Computer algebra independent integration tests

3-Logarithms/3.5-Logarithm-functions

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [314]. This is test number [64].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (314)	% 0. (0)
Mathematica	% 100. (314)	% 0. (0)
Maple	% 75.16 (236)	% 24.84 (78)
Maxima	% 64.65 (203)	% 35.35 (111)
Fricas	% 88.54 (278)	% 11.46 (36)
Sympy	% 35.03 (110)	% 64.97 (204)
Giac	% 58.92 (185)	% 41.08 (129)

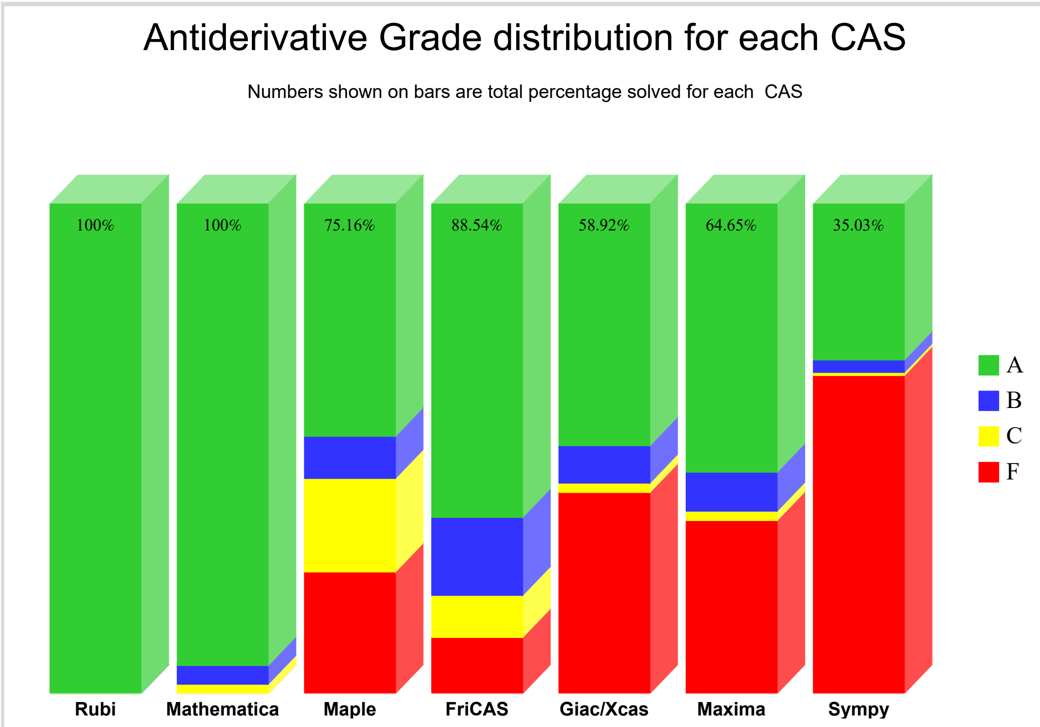
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

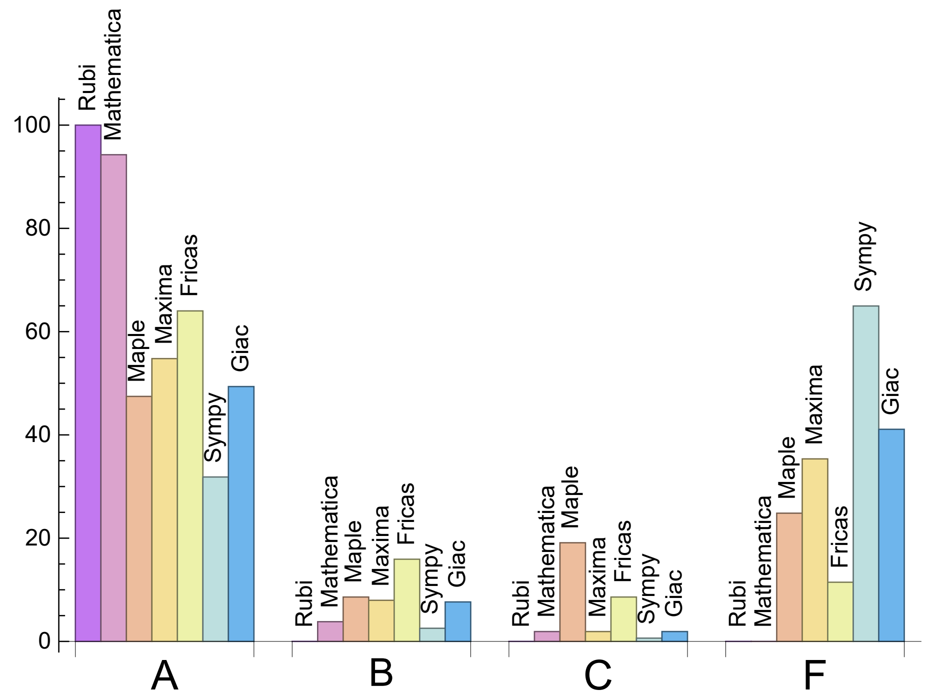
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	94.27	3.82	1.91	0.
Maple	47.45	8.6	19.11	24.84
Maxima	54.78	7.96	1.91	35.35
Fricas	64.01	15.92	8.6	11.46
Sympy	31.85	2.55	0.64	64.97
Giac	49.36	7.64	1.91	41.08

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	69.56	0.92	41.	1.
Mathematica	0.72	79.27	1.14	42.	1.
Maple	1.48	7210.49	23.15	43.5	1.17
Maxima	1.17	57.15	1.58	45.	1.32
Fricas	2.37	274.5	4.53	127.	3.24
Sympy	15.16	52.93	1.72	25.	1.
Giac	1.17	146.4	1.81	42.	1.35

1.4 list of integrals that has no closed form antiderivative

$\{1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{40, 100, 280, 281\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

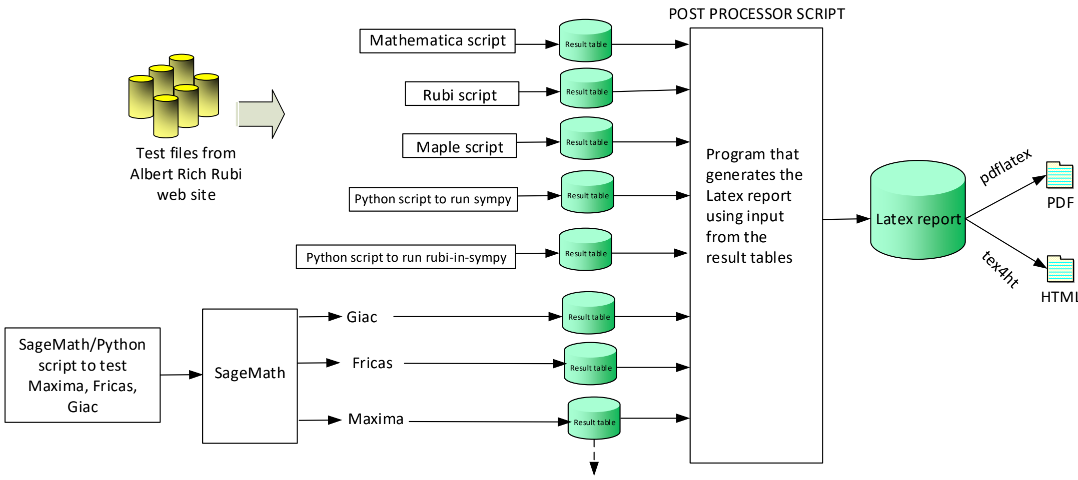
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { 40, 41, 42, 43, 44, 45, 134, 189, 278, 279, 280, 281 }

C grade: { 108, 109, 110, 111, 112, 276 }

F grade: { }

2.1.3 Maple

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 19, 25, 30, 34, 35, 36, 37, 39, 51, 55, 56, 58, 64, 75, 81, 86, 98, 104, 105, 113, 114, 115, 116, 117, 121, 122, 126, 127, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 168, 171, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 207, 209, 210, 211, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 296, 298, 299, 308, 309, 313, 314 }

B grade: { 24, 91, 118, 119, 120, 123, 124, 125, 128, 131, 160, 161, 162, 164, 165, 167, 170, 173, 174, 176, 177, 191, 208, 212, 246, 249, 278 }

C grade: { 9, 10, 11, 17, 18, 20, 21, 22, 23, 26, 27, 28, 29, 38, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 87, 88, 89, 90, 94, 95, 150, 151, 154, 155, 156, 157, 158, 159, 169, 172, 179, 182, 189, 192, 194, 201, 202, 204, 205, 215, 216, 218, 219, 276, 277, 307, 312 }

F grade: { 2, 3, 4, 16, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 92, 93, 96, 97, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 149, 152, 163, 166, 175, 178, 193, 203, 206, 217, 220, 263, 264, 265, 279, 280, 281, 293, 295, 297, 300, 301, 302, 303, 304, 305, 306, 310, 311 }

2.1.4 Maxima

A grade: { 6, 7, 8, 12, 13, 14, 15, 20, 21, 25, 27, 29, 35, 36, 37, 38, 39, 51, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 96, 105, 113, 114, 115, 116, 117, 122, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 250, 254, 259, 262, 267, 268, 270, 271, 272, 273, 274, 275, 276, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 299, 302, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314

B grade: { 22, 23, 24, 26, 28, 58, 161, 162, 163, 176, 177, 178, 182, 189, 191, 192, 193, 194, 221, 237, 246, 269, 301, 303, 304 }

C grade: { 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 5, 9, 10, 11, 16, 17, 18, 19, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 118, 119, 120, 121, 123, 124, 125, 126, 128, 149, 150, 152, 231, 232, 233, 234, 249, 251, 252, 253, 255, 256, 257, 258, 260, 261, 263, 264, 265, 266, 277, 278, 279, 280, 281, 283, 293, 300 }

2.1.5 FriCAS

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 16, 19, 20, 21, 25, 26, 27, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 121, 122, 126, 127, 129, 130, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 201, 203, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 308, 309, 313, 314 }

B grade: { 9, 10, 11, 17, 18, 22, 23, 24, 28, 89, 90, 128, 131, 135, 136, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 195, 196, 197, 198, 199, 200, 202, 218, 219, 220, 221, 222, 225, 246, 307, 312 }

C grade: { 113, 114, 115, 118, 119, 120, 123, 124, 125, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 305, 306, 310, 311 }

F grade: { 2, 3, 4, 31, 32, 33, 59, 65, 70, 76, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 193, 263, 264, 265, 266, 267, 269, 270, 277, 278, 279, 301, 302, 303, 304 }

2.1.6 Sympy

A grade: { 11, 12, 13, 14, 19, 23, 24, 25, 26, 34, 55, 60, 61, 62, 63, 64, 66, 67, 68, 69, 75, 81, 86, 117, 129, 130, 132, 137, 138, 139, 141, 142, 144, 145, 146, 147, 148, 153, 160, 182, 184, 187, 188, 190, 194, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 239, 241, 242, 243, 244, 245, 247, 248, 250, 251, 252, 253, 258, 259, 260, 261, 262, 272, 273, 274, 275, 276, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 308, 309 }

B grade: { 131, 133, 140, 189, 192, 238, 240, 246 }

C grade: { 268, 270 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 16, 17, 18, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135, 136, 143, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 185, 186, 191, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 234, 249, 254, 255, 256, 257, 263, 264, 265, 266, 267, 269, 271, 277, 278, 279, 280, 281, 284, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314 }

2.1.7 Giac

A grade: { 5, 6, 7, 8, 11, 12, 13, 14, 15, 19, 25, 26, 27, 34, 35, 36, 37, 39, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 117, 122, 127, 130, 132, 133, 135, 138, 139, 140, 142, 143, 144, 145, 146, 148, 150, 151, 152, 153, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 253, 254, 255, 257, 258, 259, 260, 261, 262, 265, 272, 273, 274, 275, 276, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 301, 302, 304 }

B grade: { 9, 10, 23, 24, 28, 82, 83, 84, 89, 90, 91, 129, 131, 136, 141, 149, 221, 222, 250, 252, 256, 294, 298, 303 }

C grade: { 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 16, 17, 18, 20, 21, 22, 29, 30, 31, 32, 33, 38, 40, 41, 42, 43, 44, 45, 52, 53, 54, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 137, 147, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 249, 263, 264, 266, 267, 268, 269, 270, 271, 277, 278, 279, 280, 281, 293, 295, 297, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	1.18	0.48	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	223	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.348	0.739	2.418	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	149	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.274	0.414	4.449	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.156	3.949	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	0	74	0	22
normalized size	1	1.	1.	1.07	0.	4.93	0.	1.47
time (sec)	N/A	0.022	0.002	0.003	0.	1.947	0.	1.301

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	0.126	8.273	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.289	0.665	6.506	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.245	1.472	49.67	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	230	61910	0	1558	0	1034
normalized size	1	1.	0.85	227.61	0.	5.73	0.	3.8
time (sec)	N/A	0.305	0.224	5.05	0.	2.162	0.	1.341

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	115	14983	0	659	0	386
normalized size	1	1.	0.92	119.86	0.	5.27	0.	3.09
time (sec)	N/A	0.166	0.105	1.375	0.	1.856	0.	1.657

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	2146	0	213	68	99
normalized size	1	1.	1.	52.34	0.	5.2	1.66	2.41
time (sec)	N/A	0.077	0.033	0.303	0.	1.956	10.166	1.278

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	43	51	18
normalized size	1	1.	1.	0.93	1.2	2.87	3.4	1.2
time (sec)	N/A	0.007	0.001	0.003	1.003	1.839	1.689	1.303

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	1.409	1.777	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	1.567	51.749	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	2.872	56.374	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	107	0	0
normalized size	1	1.	1.	0.	0.	4.12	0.	0.
time (sec)	N/A	0.171	0.089	0.5	0.	1.992	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	204	0	174	0	0
normalized size	1	1.	1.	9.27	0.	7.91	0.	0.
time (sec)	N/A	0.173	0.038	0.197	0.	2.052	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	135	0	115	0	0
normalized size	1	1.	1.	6.14	0.	5.23	0.	0.
time (sec)	N/A	0.109	0.033	0.162	0.	1.959	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	0	82	58	23
normalized size	1	1.	1.	1.06	0.	5.12	3.62	1.44
time (sec)	N/A	0.035	0.015	0.003	0.	1.999	61.902	1.315

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	213	30	51	0	0
normalized size	1	1.	1.	12.53	1.76	3.	0.	0.
time (sec)	N/A	0.185	0.233	0.164	1.56	1.824	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	28	51	0	0
normalized size	1	1.	1.	3.4	1.4	2.55	0.	0.
time (sec)	N/A	0.182	0.04	0.129	1.716	1.99	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	68	66	116	0	0
normalized size	1	1.	1.	3.09	3.	5.27	0.	0.
time (sec)	N/A	0.18	0.04	0.184	2.132	2.049	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16321	285	478	221	267
normalized size	1	1.	1.	816.05	14.25	23.9	11.05	13.35
time (sec)	N/A	0.152	0.018	1.223	1.027	1.845	13.441	1.291

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	63	100	230	117	122
normalized size	1	1.	1.9	3.15	5.	11.5	5.85	6.1
time (sec)	N/A	0.093	0.004	0.018	1.05	1.962	6.32	1.231

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	59	60	27
normalized size	1	1.	1.	1.07	1.36	4.21	4.29	1.93
time (sec)	N/A	0.009	0.002	0.003	1.015	1.882	1.676	1.241

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	428	43	84	48	38
normalized size	1	1.	1.	28.53	2.87	5.6	3.2	2.53
time (sec)	N/A	0.082	0.095	0.066	1.185	1.812	2.393	1.149

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	451	42	84	0	42
normalized size	1	1.	1.	25.06	2.33	4.67	0.	2.33
time (sec)	N/A	0.133	0.013	0.072	1.217	1.861	0.	1.451

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	451	128	254	0	413
normalized size	1	1.	1.	22.55	6.4	12.7	0.	20.65
time (sec)	N/A	0.16	0.014	0.085	1.425	2.028	0.	1.278

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	216	35	59	0	0
normalized size	1	1.	1.21	11.37	1.84	3.11	0.	0.
time (sec)	N/A	0.321	0.46	0.191	1.543	1.926	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	1.801	0.487	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	445	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.504	1.551	7.961	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	298	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.395	0.927	17.253	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	157	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.386	10.297	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	0	107	53	36
normalized size	1	1.	1.	1.04	0.	4.28	2.12	1.44
time (sec)	N/A	0.034	0.02	0.007	0.	1.862	60.999	1.313

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	4.747	35.223	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.282	7.308	7.905	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.249	66.919	91.029	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	158	42	80	0	0
normalized size	1	1.	1.	6.08	1.62	3.08	0.	0.
time (sec)	N/A	0.245	0.357	0.128	1.674	1.935	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	76.601	0.368	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	49	49	625	0	0	95	0	0
normalized size	1	1.	12.76	0.	0.	1.94	0.	0.
time (sec)	N/A	0.099	0.433	180.	0.	1.948	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	642	0	0	93	0	0
normalized size	1	1.	12.84	0.	0.	1.86	0.	0.
time (sec)	N/A	0.087	0.366	180.	0.	1.951	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	320	0	0	109	0	0
normalized size	1	1.	6.04	0.	0.	2.06	0.	0.
time (sec)	N/A	0.096	0.209	180.	0.	1.946	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	316	0	0	111	0	0
normalized size	1	1.	6.08	0.	0.	2.13	0.	0.
time (sec)	N/A	0.073	0.188	180.	0.	1.843	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	641	0	0	111	0	0
normalized size	1	1.	13.08	0.	0.	2.27	0.	0.
time (sec)	N/A	0.102	0.311	180.	0.	1.929	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	645	0	0	109	0	0
normalized size	1	1.	12.9	0.	0.	2.18	0.	0.
time (sec)	N/A	0.078	0.251	180.	0.	1.848	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	204	0	112
normalized size	1	1.	0.84	0.	0.	3.04	0.	1.67
time (sec)	N/A	0.057	0.131	180.	0.	2.052	0.	1.31

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	261	0	150
normalized size	1	1.	0.75	0.	0.	3.3	0.	1.9
time (sec)	N/A	0.061	0.157	0.185	0.	1.978	0.	1.427

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	161	0	76
normalized size	1	1.	0.89	0.	0.	2.93	0.	1.38
time (sec)	N/A	0.049	0.059	0.063	0.	1.879	0.	1.377

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	161	0	76
normalized size	1	1.	0.89	0.	0.	2.93	0.	1.38
time (sec)	N/A	0.035	0.057	0.061	0.	1.855	0.	1.319

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	144	0	57
normalized size	1	1.	0.96	0.	0.	3.2	0.	1.27
time (sec)	N/A	0.03	0.048	0.066	0.	1.975	0.	1.324

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	48	86	124	0	73
normalized size	1	1.	1.25	1.5	2.69	3.88	0.	2.28
time (sec)	N/A	0.017	0.011	0.01	1.028	1.874	0.	1.3

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	123	0	0
normalized size	1	1.	0.94	0.	0.	2.56	0.	0.
time (sec)	N/A	0.047	0.041	0.066	0.	1.895	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	136	0	0
normalized size	1	1.	0.89	0.	0.	2.47	0.	0.
time (sec)	N/A	0.046	0.042	0.053	0.	1.712	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	136	0	0
normalized size	1	1.	0.89	0.	0.	2.47	0.	0.
time (sec)	N/A	0.047	0.043	0.061	0.	1.539	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	31	81	19	35
normalized size	1	1.	1.	1.18	1.41	3.68	0.86	1.59
time (sec)	N/A	0.007	0.014	0.009	1.142	1.646	1.314	1.205

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	30	68	0	43
normalized size	1	1.	1.1	1.15	1.5	3.4	0.	2.15
time (sec)	N/A	0.02	0.007	0.006	0.994	1.639	0.	1.303

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	39	0	0	126	0	49
normalized size	1	1.	0.98	0.	0.	3.15	0.	1.22
time (sec)	N/A	0.022	0.02	0.056	0.	1.633	0.	1.214

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	35	74	105	0	58
normalized size	1	1.	1.26	1.3	2.74	3.89	0.	2.15
time (sec)	N/A	0.021	0.01	0.006	1.017	1.524	0.	1.367

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	48	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.021	180.	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	117	232	134	120
normalized size	1	1.	0.86	0.	1.18	2.34	1.35	1.21
time (sec)	N/A	0.073	0.048	0.038	1.043	1.612	14.224	1.3

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	74	0	101	196	119	101
normalized size	1	1.	0.87	0.	1.19	2.31	1.4	1.19
time (sec)	N/A	0.062	0.036	0.023	1.093	1.569	7.437	1.228

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	88	171	107	88
normalized size	1	1.	0.89	0.	1.24	2.41	1.51	1.24
time (sec)	N/A	0.053	0.029	0.023	1.083	1.505	4.219	1.142

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	0	69	132	92	69
normalized size	1	1.	0.86	0.	1.21	2.32	1.61	1.21
time (sec)	N/A	0.041	0.02	0.03	1.012	1.552	2.343	1.214

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	34	49	95	56	50
normalized size	1	1.	0.94	1.03	1.48	2.88	1.7	1.52
time (sec)	N/A	0.016	0.007	0.006	1.157	1.523	1.216	1.229

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	0	108	0	0	0
normalized size	1	1.	0.94	0.	2.04	0.	0.	0.
time (sec)	N/A	0.126	0.02	0.036	1.055	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	62	113	76	63
normalized size	1	1.	0.96	0.	1.32	2.4	1.62	1.34
time (sec)	N/A	0.044	0.012	0.016	1.005	1.922	2.679	1.305

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	84	169	110	88
normalized size	1	1.	0.9	0.	1.17	2.35	1.53	1.22
time (sec)	N/A	0.05	0.032	0.023	1.031	1.843	4.976	1.27

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	77	0	101	198	133	108
normalized size	1	1.	0.9	0.	1.17	2.3	1.55	1.26
time (sec)	N/A	0.057	0.035	0.024	1.017	1.875	8.755	1.16

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	116	228	141	124
normalized size	1	1.	0.87	0.	1.16	2.28	1.41	1.24
time (sec)	N/A	0.063	0.046	0.024	1.026	1.921	16.643	1.167

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	137	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	0.186	0.306	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	190	1621	0	1013	0	298
normalized size	1	1.	0.92	7.83	0.	4.89	0.	1.44
time (sec)	N/A	0.229	0.207	0.123	0.	1.975	0.	1.234

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	151	1146	0	826	0	238
normalized size	1	1.	0.9	6.86	0.	4.95	0.	1.43
time (sec)	N/A	0.189	0.148	0.1	0.	1.965	0.	1.271

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	122	870	0	691	0	197
normalized size	1	1.	0.9	6.4	0.	5.08	0.	1.45
time (sec)	N/A	0.149	0.109	0.089	0.	1.996	0.	1.293

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	510	0	567	0	153
normalized size	1	1.	0.86	4.68	0.	5.2	0.	1.4
time (sec)	N/A	0.112	0.087	0.093	0.	1.94	0.	1.34

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	118	0	459	275	124
normalized size	1	1.	0.99	1.49	0.	5.81	3.48	1.57
time (sec)	N/A	0.061	0.061	0.013	0.	1.825	102.018	1.313

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	156	315	0	0	0	0
normalized size	1	1.	1.21	2.44	0.	0.	0.	0.
time (sec)	N/A	0.176	0.156	0.063	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	87	261	0	486	0	134
normalized size	1	1.	1.01	3.03	0.	5.65	0.	1.56
time (sec)	N/A	0.113	0.105	0.069	0.	2.352	0.	1.21

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	105	1178	0	628	0	174
normalized size	1	1.	0.87	9.74	0.	5.19	0.	1.44
time (sec)	N/A	0.154	0.238	0.089	0.	2.354	0.	1.269

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	423	0	741	0	221
normalized size	1	1.	0.89	2.84	0.	4.97	0.	1.48
time (sec)	N/A	0.2	0.366	0.085	0.	2.747	0.	1.208

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	172	3583	0	932	0	284
normalized size	1	1.	0.91	18.86	0.	4.91	0.	1.49
time (sec)	N/A	0.223	0.45	0.114	0.	2.803	0.	1.235

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	38	50	105	46	50
normalized size	1	1.	0.83	0.9	1.19	2.5	1.1	1.19
time (sec)	N/A	0.026	0.013	0.007	1.776	2.034	0.141	1.164

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	468	31895	0	2700	0	1338
normalized size	1	1.	0.96	65.76	0.	5.57	0.	2.76
time (sec)	N/A	2.058	1.905	0.23	0.	2.992	0.	1.385

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	324	16059	0	1858	0	923
normalized size	1	1.	0.96	47.51	0.	5.5	0.	2.73
time (sec)	N/A	0.516	0.504	0.196	0.	2.435	0.	1.392

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	204	7155	0	1242	0	591
normalized size	1	1.	0.9	31.66	0.	5.5	0.	2.62
time (sec)	N/A	0.319	0.378	0.148	0.	2.25	0.	1.347

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	123	1706	0	767	0	315
normalized size	1	1.	0.8	11.08	0.	4.98	0.	2.05
time (sec)	N/A	0.186	0.186	0.145	0.	2.266	0.	1.362

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	118	0	459	275	124
normalized size	1	1.	0.99	1.49	0.	5.81	3.48	1.57
time (sec)	N/A	0.062	0.059	0.015	0.	2.147	106.601	1.313

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	226	493	0	0	0	0
normalized size	1	1.	0.99	2.16	0.	0.	0.	0.
time (sec)	N/A	0.412	0.293	0.089	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	166	6540	0	950	0	383
normalized size	1	1.	1.01	39.64	0.	5.76	0.	2.32
time (sec)	N/A	0.245	0.333	0.148	0.	3.223	0.	1.395

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	215	55216	0	2867	0	1197
normalized size	1	1.	0.83	213.19	0.	11.07	0.	4.62
time (sec)	N/A	0.406	0.595	0.237	0.	17.134	0.	1.328

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	310	306209	0	6159	0	2650
normalized size	1	1.	0.87	860.14	0.	17.3	0.	7.44
time (sec)	N/A	0.621	1.198	0.303	0.	125.094	0.	1.518

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	469	1137077	0	0	0	5075
normalized size	1	1.	0.9	2190.9	0.	0.	0.	9.78
time (sec)	N/A	1.006	2.065	0.443	0.	0.	0.	2.584

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.043	0.101	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	339	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.346	0.162	0.229	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	762	762	626	610	0	0	0	0
normalized size	1	1.	0.82	0.8	0.	0.	0.	0.
time (sec)	N/A	1.452	0.941	0.135	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	782	782	663	764	0	0	0	0
normalized size	1	1.	0.85	0.98	0.	0.	0.	0.
time (sec)	N/A	1.514	0.813	0.11	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	111	0	166	0	0	0
normalized size	1	1.	0.77	0.	1.15	0.	0.	0.
time (sec)	N/A	0.284	0.064	0.037	1.232	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	587	478	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.949	0.824	0.205	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	290	279	0	0	0	0
normalized size	1	1.	0.93	0.9	0.	0.	0.	0.
time (sec)	N/A	0.478	0.16	0.021	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	323	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	0.146	0.027	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	443	443	826	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.684	0.745	0.022	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	117	0	0	424	0	181
normalized size	1	1.	0.68	0.	0.	2.47	0.	1.05
time (sec)	N/A	0.381	0.496	0.01	0.	2.515	0.	1.357

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	107	0	0	375	0	167
normalized size	1	1.	0.72	0.	0.	2.52	0.	1.12
time (sec)	N/A	0.292	0.335	0.007	0.	2.616	0.	1.36

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	102	0	0	327	0	154
normalized size	1	1.	0.8	0.	0.	2.57	0.	1.21
time (sec)	N/A	0.245	0.269	0.004	0.	2.575	0.	1.297

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	80	0	278	0	136
normalized size	1	1.	0.89	0.84	0.	2.93	0.	1.43
time (sec)	N/A	0.162	0.028	0.031	0.	2.61	0.	1.323

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.454	0.005	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	296	0	124
normalized size	1	1.	0.89	0.	0.	3.89	0.	1.63
time (sec)	N/A	0.262	0.214	0.008	0.	2.487	0.	1.35

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	82	0	0	358	0	176
normalized size	1	1.	0.81	0.	0.	3.54	0.	1.74
time (sec)	N/A	0.291	0.275	0.006	0.	2.333	0.	1.404

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	232	0	0	327	0	0
normalized size	1	1.	1.24	0.	0.	1.75	0.	0.
time (sec)	N/A	0.541	0.829	0.006	0.	2.25	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	209	0	0	286	0	0
normalized size	1	1.	1.32	0.	0.	1.81	0.	0.
time (sec)	N/A	0.431	0.641	0.007	0.	2.347	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	186	0	0	243	0	0
normalized size	1	1.	1.58	0.	0.	2.06	0.	0.
time (sec)	N/A	0.385	0.545	0.009	0.	2.182	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	240	0	0
normalized size	1	1.	1.55	0.	0.	2.11	0.	0.
time (sec)	N/A	0.309	0.434	0.006	0.	2.29	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	204	0	0	311	0	0
normalized size	1	1.	1.35	0.	0.	2.06	0.	0.
time (sec)	N/A	0.478	0.605	0.006	0.	2.19	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	111	224	0	0
normalized size	1	1.	1.	0.9	1.19	2.41	0.	0.
time (sec)	N/A	0.073	0.007	0.007	1.207	2.048	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	69	90	185	0	0
normalized size	1	1.	1.	0.9	1.17	2.4	0.	0.
time (sec)	N/A	0.058	0.005	0.006	1.129	2.057	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	52	68	143	0	0
normalized size	1	1.	1.	0.88	1.15	2.42	0.	0.
time (sec)	N/A	0.037	0.005	0.007	1.124	2.135	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	28	46	93	0	0
normalized size	1	1.	1.	0.74	1.21	2.45	0.	0.
time (sec)	N/A	0.048	0.002	0.013	1.081	2.091	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.051	0.026	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	132	645	0	323	0	0
normalized size	1	1.	1.	4.89	0.	2.45	0.	0.
time (sec)	N/A	0.097	0.013	0.069	0.	2.195	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	462	0	236	0	0
normalized size	1	1.	1.	4.71	0.	2.41	0.	0.
time (sec)	N/A	0.062	0.005	0.049	0.	2.155	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	282	0	147	0	0
normalized size	1	1.	1.	4.48	0.	2.33	0.	0.
time (sec)	N/A	0.038	0.005	0.043	0.	2.215	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	0	63	0	0
normalized size	1	1.	1.	1.03	0.	2.03	0.	0.
time (sec)	N/A	0.015	0.001	0.007	0.	2.2	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.296	0.052	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	193	1364	0	564	0	0
normalized size	1	1.	1.	7.07	0.	2.92	0.	0.
time (sec)	N/A	0.129	0.009	0.101	0.	2.211	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	156	980	0	471	0	0
normalized size	1	1.	1.	6.28	0.	3.02	0.	0.
time (sec)	N/A	0.093	0.007	0.079	0.	2.196	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	118	598	0	382	0	0
normalized size	1	1.	1.	5.07	0.	3.24	0.	0.
time (sec)	N/A	0.065	0.005	0.059	0.	2.089	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	82	0	243	0	0
normalized size	1	1.	1.	1.09	0.	3.24	0.	0.
time (sec)	N/A	0.127	0.003	0.008	0.	2.141	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.289	0.05	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	138	0	250	0	0
normalized size	1	1.	1.	3.54	0.	6.41	0.	0.
time (sec)	N/A	0.026	0.009	0.02	0.	2.148	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	23	5	30
normalized size	1	1.	1.	1.2	1.4	4.6	1.	6.
time (sec)	N/A	0.018	0.009	0.006	1.082	1.981	0.095	1.278

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	32	10	11
normalized size	1	1.	1.	0.75	0.92	2.67	0.83	0.92
time (sec)	N/A	0.026	0.004	0.006	1.117	2.041	0.739	1.293

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	33	11	108	39	42
normalized size	1	1.	1.	3.3	1.1	10.8	3.9	4.2
time (sec)	N/A	0.017	0.003	0.004	1.127	2.021	0.138	1.253

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	22	7	8
normalized size	1	1.	1.	0.88	1.	2.75	0.88	1.
time (sec)	N/A	0.014	0.002	0.004	1.074	2.022	0.432	1.217

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.02	0.012	0.004	1.605	2.018	0.137	1.271

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	22	47	0	0
normalized size	1	1.	3.	0.93	1.57	3.36	0.	0.
time (sec)	N/A	0.032	0.015	0.01	1.102	1.971	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	9	72	0	9
normalized size	1	1.	1.	0.73	0.82	6.55	0.	0.82
time (sec)	N/A	0.035	0.019	0.012	1.644	2.032	0.	1.197

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	47	0	22
normalized size	1	1.	1.	0.86	1.	6.71	0.	3.14
time (sec)	N/A	0.029	0.015	0.01	1.612	2.068	0.	1.235

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	87	131	239	17	0
normalized size	1	1.	0.95	0.78	1.18	2.15	0.15	0.
time (sec)	N/A	0.1	0.071	0.007	1.598	2.03	0.164	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	28	8	12
normalized size	1	1.	1.	0.91	1.09	2.55	0.73	1.09
time (sec)	N/A	0.028	0.004	0.004	1.096	2.088	0.29	1.304

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	35	7	12
normalized size	1	1.	1.	1.11	1.33	3.89	0.78	1.33
time (sec)	N/A	0.015	0.006	0.013	1.073	2.144	0.418	1.3

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	14	16	58	156	16
normalized size	1	1.	0.94	0.82	0.94	3.41	9.18	0.94
time (sec)	N/A	0.02	0.014	0.02	1.125	2.235	4.997	1.297

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	41	10	36
normalized size	1	1.	1.	1.08	1.33	3.42	0.83	3.
time (sec)	N/A	0.036	0.026	0.006	1.099	2.039	0.107	1.259

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	29	24	31	77	27	31
normalized size	1	1.	1.38	1.14	1.48	3.67	1.29	1.48
time (sec)	N/A	0.036	0.014	0.004	1.102	2.076	0.129	1.318

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	31	41	115	0	50
normalized size	1	1.	0.74	0.74	0.98	2.74	0.	1.19
time (sec)	N/A	0.067	0.02	0.01	1.584	1.962	0.	1.212

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	21	27	80	17	46
normalized size	1	1.	0.83	0.88	1.12	3.33	0.71	1.92
time (sec)	N/A	0.041	0.027	0.01	1.101	1.955	0.12	1.217

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	23	47	20	23
normalized size	1	1.	0.7	0.78	1.	2.04	0.87	1.
time (sec)	N/A	0.043	0.016	0.01	1.012	1.995	4.044	1.273

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	20	22	28	54	22	28
normalized size	1	1.	0.69	0.76	0.97	1.86	0.76	0.97
time (sec)	N/A	0.051	0.021	0.01	1.011	2.015	4.353	1.2

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	39	103	32	0
normalized size	1	1.	1.	1.36	1.77	4.68	1.45	0.
time (sec)	N/A	0.06	0.018	0.007	1.014	2.061	1.945	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	43	99	32	27
normalized size	1	1.	0.81	0.89	1.59	3.67	1.19	1.
time (sec)	N/A	0.049	0.041	0.009	1.042	2.012	0.121	1.358

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	109	0	92
normalized size	1	1.	1.	0.	0.	4.04	0.	3.41
time (sec)	N/A	0.026	0.011	0.181	0.	2.138	0.	1.598

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	71	0	80	0	32
normalized size	1	1.	1.	2.63	0.	2.96	0.	1.19
time (sec)	N/A	0.035	0.006	0.245	0.	2.175	0.	1.366

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	27	43	0	36
normalized size	1	1.	1.	0.84	1.08	1.72	0.	1.44
time (sec)	N/A	0.02	0.008	0.031	1.047	2.098	0.	1.351

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	101	0	47
normalized size	1	1.	1.	0.	0.	3.48	0.	1.62
time (sec)	N/A	0.038	0.006	0.102	0.	2.094	0.	1.462

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	7	8	3	7
normalized size	1	1.	0.68	0.94	0.23	0.26	0.1	0.23
time (sec)	N/A	0.016	0.008	0.012	1.007	1.886	0.071	1.272

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	80	77	149	0	138
normalized size	1	1.	0.86	2.29	2.2	4.26	0.	3.94
time (sec)	N/A	0.078	0.042	0.107	1.157	2.284	0.	1.301

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	107	211	0	166
normalized size	1	1.	0.76	2.	1.62	3.2	0.	2.52
time (sec)	N/A	0.114	0.077	0.128	1.208	2.216	0.	1.315

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	66	162	149	304	0	613
normalized size	1	1.	0.74	1.82	1.67	3.42	0.	6.89
time (sec)	N/A	0.518	0.105	0.129	1.254	2.276	0.	1.338

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	79	74	149	0	146
normalized size	1	1.	0.86	2.26	2.11	4.26	0.	4.17
time (sec)	N/A	0.061	0.043	0.082	1.183	2.486	0.	1.344

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	103	211	0	165
normalized size	1	1.	0.76	2.	1.56	3.2	0.	2.5
time (sec)	N/A	0.121	0.088	0.079	1.204	2.749	0.	1.386

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	162	146	302	0	668
normalized size	1	1.	0.75	1.84	1.66	3.43	0.	7.59
time (sec)	N/A	0.468	0.151	0.118	1.254	2.606	0.	1.219

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	19	7	20	5	7
normalized size	1	1.	1.	3.8	1.4	4.	1.	1.4
time (sec)	N/A	0.033	0.021	0.05	1.169	2.248	12.887	1.194

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	42	76	117	396	0	0
normalized size	1	1.	0.89	1.62	2.49	8.43	0.	0.
time (sec)	N/A	0.057	0.015	0.033	2.177	2.224	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	88	120	362	0	0
normalized size	1	1.	0.96	1.96	2.67	8.04	0.	0.
time (sec)	N/A	0.059	0.017	0.032	1.926	2.107	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	123	433	0	0
normalized size	1	1.	1.	0.	2.37	8.33	0.	0.
time (sec)	N/A	0.06	0.024	0.177	2.14	2.112	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	107	81	396	0	0
normalized size	1	1.	1.	2.28	1.72	8.43	0.	0.
time (sec)	N/A	0.053	0.008	0.03	2.202	2.01	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	118	84	355	0	0
normalized size	1	1.	0.96	2.62	1.87	7.89	0.	0.
time (sec)	N/A	0.056	0.022	0.034	2.22	2.069	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	88	433	0	0
normalized size	1	1.	1.	0.	1.69	8.33	0.	0.
time (sec)	N/A	0.058	0.023	0.108	2.458	2.255	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	75	82	57	587	0	0
normalized size	1	1.	1.47	1.61	1.12	11.51	0.	0.
time (sec)	N/A	0.044	0.011	0.03	1.628	2.012	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	59	568	0	0
normalized size	1	1.	1.53	1.67	1.2	11.59	0.	0.
time (sec)	N/A	0.048	0.011	0.056	1.525	2.346	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	81	6782	65	625	0	0
normalized size	1	1.	1.45	121.11	1.16	11.16	0.	0.
time (sec)	N/A	0.048	0.011	17.115	1.538	2.113	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	75	82	58	462	0	0
normalized size	1	1.	1.47	1.61	1.14	9.06	0.	0.
time (sec)	N/A	0.046	0.011	0.036	1.53	2.182	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	59	450	0	0
normalized size	1	1.	1.53	1.67	1.2	9.18	0.	0.
time (sec)	N/A	0.047	0.012	0.043	1.529	2.13	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	81	6531	66	498	0	0
normalized size	1	1.	1.45	116.62	1.18	8.89	0.	0.
time (sec)	N/A	0.049	0.013	2.596	1.52	2.144	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	118	81	396	0	0
normalized size	1	1.	1.	2.57	1.76	8.61	0.	0.
time (sec)	N/A	0.052	0.006	0.033	2.065	2.201	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	118	82	355	0	0
normalized size	1	1.	0.96	2.62	1.82	7.89	0.	0.
time (sec)	N/A	0.054	0.02	0.105	1.932	2.302	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	88	435	0	0
normalized size	1	1.	1.	0.	1.73	8.53	0.	0.
time (sec)	N/A	0.054	0.022	0.118	2.427	2.154	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	89	117	396	0	0
normalized size	1	1.	0.89	1.93	2.54	8.61	0.	0.
time (sec)	N/A	0.053	0.012	0.033	2.163	2.19	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	88	117	365	0	0
normalized size	1	1.	0.93	1.96	2.6	8.11	0.	0.
time (sec)	N/A	0.058	0.017	0.035	1.955	2.01	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	123	435	0	0
normalized size	1	1.	1.	0.	2.41	8.53	0.	0.
time (sec)	N/A	0.06	0.022	0.154	2.216	2.137	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	13	72	23	51	0	18
normalized size	1	1.	0.62	3.43	1.1	2.43	0.	0.86
time (sec)	N/A	0.021	0.004	0.046	1.016	2.241	0.	1.348

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	27	0	8
normalized size	1	1.	1.	1.17	1.33	4.5	0.	1.33
time (sec)	N/A	0.016	0.014	0.013	1.004	2.244	0.	1.26

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	35	8	15	0	8
normalized size	1	1.	0.68	0.95	0.22	0.41	0.	0.22
time (sec)	N/A	0.028	0.035	0.02	1.01	2.214	0.	1.297

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	61	127	68	15	16
normalized size	1	1.	1.	5.08	10.58	5.67	1.25	1.33
time (sec)	N/A	0.023	0.017	0.04	1.5	2.197	137.984	1.216

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	26	0	9
normalized size	1	1.	1.	0.89	1.	2.89	0.	1.
time (sec)	N/A	0.016	0.004	0.011	0.996	2.193	0.	1.163

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	22	92	17	22
normalized size	1	1.	0.75	0.85	1.1	4.6	0.85	1.1
time (sec)	N/A	0.039	0.015	0.007	0.996	2.264	5.832	1.174

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	33	32	57	189	0	57
normalized size	1	1.	0.66	0.64	1.14	3.78	0.	1.14
time (sec)	N/A	0.026	0.01	0.031	1.078	2.609	0.	1.478

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	26	0	9
normalized size	1	1.	1.	1.17	1.33	4.33	0.	1.5
time (sec)	N/A	0.022	0.008	0.007	0.998	2.166	0.	1.305

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	27	8	9
normalized size	1	1.	1.	0.89	1.	3.	0.89	1.
time (sec)	N/A	0.015	0.003	0.009	0.994	2.159	7.459	1.256

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	41	10	14
normalized size	1	1.	1.	1.1	1.4	4.1	1.	1.4
time (sec)	N/A	0.008	0.007	0.004	0.993	2.262	0.947	1.195

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	43	73	146	100	223	36
normalized size	1	1.	3.07	5.21	10.43	7.14	15.93	2.57
time (sec)	N/A	0.02	0.011	0.017	1.007	2.396	3.287	1.197

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	39	10	15
normalized size	1	1.	1.	1.09	1.36	3.55	0.91	1.36
time (sec)	N/A	0.009	0.003	0.005	1.008	2.125	0.946	1.174

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	59	146	140	462	0	0
normalized size	1	1.	0.8	1.97	1.89	6.24	0.	0.
time (sec)	N/A	0.109	0.056	0.033	2.381	2.622	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	47	134	242	169	445	55
normalized size	1	1.	1.18	3.35	6.05	4.22	11.12	1.38
time (sec)	N/A	0.067	0.04	0.032	1.005	2.25	15.177	1.299

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	88	0	188	0	0	0
normalized size	1	1.	1.11	0.	2.38	0.	0.	0.
time (sec)	N/A	0.102	0.032	0.017	1.065	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	72	109	68	17	26
normalized size	1	1.	1.	4.8	7.27	4.53	1.13	1.73
time (sec)	N/A	0.021	0.014	0.055	1.501	2.224	106.402	1.308

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	49	393	0	70
normalized size	1	1.	0.86	1.66	1.4	11.23	0.	2.
time (sec)	N/A	0.079	0.044	0.035	1.163	2.099	0.	1.321

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	90	859	0	123
normalized size	1	1.	0.76	1.47	1.36	13.02	0.	1.86
time (sec)	N/A	0.143	0.099	0.036	1.154	1.86	0.	1.373

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	67	116	149	1669	0	130
normalized size	1	1.	0.75	1.3	1.67	18.75	0.	1.46
time (sec)	N/A	0.517	0.1	0.034	1.249	1.94	0.	1.331

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	50	393	0	73
normalized size	1	1.	0.86	1.66	1.43	11.23	0.	2.09
time (sec)	N/A	0.07	0.038	0.019	1.189	1.788	0.	1.268

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	90	859	0	120
normalized size	1	1.	0.76	1.47	1.36	13.02	0.	1.82
time (sec)	N/A	0.132	0.065	0.024	1.181	1.974	0.	1.319

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	66	116	150	1669	0	131
normalized size	1	1.	0.75	1.32	1.7	18.97	0.	1.49
time (sec)	N/A	0.483	0.141	0.035	1.23	2.001	0.	1.319

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	295	58	197	0	0
normalized size	1	1.	0.92	7.56	1.49	5.05	0.	0.
time (sec)	N/A	0.059	0.02	0.126	1.285	2.021	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	454	58	246	0	0
normalized size	1	1.	0.94	12.97	1.66	7.03	0.	0.
time (sec)	N/A	0.06	0.018	0.157	1.254	2.014	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	63	225	0	0
normalized size	1	1.	0.98	0.	1.43	5.11	0.	0.
time (sec)	N/A	0.06	0.027	0.104	1.2	1.908	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	321	43	219	0	0
normalized size	1	1.	0.92	8.23	1.1	5.62	0.	0.
time (sec)	N/A	0.056	0.018	0.109	1.709	2.002	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	478	43	267	0	0
normalized size	1	1.	0.94	13.66	1.23	7.63	0.	0.
time (sec)	N/A	0.056	0.018	0.148	1.702	1.944	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	49	247	0	0
normalized size	1	1.	0.98	0.	1.11	5.61	0.	0.
time (sec)	N/A	0.057	0.028	0.078	1.679	2.082	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	73	373	0	0
normalized size	1	1.	0.9	0.62	1.87	9.56	0.	0.
time (sec)	N/A	0.044	0.007	0.014	1.572	1.972	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	70	76	375	0	0
normalized size	1	1.	1.2	1.71	1.85	9.15	0.	0.
time (sec)	N/A	0.044	0.007	0.027	1.599	2.038	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	77	454	0	0
normalized size	1	1.	1.27	1.27	2.08	12.27	0.	0.
time (sec)	N/A	0.047	0.01	0.03	1.548	1.969	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	82	412	0	0
normalized size	1	1.	1.2	0.93	1.78	8.96	0.	0.
time (sec)	N/A	0.049	0.01	0.023	1.892	1.932	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	66	373	0	0
normalized size	1	1.	0.9	0.62	1.69	9.56	0.	0.
time (sec)	N/A	0.046	0.007	0.013	1.59	1.981	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	70	69	375	0	0
normalized size	1	1.	1.2	1.71	1.68	9.15	0.	0.
time (sec)	N/A	0.044	0.008	0.014	1.557	1.932	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	80	454	0	0
normalized size	1	1.	1.27	1.27	2.16	12.27	0.	0.
time (sec)	N/A	0.048	0.01	0.017	1.543	2.136	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	82	412	0	0
normalized size	1	1.	1.2	0.93	1.78	8.96	0.	0.
time (sec)	N/A	0.049	0.012	0.021	1.744	1.928	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	314	42	306	0	0
normalized size	1	1.	0.97	8.26	1.11	8.05	0.	0.
time (sec)	N/A	0.053	0.015	0.111	1.709	1.943	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	480	43	363	0	0
normalized size	1	1.	0.94	13.71	1.23	10.37	0.	0.
time (sec)	N/A	0.053	0.018	0.142	1.673	2.041	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	49	332	0	0
normalized size	1	1.	1.	0.	1.14	7.72	0.	0.
time (sec)	N/A	0.055	0.023	0.085	1.655	2.009	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	293	50	285	0	0
normalized size	1	1.	0.97	7.71	1.32	7.5	0.	0.
time (sec)	N/A	0.054	0.016	0.102	1.215	1.88	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	456	61	339	0	0
normalized size	1	1.	0.94	13.03	1.74	9.69	0.	0.
time (sec)	N/A	0.058	0.017	0.135	1.215	1.935	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	63	311	0	0
normalized size	1	1.	1.	0.	1.47	7.23	0.	0.
time (sec)	N/A	0.064	0.022	0.106	1.209	2.003	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	33	32	151	782	0	127
normalized size	1	1.	0.66	0.64	3.02	15.64	0.	2.54
time (sec)	N/A	0.029	0.011	0.034	1.071	1.916	0.	1.344

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	16	228	14	42
normalized size	1	1.	1.	1.08	1.23	17.54	1.08	3.23
time (sec)	N/A	0.019	0.008	0.013	1.025	1.797	1.001	1.251

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	11	14	18	32	60	18
normalized size	1	1.	0.65	0.82	1.06	1.88	3.53	1.06
time (sec)	N/A	0.007	0.002	0.007	1.029	1.815	1.88	1.313

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	32	59	27	32
normalized size	1	1.	0.96	0.93	1.19	2.19	1.	1.19
time (sec)	N/A	0.021	0.004	0.001	1.065	1.842	0.126	1.331

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	61	0	4
normalized size	1	1.	1.	1.33	1.33	20.33	0.	1.33
time (sec)	N/A	0.034	0.018	0.008	1.507	1.77	0.	1.321

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	23	55	22	30
normalized size	1	1.	1.25	0.96	0.96	2.29	0.92	1.25
time (sec)	N/A	0.023	0.001	0.003	1.046	1.787	0.105	1.347

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	32	31	53	29	31
normalized size	1	1.	0.92	1.28	1.24	2.12	1.16	1.24
time (sec)	N/A	0.01	0.004	0.001	1.021	1.748	0.319	1.311

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	32	61	22	41
normalized size	1	1.	0.88	0.91	0.94	1.79	0.65	1.21
time (sec)	N/A	0.014	0.006	0.001	1.064	1.855	0.113	1.256

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	38	70	27	57
normalized size	1	1.	1.	0.98	0.95	1.75	0.68	1.42
time (sec)	N/A	0.016	0.008	0.003	1.043	1.863	0.118	1.214

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	69	26	34
normalized size	1	1.	1.	0.84	1.1	2.23	0.84	1.1
time (sec)	N/A	0.029	0.008	0.005	1.565	1.848	0.134	1.341

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	57	20	30
normalized size	1	1.	1.	0.88	0.	2.19	0.77	1.15
time (sec)	N/A	0.004	0.016	0.004	0.	1.739	5.467	1.184

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	57	20	30
normalized size	1	1.	1.	0.88	0.	2.19	0.77	1.15
time (sec)	N/A	0.004	0.016	0.004	0.	1.95	9.581	1.231

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	57	20	30
normalized size	1	1.	1.	0.88	0.	2.19	0.77	1.15
time (sec)	N/A	0.005	0.018	0.001	0.	1.847	11.131	1.2

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	0	92	0	54
normalized size	1	1.	1.	1.21	0.	2.14	0.	1.26
time (sec)	N/A	0.012	0.024	0.006	0.	1.883	0.	1.218

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	18	45	66	18
normalized size	1	1.	0.71	0.67	0.86	2.14	3.14	0.86
time (sec)	N/A	0.007	0.002	0.004	1.063	1.934	3.095	1.319

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	32	46	10	19
normalized size	1	1.	0.92	1.	2.46	3.54	0.77	1.46
time (sec)	N/A	0.005	0.003	0.013	1.026	1.894	0.418	1.218

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	19	14	3	8
normalized size	1	1.	1.	1.17	3.17	2.33	0.5	1.33
time (sec)	N/A	0.01	0.001	0.001	1.091	1.713	0.09	1.319

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	38	34	45	130	158	45
normalized size	1	1.	1.19	1.06	1.41	4.06	4.94	1.41
time (sec)	N/A	0.205	0.014	0.005	1.068	1.81	1.052	1.317

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	49	15	22
normalized size	1	1.	1.	1.06	1.38	3.06	0.94	1.38
time (sec)	N/A	0.007	0.002	0.004	1.498	1.804	0.133	1.234

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	39	70	34	31
normalized size	1	1.	1.	1.1	1.95	3.5	1.7	1.55
time (sec)	N/A	0.005	0.007	0.007	0.991	1.808	0.505	1.212

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	44	45	97	32	47
normalized size	1	1.	0.89	1.26	1.29	2.77	0.91	1.34
time (sec)	N/A	0.006	0.005	0.01	1.007	1.777	0.161	1.199

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	16	41	15	23
normalized size	1	1.	1.	1.17	1.33	3.42	1.25	1.92
time (sec)	N/A	0.003	0.002	0.007	1.039	1.844	0.106	1.244

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	27	39	38	82	27	39
normalized size	1	1.	0.75	1.08	1.06	2.28	0.75	1.08
time (sec)	N/A	0.015	0.009	0.014	1.101	1.799	0.131	1.338

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	57	135	48	58
normalized size	1	1.	1.	0.98	1.06	2.5	0.89	1.07
time (sec)	N/A	0.039	0.013	0.016	1.056	1.868	0.15	1.272

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	33	55	70	96	41	42
normalized size	1	1.	0.94	1.57	2.	2.74	1.17	1.2
time (sec)	N/A	0.015	0.021	0.003	1.091	1.843	0.34	1.315

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	44	82	100	139	63	42
normalized size	1	1.	1.26	2.34	2.86	3.97	1.8	1.2
time (sec)	N/A	0.025	0.017	0.001	1.061	1.763	0.365	1.327

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	30	10	18
normalized size	1	1.	1.	0.93	1.2	2.	0.67	1.2
time (sec)	N/A	0.018	0.002	0.002	1.09	1.782	0.292	1.181

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	32	42	47	26	42
normalized size	1	1.	0.68	1.03	1.35	1.52	0.84	1.35
time (sec)	N/A	0.022	0.006	0.003	0.998	1.821	0.345	1.311

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	30	96	0	112	0	0
normalized size	1	1.	0.68	2.18	0.	2.55	0.	0.
time (sec)	N/A	0.03	0.014	0.013	0.	1.837	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	32	51	34	61
normalized size	1	1.	1.	1.06	1.78	2.83	1.89	3.39
time (sec)	N/A	0.011	0.017	0.004	1.081	1.919	1.1	1.216

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	324	126	35
normalized size	1	1.	1.	0.75	0.	10.12	3.94	1.09
time (sec)	N/A	0.018	0.024	0.007	0.	1.901	8.803	1.359

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	112	120	0	1335	340	328
normalized size	1	1.	0.78	0.83	0.	9.27	2.36	2.28
time (sec)	N/A	0.094	0.049	0.006	0.	2.014	79.709	1.305

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	167	168	0	502	257	271
normalized size	1	1.	0.74	0.74	0.	2.21	1.13	1.19
time (sec)	N/A	0.162	0.064	0.008	0.	2.013	66.928	1.432

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	35	45	77	0	72
normalized size	1	1.	1.26	1.3	1.67	2.85	0.	2.67
time (sec)	N/A	0.023	0.015	0.006	1.045	1.895	0.	1.301

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	47	43	0	366	0	51
normalized size	1	1.	1.18	1.08	0.	9.15	0.	1.27
time (sec)	N/A	0.027	0.024	0.006	0.	1.965	0.	1.276

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	136	0	406	0	431
normalized size	1	1.	0.89	0.91	0.	2.72	0.	2.89
time (sec)	N/A	0.109	0.05	0.006	0.	1.96	0.	1.39

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	211	181	0	505	0	365
normalized size	1	1.	0.91	0.78	0.	2.17	0.	1.57
time (sec)	N/A	0.183	0.066	0.01	0.	1.975	0.	1.335

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	0	76	22	26
normalized size	1	1.	1.	0.91	0.	3.45	1.	1.18
time (sec)	N/A	0.018	0.029	0.007	0.	1.727	0.165	1.799

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	50	127	22	50
normalized size	1	1.	1.02	0.93	1.22	3.1	0.54	1.22
time (sec)	N/A	0.058	0.089	0.005	1.525	1.849	0.179	1.232

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	0	127	22	50
normalized size	1	1.	1.02	0.93	0.	3.1	0.54	1.22
time (sec)	N/A	0.033	0.073	0.005	0.	1.766	0.196	1.31

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	40	0	136	19	53
normalized size	1	1.	1.05	0.95	0.	3.24	0.45	1.26
time (sec)	N/A	0.046	0.089	0.006	0.	1.948	0.179	1.341

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	23	58	27	30
normalized size	1	1.	1.	0.84	0.72	1.81	0.84	0.94
time (sec)	N/A	0.021	0.002	0.003	0.993	1.754	0.131	1.352

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	59	0	0	0	0	0
normalized size	1	1.	1.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.011	0.025	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	60	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.012	0.016	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	0	0	0	0	43
normalized size	1	1.	1.44	0.	0.	0.	0.	1.
time (sec)	N/A	0.031	0.01	0.045	0.	0.	0.	1.275

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	102	0	0	0	0
normalized size	1	1.	0.99	0.84	0.	0.	0.	0.
time (sec)	N/A	0.145	0.075	0.01	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	73	114	150	0	0	0
normalized size	1	1.	0.9	1.41	1.85	0.	0.	0.
time (sec)	N/A	0.099	0.029	0.026	1.011	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	5	16	22	10	0
normalized size	1	1.	1.	0.56	1.78	2.44	1.11	0.
time (sec)	N/A	0.009	0.002	0.001	1.036	2.068	1.925	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	35	77	111	0	0	0
normalized size	1	1.	0.81	1.79	2.58	0.	0.	0.
time (sec)	N/A	0.073	0.018	0.003	1.033	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	31	66	0	192	0
normalized size	1	1.	1.03	1.03	2.2	0.	6.4	0.
time (sec)	N/A	0.099	0.01	0.013	1.029	0.	83.659	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	22	0	0
normalized size	1	1.	1.	1.14	1.29	3.14	0.	0.
time (sec)	N/A	0.027	0.109	0.017	1.265	2.194	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	63	57	77	151	46	80
normalized size	1	1.	0.93	0.84	1.13	2.22	0.68	1.18
time (sec)	N/A	0.251	0.032	0.021	1.689	2.14	0.228	1.375

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	31	103	24	31
normalized size	1	1.	1.	0.83	1.07	3.55	0.83	1.07
time (sec)	N/A	0.018	0.012	0.002	1.035	2.236	4.507	1.345

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	24	43	14	26
normalized size	1	1.	1.	1.56	1.33	2.39	0.78	1.44
time (sec)	N/A	0.004	0.002	0.011	1.077	1.94	0.106	1.309

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	21	35	34	53	15	30
normalized size	1	1.	0.78	1.3	1.26	1.96	0.56	1.11
time (sec)	N/A	0.005	0.003	0.007	1.007	2.103	0.119	1.323

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	60	112	73	157	41	76
normalized size	1	1.	1.05	1.96	1.28	2.75	0.72	1.33
time (sec)	N/A	0.057	0.046	0.033	1.583	2.11	0.194	1.372

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	62	249	0	0	0	0
normalized size	1	1.	1.03	4.15	0.	0.	0.	0.
time (sec)	N/A	0.081	0.006	0.092	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	239	158	0	0	0	0
normalized size	1	1.	3.92	2.59	0.	0.	0.	0.
time (sec)	N/A	0.105	0.044	0.02	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	373	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.194	180.	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	92	0	0
normalized size	1	1.	4.62	0.	0.	3.17	0.	0.
time (sec)	N/A	0.034	0.617	0.043	0.	2.194	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	92	0	0
normalized size	1	1.	4.62	0.	0.	3.17	0.	0.
time (sec)	N/A	0.034	0.536	0.021	0.	2.185	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	14	23	8	14
normalized size	1	1.	1.	0.88	0.82	1.35	0.47	0.82
time (sec)	N/A	0.003	0.003	0.003	1.084	1.931	0.088	1.295

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	27	0	45	34	22
normalized size	1	1.	0.93	1.	0.	1.67	1.26	0.81
time (sec)	N/A	0.009	0.024	0.007	0.	2.083	2.811	1.373

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	31	25	34	35	55	0	35
normalized size	1	1.24	1.	1.36	1.4	2.2	0.	1.4
time (sec)	N/A	0.053	0.016	0.003	1.052	2.059	0.	1.196

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	22	26	32	53	26	32
normalized size	1	1.	0.85	1.	1.23	2.04	1.	1.23
time (sec)	N/A	0.035	0.015	0.016	1.134	2.15	6.917	1.307

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	6.503	0.01	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	5.362	0.01	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.006	0.009	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.038	0.01	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	6.469	0.01	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	12	34	10	15
normalized size	1	1.	1.	0.92	0.92	2.62	0.77	1.15
time (sec)	N/A	0.024	0.008	0.003	1.197	2.125	0.116	1.335

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	10	11	14	35	8	19
normalized size	1	1.	1.11	1.22	1.56	3.89	0.89	2.11
time (sec)	N/A	0.072	0.051	0.01	1.193	2.157	0.115	1.348

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	0	68	15	0
normalized size	1	1.	1.	0.	0.	5.23	1.15	0.
time (sec)	N/A	0.142	0.039	0.013	0.	2.14	2.144	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	107	90	138	53	157
normalized size	1	1.	0.79	1.6	1.34	2.06	0.79	2.34
time (sec)	N/A	0.074	0.031	0.046	1.019	2.256	161.722	1.341

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	92	139	53	0
normalized size	1	1.	1.06	0.	1.84	2.78	1.06	0.
time (sec)	N/A	0.057	0.037	180.	1.035	2.182	160.093	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	24	53	17	26
normalized size	1	1.	0.9	1.05	1.14	2.52	0.81	1.24
time (sec)	N/A	0.008	0.003	0.006	1.01	2.038	0.108	1.252

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	92	139	53	0
normalized size	1	1.	1.06	0.	1.84	2.78	1.06	0.
time (sec)	N/A	0.049	0.036	180.	0.994	2.086	154.442	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	108	90	138	53	157
normalized size	1	1.	0.93	1.57	1.3	2.	0.77	2.28
time (sec)	N/A	0.052	0.027	0.036	1.059	2.196	153.419	1.382

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	11	12	15	10	0
normalized size	1	1.	1.	1.22	1.33	1.67	1.11	0.
time (sec)	N/A	0.022	0.013	0.013	1.232	2.143	0.298	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	50	0	0
normalized size	1	1.	1.	0.	0.	1.92	0.	0.
time (sec)	N/A	0.025	0.014	0.043	0.	2.108	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	72	0	146	0	0	120
normalized size	1	1.	1.2	0.	2.43	0.	0.	2.
time (sec)	N/A	0.069	0.11	0.02	1.223	0.	0.	1.357

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	127	0	0	100
normalized size	1	1.	1.11	0.	1.98	0.	0.	1.56
time (sec)	N/A	0.068	0.091	0.02	1.219	0.	0.	1.266

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	211	0	0	174
normalized size	1	1.	1.16	0.	3.06	0.	0.	2.52
time (sec)	N/A	0.069	0.146	0.023	1.271	0.	0.	1.367

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	79	0	176	0	0	143
normalized size	1	1.	1.11	0.	2.48	0.	0.	2.01
time (sec)	N/A	0.079	0.13	0.02	1.325	0.	0.	1.418

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	95	0	127	848	0	0
normalized size	1	1.	0.97	0.	1.3	8.65	0.	0.
time (sec)	N/A	0.29	0.059	0.273	2.041	2.592	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	95	662	0	0
normalized size	1	1.	0.99	0.	1.19	8.28	0.	0.
time (sec)	N/A	0.182	0.039	0.207	2.03	2.518	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	47	368	58	427	0	0
normalized size	1	1.	0.9	7.08	1.12	8.21	0.	0.
time (sec)	N/A	0.062	0.031	0.167	1.946	2.389	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	2.255	0.417	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.345	1.93	0.434	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	100	0	127	869	0	0
normalized size	1	1.	0.97	0.	1.23	8.44	0.	0.
time (sec)	N/A	0.203	0.063	0.367	2.075	2.586	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	C	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	95	682	0	0
normalized size	1	1.	0.96	0.	1.12	8.02	0.	0.
time (sec)	N/A	0.139	0.067	0.329	2.089	2.506	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	56	583	58	447	0	0
normalized size	1	1.	0.98	10.23	1.02	7.84	0.	0.
time (sec)	N/A	0.064	0.03	0.244	1.81	2.421	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.79	0.605	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	2.265	0.622	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [207] had the largest ratio of [1.667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	0	0	0.	0	0.
2	A	10	5	1.	32	0.156
3	A	8	5	1.	32	0.156
4	A	6	5	1.	30	0.167
5	A	2	2	1.	14	0.143
6	A	0	0	0.	0	0.
7	A	0	0	0.	0	0.
8	A	0	0	0.	0	0.
9	A	13	5	1.	28	0.179
10	A	8	5	1.	28	0.179
11	A	5	4	1.	26	0.154
12	A	1	1	1.	10	0.1
13	A	0	0	0.	0	0.
14	A	0	0	0.	0	0.
15	A	0	0	0.	0	0.
16	A	1	1	1.	43	0.023
17	A	1	1	1.	43	0.023
18	A	1	1	1.	41	0.024
19	A	4	3	1.	25	0.12
20	A	1	1	1.	43	0.023
21	A	1	1	1.	43	0.023
22	A	1	1	1.	43	0.023
23	A	3	2	1.	39	0.051

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	3	2	1.	37	0.054
25	A	2	1	1.	15	0.067
26	A	2	2	1.	34	0.059
27	A	3	2	1.	37	0.054
28	A	3	2	1.	39	0.051
29	A	2	2	1.	45	0.044
30	A	0	0	0.	0	0.
31	A	9	4	1.	40	0.1
32	A	7	4	1.	40	0.1
33	A	5	4	1.	38	0.105
34	A	4	3	1.	22	0.136
35	A	0	0	0.	0	0.
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	1	1	1.	60	0.017
39	A	0	0	0.	0	0.
40	A	1	1	1.	39	0.026
41	A	1	1	1.	40	0.025
42	A	1	1	1.	41	0.024
43	A	1	1	1.	42	0.024
44	A	1	1	1.	40	0.025
45	A	1	1	1.	41	0.024
46	A	3	3	1.	19	0.158
47	A	3	3	1.	21	0.143
48	A	3	3	1.	19	0.158
49	A	3	3	1.	17	0.176
50	A	4	3	1.	15	0.2
51	A	1	1	1.	19	0.053
52	A	3	3	1.	19	0.158
53	A	3	3	1.	19	0.158
54	A	3	3	1.	19	0.158
55	A	2	2	1.	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	1	1	1.	13	0.077
57	A	3	3	1.	11	0.273
58	A	1	1	1.	15	0.067
59	A	3	3	1.	18	0.167
60	A	3	2	1.	18	0.111
61	A	3	2	1.	18	0.111
62	A	3	2	1.	18	0.111
63	A	3	2	1.	16	0.125
64	A	3	2	1.	14	0.143
65	A	7	6	1.	18	0.333
66	A	3	2	1.	18	0.111
67	A	3	2	1.	18	0.111
68	A	3	2	1.	18	0.111
69	A	3	2	1.	18	0.111
70	A	5	3	1.	19	0.158
71	A	7	6	1.	19	0.316
72	A	7	6	1.	19	0.316
73	A	7	6	1.	19	0.316
74	A	7	6	1.	17	0.353
75	A	6	6	1.	15	0.4
76	A	7	4	1.	19	0.21
77	A	7	6	1.	19	0.316
78	A	7	6	1.	19	0.316
79	A	7	6	1.	19	0.316
80	A	7	6	1.	19	0.316
81	A	6	6	1.	7	0.857
82	A	7	6	1.	23	0.261
83	A	7	6	1.	23	0.261
84	A	7	6	1.	23	0.261
85	A	7	6	1.	21	0.286
86	A	6	6	1.	15	0.4
87	A	9	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	7	6	1.	23	0.261
89	A	7	6	1.	23	0.261
90	A	7	6	1.	23	0.261
91	A	7	6	1.	23	0.261
92	A	6	7	1.	25	0.28
93	A	8	9	1.	32	0.281
94	A	20	6	1.	25	0.24
95	A	20	6	1.	28	0.214
96	A	14	10	1.	16	0.625
97	A	27	14	1.	17	0.824
98	A	28	12	1.	21	0.571
99	A	27	14	1.	9	1.556
100	A	34	16	1.	13	1.231
101	A	25	11	1.	21	0.524
102	A	20	10	1.	21	0.476
103	A	16	10	1.	19	0.526
104	A	13	9	1.	17	0.529
105	A	0	0	0.	0	0.
106	A	19	11	1.	21	0.524
107	A	20	12	1.	21	0.571
108	A	15	12	1.	23	0.522
109	A	13	12	1.	23	0.522
110	A	12	10	1.	23	0.435
111	A	15	13	1.	23	0.565
112	A	18	13	1.	23	0.565
113	A	6	5	1.	12	0.417
114	A	5	5	1.	12	0.417
115	A	4	4	1.	10	0.4
116	A	4	4	1.	8	0.5
117	A	0	0	0.	0	0.
118	A	5	4	1.	20	0.2
119	A	4	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	3	3	1.	18	0.167
121	A	2	2	1.	16	0.125
122	A	0	0	0.	0	0.
123	A	6	5	1.	20	0.25
124	A	5	5	1.	20	0.25
125	A	4	4	1.	18	0.222
126	A	4	4	1.	16	0.25
127	A	0	0	0.	0	0.
128	A	3	3	1.	16	0.188
129	A	2	2	1.	10	0.2
130	A	2	2	1.	12	0.167
131	A	2	2	1.	10	0.2
132	A	2	2	1.	10	0.2
133	A	2	1	1.	12	0.083
134	A	3	2	1.	14	0.143
135	A	2	1	1.	16	0.062
136	A	2	1	1.	14	0.071
137	A	7	6	1.	16	0.375
138	A	2	1	1.	15	0.067
139	A	2	2	1.	8	0.25
140	A	3	2	1.	9	0.222
141	A	3	2	1.	16	0.125
142	A	3	2	1.	16	0.125
143	A	4	3	1.	18	0.167
144	A	3	2	1.	16	0.125
145	A	3	2	1.	14	0.143
146	A	3	2	1.	16	0.125
147	A	4	4	1.	16	0.25
148	A	3	1	1.	20	0.05
149	A	3	2	1.	14	0.143
150	A	3	2	1.	14	0.143
151	A	3	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	3	2	1.	16	0.125
153	A	4	3	1.	10	0.3
154	A	5	6	1.	9	0.667
155	A	5	6	1.	11	0.546
156	A	15	8	1.	11	0.727
157	A	5	6	1.	9	0.667
158	A	7	8	1.	11	0.727
159	A	15	8	1.	11	0.727
160	A	4	3	1.	12	0.25
161	A	5	5	1.	5	1.
162	A	6	6	1.	7	0.857
163	A	6	6	1.	7	0.857
164	A	5	5	1.	5	1.
165	A	6	6	1.	7	0.857
166	A	6	6	1.	7	0.857
167	A	7	5	1.	5	1.
168	A	8	6	1.	7	0.857
169	A	8	6	1.	7	0.857
170	A	7	5	1.	5	1.
171	A	8	6	1.	7	0.857
172	A	8	6	1.	7	0.857
173	A	5	5	1.	5	1.
174	A	6	6	1.	7	0.857
175	A	6	6	1.	7	0.857
176	A	5	5	1.	5	1.
177	A	6	6	1.	7	0.857
178	A	6	6	1.	7	0.857
179	A	3	3	1.	16	0.188
180	A	3	2	1.	10	0.2
181	A	5	4	1.	10	0.4
182	A	3	4	1.	8	0.5
183	A	2	3	1.	6	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	4	4	1.	10	0.4
185	A	2	2	1.	35	0.057
186	A	3	3	1.	8	0.375
187	A	2	3	1.	6	0.5
188	A	2	2	1.	6	0.333
189	A	4	5	1.	6	0.833
190	A	2	2	1.	6	0.333
191	A	10	9	1.	8	1.125
192	A	7	7	1.	8	0.875
193	A	8	8	1.	7	1.143
194	A	3	4	1.	8	0.5
195	A	5	6	1.	9	0.667
196	A	7	8	1.	11	0.727
197	A	15	8	1.	11	0.727
198	A	5	6	1.	9	0.667
199	A	7	8	1.	11	0.727
200	A	15	8	1.	11	0.727
201	A	5	5	1.	5	1.
202	A	6	6	1.	7	0.857
203	A	6	6	1.	7	0.857
204	A	5	5	1.	5	1.
205	A	6	6	1.	7	0.857
206	A	6	6	1.	7	0.857
207	A	7	5	1.	3	1.667
208	A	7	5	1.	5	1.
209	A	8	6	1.	7	0.857
210	A	8	6	1.	7	0.857
211	A	7	5	1.	3	1.667
212	A	7	5	1.	5	1.
213	A	8	6	1.	7	0.857
214	A	8	6	1.	7	0.857
215	A	5	5	1.	5	1.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	6	6	1.	7	0.857
217	A	6	6	1.	7	0.857
218	A	5	5	1.	5	1.
219	A	6	6	1.	7	0.857
220	A	6	6	1.	7	0.857
221	A	2	2	1.	35	0.057
222	A	3	3	1.	8	0.375
223	A	1	1	1.	8	0.125
224	A	3	3	1.	10	0.3
225	A	2	1	1.	16	0.062
226	A	3	3	1.	9	0.333
227	A	2	2	1.	10	0.2
228	A	3	2	1.	10	0.2
229	A	3	2	1.	12	0.167
230	A	4	3	1.	8	0.375
231	A	2	2	1.	12	0.167
232	A	2	2	1.	12	0.167
233	A	2	2	1.	14	0.143
234	A	6	6	1.	14	0.429
235	A	1	1	1.	8	0.125
236	A	2	2	1.	4	0.5
237	A	1	1	1.	10	0.1
238	A	3	1	1.	11	0.091
239	A	3	3	1.	6	0.5
240	A	2	2	1.	8	0.25
241	A	2	2	1.	14	0.143
242	A	2	2	1.	6	0.333
243	A	4	3	1.	10	0.3
244	A	4	3	1.	14	0.214
245	A	2	2	1.	12	0.167
246	A	2	2	1.	14	0.143
247	A	2	2	1.	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	2	2	1.	14	0.143
249	A	2	2	1.	14	0.143
250	A	2	1	1.	15	0.067
251	A	2	1	1.	17	0.059
252	A	7	6	1.	17	0.353
253	A	10	6	1.	17	0.353
254	A	3	1	1.	17	0.059
255	A	3	2	1.	17	0.118
256	A	8	7	1.	17	0.412
257	A	11	7	1.	17	0.412
258	A	3	2	1.	18	0.111
259	A	5	4	1.	26	0.154
260	A	5	4	1.	21	0.19
261	A	7	5	1.	27	0.185
262	A	2	2	1.	10	0.2
263	A	3	3	1.	12	0.25
264	A	3	3	1.	12	0.25
265	A	3	3	1.	12	0.25
266	A	8	7	1.	15	0.467
267	A	7	7	1.	15	0.467
268	A	1	1	1.	8	0.125
269	A	6	6	1.	19	0.316
270	A	5	4	1.	19	0.21
271	A	2	1	1.	14	0.071
272	A	8	7	1.	24	0.292
273	A	4	3	1.	8	0.375
274	A	2	2	1.	9	0.222
275	A	2	2	1.	10	0.2
276	A	8	6	1.	22	0.273
277	A	5	6	1.	18	0.333
278	A	5	6	1.	20	0.3
279	A	5	6	1.	25	0.24

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	1	1	1.	39	0.026
281	A	1	1	1.	39	0.026
282	A	2	2	1.	8	0.25
283	A	3	3	1.	10	0.3
284	A	5	5	1.24	12	0.417
285	A	3	4	1.	10	0.4
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	0	0	0.	0	0.
291	A	2	2	1.	14	0.143
292	A	2	2	1.	16	0.125
293	A	8	6	1.	14	0.429
294	A	5	3	1.	14	0.214
295	A	6	4	1.	14	0.286
296	A	4	4	1.	12	0.333
297	A	5	3	1.	14	0.214
298	A	7	4	1.	14	0.286
299	A	2	1	1.	14	0.071
300	A	3	3	1.	6	0.5
301	A	4	4	1.	13	0.308
302	A	4	4	1.	14	0.286
303	A	4	4	1.	17	0.235
304	A	4	4	1.	18	0.222
305	A	13	13	1.	10	1.3
306	A	12	12	1.	8	1.5
307	A	7	6	1.	6	1.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	14	12	1.	13	0.923
311	A	13	11	1.	11	1.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	7	6	1.	9	0.667
313	A	0	0	0.	0	0.
314	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=75

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{bn(p+1)q}-\frac{am\text{CannotIntegrate}\left(x^{m-1}\left(ax^m+b\log^q(cx^n)\right)^p,x\right)}{bnq}$$

```
[Out] -((a*m*CannotIntegrate[x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p, x])/(b*n*q))
+ (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(b*n*(1 + p)*q)
```

Rubi [A] time = 0.24677, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(b*n*(1 + p)*q) - (a*m*Defer[Int][x^(-1 +
m)*(a*x^m + b*Log[c*x^n]^q)^p, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{\left(ax^m+b\log^q(cx^n)\right)^{1+p}}{bn(1+p)q}-\frac{(am)\int x^{-1+m}\left(ax^m+b\log^q(cx^n)\right)^p\,dx}{bnq}$$

Mathematica [A] time = 1.18046, size = 0, normalized size = 0.

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x, x]
```

Maple [A] time = 0.48, size = 0, normalized size = 0.

$$\int \frac{(\ln(cx^n))^{-1+q}\left(ax^m+b(\ln(cx^n))^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^p \log\left(cx^n\right)^{q-1}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="fricas
")
```

```
[Out] integral((a*x^m + b*log(c*x^n)^q)^p*log(c*x^n)^(q - 1)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.2 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=231

$$-\frac{3a^2b4^{-q}x^{2m}\left(cx^n\right)^{-\frac{2m}{n}}\log^{2q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\operatorname{Gamma}\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{a^33^{-q}x^{3m}\left(cx^n\right)^{-\frac{3m}{n}}\log^q\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-}}{n}$$

```
[Out] (b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/n
)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a^2
*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x^n)
^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*Log[c
*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)
```

Rubi [A] time = 0.348353, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2539, 2310, 2181, 2302, 30}

$$-\frac{3a^2b4^{-q}x^{2m}\left(cx^n\right)^{-\frac{2m}{n}}\log^{2q}\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\operatorname{Gamma}\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{a^33^{-q}x^{3m}\left(cx^n\right)^{-\frac{3m}{n}}\log^q\left(cx^n\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```


```
[Out] (b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/n
)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a^2
*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x^n)
^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*Log[c
*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)
```


Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx = \int \left(a^3x^{-1+3m}\log^{-1+q}(cx^n)+3a^2bx^{-1+2m}\log^{-1+2q}(cx^n)+3ab^2x^{-1+m}\log^{-1+$$

$$= a^3\int x^{-1+3m}\log^{-1+q}(cx^n)\,dx+\left(3a^2b\right)\int x^{-1+2m}\log^{-1+2q}(cx^n)\,dx+\left(3ab^2\right)$$

$$= \frac{b^3\,\mathrm{Subst}\left(\int x^{-1+4q}\,dx, x, \log(cx^n)\right)}{n}+\frac{\left(a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\right)\mathrm{Subst}\left(\int e^{\frac{3mx}{n}}x^{-1+q}\right.}{n}$$

$$= \frac{b^3\log^{4q}(cx^n)}{4nq}-\frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3}}{n}$$

Mathematica [A] time = 0.738538, size = 223, normalized size = 0.97

$$\frac{\log^q(cx^n)\left(-3a^2b4^{1-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\mathrm{Gamma}\left(2q,-\frac{2m\log(cx^n)}{n}\right)-4a^33^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n}{n}\right.\right.}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^3*Log[c*x^n]^(3*q))/q - (12*a*b^2*x^m*Gamma[3*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q))/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q))
- (3*4^(1 - q)*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/
((c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (4*a^3*x^(3*m)*Gamma[q, (
-3*m*Log[c*x^n])/n])/(3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)))/(4*n
)
```

Maple [F] time = 2.418, size = 0, normalized size = 0.

$$\int \frac{(\ln (cx^n))^{-1+q} \left(ax^m + b (\ln (cx^n))^q\right)^3}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,ab^2x^m \log (cx^n)^{2\,q} \log (cx^n)^{q-1} + 3\,a^2bx^{2\,m} \log (cx^n)^{q-1} \log (cx^n)^q + a^3x^{3\,m} \log (cx^n)^{q-1} + b^3 \log (cx^n)^{3\,q} \log (cx}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="fricas
")
```

```
[Out] integral((3*a*b^2*x^m*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1) + 3*a^2*b*x^(2*m)
*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^3*x^(3*m)*log(c*x^n)^(q - 1) + b^3*log
(c*x^n)^(3*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^3 \log\left(cx^n\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*log(c*x^n)^(q - 1)/x, x)
```

3.3 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=156

$$-\frac{a^2 2^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \operatorname{Gamma}\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{2abx^m (cx^n)^{-\frac{m}{n}} \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-2q} \operatorname{Gam}}{n}$$

```
[Out] (b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)]
*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x^(
2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n)*(
-((m*Log[c*x^n])/n))^q)
```

Rubi [A] time = 0.273688, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2539, 2310, 2181, 2302, 30}

$$-\frac{a^2 2^{-q} x^{2m} (cx^n)^{-\frac{2m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} \operatorname{Gamma}\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{2abx^m (cx^n)^{-\frac{m}{n}} \log^{2q}(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-2q} \operatorname{Gam}}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```


```
[Out] (b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)]
*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x^(
2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n)*(
-((m*Log[c*x^n])/n))^q)
```


Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
```

```
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = \int \left(a^2x^{-1+2m}\log^{-1+q}(cx^n)+2abx^{-1+m}\log^{-1+2q}(cx^n)+\frac{b^2\log^{-1+3q}(cx^n)}{x}\right) d$$

$$= a^2\int x^{-1+2m}\log^{-1+q}(cx^n)\,dx+(2ab)\int x^{-1+m}\log^{-1+2q}(cx^n)\,dx+b^2\int \frac{\log}{}$$

$$= \frac{b^2\,\text{Subst}\left(\int x^{-1+3q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\right)\text{Subst}\left(\int e^{\frac{2mx}{n}}x^{-1+q}\right.}{n}$$

$$= \frac{b^2\log^{3q}(cx^n)}{3nq}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n}$$

Mathematica [A] time = 0.413649, size = 149, normalized size = 0.96

$$\frac{\log^q(cx^n)\left(-3a^22^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\text{Gamma}\left(q,-\frac{2m\log(cx^n)}{n}\right)-6abx^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\text{Ga}\right.}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^2*Log[c*x^n]^(2*q))/q - (6*a*b*x^m*Gamma[2*q, -((m*Log[c*
x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (3*a^
2*x^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n])/(2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q)))/(3*n)
```

Maple [F] time = 4.449, size = 0, normalized size = 0.

$$\int \frac{(\ln (cx^n))^{-1+q} \left(ax^m + b (\ln (cx^n))^q\right)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\, abx^m \log (cx^n)^{q-1} \log (cx^n)^q + a^2 x^{2\,m} \log (cx^n)^{q-1} + b^2 \log (cx^n)^{2\,q} \log (cx^n)^{q-1}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*x^m*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^2*x^(2*m)*log(c*x^n
)^(q - 1) + b^2*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^2 \log\left(cx^n\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*log(c*x^n)^(q - 1)/x, x)
```

3.4 $\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=81

$$\frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\operatorname{Gamma}\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

```
[Out] (b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x
^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rubi [A] time = 0.157463, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2539, 2310, 2181, 2302, 30}

$$\frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\operatorname{Gamma}\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x
^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx &= \int\left(ax^{-1+m}\log^{-1+q}(cx^n)+\frac{b\log^{-1+2q}(cx^n)}{x}\right)dx\\
&= a\int x^{-1+m}\log^{-1+q}(cx^n)\,dx+b\int\frac{\log^{-1+2q}(cx^n)}{x}\,dx\\
&= \frac{b\,\mathrm{Subst}\left(\int x^{-1+2q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(ax^m(cx^n)^{-\frac{m}{n}}\right)\mathrm{Subst}\left(\int e^{\frac{mx}{n}}x^{-1+q}\,dx,x\right.}{n}\\
&= \frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}
\end{aligned}$$

Mathematica [A] time = 0.155532, size = 77, normalized size = 0.95

$$\frac{\log^q(cx^n)\left(\frac{b\log^q(cx^n)}{q}-2ax^m(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\mathrm{Gamma}\left(q,-\frac{m\log(cx^n)}{n}\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (Log[c*x^n]^q*((b*Log[c*x^n]^q)/q - (2*a*x^m*Gamma[q, -((m*Log[c*x^n])/n)])
/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)))/(2*n)
```

Maple [F] time = 3.949, size = 0, normalized size = 0.

$$\int \frac{(\ln(cx^n))^{-1+q} \left(ax^m + b(\ln(cx^n))^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ax^m \log(cx^n)^{q-1} + b\log(cx^n)^{q-1}\log(cx^n)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^m*log(c*x^n)^(q - 1) + b*log(c*x^n)^(q - 1)*log(c*x^n)^q)/x,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)\log\left(cx^n\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*log(c*x^n)^(q - 1)/x, x)
```

3.5 $\int \frac{\log^{-1+q}(cx^n)}{x} dx$

Optimal. Leaf size=15

$$\frac{\log^q(cx^n)}{nq}$$

[Out] Log[c*x^n]^q/(n*q)

Rubi [A] time = 0.0217537, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 30}

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

[In] Int[Log[c*x^n]^(-1 + q)/x,x]

[Out] Log[c*x^n]^q/(n*q)

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^{-1+q}(cx^n)}{x} dx &= \frac{\text{Subst}\left(\int x^{-1+q} dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\log^q(cx^n)}{nq}\end{aligned}$$

Mathematica [A] time = 0.0020717, size = 15, normalized size = 1.

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

Maple [A] time = 0.003, size = 16, normalized size = 1.1

$$\frac{(\ln(cx^n))^q}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x,x)
```

```
[Out] ln(c*x^n)^q/n/q
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94719, size = 74, normalized size = 4.93

$$\frac{(n\log(x) + \log(c))(n\log(x) + \log(c))^{q-1}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c))*(n*log(x) + log(c))^(q - 1)/(n*q)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(cx^{n}\right)^{q-1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x,x)
```

```
[Out] Integral(log(c*x**n)**(q - 1)/x, x)
```

Giac [A] time = 1.30102, size = 22, normalized size = 1.47

$$\frac{\left(n \log \left(x\right) + \log \left(c\right)\right)^{q}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(c))^q/(n*q)
```

3.6 $\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=67

$$\frac{\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{ax^m+b\log^q(cx^n)},x\right)}{bnq}$$

```
[Out] -((a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q), x])/(b*n*q)) +
Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q)
```

Rubi [A] time = 0.244673, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^q), x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\frac{\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx}{bnq}$$

Mathematica [A] time = 0.126489, size = 0, normalized size = 0.

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [A] time = 8.273, size = 0, normalized size = 0.

$$\int \frac{(\ln (c x^n))^{-1+q}}{x \left(a x^m + b (\ln (c x^n))^q\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-a \int \frac{x^m}{abxx^m \log (c) + abxx^m \log (x^n) + \left(b^2 x \log (c) + b^2 x \log (x^n)\right)(\log (c) + \log (x^n))^q}\, dx + \frac{\log (\log (c) + \log (x^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="maxima")
```

```
[Out] -a*integrate(x^m/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b
^2*x*log(x^n))*(log(c) + log(x^n))^q), x) + log(log(c) + log(x^n))/(b*n)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log (c x^n)^{q-1}}{a x x^m + b x \log (c x^n)^q}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log (cx^n)^{q-1}}{\left(ax^m + b\log (cx^n)^q\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

3.7 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=69

$$-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{(ax^m+b\log^q(cx^n))^2},x\right)}{bnq}-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}$$

```
[Out] -((a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2, x])/(b*n*q))
- 1/(b*n*q*(a*x^m + b*Log[c*x^n]^q))
```

Rubi [A] time = 0.288923, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(1/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^q)^2, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=-\frac{1}{bnq\left(ax^m+b\log^q(cx^n)\right)}-\frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2}\,dx}{bnq}$$

Mathematica [A] time = 0.665371, size = 0, normalized size = 0.

$$\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2), x]
```

Maple [A] time = 6.506, size = 0, normalized size = 0.

$$\int \frac{(\ln (cx^n))^{-1+q}}{x\left(ax^m + b(\ln (cx^n))^q\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{1}{abmx^m \log (x^n) - \left(nq - m\log (c)\right)abx^m + \left(b^2 m \log (x^n) - \left(nq - m\log (c)\right)b^2\right)(\log (c) + \log (x^n))^q} + \int -\frac{}{abm^2xx^m \log (x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="maxima
")
```

```
[Out] 1/(a*b*m*x^m*log(x^n) - (n*q - m*log(c))*a*b*x^m + (b^2*m*log(x^n) - (n*q -
 m*log(c))*b^2)*(log(c) + log(x^n))^q) + integrate(-(m*n*(q - 1) - m^2*log(
c) - m^2*log(x^n))/(a*b*m^2*x*x^m*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*a*b*x
*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b*x*x^m + (b^2*
m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n
*q*log(c) + m^2*log(c)^2)*b^2*x)*(log(c) + log(x^n))^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(cx^{n}\right)^{q-1}}{2\,abxx^{m}\log\left(cx^{n}\right)^{q}+a^{2}xx^{2\,m}+b^{2}x\log\left(cx^{n}\right)^{2\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x^(2*m) + b^2
*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

3.8 $\int \frac{\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=71

$$-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q(cx^n)\right)^3},x\right)}{bnq}-\frac{1}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] -((a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3, x])/(b*n*q))
- 1/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2)
```

Rubi [A] time = 0.245109, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m
 + b*Log[c*x^n]^q)^3, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q(cx^n)\right)^3}\,dx}{bnq}$$

Mathematica [A] time = 1.47245, size = 0, normalized size = 0.

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3), x]
```

Maple [A] time = 49.67, size = 0, normalized size = 0.

$$\int \frac{(\ln (c x^n))^{-1+q}}{x \left(a x^m + b (\ln (c x^n))^q\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a*m^2*x^m*log(x^n)^2 + (2*m^2*log(c) + m*n)*a*x^m*log(x^n) - (n^2*q^2
 - m^2*log(c)^2 - m*n*log(c))*a*x^m + (2*b*m^2*log(x^n)^2 - (m*n*(2*q - 1)
- 4*m^2*log(c))*b*log(x^n) - (m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*b)*(lo
g(c) + log(x^n))^q)/(a^3*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c)
)*a^3*b*x^(3*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2
)*a^3*b*x^(3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)
^2 - m^3*log(c)^3)*a^3*b*x^(3*m) + (a*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q -
 m^3*log(c))*a*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*l
og(c)^2)*a*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log
(c)^2 - m^3*log(c)^3)*a*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^2*b^2*m^3
*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^2*x^(2*m)*log(x^n)^2 +
 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^2*x^(2*m)*log(x^n) -
```

```
 (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^2*b^2
*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(m^3*n*(2*q - 3)*log(c)^2
 - 2*m^4*log(c)^3 - 2*m^4*log(x^n)^3 + 2*(q^2 - 1)*m^2*n^2*log(c) - (2*q^3
- 3*q^2 + q)*m*n^3 + (m^3*n*(2*q - 3) - 6*m^4*log(c))*log(x^n)^2 + 2*(m^3*n
*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 - 1)*m^2*n^2)*log(x^n))/(a^2*b*m^
4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b*x*x^(2*m)*log(x^n)^
3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b*x*x^(2*m)*log(x
^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(
c)^3)*a^2*b*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*
q^2*log(c)^2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b*x*x^(2*m) + (a*b^2*
m^4*x*x^m*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a*b^2*x*x^m*log(x^n)^3 + 6*
(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a*b^2*x*x^m*log(x^n)^2 - 4*
(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a*b^
2*x*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 -
 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(cx^{n}\right)^{q-1}}{3\,ab^{2}xx^{m}\log\left(cx^{n}\right)^{2\,q}+3\,a^{2}bxx^{2\,m}\log\left(cx^{n}\right)^{q}+a^{3}xx^{3\,m}+b^{3}x\log\left(cx^{n}\right)^{3\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3*a^2*b*x*x^(
2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log (cx^n)^{q-1}}{\left(ax^m + b\log (cx^n)^q\right)^3 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

3.9 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=272

$$\frac{9a^2bn^2x^{2m}\log\left(cx^n\right)}{4m^3}-\frac{9a^2bnx^{2m}\log^2\left(cx^n\right)}{4m^2}+\frac{3a^2bx^{2m}\log^3\left(cx^n\right)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{a^3x^{3m}\log\left(cx^n\right)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{60ab^2n^2}{}$$

```
[Out] (-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))/
(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c*x
^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c*x^
n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^m*Lo
g[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x^m*Lo
g[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Rubi [A] time = 0.305071, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2539, 2304, 2305, 2302, 30}

$$\frac{9a^2bn^2x^{2m}\log\left(cx^n\right)}{4m^3}-\frac{9a^2bnx^{2m}\log^2\left(cx^n\right)}{4m^2}+\frac{3a^2bx^{2m}\log^3\left(cx^n\right)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{a^3x^{3m}\log\left(cx^n\right)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{60ab^2n^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```


```
[Out] (-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))/
(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c*x
^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c*x^
n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^m*Lo
g[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x^m*Lo
g[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```


Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log(cx^n)\left(ax^m + b\log^2(cx^n)\right)^3}{x}\,dx = \int\left(a^3x^{-1+3m}\log(cx^n) + 3a^2bx^{-1+2m}\log^3(cx^n) + 3ab^2x^{-1+m}\log^5(cx^n) + \frac{b^3\log^7(\ldots}{x}\ldots\right.$$

$$= a^3\int x^{-1+3m}\log(cx^n)\,dx + \left(3a^2b\right)\int x^{-1+2m}\log^3(cx^n)\,dx + \left(3ab^2\right)\int x^{-1+m}\log^5\ldots$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} + \frac{3ab^2x^m\log^5(cx^n)}{m} + \frac{b^3\,\mathrm{Subs}\ldots}{\ldots}$$

$$= -\frac{a^3nx^{3m}}{9m^2} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2} + \frac{3a^2bx^{2m}\log^3(cx^n)}{2m} - \frac{15ab^2\ldots}{\ldots}$$

$$= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{9a^2bnx^{2m}\log\ldots}{4m^2}$$

$$= -\frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3} + \frac{a^3x^{3m}\log(cx^n)}{3m} - \frac{180ab^2n^3x^m\,\mathrm{l}\ldots}{m^4}$$

$$= -\frac{360ab^2n^5x^m}{m^6} - \frac{9a^2bn^3x^{2m}}{8m^4} - \frac{a^3nx^{3m}}{9m^2} + \frac{360ab^2n^4x^m\log(cx^n)}{m^5} + \frac{9a^2bn^2x^{2m}\log\ldots}{4m^3}$$

Mathematica [A] time = 0.223594, size = 230, normalized size = 0.85

$$\frac{ax^m\log(cx^n)\left(4a^2m^4x^{2m}+27abm^2n^2x^m+4320b^2n^4\right)}{12m^5}-\frac{anx^m\left(8a^2m^4x^{2m}+81abm^2n^2x^m+25920b^2n^4\right)}{72m^6}-\frac{15ab^2nx^m\text{lo}}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] -(a*n*x^m*(25920*b^2*n^4 + 81*a*b*m^2*n^2*x^m + 8*a^2*m^4*x^(2*m)))/(72*m^6
) + (a*x^m*(4320*b^2*n^4 + 27*a*b*m^2*n^2*x^m + 4*a^2*m^4*x^(2*m))*Log[c*x^
n])/(12*m^5) - (9*a*b*n*x^m*(80*b*n^2 + a*m^2*x^m)*Log[c*x^n]^2)/(4*m^4) +
(3*a*b*x^m*(40*b*n^2 + a*m^2*x^m)*Log[c*x^n]^3)/(2*m^3) - (15*a*b^2*n*x^m*L
og[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Maple [C] time = 5.05, size = 61910, normalized size = 227.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^3/x,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16194, size = 1558, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/72*(9*b^3*m^6*n^7*log(x)^8 + 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m^6
*n^5*log(c)^2*log(x)^6 + 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*n^
3*log(c)^4*log(x)^4 + 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*log
(c)^6*log(x)^2 + 72*b^3*m^6*log(c)^7*log(x) + 8*(3*a^3*m^5*n*log(x) + 3*a^3
*m^5*log(c) - a^3*m^4*n)*x^(3*m) + 27*(4*a^2*b*m^5*n^3*log(x)^3 + 4*a^2*b*m
^5*log(c)^3 - 6*a^2*b*m^4*n*log(c)^2 + 6*a^2*b*m^3*n^2*log(c) - 3*a^2*b*m^2
*n^3 + 6*(2*a^2*b*m^5*n^2*log(c) - a^2*b*m^4*n^3)*log(x)^2 + 6*(2*a^2*b*m^5
*n*log(c)^2 - 2*a^2*b*m^4*n^2*log(c) + a^2*b*m^3*n^3)*log(x))*x^(2*m) + 216
*(a*b^2*m^5*n^5*log(x)^5 + a*b^2*m^5*log(c)^5 - 5*a*b^2*m^4*n*log(c)^4 + 20
*a*b^2*m^3*n^2*log(c)^3 - 60*a*b^2*m^2*n^3*log(c)^2 + 120*a*b^2*m*n^4*log(c
) - 120*a*b^2*n^5 + 5*(a*b^2*m^5*n^4*log(c) - a*b^2*m^4*n^5)*log(x)^4 + 10*
(a*b^2*m^5*n^3*log(c)^2 - 2*a*b^2*m^4*n^4*log(c) + 2*a*b^2*m^3*n^5)*log(x)^
3 + 10*(a*b^2*m^5*n^2*log(c)^3 - 3*a*b^2*m^4*n^3*log(c)^2 + 6*a*b^2*m^3*n^4
*log(c) - 6*a*b^2*m^2*n^5)*log(x)^2 + 5*(a*b^2*m^5*n*log(c)^4 - 4*a*b^2*m^4
*n^2*log(c)^3 + 12*a*b^2*m^3*n^3*log(c)^2 - 24*a*b^2*m^2*n^4*log(c) + 24*a*
b^2*m*n^5)*log(x))*x^m)/m^6
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**3/x,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.34126, size = 1034, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/8*b^3*n^7*log(x)^8 + b^3*n^6*log(c)*log(x)^7 + 7/2*b^3*n^5*log(c)^2*log(x
)^6 + 7*b^3*n^4*log(c)^3*log(x)^5 + 35/4*b^3*n^3*log(c)^4*log(x)^4 + 7*b^3*
n^2*log(c)^5*log(x)^3 + 3*a*b^2*n^5*x^m*log(x)^5/m + 7/2*b^3*n*log(c)^6*log
(x)^2 + 15*a*b^2*n^4*x^m*log(c)*log(x)^4/m + b^3*log(c)^7*log(x) + 30*a*b^2
*n^3*x^m*log(c)^2*log(x)^3/m - 15*a*b^2*n^5*x^m*log(x)^4/m^2 + 30*a*b^2*n^2
*x^m*log(c)^3*log(x)^2/m - 60*a*b^2*n^4*x^m*log(c)*log(x)^3/m^2 + 15*a*b^2*
n*x^m*log(c)^4*log(x)/m - 90*a*b^2*n^3*x^m*log(c)^2*log(x)^2/m^2 + 3/2*a^2*
b*n^3*x^(2*m)*log(x)^3/m + 60*a*b^2*n^5*x^m*log(x)^3/m^3 + 3*a*b^2*x^m*log(
c)^5/m - 60*a*b^2*n^2*x^m*log(c)^3*log(x)/m^2 + 9/2*a^2*b*n^2*x^(2*m)*log(c
)*log(x)^2/m + 180*a*b^2*n^4*x^m*log(c)*log(x)^2/m^3 - 15*a*b^2*n*x^m*log(c
)^4/m^2 + 9/2*a^2*b*n*x^(2*m)*log(c)^2*log(x)/m + 180*a*b^2*n^3*x^m*log(c)^
2*log(x)/m^3 - 9/4*a^2*b*n^3*x^(2*m)*log(x)^2/m^2 - 180*a*b^2*n^5*x^m*log(x
)^2/m^4 + 3/2*a^2*b*x^(2*m)*log(c)^3/m + 60*a*b^2*n^2*x^m*log(c)^3/m^3 - 9/
2*a^2*b*n^2*x^(2*m)*log(c)*log(x)/m^2 - 360*a*b^2*n^4*x^m*log(c)*log(x)/m^4
 - 9/4*a^2*b*n*x^(2*m)*log(c)^2/m^2 - 180*a*b^2*n^3*x^m*log(c)^2/m^4 + 1/3*
a^3*n*x^(3*m)*log(x)/m + 9/4*a^2*b*n^3*x^(2*m)*log(x)/m^3 + 360*a*b^2*n^5*x
^m*log(x)/m^5 + 1/3*a^3*x^(3*m)*log(c)/m + 9/4*a^2*b*n^2*x^(2*m)*log(c)/m^3
 + 360*a*b^2*n^4*x^m*log(c)/m^5 - 1/9*a^3*n*x^(3*m)/m^2 - 9/8*a^2*b*n^3*x^(
2*m)/m^4 - 360*a*b^2*n^5*x^m/m^6
```

3.10 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=125

$$\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

```
[Out] (-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^n
])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 +
(2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rubi [A] time = 0.165895, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2539, 2304, 2305, 2302, 30}

$$\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] (-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^n
])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 +
(2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx = \int\left(a^2x^{-1+2m}\log(cx^n)+2abx^{-1+m}\log^3(cx^n)+\frac{b^2\log^5(cx^n)}{x}\right)dx$$
$$= a^2\int x^{-1+2m}\log(cx^n)\,dx+(2ab)\int x^{-1+m}\log^3(cx^n)\,dx+b^2\int\frac{\log^5(cx^n)}{x}\,dx$$
$$= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}+\frac{2abx^m\log^3(cx^n)}{m}+\frac{b^2\,\mathrm{Subst}\left(\int x^5\,dx,x,\log(cx^n\right.}{n}$$
$$= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}+\frac{b^2\log^6(}{6n}$$
$$= -\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx^m\log^2(}{m^2}$$

Mathematica [A] time = 0.104551, size = 115, normalized size = 0.92

$$\frac{ax^m\log(cx^n)\left(am^2x^m+24bn^2\right)}{2m^3}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx^n)}{m}-\frac{anx^m\left(am^2x^m+48bn^2\right)}{4m^4}+\frac{b^2\log^6(cx^n)}{6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] -(a*n*x^m*(48*b*n^2 + a*m^2*x^m))/(4*m^4) + (a*x^m*(24*b*n^2 + a*m^2*x^m)*L
og[c*x^n])/(2*m^3) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 + (2*a*b*x^m*Log[c*x^n]
```

```
^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Maple [C] time = 1.375, size = 14983, normalized size = 119.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^2/x,x)

[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.85593, size = 659, normalized size = 5.27

$$\frac{2\,b^2m^4n^5\log(x)^6+12\,b^2m^4n^4\log(c)\log(x)^5+30\,b^2m^4n^3\log(c)^2\log(x)^4+40\,b^2m^4n^2\log(c)^3\log(x)^3+30\,b^2m^4n\log(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="fricas")

[Out] 1/12*(2*b^2*m^4*n^5*log(x)^6 + 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*
n^3*log(c)^2*log(x)^4 + 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log
(c)^4*log(x)^2 + 12*b^2*m^4*log(c)^5*log(x) + 3*(2*a^2*m^3*n*log(x) + 2*a^2
*m^3*log(c) - a^2*m^2*n)*x^(2*m) + 24*(a*b*m^3*n^3*log(x)^3 + a*b*m^3*log(c
)^3 - 3*a*b*m^2*n*log(c)^2 + 6*a*b*m*n^2*log(c) - 6*a*b*n^3 + 3*(a*b*m^3*n^
2*log(c) - a*b*m^2*n^3)*log(x)^2 + 3*(a*b*m^3*n*log(c)^2 - 2*a*b*m^2*n^2*lo
```

```
g(c) + 2*a*b*m*n^3)*log(x))*x^m)/m^4
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**2/x,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.65729, size = 386, normalized size = 3.09

$$\frac{1}{6}\,b^2n^5\log\left(x\right)^6+b^2n^4\log\left(c\right)\log\left(x\right)^5+\frac{5}{2}\,b^2n^3\log\left(c\right)^2\log\left(x\right)^4+\frac{10}{3}\,b^2n^2\log\left(c\right)^3\log\left(x\right)^3+\frac{5}{2}\,b^2n\log\left(c\right)^4\log\left(x\right)^2+b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*n^5*log(x)^6 + b^2*n^4*log(c)*log(x)^5 + 5/2*b^2*n^3*log(c)^2*log(x
)^4 + 10/3*b^2*n^2*log(c)^3*log(x)^3 + 5/2*b^2*n*log(c)^4*log(x)^2 + b^2*lo
g(c)^5*log(x) + 2*a*b*n^3*x^m*log(x)^3/m + 6*a*b*n^2*x^m*log(c)*log(x)^2/m
+ 6*a*b*n*x^m*log(c)^2*log(x)/m - 6*a*b*n^3*x^m*log(x)^2/m^2 + 2*a*b*x^m*lo
g(c)^3/m - 12*a*b*n^2*x^m*log(c)*log(x)/m^2 - 6*a*b*n*x^m*log(c)^2/m^2 + 1/
2*a^2*n*x^(2*m)*log(x)/m + 12*a*b*n^3*x^m*log(x)/m^3 + 1/2*a^2*x^(2*m)*log(
c)/m + 12*a*b*n^2*x^m*log(c)/m^3 - 1/4*a^2*n*x^(2*m)/m^2 - 12*a*b*n^3*x^m/m
^4
```

3.11 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=41

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rubi [A] time = 0.0771009, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2539, 2304, 2302, 30}

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rule 2539

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx &= \int \left(ax^{-1+m}\log(cx^n)+\frac{b\log^3(cx^n)}{x}\right)dx \\
&= a\int x^{-1+m}\log(cx^n)\,dx+b\int \frac{\log^3(cx^n)}{x}\,dx \\
&= -\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\,\mathrm{Subst}\left(\int x^3\,dx,x,\log(cx^n)\right)}{n} \\
&= -\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\log^4(cx^n)}{4n}
\end{aligned}
$$

Mathematica [A] time = 0.0334815, size = 41, normalized size = 1.

$$
\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Maple [C] time = 0.303, size = 2146, normalized size = 52.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)/x,x)
```

```
[Out] -a*n*x^m/m^2+(-3/2*b*n*ln(x)^2-3/2*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^
n)*ln(x)+3/2*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(x)+3/2*I*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)^2*ln(x)-3/2*I*Pi*b*csgn(I*c*x^n)^3*ln(x)+3*ln(c)*b*ln(x))*ln(
```

x^n)^2-1/2*I/m*Pi*a*csgn(I*c*x^n)^3*x^m+ln(c)^3*ln(x)*b-1/4*b*n^3*ln(x)^4+1
/4*(-3*Pi^2*b*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*ln(x)*m+6*Pi^2*b*cs
gn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3*ln(x)*m-3*Pi^2*b*csgn(I*c)^2*csgn(I*c
*x^n)^4*ln(x)*m+6*Pi^2*b*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3*ln(x)*m-12
*Pi^2*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4*ln(x)*m+6*Pi^2*b*csgn(I*c)*cs
gn(I*c*x^n)^5*ln(x)*m-3*Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4*ln(x)*m+6*Pi^2
*b*csgn(I*x^n)*csgn(I*c*x^n)^5*ln(x)*m-3*Pi^2*b*csgn(I*c*x^n)^6*ln(x)*m-12*
I*ln(c)*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(x)*m+6*I*ln(x)^2*Pi*b*n
*csgn(I*c*x^n)^3*m-12*I*ln(c)*Pi*b*csgn(I*c*x^n)^3*ln(x)*m-6*I*ln(x)^2*Pi*b
*n*csgn(I*c)*csgn(I*c*x^n)^2*m+12*I*ln(c)*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*
ln(x)*m+6*I*ln(x)^2*Pi*b*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*m-6*I*ln(x)^
2*Pi*b*n*csgn(I*x^n)*csgn(I*c*x^n)^2*m+12*I*ln(c)*Pi*b*csgn(I*c)*csgn(I*c*x
^n)^2*ln(x)*m+4*b*n^2*ln(x)^3*m-12*ln(x)^2*ln(c)*b*n*m+12*ln(c)^2*b*ln(x)*m
+4*a*x^m)/m*ln(x^n)+1/m*ln(c)*a*x^m-1/2*I/m*Pi*a*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)*x^m-1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-3/
2*I*ln(x)^2*ln(c)*Pi*b*n*csgn(I*c)*csgn(I*c*x^n)^2-3/2*I*ln(x)^2*ln(c)*Pi*b
*n*csgn(I*x^n)*csgn(I*c*x^n)^2-3/2*I*ln(c)^2*Pi*b*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)*ln(x)+b*ln(x)*ln(x^n)^3+1/8*I*ln(x)*csgn(I*c*x^n)^9*b*Pi^3+3/2*c
sgn(I*c*x^n)^4*csgn(I*x^n)*csgn(I*c)*n*b*Pi^2*ln(x)^2-3/4*ln(x)*csgn(I*c*x^
n)^2*csgn(I*x^n)^2*csgn(I*c)^2*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c*x^n)^3*csgn(
I*x^n)*csgn(I*c)^2*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c*x^n)^3*csgn(I*x^n)^2*csg
n(I*c)*b*Pi^2*ln(c)-3*ln(x)*csgn(I*c*x^n)^4*csgn(I*x^n)*csgn(I*c)*b*Pi^2*ln
(c)+3/8*csgn(I*c*x^n)^2*csgn(I*x^n)^2*csgn(I*c)^2*n*b*Pi^2*ln(x)^2-3/4*csgn
(I*c*x^n)^3*csgn(I*x^n)*csgn(I*c)^2*n*b*Pi^2*ln(x)^2+ln(x)^3*ln(c)*b*n^2-3/
2*ln(x)^2*ln(c)^2*b*n+1/2*I/m*Pi*a*csgn(I*c)*csgn(I*c*x^n)^2*x^m+1/2*I/m*Pi
*a*csgn(I*x^n)*csgn(I*c*x^n)^2*x^m+3/8*I*Pi^3*b*csgn(I*c)^3*csgn(I*x^n)*csg
n(I*c*x^n)^5*ln(x)-3/8*I*Pi^3*b*csgn(I*c)^2*csgn(I*x^n)^3*csgn(I*c*x^n)^4*l
n(x)+9/8*I*Pi^3*b*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^5*ln(x)-9/8*I*Pi^
3*b*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*c)*cs
gn(I*x^n)^3*csgn(I*c*x^n)^5*ln(x)-9/8*I*Pi^3*b*csgn(I*c)*csgn(I*x^n)^2*csgn
(I*c*x^n)^6*ln(x)+9/8*I*Pi^3*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^7*ln(x)+
1/2*I*ln(x)^3*Pi*b*n^2*csgn(I*c)*csgn(I*c*x^n)^2+1/2*I*ln(x)^3*Pi*b*n^2*csg
n(I*x^n)*csgn(I*c*x^n)^2+3/2*I*ln(x)^2*ln(c)*Pi*b*n*csgn(I*c*x^n)^3+3/2*I*l
n(c)^2*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(x)+3/2*I*ln(c)^2*Pi*b*csgn(I*x^n)*
csgn(I*c*x^n)^2*ln(x)+1/8*I*ln(x)*csgn(I*c*x^n)^3*csgn(I*x^n)^3*csgn(I*c)^3
*b*Pi^3-3/8*I*Pi^3*b*csgn(I*c)^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4*ln(x)-3/4*cs
gn(I*c*x^n)^5*csgn(I*c)*n*b*Pi^2*ln(x)^2+3/8*csgn(I*c*x^n)^4*csgn(I*x^n)^2*
n*b*Pi^2*ln(x)^2-3/4*csgn(I*c*x^n)^5*csgn(I*x^n)*n*b*Pi^2*ln(x)^2-3/4*ln(x)
*csgn(I*c*x^n)^4*csgn(I*c)^2*b*Pi^2*ln(c)+3/2*ln(x)*csgn(I*c*x^n)^5*csgn(I*
c)*b*Pi^2*ln(c)-3/4*ln(x)*csgn(I*c*x^n)^4*csgn(I*x^n)^2*b*Pi^2*ln(c)+3/2*ln
(x)*csgn(I*c*x^n)^5*csgn(I*x^n)*b*Pi^2*ln(c)+3/8*csgn(I*c*x^n)^4*csgn(I*c)^
2*n*b*Pi^2*ln(x)^2-3/4*csgn(I*c*x^n)^3*csgn(I*x^n)^2*csgn(I*c)*n*b*Pi^2*ln(
x)^2+3/2*I*ln(x)^2*ln(c)*Pi*b*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3/8*csg
n(I*c*x^n)^6*n*b*Pi^2*ln(x)^2-3/4*ln(x)*csgn(I*c*x^n)^6*b*Pi^2*ln(c)-1/2*I*
ln(x)^3*Pi*b*n^2*csgn(I*c*x^n)^3-3/2*I*ln(c)^2*Pi*b*csgn(I*c*x^n)^3*ln(x)-1

```
/8*I*Pi^3*b*csgn(I*c)^3*csgn(I*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*c)^2*csgn
(I*c*x^n)^7*ln(x)-3/8*I*Pi^3*b*csgn(I*c)*csgn(I*c*x^n)^8*ln(x)-1/8*I*Pi^3*b
*csgn(I*x^n)^3*csgn(I*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*x^n)^2*csgn(I*c*x^
n)^7*ln(x)-3/8*I*Pi^3*b*csgn(I*x^n)*csgn(I*c*x^n)^8*ln(x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95644, size = 213, normalized size = 5.2

$$\frac{bm^2n^3\log(x)^4 + 4\,bm^2n^2\log(c)\log(x)^3 + 6\,bm^2n\log(c)^2\log(x)^2 + 4\,bm^2\log(c)^3\log(x) + 4\,(amn\log(x) + am\log(c}{4\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(b*m^2*n^3*log(x)^4 + 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*
log(x)^2 + 4*b*m^2*log(c)^3*log(x) + 4*(a*m*n*log(x) + a*m*log(c) - a*n)*x^
m)/m^2
```

Sympy [A] time = 10.1664, size = 68, normalized size = 1.66

$$-an\left(\begin{cases}\frac{\begin{cases}\frac{x^m}{m} & \text{for}\ m \neq 0\\ \log(x) & \text{otherwise}\end{cases}}{m} & \text{for}\ m > -\infty \wedge m < \infty \wedge m \neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right) + a\left(\begin{cases}\frac{x^m}{m} & \text{for}\ m-1 \neq -1\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n) - b\left(\begin{cases}-\log\\ -\frac{\log(c}{4n}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)/x,x)
```

```
[Out] -a*n*Piecewise((Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/m, (m > -oo)
& (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a*Piecewise((x**m/m, Ne(m -
1, -1)), (log(x), True))*log(c*x**n) - b*Piecewise((-log(c)**3*log(x), Eq(n
, 0)), (-log(c*x**n)**4/(4*n), True))
```

Giac [A] time = 1.27757, size = 99, normalized size = 2.41

$$\frac{1}{4}bn^3\log(x)^4 + bn^2\log(c)\log(x)^3 + \frac{3}{2}bn\log(c)^2\log(x)^2 + b\log(c)^3\log(x) + \frac{anx^m\log(x)}{m} + \frac{ax^m\log(c)}{m} - \frac{anx^m}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*n^3*log(x)^4 + b*n^2*log(c)*log(x)^3 + 3/2*b*n*log(c)^2*log(x)^2 + b*
log(c)^3*log(x) + a*n*x^m*log(x)/m + a*x^m*log(c)/m - a*n*x^m/m^2
```

3.12 $\int \frac{\log(cx^n)}{x} dx$

Optimal. Leaf size=15

$$\frac{\log^2(cx^n)}{2n}$$

```
[Out] Log[c*x^n]^2/(2*n)
```

Rubi [A] time = 0.0074127, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2301}

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*x^n]/x,x]
```

```
[Out] Log[c*x^n]^2/(2*n)
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\log(cx^n)}{x} dx = \frac{\log^2(cx^n)}{2n}$$

Mathematica [A] time = 0.0008967, size = 15, normalized size = 1.

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*x^n]/x,x]
```

```
[Out] Log[c*x^n]^2/(2*n)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{(\ln (cx^n))^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x,x)
```

```
[Out] 1/2*ln(c*x^n)^2/n
```

Maxima [A] time = 1.00267, size = 18, normalized size = 1.2

$$\frac{\log (cx^n)^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^n)^2/n
```

Fricas [A] time = 1.83902, size = 43, normalized size = 2.87

$$\frac{1}{2} n \log (x)^2 + \log (c) \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

Sympy [A] time = 1.68928, size = 51, normalized size = 3.4

$$\begin{cases} \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix} \middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix} \middle| cx^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x,x)
```

```
[Out] Piecewise((log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(x**(-n)/c)**2/(2*n)
, 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n
+ meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

Giac [A] time = 1.30343, size = 18, normalized size = 1.2

$$\frac{1}{2}\,n\log(x)^2 + \log(c)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

3.13 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$

Optimal. Leaf size=66

$$\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{ax^m+b\log^2(cx^n)},x\right)}{2bn}$$

```
[Out] -(a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^2), x])/(2*b*n) + Lo
g[a*x^m + b*Log[c*x^n]^2]/(2*b*n)
```

Rubi [A] time = 0.191758, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^2]/(2*b*n) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2), x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx=\frac{\log\left(ax^m+b\log^2(cx^n)\right)}{2bn}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^2(cx^n)}\,dx}{2bn}$$

Mathematica [A] time = 1.40854, size = 0, normalized size = 0.

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)), x]
```

Maple [A] time = 1.777, size = 0, normalized size = 0.

$$\int \frac{\ln(cx^n)}{x\left(ax^m + b(\ln(cx^n))^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log(cx^n)}{bx\log(cx^n)^2 + axx^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b*x*log(c*x^n)^2 + a*x*x^m), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log\left(cx^n\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2),x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(cx^n\right)}{\left(b\log\left(cx^n\right)^2+ax^m\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

3.14 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=67

$$-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^2},x\right)}{2bn}-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}$$

```
[Out] -(a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^2)^2, x])/(2*b*n) -
1/(2*b*n*(a*x^m + b*Log[c*x^n]^2))
```

Rubi [A] time = 0.196432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] -1/(2*b*n*(a*x^m + b*Log[c*x^n]^2)) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2)^2, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx=-\frac{1}{2bn\left(ax^m+b\log^2(cx^n)\right)}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2}\,dx}{2bn}$$

Mathematica [A] time = 1.56721, size = 0, normalized size = 0.

$$\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2), x]
```

Maple [A] time = 51.749, size = 0, normalized size = 0.

$$\int \frac{\ln(cx^n)}{x\left(ax^m + b(\ln(cx^n))^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{m\log(c) + m\log(x^n) + 2\,n}{4\,b^2n^2\log(c)^2 + a^2m^2x^{2\,m} + \left(m^2\log(c)^2 + 4\,n^2\right)abx^m + \left(abm^2x^m + 4\,b^2n^2\right)\log(x^n)^2 + 2\left(abm^2x^m\log(c) + 4\,b^2n^2\log(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="maxima")
```

```
[Out] -(m*log(c) + m*log(x^n) + 2*n)/(4*b^2*n^2*log(c)^2 + a^2*m^2*x^(2*m) + (m^2
*log(c)^2 + 4*n^2)*a*b*x^m + (a*b*m^2*x^m + 4*b^2*n^2)*log(x^n)^2 + 2*(a*b*
m^2*x^m*log(c) + 4*b^2*n^2*log(c))*log(x^n)) - integrate((a*m^4*x^m*log(x^n
) + 4*b*m*n^3 + (m^4*log(c) + 3*m^3*n)*a*x^m)/(16*b^3*n^4*x*log(c)^2 + a^3*
m^4*x*x^(3*m) + (m^4*log(c)^2 + 8*m^2*n^2)*a^2*b*x*x^(2*m) + 8*(m^2*n^2*log
(c)^2 + 2*n^4)*a*b^2*x*x^m + (a^2*b*m^4*x*x^(2*m) + 8*a*b^2*m^2*n^2*x*x^m +
 16*b^3*n^4*x)*log(x^n)^2 + 2*(a^2*b*m^4*x*x^(2*m)*log(c) + 8*a*b^2*m^2*n^2
*x*x^m*log(c) + 16*b^3*n^4*x*log(c))*log(x^n)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(cx^{n}\right)}{b^{2}x\log\left(cx^{n}\right)^{4}+2\,abxx^{m}\log\left(cx^{n}\right)^{2}+a^{2}xx^{2\,m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^2*x*log(c*x^n)^4 + 2*a*b*x*x^m*log(c*x^n)^2 + a^2*x*
x^(2*m)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(cx^{n}\right)}{x\left(ax^{m}+b\log\left(cx^{n}\right)^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**2,x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(cx^{n}\right)}{\left(b\log\left(cx^{n}\right)^{2}+ax^{m}\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^2*x), x)
```

3.15 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{am\text{CannotIntegrate}\left(\frac{x^{m-1}}{\left(ax^m+b\log^2(cx^n)\right)^3},x\right)}{2bn}-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}$$

```
[Out] -(a*m*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^2)^3, x])/(2*b*n) -
1/(4*b*n*(a*x^m + b*Log[c*x^n]^2)^2)
```

Rubi [A] time = 0.177491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] -1/(4*b*n*(a*x^m + b*Log[c*x^n]^2)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^2)^3, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx=-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^3}\,dx}{2bn}$$

Mathematica [A] time = 2.8723, size = 0, normalized size = 0.

