6912*c^4)
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^3\ln(-cx+1)\,\text{polylog}(2,cx)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x^3*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.12854, size = 508, normalized size = 1.69

$$\frac{9\,c^4\left(\frac{3\,c^3x^4+4\,c^2x^3+6\,cx^2+12\,x}{c^4}+\frac{12\,\log(cx-1)}{c^5}\right)+24\,c^3\left(\frac{2\,c^2x^3+3\,cx^2+6\,x}{c^3}+\frac{6\,\log(cx-1)}{c^4}\right)+108\,c^2\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right)+432\,c\left(\frac{x}{c}+\frac{\log(\cdots}{\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/6912*(9*c^4*((3*c^3*x^4 + 4*c^2*x^3 + 6*c*x^2 + 12*x)/c^4 + 12*log(c*x -
1)/c^5) + 24*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) + 1
08*c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 432*c*(x/c + log(c*x - 1)
```

```
/c^2) + 2*(27*c^4*x^4 + 92*c^3*x^3 + 300*c^2*x^2 + 1680*c*x - 72*(3*c^4*x^4
 + 4*c^3*x^3 + 6*c^2*x^2 + 12*c*x + 12*log(-c*x + 1))*dilog(c*x) - 12*(9*c^
4*x^4 + 14*c^3*x^3 + 27*c^2*x^2 + 90*c*x - 140)*log(-c*x + 1))/c - 1728*(lo
g(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*
x + 1))/c)/c^3 + 1/192*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*c^3*x^3 - 6*c
^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*log(-c*x + 1)/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^3\text{Li}_2\left(cx\right)\log\left(-cx+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x^3*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\text{Li}_2\left(cx\right)\log\left(-cx+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x^3*dilog(c*x)*log(-c*x + 1), x)
```

3.162 $\int x^2 \log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=258

$$-\frac{x\text{PolyLog}(2,cx)}{3c^2}+\frac{2\text{PolyLog}(3,1-cx)}{3c^3}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{3c^3}-\frac{2\log(1-cx)\text{PolyLog}(2,1-cx)}{3c^3}-\frac{1}{9}x^3\text{PolyL}$$

```
[Out] (31*x)/(36*c^2) + (11*x^2)/(72*c) + x^3/27 + (11*Log[1 - c*x])/(36*c^3) - (
7*x^2*Log[1 - c*x])/(36*c) - (x^3*Log[1 - c*x])/9 + (5*(1 - c*x)*Log[1 - c*
x])/(9*c^3) - Log[1 - c*x]^2/(9*c^3) + (x^3*Log[1 - c*x]^2)/9 - (Log[c*x]*L
og[1 - c*x]^2)/(3*c^3) - (x*PolyLog[2, c*x])/(3*c^2) - (x^2*PolyLog[2, c*x]
)/(6*c) - (x^3*PolyLog[2, c*x])/9 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3)
+ (x^3*Log[1 - c*x]*PolyLog[2, c*x])/3 - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x
])/(3*c^3) + (2*PolyLog[3, 1 - c*x])/(3*c^3)
```

Rubi [A] time = 0.404814, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6591, 2395, 43, 6603, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x\text{PolyLog}(2,cx)}{3c^2}+\frac{2\text{PolyLog}(3,1-cx)}{3c^3}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{3c^3}-\frac{2\log(1-cx)\text{PolyLog}(2,1-cx)}{3c^3}-\frac{1}{9}x^3\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] (31*x)/(36*c^2) + (11*x^2)/(72*c) + x^3/27 + (11*Log[1 - c*x])/(36*c^3) - (
7*x^2*Log[1 - c*x])/(36*c) - (x^3*Log[1 - c*x])/9 + (5*(1 - c*x)*Log[1 - c*
x])/(9*c^3) - Log[1 - c*x]^2/(9*c^3) + (x^3*Log[1 - c*x]^2)/9 - (Log[c*x]*L
og[1 - c*x]^2)/(3*c^3) - (x*PolyLog[2, c*x])/(3*c^2) - (x^2*PolyLog[2, c*x]
)/(6*c) - (x^3*PolyLog[2, c*x])/9 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3)
+ (x^3*Log[1 - c*x]*PolyLog[2, c*x])/3 - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x
])/(3*c^3) + (2*PolyLog[3, 1 - c*x])/(3*c^3)
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
```

```
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\log(1-cx)\mathrm{Li}_2(cx)\,dx = \frac{1}{3}x^3\log(1-cx)\mathrm{Li}_2(cx)+\frac{1}{3}\int x^2\log^2(1-cx)\,dx+\frac{1}{3}c\int\left(-\frac{\mathrm{Li}_2(cx)}{c^3}-\frac{x\mathrm{Li}_2(cx)}{c^2}-\frac{x^2\mathrm{Li}_2(c\right.$$

$$=\frac{1}{9}x^3\log^2(1-cx)+\frac{1}{3}x^3\log(1-cx)\mathrm{Li}_2(cx)-\frac{1}{3}\int x^2\mathrm{Li}_2(cx)\,dx-\frac{\int\mathrm{Li}_2(cx)\,dx}{3c^2}-\frac{\int\frac{\mathrm{Li}_2(cx)}{-1+cx}}{3c^2}$$

$$=\frac{1}{9}x^3\log^2(1-cx)-\frac{x\mathrm{Li}_2(cx)}{3c^2}-\frac{x^2\mathrm{Li}_2(cx)}{6c}-\frac{1}{9}x^3\mathrm{Li}_2(cx)-\frac{\log(1-cx)\mathrm{Li}_2(cx)}{3c^3}+\frac{1}{3}x^3\log(1$$

$$=-\frac{x^2\log(1-cx)}{12c}-\frac{1}{27}x^3\log(1-cx)+\frac{1}{9}x^3\log^2(1-cx)-\frac{\log(cx)\log^2(1-cx)}{3c^3}-\frac{x\mathrm{Li}_2(cx)}{3c^2}$$

$$=\frac{x}{3c^2}-\frac{7x^2\log(1-cx)}{36c}-\frac{1}{9}x^3\log(1-cx)+\frac{(1-cx)\log(1-cx)}{3c^3}+\frac{1}{9}x^3\log^2(1-cx)-\frac{\log(c}{}$$

$$=\frac{73x}{108c^2}+\frac{13x^2}{216c}+\frac{x^3}{81}+\frac{13\log(1-cx)}{108c^3}-\frac{7x^2\log(1-cx)}{36c}-\frac{1}{9}x^3\log(1-cx)+\frac{5(1-cx)\log(}{9c^3}$$

$$=\frac{31x}{36c^2}+\frac{11x^2}{72c}+\frac{x^3}{27}+\frac{11\log(1-cx)}{36c^3}-\frac{7x^2\log(1-cx)}{36c}-\frac{1}{9}x^3\log(1-cx)+\frac{5(1-cx)\log(1}{9c^3}$$

Mathematica [A] time = 0.357232, size = 192, normalized size = 0.74

$$\frac{12\left(6\left(c^3x^3-1\right)\log(1-cx)-cx\left(2c^2x^2+3cx+6\right)\right)\mathrm{PolyLog}(2,cx)+144\mathrm{PolyLog}(3,1-cx)-144\log(1-cx)\mathrm{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (186*c*x + 33*c^2*x^2 + 8*c^3*x^3 + 186*Log[1 - c*x] - 120*c*x*Log[1 - c*x]
 - 42*c^2*x^2*Log[1 - c*x] - 24*c^3*x^3*Log[1 - c*x] - 24*Log[1 - c*x]^2 +
24*c^3*x^3*Log[1 - c*x]^2 - 72*Log[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(6 + 3*c
*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] - 144*Log
[1 - c*x]*PolyLog[2, 1 - c*x] + 144*PolyLog[3, 1 - c*x])/(216*c^3)
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int x^2 \ln(-cx+1) \operatorname{polylog}(2, cx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x^2*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.11877, size = 400, normalized size = 1.55

$$\frac{4\,c^3\left(\frac{2\,c^2x^3+3\,cx^2+6\,x}{c^3}+\frac{6\,\log(cx-1)}{c^4}\right)+18\,c^2\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right)+72\,c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{16\,c^3x^3+69\,c^2x^2+426\,cx-36\,\left(2\,c^3x^3+3\,c^2x^2+6\,cx+\right.}{}}{648\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/648*(4*c^3*((2*c^2*x^3 + 3*c*x^2 + 6*x)/c^3 + 6*log(c*x - 1)/c^4) + 18*c^
2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 72*c*(x/c + log(c*x - 1)/c^2)
+ (16*c^3*x^3 + 69*c^2*x^2 + 426*c*x - 36*(2*c^3*x^3 + 3*c^2*x^2 + 6*c*x +
6*log(-c*x + 1))*dilog(c*x) - 6*(8*c^3*x^3 + 15*c^2*x^2 + 48*c*x - 71)*log(
-c*x + 1))/c - 216*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x +
 1) - 2*polylog(3, -c*x + 1))/c)/c^2 + 1/54*(18*c^3*x^3*dilog(c*x) - 2*c^3*
x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*log(-c*x + 1)/c^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^2 \mathrm{Li}_2(cx) \log(-cx+1), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x^2*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \mathrm{Li}_2(cx) \log(-cx+1)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x^2*dilog(c*x)*log(-c*x + 1), x)
```

3.163 $\int x\log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=262

$$\frac{\text{PolyLog}(3,1-cx)}{c^2}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{2c^2}-\frac{\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2}-\frac{1}{4}x^2\text{PolyLog}(2,cx)+\frac{1}{2}x^2\log(1-c$$

```
[Out] (13*x)/(8*c) + x^2/16 + (1 - c*x)^2/(8*c^2) + Log[1 - c*x]/(8*c^2) - (x^2*L
og[1 - c*x])/8 + (3*(1 - c*x)*Log[1 - c*x])/(2*c^2) - ((1 - c*x)^2*Log[1 -
c*x])/(4*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + ((1 - c*x)^2*Log[1 - c
*x]^2)/(4*c^2) - (Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - (x*PolyLog[2, c*x])/(2
*c) - (x^2*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x
^2*Log[1 - c*x]*PolyLog[2, c*x])/2 - (Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
 + PolyLog[3, 1 - c*x]/c^2
```

Rubi [A] time = 0.254646, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6591, 2395, 43, 6603, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{\text{PolyLog}(3,1-cx)}{c^2}-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{2c^2}-\frac{\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2}-\frac{1}{4}x^2\text{PolyLog}(2,cx)+\frac{1}{2}x^2\log(1-c$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] (13*x)/(8*c) + x^2/16 + (1 - c*x)^2/(8*c^2) + Log[1 - c*x]/(8*c^2) - (x^2*L
og[1 - c*x])/8 + (3*(1 - c*x)*Log[1 - c*x])/(2*c^2) - ((1 - c*x)^2*Log[1 -
c*x])/(4*c^2) - ((1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + ((1 - c*x)^2*Log[1 - c
*x]^2)/(4*c^2) - (Log[c*x]*Log[1 - c*x]^2)/(2*c^2) - (x*PolyLog[2, c*x])/(2
*c) - (x^2*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x
^2*Log[1 - c*x]*PolyLog[2, c*x])/2 - (Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
 + PolyLog[3, 1 - c*x]/c^2
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx)+\frac{1}{2}\int x\log^2(1-cx)\,dx+\frac{1}{2}c\int\left(-\frac{\text{Li}_2(cx)}{c^2}-\frac{x\text{Li}_2(cx)}{c}-\frac{\text{Li}_2(cx)}{c^2(-1+c$$

$$= \frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx)+\frac{1}{2}\int\left(\frac{\log^2(1-cx)}{c}-\frac{(1-cx)\log^2(1-cx)}{c}\right)dx-\frac{1}{2}\int x\text{Li}_2(cx)\,dx$$

$$= -\frac{x\text{Li}_2(cx)}{2c}-\frac{1}{4}x^2\text{Li}_2(cx)-\frac{\log(1-cx)\text{Li}_2(cx)}{2c^2}+\frac{1}{2}x^2\log(1-cx)\text{Li}_2(cx)-\frac{1}{4}\int x\log(1-c$$

$$= -\frac{1}{8}x^2\log(1-cx)-\frac{\log(cx)\log^2(1-cx)}{2c^2}-\frac{x\text{Li}_2(cx)}{2c}-\frac{1}{4}x^2\text{Li}_2(cx)-\frac{\log(1-cx)\text{Li}_2(cx)}{2c^2}+\frac{1}{2}$$

$$= \frac{x}{2c}-\frac{1}{8}x^2\log(1-cx)+\frac{(1-cx)\log(1-cx)}{2c^2}-\frac{(1-cx)\log^2(1-cx)}{2c^2}+\frac{(1-cx)^2\log^2(1-cx)}{4c^2}$$

$$= \frac{13x}{8c}+\frac{x^2}{16}+\frac{(1-cx)^2}{8c^2}+\frac{\log(1-cx)}{8c^2}-\frac{1}{8}x^2\log(1-cx)+\frac{3(1-cx)\log(1-cx)}{2c^2}-\frac{(1-cx)^2\,\text{l}}{4}$$

$$= \frac{13x}{8c}+\frac{x^2}{16}+\frac{(1-cx)^2}{8c^2}+\frac{\log(1-cx)}{8c^2}-\frac{1}{8}x^2\log(1-cx)+\frac{3(1-cx)\log(1-cx)}{2c^2}-\frac{(1-cx)^2\,\text{l}}{4}$$

Mathematica [A] time = 0.30023, size = 160, normalized size = 0.61

$$\frac{\left(8\left(c^2x^2-1\right)\log(1-cx)-4cx(cx+2)\right)\text{PolyLog}(2,cx)+16\text{PolyLog}(3,1-cx)-16\log(1-cx)\text{PolyLog}(2,1-cx)+3c^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*Log[1 - c*x] - 6*c^2*x
^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 - c*x]^2 - 8*Log[c*x]*
Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2)*Log[1 - c*x])*PolyLog
[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*PolyLog[3, 1 - c*x])/(1
6*c^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x\ln(-cx+1)\,\text{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.15223, size = 300, normalized size = 1.15

$$\frac{c^2\left(\frac{cx^2+2x}{c^2}+\frac{2\log(cx-1)}{c^3}\right)+4c\left(\frac{x}{c}+\frac{\log(cx-1)}{c^2}\right)+\frac{2\left(c^2x^2+8cx-2\left(c^2x^2+2cx+2\log(-cx+1)\right)\text{Li}_2(cx)-2\left(c^2x^2+3cx-4\right)\log(-cx+1)\right)}{c}-\frac{8\left(\log(cx)\log(-\right.}{}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] 1/16*(c^2*((c*x^2 + 2*x)/c^2 + 2*log(c*x - 1)/c^3) + 4*c*(x/c + log(c*x - 1
)/c^2) + 2*(c^2*x^2 + 8*c*x - 2*(c^2*x^2 + 2*c*x + 2*log(-c*x + 1))*dilog(c
*x) - 2*(c^2*x^2 + 3*c*x - 4)*log(-c*x + 1))/c - 8*(log(c*x)*log(-c*x + 1)^
2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1))/c)/c + 1/8*(4
```

```
*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*log(
-c*x + 1)/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(x*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(-cx+1\right)\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(x*log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(x*dilog(c*x)*log(-c*x + 1), x)
```

3.164 $\int \log(1-cx)\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=132

$$-x\text{PolyLog}(2,cx)+\frac{2\text{PolyLog}(3,1-cx)}{c}+x\log(1-cx)\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{c}-\frac{2\log(1-cx)\text{Po}}{}$$

```
[Out] 3*x + (3*(1 - c*x)*Log[1 - c*x])/c - ((1 - c*x)*Log[1 - c*x]^2)/c - (Log[c*
x]*Log[1 - c*x]^2)/c - x*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/c
 + x*Log[1 - c*x]*PolyLog[2, c*x] - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c
+ (2*PolyLog[3, 1 - c*x])/c
```

Rubi [A] time = 0.211916, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {6586, 2389, 2295, 6600, 2296, 6688, 6742, 6596, 2396, 2433, 2374, 6589}

$$-x\text{PolyLog}(2,cx)+\frac{2\text{PolyLog}(3,1-cx)}{c}+x\log(1-cx)\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{PolyLog}(2,cx)}{c}-\frac{2\log(1-cx)\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] 3*x + (3*(1 - c*x)*Log[1 - c*x])/c - ((1 - c*x)*Log[1 - c*x]^2)/c - (Log[c*
x]*Log[1 - c*x]^2)/c - x*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/c
 + x*Log[1 - c*x]*PolyLog[2, c*x] - (2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c
+ (2*PolyLog[3, 1 - c*x])/c
```


Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log(1-cx)\mathrm{Li}_2(cx)\,dx = x\log(1-cx)\mathrm{Li}_2(cx) + c\int\left(-\frac{1}{c}-\frac{1}{c(-1+cx)}\right)\mathrm{Li}_2(cx)\,dx + \int \log^2(1-cx)\,dx$$

$$= x\log(1-cx)\mathrm{Li}_2(cx) - \frac{\mathrm{Subst}\left(\int \log^2(x)\,dx, x, 1-cx\right)}{c} + c\int \frac{x\mathrm{Li}_2(cx)}{1-cx}\,dx$$

$$= -\frac{(1-cx)\log^2(1-cx)}{c} + x\log(1-cx)\mathrm{Li}_2(cx) + \frac{2\,\mathrm{Subst}(\int \log(x)\,dx, x, 1-cx)}{c} + c\int\left(-\frac{\mathrm{Li}_2}{c}\right.$$

$$= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} + x\log(1-cx)\mathrm{Li}_2(cx) - \int \mathrm{Li}_2(cx)\,dx - \int$$

$$= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - x\mathrm{Li}_2(cx) - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{c} + x\log(1$$

$$= 2x + \frac{2(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\mathrm{Li}_2(cx) - \frac{\log(1}{}$$

$$= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\mathrm{Li}_2(cx) - \frac{\log(1}{}$$

$$= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\mathrm{Li}_2(cx) - \frac{\log(1}{}$$

$$= 3x + \frac{3(1-cx)\log(1-cx)}{c} - \frac{(1-cx)\log^2(1-cx)}{c} - \frac{\log(cx)\log^2(1-cx)}{c} - x\mathrm{Li}_2(cx) - \frac{\log(1}{}$$

Mathematica [A] time = 0.0230911, size = 119, normalized size = 0.9

$$\frac{2\mathrm{PolyLog}(3,1-cx) - 2\log(1-cx)\mathrm{PolyLog}(2,1-cx) + ((cx-1)\log(1-cx)-cx)\mathrm{PolyLog}(2,cx) + 3cx + cx\log^2(1-}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*Lo
g[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x])
*PolyLog[2, c*x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*
x])/c
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \ln(-cx+1)\,\mathrm{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.10237, size = 190, normalized size = 1.44

$$c\left(\frac{x}{c}+\frac{\log\left(cx-1\right)}{c^2}\right)+\frac{\left(cx\mathrm{Li}_2\left(cx\right)-cx+\left(cx-1\right)\log\left(-cx+1\right)\right)\log\left(-cx+1\right)}{c}-\frac{\log\left(cx\right)\log\left(-cx+1\right)^2+2\,\mathrm{Li}_2\left(-cx+1\right)\mathrm{l}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] c*(x/c + log(c*x - 1)/c^2) + (c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1
))*log(-c*x + 1)/c - (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x
 + 1) - 2*polylog(3, -c*x + 1))/c + (2*c*x - (c*x + log(-c*x + 1))*dilog(c*
x) - 2*(c*x - 1)*log(-c*x + 1))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(-cx+1\right)\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \mathrm{Li}_2(cx) \log(-cx + 1) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1), x)
```

3.165 $\int \frac{\log(1-cx)\textbf{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

```
[Out] -PolyLog[2, c*x]^2/2
```

Rubi [A] time = 0.0257549, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6589, 6601}

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]

[Out] -PolyLog[2, c*x]^2/2
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rubi steps

$$\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx = -\frac{1}{2}\text{Li}_2(cx)^2$$

Mathematica [A] time = 0.0091507, size = 11, normalized size = 1.

$$-\frac{1}{2}\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

```
[Out] -PolyLog[2, c*x]^2/2
```

Maple [A] time = 0.046, size = 10, normalized size = 0.9

$$-\frac{(\mathrm{polylog}(2,cx))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] -1/2*polylog(2,c*x)^2
```

Maxima [A] time = 0.966354, size = 11, normalized size = 1.

$$-\frac{1}{2}\,\mathrm{Li}_2\,(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*dilog(c*x)^2
```

Fricas [A] time = 2.53436, size = 26, normalized size = 2.36

$$-\frac{1}{2}\,\mathrm{Li}_2\,(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*dilog(c*x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(-cx+1)\operatorname{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{Li}_2(cx)\log(-cx+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x, x)
```

3.166 $\int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=111

$$-2c\text{PolyLog}(2,cx)-c\text{PolyLog}(3,cx)-2c\text{PolyLog}(3,1-cx)+c\log(1-cx)\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{PolyLog}(2,}{x}$$

```
[Out] ((1 - c*x)*Log[1 - c*x]^2)/x + c*Log[c*x]*Log[1 - c*x]^2 - 2*c*PolyLog[2, c
*x] + c*Log[1 - c*x]*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x + 2
*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1
- c*x]
```

Rubi [A] time = 0.163861, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {6591, 2395, 36, 29, 31, 6603, 2397, 2391, 6589, 6596, 2396, 2433, 2374}

$$-2c\text{PolyLog}(2,cx)-c\text{PolyLog}(3,cx)-2c\text{PolyLog}(3,1-cx)+c\log(1-cx)\text{PolyLog}(2,cx)-\frac{\log(1-cx)\text{PolyLog}(2,}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] ((1 - c*x)*Log[1 - c*x]^2)/x + c*Log[c*x]*Log[1 - c*x]^2 - 2*c*PolyLog[2, c
*x] + c*Log[1 - c*x]*PolyLog[2, c*x] - (Log[1 - c*x]*PolyLog[2, c*x])/x + 2
*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1
- c*x]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x^2}\,dx = -\frac{\log(1-cx)\mathrm{Li}_2(cx)}{x} - c\int \left(\frac{\mathrm{Li}_2(cx)}{x} - \frac{c\mathrm{Li}_2(cx)}{-1+cx}\right) dx - \int \frac{\log^2(1-cx)}{x^2}\,dx$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x} - c\int \frac{\mathrm{Li}_2(cx)}{x}\,dx + (2c)\int \frac{\log(1-cx)}{x}\,dx + c^2\int$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} - 2c\mathrm{Li}_2(cx) + c\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x} - c\mathrm{Li}_3(cx) + c$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} + c\log(cx)\log^2(1-cx) - 2c\mathrm{Li}_2(cx) + c\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)}{x}$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} + c\log(cx)\log^2(1-cx) - 2c\mathrm{Li}_2(cx) + c\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)}{x}$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} + c\log(cx)\log^2(1-cx) - 2c\mathrm{Li}_2(cx) + c\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)}{x}$$
$$= \frac{(1-cx)\log^2(1-cx)}{x} + c\log(cx)\log^2(1-cx) - 2c\mathrm{Li}_2(cx) + c\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)}{x}$$

Mathematica [A] time = 0.135536, size = 115, normalized size = 1.04

$$-c\mathrm{PolyLog}(3,cx) - 2c\mathrm{PolyLog}(3,1-cx) + \frac{(cx-1)\log(1-cx)\mathrm{PolyLog}(2,cx)}{x} + 2c(\log(1-cx)+1)\mathrm{PolyLog}(2,1-cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] 2*c*Log[c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x
]*Log[1 - c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + L
og[1 - c*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c
*x]
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

Maxima [A] time = 1.16597, size = 153, normalized size = 1.38

$$\left(\log\left(cx\right)\log\left(-cx+1\right)^{2}+2\,\mathrm{Li}_{2}\left(-cx+1\right)\log\left(-cx+1\right)-2\,\mathrm{Li}_{3}(-cx+1)\right)c+2\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_{2}\left(-cx+1\right)\right)c-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

```
[Out] (log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog(3,
-c*x + 1))*c + 2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c - c*polylog(3
, c*x) + ((c*x - 1)*dilog(c*x)*log(-c*x + 1) - (c*x - 1)*log(-c*x + 1)^2)/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_{2}\left(cx\right)\log\left(-cx+1\right)}{x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(-cx+1\right)\mathrm{Li}_{2}\left(cx\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.167 $\int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{2}c^2\text{PolyLog}(2,cx)-\frac{1}{2}c^2\text{PolyLog}(3,cx)-c^2\text{PolyLog}(3,1-cx)+\frac{1}{2}c^2\log(1-cx)\text{PolyLog}(2,cx)+c^2\log(1-cx)\text{Po}$$

```
[Out] -(c^2*Log[x]) + c^2*Log[1 - c*x] - (c*Log[1 - c*x])/x - (c^2*Log[1 - c*x]^2
)/4 + Log[1 - c*x]^2/(4*x^2) + (c^2*Log[c*x]*Log[1 - c*x]^2)/2 - (c^2*PolyL
og[2, c*x])/2 + (c*PolyLog[2, c*x])/(2*x) + (c^2*Log[1 - c*x]*PolyLog[2, c*
x])/2 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*x^2) + c^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x] - (c^2*PolyLog[3, c*x])/2 - c^2*PolyLog[3, 1 - c*x]
```

Rubi [A] time = 0.281613, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.062, Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}c^2\text{PolyLog}(2,cx)-\frac{1}{2}c^2\text{PolyLog}(3,cx)-c^2\text{PolyLog}(3,1-cx)+\frac{1}{2}c^2\log(1-cx)\text{PolyLog}(2,cx)+c^2\log(1-cx)\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```


```
[Out] -(c^2*Log[x]) + c^2*Log[1 - c*x] - (c*Log[1 - c*x])/x - (c^2*Log[1 - c*x]^2
)/4 + Log[1 - c*x]^2/(4*x^2) + (c^2*Log[c*x]*Log[1 - c*x]^2)/2 - (c^2*PolyL
og[2, c*x])/2 + (c*PolyLog[2, c*x])/(2*x) + (c^2*Log[1 - c*x]*PolyLog[2, c*
x])/2 - (Log[1 - c*x]*PolyLog[2, c*x])/(2*x^2) + c^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x] - (c^2*PolyLog[3, c*x])/2 - c^2*PolyLog[3, 1 - c*x]
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

```
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\log(1-cx)\text{Li}_2(cx)}{x^3}\,dx = -\frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2}\int \frac{\log^2(1-cx)}{x^3}\,dx - \frac{1}{2}c\int\left(\frac{\text{Li}_2(cx)}{x^2} + \frac{c\text{Li}_2(cx)}{x} - \frac{c^2\text{Li}_2(cx)}{-1+cx}\right)dx$$

$$= \frac{\log^2(1-cx)}{4x^2} - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} + \frac{1}{2}c\int \frac{\log(1-cx)}{x^2(1-cx)}\,dx - \frac{1}{2}c\int \frac{\text{Li}_2(cx)}{x^2}\,dx - \frac{1}{2}c^2\int \frac{\text{Li}_2(cx}{x}$$

$$= \frac{\log^2(1-cx)}{4x^2} + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2\log(1-cx)\text{Li}_2(cx) - \frac{\log(1-cx)\text{Li}_2(cx)}{2x^2} - \frac{1}{2}c^2\text{Li}_3(cx) + \frac{1}{2}c\int$$

$$= -\frac{c\log(1-cx)}{2x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx)\log^2(1-cx) + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2\log(1-cx)\text{Li}_2(cx)$$

$$= -\frac{c\log(1-cx)}{x} + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx)\log^2(1-cx) - \frac{1}{2}c^2\text{Li}_2(cx) + \frac{c\text{Li}_2(cx)}{2x} + \frac{1}{2}c^2\log($$

$$= -\frac{1}{2}c^2\log(x) + \frac{1}{2}c^2\log(1-cx) - \frac{c\log(1-cx)}{x} - \frac{1}{4}c^2\log^2(1-cx) + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx$$

$$= -c^2\log(x) + c^2\log(1-cx) - \frac{c\log(1-cx)}{x} - \frac{1}{4}c^2\log^2(1-cx) + \frac{\log^2(1-cx)}{4x^2} + \frac{1}{2}c^2\log(cx)\log$$

Mathematica [A] time = 0.274997, size = 185, normalized size = 0.97

$$\frac{1}{4}\left(\frac{2\left(\left(c^2x^2-1\right)\log(1-cx)+cx\right)\text{PolyLog}(2,cx)}{x^2} - 2c^2\text{PolyLog}(3,cx) - 4c^2\text{PolyLog}(3,1-cx) + 2c^2(2\log(1-cx)+1)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c*x] - (4*c*Log[1 - c*x])/x
 + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]^2 + Log[1 - c*x]^2/x^2 +
2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c^2*x^2)*Log[1 - c*x])*Poly
Log[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 2*c^2*P
olyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x])/4
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln\left(-cx+1\right)\mathrm{polylog}\left(2,cx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

Maxima [A] time = 1.66414, size = 219, normalized size = 1.15

$$\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)^2+2\,\mathrm{Li}_2\left(-cx+1\right)\log\left(-cx+1\right)-2\,\mathrm{Li}_3(-cx+1)\right)c^2+\frac{1}{2}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2\left(-cx+1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^2 + 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^2 - c
^2*log(x) - 1/2*c^2*polylog(3, c*x) - 1/4*((c^2*x^2 - 1)*log(-c*x + 1)^2 -
2*(c*x + (c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x) - 4*(c^2*x^2 - c*x)*log(-c
*x + 1))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(-cx+1\right) \mathrm{Li}_2\left(cx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

```
[Out] Integral(log(-c*x + 1)*polylog(2, c*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{Li}_2\left(cx\right)\log \left(-cx+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.168 $\int \frac{\log(1-cx)\textbf{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=245

$$-\frac{2}{9}c^3\text{PolyLog}(2,cx)-\frac{1}{3}c^3\text{PolyLog}(3,cx)-\frac{2}{3}c^3\text{PolyLog}(3,1-cx)+\frac{c^2\text{PolyLog}(2,cx)}{3x}+\frac{1}{3}c^3\log(1-cx)\text{PolyLog}(2,$$

```
[Out] (7*c^2)/(36*x) - (3*c^3*Log[x])/4 + (3*c^3*Log[1 - c*x])/4 - (7*c*Log[1 - c
*x])/(36*x^2) - (5*c^2*Log[1 - c*x])/(9*x) - (c^3*Log[1 - c*x]^2)/9 + Log[1
 - c*x]^2/(9*x^3) + (c^3*Log[c*x]*Log[1 - c*x]^2)/3 - (2*c^3*PolyLog[2, c*x
])/9 + (c*PolyLog[2, c*x])/(6*x^2) + (c^2*PolyLog[2, c*x])/(3*x) + (c^3*Log
[1 - c*x]*PolyLog[2, c*x])/3 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*x^3) + (2*
c^3*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*PolyLog[3, c*x])/3 - (2*c^3*
PolyLog[3, 1 - c*x])/3
```

Rubi [A] time = 0.354818, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.062, Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{2}{9}c^3\text{PolyLog}(2,cx)-\frac{1}{3}c^3\text{PolyLog}(3,cx)-\frac{2}{3}c^3\text{PolyLog}(3,1-cx)+\frac{c^2\text{PolyLog}(2,cx)}{3x}+\frac{1}{3}c^3\log(1-cx)\text{PolyLog}(2,$$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```


```
[Out] (7*c^2)/(36*x) - (3*c^3*Log[x])/4 + (3*c^3*Log[1 - c*x])/4 - (7*c*Log[1 - c
*x])/(36*x^2) - (5*c^2*Log[1 - c*x])/(9*x) - (c^3*Log[1 - c*x]^2)/9 + Log[1
 - c*x]^2/(9*x^3) + (c^3*Log[c*x]*Log[1 - c*x]^2)/3 - (2*c^3*PolyLog[2, c*x
])/9 + (c*PolyLog[2, c*x])/(6*x^2) + (c^2*PolyLog[2, c*x])/(3*x) + (c^3*Log
[1 - c*x]*PolyLog[2, c*x])/3 - (Log[1 - c*x]*PolyLog[2, c*x])/(3*x^3) + (2*
c^3*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*PolyLog[3, c*x])/3 - (2*c^3*
PolyLog[3, 1 - c*x])/3
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
```

```
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x^4}\,dx = -\frac{\log(1-cx)\mathrm{Li}_2(cx)}{3x^3} - \frac{1}{3}\int \frac{\log^2(1-cx)}{x^4}\,dx - \frac{1}{3}c\int \left(\frac{\mathrm{Li}_2(cx)}{x^3} + \frac{c\mathrm{Li}_2(cx)}{x^2} + \frac{c^2\mathrm{Li}_2(cx)}{x} - \frac{c^3\mathrm{Li}\ldots}{-1\ldots}\right.$$

$$= \frac{\log^2(1-cx)}{9x^3} - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{3x^3} + \frac{1}{9}(2c)\int \frac{\log(1-cx)}{x^3(1-cx)}\,dx - \frac{1}{3}c\int \frac{\mathrm{Li}_2(cx)}{x^3}\,dx - \frac{1}{3}c^2\int \frac{\mathrm{Li}_2\ldots}{x\ldots}$$

$$= \frac{\log^2(1-cx)}{9x^3} + \frac{c\mathrm{Li}_2(cx)}{6x^2} + \frac{c^2\mathrm{Li}_2(cx)}{3x} + \frac{1}{3}c^3\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{3x^3} - \frac{1}{3}c^3\mathrm{Li}_3\ldots$$

$$= -\frac{c\log(1-cx)}{12x^2} - \frac{c^2\log(1-cx)}{3x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3\log(cx)\log^2(1-cx) + \frac{c\mathrm{Li}_2(cx)}{6x^2} + \frac{c^2\mathrm{Li}_2\ldots}{3x\ldots}$$

$$= -\frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} + \frac{\log^2(1-cx)}{9x^3} + \frac{1}{3}c^3\log(cx)\log^2(1-cx) - \frac{2}{9}c^3\mathrm{Li}_2(cx) + \ldots$$

$$= \frac{c^2}{12x} - \frac{5}{12}c^3\log(x) + \frac{5}{12}c^3\log(1-cx) - \frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} - \frac{1}{9}c^3\log^2(1-cx) + \ldots$$

$$= \frac{7c^2}{36x} - \frac{3}{4}c^3\log(x) + \frac{3}{4}c^3\log(1-cx) - \frac{7c\log(1-cx)}{36x^2} - \frac{5c^2\log(1-cx)}{9x} - \frac{1}{9}c^3\log^2(1-cx) + \ldots$$

Mathematica [A] time = 0.23515, size = 246, normalized size = 1.

$$-12c^3x^3\text{PolyLog}(3,cx)-24c^3x^3\text{PolyLog}(3,1-cx)+8c^3x^3(3\log(1-cx)+1)\text{PolyLog}(2,1-cx)+6\left(2\left(c^3x^3-1\right)\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[x] - 12*c^3*x^3*Log[c*x] - 7*c*x*Lo
g[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] + 27*c^3*x^3*Log[1 - c*x] + 8*c^3*x^3*
Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]^2 - 4*c^3*x^3*Log[1 - c*x]^2 + 12*c^
3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*x*(1 + 2*c*x) + 2*(-1 + c^3*x^3)*Log[1
 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3*(1 + 3*Log[1 - c*x])*PolyLog[2, 1 - c*
x] - 12*c^3*x^3*PolyLog[3, c*x] - 24*c^3*x^3*PolyLog[3, 1 - c*x])/(36*x^3)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

Maxima [A] time = 1.65255, size = 254, normalized size = 1.04

$$\frac{1}{3}\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)c^3+\frac{2}{9}\left(\log(cx)\log(-cx+1)+\mathrm{Li}_2(-cx+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^3 + 2/9*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^3 - 3
/4*c^3*log(x) - 1/3*c^3*polylog(3, c*x) + 1/36*(7*c^2*x^2 - 4*(c^3*x^3 - 1)
```

```
*log(-c*x + 1)^2 + 6*(2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1))*dilo
g(c*x) + (27*c^3*x^3 - 20*c^2*x^2 - 7*c*x)*log(-c*x + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\,(cx)\log\,(-cx+1)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\,(cx)\log\,(-cx+1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.169 $\int \frac{\log(1-cx)\mathbf{PolyLog}(2,cx)}{x^5}\,dx$

Optimal. Leaf size=287

$$\frac{c^2\text{PolyLog}(2,cx)}{8x^2}-\frac{1}{8}c^4\text{PolyLog}(2,cx)-\frac{1}{4}c^4\text{PolyLog}(3,cx)-\frac{1}{2}c^4\text{PolyLog}(3,1-cx)+\frac{c^3\text{PolyLog}(2,cx)}{4x}+\frac{1}{4}c^4\log($$

```
[Out] (5*c^2)/(144*x^2) + (7*c^3)/(36*x) - (41*c^4*Log[x])/72 + (41*c^4*Log[1 - c
*x])/72 - (5*c*Log[1 - c*x])/(72*x^3) - (c^2*Log[1 - c*x])/(8*x^2) - (3*c^3
*Log[1 - c*x])/(8*x) - (c^4*Log[1 - c*x]^2)/16 + Log[1 - c*x]^2/(16*x^4) +
(c^4*Log[c*x]*Log[1 - c*x]^2)/4 - (c^4*PolyLog[2, c*x])/8 + (c*PolyLog[2, c
*x])/(12*x^3) + (c^2*PolyLog[2, c*x])/(8*x^2) + (c^3*PolyLog[2, c*x])/(4*x)
 + (c^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(4
*x^4) + (c^4*Log[1 - c*x]*PolyLog[2, 1 - c*x])/2 - (c^4*PolyLog[3, c*x])/4
- (c^4*PolyLog[3, 1 - c*x])/2
```

Rubi [A] time = 0.449191, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.062, Rules used = {6591, 2395, 44, 6603, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$\frac{c^2\text{PolyLog}(2,cx)}{8x^2}-\frac{1}{8}c^4\text{PolyLog}(2,cx)-\frac{1}{4}c^4\text{PolyLog}(3,cx)-\frac{1}{2}c^4\text{PolyLog}(3,1-cx)+\frac{c^3\text{PolyLog}(2,cx)}{4x}+\frac{1}{4}c^4\log($$

Antiderivative was successfully verified.

```
[In] Int[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```



```
[Out] (5*c^2)/(144*x^2) + (7*c^3)/(36*x) - (41*c^4*Log[x])/72 + (41*c^4*Log[1 - c
*x])/72 - (5*c*Log[1 - c*x])/(72*x^3) - (c^2*Log[1 - c*x])/(8*x^2) - (3*c^3
*Log[1 - c*x])/(8*x) - (c^4*Log[1 - c*x]^2)/16 + Log[1 - c*x]^2/(16*x^4) +
(c^4*Log[c*x]*Log[1 - c*x]^2)/4 - (c^4*PolyLog[2, c*x])/8 + (c*PolyLog[2, c
*x])/(12*x^3) + (c^2*PolyLog[2, c*x])/(8*x^2) + (c^3*PolyLog[2, c*x])/(4*x)
 + (c^4*Log[1 - c*x]*PolyLog[2, c*x])/4 - (Log[1 - c*x]*PolyLog[2, c*x])/(4
*x^4) + (c^4*Log[1 - c*x]*PolyLog[2, 1 - c*x])/2 - (c^4*PolyLog[3, c*x])/4
- (c^4*PolyLog[3, 1 - c*x])/2
```


Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
```

```
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x^5}\,dx = -\frac{\log(1-cx)\mathrm{Li}_2(cx)}{4x^4} - \frac{1}{4}\int \frac{\log^2(1-cx)}{x^5}\,dx - \frac{1}{4}c\int\left(\frac{\mathrm{Li}_2(cx)}{x^4} + \frac{c\mathrm{Li}_2(cx)}{x^3} + \frac{c^2\mathrm{Li}_2(cx)}{x^2} + \frac{c^3\mathrm{Li}\right.$$

$$= \frac{\log^2(1-cx)}{16x^4} - \frac{\log(1-cx)\mathrm{Li}_2(cx)}{4x^4} + \frac{1}{8}c\int \frac{\log(1-cx)}{x^4(1-cx)}\,dx - \frac{1}{4}c\int \frac{\mathrm{Li}_2(cx)}{x^4}\,dx - \frac{1}{4}c^2\int \frac{\mathrm{Li}_2(cx}{x^3}$$

$$= \frac{\log^2(1-cx)}{16x^4} + \frac{c\mathrm{Li}_2(cx)}{12x^3} + \frac{c^2\mathrm{Li}_2(cx)}{8x^2} + \frac{c^3\mathrm{Li}_2(cx)}{4x} + \frac{1}{4}c^4\log(1-cx)\mathrm{Li}_2(cx) - \frac{\log(1-cx)\mathrm{Li}_2(}{4x^4}$$

$$= -\frac{c\log(1-cx)}{36x^3} - \frac{c^2\log(1-cx)}{16x^2} - \frac{c^3\log(1-cx)}{4x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4\log(cx)\log^2(1-cx) +$$

$$= -\frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-cx)}{8x^2} - \frac{3c^3\log(1-cx)}{8x} + \frac{\log^2(1-cx)}{16x^4} + \frac{1}{4}c^4\log(cx)\log^2(1-cx)$$

$$= \frac{c^2}{72x^2} + \frac{13c^3}{144x} - \frac{49}{144}c^4\log(x) + \frac{49}{144}c^4\log(1-cx) - \frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-cx)}{8x^2} - \frac{3c^3\log(}{8}$$

$$= \frac{5c^2}{144x^2} + \frac{7c^3}{36x} - \frac{41}{72}c^4\log(x) + \frac{41}{72}c^4\log(1-cx) - \frac{5c\log(1-cx)}{72x^3} - \frac{c^2\log(1-cx)}{8x^2} - \frac{3c^3\log(1}{8x}$$

Mathematica [A] time = 0.235598, size = 277, normalized size = 0.97

$$\frac{-36c^4x^4\text{PolyLog}(3,cx)-72c^4x^4\text{PolyLog}(3,1-cx)+18c^4x^4(4\log(1-cx)+1)\text{PolyLog}(2,1-cx)+6\left(cx\left(6c^2x^2+3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] (5*c^2*x^2 + 28*c^3*x^3 - 18*c^4*x^4 - 49*c^4*x^4*Log[x] - 33*c^4*x^4*Log[c
*x] - 10*c*x*Log[1 - c*x] - 18*c^2*x^2*Log[1 - c*x] - 54*c^3*x^3*Log[1 - c*
x] + 82*c^4*x^4*Log[1 - c*x] + 18*c^4*x^4*Log[c*x]*Log[1 - c*x] + 9*Log[1 -
 c*x]^2 - 9*c^4*x^4*Log[1 - c*x]^2 + 36*c^4*x^4*Log[c*x]*Log[1 - c*x]^2 + 6
*(c*x*(2 + 3*c*x + 6*c^2*x^2) + 6*(-1 + c^4*x^4)*Log[1 - c*x])*PolyLog[2, c
*x] + 18*c^4*x^4*(1 + 4*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 36*c^4*x^4*Poly
Log[3, c*x] - 72*c^4*x^4*PolyLog[3, 1 - c*x])/(144*x^4)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln(-cx+1)\,\text{polylog}(2,cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

```
[Out] int(ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

Maxima [A] time = 1.73984, size = 289, normalized size = 1.01

$$\frac{1}{4}\left(\log(cx)\log(-cx+1)^2+2\,\text{Li}_2(-cx+1)\log(-cx+1)-2\,\text{Li}_3(-cx+1)\right)c^4+\frac{1}{8}\left(\log(cx)\log(-cx+1)+\text{Li}_2(-cx+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylog
(3, -c*x + 1))*c^4 + 1/8*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*c^4 - 4
```

```
1/72*c^4*log(x) - 1/4*c^4*polylog(3, c*x) + 1/144*(28*c^3*x^3 + 5*c^2*x^2 -
 9*(c^4*x^4 - 1)*log(-c*x + 1)^2 + 6*(6*c^3*x^3 + 3*c^2*x^2 + 2*c*x + 6*(c^
4*x^4 - 1)*log(-c*x + 1))*dilog(c*x) + 2*(41*c^4*x^4 - 27*c^3*x^3 - 9*c^2*x
^2 - 5*c*x)*log(-c*x + 1))/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{Li}_2\left(cx\right)\log\left(-cx+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{Li}_2\left(cx\right)\log\left(-cx+1\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate(dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.170 $\int x^2(g + h\log(1 - cx))\mathbf{PolyLog}(2, cx)\,dx$

Optimal. Leaf size=423

$$-\frac{hx\text{PolyLog}(2,cx)}{3c^2}+\frac{2h\text{PolyLog}(3,1-cx)}{3c^3}-\frac{h\log(1-cx)\text{PolyLog}(2,cx)}{3c^3}-\frac{2h\log(1-cx)\text{PolyLog}(2,1-cx)}{3c^3}+\frac{1}{3}x^3$$

```
[Out] (121*h*x)/(108*c^2) + (13*h*x^2)/(216*c) + (h*x^3)/81 + (h*(1 - c*x)^2)/(6*
c^3) - (2*h*(1 - c*x)^3)/(81*c^3) + (13*h*Log[1 - c*x])/(108*c^3) - (h*x^2*
Log[1 - c*x])/(12*c) - (h*x^3*Log[1 - c*x])/27 + (h*(1 - c*x)*Log[1 - c*x])
/(3*c^3) + (h*Log[1 - c*x]^2)/(9*c^3) - (h*Log[c*x]*Log[1 - c*x]^2)/(3*c^3)
 + (x^3*Log[1 - c*x]*(g + h*Log[1 - c*x]))/9 + ((1 - c*x)*(g + 2*h*Log[1 -
c*x]))/(3*c^3) - ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/(6*c^3) + ((1 - c*x)^
3*(g + 2*h*Log[1 - c*x]))/(27*c^3) - (Log[1 - c*x]*(g + 2*h*Log[1 - c*x]))/
(9*c^3) - (h*x*PolyLog[2, c*x])/(3*c^2) - (h*x^2*PolyLog[2, c*x])/(6*c) - (
h*x^3*PolyLog[2, c*x])/9 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3) + (x^3*
(g + h*Log[1 - c*x])*PolyLog[2, c*x])/3 - (2*h*Log[1 - c*x]*PolyLog[2, 1 -
c*x])/(3*c^3) + (2*h*PolyLog[3, 1 - c*x])/(3*c^3)
```

Rubi [A] time = 0.611175, antiderivative size = 366, normalized size of antiderivative = 0.87, number of steps used = 37, number of rules used = 20, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {6603, 2439, 2410, 2389, 2295, 2395, 43, 2390, 2301, 2411, 2334, 12, 14, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$-\frac{hx\text{PolyLog}(2,cx)}{3c^2}+\frac{2h\text{PolyLog}(3,1-cx)}{3c^3}-\frac{h\log(1-cx)\text{PolyLog}(2,cx)}{3c^3}-\frac{2h\log(1-cx)\text{PolyLog}(2,1-cx)}{3c^3}+\frac{1}{3}x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (107*h*x)/(108*c^2) + (23*h*x^2)/(216*c) + (2*h*x^3)/81 + (h*(1 - c*x)^2)/(
12*c^3) - (h*(1 - c*x)^3)/(81*c^3) + (23*h*Log[1 - c*x])/(108*c^3) - (5*h*x
^2*Log[1 - c*x])/(36*c) - (2*h*x^3*Log[1 - c*x])/27 + (4*h*(1 - c*x)*Log[1
- c*x])/(9*c^3) - (h*Log[c*x]*Log[1 - c*x]^2)/(3*c^3) + (x^3*Log[1 - c*x]*(
g + h*Log[1 - c*x]))/9 + (((18*(1 - c*x))/c^3 - (9*(1 - c*x)^2)/c^3 + (2*(1
 - c*x)^3)/c^3 - (6*Log[1 - c*x])/c^3)*(g + h*Log[1 - c*x]))/54 - (h*x*Poly
Log[2, c*x])/(3*c^2) - (h*x^2*PolyLog[2, c*x])/(6*c) - (h*x^3*PolyLog[2, c*
x])/9 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(3*c^3) + (x^3*(g + h*Log[1 - c*x]
)*PolyLog[2, c*x])/3 - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + (2*
h*PolyLog[3, 1 - c*x])/(3*c^3)
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
```

```
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
```

```
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(g+h\log(1-cx))\mathrm{Li}_2(cx)\,dx = \frac{1}{3}x^3(g+h\log(1-cx))\mathrm{Li}_2(cx)+\frac{1}{3}\int x^2\log(1-cx)(g+h\log(1-cx))\,dx+\frac{1}{3}(ch)\int$$

$$=\frac{1}{9}x^3\log(1-cx)(g+h\log(1-cx))+\frac{1}{3}x^3(g+h\log(1-cx))\mathrm{Li}_2(cx)+\frac{1}{9}c\int\frac{x^3(g+}{}$$

$$=\frac{1}{9}x^3\log(1-cx)(g+h\log(1-cx))-\frac{hx\mathrm{Li}_2(cx)}{3c^2}-\frac{hx^2\mathrm{Li}_2(cx)}{6c}-\frac{1}{9}hx^3\mathrm{Li}_2(cx)-\frac{h\log}{}$$

$$=-\frac{hx^2\log(1-cx)}{12c}-\frac{1}{27}hx^3\log(1-cx)-\frac{h\log(cx)\log^2(1-cx)}{3c^3}+\frac{1}{9}x^3\log(1-cx)(g$$

$$=\frac{hx}{3c^2}-\frac{5hx^2\log(1-cx)}{36c}-\frac{2}{27}hx^3\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{3c^3}-\frac{h\log(cx)\log}{3c^3}$$

$$=\frac{61hx}{108c^2}+\frac{13hx^2}{216c}+\frac{hx^3}{81}+\frac{13h\log(1-cx)}{108c^3}-\frac{5hx^2\log(1-cx)}{36c}-\frac{2}{27}hx^3\log(1-cx)+$$

$$=\frac{107hx}{108c^2}+\frac{23hx^2}{216c}+\frac{2hx^3}{81}+\frac{h(1-cx)^2}{12c^3}-\frac{h(1-cx)^3}{81c^3}+\frac{23h\log(1-cx)}{108c^3}-\frac{5hx^2\log(1}{36c}$$

$$=\frac{107hx}{108c^2}+\frac{23hx^2}{216c}+\frac{2hx^3}{81}+\frac{h(1-cx)^2}{12c^3}-\frac{h(1-cx)^3}{81c^3}+\frac{23h\log(1-cx)}{108c^3}-\frac{5hx^2\log(1}{36c}$$

Mathematica [A] time = 0.46092, size = 252, normalized size = 0.6

$$\frac{g\left(18c^3x^3\mathrm{PolyLog}(2,cx)-cx\left(2c^2x^2+3cx+6\right)+6\left(c^3x^3-1\right)\log(1-cx)\right)}{54c^3}+\frac{h\left(12\left(6\left(c^3x^3-1\right)\log(1-cx)-cx\left(2c^2x^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (g*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Log[1 - c*x] + 18*c^3
*x^3*PolyLog[2, c*x]))/(54*c^3) + (h*(186*c*x + 33*c^2*x^2 + 8*c^3*x^3 + 18
6*Log[1 - c*x] - 120*c*x*Log[1 - c*x] - 42*c^2*x^2*Log[1 - c*x] - 24*c^3*x^
3*Log[1 - c*x] - 24*Log[1 - c*x]^2 + 24*c^3*x^3*Log[1 - c*x]^2 - 72*Log[c*x
]*Log[1 - c*x]^2 + 12*(-(c*x*(6 + 3*c*x + 2*c^2*x^2)) + 6*(-1 + c^3*x^3)*Lo
g[1 - c*x])*PolyLog[2, c*x] - 144*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 144*Po
lyLog[3, 1 - c*x]))/(216*c^3)
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int x^2 \left(g + h \ln\left(-cx + 1\right)\right) \operatorname{polylog}\left(2, cx\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int(x^2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{18} h \left(\frac{\left(2 c^3 x^3 + 3 c^2 x^2 + 6 cx - 6 \left(c^3 x^3 - 1\right) \log\left(-cx + 1\right)\right) \mathrm{Li}_2\left(cx\right)}{c^3} - \frac{\frac{4}{9} c^3 x^3 - \frac{1}{9} \left(6 x^3 \log\left(-cx + 1\right) - c\left(\frac{2 c^2 x^3 + 3 cx^2 + 6 x}{c^3} + \frac{6}{}\right.\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/18*h*((2*c^3*x^3 + 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 - 1)*log(-c*x + 1))*di
log(c*x)/c^3 - integrate((6*(c^3*x^3 - 1)*log(-c*x + 1)^2 - (2*c^3*x^3 + 3*
c^2*x^2 + 6*c*x)*log(-c*x + 1))/x, x)/c^3) + 1/54*(18*c^3*x^3*dilog(c*x) -
2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*g/c^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(hx^2\text{Li}_2\left(cx\right)\log\left(-cx+1\right)+gx^2\text{Li}_2\left(cx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*x^2*dilog(c*x)*log(-c*x + 1) + g*x^2*dilog(c*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(h\log\left(-cx+1\right)+g\right)x^2\text{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*x^2*dilog(c*x), x)
```

3.171 $\int x(g + h\log(1 - cx))\textbf{PolyLog}(2, cx)\,dx$

Optimal. Leaf size=330

$$\frac{h\text{PolyLog}(3,1-cx)}{c^2} - \frac{h\log(1-cx)\text{PolyLog}(2,cx)}{2c^2} - \frac{h\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2} + \frac{1}{2}x^2\text{PolyLog}(2,cx)(h\log(1-c$$

```
[Out] (13*h*x)/(8*c) + (h*x^2)/16 + (h*(1 - c*x)^2)/(8*c^2) + (h*Log[1 - c*x])/(8
*c^2) - (h*x^2*Log[1 - c*x])/8 + (h*(1 - c*x)*Log[1 - c*x])/(2*c^2) + (h*Lo
g[1 - c*x]^2)/(4*c^2) - (h*Log[c*x]*Log[1 - c*x]^2)/(2*c^2) + (x^2*Log[1 -
c*x]*(g + h*Log[1 - c*x]))/4 + ((1 - c*x)*(g + 2*h*Log[1 - c*x]))/(2*c^2) -
 ((1 - c*x)^2*(g + 2*h*Log[1 - c*x]))/(8*c^2) - (Log[1 - c*x]*(g + 2*h*Log[
1 - c*x]))/(4*c^2) - (h*x*PolyLog[2, c*x])/(2*c) - (h*x^2*PolyLog[2, c*x])/
4 - (h*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x^2*(g + h*Log[1 - c*x])*Po
lyLog[2, c*x])/2 - (h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + (h*PolyLog[3,
 1 - c*x])/c^2
```

Rubi [A] time = 0.44859, antiderivative size = 287, normalized size of antiderivative = 0.87, number of steps used = 30, number of rules used = 20, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {6603, 2439, 2410, 2389, 2295, 2395, 43, 2390, 2301, 2411, 2334, 12, 14, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$\frac{h\text{PolyLog}(3,1-cx)}{c^2} - \frac{h\log(1-cx)\text{PolyLog}(2,cx)}{2c^2} - \frac{h\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2} + \frac{1}{2}x^2\text{PolyLog}(2,cx)(h\log(1-c$$

Antiderivative was successfully verified.

```
[In] Int[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (3*h*x)/(2*c) + (h*x^2)/8 + (h*(1 - c*x)^2)/(16*c^2) + (h*Log[1 - c*x])/(4*
c^2) - (h*x^2*Log[1 - c*x])/4 + (3*h*(1 - c*x)*Log[1 - c*x])/(4*c^2) - (h*L
og[c*x]*Log[1 - c*x]^2)/(2*c^2) + (x^2*Log[1 - c*x]*(g + h*Log[1 - c*x]))/4
 + (((4*(1 - c*x))/c^2 - (1 - c*x)^2/c^2 - (2*Log[1 - c*x])/c^2)*(g + h*Log
[1 - c*x]))/8 - (h*x*PolyLog[2, c*x])/(2*c) - (h*x^2*PolyLog[2, c*x])/4 - (
h*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + (x^2*(g + h*Log[1 - c*x])*PolyLog
[2, c*x])/2 - (h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2 + (h*PolyLog[3, 1 -
c*x])/c^2
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
```

```
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a
 + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /;
FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1
] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(g+h\log(1-cx))\text{Li}_2(cx)\,dx=\frac{1}{2}x^2(g+h\log(1-cx))\text{Li}_2(cx)+\frac{1}{2}\int x\log(1-cx)(g+h\log(1-cx))\,dx+\frac{1}{2}(ch)\int\Big($$

$$=\frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx))+\frac{1}{2}x^2(g+h\log(1-cx))\text{Li}_2(cx)+\frac{1}{4}c\int\frac{x^2(g+h\,l}{1-}$$

$$=\frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx))-\frac{hx\text{Li}_2(cx)}{2c}-\frac{1}{4}hx^2\text{Li}_2(cx)-\frac{h\log(1-cx)\text{Li}_2(cx)}{2c^2}$$

$$=-\frac{1}{8}hx^2\log(1-cx)-\frac{h\log(cx)\log^2(1-cx)}{2c^2}+\frac{1}{4}x^2\log(1-cx)(g+h\log(1-cx))+\frac{1}{8}\Big($$

$$=\frac{hx}{2c}-\frac{1}{4}hx^2\log(1-cx)+\frac{h(1-cx)\log(1-cx)}{2c^2}-\frac{h\log(cx)\log^2(1-cx)}{2c^2}+\frac{1}{4}x^2\log(1-$$

$$=\frac{7hx}{8c}+\frac{hx^2}{16}+\frac{h\log(1-cx)}{8c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-cx)\log(1-cx)}{4c^2}-\frac{h\log^2(1-}{8c^2}$$

$$=\frac{3hx}{2c}+\frac{hx^2}{8}+\frac{h(1-cx)^2}{16c^2}+\frac{h\log(1-cx)}{4c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-cx)\log(1-cx)}{4c^2}$$

$$=\frac{3hx}{2c}+\frac{hx^2}{8}+\frac{h(1-cx)^2}{16c^2}+\frac{h\log(1-cx)}{4c^2}-\frac{1}{4}hx^2\log(1-cx)+\frac{3h(1-cx)\log(1-cx)}{4c^2}$$

Mathematica [A] time = 0.339235, size = 211, normalized size = 0.64

$$\frac{g\left(4c^2x^2\text{PolyLog}(2,cx)+2\left(c^2x^2-1\right)\log(1-cx)-cx(cx+2)\right)}{8c^2}+\frac{h\left(\left(8\left(c^2x^2-1\right)\log(1-cx)-4cx(cx+2)\right)\text{PolyLog}(2,\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] (g*(-(c*x*(2 + c*x)) + 2*(-1 + c^2*x^2)*Log[1 - c*x] + 4*c^2*x^2*PolyLog[2,
 c*x]))/(8*c^2) + (h*(-14 + 22*c*x + 3*c^2*x^2 + 22*Log[1 - c*x] - 16*c*x*L
```

```
og[1 - c*x] - 6*c^2*x^2*Log[1 - c*x] - 4*Log[1 - c*x]^2 + 4*c^2*x^2*Log[1 -
 c*x]^2 - 8*Log[c*x]*Log[1 - c*x]^2 + (-4*c*x*(2 + c*x) + 8*(-1 + c^2*x^2)*
Log[1 - c*x])*PolyLog[2, c*x] - 16*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*Po
lyLog[3, 1 - c*x]))/(16*c^2)
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int x\left(g+h\ln\left(-cx+1\right)\right)\operatorname{polylog}\left(2,cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}h\left(\frac{\left(c^2x^2+2\,cx-2\left(c^2x^2-1\right)\log\left(-cx+1\right)\right)\mathrm{Li}_2\left(cx\right)}{c^2}-\frac{\frac{1}{2}\,c^2x^2-\frac{1}{4}\left(2\,x^2\log\left(-cx+1\right)-c\left(\frac{cx^2+2\,x}{c^2}+\frac{2\,\log(cx-1)}{c^3}\right)\right)c^2+\left(c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/4*h*((c^2*x^2 + 2*c*x - 2*(c^2*x^2 - 1)*log(-c*x + 1))*dilog(c*x)/c^2 -
integrate((2*(c^2*x^2 - 1)*log(-c*x + 1)^2 - (c^2*x^2 + 2*c*x)*log(-c*x + 1
))/x, x)/c^2) + 1/8*(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 -
1)*log(-c*x + 1))*g/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(hx\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right)+gx\mathrm{Li}_2\left(cx\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*x*dilog(c*x)*log(-c*x + 1) + g*x*dilog(c*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(h\log\left(-cx+1\right)+g\right)x\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*x*dilog(c*x), x)
```

3.172 $\int (g+h\log(1-cx))\mathbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=167

$$x\text{PolyLog}(2,cx)(h\log(1-cx)+g)-hx\text{PolyLog}(2,cx)+\frac{2h\text{PolyLog}(3,1-cx)}{c}-\frac{h\log(1-cx)\text{PolyLog}(2,cx)}{c}-\frac{2h\log}{}$$

```
[Out] -(g*x) + 3*h*x - (g*(1 - c*x)*Log[1 - c*x])/c + (3*h*(1 - c*x)*Log[1 - c*x]
)/c - (h*(1 - c*x)*Log[1 - c*x]^2)/c - (h*Log[c*x]*Log[1 - c*x]^2)/c - h*x*
PolyLog[2, c*x] - (h*Log[1 - c*x]*PolyLog[2, c*x])/c + x*(g + h*Log[1 - c*x
])*PolyLog[2, c*x] - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*h*PolyLo
g[3, 1 - c*x])/c
```

Rubi [A] time = 0.210957, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6600, 2364, 2360, 2295, 2296, 6688, 6742, 6586, 2389, 6596, 2396, 2433, 2374, 6589}

$$x\text{PolyLog}(2,cx)(h\log(1-cx)+g)-hx\text{PolyLog}(2,cx)+\frac{2h\text{PolyLog}(3,1-cx)}{c}-\frac{h\log(1-cx)\text{PolyLog}(2,cx)}{c}-\frac{2h\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```



```
[Out] -(g*x) + 3*h*x - (g*(1 - c*x)*Log[1 - c*x])/c + (3*h*(1 - c*x)*Log[1 - c*x]
)/c - (h*(1 - c*x)*Log[1 - c*x]^2)/c - (h*Log[c*x]*Log[1 - c*x]^2)/c - h*x*
PolyLog[2, c*x] - (h*Log[1 - c*x]*PolyLog[2, c*x])/c + x*(g + h*Log[1 - c*x
])*PolyLog[2, c*x] - (2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*h*PolyLo
g[3, 1 - c*x])/c
```


Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 2364

```
Int[((a_.) + Log[v_]*(b_.))^(p_.)*((c_.) + Log[v_]*(d_.))^(q_.), x_Symbol]
:> Dist[1/Coeff[v, x, 1], Subst[Int[(a + b*Log[x])^p*(c + d*Log[x])^q, x],
x, v], x] /; FreeQ[{a, b, c, d, p, q}, x] && LinearQ[v, x] && NeQ[Coeff[v,
x, 0], 0]
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (g+h\log(1-cx))\mathrm{Li}_2(cx)\,dx = x(g+h\log(1-cx))\mathrm{Li}_2(cx)+(ch)\int\left(-\frac{1}{c}-\frac{1}{c(-1+cx)}\right)\mathrm{Li}_2(cx)\,dx+\int\log(1-cx)(g+$$

$$= x(g+h\log(1-cx))\mathrm{Li}_2(cx)-\frac{\mathrm{Subst}(\int\log(x)(g+h\log(x))\,dx,x,1-cx)}{c}+(ch)\int\frac{x\mathrm{Li}_2}{1-}$$

$$= x(g+h\log(1-cx))\mathrm{Li}_2(cx)-\frac{\mathrm{Subst}\left(\int\left(g\log(x)+h\log^2(x)\right)\,dx,x,1-cx\right)}{c}+(ch)\int\Bigg($$

$$= x(g+h\log(1-cx))\mathrm{Li}_2(cx)-\frac{g\,\mathrm{Subst}(\int\log(x)\,dx,x,1-cx)}{c}-h\int\mathrm{Li}_2(cx)\,dx-h\int\frac{\mathrm{L}}{-}$$

$$= -gx-\frac{g(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-hx\mathrm{Li}_2(cx)-\frac{h\log(1-cx)\mathrm{Li}_2(cx}{c}$$

$$= -gx+2hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{2h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h}{-}$$

$$= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h}{-}$$

$$= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h}{-}$$

$$= -gx+3hx-\frac{g(1-cx)\log(1-cx)}{c}+\frac{3h(1-cx)\log(1-cx)}{c}-\frac{h(1-cx)\log^2(1-cx)}{c}-\frac{h}{-}$$

Mathematica [A] time = 0.0682036, size = 149, normalized size = 0.89

$$g\left(x\mathrm{PolyLog}(2,cx)+\left(x-\frac{1}{c}\right)\log(1-cx)-x\right)+\frac{h\left(2\mathrm{PolyLog}(3,1-cx)-2\log(1-cx)\mathrm{PolyLog}(2,1-cx)+((cx-1)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*Log[1 - c*x])*PolyLog[2, c*x],x]
```

```
[Out] g*(-x + (-c^(-1) + x)*Log[1 - c*x] + x*PolyLog[2, c*x]) + (h*(-2 + 3*c*x +
3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*x]^2 + c*x*Log[1 - c*x]^2 -
 Log[c*x]*Log[1 - c*x]^2 + (-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2, c*
x] - 2*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int\left(g+h\ln(-cx+1)\right)\mathrm{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-h\left(\frac{(cx-(cx-1)\log(-cx+1))\text{Li}_2(cx)}{c}-\frac{(cx-1)\left(\log(-cx+1)^2-2\log(-cx+1)+2\right)+cx-(cx-1)\log(-cx+1)-}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -h*((c*x - (c*x - 1)*log(-c*x + 1))*dilog(c*x)/c - integrate(-(c*x*log(-c*x
 + 1) - (c*x - 1)*log(-c*x + 1)^2)/x, x)/c) + (c*x*dilog(c*x) - c*x + (c*x
- 1)*log(-c*x + 1))*g/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(h\text{Li}_2(cx)\log(-cx+1)+g\text{Li}_2(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral(h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g+h\log(-cx+1)\right)\text{Li}_2(cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x),x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(h\log\left(-cx+1\right)+g\right)\mathrm{Li}_2\left(cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x), x)
```

3.173 $\int \frac{(g+h\log(1-cx))\textbf{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=20

$$g\text{PolyLog}(3,cx)-\frac{1}{2}h\text{PolyLog}(2,cx)^2$$

```
[Out] -(h*PolyLog[2, c*x]^2)/2 + g*PolyLog[3, c*x]
```

Rubi [A] time = 0.0575455, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6602, 6589, 6601}

$$g\text{PolyLog}(3,cx)-\frac{1}{2}h\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
```

```
[Out] -(h*PolyLog[2, c*x]^2)/2 + g*PolyLog[3, c*x]
```

Rule 6602

```
Int[((Log[1 + (e_.)*(x_)]*(h_.) + (g_))*PolyLog[2, (c_.)*(x_)])/(x_), x_Sym
bol] :> Dist[g, Int[PolyLog[2, c*x]/x, x], x] + Dist[h, Int[(Log[1 + e*x]*P
olyLog[2, c*x])/x, x], x] /; FreeQ[{c, e, g, h}, x] && EqQ[c + e, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{x}\,dx = g\int \frac{\text{Li}_2(cx)}{x}\,dx + h\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx$$
$$= -\frac{1}{2}h\text{Li}_2(cx)^2 + g\text{Li}_3(cx)$$

Mathematica [A] time = 0.0110358, size = 20, normalized size = 1.

$$g\text{PolyLog}(3,cx) - \frac{1}{2}h\text{PolyLog}(2,cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x,x]
```

```
[Out] -(h*PolyLog[2, c*x]^2)/2 + g*PolyLog[3, c*x]
```

Maple [A] time = 0.155, size = 19, normalized size = 1.

$$-\frac{h\left(\text{polylog}\left(2,cx\right)\right)^2}{2} + g\text{polylog}\left(3,cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)
```

```
[Out] -1/2*h*polylog(2,c*x)^2+g*polylog(3,c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left(-cx+1\right)+g\right)\text{Li}_2\left(cx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h\text{Li}_2(cx)\log(-cx+1)+g\text{Li}_2(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="fricas")
```

```
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(g+h\log(-cx+1))\,\text{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x,x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(h\log(-cx+1)+g)\text{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x, x)
```

3.174 $\int \frac{(g+h\log(1-cx))\mathbf{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}-2ch\text{PolyLog}\left(2,\frac{1}{1-cx}\right)-ch\text{PolyLog}(3,cx)-2ch\text{PolyLog}(3,1-cx)+ch\log(1-$$

```
[Out] c*h*Log[c*x]*Log[1 - c*x]^2 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/x + c*(g
+ 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)] + c*h*Log[1 - c*x]*PolyLog[2, c
*x] - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x - 2*c*h*PolyLog[2, (1 - c*x)
^(-1)] + 2*c*h*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*h*PolyLog[3, c*x] - 2*c
*h*PolyLog[3, 1 - c*x]
```

Rubi [A] time = 0.355162, antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {6603, 2439, 2410, 2391, 2390, 2301, 2411, 2344, 2316, 2315, 6589, 6596, 2396, 2433, 2374}

$$-\frac{\text{PolyLog}(2,cx)(h\log(1-cx)+g)}{x}-2ch\text{PolyLog}(2,cx)-ch\text{PolyLog}(3,cx)-2ch\text{PolyLog}(3,1-cx)+ch\log(1-cx)\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

```
[Out] c*g*Log[x] - (c*h*Log[1 - c*x]^2)/2 + c*h*Log[c*x]*Log[1 - c*x]^2 + (Log[1
- c*x]*(g + h*Log[1 - c*x]))/x - (c*(g + h*Log[1 - c*x])^2)/(2*h) - 2*c*h*P
olyLog[2, c*x] + c*h*Log[1 - c*x]*PolyLog[2, c*x] - ((g + h*Log[1 - c*x])*P
olyLog[2, c*x])/x + 2*c*h*Log[1 - c*x]*PolyLog[2, 1 - c*x] - c*h*PolyLog[3,
 c*x] - 2*c*h*PolyLog[3, 1 - c*x]
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
```

```
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x^2}\,dx = -\frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x} - (ch)\int\left(\frac{\mathrm{Li}_2(cx)}{x} - \frac{c\mathrm{Li}_2(cx)}{-1+cx}\right)dx - \int \frac{\log(1-cx)(g+}{x^2}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{x} - \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x} + c\int \frac{g+h\log(1-cx)}{x(1-cx)}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{x} + ch\log(1-cx)\mathrm{Li}_2(cx) - \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x} -$$

$$= ch\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx))}{x} + ch\log(1-cx)\mathrm{Li}_2(cx) - \frac{(g}{}$$

$$= cg\log(x) + ch\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx))}{x} - \frac{c(g+h\log(1-}{2h}$$

$$= cg\log(x) - \frac{1}{2}ch\log^2(1-cx) + ch\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx}{x}$$

$$= cg\log(x) - \frac{1}{2}ch\log^2(1-cx) + ch\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx}{x}$$

Mathematica [A] time = 0.172119, size = 150, normalized size = 0.96

$$\frac{g(-\mathrm{PolyLog}(2,cx) + cx\log(x) + (1-cx)\log(1-cx))}{x} + h\left(-c\mathrm{PolyLog}(3,cx) - 2c\mathrm{PolyLog}(3,1-cx) + \frac{(cx-1)\log(1-}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^2,x]
```

```
[Out] (g*(c*x*Log[x] + (1 - c*x)*Log[1 - c*x] - PolyLog[2, c*x]))/x + h*(2*c*Log[
c*x]*Log[1 - c*x] - c*Log[1 - c*x]^2 + Log[1 - c*x]^2/x + c*Log[c*x]*Log[1
- c*x]^2 + ((-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x + 2*c*(1 + Log[1 - c
```

```
*x])*PolyLog[2, 1 - c*x] - c*PolyLog[3, c*x] - 2*c*PolyLog[3, 1 - c*x])
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int \frac{\left(g + h\ln\left(-cx + 1\right)\right)\operatorname{polylog}\left(2, cx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(c\log\left(x\right) - \frac{\left(cx - 1\right)\log\left(-cx + 1\right) + \operatorname{Li}_2\left(cx\right)}{x}\right)g + h\int \frac{\operatorname{Li}_2\left(cx\right)\log\left(-cx + 1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

```
[Out] (c*log(x) - ((c*x - 1)*log(-c*x + 1) + dilog(c*x))/x)*g + h*integrate(dilog
(c*x)*log(-c*x + 1)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{h\operatorname{Li}_2\left(cx\right)\log\left(-cx + 1\right) + g\operatorname{Li}_2\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g + h\log(-cx + 1))\operatorname{Li}_2(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**2,x)
```

```
[Out] Integral((g + h*log(-c*x + 1))*polylog(2, c*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(h\log(-cx + 1) + g)\operatorname{Li}_2(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^2, x)
```

3.175 $\int \frac{(g+h\log(1-cx))\mathbf{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=266

$$-\frac{1}{2}c^2h\text{PolyLog}\left(2,\frac{1}{1-cx}\right)-\frac{1}{2}c^2h\text{PolyLog}(3,cx)-c^2h\text{PolyLog}(3,1-cx)+\frac{1}{2}c^2h\log(1-cx)\text{PolyLog}(2,cx)+c^2h\log($$

```
[Out] -(c^2*h*Log[x]) + (c^2*h*Log[1 - c*x])/2 - (c*h*Log[1 - c*x])/(2*x) + (c^2*
h*Log[c*x]*Log[1 - c*x]^2)/2 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/(4*x^2)
- (c*(1 - c*x)*(g + 2*h*Log[1 - c*x]))/(4*x) + (c^2*(g + 2*h*Log[1 - c*x])*
Log[1 - (1 - c*x)^(-1)])/4 + (c*h*PolyLog[2, c*x])/(2*x) + (c^2*h*Log[1 - c
*x]*PolyLog[2, c*x])/2 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(2*x^2) - (
c^2*h*PolyLog[2, (1 - c*x)^(-1)])/2 + c^2*h*Log[1 - c*x]*PolyLog[2, 1 - c*x
] - (c^2*h*PolyLog[3, c*x])/2 - c^2*h*PolyLog[3, 1 - c*x]
```

Rubi [A] time = 0.496284, antiderivative size = 278, normalized size of antiderivative = 1.05, number of steps used = 31, number of rules used = 22, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.1, Rules used = {6603, 2439, 2410, 2395, 36, 29, 31, 2391, 2390, 2301, 2411, 2347, 2344, 2316, 2315, 2314, 6591, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}c^2h\text{PolyLog}(2,cx)-\frac{1}{2}c^2h\text{PolyLog}(3,cx)-c^2h\text{PolyLog}(3,1-cx)+\frac{1}{2}c^2h\log(1-cx)\text{PolyLog}(2,cx)+c^2h\log(1-c$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (c^2*g*Log[x])/4 - c^2*h*Log[x] + (3*c^2*h*Log[1 - c*x])/4 - (3*c*h*Log[1 -
 c*x])/(4*x) - (c^2*h*Log[1 - c*x]^2)/8 + (c^2*h*Log[c*x]*Log[1 - c*x]^2)/2
 - (c*(1 - c*x)*(g + h*Log[1 - c*x]))/(4*x) + (Log[1 - c*x]*(g + h*Log[1 -
c*x]))/(4*x^2) - (c^2*(g + h*Log[1 - c*x])^2)/(8*h) - (c^2*h*PolyLog[2, c*x
])/2 + (c*h*PolyLog[2, c*x])/(2*x) + (c^2*h*Log[1 - c*x]*PolyLog[2, c*x])/2
 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(2*x^2) + c^2*h*Log[1 - c*x]*Poly
Log[2, 1 - c*x] - (c^2*h*PolyLog[3, c*x])/2 - c^2*h*PolyLog[3, 1 - c*x]
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
```

, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{x^3}\,dx = -\frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{2x^2} - \frac{1}{2}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^3}\,dx - \frac{1}{2}(ch)\int\Bigg(\frac{\mathrm{Li}_2(}{x^2}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{4x^2} - \frac{(g+h\log(1-cx))\mathrm{Li}_2(cx)}{2x^2} + \frac{1}{4}c\int\frac{g+h\log(1-cx)}{x^2(1-cx)}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{4x^2} + \frac{ch\mathrm{Li}_2(cx)}{2x} + \frac{1}{2}c^2h\log(1-cx)\mathrm{Li}_2(cx) - \frac{(g+h\log(1-}{2x}$$

$$= -\frac{ch\log(1-cx)}{2x} + \frac{1}{2}c^2h\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(1-cx))}{4x^2} + \frac{ch\mathrm{Li}_2(}{2x}$$

$$= -\frac{3ch\log(1-cx)}{4x} + \frac{1}{2}c^2h\log(cx)\log^2(1-cx) - \frac{c(1-cx)(g+h\log(1-cx))}{4x} + \frac{\log(1-}{}$$

$$= \frac{1}{4}c^2g\log(x) - \frac{3}{4}c^2h\log(x) + \frac{1}{2}c^2h\log(1-cx) - \frac{3ch\log(1-cx)}{4x} - \frac{1}{8}c^2h\log^2(1-cx) +$$

$$= \frac{1}{4}c^2g\log(x) - c^2h\log(x) + \frac{3}{4}c^2h\log(1-cx) - \frac{3ch\log(1-cx)}{4x} - \frac{1}{8}c^2h\log^2(1-cx) + \frac{1}{2}$$

Mathematica [A] time = 0.256103, size = 238, normalized size = 0.89

$$\frac{g\left(-2\mathrm{PolyLog}(2,cx)+c^2x^2\log(x)-c^2x^2\log(1-cx)-cx+\log(1-cx)\right)}{4x^2} + \frac{1}{4}h\Bigg(\frac{2\left(\left(c^2x^2-1\right)\log(1-cx)+cx\right)\mathrm{PolyLog}(}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (g*(-(c*x) + c^2*x^2*Log[x] + Log[1 - c*x] - c^2*x^2*Log[1 - c*x] - 2*PolyL
og[2, c*x]))/(4*x^2) + (h*(-2*c^2*Log[x] - 2*c^2*Log[c*x] + 4*c^2*Log[1 - c
*x] - (4*c*Log[1 - c*x])/x + 2*c^2*Log[c*x]*Log[1 - c*x] - c^2*Log[1 - c*x]
^2 + Log[1 - c*x]^2/x^2 + 2*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(c*x + (-1 + c
^2*x^2)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*c^2*(1 + 2*Log[1 - c*x])*Pol
yLog[2, 1 - c*x] - 2*c^2*PolyLog[3, c*x] - 4*c^2*PolyLog[3, 1 - c*x]))/4
```

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int \frac{\left(g + h\ln\left(-cx+1\right)\right)\text{polylog}\left(2, cx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(c^2\log\left(x\right) - \frac{cx + \left(c^2x^2 - 1\right)\log\left(-cx+1\right) + 2\,\text{Li}_2\left(cx\right)}{x^2}\right)g + h\int \frac{\text{Li}_2\left(cx\right)\log\left(-cx+1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*log(x) - (c*x + (c^2*x^2 - 1)*log(-c*x + 1) + 2*dilog(c*x))/x^2)*g
 + h*integrate(dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{h\text{Li}_2\left(cx\right)\log\left(-cx+1\right) + g\text{Li}_2\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left(-cx+1\right)+g\right)\mathrm{Li}_2\left(cx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^3, x)
```