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3), x]
```

Maple [A] time = 56.374, size = 0, normalized size = 0.

$$\int \frac{\ln(cx^n)}{x\left(ax^m + b(\ln(cx^n))^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(24*b^3*m*n^4*log(c)^3 - 5*a^3*m^4*n*x^(3*m) - (m^5*log(c)^3 + 7*m^4*n
*log(c)^2 - 18*m^3*n^2*log(c) - 4*m^2*n^3)*a^2*b*x^(2*m) + 2*(5*m^3*n^2*log
(c)^3 - 6*m^2*n^3*log(c)^2 + 20*m*n^4*log(c) + 16*n^5)*a*b^2*x^m - (a^2*b*m
^5*x^(2*m) - 10*a*b^2*m^3*n^2*x^m - 24*b^3*m*n^4)*log(x^n)^3 + (72*b^3*m*n^
4*log(c) - (3*m^5*log(c) + 7*m^4*n)*a^2*b*x^(2*m) + 6*(5*m^3*n^2*log(c) - 2
*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 + (72*b^3*m*n^4*log(c)^2 - (3*m^5*log(c)^2
+ 14*m^4*n*log(c) - 18*m^3*n^2)*a^2*b*x^(2*m) + 2*(15*m^3*n^2*log(c)^2 - 12
*m^2*n^3*log(c) + 20*m*n^4)*a*b^2*x^m)*log(x^n))/(64*a*b^5*n^6*x^m*log(c)^4
 + a^6*m^6*x^(6*m) + 2*(m^6*log(c)^2 + 6*m^4*n^2)*a^5*b*x^(5*m) + (m^6*log(
c)^4 + 24*m^4*n^2*log(c)^2 + 48*m^2*n^4)*a^4*b^2*x^(4*m) + 4*(3*m^4*n^2*log
(c)^4 + 24*m^2*n^4*log(c)^2 + 16*n^6)*a^3*b^3*x^(3*m) + 16*(3*m^2*n^4*log(c
)^4 + 8*n^6*log(c)^2)*a^2*b^4*x^(2*m) + (a^4*b^2*m^6*x^(4*m) + 12*a^3*b^3*m
^4*n^2*x^(3*m) + 48*a^2*b^4*m^2*n^4*x^(2*m) + 64*a*b^5*n^6*x^m)*log(x^n)^4
+ 4*(a^4*b^2*m^6*x^(4*m)*log(c) + 12*a^3*b^3*m^4*n^2*x^(3*m)*log(c) + 48*a^
```

```
2*b^4*m^2*n^4*x^(2*m)*log(c) + 64*a*b^5*n^6*x^m*log(c))*log(x^n)^3 + 2*(192
*a*b^5*n^6*x^m*log(c)^2 + a^5*b*m^6*x^(5*m) + 3*(m^6*log(c)^2 + 4*m^4*n^2)*
a^4*b^2*x^(4*m) + 12*(3*m^4*n^2*log(c)^2 + 4*m^2*n^4)*a^3*b^3*x^(3*m) + 16*
(9*m^2*n^4*log(c)^2 + 4*n^6)*a^2*b^4*x^(2*m))*log(x^n)^2 + 4*(64*a*b^5*n^6*
x^m*log(c)^3 + a^5*b*m^6*x^(5*m)*log(c) + (m^6*log(c)^3 + 12*m^4*n^2*log(c)
)*a^4*b^2*x^(4*m) + 12*(m^4*n^2*log(c)^3 + 4*m^2*n^4*log(c))*a^3*b^3*x^(3*m
) + 16*(3*m^2*n^4*log(c)^3 + 4*n^6*log(c))*a^2*b^4*x^(2*m))*log(x^n)) + int
egrate(1/2*((2*m^8*log(c) + 15*m^7*n)*a^3*x^(3*m) - 2*(17*m^6*n^2*log(c) -
m^5*n^3)*a^2*b*x^(2*m) - 32*(3*m^4*n^4*log(c) + 2*m^3*n^5)*a*b^2*x^m - 96*(
m^2*n^6*log(c) + m*n^7)*b^3 + 2*(a^3*m^8*x^(3*m) - 17*a^2*b*m^6*n^2*x^(2*m)
 - 48*a*b^2*m^4*n^4*x^m - 48*b^3*m^2*n^6)*log(x^n))/(256*a*b^5*n^8*x*x^m*lo
g(c)^2 + a^6*m^8*x*x^(6*m) + (m^8*log(c)^2 + 16*m^6*n^2)*a^5*b*x*x^(5*m) +
16*(m^6*n^2*log(c)^2 + 6*m^4*n^4)*a^4*b^2*x*x^(4*m) + 32*(3*m^4*n^4*log(c)^
2 + 8*m^2*n^6)*a^3*b^3*x*x^(3*m) + 256*(m^2*n^6*log(c)^2 + n^8)*a^2*b^4*x*x
^(2*m) + (a^5*b*m^8*x*x^(5*m) + 16*a^4*b^2*m^6*n^2*x*x^(4*m) + 96*a^3*b^3*m
^4*n^4*x*x^(3*m) + 256*a^2*b^4*m^2*n^6*x*x^(2*m) + 256*a*b^5*n^8*x*x^m)*log
(x^n)^2 + 2*(a^5*b*m^8*x*x^(5*m)*log(c) + 16*a^4*b^2*m^6*n^2*x*x^(4*m)*log(
c) + 96*a^3*b^3*m^4*n^4*x*x^(3*m)*log(c) + 256*a^2*b^4*m^2*n^6*x*x^(2*m)*lo
g(c) + 256*a*b^5*n^8*x*x^m*log(c))*log(x^n)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(cx^n\right)}{b^3x\log\left(cx^n\right)^6+3\,ab^2xx^m\log\left(cx^n\right)^4+3\,a^2bxx^{2\,m}\log\left(cx^n\right)^2+a^3xx^{3\,m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^3*x*log(c*x^n)^6 + 3*a*b^2*x*x^m*log(c*x^n)^4 + 3*a^
2*b*x*x^(2*m)*log(c*x^n)^2 + a^3*x*x^(3*m)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(cx^n)}{\left(b\log(cx^n)^2 + ax^m\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^3*x), x)
```

3.16 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=26

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{p+1}$$

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.171346, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$\frac{\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{\left(ax^m+b\log^q(cx^n)\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0893974, size = 26, normalized size = 1.

$$\frac{\left(ax^m + b\log^q\left(cx^n\right)\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{\left(amx^m + bnq\left(\ln\left(cx^n\right)\right)^{-1+q}\right)\left(ax^m + b\left(\ln\left(cx^n\right)\right)^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.99164, size = 107, normalized size = 4.12

$$\frac{\left((n\log(x) + \log(c))^q b + ax^m\right)\left((n\log(x) + \log(c))^q b + ax^m\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="fricas")
```

```
[Out] ((n*log(x) + log(c))^q*b + a*x^m)*((n*log(x) + log(c))^q*b + a*x^m)^p/(p +
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x
,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.17 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\left(ax^m+b\log^q(cx^n)\right)^3$$

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Rubi [A] time = 0.172719, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$\frac{1}{3}\left(ax^m+b\log^q(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx=\frac{1}{3}\left(ax^m+b\log^q(cx^n)\right)^3$$

Mathematica [A] time = 0.0381848, size = 22, normalized size = 1.

$$\frac{1}{3}\left(ax^m+b\log^q(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Maple [C] time = 0.197, size = 204, normalized size = 9.3

$$\frac{a^3 (x^m)^3}{3} + \frac{b^3 \left(\left(\ln(c) + \ln(x^n) - \frac{i}{2}\pi \operatorname{csgn}(icx^n)(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ic))(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ix^n))\right)^q\right)^3}{3} + ab^2x^m \left(\left(\ln(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] 1/3*a^3*(x^m)^3+1/3*b^3*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^3+a*b^2*x^m*((ln(c)+ln(x^n)
-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x
^n)))^q)^2+a^2*b*(x^m)^2*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05207, size = 174, normalized size = 7.91

$$(n\log(x) + \log(c))^q a^2bx^{2m} + (n\log(x) + \log(c))^{2q} ab^2x^m + \frac{1}{3}(n\log(x) + \log(c))^{3q} b^3 + \frac{1}{3}a^3x^{3m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a^2*b*x^(2*m) + (n*log(x) + log(c))^(2*q)*a*b^2*x^m +
 1/3*(n*log(x) + log(c))^(3*q)*b^3 + 1/3*a^3*x^(3*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bnq\log\left(cx^{n}\right)^{q-1}+amx^{m}\right)\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)^2/x
, x)
```

3.18 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Rubi [A] time = 0.109131, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2544}

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m+bnq\log^{-1+q}\left(cx^n\right)\right)\left(ax^m+b\log^q\left(cx^n\right)\right)}{x}\,dx=\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Mathematica [A] time = 0.032603, size = 22, normalized size = 1.

$$\frac{1}{2}\left(ax^m+b\log^q\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/
x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Maple [C] time = 0.162, size = 135, normalized size = 6.1

$$\frac{a^2 (x^m)^2}{2} + \frac{b^2 \left(\left(\ln(c) + \ln(x^n) - \frac{i}{2}\pi \operatorname{csgn}(icx^n)(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ic))(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ix^n))\right)^q\right)^2}{2} + abx^m \Big(\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] 1/2*a^2*(x^m)^2+1/2*b^2*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2+a*b*x^m*(ln(c)+ln(x^n)-1/
2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)
))^q
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.95908, size = 115, normalized size = 5.23

$$(n\log(x) + \log(c))^q abx^m + \frac{1}{2}(n\log(x) + \log(c))^{2q} b^2 + \frac{1}{2}a^2 x^{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a*b*x^m + 1/2*(n*log(x) + log(c))^(2*q)*b^2 + 1/2*a^2
*x^(2*m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bnq\log\left(cx^n\right)^{q-1} + amx^m\right)\left(ax^m + b\log\left(cx^n\right)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)/x,
x)
```

3.19 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=16

$$ax^m + b\log^q(cx^n)$$

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Rubi [A] time = 0.034799, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {14, 2302, 30}

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx = \int \left(amx^{-1+m} + \frac{bnq\log^{-1+q}(cx^n)}{x}\right)dx$$
$$= ax^m + (bnq)\int \frac{\log^{-1+q}(cx^n)}{x}\,dx$$
$$= ax^m + (bq)\,\mathrm{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right)$$
$$= ax^m + b\log^q(cx^n)$$

Mathematica [A] time = 0.0146627, size = 16, normalized size = 1.

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Maple [A] time = 0.003, size = 17, normalized size = 1.1

$$ax^m + b(\ln(cx^n))^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x,x)
```

```
[Out] a*x^m+b*ln(c*x^n)^q
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9989, size = 82, normalized size = 5.12

$$(bn\log(x) + b\log(c))(n\log(x) + \log(c))^{q-1} + ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

```
[Out] (b*n*log(x) + b*log(c))*(n*log(x) + log(c))^(q - 1) + a*x^m
```

Sympy [A] time = 61.9015, size = 58, normalized size = 3.62

$$am\left(\begin{cases}\frac{x^m}{m} & \text{for } m \neq 0 \\ \log(x) & \text{otherwise}\end{cases}\right) + bnq\left(\begin{cases}\frac{\log(x)}{\log(c)} & \text{for } n = 0 \wedge q = 0 \\ \frac{\log(c)^q \log(x)}{\log(c)} & \text{for } n = 0 \\ \frac{\log(n\log(x)+\log(c))}{n} & \text{for } q = 0 \\ \frac{(n\log(x)+\log(c))^q}{nq} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] a*m*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + b*n*q*Piecewise((log(x)
/log(c), Eq(n, 0) & Eq(q, 0)), (log(c)**q*log(x)/log(c), Eq(n, 0)), (log(n*
log(x) + log(c))/n, Eq(q, 0)), ((n*log(x) + log(c))**q/(n*q), True))
```

Giac [A] time = 1.31518, size = 23, normalized size = 1.44

$$(n\log(x) + \log(c))^q b + ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(c))^q*b + a*x^m
```

3.20 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=17

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Rubi [A] time = 0.18538, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2541}

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Mathematica [A] time = 0.233164, size = 17, normalized size = 1.

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Maple [C] time = 0.164, size = 213, normalized size = 12.5

$$q\ln\left(\ln(x^n) - \frac{i}{2}\left(\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n) - \pi\,\mathrm{csgn}(ic)\,(\mathrm{csgn}(icx^n))^2 - \pi\,\mathrm{csgn}(ix^n)\,(\mathrm{csgn}(icx^n))^2 + \pi\,(\mathrm{csgn}(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] q*ln(ln(x^n)-1/2*I*(Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-Pi*csgn(I*c)*csg
n(I*c*x^n)^2-Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+Pi*csgn(I*c*x^n)^3+2*I*ln(c)))-
q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn
(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c
*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+1/b*a*x^m)
```

Maxima [A] time = 1.55993, size = 30, normalized size = 1.76

$$\log\left(\frac{ax^m + b(\log(c) + \log(x^n))^q}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="maxima")
```

```
[Out] log((a*x^m + b*(log(c) + log(x^n))^q)/b)
```

Fricas [A] time = 1.82366, size = 51, normalized size = 3.

$$\log\left((n\log(x) + \log(c))^q b + ax^m\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="fricas")
```

```
[Out] log((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnq \log (cx^n)^{q-1} + amx^m}{\left(ax^m + b \log (cx^n)^q\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)*x)
, x)
```

3.21 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rubi [A] time = 0.181982, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=-\frac{1}{ax^m+b\log^q(cx^n)}$$

Mathematica [A] time = 0.0401056, size = 20, normalized size = 1.

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Maple [C] time = 0.129, size = 68, normalized size = 3.4

$$-\left(ax^m + b\left(\ln(c) + \ln(x^n) - \frac{i}{2}\pi \operatorname{csgn}(icx^n)(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ic))(-\operatorname{csgn}(icx^n) + \operatorname{csgn}(ix^n))\right)^q\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] -1/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c)
)*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Maxima [A] time = 1.71604, size = 28, normalized size = 1.4

$$-\frac{1}{ax^m + b(\log(c) + \log(x^n))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="maxima")
```

```
[Out] -1/(a*x^m + b*(log(c) + log(x^n))^q)
```

Fricas [A] time = 1.99025, size = 51, normalized size = 2.55

$$-\frac{1}{(n\log(x) + \log(c))^q b + ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="fricas")
```

```
[Out] -1/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnq \log(cx^n)^{q-1} + amx^m}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^2*
x), x)
```

3.22 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] -1/(2*(a*x^m + b*Log[c*x^n]^q)^2)
```

Rubi [A] time = 0.180028, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2544}

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/(2*(a*x^m + b*Log[c*x^n]^q)^2)
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Mathematica [A] time = 0.0401071, size = 22, normalized size = 1.

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^3),x]
```

```
[Out] -1/(2*(a*x^m + b*Log[c*x^n]^q)^2)
```

Maple [C] time = 0.184, size = 68, normalized size = 3.1

$$-\frac{1}{2\left(ax^m+b\left(\ln\left(c\right)+\ln\left(x^n\right)-i/2\pi\,\mathrm{csgn}\left(icx^n\right)\left(-\mathrm{csgn}\left(icx^n\right)+\mathrm{csgn}\left(ic\right)\right)\left(-\mathrm{csgn}\left(icx^n\right)+\mathrm{csgn}\left(ix^n\right)\right)\right)^q\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] -1/2/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*
c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2
```

Maxima [B] time = 2.132, size = 66, normalized size = 3.

$$-\frac{1}{2\left(a^2x^{2\,m}+b^2(\log\left(c\right)+\log\left(x^n\right))^{2\,q}+2\,abe^{\left(m\log(x)+q\log(\log(c)+\log(x^n))\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2/(a^2*x^(2*m) + b^2*(log(c) + log(x^n))^(2*q) + 2*a*b*e^(m*log(x) + q*l
og(log(c) + log(x^n))))
```

Fricas [B] time = 2.04863, size = 116, normalized size = 5.27

$$-\frac{1}{2\left(2\left(n\log\left(x\right)+\log\left(c\right)\right)^{q}abx^{m}+\left(n\log\left(x\right)+\log\left(c\right)\right)^{2q}b^{2}+a^{2}x^{2m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/2/(2*(n*log(x) + log(c))^q*a*b*x^m + (n*log(x) + log(c))^(2*q)*b^2 + a^2
*x^(2*m))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnq\log\left(cx^{n}\right)^{q-1}+amx^{m}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^3*
x), x)
```

3.23 $\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Rubi [A] time = 0.151753, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2561, 2544}

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2\,dx=\int \frac{(ax+2bn\log(cx^n))\left(ax^2+bx\log^2(cx^n)\right)^2}{x^3}\,dx$$
$$=\int \frac{(ax+2bn\log(cx^n))\left(ax+b\log^2(cx^n)\right)^2}{x}\,dx$$
$$=\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Mathematica [A] time = 0.0175154, size = 20, normalized size = 1.

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Maple [C] time = 1.223, size = 16321, normalized size = 816.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+2*b*n*ln(c*x^n)/x^3)*(a*x^2+b*x*ln(c*x^n)^2)^2,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.02688, size = 285, normalized size = 14.25

$$\frac{1}{3}b^3\log(cx^n)^6+4\,ab^2nx\log(cx^n)^3+ab^2x\log(cx^n)^4-\frac{1}{2}a^2bn^2x^2+a^2bnx^2\log(cx^n)+a^2bx^2\log(cx^n)^2+\frac{1}{3}a^3x^3-12\,(n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="maxima")
```

```
[Out] 1/3*b^3*log(c*x^n)^6 + 4*a*b^2*n*x*log(c*x^n)^3 + a*b^2*x*log(c*x^n)^4 - 1/
2*a^2*b*n^2*x^2 + a^2*b*n*x^2*log(c*x^n) + a^2*b*x^2*log(c*x^n)^2 + 1/3*a^3
*x^3 - 12*(n*x*log(c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*a*b^2*n + 1/2*(
n^2*x^2 - 2*n*x^2*log(c*x^n))*a^2*b - 4*(n*x*log(c*x^n)^3 - 3*(n*x*log(c*x^
n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*n)*a*b^2
```

Fricas [B] time = 1.84464, size = 478, normalized size = 23.9

$$\frac{1}{3}\,b^3n^6\log\left(x\right)^6 + 2\,b^3n^5\log\left(c\right)\log\left(x\right)^5 + ab^2x\log\left(c\right)^4 + a^2bx^2\log\left(c\right)^2 + \frac{1}{3}\,a^3x^3 + \left(5\,b^3n^4\log\left(c\right)^2 + ab^2n^4x\right)\log\left(x\right)^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="fricas")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + a*b^2*x*log(c)^4 + a^2*b
*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2 + a*b^2*n^4*x)*log(x)^4 +
 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*log(x)^3 + (5*b^3*n^2*log(
c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*log(x)^2 + 2*(b^3*n*log(c)^5
 + 2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Sympy [A] time = 13.4414, size = 221, normalized size = 11.05

$$\frac{a^3x^3}{3} + a^2bn^2x^2\log\left(x\right)^2 - a^2bn^2x^2\log\left(x\right) + 2a^2bnx^2\log\left(c\right)\log\left(x\right) - a^2bnx^2\log\left(c\right) + a^2bnx^2\log\left(cx^n\right) + a^2bx^2\log\left(c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+2*b*n*ln(c*x**n)/x**3)*(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] a**3*x**3/3 + a**2*b*n**2*x**2*log(x)**2 - a**2*b*n**2*x**2*log(x) + 2*a**2
*b*n*x**2*log(c)*log(x) - a**2*b*n*x**2*log(c) + a**2*b*n*x**2*log(c*x**n)
+ a**2*b*x**2*log(c)**2 + a*b**2*n**4*x*log(x)**4 + 4*a*b**2*n**3*x*log(c)*
log(x)**3 + 6*a*b**2*n**2*x*log(c)**2*log(x)**2 + 4*a*b**2*n*x*log(c)**3*lo
g(x) + a*b**2*x*log(c)**4 - 2*b**3*n*Piecewise((-log(c)**5*log(x), Eq(n, 0)
), (-log(c*x**n)**6/(6*n), True))
```

Giac [B] time = 1.29055, size = 267, normalized size = 13.35

$$\frac{1}{3}\,b^3n^6\log\left(x\right)^6+2\,b^3n^5\log\left(c\right)\log\left(x\right)^5+2\,b^3n\log\left(c\right)^5\log\left(x\right)+ab^2x\log\left(c\right)^4+a^2bx^2\log\left(c\right)^2+\frac{1}{3}\,a^3x^3+\left(5\,b^3n^4\log\left(c\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="giac")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + 2*b^3*n*log(c)^5*log(x)
+ a*b^2*x*log(c)^4 + a^2*b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2
 + a*b^2*n^4*x)*log(x)^4 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*
log(x)^3 + (5*b^3*n^2*log(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*lo
g(x)^2 + 2*(2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

3.24 $\int \left(\frac{a}{x}+\frac{2 b n \log (c x^n)}{x^2}\right) \left(a x^2+b x \log ^2(c x^n)\right) \, dx$

Optimal. Leaf size=20

$$\frac{1}{2} \left(a x+b \log ^2(c x^n)\right)^2$$

```
[Out] (a*x + b*Log[c*x^n]^2)^2/2
```

Rubi [A] time = 0.0929345, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2561, 2544}

$$\frac{1}{2} \left(a x+b \log ^2(c x^n)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^2/2
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2(cx^n)\right)\,dx=\int \frac{(ax+2bn\log(cx^n))\left(ax^2+bx\log^2(cx^n)\right)}{x^2}\,dx$$
$$=\int \frac{(ax+2bn\log(cx^n))\left(ax+b\log^2(cx^n)\right)}{x}\,dx$$
$$=\frac{1}{2}\left(ax+b\log^2(cx^n)\right)^2$$

Mathematica [A] time = 0.0040864, size = 38, normalized size = 1.9

$$\frac{a^2x^2}{2}+abx\log^2(cx^n)+\frac{1}{2}b^2\log^4(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a^2*x^2)/2 + a*b*x*Log[c*x^n]^2 + (b^2*Log[c*x^n]^4)/2
```

Maple [B] time = 0.018, size = 63, normalized size = 3.2

$$\frac{a^2x^2}{2}+abx\left(\ln\left(ce^{n\ln(x)}\right)\right)^2-2\,bnax\ln\left(ce^{n\ln(x)}\right)+\frac{b^2(\ln(cx^n))^4}{2}+2\,\ln(cx^n)\,abnx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+2*b*n*ln(c*x^n)/x^2)*(a*x^2+b*x*ln(c*x^n)^2),x)
```

```
[Out] 1/2*a^2*x^2+a*b*x*ln(c*exp(n*ln(x)))^2-2*b*n*a*x*ln(c*exp(n*ln(x)))+1/2*b^2
*ln(c*x^n)^4+2*ln(c*x^n)*a*b*n*x
```

Maxima [B] time = 1.04964, size = 100, normalized size = 5.

$$\frac{1}{2}\,b^2\log(cx^n)^4-2\,abn^2x+2\,abnx\log(cx^n)+abx\log(cx^n)^2+\frac{1}{2}\,a^2x^2+2\left(n^2x-nx\log(cx^n)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"maxima")
```

```
[Out] 1/2*b^2*log(c*x^n)^4 - 2*a*b*n^2*x + 2*a*b*n*x*log(c*x^n) + a*b*x*log(c*x^n
)^2 + 1/2*a^2*x^2 + 2*(n^2*x - n*x*log(c*x^n))*a*b
```

Fricas [B] time = 1.96197, size = 230, normalized size = 11.5

$$\frac{1}{2}\,b^2n^4\log\left(x\right)^4+2\,b^2n^3\log\left(c\right)\log\left(x\right)^3+abx\log\left(c\right)^2+\frac{1}{2}\,a^2x^2+\left(3\,b^2n^2\log\left(c\right)^2+abn^2x\right)\log\left(x\right)^2+2\left(b^2n\log\left(c\right)^3+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"fricas")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + a*b*x*log(c)^2 + 1/2*a^2
*x^2 + (3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 2*(b^2*n*log(c)^3 + a*b*
n*x*log(c))*log(x)
```

Sympy [A] time = 6.31994, size = 117, normalized size = 5.85

$$\frac{a^2x^2}{2}+abn^2x\log\left(x\right)^2-2abn^2x\log\left(x\right)+2abnx\log\left(c\right)\log\left(x\right)-2abnx\log\left(c\right)+2abnx\log\left(cx^n\right)+abx\log\left(c\right)^2-2b^2n\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*ln(c*x**n)/x**2)*(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] a**2*x**2/2 + a*b*n**2*x*log(x)**2 - 2*a*b*n**2*x*log(x) + 2*a*b*n*x*log(c)
*log(x) - 2*a*b*n*x*log(c) + 2*a*b*n*x*log(c*x**n) + a*b*x*log(c)**2 - 2*b*
*2*n*Piecewise((-log(c)**3*log(x), Eq(n, 0)), (-log(c*x**n)**4/(4*n), True)
)
```

Giac [B] time = 1.23081, size = 122, normalized size = 6.1

$$\frac{1}{2}\,b^2n^4\log\left(x\right)^4+2\,b^2n^3\log\left(c\right)\log\left(x\right)^3+2\,b^2n\log\left(c\right)^3\log\left(x\right)+2\,abnx\log\left(c\right)\log\left(x\right)+abx\log\left(c\right)^2+\frac{1}{2}\,a^2x^2+\left(3\,b^2n^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"giac")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + 2*b^2*n*log(c)^3*log(x)
+ 2*a*b*n*x*log(c)*log(x) + a*b*x*log(c)^2 + 1/2*a^2*x^2 + (3*b^2*n^2*log(c
)^2 + a*b*n^2*x)*log(x)^2
```

3.25 $\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx$

Optimal. Leaf size=14

$$ax+b\log^2(cx^n)$$

```
[Out] a*x + b*Log[c*x^n]^2
```

Rubi [A] time = 0.0092727, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2301}

$$ax+b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx &= ax+(2bn)\int \frac{\log(cx^n)}{x}\,dx\\ &= ax+b\log^2(cx^n)\end{aligned}$$

Mathematica [A] time = 0.0018801, size = 14, normalized size = 1.

$$ax+b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Maple [A] time = 0.003, size = 15, normalized size = 1.1

$$ax + b(\ln(cx^n))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+2*b*n*ln(c*x^n)/x,x)
```

```
[Out] a*x+b*ln(c*x^n)^2
```

Maxima [A] time = 1.01534, size = 19, normalized size = 1.36

$$b\log(cx^n)^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="maxima")
```

```
[Out] b*log(c*x^n)^2 + a*x
```

Fricas [A] time = 1.88183, size = 59, normalized size = 4.21

$$bn^2\log(x)^2 + 2bn\log(c)\log(x) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="fricas")
```

```
[Out] b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + a*x
```

Sympy [A] time = 1.67615, size = 60, normalized size = 4.29

$$ax + 2bn\left(\begin{cases} \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*ln(c*x**n)/x,x)
```

```
[Out] a*x + 2*b*n*Piecewise((log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (log(x**(-n)
/c)**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ())
, c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

Giac [A] time = 1.24135, size = 27, normalized size = 1.93

$$\left(n\log(x)^2 + 2\log(c)\log(x)\right)bn + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="giac")
```

```
[Out] (n*log(x)^2 + 2*log(c)*log(x))*b*n + a*x
```

3.26 $\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=15

$$\log\left(ax+b\log^2(cx^n)\right)$$

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Rubi [A] time = 0.0820021, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2561, 2541}

$$\log\left(ax+b\log^2(cx^n)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx &= \int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)}\,dx\\ &= \log\left(ax+b\log^2(cx^n)\right)\end{aligned}$$

Mathematica [A] time = 0.0953352, size = 15, normalized size = 1.

$$\log \left(ax + b\log^2 (cx^n)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Maple [C] time = 0.066, size = 428, normalized size = 28.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+2*b*n*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2),x)
```

```
[Out] ln(ln(x^n)^2+(-I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*csgn(I*c)*csgn
(I*c*x^n)^2+I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*c*x^n)^3+2*ln(c))*
ln(x^n)-1/4*(b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-2*b*Pi^2*csgn
(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4-2*b*
Pi^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+4*b*Pi^2*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)^4-2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5+b*Pi^2*csgn(I*x^n)^2*cs
gn(I*c*x^n)^4-2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5+b*Pi^2*csgn(I*c*x^n)^6+4
*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*b*ln(c)*Pi*csgn(I*c)*
csgn(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(c)*Pi*c
sgn(I*c*x^n)^3-4*b*ln(c)^2-4*a*x)/b)
```

Maxima [B] time = 1.18479, size = 43, normalized size = 2.87

$$\log \left(\frac{b \log (c)^2 + 2\,b \log (c) \log (x^n) + b \log (x^n)^2 + ax}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="max
ima")
```

```
[Out] log((b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)/b)
```

Fricas [A] time = 1.81171, size = 84, normalized size = 5.6

$$\log \left(bn^2 \log (x)^2 + 2\,bn \log (c) \log (x) + b \log (c)^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="fri
cas")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [A] time = 2.39266, size = 48, normalized size = 3.2

$$\begin{cases} \log \left(x + \frac{bn^2 \log (x)^2}{a} + \frac{2bn \log (c) \log (x)}{a} + \frac{b \log (c)^2}{a}\right) & \text{for}\ a \neq 0 \\ 2 \log (n \log (x) + \log (c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((log(x + b*n**2*log(x)**2/a + 2*b*n*log(c)*log(x)/a + b*log(c)**2
/a), Ne(a, 0)), (2*log(n*log(x) + log(c)), True))
```

Giac [A] time = 1.14868, size = 38, normalized size = 2.53

$$\log \left(bn^2 \log (x)^2 + 2\,bn \log (c) \log (x) + b \log (c)^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="gia
c")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

3.27 $\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{ax+b\log^2(cx^n)}$$

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Rubi [A] time = 0.133133, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2561, 2544}

$$-\frac{1}{ax+b\log^2(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{ax^2 + 2bnx \log(cx^n)}{\left(ax^2 + bx \log^2(cx^n)\right)^2} dx = \int \frac{x(ax + 2bn \log(cx^n))}{\left(ax^2 + bx \log^2(cx^n)\right)^2} dx$$
$$= \int \frac{ax + 2bn \log(cx^n)}{x\left(ax + b \log^2(cx^n)\right)^2} dx$$
$$= -\frac{1}{ax + b \log^2(cx^n)}$$

Mathematica [A] time = 0.0133318, size = 18, normalized size = 1.

$$-\frac{1}{ax + b \log^2(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Maple [C] time = 0.072, size = 451, normalized size = 25.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b*n*x*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^2,x)
```

```
[Out] -4/(-b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+2*b*Pi^2*csgn(I*c)^2*
csgn(I*x^n)*csgn(I*c*x^n)^3-b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4+2*b*Pi^2*csg
n(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*b*Pi^2*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)^4+2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x
^n)^4+2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*c*x^n)^6+4*I*b*ln(
c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-4*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(
I*c)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*c)*csgn(I*c*x^n)^2-4*I*b*ln(x^
n)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^3-4*
I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3+4*b*ln(c)^2+8*b*ln(c)*ln(x^n)+4*b*ln(x^n)^2+
4*a*x)
```

Maxima [A] time = 1.21717, size = 42, normalized size = 2.33

$$-\frac{1}{b\log\left(c\right)^{2}+2\,b\log\left(c\right)\log\left(x^{n}\right)+b\log\left(x^{n}\right)^{2}+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="maxima")
```

```
[Out] -1/(b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)
```

Fricas [A] time = 1.86091, size = 84, normalized size = 4.67

$$-\frac{1}{bn^{2}\log\left(x\right)^{2}+2\,bn\log\left(c\right)\log\left(x\right)+b\log\left(c\right)^{2}+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="fricas")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b*n*x*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.45101, size = 42, normalized size = 2.33

$$-\frac{1}{bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="giac")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

3.28 $\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

```
[Out] -1/(2*(a*x + b*Log[c*x^n]^2)^2)
```

Rubi [A] time = 0.159982, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2561, 2544}

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/(2*(a*x + b*Log[c*x^n]^2)^2)
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2544

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx = \int \frac{x^2\,(ax+2bn\log(cx^n))}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx$$
$$= \int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^3}\,dx$$
$$= -\frac{1}{2\left(ax+b\log^2(cx^n)\right)^2}$$

Mathematica [A] time = 0.0141217, size = 20, normalized size = 1.

$$-\frac{1}{2\left(ax+b\log^2(cx^n)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/(2*(a*x + b*Log[c*x^n]^2)^2)
```

Maple [C] time = 0.085, size = 451, normalized size = 22.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b*n*x^2*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^3,x)
```

```
[Out] -8/(-b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+2*b*Pi^2*csgn(I*c)^2*
csgn(I*x^n)*csgn(I*c*x^n)^3-b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4+2*b*Pi^2*csg
n(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*b*Pi^2*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)^4+2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x
^n)^4+2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*c*x^n)^6+4*I*b*ln(
c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-4*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(
I*c)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*c)*csgn(I*c*x^n)^2-4*I*b*ln(x^
n)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^3-4*
I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3+4*b*ln(c)^2+8*b*ln(c)*ln(x^n)+4*b*ln(x^n)^2+
```

```
4*a*x)^2
```

Maxima [B] time = 1.42503, size = 128, normalized size = 6.4

$$-\frac{1}{2\left(b^2\log\left(c\right)^4+4\,b^2\log\left(c\right)\log\left(x^n\right)^3+b^2\log\left(x^n\right)^4+2\,abx\log\left(c\right)^2+a^2x^2+2\left(3\,b^2\log\left(c\right)^2+abx\right)\log\left(x^n\right)^2+4\left(b^2\log\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="maxima")
```

```
[Out] -1/2/(b^2*log(c)^4 + 4*b^2*log(c)*log(x^n)^3 + b^2*log(x^n)^4 + 2*a*b*x*log
(c)^2 + a^2*x^2 + 2*(3*b^2*log(c)^2 + a*b*x)*log(x^n)^2 + 4*(b^2*log(c)^3 +
 a*b*x*log(c))*log(x^n))
```

Fricas [B] time = 2.02759, size = 254, normalized size = 12.7

$$-\frac{1}{2\left(b^2n^4\log\left(x\right)^4+4\,b^2n^3\log\left(c\right)\log\left(x\right)^3+b^2\log\left(c\right)^4+2\,abx\log\left(c\right)^2+a^2x^2+2\left(3\,b^2n^2\log\left(c\right)^2+abn^2x\right)\log\left(x\right)^2+4\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="fricas")
```

```
[Out] -1/2/(b^2*n^4*log(x)^4 + 4*b^2*n^3*log(c)*log(x)^3 + b^2*log(c)^4 + 2*a*b*x
*log(c)^2 + a^2*x^2 + 2*(3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 4*(b^2*
n*log(c)^3 + a*b*n*x*log(c))*log(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b*n*x**2*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27848, size = 413, normalized size = 20.65

$$-\frac{}{2\left(4\,ab^3n^6x\log\left(x\right)^4+16\,ab^3n^5x\log\left(c\right)\log\left(x\right)^3+a^2b^2n^4x^2\log\left(x\right)^4+24\,ab^3n^4x\log\left(c\right)^2\log\left(x\right)^2+4\,a^2b^2n^3x^2\log\left(c\right)\log\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="giac")
```

```
[Out] -1/2*(4*a*b*n^2*x + a^2*x^2)/(4*a*b^3*n^6*x*log(x)^4 + 16*a*b^3*n^5*x*log(c
)*log(x)^3 + a^2*b^2*n^4*x^2*log(x)^4 + 24*a*b^3*n^4*x*log(c)^2*log(x)^2 +
4*a^2*b^2*n^3*x^2*log(c)*log(x)^3 + 16*a*b^3*n^3*x*log(c)^3*log(x) + 8*a^2*
b^2*n^4*x^2*log(x)^2 + 6*a^2*b^2*n^2*x^2*log(c)^2*log(x)^2 + 4*a*b^3*n^2*x*
log(c)^4 + 16*a^2*b^2*n^3*x^2*log(c)*log(x) + 4*a^2*b^2*n*x^2*log(c)^3*log(
x) + 2*a^3*b*n^2*x^3*log(x)^2 + 8*a^2*b^2*n^2*x^2*log(c)^2 + a^2*b^2*x^2*lo
g(c)^4 + 4*a^3*b*n*x^3*log(c)*log(x) + 4*a^3*b*n^2*x^3 + 2*a^3*b*x^3*log(c)
^2 + a^4*x^4)
```

3.29 $\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx$

Optimal. Leaf size=19

$$\log\left(ax^{m-1}+b\log^q\left(cx^n\right)\right)$$

```
[Out] Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Rubi [A] time = 0.320846, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2561, 2541}

$$\log\left(ax^{m-1}+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*Log[c*
x^n]^q),x]
```

```
[Out] Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Rule 2561

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rule 2541

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[(e*Log[a*x^
m + b*Log[c*x^n]^q])/(b*n*q), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{ax^m+bx\log^q\left(cx^n\right)}\,dx &= \int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)}\,dx\\ &= \log\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A] time = 0.460488, size = 23, normalized size = 1.21

$$\log\left(ax^m + bx\log^q(cx^n)\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*
Log[c*x^n]^q),x]
```

```
[Out] -Log[x] + Log[a*x^m + b*x*Log[c*x^n]^q]
```

Maple [C] time = 0.191, size = 216, normalized size = 11.4

$$q\ln\left(\ln(x^n) - \frac{i}{2}\left(\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n) - \pi\,\mathrm{csgn}(ic)\,(\mathrm{csgn}(icx^n))^2 - \pi\,\mathrm{csgn}(ix^n)\,(\mathrm{csgn}(icx^n))^2 + \pi\,(\mathrm{csgn}(ic$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(-1+m)*x^(-1+m)+b*n*q*ln(c*x^n)^(-1+q))/(a*x^m+b*x*ln(c*x^n)^q),x)
```

```
[Out] q*ln(ln(x^n)-1/2*I*(Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-Pi*csgn(I*c)*csg
n(I*c*x^n)^2-Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+Pi*csgn(I*c*x^n)^3+2*I*ln(c)))-
q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn
(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c
*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+a*x^m/x/b)
```

Maxima [A] time = 1.54302, size = 35, normalized size = 1.84

$$\log\left(\frac{bx(\log(c) + \log(x^n))^q + ax^m}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="maxima")
```

```
[Out] log((b*x*(log(c) + log(x^n))^q + a*x^m)/(b*x))
```

Fricas [A] time = 1.92569, size = 59, normalized size = 3.11

$$\log\left(\frac{(n\log(x)+\log(c))^q bx + ax^m}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="fricas")
```

```
[Out] log(((n*log(x) + log(c))^q*b*x + a*x^m)/x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x**(-1+m)+b*n*q*ln(c*x**n)**(-1+q))/(a*x**m+b*x*ln(c*x*
*n)**q),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bnq\log(cx^n)^{q-1} + a(m-1)x^{m-1}}{bx\log(cx^n)^q + ax^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*(m - 1)*x^(m - 1))/(b*x*log(c*x^n)^
q + a*x^m), x)
```

3.30 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=80

$$\left(d-\frac{aem}{bnq}\right)\text{CannotIntegrate}\left(x^{m-1}\left(ax^m+b\log^q(cx^n)\right)^p,x\right)+\frac{e\left(ax^m+b\log^q(cx^n)\right)^{p+1}}{bn(p+1)q}$$

```
[Out] (d - (a*e*m)/(b*n*q))*CannotIntegrate[x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p
, x] + (e*(a*x^m + b*Log[c*x^n]^q)^(1 + p))/(b*n*(1 + p)*q)
```

Rubi [A] time = 0.230662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (e*(a*x^m + b*Log[c*x^n]^q)^(1 + p))/(b*n*(1 + p)*q) + (d - (a*e*m)/(b*n*q)
)*Defer[Int][x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p, x]
```

Rubi steps

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{e\left(ax^m+b\log^q(cx^n)\right)^{1+p}}{bn(1+p)q}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m+b\log^q(cx$$

Mathematica [A] time = 1.80076, size = 0, normalized size = 0.

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x, x
]
```

Maple [A] time = 0.487, size = 0, normalized size = 0.

$$\int \frac{\left(dx^{m}+e\left(\ln \left(cx^{n}\right)\right)^{-1+q}\right)\left(ax^{m}+b\left(\ln \left(cx^{n}\right)\right)^{q}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^{m}+e\log\left(cx^{n}\right)^{q-1}\right)\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{p}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.31 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=331

$$-\frac{a^2 3^{-q} x^{3m} (cx^n)^{-\frac{3m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} (aem-bdnq)\mathrm{Gamma}\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq} - \frac{b^2 x^m (cx^n)^{-\frac{m}{n}} \log^{3q}(cx^n)\left(-\frac{m}{\cdots}\right.}{\cdots}$$

```
[Out] -(a^3*(a*e*m - b*d*n*q)*x^(4*m))/(4*b*m*n*q) - (b^2*(a*e*m - b*d*n*q)*x^m*G
amma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(m*n*q*(c*x^n)^(m/n)*(
-((m*Log[c*x^n])/n))^(3*q)) - (3*2^(-1 - 2*q)*a*b*(a*e*m - b*d*n*q)*x^(2*m)
*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^((2*m
)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*(a*e*m - b*d*n*q)*x^(3*m)*Gamma[1
+ q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*
Log[c*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q)
```

Rubi [A] time = 0.503742, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2545, 6742, 2310, 2181}

$$-\frac{a^2 3^{-q} x^{3m} (cx^n)^{-\frac{3m}{n}} \log^q(cx^n) \left(-\frac{m\log(cx^n)}{n}\right)^{-q} (aem-bdnq)\mathrm{Gamma}\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq} - \frac{b^2 x^m (cx^n)^{-\frac{m}{n}} \log^{3q}(cx^n)\left(-\frac{m}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] -(a^3*(a*e*m - b*d*n*q)*x^(4*m))/(4*b*m*n*q) - (b^2*(a*e*m - b*d*n*q)*x^m*G
amma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(m*n*q*(c*x^n)^(m/n)*(
-((m*Log[c*x^n])/n))^(3*q)) - (3*2^(-1 - 2*q)*a*b*(a*e*m - b*d*n*q)*x^(2*m)
*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^((2*m
)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*(a*e*m - b*d*n*q)*x^(3*m)*Gamma[1
+ q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*
Log[c*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q)
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
```

```
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^3}{x}\,dx = \frac{e\left(ax^m + b\log^q(cx^n)\right)^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m + b\log^q(cx^n)\right)$$

$$= \frac{e\left(ax^m + b\log^q(cx^n)\right)^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int\left(a^3x^{-1+4m} + 3a^2bx^{-1+3m}\right.$$

$$= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^4}{4bnq} - \left(3a^2b\left(-d + \frac{aem}{bnq}\right)\right)\int$$

$$= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^4}{4bnq} - \frac{\left(3a^2b\left(-d + \frac{aem}{bnq}\right)x^{3m}\right.}{}$$

$$= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} - \frac{b^2(aem - bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1 + 3q, -\frac{m\log(cx^n)}{n}\right.}{mnq}$$

Mathematica [A] time = 1.55066, size = 445, normalized size = 1.34

$$3^{-q}4^{-q-1}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\left(\left(-\frac{m\log(cx^n)}{n}\right)^q\left(4^q\left(-\frac{m\log(cx^n)}{n}\right)^q\left(4a^2bdnqx^{3m}\log^q(cx^n)\,\text{Gamma}\left(q+1,-\frac{3m\log(cx^n)}{n}\right)\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (4^(-1 - q)*(-(12^(1 + q)*a*b^2*e*m*q*x^m*(c*x^n)^((2*m)/n)*Gamma[3*q, -((m
*Log[c*x^n])/n)]*Log[c*x^n]^(3*q)) + 3^q*4^(1 + q)*b^3*d*n*q*x^m*(c*x^n)^((
2*m)/n)*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q) + (-((m*Log[c*
x^n])/n))^q*(-4*3^(1 + q)*a^2*b*e*m*q*x^(2*m)*(c*x^n)^(m/n)*Gamma[2*q, (-2*
m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 2*3^(1 + q)*a*b^2*d*n*q*x^(2*m)*(c*x^n)
^(m/n)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 4^q*(-((m*Log
[c*x^n])/n))^q*(-4*a^3*e*m*q*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*Log[c*x^
n]^q + 4*a^2*b*d*n*q*x^(3*m)*Gamma[1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q
 + 3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q*(a^3*d*n*q*x^(4*m) + b^3*e
*m*Log[c*x^n]^(4*q))))))/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^(3*q))
```

Maple [F] time = 7.961, size = 0, normalized size = 0.

$$\int \frac{\left(dx^m + e(\ln(cx^n))^{-1+q}\right)\left(ax^m + b(\ln(cx^n))^q\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3ex^{3m}\log\left(cx^n\right)^{q-1}+a^3dx^{4m}+\left(b^3dx^m+b^3e\log\left(cx^n\right)^{q-1}\right)\log\left(cx^n\right)^{3q}+3\left(ab^2ex^m\log\left(cx^n\right)^{q-1}+ab^2dx^{2m}\right)\log}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^3*e*x^(3*m)*log(c*x^n)^(q - 1) + a^3*d*x^(4*m) + (b^3*d*x^m + b
^3*e*log(c*x^n)^(q - 1))*log(c*x^n)^(3*q) + 3*(a*b^2*e*x^m*log(c*x^n)^(q -
1) + a*b^2*d*x^(2*m))*log(c*x^n)^(2*q) + 3*(a^2*b*e*x^(2*m)*log(c*x^n)^(q -
 1) + a^2*b*d*x^(3*m))*log(c*x^n)^q)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m+b\log\left(cx^n\right)^q\right)^3\left(dx^m+e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

3.32 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=235

$$-\frac{bx^m(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\text{Gamma}\left(2q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}-\frac{a2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log}{}\right.}{}$$

```
[Out] -(a^2*(a*e*m - b*d*n*q)*x^(3*m))/(3*b*m*n*q) - (b*(a*e*m - b*d*n*q)*x^m*Gam
ma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^(m/n)*(-(
(m*Log[c*x^n])/n))^(2*q)) - (a*(a*e*m - b*d*n*q)*x^(2*m)*Gamma[1 + q, (-2*m
*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n]
)/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q)
```

Rubi [A] time = 0.395189, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2545, 6742, 2310, 2181}

$$-\frac{bx^m(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\text{Gamma}\left(2q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}-\frac{a2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] -(a^2*(a*e*m - b*d*n*q)*x^(3*m))/(3*b*m*n*q) - (b*(a*e*m - b*d*n*q)*x^m*Gam
ma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^(m/n)*(-(
(m*Log[c*x^n])/n))^(2*q)) - (a*(a*e*m - b*d*n*q)*x^(2*m)*Gamma[1 + q, (-2*m
*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n]
)/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q)
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x}\,dx = \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m + b\log^q(cx\right.$$

$$= \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(-d + \frac{aem}{bnq}\right)\int \left(a^2x^{-1+3m} + 2abx^{-1+2m}\right.$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \left(2ab\left(-d + \frac{aem}{bnq}\right)\right)\int$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^3}{3bnq} - \frac{\left(2ab\left(-d + \frac{aem}{bnq}\right)x^{2m}\right.}{}$$

$$= \frac{a^2\left(d - \frac{aem}{bnq}\right)x^{3m}}{3m} - \frac{b(aem - bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1 + 2q, -\frac{m\log(cx^n)}{n}\right.}{mnq}$$

Mathematica [A] time = 0.927323, size = 298, normalized size = 1.27

$$\frac{2^{-q}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\left(\left(-\frac{m\log(cx^n)}{n}\right)^q\left(-3a^2emqx^{2m}\log^q(cx^n)\,\text{Gamma}\left(q, -\frac{2m\log(cx^n)}{n}\right) + 3abdnqx^{2m}\log^q(cx^n)\,\text{Ga}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (-3*2^(1 + q)*a*b*e*m*q*x^m*(c*x^n)^(m/n)*Gamma[2*q, -((m*Log[c*x^n])/n)]*L
og[c*x^n]^(2*q) + 3*2^q*b^2*d*n*q*x^m*(c*x^n)^(m/n)*Gamma[1 + 2*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q) + (-((m*Log[c*x^n])/n))^q*(-3*a^2*e*m*q*x^(2*
m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 3*a*b*d*n*q*x^(2*m)*Gamma[1
 + q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q*(a^2*d*n*q*x^(3*m) + b^2*e*m*Log[c*x^n]^(3*q)))))/(3*2^q*m*n*q*
(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q))
```

Maple [F] time = 17.253, size = 0, normalized size = 0.

$$\int \frac{\left(dx^m + e\left(\ln\left(cx^n\right)\right)^{-1+q}\right)\left(ax^m + b\left(\ln\left(cx^n\right)\right)^q\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2ex^{2m}\log\left(cx^n\right)^{q-1} + a^2dx^{3m} + \left(b^2dx^m + b^2e\log\left(cx^n\right)^{q-1}\right)\log\left(cx^n\right)^{2q} + 2\left(abex^m\log\left(cx^n\right)^{q-1} + abdx^{2m}\right)\log\left(c\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^2*e*x^(2*m)*log(c*x^n)^(q - 1) + a^2*d*x^(3*m) + (b^2*d*x^m + b
^2*e*log(c*x^n)^(q - 1))*log(c*x^n)^(2*q) + 2*(a*b*e*x^m*log(c*x^n)^(q - 1)
 + a*b*d*x^(2*m))*log(c*x^n)^q)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)^2 \left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

3.33 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx$

Optimal. Leaf size=139

$$x^m\left(cx^n\right)^{-\frac{m}{n}}\log^q\left(cx^n\right)\left(-\frac{m\log\left(cx^n\right)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\text{Gamma}\left(q+1,-\frac{m\log\left(cx^n\right)}{n}\right)+\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{2bnq}-\frac{ax^{2m}(aem}{2bm}$$

```
[Out] -(a*(a*e*m - b*d*n*q)*x^(2*m))/(2*b*m*n*q) + (((b*d)/m - (a*e)/(n*q))*x^m*G
amma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x
^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q)
```

Rubi [A] time = 0.170908, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2545, 14, 2310, 2181}

$$x^m\left(cx^n\right)^{-\frac{m}{n}}\log^q\left(cx^n\right)\left(-\frac{m\log\left(cx^n\right)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\text{Gamma}\left(q+1,-\frac{m\log\left(cx^n\right)}{n}\right)+\frac{e\left(ax^m+b\log^q\left(cx^n\right)\right)^2}{2bnq}-\frac{ax^{2m}(aem}{2bm}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] -(a*(a*e*m - b*d*n*q)*x^(2*m))/(2*b*m*n*q) + (((b*d)/m - (a*e)/(n*q))*x^m*G
amma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x
^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q)
```

Rule 2545

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[(e*(a
*x^m + b*Log[c*x^n]^q)^(p + 1))/(b*n*q*(p + 1)), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)}{x}\,dx &= \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}\left(ax^m + b\log^q(cx^n\right. \\
&= \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(-d + \frac{aem}{bnq}\right)\int \left(ax^{-1+2m} + bx^{-1+m}\log^q(\right. \\
&= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \left(b\left(-d + \frac{aem}{bnq}\right)\right)\int x^{-} \\
&= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \frac{e\left(ax^m + b\log^q(cx^n)\right)^2}{2bnq} - \frac{\left(b\left(-d + \frac{aem}{bnq}\right)x^m(cx^n\right.}{} \\
&= \frac{a\left(d - \frac{aem}{bnq}\right)x^{2m}}{2m} + \left(\frac{bd}{m} - \frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+q, -\frac{m\log(cx^n)}{n}\right)\mathrm{lo}
\end{aligned}
$$

Mathematica [A] time = 0.385985, size = 157, normalized size = 1.13

$$
\frac{(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(-2aemqx^m\log^q(cx^n)\,\mathrm{Gamma}\left(q, -\frac{m\log(cx^n)}{n}\right) + 2bdnqx^m\log^q(cx^n)\,\mathrm{Gamma}\left(q+1, -\frac{m\log(cx^n)}{n}\right)\right.}{2mnq}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (-2*a*e*m*q*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + 2*b*d*n*q*x^m*
Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + (c*x^n)^(m/n)*(-((m*Log[c*
x^n])/n))^q*(a*d*n*q*x^(2*m) + b*e*m*Log[c*x^n]^(2*q)))/(2*m*n*q*(c*x^n)^(m
/n)*(-((m*Log[c*x^n])/n))^q)
```

Maple [F] time = 10.297, size = 0, normalized size = 0.

$$\int \frac{\left(dx^{m}+e\left(\ln\left(cx^{n}\right)\right)^{-1+q}\right)\left(ax^{m}+b\left(\ln\left(cx^{n}\right)\right)^{q}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{aex^{m}\log\left(cx^{n}\right)^{q-1}+adx^{2\,m}+\left(bdx^{m}+be\log\left(cx^{n}\right)^{q-1}\right)\log\left(cx^{n}\right)^{q}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="fricas")
```

```
[Out] integral((a*e*x^m*log(c*x^n)^(q - 1) + a*d*x^(2*m) + (b*d*x^m + b*e*log(c*x
^n)^(q - 1))*log(c*x^n)^q)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^m + b\log\left(cx^n\right)^q\right)\left(dx^m + e\log\left(cx^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1))/x, x)
```

3.34 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rubi [A] time = 0.0338121, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {14, 2302, 30}

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{dx^m + e\log^{-1+q}(cx^n)}{x}\,dx &= \int \left(dx^{-1+m} + \frac{e\log^{-1+q}(cx^n)}{x}\right)dx\\ &= \frac{dx^m}{m} + e\int \frac{\log^{-1+q}(cx^n)}{x}\,dx\\ &= \frac{dx^m}{m} + \frac{e\,\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{dx^m}{m} + \frac{e\log^q(cx^n)}{nq}\end{aligned}$$

Mathematica [A] time = 0.0201541, size = 25, normalized size = 1.

$$\frac{e\log^q(cx^n)}{nq} + \frac{dx^m}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Maple [A] time = 0.007, size = 26, normalized size = 1.

$$\frac{dx^m}{m} + \frac{e(\ln(cx^n))^q}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x,x)
```

```
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86243, size = 107, normalized size = 4.28

$$\frac{dnqx^m + (emn\log(x) + em\log(c))(n\log(x) + \log(c))^{q-1}}{mnq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

```
[Out] (d*n*q*x^m + (e*m*n*log(x) + e*m*log(c))*(n*log(x) + log(c))^(q - 1))/(m*n*
q)
```

Sympy [A] time = 60.9989, size = 53, normalized size = 2.12

$$d\left(\begin{cases}\frac{x^m}{m} & \text{for}\ m \neq 0\\ \log(x) & \text{otherwise}\end{cases}\right) + e\left(\begin{cases}\frac{\log(x)}{\log(c)} & \text{for}\ n = 0 \wedge q = 0\\ \frac{\log(c)^q\log(x)}{\log(c)} & \text{for}\ n = 0\\ \frac{\log(n\log(x)+\log(c))}{n} & \text{for}\ q = 0\\ \frac{(n\log(x)+\log(c))^q}{nq} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] d*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + e*Piecewise((log(x)/log(c
), Eq(n, 0) & Eq(q, 0)), (log(c)**q*log(x)/log(c), Eq(n, 0)), (log(n*log(x)
 + log(c))/n, Eq(q, 0)), ((n*log(x) + log(c))**q/(n*q), True))
```

Giac [A] time = 1.31321, size = 36, normalized size = 1.44

$$\frac{dx^m}{m} + \frac{(n\log(x) + \log(c))^q e}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
```

```
[Out] d*x^m/m + (n*log(x) + log(c))^q*e/(n*q)
```

3.35 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$

Optimal. Leaf size=72

$$\left(d-\frac{aem}{bnq}\right)\text{CannotIntegrate}\left(\frac{x^{m-1}}{ax^m+b\log^q(cx^n)},x\right)+\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}$$

```
[Out] (d - (a*e*m)/(b*n*q))*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q),
x] + (e*Log[a*x^m + b*Log[c*x^n]^q])/(b*n*q)
```

Rubi [A] time = 0.267419, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] (e*Log[a*x^m + b*Log[c*x^n]^q])/(b*n*q) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q), x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx=\frac{e\log\left(ax^m+b\log^q(cx^n)\right)}{bnq}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx$$

Mathematica [A] time = 4.74691, size = 0, normalized size = 0.

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [A] time = 35.223, size = 0, normalized size = 0.

$$\int \frac{dx^m + e(\ln(cx^n))^{-1+q}}{x\left(ax^m + b(\ln(cx^n))^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{e\log(\log(c) + \log(x^n))}{bn} + \int \frac{bdx^m\log(x^n) + (bd\log(c) - ae)x^m}{abxx^m\log(c) + abxx^m\log(x^n) + \left(b^2x\log(c) + b^2x\log(x^n)\right)(\log(c) + \log(x^n))^q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="maxima")
```

```
[Out] e*log(log(c) + log(x^n))/(b*n) + integrate((b*d*x^m*log(x^n) + (b*d*log(c)
- a*e)*x^m)/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b^2*x*
log(x^n))*(log(c) + log(x^n))^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^m + e\log(cx^n)^{q-1}}{axx^m + bx\log(cx^n)^q}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^m + e\log\left(cx^n\right)^{q-1}}{\left(ax^m + b\log\left(cx^n\right)^q\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)*x), x)
```

3.36 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=74

$$\left(d-\frac{aem}{bnq}\right)\text{CannotIntegrate}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2},x\right)-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}$$

```
[Out] (d - (a*e*m)/(b*n*q))*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2
, x] - e/(b*n*q*(a*x^m + b*Log[c*x^n]^q))
```

Rubi [A] time = 0.282174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(e/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) + (d - (a*e*m)/(b*n*q))*Defer[Int][x^
(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A] time = 7.30777, size = 0, normalized size = 0.

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2), x
]
```

Maple [A] time = 7.905, size = 0, normalized size = 0.

$$\int \frac{dx^{m} + e\left(\ln\left(cx^{n}\right)\right)^{-1+q}}{x\left(ax^{m} + b\left(\ln\left(cx^{n}\right)\right)^{q}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{bd\log\left(c\right) + bd\log\left(x^{n}\right) - ae}{a^{2}bmx^{m}\log\left(x^{n}\right) - \left(nq - m\log\left(c\right)\right)a^{2}bx^{m} + \left(ab^{2}m\log\left(x^{n}\right) - \left(nq - m\log\left(c\right)\right)ab^{2}\right)\left(\log\left(c\right) + \log\left(x^{n}\right)\right)^{q}} + \int -\frac{}{a^{2}bm^{2}xx^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="maxima")
```

```
[Out] -(b*d*log(c) + b*d*log(x^n) - a*e)/(a^2*b*m*x^m*log(x^n) - (n*q - m*log(c))
*a^2*b*x^m + (a*b^2*m*log(x^n) - (n*q - m*log(c))*a*b^2)*(log(c) + log(x^n)
)^q) + integrate(-((e*m*n*(q - 1) - e*m^2*log(c))*a + (d*m*n*q*log(c) - (q^
2 - q)*d*n^2)*b + (b*d*m*n*q - a*e*m^2)*log(x^n))/(a^2*b*m^2*x*x^m*log(x^n)
^2 - 2*(m*n*q - m^2*log(c))*a^2*b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c
) + m^2*log(c)^2)*a^2*b*x*x^m + (a*b^2*m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*lo
g(c))*a*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b^2*x)
*(log(c) + log(x^n))^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{dx^{m}+e\log\left(cx^{n}\right)^{q-1}}{2\,abxx^{m}\log\left(cx^{n}\right)^{q}+a^{2}xx^{2\,m}+b^{2}x\log\left(cx^{n}\right)^{2\,q}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x
^(2*m) + b^2*x*log(c*x^n)^(2*q)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^{m}+e\log\left(cx^{n}\right)^{q-1}}{\left(ax^{m}+b\log\left(cx^{n}\right)^{q}\right)^{2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

3.37 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^3}\,dx$

Optimal. Leaf size=76

$$\left(d-\frac{aem}{bnq}\right)\text{CannotIntegrate}\left(\frac{x^{m-1}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3},x\right)-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] (d - (a*e*m)/(b*n*q))*CannotIntegrate[x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3
, x] - e/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2)
```

Rubi [A] time = 0.249227, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -e/(2*b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Mathematica [A] time = 66.9193, size = 0, normalized size = 0.

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3), x
]
```

Maple [A] time = 91.029, size = 0, normalized size = 0.

$$\int \frac{dx^{m}+e\left(\ln\left(cx^{n}\right)\right)^{-1+q}}{x\left(ax^{m}+b\left(\ln\left(cx^{n}\right)\right)^{q}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="maxima")
```

```
[Out] -1/2*(a*b*d*m^2*x^m*log(x^n)^3 + (a^2*e*m^2 - (4*d*m*n*q - 3*d*m^2*log(c))*
a*b)*x^m*log(x^n)^2 + ((2*e*m^2*log(c) + e*m*n)*a^2 - (8*d*m*n*q*log(c) - 3
*d*m^2*log(c)^2 - (3*q^2 - q)*d*n^2)*a*b)*x^m*log(x^n) - ((e*n^2*q^2 - e*m^
2*log(c)^2 - e*m*n*log(c))*a^2 + (4*d*m*n*q*log(c)^2 - d*m^2*log(c)^3 - (3*
q^2 - q)*d*n^2*log(c))*a*b)*x^m - ((e*m*n*(2*q - 1)*log(c) - 2*e*m^2*log(c)
^2)*a*b + (2*d*m*n*q*log(c)^2 - (2*q^2 - q)*d*n^2*log(c))*b^2 + 2*(b^2*d*m*
n*q - a*b*e*m^2)*log(x^n)^2 + ((e*m*n*(2*q - 1) - 4*e*m^2*log(c))*a*b + (4*
d*m*n*q*log(c) - (2*q^2 - q)*d*n^2)*b^2)*log(x^n))*(log(c) + log(x^n))^q)/(
a^4*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^4*b*x^(3*m)*log(x
^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^4*b*x^(3*m)*log(x
^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^
4*b*x^(3*m) + (a^2*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^
3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^3*
```

```
x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log
(c)^3)*a^2*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^3*b^2*m^3*x^(2*m)*log(
x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^3*b^2*x^(2*m)*log(x^n)^2 + 3*(m*n^2*q^2
 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^3*b^2*x^(2*m)*log(x^n) - (n^3*q^3 - 3
*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^3*b^2*x^(2*m))*(lo
g(c) + log(x^n))^q) - integrate(-1/2*(2*(b*d*m^3*n*q - a*e*m^4)*log(x^n)^3
+ ((e*m^3*n*(2*q - 3) - 6*e*m^4*log(c))*a + (6*d*m^3*n*q*log(c) - (2*q^2 -
3*q)*d*m^2*n^2)*b)*log(x^n)^2 + (e*m^3*n*(2*q - 3)*log(c)^2 - 2*e*m^4*log(c
)^3 + 2*(q^2 - 1)*e*m^2*n^2*log(c) - (2*q^3 - 3*q^2 + q)*e*m*n^3)*a + (2*d*
m^3*n*q*log(c)^3 - (2*q^2 - 3*q)*d*m^2*n^2*log(c)^2 - 2*(q^3 - q)*d*m*n^3*l
og(c) + (2*q^4 - 3*q^3 + q^2)*d*n^4)*b + 2*((e*m^3*n*(2*q - 3)*log(c) - 3*e
*m^4*log(c)^2 + (q^2 - 1)*e*m^2*n^2)*a + (3*d*m^3*n*q*log(c)^2 - (2*q^2 - 3
*q)*d*m^2*n^2*log(c) - (q^3 - q)*d*m*n^3)*b)*log(x^n))/(a^3*b*m^4*x*x^(2*m)
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^3*b*x*x^(2*m)*log(x^n)^3 + 6*(m^2*
n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^3*b*x*x^(2*m)*log(x^n)^2 - 4*(
m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^3*b
*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^
2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^3*b*x*x^(2*m) + (a^2*b^2*m^4*x*x^m
*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b^2*x*x^m*log(x^n)^3 + 6*(m^2*n^
2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b^2*x*x^m*log(x^n)^2 - 4*(m*n^
3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^2*b^2*x
*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 - 4*
m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^m + e\log(cx^n)^{q-1}}{3\,ab^2xx^m\log(cx^n)^{2\,q} + 3\,a^2bxx^{2\,m}\log(cx^n)^q + a^3xx^{3\,m} + b^3x\log(cx^n)^{3\,q}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + e*log(c*x^n)^(q - 1))/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3
*a^2*b*x*x^(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^m + e\log{(cx^n)}^{q-1}}{\left(ax^m + b\log{(cx^n)}^q\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + e*log(c*x^n)^(q - 1))/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

3.38 $\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=26

$$\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rubi [A] time = 0.245221, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {2546}

$$\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/(x*(a*
x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rule 2546

```
Int[(Log[(c_.)*(x_)^(n_.)]^(q_.)*(f_.) + (d_.)*(x_)^(m_.) + Log[(c_.)*(x_)^
(n_.)]*(e_.)*(x_)^(m_.))/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x
_)^(m_.))^2), x_Symbol] :> Simp[(d*Log[c*x^n])/(a*n*(a*x^m + b*Log[c*x^n]^q
)), x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[e*n + d*m, 0] && EqQ
[a*f + b*d*(q - 1), 0]
```

Rubi steps

$$\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x\left(ax^m+b\log^q(cx^n)\right)^2}\,dx=\frac{d\log(cx^n)}{ax^m+b\log^q(cx^n)}$$

Mathematica [A] time = 0.356638, size = 26, normalized size = 1.

$$\frac{d\log(cx^n)}{ax^m + b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/
(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Maple [C] time = 0.128, size = 158, normalized size = 6.1

$$\frac{d\left(2\ \ln(c) + 2\ \ln(x^n) - i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n) + i\pi\,\mathrm{csgn}(ic)\,(\mathrm{csgn}(icx^n))^2 + i\pi\,\mathrm{csgn}(ix^n)\,(\mathrm{csgn}(icx^n))^2 - i\right.}{2\,ax^m + 2\,b\,(\ln(c) + \ln(x^n) - i/2\pi\,\mathrm{csgn}(icx^n)\,(-\mathrm{csgn}(icx^n) + \mathrm{csgn}(ic))\,(-\mathrm{csgn}(icx^n) + \mathrm{csgn}(ix^n)))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*n*x^m-a*d*m*x^m*ln(c*x^n)-b*d*n*(-1+q)*ln(c*x^n)^q)/x/(a*x^m+b*ln(
c*x^n)^q)^2,x)
```

```
[Out] 1/2*(2*ln(c)+2*ln(x^n)-I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*csgn(I
*c)*csgn(I*c*x^n)^2+I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*c*x^n)^3)*
d/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))
*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Maxima [A] time = 1.67436, size = 42, normalized size = 1.62

$$\frac{d\log(c) + d\log(x^n)}{ax^m + b(\log(c) + \log(x^n))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] (d*log(c) + d*log(x^n))/(a*x^m + b*(log(c) + log(x^n))^q)
```

Fricas [A] time = 1.93464, size = 80, normalized size = 3.08

$$\frac{dn\log(x) + d\log(c)}{(n\log(x) + \log(c))^q b + ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] (d*n*log(x) + d*log(c))/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x**m-a*d*m*x**m*ln(c*x**n)-b*d*n*(-1+q)*ln(c*x**n)**q)/x/(
a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bdn\left(q-1\right)\log(cx^n)^q + admx^m\log(cx^n) - adnx^m}{\left(ax^m + b\log(cx^n)^q\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(-(b*d*n*(q - 1)*log(c*x^n)^q + a*d*m*x^m*log(c*x^n) - a*d*n*x^m)/
((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

3.39 $\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$

Optimal. Leaf size=60

$$\frac{\log(cx^n)}{a\left(ax+b\log^q(cx^n)\right)}-\frac{n(1-q)\text{CannotIntegrate}\left(\frac{1}{x\left(ax+b\log^q(cx^n)\right)},x\right)}{a}$$

```
[Out] -((n*(1 - q)*CannotIntegrate[1/(x*(a*x + b*Log[c*x^n]^q)), x])/a) + Log[c*x
^n]/(a*(a*x + b*Log[c*x^n]^q))
```

Rubi [A] time = 0.152569, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^q)) - (n*(1 - q)*Defer[Int][1/(x*(a*x + b
*Log[c*x^n]^q)), x])/a
```

Rubi steps

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx=\frac{\log(cx^n)}{a\left(ax+b\log^q(cx^n)\right)}-\frac{(n(1-q))\int \frac{1}{x\left(ax+b\log^q(cx^n)\right)}\,dx}{a}$$

Mathematica [A] time = 76.6005, size = 0, normalized size = 0.

$$\int \frac{nq-\log(cx^n)}{\left(ax+b\log^q(cx^n)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2, x]
```

Maple [A] time = 0.368, size = 0, normalized size = 0.

$$\int \frac{nq - \ln(cx^n)}{\left(ax + b(\ln(cx^n))^q\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$n(q-1)\int \frac{1}{a^2x^2 + abx(\log(c) + \log(x^n))^q}\,dx + \frac{\log(c) + \log(x^n)}{a^2x + ab(\log(c) + \log(x^n))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] n*(q - 1)*integrate(1/(a^2*x^2 + a*b*x*(log(c) + log(x^n))^q), x) + (log(c)
 + log(x^n))/(a^2*x + a*b*(log(c) + log(x^n))^q)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{nq - \log(cx^n)}{a^2x^2 + 2\,abx\log(cx^n)^q + b^2\log(cx^n)^{2\,q}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] integral((n*q - log(c*x^n))/(a^2*x^2 + 2*a*b*x*log(c*x^n)^q + b^2*log(c*x^n
)^(2*q)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-ln(c*x**n))/(a*x+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{nq - \log(cx^n)}{\left(ax + b\log(cx^n)^q\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate((n*q - log(c*x^n))/(a*x + b*log(c*x^n)^q)^2, x)
```

3.40 $\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

```
[Out] -(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)])/(2
*e)
```

Rubi [A] time = 0.099256, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2447}

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)])/(2
*e)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] time = 0.432738, size = 625, normalized size = 12.76

$$\frac{2\text{PolyLog}\left(2,\frac{\sqrt{-d}+\sqrt{e}x}{\frac{\sqrt{e}}{\sqrt{-\frac{e}{d}}}+\sqrt{-d}}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{PolyLog}\left(2,\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{-d}\sqrt{e}x+d}{2d}\right)-2\text{PolyLog}\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d*Sqrt[-(e/d)] + e*x)/(Sqrt[-d]*
Sqrt[e] + d*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(e + d*(-(e/d)
)^(3/2)*x)/(e + Sqrt[-d]*Sqrt[e]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] - Sqrt[e]*
x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] + Sqrt[e]
*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] + 2*PolyLog[2, (Sqrt[-d]
+ Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e]/Sqrt[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]
*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2,
 (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2
)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[-d]*Sqrt[e] + d*Sqrt[-(e/d
)])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(2\,\frac{x}{ex^2+d}\left(ex+d\sqrt{-\frac{e}{d}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94846, size = 95, normalized size = 1.94

$$-\frac{\sqrt{-\frac{e}{d}}\operatorname{Li}_2\left(-\frac{2\left(ex^2+dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(-e/d)*dilog(-2*(e*x^2 + d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.41 $\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}+1\right)}{2e}$$

```
[Out] (Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2*
e)
```

Rubi [A] time = 0.0872988, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2447}

$$\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2*
e)
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] time = 0.366076, size = 642, normalized size = 12.84

$$-2\mathrm{PolyLog}\left(2,\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{-d}\sqrt{-\frac{e}{d}}+\sqrt{e}}\right)+2\mathrm{PolyLog}\left(2,\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{-d}\sqrt{-\frac{e}{d}}-\sqrt{e}}\right)+2\mathrm{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\mathrm{PolyLog}\left(2,\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[e]*(1 + Sqrt[-(e/d)]*x))/(S
qrt[e] - Sqrt[-d]*Sqrt[-(e/d)])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[e]
*(1 + Sqrt[-(e/d)]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d]
 - Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Log[Sqrt[-d]
 + Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*PolyLog[2,
(Sqrt[-(e/d)]*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] +
2*PolyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] + Sqrt[e]*x))/(-Sqrt[e] + Sqrt[-d]*Sqr
t[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sq
rt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2
*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(-2\frac{x}{ex^2+d}\left(-ex+d\sqrt{-\frac{e}{d}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95143, size = 93, normalized size = 1.86

$$\frac{\sqrt{-\frac{e}{d}}\operatorname{Li}_2\left(-\frac{2\left(ex^2-dx\sqrt{-\frac{e}{d}}\right)}{ex^2+d}+1\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(-e/d)*dilog(-2*(e*x^2 - d*x*sqrt(-e/d))/(e*x^2 + d) + 1)/e
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError
```

3.42 $\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{d+ex^2}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] -PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-
d]*Sqrt[e])
```

Rubi [A] time = 0.0964644, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{d+ex^2}+1\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-
d]*Sqrt[e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\text{Li}_2\left(1+\frac{2\sqrt{e}x\left(\sqrt{-d}-\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [B] time = 0.208992, size = 320, normalized size = 6.04

$$\frac{2\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)+2\text{PolyLog}\left(2,\frac{\sqrt{-d}\sqrt{e}x+d}{2d}\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\log\left(\frac{2\left(ex^2-\sqrt{-d}\sqrt{e}x\right)}{d+ex^2}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x
]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + 2*Log[(d*Sqrt[e]*
x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*
Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2)] - 2*L
og[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2
)] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*PolyLog[2, (d + Sqrt[-d]*Sq
rt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*S
qrt[e])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(2\,\frac{x}{ex^2+d}\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94567, size = 109, normalized size = 2.06

$$\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-\sqrt{-d}\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-d)*dilog(-2*(e*x^2 - sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqrt
(e))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] sage2
```

3.43 $\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=52

$$\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{-d}+\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-d
]*Sqrt[e])
```

Rubi [A] time = 0.0734183, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}x\left(\sqrt{-d}+\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-d
]*Sqrt[e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\text{Li}_2\left(1-\frac{2\sqrt{e}x\left(\sqrt{-d}+\sqrt{e}x\right)}{d+ex^2}\right)}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [B] time = 0.188232, size = 316, normalized size = 6.08

$$2\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{PolyLog}\left(2,\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)+2\log\left(\frac{2\left(\sqrt{-d}\sqrt{e}x+ex^2\right)}{d+ex^2}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),
x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - 2*
Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] - 2*Log[
Sqrt[-d] + Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] + 2
*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*
x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e]
)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(-2\frac{x}{ex^2+d}\left(-ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84334, size = 111, normalized size = 2.13

$$-\frac{\sqrt{-d}\operatorname{Li}_2\left(-\frac{2\left(ex^2+\sqrt{-d}\sqrt{e}x\right)}{ex^2+d}+1\right)}{2\,d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="fricas")
```

```
[Out] -1/2*sqrt(-d)*dilog(-2*(e*x^2 + sqrt(-d)*sqrt(e)*x)/(e*x^2 + d) + 1)/(d*sqr
t(e))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="giac")
```

```
[Out] sage2
```

3.44 $\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$\frac{\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt[
-e])
```

Rubi [A] time = 0.102114, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2447}

$$\frac{\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt[
-e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\text{Li}_2\left(1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] time = 0.310881, size = 641, normalized size = 13.08

$$2\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{PolyLog}\left(2,\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{-d}\sqrt{e}x+d}{2d}\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] + e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] + e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] +
 Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] + 2*PolyLog[2,
1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] +
2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*
x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[d]*Sqrt[-e] +
Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(-(Sqrt[d]*Sqrt[
-e]) + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(2\,\frac{x\left(ex+\sqrt{d}\sqrt{-e}\right)}{ex^2+d}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92881, size = 111, normalized size = 2.27

$$-\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2+\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="fricas")
```

```
[Out] -1/2*sqrt(-e)*dilog(-2*(e*x^2 + sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(
d)*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="giac")
```

```
[Out] Timed out
```

3.45 $\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\text{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] -PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt
[-e])
```

Rubi [A] time = 0.0783351, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2447}

$$-\frac{\text{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt
[-e])
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\text{Li}_2\left(1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] time = 0.251433, size = 645, normalized size = 12.9

$$2\text{PolyLog}\left(2,\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)-2\text{PolyLog}\left(2,\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\text{PolyLog}\left(2,\frac{\sqrt{-d}\sqrt{e}x+d}{2d}\right)-2\text{PolyLog}\left(2,\frac{d\sqrt{e}x}{(-d)^{3/2}}+1\right)-2\text{PolyLog}($$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] - e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] - e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d
] + Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] + 2*PolyL
og[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*
d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sq
rt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (-(Sqrt[-d]*Sqrt[e]) + e*x)/(Sqrt[d]*Sq
rt[-e] - Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(Sqrt[d
]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{ex^2+d}\ln\left(-2\,\frac{x\left(-ex+\sqrt{d}\sqrt{-e}\right)}{ex^2+d}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84806, size = 109, normalized size = 2.18

$$\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(ex^2-\sqrt{d}\sqrt{-e}x\right)}{ex^2+d}+1\right)}{2\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-e)*dilog(-2*(e*x^2 - sqrt(d)*sqrt(-e)*x)/(e*x^2 + d) + 1)/(sqrt(d
)*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{2\left(ex-\sqrt{d}\sqrt{-e}\right)x}{ex^2+d}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] integrate(log(2*(e*x - sqrt(d)*sqrt(-e))*x/(e*x^2 + d))/(e*x^2 + d), x)
```

3.46 $\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx$

Optimal. Leaf size=67

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\mathrm{Ei}((m+1)\log(dx))}{e(m+1)}$$

```
[Out] -((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
+ m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Rubi [A] time = 0.0572072, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\mathrm{Ei}((m+1)\log(dx))}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] -((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
+ m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int (ex)^m \left(a+b\log\left(c\log^p(dx)\right)\right)\,dx &= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{(bp)\int\frac{(ex)^m}{\log(dx)}\,dx}{1+m}\\
&= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}-\frac{\left(bp(dx)^{-1-m}(ex)^{1+m}\right)\operatorname{Subst}\left(\int\frac{e^{(1+m)x}}{x}\,dx,x,\log\right.}{e(1+m)}\\
&= -\frac{bp(dx)^{-1-m}(ex)^{1+m}\operatorname{Ei}((1+m)\log(dx))}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}
\end{aligned}
$$

Mathematica [A] time = 0.13092, size = 56, normalized size = 0.84

$$\frac{(dx)^{-m}(ex)^m\left(dx(dx)^m\left(a+b\log\left(c\log^p(dx)\right)\right)-bp\operatorname{Ei}((m+1)\log(dx))\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] ((e*x)^m*(-(b*p*ExpIntegralEi[(1 + m)*Log[d*x]]) + d*x*(d*x)^m*(a + b*Log[c
*Log[d*x]^p])))/(d*(1 + m)*(d*x)^m)
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int (ex)^m\left(a+b\ln\left(c\left(\ln(dx)\right)^p\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.05197, size = 204, normalized size = 3.04

$$\frac{bdpxe^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}\log\left(\log\left(dx\right)\right)-bp\left(\frac{e}{d}\right)^{m}\operatorname{Ei}\left((m+1)\log\left(dx\right)\right)+\left(bdx\log\left(c\right)+adx\right)e^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="fricas")
```

```
[Out] (b*d*p*x*e^(m*log(d*x) + m*log(e/d))*log(log(d*x)) - b*p*(e/d)^m*Ei((m + 1)
*log(d*x)) + (b*d*x*log(c) + a*d*x)*e^(m*log(d*x) + m*log(e/d)))/(d*m + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^{m}\left(a+b\log\left(c\log\left(dx\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x)**p)),x)
```

```
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x)**p)), x)
```

Giac [A] time = 1.31031, size = 112, normalized size = 1.67

$$\frac{bpxx^{m}e^{m}\log\left(\log\left(d\right)+\log\left(x\right)\right)}{m+1}+\frac{bxx^{m}e^{m}\log\left(c\right)}{m+1}+\frac{axx^{m}e^{m}}{m+1}-\frac{bp\operatorname{Ei}\left(m\log\left(d\right)+m\log\left(x\right)+\log\left(d\right)+\log\left(x\right)\right)e^{m}}{dd^{m}m+dd^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="giac")
```

```
[Out] b*p*x*x^m*e^m*log(log(d) + log(x))/(m + 1) + b*x*x^m*e^m*log(c)/(m + 1) + a
*x*x^m*e^m/(m + 1) - b*p*Ei(m*log(d) + m*log(x) + log(d) + log(x))*e^m/(d*d
^m*m + d*d^m)
```

3.47 $\int (ex)^m \left(a + b\log\left(c\log^p\left(dx^n\right)\right)\right)\, dx$

Optimal. Leaf size=79

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1}\left(dx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)\log\left(dx^n\right)}{n}\right)}{e(m+1)}$$

```
[Out] -((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*x
^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m))
```

Rubi [A] time = 0.0608507, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2522, 2310, 2178}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1}\left(dx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)\log\left(dx^n\right)}{n}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*x
^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m))
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx = \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)} - \frac{(bnp)\int \frac{(ex)^m}{\log(dx^n)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)} - \frac{\left(bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{x}\,dx, x\right.}{e(1+m)}$$

$$= -\frac{bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)} + \frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}$$

Mathematica [A] time = 0.156762, size = 59, normalized size = 0.75

$$\frac{x(ex)^m\left(a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{-\frac{m+1}{n}}\text{Ei}\left(\frac{(m+1)\log(dx^n)}{n}\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x*(e*x)^m*(a - (b*p*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(d*x^n)^((1 + m
)/n) + b*Log[c*Log[d*x^n]^p]))/(1 + m)
```

Maple [F] time = 0.185, size = 0, normalized size = 0.

$$\int (ex)^m\left(a+b\ln\left(c\left(\ln\left(dx^n\right)\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97843, size = 261, normalized size = 3.3

$$\frac{bpxe^{(m\log(e)+m\log(x))}\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\text{Ei}\left(\frac{(m+1)n\log(x)+(m+1)\log(d)}{n}\right)e^{\left(\frac{mn\log(e)-(m+1)\log(d)}{n}\right)}+(bx\log\left(c\right)+ax)e^{(m\log(e}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(m*log(e) + m*log(x))*log(n*log(x) + log(d)) - b*p*Ei(((m + 1)*n*l
og(x) + (m + 1)*log(d))/n)*e^((m*n*log(e) - (m + 1)*log(d))/n) + (b*x*log(c
) + a*x)*e^(m*log(e) + m*log(x)))/(m + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ex)^m \left(a+b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A] time = 1.42738, size = 150, normalized size = 1.9

$$\frac{bpxx^me^m\log\left(n\log\left(x\right)+\log\left(d\right)\right)}{m+1}-\frac{bnp\text{Ei}\left(m\log\left(x\right)+\frac{m\log(d)}{n}+\frac{\log(d)}{n}+\log\left(x\right)\right)e^m}{d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}mn+d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}n}+\frac{bxx^me^m\log\left(c\right)}{m+1}+\frac{axx^me^m}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] b*p*x*x^m*e^m*log(n*log(x) + log(d))/(m + 1) - b*n*p*Ei(m*log(x) + m*log(d)
/n + log(d)/n + log(x))*e^m/(d^(m/n)*d^(1/n)*m*n + d^(m/n)*d^(1/n)*n) + b*x
*x^m*e^m*log(c)/(m + 1) + a*x*x^m*e^m/(m + 1)
```

3.48 $\int x^2 \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx$

Optimal. Leaf size=55

$$\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}bpx^3\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)$$

```
[Out] -(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(3*(d*x^n)^(3/n)) + (x^3*(a + b*
Log[c*Log[d*x^n]^p]))/3
```

Rubi [A] time = 0.0487893, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{1}{3}x^3\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{3}bpx^3\left(dx^n\right)^{-3/n}\text{Ei}\left(\frac{3\log\left(dx^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(3*(d*x^n)^(3/n)) + (x^3*(a + b*
Log[c*Log[d*x^n]^p]))/3
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int x^2 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right) \, dx = \frac{1}{3} x^3 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{3} (b n p) \int \frac{x^2}{\log \left(d x^n\right)} \, dx$$
$$= \frac{1}{3} x^3 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)-\frac{1}{3} \left(b p x^3 \left(d x^n\right)^{-3/n}\right) \text{Subst}\left(\int \frac{e^{\frac{3 x}{n}}}{x} \, dx,x,\log \left(d x^n\right)\right)$$
$$= -\frac{1}{3} b p x^3 \left(d x^n\right)^{-3/n} \text{Ei}\left(\frac{3 \log \left(d x^n\right)}{n}\right)+\frac{1}{3} x^3 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)\right)$$

Mathematica [A] time = 0.05883, size = 49, normalized size = 0.89

$$\frac{1}{3} x^3 \left(a+b \log \left(c \log ^p\left(d x^n\right)\right)-b p \left(d x^n\right)^{-3/n} \text{Ei}\left(\frac{3 \log \left(d x^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x^3*(a - (b*p*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + b*Log[c*Log
[d*x^n]^p]))/3
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int x^2 \left(a+b \ln \left(c \left(\ln \left(d x^n\right)\right)^p\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a x^3+\frac{1}{3} \left(x^3 \log (c)+x^3 \log \left(\left(\log (d)+\log \left(x^n\right)\right)^p\right)-3 n p \int \frac{x^2}{3 \left(\log (d)+\log \left(x^n\right)\right)} \, dx\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/3*(x^3*log(c) + x^3*log((log(d) + log(x^n))^p) - 3*n*p*integr
ate(1/3*x^2/(log(d) + log(x^n)), x))*b
```

Fricas [A] time = 1.87874, size = 161, normalized size = 2.93

$$\frac{bd^{\frac{3}{n}}px^3\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\operatorname{log\_integral}\left(d^{\frac{3}{n}}x^3\right)+\left(bx^3\log\left(c\right)+ax^3\right)d^{\frac{3}{n}}}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log(n*log(x) + log(d)) - b*p*log_integral(d^(3/n)*x^3)
 + (b*x^3*log(c) + a*x^3)*d^(3/n))/d^(3/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A] time = 1.37692, size = 76, normalized size = 1.38

$$\frac{1}{3}\,bpx^3\log\left(n\log\left(x\right)+\log\left(d\right)\right)+\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3-\frac{bp\operatorname{Ei}\left(\frac{3\,\log(d)}{n}+3\,\log\left(x\right)\right)}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/3*b*p*x^3*log(n*log(x) + log(d)) + 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/3*b*p
*Ei(3*log(d)/n + 3*log(x))/d^(3/n)
```

3.49 $\int x \left(a+b\log \left(c\log^p\left(dx^n\right)\right)\right) \, dx$

Optimal. Leaf size=55

$$\frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{2}bpx^2\left(dx^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2\log\left(dx^n\right)}{n}\right)$$

```
[Out] -(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(2*(d*x^n)^(2/n)) + (x^2*(a + b*
Log[c*Log[d*x^n]^p]))/2
```

Rubi [A] time = 0.0349517, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2522, 2310, 2178}

$$\frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{2}bpx^2\left(dx^n\right)^{-2/n}\operatorname{Ei}\left(\frac{2\log\left(dx^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(2*(d*x^n)^(2/n)) + (x^2*(a + b*
Log[c*Log[d*x^n]^p]))/2
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)dx &= \frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{2}(bnp)\int\frac{x}{\log\left(dx^n\right)}dx\\ &= \frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)-\frac{1}{2}\left(bpx^2\left(dx^n\right)^{-2/n}\right)\mathrm{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{x}dx,x,\log\left(dx^n\right)\right)\\ &= -\frac{1}{2}bpx^2\left(dx^n\right)^{-2/n}\mathrm{Ei}\left(\frac{2\log\left(dx^n\right)}{n}\right)+\frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0567191, size = 49, normalized size = 0.89

$$\frac{1}{2}x^2\left(a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{-2/n}\mathrm{Ei}\left(\frac{2\log\left(dx^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x^2*(a - (b*p*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + b*Log[c*Log
[d*x^n]^p]))/2
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int x\left(a+b\ln\left(c\left(\ln\left(dx^n\right)\right)^p\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ax^2-\frac{1}{2}\left(2np\int\frac{x}{2\left(\log\left(d\right)+\log\left(x^n\right)\right)}dx-x^2\log\left(c\right)-x^2\log\left(\left(\log\left(d\right)+\log\left(x^n\right)\right)^p\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 1/2*(2*n*p*integrate(1/2*x/(log(d) + log(x^n)), x) - x^2*log(c)
 - x^2*log((log(d) + log(x^n))^p))*b
```

Fricas [A] time = 1.85521, size = 161, normalized size = 2.93

$$\frac{bd^{\frac{2}{n}}px^2\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\text{log\_integral}\left(d^{\frac{2}{n}}x^2\right)+\left(bx^2\log\left(c\right)+ax^2\right)d^{\frac{2}{n}}}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log(n*log(x) + log(d)) - b*p*log_integral(d^(2/n)*x^2)
 + (b*x^2*log(c) + a*x^2)*d^(2/n))/d^(2/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+b\log\left(c\log\left(dx^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A] time = 1.31949, size = 76, normalized size = 1.38

$$\frac{1}{2}\,bpx^2\log\left(n\log\left(x\right)+\log\left(d\right)\right)+\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{2}\,ax^2-\frac{bp\text{Ei}\left(\frac{2\,\log(d)}{n}+2\,\log\left(x\right)\right)}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/2*b*p*x^2*log(n*log(x) + log(d)) + 1/2*b*x^2*log(c) + 1/2*a*x^2 - 1/2*b*p
*Ei(2*log(d)/n + 2*log(x))/d^(2/n)
```

3.50 $\int \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx$

Optimal. Leaf size=45

$$ax+bx\log\left(c\log^p\left(dx^n\right)\right)-bpx\left(dx^n\right)^{-1/n}\operatorname{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)$$

```
[Out] a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d*
x^n]^p]
```

Rubi [A] time = 0.0300297, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2520, 2300, 2178}

$$ax+bx\log\left(c\log^p\left(dx^n\right)\right)-bpx\left(dx^n\right)^{-1/n}\operatorname{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d*
x^n]^p]
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)dx = ax+b\int \log\left(c\log^p\left(dx^n\right)\right)dx$$
$$= ax+bx\log\left(c\log^p\left(dx^n\right)\right)-(bnp)\int \frac{1}{\log\left(dx^n\right)}dx$$
$$= ax+bx\log\left(c\log^p\left(dx^n\right)\right)-\left(bpx\left(dx^n\right)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}dx,x,\log\left(dx^n\right)\right)$$
$$= ax-bpx\left(dx^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)+bx\log\left(c\log^p\left(dx^n\right)\right)$$

Mathematica [A] time = 0.0478073, size = 43, normalized size = 0.96

$$x\left(a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] x*(a - (b*p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*Log[c*Log[d*x^n
]^p])
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int a+b\ln\left(c\left(\ln\left(dx^n\right)\right)^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*ln(d*x^n)^p),x)
```

```
[Out] int(a+b*ln(c*ln(d*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(np\int \frac{1}{\log(d)+\log\left(x^n\right)}dx-x\log(c)-x\log\left(\left(\log(d)+\log\left(x^n\right)\right)^p\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="maxima")
```

```
[Out] -(n*p*integrate(1/(log(d) + log(x^n)), x) - x*log(c) - x*log((log(d) + log(
x^n))^p))*b + a*x
```

Fricas [A] time = 1.97502, size = 144, normalized size = 3.2

$$\frac{bd^{\left(\frac{1}{n}\right)}px\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\mathrm{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)+\left(bx\log\left(c\right)+ax\right)d^{\left(\frac{1}{n}\right)}}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log(n*log(x) + log(d)) - b*p*log_integral(d^(1/n)*x) + (b*x*
log(c) + a*x)*d^(1/n))/d^(1/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*ln(d*x**n)**p),x)
```

```
[Out] Integral(a + b*log(c*log(d*x**n)**p), x)
```

Giac [A] time = 1.32369, size = 57, normalized size = 1.27

$$\left(px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+x\log\left(c\right)-\frac{p\mathrm{Ei}\left(\frac{\log(d)}{n}+\log\left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="giac")
```

```
[Out] (p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n))*b
 + a*x
```

3.51 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}-bp\log(x)$$

```
[Out] -(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n
```

Rubi [A] time = 0.0171733, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2521}

$$\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}-bp\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

```
[Out] -(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x}\,dx=-bp\log(x)+\frac{\log(dx^n)\left(a+b\log\left(c\log^p(dx^n)\right)\right)}{n}$$

Mathematica [A] time = 0.0113264, size = 40, normalized size = 1.25

$$a\log(x)+\frac{b\log(dx^n)\log\left(c\log^p(dx^n)\right)}{n}-\frac{bp\log(dx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x,x]
```

```
[Out] a*Log[x] - (b*p*Log[d*x^n])/n + (b*Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Maple [A] time = 0.01, size = 48, normalized size = 1.5

$$\frac{\ln(dx^n)\ln\left(c(\ln(dx^n))^p\right)b}{n}-\frac{\ln(dx^n)\,bp}{n}+\frac{\ln(dx^n)\,a}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x,x)
```

```
[Out] 1/n*ln(c*ln(d*x^n)^p)*ln(d*x^n)*b-1/n*ln(d*x^n)*b*p+1/n*a*ln(d*x^n)
```

Maxima [A] time = 1.02756, size = 86, normalized size = 2.69

$$b\log\left(c\log(dx^n)^p\right)\log(x)-\left(p\log(x)\log(\log(dx^n))-\frac{(\log(dx^n)\log(\log(dx^n))-\log(dx^n))p}{n}\right)b+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="maxima")
```

```
[Out] b*log(c*log(d*x^n)^p)*log(x) - (p*log(x)*log(log(d*x^n)) - (log(d*x^n)*log(
log(d*x^n)) - log(d*x^n))*p/n)*b + a*log(x)
```

Fricas [A] time = 1.8738, size = 124, normalized size = 3.88

$$\frac{\left(bnp\log(x)+bp\log(d)\right)\log(n\log(x)+\log(d))-\left(bnp-bn\log(c)-an\right)\log(x)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="fricas")
```

```
[Out] ((b*n*p*log(x) + b*p*log(d))*log(n*log(x) + log(d)) - (b*n*p - b*n*log(c) -
 a*n)*log(x))/n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \log \left(c \log \left(dx^{n}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x, x)
```

Giac [A] time = 1.30007, size = 73, normalized size = 2.28

$$\frac{\left((n \log(x) + \log(d)) \log(n \log(x) + \log(d)) - n \log(x) - \log(d)\right)bp + (n \log(x) + \log(d))b \log(c) + (n \log(x) + \log(d))}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="giac")
```

```
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*b*p + (n*
log(x) + log(d))*b*log(c) + (n*log(x) + log(d))*a)/n
```

3.52 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{x}$$

```
[Out] (b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d*
x^n]^p])/x
```

Rubi [A] time = 0.046958, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{\frac{1}{n}}\,\mathrm{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

```
[Out] (b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d*
x^n]^p])/x
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^2}\,dx = -\frac{a+b\log\left(c\log^p(dx^n)\right)}{x}+(bnp)\int \frac{1}{x^2\log(dx^n)}\,dx$$

$$= -\frac{a+b\log\left(c\log^p(dx^n)\right)}{x}+\frac{\left(bp(dx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{x}\,dx,x,\log(dx^n)\right)}{x}$$

$$= \frac{bp(dx^n)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{x}$$

Mathematica [A] time = 0.0405497, size = 45, normalized size = 0.94

$$-\frac{a+b\log\left(c\log^p(dx^n)\right)-bp(dx^n)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]

[Out] -((a - b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)] + b*Log[c*Log[d*x^
n]^p])/x)
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c(\ln(dx^n))^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)

[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(np\int \frac{1}{x^2\log(d)+x^2\log(x^n)}\,dx-\frac{\log(c)+\log\left((\log(d)+\log(x^n))^p\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="maxima")
```

```
[Out] (n*p*integrate(1/(x^2*log(d) + x^2*log(x^n)), x) - (log(c) + log((log(d) +
log(x^n))^p))/x)*b - a/x
```

Fricas [A] time = 1.89519, size = 123, normalized size = 2.56

$$\frac{bd^{\left(\frac{1}{n}\right)}px\,\mathrm{log\_integral}\left(\frac{1}{d^{\left(\frac{1}{n}\right)}x}\right)-bp\log(n\log(x)+\log(d))-b\log(c)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log_integral(1/(d^(1/n)*x)) - b*p*log(n*log(x) + log(d)) - b
*log(c) - a)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\log\left(c\log(dx^n)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**2,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(c\log\left(dx^n\right)^p\right)+a}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^2, x)
```

3.53 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{bp\,(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

```
[Out] (b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[c
*Log[d*x^n]^p])/(2*x^2)
```

Rubi [A] time = 0.0462649, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] (b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[c
*Log[d*x^n]^p])/(2*x^2)
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{1}{2}(bnp)\int \frac{1}{x^3\log\left(dx^n\right)}\,dx$$

$$= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{\left(bp\left(dx^n\right)^{2/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{2x^2}$$

$$= \frac{bp\left(dx^n\right)^{2/n}\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

Mathematica [A] time = 0.0423787, size = 49, normalized size = 0.89

$$-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{2/n}\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] -(a - b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n] + b*Log[c*Log[d*x^
n]^p])/(2*x^2)
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(\ln\left(dx^n\right)\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(2\,np\int\frac{1}{2\left(x^3\log\left(d\right)+x^3\log\left(x^n\right)\right)}\,dx-\frac{\log\left(c\right)+\log\left(\left(\log\left(d\right)+\log\left(x^n\right)\right)^p\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*n*p*integrate(1/2/(x^3*log(d) + x^3*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^2)*b - 1/2*a/x^2
```

Fricas [A] time = 1.71174, size = 136, normalized size = 2.47

$$\frac{bd^{\frac{2}{n}}px^2\,\mathrm{log\_integral}\left(\frac{1}{d^{\frac{2}{n}}x^2}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log_integral(1/(d^(2/n)*x^2)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\log\left(c\log\left(dx^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**3,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \log \left(c \log \left(dx^n\right)^p\right) + a}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^3, x)
```

3.54 $\int \frac{a+b\log\left(c\log^p(dx^n)\right)}{x^4}\,dx$

Optimal. Leaf size=55

$$\frac{bp\,(dx^n)^{3/n}\,\text{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

```
[Out] (b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[c
*Log[d*x^n]^p])/(3*x^3)
```

Rubi [A] time = 0.0466476, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2522, 2310, 2178}

$$\frac{bp\,(dx^n)^{3/n}\,\text{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p(dx^n)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] (b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[c
*Log[d*x^n]^p])/(3*x^3)
```

Rule 2522

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(m + 1
)), x] - Dist[(b*n*p)/(m + 1), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^4}\,dx &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}+\frac{1}{3}(bnp)\int \frac{1}{x^4\log\left(dx^n\right)}\,dx\\ &= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}+\frac{\left(bp\left(dx^n\right)^{3/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{3x^3}\\ &= \frac{bp\left(dx^n\right)^{3/n}\text{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.0433396, size = 49, normalized size = 0.89

$$-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)-bp\left(dx^n\right)^{3/n}\text{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] -(a - b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n] + b*Log[c*Log[d*x^
n]^p])/(3*x^3)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{a+b\ln\left(c\left(\ln\left(dx^n\right)\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}\left(3\,np\int\frac{1}{3\left(x^4\log\left(d\right)+x^4\log\left(x^n\right)\right)}\,dx-\frac{\log\left(c\right)+\log\left(\left(\log\left(d\right)+\log\left(x^n\right)\right)^p\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*n*p*integrate(1/3/(x^4*log(d) + x^4*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^3)*b - 1/3*a/x^3
```

Fricas [A] time = 1.53899, size = 136, normalized size = 2.47

$$\frac{bd^{\frac{3}{n}}px^3\operatorname{log\_integral}\left(\frac{1}{d^{\frac{3}{n}}x^3}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log_integral(1/(d^(3/n)*x^3)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\log\left(c\log\left(dx^n\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**4,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\log\left(c\log\left(dx^n\right)^p\right)+a}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^4, x)
```

3.55 $\int \log \left(c \log ^p(dx)\right) dx$

Optimal. Leaf size=22

$$x \log \left(c \log ^p(dx)\right)-\frac{p \operatorname{li}(dx)}{d}$$

```
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rubi [A] time = 0.0067795, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2520, 2298}

$$x \log \left(c \log ^p(dx)\right)-\frac{p \operatorname{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p],x]
```

```
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\int \log \left(c \log ^p(dx)\right) dx &= x \log \left(c \log ^p(dx)\right)-p \int \frac{1}{\log (dx)} dx\\ &= x \log \left(c \log ^p(dx)\right)-\frac{p \operatorname{li}(dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0140017, size = 22, normalized size = 1.

$$x \log \left(c \log^p(dx)\right) - \frac{p\text{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p],x]
```

```
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Maple [A] time = 0.009, size = 26, normalized size = 1.2

$$x \ln \left(c (\ln (dx))^p\right) + \frac{p\text{Ei} (1, -\ln (dx))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p),x)
```

```
[Out] x*ln(c*ln(d*x)^p)+p/d*Ei(1,-ln(d*x))
```

Maxima [A] time = 1.14172, size = 31, normalized size = 1.41

$$x \log \left(c \log (dx)^p\right) - \frac{p\text{Ei} (\log (dx))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="maxima")
```

```
[Out] x*log(c*log(d*x)^p) - p*Ei(log(d*x))/d
```

Fricas [A] time = 1.64578, size = 81, normalized size = 3.68

$$\frac{dpx \log (\log (dx)) + dx \log (c) - p \log\_integral (dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="fricas")
```

```
[Out] (d*p*x*log(log(d*x)) + d*x*log(c) - p*log_integral(d*x))/d
```

Sympy [A] time = 1.31437, size = 19, normalized size = 0.86

$$x\log\left(c\log\left(dx\right)^{p}\right)-\frac{p\operatorname{li}\left(dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p),x)
```

```
[Out] x*log(c*log(d*x)**p) - p*li(d*x)/d
```

Giac [A] time = 1.20494, size = 35, normalized size = 1.59

$$px\log\left(\log\left(d\right)+\log\left(x\right)\right)+x\log\left(c\right)-\frac{p\operatorname{Ei}\left(\log\left(d\right)+\log\left(x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="giac")
```

```
[Out] p*x*log(log(d) + log(x)) + x*log(c) - p*Ei(log(d) + log(x))/d
```

3.56 $\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx$

Optimal. Leaf size=20

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(x)$$

```
[Out] -(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rubi [A] time = 0.0198224, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2521}

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p]/x,x]
```

```
[Out] -(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx=-p\log(x)+\log(dx)\log\left(c\log^p(dx)\right)$$

Mathematica [A] time = 0.0065151, size = 22, normalized size = 1.1

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p]/x,x]
```

```
[Out] -(p*Log[d*x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Maple [A] time = 0.006, size = 23, normalized size = 1.2

$$\ln(dx)\ln\left(c(\ln(dx))^p\right) - p\ln(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p)/x,x)
```

```
[Out] ln(d*x)*ln(c*ln(d*x)^p)-p*ln(d*x)
```

Maxima [A] time = 0.993782, size = 30, normalized size = 1.5

$$-p\log(dx) + \log(dx)\log\left(c\log(dx)^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="maxima")
```

```
[Out] -p*log(d*x) + log(d*x)*log(c*log(d*x)^p)
```

Fricas [A] time = 1.63894, size = 68, normalized size = 3.4

$$p\log(dx)\log(\log(dx)) - \left(p - \log(c)\right)\log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="fricas")
```

```
[Out] p*log(d*x)*log(log(d*x)) - (p - log(c))*log(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c \log \left(dx\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p)/x,x)
```

```
[Out] Integral(log(c*log(d*x)**p)/x, x)
```

Giac [A] time = 1.30266, size = 43, normalized size = 2.15

$$((\log(d) + \log(x))\log(\log(d) + \log(x)) - \log(d) - \log(x))p + (\log(d) + \log(x))\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="giac")
```

```
[Out] ((log(d) + log(x))*log(log(d) + log(x)) - log(d) - log(x))*p + (log(d) + lo
g(x))*log(c)
```

3.57 $\int \log\left(c\log^p\left(dx^n\right)\right)dx$

Optimal. Leaf size=40

$$x\log\left(c\log^p\left(dx^n\right)\right)-px\left(dx^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)$$

```
[Out] -((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p]
```

Rubi [A] time = 0.0220154, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2520, 2300, 2178}

$$x\log\left(c\log^p\left(dx^n\right)\right)-px\left(dx^n\right)^{-1/n}\text{Ei}\left(\frac{\log\left(dx^n\right)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p],x]
```

```
[Out] -((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p]
```

Rule 2520

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \log\left(c\log^p(dx^n)\right)dx = x\log\left(c\log^p(dx^n)\right) - (np)\int \frac{1}{\log(dx^n)}dx$$
$$= x\log\left(c\log^p(dx^n)\right) - \left(px(dx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}dx, x, \log(dx^n)\right)$$
$$= -px(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right) + x\log\left(c\log^p(dx^n)\right)$$

Mathematica [A] time = 0.020484, size = 39, normalized size = 0.98

$$x\left(\log\left(c\log^p(dx^n)\right) - p(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p],x]
```

```
[Out] x*(-((p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + Log[c*Log[d*x^n]^p])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \ln\left(c(\ln(dx^n))^p\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p),x)
```

```
[Out] int(ln(c*ln(d*x^n)^p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-np\int \frac{1}{\log(d) + \log(x^n)}dx + x\log(c) + x\log\left((\log(d) + \log(x^n))^p\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="maxima")
```

```
[Out] -n*p*integrate(1/(log(d) + log(x^n)), x) + x*log(c) + x*log((log(d) + log(x
^n))^p)
```

Fricas [A] time = 1.63322, size = 126, normalized size = 3.15

$$\frac{d^{\left(\frac{1}{n}\right)}px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+d^{\left(\frac{1}{n}\right)}x\log\left(c\right)-p\operatorname{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="fricas")
```

```
[Out] (d^(1/n)*p*x*log(n*log(x) + log(d)) + d^(1/n)*x*log(c) - p*log_integral(d^(
1/n)*x))/d^(1/n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(c\log\left(dx^{n}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p),x)
```

```
[Out] Integral(log(c*log(d*x**n)**p), x)
```

Giac [A] time = 1.21386, size = 49, normalized size = 1.22

$$px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+x\log\left(c\right)-\frac{p\operatorname{Ei}\left(\frac{\log(d)}{n}+\log\left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="giac")
```

```
[Out] p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n)
```

3.58 $\int \frac{\log\left(c\log^p(dx^n)\right)}{x}\,dx$

Optimal. Leaf size=27

$$\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}-p\log(x)$$

```
[Out] -(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rubi [A] time = 0.0211121, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2521}

$$\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p]/x,x]
```

```
[Out] -(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=-p\log(x)+\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

Mathematica [A] time = 0.0101253, size = 34, normalized size = 1.26

$$\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}-\frac{p\log\left(dx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p]/x,x]
```

```
[Out] -((p*Log[d*x^n])/n) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Maple [A] time = 0.006, size = 35, normalized size = 1.3

$$\frac{\ln\left(c\left(\ln\left(dx^{n}\right)\right)^{p}\right)\ln\left(dx^{n}\right)}{n}-\frac{p\ln\left(dx^{n}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p)/x,x)
```

```
[Out] ln(d*x^n)*ln(c*ln(d*x^n)^p)/n-1/n*p*ln(d*x^n)
```

Maxima [B] time = 1.01719, size = 74, normalized size = 2.74

$$-p\log\left(x\right)\log\left(\log\left(dx^{n}\right)\right)+\log\left(c\log\left(dx^{n}\right)^{p}\right)\log\left(x\right)+\frac{\left(\log\left(dx^{n}\right)\log\left(\log\left(dx^{n}\right)\right)-\log\left(dx^{n}\right)\right)p}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -p*log(x)*log(log(d*x^n)) + log(c*log(d*x^n)^p)*log(x) + (log(d*x^n)*log(lo
g(d*x^n)) - log(d*x^n))*p/n
```

Fricas [A] time = 1.52428, size = 105, normalized size = 3.89

$$\frac{\left(np\log\left(x\right)+p\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right)-\left(np-n\log\left(c\right)\right)\log\left(x\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] ((n*p*log(x) + p*log(d))*log(n*log(x) + log(d)) - (n*p - n*log(c))*log(x))/
n
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(c \log \left(dx^{n}\right)^{p}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*log(d*x**n)**p)/x, x)
```

Giac [A] time = 1.36748, size = 58, normalized size = 2.15

$$\frac{\left(\left(n \log\left(x\right) + \log\left(d\right)\right) \log\left(n \log\left(x\right) + \log\left(d\right)\right) - n \log\left(x\right) - \log\left(d\right)\right)p + \left(n \log\left(x\right) + \log\left(d\right)\right) \log\left(c\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*p + (n*lo
g(x) + log(d))*log(c))/n
```

3.59 $\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}\log\left(d\left(bx+cx^2\right)^n\right)}{m+1}+\frac{nx^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{cx}{b}\right)}{(m+1)^2}-\frac{2nx^{m+1}}{(m+1)^2}$$

```
[Out] (-2*n*x^(1 + m))/(1 + m)^2 + (n*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)^2 + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1 + m)
```

Rubi [A] time = 0.059421, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2525, 80, 64}

$$\frac{x^{m+1}\log\left(d\left(bx+cx^2\right)^n\right)}{m+1}+\frac{nx^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{cx}{b}\right)}{(m+1)^2}-\frac{2nx^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (-2*n*x^(1 + m))/(1 + m)^2 + (n*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)^2 + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1 + m)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^m \log\left(d\left(bx+cx^2\right)^n\right)dx &= \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^m(b+2cx)}{b+cx}\,dx}{1+m}\\ &= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} + \frac{(bn)\int \frac{x^m}{b+cx}\,dx}{1+m}\\ &= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.0209516, size = 48, normalized size = 0.73

$$\frac{x^{m+1}\left((m+1)\log\left(d(x(b+cx))^n\right)+n\,{}_2F_1\left(1,m+1;m+2;-\frac{cx}{b}\right)-2n\right)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (x^(1 + m)*(-2*n + n*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*x)/b)] + (1 +
m)*Log[d*(x*(b + c*x))^n]))/(1 + m)^2
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int x^m \ln\left(d\left(cx^2+bx\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m\log\left(\left(cx^2+bx\right)^nd\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*log((c*x^2 + b*x)^n*d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^m\log\left(d\left(bx+cx^2\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Integral(x**m*log(d*(b*x + c*x**2)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \log \left(\left(cx^2 + bx \right)^n d \right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*log((c*x^2 + b*x)^n*d), x)
```

3.60 $\int x^4 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=99

$$\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}-\frac{b^4nx}{5c^4}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

```
[Out] -(b^4*n*x)/(5*c^4) + (b^3*n*x^2)/(10*c^3) - (b^2*n*x^3)/(15*c^2) + (b*n*x^4
)/(20*c) - (2*n*x^5)/25 + (b^5*n*Log[b + c*x])/(5*c^5) + (x^5*Log[d*(b*x +
c*x^2)^n])/5
```

Rubi [A] time = 0.0731365, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}-\frac{b^4nx}{5c^4}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -(b^4*n*x)/(5*c^4) + (b^3*n*x^2)/(10*c^3) - (b^2*n*x^3)/(15*c^2) + (b*n*x^4
)/(20*c) - (2*n*x^5)/25 + (b^5*n*Log[b + c*x])/(5*c^5) + (x^5*Log[d*(b*x +
c*x^2)^n])/5
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int x^4 \log\left(d\left(bx+cx^2\right)^n\right) dx &= \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{5}n\int \frac{x^4(b+2cx)}{b+cx}\,dx \\
&= \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{5}n\int\left(\frac{b^4}{c^4}-\frac{b^3x}{c^3}+\frac{b^2x^2}{c^2}-\frac{bx^3}{c}+2x^4-\frac{b^5}{c^4(b+cx)}\right)dx \\
&= -\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right)
\end{aligned}
$$

Mathematica [A] time = 0.0484504, size = 85, normalized size = 0.86

$$
\frac{cnx\left(-20b^2c^2x^2+30b^3cx-60b^4+15bc^3x^3-24c^4x^4\right)+60b^5n\log(b+cx)+60c^5x^5\log\left(d(x(b+cx))^n\right)}{300c^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c^2*x^2 + 15*b*c^3*x^3 - 24*c^4*x^4)
+ 60*b^5*n*Log[b + c*x] + 60*c^5*x^5*Log[d*(x*(b + c*x))^n])/(300*c^5)
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$
\int x^4\ln\left(d\left(cx^2+bx\right)^n\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A] time = 1.04288, size = 117, normalized size = 1.18

$$
\frac{1}{5}x^5\log\left(\left(cx^2+bx\right)^n d\right)+\frac{1}{300}n\left(\frac{60\,b^5\log(cx+b)}{c^5}-\frac{24\,c^4x^5-15\,bc^3x^4+20\,b^2c^2x^3-30\,b^3cx^2+60\,b^4x}{c^4}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((c*x^2 + b*x)^n*d) + 1/300*n*(60*b^5*log(c*x + b)/c^5 - (24*c^4
*x^5 - 15*b*c^3*x^4 + 20*b^2*c^2*x^3 - 30*b^3*c*x^2 + 60*b^4*x)/c^4)
```

Fricas [A] time = 1.6118, size = 232, normalized size = 2.34

$$\frac{60\,c^5nx^5\log\left(cx^2+bx\right)-24\,c^5nx^5+60\,c^5x^5\log\left(d\right)+15\,bc^4nx^4-20\,b^2c^3nx^3+30\,b^3c^2nx^2-60\,b^4cnx+60\,b^5n\log\left(cx+\right.}{300\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/300*(60*c^5*n*x^5*log(c*x^2 + b*x) - 24*c^5*n*x^5 + 60*c^5*x^5*log(d) + 1
5*b*c^4*n*x^4 - 20*b^2*c^3*n*x^3 + 30*b^3*c^2*n*x^2 - 60*b^4*c*n*x + 60*b^5
*n*log(c*x + b))/c^5
```

Sympy [A] time = 14.2238, size = 134, normalized size = 1.35

$$\begin{cases} \frac{b^5n\log\left(b+cx\right)}{5c^5}-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}+\frac{nx^5\log\left(bx+cx^2\right)}{5}-\frac{2nx^5}{25}+\frac{x^5\log\left(d\right)}{5} & \text{for}\ c\neq 0 \\ \frac{nx^5\log\left(b\right)}{5}+\frac{nx^5\log\left(x\right)}{5}-\frac{nx^5}{25}+\frac{x^5\log\left(d\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**5*n*log(b + c*x)/(5*c**5) - b**4*n*x/(5*c**4) + b**3*n*x**2/(
10*c**3) - b**2*n*x**3/(15*c**2) + b*n*x**4/(20*c) + n*x**5*log(b*x + c*x**
2)/5 - 2*n*x**5/25 + x**5*log(d)/5, Ne(c, 0)), (n*x**5*log(b)/5 + n*x**5*lo
g(x)/5 - n*x**5/25 + x**5*log(d)/5, True))
```

Giac [A] time = 1.30019, size = 120, normalized size = 1.21

$$\frac{1}{5}\,nx^5\log\left(cx^2+bx\right)-\frac{1}{25}\,(2\,n-5\,\log\left(d\right))x^5+\frac{bnx^4}{20\,c}-\frac{b^2nx^3}{15\,c^2}+\frac{b^3nx^2}{10\,c^3}-\frac{b^4nx}{5\,c^4}+\frac{b^5n\log\left(cx+b\right)}{5\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c - 1
/15*b^2*n*x^3/c^2 + 1/10*b^3*n*x^2/c^3 - 1/5*b^4*n*x/c^4 + 1/5*b^5*n*log(c*
x + b)/c^5
```

3.61 $\int x^3 \log\left(d\left(bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=85

$$-\frac{b^2nx^2}{8c^2}+\frac{b^3nx}{4c^3}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

```
[Out] (b^3*n*x)/(4*c^3) - (b^2*n*x^2)/(8*c^2) + (b*n*x^3)/(12*c) - (n*x^4)/8 - (b
^4*n*Log[b + c*x])/(4*c^4) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Rubi [A] time = 0.0616151, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{b^2nx^2}{8c^2}+\frac{b^3nx}{4c^3}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b^3*n*x)/(4*c^3) - (b^2*n*x^2)/(8*c^2) + (b*n*x^3)/(12*c) - (n*x^4)/8 - (b
^4*n*Log[b + c*x])/(4*c^4) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int x^3 \log\left(d\left(bx+cx^2\right)^n\right) dx &= \frac{1}{4}x^4 \log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{4}n\int \frac{x^3(b+2cx)}{b+cx}\,dx \\
&= \frac{1}{4}x^4 \log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{4}n\int \left(-\frac{b^3}{c^3}+\frac{b^2x}{c^2}-\frac{bx^2}{c}+2x^3+\frac{b^4}{c^3(b+cx)}\right) dx \\
&= \frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)
\end{aligned}
$$

Mathematica [A] time = 0.0357714, size = 74, normalized size = 0.87

$$
\frac{cnx\left(-3b^2cx+6b^3+2bc^2x^2-3c^3x^3\right)-6b^4n\log(b+cx)+6c^4x^4\log\left(d(x(b+cx))^n\right)}{24c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*x^3) - 6*b^4*n*Log[b + c*x]
 + 6*c^4*x^4*Log[d*(x*(b + c*x))^n])/(24*c^4)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$
\int x^3 \ln\left(d\left(cx^2+bx\right)^n\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^3*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A] time = 1.09299, size = 101, normalized size = 1.19

$$
\frac{1}{4}x^4\log\left(\left(cx^2+bx\right)^n d\right)-\frac{1}{24}n\left(\frac{6\,b^4\log(cx+b)}{c^4}+\frac{3\,c^3x^4-2\,bc^2x^3+3\,b^2cx^2-6\,b^3x}{c^3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((c*x^2 + b*x)^n*d) - 1/24*n*(6*b^4*log(c*x + b)/c^4 + (3*c^3*x^
4 - 2*b*c^2*x^3 + 3*b^2*c*x^2 - 6*b^3*x)/c^3)
```

Fricas [A] time = 1.56903, size = 196, normalized size = 2.31

$$\frac{6\,c^4nx^4\log\left(cx^2+bx\right)-3\,c^4nx^4+6\,c^4x^4\log\left(d\right)+2\,bc^3nx^3-3\,b^2c^2nx^2+6\,b^3cnx-6\,b^4n\log\left(cx+b\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^4*n*x^4*log(c*x^2 + b*x) - 3*c^4*n*x^4 + 6*c^4*x^4*log(d) + 2*b*c
^3*n*x^3 - 3*b^2*c^2*n*x^2 + 6*b^3*c*n*x - 6*b^4*n*log(c*x + b))/c^4
```

Sympy [A] time = 7.43731, size = 119, normalized size = 1.4

$$\begin{cases} -\frac{b^4n\log\left(b+cx\right)}{4c^4}+\frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}+\frac{nx^4\log\left(bx+cx^2\right)}{4}-\frac{nx^4}{8}+\frac{x^4\log\left(d\right)}{4} & \text{for}\ c\neq 0 \\ \frac{nx^4\log\left(b\right)}{4}+\frac{nx^4\log\left(x\right)}{4}-\frac{nx^4}{16}+\frac{x^4\log\left(d\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**4*n*log(b + c*x)/(4*c**4) + b**3*n*x/(4*c**3) - b**2*n*x**2/
(8*c**2) + b*n*x**3/(12*c) + n*x**4*log(b*x + c*x**2)/4 - n*x**4/8 + x**4*l
og(d)/4, Ne(c, 0)), (n*x**4*log(b)/4 + n*x**4*log(x)/4 - n*x**4/16 + x**4*l
og(d)/4, True))
```

Giac [A] time = 1.22759, size = 101, normalized size = 1.19

$$\frac{1}{4}\,nx^4\log\left(cx^2+bx\right)-\frac{1}{8}\,(n-2\,\log\left(d\right))x^4+\frac{bnx^3}{12\,c}-\frac{b^2nx^2}{8\,c^2}+\frac{b^3nx}{4\,c^3}-\frac{b^4n\log\left(cx+b\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c - 1/8*
b^2*n*x^2/c^2 + 1/4*b^3*n*x/c^3 - 1/4*b^4*n*log(c*x + b)/c^4
```

3.62 $\int x^2 \log\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=71

$$-\frac{b^2nx}{3c^2}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

```
[Out] -(b^2*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (b^3*n*Log[b + c*x])/(
3*c^3) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Rubi [A] time = 0.0526, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{b^2nx}{3c^2}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -(b^2*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (b^3*n*Log[b + c*x])/(
3*c^3) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x^2 \log\left(d\left(bx+cx^2\right)^n\right)dx &= \frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\frac{x^2(b+2cx)}{b+cx}\,dx\\ &= \frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{3}n\int\left(\frac{b^2}{c^2}-\frac{bx}{c}+2x^2-\frac{b^3}{c^2(b+cx)}\right)dx\\ &= -\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.0288719, size = 63, normalized size = 0.89

$$\frac{cnx\left(-6b^2+3bcx-4c^2x^2\right)+6b^3n\log(b+cx)+6c^3x^3\log\left(d(x(b+cx))^n\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c^2*x^2) + 6*b^3*n*Log[b + c*x] + 6*c^3*x^3*Lo
g[d*(x*(b + c*x))^n])/(18*c^3)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int x^2\ln\left(d\left(cx^2+bx\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A] time = 1.08318, size = 88, normalized size = 1.24

$$\frac{1}{3}x^3\log\left(\left(cx^2+bx\right)^n d\right)+\frac{1}{18}n\left(\frac{6\,b^3\log\left(cx+b\right)}{c^3}-\frac{4\,c^2x^3-3\,bcx^2+6\,b^2x}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((c*x^2 + b*x)^n*d) + 1/18*n*(6*b^3*log(c*x + b)/c^3 - (4*c^2*x^
3 - 3*b*c*x^2 + 6*b^2*x)/c^2)
```

Fricas [A] time = 1.50534, size = 171, normalized size = 2.41

$$\frac{6\,c^3nx^3\log\left(cx^2+bx\right)-4\,c^3nx^3+6\,c^3x^3\log\left(d\right)+3\,bc^2nx^2-6\,b^2cnx+6\,b^3n\log\left(cx+b\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/18*(6*c^3*n*x^3*log(c*x^2 + b*x) - 4*c^3*n*x^3 + 6*c^3*x^3*log(d) + 3*b*c
^2*n*x^2 - 6*b^2*c*n*x + 6*b^3*n*log(c*x + b))/c^3
```

Sympy [A] time = 4.21884, size = 107, normalized size = 1.51

$$\begin{cases}\frac{b^3n\log\left(b+cx\right)}{3c^3}-\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}+\frac{nx^3\log\left(bx+cx^2\right)}{3}-\frac{2nx^3}{9}+\frac{x^3\log\left(d\right)}{3} & \text{for } c\neq 0\\ \frac{nx^3\log\left(b\right)}{3}+\frac{nx^3\log\left(x\right)}{3}-\frac{nx^3}{9}+\frac{x^3\log\left(d\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**3*n*log(b + c*x)/(3*c**3) - b**2*n*x/(3*c**2) + b*n*x**2/(6*c
) + n*x**3*log(b*x + c*x**2)/3 - 2*n*x**3/9 + x**3*log(d)/3, Ne(c, 0)), (n*
x**3*log(b)/3 + n*x**3*log(x)/3 - n*x**3/9 + x**3*log(d)/3, True))
```

Giac [A] time = 1.1419, size = 88, normalized size = 1.24

$$\frac{1}{3}\,nx^3\log\left(cx^2+bx\right)-\frac{1}{9}\,(2\,n-3\,\log\left(d\right))x^3+\frac{bnx^2}{6\,c}-\frac{b^2nx}{3\,c^2}+\frac{b^3n\log\left(cx+b\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c - 1/3
*b^2*n*x/c^2 + 1/3*b^3*n*log(c*x + b)/c^3
```

3.63 $\int x \log \left(d \left(bx + cx^2\right)^n\right) \, dx$

Optimal. Leaf size=57

$$-\frac{b^2 n \log(b+cx)}{2c^2} + \frac{1}{2}x^2 \log\left(d\left(bx+cx^2\right)^n\right) + \frac{bnx}{2c} - \frac{nx^2}{2}$$