3.176 $\int \frac{(g+h\log(1-cx))\mathbf{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=340

$$-\frac{2}{9}c^3h\text{PolyLog}\left(2,\frac{1}{1-cx}\right)-\frac{1}{3}c^3h\text{PolyLog}(3,cx)-\frac{2}{3}c^3h\text{PolyLog}(3,1-cx)+\frac{c^2h\text{PolyLog}(2,cx)}{3x}+\frac{1}{3}c^3h\log(1-cx)$$

```
[Out] (7*c^2*h)/(36*x) - (3*c^3*h*Log[x])/4 + (19*c^3*h*Log[1 - c*x])/36 - (c*h*L
og[1 - c*x])/(12*x^2) - (c^2*h*Log[1 - c*x])/(3*x) + (c^3*h*Log[c*x]*Log[1
- c*x]^2)/3 + (Log[1 - c*x]*(g + h*Log[1 - c*x]))/(9*x^3) - (c*(g + 2*h*Log
[1 - c*x]))/(18*x^2) - (c^2*(1 - c*x)*(g + 2*h*Log[1 - c*x]))/(9*x) + (c^3*
(g + 2*h*Log[1 - c*x])*Log[1 - (1 - c*x)^(-1)])/9 + (c*h*PolyLog[2, c*x])/(
6*x^2) + (c^2*h*PolyLog[2, c*x])/(3*x) + (c^3*h*Log[1 - c*x]*PolyLog[2, c*x
])/3 - ((g + h*Log[1 - c*x])*PolyLog[2, c*x])/(3*x^3) - (2*c^3*h*PolyLog[2,
 (1 - c*x)^(-1)])/9 + (2*c^3*h*Log[1 - c*x]*PolyLog[2, 1 - c*x])/3 - (c^3*h
*PolyLog[3, c*x])/3 - (2*c^3*h*PolyLog[3, 1 - c*x])/3
```

Rubi [A] time = 0.650804, antiderivative size = 351, normalized size of antiderivative = 1.03, number of steps used = 42, number of rules used = 24, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.2, Rules used = {6603, 2439, 2410, 2395, 44, 36, 29, 31, 2391, 2390, 2301, 2411, 2347, 2344, 2316, 2315, 2314, 2319, 6591, 6589, 6596, 2396, 2433, 2374}

$$-\frac{2}{9}c^3h\text{PolyLog}(2,cx)-\frac{1}{3}c^3h\text{PolyLog}(3,cx)-\frac{2}{3}c^3h\text{PolyLog}(3,1-cx)+\frac{c^2h\text{PolyLog}(2,cx)}{3x}+\frac{1}{3}c^3h\log(1-cx)\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*c^2*h)/(36*x) + (c^3*g*Log[x])/9 - (3*c^3*h*Log[x])/4 + (23*c^3*h*Log[1
- c*x])/36 - (5*c*h*Log[1 - c*x])/(36*x^2) - (4*c^2*h*Log[1 - c*x])/(9*x) -
 (c^3*h*Log[1 - c*x]^2)/18 + (c^3*h*Log[c*x]*Log[1 - c*x]^2)/3 - (c*(g + h*
Log[1 - c*x]))/(18*x^2) - (c^2*(1 - c*x)*(g + h*Log[1 - c*x]))/(9*x) + (Log
[1 - c*x]*(g + h*Log[1 - c*x]))/(9*x^3) - (c^3*(g + h*Log[1 - c*x])^2)/(18*
h) - (2*c^3*h*PolyLog[2, c*x])/9 + (c*h*PolyLog[2, c*x])/(6*x^2) + (c^2*h*P
olyLog[2, c*x])/(3*x) + (c^3*h*Log[1 - c*x]*PolyLog[2, c*x])/3 - ((g + h*Lo
g[1 - c*x])*PolyLog[2, c*x])/(3*x^3) + (2*c^3*h*Log[1 - c*x]*PolyLog[2, 1 -
 c*x])/3 - (c^3*h*PolyLog[3, c*x])/3 - (2*c^3*h*PolyLog[3, 1 - c*x])/3
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{x^4}\,dx = -\frac{(g+h\log(1-cx))\text{Li}_2(cx)}{3x^3} - \frac{1}{3}\int \frac{\log(1-cx)(g+h\log(1-cx))}{x^4}\,dx - \frac{1}{3}(ch)\int \Bigg(\frac{\text{Li}_2(}{x^3}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{9x^3} - \frac{(g+h\log(1-cx))\text{Li}_2(cx)}{3x^3} + \frac{1}{9}c\int \frac{g+h\log(1-cx)}{x^3(1-cx)}$$

$$= \frac{\log(1-cx)(g+h\log(1-cx))}{9x^3} + \frac{ch\text{Li}_2(cx)}{6x^2} + \frac{c^2h\text{Li}_2(cx)}{3x} + \frac{1}{3}c^3h\log(1-cx)\text{Li}_2(cx) -$$

$$= -\frac{ch\log(1-cx)}{12x^2} - \frac{c^2h\log(1-cx)}{3x} + \frac{1}{3}c^3h\log(cx)\log^2(1-cx) + \frac{\log(1-cx)(g+h\log(}{9x^3}$$

$$= -\frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(1-cx)}{9x} + \frac{1}{3}c^3h\log(cx)\log^2(1-cx) - \frac{c(g+h\log(1-cx))}{18x^2}$$

$$= \frac{c^2h}{12x} - \frac{5}{12}c^3h\log(x) + \frac{5}{12}c^3h\log(1-cx) - \frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(1-cx)}{9x} - \frac{1}{18}c^3h$$

$$= \frac{7c^2h}{36x} + \frac{1}{9}c^3g\log(x) - \frac{3}{4}c^3h\log(x) + \frac{23}{36}c^3h\log(1-cx) - \frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(1}{9x}$$

$$= \frac{7c^2h}{36x} + \frac{1}{9}c^3g\log(x) - \frac{3}{4}c^3h\log(x) + \frac{23}{36}c^3h\log(1-cx) - \frac{5ch\log(1-cx)}{36x^2} - \frac{4c^2h\log(1}{9x}$$

Mathematica [A] time = 0.200583, size = 301, normalized size = 0.89

$$\frac{h\left(-12c^3x^3\text{PolyLog}(3,cx) - 24c^3x^3\text{PolyLog}(3,1-cx) + 8c^3x^3(3\log(1-cx)+1)\text{PolyLog}(2,1-cx) + 6\left(2\left(c^3x^3-1\right)\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*Log[1 - c*x])*PolyLog[2, c*x])/x^4,x]
```

```
[Out] -(g*(c*x*(1 + 2*c*x) - 2*c^3*x^3*Log[x] + 2*(-1 + c^3*x^3)*Log[1 - c*x] + 6
*PolyLog[2, c*x]))/(18*x^3) + (h*(7*c^2*x^2 - 4*c^3*x^3 - 15*c^3*x^3*Log[x]
 - 12*c^3*x^3*Log[c*x] - 7*c*x*Log[1 - c*x] - 20*c^2*x^2*Log[1 - c*x] + 27*
c^3*x^3*Log[1 - c*x] + 8*c^3*x^3*Log[c*x]*Log[1 - c*x] + 4*Log[1 - c*x]^2 -
 4*c^3*x^3*Log[1 - c*x]^2 + 12*c^3*x^3*Log[c*x]*Log[1 - c*x]^2 + 6*(c*x*(1
+ 2*c*x) + 2*(-1 + c^3*x^3)*Log[1 - c*x])*PolyLog[2, c*x] + 8*c^3*x^3*(1 +
3*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 12*c^3*x^3*PolyLog[3, c*x] - 24*c^3*x
```

```
^3*PolyLog[3, 1 - c*x]))/(36*x^3)
```

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{(g + h\ln(-cx+1))\,\mathrm{polylog}(2, cx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

```
[Out] int((g+h*ln(-c*x+1))*polylog(2,c*x)/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{18}\left(2\,c^3\log(x) - \frac{2\,c^2x^2 + cx + 2\left(c^3x^3 - 1\right)\log(-cx+1) + 6\,\mathrm{Li}_2(cx)}{x^3}\right)g + h\int \frac{\mathrm{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/18*(2*c^3*log(x) - (2*c^2*x^2 + c*x + 2*(c^3*x^3 - 1)*log(-c*x + 1) + 6*d
ilog(c*x))/x^3)*g + h*integrate(dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{h\mathrm{Li}_2(cx)\log(-cx+1) + g\mathrm{Li}_2(cx)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((h*dilog(c*x)*log(-c*x + 1) + g*dilog(c*x))/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(-c*x+1))*polylog(2,c*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left(-cx+1\right)+g\right)\mathrm{Li}_2\left(cx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(-c*x+1))*polylog(2,c*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((h*log(-c*x + 1) + g)*dilog(c*x)/x^4, x)
```

3.177 $\int x^2 \left(g + h \log \left(f(d+ex)^n\right)\right) \textbf{PolyLog}(2, c(a+bx))\, dx$

Optimal. Leaf size=2995

result too large to display

```
[Out] -(a^2*g*x)/(3*b^2) + (a*(1 - a*c)*g*x)/(6*b^2*c) - ((1 - a*c)^2*g*x)/(9*b^2
*c^2) + (7*a^2*h*n*x)/(9*b^2) - (11*a*(1 - a*c)*h*n*x)/(36*b^2*c) + (5*(1 -
 a*c)^2*h*n*x)/(27*b^2*c^2) + (13*d^2*h*n*x)/(27*e^2) + (5*a*d*h*n*x)/(12*b
*e) - (7*(1 - a*c)*d*h*n*x)/(36*b*c*e) - (a*h*n*x^2)/(9*b) + (7*(1 - a*c)*h
*n*x^2)/(108*b*c) - (19*d*h*n*x^2)/(216*e) + (h*n*x^3)/27 - (5*a*(1 - a*c)^
2*h*n*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + (2*(1 - a*c)^3*h*n*Log[1 - a*c -
 b*c*x])/(27*b^3*c^3) - (5*(1 - a*c)^2*d*h*n*Log[1 - a*c - b*c*x])/(36*b^2*
c^2*e) + (5*a*h*n*x^2*Log[1 - a*c - b*c*x])/(36*b) + (5*d*h*n*x^2*Log[1 - a
*c - b*c*x])/(36*e) - (2*h*n*x^3*Log[1 - a*c - b*c*x])/27 + (4*a^2*h*n*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b^3*c) + (4*d^2*h*n*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(9*b*c*e^2) + (a*d*h*n*(1 - a*c - b*c*x)*Log[1 - a*
c - b*c*x])/(3*b^2*c*e) - (d^3*h*n*Log[d + e*x])/(27*e^3) - (a*d^2*h*n*Log[
d + e*x])/(12*b*e^2) + ((1 - a*c)*d^2*h*n*Log[d + e*x])/(18*b*c*e^2) + (d^3
*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(9*e^3)
 + (a*d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]
)/(6*b*e^2) + (a^2*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d +
e - a*c*e)])/(3*b^2*e) - (a^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(3*b^2*e) + (
a*(1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(6*b^2*c*e) - ((1 - a*c)^2*h*(d
 + e*x)*Log[f*(d + e*x)^n])/(9*b^2*c^2*e) + (a*x^2*(g + h*Log[f*(d + e*x)^n
]))/(12*b) - ((1 - a*c)*x^2*(g + h*Log[f*(d + e*x)^n]))/(18*b*c) - (x^3*(g
+ h*Log[f*(d + e*x)^n]))/27 + (a^2*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
 e*x)^n]))/(3*b^2) - (a*x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n])
)/(6*b) + (x^3*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/9 - (a^2*(1
 - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*
x)^n]))/(3*b^3*c) + (a*(1 - a*c)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a
*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*b^3*c^2) - ((1 - a*c)^3*Log[(e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(9*b^3*c^3) -
 (a^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Lo
g[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d -
 a*e)*(1 - c*(a + b*x)))]^2)/(6*b^3) + (d^3*h*n*(Log[c*(a + b*x)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*
(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*e^3)
- (a^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*b^3) + (d
^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*e^3) + (a^3*h
*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x)
)/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*b^3) - (d^
3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e
*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*e^3) +
```

```
(a^3*g*PolyLog[2, c*(a + b*x)])/(3*b^3) - (a^3*h*n*PolyLog[2, c*(a + b*x)])
/(9*b^3) - (a*d^2*h*n*PolyLog[2, c*(a + b*x)])/(3*b*e^2) - (a^2*d*h*n*PolyL
og[2, c*(a + b*x)])/(6*b^2*e) - (d^2*h*n*x*PolyLog[2, c*(a + b*x)])/(3*e^2)
 + (d*h*n*x^2*PolyLog[2, c*(a + b*x)])/(6*e) - (h*n*x^3*PolyLog[2, c*(a + b
*x)])/9 + (d^3*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*e^3) - (a^3*h*(
n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*b^3) + (x^
3*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/3 + (d^3*h*n*PolyLog[
2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(9*e^3) + (a*d^2*h*n*PolyLog
[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*b*e^2) + (a^2*d*h*n*Poly
Log[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(3*b^2*e) - (a^3*h*n*(Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*Po
lyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*b^3) + (d^3*h*n*(Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
 e*x))/(b*d - a*e)])/(3*e^3) - (a^2*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x)
)/(b*c*d + e - a*c*e)])/(3*b^3*c) + (a*(1 - a*c)^2*h*n*PolyLog[2, (b*c*(d +
 e*x))/(b*c*d + e - a*c*e)])/(6*b^3*c^2) - ((1 - a*c)^3*h*n*PolyLog[2, (b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/(9*b^3*c^3) - (a^3*h*n*(Log[d + e*x] - Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x
)])/(3*b^3) + (d^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(3*e^3) + (a^3*h*n*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/
(b*c*(d + e*x)))])/(3*b^3) - (d^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(3*e^3)
 - (a^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, (
(b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(3*b^3) + (d^3*h*n*Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a
 + b*x)))/(b*(d + e*x))])/(3*e^3) + (a^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d
- a*e)])/(3*b^3) - (d^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(3*e^3)
+ (a^3*h*n*PolyLog[3, 1 - c*(a + b*x)])/(3*b^3) - (d^3*h*n*PolyLog[3, 1 - c
*(a + b*x)])/(3*e^3) + (a^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d
+ e*x)))])/(3*b^3) - (d^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d +
e*x)))])/(3*e^3) - (a^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(
d + e*x))])/(3*b^3) + (d^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(
b*(d + e*x))])/(3*e^3)
```

Rubi [A] time = 4.59961, antiderivative size = 2995, normalized size of antiderivative = 1., number of steps used = 108, number of rules used = 20, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.741, Rules used = {6603, 2430, 43, 2416, 2389, 2295, 2394, 2393, 2391, 2439, 2395, 2440, 2438, 2437, 2435, 6595, 2444, 2421, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -(a^2*g*x)/(3*b^2) + (a*(1 - a*c)*g*x)/(6*b^2*c) - ((1 - a*c)^2*g*x)/(9*b^2
*c^2) + (7*a^2*h*n*x)/(9*b^2) - (11*a*(1 - a*c)*h*n*x)/(36*b^2*c) + (5*(1 -
 a*c)^2*h*n*x)/(27*b^2*c^2) + (13*d^2*h*n*x)/(27*e^2) + (5*a*d*h*n*x)/(12*b
*e) - (7*(1 - a*c)*d*h*n*x)/(36*b*c*e) - (a*h*n*x^2)/(9*b) + (7*(1 - a*c)*h
*n*x^2)/(108*b*c) - (19*d*h*n*x^2)/(216*e) + (h*n*x^3)/27 - (5*a*(1 - a*c)^
2*h*n*Log[1 - a*c - b*c*x])/(36*b^3*c^2) + (2*(1 - a*c)^3*h*n*Log[1 - a*c -
 b*c*x])/(27*b^3*c^3) - (5*(1 - a*c)^2*d*h*n*Log[1 - a*c - b*c*x])/(36*b^2*
c^2*e) + (5*a*h*n*x^2*Log[1 - a*c - b*c*x])/(36*b) + (5*d*h*n*x^2*Log[1 - a
*c - b*c*x])/(36*e) - (2*h*n*x^3*Log[1 - a*c - b*c*x])/27 + (4*a^2*h*n*(1 -
 a*c - b*c*x)*Log[1 - a*c - b*c*x])/(9*b^3*c) + (4*d^2*h*n*(1 - a*c - b*c*x
)*Log[1 - a*c - b*c*x])/(9*b*c*e^2) + (a*d*h*n*(1 - a*c - b*c*x)*Log[1 - a*
c - b*c*x])/(3*b^2*c*e) - (d^3*h*n*Log[d + e*x])/(27*e^3) - (a*d^2*h*n*Log[
d + e*x])/(12*b*e^2) + ((1 - a*c)*d^2*h*n*Log[d + e*x])/(18*b*c*e^2) + (d^3
*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(9*e^3)
 + (a*d^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]
)/(6*b*e^2) + (a^2*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d +
e - a*c*e)])/(3*b^2*e) - (a^2*h*(d + e*x)*Log[f*(d + e*x)^n])/(3*b^2*e) + (
a*(1 - a*c)*h*(d + e*x)*Log[f*(d + e*x)^n])/(6*b^2*c*e) - ((1 - a*c)^2*h*(d
 + e*x)*Log[f*(d + e*x)^n])/(9*b^2*c^2*e) + (a*x^2*(g + h*Log[f*(d + e*x)^n
]))/(12*b) - ((1 - a*c)*x^2*(g + h*Log[f*(d + e*x)^n]))/(18*b*c) - (x^3*(g
+ h*Log[f*(d + e*x)^n]))/27 + (a^2*x*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
 e*x)^n]))/(3*b^2) - (a*x^2*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n])
)/(6*b) + (x^3*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/9 - (a^2*(1
 - a*c)*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*
x)^n]))/(3*b^3*c) + (a*(1 - a*c)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a
*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*b^3*c^2) - ((1 - a*c)^3*Log[(e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(9*b^3*c^3) -
 (a^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Lo
g[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d -
 a*e)*(1 - c*(a + b*x)))]^2)/(6*b^3) + (d^3*h*n*(Log[c*(a + b*x)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*
(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*e^3)
- (a^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*b^3) + (d
^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*e^3) + (a^3*h
*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x)
)/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*b^3) - (d^
3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e
*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/(6*e^3) +
(a^3*g*PolyLog[2, c*(a + b*x)])/(3*b^3) - (a^3*h*n*PolyLog[2, c*(a + b*x)])
/(9*b^3) - (a*d^2*h*n*PolyLog[2, c*(a + b*x)])/(3*b*e^2) - (a^2*d*h*n*PolyL
og[2, c*(a + b*x)])/(6*b^2*e) - (d^2*h*n*x*PolyLog[2, c*(a + b*x)])/(3*e^2)
 + (d*h*n*x^2*PolyLog[2, c*(a + b*x)])/(6*e) - (h*n*x^3*PolyLog[2, c*(a + b
```

```
*x)])/9 + (d^3*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*e^3) - (a^3*h*(
n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*b^3) + (x^
3*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/3 + (d^3*h*n*PolyLog[
2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(9*e^3) + (a*d^2*h*n*PolyLog
[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*b*e^2) + (a^2*d*h*n*Poly
Log[2, (e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(3*b^2*e) - (a^3*h*n*(Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*Po
lyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*b^3) + (d^3*h*n*(Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
 e*x))/(b*d - a*e)])/(3*e^3) - (a^2*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x)
)/(b*c*d + e - a*c*e)])/(3*b^3*c) + (a*(1 - a*c)^2*h*n*PolyLog[2, (b*c*(d +
 e*x))/(b*c*d + e - a*c*e)])/(6*b^3*c^2) - ((1 - a*c)^3*h*n*PolyLog[2, (b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/(9*b^3*c^3) - (a^3*h*n*(Log[d + e*x] - Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x
)])/(3*b^3) + (d^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(3*e^3) + (a^3*h*n*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/
(b*c*(d + e*x)))])/(3*b^3) - (d^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(3*e^3)
 - (a^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, (
(b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(3*b^3) + (d^3*h*n*Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a
 + b*x)))/(b*(d + e*x))])/(3*e^3) + (a^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d
- a*e)])/(3*b^3) - (d^3*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(3*e^3)
+ (a^3*h*n*PolyLog[3, 1 - c*(a + b*x)])/(3*b^3) - (d^3*h*n*PolyLog[3, 1 - c
*(a + b*x)])/(3*e^3) + (a^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d
+ e*x)))])/(3*b^3) - (d^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d +
e*x)))])/(3*e^3) - (a^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(
d + e*x))])/(3*b^3) + (d^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(
b*(d + e*x))])/(3*e^3)
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
```

```
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2438

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
```

```
x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int x^2\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))\,dx=\frac{1}{3}x^3\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))+\frac{1}{3}b\int\left(\frac{a^2\log(1-ac-bc$$

$$=\frac{1}{3}x^3\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))+\frac{1}{3}\int x^2\log(1-ac-bcx)$$

$$=\frac{a^2x\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{3b^2}-\frac{ax^2\log(1-ac-bcx)\left(\right.}{6}$$

$$=\frac{a^2x\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{3b^2}-\frac{ax^2\log(1-ac-bcx)\left(\right.}{6}$$

$$=\frac{a^2x\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{3b^2}-\frac{ax^2\log(1-ac-bcx)\left(\right.}{6}$$

$$=-\frac{a^2gx}{3b^2}+\frac{a(1-ac)gx}{6b^2c}-\frac{(1-ac)^2gx}{9b^2c^2}+\frac{d^2hnx}{3e^2}+\frac{5ahnx^2\log(1-ac-bc}{36b}$$

$$=-\frac{a^2gx}{3b^2}+\frac{a(1-ac)gx}{6b^2c}-\frac{(1-ac)^2gx}{9b^2c^2}+\frac{4a^2hnx}{9b^2}+\frac{4d^2hnx}{9e^2}+\frac{5ahnx^2\log}{3}$$

$$=-\frac{a^2gx}{3b^2}+\frac{a(1-ac)gx}{6b^2c}-\frac{(1-ac)^2gx}{9b^2c^2}+\frac{7a^2hnx}{9b^2}-\frac{11a(1-ac)hnx}{36b^2c}+\frac{(1-}{}$$

Mathematica [A] time = 11.4813, size = 2610, normalized size = 0.87

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(12 + 66*a^2*c^2 +
6*b*c*x + 4*b^2*c^2*x^2 - 3*a*c*(14 + 5*b*c*x))) + 6*(-2 + 11*a^3*c^3 + 2*b
^3*c^3*x^3 + 6*a^2*c^2*(-3 + b*c*x) + a*(9*c - 3*b^2*c^3*x^2))*Log[1 - a*c
- b*c*x] + 36*c^3*(a^3 + b^3*x^3)*PolyLog[2, c*(a + b*x)]))/(108*b^3*c^3) +
 (h*n*(36*b^3*c^3*(e*x*(-6*d^2 + 3*d*e*x - 2*e^2*x^2) + 6*(d^3 + e^3*x^3)*L
og[d + e*x])*PolyLog[2, c*(a + b*x)] - 216*b^2*c^2*d^2*e*(1 - a*c - b*c*x +
 (-1 + a*c + b*c*x - a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - a*c*PolyL
og[2, 1 - a*c - b*c*x]) - 27*b*c*d*e^2*(c*(-4*a^2*c + a*(4 - 6*b*c*x) + b*x
*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2*c^2*x^2 + 4*a*c*(-2 + b*c*x) - 4*a^2
*c^2*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 4*a^2*c^2*PolyLog[2, 1 - a*c
- b*c*x]) - 2*e^3*(-(c*(36*a^3*c^2 - 3*a*b*c*x*(14 + 5*b*c*x) + 6*a^2*c*(-6
 + 11*b*c*x) + 2*b*x*(6 + 3*b*c*x + 2*b^2*c^2*x^2))) - 6*(2 - 11*a^3*c^3 -
2*b^3*c^3*x^3 - 6*a^2*c^2*(-3 + b*c*x) + 3*a*c*(-3 + b^2*c^2*x^2) + 6*a^3*c
^3*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] - 36*a^3*c^3*PolyLog[2, 1 - a*c -
 b*c*x]) + 216*b^3*c^3*d^3*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e
*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e
*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]
))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d +
e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] +
 (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)
] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] +
 Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-
((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c
*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*
d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b
*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) -
PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyL
og[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/
((b*d - a*e)*(-1 + a*c + b*c*x)))]]) + 2*(b*c*(e*(48*(-1 + a*c)^2*e^2*x + 3*
b*c*(-1 + a*c)*(12*d^2 + 12*d*e*x - 5*e^2*x^2) + b^2*c^2*x*(48*d^2 - 15*d*e
*x + 8*e^2*x^2)) - 6*(d + e*x)*(6*(-1 + a*c)^2*e^2 + 3*b*c*(-1 + a*c)*e*(d
- e*x) + 2*b^2*c^2*(d^2 - d*e*x + e^2*x^2))*Log[d + e*x]) + 6*Log[1 - a*c -
 b*c*x]*(-(e*(-1 + a*c + b*c*x)*(2*(-1 + a*c)^2*e^2 + b*c*(-1 + a*c)*e*(3*d
 - 2*e*x) + b^2*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2))) + 6*e^3*(-1 + 3*a*c - 3
*a^2*c^2 + a^3*c^3 + b^3*c^3*x^3)*Log[d + e*x] + 6*(b^3*c^3*d^3 - (-1 + a*c
)^3*e^3)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + 36*(b^3*c^3*d^3 - (-1
+ a*c)^3*e^3)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)]
- 108*a^2*c^2*e^2*(e - a*c*e - 2*b*c*e*x + b*c*d*Log[d + e*x] + b*c*e*x*Log
[d + e*x] - Log[1 - a*c - b*c*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b
```

```
*c*x)*Log[d + e*x] + (b*c*d + e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a
*c*e)]) - (b*c*d + e - a*c*e)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) +
 (-1 + a*c)*e)]) - 27*a*c*e*(b*c*(e*(d*(2 - 2*a*c - 3*b*c*x) + e*x*(3 - 3*a
*c + b*c*x)) + (d + e*x)*(2*(-1 + a*c)*e + b*c*(d - e*x))*Log[d + e*x]) + L
og[1 - a*c - b*c*x]*(e*(-1 + a*c + b*c*x)*((-1 + a*c)*e + b*c*(2*d - e*x))
- 2*e^2*(1 - 2*a*c + a^2*c^2 - b^2*c^2*x^2)*Log[d + e*x] + 2*(-(b^2*c^2*d^2
) + (-1 + a*c)^2*e^2)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + 2*(-(b^2*
c^2*d^2) + (-1 + a*c)^2*e^2)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) +
(-1 + a*c)*e)]) - 108*a^3*c^3*e^3*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Lo
g[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b
*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d
- a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(
b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*
x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a
 + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c
*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x]
- Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*
c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 +
 a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x
))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 +
a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)
))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)]
- PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d +
 e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]))))/(648*b^3*c^3*e^3)
```

Maple [F] time = 0.493, size = 0, normalized size = 0.

$$\int x^2 \left(g + h\ln\left(f\,(ex+d)^n\right)\right) \operatorname{polylog}\left(2, c\,(bx+a)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] int(x^2*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(6\,e^3hx^3\log\left((ex+d)^n\right) + 3\,de^2hnx^2 - 6\,d^2ehnx + 6\,d^3hn\log\left(ex+d\right) - 2\left(e^3hn - 3\,e^3h\log\left(f\right) - 3\,e^3g\right)x^3\right)\mathrm{Li}_2\left(bcx+ac\right)}{18\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="max
ima")
```

```
[Out] 1/18*(6*e^3*h*x^3*log((e*x + d)^n) + 3*d*e^2*h*n*x^2 - 6*d^2*e*h*n*x + 6*d^
3*h*n*log(e*x + d) - 2*(e^3*h*n - 3*e^3*h*log(f) - 3*e^3*g)*x^3)*dilog(b*c*
x + a*c)/e^3 + integrate(1/18*(6*b*e^3*h*x^3*log(-b*c*x - a*c + 1)*log((e*x
 + d)^n) + (3*b*d*e^2*h*n*x^2 - 6*b*d^2*e*h*n*x + 6*b*d^3*h*n*log(e*x + d)
- 2*(b*e^3*h*n - 3*b*e^3*h*log(f) - 3*b*e^3*g)*x^3)*log(-b*c*x - a*c + 1))/
(b*e^3*x + a*e^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(hx^2\text{Li}_2\left(bcx+ac\right)\log\left((ex+d)^n f\right)+gx^2\text{Li}_2\left(bcx+ac\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fri
cas")
```

```
[Out] integral(h*x^2*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*x^2*dilog(b*c*x +
a*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(h\log\left((ex+d)^n f\right)+g\right)x^2\text{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*x^2*dilog((b*x + a)*c), x)
```

3.178 $\int x \left(g + h \log \left(f(d + ex)^n\right)\right) \mathbf{PolyLog}(2, c(a+bx))\, dx$

Optimal. Leaf size=2252

result too large to display

```
[Out] (a*g*x)/(2*b) - ((1 - a*c)*g*x)/(4*b*c) - (5*a*h*n*x)/(4*b) + ((1 - a*c)*h*
n*x)/(2*b*c) - (7*d*h*n*x)/(8*e) + (3*h*n*x^2)/16 + ((1 - a*c)^2*h*n*Log[1
- a*c - b*c*x])/(4*b^2*c^2) - (h*n*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*h*n*(
1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b^2*c) - (3*d*h*n*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b*c*e) + (d^2*h*n*Log[d + e*x])/(8*e^2) - (d^2*
h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*e^2)
- (a*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(
2*b*e) + (a*h*(d + e*x)*Log[f*(d + e*x)^n])/(2*b*e) - ((1 - a*c)*h*(d + e*x
)*Log[f*(d + e*x)^n])/(4*b*c*e) - (x^2*(g + h*Log[f*(d + e*x)^n]))/8 - (a*x
*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*b) + (x^2*Log[1 - a*c
- b*c*x]*(g + h*Log[f*(d + e*x)^n]))/4 + (a*(1 - a*c)*Log[(e*(1 - a*c - b*c
*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*b^2*c) - ((1 - a*c
)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(4*b^2*c^2) + (a^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c
*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*b^2) - (d^2*h*n*(Log[c*(a
+ b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*
e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))]^2)/(4*e^2) + (a^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*
x)])/(2*b^2) - (d^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(2*e^2) - (a^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*
(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])
^2)/(4*b^2) + (d^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))
])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x
)])^2)/(4*e^2) - (a^2*g*PolyLog[2, c*(a + b*x)])/(2*b^2) + (a^2*h*n*PolyLog
[2, c*(a + b*x)])/(4*b^2) + (a*d*h*n*PolyLog[2, c*(a + b*x)])/(2*b*e) + (d*
h*n*x*PolyLog[2, c*(a + b*x)])/(2*e) - (h*n*x^2*PolyLog[2, c*(a + b*x)])/4
- (d^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*e^2) + (a^2*h*(n*Log[d
+ e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*(g + h
*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (d^2*h*n*PolyLog[2, (e*(1
 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*PolyLog[2, (e*(1
- a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a^2*h*n*(Log[(b*(d + e*x))
/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d
+ e*x))/(b*d - a*e)])/(2*b^2) - (d^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)])/(2*e^2) + (a*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*
c*e)])/(2*b^2*c) - ((1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(4*b^2*c^2) + (a^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d -
```

```
a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*b^2) - (d^2*h*n*(
Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[
2, 1 - c*(a + b*x)])/(2*e^2) - (a^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
 c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^
2) + (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
 -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*h*n*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a +
b*x)))/(b*(d + e*x))])/(2*b^2) - (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]
)/(2*e^2) - (a^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*b^2) + (d^2*
h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*e^2) - (a^2*h*n*PolyLog[3, 1
- c*(a + b*x)])/(2*b^2) + (d^2*h*n*PolyLog[3, 1 - c*(a + b*x)])/(2*e^2) - (
a^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^2) + (d^
2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*
h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*b^2) - (d
^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*e^2)
```

Rubi [A] time = 2.90446, antiderivative size = 2252, normalized size of antiderivative = 1., number of steps used = 67, number of rules used = 20, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {6603, 2430, 43, 2416, 2389, 2295, 2394, 2393, 2391, 2439, 2395, 2440, 2438, 2437, 2435, 6595, 2444, 2421, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] (a*g*x)/(2*b) - ((1 - a*c)*g*x)/(4*b*c) - (5*a*h*n*x)/(4*b) + ((1 - a*c)*h*
n*x)/(2*b*c) - (7*d*h*n*x)/(8*e) + (3*h*n*x^2)/16 + ((1 - a*c)^2*h*n*Log[1
- a*c - b*c*x])/(4*b^2*c^2) - (h*n*x^2*Log[1 - a*c - b*c*x])/4 - (3*a*h*n*(
1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(4*b^2*c) - (3*d*h*n*(1 - a*c - b*c*
x)*Log[1 - a*c - b*c*x])/(4*b*c*e) + (d^2*h*n*Log[d + e*x])/(8*e^2) - (d^2*
h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(4*e^2)
- (a*d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(
2*b*e) + (a*h*(d + e*x)*Log[f*(d + e*x)^n])/(2*b*e) - ((1 - a*c)*h*(d + e*x
)*Log[f*(d + e*x)^n])/(4*b*c*e) - (x^2*(g + h*Log[f*(d + e*x)^n]))/8 - (a*x
*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(2*b) + (x^2*Log[1 - a*c
- b*c*x]*(g + h*Log[f*(d + e*x)^n]))/4 + (a*(1 - a*c)*Log[(e*(1 - a*c - b*c
*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(2*b^2*c) - ((1 - a*c
)^2*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(4*b^2*c^2) + (a^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c
```

```
*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(
d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*b^2) - (d^2*h*n*(Log[c*(a
+ b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*
e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))]^2)/(4*e^2) + (a^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*
x)])/(2*b^2) - (d^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(2*e^2) - (a^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*
(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])
^2)/(4*b^2) + (d^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))
])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x
)])^2)/(4*e^2) - (a^2*g*PolyLog[2, c*(a + b*x)])/(2*b^2) + (a^2*h*n*PolyLog
[2, c*(a + b*x)])/(4*b^2) + (a*d*h*n*PolyLog[2, c*(a + b*x)])/(2*b*e) + (d*
h*n*x*PolyLog[2, c*(a + b*x)])/(2*e) - (h*n*x^2*PolyLog[2, c*(a + b*x)])/4
- (d^2*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(2*e^2) + (a^2*h*(n*Log[d
+ e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*b^2) + (x^2*(g + h
*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/2 - (d^2*h*n*PolyLog[2, (e*(1
 - a*c - b*c*x))/(b*c*d + e - a*c*e)])/(4*e^2) - (a*d*h*n*PolyLog[2, (e*(1
- a*c - b*c*x))/(b*c*d + e - a*c*e)])/(2*b*e) + (a^2*h*n*(Log[(b*(d + e*x))
/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d
+ e*x))/(b*d - a*e)])/(2*b^2) - (d^2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)])/(2*e^2) + (a*(1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*
c*e)])/(2*b^2*c) - ((1 - a*c)^2*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(4*b^2*c^2) + (a^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d -
a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*b^2) - (d^2*h*n*(
Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[
2, 1 - c*(a + b*x)])/(2*e^2) - (a^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
 c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^
2) + (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
 -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*h*n*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a +
b*x)))/(b*(d + e*x))])/(2*b^2) - (d^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1
 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))]
)/(2*e^2) - (a^2*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*b^2) + (d^2*
h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*e^2) - (a^2*h*n*PolyLog[3, 1
- c*(a + b*x)])/(2*b^2) + (d^2*h*n*PolyLog[3, 1 - c*(a + b*x)])/(2*e^2) - (
a^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*b^2) + (d^
2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(2*e^2) + (a^2*
h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*b^2) - (d
^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*e^2)
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
```

```
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2438

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
```

```
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rule 2421

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_)^(q_.), x_Symbol] :> In
```

```
t[ExpandToSum[u, x]^q*(a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a,
 b, c, n, p, q}, x] && BinomialQ[u, x] && LinearQ[v, x] &&  !(BinomialMatch
Q[u, x] && LinearMatchQ[v, x])
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int x\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))\,dx=\frac{1}{2}x^2\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))+\frac{1}{2}b\int\left(-\frac{a\log(1-ac-bcx\right.$$

$$=\frac{1}{2}x^2\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))+\frac{1}{2}\int x\log(1-ac-bcx)\left(g\right.$$

$$=-\frac{ax\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{2b}+\frac{1}{4}x^2\log(1-ac-bcx)\left(\right.$$

$$=-\frac{ax\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{2b}+\frac{1}{4}x^2\log(1-ac-bcx)\left(\right.$$

$$=-\frac{ax\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{2b}+\frac{1}{4}x^2\log(1-ac-bcx)\left(\right.$$

$$=\frac{agx}{2b}-\frac{(1-ac)gx}{4bc}-\frac{dhnx}{2e}-\frac{dhn(1-ac-bcx)\log(1-ac-bcx)}{2bce}-\frac{d^2hn}{}$$

$$=\frac{agx}{2b}-\frac{(1-ac)gx}{4bc}-\frac{3ahnx}{4b}-\frac{3dhnx}{4e}-\frac{3ahn(1-ac-bcx)\log(1-ac-b}{4b^2c}$$

$$=\frac{agx}{2b}-\frac{(1-ac)gx}{4bc}-\frac{5ahnx}{4b}+\frac{(1-ac)hnx}{4bc}-\frac{7dhnx}{8e}+\frac{1}{16}hnx^2-\frac{3ahn(1}{}$$

Mathematica [A] time = 8.83211, size = 1996, normalized size = 0.89

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x*(2 - 6*a*c + b*c*x)
) + (-2 - 6*a^2*c^2 + 2*b^2*c^2*x^2 - 4*a*c*(-2 + b*c*x))*Log[1 - a*c - b*c
*x] - 4*c^2*(a^2 - b^2*x^2)*PolyLog[2, c*(a + b*x)]))/(8*b^2*c^2) + (h*n*(4
*b^2*c^2*(e*x*(2*d - e*x) - 2*(d^2 - e^2*x^2)*Log[d + e*x])*PolyLog[2, c*(a
```

```
 + b*x)] + 8*b*c*d*e*(1 - a*c - b*c*x + (-1 + a*c + b*c*x - a*c*Log[c*(a +
b*x)])*Log[1 - a*c - b*c*x] - a*c*PolyLog[2, 1 - a*c - b*c*x]) + e^2*(c*(-4
*a^2*c + a*(4 - 6*b*c*x) + b*x*(2 + b*c*x)) + (2 + 6*a^2*c^2 - 2*b^2*c^2*x^
2 + 4*a*c*(-2 + b*c*x) - 4*a^2*c^2*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] -
 4*a^2*c^2*PolyLog[2, 1 - a*c - b*c*x]) - 8*b^2*c^2*d^2*(Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(
-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + L
og[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/
(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d -
 a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
 b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d
- a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)
]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)
))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*
x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e
)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*
c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a
*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d
 + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))]
+ PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) + 2*(b*c*(
e*(d*(2 - 2*a*c - 3*b*c*x) + e*x*(3 - 3*a*c + b*c*x)) + (d + e*x)*(2*(-1 +
a*c)*e + b*c*(d - e*x))*Log[d + e*x]) + Log[1 - a*c - b*c*x]*(e*(-1 + a*c +
 b*c*x)*((-1 + a*c)*e + b*c*(2*d - e*x)) - 2*e^2*(1 - 2*a*c + a^2*c^2 - b^2
*c^2*x^2)*Log[d + e*x] + 2*(-(b^2*c^2*d^2) + (-1 + a*c)^2*e^2)*Log[(b*c*(d
+ e*x))/(b*c*d + e - a*c*e)]) + 2*(-(b^2*c^2*d^2) + (-1 + a*c)^2*e^2)*PolyL
og[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)] + 4*a*c*e*(e - a*c*
e - 2*b*c*e*x + b*c*d*Log[d + e*x] + b*c*e*x*Log[d + e*x] - Log[1 - a*c - b
*c*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b*c*x)*Log[d + e*x] + (b*c*d
 + e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) - (b*c*d + e - a*c*
e)*PolyLog[2, (e*(-1 + a*c + b*c*x))/(-(b*c*d) + (-1 + a*c)*e)]) + 4*a^2*c^
2*e^2*(Log[c*(a + b*x)]*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*
x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2
*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b
*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Lo
g[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))
/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e -
 a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c
*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d
- a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c -
b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2,
 (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2,
-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c -
b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))
/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*
```

```
c + b*c*x)))])))))/(16*b^2*c^2*e^2)
```