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b^2*n*Log[b + c*x])/(2*c^2) + (x^2*Log[d*(b*x
+ c*x^2)^n])/2
```

Rubi [A] time = 0.040766, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2525, 77}

$$-\frac{b^2 n \log(b+cx)}{2c^2} + \frac{1}{2}x^2 \log\left(d\left(bx+cx^2\right)^n\right) + \frac{bnx}{2c} - \frac{nx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b^2*n*Log[b + c*x])/(2*c^2) + (x^2*Log[d*(b*x
+ c*x^2)^n])/2
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int x\log\left(d\left(bx+cx^2\right)^n\right)dx &= \frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x(b+2cx)}{b+cx}dx\\ &= \frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{b^2}{c(b+cx)}\right)dx\\ &= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b^2n\log(b+cx)}{2c^2}+\frac{1}{2}x^2\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A] time = 0.0202679, size = 49, normalized size = 0.86

$$\frac{1}{2}x^2\log\left(d(x(b+cx))^n\right)-\frac{1}{2}n\left(\frac{b^2\log(b+cx)}{c^2}-\frac{bx}{c}+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2))/2 + (x^2*Log[d*(x*(b + c*x
))^n])/2
```

Maple [F] time = 0.03, size = 0, normalized size = 0.

$$\int x\ln\left(d\left(cx^2+bx\right)^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A] time = 1.01231, size = 69, normalized size = 1.21

$$\frac{1}{2}x^2\log\left(\left(cx^2+bx\right)^n d\right)-\frac{1}{2}n\left(\frac{b^2\log\left(cx+b\right)}{c^2}+\frac{cx^2-bx}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((c*x^2 + b*x)^n*d) - 1/2*n*(b^2*log(c*x + b)/c^2 + (c*x^2 - b*x
)/c)
```

Fricas [A] time = 1.55161, size = 132, normalized size = 2.32

$$\frac{c^2nx^2\log\left(cx^2+bx\right)-c^2nx^2+c^2x^2\log(d)+bcnx-b^2n\log(cx+b)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*n*x^2*log(c*x^2 + b*x) - c^2*n*x^2 + c^2*x^2*log(d) + b*c*n*x - b^
2*n*log(c*x + b))/c^2
```

Sympy [A] time = 2.34266, size = 92, normalized size = 1.61

$$\begin{cases} -\frac{b^2n\log(b+cx)}{2c^2}+\frac{bnx}{2c}+\frac{nx^2\log\left(bx+cx^2\right)}{2}-\frac{nx^2}{2}+\frac{x^2\log(d)}{2} & \text{for}\ c\neq 0\\ \frac{nx^2\log(b)}{2}+\frac{nx^2\log(x)}{2}-\frac{nx^2}{4}+\frac{x^2\log(d)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**2*n*log(b + c*x)/(2*c**2) + b*n*x/(2*c) + n*x**2*log(b*x + c
*x**2)/2 - n*x**2/2 + x**2*log(d)/2, Ne(c, 0)), (n*x**2*log(b)/2 + n*x**2*l
og(x)/2 - n*x**2/4 + x**2*log(d)/2, True))
```

Giac [A] time = 1.21367, size = 69, normalized size = 1.21

$$\frac{1}{2}\,nx^2\log\left(cx^2+bx\right)-\frac{1}{2}\,(n-\log(d))x^2+\frac{bnx}{2\,c}-\frac{b^2n\log(cx+b)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/2*b^2*n
*log(c*x + b)/c^2
```

3.64 $\int \log \left(d \left(b x+c x^2\right)^n\right) \, dx$

Optimal. Leaf size=33

$$x \log \left(d \left(b x+c x^2\right)^n\right)+\frac{b n \log (b+c x)}{c}-2 n x$$

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(b*x + c*x^2)^n]
```

Rubi [A] time = 0.0163603, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2523, 43}

$$x \log \left(d \left(b x+c x^2\right)^n\right)+\frac{b n \log (b+c x)}{c}-2 n x$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(b*x + c*x^2)^n]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \log\left(d\left(bx+cx^2\right)^n\right)dx = x\log\left(d\left(bx+cx^2\right)^n\right) - n\int \frac{b+2cx}{b+cx}\,dx$$
$$= x\log\left(d\left(bx+cx^2\right)^n\right) - n\int \left(2-\frac{b}{b+cx}\right)dx$$
$$= -2nx + \frac{bn\log(b+cx)}{c} + x\log\left(d\left(bx+cx^2\right)^n\right)$$

Mathematica [A] time = 0.006888, size = 31, normalized size = 0.94

$$x\log\left(d(x(b+cx))^n\right) + \frac{bn\log(b+cx)}{c} - 2nx$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(x*(b + c*x))^n]
```

Maple [A] time = 0.006, size = 34, normalized size = 1.

$$-2\,nx + \frac{bn\ln(cx+b)}{c} + x\ln\left(d\left(cx^2+bx\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] -2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

Maxima [A] time = 1.15723, size = 49, normalized size = 1.48

$$-n\left(2\,x - \frac{b\log(cx+b)}{c}\right) + x\log\left(\left(cx^2+bx\right)^n d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] -n*(2*x - b*log(c*x + b)/c) + x*log((c*x^2 + b*x)^n*d)
```

Fricas [A] time = 1.52262, size = 95, normalized size = 2.88

$$\frac{cnx\log\left(cx^2+bx\right)-2\,cnx+bn\log\left(cx+b\right)+cx\log\left(d\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] (c*n*x*log(c*x^2 + b*x) - 2*c*n*x + b*n*log(c*x + b) + c*x*log(d))/c
```

Sympy [A] time = 1.21605, size = 56, normalized size = 1.7

$$\begin{cases} \frac{bn\log\left(b+cx\right)}{c}+nx\log\left(bx+cx^2\right)-2nx+x\log\left(d\right) & \text{for } c\neq 0 \\ nx\log\left(b\right)+nx\log\left(x\right)-nx+x\log\left(d\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b*n*log(b + c*x)/c + n*x*log(b*x + c*x**2) - 2*n*x + x*log(d), N
e(c, 0)), (n*x*log(b) + n*x*log(x) - n*x + x*log(d), True))
```

Giac [A] time = 1.22886, size = 50, normalized size = 1.52

$$nx\log\left(cx^2+bx\right)-(2\,n-\log\left(d\right))x+\frac{bn\log\left(cx+b\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x) - (2*n - log(d))*x + b*n*log(c*x + b)/c
```

3.65 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx$

Optimal. Leaf size=53

$$-n\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\log(x)\log\left(\frac{cx}{b}+1\right)-\frac{1}{2}n\log^2(x)$$

```
[Out] -(n*Log[x]^2)/2 - n*Log[x]*Log[1 + (c*x)/b] + Log[x]*Log[d*(b*x + c*x^2)^n]
 - n*PolyLog[2, -((c*x)/b)]
```

Rubi [A] time = 0.125706, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2524, 1593, 2357, 2301, 2317, 2391}

$$-n\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\log(x)\log\left(\frac{cx}{b}+1\right)-\frac{1}{2}n\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]^2)/2 - n*Log[x]*Log[1 + (c*x)/b] + Log[x]*Log[d*(b*x + c*x^2)^n]
 - n*PolyLog[2, -((c*x)/b)]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\int \frac{(b+2cx)\log(x)}{bx+cx^2}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx\\
&= \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\int \frac{\log(x)}{x}\,dx - (cn)\int \frac{\log(x)}{b+cx}\,dx\\
&= -\frac{1}{2}n\log^2(x) - n\log(x)\log\left(1+\frac{cx}{b}\right) + \log(x)\log\left(d\left(bx+cx^2\right)^n\right) + n\int \frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx\\
&= -\frac{1}{2}n\log^2(x) - n\log(x)\log\left(1+\frac{cx}{b}\right) + \log(x)\log\left(d\left(bx+cx^2\right)^n\right) - n\text{Li}_2\left(-\frac{cx}{b}\right)
\end{aligned}$$

Mathematica [A] time = 0.0203252, size = 50, normalized size = 0.94

$$\log(x)\log\left(d(x(b+cx))^n\right) - n\left(\text{PolyLog}\left(2,-\frac{cx}{b}\right) + \log(x)\log\left(\frac{b+cx}{b}\right) + \frac{\log^2(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] Log[x]*Log[d*(x*(b + c*x))^n] - n*(Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + P
olyLog[2, -((c*x)/b)])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+bx\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x,x)
```

Maxima [A] time = 1.0547, size = 108, normalized size = 2.04

$$-n\log\left(cx^2+bx\right)\log\left(x\right)+\frac{1}{2}\left(2\,\log\left(cx^2+bx\right)\log\left(x\right)-2\,\log\left(\frac{cx}{b}+1\right)\log\left(x\right)-\log\left(x\right)^2-2\,\mathrm{Li}_2\left(-\frac{cx}{b}\right)\right)n+\log\left(\left(cx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="maxima")
```

```
[Out] -n*log(c*x^2 + b*x)*log(x) + 1/2*(2*log(c*x^2 + b*x)*log(x) - 2*log(c*x/b +
 1)*log(x) - log(x)^2 - 2*dilog(-c*x/b))*n + log((c*x^2 + b*x)^n*d)*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(cx^2+bx\right)^n d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x)^n*d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(d \left(bx + cx^2\right)^n\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x,x)
```

```
[Out] Integral(log(d*(b*x + c*x**2)**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(\left(cx^2 + bx\right)^n d\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x)^n*d)/x, x)
```

3.66 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(b*x + c*x^2)^n]/x
```

Rubi [A] time = 0.0438773, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(b*x + c*x^2)^n]/x
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\int \frac{\log \left(d \left(bx+cx^2\right)^n\right)}{x^2} \, dx = -\frac{\log \left(d \left(bx+cx^2\right)^n\right)}{x}+n \int \frac{b+2cx}{x^2(b+cx)} \, dx$$
$$= -\frac{\log \left(d \left(bx+cx^2\right)^n\right)}{x}+n \int \left(\frac{1}{x^2}+\frac{c}{bx}-\frac{c^2}{b(b+cx)}\right) \, dx$$
$$= -\frac{n}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{\log \left(d \left(bx+cx^2\right)^n\right)}{x}$$

Mathematica [A] time = 0.0120372, size = 45, normalized size = 0.96

$$-\frac{\log \left(d(x(b+cx))^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^2,x]
```

```
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(x*(b + c*x))^n]/x
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{\ln \left(d \left(cx^2+bx\right)^n\right)}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
```

Maxima [A] time = 1.00471, size = 62, normalized size = 1.32

$$-n\left(\frac{c\log(cx+b)}{b}-\frac{c\log(x)}{b}+\frac{1}{x}\right)-\frac{\log \left(\left(cx^2+bx\right)^n d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="maxima")
```

```
[Out] -n*(c*log(c*x + b)/b - c*log(x)/b + 1/x) - log((c*x^2 + b*x)^n*d)/x
```

Fricas [A] time = 1.92155, size = 113, normalized size = 2.4

$$-\frac{cnx\log(cx+b)-cnx\log(x)+bn\log\left(cx^2+bx\right)+bn+b\log(d)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="fricas")
```

```
[Out] -(c*n*x*log(c*x + b) - c*n*x*log(x) + b*n*log(c*x^2 + b*x) + b*n + b*log(d)
)/(b*x)
```

Sympy [A] time = 2.67932, size = 76, normalized size = 1.62

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{x}-\frac{n}{x}-\frac{\log(d)}{x}-\frac{2cn\log(b+cx)}{b}+\frac{cn\log\left(bx+cx^2\right)}{b} & \text{for } b\neq 0 \\ -\frac{n\log(c)}{x}-\frac{2n\log(x)}{x}-\frac{2n}{x}-\frac{\log(d)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**2,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/x - n/x - log(d)/x - 2*c*n*log(b + c*x)/b +
 c*n*log(b*x + c*x**2)/b, Ne(b, 0)), (-n*log(c)/x - 2*n*log(x)/x - 2*n/x -
log(d)/x, True))
```

Giac [A] time = 1.30503, size = 63, normalized size = 1.34

$$-\frac{cn\log(cx+b)}{b}+\frac{cn\log(x)}{b}-\frac{n\log\left(cx^2+bx\right)}{x}-\frac{n+\log(d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="giac")

[Out] -c*n*log(c*x + b)/b + c*n*log(x)/b - n*log(c*x^2 + b*x)/x - (n + log(d))/x
```

3.67 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3} dx$

Optimal. Leaf size=72

$$-\frac{c^2 n \log(x)}{2b^2}+\frac{c^2 n \log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}-\frac{cn}{2bx}-\frac{n}{4x^2}$$

```
[Out] -n/(4*x^2) - (c*n)/(2*b*x) - (c^2*n*Log[x])/(2*b^2) + (c^2*n*Log[b + c*x])/
(2*b^2) - Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Rubi [A] time = 0.0497263, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$-\frac{c^2 n \log(x)}{2b^2}+\frac{c^2 n \log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}-\frac{cn}{2bx}-\frac{n}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -n/(4*x^2) - (c*n)/(2*b*x) - (c^2*n*Log[x])/(2*b^2) + (c^2*n*Log[b + c*x])/
(2*b^2) - Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}+\frac{1}{2}n\int \frac{b+2cx}{x^3(b+cx)}\,dx$$
$$= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}+\frac{1}{2}n\int \left(\frac{1}{x^3}+\frac{c}{bx^2}-\frac{c^2}{b^2x}+\frac{c^3}{b^2(b+cx)}\right)dx$$
$$= -\frac{n}{4x^2}-\frac{cn}{2bx}-\frac{c^2n\log(x)}{2b^2}+\frac{c^2n\log(b+cx)}{2b^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}$$

Mathematica [A] time = 0.0320771, size = 65, normalized size = 0.9

$$\frac{1}{2}n\left(-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}-\frac{c}{bx}-\frac{1}{2x^2}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] (n*(-1/(2*x^2) - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
Log[d*(x*(b + c*x))^n]/(2*x^2)
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+bx\right)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

Maxima [A] time = 1.03116, size = 84, normalized size = 1.17

$$\frac{1}{4}n\left(\frac{2c^2\log\left(cx+b\right)}{b^2}-\frac{2c^2\log\left(x\right)}{b^2}-\frac{2cx+b}{bx^2}\right)-\frac{\log\left(\left(cx^2+bx\right)^nd\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*n*(2*c^2*log(c*x + b)/b^2 - 2*c^2*log(x)/b^2 - (2*c*x + b)/(b*x^2)) - 1
/2*log((c*x^2 + b*x)^n*d)/x^2
```

Fricas [A] time = 1.8428, size = 169, normalized size = 2.35

$$\frac{2c^2nx^2\log\left(cx+b\right)-2c^2nx^2\log\left(x\right)-2bcnx-2b^2n\log\left(cx^2+bx\right)-b^2n-2b^2\log\left(d\right)}{4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^2*n*x^2*log(c*x + b) - 2*c^2*n*x^2*log(x) - 2*b*c*n*x - 2*b^2*n*lo
g(c*x^2 + b*x) - b^2*n - 2*b^2*log(d))/(b^2*x^2)
```

Sympy [A] time = 4.9759, size = 110, normalized size = 1.53

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{2x^2}-\frac{n}{4x^2}-\frac{\log\left(d\right)}{2x^2}-\frac{cn}{2bx}+\frac{c^2n\log\left(b+cx\right)}{b^2}-\frac{c^2n\log\left(bx+cx^2\right)}{2b^2} & \text{for}\ b\neq 0 \\ -\frac{n\log\left(c\right)}{2x^2}-\frac{n\log\left(x\right)}{x^2}-\frac{n}{2x^2}-\frac{\log\left(d\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**3,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(2*x**2) - n/(4*x**2) - log(d)/(2*x**2) - c
*n/(2*b*x) + c**2*n*log(b + c*x)/b**2 - c**2*n*log(b*x + c*x**2)/(2*b**2),
Ne(b, 0)), (-n*log(c)/(2*x**2) - n*log(x)/x**2 - n/(2*x**2) - log(d)/(2*x**
2), True))
```

Giac [A] time = 1.26964, size = 88, normalized size = 1.22

$$\frac{c^2n\log\left(cx+b\right)}{2\,b^2}-\frac{c^2n\log\left(x\right)}{2\,b^2}-\frac{n\log\left(cx^2+bx\right)}{2\,x^2}-\frac{2\,cnx+bn+2\,b\log\left(d\right)}{4\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c^2*n*log(c*x + b)/b^2 - 1/2*c^2*n*log(x)/b^2 - 1/2*n*log(c*x^2 + b*x)/
x^2 - 1/4*(2*c*n*x + b*n + 2*b*log(d))/(b*x^2)
```

3.68 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=86

$$\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}-\frac{cn}{6bx^2}-\frac{n}{9x^3}$$

```
[Out] -n/(9*x^3) - (c*n)/(6*b*x^2) + (c^2*n)/(3*b^2*x) + (c^3*n*Log[x])/(3*b^3) -
 (c^3*n*Log[b + c*x])/(3*b^3) - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```

Rubi [A] time = 0.0571364, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}-\frac{cn}{6bx^2}-\frac{n}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^4,x]
```


```
[Out] -n/(9*x^3) - (c*n)/(6*b*x^2) + (c^2*n)/(3*b^2*x) + (c^3*n*Log[x])/(3*b^3) -
 (c^3*n*Log[b + c*x])/(3*b^3) - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}+\frac{1}{3}n\int \frac{b+2cx}{x^4(b+cx)}\,dx\\
&= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}+\frac{1}{3}n\int \left(\frac{1}{x^4}+\frac{c}{bx^3}-\frac{c^2}{b^2x^2}+\frac{c^3}{b^3x}-\frac{c^4}{b^3(b+cx)}\right)dx\\
&= -\frac{n}{9x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0352008, size = 77, normalized size = 0.9

$$\frac{1}{3}n\left(\frac{c^2}{b^2x}+\frac{c^3\log(x)}{b^3}-\frac{c^3\log(b+cx)}{b^3}-\frac{c}{2bx^2}-\frac{1}{3x^3}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] (n*(-1/(3*x^3) - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
+ c*x])/b^3))/3 - Log[d*(x*(b + c*x))^n]/(3*x^3)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+bx\right)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

Maxima [A] time = 1.0167, size = 101, normalized size = 1.17

$$-\frac{1}{18}\,n\left(\frac{6\,c^3\log\left(cx+b\right)}{b^3}-\frac{6\,c^3\log\left(x\right)}{b^3}-\frac{6\,c^2x^2-3\,bcx-2\,b^2}{b^2x^3}\right)-\frac{\log\left(\left(cx^2+bx\right)^n d\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*n*(6*c^3*log(c*x + b)/b^3 - 6*c^3*log(x)/b^3 - (6*c^2*x^2 - 3*b*c*x -
 2*b^2)/(b^2*x^3)) - 1/3*log((c*x^2 + b*x)^n*d)/x^3
```

Fricas [A] time = 1.87479, size = 198, normalized size = 2.3

$$-\frac{6\,c^3nx^3\log\left(cx+b\right)-6\,c^3nx^3\log\left(x\right)-6\,bc^2nx^2+3\,b^2cnx+6\,b^3n\log\left(cx^2+bx\right)+2\,b^3n+6\,b^3\log\left(d\right)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*c^3*n*x^3*log(c*x + b) - 6*c^3*n*x^3*log(x) - 6*b*c^2*n*x^2 + 3*b^
2*c*n*x + 6*b^3*n*log(c*x^2 + b*x) + 2*b^3*n + 6*b^3*log(d))/(b^3*x^3)
```

Sympy [A] time = 8.75475, size = 133, normalized size = 1.55

$$\begin{cases} -\frac{n\log\left(bx+cx^2\right)}{3x^3}-\frac{n}{9x^3}-\frac{\log\left(d\right)}{3x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}-\frac{2c^3n\log\left(b+cx\right)}{3b^3}+\frac{c^3n\log\left(bx+cx^2\right)}{3b^3} & \text{for}\: b \neq 0 \\ -\frac{n\log\left(c\right)}{3x^3}-\frac{2n\log\left(x\right)}{3x^3}-\frac{2n}{9x^3}-\frac{\log\left(d\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**4,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(3*x**3) - n/(9*x**3) - log(d)/(3*x**3) - c
*n/(6*b*x**2) + c**2*n/(3*b**2*x) - 2*c**3*n*log(b + c*x)/(3*b**3) + c**3*n
*log(b*x + c*x**2)/(3*b**3), Ne(b, 0)), (-n*log(c)/(3*x**3) - 2*n*log(x)/(3
```

```
*x**3) - 2*n/(9*x**3) - log(d)/(3*x**3), True))
```

Giac [A] time = 1.16039, size = 108, normalized size = 1.26

$$-\frac{c^3 n \log(cx+b)}{3\,b^3} + \frac{c^3 n \log(x)}{3\,b^3} - \frac{n \log\left(cx^2+bx\right)}{3\,x^3} + \frac{6\,c^2 n x^2 - 3\,bcnx - 2\,b^2 n - 6\,b^2 \log(d)}{18\,b^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*c^3*n*log(c*x + b)/b^3 + 1/3*c^3*n*log(x)/b^3 - 1/3*n*log(c*x^2 + b*x)
/x^3 + 1/18*(6*c^2*n*x^2 - 3*b*c*n*x - 2*b^2*n - 6*b^2*log(d))/(b^2*x^3)
```

3.69 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5} dx$

Optimal. Leaf size=100

$$\frac{c^2n}{8b^2x^2} - \frac{c^3n}{4b^3x} - \frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4} - \frac{cn}{12bx^3} - \frac{n}{16x^4}$$

```
[Out] -n/(16*x^4) - (c*n)/(12*b*x^3) + (c^2*n)/(8*b^2*x^2) - (c^3*n)/(4*b^3*x) -
(c^4*n*Log[x])/(4*b^4) + (c^4*n*Log[b + c*x])/(4*b^4) - Log[d*(b*x + c*x^2)
^n]/(4*x^4)
```

Rubi [A] time = 0.0632197, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2525, 77}

$$\frac{c^2n}{8b^2x^2} - \frac{c^3n}{4b^3x} - \frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4} - \frac{cn}{12bx^3} - \frac{n}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -n/(16*x^4) - (c*n)/(12*b*x^3) + (c^2*n)/(8*b^2*x^2) - (c^3*n)/(4*b^3*x) -
(c^4*n*Log[x])/(4*b^4) + (c^4*n*Log[b + c*x])/(4*b^4) - Log[d*(b*x + c*x^2)
^n]/(4*x^4)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \frac{b+2cx}{x^5(b+cx)}\,dx\\ &= -\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}+\frac{1}{4}n\int \left(\frac{1}{x^5}+\frac{c}{bx^4}-\frac{c^2}{b^2x^3}+\frac{c^3}{b^3x^2}-\frac{c^4}{b^4x}+\frac{c^5}{b^4(b+cx)}\right)dx\\ &= -\frac{n}{16x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}-\frac{c^4n\log(x)}{4b^4}+\frac{c^4n\log(b+cx)}{4b^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0462707, size = 87, normalized size = 0.87

$$-\frac{bn\left(4b^2cx+3b^3-6bc^2x^2+12c^3x^3\right)+12b^4\log\left(d(x(b+cx))^n\right)-12c^4nx^4\log(b+cx)+12c^4nx^4\log(x)}{48b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -(b*n*(3*b^3 + 4*b^2*c*x - 6*b*c^2*x^2 + 12*c^3*x^3) + 12*c^4*n*x^4*Log[x]
- 12*c^4*n*x^4*Log[b + c*x] + 12*b^4*Log[d*(x*(b + c*x))^n])/(48*b^4*x^4)
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+bx\right)^n\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

Maxima [A] time = 1.02596, size = 116, normalized size = 1.16

$$\frac{1}{48}\,n\left(\frac{12\,c^{4}\log\left(cx+b\right)}{b^{4}}-\frac{12\,c^{4}\log\left(x\right)}{b^{4}}-\frac{12\,c^{3}x^{3}-6\,bc^{2}x^{2}+4\,b^{2}cx+3\,b^{3}}{b^{3}x^{4}}\right)-\frac{\log\left(\left(cx^{2}+bx\right)^{n}d\right)}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*n*(12*c^4*log(c*x + b)/b^4 - 12*c^4*log(x)/b^4 - (12*c^3*x^3 - 6*b*c^2
*x^2 + 4*b^2*c*x + 3*b^3)/(b^3*x^4)) - 1/4*log((c*x^2 + b*x)^n*d)/x^4
```

Fricas [A] time = 1.92054, size = 228, normalized size = 2.28

$$\frac{12\,c^{4}nx^{4}\log\left(cx+b\right)-12\,c^{4}nx^{4}\log\left(x\right)-12\,bc^{3}nx^{3}+6\,b^{2}c^{2}nx^{2}-4\,b^{3}cnx-12\,b^{4}n\log\left(cx^{2}+bx\right)-3\,b^{4}n-12\,b^{4}\log\left(d\right)}{48\,b^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(12*c^4*n*x^4*log(c*x + b) - 12*c^4*n*x^4*log(x) - 12*b*c^3*n*x^3 + 6*
b^2*c^2*n*x^2 - 4*b^3*c*n*x - 12*b^4*n*log(c*x^2 + b*x) - 3*b^4*n - 12*b^4*
log(d))/(b^4*x^4)
```

Sympy [A] time = 16.6428, size = 141, normalized size = 1.41

$$\begin{cases}-\frac{n\log\left(bx+cx^{2}\right)}{4x^{4}}-\frac{n}{16x^{4}}-\frac{\log\left(d\right)}{4x^{4}}-\frac{cn}{12bx^{3}}+\frac{c^{2}n}{8b^{2}x^{2}}-\frac{c^{3}n}{4b^{3}x}+\frac{c^{4}n\log\left(b+cx\right)}{2b^{4}}-\frac{c^{4}n\log\left(bx+cx^{2}\right)}{4b^{4}} & \text{for}\ b\neq 0\\-\frac{n\log\left(c\right)}{4x^{4}}-\frac{n\log\left(x\right)}{2x^{4}}-\frac{n}{8x^{4}}-\frac{\log\left(d\right)}{4x^{4}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**5,x)
```

```
[Out] Piecewise((-n*log(b*x + c*x**2)/(4*x**4) - n/(16*x**4) - log(d)/(4*x**4) -
c*n/(12*b*x**3) + c**2*n/(8*b**2*x**2) - c**3*n/(4*b**3*x) + c**4*n*log(b +
 c*x)/(2*b**4) - c**4*n*log(b*x + c*x**2)/(4*b**4), Ne(b, 0)), (-n*log(c)/(
4*x**4) - n*log(x)/(2*x**4) - n/(8*x**4) - log(d)/(4*x**4), True))
```

Giac [A] time = 1.16691, size = 124, normalized size = 1.24

$$\frac{c^4 n \log(cx+b)}{4\,b^4} - \frac{c^4 n \log(x)}{4\,b^4} - \frac{n \log\left(cx^2+bx\right)}{4\,x^4} - \frac{12\,c^3 n x^3 - 6\,bc^2 n x^2 + 4\,b^2 c n x + 3\,b^3 n + 12\,b^3 \log(d)}{48\,b^3 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^4*n*log(c*x + b)/b^4 - 1/4*c^4*n*log(x)/b^4 - 1/4*n*log(c*x^2 + b*x)/
x^4 - 1/48*(12*c^3*n*x^3 - 6*b*c^2*n*x^2 + 4*b^2*c*n*x + 3*b^3*n + 12*b^3*l
og(d))/(b^3*x^4)
```

3.70 $\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=157

$$-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}$$

```
[Out] (-2*c*n*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) - (2*c*n*x^(2 + m)*H
ypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b +
Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Rubi [A] time = 0.220772, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2525, 830, 64}

$$-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*c*n*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) - (2*c*n*x^(2 + m)*H
ypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b +
Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}
\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)dx &= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^{1+m}(b+2cx)}{a+bx+cx^2}\,dx}{1+m} \\
&= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \left(\frac{2cx^{1+m}}{b-\sqrt{b^2-4ac}+2cx}+\frac{2cx^{1+m}}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{1+m} \\
&= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{(2cn)\int \frac{x^{1+m}}{b-\sqrt{b^2-4ac}+2cx}\,dx}{1+m} - \frac{(2cn)\int \frac{x^{1+m}}{b+\sqrt{b^2-4ac}+2cx}\,dx}{1+m} \\
&= -\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} - \frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b+\sqrt{b^2-4}}\right.}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(2+m)}
\end{aligned}$$

Mathematica [A] time = 0.186494, size = 137, normalized size = 0.87

$$-\frac{x^{m+1}\left(nx\left(\sqrt{b^2-4ac}+b\right)\,{}_2F_1\left(1,m+2;m+3;\frac{2cx}{\sqrt{b^2-4ac}-b}\right)+nx\left(b-\sqrt{b^2-4ac}\right)\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)-2a(\right.}{2a\left(m^2+3m+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -(x^(1 + m)*((b + Sqrt[b^2 - 4*a*c])*n*x*Hypergeometric2F1[1, 2 + m, 3 + m,
 (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*n*x*Hypergeome
tric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])] - 2*a*(2 + m)*Lo
```

```
g[d*(a + x*(b + c*x))^n]))/(2*a*(2 + 3*m + m^2))
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int x^m \ln\left(d\left(cx^2+bx+a\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \log\left(\left(cx^2+bx+a\right)^n d\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \log\left(\left(cx^2 + bx + a\right)^n d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

3.71 $\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=207

$$\frac{bn\left(5a^2c^2-5ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{10c^5}-\frac{nx\left(2a^2c^2-4ab^2c+b^4\right)}{5c^4}+\frac{n\sqrt{b^2-4ac}\left(a^2c^2-3ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a}}\right.}{5c^5}$$

```
[Out] -((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x)/(5*c^4) + (b*(b^2 - 3*a*c)*n*x^2)/(10*
c^3) - ((b^2 - 2*a*c)*n*x^3)/(15*c^2) + (b*n*x^4)/(20*c) - (2*n*x^5)/25 + (
Sqrt[b^2 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(5*c^5) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + b*x + c*x
^2])/(10*c^5) + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Rubi [A] time = 0.229487, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{bn\left(5a^2c^2-5ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{10c^5}-\frac{nx\left(2a^2c^2-4ab^2c+b^4\right)}{5c^4}+\frac{n\sqrt{b^2-4ac}\left(a^2c^2-3ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a}}\right.}{5c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x)/(5*c^4) + (b*(b^2 - 3*a*c)*n*x^2)/(10*
c^3) - ((b^2 - 2*a*c)*n*x^3)/(15*c^2) + (b*n*x^4)/(20*c) - (2*n*x^5)/25 + (
Sqrt[b^2 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(5*c^5) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + b*x + c*x
^2])/(10*c^5) + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int x^4 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{5}n\int \frac{x^5(b+2cx)}{a+bx+cx^2}dx$$
$$= \frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{5}n\int\left(\frac{b^4-4ab^2c+2a^2c^2}{c^4} - \frac{b\left(b^2-3ac\right)x}{c^3} + \frac{\left(b^2-2a\right.}{c^2}\right.$$
$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{1}{5}x$$
$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{1}{5}x$$
$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{b\left(\right.}{}$$
$$= -\frac{\left(b^4-4ab^2c+2a^2c^2\right)nx}{5c^4} + \frac{b\left(b^2-3ac\right)nx^2}{10c^3} - \frac{\left(b^2-2ac\right)nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{\sqrt{b}}{}$$

Mathematica [A] time = 0.20661, size = 190, normalized size = 0.92

$$\frac{cnx\left(-8c^2\left(15a^2-5acx^2+3c^2x^4\right)-20b^2c\left(cx^2-12a\right)+15bc^2x\left(cx^2-6a\right)+30b^3cx-60b^4\right)+30bn\left(5a^2c^2-5ab^2c+b\right.}{300c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c*(-12*a + c*x^2) + 15*b*c^2*x*(-6*a
+ c*x^2) - 8*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)) + 60*Sqrt[b^2 - 4*a*c]*(
b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 30*b*
(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + x*(b + c*x)] + 60*c^5*x^5*Log[d*(a
+ x*(b + c*x))^n])/(300*c^5)
```

Maple [C] time = 0.123, size = 1621, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/5*x^5*ln((c*x^2+b*x+a)^n)-1/10*I*Pi*x^5*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)
*csgn(I*d*(c*x^2+b*x+a)^n)+1/10*I*Pi*x^5*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n
)^2+1/10*I*Pi*x^5*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/10*
I*Pi*x^5*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/5*ln(d)*x^5-2/25*n*x^5+1/20*b*n*x^4/
c+2/15/c*a*n*x^3-1/15*b^2*n*x^3/c^2-3/10/c^2*a*b*n*x^2+1/10*b^3*n*x^2/c^3+1
/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^4*b^2
*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*c^5+2
5*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a^2*b
-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^4*b
^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*c^5
+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a*b
^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
b^5+1/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
a^2*b-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*
a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^
5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b
)*a*b^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+
25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4
*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2
)*b)*b^5-2/5/c^2*a^2*n*x+4/5/c^3*a*b^2*n*x-1/5*b^4*n*x/c^4+1/10/c^5*n*ln(4*
a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^
4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*c^5+25*a^4*b^2*c^4-
50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^4*
b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)-1/10/c^5*n*ln(
4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*
b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5+25*a^4*b^2*c^
4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97479, size = 1013, normalized size = 4.89

$$\left[-\frac{24\,c^5nx^5-60\,c^5x^5\log\left(d\right)-15\,bc^4nx^4+20\left(b^2c^3-2\,ac^4\right)nx^3-30\left(b^3c^2-3\,abc^3\right)nx^2-30\left(b^4-3\,ab^2c+a^2c^2\right)\sqrt{b^2-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4 + 20*(b^2*c^3 -
2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 30*(b^4 - 3*a*b^2*c + a^2
*c^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*n*x - 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2
+ b*x + a))/c^5, -1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4
+ 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 60*(b^4 -
 3*a*b^2*c + a^2*c^2)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x - 30*(2*c^
5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2 + b*x + a))/c^5]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23399, size = 298, normalized size = 1.44

$$\frac{1}{5}\,nx^5\log\left(cx^2+bx+a\right)-\frac{1}{25}\left(2\,n-5\,\log\left(d\right)\right)x^5+\frac{bnx^4}{20\,c}-\frac{\left(b^2n-2\,acn\right)x^3}{15\,c^2}+\frac{\left(b^3n-3\,abcn\right)x^2}{10\,c^3}-\frac{\left(b^4n-4\,ab^2cn+2\,a^2\right.}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x + a) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c
 - 1/15*(b^2*n - 2*a*c*n)*x^3/c^2 + 1/10*(b^3*n - 3*a*b*c*n)*x^2/c^3 - 1/5*
(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*x/c^4 + 1/10*(b^5*n - 5*a*b^3*c*n + 5*a
^2*b*c^2*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n - 7*a*b^4*c*n + 13*a^2*b^
2*c^2*n - 4*a^3*c^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^5)
```

3.72 $\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=167

$$-\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8c^4}-\frac{nx^2\left(b^2-2ac\right)}{8c^2}+\frac{bnx\left(b^2-3ac\right)}{4c^3}-\frac{bn\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}$$

```
[Out] (b*(b^2 - 3*a*c)*n*x)/(4*c^3) - ((b^2 - 2*a*c)*n*x^2)/(8*c^2) + (b*n*x^3)/(
12*c) - (n*x^4)/8 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*
x + c*x^2])/(8*c^4) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Rubi [A] time = 0.188672, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8c^4}-\frac{nx^2\left(b^2-2ac\right)}{8c^2}+\frac{bnx\left(b^2-3ac\right)}{4c^3}-\frac{bn\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*(b^2 - 3*a*c)*n*x)/(4*c^3) - ((b^2 - 2*a*c)*n*x^2)/(8*c^2) + (b*n*x^3)/(
12*c) - (n*x^4)/8 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*
x + c*x^2])/(8*c^4) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int \frac{x^4(b+2cx)}{a+bx+cx^2}\,dx$$
$$= \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int\left(-\frac{b\left(b^2-3ac\right)}{c^3} + \frac{\left(b^2-2ac\right)x}{c^2} - \frac{bx^2}{c} + 2x^3 + \frac{ab}{}\right.$$
$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{n\int}{}$$
$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} + \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{(b(}{}$$
$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log\left(a+bx\right.}{8c^4}$$
$$= \frac{b\left(b^2-3ac\right)nx}{4c^3} - \frac{\left(b^2-2ac\right)nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{b\sqrt{b^2-4ac}\left(b^2-2ac\right)n\tanh^{-1}\left(\right.}{4c^4}$$

Mathematica [A] time = 0.147535, size = 151, normalized size = 0.9

$$\frac{-3n\left(2a^2c^2-4ab^2c+b^4\right)\log(a+x(b+cx)) + cnx\left(2bc\left(cx^2-9a\right)-3c^2x\left(cx^2-2a\right)-3b^2cx+6b^3\right) - 6bn\sqrt{b^2-4ac}\left(b^2\right.}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c*(-9*a + c*x^2) - 3*c^2*x*(-2*a + c*x^2))
- 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + x*(b + c*x)] + 6*c^4*x^4*Lo
g[d*(a + x*(b + c*x))^n])/(24*c^4)
```

Maple [C] time = 0.1, size = 1146, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/4*x^4*ln((c*x^2+b*x+a)^n)+1/8*I*Pi*x^4*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(
c*x^2+b*x+a)^n)^2+1/8*I*Pi*x^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/8*I*
Pi*x^4*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/8*I*Pi*x^4*csgn(I*d)*csgn(I*(c*x^2+b*x
+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/4*ln(d)*x^4-1/8*n*x^4+1/12*b*n*x^3/c+1/4
/c*a*n*x^2-1/8*b^2*n*x^2/c^2-1/4/c^2*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5+2*(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^3+20*a^2
*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5
+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^
3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/8/c^4*n*ln(8*a^2*b*c^2-6*a
*b^3*c+b^5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*b^4-1/4/c^2*n*ln(8*a^2*b
*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c
*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln
(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)
^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/
8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*
b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b
)*b^4-3/4/c^2*a*b*n*x+1/4*b^3*n*x/c^3-1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^
5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8
*a*b^6*c+b^8)^(1/2)+1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-
8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96504, size = 826, normalized size = 4.95

$$\left[-\frac{3 c^4 n x^4 - 6 c^4 x^4 \log(d) - 2 b c^3 n x^3 + 3 \left(b^2 c^2 - 2 a c^3\right) n x^2 + 3 \left(b^3 - 2 abc\right) \sqrt{b^2 - 4 ac} n \log\left(\frac{2 c^2 x^2 + 2 bcx + b^2 - 2 ac + \sqrt{b^2 - 4 ac}(2 cx +}{cx^2 + bx + a}\right.}{24 c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c
^3)*n*x^2 + 3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*(b^3*
c - 3*a*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(
c*x^2 + b*x + a))/c^4, -1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^
3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 + 6*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 6*(b^3*c - 3*a*b*c^2)
*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(c*x^2 + b*x +
a))/c^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27076, size = 238, normalized size = 1.43

$$\frac{1}{4}\,nx^4\log\left(cx^2+bx+a\right)-\frac{1}{8}\,(n-2\,\log\left(d\right))x^4+\frac{bnx^3}{12\,c}-\frac{\left(b^2n-2\,acn\right)x^2}{8\,c^2}+\frac{\left(b^3n-3\,abcn\right)x}{4\,c^3}-\frac{\left(b^4n-4\,ab^2cn+2\,a^2c^2n\right.}{8\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x + a) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c -
1/8*(b^2*n - 2*a*c*n)*x^2/c^2 + 1/4*(b^3*n - 3*a*b*c*n)*x/c^3 - 1/8*(b^4*n
- 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n - 6*a*b^
3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^4)
```

3.73 $\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=136

$$\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6c^3}-\frac{nx\left(b^2-2ac\right)}{3c^2}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -((b^2 - 2*a*c)*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (Sqrt[b^2 -
4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) + (b*(
b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*c^3) + (x^3*Log[d*(a + b*x + c*x^2)
^n])/3
```

Rubi [A] time = 0.149037, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6c^3}-\frac{nx\left(b^2-2ac\right)}{3c^2}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```


```
[Out] -((b^2 - 2*a*c)*n*x)/(3*c^2) + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (Sqrt[b^2 -
4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) + (b*(
b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*c^3) + (x^3*Log[d*(a + b*x + c*x^2)
^n])/3
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int \frac{x^3(b+2cx)}{a+bx+cx^2}dx$$

$$= \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int\left(\frac{b^2-2ac}{c^2} - \frac{bx}{c} + 2x^2 - \frac{a\left(b^2-2ac\right)+b\left(b^2-3a\right.}{c^2\left(a+bx+cx^2\right)}\right.$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{n\int\frac{a\left(b^2-2ac\right)+b\left(b^2-3ac\right)}{a+bx+cx^2}}{3c^2}$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b\left(b^2-3ac\right)n\right)\int\frac{b}{a+b}}{6c^3}$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{b\left(b^2-3ac\right)n\log\left(a+bx+cx^2\right)}{6c^3} + \frac{1}{3}x^3\log\left(d\left(a+\right.\right.$$

$$= -\frac{\left(b^2-2ac\right)nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{\sqrt{b^2-4ac}\left(b^2-ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3} + \frac{b\left(b^2-3a\right.}{}$$

Mathematica [A] time = 0.109442, size = 122, normalized size = 0.9

$$\frac{cnx\left(-4c\left(cx^2-3a\right)-6b^2+3bcx\right)+3bn\left(b^2-3ac\right)\log(a+x(b+cx))+6n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+6c^3x^3}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c*(-3*a + c*x^2)) + 6*Sqrt[b^2 - 4*a*c]*(b^2 -
 a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*b*(b^2 - 3*a*c)*n*Log[a
+ x*(b + c*x)] + 6*c^3*x^3*Log[d*(a + x*(b + c*x))^n])/(18*c^3)
```

Maple [C] time = 0.089, size = 870, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/3*x^3*ln((c*x^2+b*x+a)^n)-1/6*I*Pi*x^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*
csgn(I*d*(c*x^2+b*x+a)^n)+1/6*I*Pi*x^3*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^
2+1/6*I*Pi*x^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/6*I*Pi
*x^3*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/3*ln(d)*x^3-2/9*n*x^3+1/6*b*n*x^2/c-1/2/
c^2*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6
)^(1/2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n
*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/
2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3-1/2/c^2*n*ln(-
4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*
x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n*ln(-4*a^2
*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x+(-4
*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3+2/3/c*a*n*x-1/3*b^2*n*x/
c^2+1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b
^4*c+b^6)^(1/2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a
^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)-1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c
-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x+(-4*a^3*c^3+9*a^2
*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9961, size = 691, normalized size = 5.08

$$\left[-\frac{4\,c^3nx^3 - 6\,c^3x^3\log\left(d\right) - 3\,bc^2nx^2 + 3\left(b^2 - ac\right)\sqrt{b^2 - 4\,ac}n\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac - \sqrt{b^2 - 4\,ac}(2\,cx + b)}{cx^2 + bx + a}\right) + 6\left(b^2c - 2\,ac^2\right)nx}{18\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*n*x^3 - 6*c^3*x^3*log(d) - 3*b*c^2*n*x^2 + 3*(b^2 - a*c)*sqrt
(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2*c^3*n*x^3
+ (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3, -1/18*(4*c^3*n*x^3 - 6*c^3*
x^3*log(d) - 3*b*c^2*n*x^2 - 6*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqr
t(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2
*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29281, size = 197, normalized size = 1.45

$$\frac{1}{3}\,nx^3\log\left(cx^2+bx+a\right)-\frac{1}{9}\,(2\,n-3\,\log\left(d\right))x^3+\frac{bnx^2}{6\,c}-\frac{\left(b^2n-2\,acn\right)x}{3\,c^2}+\frac{\left(b^3n-3\,abcn\right)\log\left(cx^2+bx+a\right)}{6\,c^3}-\frac{\left(b^4n-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x + a) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c -
 1/3*(b^2*n - 2*a*c*n)*x/c^2 + 1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)
/c^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.74 $\int x \log \left(d \left(a+b x+c x^2\right)^n\right) \, dx$

Optimal. Leaf size=109

$$-\frac{n \left(b^2-2 a c\right) \log \left(a+b x+c x^2\right)}{4 c^2}-\frac{b n \sqrt{b^2-4 a c} \tanh ^{-1}\left(\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)}{2 c^2}+\frac{1}{2} x^2 \log \left(d \left(a+b x+c x^2\right)^n\right)+\frac{b n x}{2 c}-\frac{n x^2}{2}$$

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(2*c^2) - ((b^2 - 2*a*c)*n*Log[a + b*x + c*x^2])/(4*c^2) +
(x^2*Log[d*(a + b*x + c*x^2)^n])/2
```

Rubi [A] time = 0.112237, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n \left(b^2-2 a c\right) \log \left(a+b x+c x^2\right)}{4 c^2}-\frac{b n \sqrt{b^2-4 a c} \tanh ^{-1}\left(\frac{b+2 c x}{\sqrt{b^2-4 a c}}\right)}{2 c^2}+\frac{1}{2} x^2 \log \left(d \left(a+b x+c x^2\right)^n\right)+\frac{b n x}{2 c}-\frac{n x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(2*c^2) - ((b^2 - 2*a*c)*n*Log[a + b*x + c*x^2])/(4*c^2) +
(x^2*Log[d*(a + b*x + c*x^2)^n])/2
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int x\log\left(d\left(a+bx+cx^2\right)^n\right)dx=\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x^2(b+2cx)}{a+bx+cx^2}\,dx$$
$$=\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{ab+\left(b^2-2ac\right)x}{c\left(a+bx+cx^2\right)}\right)dx$$
$$=\frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{ab+\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{2c}$$
$$=\frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(b\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{4c^2}-\frac{\left(\left(b^2-2a\right.\right.}{}$$
$$=\frac{bnx}{2c}-\frac{nx^2}{2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{\left(b\left(b^2\right.\right.}{}$$
$$=\frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2$$

Mathematica [A] time = 0.0870621, size = 94, normalized size = 0.86

$$-\frac{n\left(b^2-2ac\right)\log(a+x(b+cx))+2bn\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2cx\left(cx\log\left(d(a+x(b+cx))^n\right)+n(b-cx)\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -(2*b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + (b^2 - 2
*a*c)*n*Log[a + x*(b + c*x)] - 2*c*x*(n*(b - c*x) + c*x*Log[d*(a + x*(b + c
*x))^n]))/(4*c^2)
```

Maple [C] time = 0.093, size = 510, normalized size = 4.7

$$\frac{x^2\ln\left(\left(cx^2+bx+a\right)^n\right)}{2}+\frac{i}{4}\left(\operatorname{csgn}\left(id\left(cx^2+bx+a\right)^n\right)\right)^2\operatorname{csgn}\left(i\left(cx^2+bx+a\right)^n\right)x^2\pi-\frac{i}{4}\pi x^2\operatorname{csgn}(id)\operatorname{csgn}\left(i\left(cx^2+b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] 1/2*x^2*ln((c*x^2+b*x+a)^n)+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*x^2
+b*x+a)^n)*x^2*Pi-1/4*I*Pi*x^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(
c*x^2+b*x+a)^n)-1/4*I*Pi*x^2*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/4*I*csgn(I*d*(c*
x^2+b*x+a)^n)^2*csgn(I*d)*x^2*Pi+1/2*ln(d)*x^2-1/2*n*x^2+1/2/c*n*ln(2*(-4*a
*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)*a-1/4/c^2*n*ln(
2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)*b^2+1/2/
c*n*ln(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*
a-1/4/c^2*n*ln(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(
1/2)*b)*b^2+1/2*b*n*x/c-1/4/c^2*n*ln(2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b
^3+(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^(1/2)+1/4/c^2*n*ln(-2*(-4*a*b
^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94003, size = 567, normalized size = 5.2

$$\left[-\frac{2\,c^2nx^2 - 2\,c^2x^2\log\left(d\right) - 2\,bcnx - \sqrt{b^2 - 4\,ac}bn\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right) - \left(2\,c^2nx^2 - \left(b^2 - 2\,ac\right)n\right)\log\left(c\right.}{4\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x - sqrt(b^2 - 4*a*c)*b*n*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x
^2 + b*x + a)) - (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2,
 -1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x + 2*sqrt(-b^2 + 4*a*c)*b*
n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c^2*n*x^2 - (b
```

```
^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33988, size = 153, normalized size = 1.4

$$\frac{1}{2}\,nx^2\log\left(cx^2+bx+a\right)-\frac{1}{2}\,(n-\log\left(d\right))x^2+\frac{bnx}{2\,c}-\frac{\left(b^2n-2\,acn\right)\log\left(cx^2+bx+a\right)}{4\,c^2}+\frac{\left(b^3n-4\,abcn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x + a) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/4*(
b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/c^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.75 $\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=79

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rubi [A] time = 0.0607216, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2523, 773, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{2c}\\
&= -2nx+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(\left(b^2-4ac\right)n\right)\mathrm{Subst}\left(\int\frac{1}{b^2\cdots}\right.}{c}\\
&= -2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)\cdots\right.
\end{aligned}
$$

Mathematica [A] time = 0.0607476, size = 78, normalized size = 0.99

$$
\frac{2n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2cx\left(\log\left(d(a+x(b+cx))^n\right)-2n\right)+bn\log(a+x(b+cx))}{2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A] time = 0.013, size = 118, normalized size = 1.5

$$x\ln\left(d\left(cx^2+bx+a\right)^n\right)-2nx+\frac{bn\ln\left(cx^2+bx+a\right)}{2c}+4\frac{na}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)-\frac{b^2n}{c}\arctan\left((2cx+b)\frac{1}{\sqrt{4a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+4*n/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a-n/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^2/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82496, size = 459, normalized size = 5.81

$$\left[-\frac{4cnx-2cx\log(d)-\sqrt{b^2-4ac}n\log\left(\frac{2c^2x^2+2bcx+b^2-2ac+\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)-(2cnx+bn)\log\left(cx^2+bx+a\right)}{2c},-\frac{4cnx-2c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [A] time = 102.018, size = 275, normalized size = 3.48

$$\begin{cases} \frac{bn\log\left(\frac{b^2}{4c}+bx+cx^2\right)}{2c} + nx\log\left(\frac{b^2}{4c}+bx+cx^2\right) - 2nx + x\log(d) \\ \frac{an\log(a+bx)}{b} + nx\log(a+bx) - nx + x\log(d) \\ \frac{2an\log\left(a+bx+cx^2\right)}{\sqrt{-4ac+b^2}} - \frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}} - \frac{b^2n\log\left(a+bx+cx^2\right)}{2c\sqrt{-4ac+b^2}} + \frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + nx\log\left(a+bx+c\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((b*n*log(b**2/(4*c) + b*x + c*x**2)/(2*c) + n*x*log(b**2/(4*c) +
b*x + c*x**2) - 2*n*x + x*log(d), Eq(a, b**2/(4*c))), (a*n*log(a + b*x)/b +
 n*x*log(a + b*x) - n*x + x*log(d), Eq(c, 0)), (2*a*n*log(a + b*x + c*x**2)
/sqrt(-4*a*c + b**2) - 4*a*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/s
qrt(-4*a*c + b**2) - b**2*n*log(a + b*x + c*x**2)/(2*c*sqrt(-4*a*c + b**2))
 + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-4*a*c + b**
2)) + b*n*log(a + b*x + c*x**2)/(2*c) + n*x*log(a + b*x + c*x**2) - 2*n*x +
 x*log(d), True))
```

Giac [A] time = 1.31342, size = 124, normalized size = 1.57

$$nx\log\left(cx^2+bx+a\right) - (2n-\log(d))x + \frac{bn\log\left(cx^2+bx+a\right)}{2c} - \frac{\left(b^2n-4acn\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4acc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

3.76 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$

Optimal. Leaf size=129

$$-n\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\text{PolyLog}\left(2,-\frac{2cx}{\sqrt{b^2-4ac}+b}\right)-n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)-n\log(x)\log\left(\right.$$

```
[Out] -(n*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) - n*Log[x]*Log[1 + (2*
c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*PolyL
og[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])]
```

Rubi [A] time = 0.175996, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2524, 2357, 2317, 2391}

$$-n\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\text{PolyLog}\left(2,-\frac{2cx}{\sqrt{b^2-4ac}+b}\right)-n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)-n\log(x)\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) - n*Log[x]*Log[1 + (2*
c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*PolyL
og[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx = \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - n\int \frac{(b+2cx)\log(x)}{a+bx+cx^2}\,dx$$
$$= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - n\int \left(\frac{2c\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log(x)}{b+\sqrt{b^2-4ac}+2cx}\right) dx$$
$$= \log(x)\log\left(d\left(a+bx+cx^2\right)^n\right) - (2cn)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx - (2cn)\int \frac{\log(x}{b+\sqrt{b^2-4a}$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) + \log(x)\log\left(d\left(a\right.\right.$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right) + \log(x)\log\left(d\left(a\right.\right.$$

Mathematica [A] time = 0.156441, size = 156, normalized size = 1.21

$$-n\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\text{PolyLog}\left(2,-\frac{2cx}{\sqrt{b^2-4ac}+b}\right) - n\log(x)\log\left(\frac{-\sqrt{b^2-4ac}+b+2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) -
n*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log
[x]*Log[d*(a + x*(b + c*x))^n] - n*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*
c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])]
```

Maple [C] time = 0.063, size = 315, normalized size = 2.4

$$\ln(x)\ln\left(\left(cx^2+bx+a\right)^n\right)-\ln(x)\ln\left(\left(-b-2\,cx+\sqrt{-4\,ac+b^2}\right)\left(-b+\sqrt{-4\,ac+b^2}\right)^{-1}\right)n-\ln(x)\ln\left(\left(b+2\,cx+\sqrt{-4\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x,x)
```

```
[Out] ln(x)*ln((c*x^2+b*x+a)^n)-ln(x)*ln((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*
c+b^2)^(1/2)))*n-ln(x)*ln((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2
)))*n-dilog((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2)))*n-dilog(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))*n-1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/2*I*ln(x)*Pi*csgn(I*(c*x^2+b*x+a)^n)*csg
n(I*d*(c*x^2+b*x+a)^n)^2-1/2*I*ln(x)*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(x)*l
n(d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(d\left(a + bx + cx^2\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2 + bx + a\right)^n d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

3.77 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*n*Log[x
])/a - (b*n*Log[a + b*x + c*x^2])/(2*a) - Log[d*(a + b*x + c*x^2)^n]/x
```

Rubi [A] time = 0.113233, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*n*Log[x
])/a - (b*n*Log[a + b*x + c*x^2])/(2*a) - Log[d*(a + b*x + c*x^2)^n]/x
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^2}\,dx &= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+n\int \frac{b+2cx}{x\left(a+bx+cx^2\right)}\,dx\\ &= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+n\int\left(\frac{b}{ax}+\frac{-b^2+2ac-bcx}{a\left(a+bx+cx^2\right)}\right)dx\\ &= \frac{bn\log(x)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+\frac{n\int \frac{-b^2+2ac-bcx}{a+bx+cx^2}\,dx}{a}\\ &= \frac{bn\log(x)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{\left(\left(b^2-4ac\right)n\right)\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\ &= \frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}+\frac{\left(\left(b^2-4ac\right)n\right)\mathrm{Subst}\left(\int\right.}{}\\ &= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}+\frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d\left(a+bx+c\right.\right.}{x}\end{aligned}$$

Mathematica [A] time = 0.10475, size = 87, normalized size = 1.01

$$\frac{2n\sqrt{4ac-b^2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)-\frac{2a\log(d(a+x(b+cx))^n)}{x}-bn\log(a+x(b+cx))+2bn\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*b*n*Log[
x] - b*n*Log[a + x*(b + c*x)] - (2*a*Log[d*(a + x*(b + c*x))^n])/x)/(2*a)
```

Maple [C] time = 0.069, size = 261, normalized size = 3.

$$-\frac{\ln\left(\left(cx^2+bx+a\right)^n\right)}{x}-\frac{-i\pi\,\mathrm{csgn}\left(id\right)\mathrm{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)+i\pi\,\mathrm{csgn}\left(id\right)\left(\mathrm{csgn}\left(id\left(cx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^2,x)
```

```
[Out] -1/x*ln((c*x^2+b*x+a)^n)-1/2*(-I*Pi*a*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csg
n(I*d*(c*x^2+b*x+a)^n)+I*Pi*a*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+I*Pi*a*
csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a*csgn(I*d*(c*x^2+
b*x+a)^n)^3-2*b*n*ln(x)*x-2*sum(_R*ln(((6*a*c-2*b^2)*_R^2+_R*b*c*n+4*c^2*n^
2)*x-a*b*_R^2+(-2*a*c*n+b^2*n)*_R+2*b*c*n^2),_R=RootOf(_Z^2*a+_Z*b*n+c*n^2)
)*a*x+2*ln(d)*a)/a/x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35201, size = 486, normalized size = 5.65

$$\left[\frac{2\,bnx\log\left(x\right)+\sqrt{b^2-4\,ac}nx\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(bnx+2\,an)\log\left(cx^2+bx+a\right)-2\,a\log\left(d\right)}{2\,ax}, \frac{2\,bnx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*n*x*log(x) + sqrt(b^2 - 4*a*c)*n*x*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b*n*x + 2*a
*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x), 1/2*(2*b*n*x*log(x) + 2*sqrt(
-b^2 + 4*a*c)*n*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20994, size = 134, normalized size = 1.56

$$-\frac{bn\log\left(cx^2+bx+a\right)}{2\,a}+\frac{bn\log\left(x\right)}{a}-\frac{n\log\left(cx^2+bx+a\right)}{x}-\frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a}-\frac{\log\left(d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*n*log(c*x^2 + b*x + a)/a + b*n*log(x)/a - n*log(c*x^2 + b*x + a)/x -
 (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - log(d)/x
```

3.78 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=121

$$\frac{n\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{bn\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}-\frac{bn}{2ax}$$

```
[Out] -(b*n)/(2*a*x) - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(2*a^2) - ((b^2 - 2*a*c)*n*Log[x])/(2*a^2) + ((b^2 - 2*a*c)*n*Log[a +
b*x + c*x^2])/(4*a^2) - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Rubi [A] time = 0.154217, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(b^2-2ac\right)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{bn\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}-\frac{bn}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -(b*n)/(2*a*x) - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(2*a^2) - ((b^2 - 2*a*c)*n*Log[x])/(2*a^2) + ((b^2 - 2*a*c)*n*Log[a +
b*x + c*x^2])/(4*a^2) - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{1}{2}n\int \frac{b+2cx}{x^2\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{1}{2}n\int \left(\frac{b}{ax^2} + \frac{-b^2+2ac}{a^2x} + \frac{b\left(b^2-3ac\right)+c\left(b^2-2ac\right)x}{a^2\left(a+bx+cx^2\right)}\right)dx$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{n\int \frac{b\left(b^2-3ac\right)+c\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{2a^2}$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2} + \frac{\left(b\left(b^2-4ac\right)n\right)\int \frac{1}{a+bx+cx^2}\,dx}{4a^2} +$$

$$= -\frac{bn}{2ax} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} + \frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4a^2} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}$$

$$= -\frac{bn}{2ax} - \frac{b\sqrt{b^2-4ac}n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2} - \frac{\left(b^2-2ac\right)n\log(x)}{2a^2} + \frac{\left(b^2-2ac\right)n\log\left(a+bx\right.}{4a^2}$$

Mathematica [A] time = 0.237582, size = 105, normalized size = 0.87

$$-\frac{\frac{nx\left(2x\log(x)\left(b^2-2ac\right)-x\left(b^2-2ac\right)\log(a+x(b+cx))+2bx\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2ab\right)}{a^2}+2\log\left(d(a+x(b+cx))^n\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -((n*x*(2*a*b + 2*b*Sqrt[b^2 - 4*a*c]*x*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] + 2*(b^2 - 2*a*c)*x*Log[x] - (b^2 - 2*a*c)*x*Log[a + x*(b + c*x)]))/a^2
 + 2*Log[d*(a + x*(b + c*x))^n])/(4*x^2)
```

Maple [C] time = 0.089, size = 1178, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^3,x)
```

```
[Out] -1/2/x^2*ln((c*x^2+b*x+a)^n)-1/4*(-I*Pi*a^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^2*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^2*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^2*csgn
(I*d*(c*x^2+b*x+a)^n)^3+2*n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^
2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^
(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2
*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4
*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-
2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5+5*(-4*a*b
^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*b^2*x^2+2*n*ln((-16
*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*
c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*
a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3
)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a
^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*
a^3*b*c^2+11*b^3*c*a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2
*c+b^4)^(1/2)*a*b^3)*b^2*x^2-4*n*ln(x)*c*a*x^2+2*n*ln(x)*b^2*x^2-n*ln((-16*
a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c
+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a
^2-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)
*(-4*a*b^2*c+b^4)^(1/2)*x^2+n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*
b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4
)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a
^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*(-4*a*b^2*c+b^4)^(1/2)*x^2+2*a*b*n*x
+2*ln(d)*a^2)/a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.35442, size = 628, normalized size = 5.19

$$\left[\frac{\sqrt{b^2-4\,ac}bnx^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^2-2\,ac\right)nx^2\log\left(x\right)-2\,abnx-2\,a^2\log\left(d\right)+\left(\left(b^2-2\,ac\right)nx^2\right.}{4\,a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*n*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2 - 2*a*c)*n*x^2*log
(x) - 2*a*b*n*x - 2*a^2*log(d) + ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2
+ b*x + a))/(a^2*x^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*n*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2 - 2*a*c)*n*x^2*log(x) + 2*a*b
*n*x + 2*a^2*log(d) - ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2 + b*x + a))
/(a^2*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26885, size = 174, normalized size = 1.44

$$\frac{\left(b^2n-2\,acn\right)\log\left(cx^2+bx+a\right)}{4\,a^2}-\frac{n\log\left(cx^2+bx+a\right)}{2\,x^2}-\frac{\left(b^2n-2\,acn\right)\log\left(x\right)}{2\,a^2}+\frac{\left(b^3n-4\,abcn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}-\frac{bn}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/a^2 - 1/2*n*log(c*x^2 + b*x + a)
/x^2 - 1/2*(b^2*n - 2*a*c*n)*log(x)/a^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan((2
*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*n*x + a*log
(d))/(a*x^2)
```

3.79 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{n\left(b^2-2ac\right)}{3a^2x}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}-\frac{\log}{}$$

```
[Out] -(b*n)/(6*a*x^2) + ((b^2 - 2*a*c)*n)/(3*a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2 -
a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*a^3) + (b*(b^2 - 3*a*c)*n
*Log[x])/(3*a^3) - (b*(b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*a^3) - Log[d
*(a + b*x + c*x^2)^n]/(3*x^3)
```

Rubi [A] time = 0.199732, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{bn\left(b^2-3ac\right)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{n\left(b^2-2ac\right)}{3a^2x}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3}+\frac{n\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```



```
[Out] -(b*n)/(6*a*x^2) + ((b^2 - 2*a*c)*n)/(3*a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2 -
a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*a^3) + (b*(b^2 - 3*a*c)*n
*Log[x])/(3*a^3) - (b*(b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*a^3) - Log[d
*(a + b*x + c*x^2)^n]/(3*x^3)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3} + \frac{1}{3}n\int \frac{b+2cx}{x^3\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3} + \frac{1}{3}n\int\left(\frac{b}{ax^3} + \frac{-b^2+2ac}{a^2x^2} + \frac{b^3-3abc}{a^3x} + \frac{-b^4+4ab^2c-2a^2c^2}{a^3\left(a+bx\right.}\right.$$

$$= -\frac{bn}{6ax^2} + \frac{\left(b^2-2ac\right)n}{3a^2x} + \frac{b\left(b^2-3ac\right)n\log(x)}{3a^3} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3} + n\int \frac{-b^4+4ab^2c-}{a}$$

$$= -\frac{bn}{6ax^2} + \frac{\left(b^2-2ac\right)n}{3a^2x} + \frac{b\left(b^2-3ac\right)n\log(x)}{3a^3} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3} - \frac{\left(b\left(b^2-3ac\right)\right.}{6}$$

$$= -\frac{bn}{6ax^2} + \frac{\left(b^2-2ac\right)n}{3a^2x} + \frac{b\left(b^2-3ac\right)n\log(x)}{3a^3} - \frac{b\left(b^2-3ac\right)n\log\left(a+bx+cx^2\right)}{6a^3} - \frac{\log\left(\right.}{}$$

$$= -\frac{bn}{6ax^2} + \frac{\left(b^2-2ac\right)n}{3a^2x} + \frac{\sqrt{b^2-4ac}\left(b^2-ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3} + \frac{b\left(b^2-3ac\right)n\log(x)}{3a^3}$$

Mathematica [A] time = 0.366032, size = 132, normalized size = 0.89

$$-\frac{\frac{nx\left(a^2b-2bx^2\log(x)\left(b^2-3ac\right)+bx^2\left(b^2-3ac\right)\log(a+x(b+cx))-2x^2\sqrt{b^2-4ac}\left(b^2-ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2ax\left(b^2-2ac\right)\right)}{a^3}+2\log\left(d(a+x(b+cx))^n\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -((n*x*(a^2*b - 2*a*(b^2 - 2*a*c)*x - 2*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*x^2*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*b*(b^2 - 3*a*c)*x^2*Log[x] + b*(b
^2 - 3*a*c)*x^2*Log[a + x*(b + c*x)]))/a^3 + 2*Log[d*(a + x*(b + c*x))^n])/
(6*x^3)
```

Maple [C] time = 0.085, size = 423, normalized size = 2.8

$$-\frac{\ln\left(\left(cx^2+bx+a\right)^n\right)}{3\,x^3} - \frac{-i\pi\,a^3\mathrm{csgn}\,(id)\,\mathrm{csgn}\left(i\left(cx^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(cx^2+bx+a\right)^n\right)+i\pi\,a^3\mathrm{csgn}\,(id)\left(\mathrm{csgn}\left(id\left(cx\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^4,x)
```

```
[Out] -1/3/x^3*ln((c*x^2+b*x+a)^n)-1/6*(-I*Pi*a^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^3*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^3*csgn
(I*d*(c*x^2+b*x+a)^n)^3+6*ln(x)*a*b*c*n*x^3-2*ln(x)*b^3*n*x^3-2*sum(_R*ln((
(6*a^5*c-2*a^4*b^2)*_R^2+(-7*a^3*b*c^2*n+2*a^2*b^3*c*n)*_R+4*a^2*c^4*n^2-4*
a*b^2*c^3*n^2+b^4*c^2*n^2)*x-a^5*b*_R^2+(2*a^4*c^2*n-4*a^3*b^2*c*n+a^2*b^4*
n)*_R+6*a^2*b*c^3*n^2-5*a*b^3*c^2*n^2+b^5*c*n^2),_R=RootOf(a^3*_Z^2+(-3*a*b
*c*n+b^3*n)*_Z+c^3*n^2))*a^3*x^3+4*a^2*c*n*x^2-2*a*b^2*n*x^2+a^2*b*n*x+2*ln
(d)*a^3)/a^3/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74687, size = 741, normalized size = 4.97

$$\left[-\frac{\left(b^2-ac\right)\sqrt{b^2-4\,ac}nx^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\left(b^3-3\,abc\right)nx^3\log\left(x\right)+a^2bnx-2\left(ab^2-2\,a^2c\right)nx^2}{6\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3 - 3*a*b
*c)*n*x^3*log(x) + a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2 + 2*a^3*log(d) + (
(b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 + b*x + a))/(a^3*x^3), 1/6*(2*(b
^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + 2*(b^3 - 3*a*b*c)*n*x^3*log(x) - a^2*b*n*x + 2*(a*b^2 - 2*a
```

```
^2*c)*n*x^2 - 2*a^3*log(d) - ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 +
b*x + a))/(a^3*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20834, size = 221, normalized size = 1.48

$$-\frac{\left(b^3n-3\,abcn\right)\log\left(cx^2+bx+a\right)}{6\,a^3}-\frac{n\log\left(cx^2+bx+a\right)}{3\,x^3}+\frac{\left(b^3n-3\,abcn\right)\log\left(x\right)}{3\,a^3}-\frac{\left(b^4n-5\,ab^2cn+4\,a^2c^2n\right)\arctan\left(\right.}{3\sqrt{-b^2+4\,ac}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)/a^3 - 1/3*n*log(c*x^2 + b*x +
 a)/x^3 + 1/3*(b^3*n - 3*a*b*c*n)*log(x)/a^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4
*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3)
 + 1/6*(2*b^2*n*x^2 - 4*a*c*n*x^2 - a*b*n*x - 2*a^2*log(d))/(a^2*x^3)
```

3.80 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal. Leaf size=190

$$\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8a^4}-\frac{n\log(x)\left(2a^2c^2-4ab^2c+b^4\right)}{4a^4}+\frac{n\left(b^2-2ac\right)}{8a^2x^2}-\frac{bn\left(b^2-3ac\right)}{4a^3x}-\frac{bn\sqrt{b^2-4a}}{}$$

```
[Out] -(b*n)/(12*a*x^3) + ((b^2 - 2*a*c)*n)/(8*a^2*x^2) - (b*(b^2 - 3*a*c)*n)/(4*
a^3*x) - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(4*a^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[x])/(4*a^4) + ((b
^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*x + c*x^2])/(8*a^4) - Log[d*(a + b*
x + c*x^2)^n]/(4*x^4)
```

Rubi [A] time = 0.222803, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8a^4}-\frac{n\log(x)\left(2a^2c^2-4ab^2c+b^4\right)}{4a^4}+\frac{n\left(b^2-2ac\right)}{8a^2x^2}-\frac{bn\left(b^2-3ac\right)}{4a^3x}-\frac{bn\sqrt{b^2-4a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```


```
[Out] -(b*n)/(12*a*x^3) + ((b^2 - 2*a*c)*n)/(8*a^2*x^2) - (b*(b^2 - 3*a*c)*n)/(4*
a^3*x) - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(4*a^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[x])/(4*a^4) + ((b
^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*x + c*x^2])/(8*a^4) - Log[d*(a + b*
x + c*x^2)^n]/(4*x^4)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4} + \frac{1}{4}n\int \frac{b+2cx}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4} + \frac{1}{4}n\int\left(\frac{b}{ax^4} + \frac{-b^2+2ac}{a^2x^3} + \frac{b^3-3abc}{a^3x^2} + \frac{-b^4+4ab^2c-2a^2c}{a^4x}\right.$$

$$= -\frac{bn}{12ax^3} + \frac{\left(b^2-2ac\right)n}{8a^2x^2} - \frac{b\left(b^2-3ac\right)n}{4a^3x} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4} - \frac{\log\left(d\left(a+\right.\right.}{4x}$$

$$= -\frac{bn}{12ax^3} + \frac{\left(b^2-2ac\right)n}{8a^2x^2} - \frac{b\left(b^2-3ac\right)n}{4a^3x} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4} - \frac{\log\left(d\left(a+\right.\right.}{4x}$$

$$= -\frac{bn}{12ax^3} + \frac{\left(b^2-2ac\right)n}{8a^2x^2} - \frac{b\left(b^2-3ac\right)n}{4a^3x} - \frac{\left(b^4-4ab^2c+2a^2c^2\right)n\log(x)}{4a^4} + \frac{\left(b^4-4ab^2c\right.}{}$$

$$= -\frac{bn}{12ax^3} + \frac{\left(b^2-2ac\right)n}{8a^2x^2} - \frac{b\left(b^2-3ac\right)n}{4a^3x} - \frac{b\sqrt{b^2-4ac}\left(b^2-2ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4} -$$

Mathematica [A] time = 0.450394, size = 172, normalized size = 0.91

$$-\frac{\frac{nx\left(6x^3\log(x)\left(2a^2c^2-4ab^2c+b^4\right)-3x^3\left(2a^2c^2-4ab^2c+b^4\right)\log(a+x(b+cx))-3a^2x\left(b^2-2ac\right)+2a^3b+6abx^2\left(b^2-3ac\right)+6bx^3\sqrt{b^2-4ac}\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\right)}{a^4}+}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -((n*x*(2*a^3*b - 3*a^2*(b^2 - 2*a*c)*x + 6*a*b*(b^2 - 3*a*c)*x^2 + 6*b*Sqr
t[b^2 - 4*a*c]*(b^2 - 2*a*c)*x^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 6
*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[x] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)
*x^3*Log[a + x*(b + c*x)]))/a^4 + 6*Log[d*(a + x*(b + c*x))^n])/(24*x^4)
```

Maple [C] time = 0.114, size = 3583, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^5,x)

[Out] -1/4/x^4*ln((c*x^2+b*x+a)^n)-1/24*(-6*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-
69*a^2*b^6*c^2+20*a*b^8*c-2*b^10+6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*
c+b^8)^(1/2)*a^3*c^3-19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2
)*a^2*b^2*c^2+12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4
*c-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c
^4+74*a^4*b^3*c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9-7*(-16*a^3*b^2*c^3+20
*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*b*c^2+9*(-16*a^3*b^2*c^3+20*a^2*b^4*c
^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^3*c-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6
*c+b^8)^(1/2)*a*b^5)*c^2*a^2*x^4+12*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69
*a^2*b^6*c^2+20*a*b^8*c-2*b^10+6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+
b^8)^(1/2)*a^3*c^3-19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*
a^2*b^2*c^2+12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c
-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4
+74*a^4*b^3*c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9-7*(-16*a^3*b^2*c^3+20*a
^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*b*c^2+9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2
-8*a*b^6*c+b^8)^(1/2)*a^2*b^3*c-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c
+b^8)^(1/2)*a*b^5)*b^2*c*a*x^4-3*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^
2*b^6*c^2+20*a*b^8*c-2*b^10+6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8
)^(1/2)*a^3*c^3-19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2
*b^2*c^2+12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c-2*
(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74
*a^4*b^3*c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9-7*(-16*a^3*b^2*c^3+20*a^2*
b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*b*c^2+9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*
a*b^6*c+b^8)^(1/2)*a^2*b^3*c-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^
8)^(1/2)*a*b^5)*b^4*x^4-6*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^2*b^6*c
^2+20*a*b^8*c-2*b^10-6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)
*a^3*c^3+19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^2*c^
2-12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c+2*(-16*a^
3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74*a^4*b^
3*c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9+7*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2
-8*a*b^6*c+b^8)^(1/2)*a^3*b*c^2-9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c
+b^8)^(1/2)*a^2*b^3*c+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2
)*a*b^5)*c^2*a^2*x^4+12*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^2*b^6*c^2
+20*a*b^8*c-2*b^10-6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a
^3*c^3+19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^2*c^2-
12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c+2*(-16*a^3*
b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74*a^4*b^3*
c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9+7*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8
*a*b^6*c+b^8)^(1/2)*a^3*b*c^2-9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b
^8)^(1/2)*a^2*b^3*c+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*
a*b^5)*b^2*c*a*x^4-3*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^2*b^6*c^2+20
*a*b^8*c-2*b^10-6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*
```

```
c^3+19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^2*c^2-12*
(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c+2*(-16*a^3*b^2
*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74*a^4*b^3*c^3
-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9+7*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*
b^6*c+b^8)^(1/2)*a^3*b*c^2-9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)
^(1/2)*a^2*b^3*c+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b
^5)*b^4*x^4+12*n*ln(x)*c^2*a^2*x^4-24*n*ln(x)*b^2*c*a*x^4+6*n*ln(x)*b^4*x^4
+3*I*Pi*a^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2-3*I*Pi*a^4*csgn(I*d)*csgn
(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+3*I*Pi*a^4*csgn(I*(c*x^2+b*x+
a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-3*I*Pi*a^4*csgn(I*d*(c*x^2+b*x+a)^n)^3-18
*a^2*b*c*n*x^3+6*a*b^3*n*x^3-3*n*ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^2*
b^6*c^2+20*a*b^8*c-2*b^10+6*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^
(1/2)*a^3*c^3-19*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b
^2*c^2+12*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c-2*(-
16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74*a
^4*b^3*c^3-61*a^3*b^5*c^2+19*b^7*c*a^2-2*a*b^9-7*(-16*a^3*b^2*c^3+20*a^2*b^
4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*b*c^2+9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*
b^6*c+b^8)^(1/2)*a^2*b^3*c-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)
^(1/2)*a*b^5)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*x^4+3*n*
ln((-40*a^4*b^2*c^4+94*a^3*b^4*c^3-69*a^2*b^6*c^2+20*a*b^8*c-2*b^10-6*(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*c^3+19*(-16*a^3*b^2*c^3
+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^2*c^2-12*(-16*a^3*b^2*c^3+20*a^2
*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^4*c+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a
*b^6*c+b^8)^(1/2)*b^6)*x-24*a^5*b*c^4+74*a^4*b^3*c^3-61*a^3*b^5*c^2+19*b^7*
c*a^2-2*a*b^9+7*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^3*b*
c^2-9*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a^2*b^3*c+2*(-16
*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*a*b^5)*(-16*a^3*b^2*c^3+20
*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*x^4+6*a^3*c*n*x^2-3*a^2*b^2*n*x^2+2*a^3*b
*n*x+6*ln(d)*a^4)/a^4/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.80349, size = 932, normalized size = 4.91

$$\left[-\frac{3\left(b^3-2\,abc\right)\sqrt{b^2-4\,ac}nx^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+6\left(b^4-4\,ab^2c+2\,a^2c^2\right)nx^4\log\left(x\right)+2\,a^3bnx+6\left(\right.}{24\,a^4x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*x^4*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4
- 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)
*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^2 - 3*((b^4 - 4*a*b^2*c +
 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))/(a^4*x^4), -1/24*(6*(b^3
 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*
x + 6*(a*b^3 - 3*a^2*b*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^
2 - 3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))
/(a^4*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23513, size = 284, normalized size = 1.49

$$\frac{\left(b^4n-4\,ab^2cn+2\,a^2c^2n\right)\log\left(cx^2+bx+a\right)}{8\,a^4}-\frac{n\log\left(cx^2+bx+a\right)}{4\,x^4}-\frac{\left(b^4n-4\,ab^2cn+2\,a^2c^2n\right)\log\left(x\right)}{4\,a^4}+\frac{\left(b^5n-6\,ab^3cn\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/a^4 - 1/4*n*lo
g(c*x^2 + b*x + a)/x^4 - 1/4*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(x)/a^4
 + 1/4*(b^5*n - 6*a*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/24*(6*b^3*n*x^3 - 18*a*b*c*n*x^3 - 3*
a*b^2*n*x^2 + 6*a^2*c*n*x^2 + 2*a^2*b*n*x + 6*a^3*log(d))/(a^3*x^4)
```

3.81 $\int \log\left(1+x+x^2\right) dx$

Optimal. Leaf size=42

$$x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 + x
 + x^2]
```

Rubi [A] time = 0.0263058, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2523, 773, 634, 618, 204, 628}

$$x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2],x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 + x
 + x^2]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(1+x+x^2\right)\,dx &= x\log\left(1+x+x^2\right)-\int \frac{x(1+2x)}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)-\int \frac{-2-x}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -2x+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.012705, size = 35, normalized size = 0.83

$$\left(x+\frac{1}{2}\right)\log\left(x^2+x+1\right)-2x+\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + x^2],x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (1/2 + x)*Log[1 + x + x^2]
```

Maple [A] time = 0.007, size = 38, normalized size = 0.9

$$-2\,x+\frac{\ln\left(x^{2}+x+1\right)}{2}+x\ln\left(x^{2}+x+1\right)+\arctan\left(\frac{(1+2\,x)\,\sqrt{3}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1),x)
```

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.77586, size = 50, normalized size = 1.19

$$x\log\left(x^{2}+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x+\frac{1}{2}\,\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

Fricas [A] time = 2.03413, size = 105, normalized size = 2.5

$$\frac{1}{2}\,(2\,x+1)\log\left(x^{2}+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 1)*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*
x
```

Sympy [A] time = 0.140535, size = 46, normalized size = 1.1

$$x\log\left(x^2+x+1\right)-2x+\frac{\log\left(x^2+x+1\right)}{2}+\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1),x)
```

```
[Out] x*log(x**2 + x + 1) - 2*x + log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/
3 + sqrt(3)/3)
```

Giac [A] time = 1.16446, size = 50, normalized size = 1.19

$$x\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="giac")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

3.82 $\int (d+ex)^4 \log \left(d \left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=485

$$-\frac{n(2cd-be)\left(c^2e^2\left(5a^2e^2+10abde+4b^2d^2\right)-b^2ce^3(5ae+3bd)-2c^3d^2e(5ae+bd)+b^4e^4+c^4d^4\right)\log\left(a+bx+cx^2\right)}{10c^5e}-\frac{n}{}$$

```
[Out] -((10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3*(5*b*d + 4
*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*n*x)/(5*c^4) - (e*(2
0*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a*e))*n*x
^2)/(10*c^3) - (e^2*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*n*x^3)/(15
*c^2) - (e^3*(10*c*d - b*e)*n*x^4)/(20*c) - (2*e^4*n*x^5)/25 + (Sqrt[b^2 -
4*a*c]*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(5*c^5) - ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d
^2*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2))*n*Log[a + b*x + c*x^2])/(10*c^5*e) + ((d + e*x)^5*Log[d
*(a + b*x + c*x^2)^n])/(5*e)
```

Rubi [A] time = 2.05768, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n(2cd-be)\left(c^2e^2\left(5a^2e^2+10abde+4b^2d^2\right)-b^2ce^3(5ae+3bd)-2c^3d^2e(5ae+bd)+b^4e^4+c^4d^4\right)\log\left(a+bx+cx^2\right)}{10c^5e}-\frac{n}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```


```
[Out] -((10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3*(5*b*d + 4
*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*n*x)/(5*c^4) - (e*(2
0*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a*e))*n*x
^2)/(10*c^3) - (e^2*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*n*x^3)/(15
*c^2) - (e^3*(10*c*d - b*e)*n*x^4)/(20*c) - (2*e^4*n*x^5)/25 + (Sqrt[b^2 -
4*a*c]*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(5*c^5) - ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d
^2*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2))*n*Log[a + b*x + c*x^2])/(10*c^5*e) + ((d + e*x)^5*Log[d
```

```
*(a + b*x + c*x^2)^n])/(5*e)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \frac{(b+2cx)(d+ex)^5}{a+bx+cx^2}dx}{5e}$$

$$= \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \left(\frac{e\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+\right.}{c^4}\right.}{}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abd\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abd\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abd\right.\right.}{5c^4}$$

$$= -\frac{\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abd\right.\right.}{5c^4}$$

Mathematica [A] time = 1.90509, size = 468, normalized size = 0.96

$$\frac{(d+ex)^5 \log\left(d(a+x(b+cx))^n\right) - \frac{n\left(60cex\left(c^2e^2\left(2a^2e^2+15abde+10b^2d^2\right)-b^2ce^3(4ae+5bd)-10c^3d^2e(2ae+bd)+b^4e^4+10c^4d^4\right)+30(2cd-be)\left(c^2e^2\left(5a^2e^2+1\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-(n*(60*c*e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3
*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x + 30*c^
2*e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a
*e))*x^2 + 20*c^3*e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*x^3 + 15
*c^4*e^4*(10*c*d - b*e)*x^4 + 24*c^5*e^5*x^5 - 60*Sqrt[b^2 - 4*a*c]*e*(5*c^
4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d + 3*a*e) + c^
2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]] + 30*(2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2*e*(b*d + 5*a*e) -
b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b*d*e + 5*a^2*e^2))*L
og[a + x*(b + c*x)]))/(60*c^5) + (d + e*x)^5*Log[d*(a + x*(b + c*x))^n])/(5
*e)
```

Maple [C] time = 0.23, size = 31895, normalized size = 65.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.99206, size = 2700, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*e^4*n*x^5 + 15*(10*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^
5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3
*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)
*e^4)*n*x^2 - 30*(5*c^4*d^4 - 10*b*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2
 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(b^2
- 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2*c^3
- 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*c^2 +
 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c^5*d^
2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*(b^2*
c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4*a*b^
```

```
2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log(c*x^
2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3 + 10*
c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5, -1/300*(24*c^5*e^4*n*x^5 + 15*(10
*c^5*d*e^3 - b*c^4*e^4)*n*x^4 + 20*(20*c^5*d^2*e^2 - 5*b*c^4*d*e^3 + (b^2*c
^3 - 2*a*c^4)*e^4)*n*x^3 + 30*(20*c^5*d^3*e - 10*b*c^4*d^2*e^2 + 5*(b^2*c^3
 - 2*a*c^4)*d*e^3 - (b^3*c^2 - 3*a*b*c^3)*e^4)*n*x^2 - 60*(5*c^4*d^4 - 10*b
*c^3*d^3*e + 10*(b^2*c^2 - a*c^3)*d^2*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*e^3 + (
b^4 - 3*a*b^2*c + a^2*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*
a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 60*(10*c^5*d^4 - 10*b*c^4*d^3*e + 10*(b^2
*c^3 - 2*a*c^4)*d^2*e^2 - 5*(b^3*c^2 - 3*a*b*c^3)*d*e^3 + (b^4*c - 4*a*b^2*
c^2 + 2*a^2*c^3)*e^4)*n*x - 30*(2*c^5*e^4*n*x^5 + 10*c^5*d*e^3*n*x^4 + 20*c
^5*d^2*e^2*n*x^3 + 20*c^5*d^3*e*n*x^2 + 10*c^5*d^4*n*x + (5*b*c^4*d^4 - 10*
(b^2*c^3 - 2*a*c^4)*d^3*e + 10*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 5*(b^4*c - 4
*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4)*n)*log
(c*x^2 + b*x + a) - 60*(c^5*e^4*x^5 + 5*c^5*d*e^3*x^4 + 10*c^5*d^2*e^2*x^3
+ 10*c^5*d^3*e*x^2 + 5*c^5*d^4*x)*log(d))/c^5]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38521, size = 1338, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*d^4*n*log(c*x^2 + b*x + a)/c - (b^2*d^4*n - 4*a*c*d^4*n)*arctan((2*c*
x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) - (b^2*d^3*n*e - 2*a*c*d^
3*n*e)*log(c*x^2 + b*x + a)/c^2 + 2*(b^3*d^3*n*e - 4*a*b*c*d^3*n*e)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2) + (b^3*d^2*n*e^2 -
```

```
 3*a*b*c*d^2*n*e^2)*log(c*x^2 + b*x + a)/c^3 - 2*(b^4*d^2*n*e^2 - 5*a*b^2*c
*d^2*n*e^2 + 4*a^2*c^2*d^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*c^3) - 1/2*(b^4*d*n*e^3 - 4*a*b^2*c*d*n*e^3 + 2*a^2*c^2*d
*n*e^3)*log(c*x^2 + b*x + a)/c^4 + (b^5*d*n*e^3 - 6*a*b^3*c*d*n*e^3 + 8*a^2
*b*c^2*d*n*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^4) + 1/300*(60*c^4*n*x^5*e^4*log(c*x^2 + b*x + a) + 300*c^4*d*n*x^4*e^3*l
og(c*x^2 + b*x + a) + 600*c^4*d^2*n*x^3*e^2*log(c*x^2 + b*x + a) + 600*c^4*
d^3*n*x^2*e*log(c*x^2 + b*x + a) - 24*c^4*n*x^5*e^4 - 150*c^4*d*n*x^4*e^3 -
 400*c^4*d^2*n*x^3*e^2 - 600*c^4*d^3*n*x^2*e + 300*c^4*d^4*n*x*log(c*x^2 +
b*x + a) + 60*c^4*x^5*e^4*log(d) + 300*c^4*d*x^4*e^3*log(d) + 600*c^4*d^2*x
^3*e^2*log(d) + 600*c^4*d^3*x^2*e*log(d) - 600*c^4*d^4*n*x + 15*b*c^3*n*x^4
*e^4 + 100*b*c^3*d*n*x^3*e^3 + 300*b*c^3*d^2*n*x^2*e^2 + 600*b*c^3*d^3*n*x*
e + 300*c^4*d^4*x*log(d) - 20*b^2*c^2*n*x^3*e^4 + 40*a*c^3*n*x^3*e^4 - 150*
b^2*c^2*d*n*x^2*e^3 + 300*a*c^3*d*n*x^2*e^3 - 600*b^2*c^2*d^2*n*x*e^2 + 120
0*a*c^3*d^2*n*x*e^2 + 30*b^3*c*n*x^2*e^4 - 90*a*b*c^2*n*x^2*e^4 + 300*b^3*c
*d*n*x*e^3 - 900*a*b*c^2*d*n*x*e^3 - 60*b^4*n*x*e^4 + 240*a*b^2*c*n*x*e^4 -
 120*a^2*c^2*n*x*e^4)/c^4 + 1/10*(b^5*n*e^4 - 5*a*b^3*c*n*e^4 + 5*a^2*b*c^2
*n*e^4)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n*e^4 - 7*a*b^4*c*n*e^4 + 13*a^
2*b^2*c^2*n*e^4 - 4*a^3*c^3*n*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
sqrt(-b^2 + 4*a*c)*c^5)
```

3.83 $\int (d+ex)^3 \log \left(d \left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=338

$$-\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8c^4e}-\frac{enx^2\left(-2ce(\ldots\right.}{}$$

```
[Out] -((8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b*d + 4*a*e
))*n*x)/(4*c^3) - (e*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*n*x^2)/(8
*c^2) - (e^2*(8*c*d - b*e)*n*x^3)/(12*c) - (e^3*n*x^4)/8 + (Sqrt[b^2 - 4*a*
c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*n*Log[a + b*x + c*x^2])/(8*c^4*e) + ((d + e*x)^4*Log[d*(a + b*x +
c*x^2)^n])/(4*e)
```

Rubi [A] time = 0.515817, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8c^4e}-\frac{enx^2\left(-2ce(\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b*d + 4*a*e
))*n*x)/(4*c^3) - (e*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*n*x^2)/(8
*c^2) - (e^2*(8*c*d - b*e)*n*x^3)/(12*c) - (e^3*n*x^4)/8 + (Sqrt[b^2 - 4*a*
c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*n*Log[a + b*x + c*x^2])/(8*c^4*e) + ((d + e*x)^4*Log[d*(a + b*x +
c*x^2)^n])/(4*e)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
```

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d+ex)^3 \log\left(d\left(a+bx+cx^2\right)^n\right) dx = \frac{(d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)}{4e} - \frac{n\int \frac{(b+2cx)(d+ex)^4}{a+bx+cx^2} dx}{4e}$$

$$= \frac{(d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)}{4e} - \frac{n\int \left(\frac{e\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)}{c^3} + \right.}{}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+b^2e^2-2\right.}{8c}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+b^2e^2-2\right.}{8c}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+b^2e^2-2\right.}{8c}$$

$$= -\frac{\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)nx}{4c^3} - \frac{e\left(12c^2d^2+b^2e^2-2\right.}{8c}$$

Mathematica [A] time = 0.504436, size = 324, normalized size = 0.96

$$(d+ex)^4 \log\left(d(a+x(b+cx))^n\right) - \frac{n\left(3\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log(a+x(b+cx))+3c^2e^2x^2\left(-2ce(ae+2b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-(n*(6*c*e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b
*d + 4*a*e))*x + 3*c^2*e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*x^2
 + 2*c^3*e^3*(8*c*d - b*e)*x^3 + 3*c^4*e^4*x^4 - 6*Sqrt[b^2 - 4*a*c]*e*(2*c
*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqr
t[b^2 - 4*a*c]] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*Log[a +
x*(b + c*x)]))/(6*c^4) + (d + e*x)^4*Log[d*(a + x*(b + c*x))^n])/(4*e)
```

Maple [C] time = 0.196, size = 16059, normalized size = 47.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.4354, size = 1858, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c^3*e^3)*n*x^3 + 3*(12*c^4*d^2
*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*n*x^2 - 3*(4*c^3*d^3 - 6*b*c^
2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a*b*c)*e^3)*sqrt(b^2 - 4*a*c)*
n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*
e^2 - (b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3
 + 12*c^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3
)*d^2*e + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*
n)*log(c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^
2 + 4*c^4*d^3*x)*log(d))/c^4, -1/24*(3*c^4*e^3*n*x^4 + 2*(8*c^4*d*e^2 - b*c
^3*e^3)*n*x^3 + 3*(12*c^4*d^2*e - 4*b*c^3*d*e^2 + (b^2*c^2 - 2*a*c^3)*e^3)*
n*x^2 - 6*(4*c^3*d^3 - 6*b*c^2*d^2*e + 4*(b^2*c - a*c^2)*d*e^2 - (b^3 - 2*a
*b*c)*e^3)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
 - 4*a*c)) + 6*(8*c^4*d^3 - 6*b*c^3*d^2*e + 4*(b^2*c^2 - 2*a*c^3)*d*e^2 - (
b^3*c - 3*a*b*c^2)*e^3)*n*x - 3*(2*c^4*e^3*n*x^4 + 8*c^4*d*e^2*n*x^3 + 12*c
```

```
^4*d^2*e*n*x^2 + 8*c^4*d^3*n*x + (4*b*c^3*d^3 - 6*(b^2*c^2 - 2*a*c^3)*d^2*e
 + 4*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3)*n)*log(
c*x^2 + b*x + a) - 6*(c^4*e^3*x^4 + 4*c^4*d*e^2*x^3 + 6*c^4*d^2*e*x^2 + 4*c
^4*d^3*x)*log(d))/c^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39151, size = 923, normalized size = 2.73

$$\frac{bd^3n\log\left(cx^2+bx+a\right)}{2c}-\frac{\left(b^2d^3n-4\,acd^3n\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}-\frac{3\left(b^2d^2ne-2\,acd^2ne\right)\log\left(cx^2+bx+a\right)}{4\,c^2}+\frac{3\left(b^3d^2ne-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*d^3*n*log(c*x^2 + b*x + a)/c - (b^2*d^3*n - 4*a*c*d^3*n)*arctan((2*c*
x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) - 3/4*(b^2*d^2*n*e - 2*a*
c*d^2*n*e)*log(c*x^2 + b*x + a)/c^2 + 3/2*(b^3*d^2*n*e - 4*a*b*c*d^2*n*e)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2) + 1/2*(b^3*d
*n*e^2 - 3*a*b*c*d*n*e^2)*log(c*x^2 + b*x + a)/c^3 - (b^4*d*n*e^2 - 5*a*b^2
*c*d*n*e^2 + 4*a^2*c^2*d*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*c^3) + 1/24*(6*c^3*n*x^4*e^3*log(c*x^2 + b*x + a) + 24*c^3*
d*n*x^3*e^2*log(c*x^2 + b*x + a) + 36*c^3*d^2*n*x^2*e*log(c*x^2 + b*x + a)
- 3*c^3*n*x^4*e^3 - 16*c^3*d*n*x^3*e^2 - 36*c^3*d^2*n*x^2*e + 24*c^3*d^3*n*
x*log(c*x^2 + b*x + a) + 6*c^3*x^4*e^3*log(d) + 24*c^3*d*x^3*e^2*log(d) + 3
6*c^3*d^2*x^2*e*log(d) - 48*c^3*d^3*n*x + 2*b*c^2*n*x^3*e^3 + 12*b*c^2*d*n*
x^2*e^2 + 36*b*c^2*d^2*n*x*e + 24*c^3*d^3*x*log(d) - 3*b^2*c*n*x^2*e^3 + 6*
a*c^2*n*x^2*e^3 - 24*b^2*c*d*n*x*e^2 + 48*a*c^2*d*n*x*e^2 + 6*b^3*n*x*e^3 -
 18*a*b*c*n*x*e^3)/c^3 - 1/8*(b^4*n*e^3 - 4*a*b^2*c*n*e^3 + 2*a^2*c^2*n*e^3
)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n*e^3 - 6*a*b^3*c*n*e^3 + 8*a^2*b*c^2
```

```
*n*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

3.84 $\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) dx$

Optimal. Leaf size=226

$$-\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e}-\frac{nx\left(-ce(2ae+3bd)+b^2e^2+6c^2d^2\right)}{3c^2}+\frac{n\sqrt{b^2-4ac}\left(-ce(\right.}{}$$

```
[Out] -((6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*n*x)/(3*c^2) - (e*(6*c*d - b*
e)*n*x^2)/(6*c) - (2*e^2*n*x^3)/9 + (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2
 - c*e*(3*b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) - (
(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[a + b*x + c*x^2
])/(6*c^3*e) + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Rubi [A] time = 0.319069, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e}-\frac{nx\left(-ce(2ae+3bd)+b^2e^2+6c^2d^2\right)}{3c^2}+\frac{n\sqrt{b^2-4ac}\left(-ce(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -((6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*n*x)/(3*c^2) - (e*(6*c*d - b*
e)*n*x^2)/(6*c) - (2*e^2*n*x^3)/9 + (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2
 - c*e*(3*b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) - (
(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[a + b*x + c*x^2
])/(6*c^3*e) + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d+ex)^2 \log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n\int\frac{(b+2cx)(d+ex)^3}{a+bx+cx^2}dx}{3e}$$

$$= \frac{(d+ex)^3\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n\int\left(\frac{e\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)}{c^2} + \frac{e^2(6cd-be)x}{c} + 2e \ldots\right.}{\ldots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+ex)^3\log \ldots}{\ldots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{(d+ex)^3\log \ldots}{\ldots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 - \frac{(2cd-be)\left(c^2 \ldots\right.}{\ldots}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3 + \frac{\sqrt{b^2-4ac}\left(3c \ldots\right.}{\ldots}$$

Mathematica [A] time = 0.377775, size = 204, normalized size = 0.9

$$\frac{(d+ex)^3\log\left(d(a+x(b+cx))^n\right) - \frac{n\left(cex\left(-3ce(4ae+6bd+bex)+6b^2e^2+2c^2\left(18d^2+9dex+2e^2x^2\right)\right)+3(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log(a+x(b+cx))-\ldots\right.}{6c^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-(n*(c*e*x*(6*b^2*e^2 - 3*c*e*(6*b*d + 4*a*e + b*e*x) + 2*c^2*(18*d^2 + 9*
d*e*x + 2*e^2*x^2)) - 6*Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b
*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c*d - b*e)*(c^2*d^
2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + x*(b + c*x)]))/(6*c^3) + (d + e*x)
^3*Log[d*(a + x*(b + c*x))^n])/(3*e)
```

Maple [C] time = 0.148, size = 7155, normalized size = 31.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25016, size = 1242, normalized size = 5.5

$$\left[-\frac{4\,c^3e^2nx^3+3\left(6\,c^3de-bc^2e^2\right)nx^2+3\left(3\,c^2d^2-3\,bcde+\left(b^2-ac\right)e^2\right)\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^2)*n*x^2 + 3*(3*c^2*d^2 -
3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x +
 b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(6*c^3
*d^2 - 3*b*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^
3*d*e*n*x^2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3
 - 3*a*b*c)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 +
 3*c^3*d^2*x)*log(d))/c^3, -1/18*(4*c^3*e^2*n*x^3 + 3*(6*c^3*d*e - b*c^2*e^
2)*n*x^2 - 6*(3*c^2*d^2 - 3*b*c*d*e + (b^2 - a*c)*e^2)*sqrt(-b^2 + 4*a*c)*n
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(6*c^3*d^2 - 3*b
*c^2*d*e + (b^2*c - 2*a*c^2)*e^2)*n*x - 3*(2*c^3*e^2*n*x^3 + 6*c^3*d*e*n*x^
2 + 6*c^3*d^2*n*x + (3*b*c^2*d^2 - 3*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c
)*e^2)*n)*log(c*x^2 + b*x + a) - 6*(c^3*e^2*x^3 + 3*c^3*d*e*x^2 + 3*c^3*d^2
*x)*log(d))/c^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34656, size = 591, normalized size = 2.62

$$\frac{bd^2n\log\left(cx^2+bx+a\right)}{2c}-\frac{\left(b^2d^2n-4\,acd^2n\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c}-\frac{\left(b^2dne-2\,acdne\right)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^3dne-4\,abc\right.}{\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*d^2*n*log(c*x^2 + b*x + a)/c - (b^2*d^2*n - 4*a*c*d^2*n)*arctan((2*c*
x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) - 1/2*(b^2*d*n*e - 2*a*c*
d*n*e)*log(c*x^2 + b*x + a)/c^2 + (b^3*d*n*e - 4*a*b*c*d*n*e)*arctan((2*c*x
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2) + 1/18*(6*c^2*n*x^3*e^2*
log(c*x^2 + b*x + a) + 18*c^2*d*n*x^2*e*log(c*x^2 + b*x + a) - 4*c^2*n*x^3*
e^2 - 18*c^2*d*n*x^2*e + 18*c^2*d^2*n*x*log(c*x^2 + b*x + a) + 6*c^2*x^3*e^
2*log(d) + 18*c^2*d*x^2*e*log(d) - 36*c^2*d^2*n*x + 3*b*c*n*x^2*e^2 + 18*b*
c*d*n*x*e + 18*c^2*d^2*x*log(d) - 6*b^2*n*x*e^2 + 12*a*c*n*x*e^2)/c^2 + 1/6
*(b^3*n*e^2 - 3*a*b*c*n*e^2)*log(c*x^2 + b*x + a)/c^3 - 1/3*(b^4*n*e^2 - 5*
a*b^2*c*n*e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*c^3)
```

3.85 $\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=154

$$-\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4c^2e}+\frac{n\sqrt{b^2-4ac}(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{(d+ex)^2\log\left(d\left(a+\right.\right.}{2e}$$

```
[Out] -((4*d - (b*e)/c)*n*x)/2 - (e*n*x^2)/2 + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((2*c^2*d^2 + b^2*e^2 -
2*c*e*(b*d + a*e))*n*Log[a + b*x + c*x^2])/(4*c^2*e) + ((d + e*x)^2*Log[d*(
a + b*x + c*x^2)^n])/(2*e)
```

Rubi [A] time = 0.186291, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2525, 800, 634, 618, 206, 628}

$$-\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4c^2e}+\frac{n\sqrt{b^2-4ac}(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{(d+ex)^2\log\left(d\left(a+\right.\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```


```
[Out] -((4*d - (b*e)/c)*n*x)/2 - (e*n*x^2)/2 + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((2*c^2*d^2 + b^2*e^2 -
2*c*e*(b*d + a*e))*n*Log[a + b*x + c*x^2])/(4*c^2*e) + ((d + e*x)^2*Log[d*(
a + b*x + c*x^2)^n])/(2*e)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)dx = \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int \frac{(b+2cx)(d+ex)^2}{a+bx+cx^2}dx}{2e}$$

$$= \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int \left(e\left(4d-\frac{be}{c}\right)+2e^2x+\frac{bcd^2-4acde+abe^2+\left(2c^2\right.}{c\left(a+b\right.}\right.}{2e}$$

$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{n\int \frac{bcd^2-4acde+a}{}}{}$$

$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e} - \frac{\left(\left(b^2-4ac\right)(2c\right.}{}$$

$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log\left(a+bx+cx^2\right)}{4c^2e}$$

$$= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{\sqrt{b^2-4ac}(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2} - \frac{\left(2c^2d^2+\right.}{}$$

Mathematica [A] time = 0.186405, size = 123, normalized size = 0.8

$$\frac{n\left(2ace+b^2(-e)+2bcd\right)\log(a+x(b+cx)) - 2n\sqrt{b^2-4ac}(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + 2cx\,(c(2d+ex)\log\,(d(a+x(b\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]] + (2*b*c*d - b^2*e + 2*a*c*e)*n*Log[a + x*(b + c*x)] + 2*c*x*(b*e*n - c*
n*(4*d + e*x) + c*(2*d + e*x)*Log[d*(a + x*(b + c*x))^n]))/(4*c^2)
```

Maple [C] time = 0.145, size = 1706, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] (1/2*e*x^2+d*x)*ln((c*x^2+b*x+a)^n)-1/4*I*Pi*e*x^2*csgn(I*d)*csgn(I*(c*x^2+
b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/2*I*Pi*d*x*csgn(I*(c*x^2+b*x+a)^n)*cs
gn(I*d*(c*x^2+b*x+a)^n)^2-1/2*I*Pi*d*x*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*cs
gn(I*d*(c*x^2+b*x+a)^n)+1/2*I*Pi*d*x*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+
1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*x^2+b*x+a)^n)*x^2*e*Pi-1/4*I*Pi
*e*x^2*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/2*I*Pi*d*x*csgn(I*d*(c*x^2+b*x+a)^n)^3
+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*d)*x^2*e*Pi+1/2*ln(d)*e*x^2-1/2*e
*n*x^2+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*c*e^2+16
*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x-(-4*
a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^
(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*
c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*
c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c
^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-
4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2
)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e
+4*b^2*c^2*d^2)^(1/2)*b)*b*d+ln(d)*d*x+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*
e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d
*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4
*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^
2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4
*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3
*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*
e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b
^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-
16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b*d+1/2/c*b*e*n*x-
2*n*d*x+1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*c*e^2
+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x-(
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e
+4*b^2*c^2*d^2)^(1/2)-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*
e+4*b^2*c^2*d^2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e
^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2663, size = 767, normalized size = 4.98

$$\left[-\frac{2\,c^2enx^2+\sqrt{b^2-4\,ac}(2\,cd-be)n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\left(4\,c^2d-bce\right)nx-\left(2\,c^2enx^2+4\,c^2dnx+\left(\right.\right.}{4\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*e*n*x^2 + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*n*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 -
2*a*c)*e)*n)*log(c*x^2 + b*x + a) - 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2,
-1/4*(2*c^2*e*n*x^2 - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*n*arctan(-sqrt(-b^
2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(4*c^2*d - b*c*e)*n*x - (2*c^2*e*
n*x^2 + 4*c^2*d*n*x + (2*b*c*d - (b^2 - 2*a*c)*e)*n)*log(c*x^2 + b*x + a) -
 2*(c^2*e*x^2 + 2*c^2*d*x)*log(d))/c^2]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36214, size = 315, normalized size = 2.05

$$\frac{bdn\log\left(cx^2+bx+a\right)}{2\,c}-\frac{\left(b^2dn-4\,acdn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,acc}}+\frac{cnx^2e\log\left(cx^2+bx+a\right)-cnx^2e+2\,cdnx\log\left(cx^2+bx\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*d*n*log(c*x^2 + b*x + a)/c - (b^2*d*n - 4*a*c*d*n)*arctan((2*c*x + b)
/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*(c*n*x^2*e*log(c*x^2 + b*
x + a) - c*n*x^2*e + 2*c*d*n*x*log(c*x^2 + b*x + a) + c*x^2*e*log(d) - 4*c*
d*n*x + b*n*x*e + 2*c*d*x*log(d))/c - 1/4*(b^2*n*e - 2*a*c*n*e)*log(c*x^2 +
 b*x + a)/c^2 + 1/2*(b^3*n*e - 4*a*b*c*n*e)*arctan((2*c*x + b)/sqrt(-b^2 +
4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.86 $\int \log \left(d \left(a+b x+c x^2\right)^n\right) \, dx$

Optimal. Leaf size=79

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rubi [A] time = 0.0617153, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2523, 773, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(d\left(a+bx+cx^2\right)^n\right)dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{2c}\\
&= -2nx+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(\left(b^2-4ac\right)n\right)\mathrm{Subst}\left(\int\frac{}{b^2-4\cdots}\right.}{c}\\
&= -2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right.
\end{aligned}$$

Mathematica [A] time = 0.0586933, size = 78, normalized size = 0.99

$$\frac{2n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2cx\left(\log\left(d(a+x(b+cx))^n\right)-2n\right)+bn\log(a+x(b+cx))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A] time = 0.015, size = 118, normalized size = 1.5

$$x\ln\left(d\left(cx^2+bx+a\right)^n\right)-2\,nx+\frac{bn\ln\left(cx^2+bx+a\right)}{2\,c}+4\,\frac{na}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-\frac{b^2n}{c}\arctan\left((2\,cx+b)\frac{}{\sqrt{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+4*n/(4*a*c-b^2)^(1/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a-n/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^2/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.14705, size = 459, normalized size = 5.81

$$\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^2-4\,ac}n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(2\,cnx+bn)\log\left(cx^2+bx+a\right)}{2\,c},-\frac{4\,cnx-2\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [A] time = 106.601, size = 275, normalized size = 3.48

$$\begin{cases} \frac{bn\log\left(\frac{b^2}{4c}+bx+cx^2\right)}{2c} + nx\log\left(\frac{b^2}{4c}+bx+cx^2\right) - 2nx + x\log(d) \\ \frac{an\log(a+bx)}{b} + nx\log(a+bx) - nx + x\log(d) \\ \frac{2an\log\left(a+bx+cx^2\right)}{\sqrt{-4ac+b^2}} - \frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}} - \frac{b^2n\log\left(a+bx+cx^2\right)}{2c\sqrt{-4ac+b^2}} + \frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} + \frac{bn\log\left(a+bx+cx^2\right)}{2c} + nx\log\left(a+bx+cx^2\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((b*n*log(b**2/(4*c) + b*x + c*x**2)/(2*c) + n*x*log(b**2/(4*c) +
b*x + c*x**2) - 2*n*x + x*log(d), Eq(a, b**2/(4*c))), (a*n*log(a + b*x)/b +
 n*x*log(a + b*x) - n*x + x*log(d), Eq(c, 0)), (2*a*n*log(a + b*x + c*x**2)
/sqrt(-4*a*c + b**2) - 4*a*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/s
qrt(-4*a*c + b**2) - b**2*n*log(a + b*x + c*x**2)/(2*c*sqrt(-4*a*c + b**2))
 + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(c*sqrt(-4*a*c + b**
2)) + b*n*log(a + b*x + c*x**2)/(2*c) + n*x*log(a + b*x + c*x**2) - 2*n*x +
 x*log(d), True))
```

Giac [A] time = 1.31276, size = 124, normalized size = 1.57

$$nx\log\left(cx^2+bx+a\right) - (2n - \log(d))x + \frac{bn\log\left(cx^2+bx+a\right)}{2c} - \frac{\left(b^2n - 4acn\right)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

3.87 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$

Optimal. Leaf size=228

$$-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-n\log(d+e$$

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + b*x + c*x^2)^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Rubi [A] time = 0.411819, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-n\log(d+e$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + b*x + c*x^2)^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log \left(d \left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{n\int \frac{(b+2cx)\log(d+ex)}{a+bx+cx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{n\int \left(\frac{2c\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\right)dx}{e}$$

$$= \frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e} - \frac{(2cn)\int \frac{\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{e} - \frac{(2cn)\int \frac{\log(d+ex)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{e}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+}{}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+}{}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+}{}$$

Mathematica [A] time = 0.292592, size = 226, normalized size = 0.99

$$-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{e\sqrt{b^2-4ac}-be+2cd}\right)}{e} - \frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e} - \frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e} - \frac{n\log(d+ex}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + x*(b + c*x))^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Maple [C] time = 0.089, size = 493, normalized size = 2.2

$$\frac{\ln(ex+d)\ln\left(\left(cx^2+bx+a\right)^n\right)}{e}-\frac{n\ln(ex+d)}{e}\ln\left(\left(-2\,(ex+d)\,c-be+2\,cd+\sqrt{-4\,ce^2a+b^2e^2}\right)\left(-be+2\,cd+\sqrt{-4\,ce^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d),x)
```

```
[Out] ln(e*x+d)/e*ln((c*x^2+b*x+a)^n)-1/e*n*ln(e*x+d)*ln((-2*(e*x+d)*c-b*e+2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*
ln(e*x+d)*ln((2*(e*x+d)*c+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((-2*(e*x+d)*c-b*e+2*c*d+(-4*a*c*e^
2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((2*(
e*x+d)*c+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e
^2)^(1/2)))-1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d
*(c*x^2+b*x+a)^n)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^
2+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-
1/2*I*ln(e*x+d)/e*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(e*x+d)/e*ln(d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2 + bx + a\right)^n d\right)}{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(e*x + d), x)
```

3.88 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx$

Optimal. Leaf size=165

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e
 + a*e^2) - ((2*c*d - b*e)*n*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^2)) + ((
2*c*d - b*e)*n*Log[a + b*x + c*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - Log[d*
(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Rubi [A] time = 0.245178, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e
 + a*e^2) - ((2*c*d - b*e)*n*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^2)) + ((
2*c*d - b*e)*n*Log[a + b*x + c*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - Log[d*
(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \frac{b+2cx}{(d+ex)\left(a+bx+cx^2\right)}\,dx}{e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{bcd-b^2e+2ace+c(2cd-be)x}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right)dx}{e}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \frac{n\int \frac{bcd-b^2e+2ace+c(2cd-be)x}{a+bx+cx^2}\,dx}{e\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} - \frac{\left(\left(b^2-4ac\right)n\right)\int \frac{1}{a+bx+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)} + \frac{((\ldots}{}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)} + \ldots$$

$$= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^{\ldots}\right.}{2e\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.333308, size = 166, normalized size = 1.01

$$-\frac{n\sqrt{4ac-b^2}\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{e(bd-ae)-cd^2} + \frac{n(be-2cd)\log(d+ex)}{e\left(e(ae-bd)+cd^2\right)} - \frac{n(be-2cd)\log(a+x(b+cx))}{2e\left(e(ae-bd)+cd^2\right)} - \frac{\log\left(d(a+x(b+cx))^n\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-(c*d^2) +
 e*(b*d - a*e))) + ((-2*c*d + b*e)*n*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d) +
a*e))) - ((-2*c*d + b*e)*n*Log[a + x*(b + c*x)])/(2*e*(c*d^2 + e*(-(b*d) +
a*e))) - Log[d*(a + x*(b + c*x))^n]/(e*(d + e*x))
```

Maple [C] time = 0.148, size = 6540, normalized size = 39.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.22275, size = 950, normalized size = 5.76

$$\left[\frac{\left(e^2nx+den\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+\left(\left(2\,cde-be^2\right)nx+\left(bde-2\,ae^2\right)n\right)\log\left(cx^2+bx+a\right)-2}{2\left(cd^3e-bd^2e^2+ade^3+\left(cd^2e^2-bde^3+ae^4\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((e^2*n*x + d*e*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((2*c*d*e - b*e
^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 + b*x + a) - 2*((2*c*d*e - b*e^2)*
n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) - 2*(c*d^2 - b*d*e + a*e^2)*log(d))
/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e^2 - b*d*e^3 + a*e^4)*x), 1/2*(2*
(e^2*n*x + d*e*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + ((2*c*d*e - b*e^2)*n*x + (b*d*e - 2*a*e^2)*n)*log(c*x^2 +
 b*x + a) - 2*((2*c*d*e - b*e^2)*n*x + (2*c*d^2 - b*d*e)*n)*log(e*x + d) -
2*(c*d^2 - b*d*e + a*e^2)*log(d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3 + (c*d^2*e
^2 - b*d*e^3 + a*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39488, size = 383, normalized size = 2.32

$$\frac{(2\,cdn-bne)\log\left(cx^2+bx+a\right)}{2\left(cd^2e-bde^2+ae^3\right)}-\frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}-\frac{2\,cdnxe\log(xe+d)+cd^2n\log\left(cx^2+bx+a\right)-b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*c*d*n - b*n*e)*log(c*x^2 + b*x + a)/(c*d^2*e - b*d*e^2 + a*e^3) - (b
^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*
e^2)*sqrt(-b^2 + 4*a*c)) - (2*c*d*n*x*e*log(x*e + d) + c*d^2*n*log(c*x^2 +
b*x + a) - b*d*n*e*log(c*x^2 + b*x + a) + 2*c*d^2*n*log(x*e + d) - b*n*x*e^
2*log(x*e + d) - b*d*n*e*log(x*e + d) + a*n*e^2*log(c*x^2 + b*x + a) + c*d^
2*log(d) - b*d*e*log(d) + a*e^2*log(d))/(c*d^2*x*e^2 + c*d^3*e - b*d*x*e^3
- b*d^2*e^2 + a*x*e^4 + a*d*e^3)
```

3.89 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3} dx$

Optimal. Leaf size=259

$$\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2}-\frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4ac}(2cd-}{2\left(ae^2-\right.}$$

```
[Out] ((2*c*d - b*e)*n)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*(c*d^2 - b*d
*e + a*e^2)^2) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*Log[d + e*x])
/(2*e*(c*d^2 - b*d*e + a*e^2)^2) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e
))*n*Log[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)^2) - Log[d*(a + b*x
 + c*x^2)^n]/(2*e*(d + e*x)^2)
```

Rubi [A] time = 0.405627, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2}-\frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4ac}(2cd-}{2\left(ae^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```


```
[Out] ((2*c*d - b*e)*n)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*(c*d^2 - b*d
*e + a*e^2)^2) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*Log[d + e*x])
/(2*e*(c*d^2 - b*d*e + a*e^2)^2) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e
))*n*Log[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)^2) - Log[d*(a + b*x
 + c*x^2)^n]/(2*e*(d + e*x)^2)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}+\frac{n\int \frac{b+2cx}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx}{2e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}+\frac{n\int\left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{-2b^2cde+4ac^2}{}\right.}{2e}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}-\frac{\log\left(d\left(a+\right.\right.}{2e(d}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}-\frac{\log\left(d\left(a+\right.\right.}{2e(d}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2}+\frac{\left(2c^2d^2+b^2\right.}{}$$

$$= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2}-\frac{\left(2c^2d^2+b^2e^2-\right.}{2e\left(\right.}$$

Mathematica [A] time = 0.59451, size = 215, normalized size = 0.83

$$\frac{\frac{n(d+ex)\left(-2(d+ex)\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)+(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log(a+x(b+cx))-2e\sqrt{b^2-4ac}(d+ex)(be-2cd)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2(2cd\right.}{\left(e(ae-bd)+cd^2\right)^2}}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)) - 2*Sqrt[b^2 - 4*
a*c]*e*(-2*c*d + b*e)*(d + e*x)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[d + e*x] + (2*c^2*d
^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[a + x*(b + c*x)]))/(c*d^2 +
 e*(-(b*d) + a*e))^2 - 2*Log[d*(a + x*(b + c*x))^n])/(4*e*(d + e*x)^2)
```

Maple [C] time = 0.237, size = 55216, normalized size = 213.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.1336, size = 2867, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 + (b^2 + 2*a*c)*d*e^3)*n*x -
 ((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 - b*d*e^3)*n*x + (2*c*d^3*e -
b*d^2*e^2)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(2*c^2*d^4 - 3*b*c*d^3*
e - a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 +
(b^2 - 2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d
*e^3)*n*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)
*n)*log(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*
e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2
*c^2*d^4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^
4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c
^2*d^6*e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*
e^3 + (c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*
d^2*e^5)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 +
 (b^2 + 2*a*c)*d^3*e^4)*x), 1/4*(2*(2*c^2*d^3*e - 3*b*c*d^2*e^2 - a*b*e^4 +
```

```
 (b^2 + 2*a*c)*d*e^3)*n*x + 2*((2*c*d*e^3 - b*e^4)*n*x^2 + 2*(2*c*d^2*e^2 -
 b*d*e^3)*n*x + (2*c*d^3*e - b*d^2*e^2)*n)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(
-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(2*c^2*d^4 - 3*b*c*d^3*e - a*b
*d*e^3 + (b^2 + 2*a*c)*d^2*e^2)*n + ((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 -
2*a*c)*e^4)*n*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n
*x + (2*b*c*d^3*e + 4*a*b*d*e^3 - 2*a^2*e^4 - (b^2 + 6*a*c)*d^2*e^2)*n)*log
(c*x^2 + b*x + a) - 2*((2*c^2*d^2*e^2 - 2*b*c*d*e^3 + (b^2 - 2*a*c)*e^4)*n*
x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 + (b^2 - 2*a*c)*d*e^3)*n*x + (2*c^2*d^
4 - 2*b*c*d^3*e + (b^2 - 2*a*c)*d^2*e^2)*n)*log(e*x + d) - 2*(c^2*d^4 - 2*b
*c*d^3*e - 2*a*b*d*e^3 + a^2*e^4 + (b^2 + 2*a*c)*d^2*e^2)*log(d))/(c^2*d^6*
e - 2*b*c*d^5*e^2 - 2*a*b*d^3*e^4 + a^2*d^2*e^5 + (b^2 + 2*a*c)*d^4*e^3 + (
c^2*d^4*e^3 - 2*b*c*d^3*e^4 - 2*a*b*d*e^6 + a^2*e^7 + (b^2 + 2*a*c)*d^2*e^5
)*x^2 + 2*(c^2*d^5*e^2 - 2*b*c*d^4*e^3 - 2*a*b*d^2*e^5 + a^2*d*e^6 + (b^2 +
 2*a*c)*d^3*e^4)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32779, size = 1197, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^2*d^2*n - 2*b*c*d*n*e + b^2*n*e^2 - 2*a*c*n*e^2)*log(c*x^2 + b*x +
 a)/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4
+ a^2*e^5) - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*n*e + 4*a*b*c*n*e)*arctan
((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*
a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d^2*n
*x^2*e^2*log(x*e + d) + 4*c^2*d^3*n*x*e*log(x*e + d) - 2*c^2*d^3*n*x*e + c^
```

```
2*d^4*n*log(c*x^2 + b*x + a) - 2*b*c*d^3*n*e*log(c*x^2 + b*x + a) + 2*c^2*d
^4*n*log(x*e + d) - 2*b*c*d*n*x^2*e^3*log(x*e + d) - 4*b*c*d^2*n*x*e^2*log(
x*e + d) - 2*b*c*d^3*n*e*log(x*e + d) - 2*c^2*d^4*n + 3*b*c*d^2*n*x*e^2 + 3
*b*c*d^3*n*e + b^2*d^2*n*e^2*log(c*x^2 + b*x + a) + 2*a*c*d^2*n*e^2*log(c*x
^2 + b*x + a) + b^2*n*x^2*e^4*log(x*e + d) - 2*a*c*n*x^2*e^4*log(x*e + d) +
 2*b^2*d*n*x*e^3*log(x*e + d) - 4*a*c*d*n*x*e^3*log(x*e + d) + b^2*d^2*n*e^
2*log(x*e + d) - 2*a*c*d^2*n*e^2*log(x*e + d) + c^2*d^4*log(d) - 2*b*c*d^3*
e*log(d) - b^2*d*n*x*e^3 - 2*a*c*d*n*x*e^3 - b^2*d^2*n*e^2 - 2*a*c*d^2*n*e^
2 - 2*a*b*d*n*e^3*log(c*x^2 + b*x + a) + b^2*d^2*e^2*log(d) + 2*a*c*d^2*e^2
*log(d) + a*b*n*x*e^4 + a*b*d*n*e^3 + a^2*n*e^4*log(c*x^2 + b*x + a) - 2*a*
b*d*e^3*log(d) + a^2*e^4*log(d))/(c^2*d^4*x^2*e^3 + 2*c^2*d^5*x*e^2 + c^2*d
^6*e - 2*b*c*d^3*x^2*e^4 - 4*b*c*d^4*x*e^3 - 2*b*c*d^5*e^2 + b^2*d^2*x^2*e^
5 + 2*a*c*d^2*x^2*e^5 + 2*b^2*d^3*x*e^4 + 4*a*c*d^3*x*e^4 + b^2*d^4*e^3 + 2
*a*c*d^4*e^3 - 2*a*b*d*x^2*e^6 - 4*a*b*d^2*x*e^5 - 2*a*b*d^3*e^4 + a^2*x^2*
e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

3.90 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4} dx$

Optimal. Leaf size=356

$$\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{n(2cd-be)\log(d+e}{3e\left(ae\right.}$$