Maple [F] time = 0.289, size = 0, normalized size = 0.

$$\int x\left(g+h\ln\left(f\left(ex+d\right)^{n}\right)\right)\mathrm{polylog}\left(2,c\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] int(x*(g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,e^{2}hx^{2}\log\left(\left(ex+d\right)^{n}\right)+2\,dehnx-2\,d^{2}hn\log\left(ex+d\right)-\left(e^{2}hn-2\,e^{2}h\log\left(f\right)-2\,e^{2}g\right)x^{2}\right)\mathrm{Li}_2\left(bcx+ac\right)}{4\,e^{2}}+\int\frac{2\,be^{2}hx^{2}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxim
a")
```

```
[Out] 1/4*(2*e^2*h*x^2*log((e*x + d)^n) + 2*d*e*h*n*x - 2*d^2*h*n*log(e*x + d) -
(e^2*h*n - 2*e^2*h*log(f) - 2*e^2*g)*x^2)*dilog(b*c*x + a*c)/e^2 + integrat
e(1/4*(2*b*e^2*h*x^2*log(-b*c*x - a*c + 1)*log((e*x + d)^n) + (2*b*d*e*h*n*
x - 2*b*d^2*h*n*log(e*x + d) - (b*e^2*h*n - 2*b*e^2*h*log(f) - 2*b*e^2*g)*x
^2)*log(-b*c*x - a*c + 1))/(b*e^2*x + a*e^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(hx\mathrm{Li}_2\left(bcx+ac\right)\log\left(\left(ex+d\right)^{n}f\right)+gx\mathrm{Li}_2\left(bcx+ac\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="frica
s")
```

```
[Out] integral(h*x*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*x*dilog(b*c*x + a*c)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(h\log\left((ex+d)^n f\right)+g\right)x\mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac"
)
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*x*dilog((b*x + a)*c), x)
```

3.179 $\int \left(g+h\log\left(f(d+ex)^n\right)\right)\mathbf{PolyLog}(2,c(a+bx))\,dx$

Optimal. Leaf size=1653

result too large to display

```
[Out] -(g*x) + 3*h*n*x - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (2*h*
n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (d*h*n*Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[-d - e*x])/e + (d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/e + (d*h*n*(Log[c*(a + b*x)] + Log[(b*c*d
 + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d
+ e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (d*
h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - a*c - b
*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])^2)/(2*e) - (h*(
d + e*x)*Log[f*(d + e*x)^n])/e + h*x*Log[1 - a*c - b*c*x]*Log[f*(d + e*x)^n
] - ((1 - a*c)*h*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*Log[f*(d +
e*x)^n])/(b*c) - (a*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
 + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*b) - (a*h*n*Log[c*(a + b*x)]*
Log[d + e*x]*Log[1 - c*(a + b*x)])/b + (a*h*n*(Log[c*(a + b*x)] - Log[-((e*
(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)] + Log[1 - c*(a + b*x)])^2)/(2*b) + (a*g*PolyLog[2, c*(a + b*x)])/b - (a*
h*n*PolyLog[2, c*(a + b*x)])/b - (a*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])
*PolyLog[2, c*(a + b*x)])/b + x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a
+ b*x)] + (d*h*n*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c -
 b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e + (d*h*n*PolyLog[2, (e*(1 - a*c -
 b*c*x))/(b*c*d + e - a*c*e)])/e - h*n*x*PolyLog[2, a*c + b*c*x] + (d*h*n*L
og[-d - e*x]*PolyLog[2, a*c + b*c*x])/e - (d*h*n*Log[(b*(d + e*x))/((b*d -
a*e)*(1 - a*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))
)])/e + (d*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]*PolyLog[2
, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (d*h*n*(Log[1 - a*c -
 b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, (b
*(d + e*x))/(b*d - a*e)])/e - (a*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]
)/b - ((1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(b*c)
 - (a*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))
])*PolyLog[2, 1 - c*(a + b*x)])/b + (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(
1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b
- (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b - (d*h*n*PolyLog[3, 1 - a*c -
 b*c*x])/e - (d*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/e
 + (d*h*n*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (a
*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/b - (d*h*n*PolyLog[3, (b*(d + e
*x))/(b*d - a*e)])/e + (a*h*n*PolyLog[3, 1 - c*(a + b*x)])/b + (a*h*n*PolyL
```

```
og[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b - (a*h*n*PolyLog[3, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b
```

Rubi [A] time = 3.18592, antiderivative size = 1653, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {6600, 2418, 2389, 2295, 2394, 2393, 2391, 6688, 43, 2416, 6742, 2430, 2440, 2437, 2435, 6595, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] -(g*x) + 3*h*n*x - (g*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (2*h*
n*(1 - a*c - b*c*x)*Log[1 - a*c - b*c*x])/(b*c) + (d*h*n*Log[c*(a + b*x)]*L
og[1 - a*c - b*c*x]*Log[-d - e*x])/e + (d*h*n*Log[1 - a*c - b*c*x]*Log[(b*c
*(d + e*x))/(b*c*d + e - a*c*e)])/e + (d*h*n*(Log[c*(a + b*x)] + Log[(b*c*d
 + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d
+ e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]^2)/(2*e) - (d*
h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log[1 - a*c - b
*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])^2)/(2*e) - (h*(
d + e*x)*Log[f*(d + e*x)^n])/e + h*x*Log[1 - a*c - b*c*x]*Log[f*(d + e*x)^n
] - ((1 - a*c)*h*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*Log[f*(d +
e*x)^n])/(b*c) - (a*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
 + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
 e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*b) - (a*h*n*Log[c*(a + b*x)]*
Log[d + e*x]*Log[1 - c*(a + b*x)])/b + (a*h*n*(Log[c*(a + b*x)] - Log[-((e*
(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)] + Log[1 - c*(a + b*x)])^2)/(2*b) + (a*g*PolyLog[2, c*(a + b*x)])/b - (a*
h*n*PolyLog[2, c*(a + b*x)])/b - (a*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])
*PolyLog[2, c*(a + b*x)])/b + x*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a
+ b*x)] + (d*h*n*(Log[-d - e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c -
 b*c*x))])*PolyLog[2, 1 - a*c - b*c*x])/e + (d*h*n*PolyLog[2, (e*(1 - a*c -
 b*c*x))/(b*c*d + e - a*c*e)])/e - h*n*x*PolyLog[2, a*c + b*c*x] + (d*h*n*L
og[-d - e*x]*PolyLog[2, a*c + b*c*x])/e - (d*h*n*Log[(b*(d + e*x))/((b*d -
a*e)*(1 - a*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x))
)])/e + (d*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))]*PolyLog[2
, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (d*h*n*(Log[1 - a*c -
 b*c*x] + Log[(b*(d + e*x))/((b*d - a*e)*(1 - a*c - b*c*x))])*PolyLog[2, (b
*(d + e*x))/(b*d - a*e)])/e - (a*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c
*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)]
```

```
)/b - ((1 - a*c)*h*n*PolyLog[2, (b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(b*c)
 - (a*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))
])*PolyLog[2, 1 - c*(a + b*x)])/b + (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(
1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b
- (a*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b - (d*h*n*PolyLog[3, 1 - a*c -
 b*c*x])/e - (d*h*n*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/e
 + (d*h*n*PolyLog[3, ((b*d - a*e)*(1 - a*c - b*c*x))/(b*(d + e*x))])/e + (a
*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/b - (d*h*n*PolyLog[3, (b*(d + e
*x))/(b*d - a*e)])/e + (a*h*n*PolyLog[3, 1 - c*(a + b*x)])/b + (a*h*n*PolyL
og[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/b - (a*h*n*PolyLog[3, ((b*
d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/b
```

Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
 b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 6595

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)], x_Symbol] :> Simp[x*Poly
Log[n, c*(a + b*x)^p], x] + (-Dist[p, Int[PolyLog[n - 1, c*(a + b*x)^p], x]
, x] + Dist[a*p, Int[PolyLog[n - 1, c*(a + b*x)^p]/(a + b*x), x], x]) /; Fr
eeQ[{a, b, c, p}, x] && GtQ[n, 0]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
```

```
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))\,dx = x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) + b\int\left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\log(1 -$$

$$= x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) + b\int \frac{x\log(1-ac-bcx)\left(g + \right.}{a+bx}$$

$$= x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) + b\int\left(\frac{gx\log(1-ac-bcx)}{a+bx} + \right.$$

$$= x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) + (bg)\int \frac{x\log(1-ac-bcx)}{a+bx}\,dx$$

$$= x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) - hnx\text{Li}_2(ac+bcx) + \frac{dhn\log(}{}$$

$$= x\left(g + h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx)) - hnx\text{Li}_2(ac+bcx) + \frac{dhn\log(}{}$$

$$= hnx + \frac{hn(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{dhn\log(c(a+bx))\log(1-a}{e}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac-bcx)\log(1}{bc}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac-bcx)\log(1}{bc}$$

$$= -gx + hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{hn(1-ac-bcx)\log(1}{bc}$$

$$= -gx + 3hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{2hn(1-ac-bcx)\log(}{bc}$$

$$= -gx + 3hnx - \frac{g(1-ac-bcx)\log(1-ac-bcx)}{bc} + \frac{2hn(1-ac-bcx)\log(}{bc}$$

Mathematica [A] time = 4.92197, size = 1546, normalized size = 0.94

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)],x]
```

```
[Out] ((g - h*n*Log[d + e*x] + h*Log[f*(d + e*x)^n])*(-(b*c*x) + (-1 + a*c + b*c*
x)*Log[1 - a*c - b*c*x] + c*(a + b*x)*PolyLog[2, c*(a + b*x)]))/(b*c) + (h*
n*((-(e*x) + (d + e*x)*Log[d + e*x])*PolyLog[2, c*(a + b*x)] + (-e + a*c*e
+ 2*b*c*e*x - b*c*d*Log[d + e*x] - b*c*e*x*Log[d + e*x] + Log[1 - a*c - b*c
*x]*(-(e*(-1 + a*c + b*c*x)) + e*(-1 + a*c + b*c*x)*Log[d + e*x] + (b*c*d +
 e - a*c*e)*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)]) + e*(-1 + a*c + b*c*x
 + (1 - a*c - b*c*x + a*c*Log[c*(a + b*x)])*Log[1 - a*c - b*c*x] + a*c*Poly
Log[2, 1 - a*c - b*c*x]) + (b*c*d + e - a*c*e)*PolyLog[2, (e*(-1 + a*c + b*
c*x))/(-(b*c*d) + (-1 + a*c)*e)] + b*c*d*(Log[c*(a + b*x)]*Log[1 - a*c - b*
c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))/(-(b*d) + a*e)])
*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] + Log[(b*(d + e*x)
)/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x))/(-(b*d) + a*e)]
)*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]^2*(L
og[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*d - a*e)*(-1 + a*
c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*x)]))/2 + (Log[d
+ e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2,
 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d + e*x))/((b*d - a*e
)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)] + Log[-((b*(
d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (b*c*(d + e*x))/(e
*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c +
 b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*(d + e*x))/(b*d -
 a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] + PolyLog[3, -(
(b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]) - a*c*e*(Log[c*(a + b*x)]
*Log[1 - a*c - b*c*x]*Log[d + e*x] + ((Log[c*(a + b*x)] - Log[(e*(a + b*x))
/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*(-2*Log[1 - a*c - b*c*x] +
 Log[(b*(d + e*x))/(b*d - a*e)]))/2 + (-Log[c*(a + b*x)] + Log[(e*(a + b*x)
)/(-(b*d) + a*e)])*Log[(b*(d + e*x))/(b*d - a*e)]*Log[-((b*(d + e*x))/((b*d
 - a*e)*(-1 + a*c + b*c*x)))] + (Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c
 + b*c*x)))]^2*(Log[c*(a + b*x)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/((b*
d - a*e)*(-1 + a*c + b*c*x))] + Log[(b*c*d + e - a*c*e)/(e - a*c*e - b*c*e*
x)]))/2 + (Log[d + e*x] - Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*
x)))])*PolyLog[2, 1 - a*c - b*c*x] + (Log[1 - a*c - b*c*x] + Log[-((b*(d +
e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))])*PolyLog[2, (b*(d + e*x))/(b*d - a
*e)] + Log[-((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]*(PolyLog[2, (
b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))] - PolyLog[2, -((b*(d + e*x))/((b*d -
```

```
 a*e)*(-1 + a*c + b*c*x)))]) - PolyLog[3, 1 - a*c - b*c*x] - PolyLog[3, (b*
(d + e*x))/(b*d - a*e)] - PolyLog[3, (b*c*(d + e*x))/(e*(-1 + a*c + b*c*x))
] + PolyLog[3, -((b*(d + e*x))/((b*d - a*e)*(-1 + a*c + b*c*x)))]))/(b*c)))
/e
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int \left(g+h\ln\left(f\left(ex+d\right)^{n}\right)\right)\mathrm{polylog}\left(2,c\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(dhn\log\left(ex+d\right)+ehx\log\left(\left(ex+d\right)^{n}\right)-\left(ehn-eh\log\left(f\right)-eg\right)x\right)\mathrm{Li}_2\left(bcx+ac\right)}{e}+\int\frac{behx\log\left(-bcx-ac+1\right)\log\left(\left(ex\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="maxima"
)
```

```
[Out] (d*h*n*log(e*x + d) + e*h*x*log((e*x + d)^n) - (e*h*n - e*h*log(f) - e*g)*x
)*dilog(b*c*x + a*c)/e + integrate((b*e*h*x*log(-b*c*x - a*c + 1)*log((e*x
+ d)^n) + (b*d*h*n*log(e*x + d) - (b*e*h*n - b*e*h*log(f) - b*e*g)*x)*log(-
b*c*x - a*c + 1))/(b*e*x + a*e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(h\mathrm{Li}_2\left(bcx+ac\right)\log\left(\left(ex+d\right)^{n}f\right)+g\mathrm{Li}_2\left(bcx+ac\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="fricas"
)
```

```
[Out] integral(h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(h\log\left((ex+d)^n f\right)+g\right)\mathrm{Li}_2\left((bx+a)c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c), x)
```

3.180 $\int \frac{\left(g+h \log \left(f (d+e x)^n\right)\right) \textbf{PolyLog}(2,c(a+b x))}{x} \, dx$

Optimal. Leaf size=29

$$\text{Unintegrable}\left(\frac{\text{PolyLog}(2,c(a+bx))\left(h\log\left(f(d+ex)^n\right)+g\right)}{x},x\right)$$

```
[Out] Unintegrable[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

Rubi [A] time = 0.0250694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
```

```
[Out] Defer[Int][((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

Rubi steps

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))}{x}\,dx=\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))}{x}\,dx$$

Mathematica [A] time = 0.552935, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x,x]
```

```
[Out] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x, x]
```

Maple [A] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\ln\left(f\left(ex+d\right)^{n}\right)\right)\operatorname{polylog}\left(2,c\left(bx+a\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left(\left(ex+d\right)^{n}f\right)+g\right)\mathrm{Li}_{2}\left(\left(bx+a\right)c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="maxim
a")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{h\mathrm{Li}_{2}\left(bcx+ac\right)\log\left(\left(ex+d\right)^{n}f\right)+g\mathrm{Li}_{2}\left(bcx+ac\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="frica
s")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/x
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h \log \left((ex+d)^n f\right) + g\right) \mathrm{Li}_2\left((bx+a)c\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x,x, algorithm="giac"
)
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x, x)
```

3.181 $\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathbf{PolyLog}(2,c(a+bx))}{x^2}\,dx$

Optimal. Leaf size=2498

result too large to display

```
[Out] -((b*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*h*n*Log[(b*c*x)
/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/a - (b*h*n*(Log[(b*c*x)/(1 -
 a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e
)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x)
)]^2)/(2*a) + (b*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*
c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(2*a) + (
b*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d
+ e*x)^n]))/a + (b*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
+ e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*a) - (e*h*n*(Log[c*(a + b*x)]
+ Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a +
b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)
/(2*d) + (e*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/d + (b*h*n*Lo
g[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/a - (e*h*n*Log[c*(a + b*x
)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/d - (b*h*n*(Log[c*(a + b*x)] - Log[-(
(e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))] + Log[1 - c*(a + b*x)])^2)/(2*a) + (e*h*n*(Log[c*(a + b*x)] - Log[-((
e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x
)))] + Log[1 - c*(a + b*x)])^2)/(2*d) + (e*h*n*(Log[1 + (b*x)/a] + Log[(1 -
 a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]
)*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*d)
+ (e*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyL
og[2, -((b*x)/a)])/d - (b*g*PolyLog[2, c*(a + b*x)])/a + (e*h*n*Log[x]*Poly
Log[2, c*(a + b*x)])/d - (e*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/d + (
b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/a - ((g
+ h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x - (b*g*PolyLog[2, 1 - (b
*c*x)/(1 - a*c)])/a - (b*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(
1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a + (b*h*(n*Log[d +
e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*Lo
g[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c
*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a
*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/a + (b*
h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*
x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/a - (e*h*n*(Log[(b*(d + e*x))/(
(b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
e*x))/(b*d - a*e)])/d - (b*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
```

```
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/a + (e*h*n*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/d -
(e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a +
 b*x)))])/d + (b*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*
(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/a - (e*h*n*(Log[d + e*x] - Log[(
b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])
/d + (e*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c
*(a + b*x)])/d - (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*
PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/a + (e*h*n*Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))
/(b*c*(d + e*x)))])/d + (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b
*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*
n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e
)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d - (e*h*n*PolyLog[3, -((b*x)/a)])/d +
 (b*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*PolyLog[3, (d*(1 - a*
c - b*c*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*PolyLog[3, -((e*(1 - a*c - b
*c*x))/(b*c*(d + e*x)))])/a - (b*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])
/a + (e*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/d + (b*h*n*PolyLog[3, 1
+ (e*x)/d])/a + (e*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/d - (e*h
*n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/d - (b*h*n*PolyLog[3, 1 - c*(a
 + b*x)])/a - (b*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/
a + (e*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/d + (b*h*n
*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*n*Poly
Log[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d
```

Rubi [A] time = 2.66747, antiderivative size = 2498, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6603, 2438, 2394, 2315, 2437, 2435, 2440, 2391, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]
```

```
[Out] -((b*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/a) - (b*h*n*Log[(b*c*x)
/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/a - (b*h*n*(Log[(b*c*x)/(1 -
 a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e
)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x)
)]^2)/(2*a) + (b*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*
c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(2*a) + (
b*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d
```

```
+ e*x)^n]))/a + (b*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d
+ e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d +
e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(2*a) - (e*h*n*(Log[c*(a + b*x)]
+ Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a +
b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)
/(2*d) + (e*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*x)])/d + (b*h*n*Lo
g[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/a - (e*h*n*Log[c*(a + b*x
)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/d - (b*h*n*(Log[c*(a + b*x)] - Log[-(
(e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*
x)))] + Log[1 - c*(a + b*x)])^2)/(2*a) + (e*h*n*(Log[c*(a + b*x)] - Log[-((
e*(a + b*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x
)))] + Log[1 - c*(a + b*x)])^2)/(2*d) + (e*h*n*(Log[1 + (b*x)/a] + Log[(1 -
 a*c)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))]
)*Log[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(2*d) + (e*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(2*d)
+ (e*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyL
og[2, -((b*x)/a)])/d - (b*g*PolyLog[2, c*(a + b*x)])/a + (e*h*n*Log[x]*Poly
Log[2, c*(a + b*x)])/d - (e*h*n*Log[d + e*x]*PolyLog[2, c*(a + b*x)])/d + (
b*h*(n*Log[d + e*x] - Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/a - ((g
+ h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x - (b*g*PolyLog[2, 1 - (b
*c*x)/(1 - a*c)])/a - (b*h*n*(Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(
1 - a*c - b*c*x))])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a + (b*h*(n*Log[d +
e*x] - Log[f*(d + e*x)^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*Lo
g[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c
*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a
*c - b*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/a + (b*
h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*
x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/a - (e*h*n*(Log[(b*(d + e*x))/(
(b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2, (b*(d +
e*x))/(b*d - a*e)])/d - (b*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/a + (e*h*n*Log[-((a*
(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/d -
(e*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(1 - c*(a +
 b*x)))])/d + (b*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*
(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/a - (e*h*n*(Log[d + e*x] - Log[(
b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])
/d + (e*h*n*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c
*(a + b*x)])/d - (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*
PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/a + (e*h*n*Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))
/(b*c*(d + e*x)))])/d + (b*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b
*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*
n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e
)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d - (e*h*n*PolyLog[3, -((b*x)/a)])/d +
 (b*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/a - (b*h*n*PolyLog[3, (d*(1 - a*
```

```
c - b*c*x))/((1 - a*c)*(d + e*x))])/a + (b*h*n*PolyLog[3, -((e*(1 - a*c - b
*c*x))/(b*c*(d + e*x)))])/a - (b*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])
/a + (e*h*n*PolyLog[3, (b*(d + e*x))/(b*d - a*e)])/d + (b*h*n*PolyLog[3, 1
+ (e*x)/d])/a + (e*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/d - (e*h
*n*PolyLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/d - (b*h*n*PolyLog[3, 1 - c*(a
 + b*x)])/a - (b*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/
a + (e*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/d + (b*h*n
*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/a - (e*h*n*Poly
Log[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/d
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2438

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
```

```
x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))}{x^2}\,dx = -\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))}{x} - b\int\left(\frac{\log(1-ac-bcx)\left(g+\right.}{ax}\right.$$

$$= -\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{Li}_2(c(a+bx))}{x} - \frac{b\int\frac{\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)\right.\right.}{x}}{a}$$

$$= \frac{ehn\log(x)\text{Li}_2(c(a+bx))}{d} - \frac{ehn\log(d+ex)\text{Li}_2(c(a+bx))}{d} - \frac{\left(g+h\log\left(\right.\right.}{}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} + \frac{ehn\log(x)\text{Li}_2(c(a+bx))}{d} - \frac{ehn\log(}{}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{bhn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log(}{a}$$

$$= -\frac{bg\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{a} - \frac{bhn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log(}{a}$$

Mathematica [F] time = 10.1921, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\text{PolyLog}(2,c(a+bx))}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2,x]
```

```
[Out] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^2, x]
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\ln\left(f\left(ex+d\right)^n\right)\right)\text{polylog}\left(2,c\left(bx+a\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left((ex+d)^n f\right)+g\right)\mathrm{Li}_2\left((bx+a)c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="max
ima")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{h\mathrm{Li}_2\left(bcx+ac\right)\log\left((ex+d)^n f\right)+g\mathrm{Li}_2\left(bcx+ac\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/x
^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left((ex+d)^n f\right)+g\right)\text{Li}_2\left((bx+a)c\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^2,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^2, x)
```

3.182 $\int \frac{\left(g+h \log \left(f(d+ex)^n\right)\right) \textbf{PolyLog}(2,c(a+bx))}{x^3} \, dx$

Optimal. Leaf size=3119

result too large to display

```
[Out] (b^2*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b*e*h*n*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(a*d) + (b^2*h*n*Log[(b*c*x)/(1 -
a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/(2*a^2) + (b*e*h*n*Log[1 - a*c - b
*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*h*n*(Log[(b*
c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d +
e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c
 - b*c*x))]^2)/(4*a^2) - (b^2*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]
)*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])
^2)/(4*a^2) - (b^2*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d +
 e*x] - Log[f*(d + e*x)^n]))/(2*a^2) + (b^2*c*Log[-((e*x)/d)]*(g + h*Log[f*
(d + e*x)^n]))/(2*a*(1 - a*c)) + (b*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
e*x)^n]))/(2*a*x) - (b^2*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(
g + h*Log[f*(d + e*x)^n]))/(2*a*(1 - a*c)) - (b^2*h*n*(Log[c*(a + b*x)] + L
og[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x
))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4
*a^2) + (e^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)
)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*d^2) - (e^2*h*n*Log[x]*Log[1 + (b*x)
/a]*Log[1 - c*(a + b*x)])/(2*d^2) - (b^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*
Log[1 - c*(a + b*x)])/(2*a^2) + (e^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[
1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x)
)/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[
1 - c*(a + b*x)])^2)/(4*a^2) - (e^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b
*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + L
og[1 - c*(a + b*x)])^2)/(4*d^2) - (e^2*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c
)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Lo
g[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2) - (e^2*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(4*d^2
) - (e^2*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, -((b*x)/a)])/(2*d^2) + (b^2*g*PolyLog[2, c*(a + b*x)])/(2*a^2) -
(b*e*h*n*PolyLog[2, c*(a + b*x)])/(2*a*d) - (e*h*n*PolyLog[2, c*(a + b*x)])
/(2*d*x) - (e^2*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(2*d^2) + (e^2*h*n*Log[
d + e*x]*PolyLog[2, c*(a + b*x)])/(2*d^2) - (b^2*h*(n*Log[d + e*x] - Log[f*
(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*a^2) - ((g + h*Log[f*(d + e*x)^n]
)*PolyLog[2, c*(a + b*x)])/(2*x^2) + (b*e*h*n*PolyLog[2, (e*(1 - a*c - b*c*
x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
])/(2*a^2) - (b*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(a*d) + (b^2*h*n*(
```

```
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(2*a^2) - (b^2*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) - (b^
2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) + (e^2*h*n*(Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(2*d^2) - (b^2*c*h*n*PolyLog[2, (b*c*(d + e*x
))/(b*c*d + e - a*c*e)])/(2*a*(1 - a*c)) + (b^2*c*h*n*PolyLog[2, 1 + (e*x)/
d])/(2*a*(1 - a*c)) + (b^2*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(2*a^2) - (e^2*h*n*L
og[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x)))
)])/(2*d^2) + (e^2*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*
c*x)/(1 - c*(a + b*x)))])/(2*d^2) - (b^2*h*n*(Log[d + e*x] - Log[(b*(d + e*
x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*a^2)
+ (e^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) - (e^2*h*n*(Log[x] + Log[-((a*(1 -
 c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a +
 b*x)))/(b*c*(d + e*x)))])/(2*a^2) - (e^2*h*n*Log[(b*(d + e*x))/((b*d - a*e
)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])
/(2*d^2) - (b^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*Poly
Log[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*a^2) + (e^2*h*n*L
og[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(
1 - c*(a + b*x)))/(b*(d + e*x))])/(2*d^2) + (e^2*h*n*PolyLog[3, -((b*x)/a)]
)/(2*d^2) - (b^2*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*
PolyLog[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/(2*a^2) - (b^2*h*n
*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) + (b^2*h*n*P
olyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) - (e^2*h*n*PolyLog[3, (b*(d +
 e*x))/(b*d - a*e)])/(2*d^2) - (b^2*h*n*PolyLog[3, 1 + (e*x)/d])/(2*a^2) -
(e^2*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/(2*d^2) + (e^2*h*n*Pol
yLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/(2*d^2) + (b^2*h*n*PolyLog[3, 1 - c*
(a + b*x)])/(2*a^2) + (b^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d +
 e*x)))])/(2*a^2) - (e^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e
*x)))])/(2*d^2) - (b^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d
 + e*x))])/(2*a^2) + (e^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b
*(d + e*x))])/(2*d^2)
```

Rubi [A] time = 3.23828, antiderivative size = 3119, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.593, Rules used = {6603, 2439, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 2438, 2437, 2435, 2440, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

```
[Out] (b^2*g*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2) - (b*e*h*n*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(a*d) + (b^2*h*n*Log[(b*c*x)/(1 -
a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])/(2*a^2) + (b*e*h*n*Log[1 - a*c - b
*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*h*n*(Log[(b*
c*x)/(1 - a*c)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d +
e - a*c*e)*x)/((1 - a*c)*(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c
 - b*c*x))]^2)/(4*a^2) - (b^2*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)]
)*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])
^2)/(4*a^2) - (b^2*h*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d +
 e*x] - Log[f*(d + e*x)^n]))/(2*a^2) + (b^2*c*Log[-((e*x)/d)]*(g + h*Log[f*
(d + e*x)^n]))/(2*a*(1 - a*c)) + (b*Log[1 - a*c - b*c*x]*(g + h*Log[f*(d +
e*x)^n]))/(2*a*x) - (b^2*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(
g + h*Log[f*(d + e*x)^n]))/(2*a*(1 - a*c)) - (b^2*h*n*(Log[c*(a + b*x)] + L
og[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x
))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4
*a^2) + (e^2*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x)
)] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/
((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(4*d^2) - (e^2*h*n*Log[x]*Log[1 + (b*x)
/a]*Log[1 - c*(a + b*x)])/(2*d^2) - (b^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*
Log[1 - c*(a + b*x)])/(2*a^2) + (e^2*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[
1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x)
)/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[
1 - c*(a + b*x)])^2)/(4*a^2) - (e^2*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b
*x))/(b*d - a*e))])*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + L
og[1 - c*(a + b*x)])^2)/(4*d^2) - (e^2*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c
)/(1 - c*(a + b*x))] - Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Lo
g[-((a*(1 - c*(a + b*x)))/(b*x))]^2)/(4*d^2) - (e^2*h*n*(Log[c*(a + b*x)] -
 Log[1 + (b*x)/a])*(Log[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(4*d^2
) - (e^2*h*n*(Log[1 - c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*P
olyLog[2, -((b*x)/a)])/(2*d^2) + (b^2*g*PolyLog[2, c*(a + b*x)])/(2*a^2) -
(b*e*h*n*PolyLog[2, c*(a + b*x)])/(2*a*d) - (e*h*n*PolyLog[2, c*(a + b*x)])
/(2*d*x) - (e^2*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(2*d^2) + (e^2*h*n*Log[
d + e*x]*PolyLog[2, c*(a + b*x)])/(2*d^2) - (b^2*h*(n*Log[d + e*x] - Log[f*
(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(2*a^2) - ((g + h*Log[f*(d + e*x)^n]
)*PolyLog[2, c*(a + b*x)])/(2*x^2) + (b*e*h*n*PolyLog[2, (e*(1 - a*c - b*c*
x))/(b*c*d + e - a*c*e)])/(2*a*d) + (b^2*g*PolyLog[2, 1 - (b*c*x)/(1 - a*c)
```

```
])/(2*a^2) - (b*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(a*d) + (b^2*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
 1 - (b*c*x)/(1 - a*c)])/(2*a^2) - (b^2*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(2*a^2) - (b^2*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) - (b^
2*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) + (e^2*h*n*(Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(2*d^2) - (b^2*c*h*n*PolyLog[2, (b*c*(d + e*x
))/(b*c*d + e - a*c*e)])/(2*a*(1 - a*c)) + (b^2*c*h*n*PolyLog[2, 1 + (e*x)/
d])/(2*a*(1 - a*c)) + (b^2*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d +
e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(2*a^2) - (e^2*h*n*L
og[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x)))
)])/(2*d^2) + (e^2*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*
c*x)/(1 - c*(a + b*x)))])/(2*d^2) - (b^2*h*n*(Log[d + e*x] - Log[(b*(d + e*
x))/((b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(2*a^2)
 + (e^2*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x))
)])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) - (e^2*h*n*(Log[x] + Log[-((a*(1 -
 c*(a + b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(2*d^2) + (b^2*h*n*Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a +
 b*x)))/(b*c*(d + e*x)))])/(2*a^2) - (e^2*h*n*Log[(b*(d + e*x))/((b*d - a*e
)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])
/(2*d^2) - (b^2*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*Poly
Log[2, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(2*a^2) + (e^2*h*n*L
og[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(
1 - c*(a + b*x)))/(b*(d + e*x))])/(2*d^2) + (e^2*h*n*PolyLog[3, -((b*x)/a)]
)/(2*d^2) - (b^2*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/(2*a^2) + (b^2*h*n*
PolyLog[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/(2*a^2) - (b^2*h*n
*PolyLog[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(2*a^2) + (b^2*h*n*P
olyLog[3, (b*(d + e*x))/(b*d - a*e)])/(2*a^2) - (e^2*h*n*PolyLog[3, (b*(d +
 e*x))/(b*d - a*e)])/(2*d^2) - (b^2*h*n*PolyLog[3, 1 + (e*x)/d])/(2*a^2) -
(e^2*h*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/(2*d^2) + (e^2*h*n*Pol
yLog[3, -((b*c*x)/(1 - c*(a + b*x)))])/(2*d^2) + (b^2*h*n*PolyLog[3, 1 - c*
(a + b*x)])/(2*a^2) + (b^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d +
 e*x)))])/(2*a^2) - (e^2*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e
*x)))])/(2*d^2) - (b^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d
 + e*x))])/(2*a^2) + (e^2*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b
*(d + e*x))])/(2*d^2)
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
```

```
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2438

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
```

```
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{\left(g + h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{x^3}\,dx = -\frac{\left(g + h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{1}{2}b\int\left(\frac{\log(1-ac-bcx)\left(g+\right.}{ax}\right.$$

$$= -\frac{\left(g + h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{2x^2} - \frac{b\int \frac{\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)\right.\right.}{x^2}}{2a}$$

$$= \frac{b\log(1-ac-bcx)\left(g + h\log\left(f(d+ex)^n\right)\right)}{2ax} - \frac{ehn\mathrm{Li}_2(c(a+bx))}{2dx} - \frac{e^2hn}{}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b\log(1-ac-bcx)\left(g+h\log\left(f(d+\right.\right.}{2ax}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log}{2a^2}$$

$$= \frac{b^2g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2} + \frac{b^2hn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log}{2a^2}$$

Mathematica [F] time = 10.4117, size = 0, normalized size = 0.

$$\int \frac{\left(g + h\log\left(f(d+ex)^n\right)\right)\mathrm{PolyLog}(2, c(a+bx))}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3,x]
```

```
[Out] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^3, x]
```

Maple [F] time = 0.521, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\ln\left(f\,(ex+d)^{n}\right)\right)\operatorname{polylog}(2,c\,(bx+a))}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left((ex+d)^{n}f\right)+g\right)\mathrm{Li}_2\left((bx+a)c\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="max
ima")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{h\mathrm{Li}_2\left(bcx+ac\right)\log\left((ex+d)^{n}f\right)+g\mathrm{Li}_2\left(bcx+ac\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/x
^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left((ex+d)^n f\right)+g\right)\mathrm{Li}_2\left((bx+a)c\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^3,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^3, x)
```

3.183 $\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathbf{PolyLog}(2,c(a+bx))}{x^4}\,dx$

Optimal. Leaf size=3733

result too large to display

```
[Out] (b^2*c*e*h*n*Log[x])/(2*a*(1 - a*c)*d) - (b^2*c*e*h*n*Log[1 - a*c - b*c*x])
/(3*a*(1 - a*c)*d) + (b*e*h*n*Log[1 - a*c - b*c*x])/(3*a*d*x) - (b^3*g*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) + (b^2*e*h*n*Log[(b*c*x)/(
1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2*d) + (b*e^2*h*n*Log[(b*c*x)/(1 - a*c
)]*Log[1 - a*c - b*c*x])/(2*a*d^2) - (b^2*c*e*h*n*Log[d + e*x])/(6*a*(1 - a
*c)*d) - (b^3*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])
/(3*a^3) - (b^2*e*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(3*a^2*d) - (b*e^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(
b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*
(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) +
(b^3*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*c - b*c*x] +
 Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(6*a^3) + (b^3*h*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n
]))/(3*a^3) - (b^2*c*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)*x) + (b^3*c
^2*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) - (b^3*c*L
og[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*(1 - a*c)) + (b*Log[1 - a
*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*
x]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*x) - (b^3*c^2*Log[(e*(1 - a*c - b*c*x
))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) + (b^
3*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(3*a^2*(1 - a*c)) + (b^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e
)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*a^3) - (e^3*h*n*(Log
[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e
- a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(
a + b*x)))]^2)/(6*d^3) + (e^3*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*
x)])/(3*d^3) + (b^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(3*a^3) - (e^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*
d^3) - (b^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/
(6*a^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(
Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^
2)/(6*d^3) + (e^3*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))]
- Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(1 - c*(a + b*
x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(L
og[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(6*d^3) + (e^3*h*n*(Log[1 -
```

c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)]
)/(3*d^3) - (b^3*g*PolyLog[2, c*(a + b*x)])/(3*a^3) + (b^2*e*h*n*PolyLog[2,
c*(a + b*x)])/(6*a^2*d) + (b*e^2*h*n*PolyLog[2, c*(a + b*x)])/(3*a*d^2) -
(e*h*n*PolyLog[2, c*(a + b*x)])/(6*d*x^2) + (e^2*h*n*PolyLog[2, c*(a + b*x)
])/(3*d^2*x) + (e^3*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(3*d^3) - (e^3*h*n*
Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*d^3) + (b^3*h*(n*Log[d + e*x] - Lo
g[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*a^3) - ((g + h*Log[f*(d + e*x
)^n])*PolyLog[2, c*(a + b*x)])/(3*x^3) - (b^2*e*h*n*PolyLog[2, (e*(1 - a*c
- b*c*x))/(b*c*d + e - a*c*e)])/(3*a^2*d) - (b*e^2*h*n*PolyLog[2, (e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*g*PolyLog[2, 1 - (b*c*x
)/(1 - a*c)])/(3*a^3) + (b^2*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^
2*d) + (b*e^2*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a*d^2) - (b^3*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
1 - (b*c*x)/(1 - a*c)])/(3*a^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(3*a^3) - (b^3*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(3*a^3) + (b^3*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(3*a^3) + (b^
3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*a^3) - (e^3*h*n*(Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(3*d^3) - (b^3*c^2*h*n*PolyLog[2, (b*c*(d + e
*x))/(b*c*d + e - a*c*e)])/(6*a*(1 - a*c)^2) + (b^3*c*h*n*PolyLog[2, (b*c*(
d + e*x))/(b*c*d + e - a*c*e)])/(3*a^2*(1 - a*c)) + (b^3*c^2*h*n*PolyLog[2,
1 + (e*x)/d])/(6*a*(1 - a*c)^2) - (b^3*c*h*n*PolyLog[2, 1 + (e*x)/d])/(3*a
^2*(1 - a*c)) - (b^3*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/
(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(3*a^3) + (e^3*h*n*Log[-((
a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(3
*d^3) - (e^3*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(
1 - c*(a + b*x)))])/(3*d^3) + (b^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((
b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(3*a^3) - (e^3
*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*Po
lyLog[2, 1 - c*(a + b*x)])/(3*d^3) + (e^3*h*n*(Log[x] + Log[-((a*(1 - c*(a
+ b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(3*d^3) - (b^3*h*n*Log[(b*(d
+ e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))])/(3*a^3) + (e^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(3*d^
3) + (b^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(3*a^3) - (e^3*h*n*Log[(b*
(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*
(a + b*x)))/(b*(d + e*x))])/(3*d^3) - (e^3*h*n*PolyLog[3, -((b*x)/a)])/(3*d
^3) + (b^3*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/(3*a^3) - (b^3*h*n*PolyLo
g[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/(3*a^3) + (b^3*h*n*PolyL
og[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(3*a^3) - (b^3*h*n*PolyLog
[3, (b*(d + e*x))/(b*d - a*e)])/(3*a^3) + (e^3*h*n*PolyLog[3, (b*(d + e*x))

```
/(b*d - a*e)])/(3*d^3) + (b^3*h*n*PolyLog[3, 1 + (e*x)/d])/(3*a^3) + (e^3*h
*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/(3*d^3) - (e^3*h*n*PolyLog[3
, -((b*c*x)/(1 - c*(a + b*x)))])/(3*d^3) - (b^3*h*n*PolyLog[3, 1 - c*(a + b
*x)])/(3*a^3) - (b^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))
)])/(3*a^3) + (e^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]
)/(3*d^3) + (b^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x
))])/(3*a^3) - (e^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d +
e*x))])/(3*d^3)
```