```
[Out] ((2*c*d - b*e)*n)/(6*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((2*c^2*d^2 +
 b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(3*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
+ (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*n*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*(c*d^2 - b*d*e + a*e^2)^3) - ((2*c*d - b*e
)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[d + e*x])/(3*e*(c*d^2 - b*d
*e + a*e^2)^3) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*L
og[a + b*x + c*x^2])/(6*e*(c*d^2 - b*d*e + a*e^2)^3) - Log[d*(a + b*x + c*x
^2)^n]/(3*e*(d + e*x)^3)
```

Rubi [A] time = 0.620757, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{n(2cd-be)\log(d+e}{3e\left(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```


```
[Out] ((2*c*d - b*e)*n)/(6*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((2*c^2*d^2 +
 b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(3*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
+ (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*n*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*(c*d^2 - b*d*e + a*e^2)^3) - ((2*c*d - b*e
)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[d + e*x])/(3*e*(c*d^2 - b*d
*e + a*e^2)^3) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*L
og[a + b*x + c*x^2])/(6*e*(c*d^2 - b*d*e + a*e^2)^3) - Log[d*(a + b*x + c*x
^2)^n]/(3*e*(d + e*x)^3)
```


Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3} + \frac{n\int \frac{b+2cx}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx}{3e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3} + \frac{n\int\left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{e(2cd-be)\left(-c^2\right.}{\left(cd^2-bd\right.}\right.}{}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(2cd-be)\left(c^2d^2+b^2e\right.}{3e\left(cd\right.}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(2cd-be)\left(c^2d^2+b^2e\right.}{3e\left(cd\right.}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{(2cd-be)\left(c^2d^2+b^2e\right.}{3e\left(cd\right.}$$

$$= \frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)} + \frac{\sqrt{b^2-4ac}\left(3c^2d^2+b^2\right.}{3}$$

Mathematica [A] time = 1.19815, size = 310, normalized size = 0.87

$$\frac{\frac{n(d+ex)\left(2(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\left(e(ae-bd)+cd^2\right)-2(d+ex)^2(2cd-be)\log(d+ex)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)+(d+ex)^2(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\right.}{\left(e(ae-bd)+cd^2\right)^3}}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((n*(d + e*x)*((2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2 + 2*(c*d^2 + e*(-
(b*d) + a*e))*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 2*Sqrt[
b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*(d + e*x)^2*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e
*(b*d + 3*a*e))*(d + e*x)^2*Log[d + e*x] + (2*c*d - b*e)*(c^2*d^2 + b^2*e^2
 - c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[a + x*(b + c*x)]))/(c*d^2 + e*(-(b*d)
 + a*e))^3 - 2*Log[d*(a + x*(b + c*x))^n])/(6*e*(d + e*x)^3)
```

Maple [C] time = 0.303, size = 306209, normalized size = 860.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 125.094, size = 6159, normalized size = 17.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*c^3*d^4*e^2 - 4*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a
*b^2 - 2*a^2*c)*e^6)*n*x^2 + (10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 +
 4*(4*b^2*c + a*c^2)*d^3*e^3 - (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c
)*d*e^5)*n*x - ((3*c^2*d^2*e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(
3*c^2*d^3*e^3 - 3*b*c*d^2*e^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2
 - 3*b*c*d^3*e^3 + (b^2 - a*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2
+ (b^2 - a*c)*d^3*e^3)*n)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (6*c^3*d^6 -
13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 - 3*(b^3 + 2*a
*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^3 - 3*b*c^2*
d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3
```

```
*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*
d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e
^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*e^5 - 2*a^3*
e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 - 6*(a*b^2 + a
^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*b*c^2*d^2*e^
4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*(2*c^3*d^4*e
^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a*b*c)*d*e^5)
*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)*d^3*e^3 - (
b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b^2*c - 2*a*c
^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3*d^6 - 3*b*c
^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2 - (b^3 + 6*a
*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e - 3*b*c^2*d^8
*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^3 - (b^3 + 6
*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*b*c^2*d^5*e^
5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3 + 6*a*b*c)*
d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b*c^2*d^6*e^4
 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b^3 + 6*a*b*c
)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3*b*c^2*d^7*e
^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4 - (b^3 + 6*a
*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x), 1/6*(2*(2*c^3*d^4*e^2 - 4*b*
c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5 + (a*b^2 - 2*a^2*c)*e^6)*n*x^2 +
(10*c^3*d^5*e - 21*b*c^2*d^4*e^2 - a^2*b*e^6 + 4*(4*b^2*c + a*c^2)*d^3*e^3
- (5*b^3 + 6*a*b*c)*d^2*e^4 + 6*(a*b^2 - a^2*c)*d*e^5)*n*x + 2*((3*c^2*d^2*
e^4 - 3*b*c*d*e^5 + (b^2 - a*c)*e^6)*n*x^3 + 3*(3*c^2*d^3*e^3 - 3*b*c*d^2*e
^4 + (b^2 - a*c)*d*e^5)*n*x^2 + 3*(3*c^2*d^4*e^2 - 3*b*c*d^3*e^3 + (b^2 - a
*c)*d^2*e^4)*n*x + (3*c^2*d^5*e - 3*b*c*d^4*e^2 + (b^2 - a*c)*d^3*e^3)*n)*s
qrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (
6*c^3*d^6 - 13*b*c^2*d^5*e - a^2*b*d*e^5 + 2*(5*b^2*c + 2*a*c^2)*d^4*e^2 -
3*(b^3 + 2*a*b*c)*d^3*e^3 + 2*(2*a*b^2 - a^2*c)*d^2*e^4)*n + ((2*c^3*d^3*e^
3 - 3*b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^
3 + 3*(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3
 - 3*a*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*
a*c^2)*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (3*b*c^2*d^5*e + 6*a^2*b*d*
e^5 - 2*a^3*e^6 - 3*(b^2*c + 4*a*c^2)*d^4*e^2 + (b^3 + 15*a*b*c)*d^3*e^3 -
6*(a*b^2 + a^2*c)*d^2*e^4)*n)*log(c*x^2 + b*x + a) - 2*((2*c^3*d^3*e^3 - 3*
b*c^2*d^2*e^4 + 3*(b^2*c - 2*a*c^2)*d*e^5 - (b^3 - 3*a*b*c)*e^6)*n*x^3 + 3*
(2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*(b^2*c - 2*a*c^2)*d^2*e^4 - (b^3 - 3*a
*b*c)*d*e^5)*n*x^2 + 3*(2*c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*(b^2*c - 2*a*c^2)
*d^3*e^3 - (b^3 - 3*a*b*c)*d^2*e^4)*n*x + (2*c^3*d^6 - 3*b*c^2*d^5*e + 3*(b
^2*c - 2*a*c^2)*d^4*e^2 - (b^3 - 3*a*b*c)*d^3*e^3)*n)*log(e*x + d) - 2*(c^3
*d^6 - 3*b*c^2*d^5*e - 3*a^2*b*d*e^5 + a^3*e^6 + 3*(b^2*c + a*c^2)*d^4*e^2
- (b^3 + 6*a*b*c)*d^3*e^3 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e -
 3*b*c^2*d^8*e^2 - 3*a^2*b*d^4*e^6 + a^3*d^3*e^7 + 3*(b^2*c + a*c^2)*d^7*e^
3 - (b^3 + 6*a*b*c)*d^6*e^4 + 3*(a*b^2 + a^2*c)*d^5*e^5 + (c^3*d^6*e^4 - 3*
b*c^2*d^5*e^5 - 3*a^2*b*d*e^9 + a^3*e^10 + 3*(b^2*c + a*c^2)*d^4*e^6 - (b^3
```

```
 + 6*a*b*c)*d^3*e^7 + 3*(a*b^2 + a^2*c)*d^2*e^8)*x^3 + 3*(c^3*d^7*e^3 - 3*b
*c^2*d^6*e^4 - 3*a^2*b*d^2*e^8 + a^3*d*e^9 + 3*(b^2*c + a*c^2)*d^5*e^5 - (b
^3 + 6*a*b*c)*d^4*e^6 + 3*(a*b^2 + a^2*c)*d^3*e^7)*x^2 + 3*(c^3*d^8*e^2 - 3
*b*c^2*d^7*e^3 - 3*a^2*b*d^3*e^7 + a^3*d^2*e^8 + 3*(b^2*c + a*c^2)*d^6*e^4
- (b^3 + 6*a*b*c)*d^5*e^5 + 3*(a*b^2 + a^2*c)*d^4*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51773, size = 2650, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*c^3*d^3*n - 3*b*c^2*d^2*n*e + 3*b^2*c*d*n*e^2 - 6*a*c^2*d*n*e^2 - b^
3*n*e^3 + 3*a*b*c*n*e^3)*log(c*x^2 + b*x + a)/(c^3*d^6*e - 3*b*c^2*d^5*e^2
+ 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b
^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*(3*b^2*c^2*d^
2*n - 12*a*c^3*d^2*n - 3*b^3*c*d*n*e + 12*a*b*c^2*d*n*e + b^4*n*e^2 - 5*a*b
^2*c*n*e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*
d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a
*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)
*sqrt(-b^2 + 4*a*c)) - 1/6*(4*c^3*d^3*n*x^3*e^3*log(x*e + d) + 12*c^3*d^4*n
*x^2*e^2*log(x*e + d) + 12*c^3*d^5*n*x*e*log(x*e + d) - 4*c^3*d^4*n*x^2*e^2
 - 10*c^3*d^5*n*x*e + 2*c^3*d^6*n*log(c*x^2 + b*x + a) - 6*b*c^2*d^5*n*e*lo
g(c*x^2 + b*x + a) + 4*c^3*d^6*n*log(x*e + d) - 6*b*c^2*d^2*n*x^3*e^4*log(x
*e + d) - 18*b*c^2*d^3*n*x^2*e^3*log(x*e + d) - 18*b*c^2*d^4*n*x*e^2*log(x*
e + d) - 6*b*c^2*d^5*n*e*log(x*e + d) - 6*c^3*d^6*n + 8*b*c^2*d^3*n*x^2*e^3
 + 21*b*c^2*d^4*n*x*e^2 + 13*b*c^2*d^5*n*e + 6*b^2*c*d^4*n*e^2*log(c*x^2 +
```

```
b*x + a) + 6*a*c^2*d^4*n*e^2*log(c*x^2 + b*x + a) + 6*b^2*c*d*n*x^3*e^5*log
(x*e + d) - 12*a*c^2*d*n*x^3*e^5*log(x*e + d) + 18*b^2*c*d^2*n*x^2*e^4*log(
x*e + d) - 36*a*c^2*d^2*n*x^2*e^4*log(x*e + d) + 18*b^2*c*d^3*n*x*e^3*log(x
*e + d) - 36*a*c^2*d^3*n*x*e^3*log(x*e + d) + 6*b^2*c*d^4*n*e^2*log(x*e + d
) - 12*a*c^2*d^4*n*e^2*log(x*e + d) + 2*c^3*d^6*log(d) - 6*b*c^2*d^5*e*log(
d) - 6*b^2*c*d^2*n*x^2*e^4 - 16*b^2*c*d^3*n*x*e^3 - 4*a*c^2*d^3*n*x*e^3 - 1
0*b^2*c*d^4*n*e^2 - 4*a*c^2*d^4*n*e^2 - 2*b^3*d^3*n*e^3*log(c*x^2 + b*x + a
) - 12*a*b*c*d^3*n*e^3*log(c*x^2 + b*x + a) - 2*b^3*n*x^3*e^6*log(x*e + d)
+ 6*a*b*c*n*x^3*e^6*log(x*e + d) - 6*b^3*d*n*x^2*e^5*log(x*e + d) + 18*a*b*
c*d*n*x^2*e^5*log(x*e + d) - 6*b^3*d^2*n*x*e^4*log(x*e + d) + 18*a*b*c*d^2*
n*x*e^4*log(x*e + d) - 2*b^3*d^3*n*e^3*log(x*e + d) + 6*a*b*c*d^3*n*e^3*log
(x*e + d) + 6*b^2*c*d^4*e^2*log(d) + 6*a*c^2*d^4*e^2*log(d) + 2*b^3*d*n*x^2
*e^5 + 5*b^3*d^2*n*x*e^4 + 6*a*b*c*d^2*n*x*e^4 + 3*b^3*d^3*n*e^3 + 6*a*b*c*
d^3*n*e^3 + 6*a*b^2*d^2*n*e^4*log(c*x^2 + b*x + a) + 6*a^2*c*d^2*n*e^4*log(
c*x^2 + b*x + a) - 2*b^3*d^3*e^3*log(d) - 12*a*b*c*d^3*e^3*log(d) - 2*a*b^2
*n*x^2*e^6 + 4*a^2*c*n*x^2*e^6 - 6*a*b^2*d*n*x*e^5 + 6*a^2*c*d*n*x*e^5 - 4*
a*b^2*d^2*n*e^4 + 2*a^2*c*d^2*n*e^4 - 6*a^2*b*d*n*e^5*log(c*x^2 + b*x + a)
+ 6*a*b^2*d^2*e^4*log(d) + 6*a^2*c*d^2*e^4*log(d) + a^2*b*n*x*e^6 + a^2*b*d
*n*e^5 + 2*a^3*n*e^6*log(c*x^2 + b*x + a) - 6*a^2*b*d*e^5*log(d) + 2*a^3*e^
6*log(d))/(c^3*d^6*x^3*e^4 + 3*c^3*d^7*x^2*e^3 + 3*c^3*d^8*x*e^2 + c^3*d^9*
e - 3*b*c^2*d^5*x^3*e^5 - 9*b*c^2*d^6*x^2*e^4 - 9*b*c^2*d^7*x*e^3 - 3*b*c^2
*d^8*e^2 + 3*b^2*c*d^4*x^3*e^6 + 3*a*c^2*d^4*x^3*e^6 + 9*b^2*c*d^5*x^2*e^5
+ 9*a*c^2*d^5*x^2*e^5 + 9*b^2*c*d^6*x*e^4 + 9*a*c^2*d^6*x*e^4 + 3*b^2*c*d^7
*e^3 + 3*a*c^2*d^7*e^3 - b^3*d^3*x^3*e^7 - 6*a*b*c*d^3*x^3*e^7 - 3*b^3*d^4*
x^2*e^6 - 18*a*b*c*d^4*x^2*e^6 - 3*b^3*d^5*x*e^5 - 18*a*b*c*d^5*x*e^5 - b^3
*d^6*e^4 - 6*a*b*c*d^6*e^4 + 3*a*b^2*d^2*x^3*e^8 + 3*a^2*c*d^2*x^3*e^8 + 9*
a*b^2*d^3*x^2*e^7 + 9*a^2*c*d^3*x^2*e^7 + 9*a*b^2*d^4*x*e^6 + 9*a^2*c*d^4*x
*e^6 + 3*a*b^2*d^5*e^5 + 3*a^2*c*d^5*e^5 - 3*a^2*b*d*x^3*e^9 - 9*a^2*b*d^2*
x^2*e^8 - 9*a^2*b*d^3*x*e^7 - 3*a^2*b*d^4*e^6 + a^3*x^3*e^10 + 3*a^3*d*x^2*
e^9 + 3*a^3*d^2*x*e^8 + a^3*d^3*e^7)
```

3.91 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx$

Optimal. Leaf size=519

$$\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8e\left(ae^2-bde+cd^2\right)^4}-\frac{n\log(d+ex}{}$$

```
[Out] ((2*c*d - b*e)*n)/(12*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + ((2*c^2*d^2
+ b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(8*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^
2) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n)/(4*e*(c*d^2
- b*d*e + a*e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(
4*(c*d^2 - b*d*e + a*e^2)^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a
*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^
2))*n*Log[d + e*x])/(4*e*(c*d^2 - b*d*e + a*e^2)^4) + ((2*c^4*d^4 + b^4*e^4
 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d
^2 + 6*a*b*d*e + a^2*e^2))*n*Log[a + b*x + c*x^2])/(8*e*(c*d^2 - b*d*e + a*
e^2)^4) - Log[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)
```

Rubi [A] time = 1.00557, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2525, 800, 634, 618, 206, 628}

$$\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8e\left(ae^2-bde+cd^2\right)^4}-\frac{n\log(d+ex}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```


```
[Out] ((2*c*d - b*e)*n)/(12*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + ((2*c^2*d^2
+ b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(8*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^
2) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n)/(4*e*(c*d^2
- b*d*e + a*e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(2*c^2*d^2
 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(
4*(c*d^2 - b*d*e + a*e^2)^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a
*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^
2))*n*Log[d + e*x])/(4*e*(c*d^2 - b*d*e + a*e^2)^4) + ((2*c^4*d^4 + b^4*e^4
```

```
 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d
^2 + 6*a*b*d*e + a^2*e^2))*n*Log[a + b*x + c*x^2])/(8*e*(c*d^2 - b*d*e + a*
e^2)^4) - Log[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx = -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}+\frac{n\int \frac{b+2cx}{(d+ex)^4\left(a+bx+cx^2\right)}\,dx}{4e}$$

$$= -\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}+\frac{n\int \left(\frac{e(-2cd+be)}{\left(cd^2-bde+ae^2\right)(d+ex)^4}+\frac{e\left(-2c^2d^2-b^2e^2+2ce(bd+ae)\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)^3}+\frac{e(2cd-be)\left(-\right.}{\left(cd^2-\right.}\right.}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+\right.}{4e\left(cd^2-bde\right.}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+\right.}{4e\left(cd^2-bde\right.}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+\right.}{4e\left(cd^2-bde\right.}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+\right.}{4e\left(cd^2-bde\right.}$$

Mathematica [A] time = 2.06492, size = 469, normalized size = 0.9

$$\frac{n(d+ex)\left(-6(d+ex)^3\log(d+ex)\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)+3(d+ex)^3\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3 + 3*(c*d^2 + e*
(-(b*d) + a*e))^2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 6*(
2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a
*e))*(d + e*x)^2 + 6*Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
 - 2*c*e*(b*d + a*e))*(d + e*x)^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] -
6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e
) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[d + e*x] +
 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*
```

```
e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[a + x*(b
+ c*x)]))/(c*d^2 + e*(-(b*d) + a*e))^4 - 6*Log[d*(a + x*(b + c*x))^n])/(24*
e*(d + e*x)^4)
```

Maple [B] time = 0.443, size = 1137077, normalized size = 2190.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.58396, size = 5075, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/8*(2*c^4*d^4*n - 4*b*c^3*d^3*n*e + 6*b^2*c^2*d^2*n*e^2 - 12*a*c^3*d^2*n*e
^2 - 4*b^3*c*d*n*e^3 + 12*a*b*c^2*d*n*e^3 + b^4*n*e^4 - 4*a*b^2*c*n*e^4 + 2
*a^2*c^2*n*e^4)*log(c*x^2 + b*x + a)/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^2*c
^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b^4*d
^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12*a^2*
b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a^4*e^9
) - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*d^2*n*e + 24*a*b*c^3*
d^2*n*e + 4*b^4*c*d*n*e^2 - 20*a*b^2*c^2*d*n*e^2 + 16*a^2*c^3*d*n*e^2 - b^5
*n*e^3 + 6*a*b^3*c*n*e^3 - 8*a^2*b*c^2*n*e^3)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 -
 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 +
6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^
6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-b^2 + 4*a*c)) - 1/24*(
12*c^4*d^4*n*x^4*e^4*log(x*e + d) + 48*c^4*d^5*n*x^3*e^3*log(x*e + d) + 72*
c^4*d^6*n*x^2*e^2*log(x*e + d) + 48*c^4*d^7*n*x*e*log(x*e + d) - 12*c^4*d^5
*n*x^3*e^3 - 42*c^4*d^6*n*x^2*e^2 - 52*c^4*d^7*n*x*e + 6*c^4*d^8*n*log(c*x^
2 + b*x + a) - 24*b*c^3*d^7*n*e*log(c*x^2 + b*x + a) + 12*c^4*d^8*n*log(x*e
 + d) - 24*b*c^3*d^3*n*x^4*e^5*log(x*e + d) - 96*b*c^3*d^4*n*x^3*e^4*log(x*
e + d) - 144*b*c^3*d^5*n*x^2*e^3*log(x*e + d) - 96*b*c^3*d^6*n*x*e^2*log(x*
e + d) - 24*b*c^3*d^7*n*e*log(x*e + d) - 22*c^4*d^8*n + 30*b*c^3*d^4*n*x^3*
e^4 + 108*b*c^3*d^5*n*x^2*e^3 + 140*b*c^3*d^6*n*x*e^2 + 62*b*c^3*d^7*n*e +
36*b^2*c^2*d^6*n*e^2*log(c*x^2 + b*x + a) + 24*a*c^3*d^6*n*e^2*log(c*x^2 +
b*x + a) + 36*b^2*c^2*d^2*n*x^4*e^6*log(x*e + d) - 72*a*c^3*d^2*n*x^4*e^6*l
og(x*e + d) + 144*b^2*c^2*d^3*n*x^3*e^5*log(x*e + d) - 288*a*c^3*d^3*n*x^3*
```

e^5*log(x*e + d) + 216*b^2*c^2*d^4*n*x^2*e^4*log(x*e + d) - 432*a*c^3*d^4*n
*x^2*e^4*log(x*e + d) + 144*b^2*c^2*d^5*n*x*e^3*log(x*e + d) - 288*a*c^3*d^
5*n*x*e^3*log(x*e + d) + 36*b^2*c^2*d^6*n*e^2*log(x*e + d) - 72*a*c^3*d^6*n
*e^2*log(x*e + d) + 6*c^4*d^8*log(d) - 24*b*c^3*d^7*e*log(d) - 36*b^2*c^2*d
^3*n*x^3*e^5 + 24*a*c^3*d^3*n*x^3*e^5 - 129*b^2*c^2*d^4*n*x^2*e^4 + 66*a*c^
3*d^4*n*x^2*e^4 - 168*b^2*c^2*d^5*n*x*e^3 + 48*a*c^3*d^5*n*x*e^3 - 75*b^2*c
^2*d^6*n*e^2 + 6*a*c^3*d^6*n*e^2 - 24*b^3*c*d^5*n*e^3*log(c*x^2 + b*x + a)
- 72*a*b*c^2*d^5*n*e^3*log(c*x^2 + b*x + a) - 24*b^3*c*d*n*x^4*e^7*log(x*e
+ d) + 72*a*b*c^2*d*n*x^4*e^7*log(x*e + d) - 96*b^3*c*d^2*n*x^3*e^6*log(x*e
+ d) + 288*a*b*c^2*d^2*n*x^3*e^6*log(x*e + d) - 144*b^3*c*d^3*n*x^2*e^5*lo
g(x*e + d) + 432*a*b*c^2*d^3*n*x^2*e^5*log(x*e + d) - 96*b^3*c*d^4*n*x*e^4*
log(x*e + d) + 288*a*b*c^2*d^4*n*x*e^4*log(x*e + d) - 24*b^3*c*d^5*n*e^3*lo
g(x*e + d) + 72*a*b*c^2*d^5*n*e^3*log(x*e + d) + 36*b^2*c^2*d^6*e^2*log(d)
+ 24*a*c^3*d^6*e^2*log(d) + 24*b^3*c*d^2*n*x^3*e^6 - 36*a*b*c^2*d^2*n*x^3*e
^6 + 84*b^3*c*d^3*n*x^2*e^5 - 96*a*b*c^2*d^3*n*x^2*e^5 + 106*b^3*c*d^4*n*x*
e^4 - 54*a*b*c^2*d^4*n*x*e^4 + 46*b^3*c*d^5*n*e^3 + 6*a*b*c^2*d^5*n*e^3 + 6
*b^4*d^4*n*e^4*log(c*x^2 + b*x + a) + 72*a*b^2*c*d^4*n*e^4*log(c*x^2 + b*x
+ a) + 36*a^2*c^2*d^4*n*e^4*log(c*x^2 + b*x + a) + 6*b^4*n*x^4*e^8*log(x*e
+ d) - 24*a*b^2*c*n*x^4*e^8*log(x*e + d) + 12*a^2*c^2*n*x^4*e^8*log(x*e + d
) + 24*b^4*d*n*x^3*e^7*log(x*e + d) - 96*a*b^2*c*d*n*x^3*e^7*log(x*e + d) +
48*a^2*c^2*d*n*x^3*e^7*log(x*e + d) + 36*b^4*d^2*n*x^2*e^6*log(x*e + d) -
144*a*b^2*c*d^2*n*x^2*e^6*log(x*e + d) + 72*a^2*c^2*d^2*n*x^2*e^6*log(x*e +
d) + 24*b^4*d^3*n*x*e^5*log(x*e + d) - 96*a*b^2*c*d^3*n*x*e^5*log(x*e + d)
+ 48*a^2*c^2*d^3*n*x*e^5*log(x*e + d) + 6*b^4*d^4*n*e^4*log(x*e + d) - 24*
a*b^2*c*d^4*n*e^4*log(x*e + d) + 12*a^2*c^2*d^4*n*e^4*log(x*e + d) - 24*b^3
*c*d^5*e^3*log(d) - 72*a*b*c^2*d^5*e^3*log(d) - 6*b^4*d*n*x^3*e^7 + 36*a^2*
c^2*d*n*x^3*e^7 - 21*b^4*d^2*n*x^2*e^6 - 12*a*b^2*c*d^2*n*x^2*e^6 + 114*a^2
*c^2*d^2*n*x^2*e^6 - 26*b^4*d^3*n*x*e^5 - 48*a*b^2*c*d^3*n*x*e^5 + 108*a^2*
c^2*d^3*n*x*e^5 - 11*b^4*d^4*n*e^4 - 36*a*b^2*c*d^4*n*e^4 + 30*a^2*c^2*d^4*
n*e^4 - 24*a*b^3*d^3*n*e^5*log(c*x^2 + b*x + a) - 72*a^2*b*c*d^3*n*e^5*log(
c*x^2 + b*x + a) + 6*b^4*d^4*e^4*log(d) + 72*a*b^2*c*d^4*e^4*log(d) + 36*a^
2*c^2*d^4*e^4*log(d) + 6*a*b^3*n*x^3*e^8 - 18*a^2*b*c*n*x^3*e^8 + 24*a*b^3*
d*n*x^2*e^7 - 60*a^2*b*c*d*n*x^2*e^7 + 36*a*b^3*d^2*n*x*e^6 - 48*a^2*b*c*d^
2*n*x*e^6 + 18*a*b^3*d^3*n*e^5 - 6*a^2*b*c*d^3*n*e^5 + 36*a^2*b^2*d^2*n*e^6
*log(c*x^2 + b*x + a) + 24*a^3*c*d^2*n*e^6*log(c*x^2 + b*x + a) - 24*a*b^3*
d^3*e^5*log(d) - 72*a^2*b*c*d^3*e^5*log(d) - 3*a^2*b^2*n*x^2*e^8 + 6*a^3*c*
n*x^2*e^8 - 12*a^2*b^2*d*n*x*e^7 + 8*a^3*c*d*n*x*e^7 - 9*a^2*b^2*d^2*n*e^6
+ 2*a^3*c*d^2*n*e^6 - 24*a^3*b*d*n*e^7*log(c*x^2 + b*x + a) + 36*a^2*b^2*d^
2*e^6*log(d) + 24*a^3*c*d^2*e^6*log(d) + 2*a^3*b*n*x*e^8 + 2*a^3*b*d*n*e^7
+ 6*a^4*n*e^8*log(c*x^2 + b*x + a) - 24*a^3*b*d*e^7*log(d) + 6*a^4*e^8*log(
d))/(c^4*d^8*x^4*e^5 + 4*c^4*d^9*x^3*e^4 + 6*c^4*d^10*x^2*e^3 + 4*c^4*d^11*
x*e^2 + c^4*d^12*e - 4*b*c^3*d^7*x^4*e^6 - 16*b*c^3*d^8*x^3*e^5 - 24*b*c^3*
d^9*x^2*e^4 - 16*b*c^3*d^10*x*e^3 - 4*b*c^3*d^11*e^2 + 6*b^2*c^2*d^6*x^4*e^
7 + 4*a*c^3*d^6*x^4*e^7 + 24*b^2*c^2*d^7*x^3*e^6 + 16*a*c^3*d^7*x^3*e^6 + 3
6*b^2*c^2*d^8*x^2*e^5 + 24*a*c^3*d^8*x^2*e^5 + 24*b^2*c^2*d^9*x*e^4 + 16*a*

c^3*d^9*x*e^4 + 6*b^2*c^2*d^10*e^3 + 4*a*c^3*d^10*e^3 - 4*b^3*c*d^5*x^4*e^8 - 12*a*b*c^2*d^5*x^4*e^8 - 16*b^3*c*d^6*x^3*e^7 - 48*a*b*c^2*d^6*x^3*e^7 - 24*b^3*c*d^7*x^2*e^6 - 72*a*b*c^2*d^7*x^2*e^6 - 16*b^3*c*d^8*x*e^5 - 48*a*b*c^2*d^8*x*e^5 - 4*b^3*c*d^9*e^4 - 12*a*b*c^2*d^9*e^4 + b^4*d^4*x^4*e^9 + 12*a*b^2*c*d^4*x^4*e^9 + 6*a^2*c^2*d^4*x^4*e^9 + 4*b^4*d^5*x^3*e^8 + 48*a*b^2*c*d^5*x^3*e^8 + 24*a^2*c^2*d^5*x^3*e^8 + 6*b^4*d^6*x^2*e^7 + 72*a*b^2*c*d^6*x^2*e^7 + 36*a^2*c^2*d^6*x^2*e^7 + 4*b^4*d^7*x*e^6 + 48*a*b^2*c*d^7*x*e^6 + 24*a^2*c^2*d^7*x*e^6 + b^4*d^8*e^5 + 12*a*b^2*c*d^8*e^5 + 6*a^2*c^2*d^8*e^5 - 4*a*b^3*d^3*x^4*e^10 - 12*a^2*b*c*d^3*x^4*e^10 - 16*a*b^3*d^4*x^3*e^9 - 48*a^2*b*c*d^4*x^3*e^9 - 24*a*b^3*d^5*x^2*e^8 - 72*a^2*b*c*d^5*x^2*e^8 - 16*a*b^3*d^6*x*e^7 - 48*a^2*b*c*d^6*x*e^7 - 4*a*b^3*d^7*e^6 - 12*a^2*b*c*d^7*e^6 + 6*a^2*b^2*d^2*x^4*e^11 + 4*a^3*c*d^2*x^4*e^11 + 24*a^2*b^2*d^3*x^3*e^10 + 16*a^3*c*d^3*x^3*e^10 + 36*a^2*b^2*d^4*x^2*e^9 + 24*a^3*c*d^4*x^2*e^9 + 24*a^2*b^2*d^5*x*e^8 + 16*a^3*c*d^5*x*e^8 + 6*a^2*b^2*d^6*e^7 + 4*a^3*c*d^6*e^7 - 4*a^3*b*d*x^4*e^12 - 16*a^3*b*d^2*x^3*e^11 - 24*a^3*b*d^3*x^2*e^10 - 16*a^3*b*d^4*x*e^9 - 4*a^3*b*d^5*e^8 + a^4*x^4*e^13 + 4*a^4*d*x^3*e^12 + 6*a^4*d^2*x^2*e^11 + 4*a^4*d^3*x*e^10 + a^4*d^4*e^9)

3.92 $\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx$

Optimal. Leaf size=175

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}+\frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}e}$$

```
[Out] (I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[c]*e) + (2*n*ArcTan[(Sqrt
[c]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*
e) + (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e)
 + (I*n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[
c]*e)
```

Rubi [A] time = 0.157902, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {205, 2470, 12, 4920, 4854, 2402, 2315}

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{c}e}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{c}e}+\frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{c}e}+\frac{2n\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[c]*e) + (2*n*ArcTan[(Sqrt
[c]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*
e) + (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e)
 + (I*n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[
c]*e)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} - (2cn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{ce}\left(a+cx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} - \frac{\left(2\sqrt{c}n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{a+cx^2}\,dx}{\sqrt{ae}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{c}x}{\sqrt{a}}}\,dx}{ae}$$

lo

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} - \frac{(2n)\int -}{}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} + \frac{(2in)\,\mathrm{Su}}{}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{ce}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}x}\right)}{\sqrt{a}\sqrt{ce}} + \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\sqrt{ce}} + \frac{in\mathrm{Li}_2\left(1\right.}{\sqrt{a}}$$

Mathematica [A] time = 0.0430713, size = 131, normalized size = 0.75

$$\frac{in\mathrm{PolyLog}\left(2,\frac{\sqrt{c}x+i\sqrt{a}}{\sqrt{c}x-i\sqrt{a}}\right)+\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(\log\left(d\left(a+cx^2\right)^n\right)+2n\log\left(\frac{2i}{-\frac{\sqrt{c}x}{\sqrt{a}}+i}\right)+in\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{ce}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[c]*x)/Sqrt[a])] + Log[d*(a + c*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[c]*x)/((-I)*Sqrt[a] + Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*e)
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+a\right)^n\right)}{cex^2+ea}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

```
[Out] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{a+cx^2}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+a)**n)/(c*e*x**2+a*e),x)
```

```
[Out] Integral(log(d*(a + c*x**2)**n)/(a + c*x**2), x)/e
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+a\right)^n d\right)}{cex^2+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + a)^n*d)/(c*e*x^2 + a*e), x)
```

3.93 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$

Optimal. Leaf size=258

$$-\frac{2n\text{PolyLog}\left(2,-\frac{\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}+1}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}+\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}}-\frac{4n\log\left(\right.}{}$$

```
[Out] (2*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2)/(Sqrt[b^2 - 4*a*c]*e) - (4*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2
*c*x)/Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e) - (2*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sqrt[b^2 - 4*a*c]*e) - (2*n*
PolyLog[2, -((1 + b/Sqrt[b^2 - 4*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c])/(1 - b/S
qrt[b^2 - 4*a*c] - (2*c*x)/Sqrt[b^2 - 4*a*c]))])/(Sqrt[b^2 - 4*a*c]*e)
```

Rubi [A] time = 0.345819, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {618, 206, 2527, 12, 6121, 5984, 5918, 2402, 2315}

$$-\frac{2n\text{PolyLog}\left(2,-\frac{\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}+1}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}+\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}}-\frac{4n\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (2*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2)/(Sqrt[b^2 - 4*a*c]*e) - (4*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2
*c*x)/Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e) - (2*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sqrt[b^2 - 4*a*c]*e) - (2*n*
PolyLog[2, -((1 + b/Sqrt[b^2 - 4*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c])/(1 - b/S
qrt[b^2 - 4*a*c] - (2*c*x)/Sqrt[b^2 - 4*a*c]))])/(Sqrt[b^2 - 4*a*c]*e)
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2527

```
Int[Log[(c_.)*(Px_)^(n_.)]/(Qx_), x_Symbol] :> With[{u = IntHide[1/Qx, x]},
 Simp[u*Log[c*Px^n], x] - Dist[n, Int[SimplifyIntegrand[(u*D[Px, x])/Px, x]
, x], x]] /; FreeQ[{c, n}, x] && QuadraticQ[{Qx, Px}, x] && EqQ[D[Px/Qx, x]
, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6121

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + (f*x)/d)^m*(-(C/d^2) + (C*x^2)/d^2)^q*(a + b*ArcTanh
[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q},
x] && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx = -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} - n\int \frac{2(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2c}{\sqrt{b^2-\cdots}}\right.}{\sqrt{b^2-4ac}e\left(a+bx+cx^2\right)}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} - \frac{(2n)\int \frac{(-b-2cx)\tanh^{-1}\left(\frac{b}{\sqrt{b^2-4ac}}+\frac{2cx}{\sqrt{b^2-4ac}}\right)}{a+bx+cx^2}\,dx}{\sqrt{b^2-4ac}e}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} + \frac{n\,\mathrm{Subst}\left(\int \frac{\sqrt{b^2-4ac}x\tanh^{-1}(x)}{-\frac{b^2-4ac}{4c}+\frac{\left(b^2-4ac\right)x^2}{4c}}\,dx, x, \frac{b}{\sqrt{b^2-4\cdots}}\right.}{ce}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} + \frac{\left(\sqrt{b^2-4ac}n\right)\mathrm{Subst}\left(\int \frac{x\tanh^{-1}(x)}{-\frac{b^2-4ac}{4c}+\frac{\left(b^2-4ac\right)x^2}{4c}}\right.}{ce}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{\sqrt{b^2-4ac}e} - \frac{(4n)\,\mathrm{Subst}\left(\int \frac{\mathrm{ta}\cdots}{}\right.}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a\cdots}}\right.}{\cdots}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a\cdots}}\right.}{\cdots}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4a\cdots}}\right.}{\cdots}$$

Mathematica [A] time = 0.162295, size = 339, normalized size = 1.31

$$-2n\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}-b-2cx}{2\sqrt{b^2-4ac}}\right)+2n\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)+2\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d(a+x(b+cx))^n\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (-(n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2) + 2*n*Log[(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x] + n*Lo
g[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2 - 2*n*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*Log[b + Sqrt[b^2 - 4
*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] - 2*n*PolyLog[2, (-b + Sqrt[b^2 -
 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + 2*n*PolyLog[2, (b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(2*Sqrt[b^2 - 4*a*c]*e)
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d\left(cx^2+bx+a\right)^n\right)}{cex^2+bex+ea}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

```
[Out] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{cex^2+bex+ae},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(c*e*x**2+b*e*x+a*e),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n d\right)}{cex^2+bex+ae}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(c*e*x^2 + b*e*x + a*e), x)
```

3.94 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx$

Optimal. Leaf size=762

$$-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+ \frac{n\text{Poly}}{}$$

```
[Out] -(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) -
(n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[
b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) + (
n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) +
 (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b + Sq
rt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e])
 + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sqrt[
e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sq
rt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt
[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sq
rt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d]
- (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (
2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 1.45202, antiderivative size = 762, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2528, 2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\sqrt{e}}+ \frac{n\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] -(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) -
```

```
(n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[
b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) + (
n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b - Sqrt
[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) +
 (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b + Sq
rt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e])
 + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sqrt[
e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sq
rt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e])]])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt
[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]])/(2*Sq
rt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d]
- (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])]])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (
2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)]])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\log\left(g\left(a+bx+cx^2\right)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\sqrt{-d}\log\left(g\left(a+bx+cx^2\right)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$= -\frac{\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{-d}-\sqrt{e}x}dx}{2\sqrt{-d}} - \frac{\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{-d}+\sqrt{e}x}dx}{2\sqrt{-d}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} -$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} -$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\sqrt{e}} -$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)}{2\sqrt{-d}\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}\right.}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [A] time = 0.940713, size = 626, normalized size = 0.82

$$\frac{-n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right) - n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right) + n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}-b\right)+2c\sqrt{-d}}\right) + n\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] (-(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x]) - n*Log[(Sqrt[e]*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)]*Log[Sqrt[-d] - Sqrt[e]*x] + n*Log[(Sqrt[e]*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt
[e]*x] + n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*Sqrt[-d] + (
b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt[e]*x] + Log[Sqrt[-d] -
 Sqrt[e]*x]*Log[g*(a + x*(b + c*x))^n] - Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a
 + x*(b + c*x))^n] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d
] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[
e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])] + n*PolyLog[2, (2*
c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])
] + n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2
 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] time = 0.135, size = 610, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x)
```

```
[Out] (ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
+1/2*n/e*sum(1/_alpha*(ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((RootOf
(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1)-x+_alpha)/RootO
f(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1))-ln(x-_alpha)*
ln((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2))-dil
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1))-dil
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2))),_a
lpha=RootOf(_Z^2*e+d))-1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*
(c*x^2+b*x+a)^n)*csgn(I*g)*csgn(I*g*(c*x^2+b*x+a)^n)+1/2*I/(d*e)^(1/2)*arct
an(e*x/(d*e)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2+
1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*g)*csgn(I*g*(c*x^2+b*x+
a)^n)^2-1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*g*(c*x^2+b*x+a)
^n)^3+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*ln(g)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{ex^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(e*x^2 + d), x)
```

3.95 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=782

$$-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\text{PolyLog}\left(2,-\frac{c\left(\sqrt{e^2-4df}+\ldots\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\ldots}\right)}{\sqrt{e^2-4df}}$$

```
[Out] -((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a*
c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^2
 - 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a
*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sq
rt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqrt[b^2
 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*
x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sq
rt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]
 + 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a
 + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d
*f])))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2 - 4*d*f] + 2*
f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 -
 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^
2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] + (n*PolyLo
g[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(
e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f]
```

Rubi [A] time = 1.51443, antiderivative size = 782, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2528, 2524, 2418, 2394, 2393, 2391}

$$-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\text{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\text{PolyLog}\left(2,-\frac{c\left(\sqrt{e^2-4df}+\ldots\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\ldots}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] -((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a*
c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^2
- 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a
*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sq
rt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqrt[b^2
 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*
x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sq
rt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]
+ 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a
+ b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d
*f])))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2 - 4*d*f] + 2*
f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 -
 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^
2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f] + (n*PolyLo
g[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(
e + Sqrt[e^2 - 4*d*f])))])/Sqrt[e^2 - 4*d*f]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx = \int\left(\frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)} - \frac{2f\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}+2fx\right)}\right)dx$$

$$= \frac{(2f)\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{e-\sqrt{e^2-4df}+2fx}dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{e+\sqrt{e^2-4df}+2fx}dx}{\sqrt{e^2-4df}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a\right.\right.}{\sqrt{e^2-4df}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a\right.\right.}{\sqrt{e^2-4df}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}+2fx\right)\log\left(g\left(a\right.\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right.}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2\right.}}\right.}{\sqrt{}}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right.}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2\right.}}\right.}{\sqrt{}}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} - \frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right.}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{e^2\right.}}\right.}{\sqrt{}}$$

Mathematica [A] time = 0.812922, size = 663, normalized size = 0.85

$$\frac{-n\text{PolyLog}\left(2,\frac{c\left(\sqrt{e^2-4df}-e-2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)+c\left(\sqrt{e^2-4df}-e\right)}\right)-n\text{PolyLog}\left(2,\frac{c\left(\sqrt{e^2-4df}-e-2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)+c\left(\sqrt{e^2-4df}-e\right)}\right)+n\text{PolyLog}\left(2,\frac{c\left(\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}-b\right)+c\left(\sqrt{e^2-4d\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (-(n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(-(c*e) + b*f - Sqrt[b^2 - 4*a
*c]*f + c*Sqrt[e^2 - 4*d*f])]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]) - n*Log[(
f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqr
t[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 - 4*d*
f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x] + n*Log[(f*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e +
Sqrt[e^2 - 4*d*f] + 2*f*x] + Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a +
x*(b + c*x))^n] - Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a + x*(b + c*x)
)^n] - n*PolyLog[2, (c*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/((b - Sqrt[b^2 - 4
*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))] - n*PolyLog[2, (c*(-e + Sqrt[e^2 -
4*d*f] - 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))]
+ n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((-b + Sqrt[b^2 - 4*a*c]
)*f + c*(e + Sqrt[e^2 - 4*d*f]))] + n*PolyLog[2, (c*(e + Sqrt[e^2 - 4*d*f]
+ 2*f*x))/(-((b + Sqrt[b^2 - 4*a*c])*f) + c*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt
[e^2 - 4*d*f]
```

Maple [C] time = 0.11, size = 764, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x)
```

```
[Out] 2*(ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e
)/(4*d*f-e^2)^(1/2))+n*sum((ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((R
ootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)
-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-
c*d,index=1))-ln(x-_alpha)*ln((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alp
ha*f-_alpha*c*e+a*f-c*d,index=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*
f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2))-dilog((RootOf(_Z^2*c*f+(2*_al
pha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)-x+_alpha)/RootOf(_Z^
2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1))-dilog((
RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2
)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f
-c*d,index=2)))/(2*_alpha*f+e),_alpha=RootOf(_Z^2*f+_Z*e+d))-I/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*g)*csgn(I*g*(c*x^2+b*x+a)^n)+I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e
^2)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2+I/(4*d*f-
```

```
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*g)*csgn(I*g*(c*x^2
+b*x+a)^n)^2-I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csg
n(I*g*(c*x^2+b*x+a)^n)^3+2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*ln(g)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{fx^2+ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(cx^2+bx+a\right)^n g\right)}{fx^2+ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + e*x + d), x)
```

3.96 $\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=144

$$-\frac{2bn^2\text{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}-bn$$

```
[Out] 8*n^2*x - (4*b*n^2*Log[b + c*x])/c - (2*b*n^2*Log[-((c*x)/b)]*Log[b + c*x])
/c - (b*n^2*Log[b + c*x]^2)/c - 4*n*x*Log[d*(b*x + c*x^2)^n] + (2*b*n*Log[b
 + c*x]*Log[d*(b*x + c*x^2)^n])/c + x*Log[d*(b*x + c*x^2)^n]^2 - (2*b*n^2*P
olyLog[2, 1 + (c*x)/b])/c
```

Rubi [A] time = 0.284003, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2523, 2528, 43, 2524, 1593, 2418, 2394, 2315, 2390, 2301}

$$-\frac{2bn^2\text{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}-bn$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]^2,x]
```


```
[Out] 8*n^2*x - (4*b*n^2*Log[b + c*x])/c - (2*b*n^2*Log[-((c*x)/b)]*Log[b + c*x])
/c - (b*n^2*Log[b + c*x]^2)/c - 4*n*x*Log[d*(b*x + c*x^2)^n] + (2*b*n*Log[b
 + c*x]*Log[d*(b*x + c*x^2)^n])/c + x*Log[d*(b*x + c*x^2)^n]^2 - (2*b*n^2*P
olyLog[2, 1 + (c*x)/b])/c
```


Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
```

```
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)dx = x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int \frac{(b+2cx)\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}dx$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int \left(2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{b\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}\right)dx$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(4n)\int \log\left(d\left(bx+cx^2\right)^n\right)dx+(2bn)\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}dx$$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)+($$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)+($$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(}{}$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{bn^2\log^2(b+cx)}{c}-4nx\log\left(d\left(bx+\right.\right.$$

Mathematica [A] time = 0.0637361, size = 111, normalized size = 0.77

$$\frac{-2bn^2\text{PolyLog}\left(2,\frac{cx}{b}+1\right)+cx\left(\log^2\left(d(x(b+cx))^n\right)-4n\log\left(d(x(b+cx))^n\right)+8n^2\right)-2bn\log(b+cx)\left(-\log\left(d(x(b+cx\right.\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] (-(b*n^2*Log[b + c*x]^2) - 2*b*n*Log[b + c*x]*(2*n + n*Log[-((c*x)/b)] - Lo
g[d*(x*(b + c*x))^n]) + c*x*(8*n^2 - 4*n*Log[d*(x*(b + c*x))^n] + Log[d*(x*
(b + c*x))^n]^2) - 2*b*n^2*PolyLog[2, 1 + (c*x)/b])/c
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \left(\ln\left(d\left(cx^2+bx\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

Maxima [A] time = 1.23172, size = 166, normalized size = 1.15

$$-\left(\frac{2\left(\log(cx+b)\log\left(-\frac{cx+b}{b}+1\right)+\mathrm{Li}_2\left(\frac{cx+b}{b}\right)\right)b}{c}+\frac{b\log(cx+b)^2-8\,cx+4\,b\log(cx+b)}{c}\right)n^2-2\,n\left(2\,x-\frac{b\log(cx+b)}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="maxima")
```

```
[Out] -(2*(log(c*x + b)*log(-(c*x + b)/b + 1) + dilog((c*x + b)/b))*b/c + (b*log(
c*x + b)^2 - 8*c*x + 4*b*log(c*x + b))/c)*n^2 - 2*n*(2*x - b*log(c*x + b)/c
)*log((c*x^2 + b*x)^n*d) + x*log((c*x^2 + b*x)^n*d)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\left(cx^2+bx\right)^n d\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x)^n*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(d\left(bx + cx^2\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)**2,x)
```

```
[Out] Integral(log(d*(b*x + c*x**2)**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\left(cx^2 + bx\right)^n d\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x)^n*d)^2, x)
```

3.97 $\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=587

$$-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}+\frac{n\left(b-\sqrt{b^2-4a\right.}\cdots}{\cdots}$$

```
[Out] 8*n^2*x - (4*Sqrt[b^2 - 4*a*c]*n^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
c - ((b - Sqrt[b^2 - 4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c
) - ((b + Sqrt[b^2 - 4*a*c])*n^2*Log[-(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sq
rt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/c - ((b + Sqrt[b^2 -
4*a*c])*n^2*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c) - ((b - Sqrt[b^2 -
4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - (2*b*n^2*Log[a + b*x + c*x^2])/c - 4*n*
x*Log[d*(a + b*x + c*x^2)^n] + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + ((b + Sqrt[b^2 - 4*a*c])*
n*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + x*Log[
d*(a + b*x + c*x^2)^n]^2 - ((b - Sqrt[b^2 - 4*a*c])*n^2*PolyLog[2, -(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - ((b + Sqrt[b^2 - 4*a*c
])*n^2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c
```

Rubi [A] time = 0.949104, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {2523, 2528, 773, 634, 618, 206, 628, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}+\frac{n\left(b-\sqrt{b^2-4a\right.}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*Sqrt[b^2 - 4*a*c]*n^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
c - ((b - Sqrt[b^2 - 4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c
) - ((b + Sqrt[b^2 - 4*a*c])*n^2*Log[-(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sq
rt[b^2 - 4*a*c])]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/c - ((b + Sqrt[b^2 -
4*a*c])*n^2*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c) - ((b - Sqrt[b^2 -
4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] +
```

```
 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - (2*b*n^2*Log[a + b*x + c*x^2])/c - 4*n*
x*Log[d*(a + b*x + c*x^2)^n] + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + ((b + Sqrt[b^2 - 4*a*c])*
n*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + x*Log[
d*(a + b*x + c*x^2)^n]^2 - ((b - Sqrt[b^2 - 4*a*c])*n^2*PolyLog[2, -(b - Sq
rt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - ((b + Sqrt[b^2 - 4*a*c
])*n^2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)dx = x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int \frac{x(b+2cx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) - (2n)\int\left(2\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{(2a+bx)\log\left(d\left(a+\right.\right.}{a+bx+cx}\right.$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int \frac{(2a+bx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx - (4n)\int \log$$

$$= -4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + (2n)\int\left(\frac{\left(b-\sqrt{b^2-4ac}\right)}{b-\sqrt{b}}\right.$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + x\log^2\left(d\left(a+bx+cx^2\right)^n\right) + \left(2\left(b-\sqrt{b^2-4ac}\right)n\right.$$

$$= 8n^2x - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log}{c}$$

$$= 8n^2x - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{\left(b-\sqrt{b^2-4ac}\right)n\log}{}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{2bn^2\log\left(a+bx+cx^2\right)}{c} - 4nx\log\left(d\left(a+b\right.\right.$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(-\frac{b-\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\log}{c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2c\right.}{2c}$$

$$= 8n^2x - \frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c} - \frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2c\right.}{2c}$$

Mathematica [A] time = 0.823967, size = 478, normalized size = 0.81

$$\frac{n\left(n\left(\sqrt{b^2-4ac}-b\right)\left(2\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}-b-2cx}{2\sqrt{b^2-4ac}}\right)+\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(\log\left(-\sqrt{b^2-4ac}+b+2cx\right)+2\log\left(\cdots\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] x*Log[d*(a + x*(b + c*x))^n]^2 + (n*(4*n*(4*c*x - 2*Sqrt[b^2 - 4*a*c]*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - b*Log[a + x*(b + c*x)]) - 8*c*x*Log[d*(
a + x*(b + c*x))^n] + 2*(b - Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] +
 2*c*x]*Log[d*(a + x*(b + c*x))^n] + 2*(b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + (-b + Sqrt[b^2 - 4*a*c]
)*n*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
 + 2*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) + 2*PolyLo
g[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b
^2 - 4*a*c])*n*(Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*(2*Log[(-b + Sqrt[b^2 -
4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]
) + 2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])))/
(2*c)
```

Maple [F] time = 0.205, size = 0, normalized size = 0.

$$\int \left(\ln\left(d\left(cx^2+bx+a\right)^n\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\log\left(\left(cx^2+bx+a\right)^n d\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(d\left(a+bx+cx^2\right)^n\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)**2,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\left(cx^2+bx+a\right)^n d\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)^2, x)
```

3.98 $\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} dx$

Optimal. Leaf size=311

$$-\text{PolyLog}\left(2,\frac{2(x+1)}{1-i\sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3-i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right)+x\log\left(x^2+x+\right.$$

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[3
] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(3
+ I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]))]
 + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1 + x
 + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log[4 +
2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - PolyLog[2
, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqrt[3])]
+ 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```

Rubi [A] time = 0.477789, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2528, 2523, 773, 634, 618, 204, 628, 2524, 2418, 2394, 2393, 2391}

$$-\text{PolyLog}\left(2,\frac{2(x+1)}{1-i\sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3-i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right)+x\log\left(x^2+x+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```


```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[3
] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(3
+ I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]))]
 + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1 + x
 + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log[4 +
2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - PolyLog[2
, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqrt[3])]
+ 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```


Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
```

```
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x^2 \log\left(1+x+x^2\right)}{2+3x+x^2}\,dx = \int \left(\log\left(1+x+x^2\right) - \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\right)dx$$

$$= \int \log\left(1+x+x^2\right)\,dx - \int \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\,dx$$

$$= x\log\left(1+x+x^2\right) - \int \frac{x(1+2x)}{1+x+x^2}\,dx - \int \left(-\frac{2\log\left(1+x+x^2\right)}{2+2x} + \frac{8\log\left(1+x+x^2\right)}{4+2x}\right)dx$$

$$= -2x + x\log\left(1+x+x^2\right) + 2\int \frac{\log\left(1+x+x^2\right)}{2+2x}\,dx - 8\int \frac{\log\left(1+x+x^2\right)}{4+2x}\,dx - \int \frac{-2-x}{1+x+}$$

$$= -2x + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+x+x^2\right) + \frac{1}{2}$$

$$= -2x + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x\right.$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}}{3+i}\right.$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}}{3+i}\right.$$

$$= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}}{3+i}\right.$$

Mathematica [A] time = 0.159707, size = 290, normalized size = 0.93

$$-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right) - \text{PolyLog}\left(2,\frac{2i(x+1)}{\sqrt{3}+i}\right) + 4\left(\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right) + \text{PolyLog}\left(2,\frac{2i(x+2)}{\sqrt{3}+3i}\right) + \left(\log\left(\frac{-2ix}{\sqrt{}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[(-I + Sqrt[3] - (2*I)*x)/(I
+ Sqrt[3])]*Log[2*(1 + x)] - Log[(I + Sqrt[3] + (2*I)*x)/(-I + Sqrt[3])]*Lo
g[2*(1 + x)] + Log[1 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2*(1 + x)]*Log
[1 + x + x^2] - 4*Log[2*(2 + x)]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/
(1 + I*Sqrt[3])] - PolyLog[2, ((2*I)*(1 + x))/(I + Sqrt[3])] + 4*((Log[(-I
```

```
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])] + Log[(I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])])*Log[2*(2 + x)] + PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])] + Poly
Log[2, ((2*I)*(2 + x))/(3*I + Sqrt[3])])
```

Maple [A] time = 0.021, size = 279, normalized size = 0.9

$$x\ln\left(x^2+x+1\right)-2x+\frac{\ln\left(x^2+x+1\right)}{2}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}+\ln(1+x)\ln\left(x^2+x+1\right)-\ln(1+x)\ln\left(\frac{-1-2x}{1+i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x^2+x+1)/(x^2+3*x+2),x)
```

```
[Out] x*ln(x^2+x+1)-2*x+1/2*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(1+
x)*ln(x^2+x+1)-ln(1+x)*ln((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-ln(1+x)*ln((1+2
*x+I*3^(1/2))/(I*3^(1/2)-1))-dilog((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-dilog(
(1+2*x+I*3^(1/2))/(I*3^(1/2)-1))-4*ln(2+x)*ln(x^2+x+1)+4*ln(2+x)*ln((-1-2*x
+I*3^(1/2))/(3+I*3^(1/2)))+4*ln(2+x)*ln((1+2*x+I*3^(1/2))/(I*3^(1/2)-3))+4*
dilog((-1-2*x+I*3^(1/2))/(3+I*3^(1/2)))+4*dilog((1+2*x+I*3^(1/2))/(I*3^(1/2
)-3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\log\left(x^2+x+1\right)}{x^2+3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\log\left(x^2+x+1\right)}{x^2+3x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x**2+x+1)/(x**2+3*x+2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \log\left(x^2 + x + 1\right)}{x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

3.99 $\int \log^2\left(1+x+x^2\right)\, dx$

Optimal. Leaf size=371

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{3}\right)\log\left(x\right.$$

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt
[3] + 2*x]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*x))/Sqrt[3]
]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/2
 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[((-I/2)*(1 + I*Sqrt[3] + 2*
x))/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3])*
Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[
3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyLog
[2, -(I - Sqrt[3] + (2*I)*x)/(2*Sqrt[3])] - (1 - I*Sqrt[3])*PolyLog[2, (I +
 Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rubi [A] time = 0.541176, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 14, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.556, Rules used = {2523, 2528, 773, 634, 618, 204, 628, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{3}\right)\log\left(x\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt
[3] + 2*x]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*x))/Sqrt[3]
]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/2
 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[((-I/2)*(1 + I*Sqrt[3] + 2*
x))/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3])*
Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[
3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyLog
[2, -(I - Sqrt[3] + (2*I)*x)/(2*Sqrt[3])] - (1 - I*Sqrt[3])*PolyLog[2, (I +
 Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log^2\left(1+x+x^2\right) dx = x\log^2\left(1+x+x^2\right) - 2\int \frac{x(1+2x)\log\left(1+x+x^2\right)}{1+x+x^2} dx$$
$$= x\log^2\left(1+x+x^2\right) - 2\int \left(2\log\left(1+x+x^2\right) - \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\right) dx$$
$$= x\log^2\left(1+x+x^2\right) + 2\int \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2} dx - 4\int \log\left(1+x+x^2\right) dx$$
$$= -4x\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) + 2\int \left(\frac{\left(1-i\sqrt{3}\right)\log\left(1+x+x^2\right)}{1-i\sqrt{3}+2x} + \frac{\left(1+i\sqrt{3}\right)\log}{1+i\sqrt{3}}\right.$$
$$= 8x - 4x\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) + 4\int \frac{-2-x}{1+x+x^2} dx + \left(2\left(1-i\sqrt{3}\right)\right)\int \frac{\log\left(1\right.}{1-i\sqrt{}}$$
$$= 8x - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + \left(1+i\sqrt{3}\right)\log\left(1+\right.$$
$$= 8x - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right)$$
$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}\right.$$
$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{3}+2x\right) - \left(1-i\sqrt{3}\right)$$
$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right.$$
$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right.$$

Mathematica [A] time = 0.146165, size = 323, normalized size = 0.87

$$-\frac{1}{2}i\left(\sqrt{3}-i\right)\left(2\text{PolyLog}\left(2,\frac{-2ix+\sqrt{3}-i}{2\sqrt{3}}\right)+\log\left(2x+i\sqrt{3}+1\right)\left(2\log\left(\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)+\log\left(2x+i\sqrt{3}+1\right)\right)\right)+\frac{1}{2}i\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1
+ x + x^2] + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1
 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2
]^2 - (I/2)*(-I + Sqrt[3])*(Log[1 + I*Sqrt[3] + 2*x]*(2*Log[(I + Sqrt[3] +
(2*I)*x)/(2*Sqrt[3])] + Log[1 + I*Sqrt[3] + 2*x]) + 2*PolyLog[2, (-I + Sqrt
[3] - (2*I)*x)/(2*Sqrt[3])]) + (I/2)*(I + Sqrt[3])*(Log[1 - I*Sqrt[3] + 2*x
]*(2*Log[(-I + Sqrt[3] - (2*I)*x)/(2*Sqrt[3])] + Log[1 - I*Sqrt[3] + 2*x])
+ 2*PolyLog[2, (I + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])])
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int \left(\ln \left(x^2+x+1\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1)^2,x)
```

```
[Out] int(ln(x^2+x+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log \left(x^2+x+1\right)^2 - \int \frac{2\left(2\,x^2+x\right)\log \left(x^2+x+1\right)}{x^2+x+1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1)^2 - integrate(2*(2*x^2 + x)*log(x^2 + x + 1)/(x^2 + x +
1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log \left(x^2+x+1\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x + 1)^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x^2 + x + 1\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x + 1)^2, x)
```

3.100 $\int \frac{\log^2(-1+x+x^2)}{x^3} dx$

Optimal. Leaf size=443

$$3\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3\text{PolyLog}$$

```
[Out] Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2]
*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((1
- Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-(1 - Sqrt[5] + 2
*x)/(2*Sqrt[5])]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqrt[5]
 + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5] + 2*x
)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 + x + x^
2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*L
og[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^
2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqrt[5])] -
((3 + Sqrt[5])*PolyLog[2, -(1 - Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - ((3 - Sqrt
[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[2, 1 + (2*x
)/(1 - Sqrt[5])]
```

Rubi [A] time = 0.683829, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 16, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.231, Rules used = {2525, 2528, 800, 632, 31, 2524, 2357, 2316, 2315, 2317, 2391, 2418, 2390, 2301, 2394, 2393}

$$3\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2]
*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((1
- Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-(1 - Sqrt[5] + 2
*x)/(2*Sqrt[5])]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqrt[5]
 + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5] + 2*x
)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 + x + x^
2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*L
og[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^
```

```
2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqrt[5])] -
((3 + Sqrt[5])*PolyLog[2, -(1 - Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - ((3 - Sqrt
[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[2, 1 + (2*x
)/(1 - Sqrt[5])]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
```

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = -\frac{\log^2\left(-1+x+x^2\right)}{2x^2} + \int \frac{(1+2x)\log\left(-1+x+x^2\right)}{x^2\left(-1+x+x^2\right)}\,dx$$
$$= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2} + \int\left(-\frac{\log\left(-1+x+x^2\right)}{x^2} - \frac{3\log\left(-1+x+x^2\right)}{x} + \frac{(4+3x)\log\left(-1+x\right.}{-1+x+x^2}\right.$$
$$= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2} - 3\int \frac{\log\left(-1+x+x^2\right)}{x}\,dx - \int \frac{\log\left(-1+x+x^2\right)}{x^2}\,dx + \int \frac{(4+3x)\log}{-1+}$$
$$= \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) - \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 3\int \frac{(1+2x)\log(x)}{-1+x+x^2}\,dx$$
$$= \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) - \frac{\log^2\left(-1+x+x^2\right)}{2x^2} + 3\int\left(\frac{2\log(x)}{1-\sqrt{5}+2x} + \right.$$
$$= \log(x) + \frac{\log\left(-1+x+x^2\right)}{x} - 3\log(x)\log\left(-1+x+x^2\right) + \frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)\log\left(\right.$$
$$= \log(x) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) + 3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right) + \frac{\log\left(-1+x+\right.}{x}$$
$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\left(1-\sqrt{5}\right)$$
$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\left(1-\sqrt{5}\right)$$
$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{4}\left(3+\sqrt{5}\right.$$
$$= \log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{4}\left(3+\sqrt{5}\right.$$

Mathematica [A] time = 0.74486, size = 826, normalized size = 1.86

$$\frac{x\left(\sqrt{5}x\log^2\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+3x\log^2\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)-2\sqrt{5}x\log\left(-2x+\sqrt{5}-1\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)-6x\log\left(-2x+\sqrt{5}-1\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] (-2*Log[-1 + x + x^2]^2 + x*(4*x*Log[x] - 12*x*Log[(1 + Sqrt[5])/2]*Log[x]
- 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] - 2*Sqrt[5]*x*Log[-1
 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] + 12*x*Log[x]*Log[1/2 - Sqrt[5]/
2 + x] - 12*x*Log[(2*x)/(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + x] + 3*x*Log[
1/2 - Sqrt[5]/2 + x]^2 + Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]^2 - 6*x*Log[-1
+ Sqrt[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] - 2*Sqrt[5]*x*Log[-1 + Sqrt[5] -
2*x]*Log[(1 + Sqrt[5])/2 + x] + 12*x*Log[x]*Log[(1 + Sqrt[5])/2 + x] + 3*x*
Log[(1 + Sqrt[5])/2 + x]^2 - Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]^2 - 2*x*Log
[1 - Sqrt[5] + 2*x] - 2*Sqrt[5]*x*Log[1 - Sqrt[5] + 2*x] + 3*x*Log[5]*Log[1
 - Sqrt[5] + 2*x] + Sqrt[5]*x*Log[5]*Log[1 - Sqrt[5] + 2*x] - 2*x*Log[1 + S
qrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[1/2 - Sqrt[5]/
2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]*Log[1
+ Sqrt[5] + 2*x] - 6*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*
Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 6*x*Log[1/2 - S
qrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] - 2*Sqrt[5]*x*Log[1/2 -
Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] + 4*Log[-1 + x + x^2] +
 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] + 2*Sqrt[5]*x*Log[-1 + Sqrt[
5] - 2*x]*Log[-1 + x + x^2] - 12*x*Log[x]*Log[-1 + x + x^2] + 6*x*Log[1 + S
qrt[5] + 2*x]*Log[-1 + x + x^2] - 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x]*Log[-1
 + x + x^2] - 4*Sqrt[5]*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(2*Sqrt[5])] - 12
*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(-1 + Sqrt[5])] + 12*x*PolyLog[2, (-2*x)
/(1 + Sqrt[5])]))/(4*x^2)
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{\left(\ln\left(x^2+x-1\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x-1)^2/x^3,x)
```

```
[Out] int(ln(x^2+x-1)^2/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(x^2+x-1\right)^2}{2\,x^2}+\int \frac{(2\,x+1)\log\left(x^2+x-1\right)}{x^4+x^3-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1)^2/x^2 + integrate((2*x + 1)*log(x^2 + x - 1)/(x^4 + x
^3 - x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(x^2+x-1\right)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x - 1)^2/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x-1)**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x^2+x-1\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x - 1)^2/x^3, x)
```

3.101 $\int x^3 \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal. Leaf size=172

$$-\frac{x^4}{32}+\frac{x^3}{192}-\frac{x^2}{1024}-\frac{1}{32}\left(x^2-x\right)^{3/2}x-\frac{1}{12}\left(x^2-x\right)^{3/2}+\frac{149(1-2x)\sqrt{x^2-x}}{2048}-\frac{683\sqrt{x^2-x}}{4096}+\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x\right.$$

```
[Out] x/4096 - x^2/1024 + x^3/192 - x^4/32 - (683*Sqrt[-x + x^2])/4096 + (149*(1
- 2*x)*Sqrt[-x + x^2])/2048 - (-x + x^2)^(3/2)/12 - (x*(-x + x^2)^(3/2))/32
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32768 - (1537*ArcTanh[x/Sqrt[-x +
 x^2]])/16384 - Log[1 + 8*x]/32768 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])
/4
```

Rubi [A] time = 0.380669, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724, 670}

$$-\frac{x^4}{32}+\frac{x^3}{192}-\frac{x^2}{1024}-\frac{1}{32}\left(x^2-x\right)^{3/2}x-\frac{1}{12}\left(x^2-x\right)^{3/2}+\frac{149(1-2x)\sqrt{x^2-x}}{2048}-\frac{683\sqrt{x^2-x}}{4096}+\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```


```
[Out] x/4096 - x^2/1024 + x^3/192 - x^4/32 - (683*Sqrt[-x + x^2])/4096 + (149*(1
- 2*x)*Sqrt[-x + x^2])/2048 - (-x + x^2)^(3/2)/12 - (x*(-x + x^2)^(3/2))/32
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32768 - (1537*ArcTanh[x/Sqrt[-x +
 x^2]])/16384 - Log[1 + 8*x]/32768 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])
/4
```


Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
```

```
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \int x^3\log\left(-1+4x+4\sqrt{-x+x^2}\right)dx$$
$$= \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)+2\int \frac{x^4}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}dx$$
$$= \frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)+2\int\left(\frac{1}{8192}-\frac{x}{1024}+\frac{x^2}{128}-\frac{x^3}{16}-\frac{1}{8192(1+8\right.$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{\log(1+8x)}{32768}+\frac{1}{4}x^4\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\int$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{85(1-2x)\sqrt{-x+x^2}}{2048}-\frac{11}{192}\left(-x+\right.$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{129(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+\right.$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+\right.$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+\right.$$
$$= \frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+\right.$$

Mathematica [A] time = 0.495725, size = 117, normalized size = 0.68

$$\frac{-3072x^4+512x^3-96x^2-8\sqrt{(x-1)x}\left(384x^3+640x^2+764x+1155\right)+24576x^4\log\left(4x+4\sqrt{(x-1)x}-1\right)+24x-6\,\mathrm{lo}}{98304}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (24*x - 96*x^2 + 512*x^3 - 3072*x^4 - 8*Sqrt[(-1 + x)*x]*(1155 + 764*x + 64
0*x^2 + 384*x^3) - 6*Log[1 + 8*x] - 4611*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]]
+ 24576*x^4*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + 3*Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/98304
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int x^3 \ln\left(-1 + 4x + 4\sqrt{(-1+x)x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log\left(4x + 4\sqrt{(x-1)x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A] time = 2.51507, size = 424, normalized size = 2.47

$$-\frac{1}{32}x^4 + \frac{1}{192}x^3 - \frac{1}{1024}x^2 + \frac{1}{4}\left(x^4 - 1\right)\log\left(4x + 4\sqrt{x^2 - x} - 1\right) - \frac{1}{12288}\left(384x^3 + 640x^2 + 764x + 1155\right)\sqrt{x^2 - x} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*x^4 + 1/192*x^3 - 1/1024*x^2 + 1/4*(x^4 - 1)*log(4*x + 4*sqrt(x^2 - x
) - 1) - 1/12288*(384*x^3 + 640*x^2 + 764*x + 1155)*sqrt(x^2 - x) + 1/4096*
x + 4095/32768*log(8*x + 1) - 2559/32768*log(-2*x + 2*sqrt(x^2 - x) + 1) +
4095/32768*log(-2*x + 2*sqrt(x^2 - x) - 1) - 4095/32768*log(-4*x + 4*sqrt(x
^2 - x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35666, size = 181, normalized size = 1.05

$$\frac{1}{4}x^4\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{32}x^4+\frac{1}{192}x^3-\frac{1}{1024}x^2-\frac{1}{12288}(4(32(3x+5)x+191)x+1155)\sqrt{x^2-x}+\frac{1}{4096}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/32*x^4 + 1/192*x^3 - 1/1024*x^
2 - 1/12288*(4*(32*(3*x + 5)*x + 191)*x + 1155)*sqrt(x^2 - x) + 1/4096*x -
1/32768*log(abs(8*x + 1)) + 1537/32768*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
 - 1/32768*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/32768*log(abs(-4*x + 4*
sqrt(x^2 - x) + 1))
```

3.102 $\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=149

$$-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{223}{}$$

```
[Out] -x/384 + x^2/96 - x^3/18 - (85*Sqrt[-x + x^2])/384 + (5*(1 - 2*x)*Sqrt[-x +
 x^2])/64 - (-x + x^2)^(3/2)/18 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/30
72 - (223*ArcTanh[x/Sqrt[-x + x^2]])/1536 + Log[1 + 8*x]/3072 + (x^3*Log[-1
 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rubi [A] time = 0.291557, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724}

$$-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{223}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -x/384 + x^2/96 - x^3/18 - (85*Sqrt[-x + x^2])/384 + (5*(1 - 2*x)*Sqrt[-x +
 x^2])/64 - (-x + x^2)^(3/2)/18 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/30
72 - (223*ArcTanh[x/Sqrt[-x + x^2]])/1536 + Log[1 + 8*x]/3072 + (x^3*Log[-1
 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
```

```
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right) dx &= \int x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right) dx \\
&= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \frac{x^3}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)} dx \\
&= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \left(-\frac{1}{1024}+\frac{x}{128}-\frac{x^2}{16}+\frac{1}{1024(1+8x)} - \right. \\
&= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}+\frac{\log(1+8x)}{3072}+\frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{144}\int \\
&= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{11}{192}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}+ \\
&= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}+ \\
&= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}- \\
&= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}-
\end{aligned}$$

Mathematica [A] time = 0.334861, size = 107, normalized size = 0.72

$$\frac{-512x^3+96x^2-8\sqrt{(x-1)x}\left(64x^2+116x+165\right)+3072x^3\log\left(4x+4\sqrt{(x-1)x}-1\right)-24x+6\log(8x+1)-669\log\left(\right.}{9216}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-24*x + 96*x^2 - 512*x^3 - 8*Sqrt[(-1 + x)*x]*(165 + 116*x + 64*x^2) + 6*L
og[1 + 8*x] - 669*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 3072*x^3*Log[-1 + 4*x
 + 4*Sqrt[(-1 + x)*x]] - 3*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/9216
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int x^2 \ln\left(-1+4\,x+4\,\sqrt{(-1+x)\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \log\left(4\,x+4\,\sqrt{(x-1)x}-1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A] time = 2.61613, size = 375, normalized size = 2.52

$$-\frac{1}{18}\,x^3+\frac{1}{96}\,x^2+\frac{1}{3}\left(x^3+1\right)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-\frac{1}{1152}\left(64\,x^2+116\,x+165\right)\sqrt{x^2-x}-\frac{1}{384}\,x-\frac{511}{3072}\,\log\left(8\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/18*x^3 + 1/96*x^2 + 1/3*(x^3 + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/115
2*(64*x^2 + 116*x + 165)*sqrt(x^2 - x) - 1/384*x - 511/3072*log(8*x + 1) +
245/1024*log(-2*x + 2*sqrt(x^2 - x) + 1) - 511/3072*log(-2*x + 2*sqrt(x^2 -
 x) - 1) + 511/3072*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36025, size = 167, normalized size = 1.12

$$\frac{1}{3}x^3\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{18}x^3+\frac{1}{96}x^2-\frac{1}{1152}(4(16x+29)x+165)\sqrt{x^2-x}-\frac{1}{384}x+\frac{1}{3072}\log(|8x+1|)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/18*x^3 + 1/96*x^2 - 1/1152*(4*
(16*x + 29)*x + 165)*sqrt(x^2 - x) - 1/384*x + 1/3072*log(abs(8*x + 1)) + 2
23/3072*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/3072*log(abs(-2*x + 2*sqrt
(x^2 - x) - 1)) - 1/3072*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

3.103 $\int x \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=127

$$-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-}}\right.$$

```
[Out] x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Arc
Tanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/12
8 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```

Rubi [A] time = 0.245347, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2537, 2535, 6742, 640, 620, 206, 612, 734, 843, 724}

$$-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```



```
[Out] x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Arc
Tanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/12
8 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```


Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x\log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$
$$= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\frac{x^2}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx$$
$$= \frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int\left(\frac{1}{128}-\frac{x}{16}-\frac{1}{128(1+8x)}-\frac{x}{12\sqrt{-x+x^2}}\right.$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{12}\int\frac{\sqrt{-x+x^2}}{-1-8x}$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1\right.$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1\right.$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{13}{48}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)-\frac{1}{256}$$
$$= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{3}{12}$$

Mathematica [A] time = 0.269203, size = 102, normalized size = 0.8

$$\frac{1}{256}\left(-32x^2+128x^2\log\left(4x+4\sqrt{(x-1)x}-1\right)-32\sqrt{(x-1)x}x+8x-72\sqrt{(x-1)x}-2\log(8x+1)-33\log\left(-2x-2\sqrt{(x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (8*x - 32*x^2 - 72*Sqrt[(-1 + x)*x] - 32*x*Sqrt[(-1 + x)*x] - 2*Log[1 + 8*x
] - 33*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 128*x^2*Log[-1 + 4*x + 4*Sqrt[(-
```

```
1 + x)*x]] + Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/256
```

Maple [F] time = 0.004, size = 0, normalized size = 0.

$$\int x \ln \left(-1+4 x+4 \sqrt{(-1+x) x}\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \log \left(4 x+4 \sqrt{(x-1) x}-1\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A] time = 2.57492, size = 327, normalized size = 2.57

$$-\frac{1}{8} x^{2}+\frac{1}{2} \left(x^{2}-1\right) \log \left(4 x+4 \sqrt{x^{2}-x}-1\right)-\frac{1}{32} \sqrt{x^{2}-x}(4 x+9)+\frac{1}{32} x+\frac{63}{256} \log (8 x+1)-\frac{31}{256} \log \left(-2 x+2 \sqrt{x^{2}-x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*x^2 + 1/2*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/32*sqrt(x^2 - x
)*(4*x + 9) + 1/32*x + 63/256*log(8*x + 1) - 31/256*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 63/256*log(-2*x + 2*sqrt(x^2 - x) - 1) - 63/256*log(-4*x + 4*sqr
t(x^2 - x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29707, size = 154, normalized size = 1.21

$$\frac{1}{2}x^2\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{8}x^2-\frac{1}{32}\sqrt{x^2-x}(4x+9)+\frac{1}{32}x-\frac{1}{256}\log\left(|8x+1|\right)+\frac{33}{256}\log\left(\left|-2x+2\sqrt{x^2-x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/8*x^2 - 1/32*sqrt(x^2 - x)*(4*
x + 9) + 1/32*x - 1/256*log(abs(8*x + 1)) + 33/256*log(abs(-2*x + 2*sqrt(x^
2 - x) + 1)) - 1/256*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/256*log(abs(-
4*x + 4*sqrt(x^2 - x) + 1))
```

3.104 $\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{x^2-x}}{2}+x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

```
[Out] -x/2 - Sqrt[-x + x^2]/2 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16 - (7*Ar
cTanh[x/Sqrt[-x + x^2]])/8 + Log[1 + 8*x]/16 + x*Log[-1 + 4*x + 4*Sqrt[-x +
 x^2]]
```

Rubi [A] time = 0.162303, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2537, 2533, 6742, 640, 620, 206, 734, 843, 724}

$$-\frac{\sqrt{x^2-x}}{2}+x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -x/2 - Sqrt[-x + x^2]/2 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16 - (7*Ar
cTanh[x/Sqrt[-x + x^2]])/8 + Log[1 + 8*x]/16 + x*Log[-1 + 4*x + 4*Sqrt[-x +
 x^2]]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2533

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
, x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] + Dist[(f^
2*(b^2 - 4*a*c))/2, Int[x/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a
*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$
$$= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int \frac{x}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx$$
$$= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int\left(-\frac{1}{16}+\frac{1}{16(1+8x)}-\frac{x}{12\sqrt{-x+x^2}}+\frac{\sqrt{-x+}}{6(1+8}\right.$$
$$= -\frac{x}{2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{2}{3}\int\frac{x}{\sqrt{-x+x^2}}\,dx+\frac{4}{3}\int$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{1}{12}\int\frac{}{(1+}$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{5}{48}\int\frac{1}{\sqrt{-x}}$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\right.$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1$$

Mathematica [A] time = 0.0282042, size = 85, normalized size = 0.89

$$\frac{1}{16}\left(-8x-8\sqrt{(x-1)x}+16x\log\left(4x+4\sqrt{(x-1)x}-1\right)+2\log(8x+1)-7\log\left(-2x-2\sqrt{(x-1)x}+1\right)-\log\left(-10x+6\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-8*x - 8*Sqrt[(-1 + x)*x] + 2*Log[1 + 8*x] - 7*Log[1 - 2*x - 2*Sqrt[(-1 +
x)*x]] + 16*x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/16
```

Maple [A] time = 0.031, size = 80, normalized size = 0.8

$$x\ln\left(-1+4x+4\sqrt{(-1+x)x}\right)-\frac{7}{16}\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)-\frac{1}{16}\mathrm{Artanh}\left(\frac{32}{3}\left(\frac{1}{8}-\frac{5x}{4}\right)\frac{1}{\sqrt{64\,(x+1/8)^2-80x-1}}\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] x*ln(-1+4*x+4*((-1+x)*x)^(1/2))-7/16*ln(-1/2+x+(x^2-x)^(1/2))-1/16*arctanh(
32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/2*(x^2-x)^(1/2)-1/2*x+1/16*
ln(1+8*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{1}{2}x+\int\frac{2x^2+x}{2\left(4x^3-5x^2+4\left(x^{\frac{5}{2}}-x^{\frac{3}{2}}\right)\sqrt{x-1}+x\right)}\,dx-\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 1/2*x + integrate(1/2*(2*x^2 + x)/
(4*x^3 - 5*x^2 + 4*(x^(5/2) - x^(3/2))*sqrt(x - 1) + x), x) - 1/2*log(sqrt(
x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A] time = 2.61002, size = 278, normalized size = 2.93

$$(x+1)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{2}x-\frac{1}{2}\sqrt{x^2-x}-\frac{7}{16}\log(8x+1)+\frac{15}{16}\log\left(-2x+2\sqrt{x^2-x}+1\right)-\frac{7}{16}\log\left(-2x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) - 7/16*l
og(8*x + 1) + 15/16*log(-2*x + 2*sqrt(x^2 - x) + 1) - 7/16*log(-2*x + 2*sqr
```

```
t(x^2 - x) - 1) + 7/16*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32307, size = 136, normalized size = 1.43

$$x\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-x}+\frac{1}{16}\,\log\left(|8\,x+1|\right)+\frac{7}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)+\frac{1}{16}\,\log\left(\left|-2\,x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) + 1/16*log(a
bs(8*x + 1)) + 7/16*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/16*log(abs(-2*
x + 2*sqrt(x^2 - x) - 1)) - 1/16*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

3.105 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$

Optimal. Leaf size=25

$$\text{CannotIntegrate}\left(\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x},x\right)$$

```
[Out] CannotIntegrate[Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x, x]
```

Rubi [A] time = 0.0763766, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
```

```
[Out] Defer[Int][Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x, x]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}\,dx$$

Mathematica [A] time = 0.453661, size = 0, normalized size = 0.

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
```

```
[Out] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x, x]
```

Maple [A] time = 0.005, size = 0, normalized size = 0.

$$\int \frac{1}{x}\ln\left(-1+4\,x+4\,\sqrt{(-1+x)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(log(4*x + 4*sqrt(x^2 - x) - 1)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

3.106 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$

Optimal. Leaf size=76

$$\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+4\log(x)-4\log(8x+1)$$

```
[Out] (4*Sqrt[-x + x^2])/x + 4*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 4*Log[x]
- 4*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Rubi [A] time = 0.262083, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2537, 2535, 6742, 640, 620, 206, 662, 664, 734, 843, 724}

$$\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+4\log(x)-4\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[-x + x^2])/x + 4*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 4*Log[x]
- 4*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^2}\,dx$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int \frac{1}{x\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int\left(-\frac{1}{2x}+\frac{4}{1+8x}-\frac{x}{12\sqrt{-x+x^2}}+\frac{\sqrt{-x+x^2}}{4x^2}-\right.$$
$$= 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{2}{3}\int \frac{x}{\sqrt{-x+x^2}}\,dx - 2\int$$
$$= \frac{4\sqrt{-x+x^2}}{x} + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{1}{3}\int \frac{1}{\sqrt{-x}}$$
$$= \frac{4\sqrt{-x+x^2}}{x} + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} + \frac{2}{3}\text{Subst}\Bigg($$
$$= \frac{4\sqrt{-x+x^2}}{x} - \frac{40}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+\right.}{x}$$
$$= \frac{4\sqrt{-x+x^2}}{x} + 4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) + 4\log(x) - 4\log(1+8x) - \frac{\log\left(-1+4x+\right.}{x}$$

Mathematica [A] time = 0.214096, size = 68, normalized size = 0.89

$$\frac{4\sqrt{(x-1)x}}{x} + 4\log(x) - 8\log(8x+1) - \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x} + 4\log\left(-10x+6\sqrt{(x-1)x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[(-1 + x)*x])/x + 4*Log[x] - 8*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[
(-1 + x)*x]]/x + 4*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]]
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\ln\left(-1+4\,x+4\,\sqrt{(-1+x)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2, x)
```

Fricas [A] time = 2.48659, size = 296, normalized size = 3.89

$$-\frac{7x\log(8x+1)+2(x+1)\log\left(4x+4\sqrt{x^2-x}-1\right)-8x\log(x)+x\log\left(-2x+2\sqrt{x^2-x}+1\right)+7x\log\left(-2x+2\sqrt{}\right.}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(7*x*log(8*x + 1) + 2*(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 8*x*log
(x) + x*log(-2*x + 2*sqrt(x^2 - x) + 1) + 7*x*log(-2*x + 2*sqrt(x^2 - x) -
1) - 7*x*log(-4*x + 4*sqrt(x^2 - x) + 1) - 8*x - 8*sqrt(x^2 - x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34975, size = 124, normalized size = 1.63

$$-\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}+\frac{4}{x-\sqrt{x^2-x}}-4\,\log\left(|8\,x+1|\right)+4\,\log\left(|x|\right)-4\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)+4\,\log\left(\left|-4\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -log(4*x + 4*sqrt((x - 1)*x) - 1)/x + 4/(x - sqrt(x^2 - x)) - 4*log(abs(8*x
 + 1)) + 4*log(abs(x)) - 4*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 4*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

3.107 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2\left(x^2-x\right)^{3/2}}{3x^3}-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

```
[Out] -2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(1
 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x +
 4*Sqrt[-x + x^2]]/(2*x^2)
```

Rubi [A] time = 0.290585, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2537, 2535, 6742, 640, 620, 206, 734, 843, 724, 650, 662, 664}

$$-\frac{2\left(x^2-x\right)^{3/2}}{3x^3}-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(1
 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x +
 4*Sqrt[-x + x^2]]/(2*x^2)
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
```

```
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 662

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + p + 1)), x]
- Dist[(c*p)/(e^2*(m + p + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^3}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \frac{1}{x^2\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int\left(-\frac{1}{2x^2}+\frac{4}{x}-\frac{32}{1+8x}-\frac{x}{12\sqrt{-x+x^2}}+\frac{256\sqrt{-}}{3(-1}\right.$$

$$= -\frac{2}{x} - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} + \frac{1}{3}\int \frac{x}{\sqrt{-x+x^2}}\,d$$

$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+\right.}{2x^2}$$

$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+\right.}{2x^2}$$

$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} + \frac{160}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right) - 16\log(x) + 16\,\mathrm{lo}$$

$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2\left(-x+x^2\right)^{3/2}}{3x^3} - 16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right) - 16\log(x) + 16\log$$

Mathematica [A] time = 0.274751, size = 82, normalized size = 0.81

$$-\frac{2\sqrt{(x-1)x}(16x-1)}{3x^2} - \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{2x^2} - \frac{2}{x} - 16\log(x) + 32\log(8x+1) - 16\log\left(-10x+6\sqrt{(x-1)x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (2*Sqrt[(-1 + x)*x]*(-1 + 16*x))/(3*x^2) - 16*Log[x] + 32*Log[1 + 8*
x] - Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/(2*x^2) - 16*Log[1 - 10*x + 6*Sqrt[
(-1 + x)*x]]
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^3, x)
```

Fricas [A] time = 2.33293, size = 358, normalized size = 3.54

$$\frac{189x^2\log(8x+1)-192x^2\log(x)+3x^2\log\left(-2x+2\sqrt{x^2-x}+1\right)+189x^2\log\left(-2x+2\sqrt{x^2-x}-1\right)-189x^2\log\left(\cdots\right.}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(189*x^2*log(8*x + 1) - 192*x^2*log(x) + 3*x^2*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 189*x^2*log(-2*x + 2*sqrt(x^2 - x) - 1) - 189*x^2*log(-4*x + 4*s
qrt(x^2 - x) + 1) - 128*x^2 + 6*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) -
8*sqrt(x^2 - x)*(16*x - 1) - 24*x)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40355, size = 176, normalized size = 1.74

$$-\frac{2}{x}-\frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{2x^2}-\frac{2\left(18\left(x-\sqrt{x^2-x}\right)^2-3x+3\sqrt{x^2-x}+1\right)}{3\left(x-\sqrt{x^2-x}\right)^3}+16\log\left(|8x+1|\right)-16\log\left(|x|\right)+16\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -2/x - 1/2*log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2 - 2/3*(18*(x - sqrt(x^2 - x
))^2 - 3*x + 3*sqrt(x^2 - x) + 1)/(x - sqrt(x^2 - x))^3 + 16*log(abs(8*x +
1)) - 16*log(abs(x)) + 16*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) - 16*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

3.108 $\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=187

$$-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2\left(x^2-x\right)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{71\left(x^2-x\right)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x}\right.}{320\sqrt{2}\sqrt{x-1}\sqrt{x}}$$

```
[Out] -Sqrt[x]/160 + x^(3/2)/60 - (2*x^(5/2))/25 - (17*Sqrt[-x + x^2])/(32*Sqrt[x
]) - (71*(-x + x^2)^(3/2))/(300*x^(3/2)) - (2*(-x + x^2)^(3/2))/(25*Sqrt[x]
) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(320*Sqrt[2]*Sqrt[-
1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(320*Sqrt[2]) + (2*x^(5/2)*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Rubi [A] time = 0.541197, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2000, 2016, 1146, 444, 50, 63}

$$-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2\left(x^2-x\right)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{71\left(x^2-x\right)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x}\right.}{320\sqrt{2}\sqrt{x-1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -Sqrt[x]/160 + x^(3/2)/60 - (2*x^(5/2))/25 - (17*Sqrt[-x + x^2])/(32*Sqrt[x
]) - (71*(-x + x^2)^(3/2))/(300*x^(3/2)) - (2*(-x + x^2)^(3/2))/(25*Sqrt[x]
) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(320*Sqrt[2]*Sqrt[-
1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(320*Sqrt[2]) + (2*x^(5/2)*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
```

```
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx = \int x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)dx$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{5}\int\frac{x^{5/2}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{5}\,\mathrm{Subst}\left(\int\frac{x^6}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\right.}\right.$$

$$= \frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{5}\,\mathrm{Subst}\left(\int\left(-\frac{1}{1024}+\frac{x^2}{128}-\frac{x^4}{16}+\frac{}{1024}\right.\right.$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}+\frac{2}{5}x^{5/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{1}{160}\,\mathrm{Subst}\left(\int\frac{}{1+}\right.$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{11\left(-x+x^2\right)^{3/2}}{60x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan^{-1}}{3}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{8\sqrt{-x+x^2}}{15\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan^{-1}}{3}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan}{}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}+\frac{\tan}{}$$

$$= -\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71\left(-x+x^2\right)^{3/2}}{300x^{3/2}}-\frac{2\left(-x+x^2\right)^{3/2}}{25\sqrt{x}}-\frac{\sqrt{-x}}{}$$

<u>Mathematica</u> [C] time = 0.828841, size = 232, normalized size = 1.24

$$\frac{-3072x^{5/2}-3072\sqrt{(x-1)x}\,x^{3/2}+640x^{3/2}+15360x^{5/2}\log\left(4x+4\sqrt{(x-1)x}-1\right)-6016\sqrt{(x-1)x}\sqrt{x}-240\sqrt{x}-\frac{11312\sqrt{(x}}{\sqrt{x}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-240*Sqrt[x] + 640*x^(3/2) - 3072*x^(5/2) - (11312*Sqrt[(-1 + x)*x])/Sqrt[
x] - 6016*Sqrt[x]*Sqrt[(-1 + x)*x] - 3072*x^(3/2)*Sqrt[(-1 + x)*x] + 60*Sqr
```

```
t[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 60*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)
*x])/(3*Sqrt[x])] - (30*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] + (15*I)*Sqrt[2]*Log[
(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 15360*x^(5/2)*Log[-1 + 4*x + 4
*Sqrt[(-1 + x)*x]] + (15*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 +
x)*x])])/38400
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \ln\left(-1+4x+4\sqrt{(-1+x)x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{5}x^{\frac{5}{2}}\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{2}{25}\left(2x^2+5\right)\sqrt{x}-\frac{2}{15}x^{\frac{3}{2}}+\int\frac{2x^{\frac{5}{2}}+x^{\frac{3}{2}}}{5\left(4x^2+4\left(x^{\frac{3}{2}}-\sqrt{x}\right)\sqrt{x-1}-5x+1\right)}dx+\frac{1}{5}\log\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 2/25*(2*x^2 + 5)*sqrt(x)
 - 2/15*x^(3/2) + integrate(1/5*(2*x^(5/2) + x^(3/2))/(4*x^2 + 4*(x^(3/2) -
 sqrt(x))*sqrt(x - 1) - 5*x + 1), x) + 1/5*log(sqrt(x) + 1) - 1/5*log(sqrt(
x) - 1)
```

Fricas [A] time = 2.25007, size = 327, normalized size = 1.75

$$\frac{3840x^{\frac{7}{2}}\log\left(4x+4\sqrt{x^2-x}-1\right)+15\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)+15\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4\left(192x^2+376x+707\right)\sqrt{x}}{9600x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9600*(3840*x^(7/2)*log(4*x + 4*sqrt(x^2 - x) - 1) + 15*sqrt(2)*x*arctan(2
*sqrt(2)*sqrt(x)) + 15*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x))
- 4*(192*x^2 + 376*x + 707)*sqrt(x^2 - x)*sqrt(x) - 4*(192*x^3 - 40*x^2 + 1
5*x)*sqrt(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \log\left(4x + 4\sqrt{(x-1)x} - 1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

3.109 $\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)dx$

Optimal. Leaf size=158

$$-\frac{2x^{3/2}}{9}-\frac{11\sqrt{x^2-x}}{12\sqrt{x}}-\frac{2\left(x^2-x\right)^{3/2}}{9x^{3/2}}+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\sqrt{x}}{12}-\frac{\tan^{-1}\left(2\sqrt{\right.}}{24\sqrt{}}$$

```
[Out] Sqrt[x]/12 - (2*x^(3/2))/9 - (11*Sqrt[-x + x^2])/(12*Sqrt[x]) - (2*(-x + x^
2)^(3/2))/(9*x^(3/2)) + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])
/(24*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(24*Sqrt[2])
 + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rubi [A] time = 0.431156, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2000, 1146, 444, 50, 63, 204}

$$-\frac{2x^{3/2}}{9}-\frac{11\sqrt{x^2-x}}{12\sqrt{x}}-\frac{2\left(x^2-x\right)^{3/2}}{9x^{3/2}}+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\sqrt{x}}{12}-\frac{\tan^{-1}\left(2\sqrt{\right.}}{24\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```


```
[Out] Sqrt[x]/12 - (2*x^(3/2))/9 - (11*Sqrt[-x + x^2])/(12*Sqrt[x]) - (2*(-x + x^
2)^(3/2))/(9*x^(3/2)) + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])
/(24*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(24*Sqrt[2])
 + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```


Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
```

```
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{16}{3}\int \frac{x^{3/2}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,d$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\operatorname{Subst}\left(\int \frac{x^4}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\right.$$
$$= \frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)+\frac{32}{3}\operatorname{Subst}\left(\int\left(\frac{1}{128}-\frac{x^2}{16}-\frac{1}{128\left(1+8x^2\right)}\right.\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}+\frac{2}{3}x^{3/2}\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1+8x^2}\,dx,x,\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{8\sqrt{-x+x^2}}{9\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+\frac{2}{3}x^{3/2}\log\left(-1\right.$$
$$= \frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2\left(-x+x^2\right)^{3/2}}{9x^{3/2}}+\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{24\sqrt{2}\sqrt{-1+x}\sqrt{x}}$$

Mathematica [C] time = 0.640654, size = 209, normalized size = 1.32

$$\frac{1}{576}\left(-128x^{3/2}+384x^{3/2}\log\left(4x+4\sqrt{(x-1)x}-1\right)-128\sqrt{(x-1)x}\sqrt{x}+48\sqrt{x}-\frac{400\sqrt{(x-1)x}}{\sqrt{x}}+6i\sqrt{2}\log\left(4(8x+1)^2\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (48*Sqrt[x] - 128*x^(3/2) - (400*Sqrt[(-1 + x)*x])/Sqrt[x] - 128*Sqrt[x]*Sq
rt[(-1 + x)*x] - 12*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 12*Sqrt[2]*ArcTan[(
2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (6*I)*Sqrt[2]*Log[4*(1 + 8*x)^2]
```

```
 - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 384*x^(3/
2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10
*x + 6*Sqrt[(-1 + x)*x])])/576
```

Maple [F] time = 0.007, size = 0, normalized size = 0.

$$\int \sqrt{x}\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3}x^{\frac{3}{2}}\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)-\frac{4}{9}x^{\frac{3}{2}}-\frac{2}{3}\sqrt{x}+\int\frac{2x^2+x}{3\left(4x^{\frac{5}{2}}+4\left(x^2-x\right)\sqrt{x-1}-5x^{\frac{3}{2}}+\sqrt{x}\right)}\,dx+\frac{1}{3}\log\left(\sqrt{x}+1\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4/9*x^(3/2) - 2/3*sqrt(x
) + integrate(1/3*(2*x^2 + x)/(4*x^(5/2) + 4*(x^2 - x)*sqrt(x - 1) - 5*x^(3
/2) + sqrt(x)), x) + 1/3*log(sqrt(x) + 1) - 1/3*log(sqrt(x) - 1)
```

Fricas [A] time = 2.34709, size = 286, normalized size = 1.81

$$\frac{96x^{\frac{5}{2}}\log\left(4x+4\sqrt{x^2-x}-1\right)-3\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)-3\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4\sqrt{x^2-x}(8x+25)\sqrt{x}-4\left(8x^2\right.}{144x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/144*(96*x^(5/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 3*sqrt(2)*x*arctan(2*sqr
t(2)*sqrt(x)) - 3*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 4*s
qrt(x^2 - x)*(8*x + 25)*sqrt(x) - 4*(8*x^2 - 3*x)*sqrt(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

3.110 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=118

$$-\frac{2\sqrt{x^2-x}}{\sqrt{x}}+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}-2\sqrt{x}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}$$

```
[Out] -2*Sqrt[x] - (2*Sqrt[-x + x^2])/Sqrt[x] - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]
*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x
]]/Sqrt[2] + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Rubi [A] time = 0.385141, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 1146, 444, 50, 63}

$$-\frac{2\sqrt{x^2-x}}{\sqrt{x}}+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}-2\sqrt{x}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - (2*Sqrt[-x + x^2])/Sqrt[x] - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]
*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x
]]/Sqrt[2] + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
```

```
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
```

```
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 16\int \frac{\sqrt{x}}{-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)}\,dx$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 32\,\mathrm{Subst}\left(\int \frac{x^2}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2\right.}\right.$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 32\,\mathrm{Subst}\left(\int\left(-\frac{1}{16}+\frac{1}{16\left(1+8x^2\right)}-\frac{x^2}{12\sqrt{-x^2+}}\right.\right.$$

$$= -2\sqrt{x} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\,\mathrm{Subst}\left(\int \frac{1}{1+8x^2}\,dx, x, \sqrt{x}\right) - \frac{8}{3}\,\mathrm{Su}$$

$$= -2\sqrt{x} - \frac{8\sqrt{-x+x^2}}{3\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{\left(16\sqrt{\right.}}{}$$

$$= -2\sqrt{x} - \frac{8\sqrt{-x+x^2}}{3\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{\left(8\sqrt{\right.}}{}$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{\left(3\sqrt{\right.}}{}$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{\left(6\sqrt{\right.}}{}$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} - \frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\,\mathrm{l}$$

Mathematica [C] time = 0.544953, size = 186, normalized size = 1.58

$$\frac{1}{8}\left(-16\sqrt{x} - \frac{16\sqrt{(x-1)x}}{\sqrt{x}} - 2i\sqrt{2}\log\left(4(8x+1)^2\right) + i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x-1)x}+1\right)\right) + 16\sqrt{x}\log\left(4x+4\sqrt{(x-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] (-16*Sqrt[x] - (16*Sqrt[(-1 + x)*x])/Sqrt[x] + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*S
qrt[x]] - 4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I
```

```
)*Sqrt[2]*Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(
-1 + x)*x])] + 16*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + I*Sqrt[2]*Lo
g[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])])/8
```

Maple [F] time = 0.009, size = 0, normalized size = 0.

$$\int \ln \left(-1+4 x+4 \sqrt{(-1+x) x}\right) \frac{1}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2 \sqrt{x} \log \left(4 \sqrt{x-1} \sqrt{x}+4 x-1\right)-4 \sqrt{x}+\int \frac{2 x^{2}+x}{4 x^{\frac{7}{2}}-5 x^{\frac{5}{2}}+4 \left(x^{3}-x^{2}\right) \sqrt{x-1}+x^{\frac{3}{2}}}\, dx+\log \left(\sqrt{x}+1\right)-\log \left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4*sqrt(x) + integrate((2*x
^2 + x)/(4*x^(7/2) - 5*x^(5/2) + 4*(x^3 - x^2)*sqrt(x - 1) + x^(3/2)), x) +
 log(sqrt(x) + 1) - log(sqrt(x) - 1)
```

Fricas [A] time = 2.18175, size = 243, normalized size = 2.06

$$\frac{\sqrt{2} x \arctan \left(2 \sqrt{2} \sqrt{x}\right)+\sqrt{2} x \arctan \left(\frac{3 \sqrt{2} \sqrt{x}}{4 \sqrt{x^{2}-x}}\right)+4 x^{\frac{3}{2}} \log \left(4 x+4 \sqrt{x^{2}-x}-1\right)-4 x^{\frac{3}{2}}-4 \sqrt{x^{2}-x} \sqrt{x}}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + sqrt(2)*x*arctan(3/4*sqrt(2)*sqr
t(x)/sqrt(x^2 - x)) + 4*x^(3/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 4*x^(3/2)
- 4*sqrt(x^2 - x)*sqrt(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] undef
```

3.111 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}-\frac{4\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}-8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)$$

```
[Out] (-4*Sqrt[2]*Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x
]*Sqrt[x]) + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 8*ArcTan[Sqrt[x]/Sqrt[-x
 + x^2]] - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Rubi [A] time = 0.308602, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2021, 2008, 1146, 444, 50, 63, 204}

$$-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}-\frac{4\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}-8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x
]*Sqrt[x]) + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 8*ArcTan[Sqrt[x]/Sqrt[-x
 + x^2]] - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
```

```
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
```

```
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] && !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{3/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 16\int \frac{1}{\sqrt{x}\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\,\text{Subst}\left(\int \frac{1}{-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-x^2+\right.}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\,\text{Subst}\left(\int\left(-\frac{1}{2\left(1+8x^2\right)} - \frac{x^2}{12\sqrt{-x^2+x^4}} + \right.\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{8}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - 8\,\text{Subst}\left(\int\right.$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + 8\,\text{Subst}\left(\right.$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 8\,\text{Subst}\left(\right.$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{(24}{}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{(48}{}$$

$$= -\frac{4\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x}}\right.$$

<u>Mathematica</u> [C] time = 0.43421, size = 177, normalized size = 1.55

$$-2i\sqrt{2}\log\left(4(8x+1)^2\right) + i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x-1)x}+1\right)\right) - \frac{2\log\left(4x+4\sqrt{(x-1)x}-1\right)}{\sqrt{x}} + i\sqrt{2}\log\left((8x+1)\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 8*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] -
4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I)*Sqrt[2]*
Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x]
)] - (2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/Sqrt[x] + I*Sqrt[2]*Log[(1 + 8*
x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])]
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)x^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)}{\sqrt{x}}-\frac{2}{\sqrt{x}}-\int\frac{2x^2+x}{4x^{\frac{9}{2}}-5x^{\frac{7}{2}}+x^{\frac{5}{2}}+4\left(x^4-x^3\right)\sqrt{x-1}}\,dx-\log\left(\sqrt{x}+1\right)+\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/sqrt(x) - 2/sqrt(x) - integrate((2*
x^2 + x)/(4*x^(9/2) - 5*x^(7/2) + x^(5/2) + 4*(x^4 - x^3)*sqrt(x - 1)), x)
- log(sqrt(x) + 1) + log(sqrt(x) - 1)
```

Fricas [A] time = 2.28961, size = 240, normalized size = 2.11

$$\frac{2\left(2\sqrt{2}x\arctan\left(2\sqrt{2}\sqrt{x}\right)+2\sqrt{2}x\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-4x\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\sqrt{x}\log\left(4x+4\sqrt{x^2-x}-1\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + 2*sqrt(2)*x*arctan(3/4*sqrt(2)*s
qrt(x)/sqrt(x^2 - x)) - 4*x*arctan(sqrt(x)/sqrt(x^2 - x)) - sqrt(x)*log(4*x
 + 4*sqrt(x^2 - x) - 1))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

3.112 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=151

$$\frac{4\sqrt{x^2-x}}{3x^{3/2}}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}+\frac{32\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}}+\frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\frac{16}{3\sqrt{x}}-\frac{32}{3}\sqrt{2}\tan$$

```
[Out] -16/(3*Sqrt[x]) + (4*Sqrt[-x + x^2])/(3*x^(3/2)) + (32*Sqrt[2]*Sqrt[-x + x^
2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(3*Sqrt[-1 + x]*Sqrt[x]) - (32*Sqrt[
2]*ArcTan[2*Sqrt[2]*Sqrt[x]])/3 + (44*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/3 - (
2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```

Rubi [A] time = 0.478361, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2537, 2535, 6733, 6742, 203, 1588, 2020, 2008, 2021, 1146, 444, 50, 63}

$$\frac{4\sqrt{x^2-x}}{3x^{3/2}}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}+\frac{32\sqrt{2}\sqrt{x^2-x}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}}+\frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\frac{16}{3\sqrt{x}}-\frac{32}{3}\sqrt{2}\tan$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```



```
[Out] -16/(3*Sqrt[x]) + (4*Sqrt[-x + x^2])/(3*x^(3/2)) + (32*Sqrt[2]*Sqrt[-x + x^
2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(3*Sqrt[-1 + x]*Sqrt[x]) - (32*Sqrt[
2]*ArcTan[2*Sqrt[2]*Sqrt[x]])/3 + (44*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/3 - (
2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```


Rule 2537

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 2535

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[
```

```
a + b*x + c*x^2]])/(g*(m + 1)), x] + Dist[(f^2*(b^2 - 4*a*c))/(2*g*(m + 1))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 1146

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{5/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{16}{3}\int \frac{1}{x^{3/2}\left(-4(1+2x)\sqrt{-x+x^2}+8\left(-x+x^2\right)\right)}\,d$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(-4\left(1+2x^2\right)\sqrt{-x^2+x^4}+8\left(-\right.\right.}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\,\mathrm{Subst}\left(\int\left(-\frac{1}{2x^2}+\frac{4}{1+8x^2}-\frac{x^2}{12\sqrt{-x^2+x^4}}\right.\right.$$

$$= -\frac{16}{3\sqrt{x}} - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{8}{9}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx, x, \sqrt{x}\right) - \frac{8}{3}\,\mathrm{S}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+\right.}{3x^{3/2}}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+\right.}{3x^{3/2}}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1\right.}{}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1\right.}{}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{32\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\right.$$

Mathematica [C] time = 0.604722, size = 204, normalized size = 1.35

$$\frac{2}{3}\left(\frac{2\sqrt{(x-1)x}}{x^{3/2}} - \frac{\log\left(4x+4\sqrt{(x-1)x}-1\right)}{x^{3/2}} - \frac{8}{\sqrt{x}} + 8i\sqrt{2}\log\left(4(8x+1)^2\right) - 4i\sqrt{2}\log\left((8x+1)\left(-10x-6\sqrt{(x-1)x}+1\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] (2*(-8/Sqrt[x] + (2*Sqrt[(-1 + x)*x])/x^(3/2) - 16*Sqrt[2]*ArcTan[2*Sqrt[2]
*Sqrt[x]] - 22*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] + 16*Sqrt[2]*ArcTan[(2*Sqrt
[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (8*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] - (4*
I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] - Log[-1 + 4*x +
4*Sqrt[(-1 + x)*x]]/x^(3/2) - (4*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqr
t[(-1 + x)*x])]))/3
```

Maple [F] time = 0.006, size = 0, normalized size = 0.

$$\int \ln\left(-1+4x+4\sqrt{(-1+x)x}\right) x^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{3\sqrt{x}} - \frac{2\log\left(4\sqrt{x-1}\sqrt{x}+4x-1\right)}{3x^{\frac{3}{2}}} - \frac{2}{9x^{\frac{3}{2}}} - \int \frac{2x^2+x}{3\left(4x^{\frac{11}{2}}-5x^{\frac{9}{2}}+x^{\frac{7}{2}}+4\left(x^5-x^4\right)\sqrt{x-1}\right)}\,dx - \frac{1}{3}\log\left(\sqrt{x}+1\right) + \frac{1}{3}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/sqrt(x) - 2/3*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/x^(3/2) - 2/9/x^(3/2
) - integrate(1/3*(2*x^2 + x)/(4*x^(11/2) - 5*x^(9/2) + x^(7/2) + 4*(x^5 -
x^4)*sqrt(x - 1)), x) - 1/3*log(sqrt(x) + 1) + 1/3*log(sqrt(x) - 1)
```

Fricas [A] time = 2.19043, size = 311, normalized size = 2.06

$$-\frac{2\left(16\sqrt{2}x^2\arctan\left(2\sqrt{2}\sqrt{x}\right)+16\sqrt{2}x^2\arctan\left(\frac{3\sqrt{2}\sqrt{x}}{4\sqrt{x^2-x}}\right)-22x^2\arctan\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+8x^{\frac{3}{2}}+\sqrt{x}\log\left(4x+4\sqrt{x^2-x}-1\right.\right.}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*sqrt(2)*x^2*arctan(2*sqrt(2)*sqrt(x)) + 16*sqrt(2)*x^2*arctan(3/4*
sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 22*x^2*arctan(sqrt(x)/sqrt(x^2 - x)) + 8*x
^(3/2) + sqrt(x)*log(4*x + 4*sqrt(x^2 - x) - 1) - 2*sqrt(x^2 - x)*sqrt(x))/
x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] undef
```

3.113 $\int x^3 \log(a + be^x)\,dx$

Optimal. Leaf size=93

$$-x^3\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-6x\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\text{PolyLog}\left(5,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log(a+be^x)-$$

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Rubi [A] time = 0.0730538, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2532, 2531, 6609, 2282, 6589}

$$-x^3\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-6x\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\text{PolyLog}\left(5,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log(a+be^x)-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \log(a+be^x)\,dx = \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) + \int x^3\log\left(1+\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3\int x^2\text{Li}_2\left(-\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6\int x\text{Li}_3\left(-\frac{be^x}{a}\right)dx$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\int \text{L}$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\,\text{Sub}$$

$$= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\text{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\text{Li}_3\left(-\frac{be^x}{a}\right) - 6x\text{Li}_4\left(-\frac{be^x}{a}\right) + 6\text{Li}_5\left(\right.$$

Mathematica [A] time = 0.0071216, size = 93, normalized size = 1.

$$-x^3\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-6x\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\text{PolyLog}\left(5,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log\left(a+be^x\right)-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Maple [A] time = 0.007, size = 84, normalized size = 0.9

$$\frac{x^4\ln\left(a+b\mathrm{e}^x\right)}{4}-\frac{x^4}{4}\ln\left(1+\frac{b\mathrm{e}^x}{a}\right)-x^3\text{polylog}\left(2,-\frac{b\mathrm{e}^x}{a}\right)+3\,x^2\text{polylog}\left(3,-\frac{b\mathrm{e}^x}{a}\right)-6\,x\text{polylog}\left(4,-\frac{b\mathrm{e}^x}{a}\right)+6\,\text{polylog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(a+b*exp(x)),x)
```

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Maxima [A] time = 1.20719, size = 111, normalized size = 1.19

$$\frac{1}{4}\,x^4\log\left(be^x+a\right)-\frac{1}{4}\,x^4\log\left(\frac{be^x}{a}+1\right)-x^3\text{Li}_2\left(-\frac{be^x}{a}\right)+3\,x^2\text{Li}_3(-\frac{be^x}{a})-6\,x\text{Li}_4(-\frac{be^x}{a})+6\,\text{Li}_5(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log(b*e^x/a + 1) - x^3*dilog(-b*e^x/a) + 3
*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylog(5, -b*e^x/
a)
```

Fricas [C] time = 2.04759, size = 224, normalized size = 2.41

$$\frac{1}{4}x^4\log\left(be^x+a\right)-\frac{1}{4}x^4\log\left(\frac{be^x+a}{a}\right)-x^3\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+3x^2\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)-6x\mathrm{polylog}\left(4,-\frac{be^x}{a}\right)+6\,\mathrm{polyl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log((b*e^x + a)/a) - x^3*dilog(-(b*e^x + a
)/a + 1) + 3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylo
g(5, -b*e^x/a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\int\frac{x^4e^x}{a+be^x}\,dx}{4}+\frac{x^4\log\left(a+be^x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**4*exp(x)/(a + b*exp(x)), x)/4 + x**4*log(a + b*exp(x))/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\log\left(be^x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(b*e^x + a), x)
```

3.114 $\int x^2 \log(a + be^x)\, dx$

Optimal. Leaf size=77

$$-x^2\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+2x\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+\frac{1}{3}x^3\log\left(a+be^x\right)-\frac{1}{3}x^3\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rubi [A] time = 0.0584256, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2532, 2531, 6609, 2282, 6589}

$$-x^2\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+2x\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+\frac{1}{3}x^3\log\left(a+be^x\right)-\frac{1}{3}x^3\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log \left(a+be^x\right) dx = \frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(1+\frac{be^x}{a}\right)+\int x^2 \log \left(1+\frac{be^x}{a}\right) dx$$

$$= \frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(1+\frac{be^x}{a}\right)-x^2\text{Li}_2\left(-\frac{be^x}{a}\right)+2\int x\text{Li}_2\left(-\frac{be^x}{a}\right) dx$$

$$= \frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(1+\frac{be^x}{a}\right)-x^2\text{Li}_2\left(-\frac{be^x}{a}\right)+2x\text{Li}_3\left(-\frac{be^x}{a}\right)-2\int \text{Li}_3\left(-\frac{be^x}{a}\right) dx$$

$$= \frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(1+\frac{be^x}{a}\right)-x^2\text{Li}_2\left(-\frac{be^x}{a}\right)+2x\text{Li}_3\left(-\frac{be^x}{a}\right)-2\,\text{Subst}\left(\int \frac{\text{Li}_3\left(-\frac{bx}{a}\right)}{x}\right.$$

$$= \frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(1+\frac{be^x}{a}\right)-x^2\text{Li}_2\left(-\frac{be^x}{a}\right)+2x\text{Li}_3\left(-\frac{be^x}{a}\right)-2\text{Li}_4\left(-\frac{be^x}{a}\right)$$

Mathematica [A] time = 0.0052255, size = 77, normalized size = 1.

$$-x^2\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+2x\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+\frac{1}{3}x^3 \log \left(a+be^x\right)-\frac{1}{3}x^3 \log \left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Maple [A] time = 0.006, size = 69, normalized size = 0.9

$$\frac{x^3 \ln(a + be^x)}{3} - \frac{x^3}{3} \ln\left(1 + \frac{be^x}{a}\right) - x^2 \mathrm{polylog}\left(2, -\frac{be^x}{a}\right) + 2\,x\mathrm{polylog}\left(3, -\frac{be^x}{a}\right) - 2\,\mathrm{polylog}\left(4, -\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a+b*exp(x)),x)
```

```
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Maxima [A] time = 1.12855, size = 90, normalized size = 1.17

$$\frac{1}{3}\,x^3 \log(be^x + a) - \frac{1}{3}\,x^3 \log\left(\frac{be^x}{a} + 1\right) - x^2 \mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 2\,x\mathrm{Li}_3(-\frac{be^x}{a}) - 2\,\mathrm{Li}_4(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log(b*e^x/a + 1) - x^2*dilog(-b*e^x/a) + 2
*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Fricas [C] time = 2.05659, size = 185, normalized size = 2.4

$$\frac{1}{3}\,x^3 \log(be^x + a) - \frac{1}{3}\,x^3 \log\left(\frac{be^x + a}{a}\right) - x^2 \mathrm{Li}_2\left(-\frac{be^x + a}{a} + 1\right) + 2\,x\mathrm{polylog}\left(3, -\frac{be^x}{a}\right) - 2\,\mathrm{polylog}\left(4, -\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log((b*e^x + a)/a) - x^2*dilog(-(b*e^x + a
)/a + 1) + 2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\int\frac{x^3e^x}{a+be^x}\,dx}{3}+\frac{x^3\log\left(a+be^x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**3*exp(x)/(a + b*exp(x)), x)/3 + x**3*log(a + b*exp(x))/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\log\left(be^x+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(b*e^x + a), x)
```

3.115 $\int x \log(a + be^x) \, dx$

Optimal. Leaf size=59

$$-x\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+\text{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rubi [A] time = 0.036611, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2532, 2531, 2282, 6589}

$$-x\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+\text{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\log(a+be^x)\,dx &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)+\int x\log\left(1+\frac{be^x}{a}\right)dx\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\int \text{Li}_2\left(-\frac{be^x}{a}\right)dx\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}dx,x,e^x\right)\\ &= \frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0045427, size = 59, normalized size = 1.

$$-x\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+\text{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log(a+be^x)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Maple [A] time = 0.007, size = 52, normalized size = 0.9

$$\frac{x^2\ln(a+be^x)}{2}-\frac{x^2}{2}\ln\left(1+\frac{be^x}{a}\right)-x\text{polylog}\left(2,-\frac{be^x}{a}\right)+\text{polylog}\left(3,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a+b*exp(x)),x)
```

```
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Maxima [A] time = 1.12416, size = 68, normalized size = 1.15

$$\frac{1}{2}x^2\log\left(be^x+a\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Li}_3(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log(b*e^x/a + 1) - x*dilog(-b*e^x/a) + pol
ylog(3, -b*e^x/a)
```

Fricas [C] time = 2.13465, size = 143, normalized size = 2.42

$$\frac{1}{2}x^2\log\left(be^x+a\right)-\frac{1}{2}x^2\log\left(\frac{be^x+a}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log((b*e^x + a)/a) - x*dilog(-(b*e^x + a)/
a + 1) + polylog(3, -b*e^x/a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\int\frac{x^2e^x}{a+be^x}\,dx}{2}+\frac{x^2\log\left(a+be^x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**2*exp(x)/(a + b*exp(x)), x)/2 + x**2*log(a + b*exp(x))/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \log \left(be^{x} + a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(b*e^x + a), x)
```

3.116 $\int \log(a+be^x)\,dx$

Optimal. Leaf size=38

$$-\mathrm{PolyLog}\left(2,-\frac{be^x}{a}\right)+x\log(a+be^x)-x\log\left(\frac{be^x}{a}+1\right)$$

```
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rubi [A] time = 0.0482474, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2280, 2190, 2279, 2391}

$$-\mathrm{PolyLog}\left(2,-\frac{be^x}{a}\right)+x\log(a+be^x)-x\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*E^x],x]
```

```
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rule 2280

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[(x*
(F^(e*(c + d*x)))^n)/(a + b*(F^(e*(c + d*x)))^n), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a+be^x)\,dx &= x\log(a+be^x) - b\int \frac{e^x x}{a+be^x}\,dx \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \int \log\left(1+\frac{be^x}{a}\right)dx \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^x\right) \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{Li}_2\left(-\frac{be^x}{a}\right)
\end{aligned}$$

Mathematica [A] time = 0.0022612, size = 38, normalized size = 1.

$$-\text{PolyLog}\left(2,-\frac{be^x}{a}\right) + x\log(a+be^x) - x\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*E^x],x]
```

```
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Maple [A] time = 0.013, size = 28, normalized size = 0.7

$$\text{dilog}\left(-\frac{be^x}{a}\right) + \ln(a+be^x)\ln\left(-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*exp(x)),x)
```

```
[Out] dilog(-b*exp(x)/a)+ln(a+b*exp(x))*ln(-b*exp(x)/a)
```

Maxima [A] time = 1.08078, size = 46, normalized size = 1.21

$$\log(be^x + a)\log\left(-\frac{be^x + a}{a} + 1\right) + \text{Li}_2\left(\frac{be^x + a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] log(b*e^x + a)*log(-(b*e^x + a)/a + 1) + dilog((b*e^x + a)/a)
```

Fricas [A] time = 2.09071, size = 93, normalized size = 2.45

$$x\log(be^x + a) - x\log\left(\frac{be^x + a}{a}\right) - \text{Li}_2\left(-\frac{be^x + a}{a} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] x*log(b*e^x + a) - x*log((b*e^x + a)/a) - dilog(-(b*e^x + a)/a + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-b\int \frac{xe^x}{a + be^x}\,dx + x\log(a + be^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x*exp(x)/(a + b*exp(x)), x) + x*log(a + b*exp(x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(be^{x}+a\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(log(b*e^x + a), x)
```

3.117 $\int \frac{\log(a+be^x)}{x} \, dx$

Optimal. Leaf size=14

$$\text{CannotIntegrate}\left(\frac{\log(a+be^x)}{x},x\right)$$

```
[Out] CannotIntegrate[Log[a + b*E^x]/x, x]
```

Rubi [A] time = 0.0320759, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\log(a+be^x)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[a + b*E^x]/x,x]
```

```
[Out] Defer[Int][Log[a + b*E^x]/x, x]
```

Rubi steps

$$\int \frac{\log(a+be^x)}{x} \, dx = \int \frac{\log(a+be^x)}{x} \, dx$$

Mathematica [A] time = 0.0514136, size = 0, normalized size = 0.

$$\int \frac{\log(a+be^x)}{x} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[a + b*E^x]/x,x]
```

```
[Out] Integrate[Log[a + b*E^x]/x, x]
```

Maple [A] time = 0.026, size = 0, normalized size = 0.

$$\int \frac{\ln \left(a+b \mathrm{e}^{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*exp(x))/x,x)
```

```
[Out] int(ln(a+b*exp(x))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(b e^{x}+a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(b*e^x + a)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log \left(b e^{x}+a\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(b*e^x + a)/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(a+b e^{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x))/x,x)
```

```
[Out] Integral(log(a + b*exp(x))/x, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log \left(be^{x} + a\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(b*e^x + a)/x, x)
```

3.118 $\int x^3 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=132

$$\frac{3x^2\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{x^3\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Rubi [A] time = 0.0969669, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2531, 6609, 2282, 6589}

$$\frac{3x^2\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{x^3\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```


```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```


Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx &= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3\int x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6\int x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{b^2c^2n^2\log^2(f)}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\int \text{Li}_4\left(\right.}{b^3}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\,\text{Subst}}{}\\
&= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Li}_5\left(-\right.}{b^4c^4n}
\end{aligned}
$$

Mathematica [A] time = 0.0133012, size = 132, normalized size = 1.

$$
\frac{3x^2\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{x^3\text{PolyLog}\left(2,-e\left(\right.\right.}{bcn\log(f)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [B] time = 0.069, size = 645, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(1+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n)+3/c^2/b^2/ln(f)^2/n*dilog(1+e*f^(b*c*n*x)
*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))*x^2-3/c^3/b^3/l
n(f)^3/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(
f^(c*(b*x+a)))^2*x-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(
f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)^3/n*pol
ylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x
+a)))^2*x-1/c/b/ln(f)/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(
b*x+a))))))*x^3+1/c^4/b^4/ln(f)^4/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c
*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^3-1/c^4/b^4/ln(f)^4/n*polylog(2,-
e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^3+
3/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*
(b*x+a))))))*x^2-6/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x)*exp(-n*(ln(
f)*b*c*x-ln(f^(c*(b*x+a))))))*x-1/4*ln(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-
ln(f^(c*(b*x+a))))))*x^4+6/c^4/b^4/ln(f)^4/n^4*polylog(5,-e*f^(b*c*n*x)*exp
(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20}\,bcnx^5\log\left(f\right)+\frac{1}{4}\,x^4\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)+bcn\int\frac{x^4}{4\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)}\,dx\log\left(f\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/20*b*c*n*x^5*log(f) + 1/4*x^4*log(e*(f^(b*c*x))^n*(f^(a*c))^n + 1) + b*c
*n*integrate(1/4*x^4/(e*(f^(b*c*x))^n*(f^(a*c))^n + 1), x)*log(f)
```

Fricas [C] time = 2.19526, size = 323, normalized size = 2.45

$$-\frac{b^3c^3n^3x^3\text{Li}_2\left(-ef^{bcnx+acn}\right)\log\left(f\right)^3 - 3\,b^2c^2n^2x^2\log\left(f\right)^2\text{polylog}\left(3,-ef^{bcnx+acn}\right) + 6\,bcnx\log\left(f\right)\text{polylog}\left(4,-ef^{bcnx}\right.}{b^4c^4n^4\log\left(f\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^3*c^3*n^3*x^3*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^3 - 3*b^2*c^2*n^2*x^
2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 6*b*c*n*x*log(f)*polylog(4,
 -e*f^(b*c*n*x + a*c*n)) - 6*polylog(5, -e*f^(b*c*n*x + a*c*n)))/(b^4*c^4*n
^4*log(f)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^4e^{bcnx\log\left(f\right)}}{ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}+1}\,dx}{4}+\frac{x^4\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/4 + x**4*log(e*(f**(c*(a + b*x))
)**n + 1)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + 1), x)
```

3.119 $\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=98

$$\frac{2x\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rubi [A] time = 0.0615725, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2531, 6609, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int x^2 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2\int x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\int \text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{b^2c^2n^2\log^2(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\,\text{Subst}\left(\int \frac{\text{Li}_3(-ex^n)}{x}dx,x,f^{c(a+bx)}\right)}{b^3c^3n^2\log^3(f)}\\
&= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}
\end{aligned}
$$

Mathematica [A] time = 0.0054603, size = 98, normalized size = 1.

$$
\frac{2x\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] time = 0.049, size = 462, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(1+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n)-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c
*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))-2/c^2/b^2/ln(f)^2/n*polylog(
2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))
*x+1/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c
*(b*x+a))))))*ln(f^(c*(b*x+a)))^2-1/c/b/ln(f)/n*dilog(1+e*f^(b*c*n*x)*exp(-
n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*x^2+2/c^2/b^2/ln(f)^2/n*dilog(1+e*f^(b*
c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))*x-1/c^3/b
^3/ln(f)^3/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))
*ln(f^(c*(b*x+a)))^2+2/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*
(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*x-1/3*ln(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*
c*x-ln(f^(c*(b*x+a))))))*x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\,bcnx^4\log\left(f\right)+bcn\int\frac{x^3}{3\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)}\,dx\log\left(f\right)+\frac{1}{3}\,x^3\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/12*b*c*n*x^4*log(f) + b*c*n*integrate(1/3*x^3/(e*(f^(b*c*x))^n*(f^(a*c))
^n + 1), x)*log(f) + 1/3*x^3*log(e*(f^(b*c*x))^n*(f^(a*c))^n + 1)
```

Fricas [C] time = 2.15546, size = 236, normalized size = 2.41

$$-\frac{b^2c^2n^2x^2\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log\left(f\right)^2-2\,bcnx\log\left(f\right)\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right)+2\,\mathrm{polylog}\left(4,-ef^{bcnx+acn}\right)}{b^3c^3n^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*n^2*x^2*dilog(-e*f^(b*c*n*x + a*c*n))*log(f)^2 - 2*b*c*n*x*log(f)
*polylog(3, -e*f^(b*c*n*x + a*c*n)) + 2*polylog(4, -e*f^(b*c*n*x + a*c*n)))
/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^3e^{bcnx\log\left(f\right)}}{ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}+1}\,dx}{3}+\frac{x^3\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/3 + x**3*log(e*(f**(c*(a + b*x))
)**n + 1)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + 1), x)
```

3.120 $\int x \log \left(1+e \left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=63

$$\frac{\text{PolyLog}\left(3,-e \left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-e \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rubi [A] time = 0.0382002, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-e \left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-e \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\int \text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-ex^n)}{x}dx,x,f^{c(a+bx)}\right)}{b^2c^2n\log^2(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\end{aligned}$$

Mathematica [A] time = 0.0047085, size = 63, normalized size = 1.

$$\frac{\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [B] time = 0.043, size = 282, normalized size = 4.5

$$\frac{x^2\ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{2}-\frac{\ln\left(1+ef^{bcnx}\mathrm{e}^{-n\left(\ln(f)bcx-\ln\left(f^{c(bx+a)}\right)\right)}\right)x^2}{2}-\frac{\text{polylog}\left(2,-ef^{bcnx}\mathrm{e}^{-n\left(\ln(f)bcx-\ln\left(f^{c(bx+a)}\right)\right)}\right)\ln\left(f^{c(bx+a)}\right.}{c^2b^2\left(\ln(f)\right)^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(1+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n)-1/2*ln(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*
x-ln(f^(c*(b*x+a))))))*x^2-1/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*exp
(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))+1/c^2/b^2/ln(f)^2/n
^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))-1/c/b/
ln(f)/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*x+1/
c^2/b^2/ln(f)^2/n*dilog(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)
)))))*ln(f^(c*(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\,bcnx^3\log\left(f\right)+bcn\int\frac{x^2}{2\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)}\,dx\log\left(f\right)+\frac{1}{2}\,x^2\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*n*x^3*log(f) + b*c*n*integrate(1/2*x^2/(e*(f^(b*c*x))^n*(f^(a*c))^
n + 1), x)*log(f) + 1/2*x^2*log(e*(f^(b*c*x))^n*(f^(a*c))^n + 1)
```

Fricas [C] time = 2.21496, size = 147, normalized size = 2.33

$$-\frac{bcnx\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)\log\left(f\right)-\mathrm{polylog}\left(3,-ef^{bcnx+acn}\right)}{b^2c^2n^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b*c*n*x*dilog(-e*f^(b*c*n*x + a*c*n))*log(f) - polylog(3, -e*f^(b*c*n*x +
 a*c*n)))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcne e^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^2e^{bcnx\log\left(f\right)}}{ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}+1}\,dx}{2}+\frac{x^2\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/2 + x**2*log(e*(f**(c*(a + b*x))
)**n + 1)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(e\left(f^{(bx+a)c}\right)^{n}+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + 1), x)
```

3.121 $\int \log \left(1+e \left(f^{c (a+b x)}\right)^n\right) \, dx$

Optimal. Leaf size=31

$$-\frac{\text{PolyLog}\left(2,-e \left(f^{c (a+b x)}\right)^n\right)}{b c n \log (f)}$$

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rubi [A] time = 0.0145746, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-e \left(f^{c (a+b x)}\right)^n\right)}{b c n \log (f)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx = \frac{\text{Subst}\left(\int \frac{\log(1+ex)}{x}\,dx, x, \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$
$$= -\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Mathematica [A] time = 0.0012386, size = 31, normalized size = 1.

$$-\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Maple [A] time = 0.007, size = 32, normalized size = 1.

$$-\frac{\text{dilog}\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{ncb\ln\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] -1/c/b/ln(f)/n*dilog(1+e*(f^(c*(b*x+a)))^n)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,bcnx^2\log\left(f\right)+bcn\int\frac{x}{e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1}\,dx\log\left(f\right)+x\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*n*x^2*log(f) + b*c*n*integrate(x/(e*(f^(b*c*x))^n*(f^(a*c))^n + 1)
, x)*log(f) + x*log(e*(f^(b*c*x))^n*(f^(a*c))^n + 1)
```

Fricas [A] time = 2.19991, size = 63, normalized size = 2.03

$$-\frac{\mathrm{Li}_2\left(-ef^{bcnx+acn}\right)}{bcn\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -dilog(-e*f^(b*c*n*x + a*c*n))/(b*c*n*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{xe^{bcnx\log\left(f\right)}}{ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}+1}\,dx+x\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(e*exp(a*c
*n*log(f))*exp(b*c*n*x*log(f)) + 1), x) + x*log(e*(f**(c*(a + b*x)))**n + 1
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(e\left(f^{(bx+a)c}\right)^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1), x)
```

3.122 $\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{x},x\right)$$

```
[Out] CannotIntegrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi [A] time = 0.0641357, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]

[Out] Defer[Int][Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [A] time = 0.295591, size = 0, normalized size = 0.

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
```

```
[Out] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [A] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{\ln\left(1 + e\left(f^{c(bx+a)}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

```
[Out] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n + 1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(e\left(f^{bcx+ac}\right)^n + 1\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log(e*(f^(b*c*x + a*c))^n + 1)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n + 1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + 1)/x, x)
```

3.123 $\int x^3 \log \left(d+e \left(f^{c(a+b x)}\right)^n\right) \, dx$

Optimal. Leaf size=193

$$\frac{3 x^2 \text{PolyLog}\left(3,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^2 c^2 n^2 \log^2(f)}-\frac{6 x \text{PolyLog}\left(4,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^3 c^3 n^3 \log^3(f)}+\frac{6 \text{PolyLog}\left(5,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^4 c^4 n^4 \log^4(f)}-\frac{x^3 \text{PolyLog}\left(2,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b c n \log(f)}$$

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Rubi [A] time = 0.129465, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2532, 2531, 6609, 2282, 6589}

$$\frac{3 x^2 \text{PolyLog}\left(3,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^2 c^2 n^2 \log^2(f)}-\frac{6 x \text{PolyLog}\left(4,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^3 c^3 n^3 \log^3(f)}+\frac{6 \text{PolyLog}\left(5,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b^4 c^4 n^4 \log^4(f)}-\frac{x^3 \text{PolyLog}\left(2,-\frac{e \left(f^{c(a+b x)}\right)^n}{d}\right)}{b c n \log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```


```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```


Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int x^3 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) d$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{3\int x^2}{}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{3x^2\mathrm{Li}_3}{b^2c}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{3x^2\mathrm{Li}_3}{b^2c}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{3x^2\mathrm{Li}_3}{b^2c}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{3x^2\mathrm{Li}_3}{b^2c}$$

Mathematica [A] time = 0.0087122, size = 193, normalized size = 1.

$$\frac{3x^2\mathrm{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{6x\mathrm{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)} + \frac{6\mathrm{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)} - \frac{x^3\mathrm{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [B] time = 0.101, size = 1364, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d+e*(f^(c*(b*x+a)))^n),x)

[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-3/2/c^2/b^2/ln(f)^2*ln(1+e*f^(b*c*n*x)*ex
p(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*x^2*ln(f^(c*(b*x+a)))^2+2/c^3/b^3/
ln(f)^3*ln(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*x*ln(
f^(c*(b*x+a)))^3-1/c/b/ln(f)*ln((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(
c*(b*x+a))))))/d)*x^3*ln(f^(c*(b*x+a)))+3/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*
x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*x^2*ln(f^(c*(b*x+a)))^2-3/c^
3/b^3/ln(f)^3*ln((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/
d)*x*ln(f^(c*(b*x+a)))^3-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*exp(-
n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)
^3/n*polylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln
(f^(c*(b*x+a)))^2*x+3/c^2/b^2/ln(f)^2/n*dilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f
)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))*x^2-3/c^3/b^3/ln(f)^3/n*d
ilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(
b*x+a)))^2*x+1/c/b/ln(f)*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*
x+a))))))*ln(f^(c*(b*x+a)))*x^3-3/2/c^2/b^2/ln(f)^2*ln(d+e*f^(b*c*n*x)*exp(
-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^2*x^2+1/c^3/b^3/ln(f
)^3*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*
x+a)))^3*x+3/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c
*x-ln(f^(c*(b*x+a)))))/d)*x^2-6/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x
)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*x-1/c^4/b^4/ln(f)^4/n*polylog(
2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)
))^3-1/c/b/ln(f)/n*dilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+
a))))))/d)*x^3+1/c^4/b^4/ln(f)^4/n*dilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c
*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))^3+1/c^4/b^4/ln(f)^4*ln((d+e*f^
(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))^4-1
/4*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*x^4-1/4/c^4/
b^4/ln(f)^4*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(
f^(c*(b*x+a)))^4-3/4/c^4/b^4/ln(f)^4*ln(1+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x
-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)))^4+6/c^4/b^4/ln(f)^4/n^4*polylog(5
,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20}\,bcnx^5\log\left(f\right)+bcdn\int\frac{x^4}{4\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+d\right)}\,dx\log\left(f\right)+\frac{1}{4}\,x^4\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/20*b*c*n*x^5*log(f) + b*c*d*n*integrate(1/4*x^4/(e*(f^(b*c*x))^n*(f^(a*c
))^n + d), x)*log(f) + 1/4*x^4*log(e*(f^(b*c*x))^n*(f^(a*c))^n + d)
```

Fricas [C] time = 2.21145, size = 564, normalized size = 2.92

$$-\frac{4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)^3-12\,b^2c^2n^2x^2\log\left(f\right)^2\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-\left(b^4c^4n^4x^4-a^4c^4n^4\right)\log\left(ef^{bc\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*c^3*n^3*x^3*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^3
- 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^4*c
^4*n^4*x^4 - a^4*c^4*n^4)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^4 + (b^4*c^
4*n^4*x^4 - a^4*c^4*n^4)*log(f)^4*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 24*b
*c*n*x*log(f)*polylog(4, -e*f^(b*c*n*x + a*c*n)/d) - 24*polylog(5, -e*f^(b*
c*n*x + a*c*n)/d))/(b^4*c^4*n^4*log(f)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^4e^{bcnx\log\left(f\right)}}{d+ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}}\,dx}{4}+\frac{x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/4 + x**4*log(d + e*(f**(c*(a + b
*x)))**n)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \log \left(e \left(f^{(bx+a)c}\right)^n + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log(e*(f^((b*x + a)*c))^n + d), x)
```

3.124 $\int x^2 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=156

$$\frac{2x\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-$$

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rubi [A] time = 0.0930105, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2532, 2531, 6609, 2282, 6589}

$$\frac{2x\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx = \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)d$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2\int x\text{L}}{}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3}{b^2c^2}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3}{b^2c^2}$$

$$=\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3}{b^2c^2}$$

Mathematica [A] time = 0.0066497, size = 156, normalized size = 1.

$$\frac{2x\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}-\frac{x^2\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] time = 0.079, size = 980, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/c^2/b^2/ln(f)^2*ln(1+e*f^(b*c*n*x)*exp(
-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*x*ln(f^(c*(b*x+a)))^2-1/c/b/ln(f)*ln
((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*x^2*ln(f^(c*(
b*x+a)))+2/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*
(b*x+a))))))/d)*x*ln(f^(c*(b*x+a)))^2-2/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b
*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)))*x+1/c/
b/ln(f)*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c
*(b*x+a)))*x^2-1/c^2/b^2/ln(f)^2*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(
f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^2*x+2/c^2/b^2/ln(f)^2/n*dilog((d+e*f^(b
*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))*x+2/c
^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*
x+a)))))/d)*x-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x)*exp(-n*(ln(f)*
b*c*x-ln(f^(c*(b*x+a)))))/d)-1/c^3/b^3/ln(f)^3*ln((d+e*f^(b*c*n*x)*exp(-n*(
ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))^3-1/3*ln(d+e*f^(b*c*n
*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*x^3+1/3/c^3/b^3/ln(f)^3*ln(d+e
*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^3-1
/c/b/ln(f)/n*dilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))
)/d)*x^2-1/c^3/b^3/ln(f)^3/n*dilog((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(
f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))^2+2/3/c^3/b^3/ln(f)^3*ln(1+e*f^(b*c*
n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)))^3+1/c^3/b
^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))
))/d)*ln(f^(c*(b*x+a)))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\,bcnx^4\log\left(f\right)+bcdn\int\frac{x^3}{3\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+d\right)}\,dx\log\left(f\right)+\frac{1}{3}\,x^3\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/12*b*c*n*x^4*log(f) + b*c*d*n*integrate(1/3*x^3/(e*(f^(b*c*x))^n*(f^(a*c
))^n + d), x)*log(f) + 1/3*x^3*log(e*(f^(b*c*x))^n*(f^(a*c))^n + d)
```

Fricas [C] time = 2.19593, size = 471, normalized size = 3.02

$$-\frac{3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)^2-6\,bcnx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{ef^{bcnx+acn}}{d}\right)-\left(b^3c^3n^3x^3+a^3c^3n^3\right)\log\left(ef^{bcnx+acn}\right.}{3\,b^3c^3n^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*c^2*n^2*x^2*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f)^2
- 6*b*c*n*x*log(f)*polylog(3, -e*f^(b*c*n*x + a*c*n)/d) - (b^3*c^3*n^3*x^3
+ a^3*c^3*n^3)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^3 + (b^3*c^3*n^3*x^3 +
 a^3*c^3*n^3)*log(f)^3*log((e*f^(b*c*n*x + a*c*n) + d)/d) + 6*polylog(4, -e
*f^(b*c*n*x + a*c*n)/d))/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^3e^{bcnx\log\left(f\right)}}{d+ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}}\,dx}{3}+\frac{x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/3 + x**3*log(d + e*(f**(c*(a + b
*x)))**n)/3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e*(f^((b*x + a)*c))^n + d), x)
```

3.125 $\int x \log \left(d+e \left(f^{c(a+b x)}\right)^n\right) \, dx$

Optimal. Leaf size=118

$$\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rubi [A] time = 0.0650263, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2532, 2531, 2282, 6589}

$$\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2532

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[((f + g*x)^(m + 1)*Log[d + e*(F^(c*(a +
b*x)))^n])/(g*(m + 1)), x] + (Int[(f + g*x)^m*Log[1 + (e*(F^(c*(a + b*x)))^
n)/d], x] - Simp[((f + g*x)^(m + 1)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(g*
(m + 1)), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\int\mathrm{Li}_2\left(-\right.}{bc}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\mathrm{Subst}\left(\right.}{}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\mathrm{Li}_3\left(-\frac{e(}{}\right.}{b^2c^2n^2}$$

Mathematica [A] time = 0.0054364, size = 118, normalized size = 1.

$$\frac{\mathrm{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\mathrm{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [B] time = 0.059, size = 598, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)+1/c/b/ln(f)*ln(d+e*f^(b*c*n*x)*exp(-n*(ln
(f)*b*c*x-ln(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))*x-1/c/b/ln(f)*ln((d+e*f^(b
*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*x*ln(f^(c*(b*x+a)))+1/c
^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*
x+a)))))/d)+1/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^
(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)))^2-1/2/c^2/b^2/ln(f)^2*ln(1+e*f^(b*c*n*
x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))/d)*ln(f^(c*(b*x+a)))^2-1/c^2/b^2
/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a)))))
/d)*ln(f^(c*(b*x+a)))-1/2*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b
*x+a))))))*x^2-1/2/c^2/b^2/ln(f)^2*ln(d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-l
n(f^(c*(b*x+a))))))*ln(f^(c*(b*x+a)))^2-1/c/b/ln(f)/n*dilog((d+e*f^(b*c*n*x
)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*x+1/c^2/b^2/ln(f)^2/n*dilog((
d+e*f^(b*c*n*x)*exp(-n*(ln(f)*b*c*x-ln(f^(c*(b*x+a))))))/d)*ln(f^(c*(b*x+a)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\,bcnx^3\log\left(f\right) + bcdn\int\frac{x^2}{2\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n + d\right)}\,dx\log\left(f\right) + \frac{1}{2}\,x^2\log\left(e\left(f^{bcx}\right)^n\left(f^{ac}\right)^n + d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*c*n*x^3*log(f) + b*c*d*n*integrate(1/2*x^2/(e*(f^(b*c*x))^n*(f^(a*c)
)^n + d), x)*log(f) + 1/2*x^2*log(e*(f^(b*c*x))^n*(f^(a*c))^n + d)
```

Fricas [C] time = 2.08905, size = 382, normalized size = 3.24

$$-\frac{2\,bcnx\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)\log\left(f\right)-\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(ef^{bcnx+acn}+d\right)\log\left(f\right)^2+\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(f\right)^2}{2\,b^2c^2n^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*n*x*dilog(-(e*f^(b*c*n*x + a*c*n) + d)/d + 1)*log(f) - (b^2*c^2
*n^2*x^2 - a^2*c^2*n^2)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f)^2 + (b^2*c^2*
n^2*x^2 - a^2*c^2*n^2)*log(f)^2*log((e*f^(b*c*n*x + a*c*n) + d)/d) - 2*poly
log(3, -e*f^(b*c*n*x + a*c*n)/d))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^2e^{bcnx\log\left(f\right)}}{d+ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}}\,dx}{2}+\frac{x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/2 + x**2*log(d + e*(f**(c*(a + b
*x)))**n)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(e\left(f^{(bx+a)c}\right)^n+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log(e*(f^((b*x + a)*c))^n + d), x)
```

3.126 $\int \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx$

Optimal. Leaf size=75

$$-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rubi [A] time = 0.127399, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2280, 2190, 2279, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rule 2280

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[(x*
(F^(e*(c + d*x)))^n)/(a + b*(F^(e*(c + d*x)))^n), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)dx &= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-(bcen\log(f))\int \frac{\left(f^{c(a+bx)}\right)^n x}{d+e\left(f^{c(a+bx)}\right)^n}dx\\ &= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)dx\\ &= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{d}\right)}{x}dx,x,\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\\ &= x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}\end{aligned}$$

Mathematica [A] time = 0.0030972, size = 75, normalized size = 1.

$$-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Maple [A] time = 0.008, size = 82, normalized size = 1.1

$$\frac{\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)}{ncb\ln\left(f\right)}\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)+\frac{1}{ncb\ln\left(f\right)}\mathrm{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^{n}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n),x)
```

```
[Out] 1/c/b/ln(f)/n*ln(d+e*(f^(c*(b*x+a)))^n)*ln(-e*(f^(c*(b*x+a)))^n/d)+1/c/b/ln
(f)/n*dilog(-e*(f^(c*(b*x+a)))^n/d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,bcnx^{2}\log\left(f\right)+bcdn\int\frac{x}{e\left(f^{bcx}\right)^{n}\left(f^{ac}\right)^{n}+d}\,dx\log\left(f\right)+x\log\left(e\left(f^{bcx}\right)^{n}\left(f^{ac}\right)^{n}+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] -1/2*b*c*n*x^2*log(f) + b*c*d*n*integrate(x/(e*(f^(b*c*x))^n*(f^(a*c))^n +
d), x)*log(f) + x*log(e*(f^(b*c*x))^n*(f^(a*c))^n + d)
```

Fricas [A] time = 2.1413, size = 243, normalized size = 3.24

$$\frac{(bcnx+acn)\log\left(ef^{bcnx+acn}+d\right)\log\left(f\right)-(bcnx+acn)\log\left(f\right)\log\left(\frac{ef^{bcnx+acn}+d}{d}\right)-\mathrm{Li}_2\left(-\frac{ef^{bcnx+acn}+d}{d}+1\right)}{bcn\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] ((b*c*n*x + a*c*n)*log(e*f^(b*c*n*x + a*c*n) + d)*log(f) - (b*c*n*x + a*c*n
)*log(f)*log((e*f^(b*c*n*x + a*c*n) + d)/d) - dilog(-(e*f^(b*c*n*x + a*c*n)
```

```
+ d)/d + 1))/(b*c*n*log(f))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-bcene^{acn\log(f)}\log(f)\int\frac{xe^{bcnx\log(f)}}{d+ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx+x\log\left(d+e\left(f^{c(a+bx)}\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(d + e*exp
(a*c*n*log(f))*exp(b*c*n*x*log(f))), x) + x*log(d + e*(f**(c*(a + b*x)))**n
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(e\left(f^{(bx+a)c}\right)^{n}+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d), x)
```

3.127 $\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x} dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)}{x},x\right)$$

```
[Out] CannotIntegrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi [A] time = 0.0624786, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]

[Out] Defer[Int][Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x} dx = \int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x} dx$$

Mathematica [A] time = 0.289213, size = 0, normalized size = 0.

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
```

```
[Out] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [A] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

```
[Out] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^{n}+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(e\left(f^{bcx+ac}\right)^{n}+d\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log(e*(f^(b*c*x + a*c))^n + d)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n)/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e\left(f^{(bx+a)c}\right)^n + d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*(f^((b*x + a)*c))^n + d)/x, x)
```

3.128 $\int \log \left(b \left(F^{e(c+dx)}\right)^n+\pi \right) \, dx$

Optimal. Leaf size=39

$$x \log(\pi)-\frac{\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Rubi [A] time = 0.0259307, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2279, 2392, 2391}

$$x \log(\pi)-\frac{\text{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)dx=\frac{\operatorname{Subst}\left(\int \frac{\log(\pi+bx)}{x}\,dx,x,\left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}$$
$$=x\log(\pi)+\frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{bx}{\pi}\right)}{x}\,dx,x,\left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}$$
$$=x\log(\pi)-\frac{\operatorname{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

Mathematica [A] time = 0.0086595, size = 39, normalized size = 1.

$$x\log(\pi)-\frac{\operatorname{PolyLog}\left(2,-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Maple [B] time = 0.02, size = 138, normalized size = 3.5

$$-\frac{1}{nde\ln(F)}\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)\ln\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)+\frac{\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)}{nde\ln(F)}\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)-\frac{1}{nde\ln(F)}\operatorname{dilog}\left(\frac{b\left(F^{e}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*(F^(e*(d*x+c)))^n+Pi),x)
```

```
[Out] -1/d/e/ln(F)/n*ln(-b*(F^(e*(d*x+c)))^n/Pi)*ln((b*(F^(e*(d*x+c)))^n+Pi)/Pi)+
1/d/e/ln(F)/n*ln(-b*(F^(e*(d*x+c)))^n/Pi)*ln(b*(F^(e*(d*x+c)))^n+Pi)-1/d/e/
ln(F)/n*dilog((b*(F^(e*(d*x+c)))^n+Pi)/Pi)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,denx^2\log(F) + \pi den\int \frac{x}{\pi + \left(F^{dex}\right)^n \left(F^{ce}\right)^n b}\,dx\log(F) + x\log\left(\pi + \left(F^{dex}\right)^n \left(F^{ce}\right)^n b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="maxima")
```

```
[Out] -1/2*d*e*n*x^2*log(F) + pi*d*e*n*integrate(x/(pi + (F^(d*e*x))^n*(F^(c*e))^
n*b), x)*log(F) + x*log(pi + (F^(d*e*x))^n*(F^(c*e))^n*b)
```

Fricas [B] time = 2.14814, size = 250, normalized size = 6.41

$$\frac{(denx + cen)\log\left(\pi + F^{denx+cen}b\right)\log(F) - (denx + cen)\log(F)\log\left(\frac{\pi+F^{denx+cen}b}{\pi}\right) - \mathrm{Li}_2\left(-\frac{\pi+F^{denx+cen}b}{\pi} + 1\right)}{den\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="fricas")
```

```
[Out] ((d*e*n*x + c*e*n)*log(pi + F^(d*e*n*x + c*e*n)*b)*log(F) - (d*e*n*x + c*e*
n)*log(F)*log((pi + F^(d*e*n*x + c*e*n)*b)/pi) - dilog(-(pi + F^(d*e*n*x +
c*e*n)*b)/pi + 1))/(d*e*n*log(F))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-bdene^{cen\log(F)}\log(F)\int \frac{xe^{denx\log(F)}}{be^{cen\log(F)}e^{denx\log(F)} + \pi}\,dx + x\log\left(b\left(F^{e(c+dx)}\right)^n + \pi\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*(F**(e*(d*x+c)))**n+pi),x)
```

```
[Out] -b*d*e*n*exp(c*e*n*log(F))*log(F)*Integral(x*exp(d*e*n*x*log(F))/(b*exp(c*e
*n*log(F))*exp(d*e*n*x*log(F)) + pi), x) + x*log(b*(F**(e*(c + d*x)))**n +
```

```
pi)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\pi + \left(F^{(dx+c)e}\right)^n b\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="giac")
```

```
[Out] integrate(log(pi + (F^((d*x + c)*e))^n*b), x)
```

3.129 $\int \frac{1}{x(3+\log(x))}\,dx$

Optimal. Leaf size=5

$$\log(\log(x)+3)$$

```
[Out] Log[3 + Log[x]]
```

Rubi [A] time = 0.0178284, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2302, 29}

$$\log(\log(x)+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(3+\log(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, 3+\log(x)\right)\\ &= \log(3+\log(x))\end{aligned}$$

Mathematica [A] time = 0.0091176, size = 5, normalized size = 1.

$$\log(\log(x)+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Maple [A] time = 0.006, size = 6, normalized size = 1.2

$$\ln(3 + \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3+ln(x)),x)
```

```
[Out] ln(3+ln(x))
```

Maxima [A] time = 1.08247, size = 7, normalized size = 1.4

$$\log(\log(x) + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="maxima")
```

```
[Out] log(log(x) + 3)
```

Fricas [A] time = 1.98059, size = 23, normalized size = 4.6

$$\log(\log(x) + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="fricas")
```

```
[Out] log(log(x) + 3)
```

Sympy [A] time = 0.095218, size = 5, normalized size = 1.

$$\log(\log(x) + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+ln(x)),x)
```

```
[Out] log(log(x) + 3)
```

Giac [B] time = 1.27814, size = 30, normalized size = 6.

$$\frac{1}{2}\log\left(\frac{1}{4}\pi^2(\operatorname{sgn}(x) - 1)^2 + (\log(|x|) + 3)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2)
```

3.130 $\int \frac{\sqrt{1+\log(x)}}{x} \, dx$

Optimal. Leaf size=12

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Rubi [A] time = 0.0256897, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 30}

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\log(x)}}{x} \, dx &= \text{Subst}\left(\int \sqrt{x} \, dx, x, 1+\log(x)\right)\\ &= \frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0037376, size = 12, normalized size = 1.

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Maple [A] time = 0.006, size = 9, normalized size = 0.8

$$\frac{2}{3}\left(1+\ln\left(x\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x,x)
```

```
[Out] 2/3*(1+ln(x))^(3/2)
```

Maxima [A] time = 1.11695, size = 11, normalized size = 0.92

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

Fricas [A] time = 2.04066, size = 32, normalized size = 2.67

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

Sympy [A] time = 0.738572, size = 10, normalized size = 0.83

$$\frac{2\left(\log\left(x\right)+1\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x,x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3
```

Giac [A] time = 1.29265, size = 11, normalized size = 0.92

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

3.131 $\int \frac{(1+\log(x))^5}{x} \, dx$

Optimal. Leaf size=10

$$\frac{1}{6}(\log(x)+1)^6$$

[Out] (1 + Log[x])^6/6

Rubi [A] time = 0.0165482, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2302, 30}

$$\frac{1}{6}(\log(x)+1)^6$$

Antiderivative was successfully verified.

[In] Int[(1 + Log[x])^5/x,x]

[Out] (1 + Log[x])^6/6

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(1+\log(x))^5}{x} \, dx &= \text{Subst}\left(\int x^5 \, dx, x, 1+\log(x)\right) \\ &= \frac{1}{6}(1+\log(x))^6 \end{aligned}$$

Mathematica [A] time = 0.0025723, size = 10, normalized size = 1.

$$\frac{1}{6}(\log(x)+1)^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])^5/x,x]
```

```
[Out] (1 + Log[x])^6/6
```

Maple [B] time = 0.004, size = 33, normalized size = 3.3

$$\frac{(\ln(x))^6}{6}+(\ln(x))^5+\frac{5\,(\ln(x))^4}{2}+\frac{10\,(\ln(x))^3}{3}+\frac{5\,(\ln(x))^2}{2}+\ln(x)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^5/x,x)
```

```
[Out] 1/6*ln(x)^6+ln(x)^5+5/2*ln(x)^4+10/3*ln(x)^3+5/2*ln(x)^2+ln(x)+1/6
```

Maxima [A] time = 1.12711, size = 11, normalized size = 1.1

$$\frac{1}{6}\,(\log(x)+1)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="maxima")
```

```
[Out] 1/6*(log(x) + 1)^6
```

Fricas [B] time = 2.02142, size = 108, normalized size = 10.8

$$\frac{1}{6}\log(x)^6+\log(x)^5+\frac{5}{2}\log(x)^4+\frac{10}{3}\log(x)^3+\frac{5}{2}\log(x)^2+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="fricas")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

Sympy [B] time = 0.138367, size = 39, normalized size = 3.9

$$\frac{\log(x)^6}{6} + \log(x)^5 + \frac{5\log(x)^4}{2} + \frac{10\log(x)^3}{3} + \frac{5\log(x)^2}{2} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**5/x,x)
```

```
[Out] log(x)**6/6 + log(x)**5 + 5*log(x)**4/2 + 10*log(x)**3/3 + 5*log(x)**2/2 +
log(x)
```

Giac [B] time = 1.25287, size = 42, normalized size = 4.2

$$\frac{1}{6}\log(x)^6 + \log(x)^5 + \frac{5}{2}\log(x)^4 + \frac{10}{3}\log(x)^3 + \frac{5}{2}\log(x)^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="giac")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

3.132 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

[Out] 2*Sqrt[Log[x]]

Rubi [A] time = 0.0139242, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2302, 30}

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[Log[x]]),x]

[Out] 2*Sqrt[Log[x]]

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\log(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A] time = 0.0015343, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)^(1/2),x)
```

```
[Out] 2*ln(x)^(1/2)
```

Maxima [A] time = 1.07431, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x))
```

Fricas [A] time = 2.02241, size = 22, normalized size = 2.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x))
```

Sympy [A] time = 0.431645, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

Giac [A] time = 1.21724, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(log(x))
```

3.133 $\int \frac{1}{x\left(1+\log^2(x)\right)} \, dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0200639, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\left(1+\log^2(x)\right)} \, dx &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \log(x)\right) \\ &= \tan^{-1}(\log(x)) \end{aligned}$$

Mathematica [A] time = 0.0116724, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.004, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.605, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 2.01827, size = 22, normalized size = 7.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] time = 0.136765, size = 15, normalized size = 5.

$$\text{RootSum}\left(4z^2+1,(i\mapsto i\log(2i+\log(x)))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A] time = 1.27133, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

```
[Out] ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Rubi [A] time = 0.0323525, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-3+x^2}}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

Mathematica [B] time = 0.0148386, size = 42, normalized size = 3.

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] -Log[1 - Log[x]/Sqrt[-3 + Log[x]^2]]/2 + Log[1 + Log[x]/Sqrt[-3 + Log[x]^2]
]/2
```

Maple [A] time = 0.01, size = 13, normalized size = 0.9

$$\ln\left(\ln(x)+\sqrt{-3+(\ln(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(-3+ln(x)^2)^(1/2))
```

Maxima [A] time = 1.10158, size = 22, normalized size = 1.57

$$\log\left(2\sqrt{\log(x)^2-3}+2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(log(x)^2 - 3) + 2*log(x))
```

Fricas [A] time = 1.97105, size = 47, normalized size = 3.36

$$-\log\left(\sqrt{\log(x)^2-3}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(log(x)^2 - 3) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\log(x)^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(log(x)**2 - 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.135 $\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

```
[Out] ArcSin[(3*Log[x])/2]/3
```

Rubi [A] time = 0.0350316, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {216}

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 - 9*Log[x]^2]),x]

[Out] ArcSin[(3*Log[x])/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4-9x^2}}\,dx, x, \log(x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0188951, size = 11, normalized size = 1.

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
```

```
[Out] ArcSin[(3*Log[x])/2]/3
```

Maple [A] time = 0.012, size = 8, normalized size = 0.7

$$\frac{1}{3}\arcsin\left(\frac{3\ln(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4-9*ln(x)^2)^(1/2),x)
```

```
[Out] 1/3*arcsin(3/2*ln(x))
```

Maxima [A] time = 1.64437, size = 9, normalized size = 0.82

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*log(x))
```

Fricas [B] time = 2.03161, size = 72, normalized size = 6.55

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9\log(x)^2+4}-2}{3\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*log(x)^2 + 4) - 2)/log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(3\log\left(x\right)-2\right)\left(3\log\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(3*log(x) - 2)*(3*log(x) + 2))), x)
```

Giac [A] time = 1.19744, size = 9, normalized size = 0.82

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*arcsin(3/2*log(x))
```

3.136 $\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx$

Optimal. Leaf size=7

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

```
[Out] ArcSinh[Log[x]/2]
```

Rubi [A] time = 0.0291678, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {215}

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 + Log[x]^2]),x]

[Out] ArcSinh[Log[x]/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \log(x)\right)\\ &= \sinh^{-1}\left(\frac{\log(x)}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0153227, size = 7, normalized size = 1.

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 + Log[x]^2]),x]
```

```
[Out] ArcSinh[Log[x]/2]
```

Maple [A] time = 0.01, size = 6, normalized size = 0.9

$$\text{Arcsinh}\left(\frac{\ln(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+ln(x)^2)^(1/2),x)
```

```
[Out] arcsinh(1/2*ln(x))
```

Maxima [A] time = 1.61202, size = 7, normalized size = 1.

$$\text{arsinh}\left(\frac{1}{2}\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*log(x))
```

Fricas [B] time = 2.06776, size = 47, normalized size = 6.71

$$-\log\left(\sqrt{\log(x)^2+4}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\log(x)^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(log(x)**2 + 4)), x)
```

Giac [B] time = 1.23523, size = 22, normalized size = 3.14

$$-\log\left(\sqrt{\log(x)^2 + 4} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

3.137 $\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx$

Optimal. Leaf size=111

$$-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}$$

```
[Out] -(ArcTan[1/Sqrt[3] - (2^(2/3)*Log[6*x])/3^(1/6)]/(2^(2/3)*3^(5/6))) + Log[2
^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) - 6^(1/3)*Log[
6*x] + 3^(2/3)*Log[6*x]^2]/(6*2^(2/3)*3^(1/3))
```

Rubi [A] time = 0.0998283, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2^(2/3)*Log[6*x])/3^(1/6)]/(2^(2/3)*3^(5/6))) + Log[2
^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) - 6^(1/3)*Log[
6*x] + 3^(2/3)*Log[6*x]^2]/(6*2^(2/3)*3^(1/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \text{Subst}\left(\int \frac{1}{2+3x^3}\,dx, x, \log(6x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}+\sqrt[3]{3}x}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}x}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{6}+2\ 3^{2/3}x}{2^{2/3}-\sqrt[3]{6}x+3^{2/3}x^2}\right.}{6\ 2^{2/3}\sqrt[3]{3}}$$

$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{2^{2/3}\sqrt[3]{3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{3}\log(6x)}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}}$$

Mathematica [A] time = 0.0709949, size = 106, normalized size = 0.95

$$\frac{\sqrt{3}\left(2\log\left(2^{2/3}\sqrt[3]{3}\log(6x)+2\right)-\log\left(\sqrt[3]{2}3^{2/3}\log^2(6x)-2^{2/3}\sqrt[3]{3}\log(6x)+2\right)\right)+6\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{3}\log(6x)-1}{\sqrt{3}}\right)}{6\ 2^{2/3}3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] (6*ArcTan[(-1 + 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]] + Sqrt[3]*(2*Log[2 + 2^(
2/3)*3^(1/3)*Log[6*x]] - Log[2 - 2^(2/3)*3^(1/3)*Log[6*x] + 2^(1/3)*3^(2/3)
*Log[6*x]^2]))/(6*2^(2/3)*3^(5/6))
```

Maple [A] time = 0.007, size = 87, normalized size = 0.8

$$\frac{\sqrt[3]{2}3^{\frac{2}{3}}}{18}\ln\left(\ln(6x)+\frac{\sqrt[3]{2}3^{\frac{2}{3}}}{3}\right)-\frac{\sqrt[3]{2}3^{\frac{2}{3}}}{36}\ln\left((\ln(6x))^2-\frac{\sqrt[3]{2}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}\sqrt[3]{3}}{3}\right)+\frac{\sqrt[3]{2}\sqrt[6]{3}}{6}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}\sqrt[3]{3}\ln(6x)-1\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*ln(6*x)^3),x)
```

```
[Out] 1/18*2^(1/3)*3^(2/3)*ln(ln(6*x)+1/3*2^(1/3)*3^(2/3))-1/36*2^(1/3)*3^(2/3)*l
n(ln(6*x)^2-1/3*2^(1/3)*3^(2/3)*ln(6*x)+1/3*2^(2/3)*3^(1/3))+1/6*2^(1/3)*3^
(1/6)*arctan(1/3*3^(1/2)*(2^(2/3)*3^(1/3)*ln(6*x)-1))
```

Maxima [A] time = 1.59835, size = 131, normalized size = 1.18

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}\log\left(6\,x\right)^{2}-3^{\frac{1}{3}}2^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\log\left(6\,x\right)+2^{\frac{1}{3}}\right)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*2^(1/3)*log(3^(2/3)*log(6*x)^2 - 3^(1/3)*2^(1/3)*log(6*x) + 2
^(2/3)) + 1/18*3^(2/3)*2^(1/3)*log(1/3*3^(2/3)*(3^(1/3)*log(6*x) + 2^(1/3))
) + 1/6*3^(1/6)*2^(1/3)*arctan(1/6*3^(1/6)*2^(2/3)*(2*3^(2/3)*log(6*x) - 3^
(1/3)*2^(1/3)))
```

Fricas [A] time = 2.02954, size = 239, normalized size = 2.15

$$-\frac{1}{72}\cdot 12^{\frac{2}{3}}\log\left(6\,\log\left(6\,x\right)^{2}-12^{\frac{2}{3}}\log\left(6\,x\right)+2\cdot 12^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 12^{\frac{2}{3}}\log\left(12^{\frac{2}{3}}+6\,\log\left(6\,x\right)\right)+\frac{1}{6}\cdot 12^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{1}{6}}\left(12^{\frac{2}{3}}\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="fricas")
```

```
[Out] -1/72*12^(2/3)*log(6*log(6*x)^2 - 12^(2/3)*log(6*x) + 2*12^(1/3)) + 1/36*12
^(2/3)*log(12^(2/3) + 6*log(6*x)) + 1/6*12^(1/6)*arctan(1/6*12^(1/6)*(12^(2
/3)*log(6*x) - 12^(1/3)))
```

Sympy [A] time = 0.164076, size = 17, normalized size = 0.15

$$\operatorname{RootSum}\left(324z^{3}-1,\left(i\mapsto i\log\left(6i+\log\left(6x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*ln(6*x)**3),x)
```

```
[Out] RootSum(324*_z**3 - 1, Lambda(_i, _i*log(6*_i + log(6*x))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.138 $\int \frac{\log(\log(6x))}{x\log(6x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log^2(\log(6x))$$

```
[Out] Log[Log[6*x]]^2/2
```

Rubi [A] time = 0.0283252, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2301}

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[6*x]]/(x*Log[6*x]),x]
```

```
[Out] Log[Log[6*x]]^2/2
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(\log(6x))}{x\log(6x)}\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \log(6x)\right)\\ &= \frac{1}{2}\log^2(\log(6x))\end{aligned}$$

Mathematica [A] time = 0.0039504, size = 11, normalized size = 1.

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[6*x]]/(x*Log[6*x]),x]
```

```
[Out] Log[Log[6*x]]^2/2
```

Maple [A] time = 0.004, size = 10, normalized size = 0.9

$$\frac{(\ln(\ln(6x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(6*x))/x/ln(6*x),x)
```

```
[Out] 1/2*ln(ln(6*x))^2
```

Maxima [A] time = 1.09572, size = 12, normalized size = 1.09

$$\frac{1}{2}\log(\log(6x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(log(6*x))^2
```

Fricas [A] time = 2.08825, size = 28, normalized size = 2.55

$$\frac{1}{2}\log(\log(6x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(log(6*x))^2
```

Sympy [A] time = 0.290454, size = 8, normalized size = 0.73

$$\frac{\log\left(\log\left(6x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(6*x))/x/ln(6*x),x)
```

```
[Out] log(log(6*x))**2/2
```

Giac [A] time = 1.30393, size = 12, normalized size = 1.09

$$\frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="giac")
```

```
[Out] 1/2*log(log(6*x))^2
```

3.139 $\int \frac{2^{\log(x)}}{x}\,dx$

Optimal. Leaf size=9

$$\frac{2^{\log(x)}}{\log(2)}$$

```
[Out] 2^Log[x]/Log[2]
```

Rubi [A] time = 0.0154783, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2274, 30}

$$\frac{x^{\log(2)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x]/x,x]
```

```
[Out] x^Log[2]/Log[2]
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2^{\log(x)}}{x}\,dx &= \int x^{-1+\log(2)}\,dx\\ &= \frac{x^{\log(2)}}{\log(2)}\end{aligned}$$

Mathematica [A] time = 0.0064506, size = 9, normalized size = 1.

$$\frac{2^{\log(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x]/x,x]
```

```
[Out] 2^Log[x]/Log[2]
```

Maple [A] time = 0.013, size = 10, normalized size = 1.1

$$\frac{2^{\ln(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x)/x,x)
```

```
[Out] 2^ln(x)/ln(2)
```

Maxima [A] time = 1.07262, size = 12, normalized size = 1.33

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="maxima")
```

```
[Out] 2^log(x)/log(2)
```

Fricas [A] time = 2.14421, size = 35, normalized size = 3.89

$$\frac{e^{(\log(2)\log(x))}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="fricas")
```

```
[Out] e^(log(2)*log(x))/log(2)
```

Sympy [A] time = 0.418147, size = 7, normalized size = 0.78

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x)/x,x)
```

```
[Out] 2**log(x)/log(2)
```

Giac [A] time = 1.29955, size = 12, normalized size = 1.33

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="giac")
```

```
[Out] 2^log(x)/log(2)
```

3.140 $\int \frac{\sin^2(\log(x))}{x}\,dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2}-\frac{1}{2}\sin(\log(x))\cos(\log(x))$$

```
[Out] Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Rubi [A] time = 0.0199459, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$\frac{\log(x)}{2}-\frac{1}{2}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]]^2/x,x]

[Out] Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(\log(x))}{x}\,dx &= \text{Subst}\left(\int \sin^2(x)\,dx, x, \log(x)\right)\\ &= -\frac{1}{2}\cos(\log(x))\sin(\log(x))+\frac{1}{2}\text{Subst}\left(\int 1\,dx, x, \log(x)\right)\\ &= \frac{\log(x)}{2}-\frac{1}{2}\cos(\log(x))\sin(\log(x))\end{aligned}$$

Mathematica [A] time = 0.013997, size = 16, normalized size = 0.94

$$\frac{\log(x)}{2}-\frac{1}{4}\sin(2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]]^2/x,x]
```

```
[Out] Log[x]/2 - Sin[2*Log[x]]/4
```

Maple [A] time = 0.02, size = 14, normalized size = 0.8

$$\frac{\ln(x)}{2}-\frac{\cos(\ln(x))\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x))^2/x,x)
```

```
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Maxima [A] time = 1.12501, size = 16, normalized size = 0.94

$$\frac{1}{2}\log(x)-\frac{1}{4}\sin(2\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

Fricas [A] time = 2.23494, size = 58, normalized size = 3.41

$$-\frac{1}{2}\cos(\log(x))\sin(\log(x))+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*cos(log(x))*sin(log(x)) + 1/2*log(x)
```

Sympy [B] time = 4.99746, size = 156, normalized size = 9.18

$$\frac{\log(x)\tan^4\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right)+4\tan^2\left(\frac{\log(x)}{2}\right)+2}+\frac{2\log(x)\tan^2\left(\frac{\log(x)}{2}\right)}{2\tan^4\left(\frac{\log(x)}{2}\right)+4\tan^2\left(\frac{\log(x)}{2}\right)+2}+\frac{\log(x)}{2\tan^4\left(\frac{\log(x)}{2}\right)+4\tan^2\left(\frac{\log(x)}{2}\right)+2}+\frac{\quad}{2\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x))**2/x,x)
```

```
[Out] log(x)*tan(log(x)/2)**4/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*l
og(x)*tan(log(x)/2)**2/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + log(
x)/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*tan(log(x)/2)**3/(2*ta
n(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) - 2*tan(log(x)/2)/(2*tan(log(x)/2)
**4 + 4*tan(log(x)/2)**2 + 2)
```

Giac [A] time = 1.29689, size = 16, normalized size = 0.94

$$\frac{1}{2}\log(x)-\frac{1}{4}\sin(2\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

3.141 $\int \frac{7-\log(x)}{x(3+\log(x))} \, dx$

Optimal. Leaf size=12

$$10\log(\log(x)+3)-\log(x)$$

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Rubi [A] time = 0.0362636, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$10\log(\log(x)+3)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{7-\log(x)}{x(3+\log(x))}\,dx = \text{Subst}\left(\int \frac{7-x}{3+x}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \left(-1+\frac{10}{3+x}\right)dx, x, \log(x)\right)$$
$$= -\log(x) + 10\log(3+\log(x))$$

Mathematica [A] time = 0.0257176, size = 12, normalized size = 1.

$$10\log(\log(x)+3) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Maple [A] time = 0.006, size = 13, normalized size = 1.1

$$-\ln(x) + 10\ln(3+\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7-ln(x))/x/(3+ln(x)),x)
```

```
[Out] -ln(x)+10*ln(3+ln(x))
```

Maxima [A] time = 1.09944, size = 16, normalized size = 1.33

$$-\log(x) + 10\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="maxima")
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Fricas [A] time = 2.03898, size = 41, normalized size = 3.42

$$-\log(x) + 10\log(\log(x) + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="fricas")
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Sympy [A] time = 0.107148, size = 10, normalized size = 0.83

$$-\log(x) + 10\log(\log(x) + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-ln(x))/x/(3+ln(x)),x)
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Giac [B] time = 1.25913, size = 36, normalized size = 3.

$$5\log\left(\frac{1}{4}\pi^2(\operatorname{sgn}(x) - 1)^2 + (\log(|x|) + 3)^2\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="giac")
```

```
[Out] 5*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2) - log(x)
```

3.142 $\int \frac{(2-\log(x))(3+\log(x))^2}{x} \, dx$

Optimal. Leaf size=21

$$\frac{5}{3}(\log(x)+3)^3 - \frac{1}{4}(\log(x)+3)^4$$

```
[Out] (5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4
```

Rubi [A] time = 0.0359142, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$\frac{5}{3}(\log(x)+3)^3 - \frac{1}{4}(\log(x)+3)^4$$

Antiderivative was successfully verified.

```
[In] Int[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

```
[Out] (5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx = \mathrm{Subst}\left(\int (2-x)(3+x)^2\,dx, x, \log(x)\right)$$
$$= \mathrm{Subst}\left(\int \left(5(3+x)^2-(3+x)^3\right)dx, x, \log(x)\right)$$
$$= \frac{5}{3}(3+\log(x))^3-\frac{1}{4}(3+\log(x))^4$$

Mathematica [A] time = 0.0136064, size = 29, normalized size = 1.38

$$-\frac{1}{4}\log^4(x)-\frac{4\log^3(x)}{3}+\frac{3\log^2(x)}{2}+18\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

```
[Out] 18*Log[x] + (3*Log[x]^2)/2 - (4*Log[x]^3)/3 - Log[x]^4/4
```

Maple [A] time = 0.004, size = 24, normalized size = 1.1

$$-\frac{(\ln(x))^4}{4}-\frac{4(\ln(x))^3}{3}+\frac{3(\ln(x))^2}{2}+18\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-ln(x))*(3+ln(x))^2/x,x)
```

```
[Out] -1/4*ln(x)^4-4/3*ln(x)^3+3/2*ln(x)^2+18*ln(x)
```

Maxima [A] time = 1.10237, size = 31, normalized size = 1.48

$$-\frac{1}{4}\log(x)^4-\frac{4}{3}\log(x)^3+\frac{3}{2}\log(x)^2+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Fricas [A] time = 2.07586, size = 77, normalized size = 3.67

$$-\frac{1}{4}\log(x)^4-\frac{4}{3}\log(x)^3+\frac{3}{2}\log(x)^2+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Sympy [A] time = 0.129428, size = 27, normalized size = 1.29

$$-\frac{\log(x)^4}{4}-\frac{4\log(x)^3}{3}+\frac{3\log(x)^2}{2}+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-ln(x))*(3+ln(x))**2/x,x)
```

```
[Out] -log(x)**4/4 - 4*log(x)**3/3 + 3*log(x)**2/2 + 18*log(x)
```

Giac [A] time = 1.31823, size = 31, normalized size = 1.48

$$-\frac{1}{4}\log(x)^4-\frac{4}{3}\log(x)^3+\frac{3}{2}\log(x)^2+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="giac")
```

```
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

3.143 $\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx$

Optimal. Leaf size=42

$$\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)+\frac{1}{8}\sqrt{\log^2(x)+1}\log(x)-\frac{1}{8}\sinh^{-1}(\log(x))$$

```
[Out] -ArcSinh[Log[x]]/8 + (Log[x]*Sqrt[1 + Log[x]^2])/8 + (Log[x]^3*Sqrt[1 + Log
[x]^2])/4
```

Rubi [A] time = 0.0673748, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {279, 321, 215}

$$\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)+\frac{1}{8}\sqrt{\log^2(x)+1}\log(x)-\frac{1}{8}\sinh^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

```
[Out] -ArcSinh[Log[x]]/8 + (Log[x]*Sqrt[1 + Log[x]^2])/8 + (Log[x]^3*Sqrt[1 + Log
[x]^2])/4
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx &= \text{Subst}\left(\int x^2\sqrt{1+x^2}\,dx, x, \log(x)\right)\\ &= \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \log(x)\right)\\ &= -\frac{1}{8}\sinh^{-1}(\log(x))+\frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}\end{aligned}$$

Mathematica [A] time = 0.020075, size = 31, normalized size = 0.74

$$\frac{1}{8}\left(\log(x)\sqrt{\log^2(x)+1}\left(2\log^2(x)+1\right)-\sinh^{-1}(\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

```
[Out] (-ArcSinh[Log[x]] + Log[x]*Sqrt[1 + Log[x]^2]*(1 + 2*Log[x]^2))/8
```

Maple [A] time = 0.01, size = 31, normalized size = 0.7

$$\frac{\ln(x)}{4}\left(1+(\ln(x))^2\right)^{\frac{3}{2}}-\frac{\ln(x)}{8}\sqrt{1+(\ln(x))^2}-\frac{\text{Arcsinh}(\ln(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2*(1+ln(x)^2)^(1/2)/x,x)
```

```
[Out] 1/4*ln(x)*(1+ln(x)^2)^(3/2)-1/8*ln(x)*(1+ln(x)^2)^(1/2)-1/8*arcsinh(ln(x))
```

Maxima [A] time = 1.58441, size = 41, normalized size = 0.98

$$\frac{1}{4}\left(\log(x)^{2}+1\right)^{\frac{3}{2}}\log(x)-\frac{1}{8}\sqrt{\log(x)^{2}+1}\log(x)-\frac{1}{8}\operatorname{arsinh}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(log(x)^2 + 1)^(3/2)*log(x) - 1/8*sqrt(log(x)^2 + 1)*log(x) - 1/8*arcsi
nh(log(x))
```

Fricas [A] time = 1.96168, size = 115, normalized size = 2.74

$$\frac{1}{8}\left(2\log(x)^{3}+\log(x)\right)\sqrt{\log(x)^{2}+1}+\frac{1}{8}\log\left(\sqrt{\log(x)^{2}+1}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*log(x)^3 + log(x))*sqrt(log(x)^2 + 1) + 1/8*log(sqrt(log(x)^2 + 1) -
 log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\log(x)^{2}+1}\log(x)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2*(1+ln(x)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(log(x)**2 + 1)*log(x)**2/x, x)
```

Giac [A] time = 1.21246, size = 50, normalized size = 1.19

$$\frac{1}{8}\left(2\log(x)^{2}+1\right)\sqrt{\log(x)^{2}+1}\log(x)+\frac{1}{8}\log\left(\sqrt{\log(x)^{2}+1}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*log(x)^2 + 1)*sqrt(log(x)^2 + 1)*log(x) + 1/8*log(sqrt(log(x)^2 + 1)
 - log(x))
```

3.144 $\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

```
[Out] 1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Rubi [A] time = 0.0405776, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

```
[Out] 1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+2x)^2}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{2(3+2x)^2} + \frac{1}{2(3+2x)}\right) dx, x, \log(x)\right)$$
$$= \frac{1}{4(3+2\log(x))} + \frac{1}{4}\log(3+2\log(x))$$

Mathematica [A] time = 0.0266525, size = 20, normalized size = 0.83

$$\frac{1}{4}\left(\log(2\log(x)+3) + \frac{1}{2\log(x)+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

```
[Out] ((3 + 2*Log[x])^(-1) + Log[3 + 2*Log[x]])/4
```

Maple [A] time = 0.01, size = 21, normalized size = 0.9

$$\frac{1}{12+8\ln(x)} + \frac{\ln(3+2\ln(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))/x/(3+2*ln(x))^2,x)
```

```
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

Maxima [A] time = 1.10067, size = 27, normalized size = 1.12

$$\frac{1}{4(2\log(x)+3)} + \frac{1}{4}\log(2\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/4/(2*log(x) + 3) + 1/4*log(2*log(x) + 3)
```

Fricas [A] time = 1.95505, size = 80, normalized size = 3.33

$$\frac{(2\ \log(x)+3)\log(2\ \log(x)+3)+1}{4\,(2\ \log(x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*log(x) + 3)*log(2*log(x) + 3) + 1)/(2*log(x) + 3)
```

Sympy [A] time = 0.120399, size = 17, normalized size = 0.71

$$\frac{\log\left(\log(x)+\frac{3}{2}\right)}{4}+\frac{1}{8\log(x)+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))/x/(3+2*ln(x))**2,x)
```

```
[Out] log(log(x) + 3/2)/4 + 1/(8*log(x) + 12)
```

Giac [A] time = 1.21718, size = 46, normalized size = 1.92

$$\frac{1}{4\,(2\ \log(x)+3)}+\frac{1}{8}\ \log\left(\pi^2(\mathrm{sgn}(x)-1)^2+(2\ \log(|x|)+3)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4/(2*log(x) + 3) + 1/8*log(pi^2*(sgn(x) - 1)^2 + (2*log(abs(x)) + 3)^2)
```

3.145 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rubi [A] time = 0.043482, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2365, 43}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right) dx, x, \log(x)\right)$$
$$= -2\sqrt{1+\log(x)} + \frac{2}{3}(1+\log(x))^{3/2}$$

Mathematica [A] time = 0.0164214, size = 16, normalized size = 0.7

$$\frac{2}{3}(\log(x)-2)\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A] time = 0.01, size = 18, normalized size = 0.8

$$\frac{2}{3}(1+\ln(x))^{\frac{3}{2}} - 2\sqrt{1+\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x)
```

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Maxima [A] time = 1.01181, size = 23, normalized size = 1.

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}} - 2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Fricas [A] time = 1.99506, size = 47, normalized size = 2.04

$$\frac{2}{3}\sqrt{\log(x)+1}(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] time = 4.04351, size = 20, normalized size = 0.87

$$\frac{2(\log(x)+1)^{\frac{3}{2}}}{3}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Giac [A] time = 1.27333, size = 23, normalized size = 1.

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

3.146 $\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

```
[Out] Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24
```

Rubi [A] time = 0.0505343, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2365, 43}

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

```
[Out] Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{-1+4x}}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{4\sqrt{-1+4x}} + \frac{1}{4}\sqrt{-1+4x}\right)dx, x, \log(x)\right)$$
$$= \frac{1}{8}\sqrt{-1+4\log(x)} + \frac{1}{24}(-1+4\log(x))^{3/2}$$

Mathematica [A] time = 0.021203, size = 20, normalized size = 0.69

$$\frac{1}{12}(2\log(x)+1)\sqrt{4\log(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
```

```
[Out] ((1 + 2*Log[x])*Sqrt[-1 + 4*Log[x]])/12
```

Maple [A] time = 0.01, size = 22, normalized size = 0.8

$$\frac{1}{24}\left(-1+4\,\ln(x)\right)^{\frac{3}{2}} + \frac{1}{8}\sqrt{-1+4\,\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(-1+4*ln(x))^(1/2),x)
```

```
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

Maxima [A] time = 1.01078, size = 28, normalized size = 0.97

$$\frac{1}{24}\left(4\,\log(x)-1\right)^{\frac{3}{2}} + \frac{1}{8}\sqrt{4\,\log(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Fricas [A] time = 2.01502, size = 54, normalized size = 1.86

$$\frac{1}{12}\sqrt{4\log(x)-1}(2\log(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(4*log(x) - 1)*(2*log(x) + 1)
```

Sympy [A] time = 4.35269, size = 22, normalized size = 0.76

$$\frac{(4\log(x)-1)^{\frac{3}{2}}}{24}+\frac{\sqrt{4\log(x)-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(-1+4*ln(x))**(1/2),x)
```

```
[Out] (4*log(x) - 1)**(3/2)/24 + sqrt(4*log(x) - 1)/8
```

Giac [A] time = 1.19953, size = 28, normalized size = 0.97

$$\frac{1}{24}(4\log(x)-1)^{\frac{3}{2}}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

3.147 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rubi [A] time = 0.0601711, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2365, 50, 63, 207}

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)\\
&= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0179884, size = 22, normalized size = 1.

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Maple [A] time = 0.007, size = 30, normalized size = 1.4

$$2\sqrt{1+\ln(x)} + \ln\left(\sqrt{1+\ln(x)}-1\right) - \ln\left(\sqrt{1+\ln(x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x/ln(x),x)
```

```
[Out] 2*(1+ln(x))^(1/2)+ln((1+ln(x))^(1/2)-1)-ln((1+ln(x))^(1/2)+1)
```

Maxima [A] time = 1.01375, size = 39, normalized size = 1.77

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Fricas [A] time = 2.06094, size = 103, normalized size = 4.68

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [A] time = 1.94517, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1} + \log\left(\sqrt{\log(x)+1}-1\right) - \log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

```
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.148 $\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx$

Optimal. Leaf size=27

$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

```
[Out] -2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Rubi [A] time = 0.0494096, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {698}

$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] -2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx &= \text{Subst}\left(\int \frac{1-4x+x^2}{(-1+x)^4}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{2}{(-1+x)^4}-\frac{2}{(-1+x)^3}+\frac{1}{(-1+x)^2}\right)dx,x,\log(x)\right)\\ &= -\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}\end{aligned}$$

Mathematica [A] time = 0.0414772, size = 22, normalized size = 0.81

$$\frac{-3\log^2(x)+9\log(x)-4}{3(\log(x)-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] (-4 + 9*Log[x] - 3*Log[x]^2)/(3*(-1 + Log[x])^3)
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$-(-1+\ln(x))^{-1}+\frac{2}{3(-1+\ln(x))^3}+(-1+\ln(x))^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-4*ln(x)+ln(x)^2)/x/(-1+ln(x))^4,x)
```

```
[Out] -1/(-1+ln(x))+2/3/(-1+ln(x))^3+1/(-1+ln(x))^2
```

Maxima [A] time = 1.04224, size = 43, normalized size = 1.59

$$-\frac{3\log(x)^2-9\log(x)+4}{3\left(\log(x)^3-3\log(x)^2+3\log(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Fricas [A] time = 2.01171, size = 99, normalized size = 3.67

$$-\frac{3\log(x)^2-9\log(x)+4}{3\left(\log(x)^3-3\log(x)^2+3\log(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Sympy [A] time = 0.120792, size = 32, normalized size = 1.19

$$\frac{-3\log(x)^2+9\log(x)-4}{3\log(x)^3-9\log(x)^2+9\log(x)-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*ln(x)+ln(x)**2)/x/(-1+ln(x))**4,x)
```

```
[Out] (-3*log(x)**2 + 9*log(x) - 4)/(3*log(x)**3 - 9*log(x)**2 + 9*log(x) - 3)
```

Giac [A] time = 1.35821, size = 27, normalized size = 1.

$$-\frac{3\,\log(x)^2-9\,\log(x)+4}{3\,(\log(x)-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x) - 1)^3
```

3.149 $\int \frac{\log^2(ax^n)^p}{x} dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Rubi [A] time = 0.026034, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log^2(ax^n)^p}{x}\,dx = \frac{\text{Subst}\left(\int \left(x^2\right)^p\,dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\left(\log^{-2p}(ax^n)\log^2(ax^n)^p\right)\text{Subst}\left(\int x^{2p}\,dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

Mathematica [A] time = 0.01064, size = 27, normalized size = 1.

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int \frac{\left((\ln(ax^n))^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^p/x,x)
```

```
[Out] int((ln(a*x^n)^2)^p/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13831, size = 109, normalized size = 4.04

$$\frac{(n\log(x)+\log(a))\left(n^2\log(x)^2+2\,n\log(a)\log(x)+\log(a)^2\right)^p}{2\,np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)^p/(2*n*p
+ n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\log(ax^n)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**p/x,x)
```

```
[Out] Integral((log(a*x**n)**2)**p/x, x)
```

Giac [B] time = 1.59817, size = 92, normalized size = 3.41

$$\frac{(n\log(x)\,\mathrm{sgn}(\log(ax^n))+\log(a)\,\mathrm{sgn}(\log(ax^n)))(n\log(x)\,\mathrm{sgn}(\log(ax^n))+\log(a)\,\mathrm{sgn}(\log(ax^n)))^{2\,p}}{n\left(2\,p+1\right)\mathrm{sgn}(\log(ax^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x)*sgn(log(a*x^n)) + log(a)*sgn(log(a*x^n)))*(n*log(x)*sgn(log(a*x^n
)) + log(a)*sgn(log(a*x^n)))^(2*p)/(n*(2*p + 1)*sgn(log(a*x^n)))
```

3.150 $\int \frac{\log^m(ax^n)^p}{x} dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))

Rubi [A] time = 0.0347011, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

[In] Int[(Log[a*x^n]^m)^p/x,x]

[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\log^m(ax^n)^p}{x} \, dx = \frac{\text{Subst}\left(\int (x^m)^p \, dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\left(\log^{-mp}(ax^n)\log^m(ax^n)^p\right)\text{Subst}\left(\int x^{mp} \, dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

Mathematica [A] time = 0.0059178, size = 27, normalized size = 1.

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [C] time = 0.245, size = 71, normalized size = 2.6

$$\frac{\left(\ln(a) + \ln(x^n) - \frac{i}{2}\pi \operatorname{csgn}(iax^n)(-\operatorname{csgn}(iax^n) + \operatorname{csgn}(ia))(-\operatorname{csgn}(iax^n) + \operatorname{csgn}(ix^n))\right)^{mp+1}}{n\left(mp+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^m)^p/x,x)
```

```
[Out] 1/n*(ln(a)+ln(x^n)-1/2*I*Pi*csgn(I*a*x^n)*(-csgn(I*a*x^n)+csgn(I*a))*(-csgn
(I*a*x^n)+csgn(I*x^n)))^(m*p+1)/(m*p+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.17485, size = 80, normalized size = 2.96

$$\frac{(n\log(x)+\log(a))(n\log(x)+\log(a))^{mp}}{mnp+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*(n*log(x) + log(a))^(m*p)/(m*n*p + n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\log(ax^n)^m\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**m)**p/x,x)
```

```
[Out] Integral((log(a*x**n)**m)**p/x, x)
```

Giac [A] time = 1.36615, size = 32, normalized size = 1.19

$$\frac{(n\log(x)+\log(a))^{mp+1}}{(mp+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(a))^(m*p + 1)/((m*p + 1)*n)
```

3.151 $\int \frac{\sqrt{\log^2(ax^n)}}{x} dx$

Optimal. Leaf size=25

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)

Rubi [A] time = 0.0200935, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a*x^n]^2]/x,x]

[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\log^2(ax^n)}}{x}\,dx = \frac{\text{Subst}\left(\int \sqrt{x^2}\,dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\sqrt{\log^2(ax^n)}\,\text{Subst}\left(\int x\,dx, x, \log(ax^n)\right)}{n\log(ax^n)}$$
$$= \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Mathematica [A] time = 0.0078259, size = 25, normalized size = 1.

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]^2]/x,x]
```

```
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Maple [C] time = 0.031, size = 21, normalized size = 0.8

$$\frac{\text{csgn}(\ln(ax^n))(\ln(ax^n))^2}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^(1/2)/x,x)
```

```
[Out] 1/2/n*csgn(ln(a*x^n))*ln(a*x^n)^2
```

Maxima [A] time = 1.04749, size = 27, normalized size = 1.08

$$-\frac{1}{2}n\log(x)^2 + \log(a)\log(x) + \log(x)\log(x^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + log(a)*log(x) + log(x)*log(x^n)
```

Fricas [A] time = 2.09794, size = 43, normalized size = 1.72

$$\frac{1}{2}n\log(x)^2 + \log(a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*n*log(x)^2 + log(a)*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(log(a*x**n)**2)/x, x)
```

Giac [A] time = 1.35093, size = 36, normalized size = 1.44

$$\frac{1}{2}n\log(x)^2\operatorname{sgn}(\log(ax^n)) + \log(a)\log(x)\operatorname{sgn}(\log(ax^n))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*log(x)^2*sgn(log(a*x^n)) + log(a)*log(x)*sgn(log(a*x^n))
```

3.152 $\int \frac{\left(b\log^m(ax^n)\right)^p}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

```
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rubi [A] time = 0.0376942, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 30}

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\left(b\log^m(ax^n)\right)^p}{x}\,dx = \frac{\text{Subst}\left(\int (bx^m)^p\,dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\left(\log^{-mp}(ax^n)\left(b\log^m(ax^n)\right)^p\right)\text{Subst}\left(\int x^{mp}\,dx, x, \log(ax^n)\right)}{n}$$
$$= \frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(1+mp)}$$

Mathematica [A] time = 0.006181, size = 29, normalized size = 1.

$$\frac{\log(ax^n)\left(b\log^m(ax^n)\right)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Log[a*x^n]^m)^p/x,x]

[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{\left(b\left(\ln(ax^n)\right)^m\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(a*x^n)^m)^p/x,x)

[Out] int((b*ln(a*x^n)^m)^p/x,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.09417, size = 101, normalized size = 3.48

$$\frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{mnp+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/(m*n*p + n)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\log(ax^n)^m\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*ln(a*x**n)**m)**p/x,x)
```

```
[Out] Integral((b*log(a*x**n)**m)**p/x, x)
```

Giac [A] time = 1.46244, size = 47, normalized size = 1.62

$$\frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{(mp+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/((m*p + 1)*n)
```

3.153 $\int \frac{1}{x \log(e^x)} \, dx$

Optimal. Leaf size=31

$$\frac{\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}-\frac{\log(x)}{x-\log\left(e^x\right)}$$

```
[Out] -(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Rubi [A] time = 0.0157068, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2160, 2157, 29}

$$\frac{\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}-\frac{\log(x)}{x-\log\left(e^x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[E^x]),x]
```

```
[Out] -(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x \log(e^x)}\,dx = -\frac{\int \frac{1}{x}\,dx}{x-\log(e^x)} + \frac{\int \frac{1}{\log(e^x)}\,dx}{x-\log(e^x)}$$
$$= -\frac{\log(x)}{x-\log(e^x)} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(e^x)\right)}{x-\log(e^x)}$$
$$= -\frac{\log(x)}{x-\log(e^x)} + \frac{\log(\log(e^x))}{x-\log(e^x)}$$

Mathematica [A] time = 0.0083698, size = 21, normalized size = 0.68

$$\frac{\log(\log(e^x)) - \log(x)}{x-\log(e^x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[E^x]),x]
```

```
[Out] (-Log[x] + Log[Log[E^x]])/(x - Log[E^x])
```

Maple [A] time = 0.012, size = 29, normalized size = 0.9

$$-\frac{\ln(\ln(e^x))}{\ln(e^x)-x} + \frac{\ln(x)}{\ln(e^x)-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(exp(x)),x)
```

```
[Out] -1/(ln(exp(x))-x)*ln(ln(exp(x)))+1/(ln(exp(x))-x)*ln(x)
```

Maxima [A] time = 1.00705, size = 7, normalized size = 0.23

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/x
```

Fricas [A] time = 1.88555, size = 8, normalized size = 0.26

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/x
```

Sympy [A] time = 0.071003, size = 3, normalized size = 0.1

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(exp(x)),x)
```

```
[Out] -1/x
```

Giac [A] time = 1.27182, size = 7, normalized size = 0.23

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="giac")
```

```
[Out] -1/x
```

3.154 $\int \log(x)\sin(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos(a)\text{CosIntegral}(bx)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

```
[Out] (Cos[a]*CosIntegral[b*x])/b - (Cos[a + b*x]*Log[x])/b - (Sin[a]*SinIntegral
[b*x])/b
```

Rubi [A] time = 0.0780172, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2638, 2554, 12, 3303, 3299, 3302}

$$\frac{\cos(a)\text{CosIntegral}(bx)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x])/b - (Cos[a + b*x]*Log[x])/b - (Sin[a]*SinIntegral
[b*x])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sin(a+bx)\,dx &= -\frac{\cos(a+bx)\log(x)}{b}+\int \frac{\cos(a+bx)}{bx}\,dx\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\int \frac{\cos(a+bx)}{x}\,dx}{b}\\ &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{b}-\frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{b}\\ &= \frac{\cos(a)\mathrm{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0423173, size = 30, normalized size = 0.86

$$\frac{\cos(a)\text{CosIntegral}(bx)-\sin(a)\text{Si}(bx)-\log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x] - Cos[a + b*x]*Log[x] - Sin[a]*SinIntegral[b*x])/b
```

Maple [C] time = 0.107, size = 80, normalized size = 2.3

$$-\frac{\cos\left(bx+a\right)\ln\left(x\right)}{b}+\frac{\frac{i}{2}\mathrm{e}^{-ia}\pi\,\mathrm{csgn}\left(bx\right)}{b}-\frac{i\mathrm{e}^{-ia}\mathrm{Si}\left(bx\right)}{b}-\frac{\mathrm{e}^{-ia}\mathrm{Ei}\left(1,-ibx\right)}{2b}-\frac{\mathrm{e}^{ia}\mathrm{Ei}\left(1,-ibx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a),x)
```

```
[Out] -cos(b*x+a)*ln(x)/b+1/2*I/b*exp(-I*a)*Pi*csgn(b*x)-I/b*exp(-I*a)*Si(b*x)-1/
2/b*exp(-I*a)*Ei(1,-I*b*x)-1/2/b*exp(I*a)*Ei(1,-I*b*x)
```

Maxima [C] time = 1.15679, size = 77, normalized size = 2.2

$$-\frac{\cos\left(bx+a\right)\log\left(x\right)}{b}-\frac{\left(E_1\left(i\,bx\right)+E_1\left(-i\,bx\right)\right)\cos\left(a\right)-\left(i\,E_1\left(i\,bx\right)-i\,E_1\left(-i\,bx\right)\right)\sin\left(a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*((exp_integral_e(1, I*b*x) + exp_integral_e(1,
 -I*b*x))*cos(a) - (I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x
))*sin(a))/b
```

Fricas [A] time = 2.28442, size = 149, normalized size = 4.26

$$\frac{\left(\mathrm{Ci}\left(bx\right)+\mathrm{Ci}\left(-bx\right)\right)\cos\left(a\right)-2\,\cos\left(bx+a\right)\log\left(x\right)-2\,\sin\left(a\right)\mathrm{Si}\left(bx\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(b*x) + cos_integral(-b*x))*cos(a) - 2*cos(b*x + a)*log(x
) - 2*sin(a)*sin_integral(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a),x)
```

```
[Out] Integral(log(x)*sin(a + b*x), x)
```

Giac [C] time = 1.30149, size = 138, normalized size = 3.94

$$-\frac{\cos(bx+a)\log(x)}{b}-\frac{\Re(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right)^2+\Re(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}a\right)^2+2\,\Im(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right)-2\,\Im(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}\right.}{2\left(b\tan\left(\frac{1}{2}a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*(real_part(cos_integral(b*x))*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(b*x))*
tan(1/2*a) - 2*imag_part(cos_integral(-b*x))*tan(1/2*a) + 4*sin_integral(b*
x)*tan(1/2*a) - real_part(cos_integral(b*x)) - real_part(cos_integral(-b*x)
))/(b*tan(1/2*a)^2 + b)
```

3.155 $\int \log(x)\sin^2(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{\sin(2a)\mathrm{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -x/2 + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rubi [A] time = 0.114001, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2635, 8, 2554, 3303, 3299, 3302}

$$\frac{\sin(2a)\mathrm{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] -x/2 + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(x)\sin^2(a+bx)\,dx &= \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int\left(\frac{1}{2} - \frac{\sin(2a+2bx)}{4bx}\right)dx \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\int\frac{\sin(2a+2bx)}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\int\frac{\sin(2bx)}{x}\,dx}{4b} + \frac{\sin(2a)\int\frac{\cos(2bx)}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\text{Ci}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\text{Si}(2bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.0766454, size = 50, normalized size = 0.76

$$\frac{\sin(2a)\text{CosIntegral}(2bx)+\cos(2a)\text{Si}(2bx)-\log(x)\sin(2(a+bx))-2bx+2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] (-2*b*x + 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b*
x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Maple [C] time = 0.128, size = 132, normalized size = 2.

$$\frac{x\ln(x)}{2}-\frac{\ln(x)\sin(2bx+2a)}{4b}-\frac{\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(bx)}{8b}+\frac{\mathrm{e}^{-2ia}\mathrm{Si}(2bx)}{4b}-\frac{\frac{i}{8}\mathrm{e}^{-2ia}\mathrm{Ei}(1,-2ibx)}{b}+\frac{a\ln(ibx)}{2b}-\frac{x}{2}-\frac{a}{2b}-\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*x*ln(x)-1/4*ln(x)/b*sin(2*b*x+2*a)-1/8/b*exp(-2*I*a)*Pi*csgn(b*x)+1/4/b
*exp(-2*I*a)*Si(2*b*x)-1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2*x-1/2*a/b-1/2/b*a*ln(a+I*(I*b*x+I*a))+1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Maxima [C] time = 1.20782, size = 107, normalized size = 1.62

$$\frac{(2bx+2a-\sin(2bx+2a))\log(x)}{4b}-\frac{4bx+(i\,\mathrm{Ei}(2ibx)-i\,\mathrm{Ei}(-2ibx))\cos(2a)+4a\log(x)-(\mathrm{Ei}(2ibx)+\mathrm{Ei}(-2ib}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (I*Ei(2*I*b*x)
 - I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) - (Ei(2*I*b*x) + Ei(-2*I*b*x))*sin
(2*a))/b
```

Fricas [A] time = 2.21566, size = 211, normalized size = 3.2

$$\frac{4bx\log(x)-4\cos(bx+a)\log(x)\sin(bx+a)-4bx+(\mathrm{Ci}(2bx)+\mathrm{Ci}(-2bx))\sin(2a)+2\cos(2a)\,\mathrm{Si}(2bx)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) - 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x + (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) + 2*cos(2*a)*sin_integral(2*b*x
))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**2, x)
```

Giac [C] time = 1.31466, size = 166, normalized size = 2.52

$$\frac{1}{4}\left(2\,x-\frac{\sin\left(2\,bx+2\,a\right)}{b}\right)\log\left(x\right)-\frac{4\,bx\tan\left(a\right)^2+\Im\left(\mathrm{Ci}\left(2\,bx\right)\right)\tan\left(a\right)^2-\Im\left(\mathrm{Ci}\left(-2\,bx\right)\right)\tan\left(a\right)^2+2\,\mathrm{Si}\left(2\,bx\right)\tan\left(a\right)^2+}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x - sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 + imag_part(cos
_integral(2*b*x))*tan(a)^2 - imag_part(cos_integral(-2*b*x))*tan(a)^2 + 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x - 2*real_part(cos_integral(2*b*x))*tan(
a) - 2*real_part(cos_integral(-2*b*x))*tan(a) - imag_part(cos_integral(2*b*
x)) + imag_part(cos_integral(-2*b*x)) - 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

3.156 $\int \log(x)\sin^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cos(a)\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\text{log}$$

```
[Out] (3*Cos[a]*CosIntegral[b*x])/(4*b) - (Cos[3*a]*CosIntegral[3*b*x])/(12*b) -
(Cos[a + b*x]*Log[x])/b + (Cos[a + b*x]^3*Log[x])/(3*b) - (3*Sin[a]*SinInte
gral[b*x])/(4*b) + (Sin[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rubi [A] time = 0.517726, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3299, 3302, 3312}

$$\frac{3\cos(a)\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\text{log}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^3,x]
```



```
[Out] (3*Cos[a]*CosIntegral[b*x])/(4*b) - (Cos[3*a]*CosIntegral[3*b*x])/(12*b) -
(Cos[a + b*x]*Log[x])/b + (Cos[a + b*x]^3*Log[x])/(3*b) - (3*Sin[a]*SinInte
gral[b*x])/(4*b) + (Sin[3*a]*SinIntegral[3*b*x])/(12*b)
```


Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \log(x)\sin^3(a+bx)\,dx = -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{3bx}\,dx$$
$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{x}\,dx}{3b}$$
$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \left(-\frac{3\cos(a+bx)}{x} + \frac{\cos^3(a+bx)}{x}\right)dx}{3b}$$
$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cos^3(a+bx)}{x}\,dx}{3b} + \frac{\int \frac{\cos(a+bx)}{x}\,dx}{b}$$
$$= -\frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\int \left(\frac{3\cos(a+bx)}{4x} + \frac{\cos(3a+3bx)}{4x}\right)dx}{3b} + \frac{\cos(a)\int \frac{\cos(}{x}}{b}$$
$$= \frac{\cos(a)\text{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\sin(a)\text{Si}(bx)}{b} - \frac{\int \frac{\cos(3a+3bx)}{x}\,dx}{12b}$$
$$= \frac{\cos(a)\text{Ci}(bx)}{b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{\sin(a)\text{Si}(bx)}{b} - \frac{\cos(a)\int \frac{\cos(bx)}{x}}{4b}$$
$$= \frac{3\cos(a)\text{Ci}(bx)}{4b} - \frac{\cos(3a)\text{Ci}(3bx)}{12b} - \frac{\cos(a+bx)\log(x)}{b} + \frac{\cos^3(a+bx)\log(x)}{3b} - \frac{3\sin(a)\text{Si}(}{4b}$$

Mathematica [A] time = 0.105273, size = 66, normalized size = 0.74

$$\frac{9\cos(a)\text{CosIntegral}(bx) - \cos(3a)\text{CosIntegral}(3bx) - 9\sin(a)\text{Si}(bx) + \sin(3a)\text{Si}(3bx) - 9\log(x)\cos(a+bx) + \log(x}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (9*Cos[a]*CosIntegral[b*x] - Cos[3*a]*CosIntegral[3*b*x] - 9*Cos[a + b*x]*L
og[x] + Cos[3*(a + b*x)]*Log[x] - 9*Sin[a]*SinIntegral[b*x] + Sin[3*a]*SinI
ntegral[3*b*x])/(12*b)
```

Maple [C] time = 0.129, size = 162, normalized size = 1.8

$$-\frac{3\cos(bx+a)\ln(x)}{4b} + \frac{\ln(x)\cos(3bx+3a)}{12b} - \frac{\frac{i}{24}e^{-3ia}\pi\,\text{csgn}(bx)}{b} + \frac{\frac{i}{12}e^{-3ia}\text{Si}(3bx)}{b} + \frac{e^{-3ia}\text{Ei}(1,-3ibx)}{24b} + \frac{\frac{3i}{8}e^{-ia}\pi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^3,x)
```

```
[Out] -3/4*cos(b*x+a)*ln(x)/b+1/12*ln(x)/b*cos(3*b*x+3*a)-1/24*I/b*exp(-3*I*a)*Pi
*csgn(b*x)+1/12*I/b*exp(-3*I*a)*Si(3*b*x)+1/24/b*exp(-3*I*a)*Ei(1,-3*I*b*x)
+3/8*I/b*exp(-I*a)*Pi*csgn(b*x)-3/4*I/b*exp(-I*a)*Si(b*x)-3/8/b*exp(-I*a)*E
i(1,-I*b*x)-3/8/b*exp(I*a)*Ei(1,-I*b*x)+1/24/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Maxima [C] time = 1.25428, size = 149, normalized size = 1.67

$$\frac{\left(\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\log\left(x\right)}{3\,b}+\frac{\left(E_{1}\left(3i\,bx\right)+E_{1}\left(-3i\,bx\right)\right)\cos\left(3\,a\right)-9\left(E_{1}\left(i\,bx\right)+E_{1}\left(-i\,bx\right)\right)\cos\left(a\right)-\left(i\,E_{1}\left(3i\,b\right.\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))*log(x)/b + 1/24*((exp_integral_e(1, 3
*I*b*x) + exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(exp_integral_e(1, I*b*
x) + exp_integral_e(1, -I*b*x))*cos(a) - (I*exp_integral_e(1, 3*I*b*x) - I*
exp_integral_e(1, -3*I*b*x))*sin(3*a) - (-9*I*exp_integral_e(1, I*b*x) + 9*
I*exp_integral_e(1, -I*b*x))*sin(a))/b
```

Fricas [A] time = 2.27604, size = 304, normalized size = 3.42

$$-\frac{\left(\mathrm{Ci}\left(3\,bx\right)+\mathrm{Ci}\left(-3\,bx\right)\right)\cos\left(3\,a\right)-9\left(\mathrm{Ci}\left(bx\right)+\mathrm{Ci}\left(-bx\right)\right)\cos\left(a\right)-8\left(\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\log\left(x\right)-2\,\sin\left(3\,a\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*((cos_integral(3*b*x) + cos_integral(-3*b*x))*cos(3*a) - 9*(cos_integ
ral(b*x) + cos_integral(-b*x))*cos(a) - 8*(cos(b*x + a)^3 - 3*cos(b*x + a))
*log(x) - 2*sin(3*a)*sin_integral(3*b*x) + 18*sin(a)*sin_integral(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x\right)\sin^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**3, x)
```

Giac [C] time = 1.33762, size = 613, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(cos(b*x + a)^3/b - 3*cos(b*x + a)/b)*log(x) + 1/24*(real_part(cos_inte
gral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan
(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1
/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 18*im
ag_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) + 18*imag_part(cos_integ
ral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 36*sin_integral(b*x)*tan(3/2*a)^2*tan(
1/2*a) + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*imag_
part(cos_integral(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + 4*sin_integral(3*b*x)*
tan(3/2*a)*tan(1/2*a)^2 + real_part(cos_integral(3*b*x))*tan(3/2*a)^2 + 9*r
eal_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*real_part(cos_integral(-b*x))*
tan(3/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2 - real_part(cos
_integral(3*b*x))*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan(1/2*a)^
2 - 9*real_part(cos_integral(-b*x))*tan(1/2*a)^2 - real_part(cos_integral(-
3*b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a) - 2*imag
_part(cos_integral(-3*b*x))*tan(3/2*a) + 4*sin_integral(3*b*x)*tan(3/2*a) -
 18*imag_part(cos_integral(b*x))*tan(1/2*a) + 18*imag_part(cos_integral(-b*
x))*tan(1/2*a) - 36*sin_integral(b*x)*tan(1/2*a) - real_part(cos_integral(3
*b*x)) + 9*real_part(cos_integral(b*x)) + 9*real_part(cos_integral(-b*x)) -
 real_part(cos_integral(-3*b*x)))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*
a)^2 + b*tan(1/2*a)^2 + b)
```

3.157 $\int \cos(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\sin(a)\text{CosIntegral}(bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}+\frac{\log(x)\sin(a+bx)}{b}$$

```
[Out] -((CosIntegral[b*x]*Sin[a])/b) + (Log[x]*Sin[a + b*x])/b - (Cos[a]*SinInteg
ral[b*x])/b
```

Rubi [A] time = 0.0611252, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2637, 2554, 12, 3303, 3299, 3302}

$$-\frac{\sin(a)\text{CosIntegral}(bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}+\frac{\log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a])/b) + (Log[x]*Sin[a + b*x])/b - (Cos[a]*SinInteg
ral[b*x])/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\log(x)\,dx &= \frac{\log(x)\sin(a+bx)}{b} - \int \frac{\sin(a+bx)}{bx}\,dx \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\int \frac{\sin(a+bx)}{x}\,dx}{b} \\
&= \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\int \frac{\sin(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\cos(bx)}{x}\,dx}{b} \\
&= -\frac{\mathrm{Ci}(bx)\sin(a)}{b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\mathrm{Si}(bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0431815, size = 30, normalized size = 0.86

$$-\frac{\sin(a)\mathrm{CosIntegral}(bx)+\cos(a)\mathrm{Si}(bx)-\log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a] - Log[x]*Sin[a + b*x] + Cos[a]*SinIntegral[b*x])
/b)
```

Maple [C] time = 0.082, size = 79, normalized size = 2.3

$$\frac{\ln(x)\sin(bx+a)}{b}+\frac{e^{-ia}\pi\,\mathrm{csgn}(bx)}{2b}-\frac{e^{-ia}\mathrm{Si}(bx)}{b}+\frac{\frac{i}{2}e^{-ia}\mathrm{Ei}(1,-ibx)}{b}-\frac{\frac{i}{2}e^{ia}\mathrm{Ei}(1,-ibx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(x),x)
```

```
[Out] ln(x)*sin(b*x+a)/b+1/2/b*exp(-I*a)*Pi*csgn(b*x)-1/b*exp(-I*a)*Si(b*x)+1/2*I
/b*exp(-I*a)*Ei(1,-I*b*x)-1/2*I/b*exp(I*a)*Ei(1,-I*b*x)
```

Maxima [C] time = 1.18322, size = 74, normalized size = 2.11

$$\frac{\log(x)\sin(bx+a)}{b}+\frac{(iE_1(ibx)-iE_1(-ibx))\cos(a)+(E_1(ibx)+E_1(-ibx))\sin(a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e
(1, -I*b*x))*cos(a) + (exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x)
)*sin(a))/b
```

Fricas [A] time = 2.48628, size = 149, normalized size = 4.26

$$\frac{2\log(x)\sin(bx+a)-(\mathrm{Ci}(bx)+\mathrm{Ci}(-bx))\sin(a)-2\cos(a)\mathrm{Si}(bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*log(x)*sin(b*x + a) - (cos_integral(b*x) + cos_integral(-b*x))*sin(a
) - 2*cos(a)*sin_integral(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x), x)
```

Giac [C] time = 1.34414, size = 146, normalized size = 4.17

$$\frac{\log(x)\sin(bx+a)}{b} + \frac{\Im(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right)^2 - \Im(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}a\right)^2 + 2\,\mathrm{Si}(bx)\tan\left(\frac{1}{2}a\right)^2 - 2\,\Re(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right) -}{2\left(b\tan\left(\frac{1}{2}a\right)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*(imag_part(cos_integral(b*x))*tan(1/2*a)^2 - im
ag_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*sin_integral(b*x)*tan(1/2*a)^2
 - 2*real_part(cos_integral(b*x))*tan(1/2*a) - 2*real_part(cos_integral(-b*
x))*tan(1/2*a) - imag_part(cos_integral(b*x)) + imag_part(cos_integral(-b*x
)) - 2*sin_integral(b*x))/(b*tan(1/2*a)^2 + b)
```

3.158 $\int \cos^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sin(2a)\text{CosIntegral}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -x/2 + (x*Log[x])/2 - (CosIntegral[2*b*x]*Sin[2*a])/(4*b) + (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) - (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rubi [A] time = 0.121475, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 3327, 3303, 3299, 3302}

$$-\frac{\sin(2a)\text{CosIntegral}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -x/2 + (x*Log[x])/2 - (CosIntegral[2*b*x]*Sin[2*a])/(4*b) + (Cos[a + b*x]*L
og[x]*Sin[a + b*x])/(2*b) - (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3327

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[ExpandToSum[
u, x]^m*(a + b*Sin[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] && L
inearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int \frac{1}{4}\left(2+\frac{\sin(2(a+bx))}{bx}\right)dx \\
&= \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{1}{4}\int \left(2+\frac{\sin(2(a+bx))}{bx}\right)dx \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\int \frac{\sin(2(a+bx))}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\int \frac{\sin(2a+2bx)}{x}\,dx}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\cos(2a)\int \frac{\sin(2bx)}{x}\,dx}{4b} - \frac{\sin(2a)\int \frac{\cos(2bx)}{x}}{4b} \\
&= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\text{Ci}(2bx)\sin(2a)}{4b} + \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \frac{\cos(2a)\text{Si}(2bx)}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.0875414, size = 50, normalized size = 0.76

$$-\frac{\sin(2a)\text{CosIntegral}(2bx)+\cos(2a)\text{Si}(2bx)-\log(x)\sin(2(a+bx))+2bx-2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -(2*b*x - 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b*
x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Maple [C] time = 0.079, size = 132, normalized size = 2.

$$\frac{x\ln(x)}{2}+\frac{\ln(x)\sin(2bx+2a)}{4b}+\frac{e^{-2ia}\pi\,\text{csgn}(bx)}{8b}-\frac{e^{-2ia}\text{Si}(2bx)}{4b}+\frac{\frac{i}{8}e^{-2ia}\text{Ei}(1,-2ibx)}{b}+\frac{a\ln(ibx)}{2b}-\frac{x}{2}-\frac{a}{2b}-\frac{a\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*ln(x),x)
```

```
[Out] 1/2*x*ln(x)+1/4*ln(x)/b*sin(2*b*x+2*a)+1/8/b*exp(-2*I*a)*Pi*csgn(b*x)-1/4/b
*exp(-2*I*a)*Si(2*b*x)+1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2*x-1/2*a/b-1/2/b*a*ln(a+I*(I*b*x+I*a))-1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Maxima [C] time = 1.20405, size = 103, normalized size = 1.56

$$\frac{(2bx+2a+\sin(2bx+2a))\log(x)}{4b}-\frac{4bx+(-i\,\text{Ei}(2ibx)+i\,\text{Ei}(-2ibx))\cos(2a)+4a\log(x)+(\text{Ei}(2ibx)+\text{Ei}(-2ib}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (-I*Ei(2*I*b*x
) + I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) + (Ei(2*I*b*x) + Ei(-2*I*b*x))*si
n(2*a))/b
```

Fricas [A] time = 2.74896, size = 211, normalized size = 3.2

$$\frac{4\,bx\log\left(x\right)+4\,\cos\left(bx+a\right)\log\left(x\right)\sin\left(bx+a\right)-4\,bx-\left(\operatorname{Ci}\left(2\,bx\right)+\operatorname{Ci}\left(-2\,bx\right)\right)\sin\left(2\,a\right)-2\,\cos\left(2\,a\right)\operatorname{Si}\left(2\,bx\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) + 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x - (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) - 2*cos(2*a)*sin_integral(2*b*x
))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x\right)\cos^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**2, x)
```

Giac [C] time = 1.38623, size = 165, normalized size = 2.5

$$\frac{1}{4}\left(2\,x+\frac{\sin\left(2\,bx+2\,a\right)}{b}\right)\log\left(x\right)-\frac{4\,bx\tan\left(a\right)^{2}-\Im\left(\operatorname{Ci}\left(2\,bx\right)\right)\tan\left(a\right)^{2}+\Im\left(\operatorname{Ci}\left(-2\,bx\right)\right)\tan\left(a\right)^{2}-2\,\operatorname{Si}\left(2\,bx\right)\tan\left(a\right)^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 - imag_part(cos
_integral(2*b*x))*tan(a)^2 + imag_part(cos_integral(-2*b*x))*tan(a)^2 - 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x + 2*real_part(cos_integral(2*b*x))*tan(
a) + 2*real_part(cos_integral(-2*b*x))*tan(a) + imag_part(cos_integral(2*b*
x)) - imag_part(cos_integral(-2*b*x)) + 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

3.159 $\int \cos^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\sin(a)\text{CosIntegral}(bx)}{4b}-\frac{\sin(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\log($$

```
[Out] (-3*CosIntegral[b*x]*Sin[a])/(4*b) - (CosIntegral[3*b*x]*Sin[3*a])/(12*b) +
 (Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - (3*Cos[a]*SinInt
egral[b*x])/(4*b) - (Cos[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rubi [A] time = 0.468172, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3299, 3302, 4430}

$$-\frac{3\sin(a)\text{CosIntegral}(bx)}{4b}-\frac{\sin(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\log($$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CosIntegral[b*x]*Sin[a])/(4*b) - (CosIntegral[3*b*x]*Sin[3*a])/(12*b) +
 (Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - (3*Cos[a]*SinInt
egral[b*x])/(4*b) - (Cos[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4430

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IGtQ[q, 0]
```

Rubi steps

$$\int \cos^3(a+bx)\log(x)\,dx = \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{6bx}\,dx$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{x}\,dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(\frac{5\sin(a+bx)}{x} + \frac{\cos(2a+2bx)\sin(a+bx)}{x}\right)dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{\cos(2a+2bx)\sin(a+bx)}{x}\,dx}{6b} - \frac{5\int \frac{\sin(a+bx)}{x}\,dx}{6b}$$
$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(-\frac{\sin(a+bx)}{2x} + \frac{\sin(3a+3bx)}{2x}\right)dx}{6b} - \frac{(5\cos(a))\int \frac{\sin(b}{x}}{6b}$$
$$= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} + \frac{\int \frac{\sin(a+bx)}{x}\,dx}{12b}$$
$$= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} + \frac{\cos(a)\int \frac{\sin(}{x}}{12b}$$
$$= -\frac{3\mathrm{Ci}(bx)\sin(a)}{4b} - \frac{\mathrm{Ci}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{3\cos(a)\mathrm{Si}(}{4b}$$

Mathematica [A] time = 0.150755, size = 66, normalized size = 0.75

$$-\frac{9\sin(a)\mathbf{CosIntegral}(bx) + \sin(3a)\mathbf{CosIntegral}(3bx) + 9\cos(a)\mathbf{Si}(bx) + \cos(3a)\mathbf{Si}(3bx) - 9\log(x)\sin(a+bx) - \log(x}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] -(9*CosIntegral[b*x]*Sin[a] + CosIntegral[3*b*x]*Sin[3*a] - 9*Log[x]*Sin[a
+ b*x] - Log[x]*Sin[3*(a + b*x)] + 9*Cos[a]*SinIntegral[b*x] + Cos[3*a]*Sin
Integral[3*b*x])/(12*b)
```

Maple [C] time = 0.118, size = 162, normalized size = 1.8

$$\frac{3\ln(x)\sin(bx+a)}{4b} + \frac{\ln(x)\sin(3bx+3a)}{12b} + \frac{e^{-3ia}\pi\,\mathrm{csgn}(bx)}{24b} - \frac{e^{-3ia}\mathrm{Si}(3bx)}{12b} + \frac{\frac{i}{24}e^{-3ia}\mathrm{Ei}(1,-3ibx)}{b} + \frac{3e^{-ia}\pi\,\mathrm{csgn}(}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*ln(x),x)
```

```
[Out] 3/4*ln(x)*sin(b*x+a)/b+1/12*ln(x)/b*sin(3*b*x+3*a)+1/24/b*exp(-3*I*a)*Pi*cs
gn(b*x)-1/12/b*exp(-3*I*a)*Si(3*b*x)+1/24*I/b*exp(-3*I*a)*Ei(1,-3*I*b*x)+3/
8/b*exp(-I*a)*Pi*csgn(b*x)-3/4/b*exp(-I*a)*Si(b*x)+3/8*I/b*exp(-I*a)*Ei(1,-
I*b*x)-3/8*I/b*exp(I*a)*Ei(1,-I*b*x)-1/24*I/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Maxima [C] time = 1.25374, size = 146, normalized size = 1.66

$$-\frac{\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)\log\left(x\right)}{3\,b}+\frac{\left(i\,E_{1}\left(3i\,bx\right)-i\,E_{1}\left(-3i\,bx\right)\right)\cos\left(3\,a\right)+\left(9i\,E_{1}\left(i\,bx\right)-9i\,E_{1}\left(-i\,bx\right)\right)\cos\left(a\right)+(}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*((I*exp_integral_e(1
, 3*I*b*x) - I*exp_integral_e(1, -3*I*b*x))*cos(3*a) + (9*I*exp_integral_e(
1, I*b*x) - 9*I*exp_integral_e(1, -I*b*x))*cos(a) + (exp_integral_e(1, 3*I*
b*x) + exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(exp_integral_e(1, I*b*x)
+ exp_integral_e(1, -I*b*x))*sin(a))/b
```

Fricas [A] time = 2.60629, size = 302, normalized size = 3.43

$$\frac{8\left(\cos\left(bx+a\right)^{2}+2\right)\log\left(x\right)\sin\left(bx+a\right)-\left(\mathrm{Ci}\left(3\,bx\right)+\mathrm{Ci}\left(-3\,bx\right)\right)\sin\left(3\,a\right)-9\left(\mathrm{Ci}\left(bx\right)+\mathrm{Ci}\left(-bx\right)\right)\sin\left(a\right)-2\,\cos\left(3\,a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(cos(b*x + a)^2 + 2)*log(x)*sin(b*x + a) - (cos_integral(3*b*x) + c
os_integral(-3*b*x))*sin(3*a) - 9*(cos_integral(b*x) + cos_integral(-b*x))*
sin(a) - 2*cos(3*a)*sin_integral(3*b*x) - 18*cos(a)*sin_integral(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x\right)\cos^{3}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**3, x)
```

Giac [C] time = 1.21928, size = 668, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*(imag_part(cos_integ
ral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(
3/2*a)^2*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/
2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 2*sin_
integral(3*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 18*sin_integral(b*x)*tan(3/2*a)
^2*tan(1/2*a)^2 - 18*real_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) -
 18*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 2*real_part(cos
_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*real_part(cos_integral(-3*b*x
))*tan(3/2*a)*tan(1/2*a)^2 + imag_part(cos_integral(3*b*x))*tan(3/2*a)^2 -
9*imag_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*imag_part(cos_integral(-b*x
))*tan(3/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2 + 2*sin_inte
gral(3*b*x)*tan(3/2*a)^2 - 18*sin_integral(b*x)*tan(3/2*a)^2 - imag_part(co
s_integral(3*b*x))*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(1/2*a)
^2 - 9*imag_part(cos_integral(-b*x))*tan(1/2*a)^2 + imag_part(cos_integral(
-3*b*x))*tan(1/2*a)^2 - 2*sin_integral(3*b*x)*tan(1/2*a)^2 + 18*sin_integra
l(b*x)*tan(1/2*a)^2 - 2*real_part(cos_integral(3*b*x))*tan(3/2*a) - 2*real_
part(cos_integral(-3*b*x))*tan(3/2*a) - 18*real_part(cos_integral(b*x))*tan
(1/2*a) - 18*real_part(cos_integral(-b*x))*tan(1/2*a) - imag_part(cos_integ
ral(3*b*x)) - 9*imag_part(cos_integral(b*x)) + 9*imag_part(cos_integral(-b*
x)) + imag_part(cos_integral(-3*b*x)) - 2*sin_integral(3*b*x) - 18*sin_inte
gral(b*x))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*a)^2 + b*tan(1/2*a)^2 +
 b)
```

3.160 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)dx$

Optimal. Leaf size=5

$$\log(x)\sin(x)$$

```
[Out] Log[x]*Sin[x]
```

Rubi [A] time = 0.0332612, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2637, 2554, 3299}

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)dx=\int \cos(x)\log(x)\,dx+\int \frac{\sin(x)}{x}\,dx$$
$$=\log(x)\sin(x)+\mathbf{Si}(x)-\int \frac{\sin(x)}{x}\,dx$$
$$=\log(x)\sin(x)$$

Mathematica [A] time = 0.0209431, size = 5, normalized size = 1.

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

Maple [B] time = 0.05, size = 19, normalized size = 3.8

$$2\,\frac{\ln(x)\tan(x/2)}{1+(\tan(x/2))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(x)+sin(x)/x,x)
```

```
[Out] 2*ln(x)*tan(1/2*x)/(1+tan(1/2*x)^2)
```

Maxima [A] time = 1.16891, size = 7, normalized size = 1.4

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*sin(x)
```

Fricas [A] time = 2.24786, size = 20, normalized size = 4.

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
```

```
[Out] log(x)*sin(x)
```

Sympy [A] time = 12.8869, size = 5, normalized size = 1.

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(x)+sin(x)/x,x)
```

```
[Out] log(x)*sin(x)
```

Giac [A] time = 1.19441, size = 7, normalized size = 1.4

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
```

```
[Out] log(x)*sin(x)
```

3.161 $\int \log(a\sin(x))\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\sin(x))+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)$$

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]] + (I/2)*PolyLog[2, E^(
(2*I)*x)]
```

Rubi [A] time = 0.0573425, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3717, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\sin(x))+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]] + (I/2)*PolyLog[2, E^(
(2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\sin(x))\,dx &= x\log(a\sin(x)) - \int x\cot(x)\,dx \\
&= \frac{ix^2}{2} + x\log(a\sin(x)) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0153099, size = 42, normalized size = 0.89

$$\frac{1}{2}i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)+x\log(a\sin(x))-x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]],x]
```

```
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[a*Sin[x]] + (I/2)*(x^2 + PolyLog[2, E^((2
*I)*x)])
```

Maple [B] time = 0.033, size = 76, normalized size = 1.6

$$-i\ln\left(e^{ix}\right)\ln\left(2\,a\sin\left(x\right)\right)-\frac{i}{2}\left(\ln\left(e^{ix}\right)\right)^2+i\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)+i\text{dilog}\left(e^{ix}+1\right)-i\text{dilog}\left(e^{ix}\right)+i\ln\left(2\right)\ln\left(e^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)),x)
```

```
[Out] -I*ln(exp(I*x))*ln(2*a*sin(x))-1/2*I*ln(exp(I*x))^2+I*ln(exp(I*x))*ln(exp(I
*x)+1)+I*dilog(exp(I*x)+1)-I*dilog(exp(I*x))+I*ln(2)*ln(exp(I*x))
```

Maxima [B] time = 2.17671, size = 117, normalized size = 2.49

$$\frac{1}{2}ix^2 - ix\arctan(\sin(x), \cos(x) + 1) + ix\arctan(\sin(x), -\cos(x) + 1) - \frac{1}{2}x\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(x), cos(x) + 1) + I*x*arctan2(sin(x), -cos(x) +
 1) - 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) + x*log(a*sin(x)) + I*dilog(-e^(I*x)) + I*dilog(e^
(I*x))
```

Fricas [B] time = 2.22386, size = 396, normalized size = 8.43

$$x\log(a\sin(x)) - \frac{1}{2}x\log(\cos(x) + i\sin(x) + 1) - \frac{1}{2}x\log(\cos(x) - i\sin(x) + 1) - \frac{1}{2}x\log(-\cos(x) + i\sin(x) + 1) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x) - I*s
in(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I*sin(
x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(x)) -
 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)),x)
```

```
[Out] Integral(log(a*sin(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)), x)
```

3.162 $\int \log\left(a\sin^2(x)\right)\,dx$

Optimal. Leaf size=45

$$i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+ix^2-2x\log\left(1-e^{2ix}\right)$$

```
[Out] I*x^2 - 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^2] + I*PolyLog[2, E^((2*I
)*x)]
```

Rubi [A] time = 0.0590161, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+ix^2-2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^2] + I*PolyLog[2, E^((2*I
)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sin^2(x)\right)\,dx &= x\log\left(a\sin^2(x)\right) - \int 2x\cot(x)\,dx \\
&= x\log\left(a\sin^2(x)\right) - 2\int x\cot(x)\,dx \\
&= ix^2 + x\log\left(a\sin^2(x)\right) + 4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) + 2\int \log\left(1-e^{2ix}\right)\,dx \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) - i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= ix^2 - 2x\log\left(1-e^{2ix}\right) + x\log\left(a\sin^2(x)\right) + i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0174469, size = 43, normalized size = 0.96

$$i\text{PolyLog}\left(2, e^{2ix}\right) + x\left(\log\left(a\sin^2(x)\right) + ix - 2\log\left(1-e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 - E^((2*I)*x)] + Log[a*Sin[x]^2]) + I*PolyLog[2, E^((2*I)*
x)]
```

Maple [B] time = 0.032, size = 88, normalized size = 2.

$$-i\left(\ln\left(e^{ix}\right)\right)^{2}-i\ln\left(e^{ix}\right)\ln\left(-a\left(e^{2ix}-1\right)^{2}e^{-2ix}\right)+2i\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)+2i\ln\left(2\right)\ln\left(e^{ix}\right)-2i\operatorname{dilog}\left(e^{ix}\right)+2i\operatorname{dilog}\left(e^{i\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^2),x)
```

```
[Out] -I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(-a*(exp(2*I*x)-1)^2*exp(-2*I*x))+2*I*ln
(exp(I*x))*ln(exp(I*x)+1)+2*I*ln(2)*ln(exp(I*x))-2*I*dilog(exp(I*x))+2*I*di
log(exp(I*x)+1)
```

Maxima [B] time = 1.92556, size = 120, normalized size = 2.67

$$ix^{2}-2ix\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+2ix\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(a\sin\left(x\right)^{2}\right)-x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) +
 1) + x*log(a*sin(x)^2) - x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I
*x))
```

Fricas [B] time = 2.10701, size = 362, normalized size = 8.04

$$x\log\left(-a\cos\left(x\right)^{2}+a\right)-x\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)-x\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)-x\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a*cos(x)^2 + a) - x*log(cos(x) + I*sin(x) + 1) - x*log(cos(x) - I*si
n(x) + 1) - x*log(-cos(x) + I*sin(x) + 1) - x*log(-cos(x) - I*sin(x) + 1) +
 I*dilog(cos(x) + I*sin(x)) - I*dilog(cos(x) - I*sin(x)) - I*dilog(-cos(x)
```

```
+ I*sin(x)) + I*dilog(-cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sin^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**2),x)
```

```
[Out] Integral(log(a*sin(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sin(x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^2), x)
```

3.163 $\int \log(a\sin^n(x))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)$$

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0598711, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sin^n(x)\right)\,dx &= x\log\left(a\sin^n(x)\right)-\int nx\cot(x)\,dx\\
&= x\log\left(a\sin^n(x)\right)-n\int x\cot(x)\,dx\\
&= \frac{1}{2}inx^2+x\log\left(a\sin^n(x)\right)+(2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+n\int \log\left(1-e^{2ix}\right)\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)-\frac{1}{2}(in)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0235813, size = 52, normalized size = 1.

$$\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\sin\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^n),x)
```

```
[Out] int(ln(a*sin(x)^n),x)
```

Maxima [B] time = 2.13966, size = 123, normalized size = 2.37

$$-\frac{1}{2}\left(-i\,x^{2}+2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*l
og(a*sin(x)^n)
```

Fricas [B] time = 2.11214, size = 433, normalized size = 8.33

$$-\frac{1}{2}\,nx\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(-\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(cos(x) + I*sin(x) + 1) - 1/2*n*x*log(cos(x) - I*sin(x) + 1) -
1/2*n*x*log(-cos(x) + I*sin(x) + 1) - 1/2*n*x*log(-cos(x) - I*sin(x) + 1) +
```

```
 n*x*log(sin(x)) + 1/2*I*n*dilog(cos(x) + I*sin(x)) - 1/2*I*n*dilog(cos(x)
- I*sin(x)) - 1/2*I*n*dilog(-cos(x) + I*sin(x)) + 1/2*I*n*dilog(-cos(x) - I
*sin(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\sin^{n}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**n),x)
```

```
[Out] Integral(log(a*sin(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\sin(x)^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^n), x)
```

3.164 $\int \log(a\cos(x))\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\cos(x))+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)$$

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Rubi [A] time = 0.0533219, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3719, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\cos(x))+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\cos(x))\,dx &= x\log(a\cos(x)) + \int x\tan(x)\,dx\\
&= \frac{ix^2}{2} + x\log(a\cos(x)) - 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \int \log\left(1+e^{2ix}\right)\,dx\\
&= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) - \frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\
&= \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \frac{1}{2}i\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0080875, size = 47, normalized size = 1.