Rubi [A] time = 4.30785, antiderivative size = 3733, normalized size of antiderivative = 1., number of steps used = 78, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6603, 2439, 44, 2416, 2395, 36, 29, 31, 2394, 2315, 2393, 2391, 2438, 2437, 2435, 2440, 6598, 6597}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

```
[Out] (b^2*c*e*h*n*Log[x])/(2*a*(1 - a*c)*d) - (b^2*c*e*h*n*Log[1 - a*c - b*c*x])
/(3*a*(1 - a*c)*d) + (b*e*h*n*Log[1 - a*c - b*c*x])/(3*a*d*x) - (b^3*g*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x])/(3*a^3) + (b^2*e*h*n*Log[(b*c*x)/(
1 - a*c)]*Log[1 - a*c - b*c*x])/(2*a^2*d) + (b*e^2*h*n*Log[(b*c*x)/(1 - a*c
)]*Log[1 - a*c - b*c*x])/(2*a*d^2) - (b^2*c*e*h*n*Log[d + e*x])/(6*a*(1 - a
*c)*d) - (b^3*h*n*Log[(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*Log[d + e*x])
/(3*a^3) - (b^2*e*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(b*c*d + e -
 a*c*e)])/(3*a^2*d) - (b*e^2*h*n*Log[1 - a*c - b*c*x]*Log[(b*c*(d + e*x))/(
b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*h*n*(Log[(b*c*x)/(1 - a*c)] + Log[(b*
c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*x)/((1 - a*c)*
(d + e*x))])*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))]^2)/(6*a^3) +
(b^3*h*n*(Log[(b*c*x)/(1 - a*c)] - Log[-((e*x)/d)])*(Log[1 - a*c - b*c*x] +
 Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])^2)/(6*a^3) + (b^3*h*Log[
(b*c*x)/(1 - a*c)]*Log[1 - a*c - b*c*x]*(n*Log[d + e*x] - Log[f*(d + e*x)^n
]))/(3*a^3) - (b^2*c*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)*x) + (b^3*c
^2*Log[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) - (b^3*c*L
og[-((e*x)/d)]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*(1 - a*c)) + (b*Log[1 - a
*c - b*c*x]*(g + h*Log[f*(d + e*x)^n]))/(6*a*x^2) - (b^2*Log[1 - a*c - b*c*
x]*(g + h*Log[f*(d + e*x)^n]))/(3*a^2*x) - (b^3*c^2*Log[(e*(1 - a*c - b*c*x
))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n]))/(6*a*(1 - a*c)^2) + (b^
3*c*Log[(e*(1 - a*c - b*c*x))/(b*c*d + e - a*c*e)]*(g + h*Log[f*(d + e*x)^n
]))/(3*a^2*(1 - a*c)) + (b^3*h*n*(Log[c*(a + b*x)] + Log[(b*c*d + e - a*c*e
```

```
)/(b*c*(d + e*x))] - Log[((b*c*d + e - a*c*e)*(a + b*x))/(b*(d + e*x))])*Lo
g[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]^2)/(6*a^3) - (e^3*h*n*(Log
[c*(a + b*x)] + Log[(b*c*d + e - a*c*e)/(b*c*(d + e*x))] - Log[((b*c*d + e
- a*c*e)*(a + b*x))/(b*(d + e*x))])*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(
a + b*x)))]^2)/(6*d^3) + (e^3*h*n*Log[x]*Log[1 + (b*x)/a]*Log[1 - c*(a + b*
x)])/(3*d^3) + (b^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])
/(3*a^3) - (e^3*h*n*Log[c*(a + b*x)]*Log[d + e*x]*Log[1 - c*(a + b*x)])/(3*
d^3) - (b^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(Log
[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^2)/
(6*a^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[-((e*(a + b*x))/(b*d - a*e))])*(
Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])^
2)/(6*d^3) + (e^3*h*n*(Log[1 + (b*x)/a] + Log[(1 - a*c)/(1 - c*(a + b*x))]
- Log[((1 - a*c)*(a + b*x))/(a*(1 - c*(a + b*x)))])*Log[-((a*(1 - c*(a + b*
x)))/(b*x))]^2)/(6*d^3) + (e^3*h*n*(Log[c*(a + b*x)] - Log[1 + (b*x)/a])*(L
og[x] + Log[-((a*(1 - c*(a + b*x)))/(b*x))])^2)/(6*d^3) + (e^3*h*n*(Log[1 -
 c*(a + b*x)] - Log[-((a*(1 - c*(a + b*x)))/(b*x))])*PolyLog[2, -((b*x)/a)]
)/(3*d^3) - (b^3*g*PolyLog[2, c*(a + b*x)])/(3*a^3) + (b^2*e*h*n*PolyLog[2,
 c*(a + b*x)])/(6*a^2*d) + (b*e^2*h*n*PolyLog[2, c*(a + b*x)])/(3*a*d^2) -
(e*h*n*PolyLog[2, c*(a + b*x)])/(6*d*x^2) + (e^2*h*n*PolyLog[2, c*(a + b*x)
])/(3*d^2*x) + (e^3*h*n*Log[x]*PolyLog[2, c*(a + b*x)])/(3*d^3) - (e^3*h*n*
Log[d + e*x]*PolyLog[2, c*(a + b*x)])/(3*d^3) + (b^3*h*(n*Log[d + e*x] - Lo
g[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(3*a^3) - ((g + h*Log[f*(d + e*x
)^n])*PolyLog[2, c*(a + b*x)])/(3*x^3) - (b^2*e*h*n*PolyLog[2, (e*(1 - a*c
- b*c*x))/(b*c*d + e - a*c*e)])/(3*a^2*d) - (b*e^2*h*n*PolyLog[2, (e*(1 - a
*c - b*c*x))/(b*c*d + e - a*c*e)])/(6*a*d^2) - (b^3*g*PolyLog[2, 1 - (b*c*x
)/(1 - a*c)])/(3*a^3) + (b^2*e*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a^
2*d) + (b*e^2*h*n*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(2*a*d^2) - (b^3*h*n*(
Log[d + e*x] - Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b*c*x))])*PolyLog[2,
 1 - (b*c*x)/(1 - a*c)])/(3*a^3) + (b^3*h*(n*Log[d + e*x] - Log[f*(d + e*x)
^n])*PolyLog[2, 1 - (b*c*x)/(1 - a*c)])/(3*a^3) - (b^3*h*n*Log[((1 - a*c)*(
d + e*x))/(d*(1 - a*c - b*c*x))]*PolyLog[2, (d*(1 - a*c - b*c*x))/((1 - a*c
)*(d + e*x))])/(3*a^3) + (b^3*h*n*Log[((1 - a*c)*(d + e*x))/(d*(1 - a*c - b
*c*x))]*PolyLog[2, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(3*a^3) + (b^
3*h*n*(Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a +
b*x)])*PolyLog[2, (b*(d + e*x))/(b*d - a*e)])/(3*a^3) - (e^3*h*n*(Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))] + Log[1 - c*(a + b*x)])*PolyLog[2
, (b*(d + e*x))/(b*d - a*e)])/(3*d^3) - (b^3*c^2*h*n*PolyLog[2, (b*c*(d + e
*x))/(b*c*d + e - a*c*e)])/(6*a*(1 - a*c)^2) + (b^3*c*h*n*PolyLog[2, (b*c*(
d + e*x))/(b*c*d + e - a*c*e)])/(3*a^2*(1 - a*c)) + (b^3*c^2*h*n*PolyLog[2,
 1 + (e*x)/d])/(6*a*(1 - a*c)^2) - (b^3*c*h*n*PolyLog[2, 1 + (e*x)/d])/(3*a
^2*(1 - a*c)) - (b^3*h*n*(Log[1 - a*c - b*c*x] + Log[((1 - a*c)*(d + e*x))/
(d*(1 - a*c - b*c*x))])*PolyLog[2, 1 + (e*x)/d])/(3*a^3) + (e^3*h*n*Log[-((
a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*x)/(a*(1 - c*(a + b*x))))])/(3
*d^3) - (e^3*h*n*Log[-((a*(1 - c*(a + b*x)))/(b*x))]*PolyLog[2, -((b*c*x)/(
1 - c*(a + b*x)))])/(3*d^3) + (b^3*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((
```

```
b*d - a*e)*(1 - c*(a + b*x)))])*PolyLog[2, 1 - c*(a + b*x)])/(3*a^3) - (e^3
*h*n*(Log[d + e*x] - Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))])*Po
lyLog[2, 1 - c*(a + b*x)])/(3*d^3) + (e^3*h*n*(Log[x] + Log[-((a*(1 - c*(a
+ b*x)))/(b*x))])*PolyLog[2, 1 - c*(a + b*x)])/(3*d^3) - (b^3*h*n*Log[(b*(d
 + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x))
)/(b*c*(d + e*x)))])/(3*a^3) + (e^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 -
 c*(a + b*x)))]*PolyLog[2, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))])/(3*d^
3) + (b^3*h*n*Log[(b*(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2,
 ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x))])/(3*a^3) - (e^3*h*n*Log[(b*
(d + e*x))/((b*d - a*e)*(1 - c*(a + b*x)))]*PolyLog[2, ((b*d - a*e)*(1 - c*
(a + b*x)))/(b*(d + e*x))])/(3*d^3) - (e^3*h*n*PolyLog[3, -((b*x)/a)])/(3*d
^3) + (b^3*h*n*PolyLog[3, 1 - (b*c*x)/(1 - a*c)])/(3*a^3) - (b^3*h*n*PolyLo
g[3, (d*(1 - a*c - b*c*x))/((1 - a*c)*(d + e*x))])/(3*a^3) + (b^3*h*n*PolyL
og[3, -((e*(1 - a*c - b*c*x))/(b*c*(d + e*x)))])/(3*a^3) - (b^3*h*n*PolyLog
[3, (b*(d + e*x))/(b*d - a*e)])/(3*a^3) + (e^3*h*n*PolyLog[3, (b*(d + e*x))
/(b*d - a*e)])/(3*d^3) + (b^3*h*n*PolyLog[3, 1 + (e*x)/d])/(3*a^3) + (e^3*h
*n*PolyLog[3, -((b*x)/(a*(1 - c*(a + b*x))))])/(3*d^3) - (e^3*h*n*PolyLog[3
, -((b*c*x)/(1 - c*(a + b*x)))])/(3*d^3) - (b^3*h*n*PolyLog[3, 1 - c*(a + b
*x)])/(3*a^3) - (b^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x))
)])/(3*a^3) + (e^3*h*n*PolyLog[3, -((e*(1 - c*(a + b*x)))/(b*c*(d + e*x)))]
)/(3*d^3) + (b^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d + e*x
))])/(3*a^3) - (e^3*h*n*PolyLog[3, ((b*d - a*e)*(1 - c*(a + b*x)))/(b*(d +
e*x))])/(3*d^3)
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] :> Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
 + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2438

```
Int[(((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(Log[(h_.)*((i_.)
 + (j_.)*(x_))^(m_.)]*(g_.) + (f_)))/(x_), x_Symbol] :> Dist[f, Int[(a + b*
Log[c*(d + e*x)^n])/x, x], x] + Dist[g, Int[(Log[h*(i + j*x)^m]*(a + b*Log[
c*(d + e*x)^n]))/x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x
] && NeQ[e*i - d*j, 0]
```

Rule 2437

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*Log[(h_.)*((i_.) + (j_.)*(x_))^(m
_.)])/(x_), x_Symbol] :> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x
], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x,
 x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i
 + j*x, h*(i + j*x)^m]
```

Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)]*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol] :> Simp
[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)]
- Log[-(((b*c - a*d)*x)/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*L
og[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-
((d*x)/c)])*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + Si
mp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1 + (b*x)/a
], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x))])*PolyLog[2, 1
 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a +
b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp
[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))]
, x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c,
```

```
d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :>
 Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
 g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 6597

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, Int[(Log[d
 + e*x]*Log[1 - a*c - b*c*x])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x
] && NeQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{x^4}\,dx = -\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{1}{3}b\int\left(\frac{\log(1-ac-bcx)\left(g+\right.}{ax^3}\right.$$

$$= -\frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{Li}_2(c(a+bx))}{3x^3} - \frac{b\int\frac{\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right.}{x^3}}{3a}$$

$$= \frac{b\log(1-ac-bcx)\left(g+h\log\left(f(d+ex)^n\right)\right)}{6ax^2} - \frac{b^2\log(1-ac-bcx)\left(g+h\right.}{3a^2x}$$

$$= -\frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} + \frac{b\log(1-ac-bcx)\left(g+h\log\left(f(d+\right.\right.}{6ax^2}$$

$$= -\frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} - \frac{b^3hn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)\log(}{3a^3}$$

$$= -\frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} + \frac{b^2ehn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2d} +$$

$$= -\frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} + \frac{b^2ehn\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{2a^2d} +$$

$$= \frac{b^2cehn\log(x)}{6a(1-ac)d} - \frac{b^3g\log\left(\frac{bcx}{1-ac}\right)\log(1-ac-bcx)}{3a^3} + \frac{b^2ehn\log\left(\frac{bcx}{1-ac}\right)\log(1}{2a^2d}$$

Mathematica [F] time = 10.5193, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\log\left(f(d+ex)^n\right)\right)\mathrm{PolyLog}(2,c(a+bx))}{x^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4,x]
```

```
[Out] Integrate[((g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/x^4, x]
```

Maple [F] time = 0.537, size = 0, normalized size = 0.

$$\int \frac{\left(g+h\ln\left(f\,(ex+d)^{n}\right)\right)\operatorname{polylog}\left(2,c\,(bx+a)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
```

```
[Out] int((g+h*ln(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h\log\left((ex+d)^{n}f\right)+g\right)\mathrm{Li}_2\left((bx+a)c\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="max
ima")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{h\mathrm{Li}_2\left(bcx+ac\right)\log\left((ex+d)^{n}f\right)+g\mathrm{Li}_2\left(bcx+ac\right)}{x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="fri
cas")
```

```
[Out] integral((h*dilog(b*c*x + a*c)*log((e*x + d)^n*f) + g*dilog(b*c*x + a*c))/x
^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*ln(f*(e*x+d)**n))*polylog(2,c*(b*x+a))/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(h \log \left((ex+d)^n f\right)+g\right) \mathrm{Li}_2\left((bx+a)c\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g+h*log(f*(e*x+d)^n))*polylog(2,c*(b*x+a))/x^4,x, algorithm="gia
c")
```

```
[Out] integrate((h*log((e*x + d)^n*f) + g)*dilog((b*x + a)*c)/x^4, x)
```

3.184 $\int x^2(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=661

$$-\frac{x^2(4ac+3b)\text{PolyLog}(2,cx)}{24c^2}-\frac{x(4ac+3b)\text{PolyLog}(2,cx)}{12c^3}+\frac{(4ac+3b)\text{PolyLog}(3,1-cx)}{6c^4}-\frac{(4ac+3b)\log(1-cx)\text{P}}{12c^4}$$

```
[Out] (53*b*x)/(192*c^3) + (11*a*x)/(27*c^2) + (49*(3*b + 4*a*c)*x)/(432*c^3) + (
29*b*x^2)/(384*c^2) + (5*a*x^2)/(54*c) + (13*(3*b + 4*a*c)*x^2)/(864*c^2) +
 (2*a*x^3)/81 + (17*b*x^3)/(576*c) + ((3*b + 4*a*c)*x^3)/(324*c) + (3*b*x^4
)/256 + (29*b*Log[1 - c*x])/(192*c^4) + (5*a*Log[1 - c*x])/(27*c^3) + (13*(
3*b + 4*a*c)*Log[1 - c*x])/(432*c^4) - (b*x^2*Log[1 - c*x])/(16*c^2) - (a*x
^2*Log[1 - c*x])/(9*c) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^2) - (2*a*x
^3*Log[1 - c*x])/27 - (b*x^3*Log[1 - c*x])/(24*c) - ((3*b + 4*a*c)*x^3*Log[
1 - c*x])/(108*c) - (3*b*x^4*Log[1 - c*x])/64 + (b*(1 - c*x)*Log[1 - c*x])/
(8*c^4) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) + ((3*b + 4*a*c)*(1 - c*x)*L
og[1 - c*x])/(12*c^4) - (b*Log[1 - c*x]^2)/(16*c^4) - (a*Log[1 - c*x]^2)/(9
*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]^2)/16 - ((3*b + 4*a*
c)*Log[c*x]*Log[1 - c*x]^2)/(12*c^4) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(1
2*c^3) - ((3*b + 4*a*c)*x^2*PolyLog[2, c*x])/(24*c^2) - ((3*b + 4*a*c)*x^3*
PolyLog[2, c*x])/(36*c) - (b*x^4*PolyLog[2, c*x])/16 - ((3*b + 4*a*c)*Log[1
 - c*x]*PolyLog[2, c*x])/(12*c^4) + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyL
og[2, c*x])/12 - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6*c^4) +
 ((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^4)
```

Rubi [A] time = 0.981897, antiderivative size = 661, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.81, Rules used = {6742, 6591, 2395, 43, 6604, 2398, 2410, 2389, 2295, 2390, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x^2(4ac+3b)\text{PolyLog}(2,cx)}{24c^2}-\frac{x(4ac+3b)\text{PolyLog}(2,cx)}{12c^3}+\frac{(4ac+3b)\text{PolyLog}(3,1-cx)}{6c^4}-\frac{(4ac+3b)\log(1-cx)\text{P}}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] (53*b*x)/(192*c^3) + (11*a*x)/(27*c^2) + (49*(3*b + 4*a*c)*x)/(432*c^3) + (
29*b*x^2)/(384*c^2) + (5*a*x^2)/(54*c) + (13*(3*b + 4*a*c)*x^2)/(864*c^2) +
 (2*a*x^3)/81 + (17*b*x^3)/(576*c) + ((3*b + 4*a*c)*x^3)/(324*c) + (3*b*x^4
)/256 + (29*b*Log[1 - c*x])/(192*c^4) + (5*a*Log[1 - c*x])/(27*c^3) + (13*(
3*b + 4*a*c)*Log[1 - c*x])/(432*c^4) - (b*x^2*Log[1 - c*x])/(16*c^2) - (a*x
```

```
^2*Log[1 - c*x])/(9*c) - ((3*b + 4*a*c)*x^2*Log[1 - c*x])/(48*c^2) - (2*a*x
^3*Log[1 - c*x])/27 - (b*x^3*Log[1 - c*x])/(24*c) - ((3*b + 4*a*c)*x^3*Log[
1 - c*x])/(108*c) - (3*b*x^4*Log[1 - c*x])/64 + (b*(1 - c*x)*Log[1 - c*x])/
(8*c^4) + (2*a*(1 - c*x)*Log[1 - c*x])/(9*c^3) + ((3*b + 4*a*c)*(1 - c*x)*L
og[1 - c*x])/(12*c^4) - (b*Log[1 - c*x]^2)/(16*c^4) - (a*Log[1 - c*x]^2)/(9
*c^3) + (a*x^3*Log[1 - c*x]^2)/9 + (b*x^4*Log[1 - c*x]^2)/16 - ((3*b + 4*a*
c)*Log[c*x]*Log[1 - c*x]^2)/(12*c^4) - ((3*b + 4*a*c)*x*PolyLog[2, c*x])/(1
2*c^3) - ((3*b + 4*a*c)*x^2*PolyLog[2, c*x])/(24*c^2) - ((3*b + 4*a*c)*x^3*
PolyLog[2, c*x])/(36*c) - (b*x^4*PolyLog[2, c*x])/16 - ((3*b + 4*a*c)*Log[1
 - c*x]*PolyLog[2, c*x])/(12*c^4) + ((4*a*x^3 + 3*b*x^4)*Log[1 - c*x]*PolyL
og[2, c*x])/12 - ((3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/(6*c^4) +
 ((3*b + 4*a*c)*PolyLog[3, 1 - c*x])/(6*c^4)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
```

```
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2(a+bx)\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{12}\left(4ax^3+3bx^4\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(\frac{(-3b-4ac)\text{Li}_2(cx)}{12c^4} - \frac{(3b+4ac)x\text{Li}_2\right.$$

$$= \frac{1}{12}\left(4ax^3+3bx^4\right)\log(1-cx)\text{Li}_2(cx) + \frac{1}{3}a\int x^2\log^2(1-cx)\,dx + \frac{1}{4}b\int x^3\log^2(1$$

$$= \frac{1}{9}ax^3\log^2(1-cx) + \frac{1}{16}bx^4\log^2(1-cx) - \frac{(3b+4ac)x\text{Li}_2(cx)}{12c^3} - \frac{(3b+4ac)x^2\text{Li}_2(c}{24c^2}$$

$$= -\frac{(3b+4ac)x^2\log(1-cx)}{48c^2} - \frac{(3b+4ac)x^3\log(1-cx)}{108c} - \frac{1}{64}bx^4\log(1-cx) + \frac{1}{9}ax^3$$

$$= \frac{(3b+4ac)x}{12c^3} - \frac{(3b+4ac)x^2\log(1-cx)}{48c^2} - \frac{(3b+4ac)x^3\log(1-cx)}{108c} - \frac{1}{64}bx^4\log(1$$

$$= \frac{bx}{64c^3} + \frac{49(3b+4ac)x}{432c^3} + \frac{bx^2}{128c^2} + \frac{13(3b+4ac)x^2}{864c^2} + \frac{bx^3}{192c} + \frac{(3b+4ac)x^3}{324c} + \frac{bx^4}{256}$$

$$= \frac{9bx}{64c^3} + \frac{2ax}{9c^2} + \frac{49(3b+4ac)x}{432c^3} + \frac{bx^2}{128c^2} + \frac{13(3b+4ac)x^2}{864c^2} + \frac{bx^3}{192c} + \frac{(3b+4ac)x^3}{324c}$$

$$= \frac{53bx}{192c^3} + \frac{11ax}{27c^2} + \frac{49(3b+4ac)x}{432c^3} + \frac{29bx^2}{384c^2} + \frac{5ax^2}{54c} + \frac{13(3b+4ac)x^2}{864c^2} + \frac{2ax^3}{81} + \frac{17b}{576}$$

Mathematica [A] time = 0.755005, size = 425, normalized size = 0.64

$$\frac{48\text{PolyLog}(2,cx)\left(12\log(1-cx)\left(4ac\left(c^3x^3-1\right)+3b\left(c^4x^4-1\right)\right)-cx\left(8ac\left(2c^2x^2+3cx+6\right)+3b\left(3c^3x^3+4c^2x^2+6c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (4260*b*c*x + 5952*a*c^2*x + 834*b*c^2*x^2 + 1056*a*c^3*x^2 + 268*b*c^3*x^3
 + 256*a*c^4*x^3 + 81*b*c^4*x^4 + 4260*b*Log[1 - c*x] + 5952*a*c*Log[1 - c*
x] - 2592*b*c*x*Log[1 - c*x] - 3840*a*c^2*x*Log[1 - c*x] - 864*b*c^2*x^2*Lo
g[1 - c*x] - 1344*a*c^3*x^2*Log[1 - c*x] - 480*b*c^3*x^3*Log[1 - c*x] - 768
*a*c^4*x^3*Log[1 - c*x] - 324*b*c^4*x^4*Log[1 - c*x] - 432*b*Log[1 - c*x]^2
 - 768*a*c*Log[1 - c*x]^2 + 768*a*c^4*x^3*Log[1 - c*x]^2 + 432*b*c^4*x^4*Lo
g[1 - c*x]^2 - 1728*b*Log[c*x]*Log[1 - c*x]^2 - 2304*a*c*Log[c*x]*Log[1 - c
*x]^2 + 48*(-(c*x*(8*a*c*(6 + 3*c*x + 2*c^2*x^2) + 3*b*(12 + 6*c*x + 4*c^2*
x^2 + 3*c^3*x^3))) + 12*(4*a*c*(-1 + c^3*x^3) + 3*b*(-1 + c^4*x^4))*Log[1 -
 c*x])*PolyLog[2, c*x] - 1152*(3*b + 4*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x
```

```
] + 3456*b*PolyLog[3, 1 - c*x] + 4608*a*c*PolyLog[3, 1 - c*x])/(6912*c^4)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int x^2 (bx + a) \ln (-cx + 1) \operatorname{polylog}(2, cx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)

[Out] int(x^2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.04004, size = 560, normalized size = 0.85

$$-\frac{1}{6912} c \left(\frac{576 \left(\log (cx) \log (-cx + 1)^2 + 2 \operatorname{Li}_2 (-cx + 1) \log (-cx + 1) - 2 \operatorname{Li}_3(-cx + 1)\right)(4 ac + 3 b)}{c^5} - \frac{81 bc^4x^4 + 4 \left(64 ac^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")

[Out] -1/6912*c*(576*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1)
- 2*polylog(3, -c*x + 1))*(4*a*c + 3*b)/c^5 - (81*b*c^4*x^4 + 4*(64*a*c^4 +
 67*b*c^3)*x^3 + 6*(176*a*c^3 + 139*b*c^2)*x^2 + 12*(496*a*c^2 + 355*b*c)*x
 - 48*(9*b*c^4*x^4 + 4*(4*a*c^4 + 3*b*c^3)*x^3 + 6*(4*a*c^3 + 3*b*c^2)*x^2
+ 12*(4*a*c^2 + 3*b*c)*x + 12*(4*a*c + 3*b)*log(-c*x + 1))*dilog(c*x) - 4*(
54*b*c^4*x^4 + 4*(32*a*c^4 + 21*b*c^3)*x^3 + 6*(40*a*c^3 + 27*b*c^2)*x^2 -
1488*a*c + 12*(64*a*c^2 + 45*b*c)*x - 1065*b)*log(-c*x + 1))/c^5) + 1/1728*
(32*(18*c^3*x^3*dilog(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1
)*log(-c*x + 1))*a/c^3 + 9*(48*c^4*x^4*dilog(c*x) - 3*c^4*x^4 - 4*c^3*x^3 -
 6*c^2*x^2 - 12*c*x + 12*(c^4*x^4 - 1)*log(-c*x + 1))*b/c^4)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(bx^3 + ax^2\right)\operatorname{Li}_2 (cx) \log (-cx + 1), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)x^2 \mathrm{Li}_2\,(cx) \log\,(-cx + 1)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x^2*dilog(c*x)*log(-c*x + 1), x)
```

3.185 $\int x(a+bx)\log(1-cx)\text{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=546

$$-\frac{x(3ac+2b)\text{PolyLog}(2,cx)}{6c^2}+\frac{(3ac+2b)\text{PolyLog}(3,1-cx)}{3c^3}-\frac{(3ac+2b)\log(1-cx)\text{PolyLog}(2,cx)}{6c^3}-\frac{(3ac+2b)\log(1$$

```
[Out] (4*b*x)/(9*c^2) + (a*x)/c + (5*(2*b + 3*a*c)*x)/(24*c^2) + (b*x^2)/(9*c) +
((2*b + 3*a*c)*x^2)/(48*c) + (b*x^3)/27 + (a*(1 - c*x)^2)/(8*c^2) + (2*b*Lo
g[1 - c*x])/(9*c^3) + ((2*b + 3*a*c)*Log[1 - c*x])/(24*c^3) - (b*x^2*Log[1
- c*x])/(9*c) - ((2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c) - (b*x^3*Log[1 - c*
x])/9 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1 - c*x)*Log[1 - c*x])/c
^2 + ((2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^3) - (a*(1 - c*x)^2*Log[1
- c*x])/(4*c^2) - (b*Log[1 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (
a*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + (a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2
) - ((2*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/(6*c^3) - ((2*b + 3*a*c)*x*Poly
Log[2, c*x])/(6*c^2) - ((2*b + 3*a*c)*x^2*PolyLog[2, c*x])/(12*c) - (b*x^3*
PolyLog[2, c*x])/9 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^3) +
 ((3*a*x^2 + 2*b*x^3)*Log[1 - c*x]*PolyLog[2, c*x])/6 - ((2*b + 3*a*c)*Log[
1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + ((2*b + 3*a*c)*PolyLog[3, 1 - c*x])
/(3*c^3)
```

Rubi [A] time = 0.71802, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 21, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.105, Rules used = {6742, 6591, 2395, 43, 6604, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2410, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x(3ac+2b)\text{PolyLog}(2,cx)}{6c^2}+\frac{(3ac+2b)\text{PolyLog}(3,1-cx)}{3c^3}-\frac{(3ac+2b)\log(1-cx)\text{PolyLog}(2,cx)}{6c^3}-\frac{(3ac+2b)\log(1$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] (4*b*x)/(9*c^2) + (a*x)/c + (5*(2*b + 3*a*c)*x)/(24*c^2) + (b*x^2)/(9*c) +
((2*b + 3*a*c)*x^2)/(48*c) + (b*x^3)/27 + (a*(1 - c*x)^2)/(8*c^2) + (2*b*Lo
g[1 - c*x])/(9*c^3) + ((2*b + 3*a*c)*Log[1 - c*x])/(24*c^3) - (b*x^2*Log[1
- c*x])/(9*c) - ((2*b + 3*a*c)*x^2*Log[1 - c*x])/(24*c) - (b*x^3*Log[1 - c*
x])/9 + (2*b*(1 - c*x)*Log[1 - c*x])/(9*c^3) + (a*(1 - c*x)*Log[1 - c*x])/c
^2 + ((2*b + 3*a*c)*(1 - c*x)*Log[1 - c*x])/(6*c^3) - (a*(1 - c*x)^2*Log[1
- c*x])/(4*c^2) - (b*Log[1 - c*x]^2)/(9*c^3) + (b*x^3*Log[1 - c*x]^2)/9 - (
a*(1 - c*x)*Log[1 - c*x]^2)/(2*c^2) + (a*(1 - c*x)^2*Log[1 - c*x]^2)/(4*c^2
```

```
) - ((2*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/(6*c^3) - ((2*b + 3*a*c)*x*Poly
Log[2, c*x])/(6*c^2) - ((2*b + 3*a*c)*x^2*PolyLog[2, c*x])/(12*c) - (b*x^3*
PolyLog[2, c*x])/9 - ((2*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(6*c^3) +
 ((3*a*x^2 + 2*b*x^3)*Log[1 - c*x]*PolyLog[2, c*x])/6 - ((2*b + 3*a*c)*Log[
1 - c*x]*PolyLog[2, 1 - c*x])/(3*c^3) + ((2*b + 3*a*c)*PolyLog[3, 1 - c*x])
/(3*c^3)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x(a+bx)\log(1-cx)\text{Li}_2(cx)\,dx = \frac{1}{6}\left(3ax^2+2bx^3\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(\frac{(-2b-3ac)\text{Li}_2(cx)}{6c^3} - \frac{(2b+3ac)x\text{Li}_2(cx)}{6c^2}\right.$$

$$= \frac{1}{6}\left(3ax^2+2bx^3\right)\log(1-cx)\text{Li}_2(cx) + \frac{1}{2}a\int x\log^2(1-cx)\,dx + \frac{1}{3}b\int x^2\log^2(1-cx)$$

$$= \frac{1}{9}bx^3\log^2(1-cx) - \frac{(2b+3ac)x\text{Li}_2(cx)}{6c^2} - \frac{(2b+3ac)x^2\text{Li}_2(cx)}{12c} - \frac{1}{9}bx^3\text{Li}_2(cx) - \frac{(2b}{}$$

$$= -\frac{(2b+3ac)x^2\log(1-cx)}{24c} - \frac{1}{27}bx^3\log(1-cx) + \frac{1}{9}bx^3\log^2(1-cx) - \frac{(2b+3ac)\log}{6}$$

$$= \frac{(2b+3ac)x}{6c^2} - \frac{(2b+3ac)x^2\log(1-cx)}{24c} - \frac{1}{27}bx^3\log(1-cx) + \frac{(2b+3ac)(1-cx)\log}{6c^3}$$

$$= \frac{bx}{27c^2} + \frac{5(2b+3ac)x}{24c^2} + \frac{bx^2}{54c} + \frac{(2b+3ac)x^2}{48c} + \frac{bx^3}{81} + \frac{b\log(1-cx)}{27c^3} + \frac{(2b+3ac)\log(}{24c^3}$$

$$= \frac{7bx}{27c^2} + \frac{ax}{c} + \frac{5(2b+3ac)x}{24c^2} + \frac{bx^2}{54c} + \frac{(2b+3ac)x^2}{48c} + \frac{bx^3}{81} + \frac{a(1-cx)^2}{8c^2} + \frac{b\log(1-cx}{27c^3}$$

$$= \frac{4bx}{9c^2} + \frac{ax}{c} + \frac{5(2b+3ac)x}{24c^2} + \frac{bx^2}{9c} + \frac{(2b+3ac)x^2}{48c} + \frac{bx^3}{27} + \frac{a(1-cx)^2}{8c^2} + \frac{2b\log(1-cx}{9c^3}$$

Mathematica [A] time = 0.610532, size = 362, normalized size = 0.66

$$\frac{12\text{PolyLog}(2,cx)\left(6\log(1-cx)\left(3ac\left(c^2x^2-1\right)+2b\left(c^3x^3-1\right)\right)-cx\left(9ac(cx+2)+2b\left(2c^2x^2+3cx+6\right)\right)\right)-144(3ac+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-378*a*c + 372*b*c*x + 594*a*c^2*x + 66*b*c^2*x^2 + 81*a*c^3*x^2 + 16*b*c^
3*x^3 + 372*b*Log[1 - c*x] + 594*a*c*Log[1 - c*x] - 240*b*c*x*Log[1 - c*x]
- 432*a*c^2*x*Log[1 - c*x] - 84*b*c^2*x^2*Log[1 - c*x] - 162*a*c^3*x^2*Log[
1 - c*x] - 48*b*c^3*x^3*Log[1 - c*x] - 48*b*Log[1 - c*x]^2 - 108*a*c*Log[1
- c*x]^2 + 108*a*c^3*x^2*Log[1 - c*x]^2 + 48*b*c^3*x^3*Log[1 - c*x]^2 - 144
*b*Log[c*x]*Log[1 - c*x]^2 - 216*a*c*Log[c*x]*Log[1 - c*x]^2 + 12*(-(c*x*(9
*a*c*(2 + c*x) + 2*b*(6 + 3*c*x + 2*c^2*x^2))) + 6*(3*a*c*(-1 + c^2*x^2) +
2*b*(-1 + c^3*x^3))*Log[1 - c*x])*PolyLog[2, c*x] - 144*(2*b + 3*a*c)*Log[1
 - c*x]*PolyLog[2, 1 - c*x] + 288*b*PolyLog[3, 1 - c*x] + 432*a*c*PolyLog[3
, 1 - c*x])/(432*c^3)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int x\left(bx+a\right)\ln\left(-cx+1\right)\operatorname{polylog}\left(2,cx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.03758, size = 466, normalized size = 0.85

$$-\frac{1}{432}c\left(\frac{72\left(\log\left(cx\right)\log\left(-cx+1\right)^2+2\operatorname{Li}_2\left(-cx+1\right)\log\left(-cx+1\right)-2\operatorname{Li}_3(-cx+1)\right)(3\,ac+2\,b)}{c^4}-\frac{16\,bc^3x^3+3\left(27\,ac^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/432*c*(72*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) -
2*polylog(3, -c*x + 1))*(3*a*c + 2*b)/c^4 - (16*b*c^3*x^3 + 3*(27*a*c^3 + 2
2*b*c^2)*x^2 + 6*(99*a*c^2 + 62*b*c)*x - 12*(4*b*c^3*x^3 + 3*(3*a*c^3 + 2*b
*c^2)*x^2 + 6*(3*a*c^2 + 2*b*c)*x + 6*(3*a*c + 2*b)*log(-c*x + 1))*dilog(c*
x) - 2*(16*b*c^3*x^3 + 6*(9*a*c^3 + 5*b*c^2)*x^2 - 297*a*c + 6*(27*a*c^2 +
16*b*c)*x - 186*b)*log(-c*x + 1))/c^4) + 1/216*(27*(4*c^2*x^2*dilog(c*x) -
c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*a/c^2 + 4*(18*c^3*x^3*dilo
g(c*x) - 2*c^3*x^3 - 3*c^2*x^2 - 6*c*x + 6*(c^3*x^3 - 1)*log(-c*x + 1))*b/c
^3)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 + ax\right)\text{Li}_2\left(cx\right)\log\left(-cx + 1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a*x)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx + a)x\text{Li}_2\left(cx\right)\log\left(-cx + 1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*x*dilog(c*x)*log(-c*x + 1), x)
```

3.186 $\int (a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)\,dx$

Optimal. Leaf size=390

$$\frac{(2ac+b)\text{PolyLog}(3,1-cx)}{c^2}-\frac{(2ac+b)\log(1-cx)\text{PolyLog}(2,cx)}{2c^2}-\frac{(2ac+b)\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2}+\frac{1}{2}\Big(2ax$$

```
[Out] 2*a*x + (9*b*x)/(8*c) + ((b + 2*a*c)*x)/(2*c) + (b*x^2)/16 + (b*(1 - c*x)^2
)/(8*c^2) + (b*Log[1 - c*x])/(8*c^2) - (b*x^2*Log[1 - c*x])/8 + (b*(1 - c*x
)*Log[1 - c*x])/c^2 + (2*a*(1 - c*x)*Log[1 - c*x])/c + ((b + 2*a*c)*(1 - c*
x)*Log[1 - c*x])/(2*c^2) - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*(1 - c
*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (b*(1 - c*x)
^2*Log[1 - c*x]^2)/(4*c^2) - ((b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/(2*c^2)
- ((b + 2*a*c)*x*PolyLog[2, c*x])/(2*c) - (b*x^2*PolyLog[2, c*x])/4 - ((b +
 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + ((2*a*x + b*x^2)*Log[1 - c*
x]*PolyLog[2, c*x])/2 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
+ ((b + 2*a*c)*PolyLog[3, 1 - c*x])/c^2
```