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2ix}\right) + x\log(a\cos(x)) + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Maple [B] time = 0.03, size = 107, normalized size = 2.3

$$-i\ln\left(e^{ix}\right)\ln\left(2a\cos\left(x\right)\right)+i\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)+i\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)+i\mathrm{dilog}\left(1+ie^{ix}\right)+i\mathrm{dilog}\left(1-ie^{ix}\right)-\frac{i}{2}\left(\ln\left(e^{ix}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)),x)
```

```
[Out] -I*ln(exp(I*x))*ln(2*a*cos(x))+I*ln(exp(I*x))*ln(1+I*exp(I*x))+I*ln(exp(I*x
))*ln(1-I*exp(I*x))+I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp(I*x))-1/2*I*ln(ex
p(I*x))^2+I*ln(2)*ln(exp(I*x))
```

Maxima [A] time = 2.2018, size = 81, normalized size = 1.72

$$\frac{1}{2}ix^2-ix\arctan\left(\sin\left(2x\right),\cos\left(2x\right)+1\right)-\frac{1}{2}x\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\cos\left(2x\right)+1\right)+x\log\left(a\cos\left(x\right)\right)+\frac{1}{2}i\,\mathrm{Li}_2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) - 1/2*x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*cos(x)) + 1/2*I*dilog(-e^(2*I*x))
```

Fricas [B] time = 2.00975, size = 396, normalized size = 8.43

$$x\log\left(a\cos\left(x\right)\right)-\frac{1}{2}x\log\left(i\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}x\log\left(i\cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{2}x\log\left(-i\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)) - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(
x) + 1) - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)) +
 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)),x)
```

```
[Out] Integral(log(a*cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)), x)
```

3.165 $\int \log \left(a \cos^2(x)\right) \, dx$

Optimal. Leaf size=45

$$i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^2(x)\right)+ix^2-2x\log\left(1+e^{2ix}\right)$$

```
[Out] I*x^2 - 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^2] + I*PolyLog[2, -E^((2*
I)*x)]
```

Rubi [A] time = 0.0558014, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^2(x)\right)+ix^2-2x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^2] + I*PolyLog[2, -E^((2*
I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cos^2(x)\right)\,dx &= x\log\left(a\cos^2(x)\right) - \int -2x\tan(x)\,dx\\
&= x\log\left(a\cos^2(x)\right) + 2\int x\tan(x)\,dx\\
&= ix^2 + x\log\left(a\cos^2(x)\right) - 4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + 2\int \log\left(1+e^{2ix}\right)\,dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) - i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + i\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0215694, size = 43, normalized size = 0.96

$$i\mathrm{PolyLog}\left(2,-e^{2ix}\right) + x\left(\log\left(a\cos^2(x)\right) + ix - 2\log\left(1+e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 + E^((2*I)*x)] + Log[a*Cos[x]^2]) + I*PolyLog[2, -E^((2*I)
*x)]
```

Maple [B] time = 0.034, size = 118, normalized size = 2.6

$$-i\left(\ln\left(e^{ix}\right)\right)^{2}-i\ln\left(e^{ix}\right)\ln\left(a\left(1+e^{2ix}\right)^{2}e^{-2ix}\right)+2i\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)+2i\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)+2i\ln\left(2\right)\ln\left(e^{ix}\right)+2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^2),x)
```

```
[Out] -I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(a*(1+exp(2*I*x))^2*exp(-2*I*x))+2*I*ln(
exp(I*x))*ln(1+I*exp(I*x))+2*I*ln(exp(I*x))*ln(1-I*exp(I*x))+2*I*ln(2)*ln(e
xp(I*x))+2*I*dilog(1+I*exp(I*x))+2*I*dilog(1-I*exp(I*x))
```

Maxima [A] time = 2.21956, size = 84, normalized size = 1.87

$$ix^{2}-2ix\arctan\left(\sin\left(2x\right),\cos\left(2x\right)+1\right)+x\log\left(a\cos\left(x\right)^{2}\right)-x\log\left(\cos\left(2x\right)^{2}+\sin\left(2x\right)^{2}+2\cos\left(2x\right)+1\right)+i\,\mathrm{Li}_{2}\left(-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*cos(x)^2) - x*log(c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + I*dilog(-e^(2*I*x))
```

Fricas [B] time = 2.06943, size = 355, normalized size = 7.89

$$x\log\left(a\cos\left(x\right)^{2}\right)-x\log\left(i\cos\left(x\right)+\sin\left(x\right)+1\right)-x\log\left(i\cos\left(x\right)-\sin\left(x\right)+1\right)-x\log\left(-i\cos\left(x\right)+\sin\left(x\right)+1\right)-x\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)^2) - x*log(I*cos(x) + sin(x) + 1) - x*log(I*cos(x) - sin(x)
+ 1) - x*log(-I*cos(x) + sin(x) + 1) - x*log(-I*cos(x) - sin(x) + 1) - I*di
log(I*cos(x) + sin(x)) + I*dilog(I*cos(x) - sin(x)) + I*dilog(-I*cos(x) + s
in(x)) - I*dilog(-I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**2),x)
```

```
[Out] Integral(log(a*cos(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^2), x)
```

3.166 $\int \log(a\cos^n(x))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)$$

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Rubi [A] time = 0.0581344, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cos^n(x)\right)\,dx &= x\log\left(a\cos^n(x)\right)+\int nx\tan(x)\,dx\\
&= x\log\left(a\cos^n(x)\right)+n\int x\tan(x)\,dx\\
&= \frac{1}{2}inx^2+x\log\left(a\cos^n(x)\right)-(2in)\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+n\int \log\left(1+e^{2ix}\right)\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)-\frac{1}{2}(in)\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0231103, size = 52, normalized size = 1.

$$\frac{1}{2}in\mathrm{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Maple [F] time = 0.108, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\cos\left(x\right)\right)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^n),x)
```

```
[Out] int(ln(a*cos(x)^n),x)
```

Maxima [A] time = 2.45841, size = 88, normalized size = 1.69

$$-\frac{1}{2}\left(-i\,x^{2}+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)-i\,\mathrm{Li}_{2}\left(-e^{(2i\,x)}\right)\right)n+x\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*cos(x)^n)
```

Fricas [B] time = 2.25467, size = 433, normalized size = 8.33

$$-\frac{1}{2}\,nx\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}\,nx\log\left(-i\,\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(I*cos(x) + sin(x) + 1) - 1/2*n*x*log(I*cos(x) - sin(x) + 1) -
1/2*n*x*log(-I*cos(x) + sin(x) + 1) - 1/2*n*x*log(-I*cos(x) - sin(x) + 1) +
 n*x*log(cos(x)) - 1/2*I*n*dilog(I*cos(x) + sin(x)) + 1/2*I*n*dilog(I*cos(x
```

```
) - sin(x)) + 1/2*I*n*dilog(-I*cos(x) + sin(x)) - 1/2*I*n*dilog(-I*cos(x) -
 sin(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos^{n}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**n),x)
```

```
[Out] Integral(log(a*cos(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cos(x)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^n), x)
```

3.167 $\int \log(a\tan(x))\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\tan(x))+2x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]] - (I/2)*PolyLog[2, -E^((2*I)*x)]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.044377, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 4419, 4183, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\tan(x))+2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]],x]
```

```
[Out] 2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]] - (I/2)*PolyLog[2, -E^((2*I)*x)]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\tan(x))\,dx &= x\log(a\tan(x)) - \int x\csc(x)\sec(x)\,dx\\ &= x\log(a\tan(x)) - 2\int x\csc(2x)\,dx\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) + \int \log\left(1-e^{2ix}\right)\,dx - \int \log\left(1+e^{2ix}\right)\,dx\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.0108714, size = 75, normalized size = 1.47

$$-\frac{1}{2}i\text{PolyLog}(2,-i\tan(x)) + \frac{1}{2}i\text{PolyLog}(2,i\tan(x)) - \frac{1}{2}i\log(-i(-\tan(x)+i))\log(a\tan(x)) + \frac{1}{2}i\log(-i(\tan(x)+i))\,\text{lo}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]],x]
```

```
[Out] (-I/2)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]] + (I/2)*Log[a*Tan[x]]*Log[(-I)*
(I + Tan[x])] - (I/2)*PolyLog[2, (-I)*Tan[x]] + (I/2)*PolyLog[2, I*Tan[x]]
```

Maple [B] time = 0.03, size = 82, normalized size = 1.6

$$-\frac{i}{2}\ln(a\tan(x))\ln\left(\frac{ia\tan(x)+a}{a}\right) + \frac{i}{2}\ln(a\tan(x))\ln\left(-\frac{ia\tan(x)-a}{a}\right) - \frac{i}{2}\text{dilog}\left(\frac{ia\tan(x)+a}{a}\right) + \frac{i}{2}\text{dilog}\left(-\frac{ia\tan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)),x)
```

```
[Out] -1/2*I*ln(a*tan(x))*ln((I*a*tan(x)+a)/a)+1/2*I*ln(a*tan(x))*ln(-(I*a*tan(x)
-a)/a)-1/2*I*dilog((I*a*tan(x)+a)/a)+1/2*I*dilog(-(I*a*tan(x)-a)/a)
```

Maxima [A] time = 1.62799, size = 57, normalized size = 1.12

$$x\log\left(a\tan\left(x\right)\right)+\frac{1}{4}\pi\log\left(\tan\left(x\right)^{2}+1\right)-x\log\left(\tan\left(x\right)\right)+\frac{1}{2}i\operatorname{Li}_{2}\left(i\tan\left(x\right)+1\right)-\frac{1}{2}i\operatorname{Li}_{2}\left(-i\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)) + 1/4*pi*log(tan(x)^2 + 1) - x*log(tan(x)) + 1/2*I*dilog(I*
tan(x) + 1) - 1/2*I*dilog(-I*tan(x) + 1)
```

Fricas [B] time = 2.01159, size = 587, normalized size = 11.51

$$x\log\left(a\tan\left(x\right)\right)-\frac{1}{2}x\log\left(\frac{2\left(\tan\left(x\right)^{2}+i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}x\log\left(\frac{2\left(\tan\left(x\right)^{2}-i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}x\log\left(-\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)) - 1/2*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*x
*log(2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*x*log(-2*(I*tan(x) - 1)/
(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/4*I*dilo
g(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(-2*(tan(x)^2 -
 I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1
) + 1) - 1/4*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(a\tan\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)),x)
```

```
[Out] Integral(log(a*tan(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\tan(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)), x)
```

3.168 $\int \log\left(a\tan^2(x)\right)\,dx$

Optimal. Leaf size=49

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^2(x)\right)+4x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^2] - I*PolyLog[2, -E^((2*I)*x)] +
 I*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0482655, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^2(x)\right)+4x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^2] - I*PolyLog[2, -E^((2*I)*x)] +
 I*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\tan^2(x)\right)\,dx &= x\log\left(a\tan^2(x)\right) - \int 2x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\tan^2(x)\right) - 2\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\tan^2(x)\right) - 4\int x\csc(2x)\,dx\\
&= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) + 2\int \log\left(1-e^{2ix}\right)\,dx - 2\int \log\left(1+e^{2ix}\right)\,dx\\
&= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}
\end{aligned}$$

$$= 4x\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^2(x)\right) - i\mathrm{Li}_2\left(-e^{2ix}\right) + i\mathrm{Li}_2\left(e^{2ix}\right)$$

Mathematica [A] time = 0.0110811, size = 75, normalized size = 1.53

$$-i\mathrm{PolyLog}(2,-i\tan(x)) + i\mathrm{PolyLog}(2,i\tan(x)) - \frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\tan^2(x)\right) + \frac{1}{2}i\log(-i(\tan(x)+i))\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^2],x]
```

```
[Out] (-I/2)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^2] + (I/2)*Log[a*Tan[x]^2]*Log[(
-I)*(I + Tan[x])] - I*PolyLog[2, (-I)*Tan[x]] + I*PolyLog[2, I*Tan[x]]
```

Maple [A] time = 0.056, size = 82, normalized size = 1.7

$$-\frac{i}{2}\ln\left(a\left(\tan\left(x\right)\right)^{2}\right)\ln\left(\tan\left(x\right)-i\right)+i\ln\left(\tan\left(x\right)-i\right)\ln\left(-i\tan\left(x\right)\right)+i\mathrm{dilog}\left(-i\tan\left(x\right)\right)+\frac{i}{2}\ln\left(a\left(\tan\left(x\right)\right)^{2}\right)\ln\left(\tan\left(x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)^2),x)
```

```
[Out] -1/2*I*ln(a*tan(x)^2)*ln(tan(x)-I)+I*ln(tan(x)-I)*ln(-I*tan(x))+I*dilog(-I*
tan(x))+1/2*I*ln(a*tan(x)^2)*ln(tan(x)+I)-I*ln(tan(x)+I)*ln(I*tan(x))-I*dil
og(I*tan(x))
```

Maxima [A] time = 1.52495, size = 59, normalized size = 1.2

$$x\log\left(a\tan\left(x\right)^{2}\right)+\frac{1}{2}\pi\log\left(\tan\left(x\right)^{2}+1\right)-2x\log\left(\tan\left(x\right)\right)+i\,\mathrm{Li}_{2}\left(i\tan\left(x\right)+1\right)-i\,\mathrm{Li}_{2}\left(-i\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)^2) + 1/2*pi*log(tan(x)^2 + 1) - 2*x*log(tan(x)) + I*dilog(I*
tan(x) + 1) - I*dilog(-I*tan(x) + 1)
```

Fricas [B] time = 2.34573, size = 568, normalized size = 11.59

$$x\log\left(a\tan\left(x\right)^{2}\right)-x\log\left(\frac{2\left(\tan\left(x\right)^{2}+i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)-x\log\left(\frac{2\left(\tan\left(x\right)^{2}-i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)+x\log\left(-\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+x\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)^2) - x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - x*log(2
*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + x*log(-2*(I*tan(x) - 1)/(tan(x)^2
+ 1)) + x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*I*dilog(-2*(tan(x)^2
 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(-2*(tan(x)^2 - I*tan(x))/(ta
n(x)^2 + 1) + 1) + 1/2*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/2*I
```

```
*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tan^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**2),x)
```

```
[Out] Integral(log(a*tan(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tan(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^2), x)
```

3.169 $\int \log(a\tan^n(x))\,dx$

Optimal. Leaf size=56

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^n(x)\right)+2nx\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] 2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^n] - (I/2)*n*PolyLog[2, -E^((2*
I)*x)] + (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0482997, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^n(x)\right)+2nx\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^n] - (I/2)*n*PolyLog[2, -E^((2*
I)*x)] + (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tan^n(x)\right)\,dx &= x\log\left(a\tan^n(x)\right) - \int nx\csc(x)\sec(x)\,dx\\
&= x\log\left(a\tan^n(x)\right) - n\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\tan^n(x)\right) - (2n)\int x\csc(2x)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) + n\int\log\left(1-e^{2ix}\right)\,dx - n\int\log\left(1+e^{2ix}\right)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}(in)\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + \frac{1}{2}(in)\,\text{Subst}\left(\int\right.\\
&= 2nx\tanh^{-1}\left(e^{2ix}\right) + x\log\left(a\tan^n(x)\right) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.0114228, size = 81, normalized size = 1.45

$$-\frac{1}{2}in\text{PolyLog}(2,-i\tan(x)) + \frac{1}{2}in\text{PolyLog}(2,i\tan(x)) - \frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\tan^n(x)\right) + \frac{1}{2}i\log(-i(\tan(x)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^n],x]
```

```
[Out] (-I/2)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^n] + (I/2)*Log[a*Tan[x]^n]*Log[(
-I)*(I + Tan[x])] - (I/2)*n*PolyLog[2, (-I)*Tan[x]] + (I/2)*n*PolyLog[2, I*
```

```
Tan[x]]
```

Maple [C] time = 17.115, size = 6782, normalized size = 121.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)^n),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.53804, size = 65, normalized size = 1.16

$$-nx\log\left(\tan\left(x\right)\right)+\frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^{2}+1\right)+2i\,\mathrm{Li}_{2}\left(i\,\tan\left(x\right)+1\right)-2i\,\mathrm{Li}_{2}\left(-i\,\tan\left(x\right)+1\right)\right)n+x\log\left(a\tan\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="maxima")
```

```
[Out] -n*x*log(tan(x)) + 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*
I*dilog(-I*tan(x) + 1))*n + x*log(a*tan(x)^n)
```

Fricas [B] time = 2.11327, size = 625, normalized size = 11.16

$$-\frac{1}{2}\,nx\log\left(\frac{2\left(\tan\left(x\right)^{2}+i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}\,nx\log\left(\frac{2\left(\tan\left(x\right)^{2}-i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}\,nx\log\left(-\frac{2\left(i\,\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}\,nx\log\left(-\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*n*x*log(2*(tan(x
)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*n*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 +
 1)) + 1/2*n*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + n*x*log(tan(x)) - 1
/4*I*n*dilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(-
```

```
2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(2*(I*tan(x) - 1
)/(tan(x)^2 + 1) + 1) - 1/4*I*n*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
 + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tan^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**n),x)
```

```
[Out] Integral(log(a*tan(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tan\left(x\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^n), x)
```

3.170 $\int \log(a\cot(x))\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\cot(x))-2x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] -2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]] + (I/2)*PolyLog[2, -E^((2*I)*x)
] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.046003, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 4419, 4183, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\cot(x))-2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]],x]
```

```
[Out] -2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]] + (I/2)*PolyLog[2, -E^((2*I)*x)
] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\cot(x))\,dx &= x\log(a\cot(x))+\int x\csc(x)\sec(x)\,dx\\ &= x\log(a\cot(x))+2\int x\csc(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2ix}\right)+x\log(a\cot(x))-\int \log\left(1-e^{2ix}\right)\,dx+\int \log\left(1+e^{2ix}\right)\,dx\\ &= -2x\tanh^{-1}\left(e^{2ix}\right)+x\log(a\cot(x))+\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= -2x\tanh^{-1}\left(e^{2ix}\right)+x\log(a\cot(x))+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.0108143, size = 75, normalized size = 1.47

$$\frac{1}{2}i\text{PolyLog}(2,-i\cot(x))-\frac{1}{2}i\text{PolyLog}(2,i\cot(x))+\frac{1}{2}i\log(-i(-\cot(x)+i))\log(a\cot(x))-\frac{1}{2}i\log(-i(\cot(x)+i))\log($$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]],x]
```

```
[Out] (I/2)*Log[(-I)*(I - Cot[x])]*Log[a*Cot[x]] - (I/2)*Log[a*Cot[x]]*Log[(-I)*(
I + Cot[x])] + (I/2)*PolyLog[2, (-I)*Cot[x]] - (I/2)*PolyLog[2, I*Cot[x]]
```

Maple [B] time = 0.036, size = 82, normalized size = 1.6

$$\frac{i}{2}\ln\left(a\cot\left(x\right)\right)\ln\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\frac{i}{2}\ln\left(a\cot\left(x\right)\right)\ln\left(-\frac{ia\cot\left(x\right)-a}{a}\right)+\frac{i}{2}\text{dilog}\left(\frac{ia\cot\left(x\right)+a}{a}\right)-\frac{i}{2}\text{dilog}\left(-\frac{ia\cot\left(x\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)),x)
```

```
[Out] 1/2*I*ln(a*cot(x))*ln((I*a*cot(x)+a)/a)-1/2*I*ln(a*cot(x))*ln(-(I*a*cot(x)-
a)/a)+1/2*I*dilog((I*a*cot(x)+a)/a)-1/2*I*dilog(-(I*a*cot(x)-a)/a)
```

Maxima [A] time = 1.53032, size = 58, normalized size = 1.14

$$-\frac{1}{4}\pi\log\left(\tan(x)^2+1\right)+x\log\left(\frac{a}{\tan(x)}\right)+x\log(\tan(x))-\frac{1}{2}i\,\mathrm{Li}_2(i\,\tan(x)+1)+\frac{1}{2}i\,\mathrm{Li}_2(-i\,\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="maxima")
```

```
[Out] -1/4*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)) + x*log(tan(x)) - 1/2*I*dilog(I
*tan(x) + 1) + 1/2*I*dilog(-I*tan(x) + 1)
```

Fricas [B] time = 2.1815, size = 462, normalized size = 9.06

$$x\log\left(\frac{a\cos(2x)+a}{\sin(2x)}\right)-\frac{1}{2}x\log(\cos(2x)+i\,\sin(2x)+1)-\frac{1}{2}x\log(\cos(2x)-i\,\sin(2x)+1)+\frac{1}{2}x\log(-\cos(2x)+i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="fricas")
```

```
[Out] x*log((a*cos(2*x) + a)/sin(2*x)) - 1/2*x*log(cos(2*x) + I*sin(2*x) + 1) - 1
/2*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) + I*sin(2*x) + 1)
 + 1/2*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*dilog(-cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log(a\cot(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)),x)
```

```
[Out] Integral(log(a*cot(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cot\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)), x)
```

3.171 $\int \log\left(a\cot^2(x)\right)\,dx$

Optimal. Leaf size=49

$$i\text{PolyLog}\left(2,-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-4x\tanh^{-1}\left(e^{2ix}\right)$$

```
[Out] -4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^2] + I*PolyLog[2, -E^((2*I)*x)]
- I*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0471132, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-4x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^2] + I*PolyLog[2, -E^((2*I)*x)]
- I*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cot^2(x)\right)\,dx &= x\log\left(a\cot^2(x)\right)-\int -2x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^2(x)\right)+2\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^2(x)\right)+4\int x\csc(2x)\,dx\\
&= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-2\int\log\left(1-e^{2ix}\right)\,dx+2\int\log\left(1+e^{2ix}\right)\,dx\\
&= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)+i\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)-i\,\text{Subst}\left(\int\frac{\log(1+}{x}\right.\\
&= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)-i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0115815, size = 75, normalized size = 1.53

$$i\text{PolyLog}(2,-i\tan(x))-i\text{PolyLog}(2,i\tan(x))-\frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\cot^2(x)\right)+\frac{1}{2}i\log(-i(\tan(x)+i))\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^2],x]
```

```
[Out] (-I/2)*Log[a*Cot[x]^2]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^2]*Log[(
-I)*(I + Tan[x])] + I*PolyLog[2, (-I)*Tan[x]] - I*PolyLog[2, I*Tan[x]]
```

Maple [A] time = 0.043, size = 82, normalized size = 1.7

$$\frac{i}{2}\ln\left(a\left(\cot\left(x\right)\right)^{2}\right)\ln\left(\cot\left(x\right)-i\right)-i\ln\left(\cot\left(x\right)-i\right)\ln\left(-i\cot\left(x\right)\right)-i\mathrm{dilog}\left(-i\cot\left(x\right)\right)-\frac{i}{2}\ln\left(a\left(\cot\left(x\right)\right)^{2}\right)\ln\left(\cot\left(x\right)+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^2),x)
```

```
[Out] 1/2*I*ln(a*cot(x)^2)*ln(cot(x)-I)-I*ln(cot(x)-I)*ln(-I*cot(x))-I*dilog(-I*c
ot(x))-1/2*I*ln(a*cot(x)^2)*ln(cot(x)+I)+I*ln(cot(x)+I)*ln(I*cot(x))+I*dilo
g(I*cot(x))
```

Maxima [A] time = 1.52934, size = 59, normalized size = 1.2

$$-\frac{1}{2}\pi\log\left(\tan\left(x\right)^{2}+1\right)+x\log\left(\frac{a}{\tan\left(x\right)^{2}}\right)+2x\log\left(\tan\left(x\right)\right)-i\,\mathrm{Li}_{2}\left(i\tan\left(x\right)+1\right)+i\,\mathrm{Li}_{2}\left(-i\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)^2) + 2*x*log(tan(x)) - I*dilog(I
*tan(x) + 1) + I*dilog(-I*tan(x) + 1)
```

Fricas [B] time = 2.13031, size = 450, normalized size = 9.18

$$x\log\left(-\frac{a\cos\left(2x\right)+a}{\cos\left(2x\right)-1}\right)-x\log\left(\cos\left(2x\right)+i\sin\left(2x\right)+1\right)-x\log\left(\cos\left(2x\right)-i\sin\left(2x\right)+1\right)+x\log\left(-\cos\left(2x\right)+i\sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-(a*cos(2*x) + a)/(cos(2*x) - 1)) - x*log(cos(2*x) + I*sin(2*x) + 1)
- x*log(cos(2*x) - I*sin(2*x) + 1) + x*log(-cos(2*x) + I*sin(2*x) + 1) + x*
log(-cos(2*x) - I*sin(2*x) + 1) - 1/2*I*dilog(cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(cos(2*x) - I*sin(2*x)) - 1/2*I*dilog(-cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\cot^2 (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**2),x)
```

```
[Out] Integral(log(a*cot(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\cot (x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^2), x)
```

3.172 $\int \log (a \cot ^n(x)) \, dx$

Optimal. Leaf size=56

$$\frac{1}{2} in \text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2} in \text{PolyLog}\left(2,e^{2ix}\right)+x \log (a \cot ^n(x))-2 n x \tanh ^{-1}\left(e^{2ix}\right)$$

```
[Out] -2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^n] + (I/2)*n*PolyLog[2, -E^((2
*I)*x)] - (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0486499, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 4419, 4183, 2279, 2391}

$$\frac{1}{2} in \text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2} in \text{PolyLog}\left(2,e^{2ix}\right)+x \log (a \cot ^n(x))-2 n x \tanh ^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^n] + (I/2)*n*PolyLog[2, -E^((2
*I)*x)] - (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cot^n(x)\right)\,dx &= x\log\left(a\cot^n(x)\right)+\int nx\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+n\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+(2n)\int x\csc(2x)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)-n\int\log\left(1-e^{2ix}\right)\,dx+n\int\log\left(1+e^{2ix}\right)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}(in)\,\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)-\frac{1}{2}(in)\,\mathrm{Subst}\Bigg(\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}in\mathrm{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0131143, size = 81, normalized size = 1.45

$$\frac{1}{2}in\mathrm{PolyLog}(2,-i\tan(x))-\frac{1}{2}in\mathrm{PolyLog}(2,i\tan(x))-\frac{1}{2}i\log(-i(-\tan(x)+i))\log\left(a\cot^n(x)\right)+\frac{1}{2}i\log(-i(\tan(x)+i)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^n],x]
```

```
[Out] (-I/2)*Log[a*Cot[x]^n]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^n]*Log[(
-I)*(I + Tan[x])] + (I/2)*n*PolyLog[2, (-I)*Tan[x]] - (I/2)*n*PolyLog[2, I*
```

```
Tan[x]]
```

Maple [C] time = 2.596, size = 6531, normalized size = 116.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^n),x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.52, size = 66, normalized size = 1.18

$$nx\log\left(\tan\left(x\right)\right) - \frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^{2} + 1\right) + 2i\,\mathrm{Li}_{2}\left(i\,\tan\left(x\right) + 1\right) - 2i\,\mathrm{Li}_{2}\left(-i\,\tan\left(x\right) + 1\right)\right)n + x\log\left(a\frac{1}{\tan\left(x\right)}^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="maxima")
```

```
[Out] n*x*log(tan(x)) - 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*I
*dilog(-I*tan(x) + 1))*n + x*log(a*(1/tan(x))^n)
```

Fricas [B] time = 2.14417, size = 498, normalized size = 8.89

$$nx\log\left(\frac{\cos\left(2\,x\right) + 1}{\sin\left(2\,x\right)}\right) - \frac{1}{2}\,nx\log\left(\cos\left(2\,x\right) + i\,\sin\left(2\,x\right) + 1\right) - \frac{1}{2}\,nx\log\left(\cos\left(2\,x\right) - i\,\sin\left(2\,x\right) + 1\right) + \frac{1}{2}\,nx\log\left(-\cos\left(2\,x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log((cos(2*x) + 1)/sin(2*x)) - 1/2*n*x*log(cos(2*x) + I*sin(2*x) + 1) -
 1/2*n*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*n*x*log(-cos(2*x) + I*sin(2*x
) + 1) + 1/2*n*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*n*dilog(cos(2*x) +
 I*sin(2*x)) + 1/4*I*n*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*n*dilog(-cos(2*
```

```
x) + I*sin(2*x)) + 1/4*I*n*dilog(-cos(2*x) - I*sin(2*x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cot^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**n),x)
```

```
[Out] Integral(log(a*cot(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\cot\left(x\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^n), x)
```

3.173 $\int \log(a\sec(x))\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\sec(x))-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)$$

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2, -E
^((2*I)*x)]
```

Rubi [A] time = 0.0518107, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\sec(x))-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]],x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2, -E
^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\sec(x))\,dx &= x\log(a\sec(x)) - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} + x\log(a\sec(x)) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \int \log\left(1+e^{2ix}\right)\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0064459, size = 46, normalized size = 1.

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right) + x\log(a\sec(x)) - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]],x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2, -E
^((2*I)*x)]
```

Maple [B] time = 0.033, size = 118, normalized size = 2.6

$$-i\ln(2)\ln\left(e^{ix}\right)-i\ln\left(e^{ix}\right)\ln\left(\frac{a e^{ix}}{1+e^{2ix}}\right)+\frac{i}{2}\left(\ln\left(e^{ix}\right)\right)^{2}-i\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)-i\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-i\mathrm{dilog}\left(1+ie^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)),x)
```

```
[Out] -I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))*ln(a*exp(I*x)/(1+exp(2*I*x)))+1/2*I*ln
(exp(I*x))^2-I*ln(exp(I*x))*ln(1+I*exp(I*x))-I*ln(exp(I*x))*ln(1-I*exp(I*x)
)-I*dilog(1+I*exp(I*x))-I*dilog(1-I*exp(I*x))
```

Maxima [A] time = 2.0649, size = 81, normalized size = 1.76

$$-\frac{1}{2}ix^{2}+ix\arctan\left(\sin(2x),\cos(2x)+1\right)+\frac{1}{2}x\log\left(\cos(2x)^{2}+\sin(2x)^{2}+2\cos(2x)+1\right)+x\log\left(a\sec(x)\right)-\frac{1}{2}i\,\mathrm{Li}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/2*x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*sec(x)) - 1/2*I*dilog(-e^(2*I*x))
```

Fricas [B] time = 2.20141, size = 396, normalized size = 8.61

$$x\log\left(\frac{a}{\cos(x)}\right)+\frac{1}{2}x\log\left(i\cos(x)+\sin(x)+1\right)+\frac{1}{2}x\log\left(i\cos(x)-\sin(x)+1\right)+\frac{1}{2}x\log\left(-i\cos(x)+\sin(x)+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)) + 1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - s
in(x) + 1) + 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(
x) + 1) + 1/2*I*dilog(I*cos(x) + sin(x)) - 1/2*I*dilog(I*cos(x) - sin(x)) -
 1/2*I*dilog(-I*cos(x) + sin(x)) + 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\sec\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)),x)
```

```
[Out] Integral(log(a*sec(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\sec\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)), x)
```

3.174 $\int \log\left(a\sec^2(x)\right) dx$

Optimal. Leaf size=45

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-ix^2+2x\log\left(1+e^{2ix}\right)$$

```
[Out] (-I)*x^2 + 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^2] - I*PolyLog[2, -E^(
(2*I)*x)]
```

Rubi [A] time = 0.0543116, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-ix^2+2x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^2],x]
```


```
[Out] (-I)*x^2 + 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^2] - I*PolyLog[2, -E^(
(2*I)*x)]
```


Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sec^2(x)\right)\,dx &= x\log\left(a\sec^2(x)\right)-\int 2x\tan(x)\,dx\\
&= x\log\left(a\sec^2(x)\right)-2\int x\tan(x)\,dx\\
&= -ix^2+x\log\left(a\sec^2(x)\right)+4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-2\int \log\left(1+e^{2ix}\right)\,dx\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)+i\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= -ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-i\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.02009, size = 43, normalized size = 0.96

$$x\left(\log\left(a\sec^2(x)\right)-ix+2\log\left(1+e^{2ix}\right)\right)-i\mathrm{PolyLog}\left(2,-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^2],x]
```

```
[Out] x*((-I)*x + 2*Log[1 + E^((2*I)*x)] + Log[a*Sec[x]^2]) - I*PolyLog[2, -E^((2
*I)*x)]
```

Maple [B] time = 0.105, size = 118, normalized size = 2.6

$$i\left(\ln\left(e^{ix}\right)\right)^{2}-i\ln\left(e^{ix}\right)\ln\left(\frac{a e^{2ix}}{\left(1+e^{2ix}\right)^{2}}\right)-2i\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)-2i\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-2i\ln\left(e^{ix}\right)\ln(2)-2i\operatorname{dilog}\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^2),x)
```

```
[Out] I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(a*exp(2*I*x)/(1+exp(2*I*x))^2)-2*I*ln(ex
p(I*x))*ln(1+I*exp(I*x))-2*I*ln(exp(I*x))*ln(1-I*exp(I*x))-2*I*ln(exp(I*x))
*ln(2)-2*I*dilog(1+I*exp(I*x))-2*I*dilog(1-I*exp(I*x))
```

Maxima [A] time = 1.93225, size = 82, normalized size = 1.82

$$-ix^{2}+2ix\arctan\left(\sin(2x),\cos(2x)+1\right)+x\log\left(a\sec(x)^{2}\right)+x\log\left(\cos(2x)^{2}+\sin(2x)^{2}+2\cos(2x)+1\right)-i\operatorname{Li}_2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*sec(x)^2) + x*log(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x))
```

Fricas [B] time = 2.30184, size = 355, normalized size = 7.89

$$x\log\left(\frac{a}{\cos(x)^{2}}\right)+x\log\left(i\cos(x)+\sin(x)+1\right)+x\log\left(i\cos(x)-\sin(x)+1\right)+x\log\left(-i\cos(x)+\sin(x)+1\right)+x\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)^2) + x*log(I*cos(x) + sin(x) + 1) + x*log(I*cos(x) - sin(x)
+ 1) + x*log(-I*cos(x) + sin(x) + 1) + x*log(-I*cos(x) - sin(x) + 1) + I*di
log(I*cos(x) + sin(x)) - I*dilog(I*cos(x) - sin(x)) - I*dilog(-I*cos(x) + s
```

```
in(x)) + I*dilog(-I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\sec^2 (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**2),x)
```

```
[Out] Integral(log(a*sec(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a\sec (x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^2), x)
```

3.175 $\int \log(a\sec^n(x))\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)$$

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*PolyL
og[2, -E^((2*I)*x)]
```

Rubi [A] time = 0.0538001, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*PolyL
og[2, -E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sec^n(x)\right)\,dx &= x\log\left(a\sec^n(x)\right)-\int nx\tan(x)\,dx\\
&= x\log\left(a\sec^n(x)\right)-n\int x\tan(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\sec^n(x)\right)+(2in)\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-n\int \log\left(1+e^{2ix}\right)\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)+\frac{1}{2}(in)\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}in\mathrm{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.022417, size = 51, normalized size = 1.

$$-\frac{1}{2}in\mathrm{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*PolyL
og[2, -E^((2*I)*x)]
```

Maple [F] time = 0.118, size = 0, normalized size = 0.

$$\int \ln \left(a (\sec (x))^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^n),x)
```

```
[Out] int(ln(a*sec(x)^n),x)
```

Maxima [A] time = 2.42739, size = 88, normalized size = 1.73

$$\frac{1}{2} \left(-i x^2 + 2 i x \arctan (\sin (2 x), \cos (2 x) + 1) + x \log \left(\cos (2 x)^2 + \sin (2 x)^2 + 2 \cos (2 x) + 1\right) - i \operatorname{Li}_2 \left(-e^{(2 i x)}\right)\right) n + x \log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*sec(x)^n)
```

Fricas [B] time = 2.15378, size = 435, normalized size = 8.53

$$n x \log \left(\frac{1}{\cos (x)}\right) + \frac{1}{2} n x \log (i \cos (x) + \sin (x) + 1) + \frac{1}{2} n x \log (i \cos (x) - \sin (x) + 1) + \frac{1}{2} n x \log (-i \cos (x) + \sin (x) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/cos(x)) + 1/2*n*x*log(I*cos(x) + sin(x) + 1) + 1/2*n*x*log(I*cos(
x) - sin(x) + 1) + 1/2*n*x*log(-I*cos(x) + sin(x) + 1) + 1/2*n*x*log(-I*cos
(x) - sin(x) + 1) + 1/2*I*n*dilog(I*cos(x) + sin(x)) - 1/2*I*n*dilog(I*cos(
```

```
x) - sin(x)) - 1/2*I*n*dilog(-I*cos(x) + sin(x)) + 1/2*I*n*dilog(-I*cos(x)
- sin(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sec^{n}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**n),x)
```

```
[Out] Integral(log(a*sec(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sec(x)^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^n), x)
```

3.176 $\int \log(a\csc(x))\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\csc(x))-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

```
[Out] (-I/2)*x^2 + x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*PolyLog[2, E^
((2*I)*x)]
```

Rubi [A] time = 0.0528519, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\csc(x))-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]],x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*PolyLog[2, E^
((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\csc(x))\,dx &= x\log(a\csc(x))+\int x\cot(x)\,dx\\
&= -\frac{ix^2}{2}+x\log(a\csc(x))-2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)+x\log(a\csc(x))-\int \log\left(1-e^{2ix}\right)\,dx\\
&= -\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)+x\log(a\csc(x))+\frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)+x\log(a\csc(x))-\frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.012003, size = 41, normalized size = 0.89

$$-\frac{1}{2}i\left(x^2+\mathrm{PolyLog}\left(2,e^{2ix}\right)\right)+x\log(a\csc(x))+x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]],x]
```

```
[Out] x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] time = 0.033, size = 89, normalized size = 1.9

$$-i\ln(2)\ln\left(e^{ix}\right)-i\ln\left(e^{ix}\right)\ln\left(\frac{iae^{ix}}{e^{2ix}-1}\right)+\frac{i}{2}\left(\ln\left(e^{ix}\right)\right)^2+i\text{dilog}\left(e^{ix}\right)-i\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-i\text{dilog}\left(e^{ix}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)),x)
```

```
[Out] -I*ln(2)*ln(exp(I*x))-I*ln(exp(I*x))*ln(I*a*exp(I*x)/(exp(2*I*x)-1))+1/2*I*
ln(exp(I*x))^2+I*dilog(exp(I*x))-I*ln(exp(I*x))*ln(exp(I*x)+1)-I*dilog(exp(
I*x)+1)
```

Maxima [B] time = 2.16309, size = 117, normalized size = 2.54

$$-\frac{1}{2}ix^2+ix\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-ix\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+\frac{1}{2}x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
+ 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) + x*log(a*csc(x)) - I*dilog(-e^(I*x)) - I*dilog(e
^(I*x))
```

Fricas [B] time = 2.18976, size = 396, normalized size = 8.61

$$x\log\left(\frac{a}{\sin\left(x\right)}\right)+\frac{1}{2}x\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)+\frac{1}{2}x\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)+\frac{1}{2}x\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/sin(x)) + 1/2*x*log(cos(x) + I*sin(x) + 1) + 1/2*x*log(cos(x) - I*s
in(x) + 1) + 1/2*x*log(-cos(x) + I*sin(x) + 1) + 1/2*x*log(-cos(x) - I*sin(
x) + 1) - 1/2*I*dilog(cos(x) + I*sin(x)) + 1/2*I*dilog(cos(x) - I*sin(x)) +
```

```
1/2*I*dilog(-cos(x) + I*sin(x)) - 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\csc(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)),x)
```

```
[Out] Integral(log(a*csc(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\csc(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)), x)
```

3.177 $\int \log\left(a\csc^2(x)\right)\,dx$

Optimal. Leaf size=45

$$-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-ix^2+2x\log\left(1-e^{2ix}\right)$$

```
[Out] (-I)*x^2 + 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*PolyLog[2, E^((
2*I)*x)]
```

Rubi [A] time = 0.057692, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-ix^2+2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*PolyLog[2, E^((
2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\csc^2(x)\right)\,dx &= x\log\left(a\csc^2(x)\right)-\int -2x\cot(x)\,dx\\
&= x\log\left(a\csc^2(x)\right)+2\int x\cot(x)\,dx\\
&= -ix^2+x\log\left(a\csc^2(x)\right)-4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-2\int \log\left(1-e^{2ix}\right)\,dx\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)+i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0171565, size = 42, normalized size = 0.93

$$-i\left(x^2+\text{PolyLog}\left(2,e^{2ix}\right)\right)+x\log\left(a\csc^2(x)\right)+2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^2],x]
```

```
[Out] 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] time = 0.035, size = 88, normalized size = 2.

$$i\left(\ln\left(e^{ix}\right)\right)^{2}-i\ln\left(e^{ix}\right)\ln\left(-\frac{a e^{2ix}}{\left(e^{2ix}-1\right)^{2}}\right)-2i\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-2i\ln\left(e^{ix}\right)\ln\left(2\right)-2i\operatorname{dilog}\left(e^{ix}+1\right)+2i\operatorname{dilog}\left(e^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^2),x)
```

```
[Out] I*ln(exp(I*x))^2-I*ln(exp(I*x))*ln(-a*exp(2*I*x)/(exp(2*I*x)-1)^2)-2*I*ln(e
xp(I*x))*ln(exp(I*x)+1)-2*I*ln(exp(I*x))*ln(2)-2*I*dilog(exp(I*x)+1)+2*I*di
log(exp(I*x))
```

Maxima [B] time = 1.95529, size = 117, normalized size = 2.6

$$-i x^{2}+2 i x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2 i x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(a\csc\left(x\right)^{2}\right)+x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x)
+ 1) + x*log(a*csc(x)^2) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(
I*x))
```

Fricas [B] time = 2.01015, size = 365, normalized size = 8.11

$$x\log\left(-\frac{a}{\cos\left(x\right)^{2}-1}\right)+x\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)+x\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)+x\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a/(cos(x)^2 - 1)) + x*log(cos(x) + I*sin(x) + 1) + x*log(cos(x) - I*
sin(x) + 1) + x*log(-cos(x) + I*sin(x) + 1) + x*log(-cos(x) - I*sin(x) + 1)
```

```
 - I*dilog(cos(x) + I*sin(x)) + I*dilog(cos(x) - I*sin(x)) + I*dilog(-cos(x
) + I*sin(x)) - I*dilog(-cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\csc^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**2),x)
```

```
[Out] Integral(log(a*csc(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\csc(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^2), x)
```

3.178 $\int \log(a\csc^n(x))\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)$$

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*PolyL
og[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0601641, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3717, 2190, 2279, 2391}

$$-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*PolyL
og[2, E^((2*I)*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\csc^n(x)\right)\,dx &= x\log\left(a\csc^n(x)\right)+\int nx\cot(x)\,dx\\
&= x\log\left(a\csc^n(x)\right)+n\int x\cot(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\csc^n(x)\right)-(2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-n\int \log\left(1-e^{2ix}\right)\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)+\frac{1}{2}(in)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0222291, size = 51, normalized size = 1.

$$-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^n],x]
```

```
[Out] (-I/2)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*PolyL
og[2, E^((2*I)*x)]
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\csc\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^n),x)
```

```
[Out] int(ln(a*csc(x)^n),x)
```

Maxima [B] time = 2.21585, size = 123, normalized size = 2.41

$$\frac{1}{2}\left(-i\,x^{2}+2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*lo
g(a*csc(x)^n)
```

Fricas [B] time = 2.13657, size = 435, normalized size = 8.53

$$nx\log\left(\frac{1}{\sin\left(x\right)}\right)+\frac{1}{2}\,nx\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+\frac{1}{2}\,nx\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+\frac{1}{2}\,nx\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/sin(x)) + 1/2*n*x*log(cos(x) + I*sin(x) + 1) + 1/2*n*x*log(cos(x)
 - I*sin(x) + 1) + 1/2*n*x*log(-cos(x) + I*sin(x) + 1) + 1/2*n*x*log(-cos(x
```

```
) - I*sin(x) + 1) - 1/2*I*n*dilog(cos(x) + I*sin(x)) + 1/2*I*n*dilog(cos(x)
 - I*sin(x)) + 1/2*I*n*dilog(-cos(x) + I*sin(x)) - 1/2*I*n*dilog(-cos(x) -
I*sin(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \csc^{n}\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**n),x)
```

```
[Out] Integral(log(a*csc(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \csc \left(x\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^n), x)
```

3.179 $\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx$

Optimal. Leaf size=21

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

```
[Out] -2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Rubi [A] time = 0.021194, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2637, 2554, 12}

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

```
[Out] -2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)dx = \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x) - \int 2\cos(x)\,dx$$
$$= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x) - 2\int \cos(x)\,dx$$
$$= -2\sin(x) + \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)$$

Mathematica [A] time = 0.0039362, size = 13, normalized size = 0.62

$$\sin(x)\log\left(\sin^2(x)\right) - 2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

```
[Out] -2*Sin[x] + Log[Sin[x]^2]*Sin[x]
```

Maple [C] time = 0.046, size = 72, normalized size = 3.4

$$i\ln(2)\,e^{ix} - i\ln(2)\,e^{-ix} - \frac{i}{2}e^{ix}\ln(2-2\cos(2x)) + \frac{i}{2}e^{-ix}\ln(2-2\cos(2x)) + ie^{ix} - ie^{-ix}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(1/2-1/2*cos(2*x)),x)
```

```
[Out] I*ln(2)*exp(I*x)-I*ln(2)*exp(-I*x)-1/2*I*exp(I*x)*ln(2-2*cos(2*x))+1/2*I*ex
p(-I*x)*ln(2-2*cos(2*x))+I*exp(I*x)-I*exp(-I*x)
```

Maxima [A] time = 1.01556, size = 23, normalized size = 1.1

$$\log\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)\sin(x) - 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="maxima")
```

```
[Out] log(-1/2*cos(2*x) + 1/2)*sin(x) - 2*sin(x)
```

Fricas [A] time = 2.24085, size = 51, normalized size = 2.43

$$\log\left(-\cos\left(x\right)^{2}+1\right)\sin\left(x\right)-2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="fricas")
```

```
[Out] log(-cos(x)^2 + 1)*sin(x) - 2*sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\frac{1}{2}-\frac{\cos\left(2x\right)}{2}\right)\cos\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(1/2-1/2*cos(2*x)),x)
```

```
[Out] Integral(log(1/2 - cos(2*x)/2)*cos(x), x)
```

Giac [A] time = 1.34837, size = 18, normalized size = 0.86

$$\log\left(\sin\left(x\right)^{2}\right)\sin\left(x\right)-2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="giac")
```

```
[Out] log(sin(x)^2)*sin(x) - 2*sin(x)
```

3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$

Optimal. Leaf size=6

$$\log(\log(e\sin(x)))$$

[Out] Log[Log[E*Sin[x]]]

Rubi [A] time = 0.0163892, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4338, 31}

$$\log(\log(\sin(x))+1)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Log[E*Sin[x]],x]

[Out] Log[1 + Log[Sin[x]]]

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\log(e\sin(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x+x\log(x)}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \log(\sin(x))\right)\\
&= \log(1+\log(\sin(x)))
\end{aligned}$$

Mathematica [A] time = 0.0135556, size = 6, normalized size = 1.

$$\log(\log(\sin(x)) + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E*Sin[x]],x]
```

```
[Out] Log[1 + Log[Sin[x]]]
```

Maple [A] time = 0.013, size = 7, normalized size = 1.2

$$\ln(\ln(E\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(E*sin(x)),x)
```

```
[Out] ln(ln(E*sin(x)))
```

Maxima [A] time = 1.00412, size = 8, normalized size = 1.33

$$\log(\log(E\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="maxima")
```

```
[Out] log(log(E*sin(x)))
```

Fricas [A] time = 2.24403, size = 27, normalized size = 4.5

$$\log(\log(E\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="fricas")
```

```
[Out] log(log(E*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\log(\sin(x)) + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(E*sin(x)),x)
```

```
[Out] Integral(cot(x)/(log(sin(x)) + 1), x)
```

Giac [A] time = 1.25981, size = 8, normalized size = 1.33

$$\log(\log(E\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(E*sin(x)),x, algorithm="giac")
```

```
[Out] log(log(E*sin(x)))
```

3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx$

Optimal. Leaf size=37

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

```
[Out] Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
+ Sin[x])
```

Rubi [A] time = 0.0281907, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4338, 2160, 2157, 29}

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[E^Sin[x]],x]
```

```
[Out] Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
+ Sin[x])
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2160

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx &= \text{Subst}\left(\int \frac{1}{x\log\left(e^{x}\right)} dx, x, \sin(x)\right) \\
&= -\frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{\log(e^{x})} dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} \\
&= \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, \log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} \\
&= -\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0347289, size = 25, normalized size = 0.68

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)-\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E^Sin[x]],x]
```

```
[Out] (Log[Log[E^Sin[x]]] - Log[Sin[x]])/(-Log[E^Sin[x]] + Sin[x])
```

Maple [A] time = 0.02, size = 35, normalized size = 1.

$$\frac{\ln(\sin(x))}{\ln\left(\mathrm{e}^{\sin(x)}\right)-\sin(x)} - \frac{\ln\left(\ln\left(\mathrm{e}^{\sin(x)}\right)\right)}{\ln\left(\mathrm{e}^{\sin(x)}\right)-\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(exp(sin(x))),x)
```

```
[Out] 1/(ln(exp(sin(x)))-sin(x))*ln(sin(x))-1/(ln(exp(sin(x)))-sin(x))*ln(ln(exp(
sin(x))))
```

Maxima [A] time = 1.00954, size = 8, normalized size = 0.22

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A] time = 2.21378, size = 15, normalized size = 0.41

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(exp(sin(x))),x)
```

```
[Out] Integral(cot(x)/log(exp(sin(x))), x)
```

Giac [A] time = 1.29746, size = 8, normalized size = 0.22

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.182 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi [A] time = 0.023436, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3767, 8, 2554, 3473}

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \log(\cos(x))\sec^2(x)\,dx = \log(\cos(x))\tan(x) + \int \tan^2(x)\,dx$$
$$= \tan(x) + \log(\cos(x))\tan(x) - \int 1\,dx$$
$$= -x + \tan(x) + \log(\cos(x))\tan(x)$$

Mathematica [A] time = 0.0165154, size = 12, normalized size = 1.

$$-x + \tan(x) + \tan(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [C] time = 0.04, size = 61, normalized size = 5.1

$$\frac{-2\,i\mathrm{e}^{2\,ix}\ln\left(2\,\cos\left(x\right)\right)}{1+\mathrm{e}^{2\,ix}} + \frac{2\,i}{1+\mathrm{e}^{2\,ix}} + i\ln\left(1+\mathrm{e}^{2\,ix}\right) - \frac{2\,i\ln\left(2\right)}{1+\mathrm{e}^{2\,ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sec(x)^2,x)
```

```
[Out] -2*I/(1+exp(2*I*x))*exp(2*I*x)*ln(2*cos(x))+2*I/(1+exp(2*I*x))+I*ln(1+exp(2
*I*x))-2*I*ln(2)/(1+exp(2*I*x))
```

Maxima [B] time = 1.49995, size = 127, normalized size = 10.58

$$-\frac{2\,\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)} - \frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)} - 2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.19744, size = 68, normalized size = 5.67

$$-\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] time = 137.984, size = 15, normalized size = 1.25

$$-x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Giac [A] time = 1.21604, size = 16, normalized size = 1.33

$$\log(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

3.183 $\int \cot(x)\log(\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\sin(x))$$

```
[Out] Log[Sin[x]]^2/2
```

Rubi [A] time = 0.0158706, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475, 4338, 2301}

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Log[Sin[x]],x]
```

```
[Out] Log[Sin[x]]^2/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \cot(x)\log(\sin(x))\,dx = \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\log^2(\sin(x))$$

Mathematica [A] time = 0.0040673, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Log[Sin[x]],x]
```

```
[Out] Log[Sin[x]]^2/2
```

Maple [A] time = 0.011, size = 8, normalized size = 0.9

$$\frac{(\ln(\sin(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*ln(sin(x)),x)
```

```
[Out] 1/2*ln(sin(x))^2
```

Maxima [A] time = 0.995908, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x))^2
```

Fricas [A] time = 2.19348, size = 26, normalized size = 2.89

$$\frac{1}{2}\log(\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x))^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*ln(sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16333, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x))^2
```

3.184 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rubi [A] time = 0.0388632, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2564, 30, 2554, 12}

$$\frac{1}{3}\sin^3(x)\log(\sin(x)) - \frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]

[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\log(\sin(x))\sin^2(x)\,dx &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\operatorname{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\
&= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0145919, size = 15, normalized size = 0.75

$$\frac{1}{9}\sin^3(x)(3\log(\sin(x)) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]

[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$-\frac{(\sin(x))^3}{9} + \frac{\ln(\sin(x))(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x)

[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Maxima [A] time = 0.995932, size = 22, normalized size = 1.1

$$\frac{1}{3}\log(\sin(x))\sin(x)^3 - \frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Fricas [A] time = 2.2643, size = 92, normalized size = 4.6

$$-\frac{1}{3}\left(\cos(x)^2 - 1\right)\log(\sin(x))\sin(x) + \frac{1}{9}\left(\cos(x)^2 - 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 5.832, size = 17, normalized size = 0.85

$$\frac{\log(\sin(x))\sin^3(x)}{3} - \frac{\sin^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Giac [A] time = 1.17397, size = 22, normalized size = 1.1

$$\frac{1}{3}\log(\sin(x))\sin(x)^3 - \frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

3.185 $\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\,dx$

Optimal. Leaf size=50

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*x
])/b
```

Rubi [A] time = 0.026015, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2637, 2554}

$$\frac{\sin(a+bx)\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*x
])/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \cos(a+bx)\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx = \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b} - \int \cos(a+bx)\,dx$$
$$= -\frac{\sin(a+bx)}{b} + \frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sin(a+bx)}{b}$$

Mathematica [A] time = 0.0101918, size = 33, normalized size = 0.66

$$\frac{\sin(a+bx)\log\left(\frac{1}{2}\sin(a+bx)\right)}{b} - \frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Sin[a + b*x]/2]*Sin[a + b*x])/b
```

Maple [A] time = 0.031, size = 32, normalized size = 0.6

$$\frac{\sin(bx+a)}{b}\ln\left(\frac{\sin(bx+a)}{2}\right) - \frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

```
[Out] ln(1/2*sin(b*x+a))/b*sin(b*x+a)-sin(b*x+a)/b
```

Maxima [A] time = 1.07812, size = 57, normalized size = 1.14

$$\frac{\log\left(\cos\left(\frac{1}{2}bx+\frac{1}{2}a\right)\sin\left(\frac{1}{2}bx+\frac{1}{2}a\right)\right)\sin(bx+a)}{b} - \frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="maxima")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

Fricas [A] time = 2.60892, size = 189, normalized size = 3.78

$$\frac{2\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="fricas")
```

```
[Out] 2*(cos(1/2*b*x + 1/2*a)*log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(
1/2*b*x + 1/2*a) - cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47756, size = 57, normalized size = 1.14

$$\frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="giac")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

3.186 $\int \frac{\tan(x)}{\log(\cos(x))} \, dx$

Optimal. Leaf size=6

$$-\log(\log(\cos(x)))$$

```
[Out] -Log[Log[Cos[x]]]
```

Rubi [A] time = 0.0215558, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4339, 2302, 29}

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Log[Cos[x]],x]
```

```
[Out] -Log[Log[Cos[x]]]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{\tan(x)}{\log(\cos(x))}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x\log(x)}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \log(\cos(x))\right)$$
$$= -\log(\log(\cos(x)))$$

Mathematica [A] time = 0.0083844, size = 6, normalized size = 1.

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Log[Cos[x]],x]
```

```
[Out] -Log[Log[Cos[x]]]
```

Maple [A] time = 0.007, size = 7, normalized size = 1.2

$$-\ln(\ln(\cos(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/ln(cos(x)),x)
```

```
[Out] -ln(ln(cos(x)))
```

Maxima [A] time = 0.998179, size = 8, normalized size = 1.33

$$-\log(\log(\cos(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="maxima")
```

```
[Out] -log(log(cos(x)))
```

Fricas [A] time = 2.1658, size = 26, normalized size = 4.33

$$-\log(\log(\cos(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="fricas")
```

```
[Out] -log(log(cos(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\log(\cos(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/ln(cos(x)),x)
```

```
[Out] Integral(tan(x)/log(cos(x)), x)
```

Giac [A] time = 1.3045, size = 9, normalized size = 1.5

$$-\log(|\log(\cos(x))|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="giac")
```

```
[Out] -log(abs(log(cos(x))))
```

3.187 $\int \log(\cos(x))\tan(x)\,dx$

Optimal. Leaf size=9

$$-\frac{1}{2}\log^2(\cos(x))$$

```
[Out] -Log[Cos[x]]^2/2
```

Rubi [A] time = 0.0148105, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475, 4339, 2301}

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Tan[x],x]
```

```
[Out] -Log[Cos[x]]^2/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4339

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \log(\cos(x))\tan(x)\,dx = -\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{2}\log^2(\cos(x))$$

Mathematica [A] time = 0.003192, size = 9, normalized size = 1.

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Tan[x],x]
```

```
[Out] -Log[Cos[x]]^2/2
```

Maple [A] time = 0.009, size = 8, normalized size = 0.9

$$-\frac{(\ln(\cos(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*tan(x),x)
```

```
[Out] -1/2*ln(cos(x))^2
```

Maxima [A] time = 0.994473, size = 9, normalized size = 1.

$$-\frac{1}{2}\log(\cos(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x))^2
```

Fricas [A] time = 2.15888, size = 27, normalized size = 3.

$$-\frac{1}{2}\log\left(\cos\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(cos(x))^2
```

Sympy [A] time = 7.45885, size = 8, normalized size = 0.89

$$-\frac{\log\left(\cos\left(x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*tan(x),x)
```

```
[Out] -log(cos(x))**2/2
```

Giac [A] time = 1.25589, size = 9, normalized size = 1.

$$-\frac{1}{2}\log\left(\cos\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x))^2
```

3.188 $\int \log(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=10

$$\cos(x) - \cos(x)\log(\cos(x))$$

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Rubi [A] time = 0.0078169, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2638, 2554}

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \log(\cos(x))\sin(x)\,dx &= -\cos(x)\log(\cos(x)) - \int \sin(x)\,dx \\ &= \cos(x) - \cos(x)\log(\cos(x)) \end{aligned}$$

Mathematica [A] time = 0.007417, size = 10, normalized size = 1.

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Maple [A] time = 0.004, size = 11, normalized size = 1.1

$$\cos(x) - \cos(x)\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sin(x),x)
```

```
[Out] cos(x)-cos(x)*ln(cos(x))
```

Maxima [A] time = 0.993473, size = 14, normalized size = 1.4

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Fricas [A] time = 2.2622, size = 41, normalized size = 4.1

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Sympy [A] time = 0.947031, size = 10, normalized size = 1.

$$-\log(\cos(x))\cos(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sin(x),x)
```

```
[Out] -log(cos(x))*cos(x) + cos(x)
```

Giac [A] time = 1.19456, size = 14, normalized size = 1.4

$$-\cos(x)\log(\cos(x))+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)*log(cos(x)) + cos(x)
```

3.189 $\int \cos(x)\log(\cos(x))\,dx$

Optimal. Leaf size=14

$$-\sin(x)+\tanh^{-1}(\sin(x))+\sin(x)\log(\cos(x))$$

```
[Out] ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Rubi [A] time = 0.0195638, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {2637, 2554, 2592, 321, 206}

$$-\sin(x)+\tanh^{-1}(\sin(x))+\sin(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Cos[x]],x]
```

```
[Out] ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\log(\cos(x))\,dx &= \log(\cos(x))\sin(x) + \int \sin(x)\tan(x)\,dx \\
&= \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right) \\
&= -\sin(x) + \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right) \\
&= \tanh^{-1}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)
\end{aligned}$$

Mathematica [B] time = 0.0114056, size = 43, normalized size = 3.07

$$-\sin(x) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right) + \sin(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Cos[x]],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x] + Log[Cos[x]]
*Sin[x]
```

Maple [C] time = 0.017, size = 73, normalized size = 5.2

$$-\frac{i}{2}\ln(2)\,e^{-ix} + \frac{i}{2}\ln(2)\,e^{ix} + \frac{i}{2}e^{-ix}\ln(2\cos(x)) - \frac{i}{2}\ln(2\cos(x))\,e^{ix} - \frac{i}{2}e^{-ix} + \frac{i}{2}e^{ix} - 2\,i\arctan\left(e^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(cos(x)),x)
```

```
[Out] -1/2*I*ln(2)*exp(-I*x)+1/2*I*ln(2)*exp(I*x)+1/2*I*exp(-I*x)*ln(2*cos(x))-1/
2*I*ln(2*cos(x))*exp(I*x)-1/2*I*exp(-I*x)+1/2*I*exp(I*x)-2*I*arctan(exp(I*x
))
```

Maxima [B] time = 1.00651, size = 146, normalized size = 10.43

$$\frac{2\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="maxima")
```

```
[Out] 2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)/
((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
+ 1)^2 + 1)*(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(
x) + 1) - 1)
```

Fricas [A] time = 2.39617, size = 100, normalized size = 7.14

$$\log(\cos(x))\sin(x)+\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="fricas")
```

```
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] time = 3.28682, size = 223, normalized size = 15.93

$$-\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}-\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(cos(x)),x)
```

```
[Out] -log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan
(x/2)**2 + 1) + 2*log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))
*tan(x/2)/(tan(x/2)**2 + 1) - log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x
/2)**2 + 1))/(tan(x/2)**2 + 1) + 2*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)*
*2 + 1) + 2*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 + 1) - 2*tan(
x/2)/(tan(x/2)**2 + 1)
```

Giac [A] time = 1.19677, size = 36, normalized size = 2.57

$$\log\left(\cos\left(x\right)\right)\sin\left(x\right) + \frac{1}{2}\,\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\,\log\left(-\sin\left(x\right) + 1\right) - \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="giac")
```

```
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

3.190 $\int \cos(x)\log(\sin(x))\,dx$

Optimal. Leaf size=11

$$\sin(x)\log(\sin(x)) - \sin(x)$$

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rubi [A] time = 0.0093442, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2637, 2554}

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]],x]

[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x) - \int \cos(x)\,dx\\ &= -\sin(x) + \log(\sin(x))\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0025112, size = 11, normalized size = 1.

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A] time = 0.005, size = 12, normalized size = 1.1

$$-\sin(x) + \ln(\sin(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x)),x)
```

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Maxima [A] time = 1.00768, size = 15, normalized size = 1.36

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Fricas [A] time = 2.12528, size = 39, normalized size = 3.55

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A] time = 0.946213, size = 10, normalized size = 0.91

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Giac [A] time = 1.17447, size = 15, normalized size = 1.36

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

3.191 $\int \log(\sin(x)) \sin^2(x) \, dx$

Optimal. Leaf size=74

$$\frac{1}{4} i \text{PolyLog}\left(2, e^{2ix}\right) + \frac{ix^2}{4} + \frac{x}{4} - \frac{1}{2} x \log\left(1 - e^{2ix}\right) + \frac{1}{2} x \log(\sin(x)) + \frac{1}{4} \sin(x) \cos(x) - \frac{1}{2} \sin(x) \cos(x) \log(\sin(x))$$

```
[Out] x/4 + (I/4)*x^2 - (x*Log[1 - E^((2*I)*x)])/2 + (x*Log[Sin[x]])/2 + (I/4)*Po
lyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (Cos[x]*Log[Sin[x]]*Sin[x])/2
```

Rubi [A] time = 0.109389, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2635, 8, 2554, 12, 6742, 3717, 2190, 2279, 2391}

$$\frac{1}{4} i \text{PolyLog}\left(2, e^{2ix}\right) + \frac{ix^2}{4} + \frac{x}{4} - \frac{1}{2} x \log\left(1 - e^{2ix}\right) + \frac{1}{2} x \log(\sin(x)) + \frac{1}{4} \sin(x) \cos(x) - \frac{1}{2} \sin(x) \cos(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] x/4 + (I/4)*x^2 - (x*Log[1 - E^((2*I)*x)])/2 + (x*Log[Sin[x]])/2 + (I/4)*Po
lyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (Cos[x]*Log[Sin[x]]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(\sin(x))\sin^2(x)\,dx &= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \int \frac{1}{2}\cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \cot(x)(x-\cos(x)\sin(x))\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \left(-\cos^2(x)+x\cot(x)\right)\,dx \\
&= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + \frac{1}{2}\int \cos^2(x)\,dx - \frac{1}{2}\int x\cot(x)\,dx \\
&= \frac{ix^2}{4} + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx + \frac{\int 1}{4} \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) \\
&= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log
\end{aligned}$$

Mathematica [A] time = 0.0555071, size = 59, normalized size = 0.8

$$\frac{1}{8}\left(2i\text{PolyLog}\left(2,e^{2ix}\right) + 2x\left(ix - 2\log\left(1-e^{2ix}\right) + 2\log(\sin(x)) + 1\right) + \sin(2x)(1-2\log(\sin(x)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] (2*x*(1 + I*x - 2*Log[1 - E^((2*I)*x)] + 2*Log[Sin[x]]) + (2*I)*PolyLog[2,
E^((2*I)*x)] + (1 - 2*Log[Sin[x]])*Sin[2*x])/8
```

Maple [B] time = 0.033, size = 146, normalized size = 2.

$$\frac{i}{8}\ln(2\sin(x))e^{2ix} - \frac{i}{16}e^{2ix} - \frac{i}{2}\ln\left(e^{ix}\right)\ln(2\sin(x)) - \frac{i}{4}\left(\ln\left(e^{ix}\right)\right)^2 + \frac{i}{2}\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right) + \frac{i}{2}\text{dilog}\left(e^{ix}+1\right) - \frac{i}{2}\text{dil}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^2,x)
```

```
[Out] 1/8*I*ln(2*sin(x))*exp(2*I*x)-1/16*I*exp(2*I*x)-1/2*I*ln(exp(I*x))*ln(2*sin
(x))-1/4*I*ln(exp(I*x))^2+1/2*I*ln(exp(I*x))*ln(exp(I*x)+1)+1/2*I*dilog(exp
```

```
(I*x)+1)-1/2*I*dilog(exp(I*x))-1/8*I*exp(-2*I*x)*ln(2*sin(x))+1/16*I*exp(-2
*I*x)-1/4*I*ln(exp(I*x))-1/8*I*ln(2)*exp(2*I*x)+1/8*I*ln(2)*exp(-2*I*x)+1/2
*I*ln(2)*ln(exp(I*x))
```

Maxima [B] time = 2.38052, size = 140, normalized size = 1.89

$$\frac{1}{4}ix^2 - \frac{1}{2}ix\arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) + \frac{1}{2}ix\arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) - \frac{1}{4}x\log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\cos\left(x\right) +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*x^2 - 1/2*I*x*arctan2(sin(x), cos(x) + 1) + 1/2*I*x*arctan2(sin(x), -
cos(x) + 1) - 1/4*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/4*x*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 1/4*(2*x - sin(2*x))*log(sin(x)) + 1/4*x
 + 1/2*I*dilog(-e^(I*x)) + 1/2*I*dilog(e^(I*x)) + 1/8*sin(2*x)
```

Fricas [B] time = 2.62214, size = 462, normalized size = 6.24

$$-\frac{1}{4}x\log\left(\cos\left(x\right) + i\sin\left(x\right) + 1\right) - \frac{1}{4}x\log\left(\cos\left(x\right) - i\sin\left(x\right) + 1\right) - \frac{1}{4}x\log\left(-\cos\left(x\right) + i\sin\left(x\right) + 1\right) - \frac{1}{4}x\log\left(-\cos\left(x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*x*log(cos(x) + I*sin(x) + 1) - 1/4*x*log(cos(x) - I*sin(x) + 1) - 1/4*
x*log(-cos(x) + I*sin(x) + 1) - 1/4*x*log(-cos(x) - I*sin(x) + 1) - 1/2*(co
s(x)*sin(x) - x)*log(sin(x)) + 1/4*cos(x)*sin(x) + 1/4*x + 1/4*I*dilog(cos(
x) + I*sin(x)) - 1/4*I*dilog(cos(x) - I*sin(x)) - 1/4*I*dilog(-cos(x) + I*s
in(x)) + 1/4*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\sin\left(x\right)\right)\sin^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**2,x)
```

```
[Out] Integral(log(sin(x))*sin(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log (\sin (x)) \sin (x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(sin(x))*sin(x)^2, x)
```

3.192 $\int \log(\sin(x))\sin^3(x)\,dx$

Optimal. Leaf size=40

$$-\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}-\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

```
[Out] (-2*ArcTanh[Cos[x]])/3 + (2*Cos[x])/3 - Cos[x]^3/9 - Cos[x]*Log[Sin[x]] + (
Cos[x]^3*Log[Sin[x]])/3
```

Rubi [A] time = 0.0671791, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {2633, 2554, 12, 4366, 459, 321, 206}

$$-\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}-\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (-2*ArcTanh[Cos[x]])/3 + (2*Cos[x])/3 - Cos[x]^3/9 - Cos[x]*Log[Sin[x]] + (
Cos[x]^3*Log[Sin[x]])/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \log(\sin(x))\sin^3(x)\,dx = -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \int \frac{1}{6}\cos(x)(-5+\cos(2x))\cot(x)\,dx$$
$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{1}{6}\int \cos(x)(-5+\cos(2x))\cot(x)\,dx$$
$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) + \frac{1}{6}\text{Subst}\left(\int \frac{2x^2\left(-3+x^2\right)}{1-x^2}\,dx, x, \cos(x)\right)$$
$$= -\cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) + \frac{1}{3}\text{Subst}\left(\int \frac{x^2\left(-3+x^2\right)}{1-x^2}\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{9}\cos^3(x) - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{2}{3}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)$$
$$= \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \text{c}\right.$$
$$= -\frac{2}{3}\tanh^{-1}(\cos(x)) + \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x))$$

Mathematica [A] time = 0.0402819, size = 47, normalized size = 1.18

$$\frac{1}{36}\left(24\left(\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)\right) + \cos(3x)(3\log(\sin(x)) - 1) - 3\cos(x)(9\log(\sin(x)) - 7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (24*(-Log[Cos[x/2]] + Log[Sin[x/2]]) + Cos[3*x]*(-1 + 3*Log[Sin[x]]) - 3*Co
s[x]*(-7 + 9*Log[Sin[x]]))/36
```

Maple [C] time = 0.032, size = 134, normalized size = 3.4

$$\frac{e^{3\,ix}\ln(2\,\sin(x))}{24} - \frac{e^{3\,ix}}{72} + \frac{7\,e^{ix}}{24} + \frac{2\,\ln\left(e^{ix}-1\right)}{3} - \frac{2\,\ln\left(e^{ix}+1\right)}{3} - \frac{3\,e^{ix}\ln(2\,\sin(x))}{8} - \frac{3\,e^{-ix}\ln(2\,\sin(x))}{8} + \frac{7\,e^{-ix}}{24} + \text{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^3,x)
```

```
[Out] 1/24*exp(3*I*x)*ln(2*sin(x))-1/72*exp(3*I*x)+7/24*exp(I*x)+2/3*ln(exp(I*x)-
1)-2/3*ln(exp(I*x)+1)-3/8*exp(I*x)*ln(2*sin(x))-3/8*exp(-I*x)*ln(2*sin(x))+
```

```
7/24*exp(-I*x)+1/24*exp(-3*I*x)*ln(2*sin(x))-1/72*exp(-3*I*x)-1/24*ln(2)*ex
p(3*I*x)+3/8*ln(2)*exp(I*x)+3/8*ln(2)*exp(-I*x)-1/24*ln(2)*exp(-3*I*x)
```

Maxima [B] time = 1.00537, size = 242, normalized size = 6.05

$$-\frac{4\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+1\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{3\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}+\frac{2\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{9\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}-\frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*(3*sin(x)^2/(cos(x) + 1)^2 + 1)*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2
 + 1)*(cos(x) + 1)))/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1)^4
 + sin(x)^6/(cos(x) + 1)^6 + 1) + 2/9*(12*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x
)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1
)^4 + sin(x)^6/(cos(x) + 1)^6 + 1) - 2/3*log(sin(x)^2/(cos(x) + 1)^2 + 1) +
 2/3*log(sin(x)^2/(cos(x) + 1)^2)
```

Fricas [A] time = 2.24953, size = 169, normalized size = 4.22

$$-\frac{1}{9}\cos(x)^3+\frac{1}{3}\left(\cos(x)^3-3\cos(x)\right)\log(\sin(x))+\frac{2}{3}\cos(x)-\frac{1}{3}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{3}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(1/2*cos(x) + 1/2) + 1/3*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 15.1769, size = 445, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**3,x)
```

```
[Out] -6*log(tan(x/2)**2 + 1)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*ta
n(x/2)**2 + 9) - 18*log(tan(x/2)**2 + 1)*tan(x/2)**4/(9*tan(x/2)**6 + 27*ta
n(x/2)**4 + 27*tan(x/2)**2 + 9) + 18*log(tan(x/2)**2 + 1)*tan(x/2)**2/(9*ta
n(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 + 1)/(
9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*log(tan(x/2))*tan
(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*log(tan
(x/2))*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) -
10*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*l
og(2)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) - 2
4*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*lo
g(2)*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) - 6*
tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9)
```

Giac [A] time = 1.29915, size = 55, normalized size = 1.38

$$-\frac{1}{9}\cos(x)^3+\frac{1}{3}\left(\cos(x)^3-3\cos(x)\right)\log(\sin(x))+\frac{2}{3}\cos(x)-\frac{1}{3}\log(\cos(x)+1)+\frac{1}{3}\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(cos(x) + 1) + 1/3*log(-cos(x) + 1)
```

3.193 $\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=79

$$i\sqrt{x}\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

```
[Out] (I/3)*x^(3/2) - x*Log[1 - E^((2*I)*Sqrt[x])] + x*Log[Sin[Sqrt[x]]] + I*Sqrt
[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*I)*Sqrt[x])]/2
```

Rubi [A] time = 0.102098, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {2548, 12, 3748, 3717, 2190, 2531, 2282, 6589}

$$i\sqrt{x}\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/3)*x^(3/2) - x*Log[1 - E^((2*I)*Sqrt[x])] + x*Log[Sin[Sqrt[x]]] + I*Sqrt
[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*I)*Sqrt[x])]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3748

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx &= x\log\left(\sin\left(\sqrt{x}\right)\right) - \int \frac{1}{2}\sqrt{x}\cot\left(\sqrt{x}\right)\,dx \\
&= x\log\left(\sin\left(\sqrt{x}\right)\right) - \frac{1}{2}\int \sqrt{x}\cot\left(\sqrt{x}\right)\,dx \\
&= x\log\left(\sin\left(\sqrt{x}\right)\right) - \text{Subst}\left(\int x^2\cot(x)\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{3}ix^{3/2} + x\log\left(\sin\left(\sqrt{x}\right)\right) + 2i\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + 2\,\text{Subst}\left(\int x\log\left(1-e^{2ix}\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right) - i\,\text{Subst}\left(\int \text{Li}_2\left(e^{2ix}\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx, x, e^{2i\sqrt{x}}\right) \\
&= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\text{Li}_2\left(e^{2i\sqrt{x}}\right) - \frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0324164, size = 88, normalized size = 1.11

$$
-i\sqrt{x}\text{PolyLog}\left(2, e^{-2i\sqrt{x}}\right) - \frac{1}{2}\text{PolyLog}\left(3, e^{-2i\sqrt{x}}\right) - \frac{1}{3}ix^{3/2} - x\log\left(1-e^{-2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + \frac{i\pi^3}{24}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/24)*Pi^3 - (I/3)*x^(3/2) - x*Log[1 - E^((-2*I)*Sqrt[x])] + x*Log[Sin[Sqr
t[x]]] - I*Sqrt[x]*PolyLog[2, E^((-2*I)*Sqrt[x])] - PolyLog[3, E^((-2*I)*Sq
rt[x])]/2
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$
\int \ln\left(\sin\left(\sqrt{x}\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x^(1/2))),x)
```

```
[Out] int(ln(sin(x^(1/2))),x)
```

Maxima [B] time = 1.06539, size = 188, normalized size = 2.38

$$-ix\arctan\left(\sin\left(\sqrt{x}\right),\cos\left(\sqrt{x}\right)+1\right)+ix\arctan\left(\sin\left(\sqrt{x}\right),-\cos\left(\sqrt{x}\right)+1\right)-\frac{1}{2}x\log\left(\cos\left(\sqrt{x}\right)^2+\sin\left(\sqrt{x}\right)^2+2\cos\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="maxima")
```

```
[Out] -I*x*arctan2(sin(sqrt(x)), cos(sqrt(x)) + 1) + I*x*arctan2(sin(sqrt(x)), -c
os(sqrt(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 + 2*cos(sqrt(x
)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 - 2*cos(sqrt(x)) + 1) +
 x*log(sin(sqrt(x))) + 1/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(I*sqrt(x))) +
2*I*sqrt(x)*dilog(e^(I*sqrt(x))) - 2*polylog(3, -e^(I*sqrt(x))) - 2*polylog
(3, e^(I*sqrt(x)))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\log\left(\sin\left(\sqrt{x}\right)\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(log(sin(sqrt(x))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x**(1/2))),x)
```

```
[Out] Integral(log(sin(sqrt(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\sin\left(\sqrt{x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(log(sin(sqrt(x))), x)
```

3.194 $\int \csc^2(x) \log(\sin(x)) \, dx$

Optimal. Leaf size=15

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rubi [A] time = 0.0205751, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3767, 8, 2554, 3473}

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \csc^2(x)\log(\sin(x))\,dx = -\cot(x)\log(\sin(x)) + \int \cot^2(x)\,dx$$
$$= -\cot(x) - \cot(x)\log(\sin(x)) - \int 1\,dx$$
$$= -x - \cot(x) - \cot(x)\log(\sin(x))$$

Mathematica [A] time = 0.0143132, size = 15, normalized size = 1.

$$-x - \cot(x) - \cot(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Maple [C] time = 0.055, size = 72, normalized size = 4.8

$$\frac{-2\,i\ln(2\,\sin(x))\,e^{2\,ix}}{e^{2\,ix}-1} - \frac{2\,i}{e^{2\,ix}-1} + i\ln\left(e^{ix}-1\right) + i\ln\left(e^{ix}+1\right) + \frac{2\,i\ln(2)}{e^{2\,ix}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*ln(sin(x)),x)
```

```
[Out] -2*I/(exp(2*I*x)-1)*ln(2*sin(x))*exp(2*I*x)-2*I/(exp(2*I*x)-1)+I*ln(exp(I*x
)-1)+I*ln(exp(I*x)+1)+2*I*ln(2)/(exp(2*I*x)-1)
```

Maxima [B] time = 1.50098, size = 109, normalized size = 7.27

$$-\frac{1}{2}\left(\frac{\cos(x)+1}{\sin(x)} - \frac{\sin(x)}{\cos(x)+1}\right)\log\left(\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right) - \frac{\cos(x)+1}{2\,\sin(x)} + \frac{\sin(x)}{2\,(\cos(x)+1)} - 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(x) + 1)/sin(x) - sin(x)/(cos(x) + 1))*log(2*sin(x)/((sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))) - 1/2*(cos(x) + 1)/sin(x) + 1/2*sin(x)/(co
s(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.22429, size = 68, normalized size = 4.53

$$-\frac{\cos(x)\log(\sin(x)) + x\sin(x) + \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x)*log(sin(x)) + x*sin(x) + cos(x))/sin(x)
```

Sympy [A] time = 106.402, size = 17, normalized size = 1.13

$$-x - \log(\sin(x))\cot(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*ln(sin(x)),x)
```

```
[Out] -x - log(sin(x))*cot(x) - cos(x)/sin(x)
```

Giac [A] time = 1.30822, size = 26, normalized size = 1.73

$$-x - \frac{\log(\sin(x))}{\tan(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="giac")
```

```
[Out] -x - log(sin(x))/tan(x) - 1/tan(x)
```

3.195 $\int \log(x)\sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{\cosh(a)\mathbf{Chi}(bx)}{b}-\frac{\sinh(a)\mathbf{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

```
[Out] -((Cosh[a]*CoshIntegral[b*x])/b) + (Cosh[a + b*x]*Log[x])/b - (Sinh[a]*Sinh
Integral[b*x])/b
```

Rubi [A] time = 0.07876, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2638, 2554, 12, 3303, 3298, 3301}

$$-\frac{\cosh(a)\mathbf{Chi}(bx)}{b}-\frac{\sinh(a)\mathbf{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x])/b) + (Cosh[a + b*x]*Log[x])/b - (Sinh[a]*Sinh
Integral[b*x])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(x)\sinh(a+bx)\,dx &= \frac{\cosh(a+bx)\log(x)}{b} - \int \frac{\cosh(a+bx)}{bx}\,dx \\
&= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b} \\
&= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} \\
&= -\frac{\cosh(a)\mathbf{Chi}(bx)}{b} + \frac{\cosh(a+bx)\log(x)}{b} - \frac{\sinh(a)\mathbf{Shi}(bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0442781, size = 30, normalized size = 0.86

$$-\frac{\cosh(a)\mathbf{Chi}(bx)+\sinh(a)\mathbf{Shi}(bx)-\log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x] - Cosh[a + b*x]*Log[x] + Sinh[a]*SinhIntegral[
b*x])/b)
```

Maple [A] time = 0.035, size = 58, normalized size = 1.7

$$\left(\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}\right)\ln\left(x\right)+\frac{e^{-a}\mathrm{Ei}\left(1,bx\right)}{2b}+\frac{e^{a}\mathrm{Ei}\left(1,-bx\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a),x)
```

```
[Out] (1/2/b*exp(b*x+a)+1/2/b*exp(-b*x-a))*ln(x)+1/2/b*exp(-a)*Ei(1,b*x)+1/2/b*ex
p(a)*Ei(1,-b*x)
```

Maxima [A] time = 1.16335, size = 49, normalized size = 1.4

$$\frac{\cosh\left(bx+a\right)\log\left(x\right)}{b}-\frac{\mathrm{Ei}\left(-bx\right)e^{(-a)}+\mathrm{Ei}\left(bx\right)e^{a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)*log(x)/b - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x)*e^a)/b
```

Fricas [B] time = 2.099, size = 393, normalized size = 11.23

$$-\frac{(\mathrm{Ei}\left(bx\right)+\mathrm{Ei}\left(-bx\right))\cosh\left(bx+a\right)\cosh\left(a\right)-\log\left(x\right)\sinh\left(bx+a\right)^{2}+(\mathrm{Ei}\left(bx\right)-\mathrm{Ei}\left(-bx\right))\cosh\left(bx+a\right)\sinh\left(a\right)-\left(\cos\right.}{2\left(b\cosh\left(bx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 + 1)*log(x)
+ ((Ei(b*x) + Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) - Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\sinh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a),x)
```

```
[Out] Integral(log(x)*sinh(a + b*x), x)
```

Giac [A] time = 1.32147, size = 70, normalized size = 2.

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)\log(x)-\frac{\text{Ei}(-bx)\,e^{(-a)}+\text{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b + e^(-b*x - a)/b)*log(x) - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x
)*e^a)/b
```

3.196 $\int \log(x) \sinh^2(a+bx) \, dx$

Optimal. Leaf size=66

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

```
[Out] x/2 - (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x]
*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rubi [A] time = 0.142901, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 5274, 3303, 3298, 3301}

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^2,x]
```


```
[Out] x/2 - (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x]
*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```


Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5274

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \log(x)\sinh^2(a+bx)\,dx = -\frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \int \frac{1}{4}\left(-2+\frac{\sinh(2(a+bx))}{bx}\right)dx$$
$$= -\frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{1}{4}\int \left(-2+\frac{\sinh(2(a+bx))}{bx}\right)dx$$
$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int \frac{\sinh(2(a+bx))}{x}\,dx}{4b}$$
$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\int \frac{\sinh(2a+2bx)}{x}\,dx}{4b}$$
$$= \frac{x}{2} - \frac{1}{2}x\log(x) + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx}{4b} - \frac{\sinh(2a)\int}{4}$$
$$= \frac{x}{2} - \frac{1}{2}x\log(x) - \frac{\text{Chi}(2bx)\sinh(2a)}{4b} + \frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b} - \frac{\cosh(2a)\text{Shi}(2b}{4b}$$

Mathematica [A] time = 0.09926, size = 50, normalized size = 0.76

$$-\frac{\sinh(2a)\text{Chi}(2bx) + \cosh(2a)\text{Shi}(2bx) - \log(x)\sinh(2(a+bx)) - 2bx + 2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] -(-2*b*x + 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2*(a
+ b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Maple [A] time = 0.036, size = 97, normalized size = 1.5

$$\left(-\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}\right)\ln(x)+\frac{e^{2a}\text{Ei}(1,-2bx)}{8b}-\frac{a\ln(bx)}{2b}+\frac{a\ln(-bx)}{2b}+\frac{x}{2}+\frac{a}{2b}-\frac{e^{-2a}\text{Ei}(1,2bx)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^2,x)
```

```
[Out] (-1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei
(1,-2*b*x)-1/2/b*a*ln(b*x)+1/2/b*a*ln(-b*x)+1/2*x+1/2*a/b-1/8/b*exp(-2*a)*E
i(1,2*b*x)
```

Maxima [A] time = 1.15357, size = 90, normalized size = 1.36

$$-\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log\left(x\right)+\frac{1}{2}\,x-\frac{\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/2*x - 1/8*Ei
(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Fricas [B] time = 1.85988, size = 859, normalized size = 13.02

$$\frac{4\,\cosh\left(bx+a\right)\log\left(x\right)\sinh\left(bx+a\right)^{3}+\log\left(x\right)\sinh\left(bx+a\right)^{4}-\left(\mathrm{Ei}\left(2\,bx\right)+\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(bx+a\right)^{2}\sinh\left(2\,a\right)+\left(4\,bx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) + (4*b*x - (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 + (4*b*x - (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x - 3*cosh(b*x + a)^2)*log(x) - (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 - (4*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 + 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) - (4*b*x - (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) + 2*(2*b*x*cosh(b*x + a) - cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x\right)\sinh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**2, x)
```

Giac [A] time = 1.37262, size = 123, normalized size = 1.86

$$-\frac{\left(4\,bx - \left(2\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 4\,a - e^{(2\,bx+2\,a)}\right)\log\left(x\right)}{8\,b} + \frac{4\,bx - \mathrm{Ei}\left(2\,bx\right)e^{(2\,a)} + \mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)} + 4\,a\log\left(x\right) - 2}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b*x - (2*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 4*a - e^(2*b*x + 2
*a))*log(x)/b + 1/8*(4*b*x - Ei(2*b*x)*e^(2*a) + Ei(-2*b*x)*e^(-2*a) + 4*a*
log(x) - 2*log(x))/b
```

3.197 $\int \log(x)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cosh(a)\mathbf{Chi}(bx)}{4b}-\frac{\cosh(3a)\mathbf{Chi}(3bx)}{12b}+\frac{3\sinh(a)\mathbf{Shi}(bx)}{4b}-\frac{\sinh(3a)\mathbf{Shi}(3bx)}{12b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a}{b}$$

```
[Out] (3*Cosh[a]*CoshIntegral[b*x])/(4*b) - (Cosh[3*a]*CoshIntegral[3*b*x])/(12*b
) - (Cosh[a + b*x]*Log[x])/b + (Cosh[a + b*x]^3*Log[x])/(3*b) + (3*Sinh[a]*
SinhIntegral[b*x])/(4*b) - (Sinh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rubi [A] time = 0.517013, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3298, 3301, 3312}

$$\frac{3\cosh(a)\mathbf{Chi}(bx)}{4b}-\frac{\cosh(3a)\mathbf{Chi}(3bx)}{12b}+\frac{3\sinh(a)\mathbf{Shi}(bx)}{4b}-\frac{\sinh(3a)\mathbf{Shi}(3bx)}{12b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x])/(4*b) - (Cosh[3*a]*CoshIntegral[3*b*x])/(12*b
) - (Cosh[a + b*x]*Log[x])/b + (Cosh[a + b*x]^3*Log[x])/(3*b) + (3*Sinh[a]*
SinhIntegral[b*x])/(4*b) - (Sinh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \log(x)\sinh^3(a+bx)\,dx = -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{3bx}\,dx$$
$$= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{x}\,dx}{3b}$$
$$= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \left(-\frac{3\cosh(a+bx)}{x} + \frac{\cosh^3(a+bx)}{x}\right)dx}{3b}$$
$$= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \frac{\cosh^3(a+bx)}{x}\,dx}{3b} + \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b}$$
$$= -\frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} - \frac{\int \left(\frac{3\cosh(a+bx)}{4x} + \frac{\cosh(3a+3bx)}{4x}\right)dx}{3b} + \frac{\cosh(a)}{}$$
$$= \frac{\cosh(a)\mathbf{Chi}(bx)}{b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} + \frac{\sinh(a)\mathbf{Shi}(bx)}{b} - \frac{\int \frac{\cosh(}{}}{1}$$
$$= \frac{\cosh(a)\mathbf{Chi}(bx)}{b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} + \frac{\sinh(a)\mathbf{Shi}(bx)}{b} - \frac{\cosh(a}{}$$
$$= \frac{3\cosh(a)\mathbf{Chi}(bx)}{4b} - \frac{\cosh(3a)\mathbf{Chi}(3bx)}{12b} - \frac{\cosh(a+bx)\log(x)}{b} + \frac{\cosh^3(a+bx)\log(x)}{3b} + \frac{3\,s}{}$$

Mathematica [A] time = 0.100467, size = 67, normalized size = 0.75

$$\frac{9\cosh(a)\mathbf{Chi}(bx) - \cosh(3a)\mathbf{Chi}(3bx) + 9\sinh(a)\mathbf{Shi}(bx) - \sinh(3a)\mathbf{Shi}(3bx) - 9\log(x)\cosh(a+bx) + \log(x)\cosh(3(a}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a]*CoshIntegral[b*x] - Cosh[3*a]*CoshIntegral[3*b*x] - 9*Cosh[a + b
*x]*Log[x] + Cosh[3*(a + b*x)]*Log[x] + 9*Sinh[a]*SinhIntegral[b*x] - Sinh[
3*a]*SinhIntegral[3*b*x])/(12*b)
```

Maple [A] time = 0.034, size = 116, normalized size = 1.3

$$\left(\frac{e^{3\,bx+3\,a}}{24\,b} - \frac{3\,e^{bx+a}}{8\,b} - \frac{3\,e^{-bx-a}}{8\,b} + \frac{e^{-3\,bx-3\,a}}{24\,b}\right)\ln(x) + \frac{e^{-3\,a}\mathrm{Ei}(1,3\,bx)}{24\,b} + \frac{e^{3\,a}\mathrm{Ei}(1,-3\,bx)}{24\,b} - \frac{3\,e^{-a}\mathrm{Ei}(1,bx)}{8\,b} - \frac{3\,e^{a}\mathrm{Ei}(1,-bx)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^3,x)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)-3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)+1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(-3*a)*Ei(1,3*b*x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)-3/8/b*exp(a)*Ei(1,-b*x)
```

Maxima [A] time = 1.24853, size = 149, normalized size = 1.67

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b} - \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} + \frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x) - \frac{\mathrm{Ei}\,(3\,bx)\,e^{(3\,a)}}{24\,b} + \frac{3\,\mathrm{Ei}\,(-bx)\,e^{(-a)}}{8\,b} - \frac{\mathrm{Ei}\,(-3\,bx)\,e^{(-3\,a)}}{24\,b} + \frac{3\,\mathrm{Ei}\,(}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b - 1/24*Ei
(-3*b*x)*e^(-3*a)/b + 3/8*Ei(b*x)*e^a/b
```

Fricas [B] time = 1.94033, size = 1669, normalized size = 18.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) - Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 - 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) - Ei(-
3*b*x))*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 - 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 + 1)*log(x) - 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
```

```
a) - 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) + Ei(-3*b
*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 - 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**3, x)
```

Giac [A] time = 1.33102, size = 130, normalized size = 1.46

$$-\frac{\left(\left(9\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}\right)\log(x)}{24\,b}-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(-a)}+\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-9\,\mathrm{Ei}(bx}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) + 9*e^(b*
x + a))*log(x)/b - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) + Ei(-3*b*x)
*e^(-3*a) - 9*Ei(b*x)*e^a)/b
```

3.198 $\int \cosh(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\sinh(a)\mathbf{Chi}(bx)}{b}-\frac{\cosh(a)\mathbf{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

```
[Out] -((CoshIntegral[b*x]*Sinh[a])/b) + (Log[x]*Sinh[a + b*x])/b - (Cosh[a]*Sinh
Integral[b*x])/b
```

Rubi [A] time = 0.070019, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2637, 2554, 12, 3303, 3298, 3301}

$$-\frac{\sinh(a)\mathbf{Chi}(bx)}{b}-\frac{\cosh(a)\mathbf{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a])/b) + (Log[x]*Sinh[a + b*x])/b - (Cosh[a]*Sinh
Integral[b*x])/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \cosh(a+bx)\log(x)\,dx &= \frac{\log(x)\sinh(a+bx)}{b} - \int \frac{\sinh(a+bx)}{bx}\,dx \\
&= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b} \\
&= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} \\
&= -\frac{\mathrm{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\mathrm{Shi}(bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.0379113, size = 30, normalized size = 0.86

$$-\frac{\sinh(a)\mathrm{Chi}(bx)+\cosh(a)\mathrm{Shi}(bx)-\log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a] - Log[x]*Sinh[a + b*x] + Cosh[a]*SinhIntegral[
b*x])/b)
```

Maple [A] time = 0.019, size = 58, normalized size = 1.7

$$\left(\frac{\mathrm{e}^{bx+a}}{2b}-\frac{\mathrm{e}^{-bx-a}}{2b}\right)\ln(x)+\frac{\mathrm{e}^{a}\mathrm{Ei}(1,-bx)}{2b}-\frac{\mathrm{e}^{-a}\mathrm{Ei}(1,bx)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(x),x)
```

```
[Out] (1/2/b*exp(b*x+a)-1/2/b*exp(-b*x-a))*ln(x)+1/2/b*exp(a)*Ei(1,-b*x)-1/2/b*ex
p(-a)*Ei(1,b*x)
```

Maxima [A] time = 1.189, size = 50, normalized size = 1.43

$$\frac{\log(x)\sinh(bx+a)}{b}+\frac{\mathrm{Ei}(-bx)e^{(-a)}-\mathrm{Ei}(bx)e^{a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sinh(b*x + a)/b + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x)*e^a)/b
```

Fricas [B] time = 1.78763, size = 393, normalized size = 11.23

$$-\frac{(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\cosh(bx+a)\cosh(a)-\log(x)\sinh(bx+a)^{2}+(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\cosh(bx+a)\sinh(a)-\left(\cos\right.}{2\,(b\cosh(bx\,+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 - 1)*log(x)
+ ((Ei(b*x) - Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) + Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log{(x)}\cosh{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x), x)
```

Giac [A] time = 1.26758, size = 73, normalized size = 2.09

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b} - \frac{e^{(-bx-a)}}{b}\right)\log{(x)} + \frac{\text{Ei}\,(-bx)\,e^{(-a)} - \text{Ei}\,(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(x) + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x
)*e^a)/b
```

3.199 $\int \cosh^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sinh(2a)\mathrm{Chi}(2bx)}{4b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

```
[Out] -x/2 + (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x
]*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rubi [A] time = 0.131783, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2635, 8, 2554, 12, 5274, 3303, 3298, 3301}

$$-\frac{\sinh(2a)\mathrm{Chi}(2bx)}{4b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -x/2 + (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x
]*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5274

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\int \frac{1}{4}\left(2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{1}{4}\int \left(2+\frac{\sinh(2(a+bx))}{bx}\right)dx\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2(a+bx))}{x}dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2a+2bx)}{x}dx}{4b}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\int \frac{\sinh(2bx)}{x}dx}{4b}-\frac{\sinh(2a)}{}\\ &= -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\text{Shi}(}{4b}\end{aligned}$$

Mathematica [A] time = 0.0653197, size = 50, normalized size = 0.76

$$-\frac{\sinh(2a)\text{Chi}(2bx)+\cosh(2a)\text{Shi}(2bx)-\log(x)\sinh(2(a+bx))+2bx-2bx\log(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -(2*b*x - 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2*(a +
 b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Maple [A] time = 0.024, size = 97, normalized size = 1.5

$$\left(\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}\right)\ln(x)+\frac{e^{2a}\text{Ei}(1,-2bx)}{8b}+\frac{a\ln(bx)}{2b}-\frac{a\ln(-bx)}{2b}-\frac{x}{2}-\frac{a}{2b}-\frac{e^{-2a}\text{Ei}(1,2bx)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*ln(x),x)
```

```
[Out] (1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei(
1,-2*b*x)+1/2/b*a*ln(b*x)-1/2/b*a*ln(-b*x)-1/2*x-1/2*a/b-1/8/b*exp(-2*a)*Ei
(1,2*b*x)
```

Maxima [A] time = 1.18093, size = 90, normalized size = 1.36

$$\frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log\left(x\right)-\frac{1}{2}\,x-\frac{\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/2*x - 1/8*Ei(
2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Fricas [B] time = 1.97401, size = 859, normalized size = 13.02

$$\frac{4\,\cosh\left(bx+a\right)\log\left(x\right)\sinh\left(bx+a\right)^{3}+\log\left(x\right)\sinh\left(bx+a\right)^{4}-\left(\mathrm{Ei}\left(2\,bx\right)+\mathrm{Ei}\left(-2\,bx\right)\right)\cosh\left(bx+a\right)^{2}\sinh\left(2\,a\right)-\left(4\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) - (4*b*x + (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 - (4*b*x + (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x + 3*cosh(b*x + a)^2)*log(x) + (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 + (4*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 - 1)*log
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) + (4*b*x + (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) - 2*(2*b*x*cosh(b*x + a) + cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x\right)\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**2, x)
```

Giac [A] time = 1.31895, size = 120, normalized size = 1.82

$$\frac{\left(4\,bx-\left(2\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+4\,a+e^{(2\,bx+2\,a)}\right)\log\left(x\right)}{8\,b}-\frac{4\,bx+\mathrm{Ei}\left(2\,bx\right)e^{(2\,a)}-\mathrm{Ei}\left(-2\,bx\right)e^{(-2\,a)}+4\,a\log\left(x\right)-2\,\mathrm{lo}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/8*(4*b*x - (2*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 4*a + e^(2*b*x + 2*
a))*log(x)/b - 1/8*(4*b*x + Ei(2*b*x)*e^(2*a) - Ei(-2*b*x)*e^(-2*a) + 4*a*l
og(x) - 2*log(x))/b
```

3.200 $\int \cosh^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\sinh(a)\mathrm{Chi}(bx)}{4b}-\frac{\sinh(3a)\mathrm{Chi}(3bx)}{12b}-\frac{3\cosh(a)\mathrm{Shi}(bx)}{4b}-\frac{\cosh(3a)\mathrm{Shi}(3bx)}{12b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(}{b}$$

```
[Out] (-3*CoshIntegral[b*x]*Sinh[a])/(4*b) - (CoshIntegral[3*b*x]*Sinh[3*a])/(12*
b) + (Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - (3*Cosh[a]
*SinhIntegral[b*x])/(4*b) - (Cosh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rubi [A] time = 0.483412, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2633, 2554, 12, 6742, 3303, 3298, 3301, 3312}

$$-\frac{3\sinh(a)\mathrm{Chi}(bx)}{4b}-\frac{\sinh(3a)\mathrm{Chi}(3bx)}{12b}-\frac{3\cosh(a)\mathrm{Shi}(bx)}{4b}-\frac{\cosh(3a)\mathrm{Shi}(3bx)}{12b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CoshIntegral[b*x]*Sinh[a])/(4*b) - (CoshIntegral[3*b*x]*Sinh[3*a])/(12*
b) + (Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - (3*Cosh[a]
*SinhIntegral[b*x])/(4*b) - (Cosh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx)\right)}{3bx}\,dx$$
$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx)\right)}{x}\,dx}{3b}$$
$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \left(\frac{3\sinh(a+bx)}{x} + \frac{\sinh^3(a+bx)}{x}\right)dx}{3b}$$
$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh^3(a+bx)}{x}\,dx}{3b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}$$
$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{i\int \left(\frac{3i\sinh(a+bx)}{4x} - \frac{i\sinh(3a+3bx)}{4x}\right)dx}{3b} - \frac{\cosh(a)}{}$$
$$= -\frac{\mathbf{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\mathbf{Shi}(bx)}{b} - \int \frac{\sinh}{}$$
$$= -\frac{\mathbf{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\mathbf{Shi}(bx)}{b} + \frac{\cosh}{}$$
$$= -\frac{3\mathbf{Chi}(bx)\sinh(a)}{4b} - \frac{\mathbf{Chi}(3bx)\sinh(3a)}{12b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{3}{}$$

Mathematica [A] time = 0.141135, size = 66, normalized size = 0.75

$$-\frac{9\sinh(a)\mathbf{Chi}(bx) + \sinh(3a)\mathbf{Chi}(3bx) + 9\cosh(a)\mathbf{Shi}(bx) + \cosh(3a)\mathbf{Shi}(3bx) - 9\log(x)\sinh(a+bx) - \log(x)\sinh(3}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] -(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] - 9*Log[x]*Si
nh[a + b*x] - Log[x]*Sinh[3*(a + b*x)] + 9*Cosh[a]*SinhIntegral[b*x] + Cosh
[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Maple [A] time = 0.035, size = 116, normalized size = 1.3

$$\left(\frac{e^{3bx+3a}}{24b} + \frac{3e^{bx+a}}{8b} - \frac{3e^{-bx-a}}{8b} - \frac{e^{-3bx-3a}}{24b}\right)\ln(x) + \frac{e^{3a}\mathrm{Ei}(1,-3bx)}{24b} - \frac{e^{-3a}\mathrm{Ei}(1,3bx)}{24b} - \frac{3e^{-a}\mathrm{Ei}(1,bx)}{8b} + \frac{3e^{a}\mathrm{Ei}(1,-bx)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*ln(x),x)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)+3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)-1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-1/24/b*exp(-3*a)*Ei(1,3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)+3/8/b*exp(a)*Ei(1,-b*x)
```

Maxima [A] time = 1.23044, size = 150, normalized size = 1.7

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log\left(x\right)-\frac{\text{Ei}\left(3\,bx\right)e^{(3\,a)}}{24\,b}+\frac{3\,\text{Ei}\left(-bx\right)e^{(-a)}}{8\,b}+\frac{\text{Ei}\left(-3\,bx\right)e^{(-3\,a)}}{24\,b}-\frac{3\,\text{Ei}\,(}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b + 1/24*Ei
(-3*b*x)*e^(-3*a)/b - 3/8*Ei(b*x)*e^a/b
```

Fricas [B] time = 2.00087, size = 1669, normalized size = 18.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 + 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) + Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 + 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) + Ei(-
3*b*x))*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 + 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 - 1)*log(x) - 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
```

```
a) + 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) - Ei(-3*b
*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 + 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**3, x)
```

Giac [A] time = 1.31934, size = 131, normalized size = 1.49

$$-\frac{\left(\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}-9\,e^{(bx+a)}\right)\log(x)}{24\,b}-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(-a)}-\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}+9\,\mathrm{Ei}(bx}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))*log(x)/b - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) - Ei(-3*b*x)
*e^(-3*a) + 9*Ei(b*x)*e^a)/b
```

3.201 $\int \log(a\sinh(x))\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\sinh(x))+\frac{x^2}{2}-x\log\left(1-e^{2x}\right)$$

```
[Out] x^2/2 - x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]] - PolyLog[2, E^(2*x)]/2
```

Rubi [A] time = 0.0594839, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\sinh(x))+\frac{x^2}{2}-x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]] - PolyLog[2, E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\sinh(x))\,dx &= x\log(a\sinh(x)) - \int x\coth(x)\,dx\\
&= \frac{x^2}{2} + x\log(a\sinh(x)) + 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \int \log\left(1-e^{2x}\right)\,dx\\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right)\\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) - \frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}$$

Mathematica [A] time = 0.0203845, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(\operatorname{PolyLog}\left(2,e^{-2x}\right) - x\left(-2\log(a\sinh(x)) + x + 2\log\left(1-e^{-2x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]])) + PolyLog[2, E^(-2*x)])/
2
```

Maple [C] time = 0.126, size = 295, normalized size = 7.6

$$-x\ln\left(e^{x}\right) + \frac{i}{2}\pi\,\mathrm{csgn}\left(i\left(e^{2x}-1\right)\right)\left(\mathrm{csgn}\left(ie^{-x}\left(e^{2x}-1\right)\right)\right)^{2}x - \frac{i}{2}\pi\left(\mathrm{csgn}\left(ia\left(e^{2x}-1\right)e^{-x}\right)\right)^{3}x - \frac{i}{2}\pi\,\mathrm{csgn}\left(i\left(e^{2x}-1\right)\right)\mathrm{cs}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)),x)
```

```
[Out] -x*ln(exp(x))+1/2*I*Pi*csgn(I*(exp(2*x)-1))*csgn(I*exp(-x)*(exp(2*x)-1))^2*
x-1/2*I*Pi*csgn(I*a*(exp(2*x)-1)*exp(-x))^3*x-1/2*I*Pi*csgn(I*(exp(2*x)-1))
*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)-1))*x+1/2*I*Pi*csgn(I*exp(-x))*cs
gn(I*exp(-x)*(exp(2*x)-1))^2*x+1/2*I*Pi*csgn(I*a*(exp(2*x)-1)*exp(-x))^2*cs
gn(I*a)*x-1/2*I*Pi*csgn(I*exp(-x)*(exp(2*x)-1))^3*x+ln(a)*x-ln(2)*x+1/2*x^2
+1/2*I*Pi*csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x))^2*x-1
/2*I*Pi*csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x))*csgn(I*
a)*x+ln(exp(x))*ln(exp(2*x)-1)+dilog(exp(x))-dilog(exp(x)+1)-ln(exp(x))*ln(
exp(x)+1)
```

Maxima [A] time = 1.2852, size = 58, normalized size = 1.49

$$\frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(e^x + 1\right) - x\log\left(-e^x + 1\right) - \mathrm{Li}_2\left(-e^x\right) - \mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(e^x + 1) - x*log(-e^x + 1) - dilog(-e^x)
 - dilog(e^x)
```

Fricas [A] time = 2.02053, size = 197, normalized size = 5.05

$$\frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) - \mathrm{Li}_2\left(\cosh\left(x\right) + \sinh\left(x\right)\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(cosh(x) + sinh(x) + 1) - x*log(-cosh(x)
- sinh(x) + 1) - dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)),x)
```

```
[Out] Integral(log(a*sinh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)), x)
```

3.202 $\int \log\left(a\sinh^2(x)\right)\,dx$

Optimal. Leaf size=35

$$-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^2(x)\right)+x^2-2x\log\left(1-e^{2x}\right)$$

```
[Out] x^2 - 2*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^2] - PolyLog[2, E^(2*x)]
```

Rubi [A] time = 0.060352, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^2(x)\right)+x^2-2x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^2] - PolyLog[2, E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sinh^2(x)\right)\,dx &= x\log\left(a\sinh^2(x)\right) - \int 2x\coth(x)\,dx \\
&= x\log\left(a\sinh^2(x)\right) - 2\int x\coth(x)\,dx \\
&= x^2 + x\log\left(a\sinh^2(x)\right) + 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + 2\int \log\left(1-e^{2x}\right)\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + \text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) - \text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0182854, size = 33, normalized size = 0.94

$$\text{PolyLog}\left(2,e^{-2x}\right) + x\left(\log\left(a\sinh^2(x)\right) - x - 2\log\left(1-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 - E^(-2*x)] + Log[a*Sinh[x]^2]) + PolyLog[2, E^(-2*x)]
```

Maple [C] time = 0.157, size = 454, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^2),x)
```

```
[Out] 2*ln(exp(x))*ln(exp(2*x)-1)-2*ln(exp(x))*ln(exp(x)+1)-2*x*ln(exp(x))-2*ln(2
)*x+1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^2*x+ln(a)*x
+1/2*I*Pi*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))^2*csgn(I*a)*x+2*dilog(exp(x))-
2*dilog(exp(x)+1)+x^2-I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x-1/2*I*Pi*csg
n(I*(exp(2*x)-1)^2)^3*x-1/2*I*Pi*csgn(I*(exp(2*x)-1))^2*csgn(I*(exp(2*x)-1)
^2)*x+1/2*I*Pi*csgn(I*(exp(2*x)-1)^2)*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^2*x+
1/2*I*Pi*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x)
)^2*x+1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x-1/2*I*Pi*csgn(I*(exp(2*x
)-1)^2)*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)*x+I*Pi*csgn(I*(e
xp(2*x)-1))*csgn(I*(exp(2*x)-1)^2)^2*x+1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*
Pi*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^3*x-1/2*I*Pi*csgn(I*exp(-2*x)*(exp(2*x)
-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))*csgn(I*a)*x-1/2*I*Pi*csgn(I*a*(ex
p(2*x)-1)^2*exp(-2*x))^3*x
```

Maxima [A] time = 1.2536, size = 58, normalized size = 1.66

$$x^2 + x\log\left(a\sinh\left(x\right)^2\right) - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2\left(-e^x\right) - 2\,\mathrm{Li}_2\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x^2 + x*log(a*sinh(x)^2) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-
e^x) - 2*dilog(e^x)
```

Fricas [B] time = 2.01435, size = 246, normalized size = 7.03

$$x^2 + x\log\left(\frac{1}{2}\,a\cosh\left(x\right)^2 + \frac{1}{2}\,a\sinh\left(x\right)^2 - \frac{1}{2}\,a\right) - 2\,x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - 2\,x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 - 1/2*a) - 2*x*log(cosh(x) +
sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x) + sinh(x))
 - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sinh^{2}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**2),x)
```

```
[Out] Integral(log(a*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \sinh(x)^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^2), x)
```

3.203 $\int \log \left(a \sinh ^n(x)\right) dx$

Optimal. Leaf size=44

$$-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)$$

```
[Out] (n*x^2)/2 - n*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^n] - (n*PolyLog[2, E^(2*
x)])/2
```

Rubi [A] time = 0.0603837, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^n] - (n*PolyLog[2, E^(2*
x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sinh^n(x)\right)\,dx &= x\log\left(a\sinh^n(x)\right)-\int nx\coth(x)\,dx\\
&= x\log\left(a\sinh^n(x)\right)-n\int x\coth(x)\,dx\\
&= \frac{nx^2}{2}+x\log\left(a\sinh^n(x)\right)+(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+n\int\log\left(1-e^{2x}\right)\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{1}{2}n\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= \frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0272594, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(n\text{PolyLog}\left(2,e^{-2x}\right)-x\left(-2\log\left(a\sinh^n(x)\right)+nx+2n\log\left(1-e^{-2x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^n],x]
```

```
[Out] (-(x*(n*x + 2*n*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]^n])) + n*PolyLog[2, E^(
-2*x)])/2
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\sinh\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^n),x)
```

```
[Out] int(ln(a*sinh(x)^n),x)
```

Maxima [A] time = 1.19971, size = 63, normalized size = 1.43

$$\frac{1}{2}\left(x^{2}-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)-2\,\mathrm{Li}_2\left(-e^{x}\right)-2\,\mathrm{Li}_2\left(e^{x}\right)\right)n+x\log\left(a\sinh\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(e
^x))*n + x*log(a*sinh(x)^n)
```

Fricas [A] time = 1.90779, size = 225, normalized size = 5.11

$$\frac{1}{2}\,nx^{2}-nx\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-nx\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+nx\log\left(\sinh\left(x\right)\right)-n\mathrm{Li}_2\left(\cosh\left(x\right)+\sinh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*n*x^2 - n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(-cosh(x) - sinh(x) + 1
) + n*x*log(sinh(x)) - n*dilog(cosh(x) + sinh(x)) - n*dilog(-cosh(x) - sinh
(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\sinh^n(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**n),x)
```

```
[Out] Integral(log(a*sinh(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\sinh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^n), x)
```

3.204 $\int \log(a\cosh(x))\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+x\log(a\cosh(x))+\frac{x^2}{2}-x\log\left(e^{2x}+1\right)$$

```
[Out] x^2/2 - x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]] - PolyLog[2, -E^(2*x)]/2
```

Rubi [A] time = 0.0556963, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+x\log(a\cosh(x))+\frac{x^2}{2}-x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]] - PolyLog[2, -E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\cosh(x))\,dx &= x\log(a\cosh(x)) - \int x\tanh(x)\,dx\\ &= \frac{x^2}{2} + x\log(a\cosh(x)) - 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \int \log\left(1+e^{2x}\right)\,dx\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0183128, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(\mathrm{PolyLog}\left(2,-e^{-2x}\right) - x\left(-2\log(a\cosh(x)) + x + 2\log\left(e^{-2x}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]])) + PolyLog[2, -E^(-2*x)])
/2
```

Maple [C] time = 0.109, size = 321, normalized size = 8.2

$$-x\ln\left(\mathrm{e}^{x}\right) + \frac{i}{2}\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(1+\mathrm{e}^{2x}\right)\right)\left(\mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)\right)^2 x + \frac{i}{2}\pi\left(\mathrm{csgn}\left(ia\left(1+\mathrm{e}^{2x}\right)\mathrm{e}^{-x}\right)\right)^2\mathrm{csgn}(ia)\,x - \frac{i}{2}\pi\,\mathrm{csgn}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)),x)
```

```
[Out] -x*ln(exp(x))+1/2*I*Pi*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*e
xp(-x))^2*x+1/2*I*Pi*csgn(I*a*(1+exp(2*x))*exp(-x))^2*csgn(I*a)*x-1/2*I*Pi*
csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*exp(-x))*csgn(I*a)*x+1/2
*I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)))^2*x-1/2*I*Pi*csgn(I
*exp(-x)*(1+exp(2*x)))^3*x+1/2*I*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(2
*x)))^2*x+ln(a)*x-ln(2)*x+1/2*x^2-1/2*I*Pi*csgn(I*a*(1+exp(2*x))*exp(-x))^3
*x-1/2*I*Pi*csgn(I*exp(-x))*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)
))*x+ln(exp(x))*ln(1+exp(2*x))-ln(exp(x))*ln(1+I*exp(x))-ln(exp(x))*ln(1-I*
exp(x))-dilog(1+I*exp(x))-dilog(1-I*exp(x))
```

Maxima [A] time = 1.70867, size = 43, normalized size = 1.1

$$\frac{1}{2}x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(e^{(2x)} + 1\right) - \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(e^(2*x) + 1) - 1/2*dilog(-e^(2*x))
```

Fricas [C] time = 2.00195, size = 219, normalized size = 5.62

$$\frac{1}{2}x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(i\cosh\left(x\right) + i\sinh\left(x\right) + 1\right) - x\log\left(-i\cosh\left(x\right) - i\sinh\left(x\right) + 1\right) - \operatorname{Li}_2\left(i\cosh\left(x\right) + i\,\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(I*cosh(x) + I*sinh(x) + 1) - x*log(-I*co
sh(x) - I*sinh(x) + 1) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I*cosh(x) -
I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)),x)
```

```
[Out] Integral(log(a*cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\cosh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)), x)
```

3.205 $\int \log \left(a \cosh^2(x)\right) dx$

Optimal. Leaf size=35

$$-\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+x^2-2x\log\left(e^{2x}+1\right)$$

```
[Out] x^2 - 2*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^2] - PolyLog[2, -E^(2*x)]
```

Rubi [A] time = 0.0562567, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+x^2-2x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^2] - PolyLog[2, -E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cosh^2(x)\right)\,dx &= x\log\left(a\cosh^2(x)\right)-\int 2x\tanh(x)\,dx\\
&= x\log\left(a\cosh^2(x)\right)-2\int x\tanh(x)\,dx\\
&= x^2+x\log\left(a\cosh^2(x)\right)-4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+2\int \log\left(1+e^{2x}\right)\,dx\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01751, size = 33, normalized size = 0.94

$$\text{PolyLog}\left(2,-e^{-2x}\right)+x\left(\log\left(a\cosh^2(x)\right)-x-2\log\left(e^{-2x}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 + E^(-2*x)] + Log[a*Cosh[x]^2]) + PolyLog[2, -E^(-2*x)]
```

Maple [C] time = 0.148, size = 478, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^2),x)
```

```
[Out] -2*x*ln(exp(x))-2*ln(2)*x+2*ln(exp(x))*ln(1+exp(2*x))-2*ln(exp(x))*ln(1+I*e
xp(x))-2*ln(exp(x))*ln(1-I*exp(x))+ln(a)*x-2*dilog(1+I*exp(x))-2*dilog(1-I*
exp(x))-1/2*I*Pi*csgn(I*(1+exp(2*x))^2)^3*x-1/2*I*Pi*csgn(I*a*(1+exp(2*x))^
2*exp(-2*x))^3*x-1/2*I*Pi*csgn(I*(1+exp(2*x)))^2*csgn(I*(1+exp(2*x))^2)*x+x
^2+1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*
csgn(I*a*(1+exp(2*x))^2*exp(-2*x))*csgn(I*a)*x+1/2*I*Pi*csgn(I*(1+exp(2*x))
^2)*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2
*x))^2)^3*x+1/2*I*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*csgn(I*a*(1+exp(2*x))
^2*exp(-2*x))^2*x+I*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1+exp(2*x))^2)^2*x-1/2*
I*Pi*csgn(I*exp(-2*x))*csgn(I*(1+exp(2*x))^2)*csgn(I*exp(-2*x)*(1+exp(2*x))
^2)*x-I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*exp(x))^2*cs
gn(I*exp(2*x))*x+1/2*I*Pi*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^2*csgn(I*a)*x+
1/2*I*Pi*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^2*x
```

Maxima [A] time = 1.70243, size = 43, normalized size = 1.23

$$x^2 + x \log\left(a \cosh(x)^2\right) - 2\,x \log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x^2 + x*log(a*cosh(x)^2) - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x))
```

Fricas [C] time = 1.94445, size = 267, normalized size = 7.63

$$x^2 + x \log\left(\frac{1}{2}\,a \cosh(x)^2 + \frac{1}{2}\,a \sinh(x)^2 + \frac{1}{2}\,a\right) - 2\,x \log\left(i\,\cosh(x) + i\,\sinh(x) + 1\right) - 2\,x \log\left(-i\,\cosh(x) - i\,\sinh(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 + 1/2*a) - 2*x*log(I*cosh(x)
+ I*sinh(x) + 1) - 2*x*log(-I*cosh(x) - I*sinh(x) + 1) - 2*dilog(I*cosh(x)
+ I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \cosh^2 (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**2),x)
```

```
[Out] Integral(log(a*cosh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \cosh (x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^2), x)
```

3.206 $\int \log \left(a \cosh ^n(x)\right) dx$

Optimal. Leaf size=44

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(e^{2x}+1\right)$$

```
[Out] (n*x^2)/2 - n*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^n] - (n*PolyLog[2, -E^(2
*x)])/2
```

Rubi [A] time = 0.0566674, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^n] - (n*PolyLog[2, -E^(2
*x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cosh^n(x)\right)\,dx &= x\log\left(a\cosh^n(x)\right)-\int nx\tanh(x)\,dx\\
&= x\log\left(a\cosh^n(x)\right)-n\int x\tanh(x)\,dx\\
&= \frac{nx^2}{2}+x\log\left(a\cosh^n(x)\right)-(2n)\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+n\int \log\left(1+e^{2x}\right)\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{1}{2}n\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.028383, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(n\text{PolyLog}\left(2,-e^{-2x}\right)-x\left(-2\log\left(a\cosh^n(x)\right)+nx+2n\log\left(e^{-2x}+1\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^n],x]
```

```
[Out] (-(x*(n*x + 2*n*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]^n])) + n*PolyLog[2, -E^
(-2*x)])/2
```

Maple [F] time = 0.078, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\cosh\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^n),x)
```

```
[Out] int(ln(a*cosh(x)^n),x)
```

Maxima [A] time = 1.67901, size = 49, normalized size = 1.11

$$\frac{1}{2}\left(x^{2} - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\cosh\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*cosh(x)^n)
```

Fricas [C] time = 2.08226, size = 247, normalized size = 5.61

$$\frac{1}{2}\,nx^{2} - nx\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - nx\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) + nx\log\left(\cosh\left(x\right)\right) - n\mathrm{Li}_2\left(i\,\cosh\left(x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*n*x^2 - n*x*log(I*cosh(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sin
h(x) + 1) + n*x*log(cosh(x)) - n*dilog(I*cosh(x) + I*sinh(x)) - n*dilog(-I*
cosh(x) - I*sinh(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \cosh ^n(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**n),x)
```

```
[Out] Integral(log(a*cosh(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \cosh (x)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^n), x)
```

3.207 $\int \log(\tanh(x))\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))$$

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[2,
 E^(2*x)]/2
```

Rubi [A] time = 0.0443632, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2548, 5461, 4182, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[2,
 E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(\tanh(x))\,dx &= x\log(\tanh(x)) - \int x\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log(\tanh(x)) - 2\int x\mathrm{csch}(2x)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right) - \frac{\mathrm{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.0066358, size = 35, normalized size = 0.9

$$\frac{1}{2}\mathrm{PolyLog}(2,1-\tanh(x)) + \frac{1}{2}\mathrm{PolyLog}(2,-\tanh(x)) + \frac{1}{2}\log(\tanh(x))\log(\tanh(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Tanh[x]],x]
```

```
[Out] (Log[Tanh[x]]*Log[1 + Tanh[x]])/2 + PolyLog[2, 1 - Tanh[x]]/2 + PolyLog[2,
-Tanh[x]]/2
```

Maple [A] time = 0.014, size = 24, normalized size = 0.6

$$\frac{\mathrm{dilog}\left(\tanh\left(x\right)\right)}{2} + \frac{\mathrm{dilog}\left(\tanh\left(x\right)+1\right)}{2} + \frac{\ln\left(\tanh\left(x\right)\right)\ln\left(\tanh\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tanh(x)),x)
```

```
[Out] 1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1)+1/2*ln(tanh(x))*ln(tanh(x)+1)
```

Maxima [A] time = 1.57164, size = 73, normalized size = 1.87

$$x\log\left(e^{(2x)}+1\right)-x\log\left(e^{x}+1\right)-x\log\left(-e^{x}+1\right)+x\log\left(\tanh\left(x\right)\right)+\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)-\mathrm{Li}_2\left(-e^{x}\right)-\mathrm{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) + x*log(tanh(x)) + 1/
2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Fricas [C] time = 1.97205, size = 373, normalized size = 9.56

$$x\log\left(\frac{\sinh\left(x\right)}{\cosh\left(x\right)}\right)-x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)+x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) + I
*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh(x
) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I
*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\log\left(\tanh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tanh(x)),x)
```

```
[Out] Integral(log(tanh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\tanh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(tanh(x)), x)
```

3.208 $\int \log(a\tanh(x))\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\tanh(x))+2x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[
2, E^(2*x)]/2
```

Rubi [A] time = 0.0442116, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 5461, 4182, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\tanh(x))+2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[
2, E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\tanh(x))\,dx &= x\log(a\tanh(x)) - \int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log(a\tanh(x)) - 2\int x\text{csch}(2x)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\right.\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.0074158, size = 49, normalized size = 1.2

$$\frac{1}{2}\text{PolyLog}(2,-\tanh(x)) - \frac{1}{2}\text{PolyLog}(2,\tanh(x)) - \frac{1}{2}\log(1-\tanh(x))\log(a\tanh(x)) + \frac{1}{2}\log(\tanh(x)+1)\log(a\tanh$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]],x]

[Out] -(Log[1 - Tanh[x]]*Log[a*Tanh[x]])/2 + (Log[a*Tanh[x]]*Log[1 + Tanh[x]])/2
+ PolyLog[2, -Tanh[x]]/2 - PolyLog[2, Tanh[x]]/2
```

Maple [B] time = 0.027, size = 70, normalized size = 1.7

$$\frac{\ln(a\tanh(x))}{2}\ln\left(\frac{a\tanh(x)+a}{a}\right) + \frac{1}{2}\text{dilog}\left(\frac{a\tanh(x)+a}{a}\right) - \frac{\ln(a\tanh(x))}{2}\ln\left(-\frac{a\tanh(x)-a}{a}\right) - \frac{1}{2}\text{dilog}\left(-\frac{a\tanh}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)),x)
```

```
[Out] 1/2*ln(a*tanh(x))*ln((a*tanh(x)+a)/a)+1/2*dilog((a*tanh(x)+a)/a)-1/2*ln(a*t
anh(x))*ln(-(a*tanh(x)-a)/a)-1/2*dilog(-(a*tanh(x)-a)/a)
```

Maxima [A] time = 1.59943, size = 76, normalized size = 1.85

$$x \log \left(a \tanh \left(x\right)\right) + x \log \left(e^{\left(2 \, x\right)} + 1\right) - x \log \left(e^{x} + 1\right) - x \log \left(-e^{x} + 1\right) + \frac{1}{2} \, \mathrm{Li}_2\left(-e^{\left(2 \, x\right)}\right) - \mathrm{Li}_2\left(-e^{x}\right) - \mathrm{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)) + x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) +
1/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Fricas [C] time = 2.03789, size = 375, normalized size = 9.15

$$x \log \left(\frac{a \sinh \left(x\right)}{\cosh \left(x\right)}\right) - x \log \left(\cosh \left(x\right) + \sinh \left(x\right) + 1\right) + x \log \left(i \, \cosh \left(x\right) + i \, \sinh \left(x\right) + 1\right) + x \log \left(-i \, \cosh \left(x\right) - i \, \sinh \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) +
 I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh
(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(
-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \tanh \left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)),x)
```

```
[Out] Integral(log(a*tanh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tanh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)), x)
```

3.209 $\int \log \left(a \tanh ^2(x)\right) \, dx$

Optimal. Leaf size=37

$$\text{PolyLog}\left(2,-e^{2x}\right)-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+4x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 4*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^2] + PolyLog[2, -E^(2*x)] - PolyLog[
2, E^(2*x)]
```

Rubi [A] time = 0.0470444, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$\text{PolyLog}\left(2,-e^{2x}\right)-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+4x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^2] + PolyLog[2, -E^(2*x)] - PolyLog[
2, E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tanh^2(x)\right)\,dx &= x\log\left(a\tanh^2(x)\right)-\int 2x\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log\left(a\tanh^2(x)\right)-2\int x\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log\left(a\tanh^2(x)\right)-4\int x\mathrm{csch}(2x)\,dx\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+2\int\log\left(1-e^{2x}\right)\,dx-2\int\log\left(1+e^{2x}\right)\,dx\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)-\mathrm{Subst}\left(\int\frac{\log(1+x)}{x}\right.\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+\mathrm{Li}_2\left(-e^{2x}\right)-\mathrm{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0095761, size = 47, normalized size = 1.27

$$\mathrm{PolyLog}(2,-\tanh(x))-\mathrm{PolyLog}(2,\tanh(x))-\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^2(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\tanh^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^2],x]
```

```
[Out] -(Log[1 - Tanh[x]]*Log[a*Tanh[x]^2])/2 + (Log[a*Tanh[x]^2]*Log[1 + Tanh[x]]
)/2 + PolyLog[2, -Tanh[x]] - PolyLog[2, Tanh[x]]
```

Maple [A] time = 0.03, size = 47, normalized size = 1.3

$$-\frac{\ln(\tanh(x)-1)\ln\left(a(\tanh(x))^2\right)}{2}+\operatorname{dilog}(\tanh(x))+\ln(\tanh(x)-1)\ln(\tanh(x))+\frac{\ln(\tanh(x)+1)\ln\left(a(\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^2),x)
```

```
[Out] -1/2*ln(tanh(x)-1)*ln(a*tanh(x)^2)+dilog(tanh(x))+ln(tanh(x)-1)*ln(tanh(x))
+1/2*ln(tanh(x)+1)*ln(a*tanh(x)^2)+dilog(tanh(x)+1)
```

Maxima [A] time = 1.54846, size = 77, normalized size = 2.08

$$x\log\left(a\tanh(x)^2\right)+2x\log\left(e^{(2x)}+1\right)-2x\log(e^x+1)-2x\log(-e^x+1)+\mathrm{Li}_2\left(-e^{(2x)}\right)-2\,\mathrm{Li}_2(-e^x)-2\,\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)^2) + 2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x
 + 1) + dilog(-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x)
```

Fricas [C] time = 1.969, size = 454, normalized size = 12.27

$$x\log\left(\frac{a\cosh(x)^2+a\sinh(x)^2-a}{\cosh(x)^2+\sinh(x)^2+1}\right)-2x\log(\cosh(x)+\sinh(x)+1)+2x\log(i\cosh(x)+i\sinh(x)+1)+2x\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 - a)/(cosh(x)^2 + sinh(x)^2 + 1)) - 2*x*lo
g(cosh(x) + sinh(x) + 1) + 2*x*log(I*cosh(x) + I*sinh(x) + 1) + 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x
) + sinh(x)) + 2*dilog(I*cosh(x) + I*sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh
(x)) - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \tanh ^{2}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**2),x)
```

```
[Out] Integral(log(a*tanh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \tanh (x)^{2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)^2), x)
```

3.210 $\int \log\left(a \tanh^n(x)\right) dx$

Optimal. Leaf size=46

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+2nx\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] 2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^n] + (n*PolyLog[2, -E^(2*x)])/2 -
(n*PolyLog[2, E^(2*x)])/2
```

Rubi [A] time = 0.0491043, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+2nx\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^n] + (n*PolyLog[2, -E^(2*x)])/2 -
(n*PolyLog[2, E^(2*x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\tanh^n(x)\right)\,dx &= x\log\left(a\tanh^n(x)\right)-\int nx\operatorname{csch}(x)\operatorname{sech}(x)\,dx\\
&= x\log\left(a\tanh^n(x)\right)-n\int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx\\
&= x\log\left(a\tanh^n(x)\right)-(2n)\int x\operatorname{csch}(2x)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+n\int\log\left(1-e^{2x}\right)\,dx-n\int\log\left(1+e^{2x}\right)\,dx\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\,\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)-\frac{1}{2}n\,\text{Subst}\left(\int\frac{\text{lo}}{}\right.\\
&= 2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0103879, size = 55, normalized size = 1.2

$$\frac{1}{2}n\text{PolyLog}(2,-\tanh(x))-\frac{1}{2}n\text{PolyLog}(2,\tanh(x))-\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^n(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^n],x]
```

```
[Out] -(Log[1 - Tanh[x]]*Log[a*Tanh[x]^n])/2 + (Log[a*Tanh[x]^n]*Log[1 + Tanh[x]]
)/2 + (n*PolyLog[2, -Tanh[x]])/2 - (n*PolyLog[2, Tanh[x]])/2
```

Maple [A] time = 0.023, size = 43, normalized size = 0.9

$$\left(\ln\left(a(\tanh(x))^{n}\right)-n\ln(\tanh(x))\right)x+\frac{n\mathrm{dilog}(\tanh(x))}{2}+\frac{n\mathrm{dilog}(\tanh(x)+1)}{2}+\frac{n\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^n),x)
```

```
[Out] (ln(a*tanh(x)^n)-n*ln(tanh(x)))*x+1/2*n*dilog(tanh(x))+1/2*n*dilog(tanh(x)+
1)+1/2*n*ln(tanh(x))*ln(tanh(x)+1)
```

Maxima [A] time = 1.89179, size = 82, normalized size = 1.78

$$\frac{1}{2}\left(2x\log\left(e^{(2x)}+1\right)-2x\log(e^{x}+1)-2x\log(-e^{x}+1)+\mathrm{Li}_2\left(-e^{(2x)}\right)-2\,\mathrm{Li}_2(-e^{x})-2\,\mathrm{Li}_2(e^{x})\right)n+x\log\left(a\tanh(x)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-e
^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*tanh(x)^n)
```

Fricas [C] time = 1.93216, size = 412, normalized size = 8.96

$$nx\log\left(\frac{\sinh(x)}{\cosh(x)}\right)-nx\log(\cosh(x)+\sinh(x)+1)+nx\log(i\cosh(x)+i\sinh(x)+1)+nx\log(-i\cosh(x)-i\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(sinh(x)/cosh(x)) - n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(I*cosh(
x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x) - I*sinh(x) + 1) - n*x*log(-cosh(x
) - sinh(x) + 1) - n*dilog(cosh(x) + sinh(x)) + n*dilog(I*cosh(x) + I*sinh(
x)) + n*dilog(-I*cosh(x) - I*sinh(x)) - n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tanh^{n}(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**n),x)
```

```
[Out] Integral(log(a*tanh(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\tanh(x)^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)^n), x)
```

3.211 $\int \log(\coth(x))\,dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))$$

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog[2
, E^(2*x)]/2
```

Rubi [A] time = 0.0458674, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2548, 5461, 4182, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Coth[x]],x]
```



```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog[2
, E^(2*x)]/2
```


Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(\coth(x))\,dx &= x\log(\coth(x)) + \int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx\\ &= x\log(\coth(x)) + 2\int x\operatorname{csch}(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \int \log\left(1-e^{2x}\right)dx + \int \log\left(1+e^{2x}\right)dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,d\right.\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\operatorname{Li}_2\left(-e^{2x}\right) + \frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.0067925, size = 35, normalized size = 0.9

$$\frac{1}{2}\operatorname{PolyLog}(2,1-\coth(x)) + \frac{1}{2}\operatorname{PolyLog}(2,-\coth(x)) + \frac{1}{2}\log(\coth(x))\log(\coth(x)+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Coth[x]],x]
```

```
[Out] (Log[Coth[x]]*Log[1 + Coth[x]])/2 + PolyLog[2, 1 - Coth[x]]/2 + PolyLog[2,
-Coth[x]]/2
```

Maple [A] time = 0.013, size = 24, normalized size = 0.6

$$\frac{\operatorname{dilog}(\coth(x))}{2} + \frac{\operatorname{dilog}(\coth(x)+1)}{2} + \frac{\ln(\coth(x))\ln(\coth(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(coth(x)),x)
```

```
[Out] 1/2*dilog(coth(x))+1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*ln(coth(x)+1)
```

Maxima [A] time = 1.5903, size = 66, normalized size = 1.69

$$-x\log\left(e^{(2x)}+1\right)+x\log\left(e^{x}+1\right)+x\log\left(-e^{x}+1\right)+x\log\left(\coth\left(x\right)\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)+\mathrm{Li}_2\left(-e^{x}\right)+\mathrm{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="maxima")
```

```
[Out] -x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) + x*log(coth(x)) - 1
/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Fricas [C] time = 1.98114, size = 373, normalized size = 9.56

$$x\log\left(\frac{\cosh\left(x\right)}{\sinh\left(x\right)}\right)+x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)-x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) + I
*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh(x
) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I
*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\coth\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(coth(x)),x)
```