Rubi [A] time = 0.44939, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 20, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {6598, 43, 2416, 2389, 2295, 2391, 2395, 6604, 2296, 2401, 2390, 2305, 2304, 6586, 6591, 6596, 2396, 2433, 2374, 6589}

$$\frac{(2ac+b)\text{PolyLog}(3,1-cx)}{c^2}-\frac{(2ac+b)\log(1-cx)\text{PolyLog}(2,cx)}{2c^2}-\frac{(2ac+b)\log(1-cx)\text{PolyLog}(2,1-cx)}{c^2}+\frac{1}{2}\Big(2ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```


```
[Out] 2*a*x + (9*b*x)/(8*c) + ((b + 2*a*c)*x)/(2*c) + (b*x^2)/16 + (b*(1 - c*x)^2
)/(8*c^2) + (b*Log[1 - c*x])/(8*c^2) - (b*x^2*Log[1 - c*x])/8 + (b*(1 - c*x
)*Log[1 - c*x])/c^2 + (2*a*(1 - c*x)*Log[1 - c*x])/c + ((b + 2*a*c)*(1 - c*
x)*Log[1 - c*x])/(2*c^2) - (b*(1 - c*x)^2*Log[1 - c*x])/(4*c^2) - (b*(1 - c
*x)*Log[1 - c*x]^2)/(2*c^2) - (a*(1 - c*x)*Log[1 - c*x]^2)/c + (b*(1 - c*x)
^2*Log[1 - c*x]^2)/(4*c^2) - ((b + 2*a*c)*Log[c*x]*Log[1 - c*x]^2)/(2*c^2)
- ((b + 2*a*c)*x*PolyLog[2, c*x])/(2*c) - (b*x^2*PolyLog[2, c*x])/4 - ((b +
 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/(2*c^2) + ((2*a*x + b*x^2)*Log[1 - c*
x]*PolyLog[2, c*x])/2 - ((b + 2*a*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c^2
+ ((b + 2*a*c)*PolyLog[3, 1 - c*x])/c^2
```


Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
```

```
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (a+bx)\log(1-cx)\mathrm{Li}_2(cx)\,dx = \frac{1}{2}\left(2ax+bx^2\right)\log(1-cx)\mathrm{Li}_2(cx)+c\int\left(\frac{(-b-2ac)\mathrm{Li}_2(cx)}{2c^2}-\frac{bx\mathrm{Li}_2(cx)}{2c}+\frac{(-b-2ac)}{2c^2(-1}\right.$$

$$=\frac{1}{2}\left(2ax+bx^2\right)\log(1-cx)\mathrm{Li}_2(cx)+a\int\log^2(1-cx)\,dx+\frac{1}{2}b\int x\log^2(1-cx)\,dx-\frac{1}{2}b$$

$$=-\frac{(b+2ac)x\mathrm{Li}_2(cx)}{2c}-\frac{1}{4}bx^2\mathrm{Li}_2(cx)-\frac{(b+2ac)\log(1-cx)\mathrm{Li}_2(cx)}{2c^2}+\frac{1}{2}\left(2ax+bx^2\right)\log$$

$$=-\frac{1}{8}bx^2\log(1-cx)-\frac{a(1-cx)\log^2(1-cx)}{c}-\frac{(b+2ac)\log(cx)\log^2(1-cx)}{2c^2}-\frac{(b+2ac}{}$$

$$=2ax+\frac{(b+2ac)x}{2c}-\frac{1}{8}bx^2\log(1-cx)+\frac{2a(1-cx)\log(1-cx)}{c}+\frac{(b+2ac)(1-cx)\log(}{2c^2}$$

$$=2ax+\frac{bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{8}bx^2\log(1-cx)+\frac{2a(1-cx)\log(1-}{c}$$

$$=2ax+\frac{9bx}{8c}+\frac{(b+2ac)x}{2c}+\frac{bx^2}{16}+\frac{b(1-cx)^2}{8c^2}+\frac{b\log(1-cx)}{8c^2}-\frac{1}{8}bx^2\log(1-cx)+\frac{b(1}{}$$

Mathematica [A] time = 0.490164, size = 285, normalized size = 0.73

$$\frac{4\mathrm{PolyLog}(2,cx)(2(cx-1)\log(1-cx)(2ac+bcx+b)-cx(4ac+bcx+2b))-16(2ac+b)\log(1-cx)\mathrm{PolyLog}(2,1-cx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[1 - c*x]*PolyLog[2, c*x],x]
```

```
[Out] (-14*b - 32*a*c + 22*b*c*x + 48*a*c^2*x + 3*b*c^2*x^2 + 22*b*Log[1 - c*x] +
 48*a*c*Log[1 - c*x] - 16*b*c*x*Log[1 - c*x] - 48*a*c^2*x*Log[1 - c*x] - 6*
b*c^2*x^2*Log[1 - c*x] - 4*b*Log[1 - c*x]^2 - 16*a*c*Log[1 - c*x]^2 + 16*a*
c^2*x*Log[1 - c*x]^2 + 4*b*c^2*x^2*Log[1 - c*x]^2 - 8*b*Log[c*x]*Log[1 - c*
x]^2 - 16*a*c*Log[c*x]*Log[1 - c*x]^2 + 4*(-(c*x*(2*b + 4*a*c + b*c*x)) + 2
*(-1 + c*x)*(b + 2*a*c + b*c*x)*Log[1 - c*x])*PolyLog[2, c*x] - 16*(b + 2*a
*c)*Log[1 - c*x]*PolyLog[2, 1 - c*x] + 16*b*PolyLog[3, 1 - c*x] + 32*a*c*Po
lyLog[3, 1 - c*x])/(16*c^2)
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int (bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

Maxima [A] time = 1.00725, size = 348, normalized size = 0.89

$$-\frac{1}{16}c\left(\frac{8\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)(2\,ac+b)}{c^3}-\frac{3\,bc^2x^2+2\left(24\,ac^2+11\,bc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="maxima")
```

```
[Out] -1/16*c*(8*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*
polylog(3, -c*x + 1))*(2*a*c + b)/c^3 - (3*b*c^2*x^2 + 2*(24*a*c^2 + 11*b*c
)*x - 4*(b*c^2*x^2 + 2*(2*a*c^2 + b*c)*x + 2*(2*a*c + b)*log(-c*x + 1))*dil
og(c*x) - 2*(2*b*c^2*x^2 - 24*a*c + 2*(8*a*c^2 + 3*b*c)*x - 11*b)*log(-c*x
+ 1))/c^3) + 1/8*(8*(c*x*dilog(c*x) - c*x + (c*x - 1)*log(-c*x + 1))*a/c +
(4*c^2*x^2*dilog(c*x) - c^2*x^2 - 2*c*x + 2*(c^2*x^2 - 1)*log(-c*x + 1))*b/
c^2)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx+a)\mathrm{Li}_2(cx)\log(-cx+1),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx+a)\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1), x)
```

3.187 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x}\,dx$

Optimal. Leaf size=153

$$-\frac{1}{2}a\text{PolyLog}(2,cx)^2 - bx\text{PolyLog}(2,cx) + \frac{2b\text{PolyLog}(3,1-cx)}{c} + bx\log(1-cx)\text{PolyLog}(2,cx) - \frac{b\log(1-cx)\text{Poly}}{c}$$

```
[Out] 3*b*x + (3*b*(1 - c*x)*Log[1 - c*x])/c - (b*(1 - c*x)*Log[1 - c*x]^2)/c - (
b*Log[c*x]*Log[1 - c*x]^2)/c - b*x*PolyLog[2, c*x] - (b*Log[1 - c*x]*PolyLo
g[2, c*x])/c + b*x*Log[1 - c*x]*PolyLog[2, c*x] - (a*PolyLog[2, c*x]^2)/2 -
 (2*b*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*b*PolyLog[3, 1 - c*x])/c
```

Rubi [A] time = 0.338379, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6742, 6586, 2389, 2295, 6589, 6605, 6601, 12, 6600, 2296, 6688, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}a\text{PolyLog}(2,cx)^2 - bx\text{PolyLog}(2,cx) + \frac{2b\text{PolyLog}(3,1-cx)}{c} + bx\log(1-cx)\text{PolyLog}(2,cx) - \frac{b\log(1-cx)\text{Poly}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```


```
[Out] 3*b*x + (3*b*(1 - c*x)*Log[1 - c*x])/c - (b*(1 - c*x)*Log[1 - c*x]^2)/c - (
b*Log[c*x]*Log[1 - c*x]^2)/c - b*x*PolyLog[2, c*x] - (b*Log[1 - c*x]*PolyLo
g[2, c*x])/c + b*x*Log[1 - c*x]*PolyLog[2, c*x] - (a*PolyLog[2, c*x]^2)/2 -
 (2*b*Log[1 - c*x]*PolyLog[2, 1 - c*x])/c + (2*b*PolyLog[3, 1 - c*x])/c
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*Poly
Log[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6600

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*PolyLog[2, (c_.)*
((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[x*(g + h*Log[f*(d + e*x)^n])*PolyL
og[2, c*(a + b*x)], x] + (Dist[b, Int[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*
c - b*c*x]*ExpandIntegrand[x/(a + b*x), x], x], x] - Dist[e*h*n, Int[PolyLo
g[2, c*(a + b*x)]*ExpandIntegrand[x/(d + e*x), x], x], x]) /; FreeQ[{a, b,
c, d, e, f, g, h, n}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x}\,dx = a\int \frac{\log(1-cx)\text{Li}_2(cx)}{x}\,dx + \int b\log(1-cx)\text{Li}_2(cx)\,dx$$
$$= -\frac{1}{2}a\text{Li}_2(cx)^2 + b\int \log(1-cx)\text{Li}_2(cx)\,dx$$
$$= bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 + b\int \log^2(1-cx)\,dx + (bc)\int\left(-\frac{1}{c} - \frac{1}{c(-1+cx)}\right.$$
$$= bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 - \frac{b\,\text{Subst}\left(\int \log^2(x)\,dx, x, 1-cx\right)}{c} + (bc)\int \frac{x\text{Li}_2}{1-}$$
$$= -\frac{b(1-cx)\log^2(1-cx)}{c} + bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}_2(cx)^2 + \frac{(2b)\,\text{Subst}(\int \log(x)}{c}$$
$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} + bx\log(1-cx)\text{Li}_2(cx) - \frac{1}{2}a\text{Li}$$
$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - bx\text{Li}_2(cx) - \frac{b\log(1-cx)\text{Li}_2(}{c}$$
$$= 2bx + \frac{2b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2(1-cx)}{c} - bx\text{Li}$$
$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2(1-cx)}{c} - bx\text{Li}$$
$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2(1-cx)}{c} - bx\text{Li}$$
$$= 3bx + \frac{3b(1-cx)\log(1-cx)}{c} - \frac{b(1-cx)\log^2(1-cx)}{c} - \frac{b\log(cx)\log^2(1-cx)}{c} - bx\text{Li}$$

Mathematica [A] time = 0.234598, size = 137, normalized size = 0.9

$$-\frac{1}{2}a\text{PolyLog}(2,cx)^2 + \frac{b\left(2\text{PolyLog}(3,1-cx) - 2\log(1-cx)\text{PolyLog}(2,1-cx) + 3cx + cx\log^2(1-cx) - \log(cx)\log^2(\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x,x]
```

```
[Out] (b*(-(c*x) + (-1 + c*x)*Log[1 - c*x])*PolyLog[2, c*x])/c - (a*PolyLog[2, c*
x]^2)/2 + (b*(-2 + 3*c*x + 3*Log[1 - c*x] - 3*c*x*Log[1 - c*x] - Log[1 - c*
x]^2 + c*x*Log[1 - c*x]^2 - Log[c*x]*Log[1 - c*x]^2 - 2*Log[1 - c*x]*PolyLo
g[2, 1 - c*x] + 2*PolyLog[3, 1 - c*x]))/c
```

Maple [F] time = 0.005, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\,\text{polylog}(2,cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\log(-cx+1)\,\text{Li}_2(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x,x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\mathrm{Li}_2\left(cx\right)\log\left(-cx+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x, x)
```

3.188 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^2}\,dx$

Optimal. Leaf size=131

$$-2ac\text{PolyLog}(2,cx)-ac\text{PolyLog}(3,cx)-2ac\text{PolyLog}(3,1-cx)+ac\log(1-cx)\text{PolyLog}(2,cx)-\frac{a\log(1-cx)\text{PolyI}}{x}$$

```
[Out] (a*(1 - c*x)*Log[1 - c*x]^2)/x + a*c*Log[c*x]*Log[1 - c*x]^2 - 2*a*c*PolyLo
g[2, c*x] + a*c*Log[1 - c*x]*PolyLog[2, c*x] - (a*Log[1 - c*x]*PolyLog[2, c
*x])/x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] -
 a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

Rubi [A] time = 0.313536, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.81, Rules used = {6742, 6591, 2395, 36, 29, 31, 6589, 6605, 6601, 12, 6603, 2397, 2391, 6596, 2396, 2433, 2374}

$$-2ac\text{PolyLog}(2,cx)-ac\text{PolyLog}(3,cx)-2ac\text{PolyLog}(3,1-cx)+ac\log(1-cx)\text{PolyLog}(2,cx)-\frac{a\log(1-cx)\text{PolyI}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] (a*(1 - c*x)*Log[1 - c*x]^2)/x + a*c*Log[c*x]*Log[1 - c*x]^2 - 2*a*c*PolyLo
g[2, c*x] + a*c*Log[1 - c*x]*PolyLog[2, c*x] - (a*Log[1 - c*x]*PolyLog[2, c
*x])/x - (b*PolyLog[2, c*x]^2)/2 + 2*a*c*Log[1 - c*x]*PolyLog[2, 1 - c*x] -
 a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*Poly
Log[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6603

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(x_)^(m_.)*PolyLo
g[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> Simp[(x^(m + 1)*(g + h*Log[f
*(d + e*x)^n])*PolyLog[2, c*(a + b*x)])/(m + 1), x] + (Dist[b/(m + 1), Int[
ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], x^(m + 1)/
(a + b*x), x], x], x] - Dist[(e*h*n)/(m + 1), Int[ExpandIntegrand[PolyLog[2
, c*(a + b*x)], x^(m + 1)/(d + e*x), x], x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, n}, x] && IntegerQ[m] && NeQ[m, -1]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\mathrm{Li}_2(cx)}{x^2}\,dx = b\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x}\,dx + \int \frac{a\log(1-cx)\mathrm{Li}_2(cx)}{x^2}\,dx$$

$$= -\frac{1}{2}b\mathrm{Li}_2(cx)^2 + a\int \frac{\log(1-cx)\mathrm{Li}_2(cx)}{x^2}\,dx$$

$$= -\frac{a\log(1-cx)\mathrm{Li}_2(cx)}{x} - \frac{1}{2}b\mathrm{Li}_2(cx)^2 - a\int \frac{\log^2(1-cx)}{x^2}\,dx - (ac)\int\left(\frac{\mathrm{Li}_2(cx)}{x} - \frac{c\mathrm{Li}_2}{-1+}\right.$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} - \frac{a\log(1-cx)\mathrm{Li}_2(cx)}{x} - \frac{1}{2}b\mathrm{Li}_2(cx)^2 - (ac)\int \frac{\mathrm{Li}_2(cx)}{x}\,dx + (2a$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} - 2ac\mathrm{Li}_2(cx) + ac\log(1-cx)\mathrm{Li}_2(cx) - \frac{a\log(1-cx)\mathrm{Li}_2(cx)}{x} -$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\mathrm{Li}_2(cx) + ac\log(1-cx)\mathrm{Li}_2(cx)$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\mathrm{Li}_2(cx) + ac\log(1-cx)\mathrm{Li}_2(cx)$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\mathrm{Li}_2(cx) + ac\log(1-cx)\mathrm{Li}_2(cx)$$

$$= \frac{a(1-cx)\log^2(1-cx)}{x} + ac\log(cx)\log^2(1-cx) - 2ac\mathrm{Li}_2(cx) + ac\log(1-cx)\mathrm{Li}_2(cx)$$

Mathematica [A] time = 0.713885, size = 135, normalized size = 1.03

$$-ac\mathrm{PolyLog}(3,cx) - 2ac\mathrm{PolyLog}(3,1-cx) + \frac{a(cx-1)\log(1-cx)\mathrm{PolyLog}(2,cx)}{x} + 2ac(\log(1-cx)+1)\mathrm{PolyLog}(2,1$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^2,x]
```

```
[Out] 2*a*c*Log[c*x]*Log[1 - c*x] - a*c*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x + a
*c*Log[c*x]*Log[1 - c*x]^2 + (a*(-1 + c*x)*Log[1 - c*x]*PolyLog[2, c*x])/x
- (b*PolyLog[2, c*x]^2)/2 + 2*a*c*(1 + Log[1 - c*x])*PolyLog[2, 1 - c*x] -
a*c*PolyLog[3, c*x] - 2*a*c*PolyLog[3, 1 - c*x]
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\log(-cx+1)\,\mathrm{Li}_2(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**2,x)
```

```
[Out] Integral((a + b*x)*log(-c*x + 1)*polylog(2, c*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^2, x)
```

3.189 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^3}\,dx$

Optimal. Leaf size=331

$$\frac{b^2\text{PolyLog}(3,1-cx)}{a}-\frac{b^2\log(1-cx)\text{PolyLog}(2,1-cx)}{a}-\frac{(a+bx)^2\log(1-cx)\text{PolyLog}(2,cx)}{2ax^2}-\frac{1}{2}c(ac+2b)\text{PolyLo}$$

```
[Out] -(a*c^2*Log[x]) + a*c^2*Log[1 - c*x] - (a*c*Log[1 - c*x])/x - (a*c^2*Log[1
- c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1 - c*x]^2)/x -
 (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2
)/(2*a) - 2*b*c*PolyLog[2, c*x] - (a*c^2*PolyLog[2, c*x])/2 + (a*c*PolyLog[
2, c*x])/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a) - ((a + b
*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x])/a + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a - (c*(2*b
+ a*c)*PolyLog[3, c*x])/2 + (b^2*PolyLog[3, 1 - c*x])/a - ((b + a*c)^2*Poly
Log[3, 1 - c*x])/a
```

Rubi [A] time = 0.540904, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 20, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.952, Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 37, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 2396, 2433, 2374, 6589, 6596}

$$\frac{b^2\text{PolyLog}(3,1-cx)}{a}-\frac{b^2\log(1-cx)\text{PolyLog}(2,1-cx)}{a}-\frac{(a+bx)^2\log(1-cx)\text{PolyLog}(2,cx)}{2ax^2}-\frac{1}{2}c(ac+2b)\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```


```
[Out] -(a*c^2*Log[x]) + a*c^2*Log[1 - c*x] - (a*c*Log[1 - c*x])/x - (a*c^2*Log[1
- c*x]^2)/4 + (a*Log[1 - c*x]^2)/(4*x^2) + (b*(1 - c*x)*Log[1 - c*x]^2)/x -
 (b^2*Log[c*x]*Log[1 - c*x]^2)/(2*a) + ((b + a*c)^2*Log[c*x]*Log[1 - c*x]^2
)/(2*a) - 2*b*c*PolyLog[2, c*x] - (a*c^2*PolyLog[2, c*x])/2 + (a*c*PolyLog[
2, c*x])/(2*x) + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a) - ((a + b
*x)^2*Log[1 - c*x]*PolyLog[2, c*x])/(2*a*x^2) - (b^2*Log[1 - c*x]*PolyLog[2
, 1 - c*x])/a + ((b + a*c)^2*Log[1 - c*x]*PolyLog[2, 1 - c*x])/a - (c*(2*b
+ a*c)*PolyLog[3, c*x])/2 + (b^2*PolyLog[3, 1 - c*x])/a - ((b + a*c)^2*Poly
Log[3, 1 - c*x])/a
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^3}\,dx = -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} + c\int\Bigg(-\frac{a\text{Li}_2(cx)}{2x^2} + \frac{(-2b-ac)\text{Li}_2(cx)}{2x} + \frac{(b+ac)^2\text{L}}{2a(-1+}$$

$$= -\frac{(a+bx)^2\log(1-cx)\text{Li}_2(cx)}{2ax^2} - \frac{1}{2}a\int\frac{\log^2(1-cx)}{x^3}\,dx - b\int\frac{\log^2(1-cx)}{x^2}\,dx - \frac{b^2}{}$$

$$= \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a} + \frac{ac\text{Li}_2(cx)}{2x} + \frac{(b}{}$$

$$= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a}$$

$$= -\frac{ac\log(1-cx)}{2x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(1-cx)}{x} - \frac{b^2\log(cx)\log^2(1-cx)}{2a}$$

$$= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} + \frac{a\log^2(1-cx)}{4x^2} + \frac{b(1-cx)\log^2(}{x}$$

$$= -\frac{1}{2}ac^2\log(x) + \frac{1}{2}ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx) + \frac{a\log^2(1-}{4x^2}$$

$$= -ac^2\log(x) + ac^2\log(1-cx) - \frac{ac\log(1-cx)}{x} - \frac{1}{4}ac^2\log^2(1-cx) + \frac{a\log^2(1-cx)}{4x^2}$$

Mathematica [A] time = 1.1877, size = 285, normalized size = 0.86

$$\frac{1}{4}\Bigg(\frac{2\text{PolyLog}(2,cx)((cx-1)\log(1-cx)(acx+a+2bx)+acx)}{x^2} + 2c\text{PolyLog}(2,1-cx)(2(ac+2b)\log(1-cx)+ac+4b$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^3,x]
```

```
[Out] (-4*a*c^2*Log[c*x] + 4*a*c^2*Log[1 - c*x] - (4*a*c*Log[1 - c*x])/x + 8*b*c*
Log[c*x]*Log[1 - c*x] + 2*a*c^2*Log[c*x]*Log[1 - c*x] - 4*b*c*Log[1 - c*x]^
2 - a*c^2*Log[1 - c*x]^2 + (a*Log[1 - c*x]^2)/x^2 + (4*b*Log[1 - c*x]^2)/x
+ 4*b*c*Log[c*x]*Log[1 - c*x]^2 + 2*a*c^2*Log[c*x]*Log[1 - c*x]^2 + (2*(a*c
*x + (-1 + c*x)*(a + 2*b*x + a*c*x)*Log[1 - c*x])*PolyLog[2, c*x])/x^2 + 2*
c*(4*b + a*c + 2*(2*b + a*c)*Log[1 - c*x])*PolyLog[2, 1 - c*x] - 4*b*c*Poly
Log[3, c*x] - 2*a*c^2*PolyLog[3, c*x] - 8*b*c*PolyLog[3, 1 - c*x] - 4*a*c^2
```

```
*PolyLog[3, 1 - c*x])/4
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^3,x)
```

Maxima [A] time = 1.16395, size = 288, normalized size = 0.87

$$-ac^2\log(x)+\frac{1}{2}\left(ac^2+2\,bc\right)\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)+\frac{1}{2}\left(ac^2+4\,bc\right)(\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="maxima")
```

```
[Out] -a*c^2*log(x) + 1/2*(a*c^2 + 2*b*c)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*
x + 1)*log(-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/2*(a*c^2 + 4*b*c)*(log(c
*x)*log(-c*x + 1) + dilog(-c*x + 1)) - 1/2*(a*c^2 + 2*b*c)*polylog(3, c*x)
- 1/4*(((a*c^2 + 4*b*c)*x^2 - 4*b*x - a)*log(-c*x + 1)^2 - 2*(a*c*x + ((a*c
^2 + 2*b*c)*x^2 - 2*b*x - a)*log(-c*x + 1))*dilog(c*x) - 4*(a*c^2*x^2 - a*c
*x)*log(-c*x + 1))/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^3, x)
```

3.190 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^4}\,dx$

Optimal. Leaf size=460

$$-\frac{1}{6}c^2(2ac+3b)\text{PolyLog}(3,cx)-\frac{1}{3}c^2(2ac+3b)\text{PolyLog}(3,1-cx)+\frac{1}{6}c^2(2ac+3b)\log(1-cx)\text{PolyLog}(2,cx)+\frac{1}{3}c^2(2a$$

```
[Out] (7*a*c^2)/(36*x) - (b*c^2*Log[x])/2 - (5*a*c^3*Log[x])/12 - (c^2*(3*b + 2*a
*c)*Log[x])/6 + (b*c^2*Log[1 - c*x])/2 + (5*a*c^3*Log[1 - c*x])/12 + (c^2*(
3*b + 2*a*c)*Log[1 - c*x])/6 - (7*a*c*Log[1 - c*x])/(36*x^2) - (b*c*Log[1 -
 c*x])/(2*x) - (2*a*c^2*Log[1 - c*x])/(9*x) - (c*(3*b + 2*a*c)*Log[1 - c*x]
)/(6*x) - (b*c^2*Log[1 - c*x]^2)/4 - (a*c^3*Log[1 - c*x]^2)/9 + (a*Log[1 -
c*x]^2)/(9*x^3) + (b*Log[1 - c*x]^2)/(4*x^2) + (c^2*(3*b + 2*a*c)*Log[c*x]*
Log[1 - c*x]^2)/6 - (b*c^2*PolyLog[2, c*x])/2 - (2*a*c^3*PolyLog[2, c*x])/9
 + (a*c*PolyLog[2, c*x])/(6*x^2) + (c*(3*b + 2*a*c)*PolyLog[2, c*x])/(6*x)
+ (c^2*(3*b + 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/6 - (((2*a)/x^3 + (3*b)/
x^2)*Log[1 - c*x]*PolyLog[2, c*x])/6 + (c^2*(3*b + 2*a*c)*Log[1 - c*x]*Poly
Log[2, 1 - c*x])/3 - (c^2*(3*b + 2*a*c)*PolyLog[3, c*x])/6 - (c^2*(3*b + 2*
a*c)*PolyLog[3, 1 - c*x])/3
```

Rubi [A] time = 0.671801, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {6742, 6591, 2395, 44, 43, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{6}c^2(2ac+3b)\text{PolyLog}(3,cx)-\frac{1}{3}c^2(2ac+3b)\text{PolyLog}(3,1-cx)+\frac{1}{6}c^2(2ac+3b)\log(1-cx)\text{PolyLog}(2,cx)+\frac{1}{3}c^2(2a$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (7*a*c^2)/(36*x) - (b*c^2*Log[x])/2 - (5*a*c^3*Log[x])/12 - (c^2*(3*b + 2*a
*c)*Log[x])/6 + (b*c^2*Log[1 - c*x])/2 + (5*a*c^3*Log[1 - c*x])/12 + (c^2*(
3*b + 2*a*c)*Log[1 - c*x])/6 - (7*a*c*Log[1 - c*x])/(36*x^2) - (b*c*Log[1 -
 c*x])/(2*x) - (2*a*c^2*Log[1 - c*x])/(9*x) - (c*(3*b + 2*a*c)*Log[1 - c*x]
)/(6*x) - (b*c^2*Log[1 - c*x]^2)/4 - (a*c^3*Log[1 - c*x]^2)/9 + (a*Log[1 -
c*x]^2)/(9*x^3) + (b*Log[1 - c*x]^2)/(4*x^2) + (c^2*(3*b + 2*a*c)*Log[c*x]*
Log[1 - c*x]^2)/6 - (b*c^2*PolyLog[2, c*x])/2 - (2*a*c^3*PolyLog[2, c*x])/9
 + (a*c*PolyLog[2, c*x])/(6*x^2) + (c*(3*b + 2*a*c)*PolyLog[2, c*x])/(6*x)
+ (c^2*(3*b + 2*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/6 - (((2*a)/x^3 + (3*b)/
```

```
x^2)*Log[1 - c*x]*PolyLog[2, c*x])/6 + (c^2*(3*b + 2*a*c)*Log[1 - c*x]*Poly
Log[2, 1 - c*x])/3 - (c^2*(3*b + 2*a*c)*PolyLog[3, c*x])/6 - (c^2*(3*b + 2*
a*c)*PolyLog[3, 1 - c*x])/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
```

```
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^4}\,dx = -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(-\frac{a\text{Li}_2(cx)}{3x^3}+\frac{(-3b-2ac)\text{Li}_2(cx)}{6x^2}-\frac{c(3b+2$$

$$= -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}\right)\log(1-cx)\text{Li}_2(cx) - \frac{1}{3}a\int\frac{\log^2(1-cx)}{x^4}\,dx - \frac{1}{2}b\int\frac{\log^2(1-cx)}{x^3}\,dx -$$

$$= \frac{a\log^2(1-cx)}{9x^3}+\frac{b\log^2(1-cx)}{4x^2}+\frac{ac\text{Li}_2(cx)}{6x^2}+\frac{c(3b+2ac)\text{Li}_2(cx)}{6x}+\frac{1}{6}c^2(3b+2ac)\,\text{lo}$$

$$= -\frac{ac\log(1-cx)}{12x^2}-\frac{c(3b+2ac)\log(1-cx)}{6x}+\frac{a\log^2(1-cx)}{9x^3}+\frac{b\log^2(1-cx)}{4x^2}+\frac{1}{6}c^2(3b$$

$$= -\frac{ac\log(1-cx)}{12x^2}-\frac{c(3b+2ac)\log(1-cx)}{6x}+\frac{a\log^2(1-cx)}{9x^3}+\frac{b\log^2(1-cx)}{4x^2}+\frac{1}{6}c^2(3b$$

$$= \frac{ac^2}{12x}-\frac{1}{12}ac^3\log(x)-\frac{1}{6}c^2(3b+2ac)\log(x)+\frac{1}{12}ac^3\log(1-cx)+\frac{1}{6}c^2(3b+2ac)\log($$

$$= \frac{ac^2}{12x}-\frac{1}{12}ac^3\log(x)-\frac{1}{6}c^2(3b+2ac)\log(x)+\frac{1}{12}ac^3\log(1-cx)+\frac{1}{6}c^2(3b+2ac)\log($$

$$= \frac{7ac^2}{36x}-\frac{1}{2}bc^2\log(x)-\frac{5}{12}ac^3\log(x)-\frac{1}{6}c^2(3b+2ac)\log(x)+\frac{1}{2}bc^2\log(1-cx)+\frac{5}{12}ac$$

Mathematica [A] time = 1.30438, size = 389, normalized size = 0.85

$$\frac{1}{36}\left(\frac{6\text{PolyLog}(2,cx)\left(\log(1-cx)\left(2ac^3x^3-2a+3bc^2x^3-3bx\right)+cx(2acx+a+3bx)\right)}{x^3}+2c^2\text{PolyLog}(2,1-cx)(6(2ac+$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^4,x]
```

```
[Out] (-7*a*c^3 + (7*a*c^2)/x - 36*b*c^2*Log[c*x] - 27*a*c^3*Log[c*x] + 36*b*c^2*
Log[1 - c*x] + 27*a*c^3*Log[1 - c*x] - (7*a*c*Log[1 - c*x])/x^2 - (36*b*c*L
og[1 - c*x])/x - (20*a*c^2*Log[1 - c*x])/x + 18*b*c^2*Log[c*x]*Log[1 - c*x]
 + 8*a*c^3*Log[c*x]*Log[1 - c*x] - 9*b*c^2*Log[1 - c*x]^2 - 4*a*c^3*Log[1 -
 c*x]^2 + (4*a*Log[1 - c*x]^2)/x^3 + (9*b*Log[1 - c*x]^2)/x^2 + 18*b*c^2*Lo
g[c*x]*Log[1 - c*x]^2 + 12*a*c^3*Log[c*x]*Log[1 - c*x]^2 + (6*(c*x*(a + 3*b
*x + 2*a*c*x) + (-2*a - 3*b*x + 3*b*c^2*x^3 + 2*a*c^3*x^3)*Log[1 - c*x])*Po
lyLog[2, c*x])/x^3 + 2*c^2*(9*b + 4*a*c + 6*(3*b + 2*a*c)*Log[1 - c*x])*Pol
yLog[2, 1 - c*x] - 18*b*c^2*PolyLog[3, c*x] - 12*a*c^3*PolyLog[3, c*x] - 36
*b*c^2*PolyLog[3, 1 - c*x] - 24*a*c^3*PolyLog[3, 1 - c*x])/36
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \frac{(bx + a) \ln (-cx + 1) \operatorname{polylog}(2, cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^4,x)
```

Maxima [A] time = 1.20069, size = 387, normalized size = 0.84

$$\frac{1}{6} \left(2 ac^3 + 3 bc^2\right)\left(\log (cx) \log (-cx + 1)^2 + 2 \operatorname{Li}_2 (-cx + 1) \log (-cx + 1) - 2 \operatorname{Li}_3(-cx + 1)\right) + \frac{1}{18} \left(4 ac^3 + 9 bc^2\right)(\log (cx) \mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*c^3 + 3*b*c^2)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-
c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/18*(4*a*c^3 + 9*b*c^2)*(log(c*x)*log
(-c*x + 1) + dilog(-c*x + 1)) - 1/4*(3*a*c^3 + 4*b*c^2)*log(x) - 1/6*(2*a*c
^3 + 3*b*c^2)*polylog(3, c*x) + 1/36*(7*a*c^2*x^2 - ((4*a*c^3 + 9*b*c^2)*x^
3 - 9*b*x - 4*a)*log(-c*x + 1)^2 + 6*(a*c*x + (2*a*c^2 + 3*b*c)*x^2 + ((2*a
*c^3 + 3*b*c^2)*x^3 - 3*b*x - 2*a)*log(-c*x + 1))*dilog(c*x) + (9*(3*a*c^3
+ 4*b*c^2)*x^3 - 7*a*c*x - 4*(5*a*c^2 + 9*b*c)*x^2)*log(-c*x + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx + a)\operatorname{Li}_2 (cx) \log (-cx + 1)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\mathrm{Li}_2(cx)\log(-cx+1)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^4, x)
```

3.191 $\int \frac{(a+bx)\log(1-cx)\textbf{PolyLog}(2,cx)}{x^5}\,dx$

Optimal. Leaf size=584

$$-\frac{1}{12}c^3(3ac+4b)\text{PolyLog}(3,cx)-\frac{1}{6}c^3(3ac+4b)\text{PolyLog}(3,1-cx)+\frac{c^2(3ac+4b)\text{PolyLog}(2,cx)}{12x}+\frac{1}{12}c^3(3ac+4b)\,\text{lo}$$

```
[Out] (5*a*c^2)/(144*x^2) + (b*c^2)/(9*x) + (19*a*c^3)/(144*x) + (c^2*(4*b + 3*a*
c))/(48*x) - (b*c^3*Log[x])/3 - (37*a*c^4*Log[x])/144 - (5*c^3*(4*b + 3*a*c
)*Log[x])/48 + (b*c^3*Log[1 - c*x])/3 + (37*a*c^4*Log[1 - c*x])/144 + (5*c^
3*(4*b + 3*a*c)*Log[1 - c*x])/48 - (5*a*c*Log[1 - c*x])/(72*x^3) - (b*c*Log
[1 - c*x])/(9*x^2) - (a*c^2*Log[1 - c*x])/(16*x^2) - (c*(4*b + 3*a*c)*Log[1
 - c*x])/(48*x^2) - (2*b*c^2*Log[1 - c*x])/(9*x) - (a*c^3*Log[1 - c*x])/(8*
x) - (c^2*(4*b + 3*a*c)*Log[1 - c*x])/(12*x) - (b*c^3*Log[1 - c*x]^2)/9 - (
a*c^4*Log[1 - c*x]^2)/16 + (a*Log[1 - c*x]^2)/(16*x^4) + (b*Log[1 - c*x]^2)
/(9*x^3) + (c^3*(4*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/12 - (2*b*c^3*PolyLo
g[2, c*x])/9 - (a*c^4*PolyLog[2, c*x])/8 + (a*c*PolyLog[2, c*x])/(12*x^3) +
 (c*(4*b + 3*a*c)*PolyLog[2, c*x])/(24*x^2) + (c^2*(4*b + 3*a*c)*PolyLog[2,
 c*x])/(12*x) + (c^3*(4*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/12 - (((3*
a)/x^4 + (4*b)/x^3)*Log[1 - c*x]*PolyLog[2, c*x])/12 + (c^3*(4*b + 3*a*c)*L
og[1 - c*x]*PolyLog[2, 1 - c*x])/6 - (c^3*(4*b + 3*a*c)*PolyLog[3, c*x])/12
 - (c^3*(4*b + 3*a*c)*PolyLog[3, 1 - c*x])/6
```

Rubi [A] time = 0.844662, antiderivative size = 584, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {6742, 6591, 2395, 44, 43, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{12}c^3(3ac+4b)\text{PolyLog}(3,cx)-\frac{1}{6}c^3(3ac+4b)\text{PolyLog}(3,1-cx)+\frac{c^2(3ac+4b)\text{PolyLog}(2,cx)}{12x}+\frac{1}{12}c^3(3ac+4b)\,\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] (5*a*c^2)/(144*x^2) + (b*c^2)/(9*x) + (19*a*c^3)/(144*x) + (c^2*(4*b + 3*a*
c))/(48*x) - (b*c^3*Log[x])/3 - (37*a*c^4*Log[x])/144 - (5*c^3*(4*b + 3*a*c
)*Log[x])/48 + (b*c^3*Log[1 - c*x])/3 + (37*a*c^4*Log[1 - c*x])/144 + (5*c^
3*(4*b + 3*a*c)*Log[1 - c*x])/48 - (5*a*c*Log[1 - c*x])/(72*x^3) - (b*c*Log
[1 - c*x])/(9*x^2) - (a*c^2*Log[1 - c*x])/(16*x^2) - (c*(4*b + 3*a*c)*Log[1
 - c*x])/(48*x^2) - (2*b*c^2*Log[1 - c*x])/(9*x) - (a*c^3*Log[1 - c*x])/(8*
```

```
x) - (c^2*(4*b + 3*a*c)*Log[1 - c*x])/(12*x) - (b*c^3*Log[1 - c*x]^2)/9 - (
a*c^4*Log[1 - c*x]^2)/16 + (a*Log[1 - c*x]^2)/(16*x^4) + (b*Log[1 - c*x]^2)
/(9*x^3) + (c^3*(4*b + 3*a*c)*Log[c*x]*Log[1 - c*x]^2)/12 - (2*b*c^3*PolyLo
g[2, c*x])/9 - (a*c^4*PolyLog[2, c*x])/8 + (a*c*PolyLog[2, c*x])/(12*x^3) +
 (c*(4*b + 3*a*c)*PolyLog[2, c*x])/(24*x^2) + (c^2*(4*b + 3*a*c)*PolyLog[2,
 c*x])/(12*x) + (c^3*(4*b + 3*a*c)*Log[1 - c*x]*PolyLog[2, c*x])/12 - (((3*
a)/x^4 + (4*b)/x^3)*Log[1 - c*x]*PolyLog[2, c*x])/12 + (c^3*(4*b + 3*a*c)*L
og[1 - c*x]*PolyLog[2, 1 - c*x])/6 - (c^3*(4*b + 3*a*c)*PolyLog[3, c*x])/12
 - (c^3*(4*b + 3*a*c)*PolyLog[3, 1 - c*x])/6
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
```

```
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{(a+bx)\log(1-cx)\text{Li}_2(cx)}{x^5}\,dx = -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\log(1-cx)\text{Li}_2(cx) + c\int\left(-\frac{a\text{Li}_2(cx)}{4x^4} + \frac{(-4b-3ac)\text{Li}_2(cx)}{12x^3} - \frac{c(4b$$

$$= -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}\right)\log(1-cx)\text{Li}_2(cx) - \frac{1}{4}a\int\frac{\log^2(1-cx)}{x^5}\,dx - \frac{1}{3}b\int\frac{\log^2(1-cx)}{x^4}\,dx$$

$$= \frac{a\log^2(1-cx)}{16x^4} + \frac{b\log^2(1-cx)}{9x^3} + \frac{ac\text{Li}_2(cx)}{12x^3} + \frac{c(4b+3ac)\text{Li}_2(cx)}{24x^2} + \frac{c^2(4b+3ac)\text{L}}{12x}$$

$$= -\frac{ac\log(1-cx)}{36x^3} - \frac{c(4b+3ac)\log(1-cx)}{48x^2} - \frac{c^2(4b+3ac)\log(1-cx)}{12x} + \frac{a\log^2(1-c}{16x^4}$$

$$= -\frac{ac\log(1-cx)}{36x^3} - \frac{c(4b+3ac)\log(1-cx)}{48x^2} - \frac{c^2(4b+3ac)\log(1-cx)}{12x} + \frac{a\log^2(1-c}{16x^4}$$

$$= \frac{ac^2}{72x^2} + \frac{ac^3}{36x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{36}ac^4\log(x) - \frac{5}{48}c^3(4b+3ac)\log(x) + \frac{1}{36}ac^4\log(1$$

$$= \frac{ac^2}{72x^2} + \frac{ac^3}{36x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{36}ac^4\log(x) - \frac{5}{48}c^3(4b+3ac)\log(x) + \frac{1}{36}ac^4\log(1$$

$$= \frac{5ac^2}{144x^2} + \frac{bc^2}{9x} + \frac{19ac^3}{144x} + \frac{c^2(4b+3ac)}{48x} - \frac{1}{3}bc^3\log(x) - \frac{37}{144}ac^4\log(x) - \frac{5}{48}c^3(4b+3$$

Mathematica [A] time = 1.51805, size = 505, normalized size = 0.86

$$-\frac{-2c^3x^4\text{PolyLog}(2,1-cx)(12(3ac+4b)\log(1-cx)+9ac+16b) - 6\text{PolyLog}(2,cx)\left(cx\left(a\left(6c^2x^2+3cx+2\right)+4bx(2c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Log[1 - c*x]*PolyLog[2, c*x])/x^5,x]
```

```
[Out] -(-5*a*c^2*x^2 - 28*b*c^2*x^3 - 28*a*c^3*x^3 + 28*b*c^3*x^4 + 33*a*c^4*x^4
+ 108*b*c^3*x^4*Log[c*x] + 82*a*c^4*x^4*Log[c*x] + 10*a*c*x*Log[1 - c*x] +
28*b*c*x^2*Log[1 - c*x] + 18*a*c^2*x^2*Log[1 - c*x] + 80*b*c^2*x^3*Log[1 -
c*x] + 54*a*c^3*x^3*Log[1 - c*x] - 108*b*c^3*x^4*Log[1 - c*x] - 82*a*c^4*x^
4*Log[1 - c*x] - 32*b*c^3*x^4*Log[c*x]*Log[1 - c*x] - 18*a*c^4*x^4*Log[c*x]
```

```
*Log[1 - c*x] - 9*a*Log[1 - c*x]^2 - 16*b*x*Log[1 - c*x]^2 + 16*b*c^3*x^4*L
og[1 - c*x]^2 + 9*a*c^4*x^4*Log[1 - c*x]^2 - 48*b*c^3*x^4*Log[c*x]*Log[1 -
c*x]^2 - 36*a*c^4*x^4*Log[c*x]*Log[1 - c*x]^2 - 6*(c*x*(4*b*x*(1 + 2*c*x) +
 a*(2 + 3*c*x + 6*c^2*x^2)) + (8*b*x*(-1 + c^3*x^3) + 6*a*(-1 + c^4*x^4))*L
og[1 - c*x])*PolyLog[2, c*x] - 2*c^3*x^4*(16*b + 9*a*c + 12*(4*b + 3*a*c)*L
og[1 - c*x])*PolyLog[2, 1 - c*x] + 48*b*c^3*x^4*PolyLog[3, c*x] + 36*a*c^4*
x^4*PolyLog[3, c*x] + 96*b*c^3*x^4*PolyLog[3, 1 - c*x] + 72*a*c^4*x^4*PolyL
og[3, 1 - c*x])/(144*x^4)
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \frac{(bx+a)\ln(-cx+1)\,\mathrm{polylog}(2,cx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