```
[Out] Integral(log(coth(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\coth \left(x\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(coth(x)), x)
```

3.212 $\int \log(a\coth(x))\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\coth(x))-2x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog
[2, E^(2*x)]/2
```

Rubi [A] time = 0.0442162, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 5461, 4182, 2279, 2391}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\coth(x))-2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog
[2, E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\coth(x))\,dx &= x\log(a\coth(x)) + \int x\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log(a\coth(x)) + 2\int x\mathrm{csch}(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \int \log\left(1-e^{2x}\right)\,dx + \int \log\left(1+e^{2x}\right)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{\log(1+x}{x}\right.\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right) + \frac{\mathrm{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A] time = 0.0076459, size = 49, normalized size = 1.2

$$\frac{1}{2}\mathrm{PolyLog}(2,-\coth(x)) - \frac{1}{2}\mathrm{PolyLog}(2,\coth(x)) - \frac{1}{2}\log(1-\coth(x))\log(a\coth(x)) + \frac{1}{2}\log(\coth(x)+1)\log(a\coth($$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]],x]
```

```
[Out] -(Log[1 - Coth[x]]*Log[a*Coth[x]])/2 + (Log[a*Coth[x]]*Log[1 + Coth[x]])/2
+ PolyLog[2, -Coth[x]]/2 - PolyLog[2, Coth[x]]/2
```

Maple [B] time = 0.014, size = 70, normalized size = 1.7

$$\frac{\ln(a\coth(x))}{2}\ln\left(\frac{a\coth(x)+a}{a}\right) + \frac{1}{2}\mathrm{dilog}\left(\frac{a\coth(x)+a}{a}\right) - \frac{\ln(a\coth(x))}{2}\ln\left(-\frac{a\coth(x)-a}{a}\right) - \frac{1}{2}\mathrm{dilog}\left(-\frac{a\coth(x)}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)),x)
```

```
[Out] 1/2*ln(a*coth(x))*ln((a*coth(x)+a)/a)+1/2*dilog((a*coth(x)+a)/a)-1/2*ln(a*c
oth(x))*ln(-(a*coth(x)-a)/a)-1/2*dilog(-(a*coth(x)-a)/a)
```

Maxima [A] time = 1.55698, size = 69, normalized size = 1.68

$$x \log\left(a \coth\left(x\right)\right) - x \log\left(e^{\left(2 x\right)} + 1\right) + x \log\left(e^{x} + 1\right) + x \log\left(-e^{x} + 1\right) - \frac{1}{2} \operatorname{Li}_2\left(-e^{\left(2 x\right)}\right) + \operatorname{Li}_2\left(-e^{x}\right) + \operatorname{Li}_2\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)) - x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) -
1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Fricas [C] time = 1.93172, size = 375, normalized size = 9.15

$$x \log\left(\frac{a \cosh\left(x\right)}{\sinh\left(x\right)}\right) + x \log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - x \log\left(i \cosh\left(x\right) + i \sinh\left(x\right) + 1\right) - x \log\left(-i \cosh\left(x\right) - i \sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) +
 I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh
(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(
-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a \coth\left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)),x)
```

```
[Out] Integral(log(a*coth(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \coth \left(x \right) \right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)), x)
```

3.213 $\int \log \left(a \coth^2(x)\right) dx$

Optimal. Leaf size=37

$$-\text{PolyLog}\left(2,-e^{2x}\right)+\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^2(x)\right)-4x\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -4*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^2] - PolyLog[2, -E^(2*x)] + PolyLog
[2, E^(2*x)]
```

Rubi [A] time = 0.0484372, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$-\text{PolyLog}\left(2,-e^{2x}\right)+\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^2(x)\right)-4x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^2] - PolyLog[2, -E^(2*x)] + PolyLog
[2, E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\coth^2(x)\right)\,dx &= x\log\left(a\coth^2(x)\right)-\int -2x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\coth^2(x)\right)+2\int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\coth^2(x)\right)+4\int x\text{csch}(2x)\,dx\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-2\int\log\left(1-e^{2x}\right)\,dx+2\int\log\left(1+e^{2x}\right)\,dx\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)+\text{Subst}\left(\int\frac{\log(1+x)}{x}\right.\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)+\text{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.009953, size = 47, normalized size = 1.27

$$-\text{PolyLog}(2,-\tanh(x))+\text{PolyLog}(2,\tanh(x))-\frac{1}{2}\log(1-\tanh(x))\log\left(a\coth^2(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\coth\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^2],x]
```

```
[Out] -(Log[a*Coth[x]^2]*Log[1 - Tanh[x]])/2 + (Log[a*Coth[x]^2]*Log[1 + Tanh[x]]
)/2 - PolyLog[2, -Tanh[x]] + PolyLog[2, Tanh[x]]
```

Maple [A] time = 0.017, size = 47, normalized size = 1.3

$$-\frac{\ln(\coth(x)-1)\ln\left(a(\coth(x))^2\right)}{2}+\mathrm{dilog}(\coth(x))+\ln(\coth(x)-1)\ln(\coth(x))+\frac{\ln(\coth(x)+1)\ln\left(a(\coth(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^2),x)
```

```
[Out] -1/2*ln(coth(x)-1)*ln(a*coth(x)^2)+dilog(coth(x))+ln(coth(x)-1)*ln(coth(x))
+1/2*ln(coth(x)+1)*ln(a*coth(x)^2)+dilog(coth(x)+1)
```

Maxima [A] time = 1.54303, size = 80, normalized size = 2.16

$$x\log\left(a\coth(x)^2\right)-2x\log\left(e^{(2x)}+1\right)+2x\log(e^x+1)+2x\log(-e^x+1)-\mathrm{Li}_2\left(-e^{(2x)}\right)+2\mathrm{Li}_2(-e^x)+2\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)^2) - 2*x*log(e^(2*x) + 1) + 2*x*log(e^x + 1) + 2*x*log(-e^x
 + 1) - dilog(-e^(2*x)) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Fricas [C] time = 2.136, size = 454, normalized size = 12.27

$$x\log\left(\frac{a\cosh(x)^2+a\sinh(x)^2+a}{\cosh(x)^2+\sinh(x)^2-1}\right)+2x\log(\cosh(x)+\sinh(x)+1)-2x\log(i\cosh(x)+i\sinh(x)+1)-2x\log(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 + a)/(cosh(x)^2 + sinh(x)^2 - 1)) + 2*x*lo
g(cosh(x) + sinh(x) + 1) - 2*x*log(I*cosh(x) + I*sinh(x) + 1) - 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x
) + sinh(x)) - 2*dilog(I*cosh(x) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh
(x)) + 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \coth^2 (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**2),x)
```

```
[Out] Integral(log(a*coth(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \coth (x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)^2), x)
```

3.214 $\int \log \left(a \coth^n(x)\right) dx$

Optimal. Leaf size=46

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^n(x)\right)-2nx\tanh^{-1}\left(e^{2x}\right)$$

```
[Out] -2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^n] - (n*PolyLog[2, -E^(2*x)])/2 +
 (n*PolyLog[2, E^(2*x)])/2
```

Rubi [A] time = 0.0494245, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 5461, 4182, 2279, 2391}

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^n(x)\right)-2nx\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^n] - (n*PolyLog[2, -E^(2*x)])/2 +
 (n*PolyLog[2, E^(2*x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\coth^n(x)\right)\,dx &= x\log\left(a\coth^n(x)\right)+\int nx\,\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log\left(a\coth^n(x)\right)+n\int x\,\mathrm{csch}(x)\mathrm{sech}(x)\,dx\\ &= x\log\left(a\coth^n(x)\right)+(2n)\int x\,\mathrm{csch}(2x)\,dx\\ &= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx+n\int\log\left(1+e^{2x}\right)\,dx\\ &= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\,\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}n\,\mathrm{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\ &= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\mathrm{Li}_2\left(-e^{2x}\right)+\frac{1}{2}n\mathrm{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0116304, size = 55, normalized size = 1.2

$$-\frac{1}{2}n\mathrm{PolyLog}(2,-\tanh(x))+\frac{1}{2}n\mathrm{PolyLog}(2,\tanh(x))-\frac{1}{2}\log(1-\tanh(x))\log\left(a\coth^n(x)\right)+\frac{1}{2}\log(\tanh(x)+1)\log\left(a\coth^n(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^n],x]
```

```
[Out] -(Log[a*Coth[x]^n]*Log[1 - Tanh[x]])/2 + (Log[a*Coth[x]^n]*Log[1 + Tanh[x]]
)/2 - (n*PolyLog[2, -Tanh[x]])/2 + (n*PolyLog[2, Tanh[x]])/2
```

Maple [A] time = 0.021, size = 43, normalized size = 0.9

$$\left(\ln\left(a\left(\coth\left(x\right)\right)^{n}\right)-n\ln\left(\coth\left(x\right)\right)\right)x+\frac{n\operatorname{dilog}\left(\coth\left(x\right)\right)}{2}+\frac{n\operatorname{dilog}\left(\coth\left(x\right)+1\right)}{2}+\frac{n\ln\left(\coth\left(x\right)\right)\ln\left(\coth\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^n),x)
```

```
[Out] (ln(a*coth(x)^n)-n*ln(coth(x)))*x+1/2*n*dilog(coth(x))+1/2*n*dilog(coth(x)+
1)+1/2*n*ln(coth(x))*ln(coth(x)+1)
```

Maxima [A] time = 1.74351, size = 82, normalized size = 1.78

$$-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2\left(-e^{x}\right)-2\,\mathrm{Li}_2\left(e^{x}\right)\right)n+x\log\left(a\coth\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-
e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*coth(x)^n)
```

Fricas [C] time = 1.92849, size = 412, normalized size = 8.96

$$nx\log\left(\frac{\cosh\left(x\right)}{\sinh\left(x\right)}\right)+nx\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-nx\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)-nx\log\left(-i\,\cosh\left(x\right)-i\,\sinh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(cosh(x)/sinh(x)) + n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(I*cosh(
x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sinh(x) + 1) + n*x*log(-cosh(x
) - sinh(x) + 1) + n*dilog(cosh(x) + sinh(x)) - n*dilog(I*cosh(x) + I*sinh(
x)) - n*dilog(-I*cosh(x) - I*sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \coth^{n}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**n),x)
```

```
[Out] Integral(log(a*coth(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \coth(x)^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)^n), x)
```

3.215 $\int \log(a\,\mathbf{sech}(x))\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+x\log(a\operatorname{sech}(x))-\frac{x^2}{2}+x\log\left(e^{2x}+1\right)$$

```
[Out] -x^2/2 + x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]] + PolyLog[2, -E^(2*x)]/2
```

Rubi [A] time = 0.0529435, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3718, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+x\log(a\operatorname{sech}(x))-\frac{x^2}{2}+x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]],x]
```

```
[Out] -x^2/2 + x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]] + PolyLog[2, -E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log(a\text{sech}(x))\,dx &= x\log(a\text{sech}(x)) + \int x\tanh(x)\,dx\\ &= -\frac{x^2}{2} + x\log(a\text{sech}(x)) + 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \int \log\left(1+e^{2x}\right)\,dx\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) - \frac{1}{2}\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\ &= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\text{sech}(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.0147477, size = 37, normalized size = 0.97

$$\frac{1}{2}\left(x\left(2\log(a\text{sech}(x)) + x + 2\log\left(e^{-2x}+1\right)\right) - \text{PolyLog}\left(2,-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]],x]
```

```
[Out] (x*(x + 2*Log[1 + E^(-2*x)] + 2*Log[a*Sech[x]]) - PolyLog[2, -E^(-2*x)])/2
```

Maple [C] time = 0.111, size = 314, normalized size = 8.3

$$x\ln\left(e^{x}\right) + \frac{i}{2}\pi\left(\text{csgn}\left(\frac{ia e^{x}}{1+e^{2x}}\right)\right)^2 \text{csgn}\left(ia\right)x + \frac{i}{2}\pi\,\text{csgn}\left(\frac{i e^{x}}{1+e^{2x}}\right)\left(\text{csgn}\left(\frac{ia e^{x}}{1+e^{2x}}\right)\right)^2 x - \frac{i}{2}\pi\left(\text{csgn}\left(\frac{ia e^{x}}{1+e^{2x}}\right)\right)^3 x + \frac{i}{2}\pi$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)),x)
```

```
[Out] x*ln(exp(x))+1/2*I*Pi*csgn(I*a/(1+exp(2*x))*exp(x))^2*csgn(I*a)*x+1/2*I*Pi*
csgn(I*exp(x)/(1+exp(2*x)))*csgn(I*a/(1+exp(2*x))*exp(x))^2*x-1/2*I*Pi*csgn
(I*a/(1+exp(2*x))*exp(x))^3*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(1+exp(
2*x)))^2*x-1/2*I*Pi*csgn(I*exp(x)/(1+exp(2*x)))^3*x+1/2*I*Pi*csgn(I/(1+exp(
2*x)))*csgn(I*exp(x)/(1+exp(2*x)))^2*x+ln(2)*x+ln(a)*x-1/2*x^2-1/2*I*Pi*csg
n(I*exp(x))*csgn(I/(1+exp(2*x)))*csgn(I*exp(x)/(1+exp(2*x)))*x-1/2*I*Pi*csg
n(I*exp(x)/(1+exp(2*x)))*csgn(I*a/(1+exp(2*x))*exp(x))*csgn(I*a)*x-ln(exp(x
))*ln(1+exp(2*x))+ln(exp(x))*ln(1+I*exp(x))+ln(exp(x))*ln(1-I*exp(x))+dilog
(1+I*exp(x))+dilog(1-I*exp(x))
```

Maxima [A] time = 1.70916, size = 42, normalized size = 1.11

$$-\frac{1}{2}x^2 + x\log\left(a\operatorname{sech}(x)\right) + x\log\left(e^{(2x)} + 1\right) + \frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x*log(a*sech(x)) + x*log(e^(2*x) + 1) + 1/2*dilog(-e^(2*x))
```

Fricas [C] time = 1.94293, size = 306, normalized size = 8.05

$$-\frac{1}{2}x^2 + x\log\left(\frac{2\left(a\cosh(x) + a\sinh(x)\right)}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}\right) + x\log\left(i\cosh(x) + i\sinh(x) + 1\right) + x\log\left(-i\cosh(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + x*log(I*cosh(x) + I*sinh(x) + 1) + x*log(-I*cosh(x) - I*
sinh(x) + 1) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a \operatorname{sech}\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)),x)
```

```
[Out] Integral(log(a*sech(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a \operatorname{sech}\left(x\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)), x)
```

3.216 $\int \log\left(a\,\mathrm{sech}^2(x)\right)\,dx$

Optimal. Leaf size=35

$$\mathrm{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\,\mathrm{sech}^2(x)\right)-x^2+2x\log\left(e^{2x}+1\right)$$

```
[Out] -x^2 + 2*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^2] + PolyLog[2, -E^(2*x)]
```

Rubi [A] time = 0.0534823, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$\mathrm{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\,\mathrm{sech}^2(x)\right)-x^2+2x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^2] + PolyLog[2, -E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\operatorname{sech}^2(x)\right)\,dx &= x\log\left(a\operatorname{sech}^2(x)\right)-\int -2x\tanh(x)\,dx\\
&= x\log\left(a\operatorname{sech}^2(x)\right)+2\int x\tanh(x)\,dx\\
&= -x^2+x\log\left(a\operatorname{sech}^2(x)\right)+4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^2(x)\right)-2\int \log\left(1+e^{2x}\right)\,dx\\
&= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^2(x)\right)-\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^2(x)\right)+\operatorname{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.017846, size = 33, normalized size = 0.94

$$x\left(\log\left(a\operatorname{sech}^2(x)\right)+x+2\log\left(e^{-2x}+1\right)\right)-\operatorname{PolyLog}\left(2,-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^2],x]
```

```
[Out] x*(x + 2*Log[1 + E^(-2*x)] + Log[a*Sech[x]^2]) - PolyLog[2, -E^(-2*x)]
```

Maple [C] time = 0.142, size = 480, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^2),x)
```

```
[Out] 2*x*ln(exp(x))+2*ln(2)*x-2*ln(exp(x))*ln(1+exp(2*x))+2*ln(exp(x))*ln(1+I*ex
p(x))+2*ln(exp(x))*ln(1-I*exp(x))+ln(a)*x+2*dilog(1+I*exp(x))+2*dilog(1-I*e
xp(x))-1/2*I*Pi*csgn(I*exp(2*x))^3*x+I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2
*x-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x-I*Pi*csgn(I*(1+exp(2*x)))*c
sgn(I*(1+exp(2*x))^2)^2*x-x^2-1/2*I*Pi*csgn(I*exp(2*x)/(1+exp(2*x))^2)^3*x+
1/2*I*Pi*csgn(I*exp(2*x)/(1+exp(2*x))^2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^
2*x+1/2*I*Pi*csgn(I*(1+exp(2*x)))^2*csgn(I*(1+exp(2*x))^2)*x+1/2*I*Pi*csgn(
I/(1+exp(2*x))^2)*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2*x-1/2*I*Pi*csgn(I*a/(1+
exp(2*x))^2*exp(2*x))^3*x+1/2*I*Pi*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^2*csgn
(I*a)*x+1/2*I*Pi*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2*x+1/2*I
*Pi*csgn(I*(1+exp(2*x))^2)^3*x-1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(1+exp(2*x)
)^2)*csgn(I*exp(2*x)/(1+exp(2*x))^2)*x-1/2*I*Pi*csgn(I*exp(2*x)/(1+exp(2*x)
)^2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x))*csgn(I*a)*x
```

Maxima [A] time = 1.67255, size = 43, normalized size = 1.23

$$-x^2 + x\log\left(a\,\mathrm{sech}\left(x\right)^2\right) + 2\,x\log\left(e^{(2\,x)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*sech(x)^2) + 2*x*log(e^(2*x) + 1) + dilog(-e^(2*x))
```

Fricas [C] time = 2.04056, size = 363, normalized size = 10.37

$$-x^2 + x\log\left(\frac{4\left(a\cosh\left(x\right) + a\sinh\left(x\right)\right)}{\cosh\left(x\right)^3 + 3\,\cosh\left(x\right)\sinh\left(x\right)^2 + \sinh\left(x\right)^3 + \left(3\,\cosh\left(x\right)^2 + 1\right)\sinh\left(x\right) + 3\,\cosh\left(x\right)}\right) + 2\,x\log\left(i\,\cosh\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + 3*cosh(x))) + 2*x*log(I*cosh(x) + I*
sinh(x) + 1) + 2*x*log(-I*cosh(x) - I*sinh(x) + 1) + 2*dilog(I*cosh(x) + I*
sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{sech}^2(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**2),x)
```

```
[Out] Integral(log(a*sech(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{sech}(x)^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^2), x)
```

3.217 $\int \log \left(a \mathbf{sech}^n(x)\right) dx$

Optimal. Leaf size=43

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\text{sech}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(e^{2x}+1\right)$$

```
[Out] -(n*x^2)/2 + n*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^n] + (n*PolyLog[2, -E^(
2*x)])/2
```

Rubi [A] time = 0.0553164, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3718, 2190, 2279, 2391}

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\text{sech}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^n],x]
```

```
[Out] -(n*x^2)/2 + n*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^n] + (n*PolyLog[2, -E^(
2*x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\operatorname{sech}^n(x)\right)\,dx &= x\log\left(a\operatorname{sech}^n(x)\right)+\int nx\tanh(x)\,dx\\
&= x\log\left(a\operatorname{sech}^n(x)\right)+n\int x\tanh(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\operatorname{sech}^n(x)\right)+(2n)\int\frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)-n\int\log\left(1+e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)-\frac{1}{2}n\operatorname{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)+\frac{1}{2}n\operatorname{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0226022, size = 43, normalized size = 1.

$$-\frac{1}{2}n\operatorname{PolyLog}\left(2,-e^{-2x}\right)+x\log\left(a\operatorname{sech}^n(x)\right)+\frac{nx^2}{2}+nx\log\left(e^{-2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^n],x]
```

```
[Out] (n*x^2)/2 + n*x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]^n] - (n*PolyLog[2, -E^(
-2*x)])/2
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \ln \left(a \left(\operatorname{sech}\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^n),x)
```

```
[Out] int(ln(a*sech(x)^n),x)
```

Maxima [A] time = 1.6552, size = 49, normalized size = 1.14

$$-\frac{1}{2} \left(x^{2} - 2\,x \log\left(e^{\left(2\,x\right)} + 1\right) - \operatorname{Li}_{2}\left(-e^{\left(2\,x\right)}\right)\right)n + x \log\left(a \operatorname{sech}\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*sech(x)^n)
```

Fricas [C] time = 2.00917, size = 332, normalized size = 7.72

$$-\frac{1}{2}\,nx^{2} + nx \log\left(\frac{2\,\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}{\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1}\right) + nx \log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) + nx \log\left(-i\,\text{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + n*x*log(I*cosh(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x)
- I*sinh(x) + 1) + n*dilog(I*cosh(x) + I*sinh(x)) + n*dilog(-I*cosh(x) - I*
```

```
sinh(x)) + x*log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{sech}^{n}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**n),x)
```

```
[Out] Integral(log(a*sech(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{sech}(x)^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^n), x)
```

3.218 $\int \log(a\,\mathbf{csch}(x))\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\,\text{csch}(x))-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)$$

```
[Out] -x^2/2 + x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]] + PolyLog[2, E^(2*x)]/2
```

Rubi [A] time = 0.0539876, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2548, 3716, 2190, 2279, 2391}

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\,\text{csch}(x))-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]],x]
```

```
[Out] -x^2/2 + x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]] + PolyLog[2, E^(2*x)]/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(a\operatorname{csch}(x))\,dx &= x\log(a\operatorname{csch}(x)) + \int x\coth(x)\,dx \\
&= -\frac{x^2}{2} + x\log(a\operatorname{csch}(x)) - 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \int \log\left(1-e^{2x}\right)\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) + \frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}$$

Mathematica [A] time = 0.0155529, size = 37, normalized size = 0.97

$$\frac{1}{2}\left(x\left(2\log(a\operatorname{csch}(x)) + x + 2\log\left(1-e^{-2x}\right)\right) - \operatorname{PolyLog}\left(2, e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]],x]
```

```
[Out] (x*(x + 2*Log[1 - E^(-2*x)] + 2*Log[a*Csch[x]]) - PolyLog[2, E^(-2*x)])/2
```

Maple [C] time = 0.102, size = 293, normalized size = 7.7

$$x\ln\left(e^{x}\right) - \frac{i}{2}\pi\operatorname{csgn}\left(\frac{ie^{x}}{e^{2x}-1}\right)\operatorname{csgn}\left(\frac{iae^{x}}{e^{2x}-1}\right)\operatorname{csgn}(ia)\,x + \frac{i}{2}\pi\operatorname{csgn}\left(\frac{ie^{x}}{e^{2x}-1}\right)\left(\operatorname{csgn}\left(\frac{iae^{x}}{e^{2x}-1}\right)\right)^2 x - \frac{i}{2}\pi\left(\operatorname{csgn}\left(\frac{ie^{x}}{e^{2x}-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)),x)
```

```
[Out] x*ln(exp(x))-1/2*I*Pi*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x)-1)*exp
(x))*csgn(I*a)*x+1/2*I*Pi*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x)-1)
*exp(x))^2*x-1/2*I*Pi*csgn(I*exp(x)/(exp(2*x)-1))^3*x-1/2*I*Pi*csgn(I*a/(ex
p(2*x)-1)*exp(x))^3*x+1/2*I*Pi*csgn(I/(exp(2*x)-1))*csgn(I*exp(x)/(exp(2*x)
-1))^2*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(exp(2*x)-1))^2*x+ln(2)*x+ln
(a)*x-1/2*x^2+1/2*I*Pi*csgn(I*a/(exp(2*x)-1)*exp(x))^2*csgn(I*a)*x-1/2*I*Pi
*csgn(I*exp(x))*csgn(I/(exp(2*x)-1))*csgn(I*exp(x)/(exp(2*x)-1))*x-ln(exp(x
))*ln(exp(2*x)-1)-dilog(exp(x))+dilog(exp(x)+1)+ln(exp(x))*ln(exp(x)+1)
```

Maxima [A] time = 1.21529, size = 50, normalized size = 1.32

$$-\frac{1}{2}x^2 + x\log(a\operatorname{csch}(x)) + x\log(e^x + 1) + x\log(-e^x + 1) + \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x*log(a*csch(x)) + x*log(e^x + 1) + x*log(-e^x + 1) + dilog(-e^x
) + dilog(e^x)
```

Fricas [B] time = 1.88009, size = 285, normalized size = 7.5

$$-\frac{1}{2}x^2 + x\log\left(\frac{2(a\cosh(x) + a\sinh(x))}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1}\right) + x\log(\cosh(x) + \sinh(x) + 1) + x\log(-\cosh(x) - \mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + x*log(cosh(x) + sinh(x) + 1) + x*log(-cosh(x) - sinh(x)
+ 1) + dilog(cosh(x) + sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\operatorname{csch}(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)),x)
```

```
[Out] Integral(log(a*csch(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(a\operatorname{csch}(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)), x)
```

3.219 $\int \log\left(a\,\mathrm{csch}^2(x)\right)\,dx$

Optimal. Leaf size=35

$$\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\,\text{csch}^2(x)\right)-x^2+2x\log\left(1-e^{2x}\right)$$

```
[Out] -x^2 + 2*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^2] + PolyLog[2, E^(2*x)]
```

Rubi [A] time = 0.0584815, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\,\text{csch}^2(x)\right)-x^2+2x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^2] + PolyLog[2, E^(2*x)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\operatorname{csch}^2(x)\right)\,dx &= x\log\left(a\operatorname{csch}^2(x)\right)-\int -2x\coth(x)\,dx\\
&= x\log\left(a\operatorname{csch}^2(x)\right)+2\int x\coth(x)\,dx\\
&= -x^2+x\log\left(a\operatorname{csch}^2(x)\right)-4\int \frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -x^2+2x\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^2(x)\right)-2\int \log\left(1-e^{2x}\right)\,dx\\
&= -x^2+2x\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^2(x)\right)-\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -x^2+2x\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^2(x)\right)+\operatorname{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0173366, size = 33, normalized size = 0.94

$$x\left(\log\left(a\operatorname{csch}^2(x)\right)+x+2\log\left(1-e^{-2x}\right)\right)-\operatorname{PolyLog}\left(2,e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^2],x]
```

```
[Out] x*(x + 2*Log[1 - E^(-2*x)] + Log[a*Csch[x]^2]) - PolyLog[2, E^(-2*x)]
```

Maple [C] time = 0.135, size = 456, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^2),x)
```

```
[Out] -2*ln(exp(x))*ln(exp(2*x)-1)+2*ln(exp(x))*ln(exp(x)+1)+2*x*ln(exp(x))+2*ln(
2)*x-1/2*I*Pi*csgn(I*exp(2*x))*csgn(I/(exp(2*x)-1)^2)*csgn(I*exp(2*x)/(exp(
2*x)-1)^2)*x+1/2*I*Pi*csgn(I/(exp(2*x)-1)^2)*csgn(I*exp(2*x)/(exp(2*x)-1)^2
)^2*x-1/2*I*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))*x+ln(a)*x-2*dilog(exp(x))+
2*dilog(exp(x)+1)-1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2)*csgn(I*a/(exp(2*
x)-1)^2*exp(2*x))*csgn(I*a)*x-x^2-1/2*I*Pi*csgn(I*exp(2*x))^3*x-1/2*I*Pi*cs
gn(I*a/(exp(2*x)-1)^2*exp(2*x))^3*x+1/2*I*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2
)*csgn(I*a/(exp(2*x)-1)^2*exp(2*x))^2*x+1/2*I*Pi*csgn(I*(exp(2*x)-1)^2)^3*x
+1/2*I*Pi*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^2*x-1/2*I*Pi*csg
n(I*exp(2*x)/(exp(2*x)-1)^2)^3*x-I*Pi*csgn(I*(exp(2*x)-1))*csgn(I*(exp(2*x)
-1)^2)^2*x+I*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2*x+1/2*I*Pi*csgn(I*(exp(2*
x)-1))^2*csgn(I*(exp(2*x)-1)^2)*x+1/2*I*Pi*csgn(I*a/(exp(2*x)-1)^2*exp(2*x)
)^2*csgn(I*a)*x
```

Maxima [A] time = 1.21493, size = 61, normalized size = 1.74

$$-x^2 + x\log\left(a\operatorname{csch}(x)^2\right) + 2x\log(e^x+1) + 2x\log(-e^x+1) + 2\operatorname{Li}_2(-e^x) + 2\operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*csch(x)^2) + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 2*dilog(
-e^x) + 2*dilog(e^x)
```

Fricas [B] time = 1.93496, size = 339, normalized size = 9.69

$$-x^2 + x\log\left(\frac{4\,(a\cosh(x) + a\sinh(x))}{\cosh(x)^3 + 3\,\cosh(x)\sinh(x)^2 + \sinh(x)^3 + 3\left(\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}\right) + 2x\log(\cosh(x) + \text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x) - cosh(x))) + 2*x*log(cosh(x) + sinh(x
) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x) + sinh(x)) + 2*d
ilog(-cosh(x) - sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\operatorname{csch}^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**2),x)
```

```
[Out] Integral(log(a*csch(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(a\operatorname{csch}(x)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^2), x)
```

3.220 $\int \log\left(a\,\mathbf{csch}^n(x)\right)\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\text{csch}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)$$

```
[Out] -(n*x^2)/2 + n*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^n] + (n*PolyLog[2, E^(2
*x)])/2
```

Rubi [A] time = 0.0641465, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2548, 12, 3716, 2190, 2279, 2391}

$$\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\text{csch}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^n],x]
```

```
[Out] -(n*x^2)/2 + n*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^n] + (n*PolyLog[2, E^(2
*x)])/2
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\operatorname{csch}^n(x)\right)\,dx &= x\log\left(a\operatorname{csch}^n(x)\right)+\int nx\coth(x)\,dx\\
&= x\log\left(a\operatorname{csch}^n(x)\right)+n\int x\coth(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\operatorname{csch}^n(x)\right)-(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)-\frac{1}{2}n\,\mathrm{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)+\frac{1}{2}n\mathrm{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0219321, size = 43, normalized size = 1.

$$-\frac{1}{2}n\mathrm{PolyLog}\left(2,e^{-2x}\right)+x\log\left(a\operatorname{csch}^n(x)\right)+\frac{nx^2}{2}+nx\log\left(1-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^n],x]
```

```
[Out] (n*x^2)/2 + n*x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]^n] - (n*PolyLog[2, E^(-
2*x)])/2
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \ln\left(a\left(\operatorname{csch}\left(x\right)\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^n),x)
```

```
[Out] int(ln(a*csch(x)^n),x)
```

Maxima [A] time = 1.20927, size = 63, normalized size = 1.47

$$-\frac{1}{2}\left(x^{2}-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)-2\,\mathrm{Li}_{2}\left(-e^{x}\right)-2\,\mathrm{Li}_{2}\left(e^{x}\right)\right)n+x\log\left(a\operatorname{csch}\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(
e^x))*n + x*log(a*csch(x)^n)
```

Fricas [B] time = 2.00326, size = 311, normalized size = 7.23

$$-\frac{1}{2}\,nx^{2}+nx\log\left(\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1}\right)+nx\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+nx\log\left(-\cosh\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(-cosh(x) - sinh
(x) + 1) + n*dilog(cosh(x) + sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
```

```
(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{csch}^{n}(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**n),x)
```

```
[Out] Integral(log(a*csch(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(a \operatorname{csch}(x)^{n}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^n), x)
```

3.221 $\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) dx$

Optimal. Leaf size=50

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

```
[Out] -(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a +
 b*x])/b
```

Rubi [A] time = 0.0286319, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2637, 2554}

$$\frac{\sinh(a+bx)\log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a +
 b*x])/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \cosh(a+bx)\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)dx = \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+bx)}{b} - \int \cosh(a+$$

$$= -\frac{\sinh(a+bx)}{b} + \frac{\log\left(\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\sinh(a+}{b}$$

Mathematica [A] time = 0.0105279, size = 33, normalized size = 0.66

$$\frac{\sinh(a+bx)\log\left(\frac{1}{2}\sinh(a+bx)\right)}{b} - \frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Sinh[a + b*x]/2]*Sinh[a + b*x])/b
```

Maple [A] time = 0.034, size = 32, normalized size = 0.6

$$\frac{\sinh(bx+a)}{b}\ln\left(\frac{\sinh(bx+a)}{2}\right) - \frac{\sinh(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

```
[Out] ln(1/2*sinh(b*x+a))/b*sinh(b*x+a)-sinh(b*x+a)/b
```

Maxima [B] time = 1.0706, size = 151, normalized size = 3.02

$$\frac{\log\left(\cosh\left(\frac{1}{2}bx+\frac{1}{2}a\right)\sinh\left(\frac{1}{2}bx+\frac{1}{2}a\right)\right)\sinh(bx+a)}{b} - \frac{b\left(\frac{2(bx+a)}{b}+\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right) - b\left(\frac{2(bx+a)}{b}-\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="maxima")
```

```
[Out] log(cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x + 1/2*a))*sinh(b*x + a)/b - 1/4*(b*(
2*(b*x + a)/b + e^(b*x + a)/b - e^(-b*x - a)/b) - b*(2*(b*x + a)/b - e^(b*x
 + a)/b + e^(-b*x - a)/b))/b
```

Fricas [B] time = 1.91609, size = 782, normalized size = 15.64

$$-\frac{\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+4\,\cosh\left(\frac{1}{2}\,bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="fricas")
```

```
[Out] -1/2*(cosh(1/2*b*x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/
2*a) + 6*cosh(1/2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x +
 1/2*a)*sinh(1/2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - (cosh(1/2*b*x +
 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/2*a) + 6*cosh(1/2*b*
x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x
 + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - 1)*log(cosh(1/2*b*x + 1/2*a)*sinh(1
/2*b*x + 1/2*a)) - 1)/(b*cosh(1/2*b*x + 1/2*a)^2 + 2*b*cosh(1/2*b*x + 1/2*a
)*sinh(1/2*b*x + 1/2*a) + b*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

```
[Out] Integral(log(sinh(a/2 + b*x/2)*cosh(a/2 + b*x/2))*cosh(a + b*x), x)
```

Giac [B] time = 1.34414, size = 127, normalized size = 2.54

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(\frac{1}{4}\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}+e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\left(e^{\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-e^{\left(-\frac{1}{2}\,bx-\frac{1}{2}\,a\right)}\right)\right)-\frac{e^{(bx+a)}-e^{(-bx-a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(1/4*(e^(1/2*b*x + 1/2*a) + e^(-1/2
*b*x - 1/2*a))*(e^(1/2*b*x + 1/2*a) - e^(-1/2*b*x - 1/2*a))) - 1/2*(e^(b*x
+ a) - e^(-b*x - a))/b
```

3.222 $\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx$

Optimal. Leaf size=13

$$\cosh(x)\log\left(\cosh^2(x)\right)-2\cosh(x)$$

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rubi [A] time = 0.0191519, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2638, 2554, 12}

$$\cosh(x)\log\left(\cosh^2(x)\right)-2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx &= \cosh(x)\log\left(\cosh^2(x)\right) - \int 2\sinh(x)\,dx\\ &= \cosh(x)\log\left(\cosh^2(x)\right) - 2\int \sinh(x)\,dx\\ &= -2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0078101, size = 13, normalized size = 1.

$$\cosh(x)\log\left(\cosh^2(x)\right) - 2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Maple [A] time = 0.013, size = 14, normalized size = 1.1

$$-2\,\cosh(x) + \cosh(x)\ln\left((\cosh(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x)^2)*sinh(x),x)
```

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Maxima [A] time = 1.02456, size = 16, normalized size = 1.23

$$2\,\cosh(x)\log(\cosh(x)) - 2\,\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="maxima")
```

```
[Out] 2*cosh(x)*log(cosh(x)) - 2*cosh(x)
```

Fricas [B] time = 1.79697, size = 228, normalized size = 17.54

$$-\frac{2\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log\left(\frac{1}{2}\cosh(x)^2 + \frac{1}{2}\sinh(x)^2 + \frac{1}{2}\right) + 4\cosh(x)\sinh}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(1/2
*cosh(x)^2 + 1/2*sinh(x)^2 + 1/2) + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + 2)/(c
osh(x) + sinh(x))
```

Sympy [A] time = 1.00068, size = 14, normalized size = 1.08

$$2\log(\cosh(x))\cosh(x) - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x)**2)*sinh(x),x)
```

```
[Out] 2*log(cosh(x))*cosh(x) - 2*cosh(x)
```

Giac [B] time = 1.25137, size = 42, normalized size = 3.23

$$\left(e^{(-x)} + e^x\right)\log\left(\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x\right) - e^{(-x)} - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="giac")
```

```
[Out] (e^(-x) + e^x)*log(1/2*e^(-x) + 1/2*e^x) - e^(-x) - e^x
```

3.223 $\int \frac{\log(x)}{\sqrt{x}} \, dx$

Optimal. Leaf size=17

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

```
[Out] -4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Rubi [A] time = 0.0066523, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[x],x]
```

```
[Out] -4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{\sqrt{x}} \, dx = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

Mathematica [A] time = 0.0018617, size = 11, normalized size = 0.65

$$2\sqrt{x}(\log(x) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*(-2 + Log[x])
```

Maple [A] time = 0.007, size = 14, normalized size = 0.8

$$-4\sqrt{x}+2\ln\left(x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^(1/2),x)
```

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Maxima [A] time = 1.02922, size = 18, normalized size = 1.06

$$2\sqrt{x}\log\left(x\right)-4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

Fricas [A] time = 1.81478, size = 32, normalized size = 1.88

$$2\sqrt{x}(\log\left(x\right)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*(log(x) - 2)
```

Sympy [A] time = 1.8802, size = 60, normalized size = 3.53

$$\begin{cases} 2\sqrt{x}\log(x) - 4\sqrt{x} & \text{for } |x| < 1 \\ -2\sqrt{x}\log\left(\frac{1}{x}\right) - 4\sqrt{x} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{3}{2}, \frac{3}{2} \\ \frac{1}{2}, \frac{1}{2} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{3}{2}, \frac{3}{2}, 1 & \\ & \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(x)*log(x) - 4*sqrt(x), Abs(x) < 1), (-2*sqrt(x)*log(1/x)
- 4*sqrt(x), 1/Abs(x) < 1), (-meijerg(((1,), (3/2, 3/2)), ((1/2, 1/2), (0,)
), x) + meijerg(((3/2, 3/2, 1), ()), ((), (1/2, 1/2, 0)), x), True))
```

Giac [A] time = 1.31282, size = 18, normalized size = 1.06

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

3.224 $\int x \log\left(2-3x^2\right) dx$

Optimal. Leaf size=27

$$-\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)$$

```
[Out] -x^2/2 - ((2 - 3*x^2)*Log[2 - 3*x^2])/6
```

Rubi [A] time = 0.0212735, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2454, 2389, 2295}

$$-\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[2 - 3*x^2],x]

[Out] -x^2/2 - ((2 - 3*x^2)*Log[2 - 3*x^2])/6
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int x\log\left(2-3x^2\right)\,dx=\frac{1}{2}\operatorname{Subst}\left(\int\log(2-3x)\,dx,x,x^2\right)$$
$$=-\left(\frac{1}{6}\operatorname{Subst}\left(\int\log(x)\,dx,x,2-3x^2\right)\right)$$
$$=-\frac{x^2}{2}-\frac{1}{6}\left(2-3x^2\right)\log\left(2-3x^2\right)$$

Mathematica [A] time = 0.0037477, size = 26, normalized size = 0.96

$$\frac{1}{6}\left(\left(3x^2-2\right)\log\left(2-3x^2\right)-3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[2 - 3*x^2],x]
```

```
[Out] (-3*x^2 + (-2 + 3*x^2)*Log[2 - 3*x^2])/6
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$-\frac{\left(-3x^2+2\right)\ln\left(-3x^2+2\right)}{6}-\frac{x^2}{2}+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-3*x^2+2),x)
```

```
[Out] -1/6*(-3*x^2+2)*ln(-3*x^2+2)-1/2*x^2+1/3
```

Maxima [A] time = 1.06472, size = 32, normalized size = 1.19

$$-\frac{1}{2}x^2+\frac{1}{6}\left(3x^2-2\right)\log\left(-3x^2+2\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Fricas [A] time = 1.84247, size = 59, normalized size = 2.19

$$-\frac{1}{2}\,x^2+\frac{1}{6}\,\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2)
```

Sympy [A] time = 0.126375, size = 27, normalized size = 1.

$$\frac{x^2\log\left(2-3x^2\right)}{2}-\frac{x^2}{2}-\frac{\log\left(3x^2-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-3*x**2+2),x)
```

```
[Out] x**2*log(2 - 3*x**2)/2 - x**2/2 - log(3*x**2 - 2)/3
```

Giac [A] time = 1.33089, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,x^2+\frac{1}{6}\,\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx$

Optimal. Leaf size=3

$$\sin^{-1}(\log(x))$$

```
[Out] ArcSin[Log[x]]
```

Rubi [A] time = 0.0335502, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {216}

$$\sin^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \log(x)\right)\\ &= \sin^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.017991, size = 3, normalized size = 1.

$$\sin^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

Maple [A] time = 0.008, size = 4, normalized size = 1.3

$$\arcsin(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1-ln(x)^2)^(1/2),x)
```

```
[Out] arcsin(ln(x))
```

Maxima [A] time = 1.50694, size = 4, normalized size = 1.33

$$\arcsin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(log(x))
```

Fricas [B] time = 1.76992, size = 61, normalized size = 20.33

$$-2 \arctan\left(\frac{\sqrt{-\log(x)^2 + 1} - 1}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-log(x)^2 + 1) - 1)/log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(\log(x)-1)(\log(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(log(x) - 1)*(log(x) + 1))), x)
```

Giac [A] time = 1.32127, size = 4, normalized size = 1.33

$$\arcsin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(log(x))
```

3.226 $\int 16x^3 \log^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x^4}{2} + 4x^4 \log^2(x) - 2x^4 \log(x)$$

```
[Out] x^4/2 - 2*x^4*Log[x] + 4*x^4*Log[x]^2
```

Rubi [A] time = 0.0228337, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 2305, 2304}

$$\frac{x^4}{2} + 4x^4 \log^2(x) - 2x^4 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[16*x^3*Log[x]^2,x]
```

```
[Out] x^4/2 - 2*x^4*Log[x] + 4*x^4*Log[x]^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int 16x^3\log^2(x)\,dx = 16\int x^3\log^2(x)\,dx$$
$$= 4x^4\log^2(x) - 8\int x^3\log(x)\,dx$$
$$= \frac{x^4}{2} - 2x^4\log(x) + 4x^4\log^2(x)$$

Mathematica [A] time = 0.0012288, size = 30, normalized size = 1.25

$$16\left(\frac{x^4}{32} + \frac{1}{4}x^4\log^2(x) - \frac{1}{8}x^4\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*x^3*Log[x]^2,x]
```

```
[Out] 16*(x^4/32 - (x^4*Log[x])/8 + (x^4*Log[x]^2)/4)
```

Maple [A] time = 0.003, size = 23, normalized size = 1.

$$\frac{x^4}{2} - 2\,x^4\ln(x) + 4\,x^4(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*x^3*ln(x)^2,x)
```

```
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Maxima [A] time = 1.04629, size = 23, normalized size = 0.96

$$\frac{1}{2}\left(8\,\log(x)^2 - 4\,\log(x) + 1\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*log(x)^2 - 4*log(x) + 1)*x^4
```

Fricas [A] time = 1.78746, size = 55, normalized size = 2.29

$$4\,x^4\log\left(x\right)^2-2\,x^4\log\left(x\right)+\frac{1}{2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="fricas")
```

```
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Sympy [A] time = 0.105006, size = 22, normalized size = 0.92

$$4x^4\log\left(x\right)^2-2x^4\log\left(x\right)+\frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x**3*ln(x)**2,x)
```

```
[Out] 4*x**4*log(x)**2 - 2*x**4*log(x) + x**4/2
```

Giac [A] time = 1.34682, size = 30, normalized size = 1.25

$$4\,x^4\log\left(x\right)^2-2\,x^4\log\left(x\right)+\frac{1}{2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="giac")
```

```
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

3.227 $\int \log\left(\sqrt{a+bx}\right) dx$

Optimal. Leaf size=25

$$\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

```
[Out] -x/2 + ((a + b*x)*Log[Sqrt[a + b*x]])/b
```

Rubi [A] time = 0.009634, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2389, 2295}

$$\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[a + b*x]],x]
```

```
[Out] -x/2 + ((a + b*x)*Log[Sqrt[a + b*x]])/b
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\sqrt{a+bx}\right) dx &= \frac{\text{Subst}\left(\int \log\left(\sqrt{x}\right) dx, x, a+bx\right)}{b}\\ &= -\frac{x}{2}+\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0044784, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(\frac{(a+bx)\log(a+bx)}{b}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[a + b*x]],x]
```

```
[Out] (-x + ((a + b*x)*Log[a + b*x])/b)/2
```

Maple [A] time = 0.001, size = 32, normalized size = 1.3

$$\frac{\ln(bx+a)\,x}{2}+\frac{\ln(bx+a)\,a}{2b}-\frac{x}{2}-\frac{a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(b*x+a),x)
```

```
[Out] 1/2*ln(b*x+a)*x+1/2/b*ln(b*x+a)*a-1/2*x-1/2*a/b
```

Maxima [A] time = 1.02115, size = 31, normalized size = 1.24

$$-\frac{bx-(bx+a)\log(bx+a)+a}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Fricas [A] time = 1.74772, size = 53, normalized size = 2.12

$$-\frac{bx-(bx+a)\log(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a))/b
```

Sympy [A] time = 0.318819, size = 29, normalized size = 1.16

$$-b\left(-\frac{a\log(a+bx)}{2b^2}+\frac{x}{2b}\right)+\frac{x\log(a+bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(b*x+a),x)
```

```
[Out] -b*(-a*log(a + b*x)/(2*b**2) + x/(2*b)) + x*log(a + b*x)/2
```

Giac [A] time = 1.3112, size = 31, normalized size = 1.24

$$-\frac{bx-(bx+a)\log(bx+a)+a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

3.228 $\int x \log\left(\sqrt{2+x}\right) dx$

Optimal. Leaf size=34

$$-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

```
[Out] x/2 - x^2/8 + (x^2*Log[Sqrt[2 + x]])/2 - Log[2 + x]
```

Rubi [A] time = 0.0135295, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2395, 43}

$$-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Sqrt[2 + x]],x]
```

```
[Out] x/2 - x^2/8 + (x^2*Log[Sqrt[2 + x]])/2 - Log[2 + x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x \log\left(\sqrt{2+x}\right) dx &= \frac{1}{2}x^2 \log\left(\sqrt{2+x}\right) - \frac{1}{4}\int \frac{x^2}{2+x} dx \\
&= \frac{1}{2}x^2 \log\left(\sqrt{2+x}\right) - \frac{1}{4}\int \left(-2+x+\frac{4}{2+x}\right) dx \\
&= \frac{x}{2} - \frac{x^2}{8} + \frac{1}{2}x^2 \log\left(\sqrt{2+x}\right) - \log(2+x)
\end{aligned}
$$

Mathematica [A] time = 0.0064352, size = 30, normalized size = 0.88

$$\frac{1}{2}\left(-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x+2) + x - 2\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Sqrt[2 + x]],x]
```

```
[Out] (x - x^2/4 - 2*Log[2 + x] + (x^2*Log[2 + x])/2)/2
```

Maple [A] time = 0.001, size = 31, normalized size = 0.9

$$\frac{\ln(2+x)(2+x)^2}{4} - \frac{x^2}{8} + \frac{x}{2} + \frac{3}{2} - (2+x)\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x*ln(2+x),x)
```

```
[Out] 1/4*ln(2+x)*(2+x)^2-1/8*x^2+1/2*x+3/2-(2+x)*ln(2+x)
```

Maxima [A] time = 1.06374, size = 32, normalized size = 0.94

$$\frac{1}{4}x^2 \log(x+2) - \frac{1}{8}x^2 + \frac{1}{2}x - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="maxima")
```

```
[Out] 1/4*x^2*log(x + 2) - 1/8*x^2 + 1/2*x - log(x + 2)
```

Fricas [A] time = 1.85485, size = 61, normalized size = 1.79

$$-\frac{1}{8}x^2 + \frac{1}{4}\left(x^2 - 4\right)\log\left(x + 2\right) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="fricas")
```

```
[Out] -1/8*x^2 + 1/4*(x^2 - 4)*log(x + 2) + 1/2*x
```

Sympy [A] time = 0.112509, size = 22, normalized size = 0.65

$$\frac{x^2\log\left(x + 2\right)}{4} - \frac{x^2}{8} + \frac{x}{2} - \log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*ln(2+x),x)
```

```
[Out] x**2*log(x + 2)/4 - x**2/8 + x/2 - log(x + 2)
```

Giac [A] time = 1.25579, size = 41, normalized size = 1.21

$$\frac{1}{4}\left(x + 2\right)^2\log\left(x + 2\right) - \frac{1}{8}\left(x + 2\right)^2 - \left(x + 2\right)\log\left(x + 2\right) + x + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="giac")
```

```
[Out] 1/4*(x + 2)^2*log(x + 2) - 1/8*(x + 2)^2 - (x + 2)*log(x + 2) + x + 2
```

3.229 $\int x \log\left(\sqrt[3]{1+3x}\right) dx$

Optimal. Leaf size=40

$$-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{3x+1}\right)+\frac{x}{18}-\frac{1}{54}\log(3x+1)$$

```
[Out] x/18 - x^2/12 + (x^2*Log[(1 + 3*x)^(1/3)])/2 - Log[1 + 3*x]/54
```

Rubi [A] time = 0.0159573, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2395, 43}

$$-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{3x+1}\right)+\frac{x}{18}-\frac{1}{54}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] x/18 - x^2/12 + (x^2*Log[(1 + 3*x)^(1/3)])/2 - Log[1 + 3*x]/54
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\log\left(\sqrt[3]{1+3x}\right)\,dx=\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\frac{x^2}{1+3x}\,dx$$
$$=\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\left(-\frac{1}{9}+\frac{x}{3}+\frac{1}{9(1+3x)}\right)dx$$
$$=\frac{x}{18}-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{54}\log(1+3x)$$

Mathematica [A] time = 0.0083492, size = 40, normalized size = 1.

$$\frac{1}{3}\left(-\frac{x^2}{4}+\frac{1}{2}x^2\log(3x+1)+\frac{x}{6}-\frac{1}{18}\log(3x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] (x/6 - x^2/4 - Log[1 + 3*x]/18 + (x^2*Log[1 + 3*x])/2)/3
```

Maple [A] time = 0.003, size = 39, normalized size = 1.

$$\frac{\ln(1+3x)(1+3x)^2}{54}-\frac{x^2}{12}+\frac{x}{18}+\frac{1}{36}-\frac{(1+3x)\ln(1+3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3*x*ln(1+3*x),x)
```

```
[Out] 1/54*ln(1+3*x)*(1+3*x)^2-1/12*x^2+1/18*x+1/36-1/27*(1+3*x)*ln(1+3*x)
```

Maxima [A] time = 1.04281, size = 38, normalized size = 0.95

$$\frac{1}{6}x^2\log(3x+1)-\frac{1}{12}x^2+\frac{1}{18}x-\frac{1}{54}\log(3x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^2*log(3*x + 1) - 1/12*x^2 + 1/18*x - 1/54*log(3*x + 1)
```

Fricas [A] time = 1.86306, size = 70, normalized size = 1.75

$$-\frac{1}{12}\,x^2+\frac{1}{54}\left(9\,x^2-1\right)\log\left(3\,x+1\right)+\frac{1}{18}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="fricas")
```

```
[Out] -1/12*x^2 + 1/54*(9*x^2 - 1)*log(3*x + 1) + 1/18*x
```

Sympy [A] time = 0.117948, size = 27, normalized size = 0.68

$$\frac{x^2\log\left(3x+1\right)}{6}-\frac{x^2}{12}+\frac{x}{18}-\frac{\log\left(3x+1\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*ln(1+3*x),x)
```

```
[Out] x**2*log(3*x + 1)/6 - x**2/12 + x/18 - log(3*x + 1)/54
```

Giac [A] time = 1.21409, size = 57, normalized size = 1.42

$$\frac{1}{54}\left(3\,x+1\right)^2\log\left(3\,x+1\right)-\frac{1}{108}\left(3\,x+1\right)^2-\frac{1}{27}\left(3\,x+1\right)\log\left(3\,x+1\right)+\frac{1}{9}\,x+\frac{1}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="giac")
```

```
[Out] 1/54*(3*x + 1)^2*log(3*x + 1) - 1/108*(3*x + 1)^2 - 1/27*(3*x + 1)*log(3*x
+ 1) + 1/9*x + 1/27
```

3.230 $\int x \log\left(x + x^3\right) dx$

Optimal. Leaf size=31

$$-\frac{3x^2}{4} + \frac{1}{2}x^2 \log\left(x^3 + x\right) + \frac{1}{2}\log\left(x^2 + 1\right)$$

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rubi [A] time = 0.0293318, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2525, 444, 43}

$$-\frac{3x^2}{4} + \frac{1}{2}x^2 \log\left(x^3 + x\right) + \frac{1}{2}\log\left(x^2 + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\log\left(x+x^3\right)dx &= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{2}\int\frac{x\left(1+3x^2\right)}{1+x^2}\,dx\\
&= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\operatorname{Subst}\left(\int\frac{1+3x}{1+x}\,dx,x,x^2\right)\\
&= \frac{1}{2}x^2\log\left(x+x^3\right)-\frac{1}{4}\operatorname{Subst}\left(\int\left(3-\frac{2}{1+x}\right)dx,x,x^2\right)\\
&= -\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.00809, size = 31, normalized size = 1.

$$-\frac{3x^2}{4}+\frac{1}{2}x^2\log\left(x^3+x\right)+\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Maple [A] time = 0.005, size = 26, normalized size = 0.8

$$-\frac{3\,x^2}{4}+\frac{\ln\left(x^2+1\right)}{2}+\frac{x^2\ln\left(x^3+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^3+x),x)
```

```
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Maxima [A] time = 1.56524, size = 34, normalized size = 1.1

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.84842, size = 69, normalized size = 2.23

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.134358, size = 26, normalized size = 0.84

$$\frac{x^2\log\left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**3+x),x)
```

```
[Out] x**2*log(x**3 + x)/2 - 3*x**2/4 + log(x**2 + 1)/2
```

Giac [A] time = 1.34124, size = 34, normalized size = 1.1

$$\frac{1}{2}x^2\log\left(x^3+x\right)-\frac{3}{4}x^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

3.231 $\int \log \left(x+\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=26

$$x \log \left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0040469, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2534, 261}

$$x \log \left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \log \left(x+\sqrt{1+x^2}\right) dx &= x \log \left(x+\sqrt{1+x^2}\right)-\int \frac{x}{\sqrt{1+x^2}} \, dx\\ &= -\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0157574, size = 26, normalized size = 1.

$$x\log\left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Maple [A] time = 0.004, size = 23, normalized size = 0.9

$$x\ln\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2+1)^(1/2)),x)
```

```
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(x+\sqrt{x^2+1}\right)-x+\arctan(x)-\int\frac{x}{x^3+\left(x^2+1\right)^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - x + arctan(x) - integrate(x/(x^3 + (x^2 + 1)^(3/
2) + x), x)
```

Fricas [A] time = 1.73915, size = 57, normalized size = 2.19

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A] time = 5.46699, size = 20, normalized size = 0.77

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] x*log(x + sqrt(x**2 + 1)) - sqrt(x**2 + 1)
```

Giac [A] time = 1.18404, size = 30, normalized size = 1.15

$$x\log\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

3.232 $\int \log \left(x+\sqrt{-1+x^2}\right) dx$

Optimal. Leaf size=26

$$x \log \left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0044315, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2534, 261}

$$x \log \left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \log \left(x+\sqrt{-1+x^2}\right) dx &= x \log \left(x+\sqrt{-1+x^2}\right)-\int \frac{x}{\sqrt{-1+x^2}} dx\\ &= -\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0164519, size = 26, normalized size = 1.

$$x\log\left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Maple [A] time = 0.004, size = 23, normalized size = 0.9

$$x\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2-1)^(1/2)),x)
```

```
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(\sqrt{x+1}\sqrt{x-1}+x\right)-x+\int\frac{x}{x^3+\left(x^2-1\right)e^{\left(\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)\right)}-x}\,dx+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x + 1)*sqrt(x - 1) + x) - x + integrate(x/(x^3 + (x^2 - 1)*e^(1/
2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A] time = 1.9504, size = 57, normalized size = 2.19

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [A] time = 9.58059, size = 20, normalized size = 0.77

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2-1)**(1/2)),x)
```

```
[Out] x*log(x + sqrt(x**2 - 1)) - sqrt(x**2 - 1)
```

Giac [A] time = 1.23056, size = 30, normalized size = 1.15

$$x\log\left(x+\sqrt{x^2-1}\right)-\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

3.233 $\int \log\left(x-\sqrt{-1+x^2}\right)\,dx$

Optimal. Leaf size=26

$$\sqrt{x^2-1}+x\log\left(x-\sqrt{x^2-1}\right)$$

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0054906, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2534, 261}

$$\sqrt{x^2-1}+x\log\left(x-\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x - Sqrt[-1 + x^2]],x]

[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \log\left(x-\sqrt{-1+x^2}\right)\,dx &= x\log\left(x-\sqrt{-1+x^2}\right)+\int \frac{x}{\sqrt{-1+x^2}}\,dx\\ &= \sqrt{-1+x^2}+x\log\left(x-\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0179168, size = 26, normalized size = 1.

$$\sqrt{x^2-1}+x\log\left(x-\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x - Sqrt[-1 + x^2]],x]
```

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Maple [A] time = 0.001, size = 23, normalized size = 0.9

$$x\ln\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2-1)^(1/2)),x)
```

```
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(-\sqrt{x+1}\sqrt{x-1}+x\right)-x-\int-\frac{x}{x^3-\left(x^2-1\right)e^{\left(\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)\right)}-x}\,dx+\frac{1}{2}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(-sqrt(x + 1)*sqrt(x - 1) + x) - x - integrate(-x/(x^3 - (x^2 - 1)*e^(
1/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A] time = 1.84725, size = 57, normalized size = 2.19

$$x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Sympy [A] time = 11.1306, size = 20, normalized size = 0.77

$$x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x-(x**2-1)**(1/2)),x)
```

```
[Out] x*log(x - sqrt(x**2 - 1)) + sqrt(x**2 - 1)
```

Giac [A] time = 1.2003, size = 30, normalized size = 1.15

$$x\log\left(x-\sqrt{x^2-1}\right)+\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

3.234 $\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 + x]
]
```

Rubi [A] time = 0.0123772, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2548, 12, 1958, 50, 54, 215}

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 + x]
]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\sqrt{x}+\sqrt{1+x}\right)dx &= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.024223, size = 43, normalized size = 1.

$$-\frac{1}{2}\sqrt{x}\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -(Sqrt[x]*Sqrt[1 + x])/2 + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 + x]
]
```

Maple [A] time = 0.006, size = 52, normalized size = 1.2

$$x\ln\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{4}\sqrt{x\,(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)+1/4*(x*(1+x))^(1/2)/x^(1/
2)/(1+x)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,x-\int\frac{x}{2\left(x^2+\left(x^{\frac{3}{2}}+\sqrt{x}\right)\sqrt{x+1}+x\right)}\,dx+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*x - integrate(1/2*x/(x^2 + (x^(3/2) + sq
rt(x))*sqrt(x + 1) + x), x) + 1/2*log(x + 1)
```

Fricas [A] time = 1.88268, size = 92, normalized size = 2.14

$$\frac{1}{2}\left(2\,x+1\right)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\sqrt{x}+\sqrt{x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Integral(log(sqrt(x) + sqrt(x + 1)), x)
```

Giac [A] time = 1.21828, size = 54, normalized size = 1.26

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\sqrt{x^2+x}-\frac{1}{4}\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

3.235 $\int \sqrt[3]{x} \log(x) \, dx$

Optimal. Leaf size=21

$$\frac{3}{4}x^{4/3}\log(x) - \frac{9x^{4/3}}{16}$$

```
[Out] (-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4
```

Rubi [A] time = 0.0067835, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{3}{4}x^{4/3}\log(x) - \frac{9x^{4/3}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*Log[x],x]
```

```
[Out] (-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x} \log(x) \, dx = -\frac{9x^{4/3}}{16} + \frac{3}{4}x^{4/3}\log(x)$$

Mathematica [A] time = 0.0023618, size = 15, normalized size = 0.71

$$\frac{3}{16}x^{4/3}(4\log(x) - 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*Log[x],x]
```

```
[Out] (3*x^(4/3)*(-3 + 4*Log[x]))/16
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$-\frac{9}{16}x^{\frac{4}{3}}+\frac{3\ln(x)}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*ln(x),x)
```

```
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Maxima [A] time = 1.06314, size = 18, normalized size = 0.86

$$\frac{3}{4}x^{\frac{4}{3}}\log(x)-\frac{9}{16}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Fricas [A] time = 1.93388, size = 45, normalized size = 2.14

$$\frac{3}{16}(4x\log(x)-3x)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="fricas")
```

```
[Out] 3/16*(4*x*log(x) - 3*x)*x^(1/3)
```

Sympy [A] time = 3.09549, size = 66, normalized size = 3.14

$$\begin{cases} \frac{3x^{\frac{4}{3}}\log(x)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } |x| < 1 \\ -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4} - \frac{9x^{\frac{4}{3}}}{16} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{7}{3}, \frac{7}{3} \\ \frac{4}{3}, \frac{4}{3} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{7}{3}, \frac{7}{3}, 1 & \\ & \frac{4}{3}, \frac{4}{3}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*ln(x),x)
```

```
[Out] Piecewise((3*x**(4/3)*log(x)/4 - 9*x**(4/3)/16, Abs(x) < 1), (-3*x**(4/3)*l
og(1/x)/4 - 9*x**(4/3)/16, 1/Abs(x) < 1), (-meijerg(((1,), (7/3, 7/3)), ((4
/3, 4/3), (0,)), x) + meijerg(((7/3, 7/3, 1), ()), ((), (4/3, 4/3, 0)), x),
 True))
```

Giac [A] time = 1.31856, size = 18, normalized size = 0.86

$$\frac{3}{4}\,x^{\frac{4}{3}}\log\left(x\right) - \frac{9}{16}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

3.236 $\int 2^{\log(x)}\,dx$

Optimal. Leaf size=13

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

```
[Out] x^(1 + Log[2])/(1 + Log[2])
```

Rubi [A] time = 0.0054808, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2274, 30}

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x],x]
```

```
[Out] x^(1 + Log[2])/(1 + Log[2])
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(x)}\,dx &= \int x^{\log(2)}\,dx\\ &= \frac{x^{1+\log(2)}}{1+\log(2)}\end{aligned}$$

Mathematica [A] time = 0.0025823, size = 12, normalized size = 0.92

$$\frac{x2^{\log(x)}}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x],x]
```

```
[Out] (2^Log[x]*x)/(1 + Log[2])
```

Maple [A] time = 0.013, size = 13, normalized size = 1.

$$\frac{x2^{\ln(x)}}{1+\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x),x)
```

```
[Out] x/(1+ln(2))*2^ln(x)
```

Maxima [A] time = 1.02621, size = 32, normalized size = 2.46

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="maxima")
```

```
[Out] 2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Fricas [A] time = 1.89423, size = 46, normalized size = 3.54

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="fricas")
```

```
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A] time = 0.417882, size = 10, normalized size = 0.77

$$\frac{2^{\log(x)}x}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x),x)
```

```
[Out] 2**log(x)*x/(log(2) + 1)
```

Giac [A] time = 1.2181, size = 19, normalized size = 1.46

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="giac")
```

```
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

3.237 $\int \frac{1-\log(x)}{x^2}\,dx$

Optimal. Leaf size=6

$$\frac{\log(x)}{x}$$

[Out] Log[x]/x

Rubi [A] time = 0.0104505, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2303}

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

[In] Int[(1 - Log[x])/x^2,x]

[Out] Log[x]/x

Rule 2303

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(b*(d*x)^(m + 1)*Log[c*x^n])/(d*(m + 1)), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Rubi steps

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log(x)}{x}$$

Mathematica [A] time = 0.0012218, size = 6, normalized size = 1.

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[x])/x^2,x]
```

```
[Out] Log[x]/x
```

Maple [A] time = 0.001, size = 7, normalized size = 1.2

$$\frac{\ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x^2,x)
```

```
[Out] ln(x)/x
```

Maxima [B] time = 1.09091, size = 19, normalized size = 3.17

$$\frac{\log(x)+1}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="maxima")
```

```
[Out] (log(x) + 1)/x - 1/x
```

Fricas [A] time = 1.71323, size = 14, normalized size = 2.33

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="fricas")
```

```
[Out] log(x)/x
```

Sympy [A] time = 0.090357, size = 3, normalized size = 0.5

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x**2,x)
```

```
[Out] log(x)/x
```

Giac [A] time = 1.31882, size = 8, normalized size = 1.33

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="giac")
```

```
[Out] log(x)/x
```

3.238 $\int \log\left(1+x+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=32

$$-x+\sqrt{x+1}+x\log\left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log(x+1)$$

```
[Out] -x + Sqrt[1 + x] + Log[1 + x]/2 + x*Log[1 + x + Sqrt[1 + x]]
```

Rubi [A] time = 0.205253, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2548}

$$-x+\sqrt{x+1}+x\log\left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] + Log[1 + x]/2 + x*Log[1 + x + Sqrt[1 + x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(1+x+\sqrt{1+x}\right)\,dx &= x\log\left(1+x+\sqrt{1+x}\right)-\int \frac{x\left(1+\frac{1}{2\sqrt{1+x}}\right)}{1+x+\sqrt{1+x}}\,dx\\ &= x\log\left(1+x+\sqrt{1+x}\right)-2\,\text{Subst}\left(\int\left(-\frac{1}{2}-\frac{1}{2x}+x\right)dx,x,\sqrt{1+x}\right)\\ &= -x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log\left(1+x+\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0138862, size = 38, normalized size = 1.19

$$-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)+(x+1)\log\left(x+\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] - Log[1 + Sqrt[1 + x]] + (1 + x)*Log[1 + x + Sqrt[1 + x]]
```

Maple [A] time = 0.005, size = 34, normalized size = 1.1

$$(1+x)\ln\left(1+x+\sqrt{1+x}\right)-x-1+\sqrt{1+x}-\ln\left(\sqrt{1+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x+(1+x)^(1/2)),x)
```

```
[Out] (1+x)*ln(1+x+(1+x)^(1/2))-x-1+(1+x)^(1/2)-ln((1+x)^(1/2)+1)
```

Maxima [A] time = 1.06802, size = 45, normalized size = 1.41

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

Fricas [A] time = 1.81003, size = 130, normalized size = 4.06

$$(x-1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}+\log\left(\sqrt{x+1}+1\right)+2\,\log\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x - 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) + log(sqrt(x + 1) + 1) +
 2*log(sqrt(x + 1))
```

Sympy [B] time = 1.05152, size = 158, normalized size = 4.94

$$\frac{x\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{x\sqrt{x+1}}{\sqrt{x+1}+1}+\frac{x\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}+\frac{\sqrt{x+1}\log\left(x+\sqrt{x}\right.}{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x+(1+x)**(1/2)),x)
```

```
[Out] x*sqrt(x + 1)*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - x*sqrt(x + 1)/(s
qrt(x + 1) + 1) + x*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - sqrt(x + 1
)*log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)*log(x + sqrt(x + 1)
+ 1)/(sqrt(x + 1) + 1) - log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + log(x + s
qrt(x + 1) + 1)/(sqrt(x + 1) + 1)
```

Giac [A] time = 1.31745, size = 45, normalized size = 1.41

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

3.239 $\int \log \left(x + x^3\right) dx$

Optimal. Leaf size=16

$$x \log \left(x^3 + x\right) - 3x + 2 \tan^{-1}(x)$$

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rubi [A] time = 0.0071385, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2523, 388, 203}

$$x \log \left(x^3 + x\right) - 3x + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \log\left(x+x^3\right)\,dx = x\log\left(x+x^3\right) - \int \frac{1+3x^2}{1+x^2}\,dx$$
$$= -3x + x\log\left(x+x^3\right) + 2\int \frac{1}{1+x^2}\,dx$$
$$= -3x + 2\tan^{-1}(x) + x\log\left(x+x^3\right)$$

Mathematica [A] time = 0.0023855, size = 16, normalized size = 1.

$$x\log\left(x^3+x\right) - 3x + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Maple [A] time = 0.004, size = 17, normalized size = 1.1

$$-3\,x + 2\,\arctan(x) + x\ln\left(x^3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^3+x),x)
```

```
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Maxima [A] time = 1.49775, size = 22, normalized size = 1.38

$$x\log\left(x^3+x\right) - 3\,x + 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="maxima")
```

```
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Fricas [A] time = 1.80446, size = 49, normalized size = 3.06

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="fricas")
```

```
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Sympy [A] time = 0.133257, size = 15, normalized size = 0.94

$$x \log \left(x^{3} + x\right) - 3x + 2 \operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**3+x),x)
```

```
[Out] x*log(x**3 + x) - 3*x + 2*atan(x)
```

Giac [A] time = 1.23416, size = 22, normalized size = 1.38

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="giac")
```

```
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

3.240 $\int 2^{\log(-8+7x)}\,dx$

Optimal. Leaf size=20

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

```
[Out] (-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))
```

Rubi [A] time = 0.004664, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2274, 32}

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[-8 + 7*x],x]
```

```
[Out] (-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))
```

Rule 2274

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(-8+7x)}\,dx &= \int (-8+7x)^{\log(2)}\,dx\\ &= \frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}\end{aligned}$$

Mathematica [A] time = 0.0067256, size = 20, normalized size = 1.

$$\frac{(7x-8)2^{\log(7x-8)}}{7+\log(128)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[-8 + 7*x],x]
```

```
[Out] (2^Log[-8 + 7*x]*(-8 + 7*x))/(7 + Log[128])
```

Maple [A] time = 0.007, size = 22, normalized size = 1.1

$$\frac{(-8+7x)\,2^{\ln(-8+7x)}}{7\ln(2)+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(-8+7*x),x)
```

```
[Out] 1/7*(-8+7*x)/(1+ln(2))*2^ln(-8+7*x)
```

Maxima [A] time = 0.991234, size = 39, normalized size = 1.95

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(7x-8)}}{7\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="maxima")
```

```
[Out] 1/7*2^((1/log(2) + 1)*log(7*x - 8))/((1/log(2) + 1)*log(2))
```

Fricas [A] time = 1.8078, size = 70, normalized size = 3.5

$$\frac{(7x-8)e^{(\log(2)\log(7x-8))}}{7(\log(2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="fricas")
```

```
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Sympy [B] time = 0.505494, size = 34, normalized size = 1.7

$$\frac{7 \cdot 2^{\log(7x-8)}x}{7\log(2)+7} - \frac{8 \cdot 2^{\log(7x-8)}}{7\log(2)+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(-8+7*x),x)
```

```
[Out] 7*2**log(7*x - 8)*x/(7*log(2) + 7) - 8*2**log(7*x - 8)/(7*log(2) + 7)
```

Giac [A] time = 1.21184, size = 31, normalized size = 1.55

$$\frac{(7x-8)e^{(\log(2)\log(7x-8))}}{7(\log(2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="giac")
```

```
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

3.241 $\int \log\left(\frac{-11+5x}{5+76x}\right) dx$

Optimal. Leaf size=35

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

```
[Out] -((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))])/5 - (861*Log[5 + 76*x])/380
```

Rubi [A] time = 0.0058722, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2486, 31}

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] -((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))])/5 - (861*Log[5 + 76*x])/380
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \log\left(\frac{-11+5x}{5+76x}\right)dx = -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right) - \frac{861}{5}\int\frac{1}{5+76x}dx$$
$$= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right) - \frac{861}{380}\log(5+76x)$$

Mathematica [A] time = 0.0049719, size = 31, normalized size = 0.89

$$\left(x-\frac{11}{5}\right)\log\left(\frac{5x-11}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] (-11/5 + x)*Log[(-11 + 5*x)/(5 + 76*x)] - (861*Log[5 + 76*x])/380
```

Maple [A] time = 0.01, size = 44, normalized size = 1.3

$$\frac{861}{380}\ln\left(-861\,(5+76\,x)^{-1}\right)+\frac{5+76\,x}{5}\ln\left(\frac{5}{76}-\frac{861}{380+5776\,x}\right)\left(\frac{5}{76}-\frac{861}{380+5776\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-11+5*x)/(5+76*x)),x)
```

```
[Out] 861/380*ln(-861/(5+76*x))+1/5*ln(5/76-861/76/(5+76*x))*(5/76-861/76/(5+76*x
))*(5+76*x)
```

Maxima [A] time = 1.00665, size = 45, normalized size = 1.29

$$x\log\left(\frac{5\,x-11}{76\,x+5}\right)-\frac{5}{76}\log\left(76\,x+5\right)-\frac{11}{5}\log\left(5\,x-11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="maxima")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Fricas [A] time = 1.77686, size = 97, normalized size = 2.77

$$x \log\left(\frac{5\,x - 11}{76\,x + 5}\right) - \frac{5}{76}\,\log\left(76\,x + 5\right) - \frac{11}{5}\,\log\left(5\,x - 11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="fricas")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Sympy [A] time = 0.160948, size = 32, normalized size = 0.91

$$x \log\left(\frac{5x - 11}{76x + 5}\right) - \frac{11 \log\left(x - \frac{11}{5}\right)}{5} - \frac{5 \log\left(x + \frac{5}{76}\right)}{76}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-11+5*x)/(5+76*x)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 11*log(x - 11/5)/5 - 5*log(x + 5/76)/76
```

Giac [A] time = 1.19923, size = 47, normalized size = 1.34

$$x \log\left(\frac{5\,x - 11}{76\,x + 5}\right) - \frac{5}{76}\,\log\left(|76\,x + 5|\right) - \frac{11}{5}\,\log\left(|5\,x - 11|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="giac")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(abs(76*x + 5)) - 11/5*log(abs(5*x -
 11))
```

3.242 $\int \log\left(\frac{1}{13+x}\right) dx$

Optimal. Leaf size=12

$$x+(x+13)\log\left(\frac{1}{x+13}\right)$$

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Rubi [A] time = 0.0026972, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2295}

$$x+(x+13)\log\left(\frac{1}{x+13}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(13 + x)^(-1)],x]
```

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{1}{13+x}\right) dx &= \text{Subst}\left(\int \log\left(\frac{1}{x}\right) dx, x, 13+x\right)\\ &= x+(13+x)\log\left(\frac{1}{13+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0019943, size = 12, normalized size = 1.

$$x + (x + 13)\log\left(\frac{1}{x + 13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(13 + x)^(-1)],x]
```

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Maple [A] time = 0.007, size = 14, normalized size = 1.2

$$(13 + x)\ln\left((13 + x)^{-1}\right) + 13 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/(13+x)),x)
```

```
[Out] (13+x)*ln(1/(13+x))+13+x
```

Maxima [A] time = 1.0395, size = 16, normalized size = 1.33

$$-(x + 13)\log(x + 13) + x + 13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="maxima")
```

```
[Out] -(x + 13)*log(x + 13) + x + 13
```

Fricas [A] time = 1.84408, size = 41, normalized size = 3.42

$$(x + 13)\log\left(\frac{1}{x + 13}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="fricas")
```

```
[Out] (x + 13)*log(1/(x + 13)) + x
```

Sympy [A] time = 0.105836, size = 15, normalized size = 1.25

$$x\log\left(\frac{1}{x+13}\right)+x-13\log\left(x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/(13+x)),x)
```

```
[Out] x*log(1/(x + 13)) + x - 13*log(x + 13)
```

Giac [A] time = 1.24363, size = 23, normalized size = 1.92

$$x\log\left(\frac{1}{x+13}\right)+x-13\log\left(|x+13|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="giac")
```

```
[Out] x*log(1/(x + 13)) + x - 13*log(abs(x + 13))
```

3.243 $\int x \log\left(\frac{1+x}{x^2}\right) dx$

Optimal. Leaf size=36

$$\frac{x^2}{4}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

```
[Out] x/2 + x^2/4 - Log[1 + x]/2 + (x^2*Log[(1 + x)/x^2])/2
```

Rubi [A] time = 0.014721, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2495, 30, 43}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[(1 + x)/x^2],x]
```

```
[Out] x/2 + x^2/4 - Log[1 + x]/2 + (x^2*Log[(1 + x)/x^2])/2
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\log\left(\frac{1+x}{x^2}\right)dx &= \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\frac{x^2}{1+x}\,dx+\int x\,dx\\ &= \frac{x^2}{2}+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)-\frac{1}{2}\int\left(-1+x+\frac{1}{1+x}\right)dx\\ &= \frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\log(1+x)+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0094705, size = 27, normalized size = 0.75

$$\frac{1}{4}\left(x\left(2x\log\left(\frac{x+1}{x^2}\right)+x+2\right)-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + x)/x^2],x]
```

```
[Out] (-2*Log[1 + x] + x*(2 + x + 2*x*Log[(1 + x)/x^2]))/4
```

Maple [A] time = 0.014, size = 39, normalized size = 1.1

$$\frac{x^2}{2}\ln\left(\frac{1+x^{-1}}{x}\right)+\frac{x^2}{4}+\frac{x}{2}+\frac{\ln\left(x^{-1}\right)}{2}-\frac{\ln\left(1+x^{-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln((1+x)/x^2),x)
```

```
[Out] 1/2*x^2*ln(1/x*(1+1/x))+1/4*x^2+1/2*x+1/2*ln(1/x)-1/2*ln(1+1/x)
```

Maxima [A] time = 1.10115, size = 38, normalized size = 1.06

$$\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Fricas [A] time = 1.79942, size = 82, normalized size = 2.28

$$\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Sympy [A] time = 0.131197, size = 27, normalized size = 0.75

$$\frac{x^2\log\left(\frac{x+1}{x^2}\right)}{2}+\frac{x^2}{4}+\frac{x}{2}-\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln((1+x)/x**2),x)
```

```
[Out] x**2*log((x + 1)/x**2)/2 + x**2/4 + x/2 - log(x + 1)/2
```

Giac [A] time = 1.33758, size = 39, normalized size = 1.08

$$\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right)+\frac{1}{4}\,x^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(abs(x + 1))
```

3.244 $\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx$

Optimal. Leaf size=54

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rubi [A] time = 0.0392872, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2495, 30, 43}

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rule 2495

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[((g + h*x)^(m + 1)*Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/(h*(m + 1)), x] + (-Dist[(b*p*r)/(h*(m
+ 1)), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[(d*q*r)/(h*(m + 1)),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx &= \frac{1}{4}x^4 \log\left(\frac{7+5x}{x^2}\right) + \frac{\int x^3\,dx}{2} - \frac{5}{4}\int \frac{x^4}{7+5x}\,dx\\ &= \frac{x^4}{8} + \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right) - \frac{5}{4}\int\left(-\frac{343}{625}+\frac{49x}{125}-\frac{7x^2}{25}+\frac{x^3}{5}+\frac{2401}{625(7+5x)}\right)dx\\ &= \frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0127958, size = 54, normalized size = 1.

$$\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right)+\frac{343x}{500}-\frac{2401\log(5x+7)}{2500}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Maple [A] time = 0.016, size = 53, normalized size = 1.

$$\frac{x^4}{4}\ln\left(\frac{1}{x}\left(7x^{-1}+5\right)\right)-\frac{2401}{2500}\ln\left(7x^{-1}+5\right)+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}+\frac{2401\ln\left(x^{-1}\right)}{2500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln((7+5*x)/x^2),x)
```

```
[Out] 1/4*x^4*ln((7/x+5)/x)-2401/2500*ln(7/x+5)+1/16*x^4+7/60*x^3-49/200*x^2+343/
500*x+2401/2500*ln(1/x)
```

Maxima [A] time = 1.05635, size = 57, normalized size = 1.06

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Fricas [A] time = 1.86796, size = 135, normalized size = 2.5

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Sympy [A] time = 0.149822, size = 48, normalized size = 0.89

$$\frac{x^4\log\left(\frac{5x+7}{x^2}\right)}{4}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}-\frac{2401\log\left(5x+7\right)}{2500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln((7+5*x)/x**2),x)
```

```
[Out] x**4*log((5*x + 7)/x**2)/4 + x**4/16 + 7*x**3/60 - 49*x**2/200 + 343*x/500
- 2401*log(5*x + 7)/2500
```

Giac [A] time = 1.27232, size = 58, normalized size = 1.07

$$\frac{1}{4}\,x^{4}\log\left(\frac{5\,x+7}{x^{2}}\right)+\frac{1}{16}\,x^{4}+\frac{7}{60}\,x^{3}-\frac{49}{200}\,x^{2}+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(|5\,x+7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(abs(5*x + 7))
```

3.245 $\int (a+bx)\log(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

```
[Out] -(a + b*x)^2/(4*b) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rubi [A] time = 0.0146922, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2390, 2304}

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[a + b*x],x]

[Out] -(a + b*x)^2/(4*b) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\log(x)\,dx, x, a+bx\right)}{b}\\ &= -\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0207654, size = 33, normalized size = 0.94

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{1}{4}x(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[a + b*x],x]
```

```
[Out] -(x*(2*a + b*x))/4 + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Maple [A] time = 0.003, size = 55, normalized size = 1.6

$$\frac{b\ln(bx+a)\,x^2}{2}+\ln(bx+a)\,xa+\frac{\ln(bx+a)\,a^2}{2\,b}-\frac{bx^2}{4}-\frac{ax}{2}-\frac{a^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(b*x+a),x)
```

```
[Out] 1/2*b*ln(b*x+a)*x^2+ln(b*x+a)*x*a+1/2/b*ln(b*x+a)*a^2-1/4*b*x^2-1/2*a*x-1/4
/b*a^2
```

Maxima [A] time = 1.09095, size = 70, normalized size = 2.

$$\frac{1}{4}\,b\left(\frac{2\,a^2\log(bx+a)}{b^2}-\frac{bx^2+2\,ax}{b}\right)+\frac{1}{2}\left(bx^2+2\,ax\right)\log(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*a^2*log(b*x + a)/b^2 - (b*x^2 + 2*a*x)/b) + 1/2*(b*x^2 + 2*a*x)*lo
g(b*x + a)
```

Fricas [A] time = 1.84343, size = 96, normalized size = 2.74

$$-\frac{b^2x^2+2\,abx-2\left(b^2x^2+2\,abx+a^2\right)\log(bx+a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/b
```

Sympy [A] time = 0.339864, size = 41, normalized size = 1.17

$$\frac{a^2\log(a+bx)}{2b} - \frac{ax}{2} - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log(a+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/(2*b) - a*x/2 - b*x**2/4 + (a*x + b*x**2/2)*log(a + b*x)
```

Giac [A] time = 1.3148, size = 42, normalized size = 1.2

$$\frac{(bx+a)^2\log(bx+a)}{2\,b} - \frac{(bx+a)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x + a)^2*log(b*x + a)/b - 1/4*(b*x + a)^2/b
```

3.246 $\int (a+bx)^2 \log(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

```
[Out] -(a + b*x)^3/(9*b) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rubi [A] time = 0.0246622, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2304}

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -(a + b*x)^3/(9*b) + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\log(x)\,dx,x,a+bx\right)}{b}\\ &= -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0167406, size = 44, normalized size = 1.26

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{1}{9}x\left(3a^2+3abx+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -(x*(3*a^2 + 3*a*b*x + b^2*x^2))/9 + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Maple [B] time = 0.001, size = 82, normalized size = 2.3

$$\frac{b^2\ln(bx+a)\,x^3}{3}+b\ln(bx+a)\,x^2a+\ln(bx+a)\,xa^2+\frac{\ln(bx+a)\,a^3}{3b}-\frac{b^2x^3}{9}-\frac{bx^2a}{3}-\frac{xa^2}{3}-\frac{a^3}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(b*x+a),x)
```

```
[Out] 1/3*b^2*ln(b*x+a)*x^3+b*ln(b*x+a)*x^2*a+ln(b*x+a)*x*a^2+1/3/b*ln(b*x+a)*a^3
-1/9*b^2*x^3-1/3*b*x^2*a-1/3*x*a^2-1/9/b*a^3
```

Maxima [B] time = 1.06081, size = 100, normalized size = 2.86

$$\frac{1}{9}\left(\frac{3\,a^3\log(bx+a)}{b^2}-\frac{b^2x^3+3\,abx^2+3\,a^2x}{b}\right)b+\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(3*a^3*log(b*x + a)/b^2 - (b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/b)*b + 1/3*(b
^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(b*x + a)
```

Fricas [B] time = 1.7631, size = 139, normalized size = 3.97

$$-\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx-3\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log(bx+a)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/b
```

Sympy [B] time = 0.365034, size = 63, normalized size = 1.8

$$\frac{a^3\log(a+bx)}{3b} - \frac{a^2x}{3} - \frac{abx^2}{3} - \frac{b^2x^3}{9} + \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\log(a+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(b*x+a),x)
```

```
[Out] a**3*log(a + b*x)/(3*b) - a**2*x/3 - a*b*x**2/3 - b**2*x**3/9 + (a**2*x + a
*b*x**2 + b**2*x**3/3)*log(a + b*x)
```

Giac [A] time = 1.32667, size = 42, normalized size = 1.2

$$\frac{(bx+a)^3\log(bx+a)}{3\,b} - \frac{(bx+a)^3}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*log(b*x + a)/b - 1/9*(b*x + a)^3/b
```

3.247 $\int \frac{\log(a+bx)}{a+bx} dx$

Optimal. Leaf size=15

$$\frac{\log^2(a+bx)}{2b}$$

```
[Out] Log[a + b*x]^2/(2*b)
```

Rubi [A] time = 0.0183944, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2301}

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(a+bx)}{a+bx}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, a+bx\right)}{b} \\ &= \frac{\log^2(a+bx)}{2b} \end{aligned}$$

Mathematica [A] time = 0.0016411, size = 15, normalized size = 1.

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x),x]

[Out] Log[a + b*x]^2/(2*b)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{(\ln(bx+a))^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a),x)

[Out] 1/2*ln(b*x+a)^2/b
```

Maxima [A] time = 1.09039, size = 18, normalized size = 1.2

$$\frac{\log(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="maxima")

[Out] 1/2*log(b*x + a)^2/b
```

Fricas [A] time = 1.78235, size = 30, normalized size = 2.

$$\frac{\log(bx+a)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

Sympy [A] time = 0.292422, size = 10, normalized size = 0.67

$$\frac{\log(a + bx)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a),x)
```

```
[Out] log(a + b*x)**2/(2*b)
```

Giac [A] time = 1.18051, size = 18, normalized size = 1.2

$$\frac{\log(bx + a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

3.248 $\int \frac{\log(a+bx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{b(a+bx)}-\frac{\log(a+bx)}{b(a+bx)}$$

```
[Out] -(1/(b*(a + b*x))) - Log[a + b*x]/(b*(a + b*x))
```

Rubi [A] time = 0.0218602, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2304}

$$-\frac{1}{b(a+bx)}-\frac{\log(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(1/(b*(a + b*x))) - Log[a + b*x]/(b*(a + b*x))
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(a+bx)}{(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\log(x)}{x^2}\,dx, x, a+bx\right)}{b}$$
$$= -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

Mathematica [A] time = 0.0060834, size = 21, normalized size = 0.68

$$-\frac{\log(a+bx)+1}{ab+b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -((1 + Log[a + b*x])/(a*b + b^2*x))
```

Maple [A] time = 0.003, size = 32, normalized size = 1.

$$-\frac{1}{b(bx+a)} - \frac{\ln(bx+a)}{b(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

Maxima [A] time = 0.998021, size = 42, normalized size = 1.35

$$-\frac{\log(bx+a)}{(bx+a)b} - \frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

Fricas [A] time = 1.82127, size = 47, normalized size = 1.52

$$-\frac{\log(bx+a)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) + 1)/(b^2*x + a*b)
```

Sympy [A] time = 0.345217, size = 26, normalized size = 0.84

$$-\frac{\log(a+bx)}{ab+b^2x}-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a)**2,x)
```

```
[Out] -log(a + b*x)/(a*b + b**2*x) - 1/(a*b + b**2*x)
```

Giac [A] time = 1.31106, size = 42, normalized size = 1.35

$$-\frac{\log(bx+a)}{(bx+a)b}-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

3.249 $\int (a+bx)^n \log(a+bx)\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)}-\frac{(a+bx)^{n+1}}{b(n+1)^2}$$

```
[Out] -((a + b*x)^(1 + n)/(b*(1 + n)^2)) + ((a + b*x)^(1 + n)*Log[a + b*x])/(b*(1
 + n))
```

Rubi [A] time = 0.0302658, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2304}

$$\frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)}-\frac{(a+bx)^{n+1}}{b(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] -((a + b*x)^(1 + n)/(b*(1 + n)^2)) + ((a + b*x)^(1 + n)*Log[a + b*x])/(b*(1
 + n))
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n \log(a+bx)\,dx = \frac{\text{Subst}\left(\int x^n \log(x)\,dx, x, a+bx\right)}{b}$$
$$= -\frac{(a+bx)^{1+n}}{b(1+n)^2} + \frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}$$

Mathematica [A] time = 0.0142109, size = 30, normalized size = 0.68

$$\frac{(a+bx)^{n+1}((n+1)\log(a+bx)-1)}{b(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] ((a + b*x)^(1 + n)*(-1 + (1 + n)*Log[a + b*x]))/(b*(1 + n)^2)
```

Maple [B] time = 0.013, size = 96, normalized size = 2.2

$$\frac{\ln(bx+a)\,x\mathrm{e}^{\ln(bx+a)n}}{1+n} + \frac{a\ln(bx+a)\,\mathrm{e}^{\ln(bx+a)n}}{b(1+n)} - \frac{x\mathrm{e}^{\ln(bx+a)n}}{n^2+2n+1} - \frac{a\mathrm{e}^{\ln(bx+a)n}}{b\left(n^2+2n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*ln(b*x+a),x)
```

```
[Out] 1/(1+n)*x*ln(b*x+a)*exp(ln(b*x+a)*n)+a/b/(1+n)*ln(b*x+a)*exp(ln(b*x+a)*n)-1
/(n^2+2*n+1)*x*exp(ln(b*x+a)*n)-a/b/(n^2+2*n+1)*exp(ln(b*x+a)*n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83709, size = 112, normalized size = 2.55

$$-\frac{\left(bx-\left(an+\left(bn+b\right)x+a\right)\log\left(bx+a\right)+a\right)\left(bx+a\right)^{n}}{bn^{2}+2\,bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - (a*n + (b*n + b)*x + a)*log(b*x + a) + a)*(b*x + a)^n/(b*n^2 + 2*b*
n + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*ln(b*x+a),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx+a\right)^{n}\log\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*log(b*x + a), x)
```

3.250 $\int \frac{1}{ax+bx\log(cx^n)} dx$

Optimal. Leaf size=18

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] time = 0.0107713, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n])^(-1),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log\left(cx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A] time = 0.0170556, size = 18, normalized size = 1.

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n])^(-1),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Maple [A] time = 0.004, size = 19, normalized size = 1.1

$$\frac{\ln\left(a+b\ln\left(cx^{n}\right)\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)),x)
```

```
[Out] ln(a+b*ln(c*x^n))/b/n
```

Maxima [A] time = 1.0808, size = 32, normalized size = 1.78

$$\frac{\log\left(\frac{b\log(c)+b\log(x^{n})+a}{b}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log((b*log(c) + b*log(x^n) + a)/b)/(b*n)
```

Fricas [A] time = 1.91867, size = 51, normalized size = 2.83

$$\frac{\log\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] log(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [A] time = 1.09954, size = 34, normalized size = 1.89

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b\log(c)} & \text{for } n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b}+n\log(x)+\log(c)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b*log(c)), Eq(n, 0)
), (log(x)/a, Eq(b, 0)), (log(a/b + n*log(x) + log(c))/(b*n), True))
```

Giac [B] time = 1.21578, size = 61, normalized size = 3.39

$$\frac{\log\left(\frac{1}{4}\left(\pi bn(\text{sgn}(x)-1)+\pi b(\text{sgn}(c)-1)\right)^2+\left(bn\log(|x|)+b\log(|c|)+a\right)^2\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
+ b*log(abs(c)) + a)^2)/(b*n)
```

3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

```
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Rubi [A] time = 0.0182284, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log^2(cx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}\end{aligned}$$

Mathematica [A] time = 0.0239537, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Maple [A] time = 0.007, size = 24, normalized size = 0.8

$$\frac{1}{n}\arctan\left(b\ln\left(cx^n\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^2),x)
```

```
[Out] 1/n/(a*b)^(1/2)*arctan(ln(c*x^n)*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bx\log\left(cx^n\right)^2 + ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^2 + a*x), x)
```

Fricas [A] time = 1.90145, size = 324, normalized size = 10.12

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2-2\,\sqrt{-ab}(n\log(x)+\log(c))-a}{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+a}\right)}{2\,abn}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(n\log(x)+\log(c))}{a}\right)}{abn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 - 2
*sqrt(-a*b)*(n*log(x) + log(c)) - a)/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x)
+ b*log(c)^2 + a))/(a*b*n), sqrt(a*b)*arctan(sqrt(a*b)*(n*log(x) + log(c))/
a)/(a*b*n)]
```

Sympy [A] time = 8.80349, size = 126, normalized size = 3.94

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b\log(c)^2} & \text{for } n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ -\frac{1}{b\left(n^2\log(x)+n\log(c)\right)} & \text{for } a = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+n\log(x)+\log(c)\right)}{2\sqrt{a}bn\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+n\log(x)+\log(c)\right)}{2\sqrt{a}bn\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(
a + b*log(c)**2), Eq(n, 0)), (log(x)/a, Eq(b, 0)), (-1/(b*(n**2*log(x) + n*
log(c))), Eq(a, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + n*log(x) + log(c))/(2*s
qrt(a)*b*n*sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + n*log(x) + log(c))/(2*s
qrt(a)*b*n*sqrt(1/b)), True))
```

Giac [A] time = 1.3592, size = 35, normalized size = 1.09

$$\frac{\arctan\left(\frac{bn\log(x)+b\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] arctan((b*n*log(x) + b*log(c))/sqrt(a*b))/(sqrt(a*b)*n)
```

3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$

Optimal. Leaf size=144

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}n}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n}$$

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Log[c*x^n])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/
3)*b^(1/3)*n)) + Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)*n) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(6*a^(2/3)
*b^(1/3)*n)
```

Rubi [A] time = 0.0944799, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}n}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Log[c*x^n])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/
3)*b^(1/3)*n)) + Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)*n) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(6*a^(2/3)
*b^(1/3)*n)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n}$$
$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{2\sqrt[3]{a}n} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}\right.}{6a^{2/}}$$
$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}n} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{a^{2/3}}$$
$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx\right.}{6a^{2/3}\sqrt[3]{b}n}$$

Mathematica [A] time = 0.0494766, size = 112, normalized size = 0.78

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\log^2(cx^n)\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log(cx^n)\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{b}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]] - 2*Log[a^
(1/3) + b^(1/3)*Log[c*x^n]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^
(2/3)*Log[c*x^n]^2])/(6*a^(2/3)*b^(1/3)*n)
```

Maple [A] time = 0.006, size = 120, normalized size = 0.8

$$\frac{1}{3bn}\ln\left(\ln(cx^n)+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{1}{6bn}\ln\left((\ln(cx^n))^2-\sqrt[3]{\frac{a}{b}}\ln(cx^n)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{\sqrt{3}}{3bn}\arctan\left(\frac{\sqrt{3}}{3}\left(2\ln(cx^n)\frac{1}{\sqrt[3]{\frac{a}{b}}}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^3),x)
```

```
[Out] 1/3/n/b/(a/b)^(2/3)*ln(ln(c*x^n)+(a/b)^(1/3))-1/6/n/b/(a/b)^(2/3)*ln(ln(c*x
^n)^2-(a/b)^(1/3)*ln(c*x^n)+(a/b)^(2/3))+1/3/n/b/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*ln(c*x^n)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bx\log\left(cx^{n}\right)^{3} + ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^3 + a*x), x)
```

Fricas [A] time = 2.01421, size = 1335, normalized size = 9.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*n^3*log(x)^3 + 6*a*
b*n^2*log(c)*log(x)^2 + 6*a*b*n*log(c)^2*log(x) + 2*a*b*log(c)^3 - a^2 + 3*
sqrt(1/3)*(2*a*b*n^2*log(x)^2 + 4*a*b*n*log(c)*log(x) + 2*a*b*log(c)^2 + (a
^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) -
 3*(a^2*b)^(1/3)*(a*n*log(x) + a*log(c)))/(b*n^3*log(x)^3 + 3*b*n^2*log(c)*
log(x)^2 + 3*b*n*log(c)^2*log(x) + b*log(c)^3 + a)) - (a^2*b)^(2/3)*log(a*b
*n^2*log(x)^2 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log
(x) + log(c)) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*l
og(c) + (a^2*b)^(2/3)))/(a^2*b*n), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/
b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*n^2*log(x)^2 + 2*a*b*n*
log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log(x) + log(c)) + (a^2*b)^
```

```
(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*log(c) + (a^2*b)^(2/3)))/
(a^2*b*n)]
```

Sympy [A] time = 79.709, size = 340, normalized size = 2.36

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^3} \\ \frac{\log(x)}{a} \\ \frac{\log(x)}{a+b\log(c)^3} \\ -\frac{1}{b\left(2n^3\log(x)^2+4n^2\log(c)\log(x)+2n\log(c)^2\right)} \\ -\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{3a^{\frac{2}{3}}b^3n\left(\frac{1}{b}\right)^{\frac{8}{3}}} + \frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}n\sqrt[3]{\frac{1}{b}}\log(x)+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log(c)+4n^2\log(x)^2+8n\log(c)\log(x)+4\log \right.}{6a^{\frac{2}{3}}b^3n\left(\frac{1}{b}\right)^{\frac{8}{3}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a
, Eq(b, 0)), (log(x)/(a + b*log(c)**3), Eq(n, 0)), (-1/(b*(2*n**3*log(x)**2
 + 4*n**2*log(c)*log(x) + 2*n*log(c)**2)), Eq(a, 0)), (-(-1)**(1/3)*log(-(-
1)**(1/3)*a**(1/3)*(1/b)**(1/3) + n*log(x) + log(c))/(3*a**(2/3)*b**3*n*(1/
b)**(8/3)) + (-1)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**
(1/3)*a**(1/3)*n*(1/b)**(1/3)*log(x) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*
log(c) + 4*n**2*log(x)**2 + 8*n*log(c)*log(x) + 4*log(c)**2)/(6*a**(2/3)*b*
*3*n*(1/b)**(8/3)) - (-1)**(1/3)*sqrt(3)*atan(-sqrt(3)/3 + 2*(-1)**(2/3)*sq
rt(3)*n*log(x)/(3*a**(1/3)*(1/b)**(1/3)) + 2*(-1)**(2/3)*sqrt(3)*log(c)/(3*
a**(1/3)*(1/b)**(1/3)))/(3*a**(2/3)*b**3*n*(1/b)**(8/3)), True))
```

Giac [B] time = 1.30493, size = 328, normalized size = 2.28

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi b(\mathrm{sgn}(c)-1)-2bn\log(x)-2b\log(|c|)-2\left(ab^2\right)^{\frac{1}{3}}}{2\sqrt{3}bn\log(x)+\pi b(\mathrm{sgn}(c)-1)+2\sqrt{3}b\log(|c|)-2\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}\right)+\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}(\pi bn(\mathrm{sg}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*n^3))^(1/3)*arctan((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n
*log(x) - 2*b*log(abs(c)) - 2*(a*b^2)^(1/3))/(2*sqrt(3)*b*n*log(x) + pi*b*(
sgn(c) - 1) + 2*sqrt(3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))) + 1/6*(1/
(a^2*b*n^3))^(1/3)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b
*n*log(abs(x)) + b*log(abs(c)) + (a*b^2)^(1/3))^2) - 1/6*(1/(a^2*b*n^3))^(1
/3)*log(1/4*(sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n*log(x) - 2*b*log(abs(c)) - 2
*(a*b^2)^(1/3))^2 + 1/4*(2*sqrt(3)*b*n*log(x) + pi*b*(sgn(c) - 1) + 2*sqrt(
3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))^2)
```

3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$

Optimal. Leaf size=227

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}+\sqrt{b}\log^2\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}+\sqrt{b}\log^2\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4
)*n) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
b^(1/4)*n) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*Log
[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rubi [A] time = 0.162432, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}+\sqrt{b}\log^2\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log\left(cx^n\right)+\sqrt{a}+\sqrt{b}\log^2\left(cx^n\right)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4
)*n) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*
b^(1/4)*n) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*Log
[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx, x, \log(cx^n)\right)}{2\sqrt{a}n} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx, x, \log(cx^n)\right)}{2\sqrt{a}n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{a}\sqrt{b}n} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{a}\sqrt{b}n} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(c\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}n} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

Mathematica [A] time = 0.06405, size = 167, normalized size = 0.74

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log}{\sqrt[4]{}}\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Log[c*x^n])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Maple [A] time = 0.008, size = 168, normalized size = 0.7

$$\frac{\sqrt{2}}{8\,na}\sqrt[4]{\frac{a}{b}}\ln\left(\left((\ln(cx^n))^2+\sqrt[4]{\frac{a}{b}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{a}{b}}\right)\left((\ln(cx^n))^2-\sqrt[4]{\frac{a}{b}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{a}{b}}\right)^{-1}\right)+\frac{\sqrt{2}}{4\,na}\sqrt[4]{\frac{a}{b}}\arctan\left(\ln(cx^n)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^4),x)
```

```
[Out] 1/8/n*(a/b)^(1/4)/a*2^(1/2)*ln((ln(c*x^n)^2+(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+(
a/b)^(1/2))/(ln(c*x^n)^2-(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+(a/b)^(1/2)))+1/4/n*
(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*ln(c*x^n)+1)-1/4/n*(a/b)^(
1/4)/a*2^(1/2)*arctan(-2^(1/2)/(a/b)^(1/4)*ln(c*x^n)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{bx\log\left(cx^n\right)^4+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^4 + a*x), x)
```

Fricas [A] time = 2.01281, size = 502, normalized size = 2.21

$$\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\sqrt{a^2n^2\sqrt{-\frac{1}{a^3bn^4}}+n^2\log\left(x\right)^2+2\,n\log\left(c\right)\log\left(x\right)+\log\left(c\right)^2}a^2bn^3\left(-\frac{1}{a^3bn^4}\right)^{\frac{3}{4}}-\left(a^2bn^4\log\left(x\right)+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*n^4))^(1/4)*arctan(sqrt(a^2*n^2*sqrt(-1/(a^3*b*n^4)) + n^2*log(x
)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^2*b*n^3*(-1/(a^3*b*n^4))^(3/4) - (a^2
*b*n^4*log(x) + a^2*b*n^3*log(c))*(-1/(a^3*b*n^4))^(3/4)) + 1/4*(-1/(a^3*b*
n^4))^(1/4)*log(a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) - 1/4*(-1/(
a^3*b*n^4))^(1/4)*log(-a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c))
```

Sympy [A] time = 66.9285, size = 257, normalized size = 1.13

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^4} & \text{for } a = 0 \wedge b = \\ -\frac{1}{b(3n^4\log(x)^3+9n^3\log(c)\log(x)^2+9n^2\log(c)^2\log(x)+3n\log(c)^3)} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b\log(c)^4} & \text{for } n = 0 \\ -\frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{4a^{\frac{3}{4}}b^3n\left(\frac{1}{b}\right)^{\frac{11}{4}}} + \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+n\log(x)+\log(c)\right)}{4a^{\frac{3}{4}}b^3n\left(\frac{1}{b}\right)^{\frac{11}{4}}} - \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}n\log(x)}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}+\frac{(-1)^{\frac{3}{4}}\log(c)}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{2a^{\frac{3}{4}}b^3n\left(\frac{1}{b}\right)^{\frac{11}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(b*(3
*n**4*log(x)**3 + 9*n**3*log(c)*log(x)**2 + 9*n**2*log(c)**2*log(x) + 3*n*l
og(c)**3)), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)**4), Eq(
n, 0)), (-(-1)**(1/4)*log(-(-1)**(1/4)*a**(1/4)*(1/b)**(1/4) + n*log(x) + l
og(c))/(4*a**(3/4)*b**3*n*(1/b)**(11/4)) + (-1)**(1/4)*log((-1)**(1/4)*a**(
1/4)*(1/b)**(1/4) + n*log(x) + log(c))/(4*a**(3/4)*b**3*n*(1/b)**(11/4)) -
(-1)**(1/4)*atan((-1)**(3/4)*n*log(x)/(a**(1/4)*(1/b)**(1/4)) + (-1)**(3/4)
*log(c)/(a**(1/4)*(1/b)**(1/4)))/(2*a**(3/4)*b**3*n*(1/b)**(11/4)), True))
```

Giac [A] time = 1.43187, size = 271, normalized size = 1.19

$$\frac{1}{4}i\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(bin\log(x)+bi\log(c)-\left(-ab^3\right)^{\frac{1}{4}}\right)-\frac{1}{4}i\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(-bin\log(x)-bi\log(c)-\left(-ab^3\right)^{\frac{1}{4}}\right)+\frac{1}{8}\left(-\frac{}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] 1/4*i*(-1/(a^3*b*n^4))^(1/4)*log(b*i*n*log(x) + b*i*log(c) - (-a*b^3)^(1/4)
) - 1/4*i*(-1/(a^3*b*n^4))^(1/4)*log(-b*i*n*log(x) - b*i*log(c) - (-a*b^3)^
(1/4)) + 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sg
n(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) + (-a*b^3)^(1/4))^2) - 1/8*
(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2
+ (b*n*log(abs(x)) + b*log(abs(c)) - (-a*b^3)^(1/4))^2)
```

3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}} dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{a} - \frac{b \log\left(a \log\left(cx^n\right) + b\right)}{a^2 n}$$

```
[Out] Log[x]/a - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Rubi [A] time = 0.023225, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{\log(x)}{a} - \frac{b \log\left(a \log\left(cx^n\right) + b\right)}{a^2 n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[x]/a - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax + \frac{bx}{\log(cx^n)}} dx &= \frac{\text{Subst}\left(\int \frac{x}{b+ax} dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right) dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\log(x)}{a} - \frac{b \log\left(b + a \log\left(cx^n\right)\right)}{a^2 n}
\end{aligned}$$

Mathematica [A] time = 0.0150973, size = 34, normalized size = 1.26

$$\frac{\log(cx^n)}{an} - \frac{b\log(a\log(cx^n)+b)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[c*x^n]/(a*n) - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Maple [A] time = 0.006, size = 35, normalized size = 1.3

$$\frac{\ln(cx^n)}{na} - \frac{b\ln(b+a\ln(cx^n))}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)),x)
```

```
[Out] 1/n/a*ln(c*x^n)-b*ln(b+a*ln(c*x^n))/a^2/n
```

Maxima [A] time = 1.04503, size = 45, normalized size = 1.67

$$\frac{\log(x)}{a} - \frac{b\log\left(\frac{a\log(c)+a\log(x^n)+b}{a}\right)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - b*log((a*log(c) + a*log(x^n) + b)/a)/(a^2*n)
```

Fricas [A] time = 1.8948, size = 77, normalized size = 2.85

$$\frac{an\log(x) - b\log(an\log(x) + a\log(c) + b)}{a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (a*n*log(x) - b*log(a*n*log(x) + a*log(c) + b))/(a^2*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.30061, size = 72, normalized size = 2.67

$$\frac{\log(x)}{a} - \frac{b \log\left(\frac{1}{4} \left(\pi a n (\operatorname{sgn}(x) - 1) + \pi a (\operatorname{sgn}(c) - 1)\right)^2 + \left(a n \log(|x|) + a \log(|c|) + b\right)^2\right)}{2 a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/2*b*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n
*log(abs(x)) + a*log(abs(c)) + b)^2)/(a^2*n)
```

3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$

Optimal. Leaf size=40

$$\frac{\log(x)}{a}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Rubi [A] time = 0.0271898, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {321, 205}

$$\frac{\log(x)}{a}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{\log(x)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \log(cx^n)\right)}{an}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n} + \frac{\log(x)}{a}$$

Mathematica [A] time = 0.024212, size = 47, normalized size = 1.18

$$\frac{\log(cx^n)}{an} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[c*x^n]/
(a*n)
```

Maple [A] time = 0.006, size = 43, normalized size = 1.1

$$\frac{\ln(cx^n)}{na} - \frac{b}{na}\arctan\left(a\ln(cx^n)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^2),x)
```

```
[Out] 1/n/a*ln(c*x^n)-1/n*b/a/(a*b)^(1/2)*arctan(a*ln(c*x^n)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b\int \frac{1}{2\,a^2x\log(c)\log(x^n) + a^2x\log(x^n)^2 + \left(a^2\log(c)^2 + ab\right)x}\,dx + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(2*a^2*x*log(c)*log(x^n) + a^2*x*log(x^n)^2 + (a^2*log(c)^2
+ a*b)*x), x) + log(x)/a
```

Fricas [A] time = 1.9646, size = 366, normalized size = 9.15

$$\left[\frac{2\,n\log\left(x\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2-2\,(an\log(x)+a\log(c))\sqrt{-\frac{b}{a}}-b}{an^2\log(x)^2+2\,an\log(c)\log(x)+a\log(c)^2+b}\right)}{2\,an},\ \frac{n\log\left(x\right)-\sqrt{\frac{b}{a}}\arctan\left(\frac{(an\log(x)+a\log(c))\sqrt{\frac{b}{a}}}{b}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*n*log(x) + sqrt(-b/a)*log((a*n^2*log(x)^2 + 2*a*n*log(c)*log(x) + a
*log(c)^2 - 2*(a*n*log(x) + a*log(c))*sqrt(-b/a) - b)/(a*n^2*log(x)^2 + 2*a
*n*log(c)*log(x) + a*log(c)^2 + b)))/(a*n), (n*log(x) - sqrt(b/a)*arctan((a
*n*log(x) + a*log(c))*sqrt(b/a)/b))/(a*n)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**2),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.27586, size = 51, normalized size = 1.27

$$\frac{\log\left(x\right)}{a}-\frac{b\arctan\left(\frac{an\log(x)+a\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] log(x)/a - b*arctan((a*n*log(x) + a*log(c))/sqrt(a*b))/(sqrt(a*b)*a*n)
```

3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2\left(cx^n\right)-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\right)}{6a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log\left(cx^n\right)+\sqrt[3]{b}\right)}{3a^{4/3}n}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n}+\frac{\log(x)}{a}$$

```
[Out] (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Log[c*x^n])/(Sqrt[3]*b^(1/3))])/(Sqrt[
3]*a^(4/3)*n) + Log[x]/a - (b^(1/3)*Log[b^(1/3) + a^(1/3)*Log[c*x^n]])/(3*a
^(4/3)*n) + (b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log
[c*x^n]^2])/(6*a^(4/3)*n)
```

Rubi [A] time = 0.109425, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2\left(cx^n\right)-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\right)}{6a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log\left(cx^n\right)+\sqrt[3]{b}\right)}{3a^{4/3}n}+\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}a^{4/3}n}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Log[c*x^n])/(Sqrt[3]*b^(1/3))])/(Sqrt[
3]*a^(4/3)*n) + Log[x]/a - (b^(1/3)*Log[b^(1/3) + a^(1/3)*Log[c*x^n]])/(3*a
^(4/3)*n) + (b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log
[c*x^n]^2])/(6*a^(4/3)*n)
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx, x, \log(cx^n)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx, x, \log(cx^n)\right)}{3an} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{3an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{6a^{4/3}n} - \frac{b^{2/3}\,\text{Subs}}{}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+a^{2/3}\log^2(cx^n)\right)}{6a^{4/3}n} - \frac{\sqrt[3]{b}\,\text{Sub}}{}$$

$$= \frac{\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}a^{4/3}n} + \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)\right.}{6a^{4/3}n}$$

Mathematica [A] time = 0.0502176, size = 132, normalized size = 0.89

$$\frac{\sqrt[3]{b}\left(\log\left(a^{2/3}\log^2(cx^n)-\sqrt[3]{a}\sqrt[3]{b}\log(cx^n)+b^{2/3}\right)-2\log\left(\sqrt[3]{a}\log(cx^n)+\sqrt[3]{b}\right)\right)+2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\log(}{6a^{4/3}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]] + 6
*a^(1/3)*Log[c*x^n] + b^(1/3)*(-2*Log[b^(1/3) + a^(1/3)*Log[c*x^n]] + Log[b
^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]))/(6*a^(4/3)*n)
```

Maple [A] time = 0.006, size = 136, normalized size = 0.9

$$\frac{\ln(cx^n)}{na} - \frac{b}{3\,na^2}\ln\left(\ln(cx^n)+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}} + \frac{b}{6\,na^2}\ln\left((\ln(cx^n))^2-\sqrt[3]{\frac{b}{a}}\ln(cx^n)+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}} - \frac{b\sqrt{3}}{3\,na^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^3),x)
```

```
[Out] 1/n/a*ln(c*x^n)-1/3/n*b/a^2/(b/a)^(2/3)*ln(ln(c*x^n)+(b/a)^(1/3))+1/6/n*b/a
^2/(b/a)^(2/3)*ln(ln(c*x^n)^2-(b/a)^(1/3)*ln(c*x^n)+(b/a)^(2/3))-1/3/n*b/a^
2/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*ln(c*x^n)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b\int \frac{1}{3\,a^2x\log(c)^2\log(x^n)+3\,a^2x\log(c)\log(x^n)^2+a^2x\log(x^n)^3+\left(a^2\log(c)^3+ab\right)x}\,dx+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(3*a^2*x*log(c)^2*log(x^n) + 3*a^2*x*log(c)*log(x^n)^2 + a^2
*x*log(x^n)^3 + (a^2*log(c)^3 + a*b)*x), x) + log(x)/a
```

Fricas [A] time = 1.9596, size = 406, normalized size = 2.72

$$\frac{6\,n\log(x)+2\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(\sqrt{3}an\log(x)+\sqrt{3}a\log(c)\right)\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(n^2\log(x)^2+2\,n\log(c)\log(x)+\log(c\right.}{6\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*n*log(x) + 2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*(sqrt(3)*a*n*log(x)
+ sqrt(3)*a*log(c))*(-b/a)^(2/3) - sqrt(3)*b)/b) - (-b/a)^(1/3)*log(n^2*log
(x)^2 + 2*n*log(c)*log(x) + log(c)^2 + (n*log(x) + log(c))*(-b/a)^(1/3) + (
-b/a)^(2/3)) + 2*(-b/a)^(1/3)*log(n*log(x) - (-b/a)^(1/3) + log(c)))/(a*n)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**3),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.39006, size = 431, normalized size = 2.89

$$\frac{1}{3}\sqrt{3}\left(-\frac{b}{a^4n^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi an^3(\operatorname{sgn}(c)-1)-2\,an^4\log(x)-2\,an^3\log(|c|)+2\,\left(-a^2b\right)^{\frac{1}{3}}n^3}{2\sqrt{3}an^4\log(x)+\pi an^3(\operatorname{sgn}(c)-1)+2\sqrt{3}an^3\log(|c|)+2\sqrt{3}\left(-a^2b\right)^{\frac{1}{3}}n^3}\right)+\frac{1}{6}\left(-\frac{b}{a^4n^3}\right)^{\frac{1}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(-b/(a^4*n^3))^(1/3)*arctan((sqrt(3)*pi*a*n^3*(sgn(c) - 1) - 2*
a*n^4*log(x) - 2*a*n^3*log(abs(c)) + 2*(-a^2*b)^(1/3)*n^3)/(2*sqrt(3)*a*n^4
*log(x) + pi*a*n^3*(sgn(c) - 1) + 2*sqrt(3)*a*n^3*log(abs(c)) + 2*sqrt(3)*(
-a^2*b)^(1/3)*n^3)) + 1/6*(-b/(a^4*n^3))^(1/3)*log(1/4*(pi*a^2*n^4*(sgn(x)
- 1) + pi*a^2*n^3*(sgn(c) - 1))^2 + (a^2*n^4*log(abs(x)) + a^2*n^3*log(abs(
c)) - (-a^2*b)^(1/3)*a*n^3)^2) - 1/6*(-b/(a^4*n^3))^(1/3)*log(1/4*(sqrt(3)*
pi*a*n^3*(sgn(c) - 1) - 2*a*n^4*log(x) - 2*a*n^3*log(abs(c)) + 2*(-a^2*b)^(
1/3)*n^3)^2 + 1/4*(2*sqrt(3)*a*n^4*log(x) + pi*a*n^3*(sgn(c) - 1) + 2*sqrt(
3)*a*n^3*log(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/3)*n^3)^2) + log(x)/a
```

3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$

Optimal. Leaf size=233

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}+\frac{\sqrt[4]{b}\tan^{-1}\left(1\right.}{2\sqrt{}}$$

```
[Out] (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[2]*a^(5/
4)*n) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[
2]*a^(5/4)*n) + Log[x]/a + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(5/4)*n) - (b^(1/4)*Log[Sqr
t[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[
2]*a^(5/4)*n)
```

Rubi [A] time = 0.182802, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {321, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}a^{5/4}n}+\frac{\sqrt[4]{b}\tan^{-1}\left(1\right.}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```


```
[Out] (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[2]*a^(5/
4)*n) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[
2]*a^(5/4)*n) + Log[x]/a + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(5/4)*n) - (b^(1/4)*Log[Sqr
t[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[
2]*a^(5/4)*n)
```


Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, \log(cx^n)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}x^2}{b+ax^4}\,dx, x, \log(cx^n)\right)}{2an} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}x^2}{b+ax^4}\,dx, x, \log(cx^n)\right)}{2an}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{2}a^{5/4}n} + \frac{\sqrt[4]{b}\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}-2x}{-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n\right)}{4\sqrt{2}a^{5/4}n}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\right.}{4\sqrt{2}a^{5/4}n}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}a^{5/4}n} + \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,\mathrm{lo}\right.}{4\sqrt{2}a}$$

Mathematica [A] time = 0.0664152, size = 211, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)+\sqrt{b}\right)+2\sqrt{2}\sqrt[4]{}}{8a^{5/4}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] - 2*Sqr
t[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] + 8*a^(1/4)*L
og[c*x^n] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n
] + Sqrt[a]*Log[c*x^n]^2] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b
^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(8*a^(5/4)*n)
```

Maple [A] time = 0.01, size = 181, normalized size = 0.8

$$\frac{\ln(cx^n)}{na}-\frac{\sqrt{2}}{8\,na}\sqrt[4]{\frac{b}{a}}\ln\left(\left((\ln(cx^n))^2+\sqrt[4]{\frac{b}{a}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{b}{a}}\right)\left((\ln(cx^n))^2-\sqrt[4]{\frac{b}{a}}\ln(cx^n)\sqrt{2}+\sqrt{\frac{b}{a}}\right)^{-1}\right)-\frac{\sqrt{2}}{4\,na}\sqrt[4]{\frac{b}{a}}\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^4),x)
```

```
[Out] 1/n/a*ln(c*x^n)-1/8/n/a*(b/a)^(1/4)*2^(1/2)*ln((ln(c*x^n)^2+(b/a)^(1/4)*ln(
c*x^n)*2^(1/2)+(b/a)^(1/2))/(ln(c*x^n)^2-(b/a)^(1/4)*ln(c*x^n)*2^(1/2)+(b/a
)^(1/2)))-1/4/n/a*(b/a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/a)^(1/4)*ln(c*x^n)+
1)+1/4/n/a*(b/a)^(1/4)*2^(1/2)*arctan(-2^(1/2)/(b/a)^(1/4)*ln(c*x^n)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b\int\frac{1}{4\,a^2x\log(c)^3\log(x^n)+6\,a^2x\log(c)^2\log(x^n)^2+4\,a^2x\log(c)\log(x^n)^3+a^2x\log(x^n)^4+\left(a^2\log(c)^4+ab\right)x}\,dx+\frac{\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(4*a^2*x*log(c)^3*log(x^n) + 6*a^2*x*log(c)^2*log(x^n)^2 + 4
*a^2*x*log(c)*log(x^n)^3 + a^2*x*log(x^n)^4 + (a^2*log(c)^4 + a*b)*x), x) +
 log(x)/a
```

Fricas [A] time = 1.97476, size = 505, normalized size = 2.17

$$-\frac{4\,a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2n^2\sqrt{-\frac{b}{a^5n^4}}+n^2\log(x)^2+2\,n\log(c)\log(x)+\log(c)^2}\,a^4n^3\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}-\left(a^4n^4\log(x)+a^4n^3\log(c)\right)\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}}{b}\right)+a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\mathrm{lo}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*(-b/(a^5*n^4))^(1/4)*arctan((sqrt(a^2*n^2*sqrt(-b/(a^5*n^4)) + n^
2*log(x)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^4*n^3*(-b/(a^5*n^4))^(3/4) - (
a^4*n^4*log(x) + a^4*n^3*log(c))*(-b/(a^5*n^4))^(3/4))/b) + a*(-b/(a^5*n^4)
)^(1/4)*log(a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - a*(-b/(a^5*n^4)
)^(1/4)*log(-a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - 4*log(x))/a
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**4),x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.3349, size = 365, normalized size = 1.57

$$\frac{1}{4}\,i\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(a^2in^5\log\left(x\right)+a^2in^4\log\left(c\right)-\left(-a^3b\right)^{\frac{1}{4}}an^4\right)-\frac{1}{4}\,i\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(-a^2in^5\log\left(x\right)-a^2in^4\log\left(c\right)-\left(-a^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] 1/4*i*(-b/(a^5*n^4))^(1/4)*log(a^2*i*n^5*log(x) + a^2*i*n^4*log(c) - (-a^3*
b)^(1/4)*a*n^4) - 1/4*i*(-b/(a^5*n^4))^(1/4)*log(-a^2*i*n^5*log(x) - a^2*i*
n^4*log(c) - (-a^3*b)^(1/4)*a*n^4) + 1/8*(-b/(a^5*n^4))^(1/4)*log(1/4*(pi*a
^2*n^5*(sgn(x) - 1) + pi*a^2*n^4*(sgn(c) - 1))^2 + (a^2*n^5*log(abs(x)) + a
^2*n^4*log(abs(c)) + (-a^3*b)^(1/4)*a*n^4)^2) - 1/8*(-b/(a^5*n^4))^(1/4)*lo
g(1/4*(pi*a^2*n^5*(sgn(x) - 1) + pi*a^2*n^4*(sgn(c) - 1))^2 + (a^2*n^5*log(
abs(x)) + a^2*n^4*log(abs(c)) - (-a^3*b)^(1/4)*a*n^4)^2) + log(x)/a
```

3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$

Optimal. Leaf size=22

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rubi [A] time = 0.0180555, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {618, 204}

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx = \text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(7x)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(7x)\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0285106, size = 22, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.007, size = 20, normalized size = 0.9

$$\frac{2\sqrt{3}}{3}\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x*ln(7*x)+x*ln(7*x)^2),x)
```

```
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\log(7x)^2+x\log(7x)+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Fricas [A] time = 1.72742, size = 76, normalized size = 3.45

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(7x\right)+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(2/3*sqrt(3)*log(7*x) + 1/3*sqrt(3))
```

Sympy [A] time = 0.164534, size = 22, normalized size = 1.

$$\operatorname{RootSum}\left(3z^{2}+1,\left(i\mapsto i\log\left(\frac{3i}{2}+\log\left(7x\right)+\frac{1}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*ln(7*x)+x*ln(7*x)**2),x)
```

```
[Out] RootSum(3*_z**2 + 1, Lambda(_i, _i*log(3*_i/2 + log(7*x) + 1/2)))
```

Giac [A] time = 1.79949, size = 26, normalized size = 1.18

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log\left(7x\right)+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(7*x) + 1))
```

3.259 $\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rubi [A] time = 0.0575292, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0890602, size = 42, normalized size = 1.02

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)-\frac{\tan^{-1}\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

```
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A] time = 0.005, size = 38, normalized size = 0.9

$$\frac{\ln\left(1-\ln(3x)+(\ln(3x))^2\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)^2),x)
```

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Maxima [A] time = 1.52494, size = 50, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log(3x)-1)\right)+\frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

Fricas [A] time = 1.84896, size = 127, normalized size = 3.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A] time = 0.178734, size = 22, normalized size = 0.54

$$\mathrm{RootSum}\left(3z^2-3z+1,\left(i\mapsto i\log(-3i+\log(3x)+1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)**2),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Giac [A] time = 1.23167, size = 50, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log(3x)-1)\right)+\frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$

Optimal. Leaf size=41

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rubi [A] time = 0.0332927, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)+\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\
&= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0729712, size = 42, normalized size = 1.02

$$\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)-\frac{\tan^{-1}\left(\frac{2\log(3x)-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

```
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A] time = 0.005, size = 38, normalized size = 0.9

$$\frac{\ln\left(1-\ln(3x)+(\ln(3x))^2\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)^3),x)
```

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(3x)^2 - 1}{x\log(3x)^3 + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="maxima")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Fricas [A] time = 1.76618, size = 127, normalized size = 3.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x) - \frac{1}{3}\sqrt{3}\right) + \frac{1}{2}\log\left(\log(3x)^2 - \log(3x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A] time = 0.196357, size = 22, normalized size = 0.54

$$\mathrm{RootSum}\left(3z^2 - 3z + 1, \left(i \mapsto i\log(-3i + \log(3x) + 1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)**3),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Giac [A] time = 1.31012, size = 50, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log(3x)-1)\right)+\frac{1}{2}\log\left(\log(3x)^2-\log(3x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[x] - Log[1 + Log[3*x] + L
og[3*x]^2]/2
```

Rubi [A] time = 0.0460033, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[x] - Log[1 + Log[3*x] + L
og[3*x]^2]/2
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{1+x+x^2}\,dx, x, \log(3x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{2+x}{1+x+x^2}\right)dx, x, \log(3x)\right)\\
&= \log(x)-\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, \log(3x)\right)\\
&= \log(x)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \log(3x)\right)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(3
\\
&= \log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)+3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(3x)\right)\\
&= -\sqrt{3}\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0885525, size = 44, normalized size = 1.05

$$-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(3x)-\sqrt{3}\tan^{-1}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x] +
 Log[3*x]^2]/2
```

Maple [A] time = 0.006, size = 40, normalized size = 1.

$$\ln(3x) - \frac{\ln\left(1 + \ln(3x) + (\ln(3x))^2\right)}{2} - \arctan\left(\frac{(1 + 2\ln(3x))\sqrt{3}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)+x*ln(3*x)^2),x)
```

```
[Out] ln(3*x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\log(3) + \log(x) + 2}{x(2\log(3) + 1)\log(x) + x\log(x)^2 + \left(\log(3)^2 + \log(3) + 1\right)x}\,dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="maxima"
)
```

```
[Out] -integrate((log(3) + log(x) + 2)/(x*(2*log(3) + 1)*log(x) + x*log(x)^2 + (l
og(3)^2 + log(3) + 1)*x), x) + log(x)
```

Fricas [A] time = 1.94789, size = 136, normalized size = 3.24

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x) + \frac{1}{3}\sqrt{3}\right) - \frac{1}{2}\log\left(\log(3x)^2 + \log(3x) + 1\right) + \log(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) + 1/3*sqrt(3)) - 1/2*log(log(3*x)^2 +
log(3*x) + 1) + log(3*x)
```

Sympy [A] time = 0.178857, size = 19, normalized size = 0.45

$$\log(x) + \operatorname{RootSum}\left(z^2 + z + 1, \left(i \mapsto i\log(-i + \log(3x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)+x*ln(3*x)**2),x)
```

```
[Out] log(x) + RootSum(_z**2 + _z + 1, Lambda(_i, _i*log(-_i + log(3*x))))
```

Giac [A] time = 1.34068, size = 53, normalized size = 1.26

$$-\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\log(3x) + 1)\right) - \frac{1}{2}\log\left(\log(3x)^2 + \log(3x) + 1\right) + \log(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) + 1)) - 1/2*log(log(3*x)^2 + log(3*
x) + 1) + log(3*x)
```

3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5} \, dx$

Optimal. Leaf size=32

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

```
[Out] -1/(32*x^4) + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rubi [A] time = 0.0211592, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2305, 2304}

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/(32*x^4) + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx = -\frac{\log^2\left(\frac{1}{x}\right)}{4x^4} - \frac{1}{2}\int \frac{\log\left(\frac{1}{x}\right)}{x^5}\,dx$$
$$= -\frac{1}{32x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

Mathematica [A] time = 0.001618, size = 32, normalized size = 1.

$$-\frac{1}{32x^4} - \frac{\log^2\left(\frac{1}{x}\right)}{4x^4} + \frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/(32*x^4) + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Maple [A] time = 0.003, size = 27, normalized size = 0.8

$$-\frac{1}{32\,x^4} + \frac{\ln\left(x^{-1}\right)}{8\,x^4} - \frac{\left(\ln\left(x^{-1}\right)\right)^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x)^2/x^5,x)
```

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Maxima [A] time = 0.993019, size = 23, normalized size = 0.72

$$-\frac{8\,\log\left(x\right)^2 + 4\,\log\left(x\right) + 1}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*log(x)^2 + 4*log(x) + 1)/x^4
```

Fricas [A] time = 1.75354, size = 58, normalized size = 1.81

$$-\frac{8\,\log\left(\frac{1}{x}\right)^{2}-4\,\log\left(\frac{1}{x}\right)+1}{32\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(8*log(1/x)^2 - 4*log(1/x) + 1)/x^4
```

Sympy [A] time = 0.131458, size = 27, normalized size = 0.84

$$-\frac{\log\left(\frac{1}{x}\right)^{2}}{4x^{4}}+\frac{\log\left(\frac{1}{x}\right)}{8x^{4}}-\frac{1}{32x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x)**2/x**5,x)
```

```
[Out] -log(1/x)**2/(4*x**4) + log(1/x)/(8*x**4) - 1/(32*x**4)
```

Giac [A] time = 1.35236, size = 30, normalized size = 0.94

$$-\frac{\log\left(x\right)^{2}}{4\,x^{4}}-\frac{\log\left(x\right)}{8\,x^{4}}-\frac{1}{32\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(x)^2/x^4 - 1/8*log(x)/x^4 - 1/32/x^4
```

3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{\frac{\pi}{2}}x\operatorname{Erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

```
[Out] -((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

Rubi [A] time = 0.0288386, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2300, 2180, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}x\operatorname{Erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx = \frac{x\,\text{Subst}\left(\int \frac{e^{x/2}}{\sqrt{-x}}\,dx, x, \log\left(ax^2\right)\right)}{2\sqrt{ax^2}}$$
$$= -\frac{x\,\text{Subst}\left(\int e^{-\frac{x^2}{2}}\,dx, x, \sqrt{-\log\left(ax^2\right)}\right)}{\sqrt{ax^2}}$$
$$= -\frac{\sqrt{\frac{\pi}{2}}x\text{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Mathematica [A] time = 0.0106863, size = 59, normalized size = 1.48

$$\frac{\sqrt{\frac{\pi}{2}}x\sqrt{\log\left(ax^2\right)}\text{Erfi}\left(\frac{\sqrt{\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}\sqrt{-\log\left(ax^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*x*Erfi[Sqrt[Log[a*x^2]]/Sqrt[2]]*Sqrt[Log[a*x^2]])/(Sqrt[a*x^2]
*Sqrt[-Log[a*x^2]])
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\ln\left(ax^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^2))^(1/2),x)
```

```
[Out] int(1/(-ln(a*x^2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log\left(ax^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$

Optimal. Leaf size=39

$$\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\text{Erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

```
[Out] Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Rubi [A] time = 0.0246292, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2300, 2180, 2204}

$$\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\text{Erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a/x^2]],x]
```

```
[Out] Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = -\left(\frac{1}{2}\left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int \frac{e^{-x/2}}{\sqrt{-x}}\,dx, x, \log\left(\frac{a}{x^2}\right)\right)\right)$$
$$= \left(\sqrt{\frac{a}{x^2}}x\right)\text{Subst}\left(\int e^{\frac{x^2}{2}}\,dx, x, \sqrt{-\log\left(\frac{a}{x^2}\right)}\right)$$
$$= \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}x\,\text{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.0124686, size = 60, normalized size = 1.54

$$-\frac{\sqrt{\frac{\pi}{2}}x\sqrt{\frac{a}{x^2}}\sqrt{\log\left(\frac{a}{x^2}\right)}\text{Erf}\left(\frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a/x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erf[Sqrt[Log[a/x^2]]/Sqrt[2]]*Sqrt[Log[a/x^2]])
/Sqrt[-Log[a/x^2]])
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a/x^2))^(1/2),x)
```

[Out] int(1/(-ln(a/x^2))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-log(a/x^2)), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-ln(a/x**2))**(1/2),x)

[Out] Integral(1/sqrt(-log(a/x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{\pi}x\,(ax^n)^{-1/n}\operatorname{Erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] -((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Rubi [A] time = 0.0308945, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2300, 2180, 2205}

$$-\frac{\sqrt{\pi}x\,(ax^n)^{-1/n}\operatorname{Erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] -((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Rule 2300

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log (ax^n)}}\,dx = \frac{\left(x (ax^n)^{-1/n}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{-x}}\,dx, x, \log (ax^n)\right)}{n}$$
$$= -\frac{\left(2x (ax^n)^{-1/n}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{-\log (ax^n)}\right)}{n}$$
$$= -\frac{\sqrt{\pi} x (ax^n)^{-1/n} \operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Mathematica [A] time = 0.0097883, size = 62, normalized size = 1.44

$$\frac{\sqrt{\pi} x (ax^n)^{-1/n} \sqrt{\log (ax^n)} \text{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}\sqrt{-\log (ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]]*Sqrt[Log[a*x^n]])/(Sqrt[n]*(a*x^
n)^n^(-1)*Sqrt[-Log[a*x^n]])
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\ln (ax^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^n))^(1/2),x)
```

```
[Out] int(1/(-ln(a*x^n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^n)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**n)), x)
```

Giac [A] time = 1.27486, size = 43, normalized size = 1.

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{\sqrt{-n\log(x)-\log(a)}}{\sqrt{n}}\right)}{a^{\left(\frac{1}{n}\right)}\sqrt{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*erf(-sqrt(-n*log(x) - log(a))/sqrt(n))/(a^(1/n)*sqrt(n))
```

3.266 $\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx$

Optimal. Leaf size=122

$$2\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log$$

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] - 2*Log[1 - (2*Sqrt[x]
)/(1 - Sqrt[5])]*Log[Sqrt[x]] + 2*Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + 2*Pol
yLog[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (2*Sqrt[x])/(1 - Sqrt
[5])]
```

Rubi [A] time = 0.14536, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2530, 2524, 2357, 2317, 2391, 2316, 2315}

$$2\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[x] - x]/x,x]
```


```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] - 2*Log[1 - (2*Sqrt[x]
)/(1 - Sqrt[5])]*Log[Sqrt[x]] + 2*Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + 2*Pol
yLog[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (2*Sqrt[x])/(1 - Sqrt
[5])]
```


Rule 2530

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
```

```
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx = 2\,\text{Subst}\left(\int \frac{\log\left(1+x-x^2\right)}{x}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \frac{(1-2x)\log(x)}{1+x-x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \left(-\frac{2\log(x)}{1-\sqrt{5}-2x} - \frac{2\log(x)}{1+\sqrt{5}-2x}\right)dx, x, \sqrt{x}\right)$$
$$= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right) + 4\,\text{Subst}\left(\int \frac{\log(x)}{1-\sqrt{5}-2x}\,dx, x, \sqrt{x}\right) + 4\,\text{Subst}\left(\int \frac{\log(x)}{1+\sqrt{5}-2x}\,d\right.$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\sqrt{x}-x\right)\text{lo}$$
$$= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right) - 2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right) + 2\log\left(1+\sqrt{x}-x\right)\text{lo}$$

Mathematica [A] time = 0.075291, size = 121, normalized size = 0.99

$$2\text{PolyLog}\left(2, \frac{-2\sqrt{x}+\sqrt{5}+1}{1+\sqrt{5}}\right) - 2\text{PolyLog}\left(2, -\frac{2\sqrt{x}}{\sqrt{5}-1}\right) - 2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right) + \left(\log\left(\sqrt{5}-1\right) - \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] + (Log[-1 + Sqrt[5]] -
 Log[-1 + Sqrt[5] + 2*Sqrt[x]])*Log[x] + Log[1 + Sqrt[x] - x]*Log[x] + 2*Po
lyLog[2, (1 + Sqrt[5] - 2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (-2*Sqrt[x
])/(-1 + Sqrt[5])]
```

Maple [A] time = 0.01, size = 102, normalized size = 0.8

$$\ln(x)\ln\left(1-x+\sqrt{x}\right) - \ln(x)\ln\left(\frac{1}{\sqrt{5}+1}\left(1+\sqrt{5}-2\sqrt{x}\right)\right) - \ln(x)\ln\left(\frac{1}{\sqrt{5}-1}\left(-1+\sqrt{5}+2\sqrt{x}\right)\right) - 2\,\text{dilog}\left(\frac{1+\sqrt{5}-2}{\sqrt{5}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-x+x^(1/2))/x,x)
```

```
[Out] ln(x)*ln(1-x+x^(1/2))-ln(x)*ln((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+1))-ln(x)*ln(
(-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))-2*dilog((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+
1))-2*dilog((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(-x+\sqrt{x}+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(log(-x + sqrt(x) + 1)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-x+x**(1/2))/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-x+\sqrt{x}+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

3.267 $\int \frac{x \log(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=81

$$-\frac{a\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```

Rubi [A] time = 0.0994792, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {43, 2416, 2389, 2295, 2394, 2393, 2391}

$$-\frac{a\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c + d*x])/(a + b*x),x]
```


```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{x \log(c+dx)}{a+bx} dx = \int \left(\frac{\log(c+dx)}{b} - \frac{a \log(c+dx)}{b(a+bx)}\right) dx$$
$$= \frac{\int \log(c+dx) dx}{b} - \frac{a \int \frac{\log(c+dx)}{a+bx} dx}{b}$$
$$= -\frac{a \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)}{b^2} + \frac{\text{Subst}(\int \log(x) dx, x, c+dx)}{bd} + \frac{(ad) \int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx} dx}{b^2}$$
$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)}{b^2} + \frac{a \,\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x} dx, x, c+dx\right)}{b^2}$$
$$= -\frac{x}{b} + \frac{(c+dx)\log(c+dx)}{bd} - \frac{a \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c+dx)}{b^2} - \frac{a \text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}$$

Mathematica [A] time = 0.0291286, size = 73, normalized size = 0.9

$$\frac{-ad\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + \log(c+dx)\left(-ad \log\left(\frac{d(a+bx)}{ad-bc}\right) + bc + bdx\right) - bdx}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] (-(b*d*x) + (b*c + b*d*x - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c + d
*x] - a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d)
```

Maple [A] time = 0.026, size = 114, normalized size = 1.4

$$\frac{\ln(dx+c)\, x}{b} + \frac{\ln(dx+c)\, c}{db} - \frac{x}{b} - \frac{c}{db} - \frac{a}{b^2}\text{dilog}\left(\frac{(dx+c)\, b + ad - bc}{ad-bc}\right) - \frac{a \ln(dx+c)}{b^2} \ln\left(\frac{(dx+c)\, b + ad - bc}{ad-bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*x+c)/(b*x+a),x)
```

```
[Out] 1/b*ln(d*x+c)*x+1/d/b*ln(d*x+c)*c-x/b-1/d/b*c-a/b^2*dilog(((d*x+c)*b+a*d-b*
c)/(a*d-b*c))-a/b^2*ln(d*x+c)*ln(((d*x+c)*b+a*d-b*c)/(a*d-b*c))
```

Maxima [A] time = 1.01102, size = 150, normalized size = 1.85

$$d\left(\frac{\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)a}{b^2d}-\frac{x}{bd}+\frac{c\log\left(dx+c\right)}{bd^2}\right)+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}\right)\log\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*((log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*a/(b^2*d) - x/(b*d) + c*log(d*x + c)/(b*d^2)) + (x/b - a*log(
b*x + a)/b^2)*log(d*x + c)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\log\left(dx+c\right)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(d*x + c)/(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*x+c)/(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log(dx + c)}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(d*x + c)/(b*x + a), x)
```

3.268 $\int \frac{\log(x)}{-1+x}\,dx$

Optimal. Leaf size=9

$$-\text{PolyLog}(2,1-x)$$

```
[Out] -PolyLog[2, 1 - x]
```

Rubi [A] time = 0.0089794, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2315}

$$-\text{PolyLog}(2,1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(-1 + x),x]

[Out] -PolyLog[2, 1 - x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log(x)}{-1+x}\,dx = -\text{Li}_2(1-x)$$

Mathematica [A] time = 0.0015296, size = 9, normalized size = 1.

$$-\text{PolyLog}(2,1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(-1 + x),x]
```

```
[Out] -PolyLog[2, 1 - x]
```

Maple [A] time = 0.001, size = 5, normalized size = 0.6

$$-\text{dilog}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(-1+x),x)
```

```
[Out] -dilog(x)
```

Maxima [A] time = 1.0358, size = 16, normalized size = 1.78

$$\log\,(x)\log\,(-x+1)+\text{Li}_2\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1) + dilog(x)
```

Fricas [A] time = 2.06785, size = 22, normalized size = 2.44

$$-\text{Li}_2\,(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="fricas")
```

```
[Out] -dilog(-x + 1)
```

Sympy [C] time = 1.92516, size = 10, normalized size = 1.11

$$-\text{Li}_2\left((x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(-1+x),x)
```

```
[Out] -polylog(2, (x - 1)*exp_polar(I*pi))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x - 1), x)
```

3.269 $\int \frac{x\log(1-a-bx)}{a+bx}\,dx$

Optimal. Leaf size=43

$$\frac{a\text{PolyLog}(2,a+bx)}{b^2}-\frac{(-a-bx+1)\log(-a-bx+1)}{b^2}-\frac{x}{b}$$

```
[Out] -(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^2
```

Rubi [A] time = 0.0725418, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {43, 2416, 2389, 2295, 2393, 2391}

$$\frac{a\text{PolyLog}(2,a+bx)}{b^2}-\frac{(-a-bx+1)\log(-a-bx+1)}{b^2}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[1 - a - b*x])/(a + b*x),x]
```

```
[Out] -(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\log(1-a-bx)}{a+bx}\,dx &= \int \left(\frac{\log(1-a-bx)}{b}-\frac{a\log(1-a-bx)}{b(a+bx)}\right)dx\\
&= \frac{\int \log(1-a-bx)\,dx}{b}-\frac{a\int \frac{\log(1-a-bx)}{a+bx}\,dx}{b}\\
&= -\frac{\text{Subst}(\int \log(x)\,dx,x,1-a-bx)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,a+bx\right)}{b^2}\\
&= -\frac{x}{b}-\frac{(1-a-bx)\log(1-a-bx)}{b^2}+\frac{a\text{Li}_2(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.0177899, size = 35, normalized size = 0.81

$$\frac{a\text{PolyLog}(2,a+bx)+(a+bx-1)\log(-a-bx+1)-bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 - a - b*x])/(a + b*x),x]
```

```
[Out] (-(b*x) + (-1 + a + b*x)*Log[1 - a - b*x] + a*PolyLog[2, a + b*x])/b^2
```

Maple [A] time = 0.003, size = 77, normalized size = 1.8

$$\frac{\ln\left(-bx-a+1\right)x}{b}+\frac{a\mathrm{dilog}\left(-bx-a+1\right)}{b^{2}}+\frac{\ln\left(-bx-a+1\right)a}{b^{2}}-\frac{x}{b}-\frac{\ln\left(-bx-a+1\right)}{b^{2}}-\frac{a}{b^{2}}+b^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-b*x-a+1)/(b*x+a),x)
```

```
[Out] 1/b*ln(-b*x-a+1)*x+1/b^2*a*dilog(-b*x-a+1)+1/b^2*ln(-b*x-a+1)*a-x/b-1/b^2*l
n(-b*x-a+1)-1/b^2*a+1/b^2
```

Maxima [B] time = 1.03289, size = 111, normalized size = 2.58

$$b\left(\frac{(\log\left(bx+a\right)\log\left(-bx-a+1\right)+\mathrm{Li}_2\left(bx+a\right))a}{b^{3}}-\frac{x}{b^{2}}+\frac{(a-1)\log\left(bx+a-1\right)}{b^{3}}\right)+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^{2}}\right)\log\left(-bx-a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*((log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))*a/b^3 - x/b^2 + (a - 1
)*log(b*x + a - 1)/b^3) + (x/b - a*log(b*x + a)/b^2)*log(-b*x - a + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x\log\left(-bx-a+1\right)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(-b*x - a + 1)/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\log\left(-a-bx+1\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-b*x-a+1)/(b*x+a),x)
```

```
[Out] Integral(x*log(-a - b*x + 1)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \log(-bx - a + 1)}{bx + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(-b*x - a + 1)/(b*x + a), x)
```

3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$

Optimal. Leaf size=30

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

```
[Out] Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rubi [A] time = 0.0988166, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2357, 2301, 2317, 2391}

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

```
[Out] Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx &= \int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx \\
&= c\int \frac{\log(x)}{b+cx}\,dx+\int \frac{\log(x)}{x}\,dx \\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)-\int \frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx \\
&= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)
\end{aligned}$$

Mathematica [A] time = 0.0102847, size = 31, normalized size = 1.03

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{b+cx}{b}\right)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

```
[Out] Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Maple [A] time = 0.013, size = 31, normalized size = 1.

$$\frac{(\ln(x))^2}{2}+\ln(x)\ln\left(\frac{cx+b}{b}\right)+\text{dilog}\left(\frac{cx+b}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] 1/2*ln(x)^2+ln(x)*ln((c*x+b)/b)+dilog((c*x+b)/b)
```

Maxima [A] time = 1.02865, size = 66, normalized size = 2.2

$$(\log(cx+b)+\log(x))\log(x)-\log(cx+b)\log(x)+\log\left(\frac{cx}{b}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\mathrm{Li}_2\left(-\frac{cx}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="maxima")
```

```
[Out] (log(c*x + b) + log(x))*log(x) - log(c*x + b)*log(x) + log(c*x/b + 1)*log(x
) - 1/2*log(x)^2 + dilog(-c*x/b)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(2\,cx+b)\log(x)}{cx^2+bx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="fricas")
```

```
[Out] integral((2*c*x + b)*log(x)/(c*x^2 + b*x), x)
```

Sympy [C] time = 83.6595, size = 192, normalized size = 6.4

$$b\left(\begin{cases}-\frac{1}{cx} & \text{for } b=0\\ \frac{\begin{cases}\log(c)\log(x)+\mathrm{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } |x|<1\\ -\log(c)\log\left(\frac{1}{x}\right)+\mathrm{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{for } \frac{1}{|x|}<1\\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1\\ 0,0 & \end{matrix}\middle| x\right)\log(c)+G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle| x\right)\log(c)+\mathrm{Li}_2\left(\frac{be^{i\pi}}{cx}\right) & \text{otherwise}\end{cases}}{b} & \text{otherwise}\end{cases}\right)-b\left(\left(\begin{cases}\frac{1}{cx} & \text{fo}\\ \frac{\log\left(\frac{b}{x}+c\right)}{b} & \text{o}\end{cases}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] b*Piecewise((-1/(c*x), Eq(b, 0)), (Piecewise((log(c)*log(x) + polylog(2, b*
exp_polar(I*pi)/(c*x)), Abs(x) < 1), (-log(c)*log(1/x) + polylog(2, b*exp_p
```

```
olar(I*pi)/(c*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*
log(c) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), True))/b, True)) - b*Piecewise((1/(c*x), Eq(b, 0)), (log
(b/x + c)/b, True))*log(x) - 2*c*Piecewise((x/b, Eq(c, 0)), (Piecewise((log
(b)*log(x) - polylog(2, c*x*exp_polar(I*pi)/b), Abs(x) < 1), (-log(b)*log(1
/x) - polylog(2, c*x*exp_polar(I*pi)/b), 1/Abs(x) < 1), (-meijerg(((), (1,
1)), ((0, 0), ()), x)*log(b) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(b
) - polylog(2, c*x*exp_polar(I*pi)/b), True))/c, True)) + 2*c*Piecewise((x/
b, Eq(c, 0)), (log(b + c*x)/c, True))*log(x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2\,cx+b)\log{(x)}}{(cx+b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] integrate((2*c*x + b)*log(x)/((c*x + b)*x), x)
```

3.271 $\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx$

Optimal. Leaf size=7

$$-\cos(x\log(x))$$

```
[Out] -Cos[x*Log[x]]
```

Rubi [A] time = 0.0269251, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {4511}

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

```
[Out] -Cos[x*Log[x]]
```

Rule 4511

```
Int[Log[(b_.)*(x_)]*Sin[Log[(b_.)*(x_)]*(a_.)*(x_)], x_Symbol] :> -Simp[Cos
[a*x*Log[b*x]]/a, x] - Int[Sin[a*x*Log[b*x]], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx &= \int \sin(x\log(x))\,dx + \int \log(x)\sin(x\log(x))\,dx\\ &= -\cos(x\log(x))\end{aligned}$$

Mathematica [A] time = 0.108954, size = 7, normalized size = 1.

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

```
[Out] -Cos[x*Log[x]]
```

Maple [A] time = 0.017, size = 8, normalized size = 1.1

$$-\cos(x\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
```

```
[Out] -cos(x*ln(x))
```

Maxima [A] time = 1.2649, size = 9, normalized size = 1.29

$$-\cos(x\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="maxima")
```

```
[Out] -cos(x*log(x))
```

Fricas [A] time = 2.19406, size = 22, normalized size = 3.14

$$-\cos(x\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="fricas")
```

```
[Out] -cos(x*log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(\log(x)+1)\sin(x\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
```

```
[Out] Integral((log(x) + 1)*sin(x*log(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(x)\sin(x\log(x)) + \sin(x\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
```

```
[Out] integrate(log(x)*sin(x*log(x)) + sin(x*log(x)), x)
```

3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2} dx$

Optimal. Leaf size=68

$$-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}+\tan^{-1}(1-x)$$

```
[Out] -x^(-1) + ArcTan[1 - x] - Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x + Log[2 -
 x]/2 + Log[x]/2 - Log[2 - 2*x + x^2]/2
```

Rubi [A] time = 0.250847, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2525, 12, 6728, 634, 617, 204, 628}

$$-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}+\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] - Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x + Log[2 -
 x]/2 + Log[x]/2 - Log[2 - 2*x + x^2]/2
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \int \frac{4(1-x)}{(2-x)x^2\left(2-2x+x^2\right)}\,dx$$
$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \frac{1-x}{(2-x)x^2\left(2-2x+x^2\right)}\,dx$$
$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \left(\frac{1}{8(-2+x)} + \frac{1}{4x^2} + \frac{1}{8x} - \frac{x}{4\left(2-2x+x^2\right)}\right)dx$$
$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \int \frac{x}{2-2x+x^2}\,dx$$
$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx - \int \frac{1}{2-2x+x^2}\,dx$$
$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log\left(2-2x+x^2\right) - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1$$
$$= -\frac{1}{x} + \tan^{-1}(1-x) - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log\left(2-2x+x^2\right)$$

Mathematica [A] time = 0.0315522, size = 63, normalized size = 0.93

$$-\frac{\log\left(-\frac{(x-2)x}{x^2-2x+2}\right)}{x} - \frac{1}{2}\log\left(x^2-2x+2\right) - \frac{1}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} + \tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] + Log[2 - x]/2 + Log[x]/2 - Log[-(((-2 + x)*x)/(2 -
 2*x + x^2))]/x - Log[2 - 2*x + x^2]/2
```

Maple [A] time = 0.021, size = 57, normalized size = 0.8

$$-\frac{1}{x}\ln\left(\frac{x(2-x)}{x^2-2x+2}\right) - x^{-1} + \frac{\ln(x)}{2} - \frac{\ln\left(x^2-2x+2\right)}{2} - \arctan(-1+x) + \frac{\ln(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x)
```

```
[Out] -1/x*ln(x*(2-x)/(x^2-2*x+2))-1/x+1/2*ln(x)-1/2*ln(x^2-2*x+2)-arctan(-1+x)+1
/2*ln(x-2)
```

Maxima [A] time = 1.6894, size = 77, normalized size = 1.13

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(x-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="maxima")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(x - 2) + 1/2*log(x)
```

Fricas [A] time = 2.14019, size = 151, normalized size = 2.22

$$-\frac{2\,x\arctan\left(x-1\right)+x\log\left(x^2-2\,x+2\right)-x\log\left(x^2-2\,x\right)+2\,\log\left(-\frac{x^2-2\,x}{x^2-2\,x+2}\right)+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*arctan(x - 1) + x*log(x^2 - 2*x + 2) - x*log(x^2 - 2*x) + 2*log(-
(x^2 - 2*x)/(x^2 - 2*x + 2)) + 2)/x
```

Sympy [A] time = 0.227864, size = 46, normalized size = 0.68

$$\frac{\log\left(x^2-2x\right)}{2}-\frac{\log\left(x^2-2x+2\right)}{2}-\operatorname{atan}\left(x-1\right)-\frac{\log\left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((1-(-1+x)**2)/(1+(-1+x)**2))/x**2,x)
```

```
[Out] log(x**2 - 2*x)/2 - log(x**2 - 2*x + 2)/2 - atan(x - 1) - log((1 - (x - 1)*
*2)/((x - 1)**2 + 1))/x - 1/x
```

Giac [A] time = 1.37482, size = 80, normalized size = 1.18

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(|x-2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="giac")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(abs(x - 2)) + 1/2*log(abs(x))
```

3.273 $\int \log\left(\sqrt{x}+x\right)\,dx$

Optimal. Leaf size=29

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rubi [A] time = 0.0180576, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2548, 376, 77}

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + x],x]
```

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \log\left(\sqrt{x}+x\right)\,dx &= x\log\left(\sqrt{x}+x\right)-\int \frac{1+2\sqrt{x}}{2+2\sqrt{x}}\,dx\\
&= x\log\left(\sqrt{x}+x\right)-2\,\text{Subst}\left(\int \frac{x(1+2x)}{2+2x}\,dx,x,\sqrt{x}\right)\\
&= x\log\left(\sqrt{x}+x\right)-2\,\text{Subst}\left(\int \left(-\frac{1}{2}+x+\frac{1}{2(1+x)}\right)dx,x,\sqrt{x}\right)\\
&= \sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)
\end{aligned}
$$

Mathematica [A] time = 0.0116458, size = 29, normalized size = 1.

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + x],x]
```

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Maple [A] time = 0.002, size = 24, normalized size = 0.8

$$-x-\ln\left(1+\sqrt{x}\right)+x\ln\left(x+\sqrt{x}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+x^(1/2)),x)
```

```
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Maxima [A] time = 1.03459, size = 31, normalized size = 1.07

$$x\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Fricas [A] time = 2.23564, size = 103, normalized size = 3.55

$$(x+1)\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-2\,\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*log(x + sqrt(x)) - x + sqrt(x) - 2*log(sqrt(x) + 1) - log(sqrt(x))
```

Sympy [A] time = 4.50717, size = 24, normalized size = 0.83

$$\sqrt{x}+x\log\left(\sqrt{x}+x\right)-x-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+x**(1/2)),x)
```

```
[Out] sqrt(x) + x*log(sqrt(x) + x) - x - log(sqrt(x) + 1)
```

Giac [A] time = 1.34515, size = 31, normalized size = 1.07

$$x\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

3.274 $\int \log \left(-\frac{x}{1+x}\right) dx$

Optimal. Leaf size=18

$$x \log \left(-\frac{x}{x+1}\right)-\log (x+1)$$

```
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Rubi [A] time = 0.0035066, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2486, 31}

$$x \log \left(-\frac{x}{x+1}\right)-\log (x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-(x/(1 + x))],x]

[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(-\frac{x}{1+x}\right) dx &= x \log \left(-\frac{x}{1+x}\right)-\int \frac{1}{1+x} dx \\
&= x \log \left(-\frac{x}{1+x}\right)-\log (1+x)
\end{aligned}$$

Mathematica [A] time = 0.0016447, size = 18, normalized size = 1.

$$x \log \left(-\frac{x}{x+1}\right) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-(x/(1 + x))],x]
```

```
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Maple [A] time = 0.011, size = 28, normalized size = 1.6

$$\ln\left((1+x)^{-1}\right) - \ln\left(-1+(1+x)^{-1}\right)\left(-1+(1+x)^{-1}\right)(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-x/(1+x)),x)
```

```
[Out] ln(1/(1+x))-ln(-1+1/(1+x))*(-1+1/(1+x))*(1+x)
```

Maxima [A] time = 1.07712, size = 24, normalized size = 1.33

$$x \log \left(-\frac{x}{x+1}\right) - \log (x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="maxima")
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Fricas [A] time = 1.94007, size = 43, normalized size = 2.39

$$x \log \left(-\frac{x}{x+1}\right) - \log (x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="fricas")

[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Sympy [A] time = 0.105937, size = 14, normalized size = 0.78

$$x\log\left(-\frac{x}{x+1}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x/(1+x)),x)

[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Giac [A] time = 1.30919, size = 26, normalized size = 1.44

$$x\log\left(-\frac{x}{x+1}\right)-\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="giac")

[Out] x*log(-x/(x + 1)) - log(abs(x + 1))
```

3.275 $\int \log\left(\frac{-1+x}{1+x}\right)\,dx$

Optimal. Leaf size=27

$$-(1-x)\log\left(-\frac{1-x}{x+1}\right)-2\log(x+1)$$

```
[Out] -((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]
```

Rubi [A] time = 0.0049812, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2486, 31}

$$-(1-x)\log\left(-\frac{1-x}{x+1}\right)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-1 + x)/(1 + x)],x]
```

```
[Out] -((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \log\left(\frac{-1+x}{1+x}\right) dx = -(1-x)\log\left(-\frac{1-x}{1+x}\right) - 2\int \frac{1}{1+x}\,dx$$
$$= -(1-x)\log\left(-\frac{1-x}{1+x}\right) - 2\log(1+x)$$

Mathematica [A] time = 0.0027194, size = 21, normalized size = 0.78

$$(x-1)\log\left(\frac{x-1}{x+1}\right) - 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-1 + x)/(1 + x)],x]
```

```
[Out] (-1 + x)*Log[(-1 + x)/(1 + x)] - 2*Log[1 + x]
```

Maple [A] time = 0.007, size = 35, normalized size = 1.3

$$2\ln\left(-2(1+x)^{-1}\right) + \ln\left(1-2(1+x)^{-1}\right)\left(1-2(1+x)^{-1}\right)(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-1+x)/(1+x)),x)
```

```
[Out] 2*ln(-2/(1+x))+ln(1-2/(1+x))*(1-2/(1+x))*(1+x)
```

Maxima [A] time = 1.00724, size = 34, normalized size = 1.26

$$x\log\left(\frac{x-1}{x+1}\right) - \log(x+1) - \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="maxima")
```

[Out] x*log((x - 1)/(x + 1)) - log(x + 1) - log(x - 1)

Fricas [A] time = 2.10258, size = 53, normalized size = 1.96

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-1+x)/(1+x)),x, algorithm="fricas")

[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)

Sympy [A] time = 0.119236, size = 15, normalized size = 0.56

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln((-1+x)/(1+x)),x)

[Out] x*log((x - 1)/(x + 1)) - log(x**2 - 1)

Giac [A] time = 1.32349, size = 30, normalized size = 1.11

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((-1+x)/(1+x)),x, algorithm="giac")

[Out] x*log((x - 1)/(x + 1)) - log(abs(x^2 - 1))

3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x+1}-\tan^{-1}(x)$$

```
[Out] -(1 + x)^(-1) - ArcTan[x] + Log[1 - x^2]/2 - Log[(1 - x^2)/(1 + x^2)]/(1 +
x) - Log[1 + x^2]/2
```

Rubi [A] time = 0.0569428, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2525, 12, 2074, 260, 635, 203}

$$\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x+1}-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) - ArcTan[x] + Log[1 - x^2]/2 - Log[(1 - x^2)/(1 + x^2)]/(1 +
x) - Log[1 + x^2]/2
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx &= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+\int \frac{4x}{-1-x+x^4+x^5}\,dx\\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+4\int \frac{x}{-1-x+x^4+x^5}\,dx\\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+4\int \left(\frac{1}{4(1+x)^2}+\frac{x}{4\left(-1+x^2\right)}+\frac{-1-x}{4\left(1+x^2\right)}\right)dx\\
&= -\frac{1}{1+x}-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+\int \frac{x}{-1+x^2}\,dx+\int \frac{-1-x}{1+x^2}\,dx\\
&= -\frac{1}{1+x}+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{1+x}-\tan^{-1}(x)+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [C] time = 0.0461456, size = 60, normalized size = 1.05

$$\frac{1}{2}\left(\log\left(1-x^2\right)-\frac{2\left(\log\left(\frac{1-x^2}{x^2+1}\right)+1\right)}{x+1}+(-1+i)\log(-x+i)-(1+i)\log(x+i)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] ((-1 + I)*Log[I - x] - (1 + I)*Log[I + x] + Log[1 - x^2] - (2*(1 + Log[(1 -
 x^2)/(1 + x^2)]))/(1 + x))/2
```

Maple [C] time = 0.033, size = 112, normalized size = 2.

$$-\frac{1}{1+x}\ln\left(\frac{-x^2+1}{x^2+1}\right)+\frac{i\ln(x-i)x-i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)-\ln(x-i)x-\ln(x+i)x+\ln\left(x^2-1\right)x-\ln}{2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-x^2+1)/(x^2+1))/(1+x)^2,x)
```

```
[Out] -ln((-x^2+1)/(x^2+1))/(1+x)+1/2*(I*ln(x-I)*x-I*ln(x+I)*x+I*ln(x-I)-I*ln(x+I
)-ln(x-I)*x-ln(x+I)*x+ln(x^2-1)*x-ln(x-I)-ln(x+I)+ln(x^2-1)-2)/(1+x)
```

Maxima [A] time = 1.58296, size = 73, normalized size = 1.28

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 2.11034, size = 157, normalized size = 2.75

$$-\frac{2\,(x+1)\arctan(x)+(x+1)\log\left(x^2+1\right)-(x+1)\log\left(x^2-1\right)+2\,\log\left(-\frac{x^2-1}{x^2+1}\right)+2}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(x) + (x + 1)*log(x^2 + 1) - (x + 1)*log(x^2 - 1) + 2
*log(-(x^2 - 1)/(x^2 + 1)) + 2)/(x + 1)
```

Sympy [A] time = 0.194229, size = 41, normalized size = 0.72

$$\frac{\log\left(x^2-1\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}(x)-\frac{4}{4x+4}-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-x**2+1)/(x**2+1))/(1+x)**2,x)
```

```
[Out] log(x**2 - 1)/2 - log(x**2 + 1)/2 - atan(x) - 4/(4*x + 4) - log((1 - x**2)/
(x**2 + 1))/(x + 1)
```

Giac [A] time = 1.37203, size = 76, normalized size = 1.33

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(|x+1|\right)+\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="giac")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

3.277 $\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx$

Optimal. Leaf size=60

$$in\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[2/(1 + I*x)] + ArcTan[x]*Log[c*(1 + x^2
)^n] + I*n*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi [A] time = 0.0807481, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {203, 2470, 4920, 4854, 2402, 2315}

$$in\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[2/(1 + I*x)] + ArcTan[x]*Log[c*(1 + x^2
)^n] + I*n*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx &= \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{x\tan^{-1}(x)}{1+x^2}\,dx\\
&= in\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2n)\int \frac{\tan^{-1}(x)}{i-x}\,dx\\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2}\,dx\\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2in)\,\mathrm{Subst}\left(\int \frac{\log(2}{1-2}\right.\\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+in\mathrm{Li}_2\left(1-\frac{2}{1+ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0062376, size = 62, normalized size = 1.03

$$
in\mathrm{PolyLog}\left(2,\frac{x+i}{x-i}\right)+\tan^{-1}(x)\log\left(c\left(x^2+1\right)^n\right)+in\tan^{-1}(x)^2+2n\log\left(\frac{2i}{-x+i}\right)\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[(2*I)/(I - x)] + ArcTan[x]*Log[c*(1 + x
^2)^n] + I*n*PolyLog[2, (I + x)/(-I + x)]
```

Maple [C] time = 0.092, size = 249, normalized size = 4.2

$$\arctan(x)\ln\left(\left(x^2+1\right)^n\right)-n\ln\left(x^2+1\right)\arctan(x)-\frac{i}{2}n\ln\left(x^2+1\right)\ln(x-i)+\frac{i}{4}n\left(\ln(x-i)\right)^2+\frac{i}{2}n\ln(x-i)\ln\left(-\frac{i}{2}(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(x^2+1)^n)/(x^2+1),x)
```

```
[Out] arctan(x)*ln((x^2+1)^n)-n*ln(x^2+1)*arctan(x)-1/2*I*n*ln(x^2+1)*ln(x-I)+1/4
*I*n*ln(x-I)^2+1/2*I*n*ln(x-I)*ln(-1/2*I*(x+I))+1/2*I*n*dilog(-1/2*I*(x+I))
+1/2*I*n*ln(x^2+1)*ln(x+I)-1/4*I*n*ln(x+I)^2-1/2*I*n*ln(x+I)*ln(1/2*I*(x-I)
)-1/2*I*n*dilog(1/2*I*(x-I))-1/2*I*arctan(x)*Pi*csgn(I*c)*csgn(I*(x^2+1)^n)
*csgn(I*c*(x^2+1)^n)+1/2*I*arctan(x)*Pi*csgn(I*c)*csgn(I*c*(x^2+1)^n)^2+1/2
*I*arctan(x)*Pi*csgn(I*(x^2+1)^n)*csgn(I*c*(x^2+1)^n)^2-1/2*I*arctan(x)*Pi*
csgn(I*c*(x^2+1)^n)^3+arctan(x)*ln(c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(c\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(x**2+1)**n)/(x**2+1),x)
```

```
[Out] Integral(log(c*(x**2 + 1)**n)/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\left(x^2+1\right)^n c\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2} \, dx$

Optimal. Leaf size=61

$$i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)+\log\left(\frac{x^2}{x^2+1}\right)\tan^{-1}(x)+i\tan^{-1}(x)^2-2\log\left(2-\frac{2}{1-ix}\right)\tan^{-1}(x)$$

```
[Out] I*ArcTan[x]^2 - 2*ArcTan[x]*Log[2 - 2/(1 - I*x)] + ArcTan[x]*Log[x^2/(1 + x
^2)] + I*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rubi [A] time = 0.1055, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {203, 2526, 12, 4924, 4868, 2447}

$$i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)+\log\left(\frac{x^2}{x^2+1}\right)\tan^{-1}(x)+i\tan^{-1}(x)^2-2\log\left(2-\frac{2}{1-ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] I*ArcTan[x]^2 - 2*ArcTan[x]*Log[2 - 2/(1 - I*x)] + ArcTan[x]*Log[x^2/(1 + x
^2)] + I*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2526

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[(u*D[RFx, x])/RFx, x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-\int \frac{2\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\
&= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2\int \frac{\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\
&= i\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2i\int \frac{\tan^{-1}(x)}{x(i+x)}\,dx\\
&= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+2\int \frac{\log\left(2-\frac{2}{1-ix}\right)}{1+x^2}\,dx\\
&= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+i\mathrm{Li}_2\left(-1+\frac{2}{1-ix}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0442744, size = 239, normalized size = 3.92

$$-\frac{1}{2}i\text{PolyLog}\left(2,-\frac{1}{2}i(-x+i)\right)+i\text{PolyLog}(2,-i(-x+i))+\frac{1}{2}i\text{PolyLog}\left(2,-\frac{1}{2}i(x+i)\right)-i\text{PolyLog}(2,-i(x+i))-\frac{1}{2}i\log$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] (-I/4)*Log[I - x]^2 + I*Log[I - x]*Log[(-I)*x] - (I/2)*Log[I - x]*Log[(-I/2
)*(I + x)] + (I/2)*Log[(-I/2)*(I - x)]*Log[I + x] - I*Log[I*x]*Log[I + x] +
 (I/4)*Log[I + x]^2 - (I/2)*Log[I - x]*Log[x^2/(1 + x^2)] + (I/2)*Log[I + x
]*Log[x^2/(1 + x^2)] - (I/2)*PolyLog[2, (-I/2)*(I - x)] + I*PolyLog[2, (-I)
*(I - x)] + (I/2)*PolyLog[2, (-I/2)*(I + x)] - I*PolyLog[2, (-I)*(I + x)]
```

Maple [B] time = 0.02, size = 158, normalized size = 2.6

$$-\frac{i}{2}\ln\left(\frac{x^2}{x^2+1}\right)\ln(x-i)-\frac{i}{4}(\ln(x-i))^2-\frac{i}{2}\ln(x-i)\ln\left(-\frac{i}{2}(x+i)\right)+i\ln(x-i)\ln(-ix)-\frac{i}{2}\text{dilog}\left(-\frac{i}{2}(x+i)\right)+i\text{dilo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2/(x^2+1))/(x^2+1),x)
```

```
[Out] -1/2*I*ln(x^2/(x^2+1))*ln(x-I)-1/4*I*ln(x-I)^2-1/2*I*ln(x-I)*ln(-1/2*I*(x+I
))+I*ln(x-I)*ln(-I*x)-1/2*I*dilog(-1/2*I*(x+I))+I*dilog(-I*x)+1/2*I*ln(x^2/
(x^2+1))*ln(x+I)+1/4*I*ln(x+I)^2+1/2*I*ln(x+I)*ln(1/2*I*(x-I))-I*ln(x+I)*ln
(I*x)+1/2*I*dilog(1/2*I*(x-I))-I*dilog(I*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2/(x**2+1))/(x**2+1),x)
```

```
[Out] Integral(log(x**2/(x**2 + 1))/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$

Optimal. Leaf size=165

$$\frac{i\mathrm{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}+\frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}-\frac{2\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (I*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (I*PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.191676, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {205, 2526, 12, 4924, 4868, 2447}

$$\frac{i\mathrm{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}}+\frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}}-\frac{2\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (I*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (ArcTan[(Sqrt[b]*x)/S
qrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (I*PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2526

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[(u*D[RFx, x])/RFx, x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
```

```
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1
 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \int \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}x\left(a+bx^2\right)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{\left(2\sqrt{a}\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x\left(a+bx^2\right)}\,dx}{\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{(2i)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{x\left(i+\frac{\sqrt{b}x}{\sqrt{a}}\right)}\,dx}{\sqrt{a}\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\int \frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{a}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{i\text{Li}_2\left(-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [B] time = 0.194038, size = 373, normalized size = 2.26

$$\frac{4\text{PolyLog}\left(2,\frac{\sqrt{b}x}{\sqrt{-a}}+1\right) - 2\text{PolyLog}\left(2,\frac{a-\sqrt{-a}\sqrt{b}x}{2a}\right) + 2\text{PolyLog}\left(2,\frac{\sqrt{-a}\sqrt{b}x+a}{2a}\right) - 4\text{PolyLog}\left(2,\frac{a\sqrt{b}x}{(-a)^{3/2}}+1\right) + 2\log\left(\sqrt{-a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (-4*Log[(Sqrt[b]*x)/Sqrt[-a]]*Log[Sqrt[-a] - Sqrt[b]*x] + Log[Sqrt[-a] - Sq
rt[b]*x]^2 + 4*Log[(a*Sqrt[b]*x)/(-a)^(3/2)]*Log[Sqrt[-a] + Sqrt[b]*x] - Lo
g[Sqrt[-a] + Sqrt[b]*x]^2 + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(a - Sqrt[-a]*S
qrt[b]*x)/(2*a)] - 2*Log[Sqrt[-a] + Sqrt[b]*x]*Log[(a + Sqrt[-a]*Sqrt[b]*x)
/(2*a)] + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] - 2*Log[Sqrt
[-a] + Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] + 4*PolyLog[2, 1 + (Sqrt[b]*x)/S
qrt[-a]] - 2*PolyLog[2, (a - Sqrt[-a]*Sqrt[b]*x)/(2*a)] + 2*PolyLog[2, (a +
 Sqrt[-a]*Sqrt[b]*x)/(2*a)] - 4*PolyLog[2, 1 + (a*Sqrt[b]*x)/(-a)^(3/2)])/(
4*Sqrt[-a]*Sqrt[b])
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{1}{b x^2+a} \ln\left(\frac{c x^2}{b x^2+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)

[Out] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(\frac{c x^2}{b x^2+a}\right)}{b x^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="fricas")

[Out] integral(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(b*x**2+a))/(b*x**2+a),x)

[Out] Integral(log(c*x**2/(a + b*x**2))/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{cx^2}{bx^2+a}\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="giac")

[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rubi [A] time = 0.0335913, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2518}

$$\frac{\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] time = 0.616548, size = 134, normalized size = 4.62

$$\frac{\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-2\left(-\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(e^{-2\tanh\ldots}\right.\right.\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] - Log[1 -
I/E^ArcTanh[a*x]] + Log[1 + I/E^ArcTanh[a*x]]) - PolyLog[2, (-I)/E^ArcTanh[
a*x]] + PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1}\ln\left(1+i\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\log\left(ax+1\right)-\log\left(-ax+1\right)\right)\log\left(ax+1\right)-\log\left(ax+1\right)^2+2\,\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(-ax+1\right)^2-4\left(\log\left(ax\right.\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) + I*sqrt(-a*x + 1)))/a - integrate(-sqrt(a*x + 1)*(log
(a*x + 1) - log(-a*x + 1))/(2*(a^2*x^2 - 1)*sqrt(a*x + 1) + (2*I*a^2*x^2 -
2*I)*sqrt(-a*x + 1)), x)
```

Fricas [A] time = 2.19388, size = 92, normalized size = 3.17

$$\frac{\mathrm{Li}_2\left(-\frac{ax+i\sqrt{ax+1}\sqrt{-ax+1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x + I*sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[Out] PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rubi [A] time = 0.034437, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2518}

$$\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2518

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] time = 0.536149, size = 134, normalized size = 4.62

$$\frac{\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-2\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(e^{-2\tanh^{-1}(a\right.\right.\right.}{4a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] + Log[1 -
I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{1}{-a^2x^2+1} \ln\left(1 - i\sqrt{-ax+1}\frac{1}{\sqrt{ax+1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\log\left(ax+1\right)-\log\left(-ax+1\right)\right)\log\left(ax+1\right)-\log\left(ax+1\right)^{2}+2\,\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(-ax+1\right)^{2}-4\left(\log\left(ax\right.\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) - I*sqrt(-a*x + 1)))/a + integrate(sqrt(a*x + 1)*(log(
a*x + 1) - log(-a*x + 1))/(2*(a^2*x^2 - 1)*sqrt(a*x + 1) - (2*I*a^2*x^2 - 2
*I)*sqrt(-a*x + 1)), x)
```

Fricas [A] time = 2.18479, size = 92, normalized size = 3.17

$$\frac{\mathrm{Li}_2\left(-\frac{ax-i\sqrt{ax+1}\sqrt{-ax+1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x - I*sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\log\left(-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}+1\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(-I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

3.282 $\int \log\left(e^{a+bx}\right)\,dx$

Optimal. Leaf size=17

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Rubi [A] time = 0.003246, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2157, 30}

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Rule 2157

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^{a+bx}\right)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \log\left(e^{a+bx}\right)\right)}{b}\\ &= \frac{\log^2\left(e^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0026791, size = 17, normalized size = 1.

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$\frac{\left(\ln\left(e^{bx+a}\right)\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(b*x+a)),x)
```

```
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Maxima [A] time = 1.08409, size = 14, normalized size = 0.82

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A] time = 1.93074, size = 23, normalized size = 1.35

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.088424, size = 8, normalized size = 0.47

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(b*x+a)),x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A] time = 1.2954, size = 14, normalized size = 0.82

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

3.283 $\int \log \left(e^{a+bx^n}\right) dx$

Optimal. Leaf size=27

$$x \log \left(e^{a+bx^n}\right) - \frac{bnx^{n+1}}{n+1}$$

```
[Out] -((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Rubi [A] time = 0.0085458, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2548, 12, 30}

$$x \log \left(e^{a+bx^n}\right) - \frac{bnx^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x^n)],x]
```

```
[Out] -((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^{a+bx^n}\right)dx &= x\log\left(e^{a+bx^n}\right) - \int bnx^n\,dx\\ &= x\log\left(e^{a+bx^n}\right) - (bn)\int x^n\,dx\\ &= -\frac{bnx^{1+n}}{1+n} + x\log\left(e^{a+bx^n}\right)\end{aligned}$$

Mathematica [A] time = 0.0237789, size = 25, normalized size = 0.93

$$x\left(\log\left(e^{a+bx^n}\right) - \frac{bnx^n}{n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x^n)],x]
```

```
[Out] x*(-((b*n*x^n)/(1 + n)) + Log[E^(a + b*x^n)])
```

Maple [A] time = 0.007, size = 27, normalized size = 1.

$$-\frac{bnx^{1+n}}{1+n} + x\ln\left(e^{a+bx^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(a+b*x^n)),x)
```

```
[Out] -b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08307, size = 45, normalized size = 1.67

$$\frac{bxx^n + (an + a)x}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [A] time = 2.81085, size = 34, normalized size = 1.26

$$\begin{cases} \frac{anx}{n+1} + \frac{ax}{n+1} + \frac{bxx^n}{n+1} & \text{for}\: n \neq -1 \\ ax + b\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(a+b*x**n)),x)
```

```
[Out] Piecewise((a*n*x/(n + 1) + a*x/(n + 1) + b*x*x**n/(n + 1), Ne(n, -1)), (a*x
 + b*log(x), True))
```

Giac [A] time = 1.37261, size = 22, normalized size = 0.81

$$ax + \frac{bx^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="giac")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

3.284 $\int e^x \log(a + be^x)\, dx$

Optimal. Leaf size=25

$$\frac{(a + be^x)\log(a + be^x)}{b} - e^x$$

```
[Out] -E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

Rubi [A] time = 0.0525146, antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2194, 2554, 12, 2248, 43}

$$e^x \log(a + be^x) + \frac{a\log(a + be^x)}{b} - e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + (a*Log[a + b*E^x])/b + E^x*Log[a + b*E^x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
```

```
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \log(a+be^x)\,dx &= e^x\log(a+be^x) - \int \frac{be^{2x}}{a+be^x}\,dx \\
&= e^x\log(a+be^x) - b\int \frac{e^{2x}}{a+be^x}\,dx \\
&= e^x\log(a+be^x) - b\,\mathrm{Subst}\left(\int \frac{x}{a+bx}\,dx, x, e^x\right) \\
&= e^x\log(a+be^x) - b\,\mathrm{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)dx, x, e^x\right) \\
&= -e^x + \frac{a\log(a+be^x)}{b} + e^x\log(a+be^x)
\end{aligned}$$

Mathematica [A] time = 0.0160492, size = 25, normalized size = 1.

$$\frac{(a+be^x)\log(a+be^x)}{b} - e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

Maple [A] time = 0.003, size = 34, normalized size = 1.4

$$e^x\ln(a+be^x) - e^x + \frac{\ln(a+be^x)\,a}{b} - \frac{a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(a+b*exp(x)),x)
```

```
[Out] exp(x)*ln(a+b*exp(x))-exp(x)+a*ln(a+b*exp(x))/b-a/b
```

Maxima [A] time = 1.05244, size = 35, normalized size = 1.4

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Fricas [A] time = 2.05948, size = 55, normalized size = 2.2

$$-\frac{be^x - (be^x + a)\log(be^x + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(a+b*exp(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19577, size = 35, normalized size = 1.4

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

3.285 $\int e^{a+bx}\log(x)\,dx$

Optimal. Leaf size=26

$$\frac{\log(x)e^{a+bx}}{b}-\frac{e^a\text{Ei}(bx)}{b}$$

```
[Out] -((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Rubi [A] time = 0.0354785, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2194, 2554, 12, 2178}

$$\frac{\log(x)e^{a+bx}}{b}-\frac{e^a\text{Ei}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Log[x],x]
```

```
[Out] -((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^{a+bx}\log(x)\,dx &= \frac{e^{a+bx}\log(x)}{b}-\int \frac{e^{a+bx}}{bx}\,dx\\ &= \frac{e^{a+bx}\log(x)}{b}-\frac{\int \frac{e^{a+bx}}{x}\,dx}{b}\\ &= -\frac{e^a\text{Ei}(bx)}{b}+\frac{e^{a+bx}\log(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.0148314, size = 22, normalized size = 0.85

$$\frac{e^a\left(e^{bx}\log(x)-\text{Ei}(bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Log[x],x]
```

```
[Out] (E^a*(-ExpIntegralEi[b*x] + E^(b*x)*Log[x]))/b
```

Maple [A] time = 0.016, size = 26, normalized size = 1.

$$\frac{e^{bx+a}\ln(x)}{b}+\frac{e^a\text{Ei}(1,-bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*ln(x),x)
```

```
[Out] exp(b*x+a)*ln(x)/b+1/b*exp(a)*Ei(1,-b*x)
```

Maxima [A] time = 1.13359, size = 32, normalized size = 1.23

$$-\frac{\text{Ei}(bx)\,e^a}{b}+\frac{e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Fricas [A] time = 2.14951, size = 53, normalized size = 2.04

$$-\frac{\mathrm{Ei}\left(bx\right)e^{a}-e^{\left(bx+a\right)}\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -(Ei(b*x)*e^a - e^(b*x + a)*log(x))/b
```

Sympy [A] time = 6.91723, size = 26, normalized size = 1.

$$\left(\begin{cases} x & \text{for}\: b = 0 \\ \frac{e^{bx}}{b} & \text{otherwise} \end{cases}\right) e^{a} \log\left(x\right) - \left(\begin{cases} x & \text{for}\: b = 0 \\ \frac{\mathrm{Ei}\left(bx\right)}{b} & \text{otherwise} \end{cases}\right) e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*ln(x),x)
```

```
[Out] Piecewise((x, Eq(b, 0)), (exp(b*x)/b, True))*exp(a)*log(x) - Piecewise((x,
Eq(b, 0)), (Ei(b*x)/b, True))*exp(a)
```

Giac [A] time = 1.30655, size = 32, normalized size = 1.23

$$-\frac{\mathrm{Ei}\left(bx\right)e^{a}}{b}+\frac{e^{\left(bx+a\right)}\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

3.286 $\int \frac{x^2}{x+\log(x)} \, dx$

Optimal. Leaf size=12

$$\text{CannotIntegrate}\left(\frac{x^2}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate[x^2/(x + Log[x]), x]
```

Rubi [A] time = 0.0359061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{x+\log(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/(x + Log[x]),x]
```

```
[Out] Defer[Int][x^2/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x^2}{x+\log(x)} \, dx = \int \frac{x^2}{x+\log(x)} \, dx$$

Mathematica [A] time = 6.5027, size = 0, normalized size = 0.

$$\int \frac{x^2}{x+\log(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/(x + Log[x]),x]
```

```
[Out] Integrate[x^2/(x + Log[x]), x]
```

Maple [A] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{x^2}{x + \ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+ln(x)),x)
```

```
[Out] int(x^2/(x+ln(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x + \log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x + log(x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{x + \log(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(x^2/(x + log(x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x + \log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x+ln(x)),x)
```

```
[Out] Integral(x**2/(x + log(x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x + \log(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x + log(x)), x)
```

3.287 $\int \frac{x}{x+\log(x)}\,dx$

Optimal. Leaf size=10

$$\text{CannotIntegrate}\left(\frac{x}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate[x/(x + Log[x]), x]
```

Rubi [A] time = 0.0220771, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/(x + Log[x]),x]
```

```
[Out] Defer[Int][x/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x}{x+\log(x)}\,dx=\int \frac{x}{x+\log(x)}\,dx$$

Mathematica [A] time = 5.36157, size = 0, normalized size = 0.

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/(x + Log[x]),x]
```

```
[Out] Integrate[x/(x + Log[x]), x]
```

Maple [A] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{x}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+ln(x)),x)
```

```
[Out] int(x/(x+ln(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x + log(x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{x+\log(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(x/(x + log(x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+ln(x)),x)
```

```
[Out] Integral(x/(x + log(x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x + \log(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(x + log(x)), x)
```

3.288 $\int \frac{1}{x+\log(x)}\,dx$

Optimal. Leaf size=8

$$\text{CannotIntegrate}\left(\frac{1}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate[(x + Log[x])^(-1), x]
```

Rubi [A] time = 0.0078645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x + Log[x])^(-1),x]
```

```
[Out] Defer[Int][(x + Log[x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log(x)}\,dx$$

Mathematica [A] time = 0.005806, size = 0, normalized size = 0.

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x + Log[x])^(-1),x]
```

```
[Out] Integrate[(x + Log[x])^(-1), x]
```

Maple [A] time = 0.009, size = 0, normalized size = 0.

$$\int (x + \ln(x))^{-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+ln(x)),x)
```

```
[Out] int(1/(x+ln(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \log(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + log(x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{x + \log(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(x + log(x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \log(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+ln(x)),x)
```

```
[Out] Integral(1/(x + log(x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x + \log(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(x + log(x)), x)
```

3.289 $\int \frac{1}{x(x+\log(x))}\,dx$

Optimal. Leaf size=12

$$\text{CannotIntegrate}\left(\frac{1}{x(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate[1/(x*(x + Log[x])), x]
```

Rubi [A] time = 0.031951, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(x+\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(x + Log[x])),x]

[Out] Defer[Int][1/(x*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x(x+\log(x))}\,dx=\int \frac{1}{x(x+\log(x))}\,dx$$

Mathematica [A] time = 0.0381998, size = 0, normalized size = 0.

$$\int \frac{1}{x(x+\log(x))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(x + Log[x])),x]

[Out] Integrate[1/(x*(x + Log[x])), x]
```

Maple [A] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x+\ln\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+ln(x)),x)
```

```
[Out] int(1/x/(x+ln(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+\log\left(x\right))x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + log(x))*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{x^2+x\log\left(x\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(x^2 + x*log(x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x+\log\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+ln(x)),x)
```

```
[Out] Integral(1/(x*(x + log(x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x + \log{(x)})x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((x + log(x))*x), x)
```

3.290 $\int \frac{1}{x^2(x+\log(x))} \, dx$

Optimal. Leaf size=12

$$\text{CannotIntegrate}\left(\frac{1}{x^2(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate[1/(x^2*(x + Log[x])), x]
```

Rubi [A] time = 0.0341406, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2(x+\log(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(x + Log[x])),x]

[Out] Defer[Int][1/(x^2*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x^2(x+\log(x))} \, dx = \int \frac{1}{x^2(x+\log(x))} \, dx$$

Mathematica [A] time = 6.46883, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(x+\log(x))} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(x + Log[x])),x]

[Out] Integrate[1/(x^2*(x + Log[x])), x]
```

Maple [A] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(x +\ln \left(x \right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+ln(x)),x)
```

```
[Out] int(1/x^2/(x+ln(x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x + \log\left(x\right))x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x + log(x))*x^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{1}{x^{3} + x^{2} \log\left(x\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(x^3 + x^2*log(x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2} \left(x + \log{\left(x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x+ln(x)),x)
```

```
[Out] Integral(1/(x**2*(x + log(x))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x + \log(x))x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((x + log(x))*x^2), x)
```

3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$

Optimal. Leaf size=13

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Rubi [A] time = 0.0243466, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {203, 260}

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x + 4*x*Log[x]^2),x]
```

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x+4x\log^2(x)}\,dx &= \text{Subst}\left(\int \frac{x}{1+4x^2}\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\log\left(1+4\log^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0082652, size = 13, normalized size = 1.

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x + 4*x*Log[x]^2),x]
```

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{\ln\left(1+4\,(\ln(x))^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(x+4*x*ln(x)^2),x)
```

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

Maxima [A] time = 1.19732, size = 12, normalized size = 0.92

$$\frac{1}{8}\log\left(\log(x)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*log(log(x)^2 + 1/4)
```

Fricas [A] time = 2.12501, size = 34, normalized size = 2.62

$$\frac{1}{8}\log\left(4\log(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

Sympy [A] time = 0.115593, size = 10, normalized size = 0.77

$$\frac{\log\left(\log(x)^2 + \frac{1}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(x+4*x*ln(x)**2),x)
```

```
[Out] log(log(x)**2 + 1/4)/8
```

Giac [A] time = 1.33486, size = 15, normalized size = 1.15

$$\frac{1}{8}\log\left(4\log(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

3.292 $\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$

Optimal. Leaf size=9

$$\log\left(\frac{\log(x)}{x}+1\right)$$

```
[Out] Log[1 + Log[x]/x]
```

Rubi [A] time = 0.0718203, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6712, 31}

$$\log\left(\frac{\log(x)}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] Log[1 + Log[x]/x]
```

Rule 6712

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, -Dist[c*q, Subst[Int
[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a,
b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integer
Q[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1-\log(x)}{x(x+\log(x))}\,dx = \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\log(x)}{x}\right)$$
$$= \log\left(1+\frac{\log(x)}{x}\right)$$

Mathematica [A] time = 0.051494, size = 10, normalized size = 1.11

$$\log(x+\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] -Log[x] + Log[x + Log[x]]
```

Maple [A] time = 0.01, size = 11, normalized size = 1.2

$$-\ln(x) + \ln(x+\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x/(x+ln(x)),x)
```

```
[Out] -ln(x)+ln(x+ln(x))
```

Maxima [A] time = 1.19336, size = 14, normalized size = 1.56

$$\log(x+\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] log(x + log(x)) - log(x)
```

Fricas [A] time = 2.15737, size = 35, normalized size = 3.89

$$\log(x + \log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] log(x + log(x)) - log(x)
```

Sympy [A] time = 0.114662, size = 8, normalized size = 0.89

$$-\log(x) + \log(x + \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x/(x+ln(x)),x)
```

```
[Out] -log(x) + log(x + log(x))
```

Giac [A] time = 1.3485, size = 19, normalized size = 2.11

$$-\log(x) + \log(-x - \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="giac")
```

```
[Out] -log(x) + log(-x - log(x))
```

3.293 $\int \frac{1+x}{\log(x)(x+\log(x))} \, dx$

Optimal. Leaf size=13

$$\text{li}(x) + \log(\log(x)) - \log(x + \log(x))$$

```
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Rubi [A] time = 0.142422, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 2353, 2298, 2302, 29, 6684}

$$\text{li}(x) + \log(\log(x)) - \log(x + \log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(Log[x]*(x + Log[x])),x]
```

```
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x}{\log(x)(x+\log(x))}\,dx &= \int \left(\frac{1+x}{x\log(x)}+\frac{-1-x}{x(x+\log(x))}\right)dx \\
&= \int \frac{1+x}{x\log(x)}\,dx+\int \frac{-1-x}{x(x+\log(x))}\,dx \\
&= -\log(x+\log(x))+\int \left(\frac{1}{\log(x)}+\frac{1}{x\log(x)}\right)dx \\
&= -\log(x+\log(x))+\int \frac{1}{\log(x)}\,dx+\int \frac{1}{x\log(x)}\,dx \\
&= -\log(x+\log(x))+\mathbf{li}(x)+\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log(x)\right) \\
&= \log(\log(x))-\log(x+\log(x))+\mathbf{li}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0385612, size = 13, normalized size = 1.

$$\mathbf{li}(x)+\log(\log(x))-\log(x+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(Log[x]*(x + Log[x])),x]
```

```
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1+x}{\ln(x)(x+\ln(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/ln(x)/(x+ln(x)),x)
```

```
[Out] int((1+x)/ln(x)/(x+ln(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{x \log(x)} \, dx - \log(x + \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(x*log(x)), x) - log(x + log(x))
```

Fricas [A] time = 2.13983, size = 68, normalized size = 5.23

$$-\log(x + \log(x)) + \log(\log(x)) + \text{log\_integral}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="fricas")
```

```
[Out] -log(x + log(x)) + log(log(x)) + log_integral(x)
```

Sympy [A] time = 2.14364, size = 15, normalized size = 1.15

$$-\log(x + \log(x)) + \log(\log(x)) + \text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/ln(x)/(x+ln(x)),x)
```

```
[Out] -log(x + log(x)) + log(log(x)) + Ei(log(x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(x+\log{(x)})\log{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/((x + log(x))*log(x)), x)
```

3.294 $\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=67

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

```
[Out] -Log[1 - Sqrt[1 + x^(-1)]]/6 + Log[1 + Sqrt[1 + x^(-1)]]/2 - Log[2 + Sqrt[1
 + x^(-1)]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Rubi [A] time = 0.0742242, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2548, 12, 2058}

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -Log[1 - Sqrt[1 + x^(-1)]]/6 + Log[1 + Sqrt[1 + x^(-1)]]/2 - Log[2 + Sqrt[1
 + x^(-1)]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(2+\sqrt{\frac{1+x}{x}}\right) - \int \frac{1}{2\left(-1-x-2x\sqrt{\frac{1+x}{x}}\right)}dx$$
$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) - \frac{1}{2}\int \frac{1}{-1-x-2x\sqrt{\frac{1+x}{x}}}dx$$
$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) + \text{Subst}\left(\int \frac{1}{2+x-2x^2-x^3}dx, x, \sqrt{\frac{1+x}{x}}\right)$$
$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) + \text{Subst}\left(\int \left(-\frac{1}{6(-1+x)} + \frac{1}{2(1+x)} - \frac{1}{3(2+x)}\right)dx, x, \sqrt{\frac{1+x}{x}}\right)$$
$$= -\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right) - \frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right) + x\log\left(2+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.0305202, size = 53, normalized size = 0.79

$$x\log\left(\sqrt{\frac{1}{x}+1}+2\right) + \frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(2\sqrt{\frac{1}{x}+1}+1\right)\right) - \tanh^{-1}\left(2\sqrt{\frac{1}{x}+1}+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ArcTanh[(1 + 2*Sqrt[1 + x^(-1)])/3]/3 - ArcTanh[3 + 2*Sqrt[1 + x^(-1)]] + x
*Log[2 + Sqrt[1 + x^(-1)]]
```

Maple [A] time = 0.046, size = 107, normalized size = 1.6

$$x\ln\left(2+\sqrt{\frac{1+x}{x}}\right) - \frac{1}{18x}\left(\sqrt{9}\ln\left(\frac{1}{9x-3}\left(4\sqrt{9}\sqrt{x^2+x}+15x+3\right)\right)\sqrt{x(1+x)} + 3\sqrt{\frac{1+x}{x}}x\ln(-3x+1) - 6\ln\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+((1+x)/x)^(1/2)),x)
```

```
[Out] x*ln(2+((1+x)/x)^(1/2))-1/18/((1+x)/x)^(1/2)/x*(9^(1/2)*ln(1/3*(4*9^(1/2)*(
x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1/2)+3*((1+x)/x)^(1/2)*x*ln(-3*x+1
```

```
)-6*ln(1/2+x+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Maxima [A] time = 1.01949, size = 90, normalized size = 1.34

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+2\right)}{\frac{x+1}{x}-1}-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 2)/((x + 1)/x - 1) - 1/3*log(sqrt((x + 1)/x) + 2) + 1
/2*log(sqrt((x + 1)/x) + 1) - 1/6*log(sqrt((x + 1)/x) - 1)
```

Fricas [A] time = 2.25612, size = 138, normalized size = 2.06

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) + 2) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/6
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] time = 161.722, size = 53, normalized size = 0.79

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{6}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - log(sqrt(1 + 1/x) - 1)/6 + log(sqrt(1 + 1/x) +
 1)/2 - log(sqrt(1 + 1/x) + 2)/3
```

Giac [B] time = 1.3405, size = 157, normalized size = 2.34

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{1}{6}\left(\frac{\log\left(\left|-2\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+x-\sqrt{x^2+x}+1\right|\right)}{\operatorname{sgn}(x)}-\frac{\log\left(\left|-2\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)-x+\sqrt{x\right.\right.}{\operatorname{sgn}(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - 1/6*(log(abs(-2*(x - sqrt(x^2 + x))*sgn(x) + x
 - sqrt(x^2 + x) + 1))/sgn(x) - log(abs(-2*(x - sqrt(x^2 + x))*sgn(x) - x +
 sqrt(x^2 + x) - 1))/sgn(x) + 2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1)))*sgn(x
) - 1/6*log(abs(3*x - 1))
```

3.295 $\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

```
[Out] -1/(2*(1 + Sqrt[1 + x^(-1)])) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Rubi [A] time = 0.056997, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2548, 12, 44, 207}

$$-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/(2*(1 + Sqrt[1 + x^(-1)])) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx &= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{2\left(-1-x-x\sqrt{\frac{1+x}{x}}\right)} dx \\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\int \frac{1}{-1-x-x\sqrt{\frac{1+x}{x}}} dx \\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-1+x)(1+x)^2} dx, x, \sqrt{\frac{1+x}{x}}\right) \\
&= x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(-1+x^2\right)}\right) dx, x, \sqrt{\frac{1+x}{x}}\right) \\
&= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{\frac{1+x}{x}}\right) \\
&= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1+x}{x}}\right)+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0373296, size = 53, normalized size = 1.06

$$\frac{1}{4}\left(-2\sqrt{\frac{1}{x}+1}x+2x+4x\log\left(\sqrt{\frac{1}{x}+1}+1\right)+\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (2*x - 2*Sqrt[1 + x^(-1)]*x + 4*x*Log[1 + Sqrt[1 + x^(-1)]] + Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x])/4
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \ln\left(1+\sqrt{\frac{1+x}{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+((1+x)/x)^(1/2)),x)
```

```
[Out] int(ln(1+((1+x)/x)^(1/2)),x)
```

Maxima [A] time = 1.03464, size = 92, normalized size = 1.84

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+1\right)}{\frac{x+1}{x}-1} - \frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)} + \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 1)/((x + 1)/x - 1) - 1/2/(sqrt((x + 1)/x) + 1) + 1/4*
log(sqrt((x + 1)/x) + 1) - 1/4*log(sqrt((x + 1)/x) - 1)
```

Fricas [A] time = 2.18177, size = 139, normalized size = 2.78

$$\frac{1}{4}(4x+1)\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}x\sqrt{\frac{x+1}{x}} + \frac{1}{2}x - \frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) + 1) - 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] time = 160.093, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}-\frac{1}{2\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 1) - log(sqrt(1 + 1/x) - 1)/4 + log(sqrt(1 + 1/x) +
 1)/4 - 1/(2*(sqrt(1 + 1/x) + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] undef
```

3.296 $\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=21

$$x\log\left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log(x+1)$$

```
[Out] x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2
```

Rubi [A] time = 0.0078685, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2453, 2448, 263, 31}

$$x\log\left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[(1 + x)/x]],x]
```

```
[Out] x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2
```

Rule 2453

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx &= \int \log\left(\sqrt{1+\frac{1}{x}}\right) dx\\ &= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)x}\,dx\\ &= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\int \frac{1}{1+x}\,dx\\ &= x\log\left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0032725, size = 19, normalized size = 0.9

$$\frac{1}{2}\left(\log(x)+(x+1)\log\left(\frac{x+1}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[(1 + x)/x]],x]
```

```
[Out] (Log[x] + (1 + x)*Log[(1 + x)/x])/2
```

Maple [A] time = 0.006, size = 22, normalized size = 1.1

$$-\frac{\ln\left(x^{-1}\right)}{2}+\frac{\ln\left(1+x^{-1}\right)\left(1+x^{-1}\right)x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln((1+x)/x),x)
```

```
[Out] -1/2*ln(1/x)+1/2*ln(1+1/x)*(1+1/x)*x
```

Maxima [A] time = 1.00957, size = 24, normalized size = 1.14

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="maxima")
```

```
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Fricas [A] time = 2.03842, size = 53, normalized size = 2.52

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Sympy [A] time = 0.107513, size = 17, normalized size = 0.81

$$\frac{x\log\left(\frac{x+1}{x}\right)}{2}+\frac{\log\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln((1+x)/x),x)
```

```
[Out] x*log((x + 1)/x)/2 + log(2*x + 2)/2
```

Giac [A] time = 1.25201, size = 26, normalized size = 1.24

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="giac")
```

```
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(abs(x + 1))
```

3.297 $\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)}+x\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

```
[Out] -1/(2*(1 - Sqrt[1 + x^(-1)])) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + Sq
rt[(1 + x)/x]]
```

Rubi [A] time = 0.0487017, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2548, 44, 207}

$$-\frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)}+x\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/(2*(1 - Sqrt[1 + x^(-1)])) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + Sq
rt[(1 + x)/x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{-2+\left(-2+2\sqrt{1+\frac{1}{x}}\right)x}\,dx$$

$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-1+x)^2(1+x)}\,dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right)dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}+x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)+x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.0362554, size = 53, normalized size = 1.06

$$\frac{1}{2}\left(\sqrt{\frac{1}{x}+1}+1\right)x+x\log\left(\sqrt{\frac{1}{x}+1}-1\right)-\frac{1}{4}\log\left(\left(2\sqrt{\frac{1}{x}+1}+2\right)x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ((1 + Sqrt[1 + x^(-1)])*x)/2 + x*Log[-1 + Sqrt[1 + x^(-1)]] - Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x]/4
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \ln\left(-1+\sqrt{\frac{1+x}{x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+((1+x)/x)^(1/2)),x)
```

```
[Out] int(ln(-1+((1+x)/x)^(1/2)),x)
```

Maxima [A] time = 0.993873, size = 92, normalized size = 1.84

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-1\right)}{\frac{x+1}{x}-1}+\frac{1}{2\left(\sqrt{\frac{x+1}{x}}-1\right)}-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 1)/((x + 1)/x - 1) + 1/2/(sqrt((x + 1)/x) - 1) - 1/4*
log(sqrt((x + 1)/x) + 1) + 1/4*log(sqrt((x + 1)/x) - 1)
```

Fricas [A] time = 2.08568, size = 139, normalized size = 2.78

$$\frac{1}{4}(4x+1)\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{1}{2}x\sqrt{\frac{x+1}{x}}+\frac{1}{2}x-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) - 1) + 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) + 1)
```

Sympy [A] time = 154.442, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}+\frac{1}{2\left(\sqrt{1+\frac{1}{x}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 1) + log(sqrt(1 + 1/x) - 1)/4 - log(sqrt(1 + 1/x) +
 1)/4 + 1/(2*(sqrt(1 + 1/x) - 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{undef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] undef
```

3.298 $\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

```
[Out] Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
+ x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Rubi [A] time = 0.0517545, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2548, 706, 31, 633}

$$\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
+ x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{-2+\left(-2+4\sqrt{1+\frac{1}{x}}\right)x}dx$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-2+x)\left(-1+x^2\right)}dx,x,\sqrt{1+\frac{1}{x}}\right)$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x}dx,x,\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{-2-x}{-1+x^2}dx,x,\right.$$

$$= -\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}dx,x,\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\text{Su}$$

$$= \frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A] time = 0.0265013, size = 64, normalized size = 0.93

$$\frac{1}{6}\left(\log\left(2-\sqrt{\frac{1}{x}+1}\right)+6x\log\left(\sqrt{\frac{1}{x}+1}-2\right)-\log\left(\sqrt{\frac{1}{x}+1}+1\right)-6\tanh^{-1}\left(3-2\sqrt{\frac{1}{x}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (-6*ArcTanh[3 - 2*Sqrt[1 + x^(-1)]] + Log[2 - Sqrt[1 + x^(-1)]] + 6*x*Log[-
2 + Sqrt[1 + x^(-1)]] - Log[1 + Sqrt[1 + x^(-1)]])/6
```

Maple [A] time = 0.036, size = 108, normalized size = 1.6

$$x\ln\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{18x}\left(3\sqrt{\frac{1+x}{x}}x\ln(-3x+1)-\sqrt{9}\ln\left(\frac{1}{9x-3}\left(4\sqrt{9}\sqrt{x^2+x}+15x+3\right)\right)\sqrt{x(1+x)}+6\ln\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2+((1+x)/x)^(1/2)),x)
```

```
[Out] x*ln(-2+((1+x)/x)^(1/2))-1/18/((1+x)/x)^(1/2)/x*(3*((1+x)/x)^(1/2)*x*ln(-3*
x+1)-9^(1/2)*ln(1/3*(4*9^(1/2)*(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1/
2)+6*ln(1/2+x+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Maxima [A] time = 1.05914, size = 90, normalized size = 1.3

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-2\right)}{\frac{x+1}{x}-1}-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 2)/((x + 1)/x - 1) - 1/6*log(sqrt((x + 1)/x) + 1) + 1
/2*log(sqrt((x + 1)/x) - 1) - 1/3*log(sqrt((x + 1)/x) - 2)
```

Fricas [A] time = 2.19618, size = 138, normalized size = 2.

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) - 2) - 1/6*log(sqrt((x + 1)/x) + 1) + 1/2
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A] time = 153.419, size = 53, normalized size = 0.77

$$x\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-2\right)}{3}+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) - log(sqrt(1 + 1/x) - 2)/3 + log(sqrt(1 + 1/x) -
 1)/2 - log(sqrt(1 + 1/x) + 1)/6
```

Giac [B] time = 1.38192, size = 157, normalized size = 2.28

$$x\log\left(\sqrt{\frac{x+1}{x}}-2\right)+\frac{1}{6}\left(\frac{\log\left(\left|-2\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+x-\sqrt{x^2+x}+1\right|\right)}{\operatorname{sgn}(x)}-\frac{\log\left(\left|-2\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)-x+\sqrt{x^{\cdots}}\right.\right.}{\operatorname{sgn}(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) + 1/6*(log(abs(-2*(x - sqrt(x^2 + x))*sgn(x) + x
 - sqrt(x^2 + x) + 1))/sgn(x) - log(abs(-2*(x - sqrt(x^2 + x))*sgn(x) - x +
 sqrt(x^2 + x) - 1))/sgn(x) + 2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1)))*sgn(x
) - 1/6*log(abs(3*x - 1))
```

3.299 $\int (x^{ax} + x^{ax}\log(x))\,dx$

Optimal. Leaf size=9

$$\frac{x^{ax}}{a}$$

```
[Out] x^(a*x)/a
```

Rubi [A] time = 0.021677, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2553}

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(a*x) + x^(a*x)*Log[x],x]
```

```
[Out] x^(a*x)/a
```

Rule 2553

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (x^{ax} + x^{ax}\log(x))\,dx &= \int x^{ax}\,dx + \int x^{ax}\log(x)\,dx\\ &= \frac{x^{ax}}{a}\end{aligned}$$

Mathematica [A] time = 0.0133085, size = 9, normalized size = 1.

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(a*x) + x^(a*x)*Log[x],x]
```

```
[Out] x^(a*x)/a
```

Maple [A] time = 0.013, size = 11, normalized size = 1.2

$$\frac{e^{ax\ln(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(a*x)+x^(a*x)*ln(x),x)
```

```
[Out] 1/a*exp(a*x*ln(x))
```

Maxima [A] time = 1.23197, size = 12, normalized size = 1.33

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="maxima")
```

```
[Out] x^(a*x)/a
```

Fricas [A] time = 2.14332, size = 15, normalized size = 1.67

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="fricas")
```

```
[Out] x^(a*x)/a
```

Sympy [A] time = 0.298313, size = 10, normalized size = 1.11

$$\begin{cases} \frac{x^{ax}}{a} & \text{for } a \neq 0 \\ x\log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(a*x)+x**(a*x)*ln(x),x)
```

```
[Out] Piecewise((x**(a*x)/a, Ne(a, 0)), (x*log(x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{ax}\log(x) + x^{ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^(a*x)*log(x) + x^(a*x), x)
```

3.300 $\int \log^m(x)^p \, dx$

Optimal. Leaf size=26

$$(-\log(x))^{-mp} \log^m(x)^p \text{Gamma}(mp+1, -\log(x))$$

```
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rubi [A] time = 0.0252434, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6720, 2299, 2181}

$$(-\log(x))^{-mp} \log^m(x)^p \text{Gamma}(mp+1, -\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^m)^p,x]
```

```
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \log^m(x)^p\,dx &= \left(\log^{-mp}(x)\log^m(x)^p\right)\int \log^{mp}(x)\,dx\\ &= \left(\log^{-mp}(x)\log^m(x)^p\right)\text{Subst}\left(\int e^x x^{mp}\,dx, x, \log(x)\right)\\ &= \Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p\end{aligned}$$

Mathematica [A] time = 0.0137504, size = 26, normalized size = 1.

$$(-\log(x))^{-mp}\log^m(x)^p\text{Gamma}(mp+1,-\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^m)^p,x]
```

```
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \left((\ln(x))^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)^m)^p,x)
```

```
[Out] int((ln(x)^m)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\log(x)^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="maxima")
```

```
[Out] integrate((log(x)^m)^p, x)
```

Fricas [A] time = 2.10844, size = 50, normalized size = 1.92

$$\cos\left(\pi mp\right)\Gamma\left(mp+1,-\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="fricas")
```

```
[Out] cos(pi*m*p)*gamma(m*p + 1, -log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\log\left(x\right)^{m}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(x)**m)**p,x)
```

```
[Out] Integral((log(x)**m)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(\log\left(x\right)^{m}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="giac")
```

```
[Out] integrate((log(x)^m)^p, x)
```

3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}(2a+b)e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

```
[Out] -((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2)*E^(a/b))
+ (x*Sqrt[a + b*Log[x]])/b
```

Rubi [A] time = 0.0692622, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2294, 2299, 2180, 2204}

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}(2a+b)e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] -((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2)*E^(a/b))
+ (x*Sqrt[a + b*Log[x]])/b
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx &= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\ &= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\,\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\ &= \frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\ &= -\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{x\sqrt{a+b\log(x)}}{b}\end{aligned}$$

Mathematica [A] time = 0.109793, size = 72, normalized size = 1.2

$$\frac{2x(a+b\log(x))-(2a+b)e^{-\frac{a}{b}}\sqrt{-\frac{a+b\log(x)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)}{2b\sqrt{a+b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*x*(a + b*Log[x]) - ((2*a + b)*Gamma[1/2, -((a + b*Log[x])/b)]*Sqrt[-((a
+ b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \ln(x)\frac{1}{\sqrt{a+b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

```
[Out] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

Maxima [B] time = 1.22265, size = 146, normalized size = 2.43

$$-\frac{\frac{2\sqrt{\pi}a\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}+\frac{\sqrt{\pi}b\,\mathrm{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) +
 sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqrt
(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))/b^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a + b*log(x)), x)
```

Giac [A] time = 1.35694, size = 120, normalized size = 2.

$$\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}}+\frac{\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b}+\frac{\sqrt{b\log(x)+a}x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi)*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(p
i)*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log
(x) + a)*x/b
```

3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\pi}(2a-b)e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

```
[Out] -((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(2*b^(3/2)) -
 (x*Sqrt[a - b*Log[x]])/b
```

Rubi [A] time = 0.0677266, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2294, 2299, 2180, 2205}

$$-\frac{\sqrt{\pi}(2a-b)e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] -((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(2*b^(3/2)) -
 (x*Sqrt[a - b*Log[x]])/b
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx &= -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(-2a+b)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\ &= -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(-2a+b)\,\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx, x, \log(x)\right)}{2b}\\ &= -\frac{x\sqrt{a-b\log(x)}}{b} - \frac{(2a-b)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a-b\log(x)}\right)}{b^2}\\ &= -\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{x\sqrt{a-b\log(x)}}{b}\end{aligned}$$

Mathematica [A] time = 0.0908824, size = 71, normalized size = 1.11

$$\frac{-(b-2a)e^{a/b}\sqrt{\frac{a}{b}-\log(x)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)-2x(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] (-((-2*a + b)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]) - 2*x*(a
 - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \ln(x)\frac{1}{\sqrt{a-b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

```
[Out] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

Maxima [A] time = 1.21922, size = 127, normalized size = 1.98

$$-\frac{2\sqrt{\pi}a\sqrt{b}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - \sqrt{\pi}b^{\frac{3}{2}}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} + 2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - sqrt(
pi)*b^(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) + 2*sqrt(-b*log(x) + a
)*b*e^((b*log(x) - a)/b + a/b))/b^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a - b*log(x)), x)
```

Giac [A] time = 1.26626, size = 100, normalized size = 1.56

$$\frac{\sqrt{\pi}a\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}}-\frac{\sqrt{-b\log\left(x\right)+a}x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) - 1/2*sqrt(pi)
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*x/
b
```

3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=69

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}(2Ab-B(2a+b))\operatorname{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

```
[Out] ((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2
)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Rubi [A] time = 0.0690086, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2294, 2299, 2180, 2204}

$$\frac{\sqrt{\pi}e^{-\frac{a}{b}}(2Ab-B(2a+b))\operatorname{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] ((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2
)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx &= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\,\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\,\mathrm{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\
&= \frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}
\end{aligned}
$$

Mathematica [A] time = 0.145826, size = 80, normalized size = 1.16

$$
\frac{e^{-\frac{a}{b}}(2Ab-B(2a+b))\sqrt{-\frac{a+b\log(x)}{b}}\,\mathrm{Gamma}\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)+2Bx(a+b\log(x))}{2b\sqrt{a+b\log(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*B*x*(a + b*Log[x]) + ((2*A*b - (2*a + b)*B)*Gamma[1/2, -((a + b*Log[x])/
b)]*Sqrt[-((a + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$
\int (A+B\ln(x))\frac{1}{\sqrt{a+b\ln(x)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

```
[Out] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

Maxima [B] time = 1.27067, size = 211, normalized size = 3.06

$$\frac{\frac{2\sqrt{\pi}A\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{b\sqrt{-\frac{1}{b}}}-\frac{\left(\frac{\sqrt{\pi}b\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(pi)*A*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) -
2*sqrt(pi)*B*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/(b*sqrt(-1/b)) -
 (sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqr
t(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))*B/b)/b
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\log(x)}{\sqrt{a + b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a + b*log(x)), x)
```

Giac [B] time = 1.36718, size = 174, normalized size = 2.52

$$-\frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}}+\frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}}+\frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}b}+\frac{\sqrt{b\log(x)+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(pi)*A*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + 1/2*sqr
t(pi)*B*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(pi)*B*
a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log(x)
 + a)*B*x/b
```

3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{\pi}e^{a/b}(2aB+2Ab-bB)\text{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

```
[Out] -((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(
2*b^(3/2)) - (B*x*Sqrt[a - b*Log[x]])/b
```

Rubi [A] time = 0.0785237, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2294, 2299, 2180, 2205}

$$-\frac{\sqrt{\pi}e^{a/b}(2aB+2Ab-bB)\text{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] -((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/(
2*b^(3/2)) - (B*x*Sqrt[a - b*Log[x]])/b
```

Rule 2294

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[(B*(d + e*x)*Sqr
t[a + b*Log[c*(d + e*x)^n]])/(b*e), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx &= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\ &= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\,\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx,x,\log(x)\right)}{2b}\\ &= -\frac{Bx\sqrt{a-b\log(x)}}{b}-\frac{(2Ab+2aB-bB)\,\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a-b\log(x)}\right)}{b^2}\\ &= -\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}\end{aligned}$$

Mathematica [A] time = 0.130054, size = 79, normalized size = 1.11

$$\frac{e^{a/b}(2aB+2Ab-bB)\sqrt{\frac{a}{b}-\log(x)}\,\mathrm{Gamma}\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)-2Bx(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]
- 2*B*x*(a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (A+B\ln(x))\frac{1}{\sqrt{a-b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

```
[Out] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

Maxima [B] time = 1.32532, size = 176, normalized size = 2.48

$$-\frac{\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}+2\sqrt{\pi}A\sqrt{b}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-\frac{\left(\sqrt{\pi}b^{\frac{3}{2}}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*B*a*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) + 2*s
qrt(pi)*A*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - (sqrt(pi)*b^(3
/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - 2*sqrt(-b*log(x) + a)*b*e^((
b*log(x) - a)/b + a/b))*B/b)/b
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\log(x)}{\sqrt{a - b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a - b*log(x)), x)
```

Giac [A] time = 1.41757, size = 143, normalized size = 2.01

$$\frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}}+\frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}}-\frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}}-\frac{\sqrt{-b\log(x)+a}Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*B*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) + sqrt(pi)*A
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - 1/2*sqrt(pi)*B*erf(-sq
rt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*B*x/b
```

3.305 $\int x^2 \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=98

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\frac{ix^4}{12}-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log$$

```
[Out] (I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])/3 + (x^
3*Log[Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (x*PolyLog[3,
 E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rubi [A] time = 0.290198, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.3, Rules used = {30, 2555, 12, 6688, 14, 3717, 2190, 2531, 6609, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\frac{ix^4}{12}-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])/3 + (x^
3*Log[Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (x*PolyLog[3,
 E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int x^2 \log(\log(x)\sin(x))\,dx &= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \int \frac{x^2(1 + x\cot(x)\log(x))}{3\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \frac{x^2(1 + x\cot(x)\log(x))}{\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^2\left(x\cot(x) + \frac{1}{\log(x)}\right)dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \left(x^3\cot(x) + \frac{x^2}{\log(x)}\right)dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^3\cot(x)\,dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\,dx \\
&= \frac{ix^4}{12} + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{2}{3}i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(x)\right) \\
&= \frac{ix^4}{12} - \frac{1}{3}\mathrm{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \int x^2\log\left(1-e^{2ix}\right)dx \\
&= \frac{ix^4}{12} - \frac{1}{3}\mathrm{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\mathrm{Li}_2\left(e^{2ix}\right) - i\int x \\
&= \frac{ix^4}{12} - \frac{1}{3}\mathrm{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\mathrm{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\mathrm{Li} \\
&= \frac{ix^4}{12} - \frac{1}{3}\mathrm{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\mathrm{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\mathrm{Li} \\
&= \frac{ix^4}{12} - \frac{1}{3}\mathrm{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\mathrm{Li}_2\left(e^{2ix}\right) - \frac{1}{2}x\mathrm{Li}
\end{aligned}
$$

Mathematica [A] time = 0.0587605, size = 95, normalized size = 0.97

$$
\frac{1}{192}i\left(-96x^2\mathrm{PolyLog}\left(2,e^{-2ix}\right) + 96ix\mathrm{PolyLog}\left(3,e^{-2ix}\right) + 48\mathrm{PolyLog}\left(4,e^{-2ix}\right) + 64i\mathrm{Ei}(3\log(x)) - 16x^4 + 64ix^3\log\left(\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - 16*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x^3*Log[1
- E^((-2*I)*x)] - (64*I)*x^3*Log[Log[x]*Sin[x]] - 96*x^2*PolyLog[2, E^((-2*
I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)])
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$
\int x^2\ln\left(\ln(x)\sin(x)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(ln(x)*sin(x)),x)
```

```
[Out] int(x^2*ln(ln(x)*sin(x)),x)
```

Maxima [A] time = 2.04077, size = 127, normalized size = 1.3

$$\frac{1}{12}(2i\pi - 4\log(2))x^3 - \frac{1}{4}ix^4 + \frac{1}{3}x^3\log(\log(x)) + ix^2\text{Li}_2\left(-e^{(ix)}\right) + ix^2\text{Li}_2\left(e^{(ix)}\right) - 2x\text{Li}_3(-e^{(ix)}) - 2x\text{Li}_3(e^{(ix)}) - \frac{1}{3}\text{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(2*I*pi - 4*log(2))*x^3 - 1/4*I*x^4 + 1/3*x^3*log(log(x)) + I*x^2*dilo
g(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*polylog
(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*polylog(4,
 e^(I*x))
```

Fricas [C] time = 2.59229, size = 848, normalized size = 8.65

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{6}x^3\log(\cos(x) + i\sin(x) + 1) - \frac{1}{6}x^3\log(\cos(x) - i\sin(x) + 1) - \frac{1}{6}x^3\log(-\cos(x) + i\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin(x)) - 1/2*I*x^2
*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin(x)) + 1/2*I*x^2
*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)) - x*polylog(3,
 cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*polylog(3, -cos(
x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x) + I*sin(x)) +
I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*sin(x)) - I*poly
log(4, -cos(x) - I*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(ln(x)*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.306 $\int x \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}ix\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\text{Ei}(2\log(x))+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

```
[Out] (I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2 + (x^2
*Log[Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2
*I)*x)]/4
```

Rubi [A] time = 0.181542, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.5, Rules used = {30, 2555, 12, 6688, 14, 3717, 2190, 2531, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\text{Ei}(2\log(x))+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2 + (x^2
*Log[Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2
*I)*x)]/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int x\log(\log(x)\sin(x))\,dx &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \int \frac{x(1+x\cot(x)\log(x))}{2\log(x)}\,dx\\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \frac{x(1+x\cot(x)\log(x))}{\log(x)}\,dx\\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x\left(x\cot(x)+\frac{1}{\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \left(x^2\cot(x)+\frac{x}{\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x^2\cot(x)\,dx - \frac{1}{2}\int \frac{x}{\log(x)}\,dx\\
&= \frac{ix^3}{6} + \frac{1}{2}x^2\log(\log(x)\sin(x)) + i\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx - \frac{1}{2}\operatorname{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(x)\right)\\
&= \frac{ix^3}{6} - \frac{1}{2}\operatorname{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \int x\log\left(1-e^{2ix}\right)dx\\
&= \frac{ix^3}{6} - \frac{1}{2}\operatorname{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\operatorname{Li}_2\left(e^{2ix}\right) - \frac{1}{2}i\int \operatorname{Li}_2\\
&= \frac{ix^3}{6} - \frac{1}{2}\operatorname{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\operatorname{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\operatorname{Subst}\\
&= \frac{ix^3}{6} - \frac{1}{2}\operatorname{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\operatorname{Li}_2\left(e^{2ix}\right) - \frac{1}{4}\operatorname{Li}_3\left(e^{2i}\right.
\end{aligned}$$

Mathematica [A] time = 0.0389492, size = 79, normalized size = 0.99

$$\frac{1}{48}\left(-24ix\operatorname{PolyLog}\left(2,e^{-2ix}\right) - 12\operatorname{PolyLog}\left(3,e^{-2ix}\right) - 24\operatorname{Ei}(2\log(x)) - 8ix^3 - 24x^2\log\left(1-e^{-2ix}\right) + 24x^2\log(\log(x)\sin(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-2*I)
*x)] + 24*x^2*Log[Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)] - 12*P
olyLog[3, E^((-2*I)*x)])/48
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int x \ln(\ln(x)\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(ln(x)*sin(x)),x)
```

```
[Out] int(x*ln(ln(x)*sin(x)),x)
```

Maxima [A] time = 2.03011, size = 95, normalized size = 1.19

$$\frac{1}{12}(3i\pi - 6\log(2))x^2 - \frac{1}{3}i x^3 + \frac{1}{2}x^2\log(\log(x)) + i x\mathrm{Li}_2\left(-e^{(ix)}\right) + i x\mathrm{Li}_2\left(e^{(ix)}\right) - \frac{1}{2}\mathrm{Ei}(2\log(x)) - \mathrm{Li}_3(-e^{(ix)}) - \mathrm{Li}_3($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*pi - 6*log(2))*x^2 - 1/3*I*x^3 + 1/2*x^2*log(log(x)) + I*x*dilog(
-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I*x)) -
polylog(3, e^(I*x))
```

Fricas [C] time = 2.51811, size = 662, normalized size = 8.28

$$\frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{4}x^2\log(\cos(x) + i\sin(x) + 1) - \frac{1}{4}x^2\log(\cos(x) - i\sin(x) + 1) - \frac{1}{4}x^2\log(-\cos(x) + i\,\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)) - 1/2*I*x*dil
og(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) + 1/2*I*x*dilog(-
cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3, cos(x) + I*sin(
x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -cos(x) + I*sin(x)
) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(\log\left(x\right)\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(x*log(log(x)*sin(x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.307 $\int \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\text{li}(x)+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)+x\log(\log(x)\sin(x))$$

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
+ (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0615273, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2549, 3717, 2190, 2279, 2391, 2298}

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\text{li}(x)+\frac{ix^2}{2}-x\log\left(1-e^{2ix}\right)+x\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
+ (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2549

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\int \log(\log(x)\sin(x))\,dx &= x\log(\log(x)\sin(x)) - \int \left(x\cot(x) + \frac{1}{\log(x)}\right) dx\\ &= x\log(\log(x)\sin(x)) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx\\ &= \frac{ix^2}{2} + x\log(\log(x)\sin(x)) - \mathbf{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathbf{li}(x) + \int \log\left(1-e^{2ix}\right) dx\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathbf{li}(x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathbf{li}(x) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A] time = 0.0307747, size = 47, normalized size = 0.9

$$\frac{1}{2}i\left(x^2 + \text{PolyLog}\left(2, e^{2ix}\right)\right) - \mathbf{li}(x) - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]*Sin[x]],x]
```

```
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[Log[x]*Sin[x]] - LogIntegral[x] + (I/2)*(
x^2 + PolyLog[2, E^((2*I)*x)])
```

Maple [C] time = 0.167, size = 368, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x)),x)
```

```
[Out] -x*ln(exp(I*x))-1/2*I*Pi*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(ex
p(2*I*x)-1))^2*x-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x+1/2*I*x^2+I*ln(e
xp(I*x))*ln(exp(I*x)+1)-I*dilog(exp(I*x))-ln(2)*x+1/2*I*Pi*csgn(I*exp(-I*x)
)*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+1/2*I*Pi*csgn(I*exp(-I*
x))*csgn(ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(I*ln(x)*sin(x))^2*x+1/2*I*Pi*csgn(
I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2*x-I*ln(exp(I*x))*ln(exp(2*I*x)
-1)-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-
1))*x+I*dilog(exp(I*x)+1)+1/2*I*Pi*csgn(ln(x)*sin(x))^3*x+1/2*I*Pi*csgn(I*l
n(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x-1/2*I*Pi*csgn(I*ln(x)*sin(x))^3*x+1/
2*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))*x-1/2*I*Pi*csgn(ln(x)*sin(x)
)*csgn(I*ln(x)*sin(x))^2*x+x*ln(ln(x))+Ei(1,-ln(x))
```

Maxima [A] time = 1.94638, size = 58, normalized size = 1.12

$$\frac{1}{2}(i\pi - 2\log(2))x - \frac{1}{2}ix^2 + x\log(\log(x)) - \mathrm{Ei}(\log(x)) + i\,\mathrm{Li}_2\left(-e^{(ix)}\right) + i\,\mathrm{Li}_2\left(e^{(ix)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - 1/2*I*x^2 + x*log(log(x)) - Ei(log(x)) + I*dilog(
-e^(I*x)) + I*dilog(e^(I*x))
```

Fricas [B] time = 2.3886, size = 427, normalized size = 8.21

$$x\log(\log(x)\sin(x)) - \frac{1}{2}x\log(\cos(x) + i\sin(x) + 1) - \frac{1}{2}x\log(\cos(x) - i\sin(x) + 1) - \frac{1}{2}x\log(-\cos(x) + i\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x)
- I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I
*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(
x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x)) - l
og_integral(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\log\left(x\right)\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(log(log(x)*sin(x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.308 $\int \frac{\log(\log(x)\sin(x))}{x}\,dx$

Optimal. Leaf size=12

$$\text{CannotIntegrate}\left(\frac{\log(\log(x)\sin(x))}{x},x\right)$$

```
[Out] CannotIntegrate[Log[Log[x]*Sin[x]]/x, x]
```

Rubi [A] time = 0.0187267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x,x]
```

```
[Out] Defer[Int][Log[Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx = \int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Mathematica [A] time = 2.25513, size = 0, normalized size = 0.

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x,x]
```

```
[Out] Integrate[Log[Log[x]*Sin[x]]/x, x]
```

Maple [A] time = 0.417, size = 0, normalized size = 0.

$$\int \frac{\ln \left(\ln \left(x \right) \sin \left(x \right)\right)}{x}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(ln(x)*sin(x))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-(\log(2)+1)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\log(x)+\frac{1}{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - integrat
e(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(log(x)*sin(x))/x, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x)\sin(x))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x,x)
```

```
[Out] Integral(log(log(x)*sin(x))/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2} dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{\cot(x)}{x}, x\right) + \text{Ei}(-\log(x)) - \frac{\log(\log(x)\sin(x))}{x}$$

```
[Out] ExpIntegralEi[-Log[x]] - Log[Log[x]*Sin[x]]/x + Unintegrable[Cot[x]/x, x]
```

Rubi [A] time = 0.344563, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] - Log[Log[x]*Sin[x]]/x + Defer[Int][Cot[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(\log(x)\sin(x))}{x^2} dx &= -\frac{\log(\log(x)\sin(x))}{x} - \int \frac{-1 - x\cot(x)\log(x)}{x^2\log(x)} dx \\
&= -\frac{\log(\log(x)\sin(x))}{x} - \int \left(-\frac{\cot(x)}{x} - \frac{1}{x^2\log(x)}\right) dx \\
&= -\frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x} dx + \int \frac{1}{x^2\log(x)} dx \\
&= -\frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x} dx + \text{Subst}\left(\int \frac{e^{-x}}{x} dx, x, \log(x)\right) \\
&= \text{Ei}(-\log(x)) - \frac{\log(\log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x} dx
\end{aligned}$$

Mathematica [A] time = 1.93027, size = 0, normalized size = 0.

$$\int \frac{\log(\log(x)\sin(x))}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x^2,x]
```

```
[Out] Integrate[Log[Log[x]*Sin[x]]/x^2, x]
```

Maple [A] time = 0.434, size = 0, normalized size = 0.

$$\int \frac{\ln(\ln(x)\sin(x))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x^2,x)
```

```
[Out] int(ln(ln(x)*sin(x))/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{x\left(\mathrm{Ei}\,(-\log(x)) + \overline{\mathrm{Ei}\,(-\log(x))}\right) - 2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2+2\,\cos(x)+1\right)x}\,dx + 2\,x\int \frac{\sin(x)}{\left(\cos(x)^2+\sin(x)^2-2\,\cos(x)+1\right)x}\,dx + 2\,\log(2) - l}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*log(2) - log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*log(log(x)))/x
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log(\log(x)\sin(x))}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(log(log(x)*sin(x))/x^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x**2,x)
```

```
[Out] Integral(log(log(x)*sin(x))/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.310 $\int x^2 \log(e^x \log(x)\sin(x))\,dx$

Optimal. Leaf size=103

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}x^3\log\left(1-e^2\right.$$

```
[Out] (-1/12 + I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])
/3 + (x^3*Log[E^x*Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (
x*PolyLog[3, E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rubi [A] time = 0.203296, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {30, 2555, 12, 14, 3717, 2190, 2531, 6609, 2282, 6589, 2309, 2178}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}x^3\log\left(1-e^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/12 + I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])
/3 + (x^3*Log[E^x*Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (
x*PolyLog[3, E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int x^2 \log\left(e^x \log(x)\sin(x)\right) dx = \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \int \frac{1}{3}x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx$$
$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx$$
$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int\left(x^3(1+\cot(x))+\frac{x^2}{\log(x)}\right)dx$$
$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3(1+\cot(x))\,dx - \frac{1}{3}\int\frac{x^2}{\log(x)}dx$$
$$= \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int\left(x^3+x^3\cot(x)\right)dx - \frac{1}{3}\text{Subst}\left(\int\frac{e^{3x}}{x}dx, x, \log(x)\right)$$
$$= -\frac{x^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{3}\int x^3\cot(x)\,dx$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{2}{3}i\int\frac{e^{2ix}x^3}{1-e^{2ix}}dx$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \int x^2 \text{l}$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{L}$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{L}$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{L}$$
$$= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right) + \frac{1}{2}ix^2\text{L}$$

Mathematica [A] time = 0.0631273, size = 100, normalized size = 0.97

$$\frac{1}{192}i\left(-96x^2\text{PolyLog}\left(2,e^{-2ix}\right)+96ix\text{PolyLog}\left(3,e^{-2ix}\right)+48\text{PolyLog}\left(4,e^{-2ix}\right)+64i\text{Ei}(3\log(x))+(-16+16i)x^4+64i$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - (16 - 16*I)*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x
^3*Log[1 - E^((-2*I)*x)] - (64*I)*x^3*Log[E^x*Log[x]*Sin[x]] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])
```

Maple [F] time = 0.367, size = 0, normalized size = 0.

$$\int x^{2} \ln \left(\mathrm{e}^{x} \ln \left(x\right) \sin \left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] int(x^2*ln(exp(x)*ln(x)*sin(x)),x)
```

Maxima [A] time = 2.07471, size = 127, normalized size = 1.23

$$\frac{1}{12} \, {\left(2 i \, \pi - 4 \, \log\left(2\right)\right)} x^{3} - \left(\frac{1}{4} i - \frac{1}{4}\right) \, x^{4} + \frac{1}{3} \, x^{3} \log\left(\log\left(x\right)\right) + i \, x^{2} \mathrm{Li}_{2}\left(-e^{\left(i \, x\right)}\right) + i \, x^{2} \mathrm{Li}_{2}\left(e^{\left(i \, x\right)}\right) - 2 \, x \mathrm{Li}_{3}(-e^{\left(i \, x\right)}) - 2 \, x \mathrm{Li}_{3}(e^{\left(i \, x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(2*I*pi - 4*log(2))*x^3 - (1/4*I - 1/4)*x^4 + 1/3*x^3*log(log(x)) + I*
x^2*dilog(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x
*polylog(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*po
lylog(4, e^(I*x))
```

Fricas [C] time = 2.58554, size = 869, normalized size = 8.44

$$-\frac{1}{12} \, x^{4} + \frac{1}{3} \, x^{3} \log\left(e^{x} \log\left(x\right) \sin\left(x\right)\right) - \frac{1}{6} \, x^{3} \log\left(\cos\left(x\right) + i \, \sin\left(x\right) + 1\right) - \frac{1}{6} \, x^{3} \log\left(\cos\left(x\right) - i \, \sin\left(x\right) + 1\right) - \frac{1}{6} \, x^{3} \log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/12*x^4 + 1/3*x^3*log(e^x*log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin
(x)) - 1/2*I*x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin
```

```
(x)) + 1/2*I*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)
) - x*polylog(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*
polylog(3, -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x
) + I*sin(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*
sin(x)) - I*polylog(4, -cos(x) - I*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.311 $\int x \log(e^x \log(x) \sin(x))\, dx$

Optimal. Leaf size=85

$$\frac{1}{2} ix\text{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{4}\text{PolyLog}\left(3, e^{2ix}\right) - \frac{1}{2}\text{Ei}(2\log(x)) + \left(-\frac{1}{6} + \frac{i}{6}\right)x^3 - \frac{1}{2}x^2\log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$

```
[Out] (-1/6 + I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2
 + (x^2*Log[E^x*Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyL
og[3, E^((2*I)*x)]/4
```

Rubi [A] time = 0.138691, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {30, 2555, 12, 14, 3717, 2190, 2531, 2282, 6589, 2309, 2178}

$$\frac{1}{2} ix\text{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{4}\text{PolyLog}\left(3, e^{2ix}\right) - \frac{1}{2}\text{Ei}(2\log(x)) + \left(-\frac{1}{6} + \frac{i}{6}\right)x^3 - \frac{1}{2}x^2\log\left(1 - e^{2ix}\right) + \frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/6 + I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2
 + (x^2*Log[E^x*Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyL
og[3, E^((2*I)*x)]/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2555

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int x\log\left(e^x\log(x)\sin(x)\right)\,dx &= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\int\frac{1}{2}x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2(1+\cot(x))+\frac{x}{\log(x)}\right)dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2(1+\cot(x))\,dx-\frac{1}{2}\int\frac{x}{\log(x)}\,dx\\
&= \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2+x^2\cot(x)\right)dx-\frac{1}{2}\text{Subst}\left(\int\frac{e^{2x}}{x}\,dx,x,\log(x)\right)\\
&= -\frac{x^3}{6}-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\cot(x)\,dx\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\int x\log\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\frac{1}{2}ix\text{Li}_2\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\frac{1}{2}ix\text{Li}_2\\
&= \left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\frac{1}{2}ix\text{Li}_2
\end{aligned}$$

Mathematica [A] time = 0.0670711, size = 82, normalized size = 0.96

$$\frac{1}{48}\left(-24ix\text{PolyLog}\left(2,e^{-2ix}\right)-12\text{PolyLog}\left(3,e^{-2ix}\right)-24\text{Ei}(2\log(x))+(-8-8i)x^3-24x^2\log\left(1-e^{-2ix}\right)+24x^2\log(e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8 + 8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-
2*I)*x)] + 24*x^2*Log[E^x*Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)
] - 12*PolyLog[3, E^((-2*I)*x)])/48
```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int x \ln \left(e^{x} \ln \left(x\right) \sin \left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] int(x*ln(exp(x)*ln(x)*sin(x)),x)
```

Maxima [A] time = 2.08907, size = 95, normalized size = 1.12

$$\frac{1}{12} \left(3 i \pi - 6 \log \left(2\right)\right) x^{2} - \left(\frac{1}{3} i - \frac{1}{3}\right) x^{3} + \frac{1}{2} x^{2} \log \left(\log \left(x\right)\right) + i x \mathrm{Li}_{2}\left(-e^{\left(i x\right)}\right) + i x \mathrm{Li}_{2}\left(e^{\left(i x\right)}\right) - \frac{1}{2} \mathrm{Ei}\left(2 \log \left(x\right)\right) - \mathrm{Li}_{3}(-e^{\left(i x\right)}) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*pi - 6*log(2))*x^2 - (1/3*I - 1/3)*x^3 + 1/2*x^2*log(log(x)) + I*
x*dilog(-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(
I*x)) - polylog(3, e^(I*x))
```

Fricas [C] time = 2.50582, size = 682, normalized size = 8.02

$$-\frac{1}{6} x^{3} + \frac{1}{2} x^{2} \log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right) - \frac{1}{4} x^{2} \log \left(\cos \left(x\right) + i \sin \left(x\right) + 1\right) - \frac{1}{4} x^{2} \log \left(\cos \left(x\right) - i \sin \left(x\right) + 1\right) - \frac{1}{4} x^{2} \log \left(-\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/6*x^3 + 1/2*x^2*log(e^x*log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)
) - 1/2*I*x*dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) +
1/2*I*x*dilog(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3,
cos(x) + I*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -co
s(x) + I*sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.312 $\int \log \left(e^x \log(x) \sin(x)\right) dx$

Optimal. Leaf size=57

$$\frac{1}{2} i \text{PolyLog}\left(2, e^{2ix}\right) - \text{li}(x) + \left(-\frac{1}{2} + \frac{i}{2}\right) x^2 - x \log\left(1 - e^{2ix}\right) + x \log\left(e^x \log(x) \sin(x)\right)$$

```
[Out] (-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - LogI
ntegral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rubi [A] time = 0.0642922, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2549, 3717, 2190, 2279, 2391, 2298}

$$\frac{1}{2} i \text{PolyLog}\left(2, e^{2ix}\right) - \text{li}(x) + \left(-\frac{1}{2} + \frac{i}{2}\right) x^2 - x \log\left(1 - e^{2ix}\right) + x \log\left(e^x \log(x) \sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - LogI
ntegral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2549

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(e^x \log(x)\sin(x)\right) dx &= x\log\left(e^x\log(x)\sin(x)\right) - \int \left(x + x\cot(x) + \frac{1}{\log(x)}\right) dx \\
&= -\frac{x^2}{2} + x\log\left(e^x\log(x)\sin(x)\right) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx \\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + \int \log\left(1-e^{2ix}\right) dx \\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) - \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\right. \\
&= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \text{li}(x) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0297676, size = 56, normalized size = 0.98

$$
\frac{1}{2}\left(i\text{PolyLog}\left(2,e^{2ix}\right) - 2\text{li}(x) + (-1+i)x^2 - 2x\log\left(1-e^{2ix}\right) + 2x\log\left(e^x\log(x)\sin(x)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] ((-1 + I)*x^2 - 2*x*Log[1 - E^((2*I)*x)] + 2*x*Log[E^x*Log[x]*Sin[x]] - 2*L
ogIntegral[x] + I*PolyLog[2, E^((2*I)*x)])/2
```

Maple [C] time = 0.244, size = 583, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] -1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp(
(1-I)*x))*ln(x))*x+1/2*I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((
1-I)*x)))^2*x-ln(2)*x-1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*
ln(x)*(exp(2*I*x)-1))*x-1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))^3*x+x*ln(ln(x
))-I*dilog(exp(I*x))+1/2*I*x^2-1/2*I*Pi*x+I*ln(exp(I*x))*ln(exp(I*x)+1)+1/2
*I*Pi*csgn(ln(x)*sin(x))^3*x+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^
2*x+1/2*I*Pi*csgn(I*ln(x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))^2*x-1/2*I*Pi*c
sgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x-I*ln(exp(I*
x))*ln(exp(2*I*x)-1)+1/2*ln(exp(x))^2+1/2*I*Pi*csgn(I*exp(-I*x))*csgn(I*ln(
x)*(exp(2*I*x)-1))*csgn(ln(x)*sin(x))*x+I*dilog(exp(I*x)+1)+1/2*I*Pi*csgn((
exp((1+I)*x)-exp((1-I)*x))*ln(x))^2*x-1/2*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-e
xp((1-I)*x)))^3*x-x*ln(exp(I*x))+Ei(1,-ln(x))+1/2*I*Pi*csgn(I*ln(x)*(exp((1
+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2*x+1/2*I*Pi*
csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))
*x+1/2*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x+1/2*I*P
i*csgn(I*ln(x))*csgn(I*ln(x)*(exp(2*I*x)-1))^2*x-1/2*I*Pi*csgn((exp((1+I)*x
)-exp((1-I)*x))*ln(x))^3*x
```

Maxima [A] time = 1.80966, size = 58, normalized size = 1.02

$$\frac{1}{2}\left(i\,\pi - 2\,\log\left(2\right)\right)x - \left(\frac{1}{2}i - \frac{1}{2}\right)x^2 + x\log\left(\log\left(x\right)\right) - \mathrm{Ei}\left(\log\left(x\right)\right) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - (1/2*I - 1/2)*x^2 + x*log(log(x)) - Ei(log(x)) +
I*dilog(-e^(I*x)) + I*dilog(e^(I*x))
```

Fricas [B] time = 2.42055, size = 447, normalized size = 7.84

$$-\frac{1}{2}x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \frac{1}{2}x\log\left(\cos(x) + i\sin(x) + 1\right) - \frac{1}{2}x\log\left(\cos(x) - i\sin(x) + 1\right) - \frac{1}{2}x\log\left(-\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(e^x*log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/
2*x*log(cos(x) - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*
log(-cos(x) - I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(
cos(x) - I*sin(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x)
- I*sin(x)) - log_integral(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.313 $\int \frac{\log(e^x \log(x) \sin(x))}{x} dx$

Optimal. Leaf size=15

$$\text{CannotIntegrate}\left(\frac{\log(e^x \log(x) \sin(x))}{x}, x\right)$$

```
[Out] CannotIntegrate[Log[E^x*Log[x]*Sin[x]]/x, x]
```

Rubi [A] time = 0.0219818, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x,x]
```

```
[Out] Defer[Int][Log[E^x*Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} dx = \int \frac{\log(e^x \log(x) \sin(x))}{x} dx$$

Mathematica [A] time = 0.790202, size = 0, normalized size = 0.

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x,x]
```

```
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x, x]
```

Maple [A] time = 0.605, size = 0, normalized size = 0.

$$\int \frac{\ln \left(e^{x} \ln \left(x\right) \sin \left(x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\left(\log\left(2\right)+1\right)\log\left(x\right)+\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)\log\left(x\right)+\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
x + integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - inte
grate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(e^x*log(x)*sin(x))/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.314 $\int \frac{\log(e^x \log(x) \sin(x))}{x^2} \, dx$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\cot(x)}{x}, x\right) + \text{Ei}(-\log(x)) + \log(x) - \frac{\log\left(e^x \log(x) \sin(x)\right)}{x}$$

```
[Out] ExpIntegralEi[-Log[x]] + Log[x] - Log[E^x*Log[x]*Sin[x]]/x + Unintegrable[C
ot[x]/x, x]
```

Rubi [A] time = 0.0688721, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] + Log[x] - Log[E^x*Log[x]*Sin[x]]/x + Defer[Int][Cot
[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx &= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \frac{1 + \cot(x) + \frac{1}{x \log(x)}}{x} \, dx \\
&= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \left(\frac{1 + \cot(x)}{x} + \frac{1}{x^2 \log(x)}\right) dx \\
&= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \frac{1 + \cot(x)}{x} \, dx + \int \frac{1}{x^2 \log(x)} \, dx \\
&= -\frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \left(\frac{1}{x} + \frac{\cot(x)}{x}\right) dx + \text{Subst}\left(\int \frac{e^{-x}}{x} \, dx, x, \log(x)\right) \\
&= \text{Ei}(-\log(x)) + \log(x) - \frac{\log\left(e^x \log(x) \sin(x)\right)}{x} + \int \frac{\cot(x)}{x} \, dx
\end{aligned}$$

Mathematica [A] time = 2.26533, size = 0, normalized size = 0.

$$\int \frac{\log\left(e^x \log(x) \sin(x)\right)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2, x]
```

Maple [A] time = 0.622, size = 0, normalized size = 0.

$$\int \frac{\ln\left(\mathrm{e}^{x} \ln\left(x\right) \sin\left(x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{x\left(\mathrm{Ei}\left(-\log\left(x\right)\right) + \overline{\mathrm{Ei}\left(-\log\left(x\right)\right)}\right) - 2\,x \int \frac{\sin(x)}{\left(\cos(x)^{2}+\sin(x)^{2}+2\,\cos(x)+1\right)x}\, dx + 2\,x \int \frac{\sin(x)}{\left(\cos(x)^{2}+\sin(x)^{2}-2\,\cos(x)+1\right)x}\, dx + 2\,x\log\left(x\right) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*x*log(x) + 2*log(2) - log(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*lo
g(log(x)))/x
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(e^{x} \log\left(x\right) \sin\left(x\right)\right)}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(log(e^x*log(x)*sin(x))/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```