```
[Out] int((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x^5,x)
```

Maxima [A] time = 1.18925, size = 460, normalized size = 0.79

$$\frac{1}{12}\left(3\,ac^4+4\,bc^3\right)\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)+\frac{1}{72}\left(9\,ac^4+16\,bc^3\right)(\log(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*c^4 + 4*b*c^3)*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(
-c*x + 1) - 2*polylog(3, -c*x + 1)) + 1/72*(9*a*c^4 + 16*b*c^3)*(log(c*x)*l
og(-c*x + 1) + dilog(-c*x + 1)) - 1/72*(41*a*c^4 + 54*b*c^3)*log(x) - 1/12*
(3*a*c^4 + 4*b*c^3)*polylog(3, c*x) + 1/144*(5*a*c^2*x^2 + 28*(a*c^3 + b*c^
2)*x^3 - ((9*a*c^4 + 16*b*c^3)*x^4 - 16*b*x - 9*a)*log(-c*x + 1)^2 + 6*(2*(
3*a*c^3 + 4*b*c^2)*x^3 + 2*a*c*x + (3*a*c^2 + 4*b*c)*x^2 + 2*((3*a*c^4 + 4*
b*c^3)*x^4 - 4*b*x - 3*a)*log(-c*x + 1))*dilog(c*x) + 2*((41*a*c^4 + 54*b*c
^3)*x^4 - (27*a*c^3 + 40*b*c^2)*x^3 - 5*a*c*x - (9*a*c^2 + 14*b*c)*x^2)*log
(-c*x + 1))/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(-c*x+1)*polylog(2,c*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx+a)\text{Li}_2(cx)\log(-cx+1)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(-c*x+1)*polylog(2,c*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*dilog(c*x)*log(-c*x + 1)/x^5, x)
```

3.192 $\int x\left(a+bx+cx^2\right)\log(1-dx)\mathbf{PolyLog}(2,dx)\,dx$

Optimal. Leaf size=900

$$\frac{1}{16}c\log^2(1-dx)x^4+\frac{3cx^4}{256}-\frac{3}{64}c\log(1-dx)x^4-\frac{1}{16}c\text{PolyLog}(2,dx)x^4+\frac{1}{9}b\log^2(1-dx)x^3+\frac{2bx^3}{81}+\frac{(3c+4bd)x^3}{324d}-\frac{2}{27}$$

```
[Out] (53*c*x)/(192*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + ((3*c + 4*b*d)*x)/(108*d
^3) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^3) + (29*c*x^2)/(384*d^2) + (5*b*
x^2)/(54*d) + ((3*c + 4*b*d)*x^2)/(216*d^2) + ((3*c + 4*b*d + 6*a*d^2)*x^2)
/(96*d^2) + (2*b*x^3)/81 + (17*c*x^3)/(576*d) + ((3*c + 4*b*d)*x^3)/(324*d)
 + (3*c*x^4)/256 + (a*(1 - d*x)^2)/(8*d^2) + (29*c*Log[1 - d*x])/(192*d^4)
+ (5*b*Log[1 - d*x])/(27*d^3) + ((3*c + 4*b*d)*Log[1 - d*x])/(108*d^4) + ((
3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x])/(48*d^4) - (c*x^2*Log[1 - d*x])/(16*d^
2) - (b*x^2*Log[1 - d*x])/(9*d) - ((3*c + 4*b*d + 6*a*d^2)*x^2*Log[1 - d*x]
)/(48*d^2) - (2*b*x^3*Log[1 - d*x])/27 - (c*x^3*Log[1 - d*x])/(24*d) - ((3*
c + 4*b*d)*x^3*Log[1 - d*x])/(108*d) - (3*c*x^4*Log[1 - d*x])/64 + (c*(1 -
d*x)*Log[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*Log[1 - d*x])/(9*d^3) + (a*(1 -
 d*x)*Log[1 - d*x])/d^2 + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*Log[1 - d*x])/
(12*d^4) - (a*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(16*d^
4) - (b*Log[1 - d*x]^2)/(9*d^3) + (b*x^3*Log[1 - d*x]^2)/9 + (c*x^4*Log[1 -
 d*x]^2)/16 - (a*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) + (a*(1 - d*x)^2*Log[1 -
 d*x]^2)/(4*d^2) - ((3*c + 4*b*d + 6*a*d^2)*Log[d*x]*Log[1 - d*x]^2)/(12*d^
4) - ((3*c + 4*b*d + 6*a*d^2)*x*PolyLog[2, d*x])/(12*d^3) - ((3*c + 4*b*d +
 6*a*d^2)*x^2*PolyLog[2, d*x])/(24*d^2) - ((3*c + 4*b*d)*x^3*PolyLog[2, d*x
])/(36*d) - (c*x^4*PolyLog[2, d*x])/16 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d
*x]*PolyLog[2, d*x])/(12*d^4) + ((6*a*x^2 + 4*b*x^3 + 3*c*x^4)*Log[1 - d*x]
*PolyLog[2, d*x])/12 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x]*PolyLog[2, 1 -
 d*x])/(6*d^4) + ((3*c + 4*b*d + 6*a*d^2)*PolyLog[3, 1 - d*x])/(6*d^4)
```

Rubi [A] time = 1.18491, antiderivative size = 900, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {6742, 6591, 2395, 43, 14, 6604, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2410, 2301, 6586, 6596, 2396, 2433, 2374, 6589}

$$\frac{1}{16}c\log^2(1-dx)x^4+\frac{3cx^4}{256}-\frac{3}{64}c\log(1-dx)x^4-\frac{1}{16}c\text{PolyLog}(2,dx)x^4+\frac{1}{9}b\log^2(1-dx)x^3+\frac{2bx^3}{81}+\frac{(3c+4bd)x^3}{324d}-\frac{2}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] (53*c*x)/(192*d^3) + (11*b*x)/(27*d^2) + (a*x)/d + ((3*c + 4*b*d)*x)/(108*d
^3) + (5*(3*c + 4*b*d + 6*a*d^2)*x)/(48*d^3) + (29*c*x^2)/(384*d^2) + (5*b*
x^2)/(54*d) + ((3*c + 4*b*d)*x^2)/(216*d^2) + ((3*c + 4*b*d + 6*a*d^2)*x^2)
/(96*d^2) + (2*b*x^3)/81 + (17*c*x^3)/(576*d) + ((3*c + 4*b*d)*x^3)/(324*d)
 + (3*c*x^4)/256 + (a*(1 - d*x)^2)/(8*d^2) + (29*c*Log[1 - d*x])/(192*d^4)
+ (5*b*Log[1 - d*x])/(27*d^3) + ((3*c + 4*b*d)*Log[1 - d*x])/(108*d^4) + ((
3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x])/(48*d^4) - (c*x^2*Log[1 - d*x])/(16*d^
2) - (b*x^2*Log[1 - d*x])/(9*d) - ((3*c + 4*b*d + 6*a*d^2)*x^2*Log[1 - d*x]
)/(48*d^2) - (2*b*x^3*Log[1 - d*x])/27 - (c*x^3*Log[1 - d*x])/(24*d) - ((3*
c + 4*b*d)*x^3*Log[1 - d*x])/(108*d) - (3*c*x^4*Log[1 - d*x])/64 + (c*(1 -
d*x)*Log[1 - d*x])/(8*d^4) + (2*b*(1 - d*x)*Log[1 - d*x])/(9*d^3) + (a*(1 -
 d*x)*Log[1 - d*x])/d^2 + ((3*c + 4*b*d + 6*a*d^2)*(1 - d*x)*Log[1 - d*x])/
(12*d^4) - (a*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(16*d^
4) - (b*Log[1 - d*x]^2)/(9*d^3) + (b*x^3*Log[1 - d*x]^2)/9 + (c*x^4*Log[1 -
 d*x]^2)/16 - (a*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) + (a*(1 - d*x)^2*Log[1 -
 d*x]^2)/(4*d^2) - ((3*c + 4*b*d + 6*a*d^2)*Log[d*x]*Log[1 - d*x]^2)/(12*d^
4) - ((3*c + 4*b*d + 6*a*d^2)*x*PolyLog[2, d*x])/(12*d^3) - ((3*c + 4*b*d +
 6*a*d^2)*x^2*PolyLog[2, d*x])/(24*d^2) - ((3*c + 4*b*d)*x^3*PolyLog[2, d*x
])/(36*d) - (c*x^4*PolyLog[2, d*x])/16 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d
*x]*PolyLog[2, d*x])/(12*d^4) + ((6*a*x^2 + 4*b*x^3 + 3*c*x^4)*Log[1 - d*x]
*PolyLog[2, d*x])/12 - ((3*c + 4*b*d + 6*a*d^2)*Log[1 - d*x]*PolyLog[2, 1 -
 d*x])/(6*d^4) + ((3*c + 4*b*d + 6*a*d^2)*PolyLog[3, 1 - d*x])/(6*d^4)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
```

```
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\left(a+bx+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)\,dx = \frac{1}{12}\left(6ax^2+4bx^3+3cx^4\right)\log(1-dx)\mathrm{Li}_2(dx)+d\int\left(\frac{\left(-3c-4bd-6ad^2\right)\mathrm{Li}_2\right.$$

$$= \frac{1}{12}\left(6ax^2+4bx^3+3cx^4\right)\log(1-dx)\mathrm{Li}_2(dx)+\frac{1}{2}a\int x\log^2(1-dx)\,dx+\frac{1}{3}b$$

$$= \frac{1}{9}bx^3\log^2(1-dx)+\frac{1}{16}cx^4\log^2(1-dx)-\frac{\left(3c+4bd+6ad^2\right)x\mathrm{Li}_2(dx)}{12d^3}-\frac{(3}{}$$

$$= -\frac{\left(3c+4bd+6ad^2\right)x^2\log(1-dx)}{48d^2}-\frac{(3c+4bd)x^3\log(1-dx)}{108d}-\frac{1}{64}cx^4\log($$

$$= \frac{\left(3c+4bd+6ad^2\right)x}{12d^3}-\frac{\left(3c+4bd+6ad^2\right)x^2\log(1-dx)}{48d^2}-\frac{(3c+4bd)x^3\log}{108d}$$

$$= \frac{cx}{64d^3}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{cx^2}{128d^2}+\frac{(3c+4bd)x^2}{216d^2}+\frac{(3}{}$$

$$= \frac{9cx}{64d^3}+\frac{2bx}{9d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{cx^2}{128d^2}+\frac{(3c+}{21}$$

$$= \frac{53cx}{192d^3}+\frac{11bx}{27d^2}+\frac{ax}{d}+\frac{(3c+4bd)x}{108d^3}+\frac{5\left(3c+4bd+6ad^2\right)x}{48d^3}+\frac{29cx^2}{384d^2}+\frac{5b}{54}$$

<u>Mathematica</u> [A] time = 1.27973, size = 583, normalized size = 0.65

$$\frac{\mathrm{PolyLog}(2,dx)\left(12\log(1-dx)\left(6ad^4x^2-6ad^2+4bd^4x^3-4bd+3c\left(d^4x^4-1\right)\right)-dx\left(4d\left(9ad(dx+2)+2b\left(2d^2x^2+3d\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] ((355*c*d*x)/4 + 124*b*d^2*x + 198*a*d^3*x + (139*c*d^2*x^2)/8 + 22*b*d^3*x
^2 + 27*a*d^4*x^2 + (67*c*d^3*x^3)/12 + (16*b*d^4*x^3)/3 + (27*c*d^4*x^4)/1
6 + (355*c*Log[1 - d*x])/4 + 124*b*d*Log[1 - d*x] + 198*a*d^2*Log[1 - d*x]
- 54*c*d*x*Log[1 - d*x] - 80*b*d^2*x*Log[1 - d*x] - 144*a*d^3*x*Log[1 - d*x
] - 18*c*d^2*x^2*Log[1 - d*x] - 28*b*d^3*x^2*Log[1 - d*x] - 54*a*d^4*x^2*Lo
g[1 - d*x] - 10*c*d^3*x^3*Log[1 - d*x] - 16*b*d^4*x^3*Log[1 - d*x] - (27*c*
d^4*x^4*Log[1 - d*x])/4 - 9*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2 - 36*a
*d^2*Log[1 - d*x]^2 + 36*a*d^4*x^2*Log[1 - d*x]^2 + 16*b*d^4*x^3*Log[1 - d*
x]^2 + 9*c*d^4*x^4*Log[1 - d*x]^2 - 36*c*Log[d*x]*Log[1 - d*x]^2 - 48*b*d*L
```

```
og[d*x]*Log[1 - d*x]^2 - 72*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d*x*(3*c*(12
 + 6*d*x + 4*d^2*x^2 + 3*d^3*x^3) + 4*d*(9*a*d*(2 + d*x) + 2*b*(6 + 3*d*x +
 2*d^2*x^2)))) + 12*(-4*b*d - 6*a*d^2 + 6*a*d^4*x^2 + 4*b*d^4*x^3 + 3*c*(-1
 + d^4*x^4))*Log[1 - d*x])*PolyLog[2, d*x] - 24*(3*c + 4*b*d + 6*a*d^2)*Log
[1 - d*x]*PolyLog[2, 1 - d*x] + 24*(3*c + 4*b*d + 6*a*d^2)*PolyLog[3, 1 - d
*x])/(144*d^4)
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int x\left(cx^2+bx+a\right)\ln\left(-dx+1\right)\operatorname{polylog}\left(2,dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] int(x*(c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

Maxima [A] time = 1.05354, size = 699, normalized size = 0.78

$$-\frac{1}{6912}\,d\left(\frac{576\left(6\,ad^2+4\,bd+3\,c\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^2+2\,\mathrm{Li}_2\left(-dx+1\right)\log\left(-dx+1\right)-2\,\mathrm{Li}_3(-dx+1)\right)}{d^5}-\frac{81\,cd^4x^4+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")
```

```
[Out] -1/6912*d*(576*(6*a*d^2 + 4*b*d + 3*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(
-d*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^5 - (81*c*d^4*x^4 + 4*(
64*b*d^4 + 67*c*d^3)*x^3 + 6*(216*a*d^4 + 176*b*d^3 + 139*c*d^2)*x^2 + 12*(
792*a*d^3 + 496*b*d^2 + 355*c*d)*x - 48*(9*c*d^4*x^4 + 4*(4*b*d^4 + 3*c*d^3
)*x^3 + 6*(6*a*d^4 + 4*b*d^3 + 3*c*d^2)*x^2 + 12*(6*a*d^3 + 4*b*d^2 + 3*c*d
)*x + 12*(6*a*d^2 + 4*b*d + 3*c)*log(-d*x + 1))*dilog(d*x) - 4*(54*c*d^4*x^
4 + 4*(32*b*d^4 + 21*c*d^3)*x^3 - 2376*a*d^2 + 6*(72*a*d^4 + 40*b*d^3 + 27*
c*d^2)*x^2 - 1488*b*d + 12*(108*a*d^3 + 64*b*d^2 + 45*c*d)*x - 1065*c)*log(
-d*x + 1))/d^5) + 1/1728*(216*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(
d^2*x^2 - 1)*log(-d*x + 1))*a/d^2 + 32*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 -
 3*d^2*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*b/d^3 + 9*(48*d^4*x^4*d
ilog(d*x) - 3*d^4*x^4 - 4*d^3*x^3 - 6*d^2*x^2 - 12*d*x + 12*(d^4*x^4 - 1)*l
```

```
og(-d*x + 1))*c/d^4)*log(-d*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^3+bx^2+ax\right)\text{Li}_2\left(dx\right)\log\left(-dx+1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")
```

```
[Out] integral((c*x^3 + b*x^2 + a*x)*dilog(d*x)*log(-d*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2+bx+a\right)x\text{Li}_2\left(dx\right)\log\left(-dx+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*x*dilog(d*x)*log(-d*x + 1), x)
```

3.193 $\int \left(a+bx+cx^2\right)\log(1-dx)\textbf{PolyLog}(2,dx)\,dx$

Optimal. Leaf size=645

$$-\frac{x\text{PolyLog}(2,dx)(3d(2ad+b)+2c)}{6d^2}+\frac{\text{PolyLog}(3,1-dx)(3d(2ad+b)+2c)}{3d^3}-\frac{\log(1-dx)\text{PolyLog}(2,dx)(3d(2ad+b)}{6d^3}$$

```
[Out] 2*a*x + (4*c*x)/(9*d^2) + (b*x)/d + ((2*c + 3*b*d)*x)/(24*d^2) + ((2*c + 3*
d*(b + 2*a*d))*x)/(6*d^2) + (c*x^2)/(9*d) + ((2*c + 3*b*d)*x^2)/(48*d) + (c
*x^3)/27 + (b*(1 - d*x)^2)/(8*d^2) + (2*c*Log[1 - d*x])/(9*d^3) + ((2*c + 3
*b*d)*Log[1 - d*x])/(24*d^3) - (c*x^2*Log[1 - d*x])/(9*d) - ((2*c + 3*b*d)*
x^2*Log[1 - d*x])/(24*d) - (c*x^3*Log[1 - d*x])/9 + (2*c*(1 - d*x)*Log[1 -
d*x])/(9*d^3) + (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x
])/d + ((2*c + 3*d*(b + 2*a*d))*(1 - d*x)*Log[1 - d*x])/(6*d^3) - (b*(1 - d
*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1 - d
*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 - d*x]
^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((2*c + 3*d*(b + 2*a*d))*L
og[d*x]*Log[1 - d*x]^2)/(6*d^3) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x
])/(6*d^2) - ((2*c + 3*b*d)*x^2*PolyLog[2, d*x])/(12*d) - (c*x^3*PolyLog[2,
 d*x])/9 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^3) +
 ((6*a*x + 3*b*x^2 + 2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 - ((2*c + 3*d
*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^3) + ((2*c + 3*d*(b +
2*a*d))*PolyLog[3, 1 - d*x])/(3*d^3)
```

Rubi [A] time = 0.818251, antiderivative size = 645, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 21, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.913, Rules used = {6742, 6586, 2389, 2295, 6591, 2395, 43, 6604, 2296, 2401, 2390, 2305, 2304, 2398, 2410, 2301, 6596, 2396, 2433, 2374, 6589}

$$-\frac{x\text{PolyLog}(2,dx)(3d(2ad+b)+2c)}{6d^2}+\frac{\text{PolyLog}(3,1-dx)(3d(2ad+b)+2c)}{3d^3}-\frac{\log(1-dx)\text{PolyLog}(2,dx)(3d(2ad+b)}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```


```
[Out] 2*a*x + (4*c*x)/(9*d^2) + (b*x)/d + ((2*c + 3*b*d)*x)/(24*d^2) + ((2*c + 3*
d*(b + 2*a*d))*x)/(6*d^2) + (c*x^2)/(9*d) + ((2*c + 3*b*d)*x^2)/(48*d) + (c
*x^3)/27 + (b*(1 - d*x)^2)/(8*d^2) + (2*c*Log[1 - d*x])/(9*d^3) + ((2*c + 3
*b*d)*Log[1 - d*x])/(24*d^3) - (c*x^2*Log[1 - d*x])/(9*d) - ((2*c + 3*b*d)*
x^2*Log[1 - d*x])/(24*d) - (c*x^3*Log[1 - d*x])/9 + (2*c*(1 - d*x)*Log[1 -
d*x])/(9*d^3) + (b*(1 - d*x)*Log[1 - d*x])/d^2 + (2*a*(1 - d*x)*Log[1 - d*x
```

```
])/d + ((2*c + 3*d*(b + 2*a*d))*(1 - d*x)*Log[1 - d*x])/(6*d^3) - (b*(1 - d
*x)^2*Log[1 - d*x])/(4*d^2) - (c*Log[1 - d*x]^2)/(9*d^3) + (c*x^3*Log[1 - d
*x]^2)/9 - (b*(1 - d*x)*Log[1 - d*x]^2)/(2*d^2) - (a*(1 - d*x)*Log[1 - d*x]
^2)/d + (b*(1 - d*x)^2*Log[1 - d*x]^2)/(4*d^2) - ((2*c + 3*d*(b + 2*a*d))*L
og[d*x]*Log[1 - d*x]^2)/(6*d^3) - ((2*c + 3*d*(b + 2*a*d))*x*PolyLog[2, d*x
])/(6*d^2) - ((2*c + 3*b*d)*x^2*PolyLog[2, d*x])/(12*d) - (c*x^3*PolyLog[2,
 d*x])/9 - ((2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/(6*d^3) +
 ((6*a*x + 3*b*x^2 + 2*c*x^3)*Log[1 - d*x]*PolyLog[2, d*x])/6 - ((2*c + 3*d
*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x])/(3*d^3) + ((2*c + 3*d*(b +
2*a*d))*PolyLog[3, 1 - d*x])/(3*d^3)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a + bx + cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)\,dx = \frac{1}{6}\left(6ax + 3bx^2 + 2cx^3\right)\log(1-dx)\mathrm{Li}_2(dx) + d\int\left(\frac{(-2c - 3d(b+2ad))\mathrm{Li}_2(dx}{6d^3}\right.$$

$$= \frac{1}{6}\left(6ax + 3bx^2 + 2cx^3\right)\log(1-dx)\mathrm{Li}_2(dx) + a\int \log^2(1-dx)\,dx + \frac{1}{2}b\int x\,\mathrm{lo}$$

$$= \frac{1}{9}cx^3\log^2(1-dx) - \frac{(2c + 3d(b+2ad))x\mathrm{Li}_2(dx)}{6d^2} - \frac{(2c+3bd)x^2\mathrm{Li}_2(dx)}{12d} - \frac{1}{9}c$$

$$= -\frac{(2c+3bd)x^2\log(1-dx)}{24d} - \frac{1}{27}cx^3\log(1-dx) + \frac{1}{9}cx^3\log^2(1-dx) - \frac{a(1-d}{}$$

$$= 2ax + \frac{(2c+3d(b+2ad))x}{6d^2} - \frac{(2c+3bd)x^2\log(1-dx)}{24d} - \frac{1}{27}cx^3\log(1-dx) +$$

$$= 2ax + \frac{cx}{27d^2} + \frac{(2c+3bd)x}{24d^2} + \frac{(2c+3d(b+2ad))x}{6d^2} + \frac{cx^2}{54d} + \frac{(2c+3bd)x^2}{48d} + \frac{c}{8}$$

$$= 2ax + \frac{7cx}{27d^2} + \frac{bx}{d} + \frac{(2c+3bd)x}{24d^2} + \frac{(2c+3d(b+2ad))x}{6d^2} + \frac{cx^2}{54d} + \frac{(2c+3bd)x}{48d}$$

$$= 2ax + \frac{4cx}{9d^2} + \frac{bx}{d} + \frac{(2c+3bd)x}{24d^2} + \frac{(2c+3d(b+2ad))x}{6d^2} + \frac{cx^2}{9d} + \frac{(2c+3bd)x^2}{48d}$$

Mathematica [A] time = 1.03267, size = 472, normalized size = 0.73

$$\frac{\mathrm{PolyLog}(2,dx)\left(6(dx-1)\log(1-dx)\left(3d(2ad+bdx+b)+2c\left(d^2x^2+dx+1\right)\right)-dx\left(9d(4ad+bdx+2b)+2c\left(2d^2x^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x],x]
```

```
[Out] (31*c*d*x + (99*b*d^2*x)/2 + 108*a*d^3*x + (11*c*d^2*x^2)/2 + (27*b*d^3*x^2
)/4 + (4*c*d^3*x^3)/3 + 31*c*Log[1 - d*x] + (99*b*d*Log[1 - d*x])/2 + 108*a
*d^2*Log[1 - d*x] - 20*c*d*x*Log[1 - d*x] - 36*b*d^2*x*Log[1 - d*x] - 108*a
*d^3*x*Log[1 - d*x] - 7*c*d^2*x^2*Log[1 - d*x] - (27*b*d^3*x^2*Log[1 - d*x]
)/2 - 4*c*d^3*x^3*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 9*b*d*Log[1 - d*x]^2
- 36*a*d^2*Log[1 - d*x]^2 + 36*a*d^3*x*Log[1 - d*x]^2 + 9*b*d^3*x^2*Log[1 -
 d*x]^2 + 4*c*d^3*x^3*Log[1 - d*x]^2 - 12*c*Log[d*x]*Log[1 - d*x]^2 - 18*b*
d*Log[d*x]*Log[1 - d*x]^2 - 36*a*d^2*Log[d*x]*Log[1 - d*x]^2 + (-(d*x*(9*d*
(2*b + 4*a*d + b*d*x) + 2*c*(6 + 3*d*x + 2*d^2*x^2))) + 6*(-1 + d*x)*(3*d*(
b + 2*a*d + b*d*x) + 2*c*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2, d*x]
```

```
 - 12*(2*c + 3*d*(b + 2*a*d))*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 12*(2*c +
3*d*(b + 2*a*d))*PolyLog[3, 1 - d*x])/(36*d^3)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right) \ln\left(-dx + 1\right) \operatorname{polylog}\left(2, dx\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

Maxima [A] time = 1.05019, size = 556, normalized size = 0.86

$$-\frac{1}{432}d\left(\frac{72\left(6\,ad^2 + 3\,bd + 2\,c\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^2 + 2\,\mathrm{Li}_2\left(-dx+1\right)\log\left(-dx+1\right) - 2\,\mathrm{Li}_3(-dx+1)\right)}{d^4} - \frac{16\,cd^3x^3 + 3\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="maxima")
```

```
[Out] -1/432*d*(72*(6*a*d^2 + 3*b*d + 2*c)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d
*x + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1))/d^4 - (16*c*d^3*x^3 + 3*(27
*b*d^3 + 22*c*d^2)*x^2 + 6*(216*a*d^3 + 99*b*d^2 + 62*c*d)*x - 12*(4*c*d^3*
x^3 + 3*(3*b*d^3 + 2*c*d^2)*x^2 + 6*(6*a*d^3 + 3*b*d^2 + 2*c*d)*x + 6*(6*a*
d^2 + 3*b*d + 2*c)*log(-d*x + 1))*dilog(d*x) - 2*(16*c*d^3*x^3 - 648*a*d^2
+ 6*(9*b*d^3 + 5*c*d^2)*x^2 - 297*b*d + 6*(72*a*d^3 + 27*b*d^2 + 16*c*d)*x
- 186*c)*log(-d*x + 1))/d^4) + 1/216*(216*(d*x*dilog(d*x) - d*x + (d*x - 1)
*log(-d*x + 1))*a/d + 27*(4*d^2*x^2*dilog(d*x) - d^2*x^2 - 2*d*x + 2*(d^2*x
^2 - 1)*log(-d*x + 1))*b/d^2 + 4*(18*d^3*x^3*dilog(d*x) - 2*d^3*x^3 - 3*d^2
*x^2 - 6*d*x + 6*(d^3*x^3 - 1)*log(-d*x + 1))*c/d^3)*log(-d*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(cx^2 + bx + a\right)\mathrm{Li}_2\left(dx\right)\log\left(-dx+1\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx + a\right)\mathrm{Li}_2\left(dx\right)\log\left(-dx + 1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1), x)
```

3.194 $\int \frac{(a+bx+cx^2)\log(1-dx)\mathbf{PolyLog}(2,dx)}{x}\,dx$

Optimal. Leaf size=402

$$-\frac{1}{2}a\text{PolyLog}(2,dx)^2+\frac{(2bd+c)\text{PolyLog}(3,1-dx)}{d^2}-\frac{(2bd+c)\log(1-dx)\text{PolyLog}(2,dx)}{2d^2}-\frac{(2bd+c)\log(1-dx)\text{Poly}}{d^2}$$

```
[Out] 2*b*x + (9*c*x)/(8*d) + ((c + 2*b*d)*x)/(2*d) + (c*x^2)/16 + (c*(1 - d*x)^2
)/(8*d^2) + (c*Log[1 - d*x])/(8*d^2) - (c*x^2*Log[1 - d*x])/8 + (c*(1 - d*x
)*Log[1 - d*x])/d^2 + (2*b*(1 - d*x)*Log[1 - d*x])/d + ((c + 2*b*d)*(1 - d*
x)*Log[1 - d*x])/(2*d^2) - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*(1 - d
*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (c*(1 - d*x)
^2*Log[1 - d*x]^2)/(4*d^2) - ((c + 2*b*d)*Log[d*x]*Log[1 - d*x]^2)/(2*d^2)
- ((c + 2*b*d)*x*PolyLog[2, d*x])/(2*d) - (c*x^2*PolyLog[2, d*x])/4 - ((c +
 2*b*d)*Log[1 - d*x]*PolyLog[2, d*x])/(2*d^2) + ((2*b*x + c*x^2)*Log[1 - d*
x]*PolyLog[2, d*x])/2 - (a*PolyLog[2, d*x]^2)/2 - ((c + 2*b*d)*Log[1 - d*x]
*PolyLog[2, 1 - d*x])/d^2 + ((c + 2*b*d)*PolyLog[3, 1 - d*x])/d^2
```

Rubi [A] time = 0.618594, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 24, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {6742, 6586, 2389, 2295, 6589, 6591, 2395, 43, 6605, 6601, 1584, 6598, 2416, 2391, 6604, 2296, 2401, 2390, 2305, 2304, 6596, 2396, 2433, 2374}

$$-\frac{1}{2}a\text{PolyLog}(2,dx)^2+\frac{(2bd+c)\text{PolyLog}(3,1-dx)}{d^2}-\frac{(2bd+c)\log(1-dx)\text{PolyLog}(2,dx)}{2d^2}-\frac{(2bd+c)\log(1-dx)\text{Poly}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

```
[Out] 2*b*x + (9*c*x)/(8*d) + ((c + 2*b*d)*x)/(2*d) + (c*x^2)/16 + (c*(1 - d*x)^2
)/(8*d^2) + (c*Log[1 - d*x])/(8*d^2) - (c*x^2*Log[1 - d*x])/8 + (c*(1 - d*x
)*Log[1 - d*x])/d^2 + (2*b*(1 - d*x)*Log[1 - d*x])/d + ((c + 2*b*d)*(1 - d*
x)*Log[1 - d*x])/(2*d^2) - (c*(1 - d*x)^2*Log[1 - d*x])/(4*d^2) - (c*(1 - d
*x)*Log[1 - d*x]^2)/(2*d^2) - (b*(1 - d*x)*Log[1 - d*x]^2)/d + (c*(1 - d*x)
^2*Log[1 - d*x]^2)/(4*d^2) - ((c + 2*b*d)*Log[d*x]*Log[1 - d*x]^2)/(2*d^2)
- ((c + 2*b*d)*x*PolyLog[2, d*x])/(2*d) - (c*x^2*PolyLog[2, d*x])/4 - ((c +
 2*b*d)*Log[1 - d*x]*PolyLog[2, d*x])/(2*d^2) + ((2*b*x + c*x^2)*Log[1 - d*
x]*PolyLog[2, d*x])/2 - (a*PolyLog[2, d*x]^2)/2 - ((c + 2*b*d)*Log[1 - d*x]
*PolyLog[2, 1 - d*x])/d^2 + ((c + 2*b*d)*PolyLog[3, 1 - d*x])/d^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*Poly
Log[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6598

```
Int[((d_.) + (e_.)*(x_))^(m_.)*PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Sy
mbol] :> Simp[((d + e*x)^(m + 1)*PolyLog[2, c*(a + b*x)])/(e*(m + 1)), x] +
 Dist[b/(e*(m + 1)), Int[((d + e*x)^(m + 1)*Log[1 - a*c - b*c*x])/(a + b*x)
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6604

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*PolyLog[2,
(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[Px, x]}, Simp[u
*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (Dist[b, Int[Expa
ndIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x], u/(a + b*x), x
], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a + b*x)], u/(d +
 e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && PolyQ[Px, x
]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
```

```
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\text{Li}_2(dx)}{x}\,dx = a\int \frac{\log(1-dx)\text{Li}_2(dx)}{x}\,dx + \int \frac{\left(bx+cx^2\right)\log(1-dx)\text{Li}_2(dx)}{x}\,dx$$

$$= -\frac{1}{2}a\text{Li}_2(dx)^2 + \int (b+cx)\log(1-dx)\text{Li}_2(dx)\,dx$$

$$= \frac{1}{2}\left(2bx+cx^2\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}a\text{Li}_2(dx)^2 + d\int \left(\frac{(-c-2bd)\text{Li}_2(dx)}{2d^2} -\right.$$

$$= \frac{1}{2}\left(2bx+cx^2\right)\log(1-dx)\text{Li}_2(dx) - \frac{1}{2}a\text{Li}_2(dx)^2 + b\int \log^2(1-dx)\,dx + \frac{1}{2}c$$

$$= -\frac{(c+2bd)x\text{Li}_2(dx)}{2d} - \frac{1}{4}cx^2\text{Li}_2(dx) - \frac{(c+2bd)\log(1-dx)\text{Li}_2(dx)}{2d^2} + \frac{1}{2}\left(2bx\right.$$

$$= -\frac{1}{8}cx^2\log(1-dx) - \frac{b(1-dx)\log^2(1-dx)}{d} - \frac{(c+2bd)\log(dx)\log^2(1-dx)}{2d^2}$$

$$= 2bx + \frac{(c+2bd)x}{2d} - \frac{1}{8}cx^2\log(1-dx) + \frac{2b(1-dx)\log(1-dx)}{d} + \frac{(c+2bd)(1}{}$$

$$= 2bx + \frac{cx}{8d} + \frac{(c+2bd)x}{2d} + \frac{cx^2}{16} + \frac{c\log(1-dx)}{8d^2} - \frac{1}{8}cx^2\log(1-dx) + \frac{2b(1-d}{}$$

$$= 2bx + \frac{9cx}{8d} + \frac{(c+2bd)x}{2d} + \frac{cx^2}{16} + \frac{c(1-dx)^2}{8d^2} + \frac{c\log(1-dx)}{8d^2} - \frac{1}{8}cx^2\log(1-$$

Mathematica [A] time = 0.355676, size = 298, normalized size = 0.74

$$\frac{-8ad^2\text{PolyLog}(2,dx)^2 + 4\text{PolyLog}(2,dx)(2(dx-1)\log(1-dx)(2bd+cdx+c) - dx(4bd+cdx+2c)) - 16(2bd+c)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x,x]
```

```
[Out] (-14*c - 32*b*d + 22*c*d*x + 48*b*d^2*x + 3*c*d^2*x^2 + 22*c*Log[1 - d*x] +
 48*b*d*Log[1 - d*x] - 16*c*d*x*Log[1 - d*x] - 48*b*d^2*x*Log[1 - d*x] - 6*
c*d^2*x^2*Log[1 - d*x] - 4*c*Log[1 - d*x]^2 - 16*b*d*Log[1 - d*x]^2 + 16*b*
d^2*x*Log[1 - d*x]^2 + 4*c*d^2*x^2*Log[1 - d*x]^2 - 8*c*Log[d*x]*Log[1 - d*
x]^2 - 16*b*d*Log[d*x]*Log[1 - d*x]^2 + 4*(-(d*x*(2*c + 4*b*d + c*d*x)) + 2
*(-1 + d*x)*(c + 2*b*d + c*d*x)*Log[1 - d*x])*PolyLog[2, d*x] - 8*a*d^2*Pol
yLog[2, d*x]^2 - 16*(c + 2*b*d)*Log[1 - d*x]*PolyLog[2, 1 - d*x] + 16*c*Pol
yLog[3, 1 - d*x] + 32*b*d*PolyLog[3, 1 - d*x])/(16*d^2)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)\ln(-dx + 1)\,\mathrm{polylog}(2, dx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)\mathrm{Li}_2(dx)\log(-dx + 1)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(cx^2 + bx + a)\mathrm{Li}_2(dx)\log(-dx + 1)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)\text{Li}_2\left(dx\right)\log\left(-dx + 1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x, x)
```

3.195 $\int \frac{(a+bx+cx^2)\log(1-dx)\textbf{PolyLog}(2,dx)}{x^2}\,dx$

Optimal. Leaf size=218

$$-2d\left(a-\frac{c}{d^2}\right)\text{PolyLog}(3,1-dx)+d\left(a-\frac{c}{d^2}\right)\log(1-dx)\text{PolyLog}(2,dx)+2d\left(a-\frac{c}{d^2}\right)\log(1-dx)\text{PolyLog}(2,1-dx)-$$

```
[Out] 3*c*x + (3*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d + (
a*(1 - d*x)*Log[1 - d*x]^2)/x + (a - c/d^2)*d*Log[d*x]*Log[1 - d*x]^2 - 2*a
*d*PolyLog[2, d*x] - c*x*PolyLog[2, d*x] + (a - c/d^2)*d*Log[1 - d*x]*PolyL
og[2, d*x] - (a/x - c*x)*Log[1 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^
2)/2 + 2*(a - c/d^2)*d*Log[1 - d*x]*PolyLog[2, 1 - d*x] - a*d*PolyLog[3, d*
x] - 2*(a - c/d^2)*d*PolyLog[3, 1 - d*x]
```

Rubi [A] time = 0.507956, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 21, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.808, Rules used = {6742, 6586, 2389, 2295, 6591, 2395, 36, 29, 31, 6589, 6605, 6601, 14, 6606, 2296, 2397, 2391, 6596, 2396, 2433, 2374}

$$-2d\left(a-\frac{c}{d^2}\right)\text{PolyLog}(3,1-dx)+d\left(a-\frac{c}{d^2}\right)\log(1-dx)\text{PolyLog}(2,dx)+2d\left(a-\frac{c}{d^2}\right)\log(1-dx)\text{PolyLog}(2,1-dx)-$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```


```
[Out] 3*c*x + (3*c*(1 - d*x)*Log[1 - d*x])/d - (c*(1 - d*x)*Log[1 - d*x]^2)/d + (
a*(1 - d*x)*Log[1 - d*x]^2)/x + (a - c/d^2)*d*Log[d*x]*Log[1 - d*x]^2 - 2*a
*d*PolyLog[2, d*x] - c*x*PolyLog[2, d*x] + (a - c/d^2)*d*Log[1 - d*x]*PolyL
og[2, d*x] - (a/x - c*x)*Log[1 - d*x]*PolyLog[2, d*x] - (b*PolyLog[2, d*x]^
2)/2 + 2*(a - c/d^2)*d*Log[1 - d*x]*PolyLog[2, 1 - d*x] - a*d*PolyLog[3, d*
x] - 2*(a - c/d^2)*d*PolyLog[3, 1 - d*x]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6586

```
Int[PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbol] :> Simp[x*PolyLo
g[n, a*(b*x^p)^q], x] - Dist[p*q, Int[PolyLog[n - 1, a*(b*x^p)^q], x], x] /
```

```
; FreeQ[{a, b, p, q}, x] && GtQ[n, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*Poly
Log[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
```

```
NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)}{x^2}\,dx = b\int \frac{\log(1-dx)\mathrm{Li}_2(dx)}{x}\,dx + \int \frac{\left(a+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)}{x^2}\,dx$$

$$= -\left(\frac{a}{x}-cx\right)\log(1-dx)\mathrm{Li}_2(dx) - \frac{1}{2}b\mathrm{Li}_2(dx)^2 + d\int\left(-\frac{c\mathrm{Li}_2(dx)}{d} - \frac{a\mathrm{Li}_2(dx)}{x} + \right.$$

$$= -\left(\frac{a}{x}-cx\right)\log(1-dx)\mathrm{Li}_2(dx) - \frac{1}{2}b\mathrm{Li}_2(dx)^2 - a\int \frac{\log^2(1-dx)}{x^2}\,dx + c\int \log^2$$

$$= \frac{a(1-dx)\log^2(1-dx)}{x} - cx\mathrm{Li}_2(dx) - \frac{\left(c-ad^2\right)\log(1-dx)\mathrm{Li}_2(dx)}{d} - \left(\frac{a}{x}-cx\right)$$

$$= -\frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-dx)\log^2(1-dx)}{x} - \frac{\left(c-ad^2\right)\log(dx)\log^2(1-}{d}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-dx)\log^2(1-dx)}{x}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-dx)\log^2(1-dx)}{x}$$

$$= 3cx + \frac{3c(1-dx)\log(1-dx)}{d} - \frac{c(1-dx)\log^2(1-dx)}{d} + \frac{a(1-dx)\log^2(1-dx)}{x}$$

<u>Mathematica</u> [A] time = 0.83377, size = 280, normalized size = 1.28

$$\frac{2\left(2x\mathrm{PolyLog}(2,1-dx)\left(\left(ad^2-c\right)\log(1-dx)+ad^2\right)-ad^2x\mathrm{PolyLog}(3,dx)-2ad^2x\mathrm{PolyLog}(3,1-dx)+2cx\mathrm{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^2,x]
```

```
[Out] (2*(-(c*d*x^2) + (a*d + c*x)*(-1 + d*x)*Log[1 - d*x])*PolyLog[2, d*x] - b*d
*x*PolyLog[2, d*x]^2 + 2*(-2*c*x + 3*c*d*x^2 + 3*c*x*Log[1 - d*x] - 3*c*d*x
^2*Log[1 - d*x] + 2*a*d^2*x*Log[d*x]*Log[1 - d*x] + a*d*Log[1 - d*x]^2 - c*
x*Log[1 - d*x]^2 - a*d^2*x*Log[1 - d*x]^2 + c*d*x^2*Log[1 - d*x]^2 - c*x*Lo
g[d*x]*Log[1 - d*x]^2 + a*d^2*x*Log[d*x]*Log[1 - d*x]^2 + 2*x*(a*d^2 + (-c
+ a*d^2)*Log[1 - d*x])*PolyLog[2, 1 - d*x] - a*d^2*x*PolyLog[3, d*x] + 2*c*
x*PolyLog[3, 1 - d*x] - 2*a*d^2*x*PolyLog[3, 1 - d*x]))/(2*d*x)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(c x^2+b x+a) \ln(-d x+1) \operatorname{polylog}(2, d x)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c x^2+b x+a) \operatorname{Li}_2(d x) \log(-d x+1)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="maxima
")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(c x^2+b x+a) \operatorname{Li}_2(d x) \log(-d x+1)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)\text{Li}_2(dx)\log(-dx + 1)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^2, x)
```

3.196 $\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\textbf{PolyLog}(2,dx)}{x^3}\,dx$

Optimal. Leaf size=343

$$\frac{b^2\text{PolyLog}(3,1-dx)}{a}-\frac{b^2\log(1-dx)\text{PolyLog}(2,1-dx)}{a}-\frac{(a+bx)^2\log(1-dx)\text{PolyLog}(2,dx)}{2ax^2}-\frac{1}{2}d(ad+2b)\text{PolyL}$$

```
[Out] -(a*d^2*Log[x]) + a*d^2*Log[1 - d*x] - (a*d*Log[1 - d*x])/x - (a*d^2*Log[1
- d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1 - d*x]^2)/x -
 (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2
)/(2*a) - 2*b*d*PolyLog[2, d*x] - (a*d^2*PolyLog[2, d*x])/2 + (a*d*PolyLog[
2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a) - ((a + b
*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (
b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a + ((b + a*d)^2*Log[1 - d*x]*PolyLog
[2, 1 - d*x])/a - (d*(2*b + a*d)*PolyLog[3, d*x])/2 + (b^2*PolyLog[3, 1 - d
*x])/a - ((b + a*d)^2*PolyLog[3, 1 - d*x])/a
```

Rubi [A] time = 0.740443, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 22, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 6589, 6605, 6601, 37, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 2396, 2433, 2374, 6596}

$$\frac{b^2\text{PolyLog}(3,1-dx)}{a}-\frac{b^2\log(1-dx)\text{PolyLog}(2,1-dx)}{a}-\frac{(a+bx)^2\log(1-dx)\text{PolyLog}(2,dx)}{2ax^2}-\frac{1}{2}d(ad+2b)\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```


```
[Out] -(a*d^2*Log[x]) + a*d^2*Log[1 - d*x] - (a*d*Log[1 - d*x])/x - (a*d^2*Log[1
- d*x]^2)/4 + (a*Log[1 - d*x]^2)/(4*x^2) + (b*(1 - d*x)*Log[1 - d*x]^2)/x -
 (b^2*Log[d*x]*Log[1 - d*x]^2)/(2*a) + ((b + a*d)^2*Log[d*x]*Log[1 - d*x]^2
)/(2*a) - 2*b*d*PolyLog[2, d*x] - (a*d^2*PolyLog[2, d*x])/2 + (a*d*PolyLog[
2, d*x])/(2*x) + ((b + a*d)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a) - ((a + b
*x)^2*Log[1 - d*x]*PolyLog[2, d*x])/(2*a*x^2) - (c*PolyLog[2, d*x]^2)/2 - (
b^2*Log[1 - d*x]*PolyLog[2, 1 - d*x])/a + ((b + a*d)^2*Log[1 - d*x]*PolyLog
[2, 1 - d*x])/a - (d*(2*b + a*d)*PolyLog[3, d*x])/2 + (b^2*PolyLog[3, 1 - d
*x])/a - ((b + a*d)^2*PolyLog[3, 1 - d*x])/a
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6605

```
Int[((g_.) + Log[1 + (e_.)*(x_)]*(h_.))*(Px_)*(x_)^(m_)*PolyLog[2, (c_.)*(x
_)], x_Symbol] :> Dist[Coeff[Px, x, -m - 1], Int[((g + h*Log[1 + e*x])*Poly
Log[2, c*x])/x, x], x] + Int[x^m*(Px - Coeff[Px, x, -m - 1]*x^(-m - 1))*(g
+ h*Log[1 + e*x])*PolyLog[2, c*x], x] /; FreeQ[{c, e, g, h}, x] && PolyQ[Px
, x] && ILtQ[m, 0] && EqQ[c + e, 0] && NeQ[Coeff[Px, x, -m - 1], 0]
```

Rule 6601

```
Int[(Log[1 + (e_.)*(x_)]*PolyLog[2, (c_.)*(x_)])/(x_), x_Symbol] :> -Simp[P
olyLog[2, c*x]^2/2, x] /; FreeQ[{c, e}, x] && EqQ[c + e, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
```

```
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\text{Li}_2(dx)}{x^3}\,dx = c\int \frac{\log(1-dx)\text{Li}_2(dx)}{x}\,dx + \int \frac{(a+bx)\log(1-dx)\text{Li}_2(dx)}{x^3}\,dx$$

$$= -\frac{(a+bx)^2\log(1-dx)\text{Li}_2(dx)}{2ax^2} - \frac{1}{2}c\text{Li}_2(dx)^2 + d\int\left(-\frac{a\text{Li}_2(dx)}{2x^2} + \frac{(-2b-ad}{2x}\right.$$

$$= -\frac{(a+bx)^2\log(1-dx)\text{Li}_2(dx)}{2ax^2} - \frac{1}{2}c\text{Li}_2(dx)^2 - \frac{1}{2}a\int \frac{\log^2(1-dx)}{x^3}\,dx - b\int$$

$$= \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2\log(dx)\log^2(1-dx)}{2a} + \frac{ad\text{Li}_2(d}{2x}$$

$$= -\frac{ad\log(1-dx)}{2x} + \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2\log(dx)\log^2}{2a}$$

$$= -\frac{ad\log(1-dx)}{2x} + \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-dx)\log^2(1-dx)}{x} - \frac{b^2\log(dx)\log^2}{2a}$$

$$= -\frac{1}{2}ad^2\log(x) + \frac{1}{2}ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} + \frac{a\log^2(1-dx)}{4x^2} + \frac{b(1-}{}$$

$$= -\frac{1}{2}ad^2\log(x) + \frac{1}{2}ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} - \frac{1}{4}ad^2\log^2(1-dx) + \frac{a\,l}{}$$

$$= -ad^2\log(x) + ad^2\log(1-dx) - \frac{ad\log(1-dx)}{x} - \frac{1}{4}ad^2\log^2(1-dx) + \frac{a\log^2}{4}$$

Mathematica [F] time = 1.98638, size = 0, normalized size = 0.

$$\int \frac{(a+bx+cx^2)\log(1-dx)\text{PolyLog}(2,dx)}{x^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3,x]
```

```
[Out] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^3, x]
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)\ln(-dx+1)\,\text{polylog}(2,dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)\text{Li}_2(dx)\log(-dx+1)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="maxima
")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2+bx+a)\text{Li}_2(dx)\log(-dx+1)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)\mathrm{Li}_2\left(dx\right)\log\left(-dx + 1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^3, x)
```

3.197 $\int \frac{(a+bx+cx^2)\log(1-dx)\mathbf{PolyLog}(2,dx)}{x^4}\,dx$

Optimal. Leaf size=515

$$-\frac{1}{6}\log(1-dx)\text{PolyLog}(2,dx)\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)-\frac{1}{6}d\text{PolyLog}(3,dx)(d(2ad+3b)+6c)-\frac{1}{3}d\text{PolyLog}(3,1-dx)(d(2ad+$$

```
[Out] (7*a*d^2)/(36*x) - (b*d^2*Log[x])/2 - (5*a*d^3*Log[x])/12 - (d^2*(3*b + 2*a
*d)*Log[x])/6 + (b*d^2*Log[1 - d*x])/2 + (5*a*d^3*Log[1 - d*x])/12 + (d^2*(
3*b + 2*a*d)*Log[1 - d*x])/6 - (7*a*d*Log[1 - d*x])/(36*x^2) - (b*d*Log[1 -
 d*x])/(2*x) - (2*a*d^2*Log[1 - d*x])/(9*x) - (d*(3*b + 2*a*d)*Log[1 - d*x]
)/(6*x) - (b*d^2*Log[1 - d*x]^2)/4 - (a*d^3*Log[1 - d*x]^2)/9 + (a*Log[1 -
d*x]^2)/(9*x^3) + (b*Log[1 - d*x]^2)/(4*x^2) + (c*(1 - d*x)*Log[1 - d*x]^2)
/x + (d*(6*c + d*(3*b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/6 - 2*c*d*PolyLog[
2, d*x] - (b*d^2*PolyLog[2, d*x])/2 - (2*a*d^3*PolyLog[2, d*x])/9 + (a*d*Po
lyLog[2, d*x])/(6*x^2) + (d*(3*b + 2*a*d)*PolyLog[2, d*x])/(6*x) + (d*(6*c
+ d*(3*b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/6 - (((2*a)/x^3 + (3*b)/x^
2 + (6*c)/x)*Log[1 - d*x]*PolyLog[2, d*x])/6 + (d*(6*c + d*(3*b + 2*a*d))*L
og[1 - d*x]*PolyLog[2, 1 - d*x])/3 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3,
d*x])/6 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3, 1 - d*x])/3
```

Rubi [A] time = 0.823474, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 20, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {6742, 6591, 2395, 44, 36, 29, 31, 14, 6606, 2398, 2410, 2391, 2390, 2301, 2397, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{6}\log(1-dx)\text{PolyLog}(2,dx)\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)-\frac{1}{6}d\text{PolyLog}(3,dx)(d(2ad+3b)+6c)-\frac{1}{3}d\text{PolyLog}(3,1-dx)(d(2ad+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```


```
[Out] (7*a*d^2)/(36*x) - (b*d^2*Log[x])/2 - (5*a*d^3*Log[x])/12 - (d^2*(3*b + 2*a
*d)*Log[x])/6 + (b*d^2*Log[1 - d*x])/2 + (5*a*d^3*Log[1 - d*x])/12 + (d^2*(
3*b + 2*a*d)*Log[1 - d*x])/6 - (7*a*d*Log[1 - d*x])/(36*x^2) - (b*d*Log[1 -
 d*x])/(2*x) - (2*a*d^2*Log[1 - d*x])/(9*x) - (d*(3*b + 2*a*d)*Log[1 - d*x]
)/(6*x) - (b*d^2*Log[1 - d*x]^2)/4 - (a*d^3*Log[1 - d*x]^2)/9 + (a*Log[1 -
d*x]^2)/(9*x^3) + (b*Log[1 - d*x]^2)/(4*x^2) + (c*(1 - d*x)*Log[1 - d*x]^2)
/x + (d*(6*c + d*(3*b + 2*a*d))*Log[d*x]*Log[1 - d*x]^2)/6 - 2*c*d*PolyLog[
2, d*x] - (b*d^2*PolyLog[2, d*x])/2 - (2*a*d^3*PolyLog[2, d*x])/9 + (a*d*Po
```

```
lyLog[2, d*x])/(6*x^2) + (d*(3*b + 2*a*d)*PolyLog[2, d*x])/(6*x) + (d*(6*c
+ d*(3*b + 2*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/6 - (((2*a)/x^3 + (3*b)/x^
2 + (6*c)/x)*Log[1 - d*x]*PolyLog[2, d*x])/6 + (d*(6*c + d*(3*b + 2*a*d))*L
og[1 - d*x]*PolyLog[2, 1 - d*x])/3 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3,
d*x])/6 - (d*(6*c + d*(3*b + 2*a*d))*PolyLog[3, 1 - d*x])/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
 - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)}{x^4}\,dx = -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)\log(1-dx)\mathrm{Li}_2(dx)+d\int\left(-\frac{a\mathrm{Li}_2(dx)}{3x^3}+\frac{(-3b-2ad)\mathrm{Li}_2(\right.$$

$$= -\frac{1}{6}\left(\frac{2a}{x^3}+\frac{3b}{x^2}+\frac{6c}{x}\right)\log(1-dx)\mathrm{Li}_2(dx)-\frac{1}{3}a\int\frac{\log^2(1-dx)}{x^4}\,dx-\frac{1}{2}b\int\frac{\log^2(}{}$$

$$= \frac{a\log^2(1-dx)}{9x^3}+\frac{b\log^2(1-dx)}{4x^2}+\frac{c(1-dx)\log^2(1-dx)}{x}+\frac{ad\mathrm{Li}_2(dx)}{6x^2}+\frac{d(3b}{}$$

$$= -\frac{ad\log(1-dx)}{12x^2}-\frac{d(3b+2ad)\log(1-dx)}{6x}+\frac{a\log^2(1-dx)}{9x^3}+\frac{b\log^2(1-dx)}{4x^2}$$

$$= -\frac{ad\log(1-dx)}{12x^2}-\frac{d(3b+2ad)\log(1-dx)}{6x}+\frac{a\log^2(1-dx)}{9x^3}+\frac{b\log^2(1-dx)}{4x^2}$$

$$= \frac{ad^2}{12x}-\frac{1}{12}ad^3\log(x)-\frac{1}{6}d^2(3b+2ad)\log(x)+\frac{1}{12}ad^3\log(1-dx)+\frac{1}{6}d^2(3b+2$$

$$= \frac{ad^2}{12x}-\frac{1}{12}ad^3\log(x)-\frac{1}{6}d^2(3b+2ad)\log(x)+\frac{1}{12}ad^3\log(1-dx)+\frac{1}{6}d^2(3b+2$$

$$= \frac{7ad^2}{36x}-\frac{1}{2}bd^2\log(x)-\frac{5}{12}ad^3\log(x)-\frac{1}{6}d^2(3b+2ad)\log(x)+\frac{1}{2}bd^2\log(1-dx$$

Mathematica [A] time = 1.63239, size = 488, normalized size = 0.95

$$\frac{1}{36}\left(\frac{6\mathrm{PolyLog}(2,dx)\left((dx-1)\log(1-dx)\left(2a\left(d^2x^2+dx+1\right)+3x(bdx+b+2cx)\right)+dx(2adx+a+3bx)\right)}{x^3}+2d\mathrm{PolyLo}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^4,x]
```

```
[Out] (-7*a*d^3 + (7*a*d^2)/x - 36*b*d^2*Log[d*x] - 27*a*d^3*Log[d*x] + 36*b*d^2*
Log[1 - d*x] + 27*a*d^3*Log[1 - d*x] - (7*a*d*Log[1 - d*x])/x^2 - (36*b*d*L
```

```
og[1 - d*x])/x - (20*a*d^2*Log[1 - d*x])/x + 72*c*d*Log[d*x]*Log[1 - d*x] +
 18*b*d^2*Log[d*x]*Log[1 - d*x] + 8*a*d^3*Log[d*x]*Log[1 - d*x] - 36*c*d*Lo
g[1 - d*x]^2 - 9*b*d^2*Log[1 - d*x]^2 - 4*a*d^3*Log[1 - d*x]^2 + (4*a*Log[1
 - d*x]^2)/x^3 + (9*b*Log[1 - d*x]^2)/x^2 + (36*c*Log[1 - d*x]^2)/x + 36*c*
d*Log[d*x]*Log[1 - d*x]^2 + 18*b*d^2*Log[d*x]*Log[1 - d*x]^2 + 12*a*d^3*Log
[d*x]*Log[1 - d*x]^2 + (6*(d*x*(a + 3*b*x + 2*a*d*x) + (-1 + d*x)*(3*x*(b +
 2*c*x + b*d*x) + 2*a*(1 + d*x + d^2*x^2))*Log[1 - d*x])*PolyLog[2, d*x])/x
^3 + 2*d*(36*c + 9*b*d + 4*a*d^2 + 6*(6*c + 3*b*d + 2*a*d^2)*Log[1 - d*x])*
PolyLog[2, 1 - d*x] - 36*c*d*PolyLog[3, d*x] - 18*b*d^2*PolyLog[3, d*x] - 1
2*a*d^3*PolyLog[3, d*x] - 72*c*d*PolyLog[3, 1 - d*x] - 36*b*d^2*PolyLog[3,
1 - d*x] - 24*a*d^3*PolyLog[3, 1 - d*x])/36
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right) \ln \left(-dx + 1\right) \operatorname{polylog}\left(2, dx\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^4,x)
```

Maxima [A] time = 1.21818, size = 431, normalized size = 0.84

$$\frac{1}{6}\left(2\,ad^3 + 3\,bd^2 + 6\,cd\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^2 + 2\,\mathrm{Li}_2\left(-dx+1\right)\log\left(-dx+1\right) - 2\,\mathrm{Li}_3(-dx+1)\right) + \frac{1}{18}\left(4\,ad^3 + 9\,bd^2 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x +
1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/18*(4*a*d^3 + 9*b*d^2 + 36*c
*d)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/4*(3*a*d^3 + 4*b*d^2)*lo
g(x) - 1/6*(2*a*d^3 + 3*b*d^2 + 6*c*d)*polylog(3, d*x) + 1/36*(7*a*d^2*x^2
- ((4*a*d^3 + 9*b*d^2 + 36*c*d)*x^3 - 36*c*x^2 - 9*b*x - 4*a)*log(-d*x + 1)
^2 + 6*(a*d*x + (2*a*d^2 + 3*b*d)*x^2 + ((2*a*d^3 + 3*b*d^2 + 6*c*d)*x^3 -
```

```
6*c*x^2 - 3*b*x - 2*a)*log(-d*x + 1))*dilog(d*x) + (9*(3*a*d^3 + 4*b*d^2)*x
^3 - 7*a*d*x - 4*(5*a*d^2 + 9*b*d)*x^2)*log(-d*x + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2 + bx + a\right)\text{Li}_2\left(dx\right)\log\left(-dx + 1\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)\text{Li}_2\left(dx\right)\log\left(-dx + 1\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^4, x)
```

3.198 $\int \frac{(a+bx+cx^2)\log(1-dx)\mathbf{PolyLog}(2,dx)}{x^5}\,dx$

Optimal. Leaf size=767

$$-\frac{1}{12}d^2\text{PolyLog}(3,dx)(d(3ad+4b)+6c)-\frac{1}{6}d^2\text{PolyLog}(3,1-dx)(d(3ad+4b)+6c)+\frac{1}{12}d^2\log(1-dx)\text{PolyLog}(2,dx$$

```
[Out] (5*a*d^2)/(144*x^2) + (b*d^2)/(9*x) + (19*a*d^3)/(144*x) + (d^2*(4*b + 3*a*
d))/(48*x) - (c*d^2*Log[x])/2 - (b*d^3*Log[x])/3 - (37*a*d^4*Log[x])/144 -
(d^3*(4*b + 3*a*d)*Log[x])/48 - (d^2*(6*c + d*(4*b + 3*a*d))*Log[x])/12 + (
c*d^2*Log[1 - d*x])/2 + (b*d^3*Log[1 - d*x])/3 + (37*a*d^4*Log[1 - d*x])/14
4 + (d^3*(4*b + 3*a*d)*Log[1 - d*x])/48 + (d^2*(6*c + d*(4*b + 3*a*d))*Log[
1 - d*x])/12 - (5*a*d*Log[1 - d*x])/(72*x^3) - (b*d*Log[1 - d*x])/(9*x^2) -
 (a*d^2*Log[1 - d*x])/(16*x^2) - (d*(4*b + 3*a*d)*Log[1 - d*x])/(48*x^2) -
(c*d*Log[1 - d*x])/(2*x) - (2*b*d^2*Log[1 - d*x])/(9*x) - (a*d^3*Log[1 - d*
x])/(8*x) - (d*(6*c + d*(4*b + 3*a*d))*Log[1 - d*x])/(12*x) - (c*d^2*Log[1
- d*x]^2)/4 - (b*d^3*Log[1 - d*x]^2)/9 - (a*d^4*Log[1 - d*x]^2)/16 + (a*Log
[1 - d*x]^2)/(16*x^4) + (b*Log[1 - d*x]^2)/(9*x^3) + (c*Log[1 - d*x]^2)/(4*
x^2) + (d^2*(6*c + d*(4*b + 3*a*d))*Log[d*x]*Log[1 - d*x]^2)/12 - (c*d^2*Po
lyLog[2, d*x])/2 - (2*b*d^3*PolyLog[2, d*x])/9 - (a*d^4*PolyLog[2, d*x])/8
+ (a*d*PolyLog[2, d*x])/(12*x^3) + (d*(4*b + 3*a*d)*PolyLog[2, d*x])/(24*x^
2) + (d*(6*c + d*(4*b + 3*a*d))*PolyLog[2, d*x])/(12*x) + (d^2*(6*c + d*(4*
b + 3*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/12 - (((3*a)/x^4 + (4*b)/x^3 + (6
*c)/x^2)*Log[1 - d*x]*PolyLog[2, d*x])/12 + (d^2*(6*c + d*(4*b + 3*a*d))*Lo
g[1 - d*x]*PolyLog[2, 1 - d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3,
 d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3, 1 - d*x])/6
```

Rubi [A] time = 1.12465, antiderivative size = 767, normalized size of antiderivative = 1., number of steps used = 61, number of rules used = 19, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.731, Rules used = {6742, 6591, 2395, 44, 14, 6606, 2398, 2410, 36, 29, 31, 2391, 2390, 2301, 6589, 6596, 2396, 2433, 2374}

$$-\frac{1}{12}d^2\text{PolyLog}(3,dx)(d(3ad+4b)+6c)-\frac{1}{6}d^2\text{PolyLog}(3,1-dx)(d(3ad+4b)+6c)+\frac{1}{12}d^2\log(1-dx)\text{PolyLog}(2,dx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

```
[Out] (5*a*d^2)/(144*x^2) + (b*d^2)/(9*x) + (19*a*d^3)/(144*x) + (d^2*(4*b + 3*a*
d))/(48*x) - (c*d^2*Log[x])/2 - (b*d^3*Log[x])/3 - (37*a*d^4*Log[x])/144 -
```

```
(d^3*(4*b + 3*a*d)*Log[x])/48 - (d^2*(6*c + d*(4*b + 3*a*d))*Log[x])/12 + (
c*d^2*Log[1 - d*x])/2 + (b*d^3*Log[1 - d*x])/3 + (37*a*d^4*Log[1 - d*x])/14
4 + (d^3*(4*b + 3*a*d)*Log[1 - d*x])/48 + (d^2*(6*c + d*(4*b + 3*a*d))*Log[
1 - d*x])/12 - (5*a*d*Log[1 - d*x])/(72*x^3) - (b*d*Log[1 - d*x])/(9*x^2) -
 (a*d^2*Log[1 - d*x])/(16*x^2) - (d*(4*b + 3*a*d)*Log[1 - d*x])/(48*x^2) -
(c*d*Log[1 - d*x])/(2*x) - (2*b*d^2*Log[1 - d*x])/(9*x) - (a*d^3*Log[1 - d*
x])/(8*x) - (d*(6*c + d*(4*b + 3*a*d))*Log[1 - d*x])/(12*x) - (c*d^2*Log[1
- d*x]^2)/4 - (b*d^3*Log[1 - d*x]^2)/9 - (a*d^4*Log[1 - d*x]^2)/16 + (a*Log
[1 - d*x]^2)/(16*x^4) + (b*Log[1 - d*x]^2)/(9*x^3) + (c*Log[1 - d*x]^2)/(4*
x^2) + (d^2*(6*c + d*(4*b + 3*a*d))*Log[d*x]*Log[1 - d*x]^2)/12 - (c*d^2*Po
lyLog[2, d*x])/2 - (2*b*d^3*PolyLog[2, d*x])/9 - (a*d^4*PolyLog[2, d*x])/8
+ (a*d*PolyLog[2, d*x])/(12*x^3) + (d*(4*b + 3*a*d)*PolyLog[2, d*x])/(24*x^
2) + (d*(6*c + d*(4*b + 3*a*d))*PolyLog[2, d*x])/(12*x) + (d^2*(6*c + d*(4*
b + 3*a*d))*Log[1 - d*x]*PolyLog[2, d*x])/12 - (((3*a)/x^4 + (4*b)/x^3 + (6
*c)/x^2)*Log[1 - d*x]*PolyLog[2, d*x])/12 + (d^2*(6*c + d*(4*b + 3*a*d))*Lo
g[1 - d*x]*PolyLog[2, 1 - d*x])/6 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3,
 d*x])/12 - (d^2*(6*c + d*(4*b + 3*a*d))*PolyLog[3, 1 - d*x])/6
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6591

```
Int[((d_.)*(x_))^(m_.)*PolyLog[n_, (a_.)*((b_.)*(x_)^(p_.))^(q_.)], x_Symbo
l] :> Simp[((d*x)^(m + 1)*PolyLog[n, a*(b*x^p)^q])/(d*(m + 1)), x] - Dist[(
p*q)/(m + 1), Int[(d*x)^m*PolyLog[n - 1, a*(b*x^p)^q], x], x] /; FreeQ[{a,
b, d, m, p, q}, x] && NeQ[m, -1] && GtQ[n, 0]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6606

```
Int[((g_.) + Log[(f_.)*((d_.) + (e_.)*(x_))^(n_.)]*(h_.))*(Px_)*(x_)^(m_.)*
PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{u = IntHide[x^m*
Px, x]}, Simp[u*(g + h*Log[f*(d + e*x)^n])*PolyLog[2, c*(a + b*x)], x] + (D
ist[b, Int[ExpandIntegrand[(g + h*Log[f*(d + e*x)^n])*Log[1 - a*c - b*c*x],
 u/(a + b*x), x], x], x] - Dist[e*h*n, Int[ExpandIntegrand[PolyLog[2, c*(a
+ b*x)], u/(d + e*x), x], x], x])] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x]
 && PolyQ[Px, x] && IntegerQ[m]
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^
n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6596

```
Int[PolyLog[2, (c_.)*((a_.) + (b_.)*(x_))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 - a*c - b*c*x]*PolyLog[2, c*(a + b*x)])/e, x] + Dist[b/e, In
t[Log[1 - a*c - b*c*x]^2/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c*(b*d - a*e) + e, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)\log(1-dx)\mathrm{Li}_2(dx)}{x^5}\,dx = -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)\log(1-dx)\mathrm{Li}_2(dx)+d\int\left(-\frac{a\mathrm{Li}_2(dx)}{4x^4}+\frac{(-4b-3ad)\mathrm{L}}{12x^3}\right.$$

$$= -\frac{1}{12}\left(\frac{3a}{x^4}+\frac{4b}{x^3}+\frac{6c}{x^2}\right)\log(1-dx)\mathrm{Li}_2(dx)-\frac{1}{4}a\int\frac{\log^2(1-dx)}{x^5}\,dx-\frac{1}{3}b\int\frac{\mathrm{lo}}{}$$

$$= \frac{a\log^2(1-dx)}{16x^4}+\frac{b\log^2(1-dx)}{9x^3}+\frac{c\log^2(1-dx)}{4x^2}+\frac{ad\mathrm{Li}_2(dx)}{12x^3}+\frac{d(4b+3ad)}{24x^2}$$

$$= -\frac{ad\log(1-dx)}{36x^3}-\frac{d(4b+3ad)\log(1-dx)}{48x^2}-\frac{d(6c+d(4b+3ad))\log(1-dx)}{12x}$$

$$= -\frac{ad\log(1-dx)}{36x^3}-\frac{d(4b+3ad)\log(1-dx)}{48x^2}-\frac{d(6c+d(4b+3ad))\log(1-dx)}{12x}$$

$$= \frac{ad^2}{72x^2}+\frac{ad^3}{36x}+\frac{d^2(4b+3ad)}{48x}-\frac{1}{36}ad^4\log(x)-\frac{1}{48}d^3(4b+3ad)\log(x)-\frac{1}{12}d$$

$$= \frac{ad^2}{72x^2}+\frac{ad^3}{36x}+\frac{d^2(4b+3ad)}{48x}-\frac{1}{36}ad^4\log(x)-\frac{1}{48}d^3(4b+3ad)\log(x)-\frac{1}{12}d$$

$$= \frac{5ad^2}{144x^2}+\frac{bd^2}{9x}+\frac{19ad^3}{144x}+\frac{d^2(4b+3ad)}{48x}-\frac{1}{2}cd^2\log(x)-\frac{1}{3}bd^3\log(x)-\frac{37}{144}ad$$

Mathematica [A] time = 1.95317, size = 621, normalized size = 0.81

$$\frac{1}{144}\left(\frac{6\mathrm{PolyLog}(2,dx)\left(dx\left(a\left(6d^2x^2+3dx+2\right)+4x(2bdx+b+3cx)\right)+2\log(1-dx)\left(3a\left(d^4x^4-1\right)+4bd^3x^4-4bx+\right.\right.}{x^4}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)*Log[1 - d*x]*PolyLog[2, d*x])/x^5,x]
```

```
[Out] (-28*b*d^3 - 33*a*d^4 + (5*a*d^2)/x^2 + (28*b*d^2)/x + (28*a*d^3)/x - 144*c
*d^2*Log[d*x] - 108*b*d^3*Log[d*x] - 82*a*d^4*Log[d*x] + 144*c*d^2*Log[1 -
d*x] + 108*b*d^3*Log[1 - d*x] + 82*a*d^4*Log[1 - d*x] - (10*a*d*Log[1 - d*x
])/x^3 - (28*b*d*Log[1 - d*x])/x^2 - (18*a*d^2*Log[1 - d*x])/x^2 - (144*c*d
*Log[1 - d*x])/x - (80*b*d^2*Log[1 - d*x])/x - (54*a*d^3*Log[1 - d*x])/x +
72*c*d^2*Log[d*x]*Log[1 - d*x] + 32*b*d^3*Log[d*x]*Log[1 - d*x] + 18*a*d^4*
Log[d*x]*Log[1 - d*x] - 36*c*d^2*Log[1 - d*x]^2 - 16*b*d^3*Log[1 - d*x]^2 -
 9*a*d^4*Log[1 - d*x]^2 + (9*a*Log[1 - d*x]^2)/x^4 + (16*b*Log[1 - d*x]^2)/
x^3 + (36*c*Log[1 - d*x]^2)/x^2 + 72*c*d^2*Log[d*x]*Log[1 - d*x]^2 + 48*b*d
^3*Log[d*x]*Log[1 - d*x]^2 + 36*a*d^4*Log[d*x]*Log[1 - d*x]^2 + (6*(d*x*(4*
x*(b + 3*c*x + 2*b*d*x) + a*(2 + 3*d*x + 6*d^2*x^2)) + 2*(-4*b*x - 6*c*x^2
+ 6*c*d^2*x^4 + 4*b*d^3*x^4 + 3*a*(-1 + d^4*x^4))*Log[1 - d*x])*PolyLog[2,
d*x])/x^4 + 2*d^2*(36*c + 16*b*d + 9*a*d^2 + 12*(6*c + 4*b*d + 3*a*d^2)*Log
[1 - d*x])*PolyLog[2, 1 - d*x] - 72*c*d^2*PolyLog[3, d*x] - 48*b*d^3*PolyLo
g[3, d*x] - 36*a*d^4*PolyLog[3, d*x] - 144*c*d^2*PolyLog[3, 1 - d*x] - 96*b
*d^3*PolyLog[3, 1 - d*x] - 72*a*d^4*PolyLog[3, 1 - d*x])/144
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+bx+a\right)\ln\left(-dx+1\right)\operatorname{polylog}\left(2,dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

```
[Out] int((c*x^2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x^5,x)
```

Maxima [A] time = 1.23365, size = 544, normalized size = 0.71

$$\frac{1}{12}\left(3\,ad^4+4\,bd^3+6\,cd^2\right)\left(\log\left(dx\right)\log\left(-dx+1\right)^2+2\,\mathrm{Li}_2\left(-dx+1\right)\log\left(-dx+1\right)-2\,\mathrm{Li}_3(-dx+1)\right)+\frac{1}{72}\left(9\,ad^4+16\,bd^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*(log(d*x)*log(-d*x + 1)^2 + 2*dilog(-d*x
 + 1)*log(-d*x + 1) - 2*polylog(3, -d*x + 1)) + 1/72*(9*a*d^4 + 16*b*d^3 +
```

```
36*c*d^2)*(log(d*x)*log(-d*x + 1) + dilog(-d*x + 1)) - 1/72*(41*a*d^4 + 54*
b*d^3 + 72*c*d^2)*log(x) - 1/12*(3*a*d^4 + 4*b*d^3 + 6*c*d^2)*polylog(3, d*
x) + 1/144*(5*a*d^2*x^2 + 28*(a*d^3 + b*d^2)*x^3 - ((9*a*d^4 + 16*b*d^3 + 3
6*c*d^2)*x^4 - 36*c*x^2 - 16*b*x - 9*a)*log(-d*x + 1)^2 + 6*(2*(3*a*d^3 + 4
*b*d^2 + 6*c*d)*x^3 + 2*a*d*x + (3*a*d^2 + 4*b*d)*x^2 + 2*((3*a*d^4 + 4*b*d
^3 + 6*c*d^2)*x^4 - 6*c*x^2 - 4*b*x - 3*a)*log(-d*x + 1))*dilog(d*x) + 2*((
41*a*d^4 + 54*b*d^3 + 72*c*d^2)*x^4 - (27*a*d^3 + 40*b*d^2 + 72*c*d)*x^3 -
5*a*d*x - (9*a*d^2 + 14*b*d)*x^2)*log(-d*x + 1))/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^2+bx+a\right)\text{Li}_2\left(dx\right)\log\left(-dx+1\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="fricas
")
```

```
[Out] integral((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*ln(-d*x+1)*polylog(2,d*x)/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^2+bx+a\right)\text{Li}_2\left(dx\right)\log\left(-dx+1\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*log(-d*x+1)*polylog(2,d*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x + a)*dilog(d*x)*log(-d*x + 1)/x^5, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```